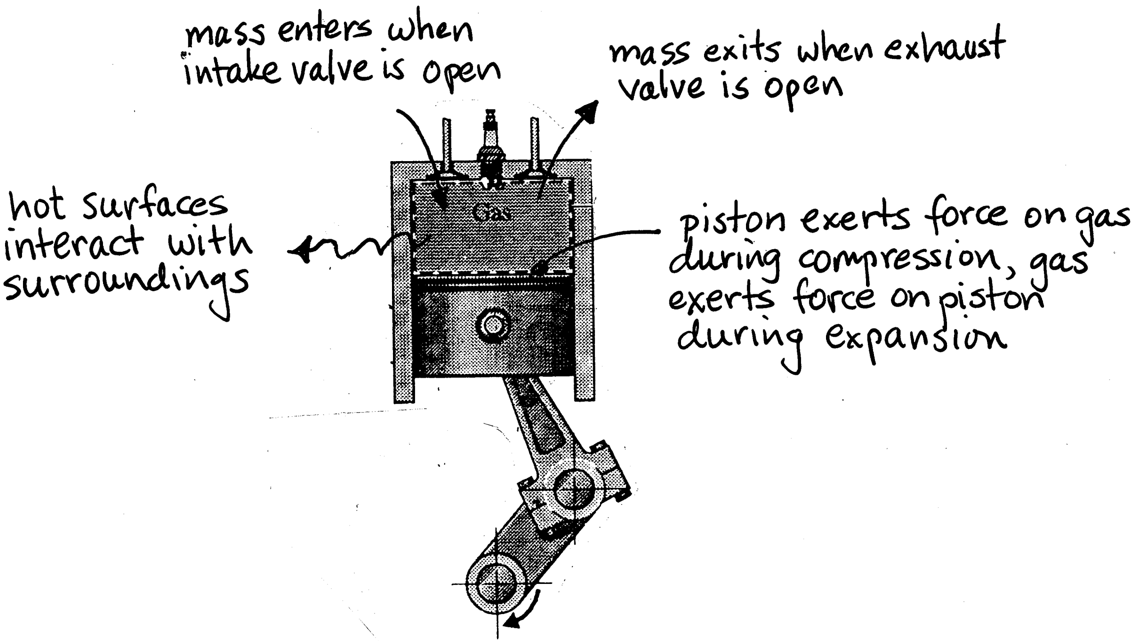
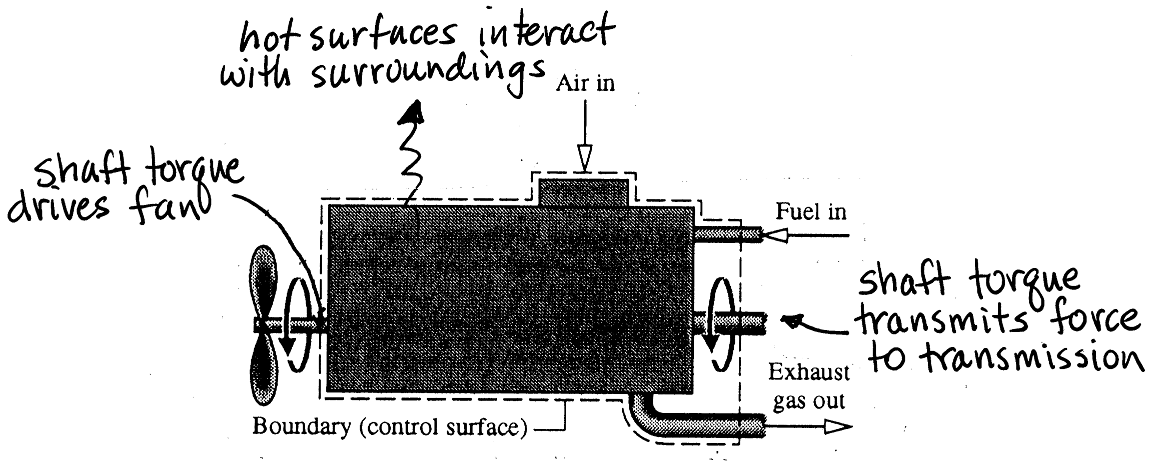
CHAPTER ONE

GETTING STARTED: INTRODUCTORY CONCEPTS AND DEFINITIONS

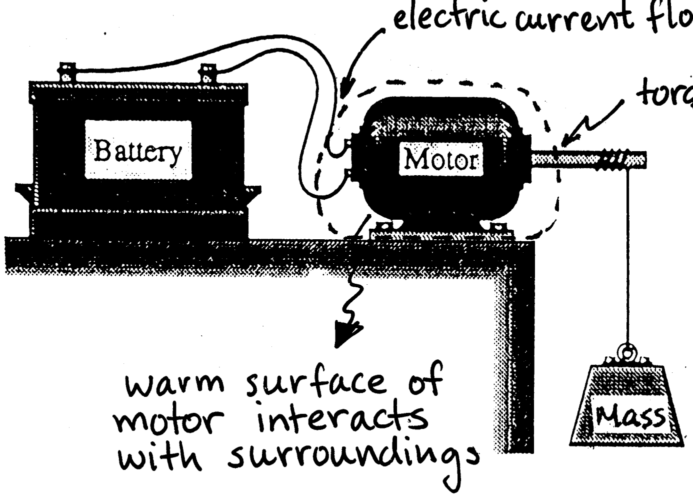
PROBLEM 1.1


mass enters when intake valve is open
mass exits when exhaust valve is open
hot surfaces interact with surroundings
Gas
piston exerts force on gas during compression, gas exerts force on piston during expansion


hot surfaces interact with surroundings
Air in
shaft torque drives fan
Fuel in
shaft torque transmits force to transmission
Exhaust gas out
Boundary (control surface)

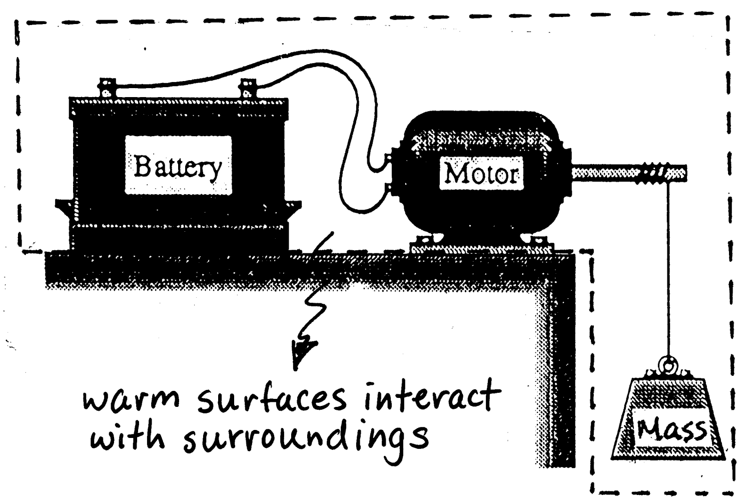
PROBLEM 1.2

Motor as system:


electric current flows
torque transmitted through shaft
Battery
Motor
Mass
warm surface of motor interacts with surroundings

- speed of rotation of armature can change
- temperature of motor parts can change

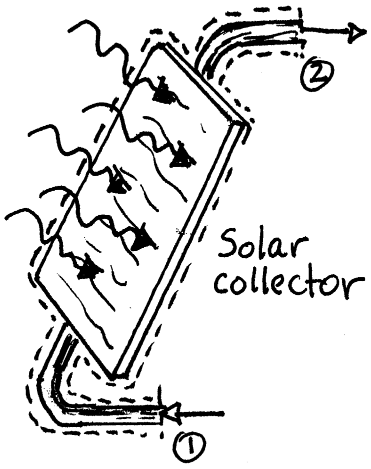
Enlarged system:



- chemical changes occur within the battery
- speed of rotation of armature can change
- temperature of motor parts and battery can change
- mass is raised within system

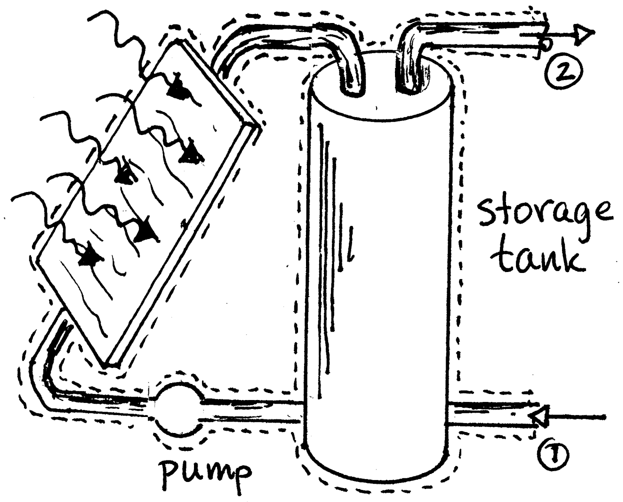
COMMENT: The shaft torque and current flow interactions become internal to the enlarged system.

PROBLEM 1.3



A control volume encloses the solar collector.

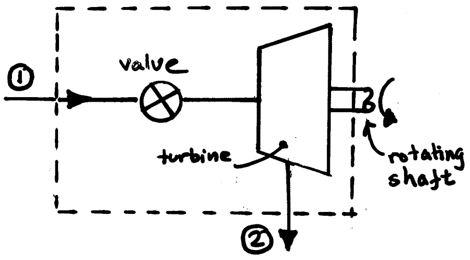
- Cool water enters the collector at ①, and hot water exits at ②.
- solar radiation impinges on the front of the collector.
- Warm surfaces of the collector interact with the surroundings.
- some of the incoming radiation is reflected away, and some is absorbed in the collector surface.



A control volume encloses the solar collector, the tank, and the interconnected piping.

- Cold water enters the tank at ①, and hot water exits at ②.
- Warm surfaces of the collector, storage tank, and interconnected piping interact with the surroundings.
- Solar radiation impinges on the front of the collector; some is reflected and some is absorbed.
- The temperature of the water in the storage tank changes with time.

PROBLEM 1.4



A control volume encloses the valve and turbine.

- Steam enters at ① and exits at ②.
- A torque is transmitted through the rotating shaft.
- Warm surfaces of the turbine interact with the surroundings.

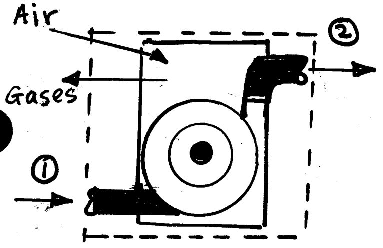
Within the control volume, steam flows across the valve and through the turbine blades.

When the generator is included in the control volume,

- Steam enters at ① and exits at ②.
- Warm surfaces of the turbine and the generator interact with the surroundings.
- electric current flows from the generator.

Note that the transmitted torque does not cross the boundary of the enlarged control volume.

PROBLEM 1.5



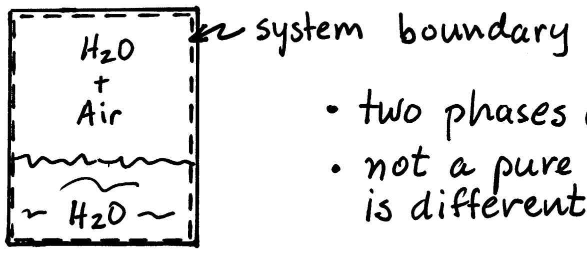
A control volume encloses the engine-driven pump.

- Water enters at ① and exits at ②
- Air for combustion of the on-board fuel enters, and combustion gases exit.
- Warm surfaces of the pump interact with the surroundings.

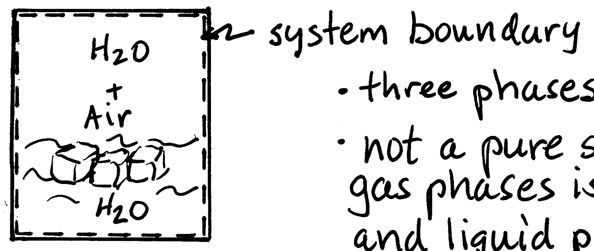
Within the pump, a piston is kept in motion within a cylinder owing to combustion of the on-board fuel. The piston motion is harnessed to pump the liquid. The amount of fuel within the system decreases with time.

When the hose and nozzle are included, a high-speed water jet exits the extended control volume at the nozzle exit,

PROBLEM 1.6

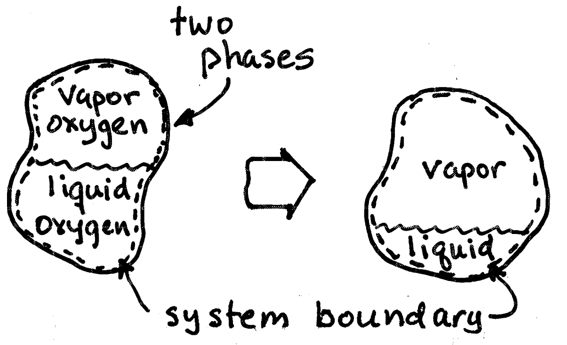


- two phases are present (liquid and gas).
- not a pure substance because composition is different in each phase.



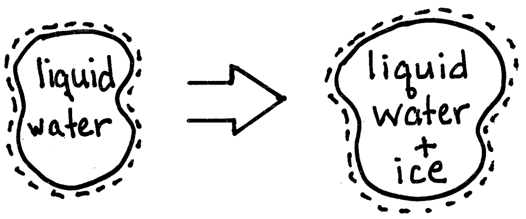
- three phases are present (solid, liquid, and gas).
- not a pure subtance because composition of gas phases is different than that of the solid and liquid phases.

PROBLEM 1.7



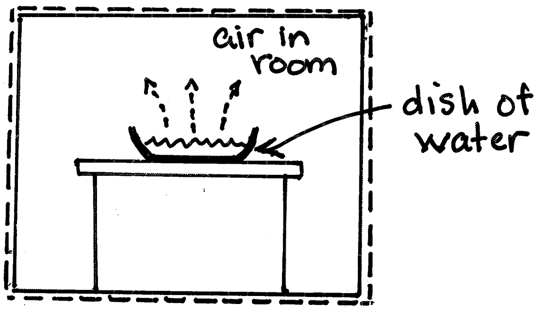
The system is a pure substance. Although the liquid is vaporized, the system remains fixed in chemical composition and is chemically homogeneous.

PROBLEM 1.8



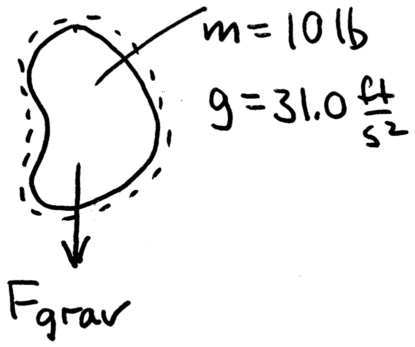
The system is a pure substance. Although the phases change, the system remains of fixed chemical composition and is chemically homogeneous.

PROBLEM 1.9



The system is not a pure substance during the process since the composition of the gas phase changes as water evaporates into the air. Once all of the water evaporates, the gas phase comes to equilibrium and the composition becomes homogeneous. At this point, the gas phase can be treated as a pure substance.

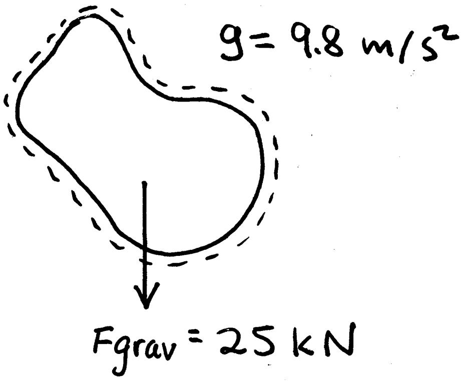
PROBLEM 1.10*



$$F_{grav} = mg = (10\ \text{lb})(31.0\ \tfrac{\text{ft}}{\text{s}^2})\left|\frac{1\ \text{lb}_f}{32.2\ \text{lb ft/s}^2}\right|$$

$$= 9.63\ \text{lb}_f \xleftarrow{\quad F_{grav}\quad}$$

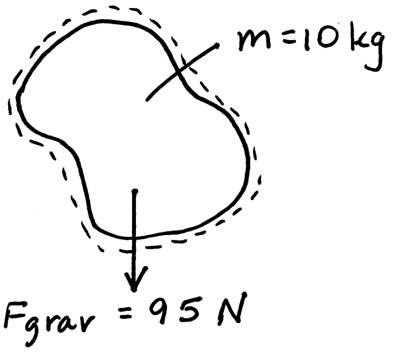
PROBLEM 1.11*



$$m = \frac{F_{grav}}{g} = \left(\frac{25\ \text{kN}}{9.8\ \text{m/s}^2}\right)\left|\frac{1000\ \text{N}}{\text{kN}}\right|\left|\frac{1\ \text{kg m/s}^2}{1\ \text{N}}\right|$$

$$= 2551\ \text{kg} \xleftarrow{\quad m\quad}$$

PROBLEM 1.12



(a) $$g_{local} = \frac{F_{grav}}{m} = \left(\frac{95\ \text{N}}{10\ \text{kg}}\right)\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|$$

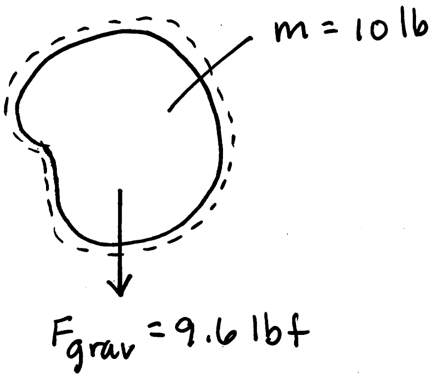
$$= 9.5\ \text{m/s}^2 \xleftarrow{\quad g_{local}\quad}$$

(b) mass is unchanged. $\xleftarrow{\quad m\quad}$

$$F_{grav} = mg = (10\ \text{kg})(9.81\ \text{m/s}^2)\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|$$

$$= 98.1\ \text{N} \xleftarrow{\quad F_{grav}\quad}$$

PROBLEM 1.13



(a) $$g_{local} = \frac{F_{grav}}{m} = \left(\frac{9.6\ \text{lbf}}{10\ \text{lb}}\right)\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|$$

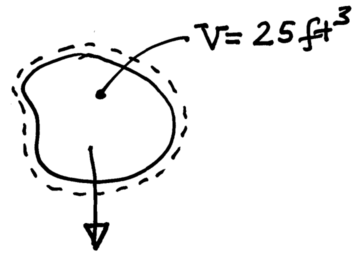
$$= 30.9\ \text{ft/s}^2 \xleftarrow{\quad g_{local}\quad}$$

(b) mass is unchanged. $\xleftarrow{\quad m\quad}$

$$F_{grav} = mg = (10\ \text{lb})(32.2\ \tfrac{\text{ft}}{\text{s}^2})\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|$$

$$= 10\ \text{lbf} \xleftarrow{\quad F_{gra}\quad}$$

PROBLEM 1.14



$F_{grav} = 3.5\ lbf$

$g_{moon} = 5.47\ ft/s^2$

$g_{mars} = 12.86\ ft/s^2$

In general, $F_{grav} = mg$. So,

$$m = \frac{F_{grav}}{g} \qquad (*)$$

Since the mass is the same on mars as on the moon,

$$\left(\frac{F_{grav}}{g}\right)_{mars} = \left(\frac{F_{grav}}{g}\right)_{moon}$$

Accordingly

$$(F_{grav})_{mars} = \left(\frac{g_{mars}}{g_{moon}}\right)(F_{grav})_{moon}$$

$$= \left(\frac{12.86\ ft/s^2}{5.47\ ft/s^2}\right)(3.5\ lbf) = 8.23\ lbf \longleftarrow$$

The density is $\rho = m/V$. Applying Eq. (*) with data on mars

$$m = \left(\frac{8.23\ lbf}{12.86\ ft/s^2}\right)\left|\frac{32.2\ lb \cdot ft/s^2}{1\ lbf}\right| = 20.61\ lb$$

Then

$$\rho = \frac{20.61\ lb}{25\ ft^3} = 0.824\ \frac{lb}{ft^3} \longleftarrow$$

PROBLEM 1.15*

Eq. 1.10 is used on both parts: $n = m/M$, Where M is from Tables A-1.

(a) $m = Mn$, $n = 10\ kmol$

Air: $m = (28.97\ kg/kmol)(10\ kmol) = 289.7\ kg$

H_2O: $m = (18.02\ kg/kmol)(10\ kmol) = 180.2\ kg$

Cu: $m = (63.54\ kg/kmol)(10\ kmol) = 635.4\ kg$

SO_2: $m = (64.06\ kg/kmol)(10\ kmol) = 640.6\ kg$

(b) $n = m/M$, $m = 20\ lb$

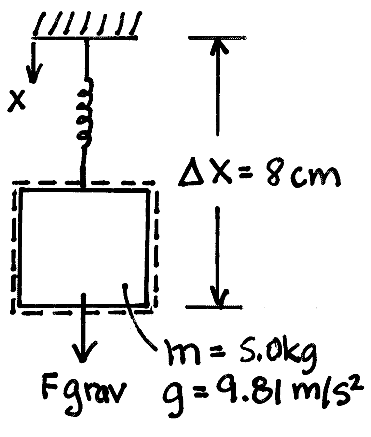
Ar: $n = (20\ lb)/(39.94\ lb/lbmol) = 0.501\ lbmol$

H_2: $n = (20\ lb)/(2.016\ lb/lbmol) = 9.921\ lbmol$

N_2: $n = (20\ lb)/(28.01\ lb/lbmol) = 0.714\ lbmol$

C: $n = (20\ lb)/(12.01\ lb/lbmol) = 1.665\ lbmol$

PROBLEM 1.16*



$F_{spring} = K(\Delta X)$ and $F_{spring} = F_{grav} = mg$

Thus

$K(\Delta X) = mg$

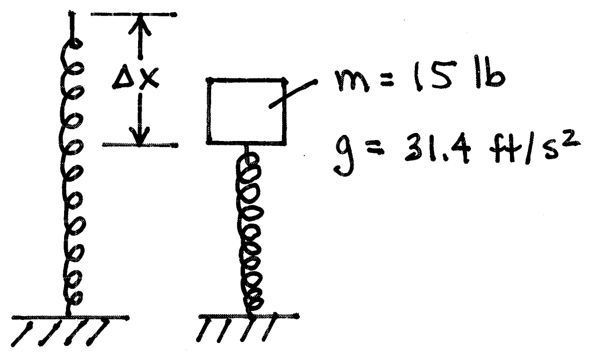
and

$$K = \frac{mg}{\Delta X}$$

$$= \frac{(5.0\ kg)(9.81\ m/s^2)}{(8\ cm)}\left|\frac{1\ N}{kg\cdot m/s^2}\right|$$

$$= 6.131\ N/cm \longleftarrow$$ proportionality constant

PROBLEM 1.17*



The spring is known to deflect 0.12 in for every 1lbf of applied force. Thus, we begin by determining the weight of the object

$$F_{grav} = mg$$

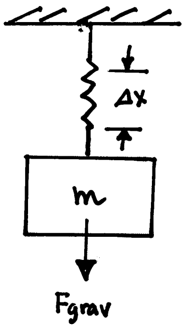
$$= (15\ lb)(31.4\ ft/s^2)\left|\frac{1\ lbf}{32.2\ ft\cdot lb/s^2}\right|$$

$$= 14.63\ lbf$$

The deflection is

$$\Delta X = (0.12\ in/lb_f)(14.63\ lb_f) = 1.76\ in \longleftarrow$$ ΔX

PROBLEM 1.18



For a linear spring, $F_{spring} = K(\Delta x)$, where Δx is the spring extension. Since $F_{spring} = F_{grav} = mg$, we have $K(\Delta x) = mg$. Since m and K are independent of location, the local acceleration of gravity is proportional to the deflection. Thus

mars:

$$\frac{g_{mars}}{g_{earth}} = \frac{(\Delta x)_{mars}}{(\Delta x)_{earth}} = \frac{0.116\,in}{0.291\,in} \Rightarrow g_{mars} = \left(\frac{0.116}{0.291}\right)\left(32.174\,\frac{ft}{s^2}\right)$$
$$= 12.825\,\frac{ft}{s^2} \longleftarrow$$

moon:

$$\frac{g_{moon}}{g_{earth}} = \frac{(\Delta x)_{moon}}{(\Delta x)_{earth}} \Rightarrow (\Delta x)_{moon} = \left(\frac{g_{moon}}{g_{earth}}\right)(\Delta x)_{earth}$$
$$= \left(\frac{5.471}{32.174}\right)(0.291\,in) = 0.049\,in \swarrow$$

PROBLEM 1.19

Deceleration occurs from 5 mi/h to rest in 0.1 s. The average acceleration magnitude is

$$|a|_{avg} = \frac{\Delta V}{\Delta t} = \left(\frac{5\,mi/h - 0}{0.1\,s}\right)\left|\frac{5280\,ft}{1\,mi}\right|\left|\frac{1\,h}{3600\,s}\right|$$
$$= 73.33\,ft/s^2$$

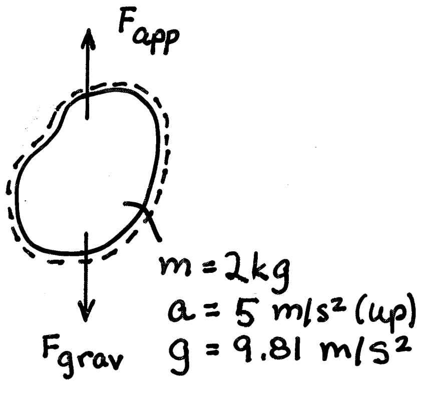
or, in g's

$$|a|_{avg} = \left(\frac{73.33\,ft/s^2}{32.2\,ft/s^2}\right) = 2.28\,g\text{'s} \longleftarrow \text{deceleration (in g's)}$$

Thus, the magnitude of the average force applied is

$$|F|_{avg} = m\,|a|_{avg} = (50\,lb)\left(73.33\,\frac{ft}{s^2}\right)\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|$$
$$= 113.9\,lbf \longleftarrow |F|_{avg}$$

PROBLEM 1.20*



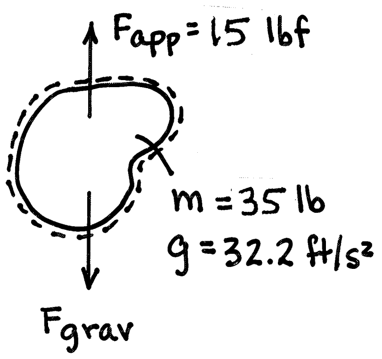
$$F_{app} - F_{grav} = ma$$

$$F_{app} = ma + mg = m(a+g)$$

$$= (2\,kg)(5 + 9.81)\frac{m}{s^2}\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|$$

$$= 29.62\ N \longleftarrow F_{app}$$

PROBLEM 1.21*



$$F_{app} - F_{grav} = ma$$

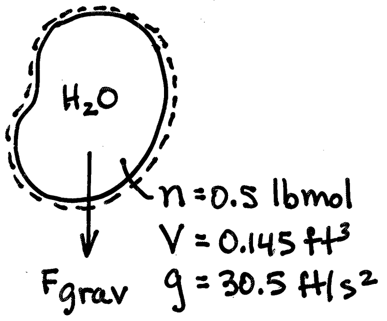
$$a = \frac{F_{app} - F_{grav}}{m} = \frac{F_{app} - mg}{m}$$

$$= \frac{F_{app}}{m} - g$$

$$= \left(\frac{15\ lbf}{35\ lb}\right)\left|\frac{32.2\ lb\cdot ft/s^2}{1\ lbf}\right| - 32.2\ ft/s^2$$

$$= -18.4\ ft/s^2 \text{ (downward)} \longleftarrow a$$

PROBLEM 1.22



From Table A-1E : $M = 18.02\ lb/lbmol$

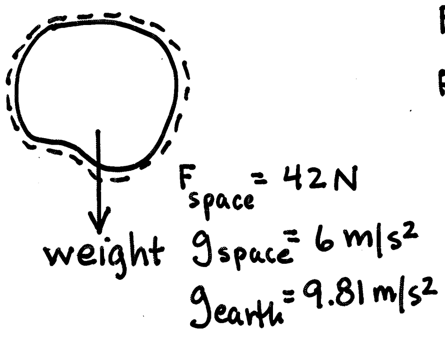
$$F_{grav} = mg = nMg$$

$$= (0.5\ lbmol)\left(18.02\frac{lb}{lbmol}\right)\left(30.5\frac{ft}{s^2}\right)\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|$$

$$= 8.534\ lbf \longleftarrow F_{grav}$$

$$\rho_{ave} = \frac{m}{V} = \frac{(0.5)(18.02)}{(0.145)} = 62.14\ lb/ft^3 \longleftarrow \rho_{ave}$$

PROBLEM 1.23

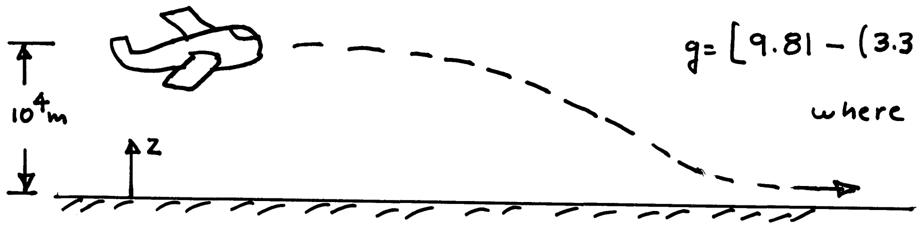


$$\left.\begin{aligned} F_{space} &= m\,g_{space} \\ F_{earth} &= m\,g_{earth} \end{aligned}\right\} \quad \frac{F_{space}}{g_{space}} = \frac{F_{earth}}{g_{earth}}$$

$$F_{earth} = F_{space}\left(\frac{g_{earth}}{g_{space}}\right) = (42\ N)\left(\frac{9.81}{6}\right)$$

$$= 68.67\ N \longleftarrow F_{earth}$$

PROBLEM 1.24



$g = [9.81 - (3.3\times10^{-6}/s^2)z]\ m/s^2$,

where z is in m.

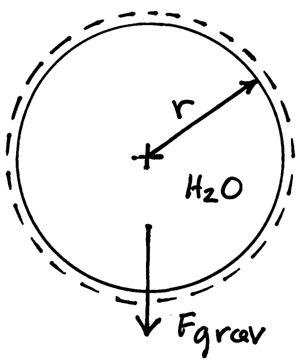
Since weight is $\mathcal{W} = mg$, the percent change in weight on landing is

$$\%\text{change} = \left[\frac{\mathcal{W}|_{z=0} - \mathcal{W}}{\mathcal{W}}\right](100)$$

$$= \left[\frac{[9.81\,m/s^2] - [(9.81\,m/s^2) - (3.3\times10^{-6}/s^2)(10^4\,m)]}{[9.81 - (3.3\times10^{-6})(10^4)]}\right](100) \doteq \left[\frac{3.3\times10^{-2}}{9.777}\right](100)$$

$$= 0.34\% \longleftarrow$$

PROBLEM 1.25



r = 30 ft

$\rho_{H_2O} = 62.4\ lb/ft^3$

$g = 32.1\ ft/s^2$

The mass of water is $m = \rho V$, where V is the volume of the spherical tank. Thus

$$V = \frac{4}{3}\pi r^3 = \left(\frac{4}{3}\right)(\pi)(30\ ft)^3 = 1.131\times10^5\ ft^3$$

and the mass is

$$m = \rho V = \left(62.4\ \frac{lb}{ft^3}\right)(1.131\times10^5)\ ft^3$$

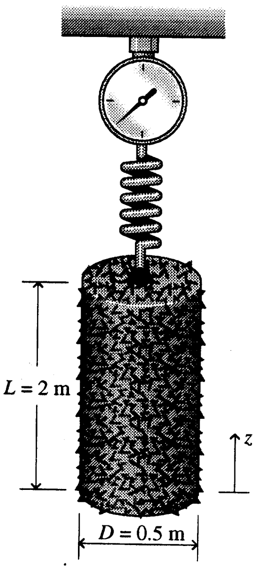
$$= 7.06\times10^6\ lb \longleftarrow \quad m$$

The weight is

$$F_{grav} = mg = (7.06\times10^6\ lb)\left(32.1\ \frac{ft}{s^2}\right)\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|$$

$$= 7.04\times10^6\ lbf \longleftarrow \quad F_{grav}$$

PROBLEM 1.26



$g = 9.78 \ \text{m/s}^2$

$\rho = 7800 - 360(z/L)^2 \ \text{kg/m}^3$, where z is in m.

The scale records the weight, $F_{grav} = mg$, where mass m is given by

$$m = \int_{Vol} \rho \, dV = \int_0^L \rho A \, dz$$

$$= A \int_0^L [7800 - 360(z/L)^2] \, dz$$

For simplicity, introduce a new variable: $\hat{z} = z/L$, so $dz = L\, d\hat{z}$ and the expression for mass reads

$$m = AL \int_0^1 [7800 - 360(\hat{z})^2] \, d\hat{z}$$

$$= AL \left[7800\hat{z} - \frac{360(\hat{z})^3}{3} \right]_0^1$$

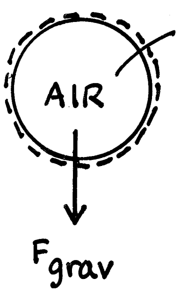
$$= \left(\frac{\pi D^2}{4}\right) L \left[7680 \frac{\text{kg}}{\text{m}^3} \right]$$

$$= \frac{\pi}{4}(0.5\ \text{m})^2 (2\ \text{m}) \left[7680 \frac{\text{kg}}{\text{m}^3} \right] = 3016\ \text{kg}$$

Finally,

$$F_{grav} = mg = (3016\ \text{kg})\left(9.78 \frac{\text{m}}{\text{s}^2}\right) \left| \frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2} \right| = 29{,}496\ \text{N} \longleftarrow$$

PROBLEM 1.27



d = 10 ft

$v = 15.1\ \text{ft}^3/\text{lb}$

$g = 31.0\ \text{ft/s}^2$

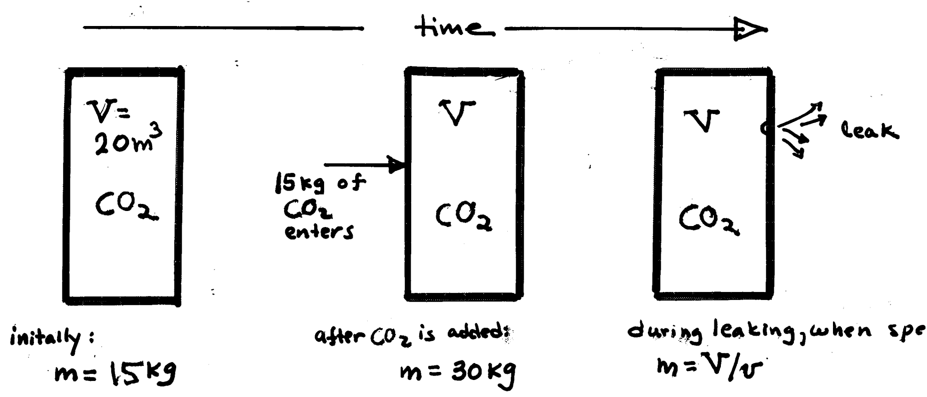
$$V = \frac{\pi d^3}{6} = \frac{\pi (10^3)\text{ft}^3}{6} = 523.6\ \text{ft}^3$$

$$m = \frac{V}{v} = \frac{523.6\ \text{ft}^3}{15.1\ \text{ft}^3/\text{lb}} = 34.68\ \text{lb}$$

$$F_{grav} = mg = (34.68\ \text{lb})\left(31.0 \frac{\text{ft}}{\text{s}^2}\right) \left| \frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2} \right|$$

$$= 33.39\ \text{lbf} \longleftarrow F_{grav}$$

PROBLEM 1.28*



(a) Using $v = V/m$

initial specific volume: $v = \dfrac{20\ m^3}{15\ kg} = 1.33\ \dfrac{m^3}{kg}$

after an additional 15 kg enters: $v = \dfrac{20\ m^3}{30\ kg} = 0.67\ \dfrac{m^3}{kg}$

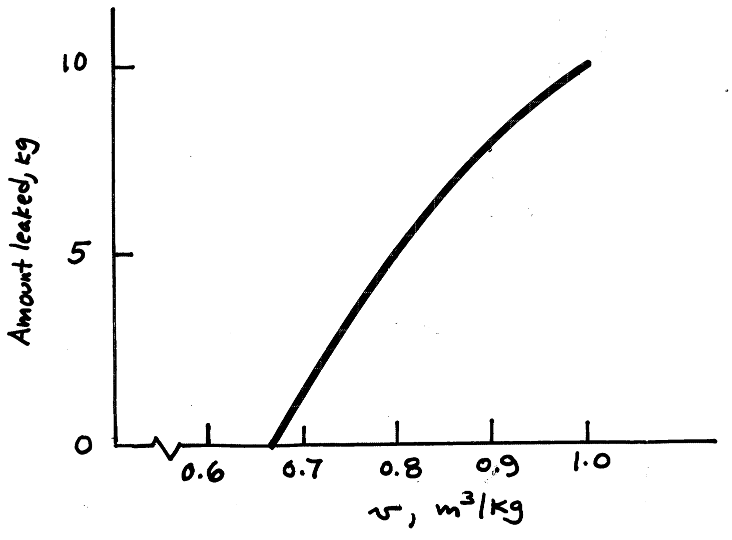
(b) Beginning with 30 kg of CO_2 in the cylinder, the amount of mass that has leaked is

$$\begin{bmatrix}\text{Amount}\\ \text{leaked}\end{bmatrix} = [30\ kg] - \begin{bmatrix}\text{mass in cylinder}\\ \text{when the specific}\\ \text{volume in the cylinder}\\ \text{is } v\end{bmatrix}$$

$$= [30\ kg] - \left[\frac{20\ m^3}{v}\right]$$

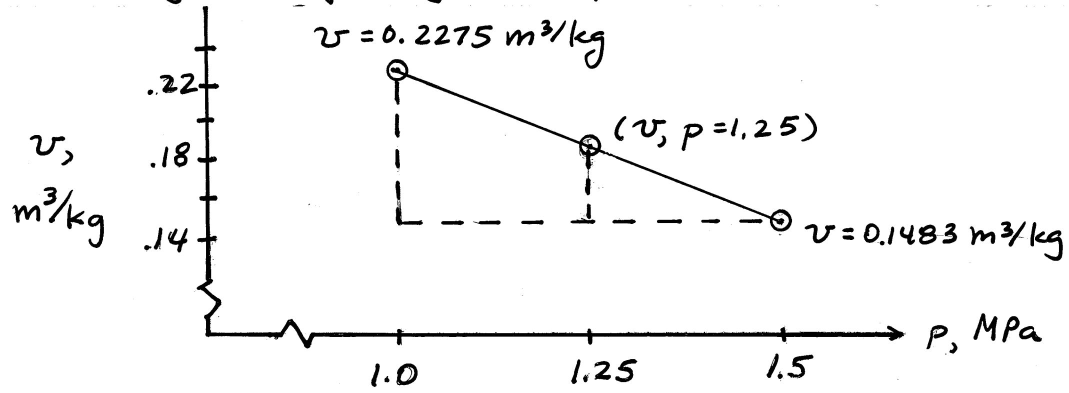
In writing this, we assume that the specific volume does not vary with location within the cylinder: the gas leaks slowly, so the specific volume within the cylinder varies only with time.

Since the specific volume v varies from 0.67 m³/kg, when 30 kg is in the cylinder, to the specified upper limit specified: 1.0 m³/kg, the plot is



PROBLEM 1.29

(a) At a temperature of 240°C, the specified pressure of 1.25 MPa falls between the tabe values of 1.0 and 1.5 MPa. To determine the specific volume corresponding to 1.25 MPa, we think of the slope of a straight line joining the adjacent table states, as follows:

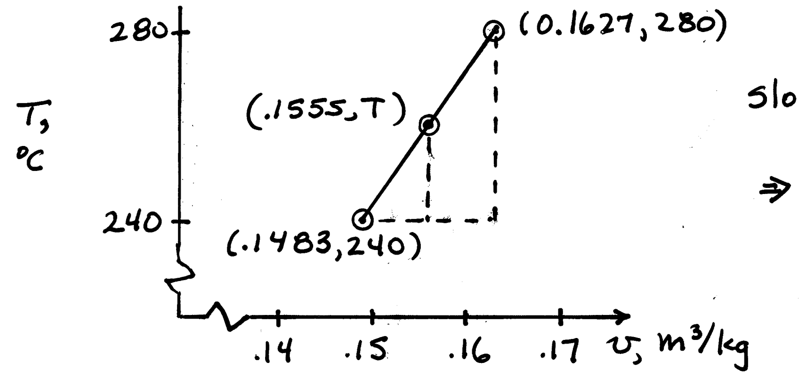


similar triangles:

$$|\text{slope}| = \frac{v - 0.1483}{1.5 - 1.25} = \frac{0.2275 - 0.1483}{1.5 - 1.0} \Rightarrow v = 0.1483 + \left(\frac{0.25}{0.50}\right)(0.2275 - 0.1483)$$

$$= 0.1879\ \text{m}^3/\text{kg} \longleftarrow \text{(a)}$$

(b) At a pressure of 1.5 MPa, the given specific volume of 0.1555 m³/kg falls between the table values of 240 and 280°C. To determine the temperature corresponding to the given specific volume, we think of the slope of a straight line joining the adjacent table states, as follows:



$$\text{slope} = \frac{T - 240}{.1555 - .1483} = \frac{280 - 240}{.1627 - .1483}$$

$$\Rightarrow T = 240 + \left[\frac{.1555 - .1483}{.1627 - .1483}\right](40)$$

$$= 260°\text{C} \longleftarrow \text{(b)}$$

(c) In this case, the specified pressure falls between the table values of 1.0 and 1.5 MPa and the specified temperature falls between the table values of 200 and 240°C. Thus, double interpolation is required.

- At 220°C, the specific volume at each pressure is simply the average over the interval:

at 1.0 MPa, 220°C; $v = \dfrac{.2060 + .2275}{2} = 0.21675$ m³/kg

at 1.5 MPa, 220°C $v = \dfrac{.1325 + .1483}{2} = 0.1404$ m³/kg

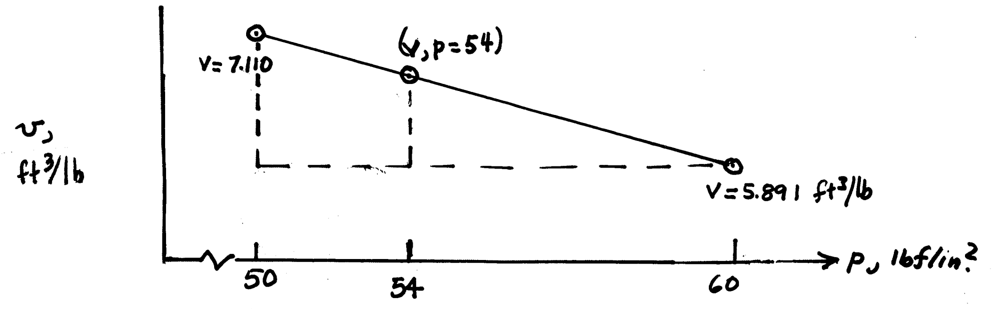
- Thus, with the same approach as in (a)

$$\frac{v - 0.1404}{1.5 - 1.4} = \frac{0.21675 - 0.1404}{1.5 - 1.0} \Rightarrow v = 0.1404 + \left(\frac{0.1}{0.5}\right)(0.21675 - 0.1404)$$

$$= 0.15567\ \text{m}^3/\text{kg} \longleftarrow \text{(c)}$$

PROBLEM 1.30

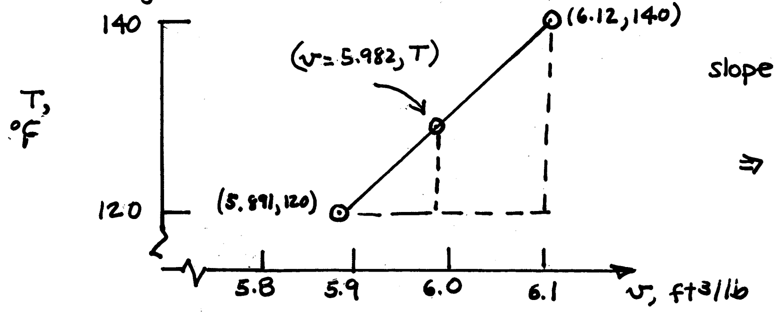
(a) At a temperature of 120°C, the specified pressure of 54 lbf/in.² falls between the table values of 50 and 60 lbf/in.² To determine the specific volume corresponding to 54 lbf/in.², we think of the slope of a straight line joining the adjacent table states, as follows:



similar triangles:

$$|\text{slope}| = \frac{v-5.891}{60-54} = \frac{7.110-5.891}{60-50} \Rightarrow v = 5.891 + \frac{6}{10}(7.110-5.891) = 6.622 \frac{\text{ft}^3}{\text{lb}} \quad \leftarrow (a)$$

(b) At a pressure of 60 lbf/in.², the given specific volume of 5.982 ft³/lb falls between the table values of 120 and 140°F. To determine the temperature corresponding to the given specific volume, we think of the slope of a straight line joining the adjacent table states, as follows:



$$\text{slope} = \frac{T-120}{5.982-5.891} = \frac{140-120}{6.12-5.891}$$

$$\Rightarrow \quad T = 120 + \left[\frac{5.982-5.891}{6.12-5.891}\right](20)$$

$$= 127.9\,°\text{F} \quad \longleftarrow (b)$$

(c) In this case, the specified pressure falls between the table values of 50 and 60 lbf/in² and the specified temperature falls between the table values of 100 and 120°F. Thus, double interpolation is required.

- At 110°F, the specific volume at each pressure is simply the average over the interval:

$$\text{at } 50\frac{\text{lbf}}{\text{in}^2},\ 110°\text{F};\quad v = \frac{7.110+6.836}{2} = 6.973\ \text{ft}^3/\text{lb}$$

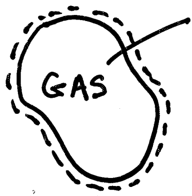
$$\text{at } 60\frac{\text{lbf}}{\text{in}^2},\ 110°\text{F};\quad v = \frac{5.891+5.659}{2} = 5.775\ \text{ft}^3/\text{lb}$$

- Then, with the same approach as in (a)

$$\frac{v-5.775}{60-58} = \frac{6.973-5.775}{60-50} \Rightarrow v = 5.775 + \frac{2}{10}\left[6.973-5.775\right]$$

$$= 6.015\ \text{ft}^3/\text{lb} \quad \longleftarrow (c)$$

PROBLEM 1.31*



$m = 5\,kg$

$pv^{1.3} = const.$

$p_1 = 1\ bar$, $v_1 = 0.2\ m^3/kg$

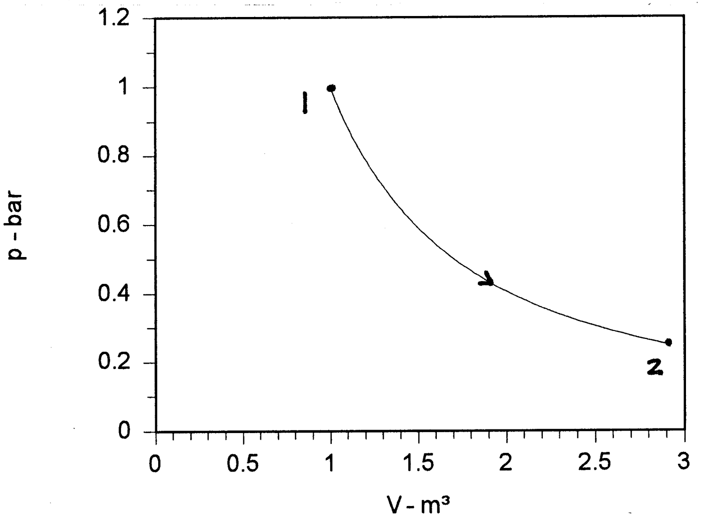
$p_2 = 0.25\ bar$

From the pressure-specific volume relation

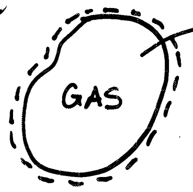
$$v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.3}} v_1 = \left(\frac{1}{0.25}\right)^{\frac{1}{1.3}} (0.2\ m^3/kg)$$

$$= 0.5810\ m^3/kg$$

$$V_2 = v_2 m = 2.905\ m^3 \longleftarrow V_2$$



PROBLEM 1.32



$m = 1\ lb$

$pV^n = const.$

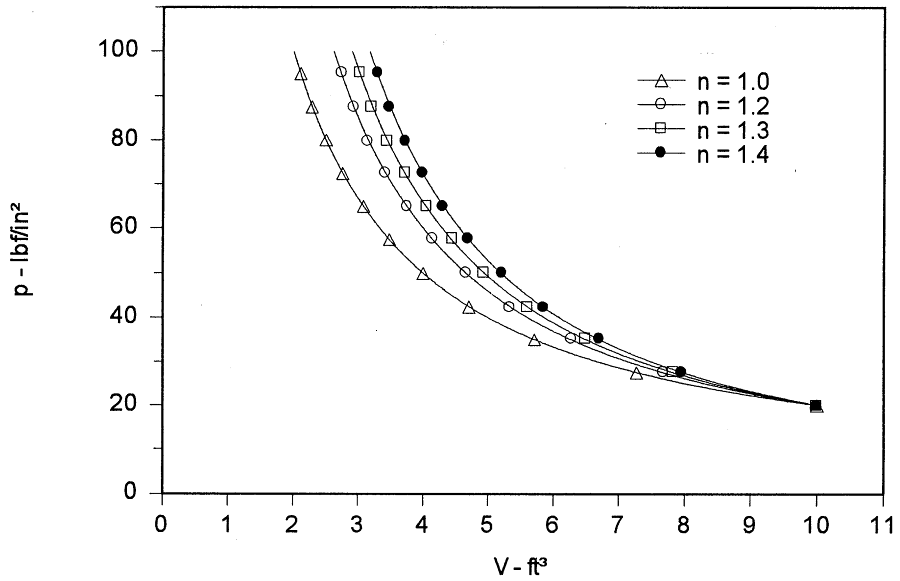
$p_1 = 20\ lbf/in^2$, $V_1 = 10\ ft^3$

$p_2 = 100\ lbf/in^2$

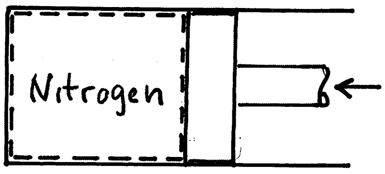
From the pressure-volume relation

$$V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} V_1 = \left(\frac{20}{100}\right)^{\frac{1}{n}} (10\ ft^3)$$

$$\left.\begin{array}{l} n = 1;\ V_2 = 2\ ft^3 \\ n = 1.2;\ V_2 = 2.615\ ft^3 \\ n = 1.3;\ V_2 = 2.900\ ft^3 \\ n = 1.4;\ V_2 = 3.168\ ft^3 \end{array}\right\} \longleftarrow V_2$$



PROBLEM 1.33*



$p_1 = 15 \text{ lbf/in}^2$
$V_1 = 2.5 \text{ ft}^3$
$p_2 = 100 \text{ lbf/in}^2$
$V_2 = 1.5 \text{ ft}^3$

The pressure-volume relation is linear during the process. Thus

$$p = p_1 + \left(\frac{p_2 - p_1}{V_2 - V_1}\right)(V - V_1)$$

Or, using given data

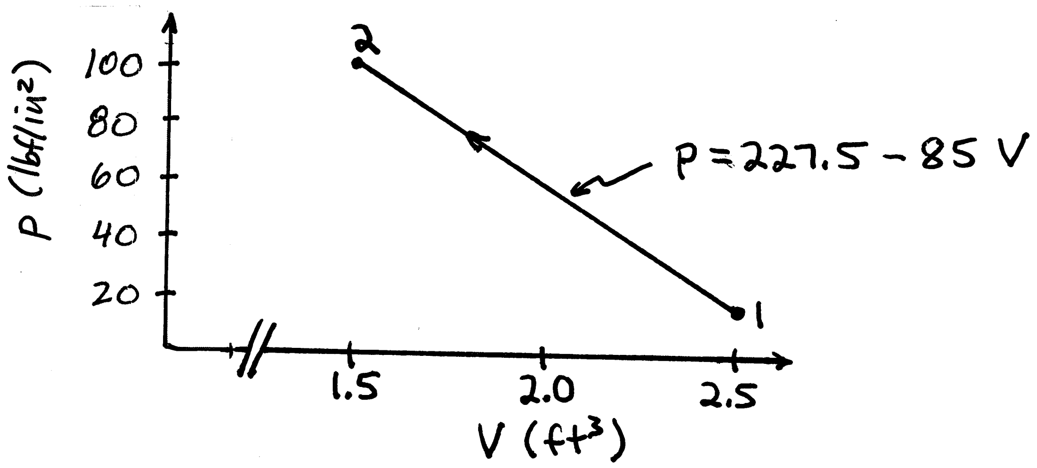
$$p = 15 \frac{\text{lbf}}{\text{in}^2} + \frac{(100-15)\text{ lbf/in}^2}{(1.5-2.5)\text{ ft}^3}(V - 2.5)\text{ ft}^3$$

$$= 227.5 - 85V$$

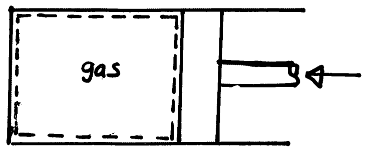
When $V = 2.1 \text{ ft}^3$

$$p = 227.5 - 85(2.1) = 49 \text{ lbf/in}^2 \longleftarrow \underline{p}$$

On p-V coordinates



PROBLEM 1.34



$p_1 = 1$ bar
$V_1 = 1$ L
$p_2 = 4$ bar

(a) The process is described by pV = constant. The constant can be evaluated using data at state 1:

$$pV = \text{constant}$$
$$= p_1 V_1$$
$$= (1\,\text{bar})(1\,\text{L}) = 1\ \text{bar}\cdot\text{L}$$

So, for every state during the process, we have the relation

$$pV = 1\ \text{bar}\cdot\text{L}$$

When $p = 3$ bar,

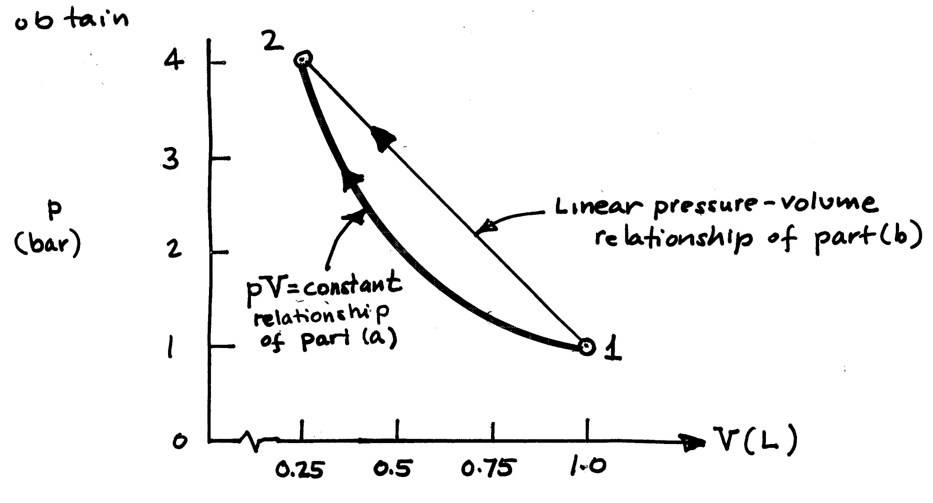
$$V = \frac{1\ \text{bar}\cdot\text{L}}{3\ \text{bar}} = 0.33\ \text{L}$$

when $p = 4$ bar, $V_2 = \frac{1\ \text{bar}\cdot\text{L}}{4\ \text{bar}} = 0.25\ \text{L}$

Plotting the relation on pressure-volume coordinates we use

$$p = \frac{1\ \text{bar}\cdot\text{L}}{V}$$

to obtain

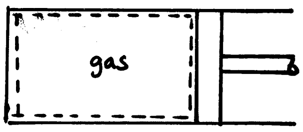


(b) For comparison, the linear pressure-volume relationship is shown on the plot above. The volume corresponding to $p = 3$ bar can be obtained simply using the slope of the straight line between 1 and 2:

$$|\text{slope}| = \frac{(4-1)\,\text{bar}}{(1.0-0.25)\,\text{L}} = \frac{(3-1)}{(1.0-V)} \Rightarrow V = 0.5\ \text{L}$$

This value also can be read from the plot.

PROBLEM 1.35



Thermodynamic cycle:

1-2: $pV = \text{constant}$

$p_1 = 1\,\text{bar},\ V_1 = 1\,m^3,\ V_2 = 0.2\,m^3$

2-3 $p = \text{constant},\ V > V_2$ (expansion), $V_3 = 1.0\,m^3$

3-1 $V = \text{constant}$

For process 1-2, $pV = \text{constant}$. The constant can be evaluated using data at state 1:

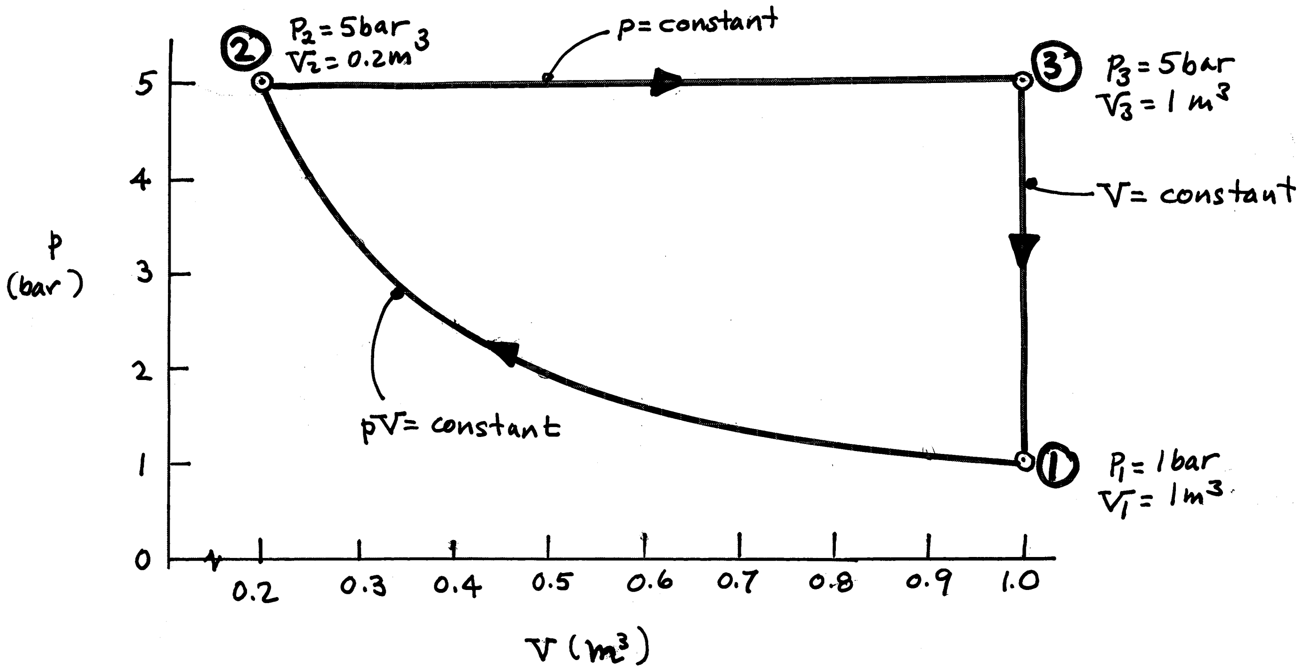
$$\begin{aligned} pV &= \text{constant} \\ &= p_1 V_1 \\ &= (1\,\text{bar})(1\,m^3) = 1\ \text{bar}\cdot m^3 \end{aligned}$$

Accordingly, on a pressure-volume plot process 1-2 is described by

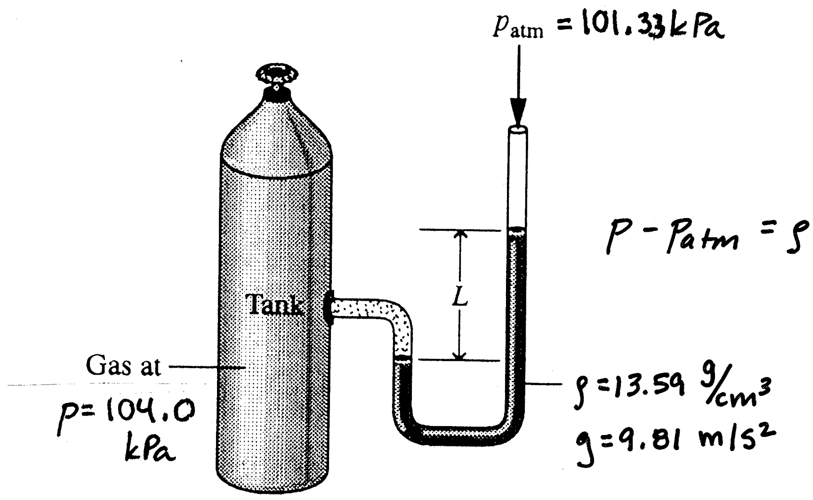
$$p = \frac{1\,\text{bar}\cdot m^3}{V}$$

In particular, when $V_2 = 0.2\,m^3$, $p = 5\,\text{bar}$.

The thermodynamic cycle takes the form



PROBLEM 1.36*

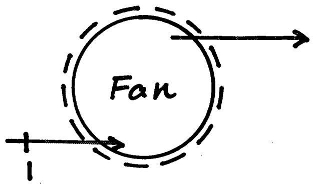


$$p - p_{atm} = \rho g L$$

(a)
$$L = \frac{p - p_{atm}}{\rho g}$$
$$= \frac{(104.0 - 101.33)\text{ kPa}}{(13.59\text{ g/cm}^3)(9.81\text{ m/s}^2)}\left|\frac{10^3\text{ N/m}^2}{1\text{ kPa}}\right|\left|\frac{10^3\text{ g}}{1\text{ kg}}\right|\left|\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right|\left|\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right|$$
$$= 0.02\text{ m} = 2\text{ cm} \longleftarrow L$$

(b) $p_{gage} = p - p_{atm} = 104.0 - 101.33 = 2.67\text{ kPa} \longleftarrow p_{gage}$

PROBLEM 1.37*

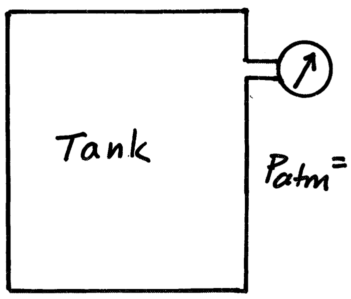


$p_{1,vac}$ = 4.2 in. H_2O
p_{atm} = 14.5 lbf/in²
g = 32.2 ft/s²
ρ_f = 49.94 lb/ft³

$$p_{1,vac} = \rho_f\, g\, L$$
$$= \left(49.94\,\frac{\text{lb}}{\text{ft}^3}\right)\left(32.2\,\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{4.2}{12}\text{ ft}\right)\left|\frac{1\text{ lbf}}{32.2\text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\text{ ft}^2}{144\text{ in}^2}\right|$$
$$= 0.1214\text{ lbf/in}^2$$

$$p_{1,abs} = p_{atm} - p_{1,vac}$$
$$= 14.5 - 0.1214 = 14.379\,\frac{\text{lbf}}{\text{in}^2} \longleftarrow p_{1,abs}$$

PROBLEM 1.38*



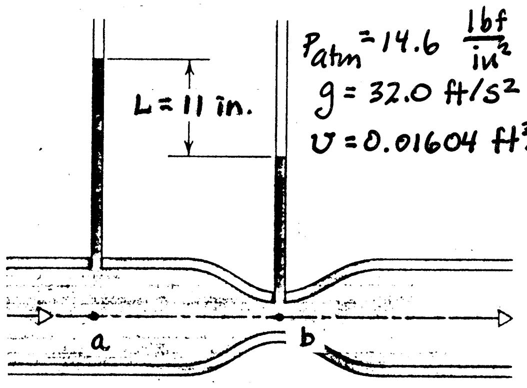
p_{atm} = 98 kPa

p_{abs} = 0.4 bar = 40 kPa

$$p_{abs} < p_{atm} \Rightarrow \text{vacuum}$$

$$p_{vac} = p_{atm} - p_{abs}$$
$$= 98 - 40 = 58\text{ kPa} \longleftarrow p_{vac}$$

PROBLEM 1.39



$p_{atm} = 14.6 \ \frac{lbf}{in^2}$

$g = 32.0 \ ft/s^2$

$v = 0.01604 \ ft^3/lb$

$$|\Delta p_{gage}| = \rho_{H_2O}\, g\, L$$

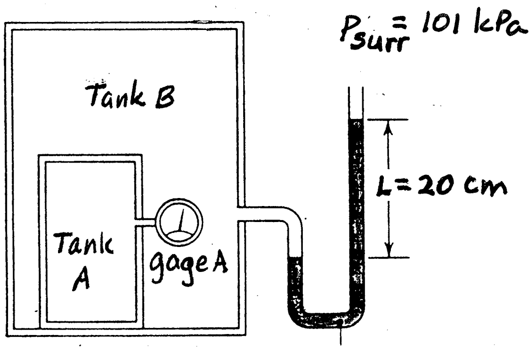
$$\rho_{H_2O} = 1/v_{H_2O} = 62.344 \ lb/ft^3$$

$$|\Delta p_{gage}| = \left(62.344 \ \frac{lb}{ft^3}\right)\left(32.0 \ \frac{ft}{s^2}\right)\left(\frac{11}{12} ft\right)\left|\frac{1 \ lbf}{32.2 \ lb\cdot ft/s^2}\right|\left|\frac{1 \ ft^2}{144 \ in^2}\right|$$

$$= 0.3944 \ lbf/in^2 \longleftarrow \Delta p_{gage}$$

(decreases)

PROBLEM 1.40



$p_{surr} = 101 \ kPa$

$p_{gage,A} = 1.4$ bars

Mercury ($\rho = 13.59 \ g/cm^3$)

$g = 9.81 \ m/s^2$

$$L = 20 \ cm = 0.2 \ m$$

$$p_{gage,B} = \rho_{Hg}\, g\, L$$

$$= \left(13.59 \ \frac{g}{cm^3}\right)\left(9.81 \ \frac{m}{s^2}\right)(0.2 \ m)\left|\frac{1 \ kg}{10^3 g}\right|\left|\frac{10^6 \ cm^3}{1 \ m^3}\right|\left|\frac{1 \ N}{1 \ kg\cdot m/s^2}\right|$$

$$= 26{,}664 \ N/m^2 = 0.2666 \ bar$$

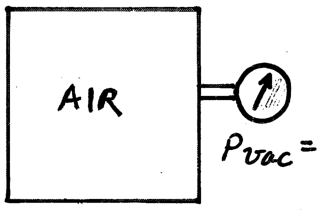
$$p_{abs,B} = p_{surr} + p_{gage,B}$$

$$= 1.01 + .2666 = 1.2766 \ bar \longleftarrow$$

$$p_{abs,A} = p_{abs,B} + p_{gage,A}$$

$$= 1.2766 + 1.4 = 2.6766 \ bar \longleftarrow$$

PROBLEM 1.41



$p_{vac} = 0.2$ bar

$p_{atm} = 750$ mm Hg

$\rho_{Hg} = 13.59 \ g/cm^3$

$g = 9.81 \ m/s^2$

$$p_{atm} = \rho_{Hg}\, g\, L_{Hg}$$

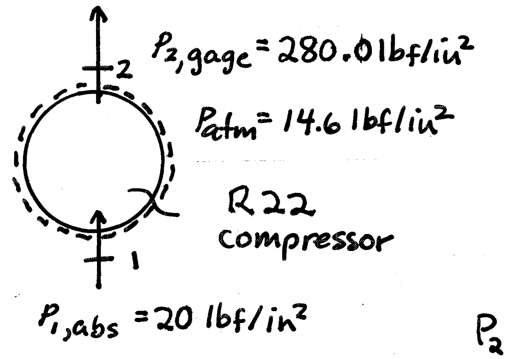
$$= \left(13.59 \ \frac{g}{cm^3}\right)(9.81 \ m/s^2)\left(\frac{750}{1000}\right) m \left|\frac{1 \ kg}{10^3 g}\right|\left|\frac{10^6 \ cm^3}{1 \ m^3}\right|\left|\frac{1 \ N}{1 \ kg\cdot m/s^2}\right|\left|\frac{1 \ bar}{10^5 \ N/m^2}\right|$$

$$= 1 \ bar$$

$$p_{abs} = p_{atm} - p_{vac}$$

$$= 1 \ bar - 0.2 \ bar = 0.8 \ bar \longleftarrow p_{abs}$$

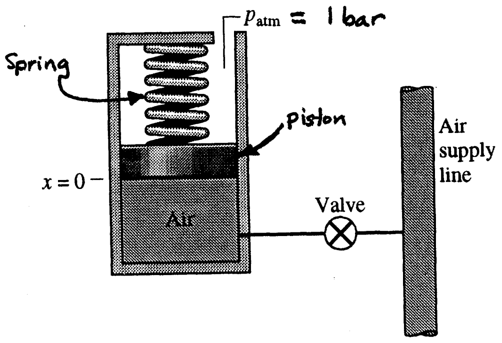
PROBLEM 1.42*


2
$p_{2,gage} = 280.0\ \text{lbf/in}^2$
$p_{atm} = 14.6\ \text{lbf/in}^2$
R22
compressor
1
$p_{1,abs} = 20\ \text{lbf/in}^2$

$$\Delta p_{abs} = p_{2,abs} - p_{1,abs}$$
$$= (p_{2,gage} + p_{atm}) - p_{1,abs}$$
$$= (280.0 + 14.6) - 20$$
$$= 274.6\ \text{lbf/in}^2 \longleftarrow \underline{\Delta p_{abs}}$$

$$p_{2,abs}/p_{1,abs} = (280.0 + 14.6)/20 = 14.73 \longleftarrow \underline{p_2/p_1}$$

PROBLEM 1.43



$g = 9.81\ \text{m/s}^2$

$F_{spring} = kx$, where $k = 10{,}000\ \text{N/m}$

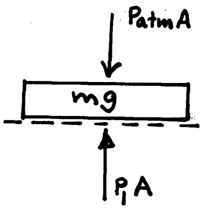
For the piston:

$m = 10\ \text{kg}$

$A = 7.8 \times 10^{-3}\ \text{m}^2$

For the air: $\Delta V = 3.9 \times 10^{-4}\ \text{m}^3$

Initially, $x = 0$ and there is no spring force acting on the piston. Also, friction between the piston and the cylinder wall can be ignored. Accordingly, the force exerted by the air within the cylinder on the bottom of piston is equal to the weight of the piston plus the force exerted by the atmosphere on the top of the piston:



$\Sigma F_x = 0$:

$$p_1 A = p_{atm} A + mg$$

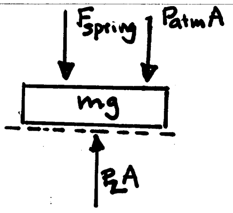
$$p_1 = p_{atm} + \frac{mg}{A}$$

$$p_1 = 1\ \text{bar} + \left[\frac{(10\ \text{kg})(9.81\ \text{m/s}^2)}{7.8 \times 10^{-3}\ \text{m}^2}\right]\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{bar}}{10^5\ \text{N/m}^2}\right|$$

$$p_1 = 1.126\ \text{bar} \longleftarrow p_1$$

Finally, the force exerted by the air within the cylinder on the bottom of the piston is equal to the weight of the piston plus the force exerted by the atmosphere on the top of the piston plus the force exerted by the spring on the top of the piston:

PROBLEM 1.43 (Cont'd)



$$\Sigma F_x = 0:$$

$$P_2 A = P_{atm} A + mg + F_{spring}$$

$$P_2 = P_{atm} + \frac{mg}{A} + \frac{F_{spring}}{A}$$

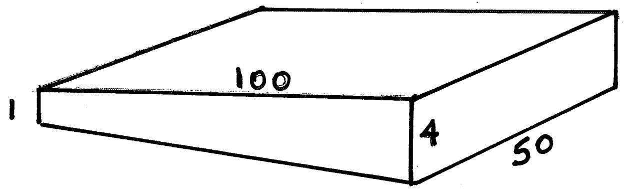
For the spring, $F_{spring} = kx$, where x is found using the increase in volume of the air:
$$x = \frac{\Delta V}{A} = \frac{3.9 \times 10^{-4}\,m^3}{7.8 \times 10^{-3}\,m^2} = 0.05\ m.$$

Collecting results

$$P_2 = P_{atm} + \frac{mg}{A} + \left[\frac{(10{,}000\,N/m)(0.05\,m)}{(7.8 \times 10^{-3}\,m^2)}\left|\frac{1\,bar}{10^5\,N/m^2}\right|\right]$$
$$= 1.126\ bar + 0.641\ bar$$
$$= 1.767\ bar \longleftarrow P_2$$

PROBLEM 1.44

$P_{atm} = 0.98$ bar



$\rho_{H_2O} = 998.2\ kg/m^3$
$g = 9.8\ m/s^2$

The total force on the bottom of the pool is the sum of the weight of the water and the downward force of atmospheric pressure on the surface of the water;

$$F_{tot} = F_{grav} + F_{atm} \qquad (a)$$

To get the weight of the water, first find the mass, as follows

$$m = \rho V = (998.2\ \tfrac{kg}{m^3})\left[(1)(100)(50) + \tfrac{1}{2}(100)(3)(50)\right] m^3$$
$$= (998.2)(12500) = 1.25 \times 10^7\ kg$$

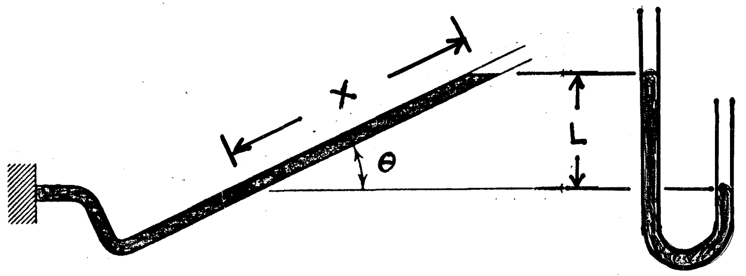
Thus
$$F_{grav} = mg = (1.25 \times 10^7\,kg)(9.81\,m/s^2)\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|\left|\frac{1\,kN}{10^3\,N}\right| = 1.226 \times 10^5\,kN$$
$$F_{atm} = P_{atm} A_{surface} = (0.98\,bar)(100 \times 50)\,m^2\left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,kN}{10^3\,N}\right| = 4.9 \times 10^5\,kN$$

Or
$$F_{tot} = 1.226 \times 10^5 + 4.9 \times 10^5 = 6.126 \times 10^5\,kN \longleftarrow F_{tot}$$

The depth at the center of the pool is $h = 2.5$ m. Thus

$$P = P_{atm} + \rho g h$$
$$= (0.98\,bar)\left|\frac{100\,kPa}{1\,bar}\right| + (998.2\,\tfrac{kg}{m^3})(9.8\,\tfrac{m}{s^2})(2.5\,m)\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|\left|\frac{1\,kPa}{10^3\,N/m^2}\right| = 122.5\,kPa \longleftarrow P$$

PROBLEM 1.45



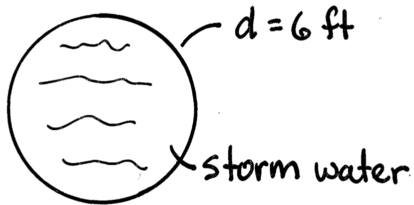
$L/x = \sin\theta$

$x = \dfrac{L}{\sin\theta}$

$\therefore\ x > L$

inclined manometer gives greater resolution

PROBLEM 1.46



$\rho = 62.2\ \text{lb/ft}^3$
$g = 32.2\ \text{ft/s}^2$

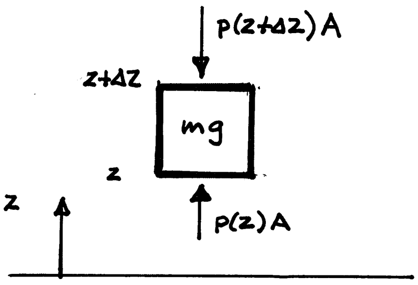
$$\Delta p = \rho g d$$

$$= \left(62.2\ \frac{\text{lb}}{\text{ft}^3}\right)\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)(6\ \text{ft})\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right|$$

$$= 2.59\ \text{lbf/in}^2 \longleftarrow \Delta p$$

PROBLEM 1.47

Consider the variation of pressure owing to the effect of gravity. For an element in a motionless gas or liquid, the forces acting are the forces of pressure on the upper and lower surfaces and the weight of the system:



$\Sigma F_z = 0$: $$0 = p(z+\Delta z)A + mg - p(z)A$$

where $$m = \rho V = \rho[A\Delta z] = \frac{[A\Delta z]}{v}$$

Rearranging $$A\left[p(z+\Delta z) - p(z)\right] = -\left(\frac{A\Delta z}{v}\right)g$$

or $$\frac{p(z+\Delta z) - p(z)}{\Delta z} = -\frac{g}{v}$$

In the limit as $\Delta z \to 0$. This gives

$$\frac{dp}{dz} = -\frac{g}{v} \qquad (*)$$

(a) Atmosphere. If $v = c/p$, where $c = 72.435 (m^3/kg)(kPa) = 72{,}435 \frac{N\cdot m}{kg}$, is inserted in Eq. (*), we have

$$\frac{dp}{dz} = -\frac{gp}{c} \Rightarrow \frac{d \ln p}{dz} = -g/c \Rightarrow \ln p = -\frac{gz}{c} + \ln K$$

or

$$p = K\exp\left(-\frac{gz}{c}\right)$$

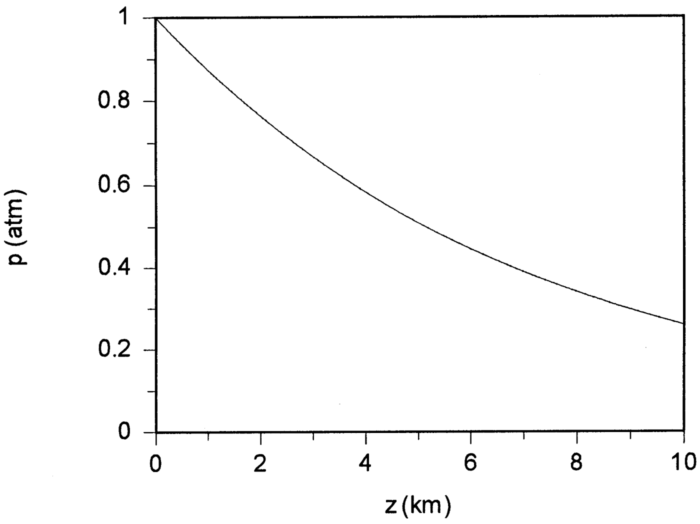
When $z=0$, $p = P_0$ (1atm), giving $p = P_0 \exp(-gz/c)$.

Inserting known values gives p in atm when z is in km

$$p = P_0 \exp\left(\frac{-(9.81 m/s^2)(z\ km)}{72{,}435 \frac{N\cdot m}{kg}} \left| \frac{10^3 m}{1 km} \right| \left| \frac{1N}{1 kg\cdot m/s^2} \right| \right) = P_0 \exp(-0.135 z)$$

This relationship is shown in the accompanying plot.

PROBLEM 1.47 (Cont'd)



(b) Ocean. Letting $\hat{z}\,(=-z)$ denote depth, Eq. (*) reads

$$\frac{dp}{d\hat{z}} = \frac{g}{v}$$

Then, assuming v is constant

$$p = \left(\frac{g}{v}\right)\hat{z} + K$$

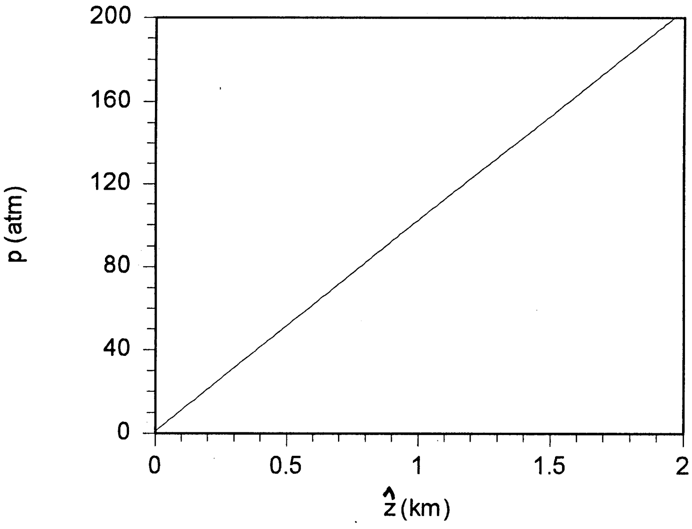
When $\hat{z}=0$, $p = p_0$ (1 atm), giving $p = \left(\frac{g}{v}\right)\hat{z} + p_0$.

Inserting known values gives p in atm when $\hat{z}$ is in km

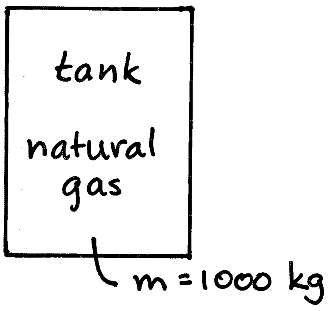
$$p = p_0 + \left[\frac{(9.81\ \text{m/s}^2)\,\hat{z}(\text{km})}{0.956\times10^{-3}\ \text{m}^3/\text{kg}}\left|\frac{10^3\ \text{m}}{1\ \text{km}}\right|\left|\frac{1\ \text{atm}}{1.01325\times10^5\ \text{N/m}^2}\right|\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\right]$$

$$= p_0 + 101.3\,\hat{z}$$

This relationship is shown in the accompanying plot.



PROBLEM 1.48



For the gas in the tank, the p-v-T relation is

$$p = [(5.18\times10^{-3})\,T/(v-0.002668)] - (8.91\times10^{-3})/v^2$$

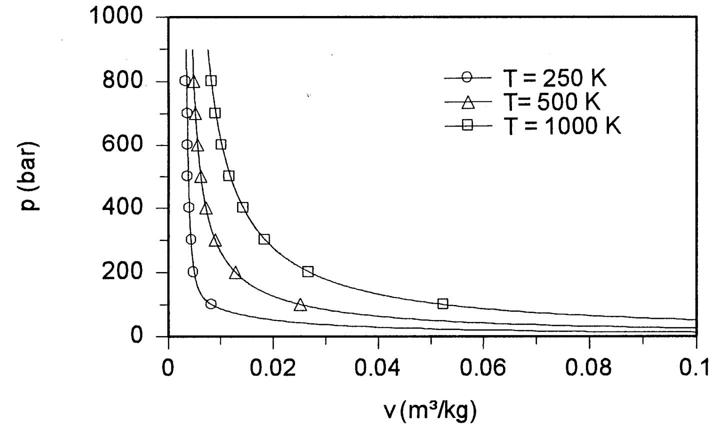
Solving iteratively for v at $p = 100$ bar, $T = 255$ K we get

$$v = 0.00884 \text{ m}^3/\text{kg}$$

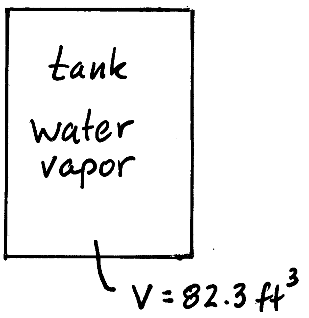
and

$$V = mv = 8.84 \text{ m}^3 \longleftarrow \quad V$$

The following plots can be constructed for T = 250, 500, and 1000 K:



PROBLEM 1.49



For the water vapor in the tank, the p-v-T relation is

$$p = [(0.5954)\,T/(v-0.2708)] - 63.36/v^2$$

where v is in ft³/lb, T is in °R, and p is in lbf/in²

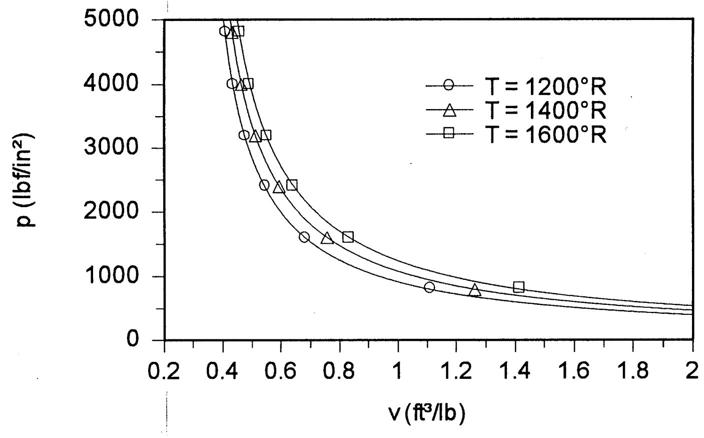
Solving iteratively for v at $p = 1500$ lbf/in², $T = 1140$°R we get

$$v = 0.686 \text{ ft}^3/\text{lb}$$

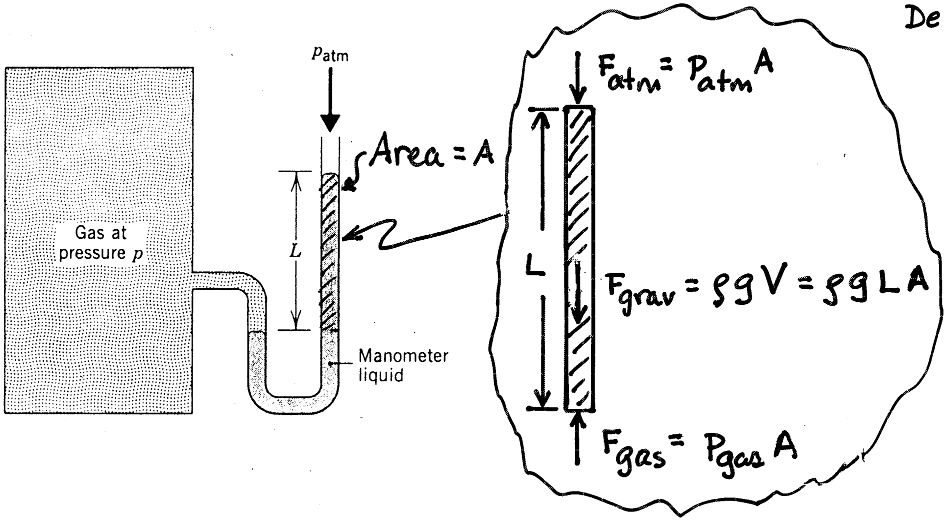
Thus, with the given value for V

$$m = V/v = 82.3/0.686 = 120 \text{ lb} \longleftarrow \quad m$$

The following plots can be constructed for T = 1200, 1400, 1600 °R:



PROBLEM 1.50



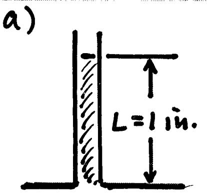
Derivation of Eq. 1.15:

$$\Sigma F(\uparrow) = 0$$

$$P_{gas}A - P_{atm}A - \rho g L A = 0$$

$$\therefore P_{gas} - P_{atm} = \rho g L \leftarrow$$

(a)



H_2O: $P_{gage} = \rho g L$

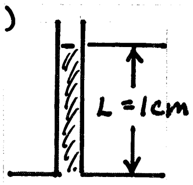
$$= \left(6.24\ \frac{lb}{ft^3}\right)\left(32.2\ \frac{ft}{s^2}\right)\left(\frac{1}{12}\ ft\right)\left|\frac{1\ lbf}{32.2\ lb \cdot ft/s^2}\right|\left|\frac{1\ ft^2}{144\ in^2}\right|$$

$$= 0.0361\ lbf/in^2 \longleftarrow P_{gage}$$

Hg: $\rho = 13.59\ \rho_{H_2O}$

$$\Rightarrow P_{gage} = 13.59\ (0.0361) = 0.491\ lbf/in^2 \longleftarrow P_{gage}$$

(b)



H_2O: $P_{gage} = \rho g L$

$$= \left(1000\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(0.01\ m)\left|\frac{1\ N}{1\ kg \cdot m/s^2}\right|\left|\frac{1\ bar}{10^5\ N/m^2}\right|$$

$$= 9.81 \times 10^{-4}\ bars \longleftarrow P_{gage}$$

Hg: $\rho_{Hg} = 13.59\ \rho_{H_2O}$

$$\Rightarrow P_{gage} = 13.59\ (9.81 \times 10^{-4}) = 0.01333\ bars \longleftarrow P_{gage}$$

PROBLEM 1.51*

Using Eq. 1.22

$$T(°F) = 1.8\ T(°C) + 32$$

(a) $\underline{\underline{T(°C) = 21}}$

$T(°F) = (1.8)(21) + 32 = 69.8$

$T(°R) = T(°F) + 459.67 = 529.47$

(b) $\underline{\underline{T(°C) = -40}}$

$T(°F) = (1.8)(-40) + 32 = -40$

$T(°R) = -40 + 459.67 = 419.67$

(c) $\underline{\underline{T(°C) = 500}}$

$T(°F) = (1.8)(500) + 32 = 932$

$T(°R) = 932 + 459.67 = 1391.67$

(d) $\underline{\underline{T(°C) = 0}}$

$T(°F) = (1.8)(0) + 32 = 32$

$T(°R) = 32 + 459.67 = 491.67$

(e) $\underline{\underline{T(°C) = 100}}$

$T(°F) = (1.8)(100) + 32 = 212$

$T(°R) = 212 + 459.67 = 671.67$

(f) $\underline{\underline{T(°C) = -273.15}}$

$T(°F) = (1.8)(-273.15) + 32 = -459.67$

$T(°R) = -459.67 + 459.67 = 0$

PROBLEM 1.52*

Using Eq. 1.22

$$T(°C) = \frac{T(°F)}{1.8} - \frac{32}{1.8}$$

$$= \frac{T(°F)}{1.8} - 17.78$$

(a) $\underline{\underline{T(°F) = 68}}$

$T(°C) = \frac{68}{1.8} - 17.78 = 20$

$T(K) = 20 + 273.15 = 293.15$

(b) $\underline{\underline{T(°F) = -40}}$

$T(°C) = \frac{-40}{1.8} - 17.78 = -40$

$T(K) = -40 + 273.15 = 233.15$

(c) $\underline{\underline{T(°F) = 500}}$

$T(°C) = \frac{500}{1.8} - 17.78 = 260$

$T(K) = 260 + 273.15 = 533.15$

(d) $\underline{\underline{T(°F) = 0}}$

$T(°C) = \frac{0}{1.8} - 17.78 = -17.78$

$T(K) = -17.78 + 273.15 = 255.37$

(e) $\underline{\underline{T(°F) = 212}}$

$T(°C) = \frac{212}{1.8} - 17.78 = 100$

$T(K) = 100 + 273.15 = 373.15$

(f) $\underline{\underline{T(°F) = -459.67}}$

$T(°C) = \frac{-459.67}{1.8} - 17.78 = -273.15$

$T(K) = 0$

PROBLEM 1.53

$$T_2(°C) - T_1(°C) = [T_2(°C) + \cancel{273.15}] - [T_1(°C) + \cancel{273.15}] = T_2(K) - T_1(K)$$

$$T_2(°F) - T_1(°F) = [T_2(°F) + \cancel{459.67}] - [T_1(°F) + \cancel{459.67}] = T_2(°R) - T_1(°R)$$

PROBLEM 1.54

The expression for resistance is

$$R = R_o \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_o}\right)\right]$$

where $R_o = 2.2\ \Omega$ and $T_o = 310$ K

Since $R = 0.31\ \Omega$ at $T = 422$ K

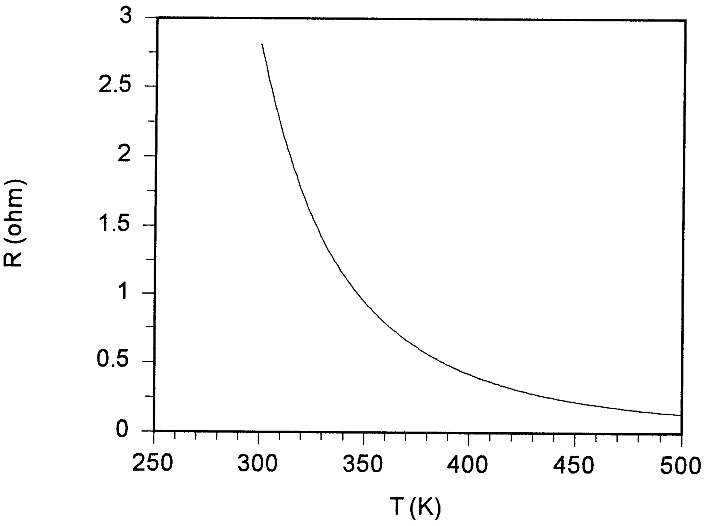
$$.31 = (2.2)\exp\left[\beta\left(\frac{1}{422} - \frac{1}{310}\right)\right]$$

Solving for β

$$\beta = \frac{\ln(.31/2.2)}{\left(\frac{1}{422} - \frac{1}{310}\right)} = 2288.9\ \text{K}$$

Thus

$$R = 2.2 \exp\left[2288.9\left(\frac{1}{T} - \frac{1}{310}\right)\right]$$



PROBLEM 1.55*

GIVEN DATA:

	T (°F)	R (Ω)
Test 1	32	51.39
Test 2	196	51.72

$R = R_o[1 + \alpha(T - T_o)]$

Let $T_o = 32°F$

From the data at $T = 32°F$

$51.39 = R_o[1 + \alpha(32-32)] \Rightarrow R_o = 51.39\ \Omega$

At $T = 196°F$, $R = 51.72$. Thus

$51.72 = 51.39[1 + \alpha(196-32)] \Rightarrow \alpha = 3.9155 \times 10^{-5}\ (°F)^{-1}$

Finally, $R = 51.39[1 + 3.9155\times10^{-5}(T-32)]$

$R = 51.326 + 2.0122\times10^{-3}\,T$

$T = 496.97\,(R - 51.326)$

$T(R=51.47) = 71.6°F \longleftarrow \underline{T}$

PROBLEM 1.56

Using Eq. 1.22, $T(°F) = 1.8\,T(°C) + 32$. For $T(°F) = T(°C) = \hat{T}$

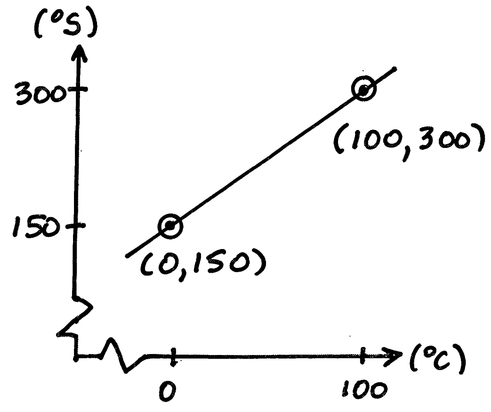
$$\hat{T} = 1.8\,\hat{T} + 32$$

Solving $\hat{T} = -40$ °F or °C $\longleftarrow \underline{\hat{T}}$

In K: $T(K) = T(°C) + 273.15 = -40 + 273.15 = 233.15 \longleftarrow \underline{\hat{T}(K)}$

In °R: $T(°R) = T(°F) + 459.67 = -40 + 459.67 = 419.67 \longleftarrow \underline{\hat{T}(°R)}$

PROBLEM 1.57



From the data, the relation can be expressed as

$$T(°S) = \left(\frac{300-150}{100-0}\right) T(°C) + 150\ °C$$
$$= 1.5\ T(°C) + 150$$

or
$$T(°C) = \frac{T(°S) - 150}{1.5}$$

Using this relation

$T = 100°S \rightarrow -33.33\ °C$ ◄——— T(°C)

$T = 400°S \rightarrow 166.67\ °C$ ◄——— T(°C)

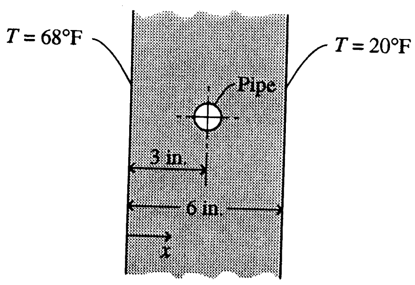
From Eq. 1.19

$$T(°S) = 1.5\,[T(K) - 273.15] + 150$$

Thus, the ratio of the size of the °S to the kelvin is

1.5 °S/K ◄——— °S/K

PROBLEM 1.58



Since the temperature variation is linear,

$$T = mx + b$$

where m is the slope and b is the value when x = 0.

Slope:
$$m = \left[\frac{20°F - 68°F}{6\ \text{in.}}\right] = -8\ °F/\text{in.}$$

When x = 0, T = 68°F

Accordingly, the temperature variation through the wall is

$$T = -(8\ °F/\text{in.})\,x + 68°F$$

Checking the temperature at the pipe: x = 3 in.

$$T = -(8°F/\text{in.})(3\ \text{in.}) + 68°F$$
$$= 44°F$$

There is no danger of freezing. ◄———

CHAPTER TWO

ENERGY AND THE FIRST LAW OF THERMODYNAMICS

PROBLEM 2.1

KNOWN: An automobile of known mass accelerates from a given velocity to another.

FIND: Determine the initial kinetic energy and the change in kinetic energy.

SCHEMATIC & GIVEN DATA:

$m = 1200$ kg
$V_1 = 50$ km/h
$V_2 = 100$ km/h

ASSUMPTIONS: (1) The automobile is the closed system. (2) The velocities and kinetic energies are relative to the road.

ANALYSIS: The initial kinetic energy (assumption 2) is

$$KE_1 = \frac{1}{2} m V_1^2$$

$$= \frac{1}{2}(1200\,\text{kg})\left(50\,\frac{\text{km}}{\text{h}}\right)^2 \left|\frac{10^3\,\text{m/km}}{3600\,\text{s/h}}\right|^2 \left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| \left|\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right|$$

$$= 115.74\ \text{kJ} \qquad \longleftarrow \underline{KE_1}$$

The change in kinetic energy is

$$KE_2 - KE_1 = \frac{1}{2} m \left[V_2^2 - V_1^2\right]$$

$$= \frac{1}{2}(1200)\left[(100)^2 - (50)^2\right] \left|\frac{10^3}{3600}\right|^2 \left|\frac{1}{10^3}\right|$$

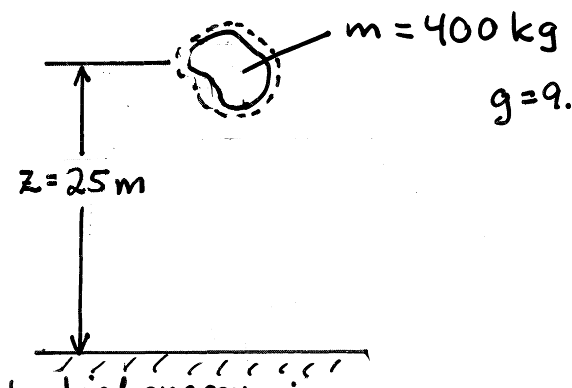
$$= 347\ \text{kJ} \qquad \longleftarrow \underline{KE_2 - KE_1}$$

PROBLEM 2.2*

KNOWN: An object of known mass is located at a specified elevation relative to the surface of the earth.

FIND: Determine gravitational potential energy of the object.

SCHEMATIC & GIVEN DATA:



$g = 9.78\ m/s^2$

ASSUMPTIONS: (1) The object is a closed system. (2) The acceleration of gravity is constant.

ANALYSIS: The gravitational potential energy is

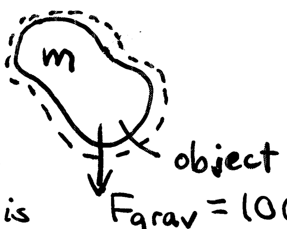
$$
\begin{aligned}
PE &= mgz \\
&= (400\,kg)(9.78\,\tfrac{m}{s^2})(25\,m)\left|\frac{1\,N}{kg\,m/s^2}\right|\left|\frac{1\,kN}{1000\,N}\right|\left|\frac{1\,kJ}{1\,kN\cdot m}\right| \\
&= 97.8\ kJ \longleftarrow \underline{\hspace{3em}}\ PE
\end{aligned}
$$

PROBLEM 2.3 *

KNOWN: An object of known weight undergoes a specified change in its kinetic energy while its potential energy increases.

FIND: Determine the final velocity.

SCHEMATIC & GIVEN DATA:



$g = 32.2\,ft/s^2$

$V_1 = 40\,ft/s$

$\Delta KE = -500\ ft\cdot lbf$

$F_{grav} = 100\ lb_f = mg$

$$m = \frac{(100\,lb_f)}{(32.2\,ft/s^2)}\left|\frac{32.2\,lb\,ft/s^2}{1\,lbf}\right| = 100\,lb$$

ASSUMPTION: The object is a closed system.

ANALYSIS: The change in kinetic energy is

① $$\Delta KE = \tfrac{1}{2} m (V_2^{\,2} - V_1^{\,2})$$

Thus, solving for V_2 and inserting values and a unit conversion factor

$$
\begin{aligned}
V_2 &= \sqrt{\frac{2\,\Delta KE}{m} + V_1^{\,2}} \\
&= \sqrt{\frac{2(-500\ ft\cdot lbf)}{(100\ lb)}\left|\frac{32.2\ lb\cdot ft/s^2}{1\ lbf}\right| + (40\,ft/s)^2} \\
&= 35.75\ ft/s \longleftarrow \underline{\hspace{3em}}\ V_2
\end{aligned}
$$

---

1. The analysis makes no use of the information related to potential energy.

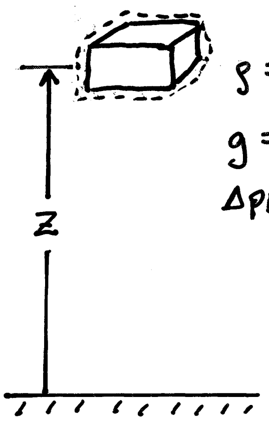
PROBLEM 2.4*

KNOWN: A brick of known volume and density experiences a given decrease in gravitational potential energy.

FIND: Determine the change in elevation.

SCHEMATIC & GIVEN DATA:

$V = 2.5 \times 3.5 \times 6\ \text{in}^3$



$\rho = 120\ \text{lb/ft}^3$

$g = 32.0\ \text{ft/s}^2$

$\Delta PE = -500\ \text{ft}\cdot\text{lbf}$

ASSUMPTIONS: (1) The brick is a closed system. (2) The acceleration of gravity is constant. (3) The density of the body is uniform throughout.

ANALYSIS:

$$V = (2.5\,\text{in})(3.5\,\text{in})(6\,\text{in})\left|\frac{1\ \text{ft}^3}{12^3\ \text{in}^3}\right| = 0.03038\ \text{ft}^3$$

Based on assumption (3)

$$m = \rho V = (120\,\text{lb/ft}^3)(0.03038\,\text{ft}^3) = 3.65\,\text{lb}$$

The change in potential energy and the elevation are related by

$$\Delta PE = mg\Delta z$$

Thus, solving for Δz

$$\Delta z = \frac{\Delta PE}{mg} = \frac{(-500\ \text{ft}\cdot\text{lbf})}{(3.65\,\text{lb})(32.0\ \text{ft/s}^2)}\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|$$

① $$= -138\ \text{ft} \leftarrow \Delta z$$

---

1. The negative sign denotes a decrease in elevation.

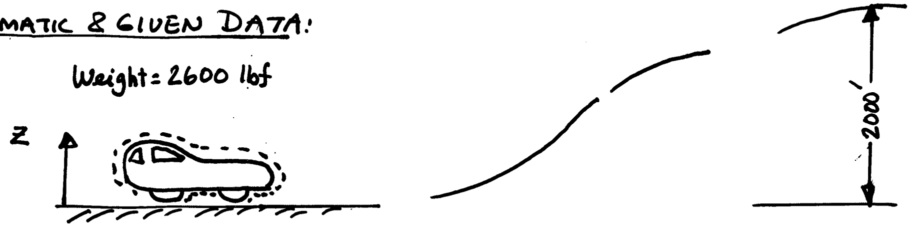
PROBLEM 2.5

KNOWN: An automobile of known weight travels from sea level to a known elevation.

FIND: Determine the change in potential energy.

SCHEMATIC & GIVEN DATA:

Weight = 2600 lbf



ASSUMPTIONS: 1. As shown in the schematic, the automobile is the closed system. 2. The acceleration of gravity is constant.

ANALYSIS: The change in potential energy is

$$\Delta PE = mg\,(z_2 - z_1)$$

The quantity mg is recognized as the vehicle weight. Thus, inserting known values

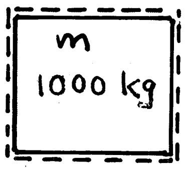
$$\Delta PE = (2600 \text{ lbf})(2000 \text{ ft}) = 5.2\times10^6 \text{ ft}\cdot\text{lbf} \longleftarrow \Delta PE$$

PROBLEM 2.6*

KNOWN: An object of known mass decelerates from a given initial velocity to a known final velocity.

FIND: Determine the change in kinetic energy of the object.

SCHEMATIC & GIVEN DATA:



$V_1 = 100$ m/s

$V_2 = 20$ m/s

ASSUMPTION: The object is a closed system.

ANALYSIS: The change in kinetic energy is

$$\Delta KE = \tfrac{1}{2} m \left[V_2^2 - V_1^2\right]$$

Inserting known values and converting units

$$\Delta KE = \frac{1}{2}(1000 \text{ kg})\left[20^2 - 100^2\right]\frac{\text{m}^2}{\text{s}^2}\left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$

① $$= -4800 \text{ kJ} \longleftarrow \Delta KE$$

---

1. The negative sign denotes a decrease in kinetic energy, as expected.

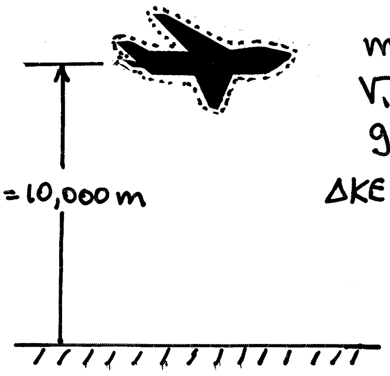
PROBLEM 2.7

KNOWN: An airplane of known mass flies with a given velocity at a given altitude, both measured relative to the surface of the earth.

FIND: Calculate (a) the kinetic and potential energies of the airplane, and (b) the final velocity for a given change in kinetic energy.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) The airplane is a closed system. (2) The acceleration of gravity is constant. (3) In part (b), there is no change in elevation.



$m = 5000$ kg
$V_1 = 150$ m/s
$g = 9.78$ m/s²
$\Delta KE = 10{,}000$ kJ

ANALYSIS: (a) The kinetic energy is

$$KE_1 = \frac{1}{2} m V_1^2 = \frac{1}{2}(5000\ \text{kg})(150^2)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 56{,}520\ \text{kJ} \longleftarrow \underline{KE_1}$$

The gravitational potential energy is

$$PE = mgz = (5000\ \text{kg})(9.78\ \text{m/s}^2)(10{,}000\ \text{m})\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 489{,}000\ \text{kJ} \longleftarrow \underline{PE}$$

(b) The change in kinetic energy is related to the initial and final velocities by

$$\Delta KE = \frac{1}{2} m (V_2^2 - V_1^2)$$

Thus, solving for V_2^2 and converting units

$$V_2^2 = \frac{2\,\Delta KE}{m} + V_1^2$$

$$= \frac{2(10{,}000\ \text{kJ})}{(5000\ \text{kg})}\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right| + (150^2)\frac{\text{m}^2}{\text{s}^2}$$

$$= 26{,}500\ \text{m}^2/\text{s}^2$$

or

① $$V_2 = 162.8\ \text{m/s} \longleftarrow \underline{V_2}$$

---

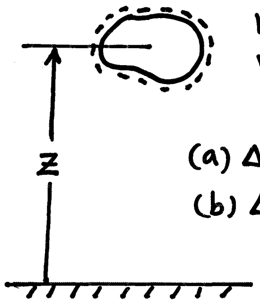
1. In part (b), the velocity increases, as expected.

PROBLEM 2.8

KNOWN: An object of known mass moves with a given velocity.

FIND: Determine (a) the final velocity for a given change in kinetic energy, and (b) the change in elevation for a given change in potential energy.

SCHEMATIC & GIVEN DATA:



$m = 1\ \text{lb}$

$V_1 = 100\ \text{ft/s}$

$g = 32.0\ \text{ft/s}^2$

(a) $\Delta KE = -100\ \text{ft}\cdot\text{lbf}$

(b) $\Delta PE = 100\ \text{ft}\cdot\text{lbf}$

ASSUMPTIONS: (1) The object is a closed system. (2) The acceleration of gravity is constant.

ANALYSIS: (a) The change in kinetic energy is related to the initial and final velocities by

$$\Delta KE = \frac{1}{2} m \left[V_2^2 - V_1^2\right]$$

Thus, solving for the final velocity V_2

$$V_2 = \sqrt{\frac{2\,\Delta KE}{m} + V_1^2}$$

Inserting values and converting units accordingly

$$V_2 = \sqrt{\frac{(2)(-100\ \text{ft}\cdot\text{lbf})}{(1\ \text{lb})}\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right| + 100^2\ \frac{\text{ft}^2}{\text{s}^2}}$$

① $$= 59.67\ \text{ft/s} \longleftarrow V_2$$

(b) The change in potential energy is related to the change in elevation by

$$\Delta PE = mg\,\Delta z$$

Thus, the change in elevation is

$$\Delta z = \frac{\Delta PE}{mg}$$

$$= \frac{(100\ \text{ft}\cdot\text{lbf})}{(1\ \text{lb})(32.0\ \text{ft/s}^2)}\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|$$

② $$= 100.6\ \text{ft} \longleftarrow \Delta z$$

---

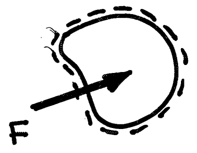
1. The velocity decreases, as expected.
2. The elevation increases, as expected.

PROBLEM 2.9*

KNOWN: An object of known mass accelerates from a given initial velocity to a given final velocity due to the action of a resultant force.

FIND: Determine the work done by the resultant force.

SCHEMATIC & GIVEN DATA:



$m = 2\ \text{kg}$
$V_1 = 200\ \text{m/s}$
$V_2 = 500\ \text{m/s}$

ASSUMPTIONS: (1) The object is a closed system. (2) The resultant force is the only interaction between the object an its surroundings.

ANALYSIS: By assumption (2), the work of the resultant force must equal the change in kinetic energy. Thus, using Eq. 2.6

① $$\text{work} = \frac{1}{2} m (V_2^2 - V_1^2)$$

$$= \frac{1}{2}(2\ \text{kg})(500^2 - 200^2)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 210\ \text{kJ} \longleftarrow \underline{\qquad\qquad\qquad \text{work}}$$

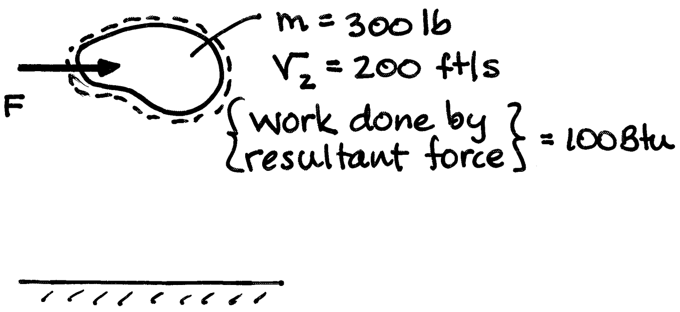
1. The increase in kinetic energy of the object is the result of energy transferred to it by the work of the resultant force.

PROBLEM 2.10

KNOWN: An object of known mass undergoes a change of kinetic energy due to the action of a resultant force. The final velocity and the work done by the force are given.

FIND: Determine the initial velocity.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The object is a closed system. (2) There is no change in elevation. (3) The resultant force is the only interaction between the object and its surroundings.

ANALYSIS: By assumption (3), the work of the resultant force must equal the change in kinetic energy. Thus, using Eq. 2.6

$$\text{work} = \frac{1}{2}m\left(V_2^2 - V_1^2\right)$$

Solving for V_1^2, inserting values, and converting units

$$V_1^2 = -\frac{2W}{m} + V_2^2$$

$$= -\frac{2\,(100\text{ Btu})}{(300\text{ lb})}\left|\frac{778\text{ ft}\cdot\text{lbf}}{1\text{ Btu}}\right|\left|\frac{32.2\text{ lb}\cdot\text{ft/s}^2}{1\text{ lbf}}\right| + 200^2\,\frac{\text{ft}^2}{\text{s}^2}$$

$$= 23{,}300\text{ ft}^2/\text{s}^2$$

or

① $V_1 = 152.6$ ft/s $\longleftarrow$ V_1

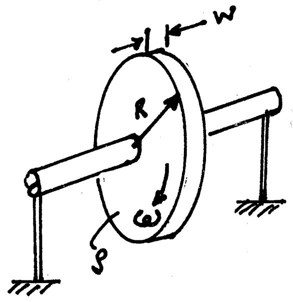
1. The increase in velocity reflects the increase in kinetic energy of the object as a result of energy transferred to it by the work of the resultant force.

PROBLEM 2.11

KNOWN: Data are provided for a disk-shaped flywheel.

FIND: (a) Obtain appropriate expressions for the moment of inertia and the kinetic energy. (b) Using given data, determine the kinetic energy and mass for a steel flywheel. (c) Using results from part (b), determine the radius and mass of an aluminum flywheel.

SCHEMATIC & GIVEN DATA:



Steel flywheel:
$\omega = 3000$ RPM
$R = 0.38$ m
$w = 0.025$ m

Aluminum flywheel:
$\omega = 3000$ RPM
$w = 0.025$ m

ASSUMPTIONS: 1. The flywheel is the closed system. 2. Motion is relative to the flywheel support structure.

ANALYSIS:

(a) Evaluating the moment of inertia

$$I = \int_{vol} \rho r^2 dV$$

For the disk, $dV = (2\pi r dr)w$. Thus, since ρ is constant

$$I = \rho(2\pi)w\int_0^R r^3 dr = \rho\pi w R^4/2 \longleftarrow I$$

The kinetic energy is

$$KE = \int_{Vol}\left(\tfrac{1}{2}\rho V^2\right)dV$$

and $V = r\omega$, so

$$KE = \int_0^R\left(\tfrac{1}{2}\rho r^2\omega^2\right)(2\pi r dr)w$$

$$= \tfrac{1}{2}\rho\omega^2(2\pi)w\int_0^R r^3 dr$$

$$= \tfrac{1}{2}\underbrace{\left(\rho\pi\tfrac{R^4}{2}w\right)}_{I}\omega^2$$

$$= \tfrac{1}{2}I\omega^2 \longleftarrow KE$$

(b) From Table A-19, the density of steel is $\rho = 8060\ \mathrm{kg/m^3}$. Thus, the mass is

$$m = \rho V = \rho\left[w\cdot\pi R^2\right]$$
$$= \left(8060\ \tfrac{kg}{m^3}\right)\left[(0.025\,m)\cdot\pi\cdot(0.38\,m)^2\right] = 91.41\,kg \longleftarrow m$$

Using the result of part (a), $KE = \frac{1}{2}I\omega^2$, where

$$I = \pi\rho w\frac{R^4}{2} = \frac{\pi}{2}\left(8060\ \tfrac{kg}{m^3}\right)(0.025\,m)(0.38\,m)^4 = 6.6\ kg\cdot m^2$$

PROBLEM 2.11 (Contd.)

Accordingly,

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2}(6.6\ kg\cdot m^2)\left(3000\ \frac{Rev}{min}\left|\frac{2\pi\ rad}{Rev}\right|\left|\frac{1\ min}{60 s}\right|\right)^2\left|\frac{1\ N}{1 kg\cdot m/s^2}\right|$$

$$= 32.57 \times 10^4\ N\cdot m \longleftarrow KE$$

(c) If ω, W, and KE are the same for the aluminum flywheel as for the steel flywheel

$$(KE)_{AL} = (KE)_{ST}$$

$$\left(\frac{1}{2} I \omega^2\right)_{AL} = \left(\frac{1}{2} I \omega^2\right)_{ST} \Rightarrow I_{AL} = I_{ST}$$

or

$$\left(\pi \rho W \frac{R^4}{2}\right)_{AL} = \left(\pi \rho W \frac{R^4}{2}\right)_{ST}$$

$$\Rightarrow (\rho R^4)_{AL} = (\rho R^4)_{ST}$$

$$R_{AL} = \left(\frac{\rho_{ST}}{\rho_{AL}}\right)^{1/4} R_{ST}$$

with ρ_{AL} from Table A-19, $\rho_{AL} = 2700\ kg/m^3$

$$R_{AL} = \left(\frac{8060}{2700}\right)^{\frac{1}{4}}(0.38 m)$$

$$= 0.5 m \longleftarrow R_{AL}$$

Then, the mass of the aluminum flywheel is

$$m = \rho V = \rho\left[W \pi R^2\right]$$

$$= \left(2700\ \frac{kg}{m^3}\right)(0.025 m)(\pi)(0.5 m)^2$$

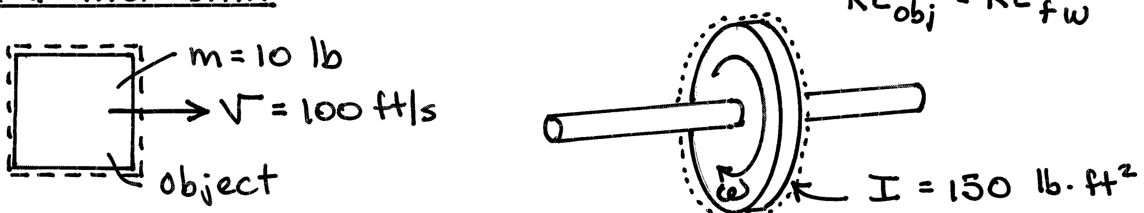
$$= 53.01\ kg \longleftarrow m$$

PROBLEM 2.12

KNOWN: An object of known mass moves along a straight line with a known velocity.

FIND: Determine the rotational speed of a flywheel whose rotational kinetic energy is equal in magnitude to the object's linear kinetic energy.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: (1) The object and the flywheel are both closed systems.

ANALYSIS: The kinetic energy of the object is

$$KE_{obj} = \frac{1}{2} m V^2$$

$$= \frac{1}{2}(10\ \text{lb})(100^2)\ \frac{\text{ft}^2}{\text{s}^2}\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right| = 1553\ \text{ft}\cdot\text{lbf}$$

For the flywheel (See the solution to Problem 2.11)

$$KE_{fw} = \frac{1}{2} I \omega^2$$

or

$$\omega = \sqrt{\frac{2\,KE_{fw}}{I}} = \sqrt{\frac{2(1553\ \text{ft}\cdot\text{lbf})}{(150\ \text{lb}\cdot\text{ft}^2)}\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|}$$

$$= 25.82\ \frac{1}{s}$$

In terms of RPM

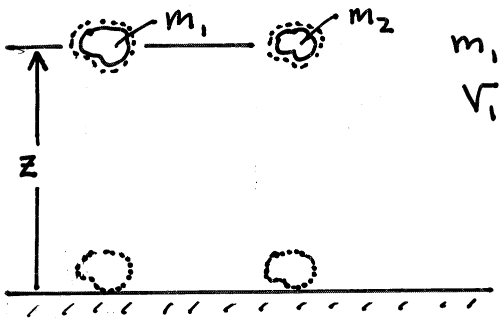
$$\omega = \left(25.82\ \frac{1}{s}\right)\left|\frac{\text{rev}}{2\pi}\right|\left|\frac{60\ s}{1\ \text{min}}\right| = 246.6\ \text{rev/min} \leftarrow \underline{\quad\omega}$$

PROBLEM 2.13

KNOWN: Two objects fall freely under the infuence of gravity from rest and the same initial elevation.

FIND: Show that the magnitudes of the velocities are equal at the moment just before they stike the earth.

SCHEMATIC & GIVEN DATA:



$m_1 \neq m_2$

$V_1 = 0$

ASSUMPTIONS: (1) An object in free fall is a closed system. (2) The acceleration of gravity is constant. (3) There is no affect of air resistance. (4) The only force acting is that due to gravity.

ANALYSIS: For an object falling freely under the infuence of gravity, Eq 2.11 applies

$$\frac{1}{2} m (V_2^2 - V_1^2) + mg(z_2 - z_1) = 0$$

For $V_1 = 0$ and $z_2 = 0$

$$\frac{1}{2} m V_2^2 = mgz_1$$

Thus $$V_2 = \sqrt{2gz_1}$$

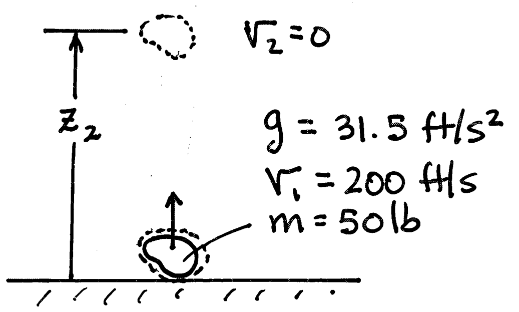
Since the final velocity doesn't depend on mass, both objects will have identical velocities at the moment just before they strike the earth.

PROBLEM 2.14

KNOWN: An object of known mass is projected upward from the surface of the earth with a known initial velocity. The only force acting on the object is the force of gravity.

FIND: Plot the velocity of the object versus elevation and determine the elevation when its velocity reaches zero.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The object is a closed system. (2) The acceleration of gravity is constant. (3) The only force acting on the object is the force of gravity.

ANALYSIS: Since the only force acting on the body is the force of gravity, Eq. 2.10 applies. Thus, the velocity and elevation are related to the initial condition by

$$\frac{1}{2}\cancel{m}V^2 + \cancel{m}gz = \frac{1}{2}\cancel{m}V_1^2 + \cancel{m}g\cancel{z_1}^{\,0}$$

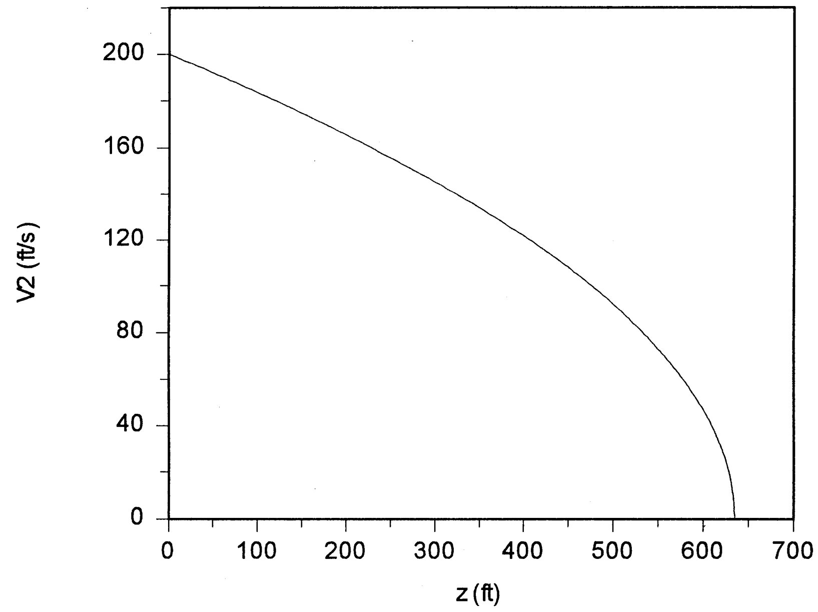
Thus, solving for V

$$V = \sqrt{V_1^2 - 2gz} \longleftarrow \qquad \text{Expression for } V \text{ in terms of } z$$

When $V_2 = 0$, z_2 is

$$z_2 = \frac{V_1^2}{2g} = \frac{200^2 \text{ ft}^2/\text{s}^2}{2(31.5 \text{ ft/s}^2)} = 634.92 \text{ ft} \longleftarrow \qquad z_2$$

Plotting the above relationship

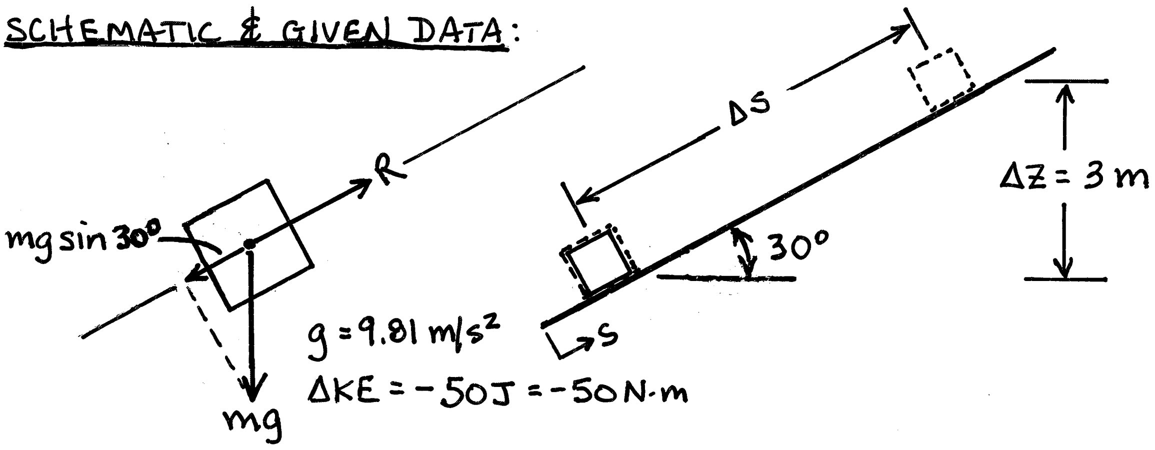


PROBLEM 2.15

KNOWN: A block of known mass moves along an inclined surface. The change in elevation and the change in kinetic energy of the block are given. The block is acted upon by a force parallel to the incline and the force of gravity.

FIND: Determine the magnitude and direction of applied force.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The block is a closed system. (2) The acceleration of gravity is constant. (3) The force R is constant. (4) There is no frictional force between the block and the incline.

ANALYSIS: Beginning with Eq. 2.6

$$\int \underline{F} \cdot d\underline{s} = \Delta KE$$

From the free body diagram, the dot product $\underline{F} \cdot d\underline{s}$ can be expressed as

$$\underline{F} \cdot d\underline{s} = (R - mg \sin 30°) ds$$

so

$$\int_{s_1}^{s_2} (R - mg \sin 30°) ds = \Delta KE$$

Noting that $mg \sin 30° ds = mg \Delta z$, the integral becomes

$$R \Delta s = \Delta KE + mg \Delta z$$

Evaluating Δs

$$\Delta s = \frac{\Delta z}{\sin 30°} = \frac{3\text{ m}}{\sin 30°} = 6\text{ m}$$

Thus

$$R = \frac{(-50\text{ N}\cdot\text{m}) + (10\text{ kg})(9.81\text{ m/s}^2)(3\text{ m}) \left| \frac{1\text{N}}{1\text{ kg}\cdot\text{m/s}^2} \right|}{(6\text{ m})}$$

$$= 40.72\text{ N} \longleftarrow R$$

The positive value denotes that the direction is the same as indicated on the above figure.

PROBLEM 2.16*

KNOWN: Beginning from rest, and object of known mass slides down an inclined plane. The length of the ramp is given.

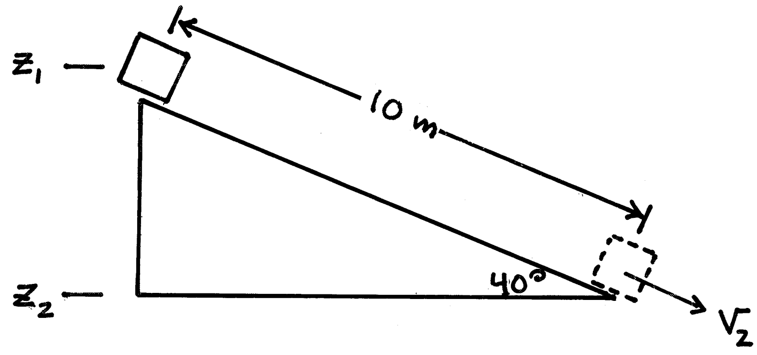
FIND: Determine the velocity of the object at the bottom of the ramp.

SCHEMATIC & GIVEN DATA:

$m = 200\ \text{kg}$

$g = 9.81\ \text{m/s}^2$

$V_1 = 0$



ASSUMPTIONS: (1) The mass is a closed system. (2) there is no friction between the mass and the ramp, and air resistance is negligible. (3) The acceleration of gravity is constant.

ANALYSIS: By assumption (2), the only force acting on the system is the force of gravity. Thus, Eq. 2.11 applies

$$① \quad \frac{1}{2}\cancel{m}(V_2^2 - \overset{0}{\cancel{V_1^2}}) + \cancel{m}g(z_2 - z_1) = 0$$

Solving for V_2

$$V_2 = \sqrt{2g(z_1 - z_2)}$$

From trigonometric relationships

$$z_1 - z_2 = (10\ \text{m}) \sin 40°$$

Thus

$$V_2 = \sqrt{2(9.81\ \text{m/s}^2)(10\ \text{m}) \sin 40°}$$

$$= 11.23\ \text{m/s} \longleftarrow V_2$$

---

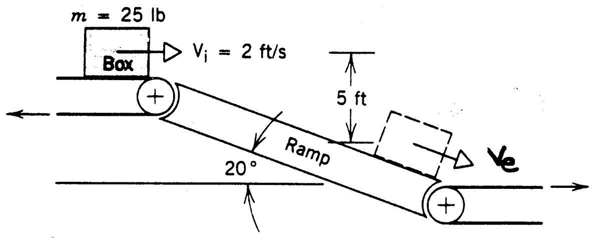
1. Even though the object travels along an inclined path, the vertical distance appears in this expression.

PROBLEM 2.17

KNOWN: A box slides down a ramp. The mass of the box and its velocity at the top of the ramp are known. The ramp geometry is also specified.

FIND: (a) In the absence of friction, determine the velocity of the box at the base of the ramp and the changes in kinetic and potential energy for the box. (b) Determine the changes in kinetic and potential energy of the box when friction is acting and the velocity at the base of the ramp is known. Compare with the results of part (a).

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The box is the closed system. 2. In part (a), friction is negligible. 3. The acceleration of gravity is 32.0 ft/s².

ANALYSIS: (a) In the absence of any resultant force acting on the system, including friction, Eq. 2.9 reduces to

$$\frac{1}{2}m(V_e^2 - V_i^2) + mg(z_e - z_i) = 0 \qquad (*)$$

Solving

$$V_e = \sqrt{V_i^2 + 2g(z_i - z_e)}$$

$$= \sqrt{\left(2\,\frac{\text{ft}}{\text{s}}\right)^2 + 2\left(32\,\frac{\text{ft}}{\text{s}^2}\right)(5\,\text{ft})} = 18\ \text{ft/s} \longleftarrow V_e$$

As shown by Eq (*), the kinetic and potential energy changes have the same magnitude but are opposite in sign. Thus

$$\Delta PE = mg(z_e - z_i) = (25\,\text{lb})\left(32.0\,\frac{\text{ft}}{\text{s}^2}\right)(-5\,\text{ft})\left|\frac{1\,\text{lbf}}{32.2\,\text{lb}\cdot\text{ft/s}^2}\right|$$

$$= -124.2\ \text{ft}\cdot\text{lbf} \longleftarrow \Delta PE$$

and $\Delta KE = +\,124.2\ \text{ft}\cdot\text{lbf}$. $\longleftarrow \Delta KE$

(b) Whether there is friction or not, the potential energy change in this case is the same as determined in part (a): −124.2 ft·lbf. If V_e = 9 ft/s, the change in kinetic energy is smaller in magnitude:

$$\Delta KE = \frac{1}{2}m\left[V_e^2 - V_i^2\right] = \frac{1}{2}(25\,\text{lb})\left[(9\,\text{ft/s})^2 - (2\,\text{ft/s})^2\right]\left|\frac{1\,\text{lbf}}{32.2\,\text{lb}\cdot\text{ft/s}^2}\right| = 29.9\ \text{ft}\cdot\text{lbf} \longleftarrow \Delta KE$$

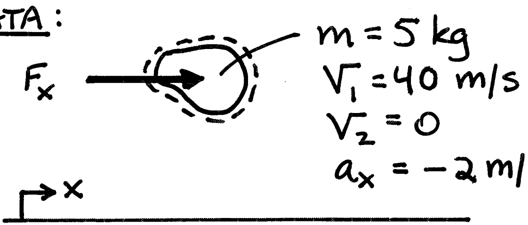
Referring again to Eq. 2.9, the decrease in potential energy in this case can be accounted for in terms of the increase in kinetic energy of the box and work to overcome friction.

PROBLEM 2.18*

KNOWN: A system of known mass and a given initial velocity experiences a constant deceleration due to the action of a resultant force and comes to rest.

FIND: Determine the length of time the force is applied and the work.

SCHEMATIC & GIVEN DATA:



$m = 5$ kg
$V_1 = 40$ m/s
$V_2 = 0$
$a_x = -2$ m/s²

ASSUMPTIONS: (1) The system is closed. (2) The horizontal deceleration is constant.

ANALYSIS: To find the time, use the fact that the acceleration is constant, as follows

$$a_x = \frac{dV}{dt} \Rightarrow dV = a_x\,dt$$

$$\int_{V_1}^{V_2} dV = \int_{t_1}^{t_2} a_x\,dt$$

or

$$\overset{0}{\cancel{V_2}} - V_1 = a_x \Delta t$$

Thus

$$\Delta t = \frac{-V_1}{a_x} = \frac{(-40 \text{ m/s})}{(-2 \text{ m/s}^2)} = 20 \text{ s} \longleftarrow \underline{\Delta t}$$

The work of the force F_x is found from Eq. 2.6

$$\text{Work} = \frac{1}{2} m (\overset{0}{\cancel{V_2^2}} - V_1^2)$$

$$= -\frac{1}{2}(5 \text{ kg})(40^2)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$

① $$= -4 \text{ kJ} \longleftarrow \underline{\text{work}}$$

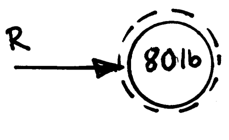
1. The negative denotes energy transfer out of the system.

PROBLEM 2.19*

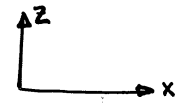
KNOWN: An object of known mass, acted on by a resultant force for a specified time, experiences a constant acceleration from a known initial velocity.

FIND: Determine the work of the resultant force.

SCHEMATIC & GIVEN DATA:



$V_1 = 0$
$a = 12$ ft/s
$\Delta t = 6.5$ s



ASSUMPTIONS: 1. The object is the closed system. 2. Motion is horizontal, so the system experiences no change in potential energy. 3. The horizontal acceleration is constant.

ANALYSIS: Applying Eq. 2.9 with assumption 2, the work of the force R is given by

$$\text{Work} = \frac{1}{2} m \left(V_2^2 - V_1^2 \right)$$

The final velocity V_2 can be determined using assumption 3

$$\frac{dV}{dt} = a \quad \text{(constant)}$$

Thus

$$dV = a\,dt \implies V_2 - V_1 = a(t_2 - t_1)$$

Or

$$V_2 = \overset{0}{\cancel{V_1}} + a(t_2 - t_1)$$

$$= 12 \tfrac{\text{ft}}{\text{s}^2}(6.5\ \text{s}) = 78\ \text{ft/s}$$

Then

$$\text{work} = \frac{1}{2}(80\ \text{lb})\left(78\ \frac{\text{ft}}{\text{s}}\right)^2 \left| \frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2} \right|$$

$$= 7558\ \text{ft}\cdot\text{lbf} \longleftarrow \text{work (ft lbf)}$$

$$= (7558\ \text{ft}\cdot\text{lbf}) \left| \frac{1\ \text{Btu}}{778.2\ \text{ft}\cdot\text{lbf}} \right| = 9.71\ \text{Btu} \longleftarrow \text{work (Btu)}$$

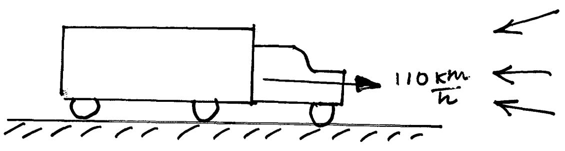
PROBLEM 2.20

KNOWN: The drag force on a truck is known as a function of velocity and other parameters.

FIND: Determine the power required by the truck to overcome drag.

SCHEMATIC & GIVEN DATA:

$C_d = 0.65$
$A = 10\ \text{m}^2$



$\rho = 1.1\ \text{kg/m}^3$

ASSUMPTION: The truck is the system.

ANALYSIS: Applying Eq. 2.13, the power required to overcome drag is

$$\dot{W}_d = F_d \cdot V = \left(\tfrac{1}{2} C_d A \rho V^2\right) V$$

$$= \tfrac{1}{2} C_d A \rho V^3$$

$$= \tfrac{1}{2}(0.65)(10\ \text{m}^2)\left(1.1\ \frac{\text{kg}}{\text{m}^3}\right)\left[\left(110\ \frac{\text{km}}{\text{h}}\right)\left|\frac{10^3\ \text{m}}{\text{km}}\right|\left|\frac{1\text{h}}{3600\text{s}}\right|\right]^3\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 102\ \frac{\text{kJ}}{\text{s}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 102\ \text{kW} \qquad \longleftarrow \dot{W}_d$$

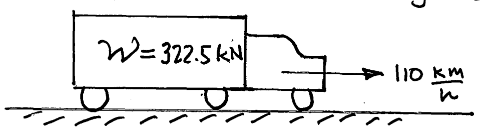
PROBLEM 2.21

KNOWN: The force associated with the rolling resistance of the tires of a truck is known as a function of the truck weight.

FIND: Determine the power required by the truck to overcome rolling resistance.

SCHEMATIC & GIVEN DATA:

$f = 0.0069$



ASSUMPTION: The truck is the system.

ANALYSIS: Applying Eq. 2.13, the power required to overcome rolling resistance is

$$\dot{W}_r = F_r \cdot V = (f\mathcal{W})V$$

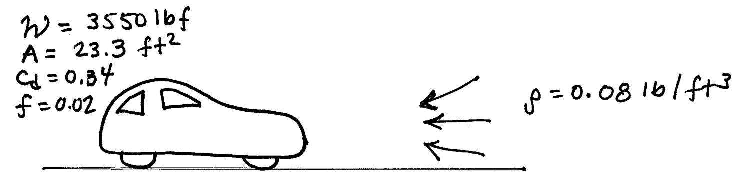
$$= (0.0069)\left(322.5\ \text{kN}\left|\frac{10^3\ \text{N}}{1\text{kN}}\right|\right)\left(110\ \frac{\text{km}}{\text{h}}\left|\frac{10^3\ \text{m}}{1\text{km}}\right|\left|\frac{1\ \text{h}}{3600\text{s}}\right|\right)\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 68\ \text{kW} \qquad \longleftarrow \dot{W}_r$$

PROBLEM 2.22

KNOWN: The drag force and the force associated with rolling resistance are known as functions of variables associated with a vehicle in motion.

FIND: (a) Determine the power required to overcome drag and rolling resistance when the vehicle is moving at 55 mi/h. (b) Plot the quantities of part (a) and their sum versus vehicle velocity ranging from 0 to 75 mi/h. Discuss the implication for vehicle fuel economy.



ASSUMPTIONS: The vehicle is the system.

ANALYSIS: Applying Eq. 2.13, the power required to overcome drag is

$$\dot{W}_d = F_d \cdot V = \left(\tfrac{1}{2} C_d A \rho V^2\right) V = \tfrac{1}{2} C_d A \rho V^3$$

$$= \tfrac{1}{2}(0.34)(23.3\,\text{ft}^2)\left(0.08\,\frac{\text{lb}}{\text{ft}^3}\right)\left[V\left(\frac{\text{mi}}{\text{h}}\right)\left|\frac{5280\,\text{ft}}{\text{mi}}\right|\left|\frac{\text{h}}{3600\,\text{s}}\right|\right]^3\left|\frac{1\,\text{lbf}}{32.2\,\text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{\text{hp}}{550\,\text{ft}\cdot\text{lbf/s}}\right|$$

$$= 5.65\times10^{-5}\,[V(\text{mi/h})]^3\ \text{hp} \qquad (*)$$

The power required to overcome rolling resistance is

$$\dot{W}_r = F_r \cdot V = (f\,\mathcal{W})\,V = (0.02)(3550\,\text{lbf})\left[V\left(\frac{\text{mi}}{\text{h}}\right)\left|\frac{5280\,\text{ft}}{\text{mi}}\right|\left|\frac{\text{h}}{3600\,\text{s}}\right|\right]\left|\frac{\text{hp}}{550\,\text{ft}\cdot\text{lbf/s}}\right|$$

$$= 0.189\,V\left(\frac{\text{mi}}{\text{h}}\right)\ \text{hp} \qquad (**)$$

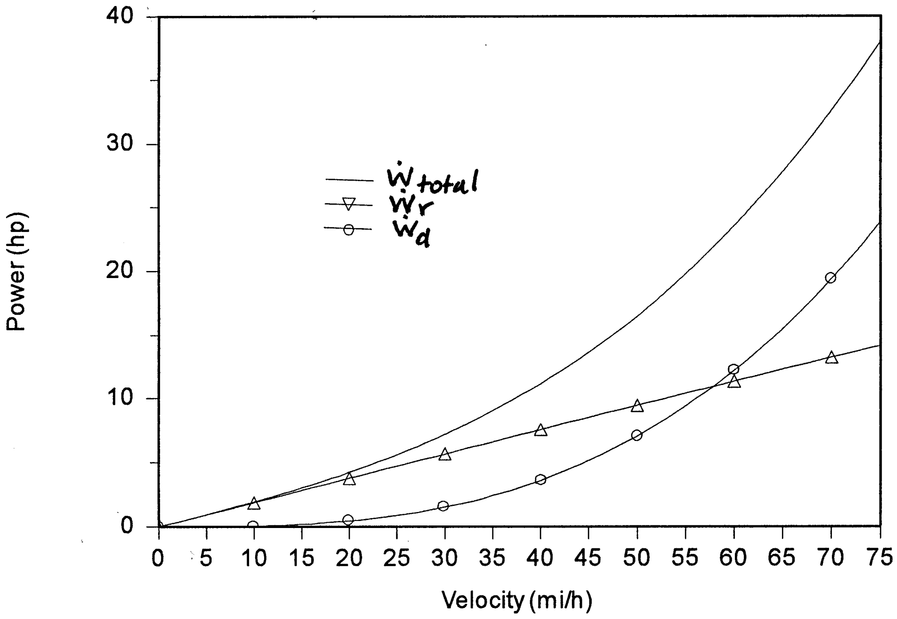
(a) When $V = 55$ mi/h, we have

$$\dot{W}_d = 5.65\times10^{-5}\,[55]^3 = 9.4\,\text{hp}$$
$$\dot{W}_r = 0.189\,[55] = 10.4\,\text{hp}$$

$\longleftarrow \dot{W}_d, \dot{W}_r$

PROBLEM 2.22 (Cont'd)

(b) Letting V range from 0 to 75 mi/h, the accompanying plots can be developed.



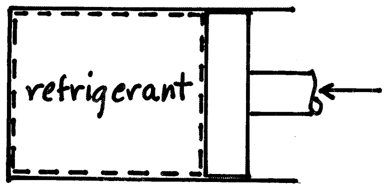
We see from the plots that up to about 58 mi/h, the power required to overcome rolling resistance is more significant than the power to overcome drag. At higher speeds, the drag effect becomes dominant because of the V^3 term in the expression for $\dot{W}_d$. Since the power to overcome these effects is developed by the engine from the fuel stored on board the vehicle, high-speed driving has an especially significant effect on fuel consumption.

PROBLEM 2.23

KNOWN: Measured data for pressure versus volume during the compression of a refrigerant are given.

FIND: (a) Determine n for a fit of the data by pV^n = constant. (b) Use the result of part (a) to evaluate the work done during the compression. (c) Evaluate the work using graphical or numerical integration of the data, (d) compare and discuss parts (b) and (c).

SCHEMATIC & GIVEN DATA:



Data Point	p (lbf/in.^{2})	V (in.^{3})
1	112	13.0
2	131	11.0
3	157	9.0
4	197	7.0
5	270	5.0
6	424	3.0

ASSUMPTIONS: 1. The refrigerant in the piston-cylinder assembly form a closed system. 2. The pressure values provided approximate the pressure at the piston face.

ANALYSIS:

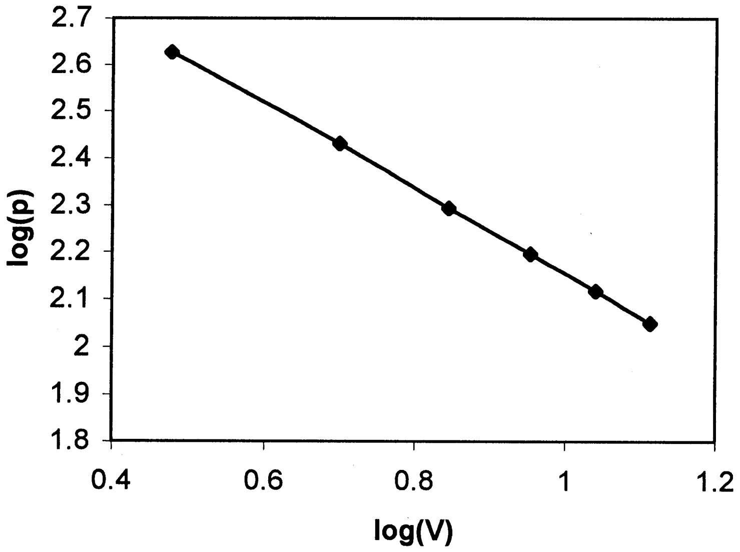
(a) One approach to find n is to begin with pV^n = constant. Taking the log of both sides of this equation

$$\log p + n \log V = \log c$$

or

$$\log p = (-n) \log V + \log c$$

Thus, $(-n)$ corresponds to the slope of a plot of $\log p$ vs. $\log V$. Using ① a spreadsheet program to obtain the plot and the least squares best fit curve:



From the curve fit

$(-n) = -0.90887$

or

$n = 0.90887 \longleftarrow n$

Thus

$pV^{0.90887}$ = constant

PROBLEM 2.23 (Cont'd)

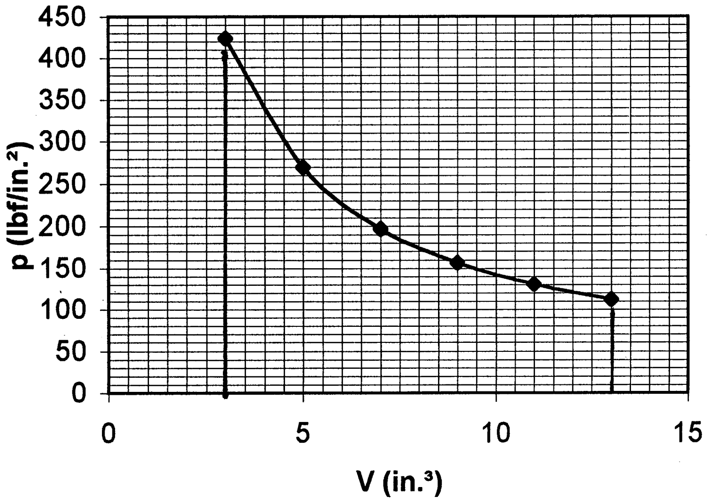
(b) Using the results of part(a) and the procedure of Example 2.1, the work is

$$W = \int_{V_1}^{V_2} p\,dV = \frac{p_2 V_2 - p_1 V_1}{(1-n)}$$

$$= \frac{(424\ \text{lbf/in}^2)(3.0\ \text{in}^3) - (112)(13.0)}{(1-0.90887)} \left|\frac{1\ \text{ft}}{12\ \text{in}}\right| \left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= -0.2163\ \text{Btu} \longleftarrow \underline{W}$$

② (c) A graphical evaluation of the work involves a plot of the tabulated data and a smooth curve drawn through the data points:



Each elemental rectangle in the plot contributes the following to the area under the curve:

$$(10\ \tfrac{\text{lbf}}{\text{in}^2})(0.5\ \text{in}^3)\left|\frac{1\ \text{ft}}{12\ \text{in}}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 5.356\times10^{-4}\ \text{Btu}$$

The number of retangles is approximately 401.1, thus

$$W \approx (401.1)(5.356\times10^{-4}) = -0.2148\ \text{Btu} \longleftarrow \underline{W}$$

(d) The results obtained in parts (b) and (c) are in good agreement. Each should be considered a plausible estimate for the reasons presented on page 40 in the discussion of actual expansion and compression processes.

---

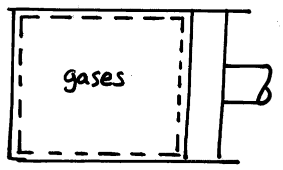
1. The software IT could be used to obtain the least squares curve fit by programming the equations for curve fitting. It is easier to use a spreadsheet program in this instance, however.
2. The only measured data are the tabulated data points, shown as filled circles. The smooth curve does not necessarily represent the actual pressure at the piston face for the corresponding volume.

PROBLEM 2.24

KNOWN: Measured pressure-volume data for an expansion of gases within the cylinder of an internal combustion engine are given.

FIND: (a) Determine n for a fit of the data by pV^n = constant. (b) Use the result of part (a) to evaluate the work done in the expansion. (c) Evaluate the work done using graphical or numerical integration of the data. (d) Compare and discuss parts (c), (d).

SCHEMATIC & GIVEN DATA:



Data Point	p (bar)	V (cm³)
1	15	300
2	12	361
3	9	459
4	6	644
5	4	903
6	2	1608

ASSUMPTIONS: 1. As shown in the schematic, the gases within the piston-cylinder form the closed system. 2. The pressure values provided approximate the pressure at the piston face.

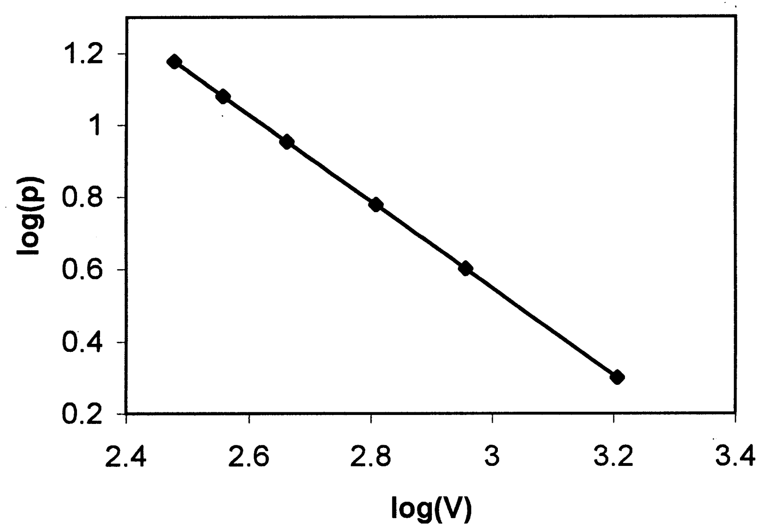
ANALYSIS: (a) One approach to find n is to begin with pV^n = constant. Taking the log of both sides of this equation

$$\log p + n \log V = \log c$$

or

$$\log p = (-n) \log V + \log c$$

① Thus, (−n) corresponds to the slope of a plot of log p vs. log V. Using a spreadsheet program to obtain the plot and the least squares best fit curve:



From the curve fit

$(-n) = -1.1996$

or

$n = 1.1996$ ← n

Thus

$pV^{1.1996}$ = constant

PROBLEM 2.24 (Cont'd)

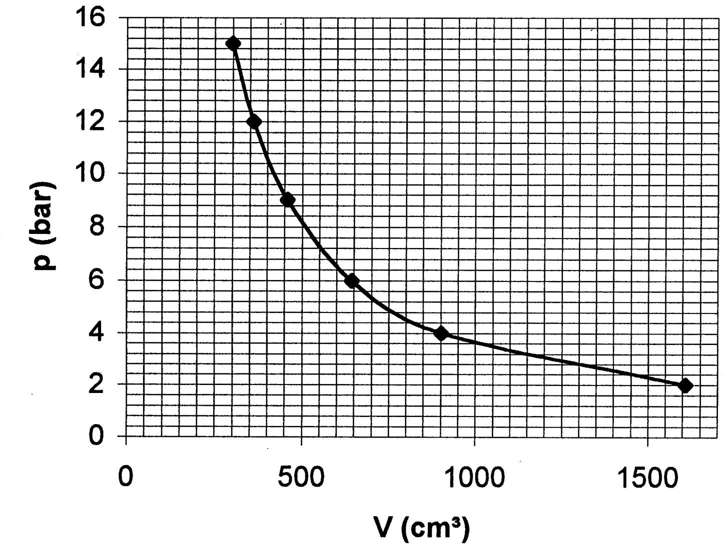
(b) Using the results of part (a) and the procedure of Example 2.1, the work is

$$W = \int_{V_1}^{V_2} p\,dV = \frac{p_2 V_2 - p_1 V_1}{(1-n)}$$

$$= \frac{(2\ \text{bar})(1608\ \text{cm}^3) - (15)(300)}{(1-1.1996)} \left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right| \left|\frac{1\ \text{m}^3}{10^6\ \text{cm}^3}\right| \left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$= 0.643\ \text{kJ}$ ◄ W

② (c) A graphical evaluation of the work involves a plot of the tabulated data and a smooth curve drawn through the data points:



Each elemental rectangle in the plot contributes the following to the area under the curve:

$$(.4\ \text{bar})(50\ \text{cm}^3) \left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right| \left|\frac{1\ \text{m}^3}{10^6\ \text{cm}^3}\right| \left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = 0.002\ \text{kJ}$$

The number of rectangles is approximately 324, thus

$W \approx (324)(0.002) = 0.648\ \text{kJ}$ ◄ W

(d) The results obtained in parts (b) and (c) are in good agreement. Each should be considered a plausible estimate for the reasons presented on page 40 in the discussion of actual expansion and compression processes.

---

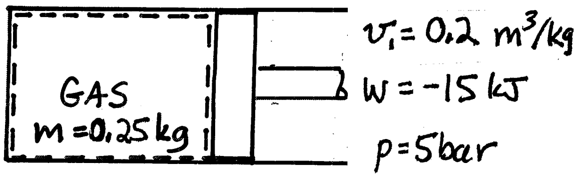
1. The software IT could be used to obtain the least squares curve fit by programming the equations for curve fitting. It is easier to use a spreadsheet program in this instance, however.
2. The only measured data are the tabulated data points, shown as filled circles. The smooth curve does not necessarily represent the actual pressure at the piston face for the corresponding volume.

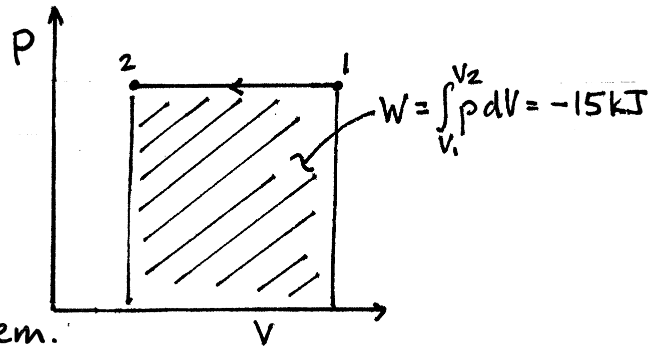
PROBLEM 2.25*

KNOWN: A known amount of gas undergoes a constant-pressure process in a piston-cylinder assembly beginning at a specified specific volume. The work is known.

FIND: Determine the final volume.

SCHEMATIC & GIVEN DATA:

① 



ASSUMPTIONS: (1) The gas is a closed system.
(2) Pressure is constant during the process.

ANALYSIS: Using Eq. 2.17

$$W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

$$= p(V_2 - m v_1)$$

Solving for V_2 and inserting values

$$V_2 = \frac{W}{p} + m v_1$$

$$= \frac{(-15\,kJ)}{(5\,bar)}\left|\frac{1\,bar}{10^5\,N/m^2}\right|\left|\frac{10^3\,N\cdot m}{1\,kJ}\right| + (0.25\,kg)(0.2\,m^3/kg)$$

$$= 0.02\,m^3 \longleftarrow V_2$$

PROBLEM 2.26*

KNOWN: Oxygen in a piston-cylinder assembly undergoes a process for which $pV^{1.3}$ = constant. The work is known.

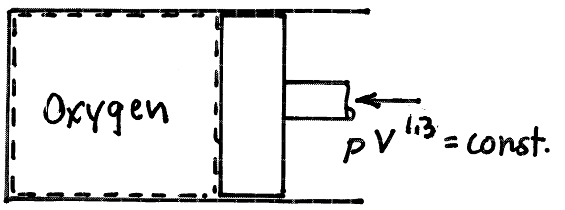
FIND: Determine the final volume and pressure.

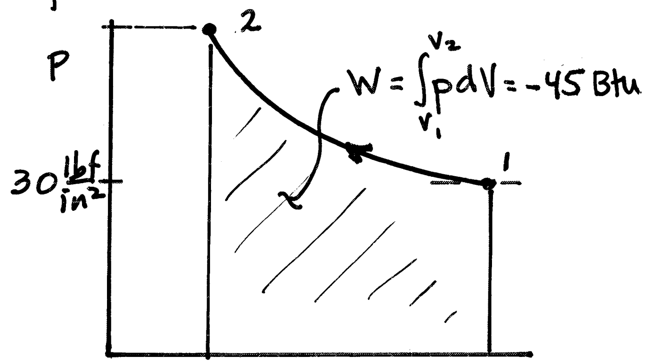
SCHEMATIC & GIVEN DATA:

$P_1 = 30\ \frac{lbf}{in^2}$

$V_1 = 25\ ft^3$

①





ASSUMPTIONS: (1) The O_2 is a closed system. (2) The process is polytropic.

ANALYSIS: To determine V_2, substitute the p-V relation into Eq. 2.17 and integrate

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \left(\frac{\text{const.}}{V^{1.3}}\right) dV = \text{const.}\ \frac{V_2^{-.3} - V_1^{-.3}}{(-.3)}$$

Now, const. $= P_1 V_1^{1.3}$. With this expression, and solving for $V_2^{1.3}$

$$V_2^{-.3} = \frac{(-.3)\,W}{P_1 V_1^{1.3}} + V_1^{-.3}$$

$$= \frac{(-.3)(-45\ \text{Btu})}{(30\ \text{lbf/in}^2)(25\ \text{ft}^3)^{1.3}} \left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right| \left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| + (25\ \text{ft}^3)^{-.3}$$

$$= 0.41776$$

$\Rightarrow V_2 = 18.35\ ft^3 \longleftarrow$ V_2

Now, we can use the p-V relation to get P_2

$$P_2 = P_1\left(\frac{V_1}{V_2}\right)^{1.3} = \left(30\ \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{25}{18.35}\right)^{1.3} = 44.85\ \text{lbf/in}^2 \longleftarrow$$ P_2

---

1. The shaded area on the p-V diagram represents the work for the polytropic process.

PROBLEM 2.27 *

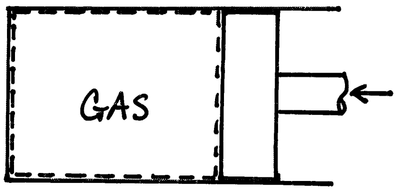
KNOWN: A gas undergoes a compression process. Pressure and volume are given at the initial and final states. Pressure and volume are related linearly during the process.

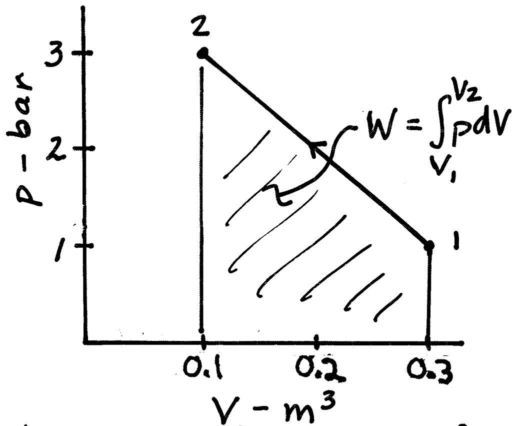
FIND: Determine the work.

SCHEMATIC & GIVEN DATA:

$p_1 = 1$ bar, $V_1 = 0.3$ m^3

$p_2 = 3$ bar, $V_2 = 0.1$ m^3





ASSUMPTIONS: (1) The gas is a closed system. (2) The compression is a quasi-equilibrium process. with a linear relation between pressure and volume.

ANALYSIS: Based on the given data, the p-V relation can be expressed as

$$p = 4 - 10\,V$$

where p is in bars and V is in m^3. The work is determined using Eq. 2.17

$$W = \int_{V_1}^{V_2} p\,dV$$

Inserting the p-V relation and integrating

$$W = \int_{V_1 = 0.3\text{ m}^3}^{V_2 = 0.1\text{ m}^3} [4 - 10\,V] \left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right| \left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| dV$$

$$= \left[4V - \left(\tfrac{10}{2}\right)V^2\right]\Bigg|_{V_1 = 0.3}^{V_2 = 0.1} \left|100\right|$$

$$= [4(0.1 - 0.3) - 5(0.1^2 - 0.3^2)]\left|100\right|$$

① $= -40$ kJ ← W

---

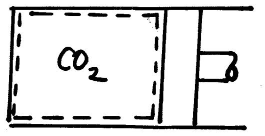
1. The negative sign for work denotes energy transfer to the system.

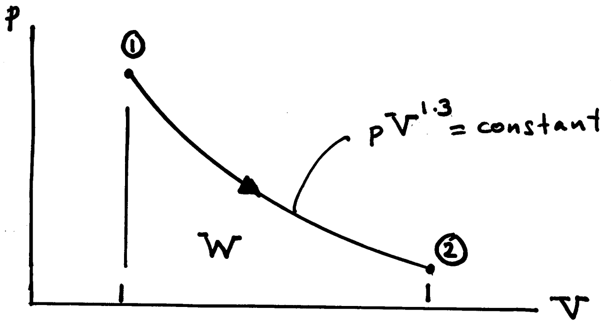
PROBLEM 2.28*

KNOWN: Carbon dioxide expands from a known initial state to a known final pressure. The pressure-volume relation is specified.

FIND: For the gas, determine the work.

SCHEMATIC & GIVEN DATA:





$p_1 = 80$ lbf/in.^{2}
$V_1 = 0.65$ ft^3
$p_2 = 20$ lbf/in.^{2}

ASSUMPTIONS: 1. The gas is a closed system. 2. The moving boundary is the only work mode. 3. The expansion is polytropic.

ANALYSIS: Using Eq. 2.17 and the procedure discussed in part (a) of Example 2.1

$$W = \int_{V_1}^{V_2} p\, dV = \int_{V_1}^{V_2} \frac{C}{V^{1.3}}\, dV = \frac{CV_2^{-0.3} - CV_1^{-0.3}}{(-0.3)}$$

The constant C can be evaluated at either end state: $C = p_1V_1^{1.3} = p_2V_2^{1.3}$, giving

$$W = \frac{(p_2V_2^{1.3})V_2^{-0.3} - (p_1V_1^{1.3})(V_1^{-0.3})}{(-0.3)} = \frac{p_2V_2 - p_1V_1}{(-0.3)}$$

To complete the calculation, the final volume is required. That is

$$p_1V_1^{1.3} = p_2V_2^{1.3} \Rightarrow V_2 = \left(\frac{p_1}{p_2}\right)^{1/1.3} V_1 = \left(\frac{80}{20}\right)^{1/1.3}(0.65 \text{ft}^3) = 1.89 \text{ ft}^3$$

Inserting values, the work is

$$W = \frac{(20 \text{ lbf/in}^2)\left|144 \text{ in}^2/1 \text{ ft}^2\right|(1.89 \text{ft}^3) - (80)|144|(0.65)}{(-0.3)}$$

① $= 6816$ ft·lbf ← W (ft·lbf)

Converting to Btu

$$W = (6816 \text{ ft}\cdot\text{lbf})\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$

$= 8.76$ Btu ← W(Btu)

---

1. In this case, the work is the area under the process line from state 1 to state 2 on the p-V diagram. Also, note that the work is positive, denoting energy transfer from the gas.

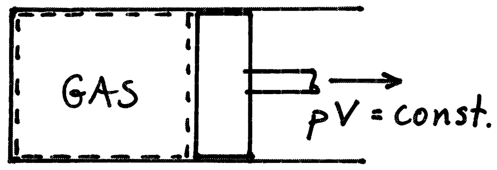
PROBLEM 2.29

KNOWN: A gas expands from a known initial state to a known final pressure. The pressure-volume relation for the process is specified.

FIND: Sketch the process on a p-V diagram and determine the work.

SCHEMATIC & GIVEN DATA:

$p_1 = 500$ kPa
$V_1 = 0.1\ m^3$
$p_2 = 100$ kPa



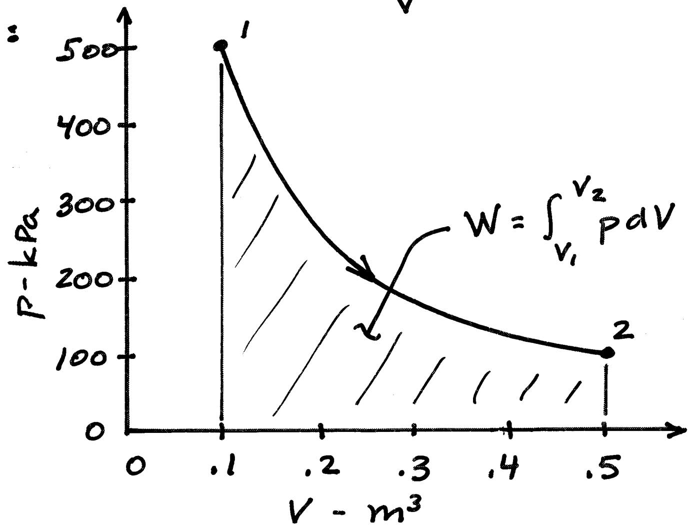
ASSUMPTIONS: (1) The gas is a closed system. (2) The expansion is polytropic.

ANALYSIS: The pressure-volume relation for the process is

$$pV = \text{const.} \Rightarrow p = \frac{\text{const.}}{V}$$

Thus, the p-V diagram is:

$$V_2 = \left(\frac{p_1}{p_2}\right) V_1 = 0.5\ m^3$$



Next, to determine the work, substitute the p-V relation into Eq. 2.17 and integrate.

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \left(\frac{\text{const.}}{V}\right) dV = p_1 V_1 \ln \frac{V_2}{V_1}$$

$$= (500\ \text{kPa})(0.1\ m^3) \ln\left(\frac{0.5}{0.1}\right) \left|\frac{10^3\ N/m^2}{1\ \text{kPa}}\right| \left|\frac{1\ \text{kJ}}{10^3\ N\cdot m}\right|$$

① $= +80.47$ kJ ◄ W

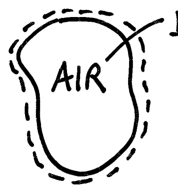
1. The work is positive for the expansion, as expected.

PROBLEM 2.30*

KNOWN: Air undergoes a polytropic process between two specified states.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:



$n = 0.2$ lbmol

$p_1 = 20$ lbf/in², $v_1 = 11.50$ ft³/lb

$p_2 = 80$ lbf/in², $v_2 = 3.98$ ft³/lb

$pV^n = \text{constant}$

ASSUMPTIONS: (1) The air is a closed system. (2) The system undergoes a polytropic process.

ANALYSIS: From the pressure-volume relation for a polytropic process

$$p_1 V_1^n = p_2 V_2^n \Rightarrow p_1 v_1^n = p_2 v_2^n$$

Solving for n

$$n = \frac{\log(p_1/p_2)}{\log(v_2/v_1)} = \frac{\log(20/80)}{\log(3.98/11.50)} = 1.307$$

Now, using Eq. 2.17 to determine work and with the molecular weight of air from Table A-1E

$$W = \int_{V_1}^{V_2} p\,dV = m\int_{v_1}^{v_2} p\,dv = m(\text{const.})\int_{v_1}^{v_2}\frac{dv}{v}$$

$$= m\left[\frac{(p_2 v_2^n)\,v_2^{1-n} - (p_1 v_1^n)\,v_1^{1-n}}{1-n}\right] = m\left(\frac{p_2 v_2 - p_1 v_1}{1-n}\right)$$

Now $m = (0.2\text{ lbmol})(28.97\text{ lb/lbmol}) = 5.794$ lb

and

$$W = (5.794\text{ lb})\left[\frac{(80\text{ lbf/in}^2)(3.98\text{ ft}^3/\text{lb}) - (20)(11.50)}{(1-1.307)}\right]\left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|$$

① $= -308.8$ Btu ← W

---

1. The negative sign for work denotes energy transfer into the system.

PROBLEM 2.31

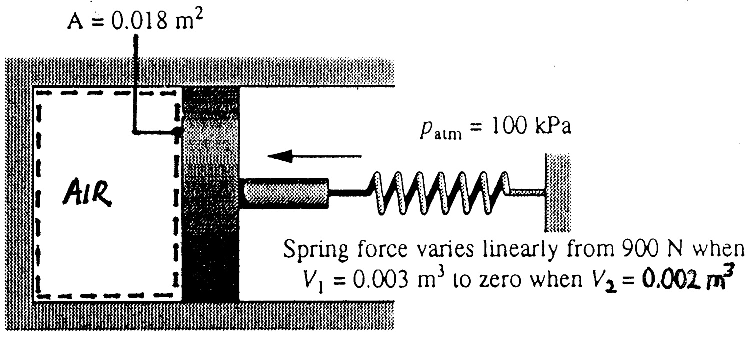
KNOWN: Warm air cools slowly in a piston-cylinder assembly from a known initial volume to a known final volume. During the process, a spring exerts a force on the piston that varies linearly from a known initial value to a final value of zero.

FIND: Determine the initial and final pressures of the air, and the work.

SCHEMATIC & GIVEN DATA:

$V_1 = 0.003\ m^3$

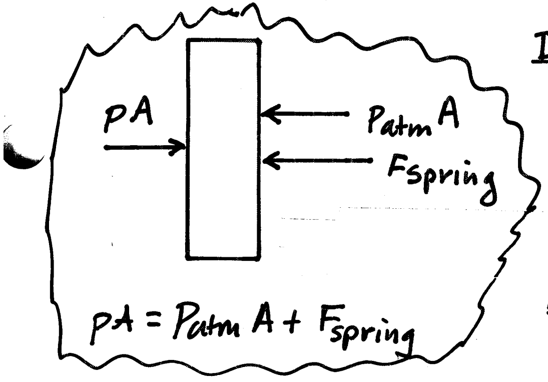
$V_2 = 0.002\ m^3$



A = 0.018 m²
AIR
p_{atm} = 100 kPa
Spring force varies linearly from 900 N when $V_1 = 0.003\ m^3$ to zero when $V_2 = 0.002\ m^3$

ASSUMPTIONS: (1) The air is a closed system. (2) The process occurs slowly, so there is no acceleration of the piston. (3) There is no friction between the piston and the cylinder wall. (4) The spring force varies linearly with volume.

ANALYSIS: The initial and final pressures of the air are determined from a free-body diagram of the piston, as follows. That is, $\Sigma F = 0$, so



Initially: $F_{spring} = 900\ N$

$$p_1 = p_{atm} + \frac{F_{spring,1}}{A}$$

$$= 100\ kPa + \frac{(900\ N)}{(0.018 m^2)}\left|\frac{1\ kPa}{10^3\ N/m^2}\right| = 150\ kPa \leftarrow p_1$$

Finally: $F_{spring} = 0 \Rightarrow p_2 = 100\ kPa \leftarrow p_2$

Now, the work is determined using Eq. 2.17, $W = \int_{V_1}^{V_2} p\,dV$, but from above

$$p = p_{atm} + \frac{F_{spring}}{A}$$

Since the spring force varies linearly from 900 N to zero as volume goes from $V_1 = 0.003\ m^3$ to $V_2 = 0.002\ m^3$

$$F_{spring} = \left(\frac{900\ N}{0.001}\right)(V - 0.002)$$

and

$$W = \int_{V_1}^{V_2}\left(p_{atm} + \frac{F_{spring}}{A}\right)dV = \int_{V_1}^{V_2}\left[100 + \left(\frac{900}{0.001}\right)\frac{(V-0.002)}{(0.018)|10^3|}\right]dV$$

$$= \int_{V_1}^{V_2}\left[\cancel{100} + 50000V - \cancel{100}\right]dV = \left(\frac{50000}{2}\right)V^2\Bigg|_{V_1 = 0.003\ m^3}^{V_2 = 0.002\ m^3}$$

① $$= -0.125\ kPa\cdot m^3\left|\frac{10^3 N/m^2}{1\ kPa}\right|\left|\frac{1\ kJ}{10^3 N\cdot m}\right| = -0.125\ kJ \leftarrow W$$

---

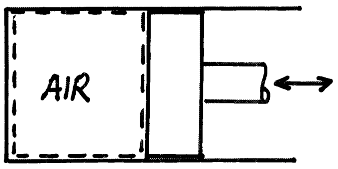
1. The negative sign denotes that the piston does work on the air as the air cools. Also, the atmosphere and the spring do work on the piston.

PROBLEM 2.32

KNOWN: Air undergoes two processes in series.

FIND: Sketch the processes on a p-v diagram and determine the work per unit mass of air.

SCHEMATIC AND GIVEN DATA:



PROCESS 1-2: $pv^{1.3} = \text{const.}$

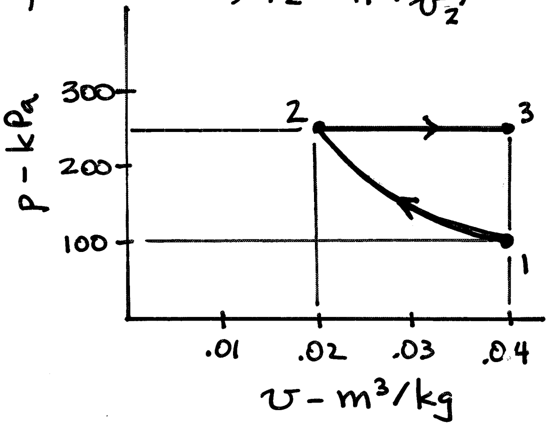
$p_1 = 100\ \text{kPa},\ v_1 = 0.04\ \text{m}^3/\text{kg}$

$v_2 = 0.02\ \text{m}^3/\text{kg}$

PROCESS 2-3: $p = \text{const.},\ v_3 = v_1$

ASSUMPTIONS: (1) The air is a closed system. (2) Both processes are quasi-equilibrium processes.

ANALYSIS: For process 1-2, $p_2 = p_1\left(\frac{v_1}{v_2}\right)^{1.3} = 246.23\ \text{kPa}$. Thus, the p-v diagram is



The work for each process is determined using Eq. 2.17

$$W = \int p\,dV = m\int p\,dv \Rightarrow \frac{W}{m} = \int p\,dv$$

Thus

$$\frac{W_{12}}{m} = \int_{v_1}^{v_2} \frac{\text{const.}}{v^{1.3}}\,dv = \left(\frac{p_2 v_2 - p_1 v_1}{1-1.3}\right)$$

$$= \left[\frac{(246.23\ \text{kPa})(0.02\ \text{m}^3/\text{kg}) - (100)(0.04)}{(1-1.3)}\right]\left|\frac{10^3\ \text{N/m}^2}{1\ \text{kPa}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= -3.082\ \text{kJ/kg}$$

$$\frac{W_{23}}{m} = \int_{v_2}^{v_3} p\,dv = p_2(v_3 - v_2)$$

$$= (246.23\ \text{kPa})(0.04 - 0.02)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^3}{10^3}\right|$$

$$= +4.9246\ \text{kJ/kg}$$

① Finally $\frac{W_{13}}{m} = \frac{W_{12}}{m} + \frac{W_{23}}{m} = +1.8426\ \frac{\text{kJ}}{\text{kg}} \longleftarrow W_{13}/m$

---

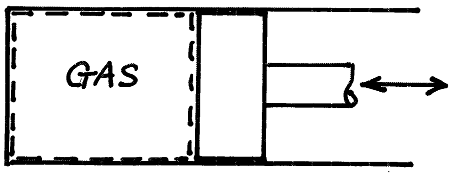
1. The result is positive, denoting that the net energy transfer by work is from the system to the surroundings.

PROBLEM 2.33

KNOWN: A gas undergoes three processes that complete a cycle.

FIND: Sketch the processes on a p-V diagram and determine the net work.

SCHEMATIC & GIVEN DATA:



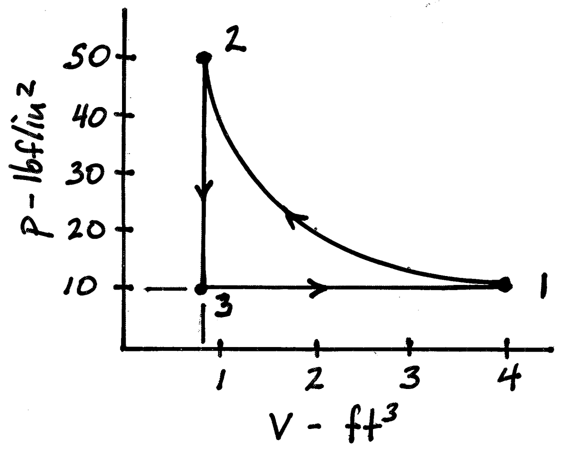
PROCESS 1-2: compression with $pV = \text{const.}$
$p_1 = 10\ \text{lbf/in}^2$, $V_1 = 4.0\ \text{ft}^3$
to $p_2 = 50\ \text{lbf/in}^2$

PROCESS 2-3: constant volume to $p_3 = p_1$

PROCESS 3-1: constant pressure

ASSUMPTIONS: (1) The gas is a closed system. (2) Processes 1-2 and 3-1 are quasi-equilibrium processes.

ANALYSIS: For process 1-2, $V_2 = (p_1/p_2)\,V_1 = 0.8\ \text{ft}^3$. Thus, the p-V diagram is



The work for process 1-2 is determined using Eq. 2.17

$$W_{12} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{\text{const.}}{V}\,dV = (p_1V_1)\ln\left(\frac{V_2}{V_1}\right)$$

$$= (10\ \text{lbf/in}^2)(4.0\ \text{ft}^3)\ln\left(\frac{0.8}{4.0}\right)\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= -11.92\ \text{Btu}$$

For process 2-3, $W_{23} = 0$. Finally, for process 3-1

$$W_{31} = \int_{V_3}^{V_1} p\,dV = p_3(V_1 - V_3) = p_1(V_1 - V_2)$$

$$= (10\ \text{lbf/in}^2)(4 - 0.8)\ \text{ft}^3\left|\frac{144}{778}\right| = 5.923\ \text{Btu}$$

① Thus $W_{net} = W_{12} + W_{23} + W_{31} = -11.92 + 0 + 5.923 = -5.997\ \text{Btu} \leftarrow W_{net}$

---

1. The negative sign denotes that the net energy transfer by work for the cycle is into the system.

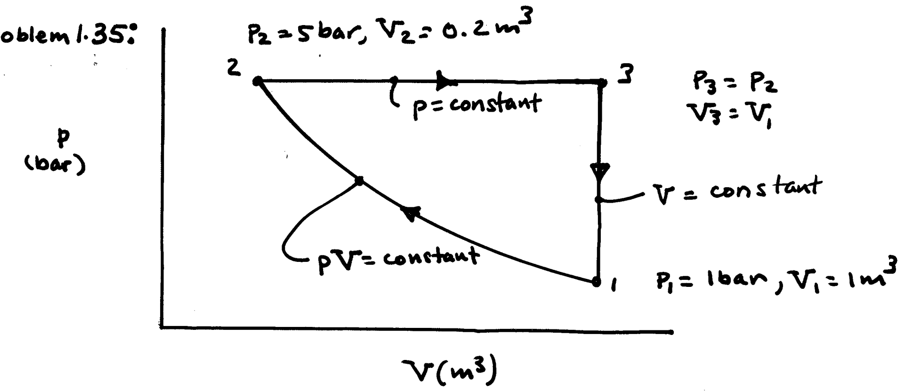
PROBLEM 2.34

KNOWN: A gas contained within a piston-cylinder assemby undergoes a thermodynamic cycle consisting of three processes.

FIND: Determine the work for each process and the net work for the cycle.

SCHEMATIC & GIVEN DATA:

From Problem 1.35:



ASSUMPTIONS: 1. The gas is the closed system. 2. Volume change is the only work mode.

ANALYSIS: Using Eq. 2.17

Process 1-2: $$W_{12} = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{C}{V}\,dV = C \ln\frac{V_2}{V_1} = p_1 V_1 \ln\frac{V_2}{V_1}$$

$$= \left[(1\text{ bar})\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|(1\text{ m}^3)\right] \ln\left[\frac{0.2}{1.0}\right] \left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

① $$= -160.9\text{ kJ} \longleftarrow W_{12}$$

Process 2-3: $$W_{23} = \int_{V_2}^{V_3} p\,dV = p_2[V_3 - V_2]$$

$$= (5\text{ bar})\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left[1\text{ m}^3 - 0.2\text{ m}^3\right]\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 400\text{ kJ} \longleftarrow W_{23}$$

Process 3-1: $$W_{31} = \int_{V_3}^{V_1} p\,dV = 0 \quad \text{since } V_1 = V_3 \longleftarrow W_{31}$$

The net work is

$$W_{net} = W_{12} + W_{23} + W_{31}$$

② $$= -160.9 + 400 + 0 = 239.1\text{ kJ} \longleftarrow W_{net}$$

---

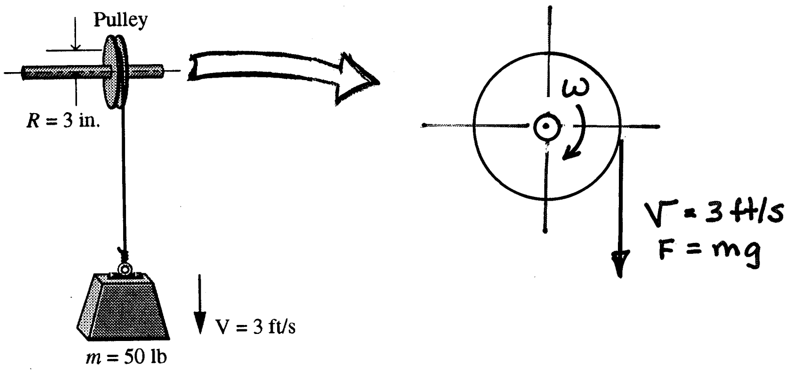
1. The minus denotes energy transfer to the gas as it is compressed.
2. The value of work depends on the details of the process between two states and is not determined only by the end states. In this case, the net work for the overall cycle is positive. By contrast, there is no net change in the properties p and V for the overall cycle. Property values are determined only by the end states. Work is not a property, and $\Sigma W\text{'s} \neq 0$.

PROBLEM 2.35

KNOWN: An object of known mass is attached to a rope wound around a pulley and falls at constant speed.

FIND: Determine the power transmitted to the pulley and the rotational speed of the pulley.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The object falls at constant speed. 2. The acceleration of gravity is constant, $g = 32.0\ \text{ft/s}^2$.

ANALYSIS: The power transmitted to the pulley can be determined using Eq. 2.13

$$\dot{W} = F \cdot V = mgV$$

$$= (50\ \text{lb})\left(32.0\ \frac{\text{ft}}{\text{s}^2}\right)\left(3\ \frac{\text{ft}}{\text{s}}\right)\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|$$

$$= \left(149.1\ \frac{\text{ft}\cdot\text{lbf}}{\text{s}}\right)\left|\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lbf/s}}\right|$$

$$= 0.27\ \text{hp} \qquad \longleftarrow \dot{W}$$

Since $V = R\omega$, we have

$$\omega = \frac{V}{R} = \frac{3\ \text{ft/s}}{(3/12)\ \text{ft}}\left|\frac{1\ \text{Rev}}{2\pi\ \text{rad}}\right|\left|\frac{60\ \text{s}}{1\ \text{min}}\right|$$

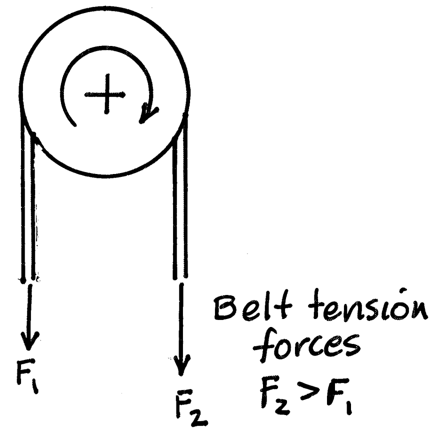
$$= 115\ \text{RPM} \qquad \longleftarrow \omega$$

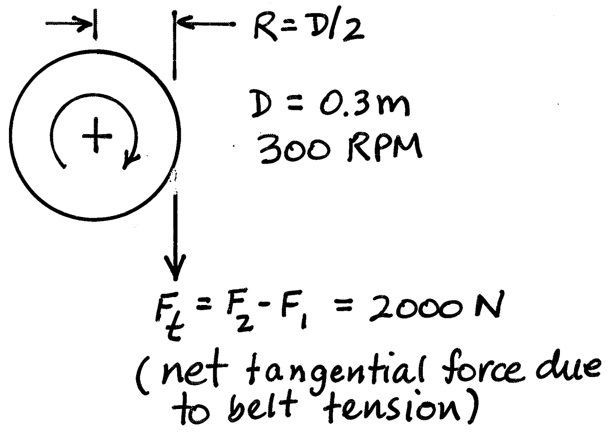
PROBLEM 2.36

KNOWN: The rotational speed and diameter of a drive shaft pulley are known. The net force applied by the belt on the pulley is also given.

FIND: Determine the applied torque and the power transmitted.

SCHEMATIC & GIVEN DATA:




(+)
R = D/2
D = 0.3 m
300 RPM
$F_t = F_2 - F_1 = 2000$ N
(net tangential force due to belt tension)

ANALYSIS: The torque is calculated using the tangential force and the radius at which it acts

$$\mathcal{T} = F_t \cdot R$$

$$= (2000\ \text{N})\left(\frac{0.3}{2}\ \text{m}\right) = 300\ \text{N}\cdot\text{m} \longleftarrow \underline{\qquad\qquad \mathcal{T}}$$

Thus, with Eq. 2.20 the power transmitted is

$$\dot{W}_{\text{shaft}} = \mathcal{T} \cdot \omega$$

$$= (300\ \text{N}\cdot\text{m})\left(300\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{1}{\text{rev}}\right)\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{1\ \text{J}}{1\ \text{N}\cdot\text{m}}\right|\left|\frac{1\ \text{kW}}{10^3\ \text{J/s}}\right|$$

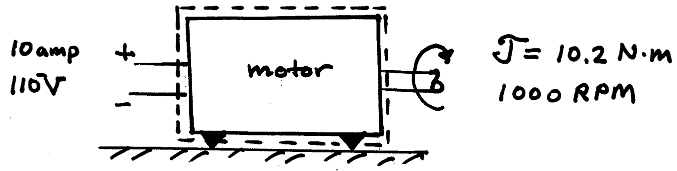
$$= 9.42\ \text{kW} \longleftarrow \underline{\qquad\qquad \dot{W}}$$

PROBLEM 2.37

KNOWN: Operating data are provided for an electric motor at steady state.

FIND: Determine the electric power required by the motor and the power developed by the output shaft. Determine the net power input to the motor. Also, determine the amounts of energy transfer by electrical work and by the shaft during 2h of operation

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS 1. As shown in the schematic, the motor is the closed system. 2. The system is at steady state.

ANALYSIS: (a) Using Eq. 2.21

$$\dot{W}_{electric} = -(\text{Voltage})(\text{current})$$
$$= -(110\text{ volts})(10\text{ amp})\left|\frac{1\text{ watt/amp}}{1\text{ volt}}\right| = -1100\text{ watt}\left|\frac{1\text{ kW}}{10^3\text{ W}}\right|$$

① $= -1.1$ kW $\longleftarrow \dot{W}_{electric}$

Using Eq. 2.20

$$\dot{W}_{shaft} = (\text{Torque})(\text{angular velocity})$$
$$= (10.2\text{ N·m})\left(1000\ \frac{\text{Rev}}{\text{min}}\left|\frac{2\pi\text{ rad}}{\text{Rev}}\right|\left|\frac{1\text{ min}}{60\text{ s}}\right|\right)\left|\frac{1\text{ kW}}{10^3\text{ N·m/s}}\right|$$
$= 1.07$ kW $\longleftarrow \dot{W}_{shaft}$

(b) $\dot{W}_{net} = \dot{W}_{electric} + \dot{W}_{shaft}$

② $= (-1.1\text{ kW}) + (1.07\text{ kW}) = -0.03$ kW $\longleftarrow \dot{W}_{net}$

(c) Integrating to find energy transfer amounts

$W_{electric} = \int_0^{2h} \dot{W}_{elec}\,dt = (-1.1\text{ kW})(2\text{ h}) = -2.2$ kW·h $\longleftarrow W_{electric}$

$W_{shaft} = \int_0^{2h} \dot{W}_{shaft}\,dt = (1.07\text{ kW})(2\text{ h}) = 2.14$ kW·h $\longleftarrow W_{shaft}$

---

1. The minus sign is needed because energy is transferred to the motor electrically.

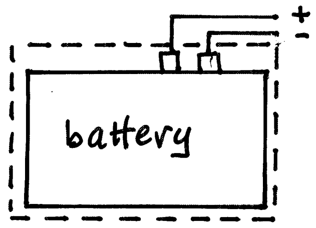
2. This value represents the portion of the electric power input that is not obtained as a shaft power output because of effects within the motor such as electrical resistance and friction.

PROBLEM 2.38

KNOWN: An electric storage battery is charged with a constant current for a known length of time.

FIND: Determine the cost of electricity to charge the battery.

SCHEMATIC & GIVEN DATA:



12 volt
2 amp
$\Delta t = 24\ \text{h}$
electricity cost = \$0.08/kw·h

ASSUMPTION: The voltage and current are constant.

ANALYSIS: The electric power is obtained using Eq. 2.21

$$|\dot{W}| = \mathcal{E}\, i$$

Since the voltage and current are constant, the electricity used in 24 h is

$$|W| = \int_{t_1}^{t_2} \dot{W}\, dt = |\dot{W}|\Delta t = \mathcal{E}\, i\, \Delta t$$

$$= (2\ \text{amp})(12\ \text{volt})\left|\frac{1\ \text{Watt/amp}}{1\ \text{volt}}\right|(24\ \text{h})\left|\frac{1\ \text{kW}}{10^3\ \text{Watt}}\right|$$

$$= 0.576\ \text{kW·h}$$

The cost is

$$\text{cost} = (0.576\ \text{kW·h})(\$0.08/\text{kW·h}) \approx \$.05 \longleftarrow \text{electric cost}$$

PROBLEM 2.39

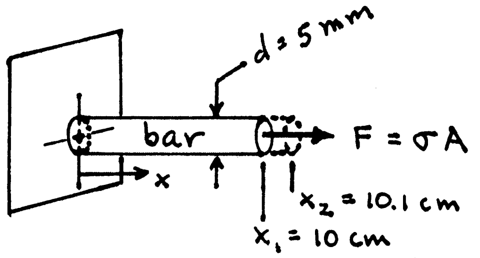
The students will need to estimate the typical power for each of the devices. They also will need to estimate how long they each device in the course of a month. The details are left to the reader.

PROBLEM 2.40

KNOWN: A solid cylindrical bar is slowly stretched. The initial and final length and the stress acting at the end of the bar are known.

FIND: Determine the work done on the bar.

SCHEMATIC & GIVEN DATA:



$\sigma = C(x - x_1)/x_1$

$C = 2 \times 10^7$ kPa

$A = \dfrac{\pi d^2}{4}$

ASSUMPTIONS: (1) The bar is a closed system. (2) The moving boundary is the only work mode. (3) The known normal stress acts at the end of the bar. (4) The change in area A is neglected.

ANALYSIS: The work done is given by Eq. 2.18

$$W = -\int_{x_1}^{x_2} \sigma A\,dx = -\frac{CA}{x_1}\int_{x_1}^{x_2}(x - x_1)\,dx$$

$$= -\frac{CA}{x_1}\left(\frac{x^2}{2} - x_1 x\right)\Big|_{x_1}^{x_2} dx$$

$$= -\frac{C\pi d^2}{4x_1}\left[\left(\frac{x_2^2 - x_1^2}{2}\right) - x_1(x_2 - x_1)\right]$$

Now, with $C = 2\times10^7$ kPa, $d = 5$ mm $= 0.005$ m, $x_1 = 10$ cm $= 0.1$ m, and $x_2 = 0.101$ m

$$W = -\frac{(2\times10^7\,\text{kPa})\,\pi\,(5\times10^{-3}\,\text{m})^2}{4\,(0.1\,\text{m})\left|\dfrac{1\,\text{kPa}}{10^3\,\text{N/m}^2}\right|}\left[\left(\frac{0.101^2 - 0.1^2}{2}\right)\text{m}^2 - (0.1)(.101 - 0.1)\,\text{m}^2\right]$$

① $= -1.963$ N·m $= -1.963$ J ◀ W

---

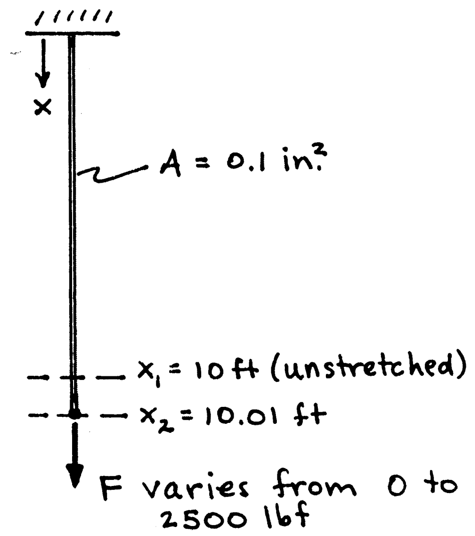
1. The negative sign denotes energy tranfer by work to the bar.

PROBLEM 2.41

KNOWN: A wire suspended vertically is stretched by an applied force.

FIND: Determine (a) the work done, and (b) the Young's modulus.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The wire is a closed system. (2) The moving boundary is the only work mode. (3) The applied force varies linearly with x. (4) The change in area A is neglected.

ANALYSIS: (a) The applied force varies with x according to

$$F = \left(\frac{2500\ \text{lbf}}{0.01\ \text{ft}}\right)(x - 10\ \text{ft}) = 2.5\times10^5\ x - 2.5\times10^6$$

where x is in ft and F is in lbf.

Thus, applying Eq. 2.12 to evaluate the work of the force F

$$W = -\int_{x_1}^{x_2} F\,dx$$

$$= -\int_{10\ \text{ft}}^{10.01\ \text{ft}} (2.5\times10^5 x - 2.5\times10^6)\,dx$$

$$= -\left[\frac{2.5\times10^5}{2}(10.01^2 - 10^2) - 2.5\times10^6(10.01 - 10)\right]$$

$= -12.5\ \text{ft}\cdot\text{lbf}$ ← W

(b) From Problem 2.40, the Young's modulus $\mathcal{E}$ can be expressed as

$$\mathcal{E} = \sigma\left(\frac{x_0}{x - x_0}\right)$$

where $\sigma = F/A$ is the normal stress, and x_0 is the unstretched length. Thus

$$\mathcal{E} = \frac{F}{A}\left(\frac{x_0}{x - x_0}\right)$$

$$= \frac{(2500\ \text{lbf})}{(0.1\ \text{in}^2)}\frac{(10\ \text{ft})}{(10.01 - 10)\ \text{ft}}$$

$= 2.5\times10^7\ \text{lbf/in}^2$ ← Young's modulus

PROBLEM 2.42

KNOWN: A wire of constant cross-sectional area and a given initial length is stretched. The stress-strain relation is known.

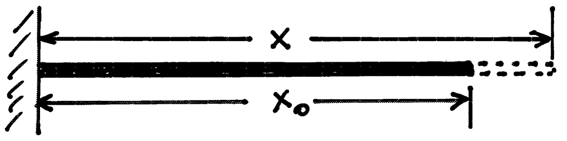
FIND: Derive an expression for the work done on the wire as a function of strain.

SCHEMATIC & GIVEN DATA:

A - area

σ - normal stress

$\varepsilon = \frac{x-x_0}{x_0}$ (strain)



ASSUMPTIONS: (1) The wire is a closed system. (2) Stress and strain are related linearly. (3) The cross-sectional area remains constant.

ANALYSIS: The work done on the wire is given by Eq. 2.18

$$W = -\int_{x_0}^{x} \sigma A\, dx$$

From the given stress-strain relation

$$\sigma = C\varepsilon = C\left(\frac{x-x_0}{x_0}\right)$$

Where C is a constant (Young's modulus). From this expression

$$d\varepsilon = \frac{dx}{x_0} \Rightarrow dx = x_0\, d\varepsilon$$

Substituting into the work expression

$$W = -\int_0^{\varepsilon} (C\varepsilon) A (x_0\, d\varepsilon) = -CAx_0 \int_0^{\varepsilon} \varepsilon\, d\varepsilon$$

Finally

$$W = -\frac{CAx_0\varepsilon^2}{2} \longleftarrow$$

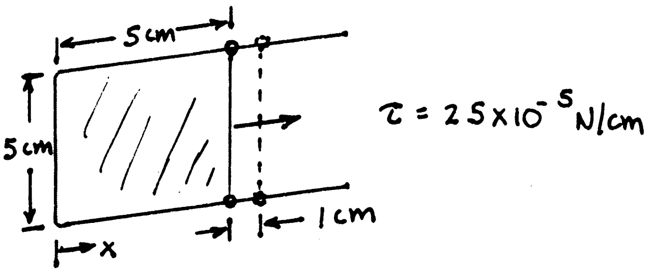
work expression.

PROBLEM 2.43

KNOWN: A soap film on a wire frame is stretched.

FIND: Determine the work done.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The film is a closed system. (2) The moving boundary is the only work mode. (3) The surface tension is constant, acting on both sides of the film.

ANALYSIS: The work is determined using Eq. 2.19

$$W = -\int_{A_1}^{A_2} \tau \, dA = -\int_{x_1}^{x_2} \tau \, 2\ell \, dx$$

For constant surface tension

$$W = -\tau \, 2\ell \, \Delta x$$

$$= -\left(25 \times 10^{-5} \frac{\text{N}}{\text{m}}\right) 2 \, (5 \text{ cm})(1 \text{ cm}) \left|\frac{1 \text{ m}}{10^2 \text{ cm}}\right| \left|\frac{1 \text{ J}}{1 \text{ N}\cdot\text{m}}\right|$$

① $= -2.5 \times 10^{-5}$ J ← W

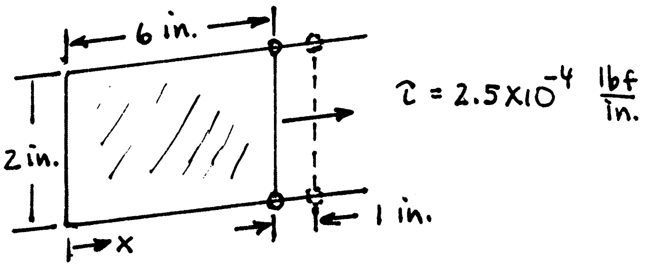
1. The negative sign denotes work done on the film. Note the small magnitude of the work required to stretch the film.

PROBLEM 2.44

KNOWN: A liquid film on a wire frame is stretched.

FIND: Determine the work done.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The film is a closed system. (2) The moving boundary is the only work mode. (3) The surface tension is constant, acting on both sides of the film.

ANALYSIS: The work is determined using Eq. 2.19

$$W = -\int_{A_1}^{A_2} \tau \, dA = -\int_{x_1}^{x_2} \tau \, 2\ell \, dx$$

For constant surface tension

$$W = -\tau \, 2\ell \, \Delta x$$

$$= -\left(2.5 \times 10^{-4} \frac{\text{lbf}}{\text{in.}}\right) 2 \, (2 \text{ in.})(1 \text{ in.}) \left|\frac{1 \text{ ft}}{12 \text{ in.}}\right|$$

① $= -8.33 \times 10^{-5}$ ft · lbf ← W

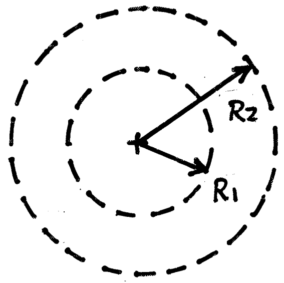
1. The negative sign denotes work done on the film. Note the small magnitude of the work required to stretch the film.

PROBLEM 2.45

KNOWN: A common baloon is being inflated.

FIND: Estimate the work required.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system consists of the air in the balloon. (2) The balloon is spherical. (3) The pressure inside the balloon varies linearly with the balloon's radius.

ANALYSIS: With the above assumptions, the work can be estimated using Eq. 2.17; $W = \int p dV$.

If the pressure is assumed vary linearly with radius, then the pressure when $R=0$ is atmospheric, and

$$p = cR + p_{atm}$$

where c is a constant. Further, for a sphere

$$V = \frac{4}{3}\pi R^3$$

and $$dV = 4\pi R^2 dR$$

Thus

$$W = \int_{R=0}^{R} (cR + p_{atm})\, 4\pi R^2 dR \qquad (cR + p_{atm})$$

$$= 4\pi \left[\frac{cR^4}{4} + \frac{p_{atm} R^3}{3}\right]$$

① $$= \frac{4\pi R^3}{3}\left[\frac{3cR}{4} + p_{atm}\right] \longleftarrow \text{work expression}$$

---

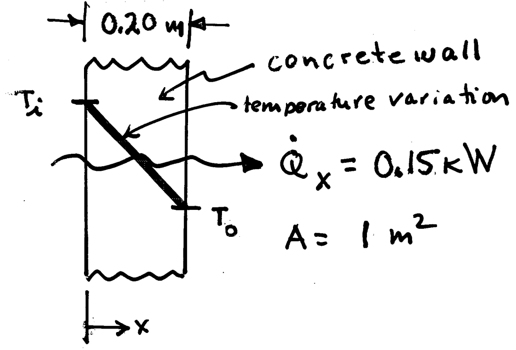
1. A more detailed analysis could be done to incorporate a more accurate stress-strain relation for the balloon material and to account more accurately for geometry.

PROBLEM 2.46*

KNOWN: Data are provided for a concrete wall at steady state.

FIND: Determine the temperature difference across the wall.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The wall is at steady state. 2. The temperature varies linearly through the wall. 3. Energy transfer is by conduction.

ANALYSIS: Using Eq. 2.31 together with assumption 2

$$\dot{Q}_x = -kA\frac{dT}{dx} \quad , \quad \frac{dT}{dx} = \frac{T_o - T_i}{L}$$

where L = 0.20 m and $(T_o - T_i)$ is the temperature difference across the wall. Thus

$$\dot{Q}_x = -kA\left[\frac{T_o - T_i}{L}\right]$$

$$\Rightarrow \quad (T_o - T_i) = -\left[\frac{\dot{Q}_x L}{kA}\right]$$

With k = 1.4 W/m·K from Table A-19

$$(T_o - T_i) = -\left[\frac{(150 \text{ W})(0.20 \text{ m})}{\left(1.4 \frac{\text{W}}{\text{m}\cdot\text{K}}\right)(1 \text{ m}^2)}\right]$$

$$= -21.4 \text{ K} \longleftarrow T_o - T_i$$

where the minus sign is required because $T_o < T_i$.

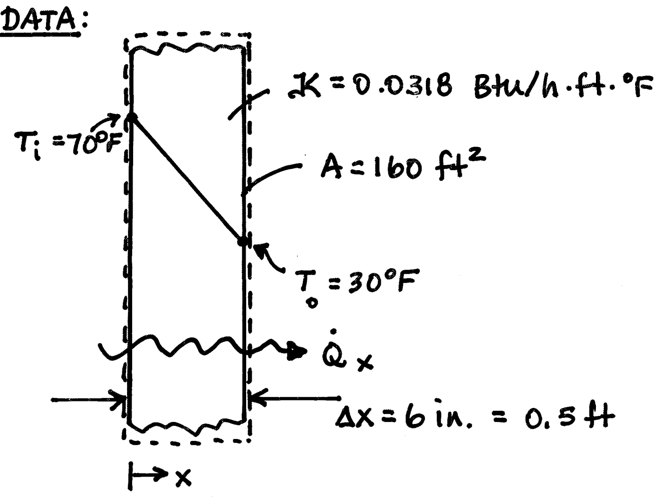
PROBLEM 2.47

KNOWN: Energy transfer by conduction occurs at steady state through a plane wall. The dimensions, thermal conductivity, and surface temperatures are specified.

FIND: Determine the rate of energy transfer by conduction.

SCHEMATIC & GIVEN DATA:

①



ASSUMPTIONS: (1) The wall is at steady state. (2) The temperature varies linearly through the wall. (3) The thermal conductivity is uniform and heat transfer is by conduction.

ANALYSIS: Using Eq. 2.31 together with assumption 2

$$\dot{Q}_x = -\kappa A \frac{dT}{dx} \quad , \quad \frac{dT}{dx} = \frac{T_o - T_i}{\Delta x}$$

Thus

$$\dot{Q}_x = -\kappa A \left(\frac{T_o - T_i}{\Delta x}\right)$$

Inserting values

$$\dot{Q}_x = -\left(0.0318 \frac{\text{Btu}}{\text{h}\cdot\text{ft}\cdot°\text{F}}\right)\left(160\ \text{ft}^2\right)\left(\frac{30-70}{0.5}\right)\frac{°\text{F}}{\text{ft}}$$

$$= 407\ \text{Btu/h} \longleftarrow \dot{Q}_x$$

---

1. The values given are for a typical insulated frame wall.

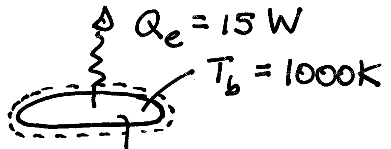
PROBLEM 2.48

KNOWN: A surface of given diameter emits thermal radiation at a known rate at a specified temperature.

FIND: Determine the emissivity of the surface and plot the rate of radiant emission vs. surface temperature.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) The Stefan-Boltzmann law applies. (2) The emissivity is constant.



$d = 2\,cm = 0.02\,m$

$\sigma = 5.67 \times 10^{-8}\,W/m^2 \cdot K^4$

ANALYSIS: Using Eq. 2.33

$$\dot{Q}_e = \varepsilon \sigma A T_b^4 \qquad (*)$$

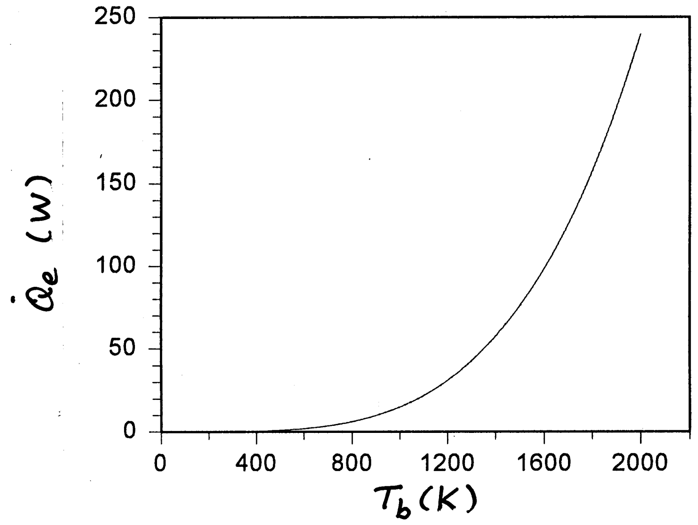
Solving for ε

$$\varepsilon = \frac{\dot{Q}_e}{\sigma A T^4} = \frac{\dot{Q}_e}{\sigma\left(\frac{\pi d^2}{4}\right)T^4}$$

$$= \frac{(15\,W)}{\left(5.67\times10^{-8}\,\frac{W}{m^2\cdot K^4}\right)\left(\frac{\pi(0.02m)^2}{4}\right)(1000\,K)^4}$$

$$= 0.842 \longleftarrow \varepsilon$$

Using the emissivity just calculated, and varying T_b from 0 to 2000K, the following plot can be constructed based on Eq. (*) above



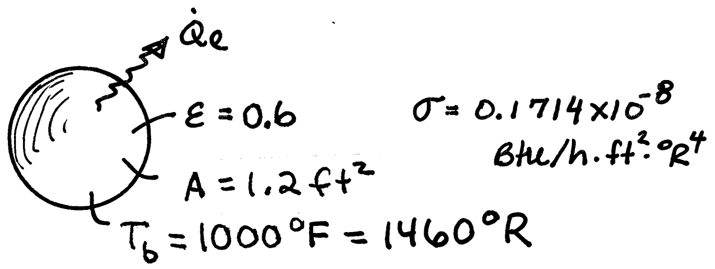
Notice the rapid growth in $\dot{Q}_e$ with T_b.

PROBLEM 2.49*

KNOWN: A sphere of known surface area, temperature, and emissivity emits thermal radiation.

FIND: Determine the rate of thermal emission.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The Stefan-Boltzmann law applies.

ANALYSIS: Using Eq. 2.33

$$\dot{Q}_e = \varepsilon \sigma A T_b^4$$

$$= (0.6)\left(0.1714 \times 10^{-8}\ \frac{\text{Btu}}{\text{hr} \cdot \text{ft}^2 \cdot {}^\circ\text{R}^4}\right)(1.2\ \text{ft}^2)(1460\ {}^\circ\text{R})^4$$

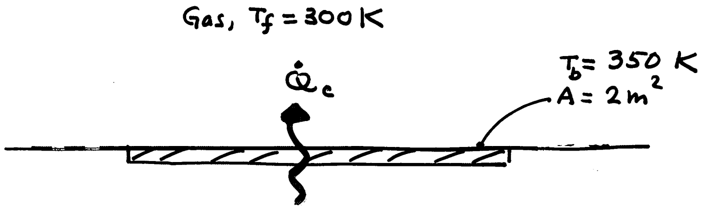
$$= 5607\ \text{Btu/h} \longleftarrow \underline{\qquad\qquad\qquad\qquad\qquad \dot{Q}_e}$$

PROBLEM 2.50

KNOWN: Data are provided for a flat surface cooled convectively by a gas.

FIND: Determine the range of the heat transfer rate for cooling by free convection. By forced convection.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: Heat transfer between the surface and the gas is by convection.

ANALYSIS: Using Eq. 2.34

$$\dot{Q}_c = hA(T_b - T_f)$$
$$= h(2\,m^2)(350 - 300)\,K$$
$$= h(100\,m^2 \cdot K) \qquad (*)$$

With data from Table 2.1

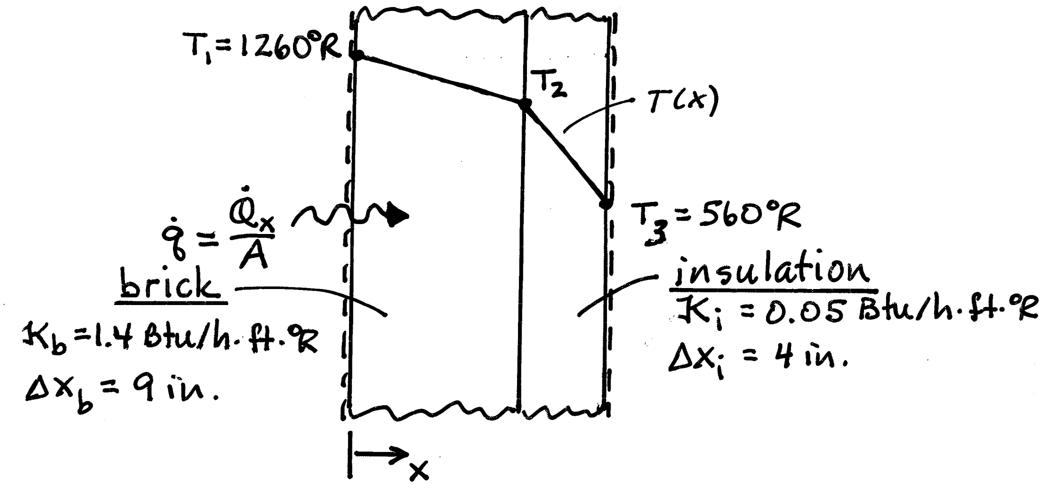
	$h\ (W/m^2 \cdot K)$	$\dot{Q}_c$ from Eq(*), in kW
free convection:	2 – 25	0.2 – 2.5
forced convection:	25 – 250	2.5 – 25

PROBLEM 2.51

KNOWN: Energy transfer occurs by conduction through a composite plane wall, consisting of two layers.

FIND: Determine the steady-state heat flux and the temperature at the interface between the layers.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The wall is a closed system at steady state. (2) The temperature distributions are linear in both layers. (3) The two layers are in perfect thermal contact. (4) The thermal conductivities of both layers are uniform.

ANALYSIS: From Eq. 2.31

$$\dot{q} = -k_b \left.\frac{dT}{dx}\right)_{brick} = -k_i \left.\frac{dT}{dx}\right)_{insul.}$$

Since the temperature distributions are linear

$$\dot{q} = k_b \frac{T_1 - T_2}{\Delta x_b} = k_i \frac{T_2 - T_3}{\Delta x_i}$$

or

$$\dot{q} = \frac{T_1 - T_2}{R_b} = \frac{T_2 - T_3}{R_i} \qquad (*)$$

where $R_b = \Delta x_b / k_b$ and $R_i = \Delta x_i / k_i$. From (*)

① $$T_2 = T_1 - \dot{q} R_b \qquad (**)$$

Combining (*) and (**)

$$\dot{q} = \frac{T_1 - T_3}{R_b + R_i} \qquad (***)$$

Inserting values

$$R_b = \frac{(9/12)\ \text{ft}}{(1.4\ \text{Btu/h·ft·°R})} = 0.5357\ \frac{\text{h·°R}}{\text{Btu}}\ ;\ R_i = \frac{4/12}{0.05} = 6.667\ \frac{\text{h·°R}}{\text{Btu}}$$

PROBLEM 2.51 (Cont'd)

Thus

$$\dot{q} = \frac{1260 - 560}{(0.5357 + 6.667)} = 97.19 \text{ Btu/h} \longleftarrow \qquad \dot{q}$$

Now, inserting values in (**)

$$T_2 = 1260 - (97.19)(0.5357)$$

$$= 1208\ ^\circ R \longleftarrow \qquad T_2$$

---

1. The form of (***) illustrates the analogy between heat conduction through a composite wall and electric current flow through a series of resistances. The temperature difference in the numerator is analogous to a voltage difference, and the value of R_b and R_i are "thermal" resistances" analogous to the electrical resistances.

PROBLEM 2.52

KNOWN: Energy transfer occurs from the inside air to the outside air through an insulated frame wall.

FIND: Determine the steady-state heat transfer rate through the wall.

SCHEMATIC & GIVEN DATA:

inside air

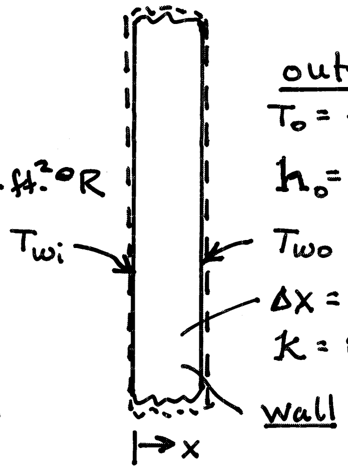
$T_i = 70°F$

$h_i = 1.5\ \text{Btu/h·ft}^2\text{·°R}$

outside air

$T_o = -10°F$

$h_o = 6\ \text{Btu/h·ft}^2\text{·°R}$



$\Delta x = 6$ in.

$k = 0.0318\ \text{Btu/h·ft·°R}$

ASSUMPTIONS: (1) The system is at steady state. (2) Newton's law of cooling applies for heat from the air to the wall. (3) The temperature distribution is linear through the wall. (4) The thermal conductivity of the wall is uniform.

ANALYSIS: For energy transfer between the inside air and the wall

$$\dot{Q}_i = h_i A (T_i - T_{wi}) \qquad (1)$$

For conduction through the wall

$$\dot{Q}_x = -kA\frac{dT}{dx} = kA\left(\frac{T_{wi} - T_{wo}}{\Delta x}\right) \qquad (2)$$

Finally, for energy transfer between the wall and the outside air

$$\dot{Q}_o = h A (T_{wo} - T_o) \qquad (3)$$

Now, from (1)

$$T_{wi} = T_i - \dot{Q}_i\left(\frac{1}{h_i A}\right) \qquad (4)$$

And, from (3)

$$T_{wo} = T_o + \dot{Q}_o\left(\frac{1}{h_o A}\right) \qquad (5)$$

Combing (2), (4), and (5), and noting that $\dot{Q}_i = \dot{Q}_x = \dot{Q}_o \equiv \dot{Q}$

① $$\dot{Q} = \frac{A(T_o - T_i)}{\left(\frac{1}{h_i} + \frac{\Delta x}{k} + \frac{1}{h_o}\right)}$$

Inserting values

$$\dot{Q} = \frac{(160\ \text{ft}^2)(530 - 450)\ °R}{\left(\frac{1}{1.5\ \frac{\text{Btu}}{\text{h·ft}^2\text{·°R}}} + \frac{.5\ \text{ft}}{.0318\ \frac{\text{Btu}}{\text{h·ft·°R}}} + \frac{1}{6\ \frac{\text{Btu}}{\text{h·ft}^2\text{·°R}}}\right)}$$

$$= \frac{(160)(80)}{(16.56)} = 772.9\ \text{Btu/h} \longleftarrow \dot{Q}$$

---

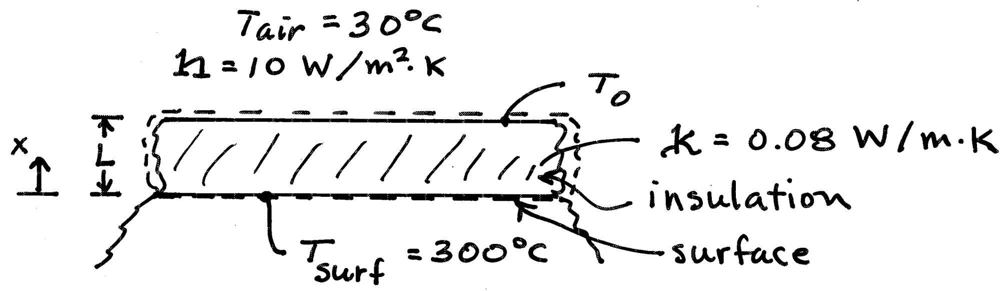
1. The quantity in the denominator is called the overall resistance, or "R-value."

PROBLEM 2.53

KNOWN: A hot surface is covered with insulation. Energy transfer occurs from outer surface of the insulation and the surrounding air.

FIND: Determine the minimum thickness of insulation to maintain the outer surface temperature below a desired value at steady state.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is at steady state. (2) Newton's law of cooling applies for heat transfer from the insulation to the air. (3) The temperature distribution through the insulation is linear. (4) The thermal conductivity of the insulation is uniform.

ANALYSIS: For energy transfer by conduction through the insulation

$$\dot{Q}_x = -kA\frac{dT}{dx} = kA\frac{(T_{surf} - T_o)}{L}$$

Further, for energy transfer from the insulation to the air

$$\dot{Q}_o = hA(T_o - T_{air})$$

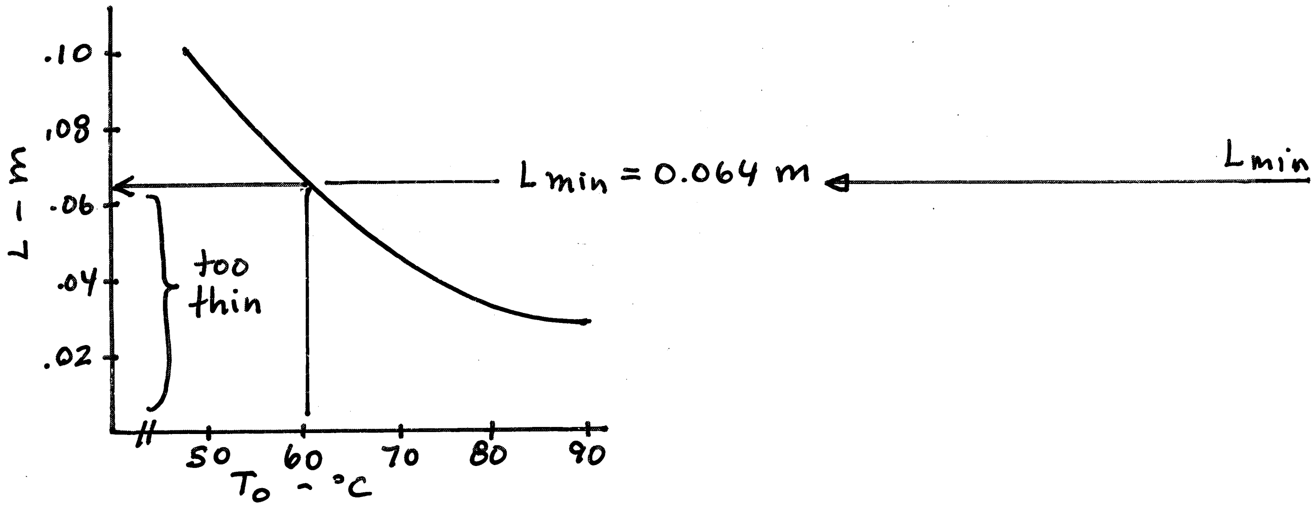
At steady state, $\dot{Q}_x = \dot{Q}_o$, so

$$\frac{k\cancel{A}}{L}(T_{surf} - T_o) = h\cancel{A}(T_o - T_{air})$$

Solving for L

$$L = \frac{k(T_{surf} - T_o)}{h(T_o - T_{air})}$$

With $k = 0.08\ W/m \cdot K$, $h = 10\ W/m^2 \cdot K$, $T_{surf} = 300°C$, and $T_{air} = 30°C$ the plot below can be constructed



PROBLEM 2.54

Process	Q	W	E_1	E_2	ΔE
a	+50	−20	−20	+50	+70
b	+50	+20	+20	+50	+30
c	−40	−60	+40	+60	+20
d	−90	−90	+50	+50	0
e	+50	+150	+20	−80	−100

Process a: $\Delta E = Q - W = (+50) - (-20) = \underline{+70}$

$E_2 - E_1 = \Delta E \Rightarrow E_1 = E_2 - \Delta E$

$= (+50) - (+70) = \underline{-20}$

Process b: $\Delta E = Q - W = (+50) - (+20) = \underline{+30}$

$E_2 - E_1 = \Delta E \Rightarrow E_2 = \Delta E + E_1$

$= (+30) + (+20) = \underline{+50}$

Process c: $W = Q - \Delta E = (-40) - (20) = \underline{-60}$

$E_2 - E_1 = \Delta E \Rightarrow E_1 = E_2 - \Delta E$

$= (+60) - (+20) = \underline{+40}$

Process d: $Q = \Delta E + W = (0) + (-90) = \underline{-90}$

$E_2 - E_1 = \Delta E \Rightarrow E_1 = E_2 - \Delta E$

$= (+50) - (0) = \underline{+50}$

Process e: $W = Q - \Delta E = (+50) - (-100) = \underline{+150}$

$E_2 - E_1 = \Delta E \Rightarrow E_2 = \Delta E + E_1$

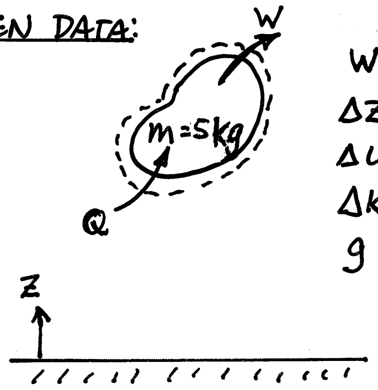
$= (-100) + (+20) = \underline{-80}$

PROBLEM 2.55*

KNOWN: A system of known mass undergoes a process for which the work is specified. In addition, the changes in specific internal energy, elevation, and kinetic energy are given.

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:



$W = -9\ \text{kJ}$
$\Delta z = 700\ \text{m}$
$\Delta u = -6\ \text{kJ/kg}$
$\Delta KE = 0$
$g = 9.6\ \text{m/s}^2$

ASSUMPTIONS: (1) The system is closed. (2) There is no change in kinetic energy. (3) The acceleration of gravity is constant.

ANALYSIS: Heat transfer is determined by applying the energy balance and assumption 2

$$\overset{0}{\cancel{\Delta KE}} + \Delta PE + \Delta U = Q - W$$

The potential energy change can be expressed $\Delta PE = mg\Delta z$, and the internal energy change is $\Delta U = m\Delta u$. Thus

$$mg\Delta z + m\Delta u = Q - W$$

Solving for Q and inserting values

$$Q = W + mg\Delta z + m\Delta u$$

$$= (-9\ \text{kJ}) + (5\ \text{kg})(9.6\ \tfrac{\text{m}}{\text{s}^2})(700\ \text{m})\left|\frac{1\ \text{N}}{1\ \text{kg m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{Nm}}\right| + (5\ \text{kg})(-6\ \tfrac{\text{kJ}}{\text{kg}})$$

① $Q = -5.4\ \text{kJ} \longleftarrow$ Q

---

1. The negative sign denotes energy transfer from the system by heat.

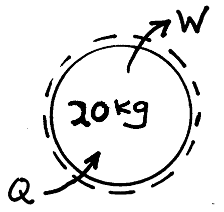
PROBLEM 2.56*

KNOWN: A system of known mass undergoes a process for which the heat transfer and work are known. The initial specific internal energy is also known.

FIND: Determine the final specific internal energy.

SCHEMATIC & GIVEN DATA:

①



$Q = -1000$ kJ
$W = -200$ kJ
$u_1 = 300$ kJ/kg

ASSUMPTIONS: 1. A closed system of known mass is under consideration. 2. Changes in kinetic and potential energy can be neglected. 3. The initial and final states are equilibrium states.

ANALYSIS: The change in internal energy can be determined from the energy balance using assumption 2

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$$

$$\Rightarrow \quad \Delta U = Q - W$$

Then, with assumption 3, $\Delta U = m(u_2 - u_1)$, so

$$m(u_2 - u_1) = Q - W$$

or

$$u_2 = u_1 + \left[\frac{Q - W}{m}\right]$$

$$= 300\frac{\text{kJ}}{\text{kg}} + \left[\frac{(-1000) - (-200)}{20 \text{ kg}}\right]\text{kJ}$$

$$= 260\frac{\text{kJ}}{\text{kg}} \longleftarrow u_2$$

---

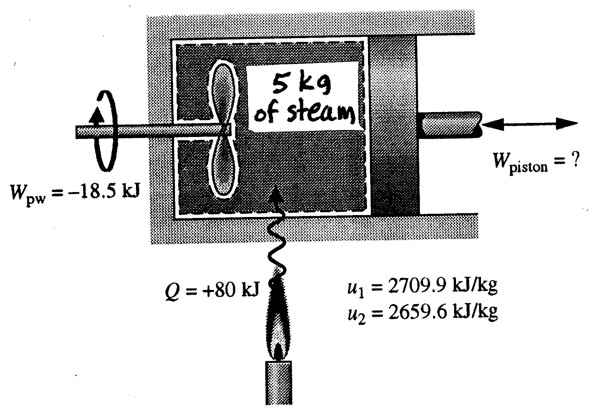
1. According to the sign conventions for Q and W, Q is negative when there is a net heat transfer of energy from the system and W is negative when there is a net work transfer of energy to the system.

PROBLEM 2.57

KNOWN: Five kg of steam undergo an expansion in a piston-cylinder assembly from state 1 to state 2. During the process there is a known heat transfer to the steam and a known work transfer of energy to the steam by a paddle wheel. The change in specific internal energy of the steam is also known.

FIND: Determine the amount of energy transfer by work from the steam to the piston during the process.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The steam is the closed system. 2. There is no change in the kinetic or potential energy from state 1 to state 2.

ANALYSIS: The net work can be determined from an energy balance. That is, with assumption 2

$$\overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} + \Delta U = Q - W$$

or

$$W = Q - \Delta U$$

The net work is the sum of the work associated with the paddlewheel W_{pw} and the work done on the piston W_{piston}:

$$W = W_{pw} + W_{piston}$$

From the given information $W_{pw} = -18.5\ \text{kJ}$, where the minus sign is required because the paddle wheel transfers energy to the system. Collecting results

$$W_{pw} + W_{piston} = Q - \Delta U$$

or

$$\begin{aligned} W_{piston} &= Q - \Delta U - W_{pw} \\ &= Q - m(u_2 - u_1) - W_{pw} \\ &= 80\ \text{kJ} - 5\ \text{kg}\,(2659.6 - 2709.9)\frac{\text{kJ}}{\text{kg}} - (-18.5\ \text{kJ}) \\ &= +350\ \text{kJ} \longleftarrow W_{piston} \end{aligned}$$

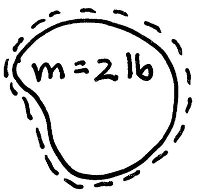
where the positive sign indicates that energy is transferred from the steam to the piston as the steam expands during the process.

PROBLEM 2.58

KNOWN: A system of known mass undergoes two processes in series.

FIND: Determine the work and heat transfer for the second process.

SCHEMATIC & GIVEN DATA:



Process 1-2: $v_1 = v_2 = 4.434$ ft³/lb, $p_1 = 100$ lbf/in², $u_1 = 1105.8$ Btu/lb, $Q_{12} = -581.36$ Btu

Process 2-3: $p_2 = p_3 = 60$ lbf/in², $v_3 = 7.82$ ft³/lb, $u_3 = 1121.4$ Btu/lb

ASSUMPTIONS: (1) A closed system is under consideration. (2) Kinetic and potential energy effects can be neglected. (3) There is no work for process 1-2. (4) The pressure is constant during process 2-3.

ANALYSIS: By assumption (4), the work for process 2-3 can be found using Eq. 2.17

$$W_{23} = \int_{V_2}^{V_3} p\,dV = p_2(V_3 - V_2) = p_2 m(v_3 - v_2)$$

$$= \left(60\,\frac{\text{lbf}}{\text{in}^2}\right)(2\text{ lb})(7.82 - 4.434)\frac{\text{ft}^3}{\text{lb}}\left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|$$

$$= 75.21\text{ Btu} \longleftarrow W_{23}$$

Use the energy balance to find Q_{23}

$$\Delta U_{23} = Q_{23} - W_{23}$$

With $\Delta U = m\Delta u$

$$Q_{23} = m(u_3 - u_2) + W_{23} \qquad (*)$$

To get u_2, use the energy balance for process 1-2

$$m(u_2 - u_1) = Q_{12} - \overset{0}{\cancel{W_{12}}}$$

$$u_2 = \frac{Q_{12}}{m} + u_1 = \frac{(-581.36\text{ Btu})}{(2\text{ lb})} + 1105.8\text{ Btu/lb}$$

$$= 815.12\text{ Btu/lb}$$

Thus, from (*)

$$Q_{23} = (2)(1121.4 - 815.12) + (75.21)$$

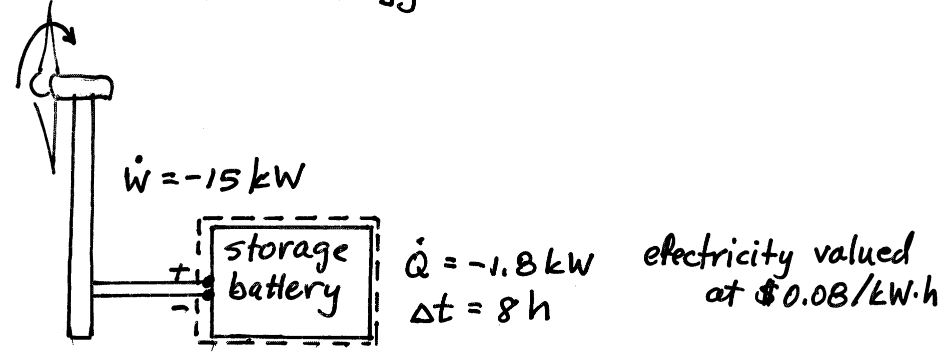
$$= 687.8\text{ Btu} \longleftarrow Q_{23}$$

PROBLEM 2.59

KNOWN: A storage battery is charged by the electric power output from a windmill. The work and heat transfer rates are known.

FIND: Determine for 8 h of operation (a) the total amount of energy stored and (b) the value of the stored energy.

SHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The battery is a closed system. (2) The work and heat transfer rates are constant.

ANALYSIS: (a) The amount of energy stored is found from an energy balance

$$\Delta E = Q - W \qquad (*)$$

To evaluate Q and W, respectively

$$Q = \int_{t_1}^{t_2} \dot{Q}\,dt = \dot{Q}\,\Delta t = (-1.8\text{ kW})(8\text{ h}) = -14.4\text{ kW·h}$$

$$= (-14.4\text{ kW·h})\left|\frac{1\text{ kJ/s}}{1\text{ kW}}\right|\left|\frac{3600\text{ s}}{1\text{ h}}\right|$$

$$= -51{,}840\text{ kJ}$$

$$W = \int_{t_1}^{t_2} \dot{W}\,dt = \dot{W}\,\Delta t = (-15\text{ kW})(8\text{ h}) = -120\text{ kW·h}$$

$$= (-120)\left|\frac{1}{1}\right|\left|\frac{3600}{1}\right| = -4.32\times10^5\text{ kJ}$$

Inserting these results in (*)

$$\Delta E = (-51{,}840) - (-4.32\times10^5) = 3.8\times10^5\text{ kJ} \longleftarrow \Delta E$$

(b) If electricity is valued at \$0.08/kW·h

$$\text{value of stored energy} = (3.8\times10^5\text{ kJ})\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|\left|\frac{1\text{ h}}{3600\text{ s}}\right|(\$0.08/\text{kW·h})$$

$$= \$\,8.45 \longleftarrow \text{value}$$

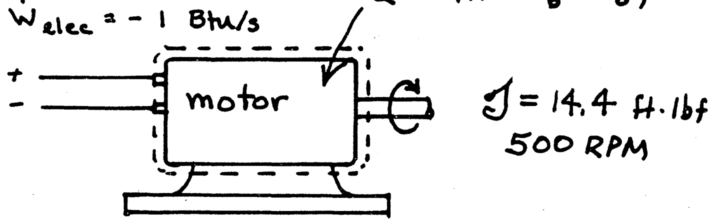
PROBLEM 2.60

KNOWN: An electric motor operates at steady state. The electric power input is given and the shaft power output can be determined. An expression is given for the heat transfer rate.

FIND: Determine the average surface temperature of the motor.

SCHEMATIC & GIVEN DATA:

① $\dot{Q} = -hA(T_b - T_0)$ $hA = 10\ \text{Btu/h}\cdot{}^\circ\text{R}$, $T_0 = 80^\circ\text{F}$



ASSUMPTIONS: (1) The motor is a closed system. (2) The system is at steady state; all operating data are constant with time. (3) The normal sign conventions for heat and work are used.

ANALYSIS: The rate form of the energy balance reduces as follows

$$\cancel{\frac{dKE}{dt}}^{\,0} + \cancel{\frac{dPE}{dt}}^{\,0} + \cancel{\frac{dU}{dt}}^{\,0} = \dot{Q} - (\dot{W}_{elec} + \dot{W}_{shaft})$$

$$\dot{Q} = \dot{W}_{elec} + \dot{W}_{shaft}$$

or, using the given expression for heat transfer rate

$$-hA(T_b - T_0) = \dot{W}_{elec} + \dot{W}_{shaft}$$

Solving for T_b

$$T_b = \frac{\dot{W}_{elec} + \dot{W}_{shaft}}{(-hA)} + T_0$$

Evaluating the work terms

$$\dot{W}_{elec} = -1\ \text{Btu/s} = -3600\ \text{Btu/h}$$

$$\dot{W}_{shaft} = \mathcal{T}\omega = (14.4\ \text{ft}\cdot\text{lbf})\left(500\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right)\left(\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right)$$

$$= 3489\ \text{Btu/h}$$

Finally

$$T_b = \frac{(-3600 + 3489.)\ \text{Btu/h}}{(-10\ \text{Btu/h}\cdot{}^\circ\text{R})} + 540^\circ\text{R} = 551^\circ\text{R}$$

$$= 91^\circ\text{F} \longleftarrow T_b$$

---

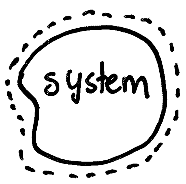
1. Note that when $T_b > T_0$, $\dot{Q}$ is negative, denoting energy transfer by heat from the motor to the surroundings.

PROBLEM 2.61

KNOWN: A closed system undergoes a process with a known heat transfer rate, and the power varies as a specified function of time.

FIND: Determine (a) the rate of change of system energy at $t = 0.6\text{h}$ and (b) the change in system energy after 2 h.

SCHEMATIC & GIVEN DATA:



$$\dot{Q} = -10 \text{ kW}$$

$$\dot{W} = \begin{cases} -8t & 0 < t \le 1\text{h} \\ -8 & t > 1\text{h} \end{cases}$$

where t is in h and $\dot{W}$ is in kW

ASSUMPTION: The system is closed.

ANALYSIS: (a) The time rate of change of energy at any time t is given by

$$\frac{dE}{dt} = \dot{Q} - \dot{W} \qquad (*)$$

At $t = 0.6$ h

$$\left.\frac{dE}{dt}\right|_{t=0.6\text{h}} = \left[\dot{Q} - (-8t)\right]_{t=0.6\text{h}} = (-10 \text{ kW}) - (-8 \cdot 0.6) \text{ kW}$$

① $$= -5.2 \text{ kW} \longleftarrow \qquad dE/dt$$

(b) The change in system energy is obtained by integrating (*) over the time period of 2 h. That is

$$\Delta E = \int_{t=0}^{t=2\text{h}} (\dot{Q} - \dot{W})\, dt$$

$$= \dot{Q}\Delta t - \left[\int_{t=0}^{t=1\text{h}} (-8t)\, dt + \int_{t=1\text{h}}^{t=2\text{h}} (-8)\, dt\right]$$

$$= (-10)(2) - \left[\frac{(-8)}{2} t^2\right]_0^1 - \left[(-8)t\right]_1^2$$

$$= -20 - [(-4)] - [(-8)(2-1)]$$

$$= -8 \text{ kW}\cdot\text{h}$$

Thus

② $$\Delta E = (-8 \text{ kW}\cdot\text{h}) \left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| \left|\frac{3600 \text{ s}}{1 \text{ h}}\right| = -28{,}800 \text{ kJ} \longleftarrow \qquad \Delta E$$

---

1. At $t = 0.6$ h, the energy of the system is decreasing at a rate of 5.2 kW because the rate of energy transfer out by heat exceeds the rate of energy transfer in by work.

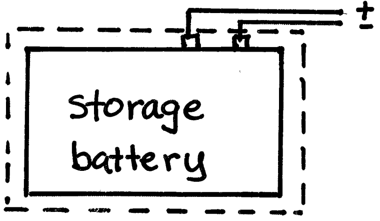
2. The negative sign denotes a net decrease of energy over the time period.

PROBLEM 2.62

KNOWN: A storage battery develops a known power output as a function of time.

FIND: Plot the power output and the change in energy of the battery, each as a function of time.

SCHEMATIC & GIVEN DATA:



$\dot{W} = 1.2 \exp(-t/60)$

(where $\dot{W}$ is in kW and t is time in s)

$\dot{Q} = 0$

ASSUMPTIONS: (1) A closed system is under consideration. (2) There is no heat transfer.

ANALYSIS: The time rate of change of system energy is

$$\frac{dE}{dt} = \cancel{\dot{Q}}^{0} - \dot{W} \Rightarrow \frac{dE}{dt} = -\dot{W}$$

or

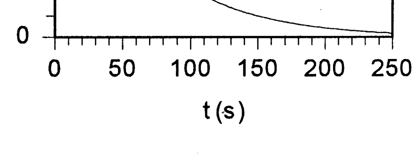
$$\frac{dE}{dt} = -1.2 \exp(-t/60)$$

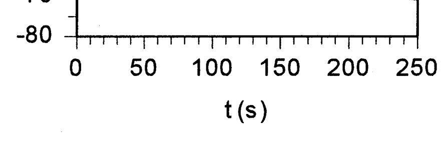
Integrating from $t=0$ to any time t

$$\Delta E = -\int_0^t 1.2 \exp(-t/60)\,dt = \frac{-1.2}{(-1/60)}\left[\exp(-t/60) - 1\right]$$

$$= 72\left[\exp(-t/60) - 1\right] \text{ kJ}$$

(a) Using software to plot $\dot{W}$ and ΔE vs. t ranging from 0 to 250 s:





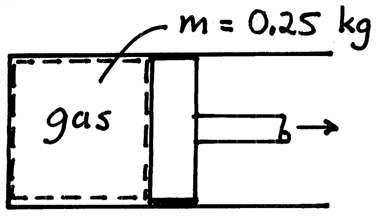
(b) In the limit as $t \to \infty$, we have $\dot{W} \to 0$ and $\Delta E \to -72$ kJ. The plots show that these limits are approached closely after about 225 s. For times greater than this, no more electric power is developed by the battery and the energy remains constant.

PROBLEM 2.63*

KNOWN: A gas of known mass expands in a piston-cylinder from a specified initial state to a known final pressure. The pressure-volume for the process is given. Also, the specific internal energy change is known.

FIND: Determine the heat transfer for the process.

SCHEMATIC & GIVEN DATA:



$p_1 = 8$ bar, $V_1 = 0.02\ \text{m}^3$
$p_2 = 2$ bar
$pV^{1.2} = \text{constant}$
$\Delta u = -55$ kJ/kg

ASSUMPTION: (1) The gas is a closed system. (2) Kinetic and potential energy effects are negligible. (3) The process is polytropic with $pV^{1.2} = \text{constant}$.

ANALYSIS: The heat transfer can be found using an energy balance. First, find the work using Eq. 2.17, as follows:

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{\text{const.}}{V^{1.2}}\,dV$$

Integrating and simplifying

$$W = \frac{p_2V_2 - p_1V_1}{(1-1.2)}$$

The volume at the final state is $V_2 = (p_1/p_2)^{1/1.2} V_1 = 0.0635\ \text{m}^3$. Thus

$$W = \frac{(2\ \text{bar})(0.0635\ \text{m}^3) - (8)(0.02)}{(1-1.2)} \left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right| \left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 16.5\ \text{kJ}$$

Now, to get the heat transfer, begin with the energy balance

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q - W$$

Solving for Q and noting that $\Delta U = m\,\Delta u$

$$Q = m\,\Delta u + W$$

$$= (0.25\ \text{kg})(-55\ \text{kJ/kg}) + (16.5\ \text{kJ})$$

① $= 2.75\ \text{kJ} \longleftarrow$ Q

---

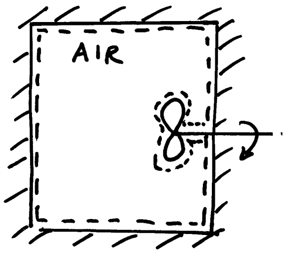
1. The heat transfer is positive, denoting energy transfer by heat to the gas as it expands.

PROBLEM 2.64*

KNOWN: Air contained in a rigid well-insulated tank receives energy at a specified rate from a paddle wheel.

FIND: Determine the specific volume at the final state, the energy transfer by work, and the change in specific internal energy of the air.

SCHEMATIC & GIVEN DATA:



$V = 0.6\,m^3$

$\dot{W} = -10\,W$, $\Delta t = 1\,h$

$m = 2\,kg$

ASSUMPTIONS: 1. The air is the closed system. 2. For the system, $Q = 0$ and there are no changes in kinetic energy and potential energy. 3. The initial and final states are equilibrium states.

ANALYSIS: (a) Since the mass and volume are each constant, the specific volume at states 1 and 2 are the same: $v_2 = v_1$. Thus

$$v_2 = V/m = (0.6\,m^3)/(2\,kg) = 0.3\,\frac{m^3}{kg} \longleftarrow \underline{v_2}$$

(b) To evaluate W, integrate

$$W = \int_0^{1h} \dot{W}\,dt = \int_0^{1h} (-10W)\,dt = (-10\,W)(1\,h)\left|\frac{1\,J/s}{1\,W}\right|\left|\frac{3600\,s}{1\,h}\right|\left|\frac{kJ}{10^3 J}\right|$$

$$= -36\,kJ \longleftarrow \underline{W}$$

(c) The change in specific internal energy can be found from an energy balance:

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = \cancel{Q} - W \Rightarrow m\Delta u = -W$$

or

$$u_2 - u_1 = \frac{(-W)}{m}$$

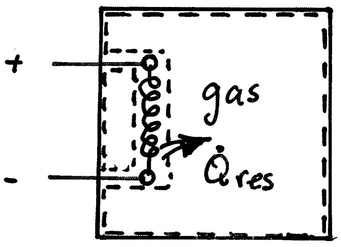
$$= \frac{-(-36\,kJ)}{2\,kg} = 18\,\frac{kJ}{kg} \longleftarrow \underline{u_2 - u_1}$$

PROBLEM 2.65

KNOWN: An electric resistor transfers energy at a constant rate to a gas contained in a rigid tank. Heat transfer between the gas and the surroundings occurs as a specified function of time.

FIND: (a) Plot the time rate of change of energy of the gas for $0 \le t \le 20$ min, (b) Determine the net change in energy of the gas after 20 min., (c) Determine the cost of the electrical input to the resistor.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{surr} = -50t$
where $\dot{Q}_{surr}$ is in Watts and t is in min.

$\dot{Q}_{res} = 1000$ W

cost of electricity = \$0.08/kW·h

ASSUMPTIONS: (1) The gas is a closed system. (2) The energy transfer from the resistor to the gas occurs at a constant rate. (3) There is no work.

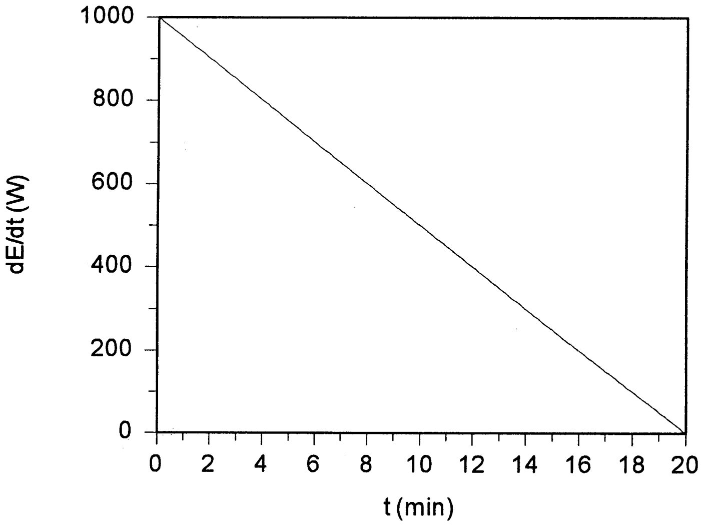
ANALYSIS: (a) The time rate of change of energy is

$$\frac{dE}{dt} = \dot{Q} - \overset{0}{\cancel{\dot{W}}} \Rightarrow \frac{dE}{dt} = \dot{Q}_{surr} + \dot{Q}_{res}$$

or

$$\frac{dE}{dt} = -50t + 1000 \qquad (t \text{ in min., } dE/dt \text{ in W}) \qquad (*)$$

Using software to plot dE/dt vs. t ranging from 0 to 20 min:



(b) Integrating (*) from $t = 0$ to 20 min

$$\Delta E = \int_0^{20} [-50t + 1000] = \left[-\left(\frac{50}{2}\right)t^2 + 1000t\right]\Big|_0^{20} = 10000 \text{ W·min}$$

$$= (10000 \text{ W·min}) \left|\frac{1 \text{ kJ/s}}{10^3 \text{ W}}\right| \left|\frac{60 \text{ s}}{1 \text{ min}}\right| = 600 \text{ kJ} \longleftarrow \Delta E$$

(c) Cost of electric input $= (1 \text{ kW})(20 \text{ min}) \left|\frac{1 \text{ h}}{60 \text{ min}}\right| (\$0.08/\text{kW·h})$

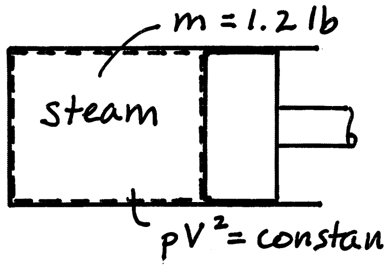
$= \$0.027 \longleftarrow$ cost of electric input

PROBLEM 2.66

KNOWN: A known mass of steam undergoes a polytropic process in a piston-cylinder assembly. The initial and final states and the heat transfer are specified.

FIND: Determine the work and the final specific volume.

SCHEMATIC & GIVEN DATA:



State 1: $p_1 = 500\ \text{lbf/in}^2$, $v_1 = 1.701\ \text{ft}^3/\text{lb}$
$u_1 = 1363.3\ \text{Btu/lb}$

State 2: $u_2 = 990.58\ \text{Btu/lb}$

$Q = -342.9\ \text{Btu}$

ASSUMPTIONS: (1) The steam is a closed system. (2) Kinetic and potential energy effects are negligible. (3) The process is polytropic with $n = 2$.

ANALYSIS: To determine the work, begin with the energy balance

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q - W$$

With $\Delta U = m(u_2 - u_1)$, and solving for W

$$\begin{aligned} W &= Q - m(u_2 - u_1) \\ &= (-342.9\ \text{Btu}) - (1.2\ \text{lb})(990.58 - 1363.3)\ \text{Btu/lb} \\ &= 104.4\ \text{Btu} \longleftarrow\!\!\!\text{———————} \ W \end{aligned}$$

To get v_2, begin with Eq. 2.17 and use the polytropic process expression

$$W = \int_{V_1}^{V_2} p\,dV = m\int_{v_1}^{v_2} p\,dv = m(p_1 v_1^2)\int_{v_1}^{v_2} \frac{dv}{v^2}$$

$$= -m(p_1 v_1^2)\left(\frac{1}{v_2} - \frac{1}{v_1}\right)$$

Solving for $1/v_2$

$$\frac{1}{v_2} = \frac{1}{v_1} - \frac{W}{m p_1 v_1^2}$$

$$= \left(\frac{1}{1.701\ \text{ft}^3/\text{lb}}\right) - \frac{(104.4\ \text{Btu})}{(1.2\ \text{lb})(500\ \text{lbf/in}^2)(1.701\ \text{ft}^3/\text{lb})^2}\left|\frac{778\ \text{ft·lbf}}{1\ \text{Btu}}\right|\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right|$$

$$= 0.2630$$

① Thus $v_2 = \dfrac{1}{0.2630} = 3.802\ \text{ft}^3/\text{lb} \longleftarrow$ ——————— v_2

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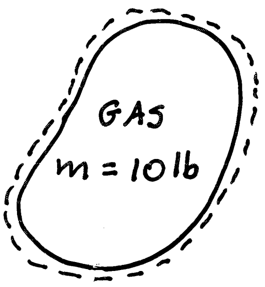
1. The increase in volume is consistent with the positive sign for work, denoting energy transfer out of the system by work.

PROBLEM 2.67

KNOWN: A known mass of gas undergoes a polytropic process between two specified states. The relationship between pressure, volume, and internal energy is known for the gas.

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:



$p_1 = 60$ lbf/in², $v_1 = 6.0$ ft³/lb
$p_2 = 20$ lbf/in²
$pv^{1.3} = \text{constant}$
For the gas
$u = (0.2651)\,pv - 95.436$
(where v is in ft³/lb, p is in lbf/in², and u is in Btu/lb)

ASSUMPTIONS: (1) The gas is a closed system. (2) Kinetic and potential energy effects are negligible. (3) The process is polytropic with $n = 1.3$.

ANALYSIS: The heat transfer is determined from an energy balance. First, determine the work beginning with Eq. 2.17

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{\text{const.}}{V^{1.3}}\,dV = \left(\frac{p_2V_2 - p_1V_1}{1-1.3}\right) = m\left(\frac{p_2v_2 - p_1v_1}{1-1.3}\right)$$

Evaluating v_2

$$v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.3}} v_1 = \left(\frac{60}{20}\right)^{\frac{1}{1.3}}(6.0) = 13.97 \text{ ft}^3/\text{lb}$$

Thus

$$W = (10\text{ lb})\left[\frac{(20 \text{ lbf/in}^2)(13.97 \text{ ft}^3/\text{lb}) - (60)(6.0)}{(1-1.3)}\right]\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$

$$= 497.3 \text{ Btu}$$

Now, writing the energy balance

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

Noting that $\Delta U = m(u_2 - u_1)$ and solving for Q

$$Q = m(u_2 - u_1) + W$$

Evaluating $(u_2 - u_1)$

$$(u_2 - u_1) = [(0.2651)(20)(13.97) - 95.436] - [(0.2651)(60)(6.0) - 95.436]$$
$$= -21.37 \text{ Btu/lb}$$

Finally

$$Q = (10\text{ lb})(-21.37 \text{ Btu/lb}) + (497.3 \text{ Btu})$$

① $= 283.6$ Btu ← Q

---

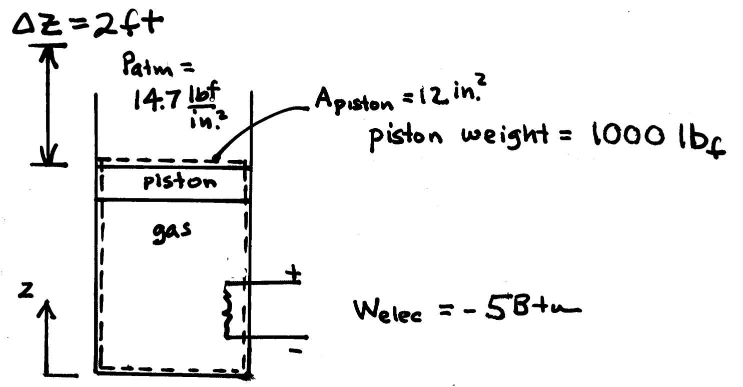
1. The signs for work and heat denote energy from the system by work to the system by heat, respectively.

PROBLEM 2.68*

KNOWN: An electrical resistor transfers energy to a gas contained in a vertical piston-cylinder assembly.

FIND: Determine the change in internal energy of the gas

SCHEMATIC & GIVEN DATA:



SMPTIONS: 1. The system is the piston plus the gas. 2. For the piston, =0, ΔKE=0. For the gas, ΔPE=0, ΔKE=0. 3. For the system, Q=0. Friction between piston and cylinder can be ignored. (5) The mass of e electrical resistor is negligible. So ΔU=0 for the resistor.

NALYSIS: An energy balance for the system reads

$$\cancel{\Delta KE}^{0} + \left[(\Delta PE)_{piston} + (\cancel{\Delta PE})^{0}_{gas}\right] + \left[(\cancel{\Delta U})^{0}_{piston} + (\Delta U)_{gas} + (\cancel{\Delta U})^{0}_{resistor}\right] = \cancel{Q}^{0} - W$$

or

$$(\Delta PE)_{piston} + (\Delta U)_{gas} = -W$$

For the piston

$$(\Delta PE)_{piston} = mg(z_2 - z_1)$$

For the system consisting of freely-moving piston (assumption 4) and gas, the net work is the sum of the electrical work and the work done at the top of the piston in displacing the surrounding atmosphere:

$$W = W_{elect} + \int_{z_1}^{z_2} (P_{atm} A_{piston})dz \Rightarrow W = W_{elect} + (P_{atm} A_{piston})(z_2 - z_1)$$

Collecting results

$$mg(z_2 - z_1) + (\Delta U)_{gas} = -\left[W_{elect} + (P_{atm} A_{piston})(z_2 - z_1)\right]$$

Thus $$(\Delta U)_{gas} = -W_{elec} - P_{atm} A_{piston}(z_2 - z_1) - mg(z_2 - z_1)$$

$$= -(-5\,Btu) - (14.7 \tfrac{lbf}{in^2})(12 in^2)(2ft)\left|\frac{1\,Btu}{778\,ft\,lbf}\right| - (1000 lbf)(2ft)\left|\frac{1\,Btu}{778\,ft\,lbf}\right|$$

$$= 1.98\,Btu \longleftarrow \underline{\Delta U}$$

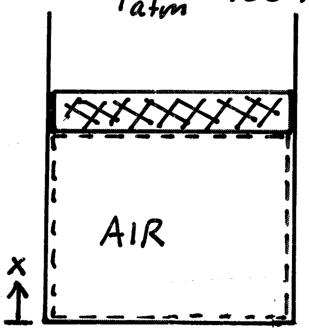
PROBLEM 2.69 *

KNOWN: A known quantity of air undergoes a process in a vertical piston-cylinder assembly. The initial and final volumes are given, and the change in specific internal energy is specified.

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:

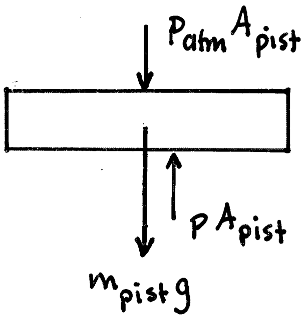
$p_{atm} = 100$ kPa



$m_{pist} = 50$ kg
$A_{pist} = 0.01\ m^2$
$m_{air} = 5g = 0.005$ kg
$V_1 = 5L = 0.005\ m^3$
$V_2 = 0.002\ m^3$
$\Delta u = -260$ kJ/kg

ASSUMPTIONS: (1) The air is a closed system. (2) Kinetic and potential energy effects are negligible for the air. (3) There is no friction between the piston and the cylinder wall. (4) The process occurs slowly with no acceleration of the piston. (5) The acceleration of gravity is constant, $g = 9.81\ m/s^2$.

ANALYSIS: For the piston, $\Sigma F_x = 0$. Thus, if p is the pressure exerted by the air



$$pA_{pist} = p_{atm}A_{pist} + m_{pist}g$$

$$p = p_{atm} + \frac{m_{pist}\,g}{A_{pist}}$$

$$= 100\ kPa + \frac{(50\ kg)(9.81\ m/s^2)}{(0.01\ m^2)}\left|\frac{1\ N}{1\ kg\cdot m/s}\right|\left|\frac{1\ kPa}{10^3\ N/m^2}\right|$$

$$= 149.05\ kPa$$

To find the work for the process, use Eq. 2.17. Noting that the pressure is constant

$$W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

$$= (149.05\ kPa)(0.002 - 0.005)\ m^3\left|\frac{10^3\ N/m^2}{1\ kPa}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= -0.447\ kJ$$

Now, the energy balance reduces to $\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + \Delta U = Q - W$. Thus, with $\Delta U = m_{air}\Delta u$

$$Q = m_{air}\Delta u + W = (0.005\ kg)(-260\ \tfrac{kJ}{kg}) + (-0.447\ kJ)$$

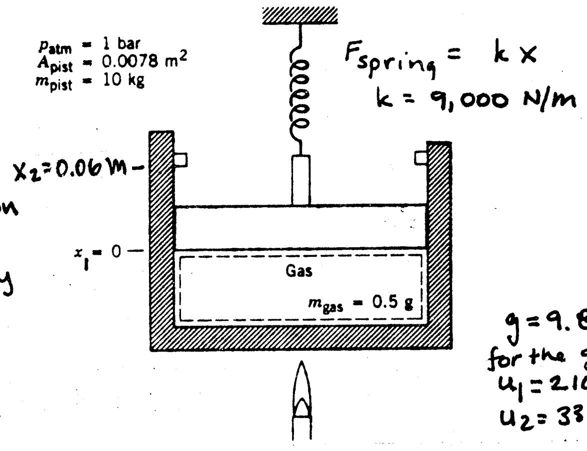
$$Q = -1.747 \longleftarrow \qquad Q$$

PROBLEM 2.70

KNOWN: A gas undergoes a process in a piston-cylinder assembly. The piston is constrained by a spring with a linear force-displacement relation.

FIND: Determine (a) the initial pressure of the gas, (b) the work done by the gas on the piston, and (c) the heat transfer.

SCHEMATIC & GIVEN DATA:



$g = 9.81\ m/s^2$

for the gas

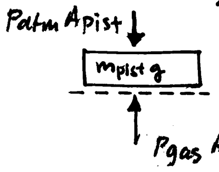
$u_1 = 210\ kJ/kg$

$u_2 = 335\ kJ/kg$

ASSUMPTIONS: (1) The gas is a closed system. (2) There is no friction between the piston and cylinder wall. (3) The process occurs slowly with no acceleration of the piston. (4) The acceleration of gravity is constant. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: (a) Initially, the spring exerts no force on the piston, which is at rest. Thus, with assumption 2 the force exerted by the gas on the bottom of the piston equals the piston weight plus the force of the atmosphere acting on the top of the piston. That is

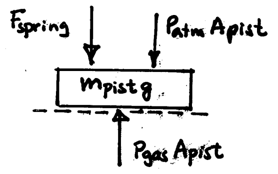
$$\Sigma F_x = 0 \Rightarrow \quad p_{gas} A_{pist} = (m_{pist}\, g) + p_{atm} A_{pist}$$



$$p_{gas} = p_{atm} + \frac{m_{pist} g}{A_{pist}} = 1\ bar \left|\frac{100\ kPa}{1\ bar}\right| + \frac{(10\ kg)(9.81\ m/s^2)}{(7.8\times 10^{-3}\ m^2)} \left|\frac{1\ N}{1\ kg\cdot m/s^2}\right| \left|\frac{1\ kPa}{10^3\ N/m^2}\right|$$

$$= 112.6\ kPa \quad \longleftarrow p_{gas}$$

(b) As the piston moves from $x = 0$ to $x = 0.06\ m$, the spring force acts. Then, with assumptions 2 and 3, $\Sigma F_x = 0$ reads



$$p_{gas} A_{pist} = (m_{pist}\, g) + p_{atm} A_{pist} + F_{spring}$$

$$= (m_{pist}\, g) + p_{atm} A_{pist} + kx$$

The work done by the gas on the piston is given by

$$W = \int_{x_1}^{x_2} (p_{gas} A_{pist})\, dx = \int_{x_1}^{x_2} [p_{atm} A_{pist} + m_{pist}\, g + kx]\, dx$$

$$= \left[[p_{atm} A_{pist} + m_{pist}\, g]\, x + \frac{kx^2}{2} \right]_0^{0.06}$$

$$= \left[\left(10^5 \frac{N}{m^2}\right)(7.8\times 10^{-3} m^2) + (10\ kg)\left(9.81 \frac{m}{s^2}\right) \left|\frac{1\ N}{1\ kg\cdot m/s^2}\right| \right] (0.06 m)$$

$$+ \left(9\times 10^3 \frac{N}{m}\right)\left(\frac{(0.06 m)^2}{2}\right)$$

$$= [780\ N + 98.1\ N](0.06\ m) + 16.2\ N\cdot m = 68.89\ N\cdot m$$

$$= 68.89\ N\cdot m \left|\frac{1\ J}{1\ N\cdot m}\right| = 68.89\ J \quad \longleftarrow W$$

(c) The energy balance for the system consisting of the gas reduces to $\Delta U = Q - W$. Then, with $\Delta U = m_{gas}(u_2 - u_1)$

$$Q = m_{gas}(u_2 - u_1) + W \Rightarrow \quad Q = (0.5\ g)(335 - 210)\frac{J}{g} + 68.89\ J = 131.39\ J \longleftarrow Q$$

PROBLEM 2.71*

KNOWN: A system undergoes a cycle consisting of four pocesses in series.

FIND: (a) Complete the table of energy values for the cycle, and (b) deteermine whether the cycle is a power cycle or a refrigeration cycle.

SCHEMATIC & GIVEN DATA:

Process	ΔU	Q	W
1–2	600	c	-600
2–3	a	d	-1300
3–4	-700	0	e
4–1	b	500	700

ASSUMPTIONS: (a) The system is a closed system. (b) Kinetic and potential energy effects can be neglected.

ANALYSIS: (a) Beginning with the process 4-1

b. $\Delta U = Q - W$; $\Delta U = 500 - 700 = -200 \Rightarrow b = -200$

Now, for the cycle, $\Sigma(\Delta U) = 0$. Thus

a. $a + 600 + (-700) + (-200) = 0 \Rightarrow a = 300$

$\Delta U = Q - W$

c. $600 = c - (-600) \Rightarrow c = 0$

d. $300 = d - (-1300) \Rightarrow d = -1000$

e. $-700 = 0 - e \Rightarrow e = 700$

(b) For the cycle

$$W_{cycle} = W_{12} + W_{23} + W_{34} + W_{41}$$
$$= (-600) + (-1300) + (700) + (700)$$
$$= -500$$

① Since $W_{cycle} < 0$, the cycle is a refrigeration cycle.

---

1. For the cycle, note that

$$Q_{cycle} = Q_{12} + Q_{23} + Q_{34} + Q_{41}$$
$$= (0) + (-1000) + (0) + (500) = -500$$

Thus $Q_{cycle} = W_{cycle}$ as expected.

PROBLEM 2.72*

KNOWN: A system undergoes a thermodynamic cycle consisting of four processes in series.

FIND: Complete the table of energy values provided for the cycle and determine whether the cycle is a power cycle or a refrigeration cycle.

SCHEMATIC + GIVEN DATA:

Process	ΔU	ΔKE	ΔPE	ΔE	Q	W
1–2	72	−5	−6	d	0	j
2–3	64	0	c	e	90	k
3–4	−97	b	0	f	h	92
4–1	a	0	3	g	i	0

ANALYSIS: For a cycle, the overall change in every property value is zero.

a. $\Sigma(\Delta U) = 0: \quad 72 + 64 - 97 + a = 0 \quad \Rightarrow a = -39$

b. $\Sigma(\Delta KE) = 0: \quad -5 + 0 + b + 0 = 0 \quad \Rightarrow b = 5$

c. $\Sigma(\Delta PE) = 0: \quad -6 + c + 0 + 3 = 0 \quad \Rightarrow c = 3$

$\Delta E = \Delta U + \Delta KE + \Delta PE$:

d. $d = 72 - 5 - 6 \quad \Rightarrow \quad d = 61$

e. $e = 64 + 0 + c \quad \Rightarrow \quad e = 67$

f. $f = -97 + b + 0 \quad \Rightarrow \quad f = -92$

g. $g = a + 0 + 3 \quad \Rightarrow \quad g = -36$

$\Delta E = Q - W$:

h. $f = h - 92 \quad \Rightarrow \quad h = 0$

i. $g = i - 0 \quad \Rightarrow \quad i = -36$

j. $d = 0 - j \quad \Rightarrow \quad j = -61$

k. $e = 90 - k \quad \Rightarrow \quad k = 23$

For the overall cycle, $W_{cycle} = Q_{cycle}$

$$\left.\begin{aligned} W_{cycle} &= -61 + 23 + 92 + 0 = 54 \\ Q_{cycle} &= 0 + 90 + 0 - 36 = 54 \end{aligned}\right\} \text{Checks the above calculations. Also, note that } \Sigma(\Delta E) = 0$$

Since $W_{cycle} > 0$, the cycle is a power cycle.

PROBLEM 2.73

KNOWN: A gas undergoes a thermodynamic cycle consisting of three processes.

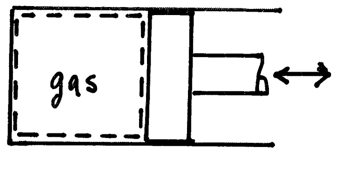
FIND: Determine the heat transfer and work for process 2-3, and whether the cycle is a power cycle or a refrigeration cycle.

SCHEMATIC & GIVEN DATA: The following data are given for each process:

Process 1-2: compression with $pV = \text{const.}$
$p_1 = 1$ bar, $V_1 = 1.6\ \text{m}^3$ to $V_2 = 0.2\text{m}^3$
$U_2 - U_1 = 0$

Process 2-3: Constant pressure to $V_3 = V_1$

Process 3-1: Constant volume, $U_1 - U_3 = -3549$ kJ



ASSUMPTIONS: (1) The gas is a closed system. (2) Neglect kinetic and potential energy changes. (3) The compression from state 1 to 2 is a polytropic process.

ANALYSIS: To find the work for Process 2-3, use Eq. 2.17, with constant pressure

$$W_{23} = \int_{V_2}^{V_3} p\,dV = p_2(V_3 - V_2)$$

Using the p-V relation for process 1-2; $p_2 = \left(\frac{V_1}{V_2}\right)p_1 = 8$ bars. Thus, with $V_3 = V_1$

$$W_{23} = (8\ \text{bars})(1.6 - 0.2)\text{m}^3 \left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = 1120\ \text{kJ} \leftarrow \underline{W_{23}}$$

The energy balance for process 2-3 reduces to

$$Q_{23} = (U_3 - U_2) + W_{23}$$

To get $U_3 - U_2$, note that for the cycle, $\Delta U]_{\text{cycle}} = 0$. Thus

$$(\overset{0}{\cancel{U_2 - U_1}}) + (U_3 - U_2) + (U_1 - U_3) = 0 \Rightarrow (U_3 - U_2) = -(U_1 - U_3) = 3549\ \text{kJ}$$

Finally, for process 2-3

$$Q_{23} = (3549\ \text{kJ}) + (1120\ \text{kJ}) = 4669\ \text{kJ} \leftarrow \underline{Q_{23}}$$

Next, for process 1-2, $\Delta U = 0$, so $Q_{12} = W_{12}$. Using Eq. 2.17

$$Q_{12} = W_{12} = \int_{V_1}^{V_2} p\,dV = p_1V_1 \ln\frac{V_2}{V_1} = (1)(1.6)\ln\left(\frac{0.2}{1.6}\right)\left|\frac{10^5}{10^3}\right| = -332.7\ \text{kJ}$$

And, for process 3-1: $W_{31} = 0$, and $Q_{31} = U_1 - U_3 = -3549$ kJ. Collecting various results

$$W_{\text{cycle}} = W_{12} + W_{23} + \overset{0}{\cancel{W_{31}}} = (-332.7) + (1120) = +787.3\ \text{kJ}$$

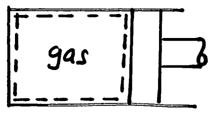
Since $W_{\text{cycle}} > 0$, the cycle is a power cycle.

PROBLEM 2.74

KNOWN: A gas undergoes a thermodynamic cycle consisting of three processes.

FIND: Sketch the cycle on a p-V diagram. Calculate W_{cycle}, Q_{23}, Q_{31}, and determine whether the cycle is a power or refrigeration cycle.

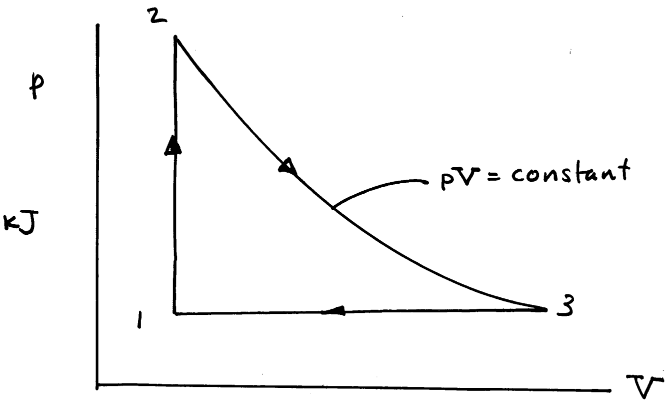
SCHEMATIC & GIVEN DATA:



1-2: $V = 0.028\,m^3$, $U_2 - U_1 = 26.4\,kJ$

2-3: $pV = \text{constant}$, $U_3 = U_2$

3-1: $p = 1.4\,bar$, $W_{31} = -10.5\,kJ$



ASSUMPTIONS: 1. The gas is the closed system. 2. For the system, $\Delta KE = \Delta PE = 0$. 3. Volume change is the only work mode.

ANALYSIS: (b) $W_{cycle} = W_{12} + W_{23} + W_{31}$. Since volume is constant and volume change is the only work mode, $W_{12} = 0$. To find W_{23}

$$W_{23} = \int_{V_2}^{V_3} p\,dV = \int_{V_2}^{V_3} \frac{c}{V}\,dV = c\ln\frac{V_3}{V_2} = p_3V_3\ln\frac{V_3}{V_2} \quad , V_2 = V_1$$

To evaluate V_3

$$W_{31} = \int_{V_3}^{V_1} p\,dV = p(V_1 - V_3) \Rightarrow V_3 = V_1 - W_{31}/p$$

or

$$V_3 = 0.028\,m^3 - \frac{(-10.5\,kJ)}{(1.4\,bar)}\left|\frac{1\,bar}{10^5\,N/m^2}\right|\left|\frac{10^3\,N\cdot m}{1\,kJ}\right| = 0.103\,m^3$$

Then

$$W_{23} = (1.4\,bar)\left|\frac{10^5\,N/m^2}{1\,bar}\right|(0.103\,m^3)\ln\left(\frac{0.103}{0.028}\right)\left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = 18.78\,kJ$$

Wcycle

Finally,

$$W_{cycle} = 0 + 18.78 + (-10.5) = 8.28\,kJ$$ The cycle is a power cycle.

(c) An energy balance for 2-3 gives Q_{23}: $\overset{0}{\cancel{\Delta U}} + \overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} = Q_{23} - W_{23} \Rightarrow Q_{23} = W_{23} \leftarrow Q_{23}$

(d) To evaluate Q_{31} begin with an energy balance together with assumption 2

$$U_1 - U_3 = Q_{31} - W_{31} \Rightarrow Q_{31} = (U_1 - U_3) + W_{31}.$$

Since there is no overall change in internal energy for the cycle, $\Sigma(\Delta U) = 0$

$$(U_2 - U_1) + (U_3 - U_2) + (U_1 - U_3) = 0 \Rightarrow$$

$$(U_1 - U_3) = -26.4 - (0) = -26.4\,kJ$$

Then

① $$Q_{31} = (-26.4) + (-10.5) = -36.9\,kJ \longleftarrow Q_{31}$$

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1. As a check, note that $Q_{cycle} = W_{cycle}$, $Q_{cycle} = Q_{12} + Q_{23} + Q_{31}$. An energy balance for 1-2 gives $Q_{12} = 26.4\,kJ$. So $Q_{cycle} = 26.4 + 18.78 + (-36.9) = 8.28\,kJ$, which checks the result of part (b).

PROBLEM 2.75

KNOWN: A closed system undergoes a cycle consisting of three processes.

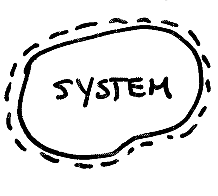
FIND: Sketch the cycle on a p-V diagram and calculate the net work for the cycle and the heat transfer for process 2-3.

SCHEMATIC & GIVEN DATA: The following data are given for each process:

Process 1-2: adiabatic compression with $pV^{1.4} = \text{const.}$ from $p_1 = 50 \text{ lbf/in}^2, V_1 = 3 \text{ ft}^3$ to $V_2 = 1 \text{ ft}^3$

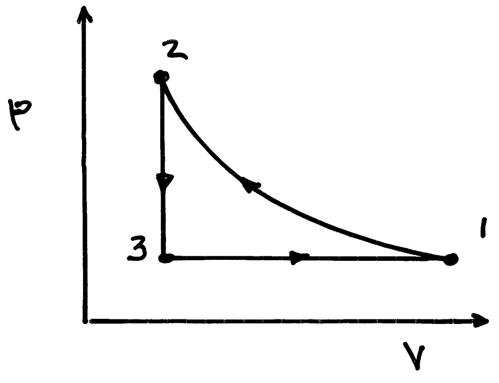
Process 2-3: constant volume

Process 3-1: Constant pressure, $U_1 - U_3 = 46.7$ Btu



ASSUMPTIONS: (1) The system is closed. (2) Kinetic and potential energy effects are negligible. (3) Process 1-2 is polytropic.

ANALYSIS: (a) Since process 1-2 is a polytropic compression, the p-V diagram for the cycle is



(b) Use Eq. 2.17 to evaluate the work for process 1-2

$$W_{12} = \int_{V_1}^{V_2} p\,dV = \text{const.} \int_{V_1}^{V_2} \frac{dV}{V^{1.4}} = p_1 V_1^{1.4} \frac{(V_2^{-.4} - V_1^{-.4})}{(-.4)}$$

$$= \left(50 \frac{\text{lbf}}{\text{in}^2}\right)(3 \text{ ft}^3)^{1.4} \left[\frac{(1 \text{ ft}^3)^{-.4} - (3 \text{ ft}^3)^{-.4}}{(-.4)}\right] \left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right| \left|\frac{1 \text{ Btu}}{778 \text{ ft-lbf}}\right|$$

$$= -38.3 \text{ Btu}$$

For process 2-3: $W_{23} = 0$

Finally, for process 3-1 use Eq. 2.17: $W_{31} = \int_{V_3}^{V_1} p\,dV = p_1 (V_1 - V_3)$

$$W_{31} = \left(50 \frac{\text{lbf}}{\text{in}^2}\right)(3-1) \text{ ft}^3 \left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right| \left|\frac{1 \text{ Btu}}{778 \text{ ft-lb}}\right| = +18.51 \text{ Btu}$$

Thus $W_{cycle} = W_{12} + W_{23} + W_{31} = -19.79$ Btu ◄ W_{cycle}

(c) For the overall cycle $Q_{cycle} = W_{cycle}$

$$\cancelto{0}{Q_{12}} + Q_{23} + Q_{31} = W_{cycle}$$

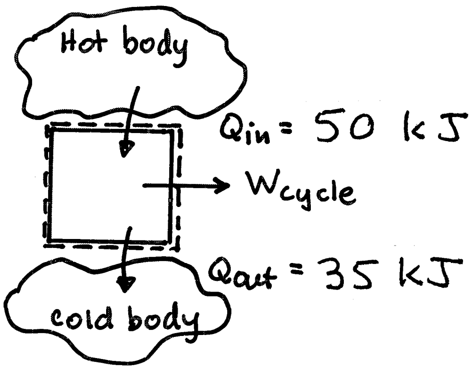
$$Q_{23} = W_{cycle} - Q_{31}$$

For process 3-1: $\Delta KE + \Delta PE + (U_1 - U_3) = Q_{31} - W_{31} \Rightarrow Q_{31} = U_1 - U_3 + W_{31}$

$$Q_{31} = 46.7 + 18.51 = +65.21 \text{ Btu}$$

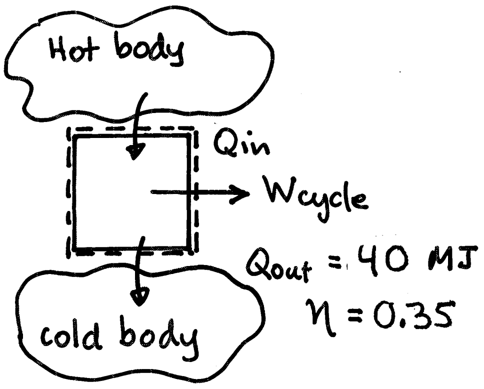
Finally $Q_{23} = W_{cycle} - Q_{31} = -19.79 - 65.21 = -85$ Btu ◄ Q_{23}

PROBLEM 2.76*



$$W_{cycle} = Q_{in} - Q_{out}$$
$$= 50 - 35 = 15 \text{ kJ} \longleftarrow \underline{W_{cycle}}$$
$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{15}{50} = 0.3 \ (30\%) \longleftarrow \underline{\eta}$$

PROBLEM 2.77*



$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

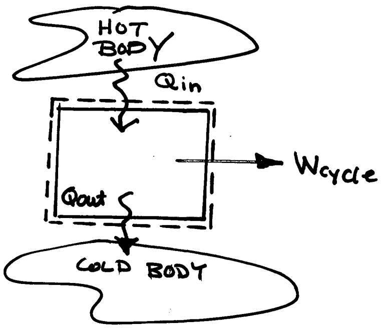
Solving for Q_{in}

$$Q_{in} = \frac{Q_{out}}{1-\eta} = \frac{40}{(1-.35)}$$
$$= 61.54 \text{ MJ} \longleftarrow \underline{Q_{in}}$$

Thus

$$W_{cycle} = \eta Q_{in} = (.35)(61.54)$$
$$= 21.54 \text{ MJ} \longleftarrow \underline{W_{cycle}}$$

PROBLEM 2.78*



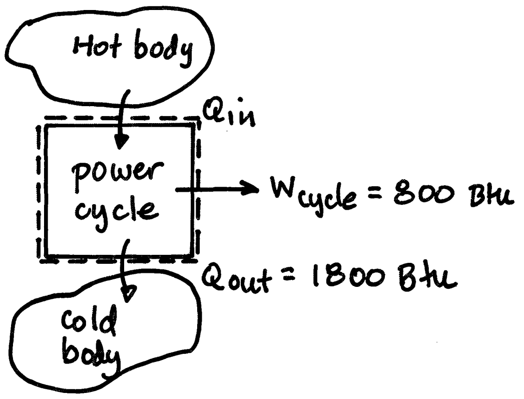
$W_{cycle} = 4000$ Btu

$Q_{out} = 8000$ Btu

Energy Balance: $W_{cycle} = Q_{cycle}$

$$= Q_{in} - Q_{out}$$
$$\Rightarrow \quad Q_{in} = W_{cycle} + Q_{out}$$
$$= 4000 + 8000$$
$$= 12{,}000 \text{ Btu}$$
$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{4000 \text{ Btu}}{12{,}000 \text{ Btu}} = 0.33 \ (33\%) \longleftarrow \underline{\eta}$$

PROBLEM 2.79



Energy balance: $W_{cycle} = Q_{in} - Q_{out}$

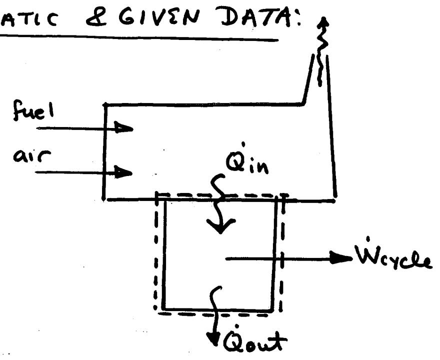
$$Q_{in} = W_{cycle} + Q_{out}$$
$$= 800 + 1800 = 2600 \text{ Btu}$$
$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{800}{2600}$$
$$= 0.308 \ (30.8\%) \longleftarrow \eta$$

PROBLEM 2.80

KNOWN: Operating data are provided for a power cycle.

FIND: Determine the net rate power is developed, the net work output annually, and the value of the net work, in \$/year.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{in} = 300$ MW

$\eta = 33.3\%$

ASSUMPTIONS: 1. The system undergoes a power cycle. 2. The cycle operates steadily for 8000 h annually. 3. The value of the net work is \$0.08/kW·h

ANALYSIS: (a) To determine $\dot{W}_{cycle}$

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} \Rightarrow \dot{W}_{cycle} = \eta \dot{Q}_{in} = (0.333)(300\text{ MW}) = 99.9 \text{ MW} \longleftarrow \eta$$

(b) With assumption 2

$$W_{cycle} = (99.9 \text{ MW}) \left| \frac{10^3 \text{ kW}}{1 \text{ MW}} \right| \left(\frac{8000 \text{ h}}{\text{year}} \right) = 799.2 \times 10^6 \ \frac{\text{kW·h}}{\text{year}} \longleftarrow W_{cycle}$$

(c) With assumption 3

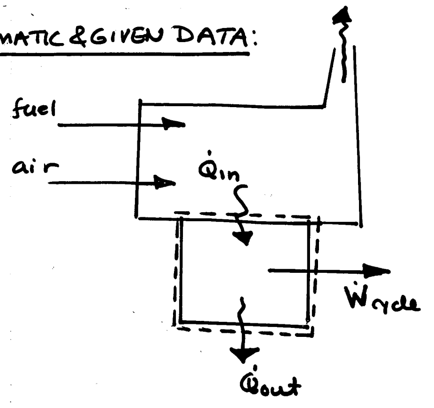
$$\dot{\$} = \left(799.2 \times 10^6 \ \frac{\text{kW·h}}{\text{year}}\right)\left(\frac{\$0.08}{\text{kW·h}}\right) = \$63.94 \times 10^6/\text{year} \longleftarrow \dot{\$}$$

PROBLEM 2.81

KNOWN: Operating data are provided for a power cycle.

FIND: Determine, in \$/year, the value of the power generated and the cost of $\dot{Q}_{in}$.

SCHEMATIC & GIVEN DATA:



$\eta = 35\%$

$\dot{W}_{cycle} = 100\,\text{MW}$

ASSUMPTIONS: 1. The system undergoes a thermodynamic cycle. 2 The power cycle operates steadily for 8000 h per year. 3. Electricity is valued at \$0.08/kW·h and $\dot{Q}_{in}$ at \$4.50/GJ

ANALYSIS: (a) Using assumptions 2,3

$$\dot{\$}_{electricity} = (100\ \text{MW})\left|\frac{10^3\ \text{kW}}{1\ \text{MW}}\right|\left(\frac{8000\ \text{h}}{\text{year}}\right)\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right) = \$64\times10^6/\text{year} \longleftarrow \dot{\$}_{electricity}$$

(b) With

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} \Rightarrow \dot{Q}_{in} = \frac{\dot{W}_{cycle}}{\eta} = \frac{100\ \text{MW}}{0.35} = 285.7\ \text{MW}$$

① $$\dot{\$}_{Q_{in}} = (285.7\ \text{MW})\left|\frac{1\ \text{GJ/s}}{10^3\ \text{MW}}\right|\left|\frac{3600\ \text{s}}{\text{h}}\right|\left(\frac{8000\ \text{h}}{\text{year}}\right)\left(\frac{\$4.50}{\text{GJ}}\right) = \$37\times10^6/\text{year} \longleftarrow \dot{\$}_{Q_{in}}$$

1. This is an operating cost. Additional operating costs include the cost for the plant, a capital cost.

PROBLEM 2.82

(a) Window air conditioner.

cold body : environment inside the building served
hot body : outside environment

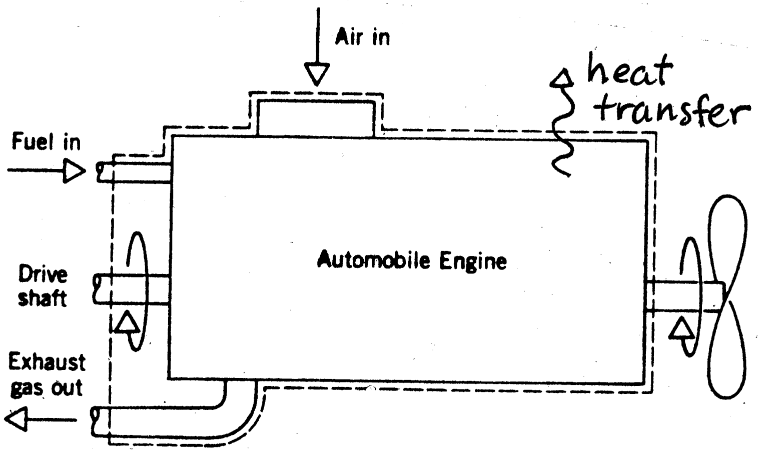
(b) Nuclear power plant.

hot body : coolant circulated through the reactor core
cold body : sea water

(c) Ground source heat pump.

hot body : environment inside the building served
cold body : ground

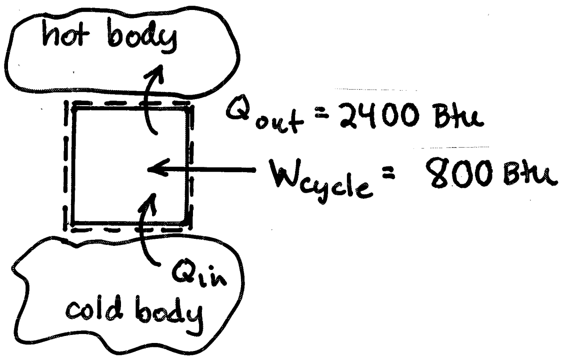
PROBLEM 2.83



ASSUMPTION: Engine operates at steady state.

An engine operating at steady state has continuous flows of fuel and air in as well as exhaust gas out. It also develops a steady power output. Although the operation of an engine does not conform to the strict definition of a thermodynamic cycle, there are similarities between an engine and a power cycle as illustrated schematically in Fig. 2.15(a). Specifically, we can think of the hot gases formed during combustion as playing the role of the hot body, and the surroundings, which interact with the engine through heat transfer and the discharge of hot exhaust gas, can be viewed as the cold body.

PROBLEM 2.84 *



$$\beta = \frac{Q_{in}}{W_{cycle}}$$

$$W_{cycle} = Q_{out} - Q_{in}$$

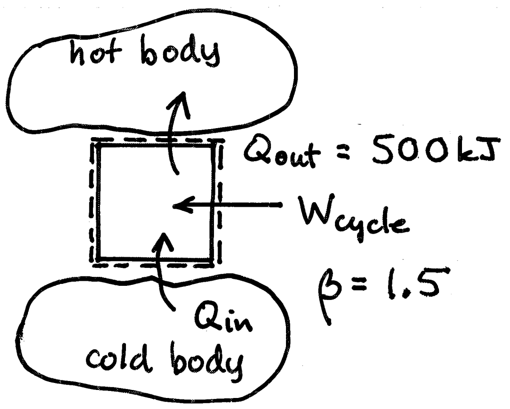
$$Q_{in} = Q_{out} - W_{cycle}$$

$$= 2400 - 800$$

$$= 1600 \text{ Btu}$$

$$\beta = \frac{1600}{800} = 2.0 \longleftarrow \beta$$

PROBLEM 2.85*



$$\beta = \frac{Q_{in}}{W_{cycle}} = \frac{Q_{in}}{Q_{out} - Q_{in}}$$

Solving for Q_{in}

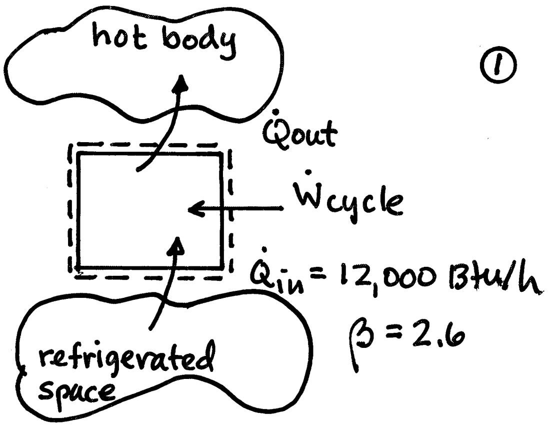
$$Q_{in} = Q_{out}\left(\frac{\beta}{1+\beta}\right)$$

$$= (500)\left(\frac{1.5}{1+1.5}\right) = 300\text{ kJ} \xleftarrow{\quad Q_{in} \quad}$$

$$W_{cycle} = \frac{Q_{in}}{\beta} = \frac{300}{1.5}$$

$$= 200\text{ kJ} \xleftarrow{\quad W_{cycle} \quad}$$

PROBLEM 2.86



① Since the cycle operates continuously

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}}$$

$$\dot{W}_{cycle} = \dot{Q}_{in}/\beta$$

$$= 12{,}000/2.6$$

$$= 4615\text{ Btu/h} \xleftarrow[\text{(Btu/h)}]{\quad \dot{W}_{cycle} \quad}$$

$$\dot{W}_{cycle} = 4615\text{ Btu/h}\left|\frac{1\text{ hp}}{2545\text{ Btu/h}}\right|$$

$$= 1{,}813\text{ hp} \xleftarrow[\text{(hp)}]{\quad \dot{W}_{cycle} \quad}$$

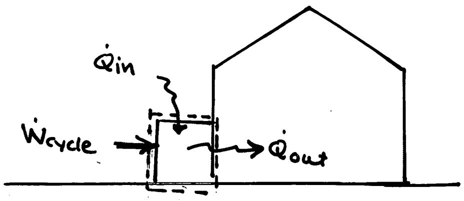
---

1. Cycles operating continuously are commonly used for space cooling.

PROBLEM 2.87

KNOWN: Operating data are provided for a heat pump.

FIND: Determine the net power required to operate the heat pump and its monthly cost.



$\gamma = 2.5$

$\dot{Q}_{out} = 20\,kW$

ASSUMPTIONS: 1. The system undergoes a heat pump cycle. 2. The cycle operates steadily for 200 h monthly. 3. Electricity is valued at \$0.08/kW·h.

ANALYSIS: (a) The coefficient of performance for the heat pump is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = \frac{\dot{Q}_{out}}{\gamma} = \frac{20\,kW}{2.5} = 8\,kW \longleftarrow \gamma$$

(b) With assumptions 2, 3

$$\dot{\$} = (8\,kW)\left(\frac{200\,h}{month}\right)\left(\frac{\$0.08}{kW\cdot h}\right) = \$128/month \longleftarrow \dot{\$}$$

PROBLEM 2.88

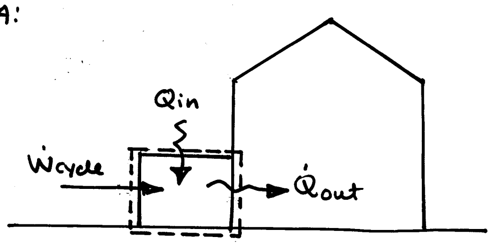
KNOWN: Operating data are provided for a heat pump.

FIND: Determine the coefficient of performance and the monthly cost to operate the heat pump.

SCHEMATIC & GIVEN DATA:

$\dot{W}_{cycle} = 7.8\,hp$

$\dot{Q}_{out} = 60{,}000\,Btu/h$



ASSUMPTIONS: 1. The system undergoes a heat pump cycle. 2. The cycle operates steadily for 200 h monthly. 3. Electricity is valued at \$0.08/kW·h

ANALYSIS: (a) The coefficient of performance for a heat pump is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_{cycle}} = \frac{60{,}000\,Btu/h}{(7.8\,hp)\left|\frac{2545\,Btu/h}{hp}\right|} = 3.02 \longleftarrow \gamma$$

(b) Using assumptions 2, 3

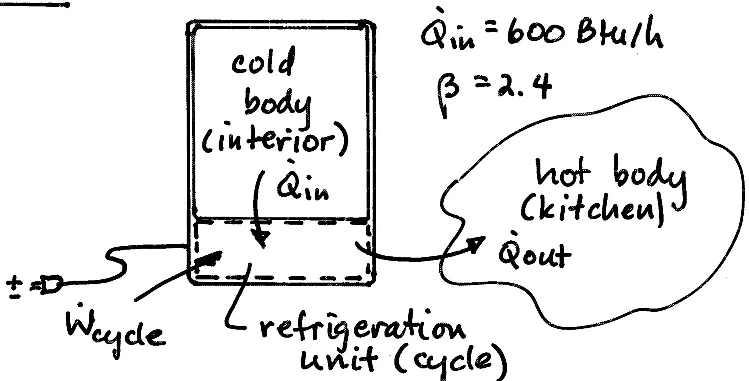
$$\dot{\$} = (7.8\,hp)\left|\frac{1\,kW}{1.341\,hp}\right|\left(\frac{200\,h}{month}\right)\left(\frac{\$0.08}{kW\cdot h}\right) = \$\,93/month \longleftarrow \dot{\$}$$

PROBLEM 2.89

KNOWN: Operating data are given for a household refrigerator.

Find: Determine the cost of operating the refrigerator for 360 h per month.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{in} = 600$ Btu/h

$\beta = 2.4$

ASSUMPTIONS: (1) The refrigeration unit operates as a refrigeration cycle. (2) The cycle operates steadily for 360 h monthly. (3) Electricity is valued at \$0.08/kW·h.

ANALYSIS: To determine the electric power input, begin with the expression for coefficient of performance: $\beta = \dot{Q}_{in}/\dot{W}_{cycle}$. Solving for the power

$$\dot{W}_{cycle} = \dot{Q}_{in}/\beta$$

$$= \left(\frac{600 \text{ Btu/h}}{2.4}\right)\left|\frac{1 \text{ kW}}{3413 \text{ Btu/h}}\right| = 0.0732 \text{ kW}$$

The monthly cost rate, $\dot{\$}$, is

$$\dot{\$} = (0.0732 \text{ kW})(360 \text{ h/month})(\$0.08/\text{kW}\cdot\text{h})$$

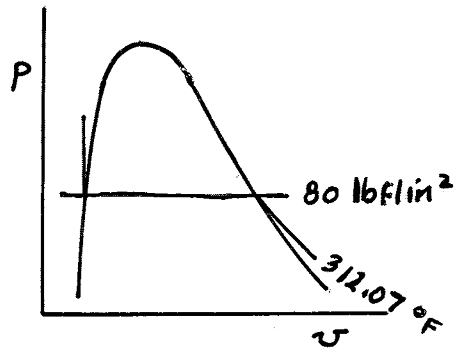
① $$= \$2.11/\text{month} \longleftarrow \dot{\$}$$

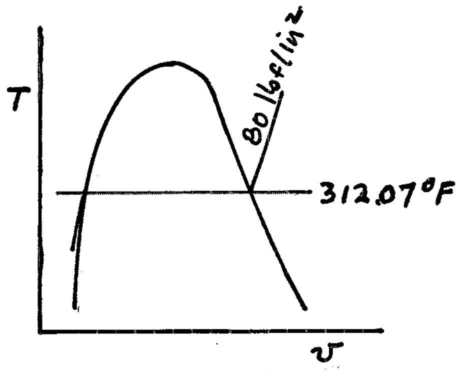
---

1. The actual electric cost to operate a refrigerator is likely to deviate significantly from this value due to such factors as the electric rate, door openings, inserting room-temperature foods, etc.

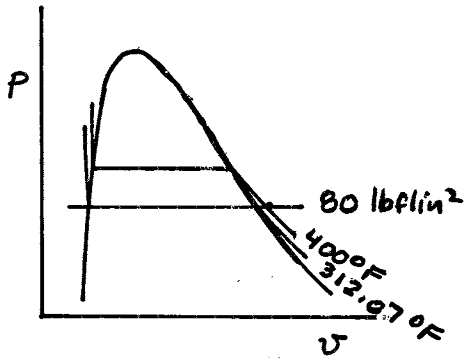
CHAPTER THREE
EVALUATING PROPERTIES

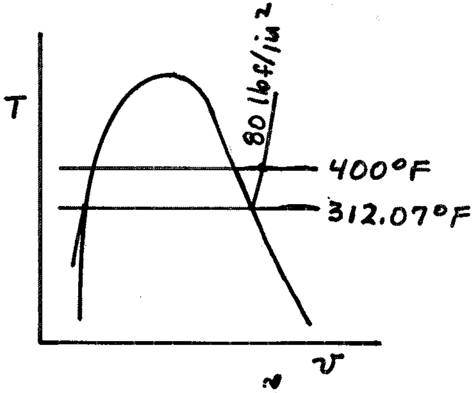
PROBLEM 3.1



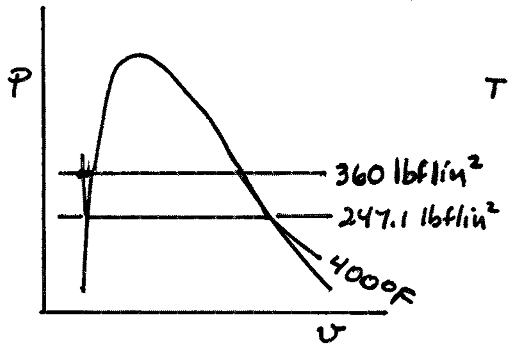


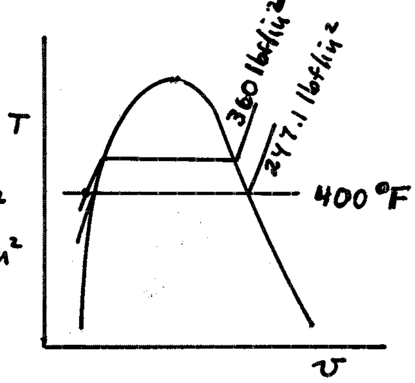
p = 80 lbf/in²
T = 312.07°F
two-phase liquid-vapor mixture



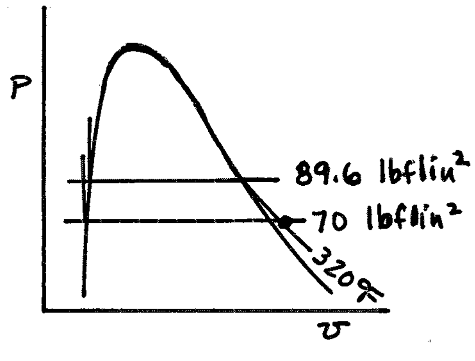


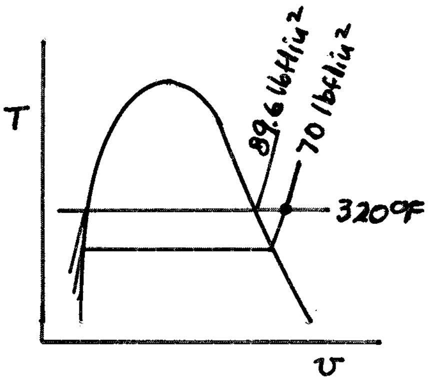
p = 80 lbf/in²
T = 400°F
superheated vapor



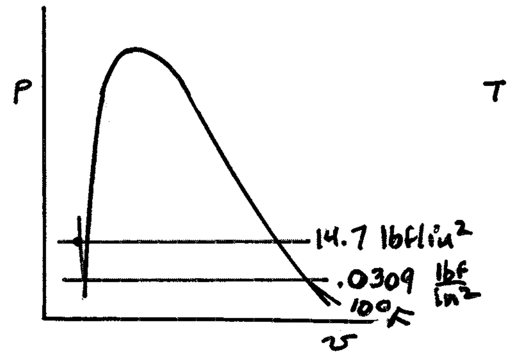


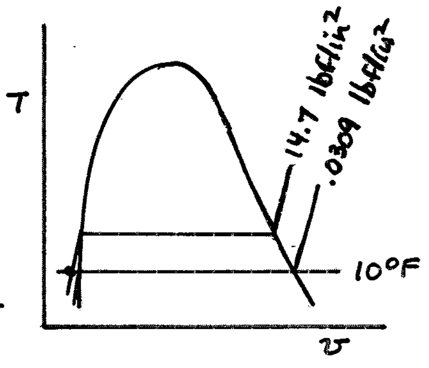
T = 400°F
p = 360 lbf/in²
subcooled (compressed) liquid





T = 320°F
p = 70 lbf/in²
superheated vapor

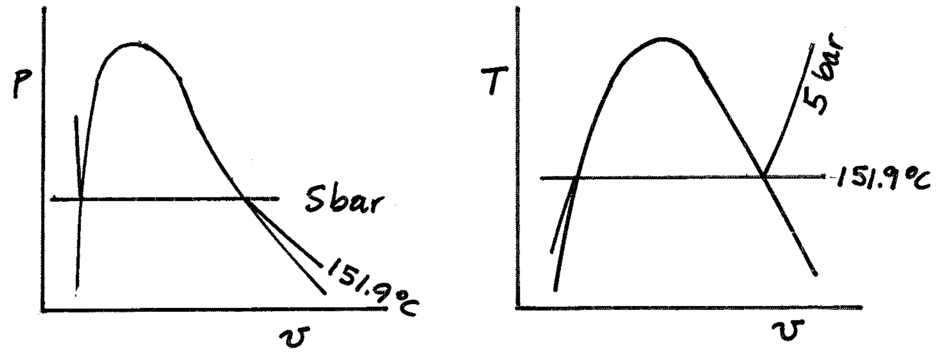




T = 10°F
p = 14.7 lbf/in²
solid

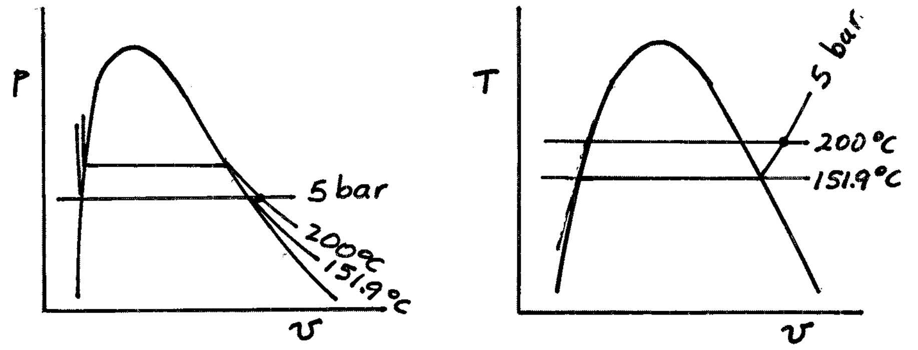
PROBLEM 3.2

(a)



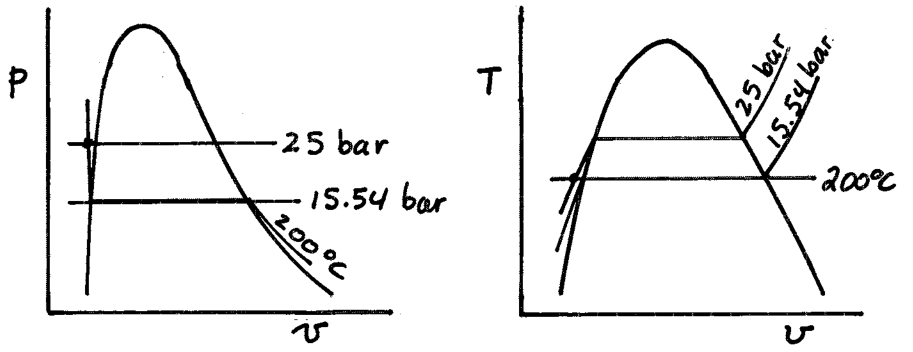
$p = 5$ bar
$T = 151.9$ °c
two-phase liquid-vapor mixture

(b)



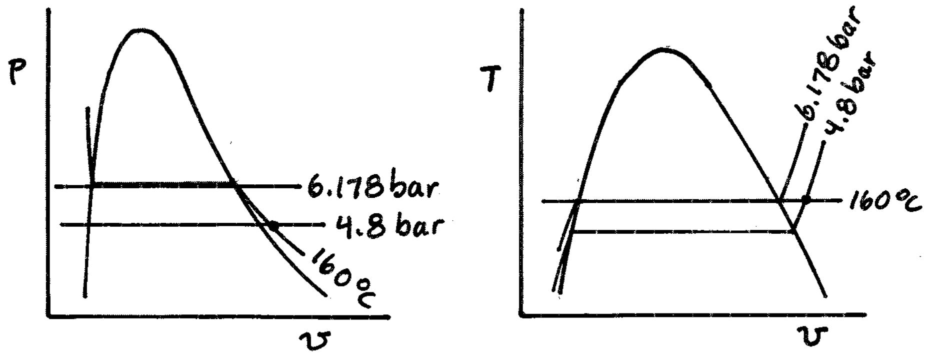
$p = 5$ bar
$T = 200$ °c
superheated vapor

(c)



$p = 2.5$ MPa $= 25$ bar
$T = 200$ °c
subcooled (compresed) liquid

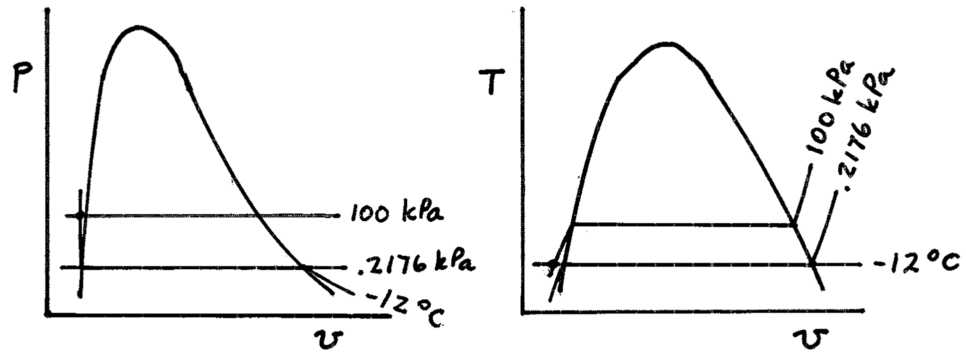
(d)



$T = 160$ °c
$p = 4.8$ bar

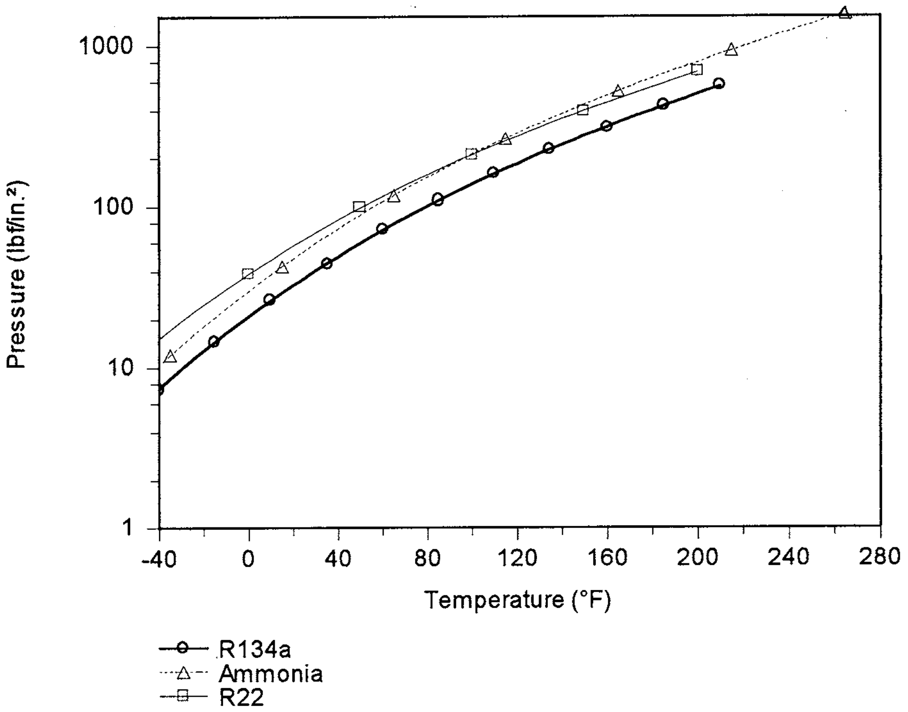
superheated vapor

(e)

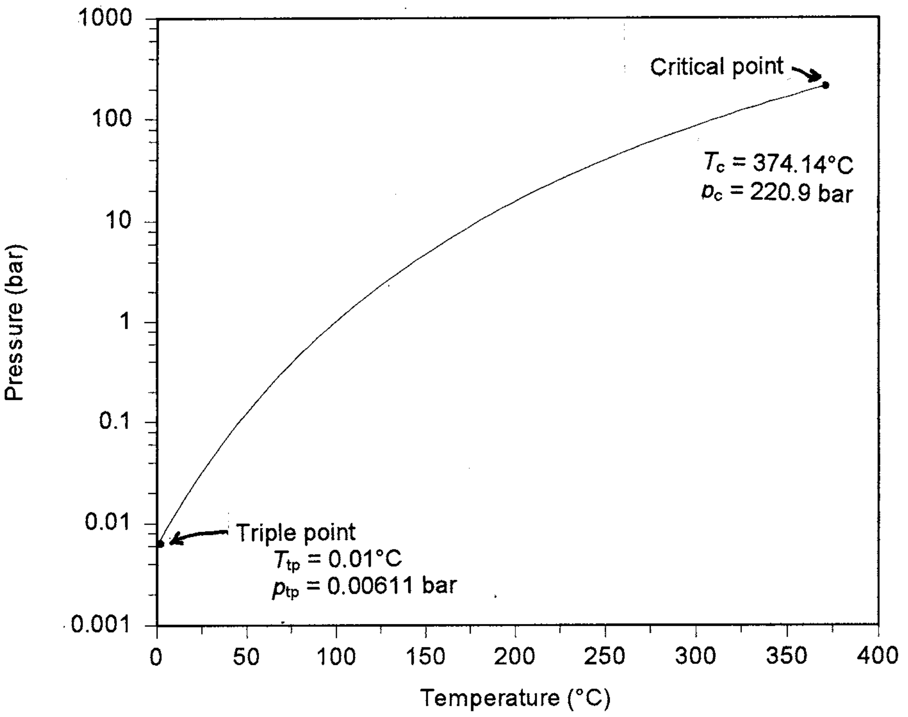


$T = -12$ °c
$p = 1$ bar $= 100$ kPa
solid

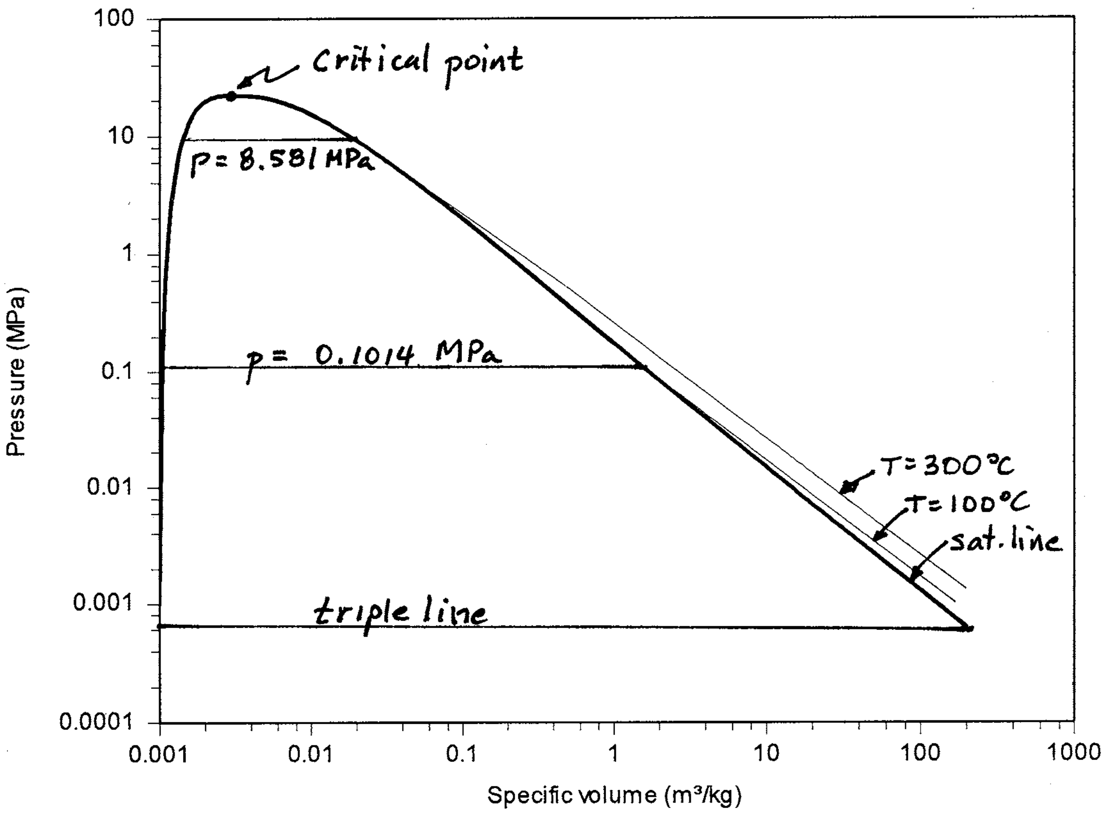
PROBLEM 3.3



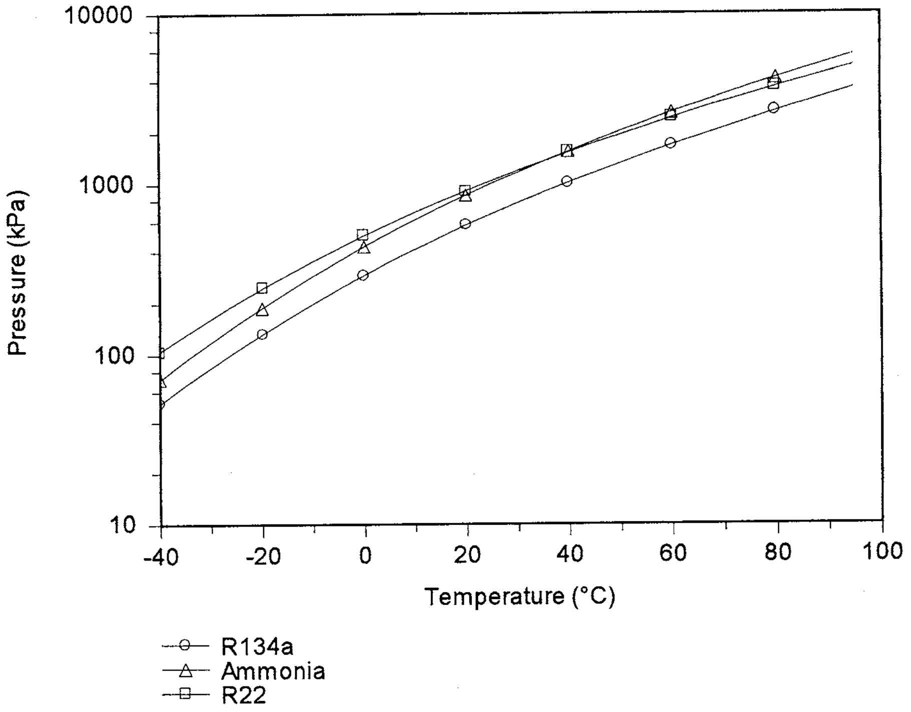
PROBLEM 3.4



PROBLEM 3.5



PROBLEM 3.6



PROBLEM 3.7*

(a) H_2O ; $T = 20°C$, $v = 20\ m^3/kg$

Table A-2: $v_f = 1.0018 \times 10^{-3}\ m^3/kg$, $v_g = 57.791\ m^3/kg$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{20 - 1.0018 \times 10^{-3}}{57.791 - 1.0018 \times 10^{-3}} = 0.346\ (34.6\%) \longleftarrow x$$

(b) Propane ; $p = 15.00$ bar, $v = 0.02997\ m^3/kg$

Table A-17: $v_g = 0.02997\ m^3/kg$

$$\Rightarrow x = 1\ \ (100\%) \longleftarrow x$$

(c) Refrigerant 134a ; $T = 60°C$, $v = 0.001\ m^3/kg$

Table A-10: $v_f = 0.9488 \times 10^{-3}\ m^3/kg$, $v_g = 0.0114\ m^3/kg$

$$x = \frac{0.001 - 0.9488 \times 10^{-3}}{0.0114 - 0.9488 \times 10^{-3}} = 0.0049\ (0.49\%) \longleftarrow x$$

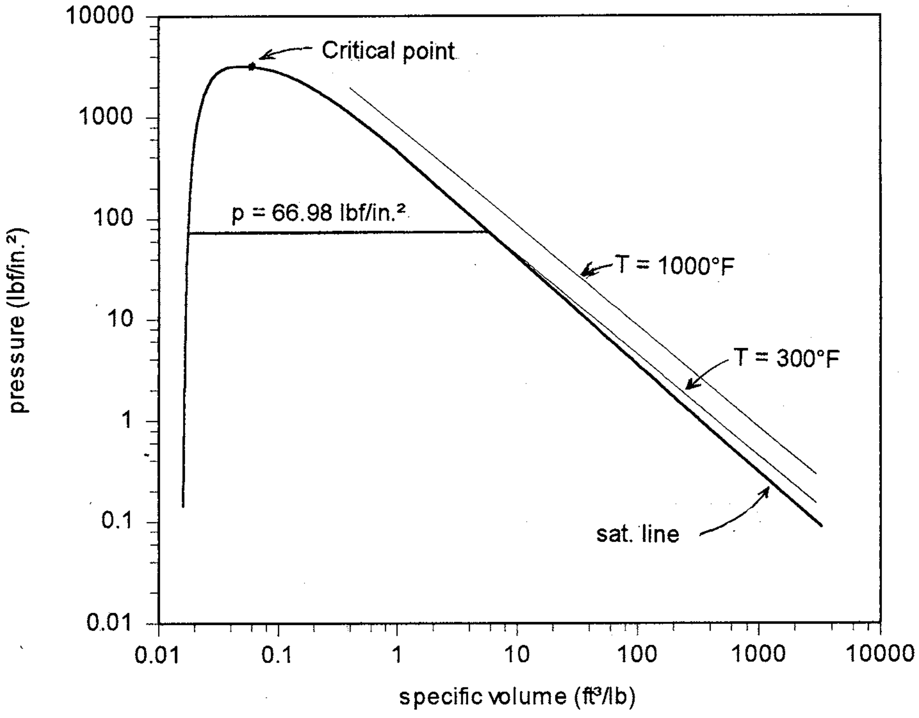
(d) Ammonia ; $p = 1\ MPa = 10$ bar, $v = 0.1\ m^3/kg$

Table A-14: $v_f = 1.6584 \times 10^{-3}\ m^3/kg$, $v_g = 0.1285\ m^3/kg$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.1 - 1.6584 \times 10^{-3}}{0.1285 - 1.6584 \times 10^{-3}}$$

$$= 0.775\ (77.5\%) \longleftarrow x$$

PROBLEM 3.8



PROBLEM 3.9*

(a) H_2O; $p = 14.696\ \text{lb}_f/\text{in}^2$, $v = 25\ \text{ft}^3/\text{lb}$

Table A-3E: $v_f = 0.01672\ \text{ft}^3/\text{lb}$, $v_g = 26.80\ \text{ft}^3/\text{lb}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{25 - 0.01672}{26.80 - 0.01672} = 0.933\ (93.3\%) \longleftarrow x$$

(b) Propane; $T = -80\,°\text{F}$, $v = 0.02653\ \text{ft}^3/\text{lb}$

Table A-16E: $v = v_f = 0.02653\ \text{ft}^3/\text{lb}$

saturated liquid $\Rightarrow x = 0\ (0\%) \longleftarrow x$

(c) Refrigerant 134a; $p = 50\ \text{lb}_f/\text{in}^2$, $v = 0.5\ \text{ft}^3/\text{lb}$

Table A-11E: $v_f = 0.01252\ \text{ft}^3/\text{lb}$, $v_g = 0.9422\ \text{ft}^3/\text{lb}$

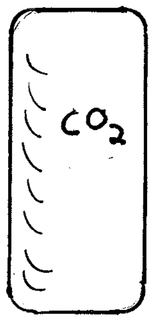
$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.5 - 0.01252}{0.9422 - 0.01252} = 0.524\ (52.4\%) \longleftarrow x$$

(d) Ammonia; $T = -40\,°\text{F}$, $v = 20\ \text{ft}^3/\text{lb}$

Table A-13E: $v_f = 0.02322\ \text{ft}^3/\text{lb}$, $v_g = 24.8672\ \text{ft}^3/\text{lb}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{20 - 0.02322}{24.8672 - 0.02322} = 0.804\ (80.4\%) \longleftarrow x$$

PROBLEM 3.10*



$T = -40\text{K}$
$m = 2\ \text{kg}$
$V = 0.05\ \text{m}^3$
$v_f = 0.896 \times 10^{-3}\ \frac{\text{m}^3}{\text{kg}}$
$v_g = 3.824 \times 10^{-2}\ \frac{\text{m}^3}{\text{kg}}$

First, find the specific volume

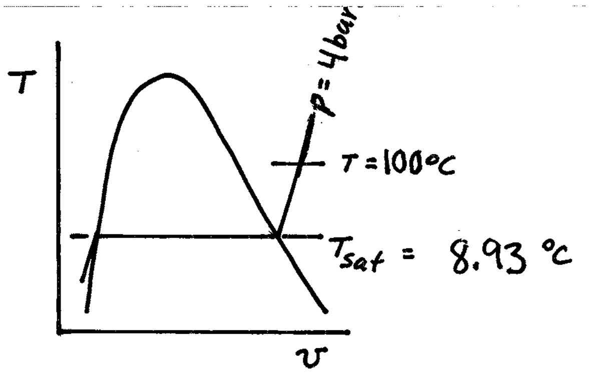
$$v = \frac{V}{m} = \frac{0.05\ \text{m}^3}{2\ \text{kg}} = 0.025\ \text{m}^3/\text{kg}$$

Now, the quality is

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.025 - 0.896 \times 10^{-3}}{3.824 \times 10^{-2} - 0.896 \times 10^{-3}}$$

$$= 0.645\ (64.5\%) \longleftarrow x$$

PROBLEM 3.11*



$p = 4\ \text{bar}$, $T = 100°\text{C}$

Table A-12: superheated vapor

$v = 0.07327\ \text{m}^3/\text{kg}$

$$m = \frac{V}{v} = \frac{(0.1\ \text{m}^3)}{(0.07327\ \text{m}^3/\text{kg})}$$

$$= 1.36\ \text{kg} \longleftarrow m$$

PROBLEM 3.12*

$v_1 = v_2 = 1.0 \text{ ft}^3/\text{lb}$

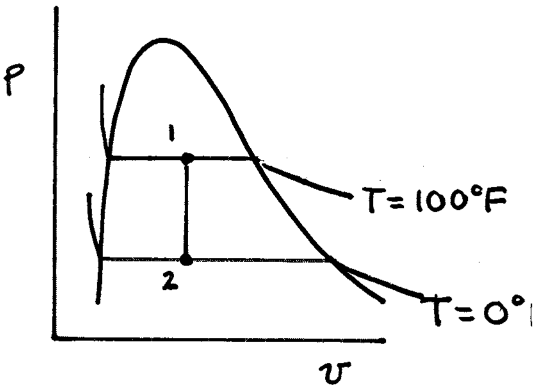


Table A-13E: $v_{f2} = 0.02419 \text{ ft}^3/\text{lb}, v_{g2} = 9.1100 \text{ ft}^3/\text{lb}$

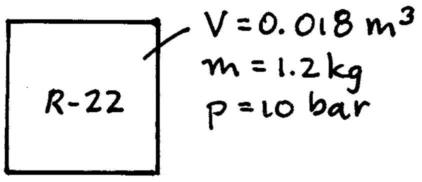
$v_2 = 1.0 \text{ ft}^3/\text{lb} \Rightarrow$

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}}$$

$$= \frac{1.0 - 0.02419}{9.1100 - 0.02419}$$

$$= 0.107 \ (10.7\%) \longleftarrow x_2$$

PROBLEM 3.13



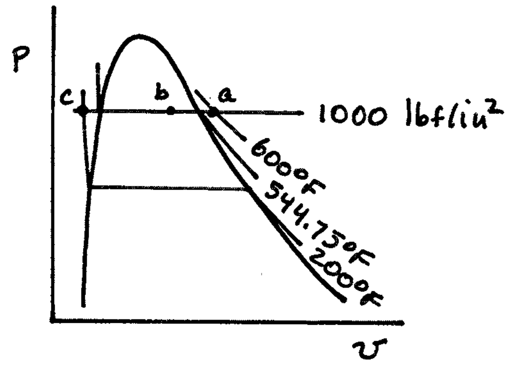
$$v = \frac{V}{m} = \frac{0.018 \text{ m}^3}{1.2 \text{ kg}}$$

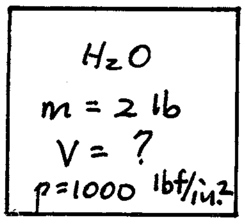
$$= 0.015 \text{ m}^3/\text{kg}$$

Table A-7: $v_f < v < v_g \Rightarrow$ two-phase

at 10 bar ; $T_{sat} = 23.40°C \longleftarrow T$

PROBLEM 3.14





(a) $T = 600°F \Rightarrow$ superheated vapor

Table A-4E: $v = 0.514 \text{ ft}^3/\text{lb}$

$V = mv = (2 \text{ lb})(0.514 \text{ ft}^3/\text{lb})$

$= 1.028 \text{ ft}^3 \longleftarrow V$

(b) $x = 0.8 \Rightarrow$ two-phase liquid-vapor mixture

Table A-3E: $v_f = 0.02159 \text{ ft}^3/\text{lb}$

$v_g = 0.446 \text{ ft}^3/\text{lb}$

$v = v_f + x(v_g - v_f) = 0.3612 \text{ ft}^3/\text{lb}$

$V = (2 \text{ lb})(0.3612 \text{ ft}^3/\text{lb})$

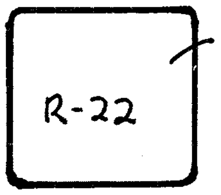
$= 0.7224 \text{ ft}^3 \longleftarrow V$

(c) $T = 200°F \Rightarrow$ subcooled (compressed) liquid

Table A-5E: $v = 0.016580 \text{ ft}^3/\text{lb}$

$V = (2)(0.016580) = 0.0332 \text{ ft}^3 \longleftarrow V$

PROBLEM 3.15*

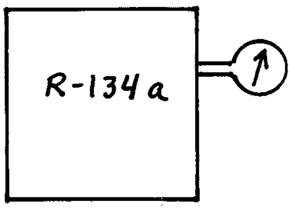


$V = 1\ \text{m}^3$
$p = 1\ \text{bar}$
$x = 0.75$

Using data from Table A-8 :

$$v = v_f + x(v_g - v_f)$$
$$= 0.7093\times10^{-3} + (0.75)(0.2152 - 0.7093\times10^{-3})$$
$$= 0.1616\ \text{m}^3/\text{kg}$$
$$m = \frac{V}{v} = \frac{(1\ \text{m}^3)}{(0.1616\ \text{m}^3/\text{kg})}$$
$$m = 6.189\ \text{kg} \longleftarrow \underline{m}$$

PROBLEM 3.16



$V = 2\ \text{ft}^3$
$m = 5\ \text{lb}$
$p_{gage} = 71.39\ \text{lbf/in}^2$
$p_{atm} = 14.4\ \text{lbf/in}^2$
$T = ?$

First determine the absolute pressure p

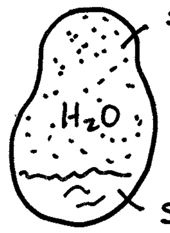
$$p = p_{gage} + p_{atm}$$
$$= 71.39 + 14.4 = 85.79\ \text{lbf/in}^2$$

Now, $v = V/m = (2\ \text{ft}^3)/(5\ \text{lb})$
$= 0.4\ \text{ft}^3/\text{lb}$

From Table A-10E at $p \approx 85.79$
$v_f < v < v_g \Rightarrow$ two-phase liquid-vapor mixture

Thus, $T = T_{sat} \approx 70\,°\text{F} \longleftarrow \underline{T}$

PROBLEM 3.17 *

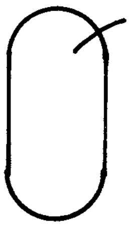


sat. vapor, $m_g = 4.2\ \text{kg}$
$p = 150\ \text{bar}$
$V = 0.2\ \text{m}^3$
sat. liquid, $m_f = 3.8\ \text{kg}$

For the two-phase liquid-vapor mixture, the specific volume is

$$v = \frac{V}{m_f + m_g}$$
$$= \frac{0.2}{(3.8 + 4.2)}$$
$$= 0.025\ \text{m}^3/\text{kg} \longleftarrow \underline{v}$$

PROBLEM 3.18



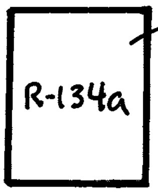
Ammonia
$V = 0.21\ \text{m}^3$
$T = 20°\text{C}$
saturated liquid

Using data from Table A-13 :

$$v = v_{f@20°C} = 1.6386\times10^{-3}\ \text{m}^3/\text{kg}$$
$$m = \frac{V}{v} = \frac{0.21}{1.6386\times10^{-3}} = 128.2\ \text{kg} \longleftarrow \underline{m}$$

at $T = 20°\text{C}$; $p_{sat} = 8.5762\ \text{bar}\left(\frac{100\ \text{kPa}}{1\ \text{bar}}\right)$
$= 857.62\ \text{kPa} \longleftarrow \underline{p}$

PROBLEM 3.19



$V = 0.006\ m^3$
$p = 180\ kPa$
$= 1.8\ bar$
$0 \le x \le 1$

First, using data from Table A-11 at $p = 1.8$ bar, the specific volume varies with x as follows:

$$v = v_f + x(v_g - v_f)$$
$$= 0.7485\times10^{-3} + x\,(0.1098 - 0.7485\times10^{-3})$$

Thus
$$m = \frac{V}{v} = \frac{0.006}{0.7485\times10^{-3} + x\,(0.10905)} \longleftarrow m$$

Now
$$m_g = x\,m \;;\; m_f = m - m_g$$
$$\frac{V_g}{V} = \frac{v_g\,m_g}{V} \;;\; \frac{V_f}{V} = 1 - \frac{V_g}{V} \longleftarrow \text{Volume fractions}$$

Sample calculation: $x = 0.9$; $m = 0.0607$ kg, $V_g/V = 0.9993$, $V_f/V = 0.0007$

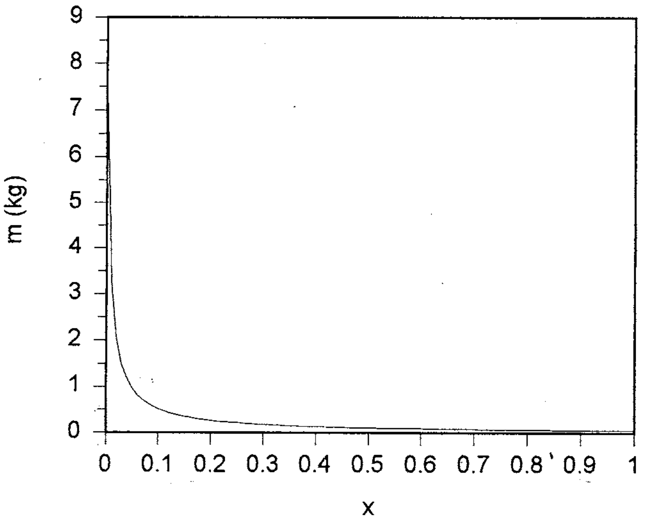
The following IT code is used to develop the plots below:

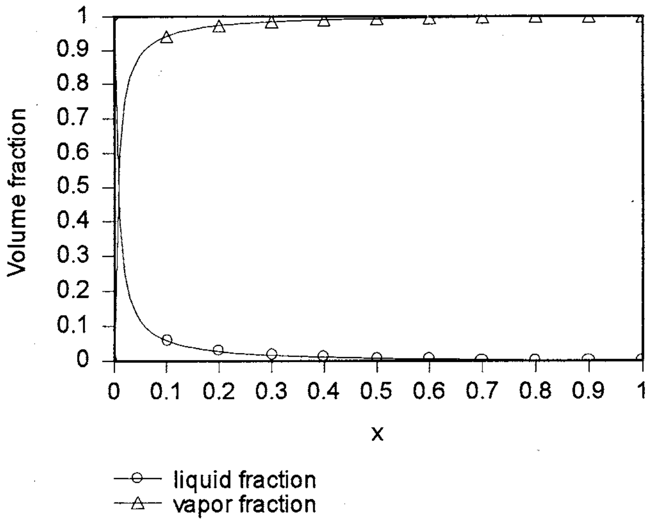
```
p = 180 // kPa
V=0.006 //m³
v = vsat_Px("R134A", p, x)
vg = vsat_Px("R134A", p, 1)
x=0.9
m=V/v
mg=x*m
mf=m*(1-x)
vfrac_g=(vg*mg)/V
vfrac_f=1-vfrac_g
mm=mf+mg
```

```
Results for sample calculation
x          0.9
m          0.06065 kg
vfrac_f    0.0007566
vfrac_g    0.9992
```

These results compare well with the results of the hand calculation above.

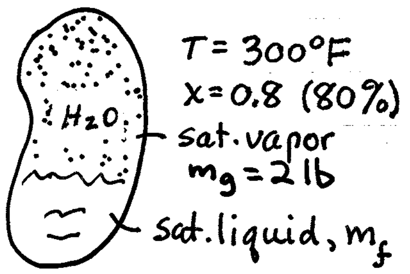
To develop data for the following plots, sweep x from 0 to 1 in steps of 0.01.





The mass contained in the tank decreases rapidly with increasing quality. The corresponding volume fractions of liquid and vapor are strong functions of quality as well.

PROBLEM 3.20*



From the definition of quality

$$m_f = \left(\frac{1-x}{x}\right) m_g = \left(\frac{1-0.8}{0.8}\right)(2\ \text{lb}) = 0.5\ \text{lb} \xleftarrow{\quad m_f \quad}$$

The total mass m is

$$m = m_f + m_g = 0.5 + 2 = 2.5\ \text{lb}$$

Using data from Table A-2E at 300 °F:

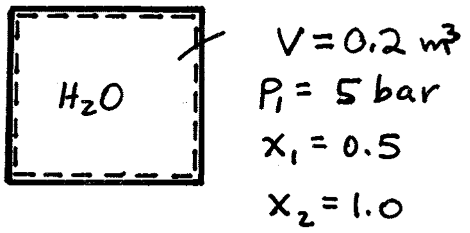
$$v = v_f + x(v_g - v_f) = 0.01745 + (.8)(6.472 - 0.01745)$$
$$= 5.1811\ \text{ft}^3/\text{lb}$$
$$V = mv = (2.5\ \text{lb})(5.1811\ \text{ft}^3/\text{lb})$$
$$= 12.95\ \text{ft}^3 \xleftarrow{\qquad\qquad V}$$

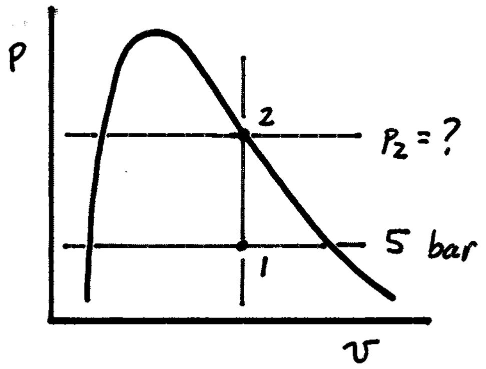
PROBLEM 3.21*

KNOWN: A two-phase liquid-vapor mixture of water, initially at a given pressure and quality, is heated in a closed rigid tank until only saturated water vapor remains. The volume is known.

FIND: Determine the mass of the water in the tank and the final pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The water is a closed system. (2) The volume and mass are constant.

ANALYSIS: Using data from Table A-3, the initial specific volume is

$$v_1 = v_{f_1} + x_1(v_{g_1} - v_{f_1})$$
$$= 1.0926 \times 10^{-3} + (.5)(0.3749 - 1.0926 \times 10^{-3})$$
$$= 0.1880\ \text{m}^3/\text{kg}$$

Thus, the mass is

$$m = \frac{V}{v_1} = \frac{(0.2\ \text{m}^3)}{(0.1880\ \text{m}^3/\text{kg})} = 1.064\ \text{kg} \xleftarrow{\qquad\qquad m}$$

By assumptions (1) and (2), $v_2 = v_1$. Thus

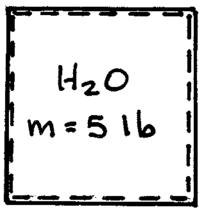
$$P_2 \approx P_{sat\,@\,v_2} = 10.5\ \text{bar} \xleftarrow{\qquad\qquad P_2}$$

PROBLEM 3.22

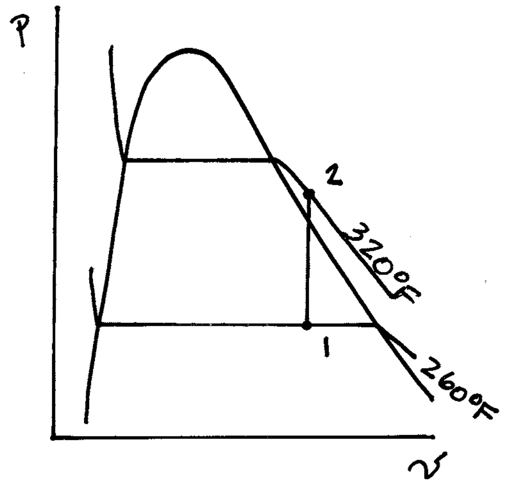
KNOWN: A specified amount of water is heated in a closed, rigid tank from a known initial state to a specified final temperature.

FIND: Determine the mass of vapor initially present and the final pressure.

SCHEMATIC & GIVEN DATA:



State 1: $T_1 = 260°F$, $x_1 = 0.6$
State 2: $T_2 = 320°F$



ASSUMPTIONS: (1) The water is the closed system. (2) the volume is constant.

ANALYSIS: The initial mass of vapor in the tank is found using the known total mass and the quality x_1 at the initial state

$$m_{g1} = x_1 m = (0.6)(5 \text{ lb}) = 3 \text{ lb} \longleftarrow m_{g1}$$

Now, since the volume and total mass are constant during the process, the specific volumes at states 1 and 2 are equal: $v_2 = v_1$. From Table A-2E at 260°F, $v_{f1} = 0.01708$ ft³/lb and $v_{g1} = 11.77$ ft³/lb. Thus

$$v_2 = v_1 = v_{f1} + x_1(v_{g1} - v_{f1})$$

$$= 0.01708 + (0.6)(11.77 - 0.01708) = 7.0688 \text{ ft}^3/\text{lb}$$

Now, from Table A-2E at 320°F, $v_g = 4.919$ ft³/lb. Since $v_2 > v_{g@320°F}$ state 2 is in the superheated vapor region. Thus, interpolating in Table A-4 at 320°F, 7.0688 ft³/lb

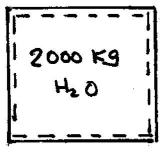
$$P_2 \approx 64.27 \text{ lbf/in.}^2 \longleftarrow P_2$$

PROBLEM 3.23

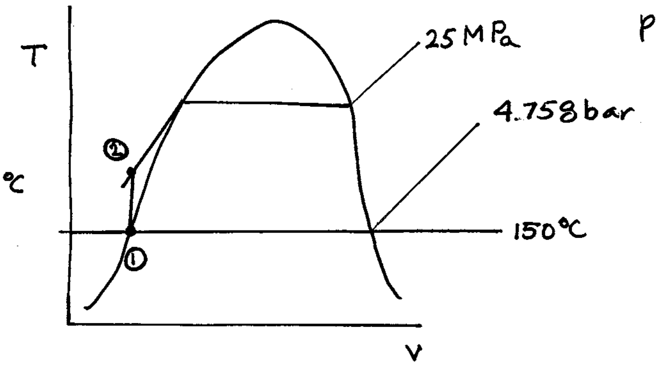
KNOWN: A specified amount of water is heated in a closed rigid tank from a known initial state to a final pressure.

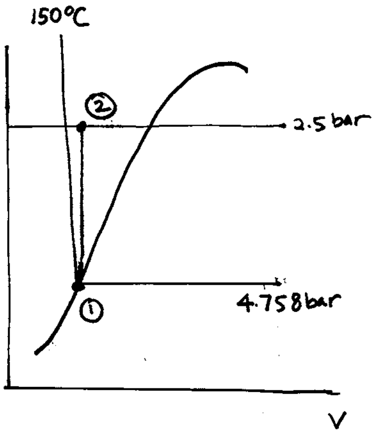
FIND: Sketch the process on T-v and p-v diagrams. Determine the volume of the tank and the temperature at the final state.

SCHEMATIC & GIVEN DATA:



State 1: Sat. liq. at 150°C
State 2: $p_2 = 2.5$ MPa





ASSUMPTIONS: 1. The water is the closed system. 2. Volume remains constant.

ANALYSIS: Since volume and total mass remain constant during the process, the specific volume at states 1, 2 are equal: $v_2 = v_1$. From Table A-2 at 150 °C

$$v_1 = v_f(150°C) = 1.0905 \times 10^{-3}\ m^3/kg$$

As shown by the T-v and p-v diagrams, state 2 is in the liquid region. Interpolation in Table A-5 gives $T_2 = 150.15°C$ ←

The total volume is

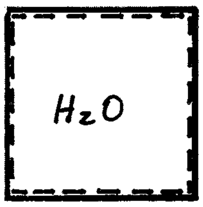
$$V = m\,v = (2000\ kg)\left(\frac{1.0905}{10^3}\ \frac{m^3}{kg}\right) = 2.181\ m^3 \leftarrow$$

PROBLEM 3.24*

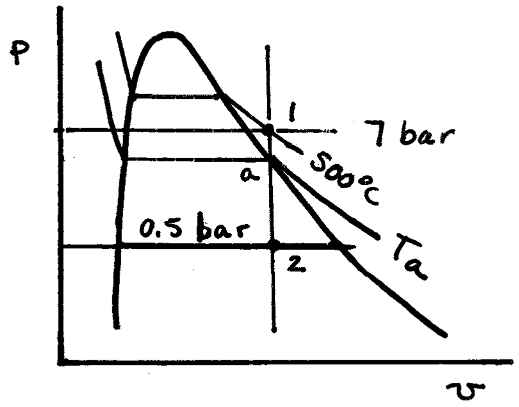
KNOWN: Steam is cooled in a closed rigid container from a known initial state to a known final pressure.

FIND: Determine the temperature at which condensation first occurs, the fraction of the total mass that condenses, and the volume occupied by saturated liquid at the final state.

SCHEMATIC & GIVEN DATA:



$P_1 = 7$ bar
$T_1 = 500°C$
$V = 1\ m^3$



ASSUMPTIONS: (1) The steam is a closed system. (2) The volume is constant.

ANALYSIS: By assumptions (1) and (2), the specific volume is constant. Thus, using data from Table A-4

$$v_1 = v_a = v_2 = 0.5070\ m^3/kg$$

Then from Table A-2

$$T_a = T_{sat@}\ v_a = 140°C \longleftarrow \qquad T_a$$

Next, the fraction of the total mass that condenses is

$$\text{fraction condensed} = \frac{m_{f2}}{m} = 1 - x_2$$

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}} = \frac{0.5070 - 1.0300 \times 10^{-3}}{3.420 - 1.0300 \times 10^{-3}} = 0.1480$$

Thus $$\text{fraction condensed} = 1 - 0.1480 = 0.8520 \longleftarrow$$

The total mass of the system is $m = \frac{V}{v_1} = \frac{(1\ m^3)}{(0.5070\ m^3/kg)}$

$$= 1.972\ kg$$

Then, the volume of the liquid at state 2 is

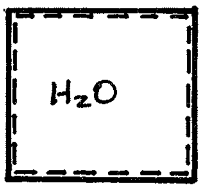
$$V_{f2} = m\,v_{f2} = (1.972\ kg)(1.0300 \times 10^{-3}\ m^3/kg)$$

$$V_{f2} = 0.00203\ m^3 \longleftarrow \qquad V_{f2}$$

PROBLEM 3.25

KNOWN: Water vapor is heated in a closed, rigid tank from satured vapor at a known temperature to a known final temperature.

FIND: Determine the initial and final pressures and sketch the process on T-v and p-v diagrams.



State 1: T_1 = 160°C
sat. vapor

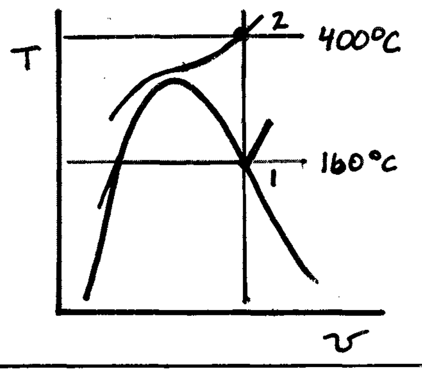
State 2: T_2 = 400°C

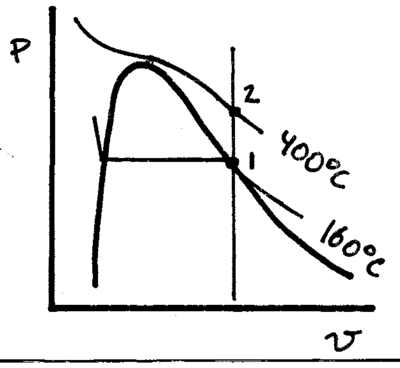
ASSUMPTIONS: (1) The water vapor is a closed system. (2) The volume is constant.

ANALYSIS: Using data from Table A-2, p_1 = 6.178 bar ← p_1

With assumptions (1) and (2), $v_2 = v_1$ = 0.3071 m³/kg. Interpolating in Table A-3, $p_2 \approx$ 9.99 bar ← p_2

Thus



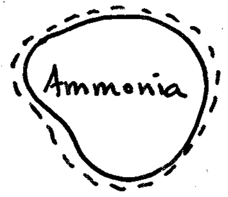


PROBLEM 3.26

KNOWN: Ammonia undergoes an isothermal process from a known initial state to the saturated vapor state.

FIND: Determine the initial and final pressures, and sketch the process on T-v and p-v diagrams.

ASSUMPTIONS: (1) The ammonia is a closed system. (2) The process is isothermal.



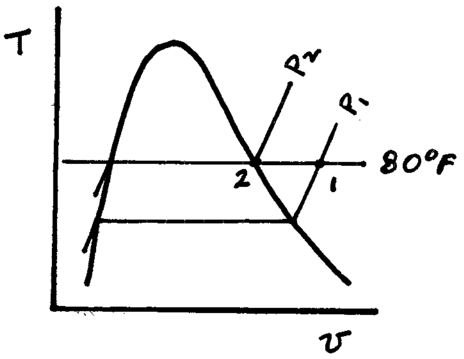
State 1: T_1 = 80°F
v_1 = 10 ft³/lb

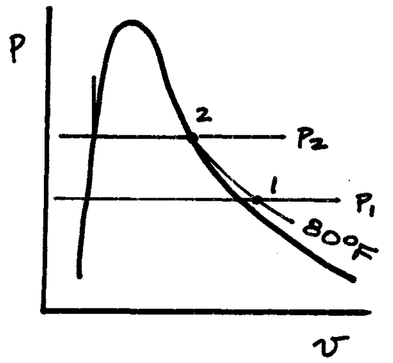
State 2: $T_2 = T_1$
Sat. vapor

ANALYSIS: Interpolating in Table A-15E, $p_1 \approx$ 33.86 lbf/in² ← p_1

Using data from Table A-13E, p_2 = 153.13 lbf/in² ← p_2

Thus



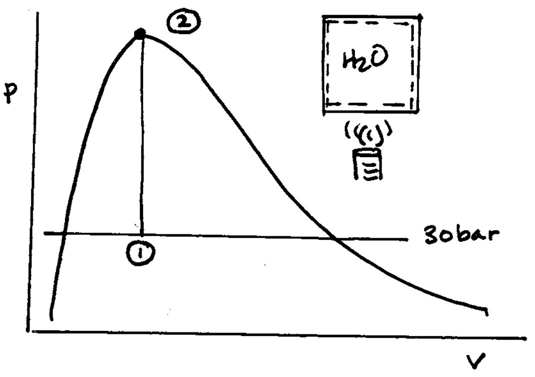


PROBLEM 3.27

KNOWN: A two-phase liquid-vapor mixture is heated at fixed volume from an initial pressure to the critical point.

FIND: Determine the quality at the initial state.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The quantity of water under consideration is the closed system. 2. Volume remains constant.

ANALYSIS: Since volume and mass remain constant, $V_2 = V_1$. From Table A-3 at the critical point, state 2, $v_2 = 3.155 \times 10^{-3}$ m³/kg. Thus, with $v_2 = v_1$, we have with data from Table A-3 at 30 bar

$$v_1 = v_f + x_1(v_g - v_f) \Rightarrow$$

$$x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{(3.155 - 1.2165)10^{-3}}{(66.68 - 1.2165)10^{-3}} = 0.0296 \quad (2.96\%)$$

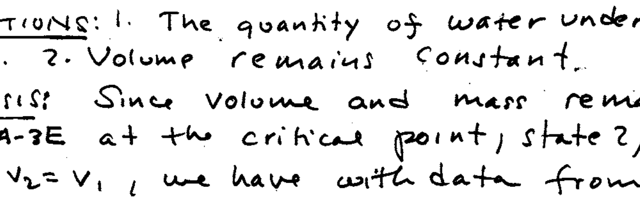
$\leftarrow x_1$

PROBLEM 3.28

KNOWN: A two-phase liquid-vapor mixture is heated at fixed volume from an initial pressure to the critical point.

FIND: Determine the quality at the initial state.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The quantity of water under consideration is the closed system. 2. Volume remains constant.

ANALYSIS: Since volume and mass remain constant, $V_2 = V_1$. From Table A-3E at the critical point, state 2, $v_2 = 0.0505$ ft³/lb. Thus, with $v_2 = v_1$, we have with data from Table A-3E at 450 lbf/in.²

$$v_1 = v_f + x_1(v_g - v_f) \Rightarrow$$

$$x_1 = \frac{v_1 - v_f}{v_g - v_f} = \frac{(0.0505 - 0.01955)}{1.033 - 0.01955} = 0.0305 \quad (3.05\%)$$

$\leftarrow x_1$

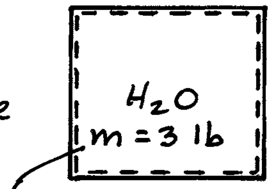
PROBLEM 3.29

KNOWN: A specified amount of water is heated in a closed rigid tank. The volume is known, and data are given for the initial and final states.

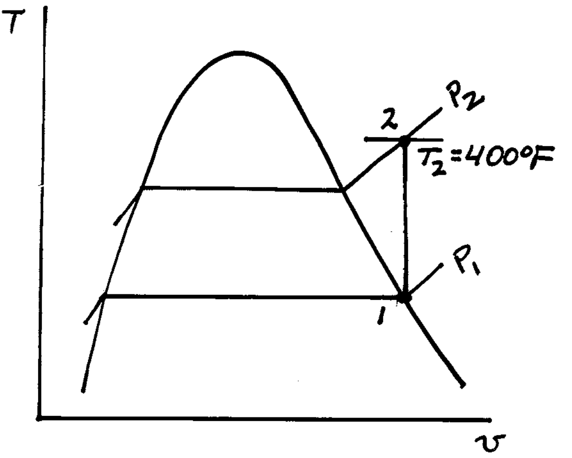
FIND: Determine the pressures at the initial and final states.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) The water is the closed system. (2) The volume is constant.



$V = 13.3$ ft³ state 1: sat. vap.
State 2: $T_2 = 400$°F



ANALYSIS: The T-v diagram is shown to the right. To determine the pressures, first fix state 2. Since the volume and mass are constant, $v_1 = v_2$. Thus

$$v_2 = V/m = (13.3 \text{ ft}^3)/(3 \text{ lb}) = 4.4333 \text{ ft}^3/\text{lb}$$

Now, interpolating in Table A-4E at 400°F, 4.4333 ft³/lb; $p_2 \approx 111.7$ lbf/in.² ← p_2

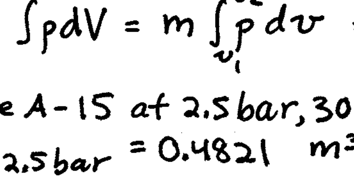
Then, from Table A-3E, at $v_g = 4.4333$ ft³/lb; $p_1 = p_{sat} \approx 100$ lbf/in.² ← p_1

PROBLEM 3.30*

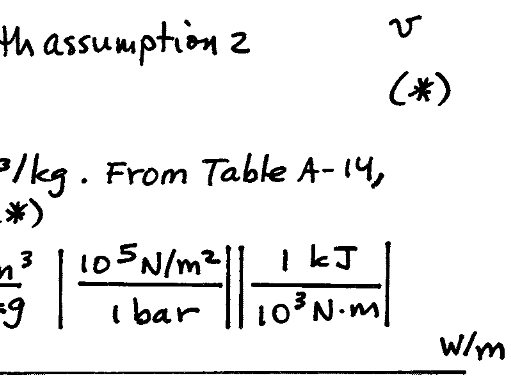
KNOWN: Ammonia undergoes a constant pressure process. The initial and final states are fixed.

FIND: Determine the work for the process, per unit mass of ammonia.

SCHEMATIC & GIVEN DATA:



$p = 2.5$ bar
$T_1 = 30$ °C
State 2: sat. vapor



ASSUMPTIONS: (1) The refrigerant is the closed system. (2) The pressure remains constant.

ANALYSIS: The work is determined using Eq. 2.17 with assumption 2

$$W = \int p\,dV = m\int_{v_1}^{v_2} p\,dv = mp(v_2 - v_1) \qquad (*)$$

From Table A-15 at 2.5 bar, 30 °C; $v_1 = 0.57745$ m³/kg. From Table A-14, $v_2 = v_{g@2.5\text{bar}} = 0.4821$ m³/kg. Thus, from (*)

$$\frac{W}{m} = p(v_2 - v_1) = (2.5 \text{ bar})(0.4821 - 0.57745)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$

$$= -23.84 \text{ kJ/kg}$$ ← W/m

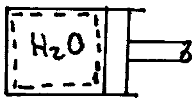
PROBLEM 3.31*

KNOWN: Water is compressed at a constant pressure between two specified states.

FIND: Determine the temperatures at the initial and final states, and the work for the process.

SCHEMATIC & GIVEN DATA:

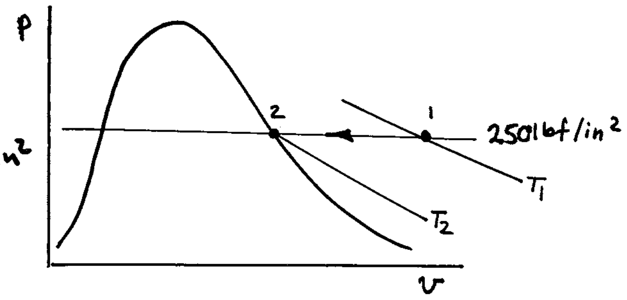
$m = 2$ lb



$p = 250$ lbf/in²

$V_1 = 6.88$ ft³

state 2: sat. vap.



ASSUMPTIONS: 1. The given quantity of water is the closed system. 2. Volume change is the only work mode. 3. The process occurs at constant pressure.

ANALYSIS: The initial specific volume is

$$v_1 = \frac{V_1}{m} = \frac{6.88 \text{ ft}^3}{2 \text{ lb}} = 3.44 \text{ ft}^3/\text{lb}$$

From Table A-4E, with $v_1 = 3.44$ ft³/lb and $p_1 = 250$ lbf/in²

$$T_1 = 1000\,°\text{F} \longleftarrow T_1$$

The final specific volume is v_g at 250 lbf/in².

From Table A-2E

$$v_2 = v_{g@250\,\text{psi}} = 1.845 \text{ ft}^3/\text{lb}$$

Thus $T_2 = 401.04$ °F $\longleftarrow T_2$

The final volume is

$$V_2 = m v_2 = (2 \text{ lb})\left(1.845 \frac{\text{ft}^3}{\text{lb}}\right) = 3.69 \text{ ft}^3$$

To evaluate the work

$$W = \int_1^2 p\,dV = p(V_2 - V_1)$$

$$= \left(250 \frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right| [3.69 - 6.88] \text{ ft}^3 \left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right|$$

$$= -147.6 \text{ Btu} \longleftarrow W$$

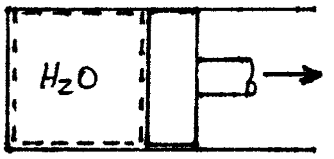
Water is compressed, thus the work is negative.

PROBLEM 3.32

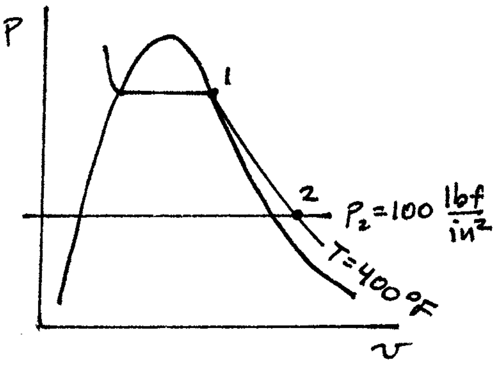
KNOWN: Water is heated in a piston-cylinder assembly at constant temperature from saturated vapor to a given final pressure.

FIND: Determine the work per unit mass by (a) numerical integration using steam table data and (b) using IT.

SCHEMATIC & GIVEN DATA:



$T = 400°F$
State 1: sat. vapor
state 2: $p_2 = 100$ lbf/in²



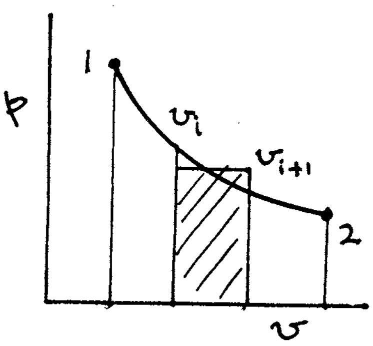
ASSUMPTIONS: (1) The water is the closed system. (2) The process occurs at constant temperature.

ANALYSIS: (a) To determine the work, use Eq. 2.17

$$\frac{W}{m} = \int_{v_1}^{v_2} p\,dv \qquad (*)$$

A numerical scheme is employed with data from Table A-4E at $T = 400°F$.

p (lbf/in²)	v (ft³/lb)
247.1	1.866 (sat. vapor, Table A-2E)
200	2.361
180	2.648
160	3.007
140	3.466
120	4.079
100	4.934



$$W/m \approx \sum_{i=1}^{6} \left(\frac{p_i + p_{i+1}}{2}\right)(v_{i+1} - v_i)$$

$$= \left[\left(\frac{247.1 + 200}{2}\right)\frac{\text{lbf}}{\text{in}^2}(2.361 - 1.866)\frac{\text{ft}^3}{\text{lb}} + \left(\frac{200+180}{2}\right)(2.648 - 2.361)\right.$$

$$+ \left(\frac{180+160}{2}\right)(3.007 - 2.648) + \left(\frac{160+140}{2}\right)(3.466 - 3.007) + \left(\frac{140+120}{2}\right)(4.079 - 3.466)$$

$$\left. + \left(\frac{120+100}{2}\right)(4.934 - 4.079)\right]\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 86.77\ \text{Btu} \longleftarrow \underset{\text{(part a)}}{W/m}$$

(b) The following code uses the integration capability of IT to integrate (*) above:

```
T=400  // °F
v=v_PT("Water",p,T)  // ft³/lb
W=conv*Integral(p,v)
conv = 144/778
/*
Using the Explore button, sweep v from 1.864 (sat. vapor)
to 4.934 in steps of 0.01
Result:  W/m = 86.48  Btu/lb
*/
```

← W/m (part b)

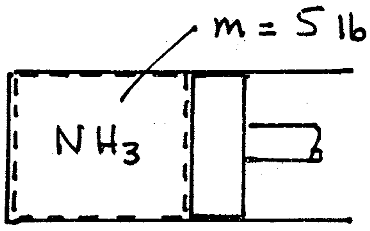
Note that the results of parts (a) and (b) are in agreement.

PROBLEM 3.33*

KNOWN: Ammonia undergoes a constant pressure process from a specified initial state to a given final quality.

FIND: Determine the work.

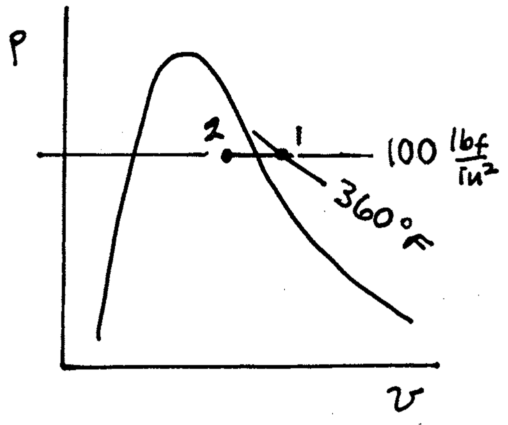
SCHEMATIC & GIVEN DATA :



$P_1 = P_2 = 100$ lbf/in²

$T_1 = 360°F$

$x_2 = 0.85$



ASSUMPTIONS: (1) The refrigerant is a closed system. (2) The process occurs at constant pressure. (3) Volume change is the only work mode.

ANALYSIS: State 1 is a superheated vapor. Referring to Table A-15E

$$v_1 = 5.0888 \text{ ft}^3/\text{lb}$$

Using the given quality at state 2 and data from Table A-14E

$$v_2 = v_f + x(v_g - v_f) = 0.02584 + (0.85)[2.9497 - 0.02584]$$
$$= 2.5111 \text{ ft}^3/\text{lb}$$

Using Eq. 2.17 to determine the work

$$W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1) = m\,p(v_2 - v_1)$$

Inserting values

$$W = (5\text{ lb})(100\text{ lbf/in}^2)(2.5111 - 5.0888)\frac{\text{ft}^3}{\text{lb}}\left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|$$

① $= -238.6$ Btu ◀ W

---

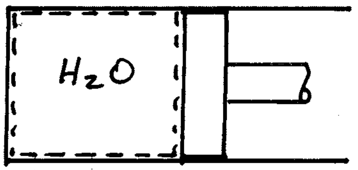
1. The refrigerant is compressed, thus the work is negative.

PROBLEM 3.34*

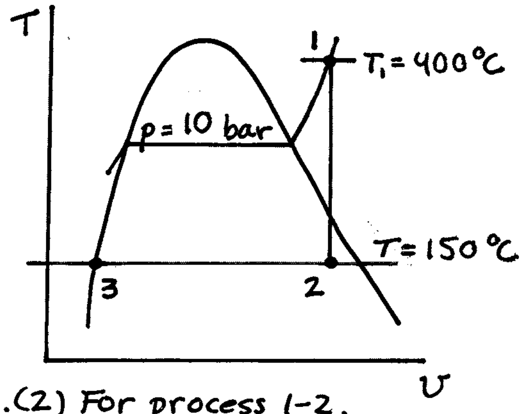
KNOWN: Water vapor undergoes a constant volume cooling process followed by isothermal condensation to saturated liquid.

FIND: Evaluate the work per unit mass.

SCHEMATIC & GIVEN DATA:



$p_1 = 10$ bar
$T_1 = 400$ °C
$T_2 = 150$ °C
State 3: Sat. liquid at 150 °C



ASSUMPTIONS: (1) The water is the closed system. (2) For process 1-2, the volume is constant. (3) For process 2-3, the temperature is constant. (4) Volume change is the only work mode.

ANALYSIS: By assumptions (2) and (4), only process 2-3 involves work. To evaluate the work, begin with Eq. 2.17

$$W = \int_{V_1}^{V_2} p\,dV \Rightarrow \frac{W}{m} = \int_{v_1}^{v_2} p\,dv$$

Next, we fix each state and obtain relevant data. Since the mass and volume are constant to process 1-2, $v_2 = v_1$. From Table A-4; $v_1 = 0.3066$ m³/kg.

With $v_2 = v_1 = 0.3066$, we see in Table A-2 at 150 °C that $v_f < v_2 < v_g$. Thus, state 2 is in the two-phase, liquid-vapor region. Since the temperature is constant for process 2-3, so is the pressure. That is

$$p_2 = p_3 = p_{sat @ 150°C} = 4.758 \text{ bar}$$

Finally, from Table A-2, $v_3 = v_{f @ 150°C} = 1.0905 \times 10^{-3}$ m³/kg. Thus

$$\frac{W}{m} = \int_{v_1}^{v_2} p\,dv = p_2(v_3 - v_2)$$

$$= (4.758 \text{ bar})(1.0905\times10^{-3} - 0.3066)\frac{m^3}{kg}\left|\frac{10^5 N/m^2}{1 \text{ bar}}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right|$$

① $$= -145.4 \text{ kJ/kg} \longleftarrow \quad W/m$$

---

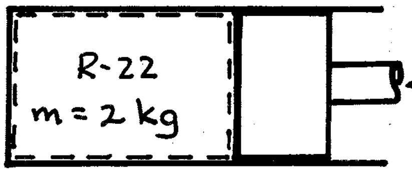
1. The negative sign denotes energy transfer to the system by work during process 2-3.

PROBLEM 3.35

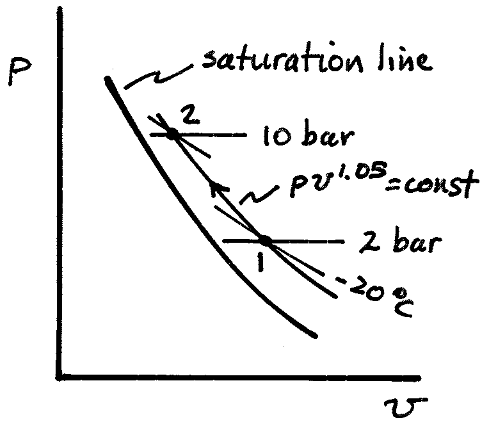
KNOWN: Refrigerant 22 undergoes a process with a known pressure-volume relation from a given initial state to a given final pressure.

FIND: Calculate the work for the process.

SCHEMATIC & GIVEN DATA:



$p_1 = 2$ bar
$T_1 = -20°C$
$p_2 = 10$ bar



ASSUMPTIONS: (1) The refrigerant is a closed system. (2) The process is polytropic, with $n = 1.05$

ANALYSIS: The work is obtained by using Eq. 2.17. First, determine the specific volumes. From Table A-9, $v_1 = 0.11520$ m³/kg, and

$$v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.05}} v_1 = \left(\frac{2}{10}\right)^{\frac{1}{1.05}} (0.11520) = 0.024875 \text{ m}^3/\text{kg}$$

The work is

$$W = \int_{V_1}^{V_2} p\,dV = m\int_{v_1}^{v_2} p\,dv = m\int_{v_1}^{v_2} \frac{const.}{v^{1.05}}\,dv$$

$$= \frac{m(p_2v_2 - p_1v_1)}{(1-1.05)}$$

$$= \frac{(2\text{ kg})[(10\text{ bar})(0.024875\text{ m}^3/\text{kg}) - (2)(0.11520)]}{(1-1.05)}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

① $$= -73.4 \text{ kJ} \longleftarrow W$$

---

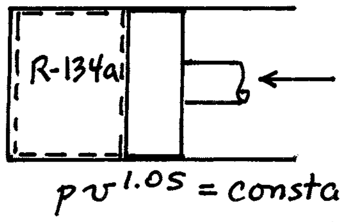
1. The negative value for work indicates that the process is a compression process

PROBLEM 3.36

KNOWN: Refrigerant 134a undergoes a polytropic process from a specified initial state to a given final temperature.

FIND: Determine the final pressure and the work per unit mass.

SCHEMATIC & GIVEN DATA:

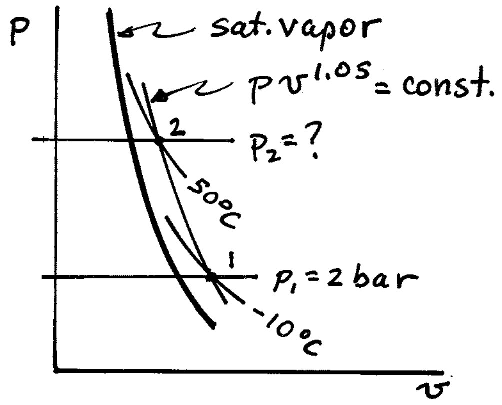


$p_1 = 200\,\text{kPa} = 2\,\text{bar}$

$T_1 = -10°\text{C}$

$T_2 = 50°\text{C}$

$pv^{1.05} = \text{constant.}$



ASSUMPTIONS: (1) The refrigerant is the closed system. (2) The process is polytropic, with $n = 1.05$.

ANALYSIS: State 1 is fixed by $T_1 = -10°\text{C}$, $p_1 = 2$ bar. From Table A-12; $v_1 = 0.09938\ \text{ft}^3/\text{lb}$. Thus, with the polytropic process expression

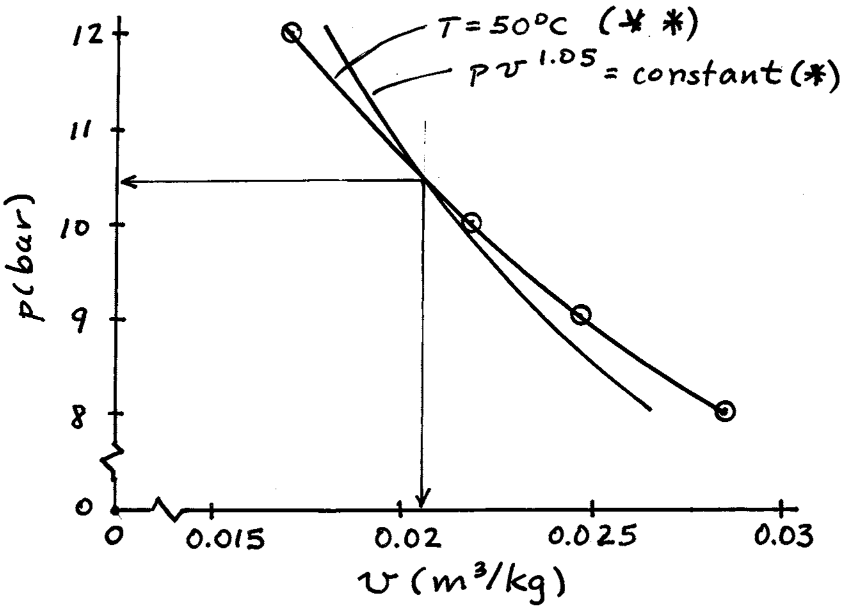
$$v_2 = \left(P_1/P_2\right)^{1/1.05} v_1 \qquad (*)$$

In Table A-12 at $T_2 = 50°\text{C}$, there is a unique v associated with each pressure, that is

$$v_{table} = v\left(p, T_2 = 50°\text{C}\right) \qquad (**)$$

The following table of data and plot are obtained using (*) and (**). Where the lines intersect, both expressions are satisfied simultaneously.

P(bar)	(*)	(**)
8	0.02654	0.02846
9	0.02372	0.02472
10	0.02146	0.02171
11	0.01960	---
12	0.01804	0.01712



From the plot:

$P_2 \approx 10.4$ bar ◄—— P_2

$v_2 \approx 0.0205\ \text{m}^3/\text{kg}$

PROBLEM 3.36 (Cont'd)

The expressions (*) and (**) can be solved much more easily using the following IT code :

```
p1 = 2  // bar
T1 = -10  // °C
T2 = 50  // °C

v1 = v_PT("R134A", p1, T1)  // m³/kg
p2 * v2^n = p1 * v1^n                    (*)
n = 1.05
v2 = v_PT("R134A", p2, T2)              (**)
```

The results from the computer solution are $p_2 = 10.38$ bar, $v_2 = 0.0207$ m³/kg, which agree well with the graphical results.

Now, the work is determined using Eq. 2.17 and the procedure of Example 2.1 for the polytropic process to get

$$\frac{W}{m} = \int_{v_1}^{v_2} p\,dv = \left(\frac{p_2 v_2 - p_1 v_1}{1-n}\right)$$

Using the values from the computer solution

$$\frac{W}{m} = \frac{(10.38\ \text{bar})(0.0207\ \text{m}^3/\text{kg}) - (2)(0.09938)}{(1-1.05)}\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

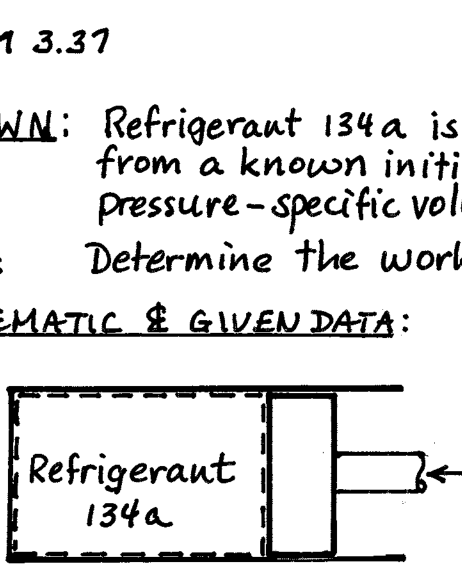
$$= -32.21\ \text{kJ} \xleftarrow{\qquad\qquad} W/m$$

PROBLEM 3.37

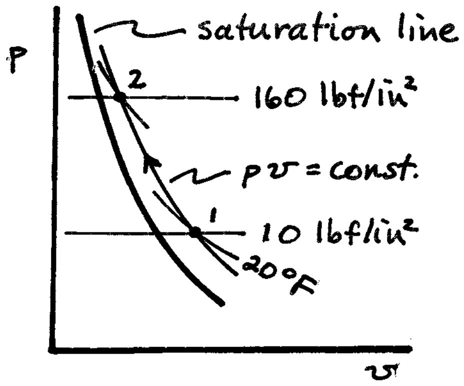
KNOWN: Refrigerant 134a is compressed in a piston-cylinder assembly from a known initial state to a known final pressure. The pressure-specific volume relationship for the process is given.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:



$m = 0.04$ lb
$p_1 = 10$ lbf/in²
$T_1 = 20$°F
$p_2 = 160$ lbf/in²
$pv =$ const.



ASSUMPTIONS: (1) The refrigerant is a closed system. (2) The process is polytropic with $pv =$ const.

ANALYSIS: The work is obtained by using Eq. 2.17. First, determine the specific volumes. From Table A-12E, $v_1 = 4.9297$ ft³/lb, and

$$v_2 = \left(\frac{p_1}{p_2}\right) v_1 = \left(\frac{10}{160}\right)(4.9297) = 0.3081 \text{ ft}^3/\text{lb}$$

The work is

$$W = \int_{V_1}^{V_2} p\,dV = m\int_{v_1}^{v_2} p\,dv = m\int_{v_1}^{v_2} \frac{\text{const.}}{v}\,dv$$

$$= m(p_1 v_1)\ln\left(\frac{v_2}{v_1}\right) = m(p_1 v_1)\ln\left(\frac{p_1}{p_2}\right)$$

$$= (0.04\text{ lb})(10\text{ lbf/in}^2)(4.9297\ \tfrac{\text{ft}^3}{\text{lb}})\ln\left(\frac{10}{160}\right)\left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|$$

① $$= -1.012\text{ Btu} \longleftarrow W$$

---

1. The work is negative for the compression, as expected.

PROBLEM 3.38 Water is the substance

(a) $p = 3$ bar, $T = 240°C$

Table A-4

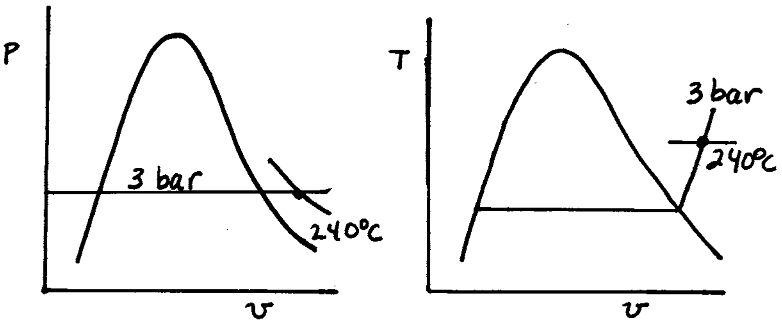
$v = 0.781\ m^3/kg$

$u = 2713.1\ kJ/kg$

IT Results

v = 0.7805 m³/kg

u = 2713 kJ/kg



(b) $p = 3$ bar, $v = 0.5\ m^3/kg$

Table A-3 ; $v_g < v < v_f$

$\Rightarrow T = 133.6°C$

$$x = \frac{v - v_f}{v_g - v_f}$$

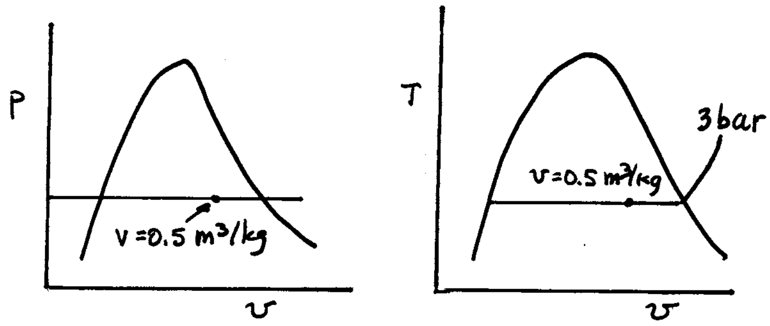
$$= \frac{0.5 - 1.0732 \times 10^{-3}}{0.6058 - 1.0732 \times 10^{-3}}$$

$$= 0.825$$

$$\therefore u = u_f + x(u_g - u_f)$$

$$= 561.15 + (.825)(2543.6 - 561.5)$$

$$= 2196.7\ kJ/kg$$



IT Results

x = 0.825

2196 kJ/kg

(c) $T = 400°C$, $p = 10$ bar

Table A-4

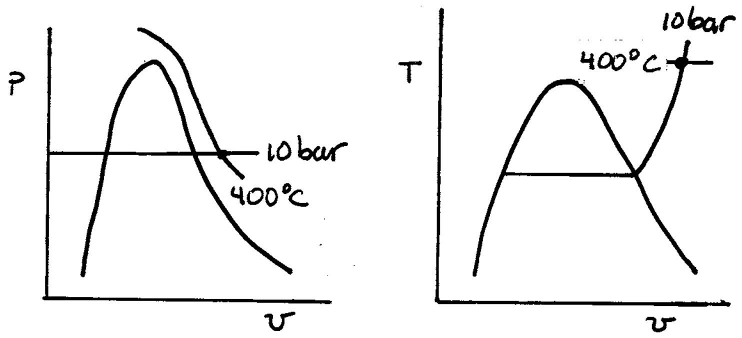
$v = 0.3066\ m^3/kg$

$h = 3263.9\ kJ/kg$

IT Results

v = 0.3066 m³/kg

h = 3263 kJ/kg



(d) $T = 320°C$, $v = 0.03\ m^3/kg$

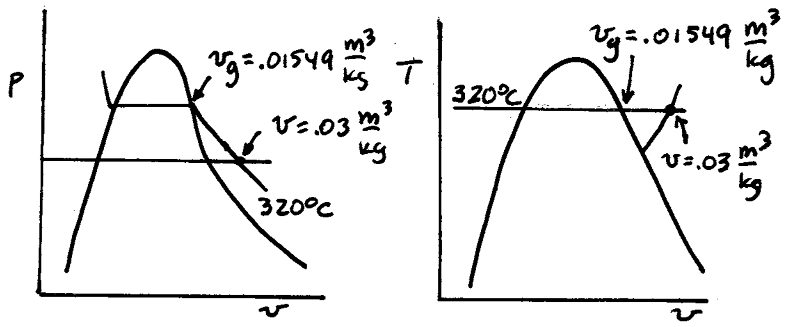
Table A-2 ; $v > v_g$ at 320°C

$\Rightarrow$ Table A-4 ; At 320°C the state falls between 60 and 80 bar. Interpolating,

$p = 74.67$ bar $= 7.467$ MPa

$h = 2678.0\ kJ/kg$

IT Results; p = 7.356 MPa, u = 2682 kJ/kg


P
$v_g = .01549\ \frac{m^3}{kg}$
$v = .03\ \frac{m^3}{kg}$
320°C
v
T
320°C
$v_g = .01549\ \frac{m^3}{kg}$
$v = .03\ \frac{m^3}{kg}$
v

(e) $p = 28\ MPa = 280$ bar, $T = 520°C$

Table A-2

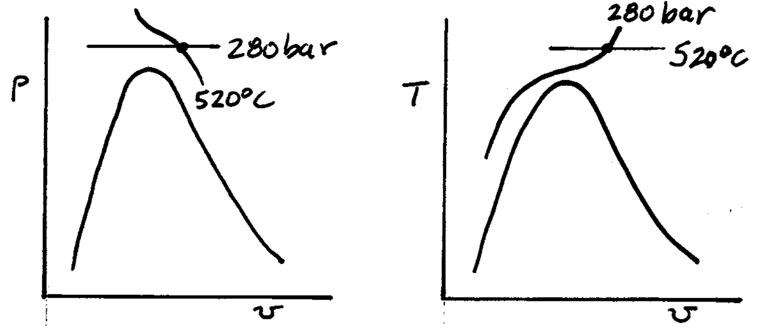
$v = 0.01020\ m^3/kg$

$h = 3192.3\ kJ/kg$

IT Results

v = 0.0102 m³/kg

h = 3192 kJ/kg



PROBLEM 3.38 (Contd.)

(f) $T = 100°C,\ x = 0.60$

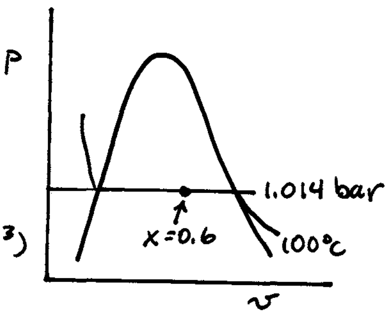
Table A-2

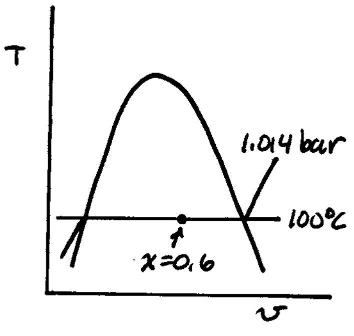
$p = 1.014$ bar

$v = v_f + x(v_g - v_f)$

$= 1.0435\times10^{-3} + (.60)(1.673 - 1.0435\times10^{-3})$

$= 1.0042\ m^3/kg$





IT Results; p = 1.013 bar, v = 1.004 m³/kg

(g) $T = 10°C,\ v = 100\ m^3/kg$

Table A-2; $v_f < v < v_g$ at 10°C.

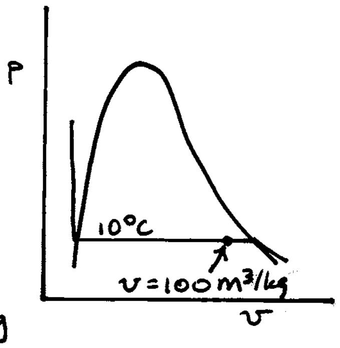
Thus, $p = 0.01228$ bar $= 1.228$ kPa

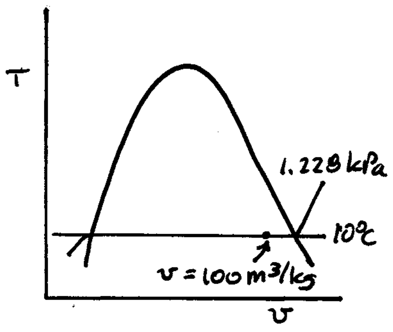
$x = \frac{v - v_f}{v_g - v_f} = \frac{100 - 1.0004\times10^{-3}}{106.379 - 1.0004\times10^{-3}}$

$= 0.94$

$h = h_f + x\,h_{fg}$

$= 42.01 + (.94)(2477.7) = 2371\ kJ/kg$





IT Results; x = 0.94, h = 2371 kJ/kg

(h) $p = 4$ MPa $= 40$ bar, $T = 160°C$

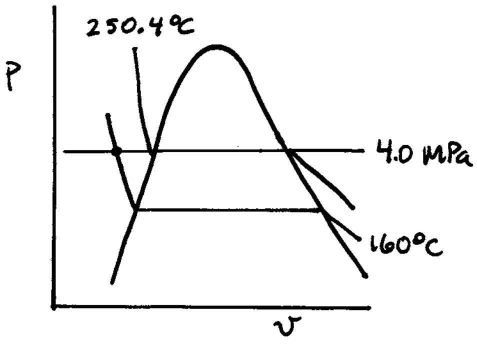
Table A-3 at 40 bar; $T_{sat} = 250.4°C$

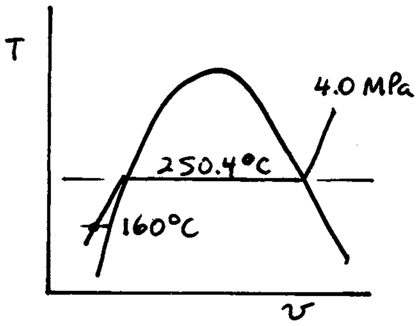
$\Rightarrow$ liquid state ($T < T_{sat}$)

Table A-5; double interpolation

	p = 2.5 MPa	p = 5.0 MPa
T = 140°C	$v = 1.0784\times10^{-3}$ $u = 587.82$	1.0768×10^{-3} 586.76
T = 180°C	$v = 1.1261\times10^{-3}$ $u = 761.16$	1.1240×10^{-3} 759.63

	at p = 4.0 MPa
140°C	$v = 1.0774\times10^{-3}$ $u = 587.18$
180°C	$v = 1.1248\times10^{-3}$ $u = 760.24$





Then, at 4.0 MPa, 160°C

$v = 1.1011\times10^{-3}\ m^3/kg$

$u = 673.71\ m^3/kg$

IT Results

v = 1.102 x 10^{-3} m³/kg
u = 670.7 kJ/kg

PROBLEM 3.39 Water is the substance.

(a) $p = 20\ \text{lbf/in}^2,\ T = 400°\text{F}$

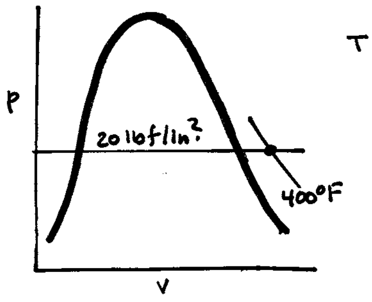
Table A-4E

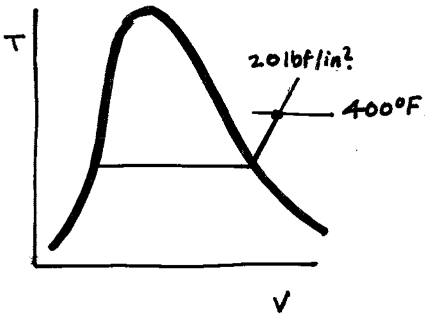
$v = 25.43\ \text{ft}^3/\text{lb}$

$u = 1145.1\ \text{Btu/lb}$

IT Results

$v = 25.43\ \text{ft}^3/\text{lb}$
$u = 1145\ \text{Btu/lb}$





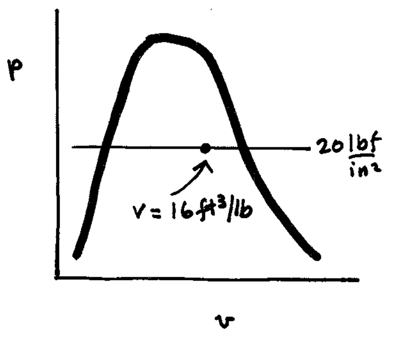
(b) $p = 20\ \text{lbf/in}^2,\ v = 16\ \text{ft}^3/\text{lb}$

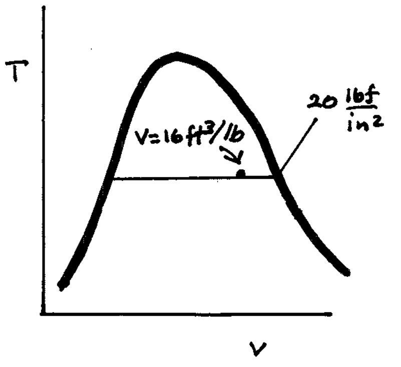
Table A-3E $v_f < v < v_g$

$\Rightarrow T = 227.96\ °\text{F}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{16 - 0.01683}{20.09 - 0.01683} = 0.796$$

$$\therefore u = u_f + x(u_g - u_f) = 196.19 + 0.796(1082 - 196.19) = 901.29\ \text{Btu/lb}$$





IT Results; T = 228 °F, x = 0.7962, u = 901.3 Btu/lb

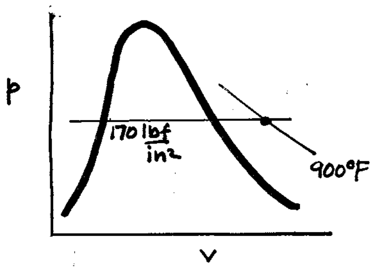
(c) $T = 900°\text{F},\ p = 170\ \text{lbf/in}^2$

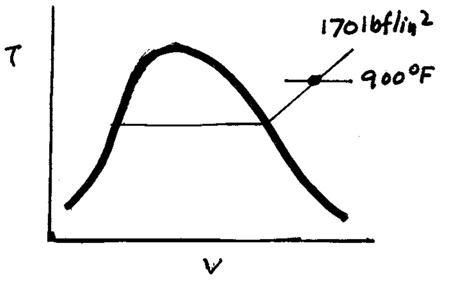
Table A-4E, interpolate at 900°F

$v = 4.734\ \text{ft}^3/\text{lb}$

$h = 1478.05\ \text{Btu/lb}$

IT Results
$v = 4.718\ \text{ft}^3/\text{lb}$
$h = 1478\ \text{Btu/lb}$





(d) $T = 600°\text{F},\ v = 0.6\ \text{ft}^3/\text{lb}$

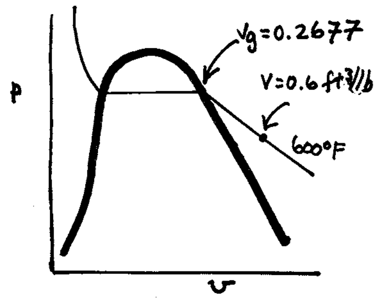
Table A-2E, $v > v_g$ at 600°F.

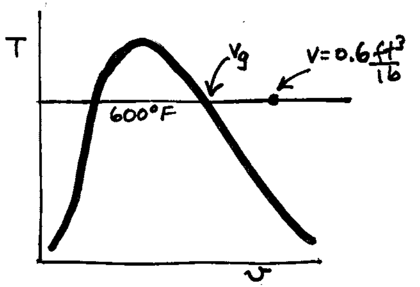
$\Rightarrow$ Table A-4E. At 600°F the state falls between 800 and 900 lbf/in².

Interpolating,

$p = 885.6\ \text{lbf/in}^2$

$u = 1163.34\ \text{Btu/lb}$





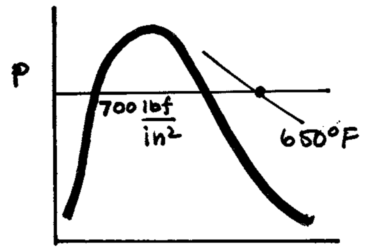
IT Results; p = 884.3 lbf/in.², u = 1163 Btu/lb

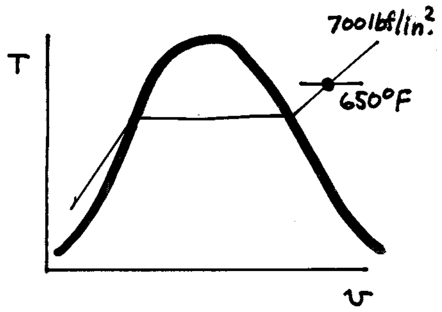
(e) $p = 700\ \text{lbf/in}^2,\ T = 650°\text{F}$

Table A-4E, interpolation at 700 lbf/in²

$v = 0.85\ \text{ft}^3/\text{lb}$

$h = 1312.3\ \text{Btu/lb}$





IT Results; v = 0.852 ft³/lb, h = 1313 Btu/lb

PROBLEM 13.39 (Contd.)

(f) $T = 400°F$, $x = 90\%$

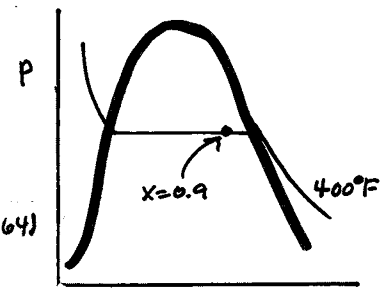
Table A-2E

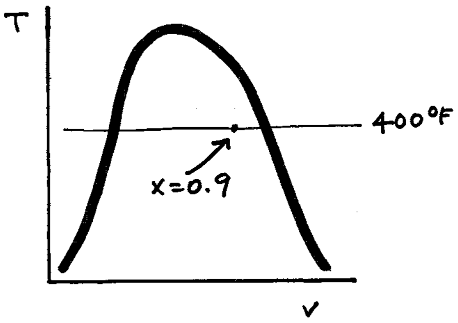
$p = 247.1$ lbf/in²

$v = v_f + x\,v_{fg}$

$= 0.01864 + .9(1.866 - 0.01864)$

$= 1.6813$ ft³/lb





IT Results; p = 247.1 lbf/in.², v = 1.681 ft³/lb

(g) $T = 40°F$, $v = 1950\ \frac{ft^3}{lb}$

Table A-2E $v_f < v < v_g$ at 40°F. Thus, $p = 0.1217$ lbf/in²,

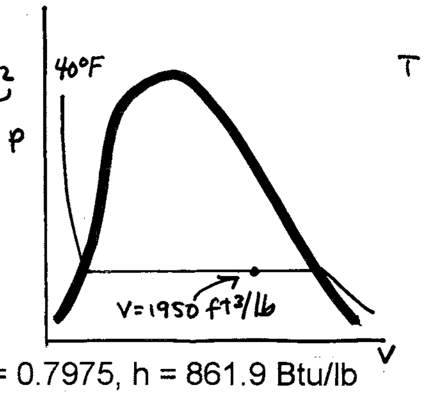
$x = \frac{v - v_f}{v_g - v_f} = \frac{1950 - 0.016}{2445 - 0.016}$

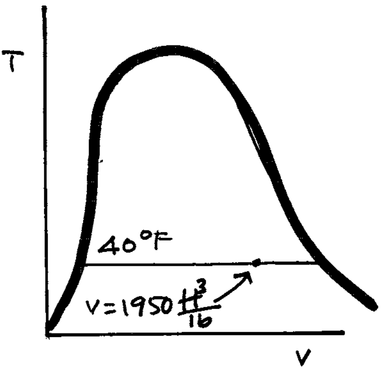
$= 0.798$

$h = h_f + x\,h_{fg}$

$= 8.02 + 0.798(1070.9)$

$= 862.6$ Btu/lb





IT Results; p = 0.1217 lbf/in.², x = 0.7975, h = 861.9 Btu/lb

---

(h) $p = 600$ lbf/in², $T = 320°F$

Table A-3E at 600 lbf/in²

$T_{sat} = 486.33°F$

⇒ liquid state

Table A-5E – double interpolation

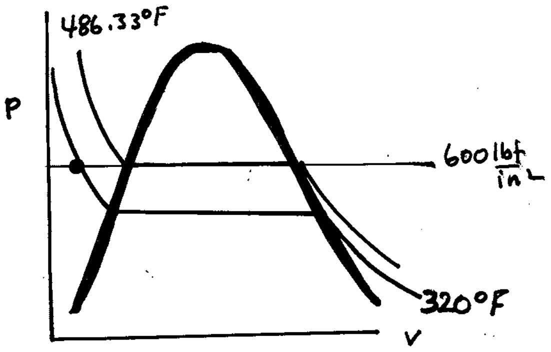
	p = 500 lbf/in²	p = 1000
T = 300°F	v = 0.017416 u = 268.92	0.017379 268.24
T = 400	v = 0.018608 u = 373.68	0.018550 372.55

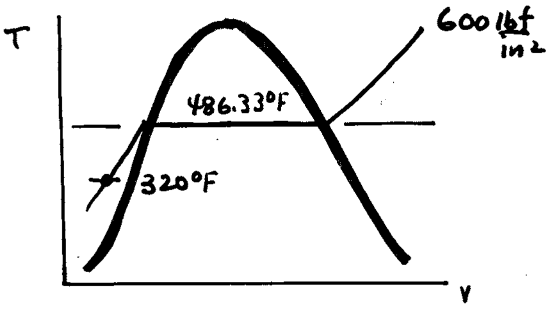
	at 600 lbf/in²
300°F	v = 0.017409 u = 268.78
400°F	v = 0.018596 u = 373.45

Then, at 600 lbf/in², 320°F

$v = 0.017646$ ft³/lb

$u = 289.71$ Btu/lb





IT Results

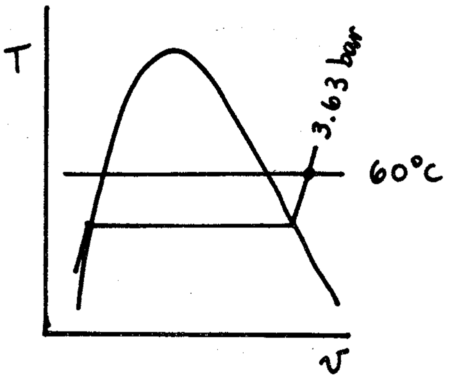
v = 0.01766 ft³/lb

u = 288.3 Btu/lb

PROBLEM 3.40

(a) Refrigerant 134a; $T = 60°C$, $v = 0.072\ m^3/kg$
from Table A-10 at 60°C; $v_g = 0.0114\ m^3/kg$
$v > v_g \Rightarrow$ superheated vapor
Interpolating in Table A-12; $p = 3.63$ bar
$= \underline{\underline{363\ kPa}}$

$h = \underline{\underline{302.1\ kJ/kg}}$

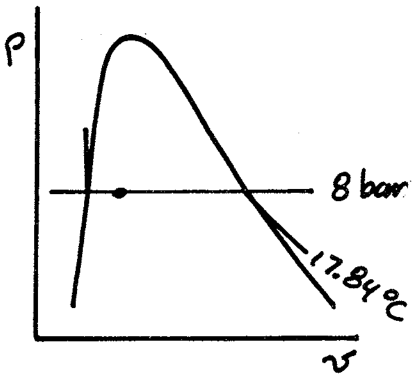


(b) Ammonia; $p = 8$ bar, $v = 0.005\ m^3/kg$
from Table A-14 at 8 bar; $v_g = 0.1596\ m^3/kg$
$v_f < v < v_g \Rightarrow$ two-phase, liquid-vapor mixture

$T = T_{sat} = \underline{17.84°C}$

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.005 - 1.6302\times10^{-3}}{0.1596 - 1.6302\times10^{-3}} = 0.02133$$

$$u = u_f + x(u_g - u_f)$$
$$= 262.64 + (0.02133)(1330.64 - 262.64)$$
$$= \underline{\underline{285.4\ kJ/kg}}$$

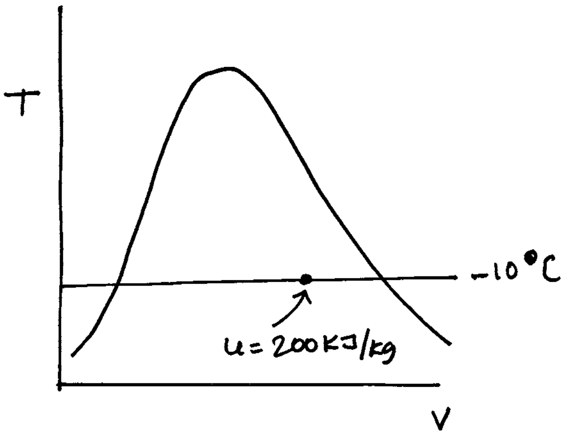


(c) Refrigerant 22; $T = -10°C$, $u = 200\ kJ/kg$
From Table A-7 at −10°C, $u_f < u < u_g$
$\Rightarrow$ 2-phase liquid-vapor mixture

$p = p_{sat} = 3.8485$ bar

$$x = \frac{u - u_f}{u_g - u_f} = \frac{200 - 33.27}{223.02 - 33.27}$$
$$= 0.879$$

$$\therefore\ v = v_f + x(v_g - v_f)$$
$$= \frac{0.7606}{10^3} + 0.879\left(0.0652 - \frac{0.7606}{10^3}\right)$$
$$= 0.0574\ m^3/kg$$



PROBLEM 3.41

(a) Refrigerant 134a; $p = 140\ \text{lbf/in}^2$, $h = 100\ \text{Btu/lb}$

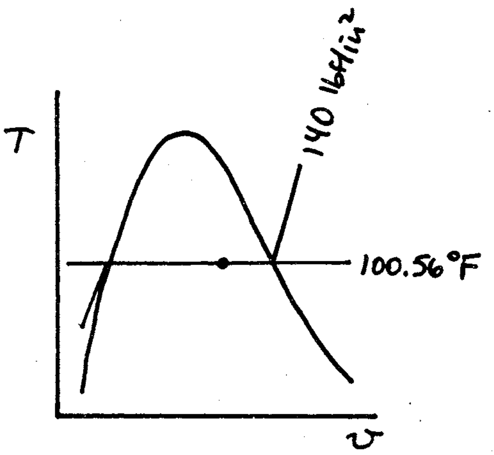
from Table A-11E; $h_f < h < h_g$

$\Rightarrow$ two-phase, liquid-vapor mixture

$$x = \frac{h - h_f}{h_{fg}} = \frac{100 - 44.43}{70.52} = 0.788$$

$$v = v_f + x\,(v_g - v_f)$$
$$= 0.01386 + (.788)(0.3358 - 0.01386)$$
$$= \underline{0.2675\ \text{ft}^3/\text{lb}}$$

$$T = T_{sat} = \underline{\underline{100.56\,°F}}$$



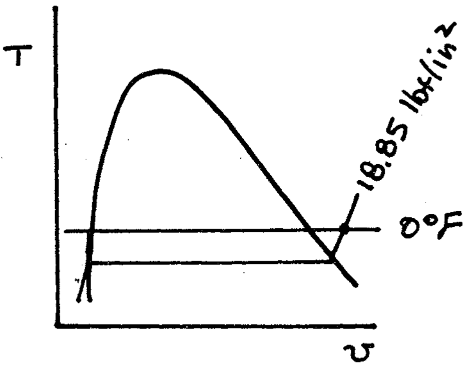
(b) Ammonia; $T = 0\,°F$, $v = 15\ \text{ft}^3/\text{lb}$

from Table A-13E; $v_g = 9.1100\ \text{ft}^3/\text{lb}$

$v > v_g \Rightarrow$ superheated vapor

Interpolating in Table A-15E: $\underline{\underline{p = 18.85\ \text{lbf/in}^2}}$

$\underline{\underline{h = 615.2\ \text{Btu/lb}}}$



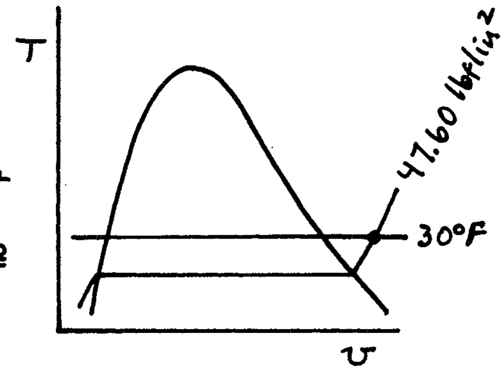
(c) Refrigerant 22; $T = 30\,°F$, $v = 1.2\ \text{ft}^3/\text{lb}$

from Table A-7E, $v_g = 0.7804\ \text{ft}^3/\text{lb}$

$v > v_g \Rightarrow$ superheated vapor

Interpolating in Table A-9E, $\underline{\underline{p = 47.60\ \text{lbf/in}^2}}$

$\underline{\underline{h = 108.80\ \text{Btu/lb}}}$

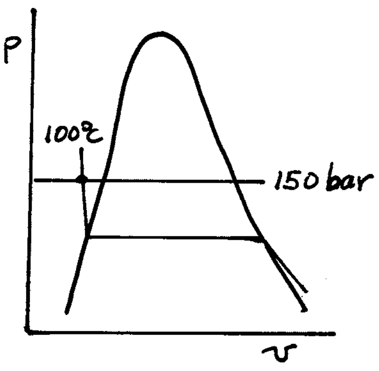


PROBLEM 3.42

(a) H_2O at $p = 15\,MPa = 150\,bar$, $T = 100°C$

from Table A-5; $\underline{v = 1.0361 \times 10^{-3}\,m^3/kg}$, $\underline{h = 430.28\,kJ/kg}$

(b) Using saturated liquid data from Table A-2;



$$v \approx v_{f\,@\,100°C} = \underline{1.0435 \times 10^{-3}\,m^3/kg}$$

Now, using Eq. 3.13 to estimate specific enthalpy

$$h \approx h_{f\,@\,100°C} + v_{f\,@\,100°C}\,(p - p_{sat\,@\,100°C})$$

$$= 419.04\,\frac{kJ}{kg} + 1.0435 \times 10^{-3}\,\frac{m^3}{kg}\,(150 - 1.014)\,bar \left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3\,N \cdot m}\right|$$

① $= 419.04 + 15.55 = \underline{434.59\,kJ/kg}$

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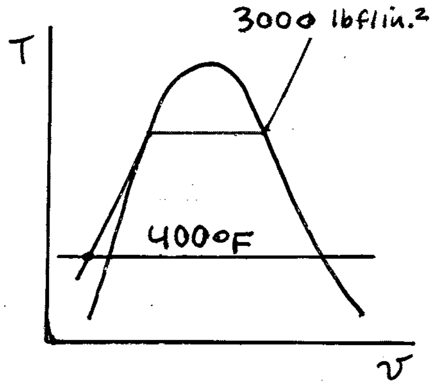
1. Note that $h_{f\,@\,100°C}$ is a plausible estimate for h in some applications (within about 2.6% of the value from Table A-5).

PROBLEM 3.43*



H_2O at $T = 400°F$, $p = 3000\,lbf/in^2$

from Table A-5E: $v = 0.018334\,ft^3/lb$

$h = 378.50\,Btu/lb$

The specific volume and enthalpy values can also be estimated using saturation data from Table A-2E and Eqs. 3.11 and 3.14, respectively.

$$v \approx v_f(400\,°F) = \underline{0.01864\,ft^3/lb}$$

and

$$h \approx h_f(400\,°F) + v_f(400°F)\,[p - p_{sat\,@\,400°F}]$$

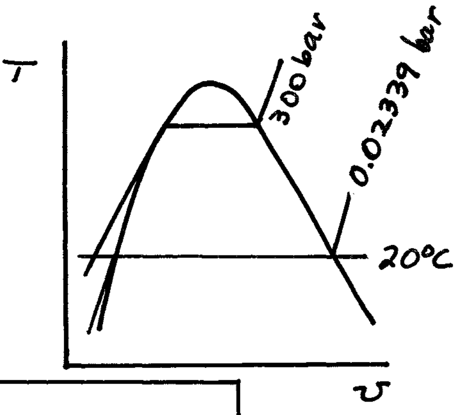
$$= 375.1\,\frac{Btu}{lb} + 0.01864\,\frac{ft^3}{lb}\left[3000\,\frac{lbf}{in^2} - 247.1\,\frac{lbf}{in^2}\right]\left|\frac{144\,in^2}{1\,ft^2}\right|\left|\frac{1\,Btu}{778\,ft \cdot lbf}\right|$$

$$= \underline{384.6\,Btu/lb}$$

PROBLEM 3.44

H_2O at $T = 20°C$, $p_{sat} \leq p \leq 300$ bar

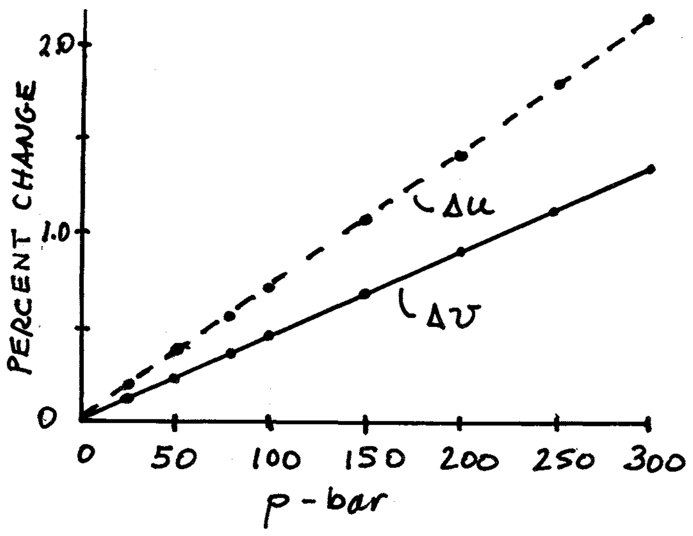
from Table A-2 : $p_{sat} = 0.02339$ bar
$v_f = 1.0018 \times 10^{-3}\ m^3/kg$
$u_f = 83.95$ kJ/kg
$h_f = 83.96$ kJ/kg

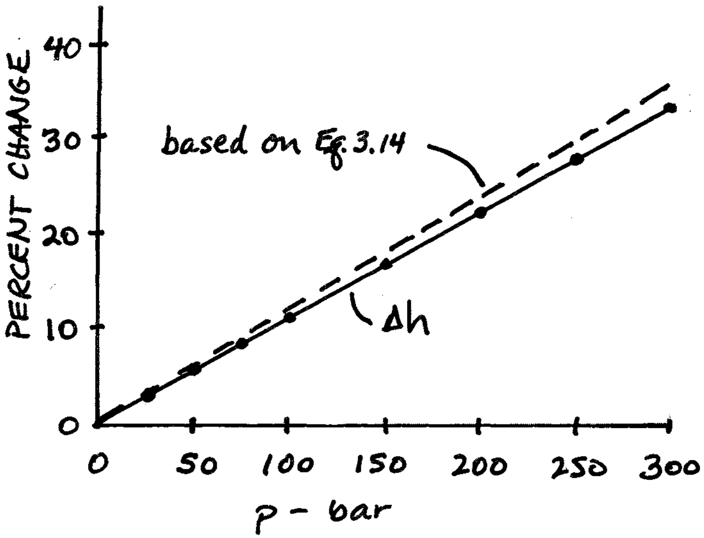


from Table A-5 :

	P - bar (T=20°C).							
	25	50	75	100	150	200	250	300
$v \times 10^3$	1.0006	.9995	.9984	.9972	.9950	.9928	.9907	.9886
u	83.80	83.65	83.50	83.36	83.06	82.77	82.47	82.17
h	86.30	88.65	90.99	93.33	97.99	102.62	107.24	111.84

	25	50	75	100	150	200	250	300
Δv (%)	0.12	0.23	0.34	0.46	0.68	0.90	1.11	1.32
Δu (%)	0.18	0.36	0.54	0.70	1.06	1.41	1.76	2.12
Δh (%)	2.79	5.59	8.37	11.12	16.69	22.22	27.73	33.21





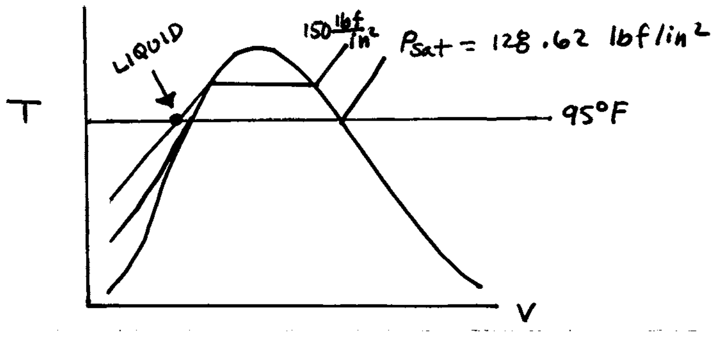
COMMENTS: The changes in v and u are small over the pressure range in this problem. Thus, v_f and u_f are reasonable approximations, respectively. However, h deviates by over 30% from h_f, and the approximation $h \approx h_f$ is not reasonable. Note that Eq. 3.14 provides a plausible estimate of h for compressed liquids when tabular data are not available.

PROBLEM 3.45

Refrigerant 134a at 95°F, 150 lbf/in²

Table A-10E at 95°F; p_{sat} = 128.62 lbf/in²

⇒ liquid state when p = 150 lbf/in²



Using Eq. 3.11, $v \approx v_f(95°F)$ = 0.01371 ft³/lb ← v

Using Eq. 3.14, $h \approx h_f(95°F)$ = 42.47 Btu/lb ← h

PROBLEM 3.46*

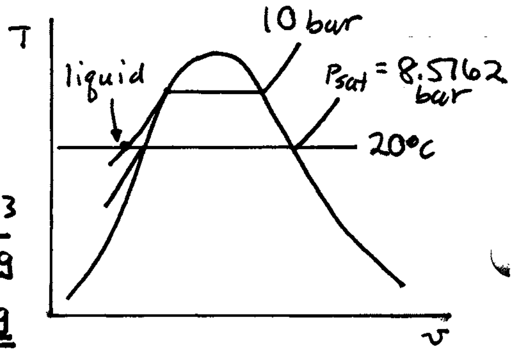
Ammonia at 20°C, 1 MPa = 10 bar

Table A-13 at 20°C; p_{sat} = 8.5762 bar

⇒ liquid state when p = 10 bar

Using Eq. 3.11; $v \approx v_f(20°C) = 1.6386 \times 10^{-3}\ \frac{m^3}{kg}$

Using Eq. 3.14; $h \approx h_f(20°C)$ = 274.26 kJ/kg



PROBLEM 3.47*

Propane at 800 kPa, 0°C

Table A-17 at 8 bar, T_{sat} = 18.33°C

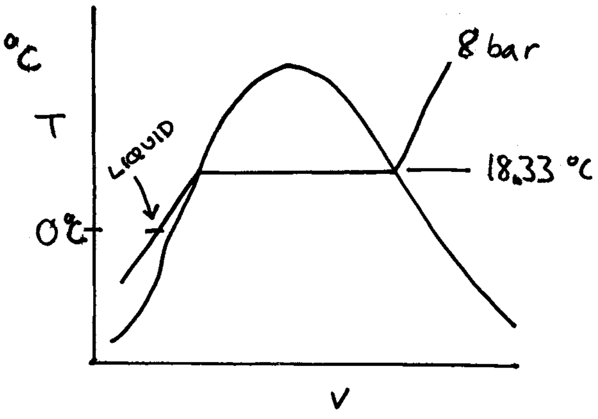
⇒ liquid state

Using Eq. 3.11, $v \approx v_f(0°C)$

$= 1.890 \times 10^{-3}\ \frac{m^3}{kg}$

Using Eq. 3.14, $h \approx h_f(0°C)$

= 95.1 kJ/kg

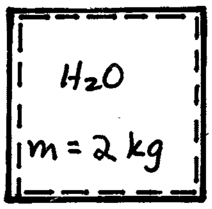


PROBLEM 3.48*

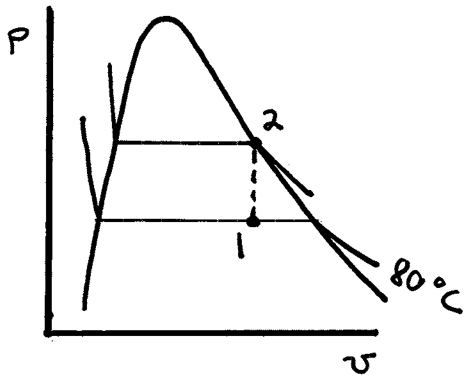
KNOWN: Saturated water vapor is contained in a rigid tank at a known initial temperature. The pressure drops as a result of heat transfer to a known final value.

FIND: Determine the amount of energy transfer by heat.

SCHEMATIC & GIVEN DATA:



$T_1 = 80°C$
$x_1 = 0.60$
state 2: sat. vapor



ASSUMPTIONS: (1) The H_2O is a closed system. (2) The volume is constant. (3) For the system, $W = 0$. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The heat transfer is determined using the energy balance

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q - \cancelto{0}{W}$$

or, with $\Delta U = m(u_2 - u_1)$

$$Q = m(u_2 - u_1)$$

Using data from Table A-2

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 334.86 + (0.60)(2482.2 - 334.86)$$
$$= 1623.26 \text{ kJ/kg}$$

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 1.0291 \times 10^{-3} + (0.60)(3.407 - 1.0291 \times 10^{-3})$$
$$= 2.0446 \text{ m}^3/\text{kg}$$

Then, using Table A-2 with $v_g = v_2 = v_1 = 2.0446 \frac{\text{m}^3}{\text{kg}}$

$$T \approx 93.5°C$$

and $u_2 = u_{g@93.5°C} = 2498.8$ kJ/kg

Finally $Q = (2 \text{ kg})(2498.8 - 1623.26) \text{ kJ/kg}$

① $= 1751.1$ kJ ← Q

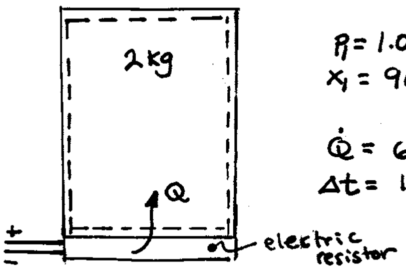
1. The positive sign for Q denotes energy transfer into the system.

PROBLEM 3.49

KNOWN: Water contained in a rigid, insulated tank is heated from a specified initial state.

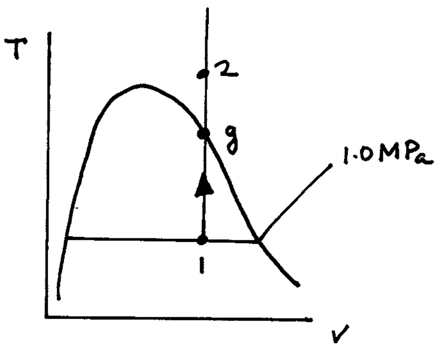
FIND: Determine the final temperature of the water.

SCHEMATIC & GIVEN DATA:



$p_1 = 1.0\text{ MPa}$
$x_1 = 90\%$

$\dot{Q} = 60\text{ W}$
$\Delta t = 1.95\text{ h}$



ASSUMPTIONS: 1. The water is the closed system. 2. For the system, W = 0, and ΔKE = ΔPE = 0. 3. Volume remains constant. 4. The initial and final states are equilibrium states.

ANALYSIS: The values of two independent intensive properties are required to fix the final state. Since the total volume and mass do not change, one of these is the final specific volume: $v_2 = v_1$. Using data from Table A-3,

$$v_1 = v_f + x(v_g - v_f) = \frac{1.1273}{10^3} + 0.9\left[0.1944 - \frac{1.1273}{10^3}\right]$$
$$= 0.1751\text{ m}^3/\text{kg}$$

The other property is u_2, found from an energy balance. With assumption 2,

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - \cancel{W}$$
$$\Rightarrow \quad m(u_2 - u_1) = Q \Rightarrow \quad u_2 = u_1 + Q/m \qquad (*)$$

With data from Table A-3

$$u_1 = u_f + x(u_g - u_f) = 761.68 + .9\ (2583.6 - 761.68)$$
$$= 2401.4\text{ kJ/kg}$$

Thus, since $Q = \int \dot{Q}dt = \dot{Q}\Delta t$,

$$u_2 = 2401.4 + \frac{(60\text{ W})(1.95\text{h})}{2\text{kg}}\left|\frac{1\text{ J/s}}{1\text{ W}}\right|\left|\frac{3600\text{s}}{1\text{h}}\right|\left|\frac{\text{kJ}}{10^3\text{ J}}\right|$$
$$= 2612\ \frac{\text{kJ}}{\text{kg}}$$

Using state g shown on the T-v diagram for reference; $v_g = v_1$. Interpolating in Table A-3 gives, $u_g = 2587$ kJ/kg. Since $u_2 > u_g$, state 2 is in the superheated vapor region, as shown on the figure.

Thus, state 2 is fixed by $u_2 = 2612$ kJ/kg, $v_2 = 0.1751$ m³/kg. Since interpolating in Table A-4 with u_2, v_2 is inconvenient, we resort to using Interactive Thermodynamics: IT to determine T_2, as follows:

PROBLEM 3.49 (Contd.)

The IT code is

```
p1 = 10  // bar
x1 = 0.9
Qdot = 60  // W
delt = 1.95  // h
m = 2  // kg
v1 = vsat_Px("Water/Steam", p1, x1)   // m³/kg
u1 = usat_Px("Water/Steam", p1, x1)  //  kJ/kg
m * (u2 - u1) = Q
Q = Qdot * delt * (3600 / 1000)
v2 = v1
v2 = v_PT("Water/Steam", p2, T2)
u2 = u_PT("Water/Steam", p2, T2)
```

IT Results

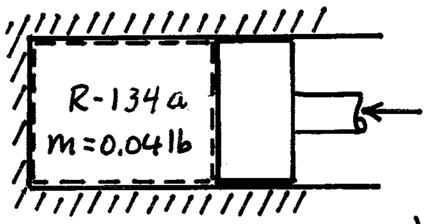
T2   198.7 °C   ←——————————— T_2
p2   11.59 bar

PROBLEM 3.50

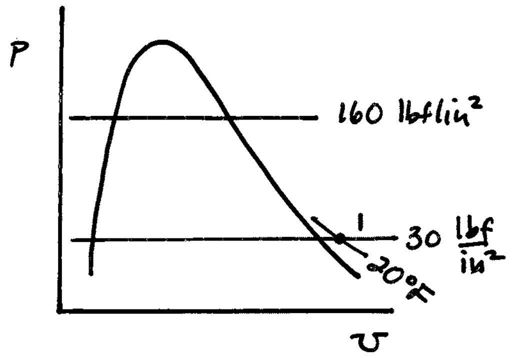
KNOWN: Refrigerant 134a is compressed from a given initial state to a given final pressure. There is no heat transfer, and the work is known.

FIND: Determine the final temperature.

SCHEMATIC & GIVEN DATA:



$P_1 = 30$ lbf/in²
$T_1 = 20$°F
$P_2 = 160$ lbf/in²
$W = -0.56$ Btu



ASSUMPTIONS: (1) The R-134a is a closed system. (2) There is no heat transfer. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The final state is fixed by determining u_2 using the energy balance.

$$\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + \Delta U = \cancel{Q}^{0} - W$$

or, with $\Delta U = m(u_2 - u_1)$

$$u_2 = -\frac{W}{m} + u_1$$

Referring to Table A-12E; $u_1 = 96.26$ Btu/lb. Thus

$$u_2 = -\frac{(-0.56 \text{ Btu})}{(0.04 \text{ lb})} + 96.26 \text{ Btu/lb}$$

$$= 110.26 \text{ Btu/lb}$$

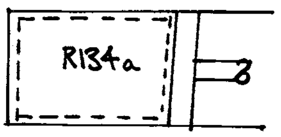
Referring again to Table A-12E; $T_2 \approx 121.6$ °F ◄ T_2

PROBLEM 3.51

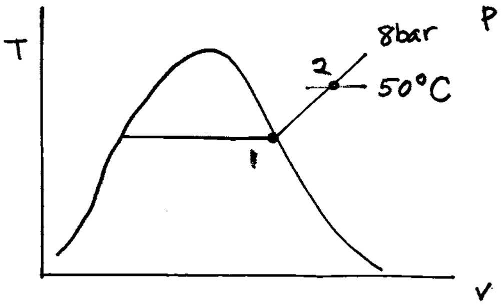
KNOWN: Refrigerant 134a in a piston-cylinder assembly undergoes a process at constant pressure between two known states.

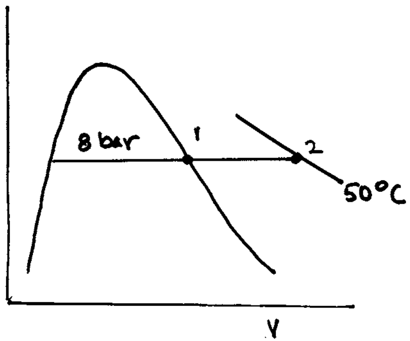
FIND: Determine the work and heat transfer, per unit mass

SCHEMATIC & GIVEN DATA:



$p = 8$ bar
state 1: sat. vapor
$T_2 = 50°C$





ASSUMPTIONS: 1. The R134a is a closed system. 2. Pressure is constant. 3. For the system $\Delta KE = \Delta PE = 0$.

ANALYSIS: With assumption 2 and data from Table A-12

$$W = \int_1^2 p\,dV = p(V_2 - V_1) = m\,p(v_2 - v_1)$$

$$\Rightarrow W/m = p(v_2 - v_1) = 8\,\text{bar}\left|\frac{10^5\,\text{N/m}^2}{1\,\text{bar}}\right|(0.02846 - 0.02547)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= 2.392\ \text{kJ/kg} \longleftarrow$$

Applying an energy balance and data from Table A-12

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$$

$$\Rightarrow \quad Q = \Delta U + W$$

$$= m(u_2 - u_1) + W$$

① $$\Rightarrow \frac{Q}{m} = (u_2 - u_1) + \frac{W}{m}$$

$$= (261.62 - 243.78) + 2.392$$

$$= 20.23\ \text{kJ/kg} \longleftarrow$$

---

1. Using $\frac{W}{m} = p(v_2 - v_1)$, this also reads

$$\frac{Q}{m} = (u_2 - u_1) + p(v_2 - v_1)$$

$$= h_2 - h_1 \qquad (\text{since } p_2 = p_1 = p)$$

with data from Table A-12

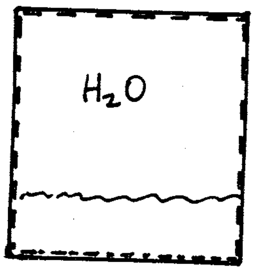
$$\frac{Q}{m} = 284.39 - 264.15 = 20.24\ \text{kJ/kg}$$

PROBLEM 3.52

KNOWN: Saturated liquid water is cooled in a closed, rigid tank to a final state where saturated liquid and vapor are present.

FIND: Determine the heat transfer for the process.

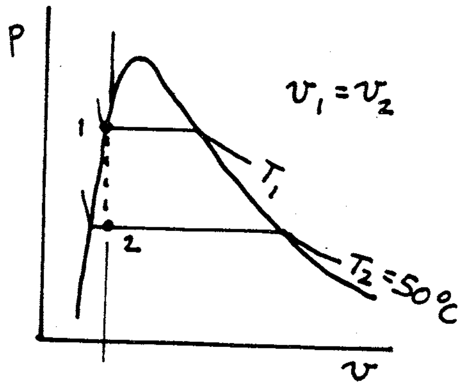
SCHEMATIC & GIVEN DATA:



$T_2 = 50°C$

$m_{f2} = 1999.97$ kg

$m_{g2} = 0.03$ kg



ASSUMPTIONS: 1. The water is the closed system. 2. The volume is constant. 3. Kinetic and potential energy changes are absent. 4. The initial and final states are equilibrium states.

ANALYSIS: The heat transfer is found using an energy balance:

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - \cancel{W}$$

$$\Rightarrow \quad Q = \Delta U = U_2 - U_1$$

where

$$\begin{cases} U_1 = m u_1 = m u_f(T_1) \\ U_2 = m_{f2} u_{f2} + m_{g2} u_{g2} \end{cases}$$

State 1 is fixed by saturated liquid and $V_1 = V_2$, where with data from Table A-2

$$v_2 = \frac{m_{f2} v_{f2} + m_{g2} v_{g2}}{m_{f2} + m_{g2}}$$

$$= \frac{(1999.97\ \text{kg})\left(\frac{1.0121}{10^3}\frac{\text{m}^3}{\text{kg}}\right) + (0.03\ \text{kg})\left(12.032\ \frac{\text{m}^3}{\text{kg}}\right)}{2000\ \text{kg}}$$

$$= 1.193 \times 10^{-3}\ \frac{\text{m}^3}{\text{kg}}$$

Using $v_1 = v_2$, and noting that state 1 is saturated liquid, Table A-2 gives $T_2 = 222°C$ and $u_1 = u_{f1} = 948.2$ kJ/kg

Thus,

$$U_1 = (2000\ \text{kg})(948.2\ \text{kJ/kg}) = 1.896 \times 10^6\ \text{kJ}$$

$$U_2 = (1999.97)(209.32) + (0.03)(2443.5) = 0.419 \times 10^6\ \text{kJ}$$

$$\Rightarrow \quad Q = (0.419 \times 10^6 - 1.896 \times 10^6)\ \text{kJ}$$

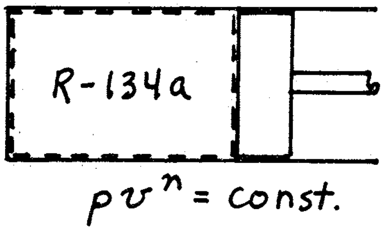
$$= -1.477 \times 10^6\ \text{kJ} \quad \longleftarrow$$

PROBLEM 3.53

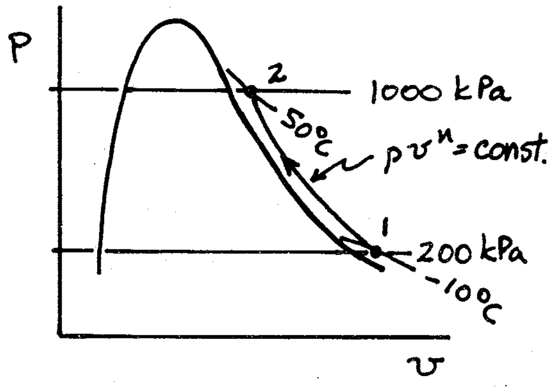
KNOWN: Refrigerant 134a undergoes a process for which pv^n = constant between two specified states.

FIND: Determine the work and heat transfer for the process, each per unit of mass.

SCHEMATIC & GIVEN DATA:



$p_1 = 200\text{ kPa}$
$T_1 = -10°C$
$p_2 = 1000\text{ kPa}$
$T_2 = 50°C$



ASSUMPTIONS: 1. The refrigerant is a closed system. 2. The process is described by pv^n = constant. 3. Kinetic and potential energy effects are negligible.

ANALYSIS: Using Eq. 2.17

$$\frac{W}{m} = \int_{v_1}^{v_2} p\,dv = \int_{v_1}^{v_2} \frac{\text{const.}}{v^n}\,dv$$

$$= \frac{p_2 v_2 - p_1 v_1}{1-n} \qquad (*)$$

To find the polytropic exponent n, we first must determine the initial and final specific volumes. Using Table A-12

$$p_1 = 200\text{ kPa},\ T_1 = -10°C \Rightarrow v_1 = 0.09938\ m^3/kg$$

$$p_2 = 1000\text{ kPa},\ T_2 = 50°C \Rightarrow v_2 = 0.02171\ m^3/kg$$

Thus, for the polytropic process

$$p_1 v_1^n = p_2 v_2^n \Rightarrow \log(p_2/p_1) = n \log(v_1/v_2)$$

$$n = \frac{\log(p_2/p_1)}{\log(v_1/v_2)} = \frac{\log(1000/200)}{\log(.09938/.02171)} = 1.058$$

Inserting values in Eq. (*)

$$\frac{W}{m} = \frac{(1000\,\text{kPa})(0.02171\ m^3/kg) - (200)(.09938)}{(1-1.058)} \left|\frac{10^3\ N/m^2}{1\,\text{kPa}}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= -31.62\ kJ/kg \longleftarrow \text{W/m}$$

The heat transfer is found by using an energy balance

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q - W$$

with $\Delta U = m(u_2 - u_1)$

$$\frac{Q}{m} = (u_2 - u_1) + \frac{W}{m}$$

From Table A-12; $u_1 = 221.50$ kJ/kg and $u_2 = 258.48$ kJ/kg. With the result from Problem 3.36 for W/m

$$\frac{Q}{m} = (258.48 - 221.50) + (-31.62 \text{ kJ/kg})$$

$$= +5.36 \text{ kJ/kg} \longleftarrow \underline{\hspace{6em} Q/m}$$

PROBLEM 3.54

KNOWN: Refrigerant 134a is compressed in a piston-cylinder assembly. The initial and final states are known, and the work is known.

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:

See solution to Problem 3.37.

ASSUMPTIONS: (1) The refrigerant is a closed system. (2) Kinetic and potential energy effects are negligible.

ANALYSIS: The heat transfer is found by using an energy balance

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q - W$$

with $\Delta U = m(u_2 - u_1)$

$$Q = m(u_2 - u_1) + W$$

From Table A-12E ; $u_1 = 97.67$ Btu/lb and $u_2 = 110.68$ Btu/lb. With the result from Problem 3.37 for W

$$Q = (0.04\ \text{lb})(110.68 - 97.67)\,\text{Btu/lb} + (-1.012\ \text{Btu})$$

① $$= -0.4916\ \text{Btu} \longleftarrow Q$$

---

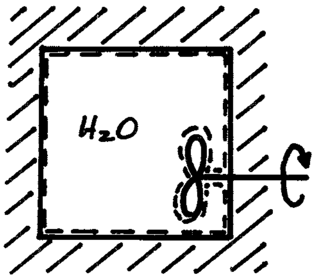
1. The heat transfer is out of the system.

PROBLEM 3.55*

KNOWN: A two-phase, liquid-vapor mixture of H_2O is stirred in a rigid, well-insulated tank until only saturated vapor remains.

FIND: Determine the amount of energy transfer by work.

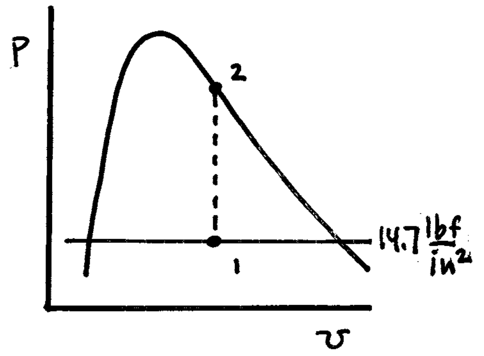
SCHEMATIC & GIVEN DATA:



$V_{f1} = 0.005\ ft^3$

$V_{g1} = 1.2\ ft^3$

$P_1 = 14.7\ lbf/in^2$



ASSUMPTIONS: (1) The H_2O is a closed system. (2) There is no heat transfer. (3) The volume is constant. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The work is determined using the energy balance

$$\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + \Delta U = \cancel{Q}^{0} - W$$

or, with $\Delta U = m(u_2 - u_1)$

$$W = m(u_1 - u_2)$$

Using data from Table A-3E

$$m_{f1} = \frac{V_{f1}}{v_{f1}} = \frac{(0.005\ ft^3)}{(0.01672\ ft^3/lb)} = 0.299\ lb$$

$$m_{g1} = \frac{V_{g1}}{v_{g1}} = \frac{(1.2\ ft^3)}{(26.80\ ft^3/lb)} = 0.045\ lb$$

$$x_1 = m_{g1}/(m_{f1} + m_{g1}) = 0.045/(0.299 + 0.045) = 0.131$$

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 180.10 + (0.131)(1077.6 - 180.10)$$
$$= 297.7\ Btu/lb$$

$$v_2 = v_1 = (V_{f1} + V_{g1})/(m_{f1} + m_{g1}) = (1.205\ ft^3)/(0.299 + 0.045)\ lb = 3.503\ \frac{f}{l}$$

Interpolating in Table A-3E with $v_g = 3.503\ ft^3/lb \Rightarrow u_2 = 1109.2\ \frac{Btu}{lb}$

Finally

$$W = (0.299 + 0.045)(297.7 - 1109.2)$$

① $= -279.16\ Btu \longleftarrow W$

---

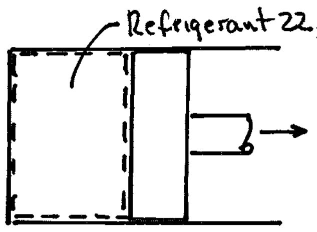
1. The negative sign for work denotes energy transfer into the system, as expected.

PROBLEM 3.56*

KNOWN: Refrigerant 22 expands in a piston-cylinder assembly from a known initial state to a known final pressure. The pressure-specific volume relation is specified.

FIND: Determine the work and heat transfer, each per unit mass.

SCHEMATIC & GIVEN DATA:

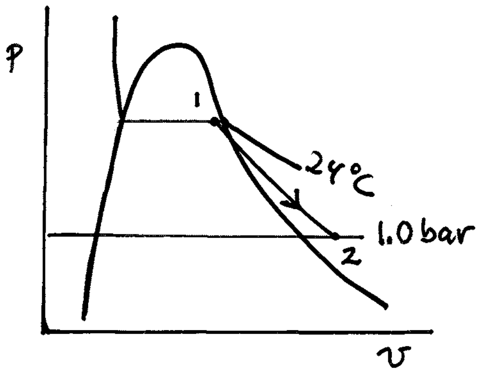


$T_1 = 24°C$

$x_1 = 0.95$

$p_2 = 1.0$ bar

$pv = \text{constant}$



ASSUMPTIONS: (1) The R22 is the closed system. (2) During the process, $pv = \text{constant}$. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The work is determined using Eq. 2.17 along with the given p-v relation

$$W = \int_{V_1}^{V_2} p\,dV = m\int_{v_1}^{v_2} p\,dv = m\int_{v_1}^{v_2} \frac{\text{const.}}{v}\,dv$$

With $p \cdot v = \text{const.} = p_1 v_1$

$$\frac{W}{m} = (p_1 v_1)\ln\left(\frac{v_2}{v_1}\right)$$

Using data from Table A-7; $v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 0.0221 \text{ m}^3/\text{kg}$, $p_1 = 10.164$ bar

Thus $$v_2 = \left(\frac{p_1}{p_2}\right)v_1 = \frac{(10.164)(0.0221)}{(1.0)} = 0.2246 \text{ m}^3/\text{kg}$$

① and $$\frac{W}{m} = (10.164 \text{ bar})\left(0.0221 \frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right| \ln\left(\frac{0.2246}{0.0221}\right) = 52.1 \frac{\text{kJ}}{\text{kg}} \longleftarrow \frac{W}{m}$$

The heat transfer is found using the energy balance. With assumption 3

$$\overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} + \Delta U = Q - W$$

and $\Delta U = m(u_2 - u_1)$

$$Q/m = (u_2 - u_1) + W/m$$

From Table A-7 at 24°C, $x_1 = 0.95$; $u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 225.84$ kJ/kg. Also, interpolating in Table A-9 at $p_2 = 1.0$ bar, $v_2 = 0.2246$ m³/kg gives $u_2 = 215.74$ kJ/kg. Finally

$$Q/m = (215.74 - 225.84)\frac{\text{kJ}}{\text{kg}} + (52.1 \text{ kJ/kg})$$

① $$= 42.0 \text{ kJ/kg} \longleftarrow Q/m$$

---

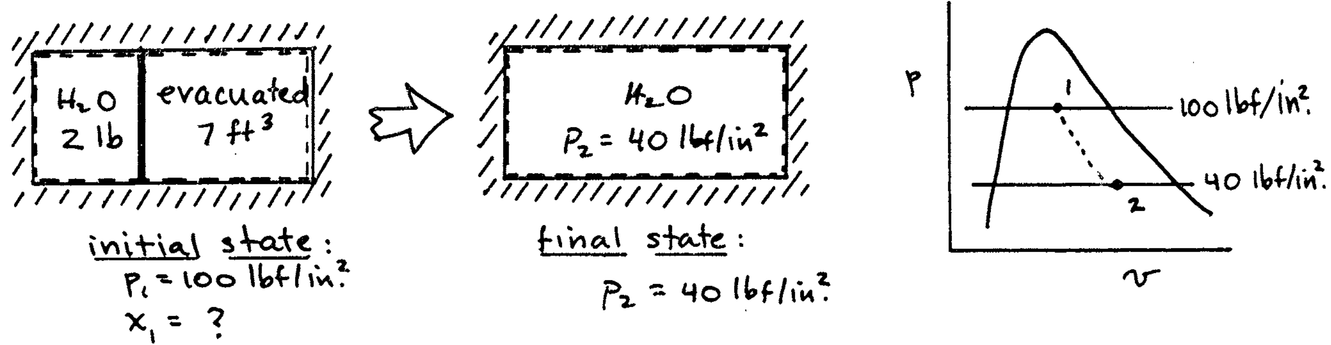
1. The respective positive signs for heat and work denote that for this process there is energy transfer by heat into the system and energy transfer by work out of the system.

PROBLEM 3.57

KNOWN: A known amount of H_2O at a specified initial pressure is confined to one side of a rigid, well insulated container by a partition. The parition is removed and the H_2O expands into the initially evacuated side, reaching a specified pressure when equilibrium is attained.

FIND: Determine the initial quality and the overall volume of the container.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The contents of the tank are the closed system. (2) The expansion occurs with no work or heat transfer. (3) The volume of the tank is constant. (4) The are no changes in kinetic or potential energy.

ANALYSIS: First, we solve this problem analytically. Then, we use the software Interactive Thermodynamics: IT. Begin with the ~~total~~ volume

$$V = m v_1 + 7 \text{ ft}^3$$

where m = 2 lb. Also, noting that $V = m v_2$, we get

$$v_2 = \frac{V}{m} = v_1 + \frac{7 \text{ ft}^3}{2 \text{ lb}} \Rightarrow v_2 = v_1 + 3.5 \text{ ft}^3/\text{lb} \qquad (1)$$

① Now, v_1 and v_2 can be expressed in terms of the qualities x_1 and x_2, respectively

$$v_1 = v_{f_1} + x_1(v_{g_1} - v_{f_1})$$

$$v_2 = v_{f_2} + x_2(v_{g_2} - v_{f_2})$$

From (1)

$$v_{f_2} + x_2(v_{g_2} - v_{f_2}) = v_{f_1} + x_1(v_{g_1} - v_{f_1}) + 3.5$$

Using data from Table A-3E

$$0.01715 + x_2(10.50 - 0.01715) = 0.01774 + x_1(4.434 - 0.01774) + 3.5$$

or

$$x_2(10.483) = x_1(4.4163) + 3.50059 \qquad (2)$$

Now, an overall energy balance reads $\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = \cancel{Q} - \cancel{W} \Rightarrow \Delta U = 0$, or $u_2 = u_1$. In terms of saturated liquid and saturated vapor data

$$u_{f_2} + x_2(u_{g_2} - u_{f_2}) = u_{f_1} + x_1(u_{g_1} - u_{f_1})$$

Inserting values from Table A-3E

PROBLEM 3.57 (Contd.)

$$236.03 + x_2(1092.3 - 236.03) = 298.3 + x_1(1105.8 - 298.3)$$

or

$$x_2(856.27) = x_1(807.5) + 62.27 \qquad (3)$$

Expressions (2) and (3) each involve the unknown qualities x_1 and x_2. Solving simutaneously

$$\left.\begin{aligned} x_1 &= 0.5006 \\ x_2 &= 0.5448 \end{aligned}\right\} \text{analytical resuts} \longleftarrow x_1$$

Using these results gives $v_1 = 2.2284$ ft³/lb, $v_2 = 5.7284$ ft³/lb, and

$$V = 11.457 \text{ ft}^3 \longleftarrow V$$

The IT code is much less involved than the analytical solution. Seven expressions are written that involve the seven unknowns: $V, v_1, v_2, x_1, x_2, u_1, u_2$. The equation solver solves automatically, without the need to further manipulate the expressions algebraically. The code follows

```
p1 = 100  // lbf/in.²
p2 = 40
m = 2  // lb
V = m * v1 + 7 // ft³
v2 = v1 + 3.5
v1 = vsat_Px("Water/Steam", p1,x1)
v2 = vsat_Px("Water/Steam", p2, x2)
u1 = usat_Px("Water/Steam", p1, x1)
u1 = u2
u2 = usat_Px("Water/Steam", p2, x2)
```

②

```
V    11.46
x1   0.5007
```

} IT Resuts

---

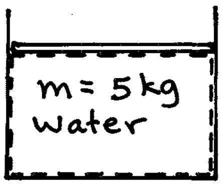
1. This assumes that state 2 is in the two-phase liquid vapor region. If state 2 is actually in the superheated vapor region, x_1 and/or x_2 would turn out to be physically unreasonable, and it would be necessary to modify the solution method.
2. There is excellent agreement between the analytical and computer results.

PROBLEM 3.58

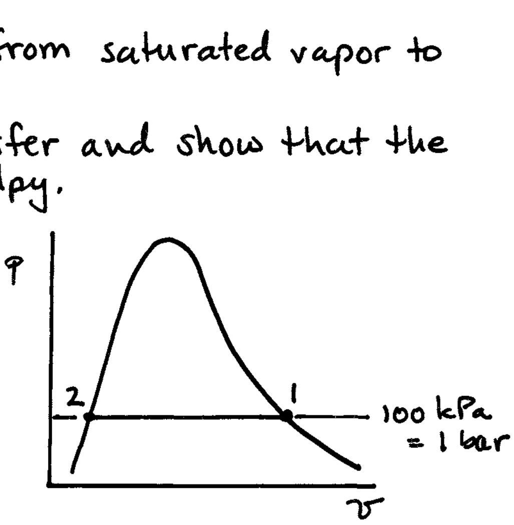
KNOWN: Water is cooled at constant pressure from saturated vapor to saturated liquid.

FIND: Determine the work and heat transfer and show that the heat transfer equals the change in enthalpy.

SCHEMATIC & GIVEN DATA:



p = 100 kPa = 1 bar



ASSUMPTIONS: (1) The water is the closed system. (2) The pressure is constant. (3) Kinetic and potential energy effects are neglected.

ANALYSIS: To determine the work, use Eq. 2.17 along with assumption 2

$$W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1) = mp(v_2 - v_1) \qquad (1)$$

Using data from Table A-3 at p = 1 bar; $v_1 = v_g = 1.694\ \text{m}^3/\text{kg}$ and $v_2 = v_f = 1.0432\times10^{-3}\ \text{m}^3/\text{kg}$. Thus

$$W = (5\ \text{kg})(100\ \text{kPa})\left|\frac{10^3\ \text{N/m}^2}{1\ \text{kPa}}\right|(1.0432\times10^{-3} - 1.694)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= -846.5\ \text{kJ} \longleftarrow W$$

The heat transfer is found using the energy balance; $\overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} + \Delta U = Q - W$

$$Q = \Delta U + W$$

or, with $\Delta U = m(u_2 - u_1)$

$$Q = m(u_2 - u_1) + W \qquad (2)$$

With $u_1 = u_g = 2506.1\ \text{kJ/kg}$ and $u_2 = u_f = 417.36\ \text{kJ/kg}$ from Table A-3

$$Q = (5\ \text{kg})(417.36 - 2506.1)\frac{\text{kJ}}{\text{kg}} + (-846.5\ \text{kJ})$$

$$= -11290\ \text{kJ} \longleftarrow Q$$

Combining (1) and (2)

$$Q = m(u_2 - u_1) + mp(v_2 - v_1)$$

$$= m[(u_2 + p_2 v_2) - (u_1 + p_1 v_1)]$$

With $h = u + pv$ and data from Table A-3

① $$Q = m[h_2 - h_1] = (5)[417.46 - 2675.5] = -11290\ \text{kJ}$$

---

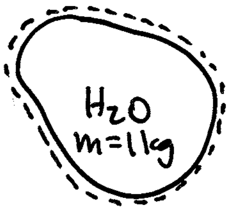
1. The result developed here for a closed system is only valid for a constant pressure process.

PROBLEM 3.59

KNOWN: Saturated solid water is heated at constant pressure to saturated liquid.

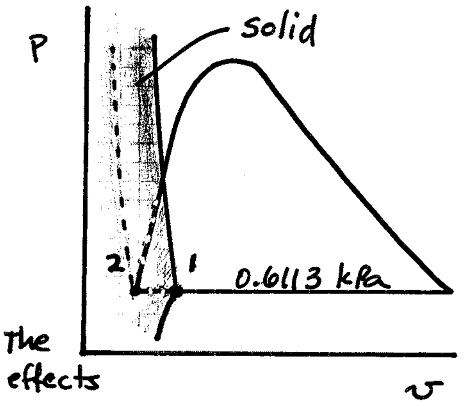
FIND: Determine the work and heat transfer. Show that the heat transfer equals the change in enthalpy.

SCHEMATIC & GIVEN DATA:



State 1: triple point
sat. solid

State 2: sat. liquid



ASSUMPTIONS: (1) The H_2O is a closed system. (2) The pressure is constant. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The work is determined using Eq. 2.17

$$W = \int_{V_1}^{V_2} p\,dV = mp(v_2 - v_1) \qquad (1)$$

From Table A-6; $v_1 = v_{i@T_{tp}} = 1.0908\times10^{-3}\ \mathrm{m^3/kg}$. Also, from Table A-2; $v_2 = v_{f@T_{tp}} = 1.002\times10^{-3}\ \mathrm{m^3/kg}$. Thus

$$W = (1\ \mathrm{kg})(0.6113\ \mathrm{kPa})(1.002\times10^{-3} - 1.0908\times10^{-3})\frac{\mathrm{m^3}}{\mathrm{kg}}\left|\frac{1\ \mathrm{kN/m^2}}{1\ \mathrm{kPa}}\right|\left|\frac{1\ \mathrm{kJ}}{1\ \mathrm{kN\cdot m}}\right|$$

$$= -5.428\times10^{-5}\ \mathrm{kJ} \longleftarrow \mathrm{W}$$

The heat transfer is found using an energy balance.

$$\Delta KE + \Delta PE + \Delta U = Q - W$$

or, with $\Delta U = m(u_2 - u_1)$

$$Q = m(u_2 - u_1) + W \qquad (2)$$

From Table A-6; $u_1 = -333.40$ kJ/kg, and from Table A-2; $u_2 = 0.00$. Thus

$$Q = (1\ \mathrm{kg})[0 - (-333.40)]\frac{\mathrm{kJ}}{\mathrm{kg}} + (-5.428\times10^{-5}\ \mathrm{kJ})$$

$$= 333.4\ \mathrm{kJ} \longleftarrow Q$$

Combining Eqs. (1) and (2)

$$Q = m(u_2 - u_1) + mp(v_2 - v_1)$$

$$= m[(u_2 + pv_2) - (u_1 + pv_1)]$$

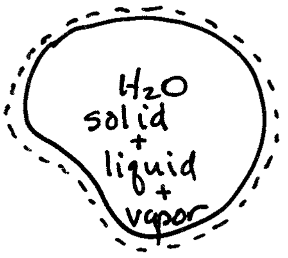
$$= m(h_2 - h_1)$$

PROBLEM 3.60

KNOWN: A three-phase, solid-liquid-vapor mixture of H_2O at the triple point is heated at constant pressure to saturated vapor.

FIND: Determine the work and heat transfer.

SCHEMATIC & GIVEN DATA:



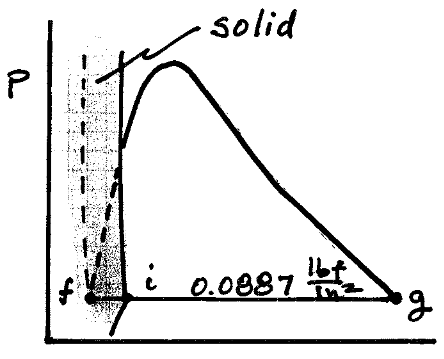
$m_{i_1} = 1$ lb

$m_{f_1} = 1$ lb

$m_{g_1} = 0.2$ lb

$m_{g_2} = 2.2$ lb

triple point



ASSUMPTIONS: (1) The H_2O is a closed system. (2) The pressure is constant. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The work is determined using Eq. 2.17

$$W = \int_{V_1}^{V_2} V dp = p(V_2 - V_1)$$

From Table A-6E; $p = 0.0887$ lbf/in², $T = 32.018$ °F. Also

$$v_i = 0.01747 \text{ ft}^3/\text{lb}, \quad v_g = 3302 \text{ ft}^3/\text{lb}$$

From Table A-2E; $v_f = 0.01602$ ft³/lb. Thus

$$V_1 = m_{i_1} v_i + m_{f_1} v_f + m_{g_1} v_g$$
$$= (1)(0.01747) + (1)(0.01602) + (.2)(3302) = 660.4 \text{ ft}^3$$
$$V_2 = (2.2)(3302) = 7264.4 \text{ ft}^3$$

and

$$W = \left(0.0887 \frac{lbf}{in^2}\right)(7264.4 - 660.4) \text{ ft}^3 \left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$
$$= 108.4 \text{ Btu} \longleftarrow W$$

To determine the heat transfer use the energy balance

$$\Delta \cancel{KE}^{0} + \Delta \cancel{PE}^{0} + \Delta U = Q - W$$

or,

$$Q = \Delta U + W$$

Again, with data from the tables

$$U_1 = m_{i_1} u_i + m_{f_1} u_f + m_{g_1} u_g = (1)(-143.34) + (1)(0) + (.2)(1021.2)$$
$$= 60.9 \text{ Btu}$$
$$U_2 = m_{g_2} u_g = (2.2)(1021.2) = 2246.6 \text{ Btu}$$

Finally

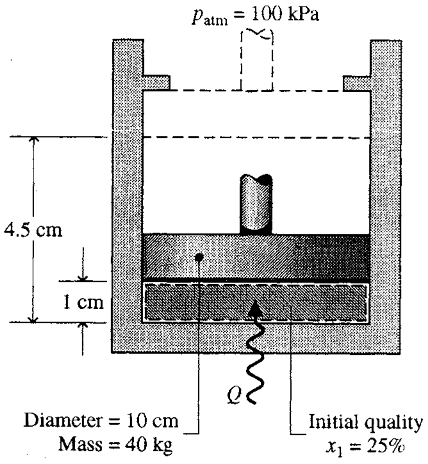
$$Q = (2246.6 - 60.9) + (108.4) = 2294.1 \text{ Btu} \longleftarrow Q$$

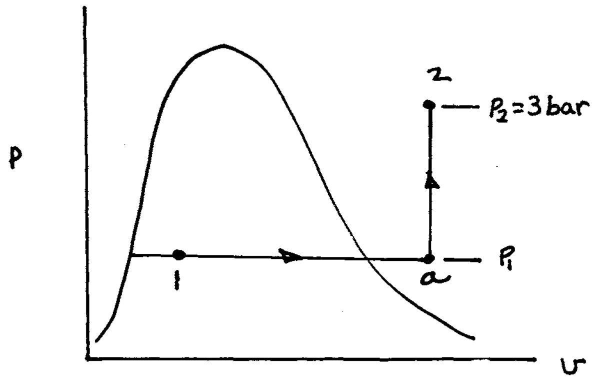
PROBLEM 3.61

KNOWN: Water contained in a piston-cylinder assemby is heated at constant pressure followed by heating at constant volume.

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The water is a closed system. 2. The pressure is constant until the piston hits the stops. 3. Friction between the piston and cylinder wall can be ignored. 4. For the system, ΔPE, ΔKE can be ignored. 5. g is constant.

ANALYSIS: To begin, fix the three states located by dots on the p-v diagram. State 1 is fixed by $x_1 = 25\%$ and P_1, which is found from a force balance on the piston: The force exerted by the water on the lower face of the piston equals the piston weight plus the force exerted on the top face by the atmosphere:

$$P_1 A = m_{pist}\, g + P_{atm} A \quad , \quad A = \pi D^2_{pist}/4$$

$$\Rightarrow \quad P_1 = \frac{m_{pist}\, g}{A} + P_{atm} = \frac{(40\,kg)(9.81\,m/s^2)}{\frac{\pi}{4}(0.1\,m)^2}\left|\frac{1\,N}{kg\cdot m/s^2}\right|\left|\frac{bar}{10^5\,N/m^2}\right| + (100\,kPa)\left|\frac{1\,bar}{10^2\,kPa}\right|$$

$$= 1.5\ bar$$

State a is fixed by $P_a = P_1 = 1.5$ bar and the specific volume, v_a. From the given geometry, $V_a = 4.5 V_1$ or $v_a = 4.5 v_1$, where

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = \frac{1.0528}{10^3} + 0.25\left(1.159 - \frac{1.0528}{10^3}\right) = 0.29054\ m^3/kg$$

$$\rightarrow \quad v_a = 4.5(0.29054) = 1.3074\ m^3/kg$$

Since $v_a > v_g(1.5\,bar)$, state a falls in the superheated vapor region, as shown in the p-v diagram. State 3 is fixed by $v_2 = v_a$ and $P_2 = 3$ bar.

The total mass of water is

$$m = \frac{V}{v_1} = \frac{\pi/4\,(0.1\,m)^2(0.01\,m)}{0.29054\ m^3/kg} = 2.703\times10^{-4}\ kg$$

PROBLEM 3.61 (Contd.)

Since there is no work associated with the constant volume portion of the process, the total work is obtained as the water undergoes the constant pressure expansion from 1 to a:

$$W = \int_1^a p\,dV = p[V_a - V_1] = p[V_2 - V_1] = mp(v_2 - v_1)$$

That is, with $v_2 = 4.5 v_1$

$$W = (2.703 \times 10^{-4}\ \text{kg})(1.5\ \text{bar})\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|(3.5)\left(0.29054\ \frac{\text{m}^3}{\text{kg}}\right)\left|\frac{1\ \text{J}}{1\ \text{N}\cdot\text{m}}\right|$$

$$= 41.23\ \text{J} \qquad \longleftarrow$$

An energy balance reads

$$\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = Q - W$$

$$\Rightarrow \quad Q = \Delta U + W$$

$$= m(u_2 - u_1) + W$$

where

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1})$$

$$= 466.94 + 0.25(2519.7 - 466.94) = 980.13\ \text{kJ/kg}$$

To find u_2, interpolate in Table A-4 at 3 bar using v_2: $u_2 = 3263.53$ kJ/kg. Then

$$Q = (2.703 \times 10^{-4}\ \text{kg})(3263.53 - 980.13)\frac{\text{kJ}}{\text{kg}}\left|\frac{10^3\ \text{J}}{\text{kJ}}\right| + 41.23\ \text{J}$$

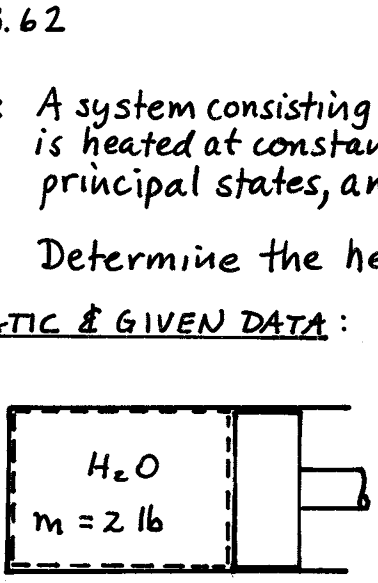
$$= 658.43\ \text{J} \qquad \longleftarrow$$

PROBLEM 3.62

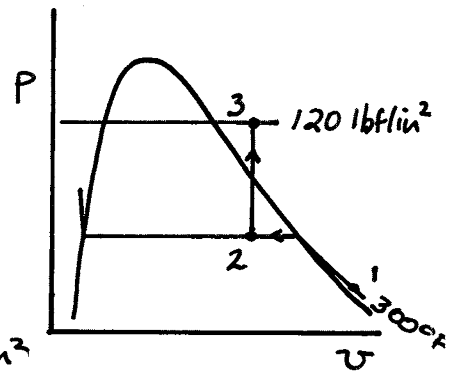
KNOWN: A system consisting of H_2O is compressed isothermally and then is heated at constant volume. Data are known at each of the principal states, and the work is known for process 1-2.

FIND: Determine the heat transfer for each process.

SCHEMATIC & GIVEN DATA:



State 1: $T_1 = 300°F$
$V_1 = 20\ ft^3$
$W_{12} = -90.8$ Btu
State 2: $T_2 = T_1$
$V_2 = 9.05\ ft^3$
State 3: $V_3 = V_2$
$P_3 = 120\ lbf/in^2$



ASSUMPTIONS: (1) The H_2O is a closed system. (2) Process 1-2 occurs at constant temperature and process 2-3 occurs at constant volume. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $T_1 = 300°F$, $v_1 = V_1/m = 10\ ft^3/lb$. From Table A-4E; $u_1 = 1104.1$ Btu/lb

State 2: $T_2 = 300°F$, $v_2 = V_2/m = 9.05/2 = 4.525\ ft^3/lb \Rightarrow$ two-phase (Table A-2E)

$$x_2 = \frac{v_2 - v_{f2}}{v_{g2} - v_{f2}} = \frac{4.525 - 0.01745}{6.472 - 0.01745} = 0.6984$$

$$u_2 = u_{f2} + x_2(u_{g2} - u_{f2}) = 269.5 + (.6984)(1100.0 - 269.5) = 849.5\ \text{Btu/lb}$$

State 3: $P_3 = 120\ lbf/in^2$, $v_3 = 4.525$. Interpolating in Table A-4E; $u_3 = 1166.3$ Btu/lb

Now, using energy balances for each process

Process 1-2: $\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q_{12} - W_{12}$

$$Q_{12} = m(u_2 - u_1) + W_{12}$$
$$= (2\ lb)(849.5 - 1104.1)\ \text{Btu/lb} + (-90.8\ \text{Btu})$$
$$= -600\ \text{Btu} \longleftarrow Q_{12}$$

Process 2-3: $\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q_{23} - \cancelto{0}{W_{23}}$

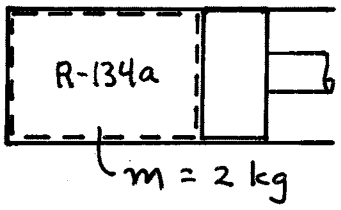
$$Q_{23} = m(u_3 - u_2)$$
$$= (2)(1166.3 - 849.5)$$
$$= 633.6\ \text{Btu} \longleftarrow Q_{23}$$

PROBLEM 3.63*

KNOWN: Refrigerant 134a undergoes a constant-pressure process. The pressure and the initial and final volumes are specified.

FIND: Determine the work and heat transfer for the process.

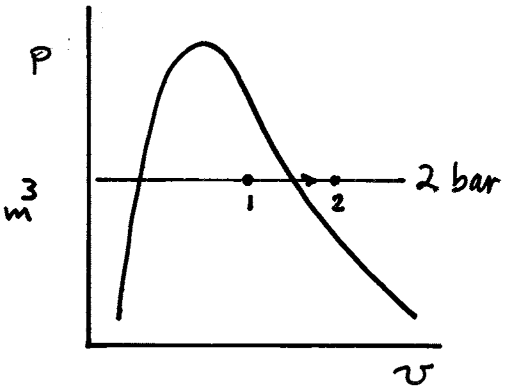
SCHEMATIC & GIVEN DATA:



$p_1 = p_2 = 2$ bar

$V_1 = 0.12\ m^3$

$V_2 = 2.0\ V_1 = 0.24\ m^3$



ASSUMPTIONS: (1) The refrigerant is a closed system. (2) The pressure is constant. (3) Kinetic and potential energy effects are constant.

ANALYSIS: The work is determined using Eq. 2.17

$$W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

$$= (2\ \text{bar})(0.24 - 0.12)\ m^3 \left|\frac{10^5\ N/m^2}{1\ \text{bar}}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 24\ kJ \longleftarrow W$$

The heat transfer is found using the energy balance

$$\overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} + \Delta U = Q - W$$

or, with $\Delta U = m(u_2 - u_1)$

$$Q = m(u_2 - u_1) + W$$

From Table A-11, with $v_1 = V_1/m = 0.12/2 = 0.06\ m^3/kg$ and $p = 2$ bar

$$x_1 = \frac{v_1 - v_{f1}}{v_{g1} - v_{f1}} = \frac{0.06 - 0.7532\times10^{-3}}{0.0993 - 0.7532\times10^{-3}} = 0.6012$$

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 36.69 + (0.6012)(221.43 - 36.69)$$

$$= 147.76\ kJ/kg$$

And, from Table A-12, at $v_2 = 0.12\ m^3/kg$ and $p = 2$ bar

$$u_2 = 255.66\ kJ/kg$$

Thus

$$Q = (2\ kg)(255.66 - 147.76)\ kJ/kg + (24\ kJ)$$

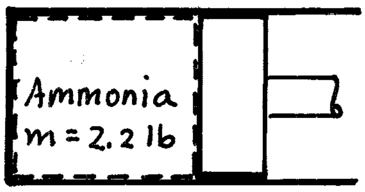
$$= 239.8\ kJ \longleftarrow Q$$

PROBLEM 3.64

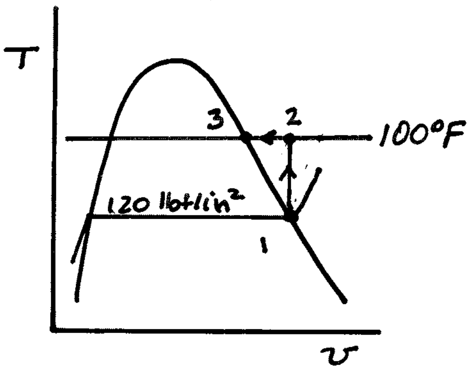
KNOWN: Ammonia undergoes two processes in series in a piston-cylinder assembly; the first at constant volume and the second at constant temperature. Data are known at each of three end states and the heat transfer is given for the second process.

FIND: Determine (a) the heat transfer for the first process, and (b) the work for the second process.

SCHEMATIC & GIVEN DATA:



$p_1 = 120$ lbf/in²
$x_1 = 1.0$
$T_2 = 100°F$
$v_2 = v_1$
$T_3 = T_2$
$x_3 = 1.0$
$Q_{23} = -98.9$ Btu



ASSUMPTIONS: (1) The ammonia is closed system. (2) The processes occur at constant volume and constant temperature, respectively. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: The heat transfer Q_{12} and work W_{23} are found using energy balances. First, fix each of the principal states.

State 1: $p_1 = 120$ lbf/in², $x_1 = 1.0 \Rightarrow u_1 = 572.73$ Btu/lb (Table A-14E)
$v_1 = 2.4745$ ft³/lb

State 2: $v_2 = v_1$, $T_2 = 100°F$; Interpolating in Table A-15E
$u_2 \approx 589.17$ Btu/lb

State 3: $T_3 = 100°F$, $x_3 = 1.0 \Rightarrow u_3 = 576.51$ Btu/lb (Table A-13E)

(a) The energy balance for process 1-2 reduces to

$$\Delta \cancel{KE}^{0} + \Delta \cancel{PE}^{0} + m(u_2 - u_1) = Q_{12} - W_{12}$$

and

$$Q_{12} = m(u_2 - u_1)$$

$$= (2.2 \text{ lb})(589.17 - 572.73) \text{ Btu/lb} = 36.17 \text{ Btu} \longleftarrow Q_{12}$$

(b) Similarly, for process 2-3

$$\Delta \cancel{KE}^{0} + \Delta \cancel{PE}^{0} + m(u_3 - u_2) = Q_{23} - W_{23}$$

$$W_{23} = Q_{23} - m(u_3 - u_2)$$

$$= (-98.9 \text{ Btu}) - (2.2 \text{ lb})(576.51 - 589.17) \text{ Btu/lb}$$

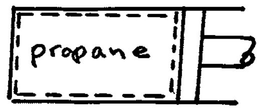
$$= -71.05 \text{ Btu} \longleftarrow W_{23}$$

PROBLEM 3.65*

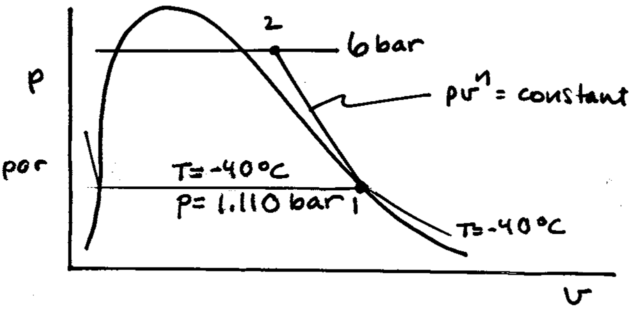
KNOWN: Propane vapor is compressed according to pv^n = constant between specified end states.

FIND: Determine the work and heat transfer for the process, each per unit mass.

SCHEMATIC & GIVEN DATA:



$T_1 = -40\,°C$, sat. vapor
$T_2 = 80\,°C$
$P_2 = 6$ bar



ASSUMPTIONS: 1. The propane is the closed system. 2. For the system, kinetic and potential energy effects can be ignored. 3. The process is described by pv^n = constant.

ANALYSIS: The work can be evaluated as

$$W = \int_1^2 p\,dV \Rightarrow \frac{W}{m} = \int_1^2 p\,dv = \int_1^2 \frac{c}{v^n}\,dv = \frac{p_2v_2 - p_1v_1}{1-n}$$

This requires n, which can be evaluated using data from Tables A-16, A-18

$$p_2 v_2^n = p_1 v_1^n \Rightarrow \ln p_2 + n \ln v_2 = \ln p_1 + n \ln v_1$$

$$\Rightarrow \quad n \ln v_1/v_2 = \ln p_2/p_1 \quad \text{or} \quad n = \frac{\ln(p_2/p_1)}{\ln(v_1/v_2)}$$

Then

$$n = \frac{\ln(6/1.110)}{\ln(0.3798/0.1045)} \Rightarrow n = 1.308$$

So,

$$\frac{W}{m} = \frac{[6\text{ bar})(0.1045\,\frac{m^3}{kg}) - (1.110)(0.3798)]}{1-1.308}\left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= -66.7\ kJ/kg$$

An energy balance reduces to $\Delta U = Q - W$, or

$$Q = \Delta U + W \Rightarrow Q/m = (u_2 - u_1) + W/m$$

$$\frac{Q}{m} = (552.7 - 381.0) + (-66.7)$$
$$= 105.0\ kJ/kg$$

PROBLEM 3.66

KNOWN: A system consisting of ammonia undergoes a cycle composed of three processes.

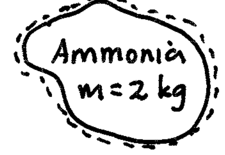
FIND: Sketch p-v and T-v diagrams. Determine the cycle net work and the heat transfer for each process.

SCHEMATIC & GIVEN DATA:

Process 1-2: constant volume from $p_1 = 10$ bar, $x_1 = 0.6$ to saturated vapor

Process 2-3: Constant temperature, $Q_{23} = 228$ kJ

Process 3-1: constant pressure



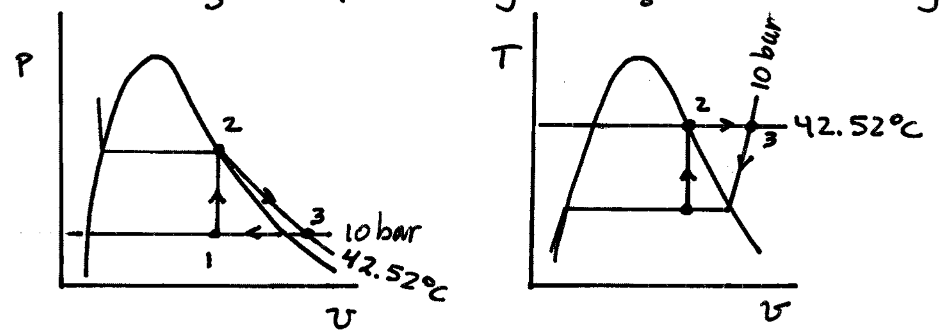
ASSUMPTIONS: (1) Closed system. (2) Neglect kinetic and potential energy effects.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 10$ bar, $x_1 = 0.6$. Thus $u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 296.10 + (.6)(1334.66 - 296.10)$
$= 919.24$ kJ/kg (Table A-14)
$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 0.07776\ \text{m}^3/\text{kg}$

State 2: Sat. vapor, $v_2 = v_1$. Interpolating in Table A-13, at $v_g = 0.07776\ \text{m}^3/\text{kg}$ gives $u_2 \approx 1341.26$ kJ/kg and $T_2 \approx 42.52°\text{C}$.

State 3: $p_3 = p_1 = 10$ bar, $T_3 = T_2 = 42.52°\text{C}$. Interpolating in Table A-15
$u_3 = 1374.95$ kJ/kg and $v_3 \approx 0.14027\ \text{m}^3/\text{kg}$



The required energy transfers are determined by using energy balances.

Process 1-2: $\overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} + m(u_2 - u_1) = Q_{12} - \overset{0}{\cancel{W_{12}}} \Rightarrow Q_{12} = m(u_2 - u_1) = 844.0\ \text{kJ} \longleftarrow Q_{12}$

Process 2-3: $\overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} + m(u_3 - u_2) = Q_{23} - W_{23} \Rightarrow W_{23} = Q_{23} - m(u_3 - u_2)$
$= 160.6$ kJ

Process 3-1: Using Eq. 2.17; $W_{31} = \int_{V_3}^{V_1} p\,dV = \int_{v_3}^{v_1} m\,p\,dv = m\,p\,(v_1 - v_3)$

$$= (2\ \text{kg})(10\ \text{bar})(0.07776 - 0.14027)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$= -125.0$ kJ

and $\overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} + m(u_1 - u_3) = Q_{31} - W_{31} \Rightarrow Q_{31} = m(u_1 - u_3) + W_{31}$
$= -1036.4\ \text{kJ} \longleftarrow Q_{31}$

① ② $W_{\text{cycle}} = W_{12} + W_{23} + W_{31} = 0 + 160.6 + (-125.0) = 35.6\ \text{kJ} \longleftarrow W_{\text{cycle}}$

---

1. The positive value for W_{cycle} indicates the cycle is a power cycle.
2. A check of the net heat transfer confirms that $Q_{\text{cycle}} = W_{\text{cycle}}$.

PROBLEM 3.67

KNOWN: One kg of water undergoes a thermodynamic cycle composed of four processes.

FIND: Determine the thermal efficiency

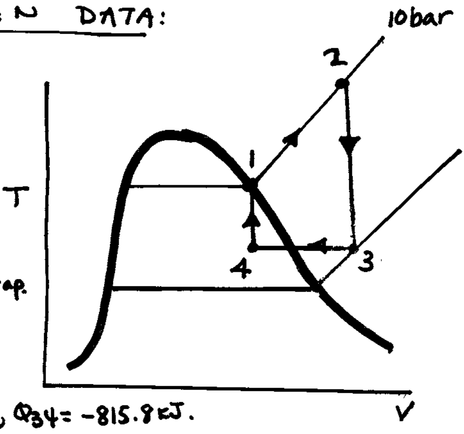
SCHEMATIC & GIVEN DATA:

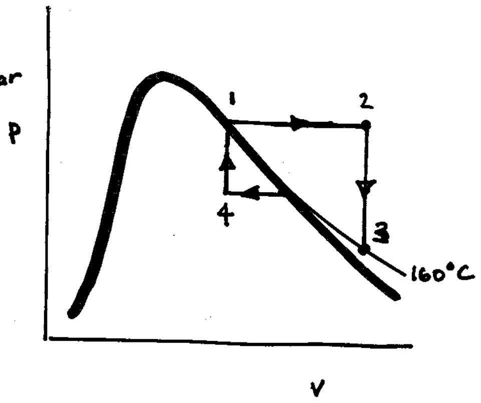
1-2: constant pressure at 10 bar from sat. vap.

2-3: constant volume to 5 bar, 160°C

3-4 isothermal compression, $Q_{34} = -815.8$ kJ.

4-1 constant volume





ASSUMPTIONS: 1. The water is the closed system. 2. Volume change is the only work mode. 3. Kinetic and potential energy effects can be ignored.

ANALYSIS: The thermal efficiency of a power cycle is $\eta = W_{cycle}/Q_{in}$ (see discussion of Eqs. 2.42, 43), where $W_{cycle} = W_{12} + W_{23} + W_{34} + W_{41}$.

Process 1-2: State 1 is fixed. State 2 is fixed by $P_2 = 10$ bar, $v_2 = v_3$. From Table A-4

$$v_3 = 0.3835 \frac{m^3}{kg}, \quad u_3 = 2575.2 \text{ kJ/kg}$$

$$\Rightarrow W_{12} = \int_1^2 p\,dV = p\,m(v_2 - v_1) = (10 \text{ bar})\left|\frac{10^5 N/m^2}{1 \text{ bar}}\right|(1 \text{ kg})(0.3835 - 0.1944)\frac{m^3}{kg}\left|\frac{1 \text{ kJ}}{10^3 N\cdot m}\right|$$

$$= 189.1 \text{ kJ}$$

An energy balance reduces to give $Q = \Delta U + W$, or

$$Q_{12} = m(u_2 - u_1) + W = (1\text{kg})[3231.8 - 2583.6] + 189.1 \text{ kJ} = 837.3 \text{ kJ}$$

where u_2 is obtained from Table A-3 using $P_2, v_2 (= v_3)$.

Process 2-3: $W_{23} = 0$

$$Q_{23} = m(u_3 - u_2) + \cancel{W_{23}}^{0} = (1\text{kg})[2575.2 - 3231.8] = -656.6 \text{ kJ}$$

Process 3-4: $Q_{34} = -815.8$ kJ (given). Then, $\Delta U = Q_{34} - W_{34}$ gives

$$W_{34} = Q_{34} - \Delta U = Q_{34} - m(u_4 - u_3)$$

State 4 is fixed by $T_4 = 160°C$, $v_4 = v_1$

$$\therefore \; x_4 = \frac{v_4 - v_f}{v_g - v_f} = \frac{0.1944 - 1.102/10^3}{0.3071 - 1.102/10^3} = 0.6317$$

$$\Rightarrow \quad u_4 = u_f + x_4(u_g - u_f) = 674.86 + (0.6317)(2568.4 - 674.86) = 1871 \text{ kJ/kg}$$

$$\Rightarrow w_{34} = -815.8 - (1)[1871 - 2575.2] = -111.6 \text{ kJ}$$

PROBLEM 3.67 (Contd.)

Process 4–1: $W_{41} = 0$, and

$$Q_{41} = \Delta U + \overset{0}{\cancel{W_{41}}} = m(u_1 - u_4)$$

$$= (1\,kg)[2583.6 - 1871]\frac{kJ}{kg} = 712.6\,kJ$$

The net work is then

$$W_{cycle} = W_{12} + W_{23} + W_{34} + W_{41}$$

① $$= 189.1 + 0 + (-111.6) + 0 = 77.5\,kJ$$

To obtain Q_{in},

$$Q_{in} = Q_{12} + Q_{41} = 837.3 + 712.6 = 1549.9\,kJ$$

Then

$$\eta = \left(\frac{77.5}{1549.9}\right) = 0.05 \quad (5\%)$$

---

① As a check, note that for every cycle $Q_{cycle} = W_{cycle}$.

$$Q_{cycle} = Q_{12} + Q_{23} + Q_{34} + Q_{41}$$

$$= 837.3 + (-656.6) + (-815.8) + 712.6 = 77.5\,kJ$$

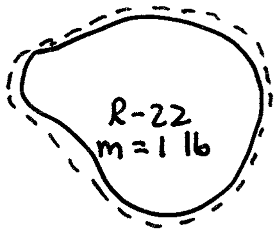
which agrees with Wcycle calculated using the work quantities.

PROBLEM 3.68

KNOWN: A system consisting of Refrigerant 22 undergoes a cycle composed of three processes.

FIND: Sketch p-v and T-v diagrams. Determine the cycle net work and the heat transfer for each process.

SCHEMATIC & GIVEN DATA:



Process 1-2: Constant pressure from $p_1 = 30$ lbf/in² $x_1 = 0.95$ to $T_2 = 40°F$

Process 2-3: isothermal with $W_{23} = -11.82$ Btu to saturated vapor

Process 3-1: Adiabatic expansion; $Q_{31} = 0$

ASSUMPTIONS: (1) The R-22 is a closed system. (2) kinetic and potential energy effects are negligible.

ANALYSIS: First, using data from the refrigerant tables, fix each state.

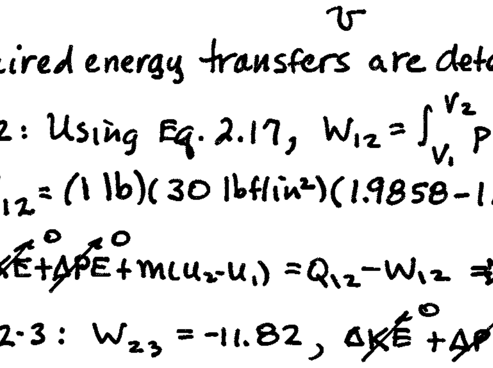
State 1: Table A-8E at $p_1 = 30$ lbf/in² and $x_1 = 0.95$

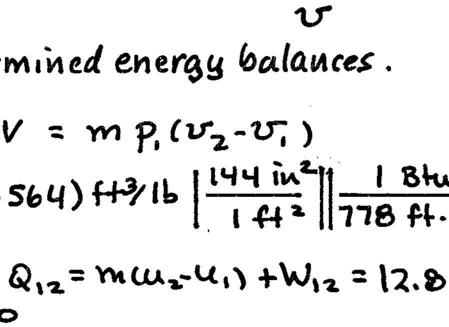
$$u_1 = u_{f_1} + x_1(u_{g_1} - u_{f_1}) = 7.38 + (.95)(93.67 - 7.38) = 89.36 \text{ Btu/lb}$$

$$v_1 = v_{f_1} + x_1(v_{g_1} - v_{f_1}) = 0.01178 + (.95)(1.7430 - 0.01178) = 1.6564 \text{ ft}^3/\text{lb}$$

State 2: from Table A-9E at $p_2 = p_1$, $T_2 = 40°F$; $u_2 = 100.40$ Btu/lb, $v_2 = 1.9858$ ft³/lb

State 3: $T_3 = 40°F$, sat. vapor. From Table A-7E; $u_3 = 98.08$ Btu/lb





The required energy transfers are determined energy balances.

Process 1-2: Using Eq. 2.17, $W_{12} = \int_{V_1}^{V_2} p\,dV = m\,p_1(v_2 - v_1)$

$$W_{12} = (1\text{ lb})(30\text{ lbf/in}^2)(1.9858 - 1.6564)\text{ ft}^3/\text{lb}\left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 1.829\text{ Btu}$$

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + m(u_2 - u_1) = Q_{12} - W_{12} \Rightarrow Q_{12} = m(u_2 - u_1) + W_{12} = 12.87\text{ Btu}$$ ← Q_{12}

Process 2-3: $W_{23} = -11.82$, $\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + m(u_3 - u_2) = Q_{23} - W_{23}$

$$Q_{23} = m(u_3 - u_2) + W_{23} = -14.14\text{ Btu}$$ ← Q_{23}

Process 3-1: $\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + m(u_1 - u_3) = \cancelto{0}{Q_{31}} - W_{31}$

$$W_{31} = -m(u_1 - u_3) = 8.72\text{ Btu}$$

① ② $W_{cycle} = W_{12} + W_{23} + W_{31} = (1.829) + (-11.82) + (8.72) = -1.27$ Btu ← W_{cycle}

---

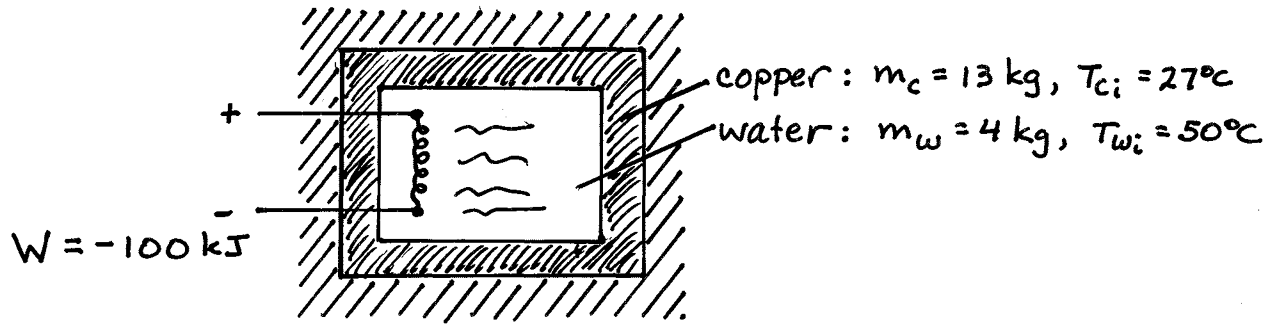
1. The negative value for W_{cycle} indicates the cycle is a refrigeration cycle.
2. A check of the net heat transfer confirms that $Q_{cycle} = W_{cycle}$.

PROBLEM 3.69

KNOWN: An electric resistor transfers energy to water in a well-insulated copper tank.

FIND: Determine the final equilibrium temperature.

SCHEMATIC & GIVEN DATA:


+
−
W = −100 kJ
copper: $m_c = 13$ kg, $T_{ci} = 27$°C
water: $m_w = 4$ kg, $T_{wi} = 50$°C

ASSUMPTIONS: (1) The copper tank and water are a closed system. (2) There is no heat transfer. (3) The copper and water behave as incompressible substances with constant specific heats. (4) Kinetic and potential energy effects are negligible. (5) No energy is stored in the resistor; $\Delta U_{res} = 0$

ANALYSIS: The energy balance reduces as follows

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = \cancelto{0}{Q} - W \Rightarrow \Delta U = -W$$

or $$\cancelto{0}{\Delta U_{res}} + \Delta U_c + \Delta U_w = -W$$

$$m_c c_c (T_f - T_{ci}) + m_w c_w (T_f - T_{wi}) = -W$$

Solving for the final temperature, T_f

$$T_f = \frac{m_c c_c T_{ci} + m_w c_w T_{wi} - W}{m_c c_c + m_w c_w}$$

Using Table A-19, the specific heat of copper is $c_c = 0.385$ kJ/kg·K. And for water at 325 K, $c_w = 4.179$ kJ/kg·K. Thus

① $$T_f = \frac{(13\text{ kg})(0.385\text{ kJ/kg·K})(27+273)\text{K} + (4)(4.179)(50+273) - (-100)}{(13\text{ kg})(0.385\text{ kJ/kg·K}) + (4)(4.179)}$$

$$= 322.3\text{ K} = 49.3\text{°C} \longleftarrow T_f$$

---

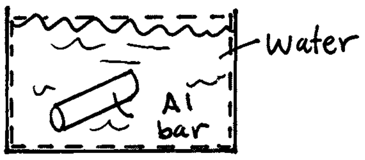
1. Note that temperature in °C could have been used directly in this expression.

PROBLEM 3.70*

KNOWN: A hot aluminum bar is placed in an open tank of cool water. The bar and the water come to equilibrium with no heat transfer with the surroundings.

FIND: Determine the final temperature.

SCHEMATIC & GIVEN DATA:



$V_a = 0.2\ ft^3$

$T_{ai} = 200\ °F$

$V_w = 4\ ft^3$

$T_{wi} = 70\ °F$

ASSUMPTIONS: (1) The Al bar and water are the closed system. (2) The aluminum and water behave as incompressible substances with constant specific heats. (3) Kinetic and potential energy effects are neglected. (4) There is no heat transfer between the tank and its surroundings, and $W=0$.

ANALYSIS: The energy balance reduces as follows:

$$\cancel{\Delta KE}+\cancel{\Delta PE}+\Delta U = \cancel{Q}-\cancel{W} \Rightarrow \Delta U = 0$$

or $\Delta U_a + \Delta U_w = 0$. Thus, if T_f is the final equilibrium temperature

① $$m_a c_a (T_f - T_{ai}) + m_w c_w (T_f - T_{wi}) = 0$$

Solving for T_f

$$T_f = \frac{m_a c_a T_{ai} + m_w c_w T_{wi}}{m_a c_a + m_w c_w} \qquad (*)$$

To evaluate m_w, use v_f @ 70°F from Table A-2E to get

$$m_w = \frac{V_w}{v_f} = \frac{4\ ft^3}{.01605\ ft^3/lb} = 249.2\ lb$$

To evaluate m_a, using data from Table A-19E

$$m_a = \rho_a V_a = (169\ \tfrac{lb}{ft^3})(0.2\ ft^3) = 33.8\ lb$$

Now, with $c_a = 0.216$ Btu/lb·°R and $c_w = 0.998$ Btu/lb·°R (at 540°R) from Table A-19E, we can evaluate (*)

$$T_f = \frac{(33.8\ lb)(0.216\ Btu/lb\cdot °R)(200+460)°R + (249.2)(0.998)(70+460)}{(33.8\ lb)(0.216\ Btu/lb\cdot °R) + (249.2)(0.998)}$$

$$= 533.7\ °R = 73.7\ °F \longleftarrow T_f$$

---

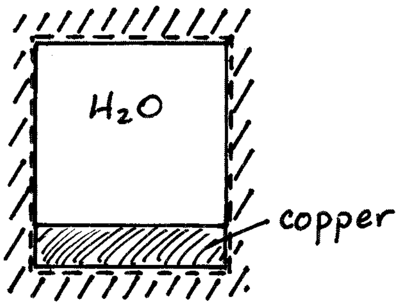
1. Note that temperature in °F could be used in this expression because it involves temperature differences.

PROBLEM 3.71

KNOWN: An isolated system is formed from saturated water vapor and a copper slab initially at different temperatures.

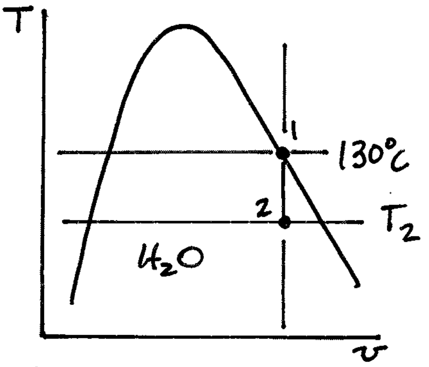
FIND: Determine the final equilibrium temperature of the system.

SCHEMATIC & GIVEN DATA:



$T_{w1} = 130°C$, $m_w = 0.2 kg$

$T_{c1} = 30°C$, $m_c = 10 kg$



ASSUMPTIONS: (1) The H_2O and copper form an isolated system. (2) The volume of the system is constant. (3) The copper behaves as an incompressible substance with constant specific heats. (4) kinetic and potential energy effects are negligible.

ANALYSIS: The energy balance reduces as follows:

$$\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + \Delta U = \cancel{Q}^{0} - \cancel{W}^{0} \Rightarrow \Delta U = 0$$

or

$$\Delta U_c + \Delta U_w = 0$$

$$m_c c (T_2 - T_{1c}) + m_w (u_{2w} - u_{1w}) = 0$$

With $u_{1w} = u_{g @ 130°C} = 2539.9$ kJ/kg and $c = 0.385$ kJ/kg·K from Table A-19

$$(10 kg)(.385 kJ/kg \cdot K)(T_2 - 30) + (.2 kg)(u_{2w} - 2539.9) = 0$$

or, solving for T_2

$$T_2 = 30 - \frac{(.2)(u_{2w} - 2539.9)}{(10)(.385)} \qquad (1)$$

The final specific volume of the water vapor, v_{2w}, is the same as the initial value; $v_1 = v_{g @ 130°C} = 0.6685$ m³/kg. The final quality of the H_2O is

$$x_2 = \frac{v_{2w} - v_{f2}}{v_{g2} - v_f}$$

and

$$u_{2w} = u_{f2} + x_2 (u_g - u_f)$$

Thus, an iterative solution is required. First, guess a trial value of T_2. Calculate x_2 and u_{2w} based on the trial T_2 and saturation data. Use (1) to calculate a corresponding value of T_2. Repeat until the trial value equals the one predicted by (1).

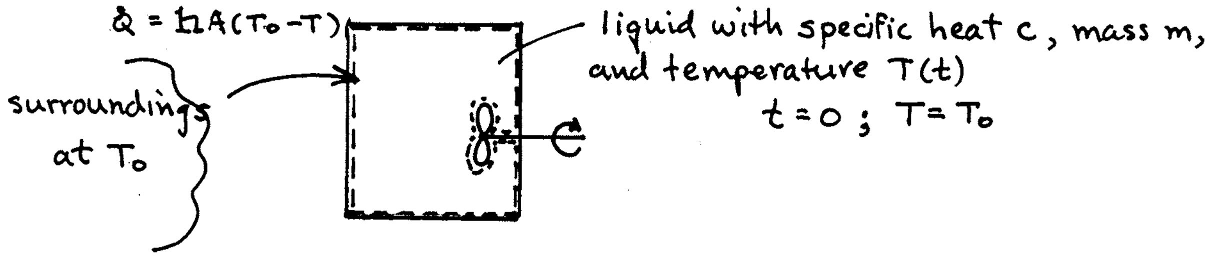
Result: $T_2 \approx 98°C$ ← ——————————— $\underline{T_2}$

PROBLEM 3.72

KNOWN: A liquid in a rigid tank is stirred by a paddle wheel and experiences energy transfer by heat with its surroundings.

FIND: Obtain a differential equation for temperature T in terms of time t and relevant parameters. Solve for T(t).

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The liquid is the closed system. (2) The liquid behaves as an incompressible substance with constant specific heat c. (3) The energy transfer from the paddle wheel to the system occurs at a constant rate. (4) There are no changes in kinetic or potential energy.

ANALYSIS: The energy rate balance is

$$\overset{0}{\cancel{\frac{dKE}{dt}}} + \overset{0}{\cancel{\frac{dPE}{dt}}} + \frac{dU}{dt} = \dot{Q} - \dot{W}$$

with $dU/dt = mc\,dT/dt$ and $\dot{Q} = hA(T_0 - T)$

$$mc\frac{dT}{dt} = hA(T_0 - T) - \dot{W} \longleftarrow \text{differential equation}$$

To solve, let $\theta = T_0 - T$; $dT = -d\theta$. Thus

$$\frac{d\theta}{dt} + \left(\frac{hA}{mc}\right)\theta - \frac{\dot{W}}{mc} = 0$$

The solution of this differential equation is of the form

$$\theta(t) = c_1 \exp\left[-\left(\frac{hA}{mc}\right)t\right] + \frac{\dot{W}}{hA}$$

at $t = 0$, $T = T_0 \Rightarrow \theta = 0$. Thus $c_1 = -\dot{W}/hA$, so

$$\theta(t) = \frac{\dot{W}}{hA}\left\{1 - \exp\left[-\left(\frac{hA}{mc}\right)t\right]\right\}$$

or

① ② $$T(t) = T_0 - \frac{\dot{W}}{hA}\left\{1 - \exp\left[-\left(\frac{hA}{mc}\right)t\right]\right\} \longleftarrow T(t)$$

---

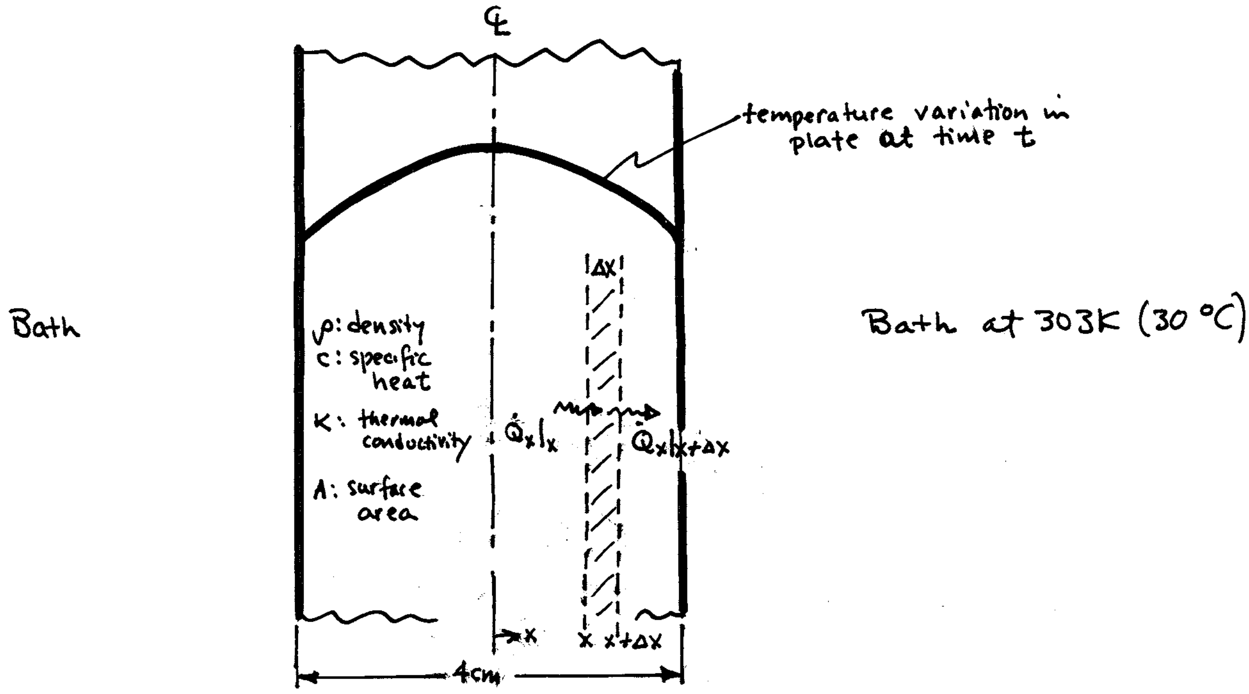
1. The solution can be verified by direct substitution into the differential equation. 2. The solution satisfies the initial condition that $T(0) = T_0$.

PROBLEM 3.73

KNOWN: A steel plate of known thickness and initial temperature is quenched in a bath whose temperature is also known.

FIND: Derive a partial differential equation for the variation of temperature within the plate as a function of time and position. Evaluate the physical parameters using data from Table A-19.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The element of thickness Δx is the closed system. 2. The incompressible model applies. 3. Specific heat and thermal conductivity are each constant. 4. There is no work and no kinetic/potential energy effects.

ANALYSIS: The energy rate balance applied to the element of thickness Δx reads

$$\cancel{\frac{dKE}{dt}} + \cancel{\frac{dPE}{dt}} + \frac{dU}{dt} = \left[\dot{Q}_x|_x - \dot{Q}_x|_{x+\Delta x} \right] - \cancel{\dot{W}}$$

The temperature within the plate varies with position and time. Thus, with $U = mu = (\rho A \Delta x)u$, where $u = u(T)$,

$$\frac{dU}{dt} = (\rho A \Delta x)\frac{du}{dT}\frac{\partial T}{\partial t} = (\rho A \Delta x)\, c\, \frac{\partial T}{\partial t}$$

Collecting results

$$\rho A c \frac{\partial T}{\partial t} = -\left[\frac{\dot{Q}_x|_{x+\Delta x} - \dot{Q}_x|_x}{\Delta x} \right]$$

In the limit as $\Delta x \to 0$

$$\rho A c \frac{\partial T}{\partial t} = -\frac{\partial \dot{Q}_x}{\partial x} \qquad (*)$$

PROBLEM 3.73 (Contd.)

When $T = T(x,t)$, Fourier's conduction model can be expressed as

$$\dot{Q}_x = -\kappa A \frac{\partial T}{\partial x} \qquad (**)$$

Combining Eqs. (*), (**)

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x}\right)$$

Since thermal conductivity is assumed constant

$$\rho c \frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2}$$

With data from Table A-19

$\rho = 8060\ \mathrm{kg/m^3}$

$c = 0.480\ \mathrm{kJ/kg \cdot K}$

$\kappa = 15.1\ \mathrm{W/m \cdot K}$

Defining

$$\alpha = \frac{\kappa}{\rho c} = \frac{\left(15.1\ \frac{W}{m\cdot K}\right)\left|\frac{1\ J/s}{1\ W}\right|}{\left(8060\ \frac{kg}{m^3}\right)\left(0.48\ \frac{kJ}{kg\cdot K}\right)\left(\frac{10^3\ J}{1\ kJ}\right)} = 3.9 \times 10^{-6}\ \frac{m^2}{s}$$

we have

① $$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \longleftarrow \text{differential equation}$$

---

1. This is a second order partial differential equation giving $T(x,t)$. The solution requires an initial condition:

$$T(x, t=0) = 673\ \mathrm{K}\ (400^\circ C)$$

and appropriate boundary conditions. For further discussion, see Fundamentals of Heat and Mass Transfer by F. P. Incropera and D. P. DeWitt.

PROBLEM 3.74*

H_2O at $p = 200$ bar , $T = 470\,°C = 743$ K

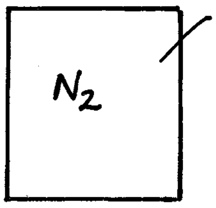
(a) $$\left.\begin{aligned} p_R &= p/p_c = 200\,\text{bar}/220.9\,\text{bar} = 0.905 \\ T_R &= T/T_c = 743\,\text{K}/647.3\,\text{K} = 1.15 \end{aligned}\right\}\ \begin{aligned} &\text{Fig A-1} \\ &Z \approx 0.79 \leftarrow\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\ \underline{Z\ \text{chart}} \end{aligned}$$

(b) Table A-4 at $p = 200$ bar, $T = 470\,°C \Rightarrow v = 0.1355\ \text{m}^3/\text{kg}$

$$Z = \frac{pv}{RT} = \frac{(200\,\text{bar})(0.01355\,\text{m}^3/\text{kg})}{\left(\frac{8.314}{18.02}\right)\frac{\text{kJ}}{\text{kg K}}(743\,\text{K})}\left|\frac{10^5\,\text{N/m}^2}{1\,\text{bar}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= 0.791 \leftarrow\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\ \underline{Z\ \text{table}}$$

PROBLEM 3.75



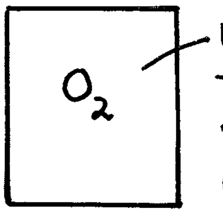
m = 40 kg
p = 17 MPa
T = 180 K
V = ?

Determine Z: $$\left.\begin{aligned} P_R &= 170/33.9 = 5.02 \\ T_R &= 180/126.2 = 1.43 \end{aligned}\right\}\ \begin{aligned} &\text{Fig. A-2} \\ &Z \approx 0.82 \end{aligned}$$

$$V = \frac{ZmRT}{p} = \frac{(.82)(40\,\text{kg})\left(\frac{8.314}{28.01}\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(180\,\text{k})}{(17\times10^6\,\text{N/m}^2)}\left|\frac{10^3\,\text{N}\cdot\text{m}}{1\,\text{kJ}}\right|$$

$$= 0.103\,\text{m}^3 \leftarrow\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\ \underline{V}$$

PROBLEM 3.76*



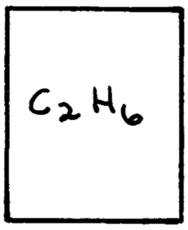
$V = 5\,\text{ft}^3$
$T = 400°R$
$p = 4000\ \text{lb}_f/\text{in}^2$
m = ?

Determine Z: $$\left.\begin{aligned} P_R &= \frac{4000/14.7}{49.8} = 5.5 \\ T_R &= 400/278 = 1.44 \end{aligned}\right\}\ \begin{aligned} &\text{Fig A-2} \\ &Z \approx 0.84 \end{aligned}$$

$$m = \frac{pV}{ZRT} = \frac{(4000\,\text{lb}_f/\text{in}^2)(5\,\text{ft}^3)}{(0.84)\left(\frac{1545}{32.00}\right)\frac{\text{ft}\cdot\text{lb}_f}{\text{lb}\,R}(400\,R)}\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right|$$

$$= 177.5\ \text{lb}\ -\!\!-\!\!-\!\!-\!\!-\!\!-\ \underline{m}$$

PROBLEM 3.77*



$T = 115°F = 575°R$
$v = 0.30\ \text{ft}^3/\text{lb}$
p = ?

Determine Z: $T_R = \frac{575}{549} = 1.05$

$$v_R' = \frac{v\,p_c}{RT_c} = \frac{(0.30\,\text{ft}^3/\text{lb})(48.2\,\text{atm})}{\left(\frac{1545}{30.07}\,\frac{\text{ft}\cdot\text{lb}_f}{\text{lb}\,°R}\right)(575°R)}\left|\frac{14.7\,\frac{\text{lb}_f}{\text{in}^2}}{1\,\text{atm}}\right|\left|\frac{144\,\text{i}}{1\,\text{ft}}\right.$$

$$= 1.04$$

Figure A-1: $Z \approx 0.74$

$$p = \frac{ZRT}{v} = \frac{(0.74)\left(\frac{1545}{30.07}\,\frac{\text{ft}\cdot\text{lb}_f}{\text{lb}\,°R}\right)(575\,°R)}{(0.30\,\text{ft}^3/\text{lb})}\left|\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right|$$

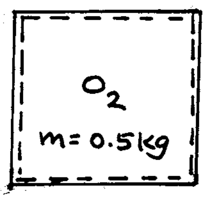
$$= 506\ \text{lb}_f/\text{in}^2 \leftarrow\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\ \underline{P}$$

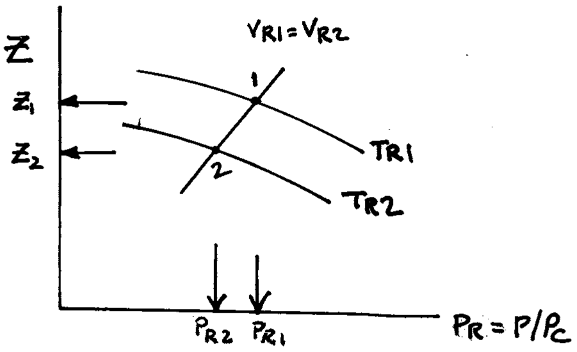
PROBLEM 3.78

KNOWN: Oxygen (O_2) contained in a closed, rigid tank is cooled to a final state where pressure is specified.

FIND: Determine the tank volume and the final temperature.

SCHEMATIC & GIVEN DATA:





P_1 = 30 bar
① T_1 = 200 K
P_2 = 20 bar

ASSUMPTIONS: 1. The oxygen is a closed system. 2. The volume of the tank is constant.

ANALYSIS: The volume of the tank can be evaluated using the compressibility factor at state 1:

$$Z_1 = \frac{P_1(V/m)}{(\bar{R}/M)\,T_1} \Rightarrow V = Z_1 \frac{m(\bar{R}/M)T_1}{P_1}$$

The value of Z_1 can be obtained by inspection of the compressibility chart using P_{R1} and T_{R1}. With T_c, P_c from Table A-1

$$P_{R1} = \frac{30\text{ bar}}{50.5\text{ bar}} = 0.59 \quad , \quad T_{R1} = \frac{200\text{K}}{154\text{ K}} = 1.3$$

From Fig A-1, Z_1 = 0.91. Thus

$$V = 0.91 \frac{(0.5\text{ kg})(8314/32\ \text{N·m/kg·K})\ 200\text{K}}{(30\times10^5\ \text{N/m}^2)}$$

$$= 7.88 \times 10^{-3}\ \text{m}^3 \longleftarrow$$

Since volume and mass are constant, $v_2 = v_1$ and thus $v_{R1}' = v_{R1}'$, as shown in the sketch above. Returning to Fig A-1 with
② P_R = 20/50.5 = 0.396 and following a line of constant v_R' from 1 to 2, we have T_{R2} = 0.96. Thus, T_2 = 0.96 T_c

$$T_2 = (0.96)(154) = 148\text{ K} \longleftarrow$$

---

1. These values for P_1, T_1, T_2 revise the values given in the first printing of the book.

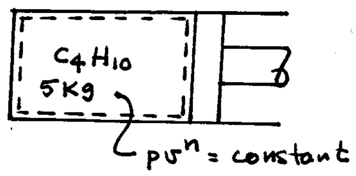
2. Alternatively, Z_2 = 0.825 can be read from the figure, from which T_2 can be evaluated as T_2 = 147 K.

PROBLEM 3.79

KNOWN: Five kg of C_4H_{10} contained in a piston-cylinder arrangement undergo a process between known states for which pv^n = constant.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:



$p_1 = 5MPa$, $T_1 = 500K$

$p_2 = 3MPa$, $T_2 = 450K$

ASSUMPTIONS: 1. The Butane is a closed system. 2. The process is described by pv^n = constant.

ANALYSIS: The work is given by

$$W = \int_1^2 p\,dV = m\int_1^2 \frac{C}{v^{n}}\,dv = \frac{m(p_2v_2 - p_1v_1)}{1-n}$$

To evaluate W requires v_1, v_2 and n. The compressibility chart can be used to obtain v_1 and v_2: From Table A-1, $p_c = 38 bar$, $T_c = 425K$, $M = 58.12$.

- $$p_{R1} = \frac{p_1}{p_c} = \frac{50\,bar}{38\,bar} = 1.32, \quad T_{R1} = \frac{T_1}{T_c} = \frac{500K}{425K} = 1.18 \Bigg\}\ \text{Fig. A-2}: \ v_R' = 0.6$$

$$\Rightarrow v_1 = v_R'\left(\frac{RT_c}{p_c}\right) = 0.6\frac{\left(\frac{8314}{58.12}\frac{N\cdot m}{kg\cdot K}\right)(425K)}{(38\times10^5 N/m^2)} = 0.0096\frac{m^3}{kg}$$

- $$p_{R2} = \frac{p_2}{p_c} = \frac{30}{38} = 0.79, \quad T_{R2} = \frac{T_2}{T_c} = \frac{450}{425} = 1.06 \Bigg\}\ \text{Fig. A-1} \quad v_{R2}' = 0.99$$

$$\Rightarrow v_2 = \frac{(0.99)(8314/58.12)(425)}{(38\times10^5)} = 0.0158\frac{m^3}{kg}$$

To find n, use $p_2v_2^n = p_1v_1^n \Rightarrow \left(\frac{v_2}{v_1}\right)^n = \frac{p_1}{p_2} \Rightarrow n = \frac{\ln(p_1/p_2)}{\ln(v_2/v_1)}$

$$\therefore n = \frac{\ln(5/3)}{\ln(0.0158/.0096)} = 1.025$$

Thus, the work is

$$W = \frac{m(p_2v_1 - p_1v_1)}{1-n} = (5\,kg)\frac{[(3\,MPa)(0.0158\,m^3/kg - (5)(0.0096)]}{1-1.025}\left|\frac{10^6 N/m^2}{1\,MPa}\right|\left|\frac{1\,kJ}{10^3 N\cdot m}\right|$$

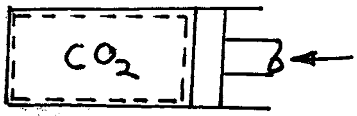
$$= 120\,kJ \longleftarrow W$$

PROBLEM 3.80*

KNOWN: Five lbmol of CO_2 undergo a constant-pressure compression process in a piston-cylinder assembly. The work, pressure, and initial gas temperature are known.

FIND: Evaluate the final temperature.

SCHEMATIC & GIVEN DATA:



$p = 320\ \text{lbf/in}^2$
$T_1 = 660\,°R$
$W = -2000\ \text{Btu}$

$n = 5\ \text{lbmol}$

ASSUMPTIONS: 1. The CO_2 is a closed system. 2. Pressure remains constant during the compression process.

ANALYSIS: The final state of the gas is fixed by pressure, 320 lbf/in², and the specific volume $\bar{v}_2$, which can be evaluated from the work.

$$W = \int_1^2 p\,dV = p(V_2 - V_1) = np(\bar{v}_2 - \bar{v}_1) \Rightarrow \bar{v}_2 = \bar{v}_1 + \frac{W}{np} \quad (*)$$

$\bar{v}_1$ can be found from the compressibility chart. From Table A-1E, $P_c = 72.9$ atm, $T_c = 548\,°R$. Then

$$P_R = \frac{(320/14.7)\,\text{atm}}{72.9\ \text{atm}} \approx 0.30 \qquad T_{R1} = \frac{660\,°R}{548\,°R} \approx 1.2\,.$$

Using these, Fig. A-1 gives $v'_{R1} = 4.0$. Then

$$\bar{v}_1 = v'_{R1}\left(\frac{\bar{R}T_c}{P_c}\right) = 4.0\left(\frac{\left(1545\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°R}\right)(548\,°R)}{(72.9 \times 14.7)\ \text{lbf/in}^2}\right)\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| = 21.95\ \text{ft}^3/\text{lbmol}$$

Returning to Eq. (*)

$$\bar{v}_2 = \bar{v}_1 + \frac{W}{np} = 21.95\,\frac{\text{ft}^3}{\text{lbmol}} + \frac{(-2000\ \text{Btu})\left|\frac{778\ \text{ft lbf}}{1\ \text{Btu}}\right|}{(5\ \text{lbmol})(320 \times 144\ \text{lbf/ft}^2)}$$

$$= 15.21\ \text{ft}^3/\text{lbmol}$$

So, at state 2, $P_{R2} = P_{R1} = 0.30$ and $v'_{R2} = \bar{v}_2 P_c/\bar{R}T_c$

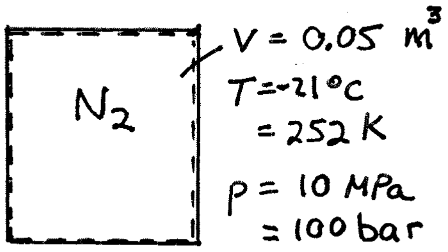
$$v'_{R2} = \frac{(15.21\ \text{ft}^3/\text{lbmol})((72.9)(14.7)(144)\ \text{lbf/ft}^2)}{\left(1545\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°R}\right)(548\,°R)} = 2.77$$

Returning to Fig. A-1, $T_{R2} = 0.95$. Thus

$$T_2 = T_{R2}T_c = (0.95)(548\,°R) = 521\,°R \quad \longleftarrow$$

PROBLEM 3.81*

(a) Using the ideal gas equation of state



$$m = \frac{pV}{RT}$$

$$= \frac{(10 MPa)(0.05\ m^3)}{\left(\frac{8.314}{28.01}\ \frac{kJ}{kg\cdot K}\right)(252\ K)} \left|\frac{10^6\ N/m^2}{1\ MPa}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 6.685\ kg \longleftarrow \underline{m(\text{ideal gas})}$$

(b) Using data from Table A-1

$$\left.\begin{aligned} P_R &= P/P_c = \frac{100\ bar}{33.9\ bar} = 2.95 \\ T_R &= T/T_c = \frac{252 K}{126 K} = 2.00 \end{aligned}\right\} \begin{aligned} &\text{Fig. A-2} \\ &v_R' \approx 0.65 \end{aligned}$$

Thus

$$v = v_{R'}\left(\frac{RT_c}{p_c}\right) = (0.65)\left[\frac{\left(\frac{8.314}{28.01}\right)(126)}{(33.9)}\right]\left(\frac{1}{10^2}\right) = 7.171 \times 10^{-3}\ m^3/kg$$

and

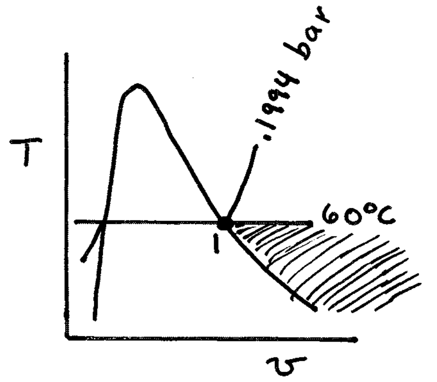
① $$m = \frac{V}{v} = \frac{0.05\ m^3}{7.171 \times 10^{-3}\ m^3/kg} = 6.973\ kg \longleftarrow \underline{m\ (\text{chart})}$$

---

1. Assuming that the chart value is correct, the ideal gas model under-predicts the mass by about 4.1%.

PROBLEM 3.82

For all states in the shaded region on the T-v diagram $p < 0.1994$ bar. Since $Z \to 1$ as $P_R \to 0$, the ideal gas model can be checked at state 1. If it is valid there it will be even better at other states in the shaded region. From Table A-2



$$v_1 = v_{g @ 60°C} = 7.671\ m^3/kg$$

$$Z_1 = \frac{P_1 v_1}{RT_1}$$

$$= \frac{(0.1994\ bar)(7.671\ m^3/kg)}{\left(\frac{8.314}{18.02}\ \frac{kJ}{kg\cdot K}\right)(60+273)\ K}\left(\frac{10^5\ N/m^2}{1\ bar}\right)\left(\frac{1\ kJ}{10^3\ N\cdot m}\right)$$

$$= 0.996 \Rightarrow \text{Ideal gas model is accurate in shaded region.}$$

PROBLEM 3.83

The ideal gas equation is accurate for ranges of pressure and temperature for which $Z \approx 1$. We might arbitrarily consider the ideal gas model to be satisfactory if $0.95 \leq Z \leq 1.05$ (5% deviation). These limits are illustrated on the compressibility chart below.

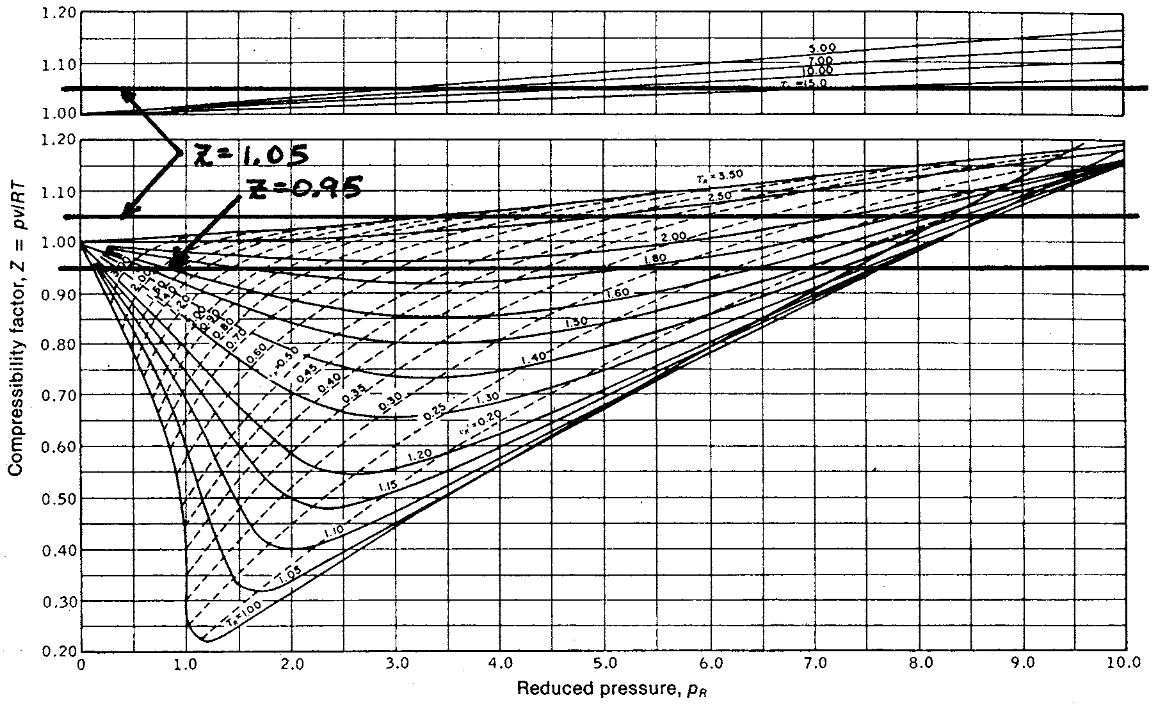


Figure A-2 Generalized compressibility chart, $p_R \leq 10.0$. *Source:* E. F. Obert, *Concepts of Thermodynamics*, McGraw-Hill, New York, 1960.

For air, Table A-1 gives; $T_c = 133$ K, $p_c = 37.7$ bar. The following observations can be made from the chart. The ideal gas model is reasonable when

- $p_R < 0.1$, for all temperatures ($p < 3.77$ bar)
- for $T_R = 2.0$ ($T = 266$ K); $p_R < 7.2$ ($p < 271$ bar)
- for $T_R = 2.5$ ($T = 333$ K); $p_R < 5.0$ ($p < 189$ bar)
- for $T_R = 3.5$ ($T = 466$ K); $p_R < 3.5$ ($p < 132$ bar)
- for $T_R > 5.0$ ($T = 665$ K); $p_R < 3.2$ ($p < 121$ bar)
- for $T_R > 15$ ($T = 1995$ K); $p_R < 7.0$ ($p < 264$ bar)
- for $T_R > 2.0$ ($T = 266$ K); $p_R < 3.2$ ($p < 121$ bar)

Thus, for many common applications, air can be reasonably modeled as an ideal gas.

PROBLEM 3.84

(a) Water vapor at 2000 lbf/in², 700°F

From Table A-4E : $v_{table} = 0.249 \text{ ft}^3/\text{lb}$

Ideal gas model :

$$v_{\substack{ideal\\gas}} = \frac{\left(\frac{1545}{18.02}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(1160°\text{R})}{(2000\ \text{lbf/in}^2)\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|} = 0.3453\ \text{ft}^3/\text{lb}$$

$$\%\ \text{error} = \frac{0.3453 - 0.249}{0.249} \times 100 = 38.7\% \longleftarrow \quad \text{(a)}$$

(b) Water vapor at 1 lbf/in², 200°F

Table A-4E : $v_{table} = 392.5 \text{ ft}^3/\text{lb}$

Ideal gas model:

$$v_{\substack{ideal\\gas}} = \frac{\left(\frac{1545}{18.02}\right)(660)}{(1)\,|144|} = 393.0\ \text{ft}^3/\text{lb}$$

$$\%\ \text{error} = \frac{393.0 - 392.5}{392.5} \times 100 = 0.13\% \longleftarrow \quad \text{(b)}$$

(c) Ammonia at 60 lbf/in², 160°F

Table A-15E : $v_{table} = 6.3458 \text{ ft}^3/\text{lb}$

Ideal gas model :

$$v_{\substack{ideal\\gas}} = \frac{\left(\frac{1545}{17.04}\right)(620)}{(60)\,(144)} = 6.5063\ \text{ft}^3/\text{lb}$$

$$\%\ \text{error} = \frac{6.5063 - 6.3458}{6.3458} \times 100 = 2.53\% \longleftarrow \quad \text{(c)}$$

(d) Air at 1atm, 2000°R

$$\left.\begin{aligned} P_R &= P/P_c = 1\ \text{atm}/37.2\ \text{atm} = 0.027 \\ T_R &= T/T_c = 2000°\text{R}/239°\text{R} = 8.37 \end{aligned}\right\} \quad \begin{aligned} &\text{Fig. A-2} \\ &Z \approx 1.0 \end{aligned}$$

Ideal gas ⟵ (d)

(e) R-22 at 300 lbf/in², 140°F

Table A-9E : $v_{table} = 0.1849 \text{ ft}^3/\text{lb}$

Ideal gas model :

$$v_{\substack{ideal\\gas}} = \frac{\left(\frac{1545}{86.48}\right)(600)}{(300)\,|144|} = 0.2481\ \text{ft}^3/\text{lb}$$

$$\%\ \text{error} = \frac{0.2481 - 0.1849}{0.1849} \times 100 = 34.2\% \longleftarrow \quad \text{(e)}$$

PROBLEM 3.85

(a) Refrigerant 134a at 80°C, 1.6 MPa

From Table A-12; $v_{table} = 0.01435\ m^3/kg$

Ideal gas model;

$$v_{\substack{ideal\\gas}} = \frac{RT}{P} = \frac{\left(\frac{8.314}{102.03}\ \frac{kJ}{kg\cdot K}\right)(353\ K)}{(1.6\ MPa)}\left|\frac{1\ MPa}{10^6\ N/m^2}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|$$

$$= 0.01798\ m^3/kg$$

$$\%\ \text{deviation} = \frac{0.01798 - 0.01435}{0.01435} \times 100 = 25.3\% \longleftarrow$$

(not applicable)

(b) Refrigerant 134a at 80°C, 0.10 MPa

From Table A-12; $v_{table} = 0.28464\ m^3/kg$

Ideal gas model;

$$v_{\substack{ideal\\gas}} = \frac{\left(\frac{8.314}{102.03}\right)(353)}{(0.10)}\left|\frac{1}{10^3}\right| = 0.28765\ m^3/kg$$

$$\%\ \text{deviation} = \frac{0.28765 - 0.28464}{0.28464} \times 100 = 1.1\% \longleftarrow$$

(Ideal gas model is applicable)

PROBLEM 3.86*

KNOWN: Oxygen is at a state fixed by pressure and specific volume.
FIND: Determine the temperature using generalized compressibility data and compare with the value calculated from the ideal gas model.
ANALYSIS: $p = 250$ bar, $v = 0.003\ m^3/kg$. With $T_c = 154$ K and $p_c = 50.5$ bar from Table A-1

$$p_R = \frac{250}{50.5} = 4.95, \quad v_R' = \frac{v\,p_c}{R T_c} = \frac{(0.003\ m^3/kg)(50.5 \times 10^5\ N/m^2)}{\left(\frac{8314}{32.00}\ \frac{N \cdot m}{kg \cdot K}\right)(154\ K)} = 0.379$$

From Fig. A-2, $T_R = 2.0$. Thus $T = T_c T_R = (154\ K)(2.0) = 308\ K \longleftarrow T$

Using the ideal gas equation of state

$$T = \frac{p v}{R} = \frac{(250 \times 10^5\ N/m^2)(0.003\ m^3/kg)}{\left(\frac{8314}{32.00}\ \frac{N \cdot m}{kg \cdot K}\right)} = 289\ K$$

This value is 6 % less than the result obtained with compressibility data.

PROBLEM 3.87

KNOWN: Air is at a state fixed by pressure, volume and total mass.
FIND: Determine the temperature. Check the validity of the ideal gas model.
ANALYSIS: $m = 5$ kg, $p = 0.3$ MPa, $V = 2.2\ m^3$ From Table A-1, $M = 28.97$, $p_c = 37.7$ bar, $T_c = 133$ K. Then

$$p_R = \frac{p}{p_c} = \frac{3\ bar}{37.7\ bar} = 0.08, \quad v_R' = \frac{v\,p_c}{R\,T_c} = \frac{(2.2\ m^3/5\ kg)(37.7 \times 10^5\ N/m^2)}{\left(\frac{8314}{28.97}\ \frac{N \cdot m}{kg \cdot K}\right)(133\ K)} = 43.46$$

Referring to Fig A-1, $Z \sim 1$. Thus, use of the ideal gas equation of state is justified.

$$T = \frac{p(V/m)}{R} = \frac{(3 \times 10^5\ N/m^2)(2.2\ m^3/5\ kg)}{(8314/28.97\ N \cdot m/kg \cdot K)} = 460\ K \longleftarrow T$$

PROBLEM 3.88*

KNOWN: A tank of known mass contains air at a specified temperature and pressure.
FIND: Determine the volume of the air. Verify that the ideal gas model applies.

ANALYSIS: $m = 10$ lb, $T = 70°F = 530°R$, $p = 30\ lb_f/in^2$. With T_c, p_c from Table A-1E
$T_c = 239°R$, $p_c = 37.2$ atm

$$T_R = \frac{T}{T_c} = \frac{530°R}{239°R} = 2.22, \quad p_R = \frac{p}{p_c} = \frac{(30/14.7)}{37.2} = 0.055$$

Referring to Fig. A-1, $Z \sim 1$. Thus, use of the ideal gas equation of state is justified: $pV = mRT$, or

$$V = \frac{mRT}{p} = \frac{(10\ lb)\left(\frac{1545}{28.97}\ \frac{ft \cdot lbf}{lb\ °R}\right)(530\ °R)}{(30 \times 144\ lb_f/ft^2)} = 65.4\ ft^3 \longleftarrow m$$

PROBLEM 3.89

For an ideal gas: $\rho = p/\left(\frac{\bar{R}}{M}\right)T = pM/\bar{R}T$

For helium and air, each at the same pressure and temperature

$$\rho_{He} = pM_{He}/\bar{R}T$$

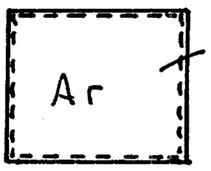
$$\rho_{air} = pM_{air}/\bar{R}T$$

Thus

$$\frac{\rho_{He}}{\rho_{air}} = \frac{M_{He}}{M_{air}} = \frac{4.003}{28.97} = 0.138$$

PROBLEM 3.90*

① Using the ideal gas model to determine the volume



n = 1 lbmol
p = 100 lbf/in²
T = 550 °R

$$V = \frac{n\bar{R}T}{p}$$

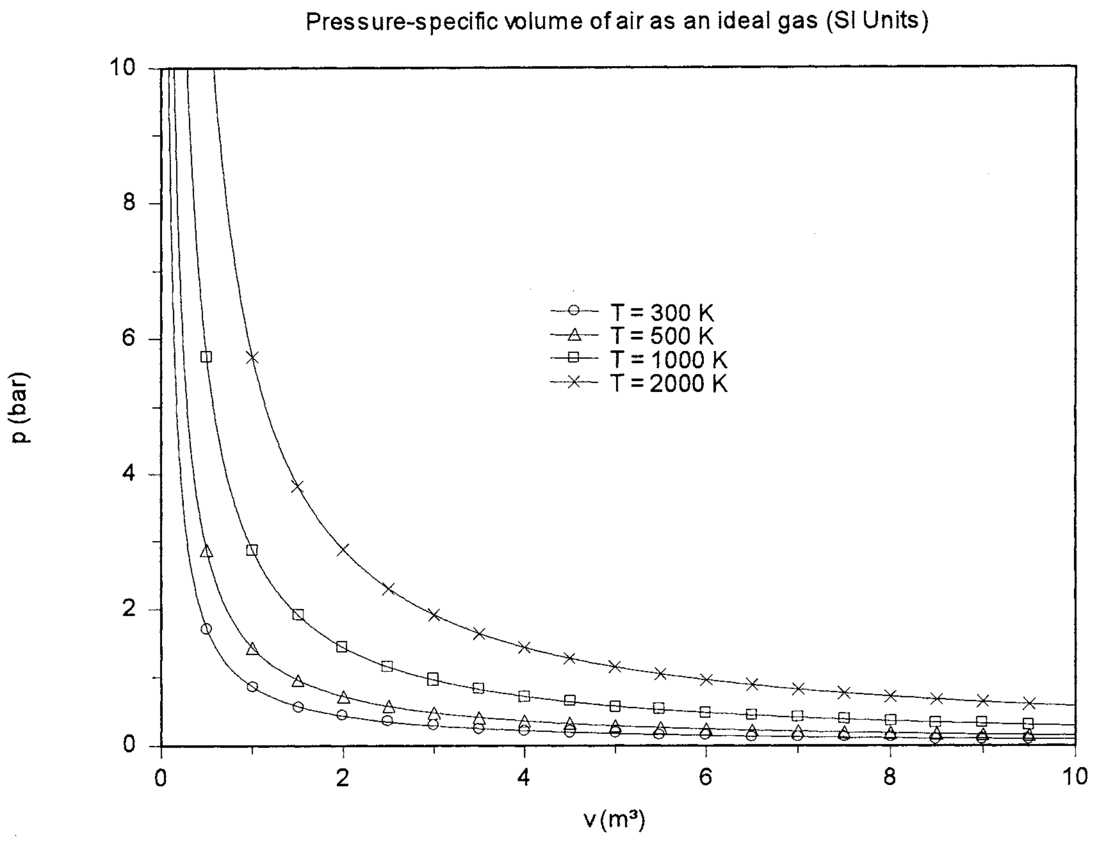
$$= \frac{(1\text{ lbmol})\left(1545\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^{\circ}\text{R}}\right)(550\ {}^{\circ}\text{R})}{(100\text{ lbf/in}^2)}\left|\frac{1\text{ ft}^2}{144\text{ in}^2}\right|$$

$$= 59.01\text{ ft}^3 \longleftarrow V$$

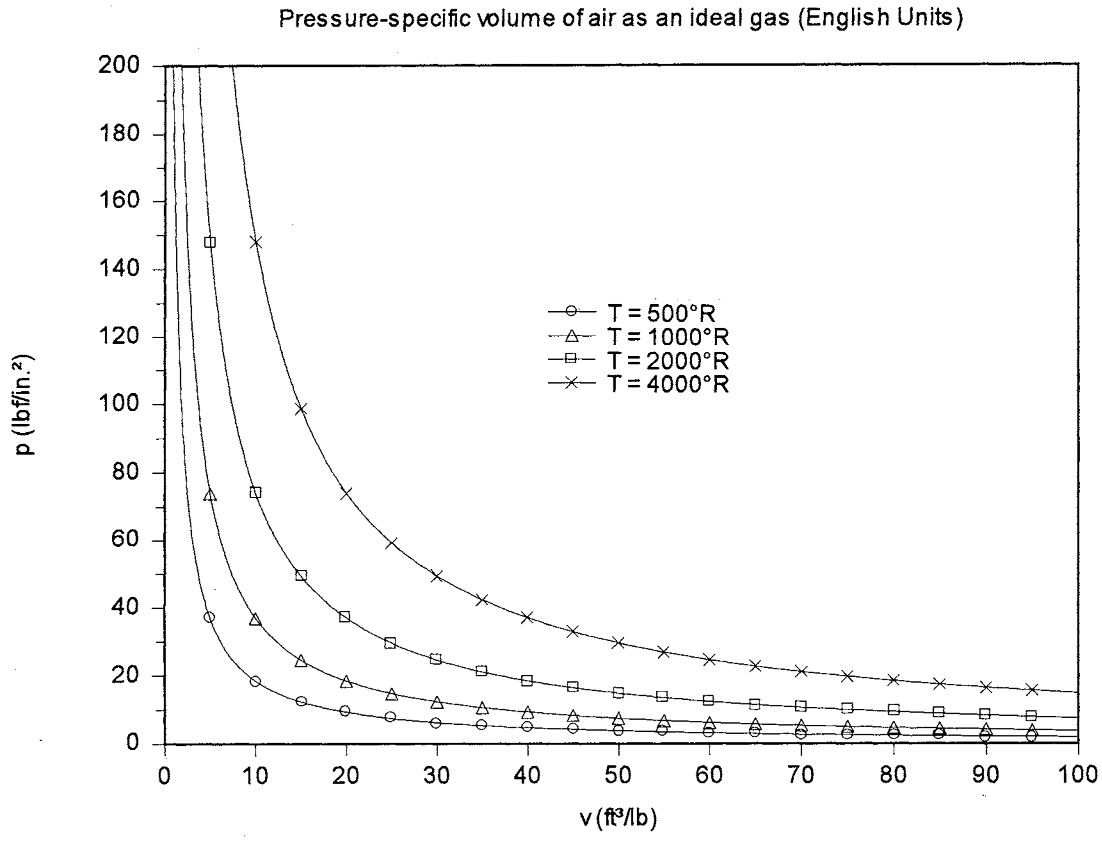
---

1. The applicability of the ideal gas model can be verified by referring to the compressibility chart.

PROBLEM 3.91



PROBLEM 3.92



PROBLEM 3.93

KNOWN: Methane undergoes a process between two states, each fixed by known values of pressure and temperature.

FIND: Determine Δh by integrating $\bar{c}_p(T)$ from Table A-1 and check the result using IT.

ANALYSIS: $T_1 = 320\text{ K}$, $P_1 = 2\text{ bar}$; $T_2 = 800\text{ K}$, $P_2 = 10\text{ bar}$. From Table A-1, $T_c = 191\text{ K}$, $P_c = 46.4\text{ bar}$. Thus

$$T_{R1} = \frac{320}{191} = 1.68 \qquad T_{R2} = \frac{800}{191} = 4.19$$

$$P_{R1} = \frac{2}{46.4} = 0.04 \qquad P_{R2} = \frac{10}{46.4} = 0.22$$

Referring to Fig. A-1 at P_{R1}, T_{R1} and P_{R2}, T_{R2}, the ideal gas model is applicable. Thus, $\Delta h = \int_{T_1}^{T_2} c_p\, dT$, where $c_p(T)$ is obtained from Table A-21. That is

$$c_p(T) = R(\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)$$

$$\Rightarrow h(T_2) - h(T_1) = \int_{T_1}^{T_2} c_p(T)\, dT = R\int_{T_1}^{T_2} (\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)\, dT$$

$$= R\left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5)\right]$$

Inserting values

$$h(T_2) - h(T_1) = \frac{\left(8.314 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)}{\left(16.04 \frac{\text{kg}}{\text{kmol}}\right)} \left[3.826(800-320) + \left(\frac{-3.979\times10^{-3}}{2}\right)(800^2 - 320^2)\right.$$

$$+ \left(\frac{24.558\times10^{-6}}{3}\right)(800^3 - 320^3) + \left(\frac{-22.733\times10^{-9}}{4}\right)(800^4 - 320^4)$$

$$\left. + \left(\frac{6.963\times10^{-12}}{5}\right)(800^5 - 320^5)\right]$$

$$= 1489.3\text{ kJ/kg} \longleftarrow \Delta h$$

IT Code

```
T1 = 320  // K
T2 = 800  // K
h1 = h_T("CH4", T1)  // kJ/kg
h2 = h_T("CH4", T2)  // kJ/kg
delh = h2 - h1
```

IT Result

Δh = 1489 kJ/kg

Alternative solution :

```
cp = cp_T("CH4", T)
dh = Integral(cp,T)
```

Using Explore button, sweep T from 320 K to 800 K in steps of 1.

The analytical and IT results compare very favorably.

PROBLEM 3.94

From Table A-21: $\bar{c}_p / \bar{R} = 2.5 = 5/2$

Now $\bar{c}_v = \bar{c}_p - \bar{R} \Rightarrow \dfrac{\bar{c}_v}{\bar{R}} = \dfrac{\bar{c}_p}{\bar{R}} - 1 = \dfrac{3}{2}$

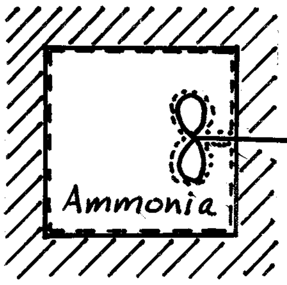
$$\Rightarrow \quad k = \frac{\bar{c}_p}{\bar{c}_v} = \frac{5/2}{3/2} = 5/3 \longleftarrow k$$

PROBLEM 3.95

KNOWN: A paddle wheel transfers energy to ammonia contained in a closed, rigid, well-insulated tank.

FIND: Determine the final temperature of the ammonia.

SCHEMATIC & GIVEN DATA:



$m = 0.12$ lb

$V = 2$ ft^3

$p_1 = 20$ lbf/in^2

$W = -20$ Btu

ASSUMPTIONS: (1) The ammonia is a closed system. (2) The ammonia behaves
① as an ideal gas. (3) There is no heat transfer. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: To determine the final temperature, begin with the energy balance

$$\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + \Delta U = \cancel{Q}^{0} - W \qquad (*)$$

Evaluating ΔU

$$\Delta U = m\,\Delta u = m\int_{T_1}^{T_2} c_v(T)\,dT$$

From Table A-21E: $c_p = R(3.591 + 0.274\times10^{-3}T + 2.576\times10^{-6}T^2 - 1.437\times10^{-9}T^3 + 0.2601\times10^{-12}T^4$

with $c_v = c_p - R$

$$\Delta U = mR\int_{T_1}^{T_2}(2.591 + 0.274\times10^{-3}T + 2.576\times10^{-6}T^3 - 1.437\times10^{-9}T^3 + 0.2601\times10^{-12}T^4)$$

$$= (0.12\text{ lb})\left(\frac{1545}{17.04}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|\left[2.591(T_2 - T_1) + \frac{0.274\times10^{-3}}{2}(T_2^2 - T_1^2)\right.$$

$$\left. + \frac{2.576\times10^{-6}}{3}(T_2^3 - T_1^3) - \frac{1.437\times10^{-9}}{4}(T_2^4 - T_1^4) + \frac{0.2601\times10^{-12}}{5}(T_2^5 - T_1^5)\right] \qquad (**)$$

The initial temperature is

$$T_1 = \frac{p_1 V}{mR} = \frac{(20\text{ lbf/in}^2)(2\text{ ft}^3)}{(.12\text{ lb})\left(\frac{1545}{17.04}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)}\left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right| = 529.4\,^\circ\text{R}$$

Combining the starred relations and inserting values, T_2 becomes the only unknown. Using an equation solver such as IT,

$T_2 = 920.1\,^\circ$R ← T_2

---

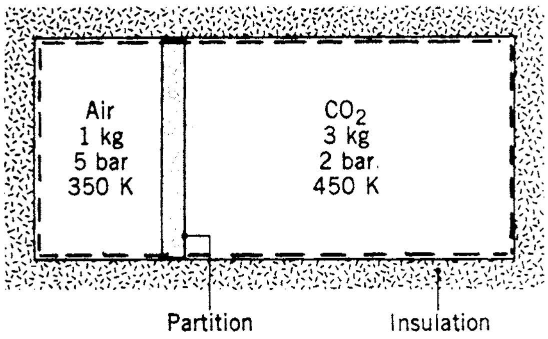
1. For state 1, $p_{R1} = (20)/(111.3 \cdot 14.7) = 0.00122$, $T_{R1} = (529.4)/(730) = 0.72$. From Fig. A-1; $Z_1 \approx 1$. Thus, assumption 2 is reasonable for state 1. By similar considerations, it can be concluded that the ideal gas model is reasonable for state 2 as well.

PROBLEM 3.96

KNOWN: Air and carbon dioxide are confined to opposite sides of a rigid, wall-insulated container. The partition moves and allows conduction from one gas to the other until equilibrium is achieved.

FIND: Determine the final temperature and pressure.

SCHEMATIC & GIVEN DATA: **



ASSUMPTIONS: (1) The contents of the container form a closed system. (2) The air and CO_2 behaves as ideal gases with constant specific heats. (3) The system is isolated, so $Q=0$ and $W=0$. (4) There is no energy stored in the partition. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: To determine the final temperature, begin with the energy balance

$$\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + \Delta U = \cancel{Q}^{0} - \cancel{W}^{0}$$

or, with $\Delta U = m_{air}\,\Delta u_{air} + m_{CO_2}\,\Delta u_{CO_2}$ and using Eq. 3.50

① $$m_{air}\, c_{v,air}\,(T_2 - T_{1,air}) + m_{CO_2}\, c_{v,CO_2}\,(T_2 - T_{1,CO_2}) = 0$$

Solving for T_2

$$T_2 = \frac{m_{air}\, c_{v,air}\, T_{1,air} + m_{CO_2}\, c_{v,CO_2}\, T_{1,CO_2}}{m_{air}\, c_{v,air} + m_{CO_2}\, c_{v,CO_2}}$$

The specific heats are evaluated using data from Table A-20 at a mean temperature of 400K ; $c_{v,air} = 0.726$ kJ/kg·K and $c_{v,CO_2} = 0.750$ kJ/kg·K. Thus, the final temperature is

$$T_2 = \frac{(1\ \text{kg})(0.726\ \text{kJ/kg·K})(350\ \text{K}) + (3)(.750)(450)}{(1\ \text{kg})(0.726\ \text{kJ/kg·K}) + (3)(.750)}$$

$$= 425.6\ \text{K} \longleftarrow T_2$$

Next, to find the final pressure, the total volume is needed. The initial volume of the air is

$$V_{1,air} = \frac{m_{air}\, R_{air}\, T_{1,air}}{P_{1,air}}$$

$$= \frac{(1\ \text{kg})\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg·K}}\right)(350\ \text{K})}{(5\ \text{bar})}\left|\frac{1\ \text{bar}}{10^5\ \text{N/m}^2}\right|\left|\frac{10^3\ \text{N·m}}{1\ \text{kJ}}\right| = 0.201\ \text{m}^3$$

PROBLEM 3.96

similarly for the carbon dioxide

$$V_{1,CO_2} = \frac{m_{CO_2} R_{CO_2} T_{1,CO_2}}{P_{1,CO_2}} = \frac{(3)\left(\frac{8.314}{44.01}\right)(450)}{(2)} \left|\frac{10^3}{10^5}\right|$$

$$= 1.275 \text{ m}^3$$

Thus

$$V_{tot} = 0.201 \text{ m}^3 + 1.275 \text{ m}^3 = 1.476 \text{ m}^3$$

Now, at the final state

$$P_2 = \frac{n_{tot} \bar{R} T_2}{V_{tot}} = \frac{(n_{air} + n_{CO_2}) \bar{R} T_2}{V_{tot}}$$

With $n = m/M$

$$P_2 = \frac{(m_{air}/M_{air} + m_{CO_2}/M_{CO_2}) \bar{R} T_2}{V_{tot}}$$

$$= \frac{\left(\frac{1 \text{ kg}}{28.97 \text{ kg/kmol}} + \frac{3}{44.01}\right)\left(8.314 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(425.6 \text{K})}{(1.476 \text{ m}^3)} \left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right| \left|\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right|$$

$$= 2.462 \text{ bar} \longleftarrow P_2$$

---

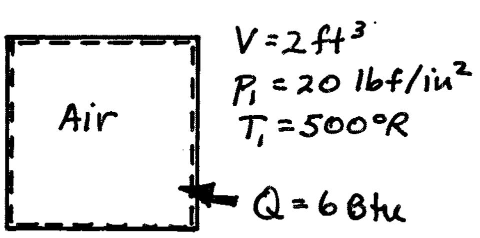
1. The assumption of constant specific heats facilitates the determination of T_2. The assumption is reasonable for the relatively small temperature range in this problem.

PROBLEM 3.97

KNOWN: Heat transfer occurs to air in a rigid tank. The initial state is specified.

FIND: Determine the final temperature and pressure using (a) a constant specific value, (b) a specific heat function, (c) the air tables.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The air is a closed system. (2) The air behaves as an ideal gas. (3) There is no work. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: To determine the final temperature, begin with the energy balance

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q - \cancelto{0}{W}$$

or

$$m(u_2 - u_1) = Q \Rightarrow u_2 - u_1 = Q/m \qquad (*)$$

Evaluating the mass

$$m = \frac{pV}{RT} = \frac{(20\ lbf/in^2)(2\ ft^3)}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot°R}\right)(500°R)}\left|\frac{144\ in^2}{1\ ft^2}\right| = 0.216\ lb$$

(a) For a constant specific heat c_v

$$u_2 - u_1 = c_v(T_2 - T_1)$$

Thus, with (*)

$$T_2 = \frac{Q}{mc_v} + T_1$$

From Table A-20E, at $T = 100°F$ (600°R), $c_v = 0.172\ Btu/lb\cdot°R$. Thus

$$T_2 = \frac{(6\ Btu)}{(.216\ lb)(.172\ Btu/lb\cdot°R)} + 500°R$$

$$= 661.5°R \longleftarrow \underline{(a)\ T_2}$$

Accordingly, for this temperature interval the value used for c_v is appropriate. The pressure P_2 is

$$P_2 = \frac{mRT_2}{V} = \frac{(.216)\left(\frac{1545}{28.97}\right)(661.5)}{(2)(144)} = 26.46\ lbf/in^2 \longleftarrow \underline{(a)\ P_2}$$

① (b) From Table A-21E

$$c_p = R(\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)$$

with $c_p = c_v + R$

$$c_v = R(\alpha - 1 + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)$$

PROBLEM 3.97 (Cont'd)

Thus

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v \, dT$$

$$= R\left[(\alpha - 1)(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5)\right]$$

Inserting values

$$u_2 - u_1 = \left(\frac{1.986 \text{ Btu/lbmol} \cdot {}^\circ R}{28.97 \text{ lb/lbmol}}\right)\left[2.653(T_2 - 500) - \left(\frac{0.742 \times 10^{-3}}{2}\right)(T_2^2 - 500^2)\right.$$

$$\left. + \left(\frac{1.017 \times 10^{-6}}{3}\right)(T_2^3 - 500^3) - \left(\frac{0.328 \times 10^{-9}}{4}\right)(T_2^4 - 500^4) + \left(\frac{0.02632 \times 10^{-12}}{5}\right)(T_2^5 - 500^5)\right] \quad (**)$$

Combining the starred relations and inserting values, T_2 becomes the only unknown. Solving iteratively

$$T_2 = 661.8 \, {}^\circ R \longleftarrow \text{(b) } T_2$$

and

$$p_2 = 26.47 \text{ lbf/in}^2 \longleftarrow \text{(b) } p_2$$

(c) From Table A-22E, $u_1 = 85.20$ Btu/lb. From (*)

$$u_2 = Q/m + u_1$$

$$= \left(\frac{6 \text{ Btu}}{0.216 \text{ lb}}\right) + 85.20 = 112.98 \text{ Btu/lb}$$

Interpolating in Table A-22E

$$T_2 = 661.8 \, {}^\circ R \longleftarrow \text{(c) } T_2$$

and

$$p_2 = 26.47 \text{ lbf/in}^2 \longleftarrow \text{(c) } p_2$$

---

① *IT solution (based on part b)*

```
T1 = 500  // °R
p1 = 20  // lbf/in.²
V = 2  // ft³
Q = 6  // Btu
delu = Q/m
m = V / v1
v1 = v_TP("Air",T1,p1)
delu = R * ((alpha-1)*(T2-T1)+(beta/2)*(T2^2- T1^2)
     +(gamma/3)*(T2^3-T1^3)+(delta/4)*(T2^4- T1^4)
     +(epsilon/5)*(T2^5-T1^5))
R = 1.986/28.97  // Btu/lb·°R
alpha = 3.653
beta = -0.742E-3
gamma = 1.017E-6
delta = -0.328E-9
epsilon = 0.02632E-12
v2 = v1
v2 = v_TP("Air",T2,p2)
```

IT Results

T_2 = 661.8°R

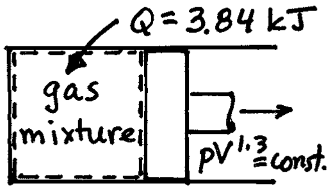
p_2 = 26.47 bar

PROBLEM 3.98

KNOWN: A gas mixture expands with a known pressure-volume relation. The initial state is fixed and the heat transfer for the process is known.

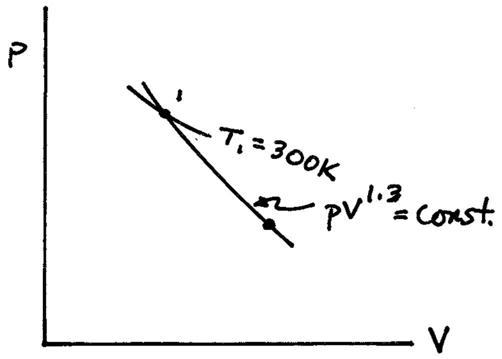
FIND: Determine (a) the final temperature, (b) the final pressure, (c) the final volume, and (d) the work.

SCHEMATIC & GIVEN DATA:



$M = 33$ kg/kmol
$p_1 = 3$ bar
$T_1 = 300$ K
$V_1 = 0.1\ m^3$

For the mixture: $c_v = 0.6 + (2.5\times10^{-4})T$
(T in K, c_v in kJ/kg·K)



ASSUMPTIONS: (1) The gas mixture is the closed system. (2) The gas mixture behaves as an ideal gas. (3) The process is polytropic with $n = 1.3$. (4) Kinetic and potential energy effects are neglected.

ANALYSIS: (a) To find the final temperature, start with the energy balance and express the internal energy change and work in terms of temperature change, as follows. First

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$$

From Eq. 3.57 for the polytropic process

$$W = \int_{V_1}^{V_2} p\,dV = \frac{mR(T_2 - T_1)}{1-n} \qquad (*)$$

Also, from the given c_v relation

$$\Delta U = m\int_{T_1}^{T_2} c_v(T)\,dT = m\left[(0.6)(T_2 - T_1) + \left(\frac{2.5\times10^{-4}}{2}\right)(T_2^2 - T_1^2)\right] \qquad (**)$$

Incorporating (*) and (**) into the energy balance

$$m\left[(0.6)(T_2 - T_1) + \left(\frac{2.5\times10^{-4}}{2}\right)(T_2^2 - T_1^2)\right] = Q - \frac{mR(T_2 - T_1)}{1-n} \qquad (***)$$

The mass is found using the ideal gas equation at state 1

$$m = \frac{p_1V_1}{RT_1} = \frac{(3\ \text{bar})(0.1\ m^3)}{\left(\frac{8.314}{33}\right)\frac{kJ}{kg\cdot K}(300K)}\left|\frac{10^5\ N/m^2}{1\ \text{bar}}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 0.397\ kg$$

Using IT with $m = 0.397$ kg, $T_1 = 300$K, $Q = 3.84$ kJ, and $R = 8.314/33$ kJ/kg·K to solve for T_2:

```
delU = Q - W
W = (m * R * (T2 - T1)) / (1 - 1.3)
delU = m * (0.6 * (T2 - T1) + ((2.5E-4) / 2) * (T2^2 - T1^2))
```

IT Result
$T_2 = 243.7$ K $\longleftarrow$ T_2

PROBLEM 3.98 (Contd.)

(b) To find p_2, use Eq. 3.56 for the polytropic process

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{n-1/n} \Rightarrow p_2 = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} p_1 = \left(\frac{243.7\text{ K}}{300\text{ K}}\right)^{\frac{1.3}{.3}} (3\text{ bar})$$

$$= 1.219\text{ bar} \longleftarrow p_2$$

(c) Again using Eq. 3.56

$$\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1} \Rightarrow V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} V_1 = \left(\frac{3\text{ bar}}{1.219\text{ bar}}\right)^{\frac{1}{1.3}} (0.1\text{ m}^3)$$

$$= 0.1999\text{ m}^3 \longleftarrow V_2$$

(d) Now, using (✱) to evaluate the work

$$W = \frac{mR(T_2 - T_1)}{(1-n)}$$

$$= \frac{(0.397\text{ kg})\left(\frac{8.314}{33}\right)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}(243.7 - 300)}{(1-1.3)}$$

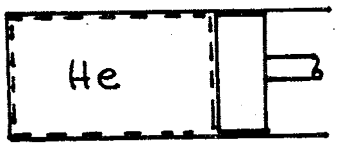
$$= 18.77\text{ kJ} \longleftarrow W$$

PROBLEM 3.99*

KNOWN: Helium gas undergoes a process for which pv^k = constant from a given initial state to a specified final pressure.

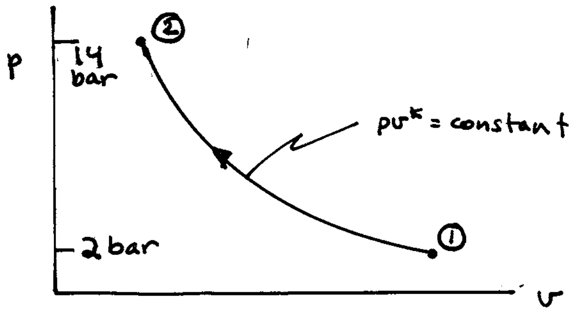
FIND: Determine the work and heat transfer, each per unit of mass.

SCHEMATIC & GIVEN DATA:



$p v^k = \text{const.}$

$P_1 = 2$ bar, $T_1 = 200$ K
$P_2 = 14$ bar



ASSUMPTIONS: 1. The helium is a closed system. 2. The process is described by pv^k = constant. 3. The helium behaves as an ideal gas. 4. Kinetic and potential energy changes are ignored.

ANALYSIS: From Table A-21, $\bar{c}_p = 2.5\bar{R}$. Using Eq. 3.45, $\bar{c}_v = \bar{c}_p - R = 1.5\bar{R}$. Thus $k = \bar{c}_p/\bar{c}_v = 2.5/1.5 = 1.667$.

With assumptions 2 and 3,

$$W = \int_1^2 p\,dV = m\int_1^2 \frac{c}{v^k}\,dv \Rightarrow \frac{W}{m} = \frac{P_2v_2 - P_1v_1}{1-k} = \frac{R(T_2-T_1)}{1-k} \qquad \text{(Eq. 3.57)}$$

To find T_2, use Eq. 3.56 with $n=k$

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = 200\,\text{K}\left(\frac{14}{1}\right)^{0.667/1.667} = 575\,\text{K}$$

Inserting values,

$$\frac{W}{m} = \frac{R(T_2-T_1)}{(1-k)}$$

$$= \frac{\left(\frac{8.314}{4.003}\right)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}(575-200)\,\text{K}}{(1-1.667)} = -1167.7\,\text{kJ} \longleftarrow \underline{\qquad W/m}$$

Next, the heat transfer is

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$$

$$Q = m(u_2-u_1) + W$$

or

$$\frac{Q}{m} = (u_2-u_1) + \frac{W}{m}$$

For a monatomic gas, c_v is constant. Thus

$$\frac{Q}{m} = c_v(T_2-T_1) + \frac{R(T_2-T_1)}{1-k}$$

With $c_v = R/(k-1)$ (Eq. 3.47b)

$$\frac{Q}{m} = \left(\frac{R}{k-1}\right)(T_2-T_1) + \left(\frac{R}{1-k}\right)(T_2-T_1) = 0 \longleftarrow \underline{\qquad Q/m}$$

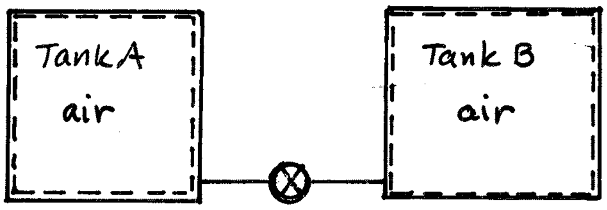
PROBLEM 3.100

KNOWN: The contents of two uninsulated tanks are allowed to mix, and equilibrium is attained at the temperature of the surroundings.

FIND: Determine the amount of energy transfer by heat and the final pressure.

SCHEMATIC & GIVEN DATA:

$m_{1A} = 1$ lb
$T_{1A} = 1440°R$
$P_{1A} = 50$ lbf/in²



$m_{1B} = 2$ lb
$T_{1B} = 900°R$
$P_{1B} = 50$ lbf/in²
$T_2 = 520°R$

ASSUMPTIONS: (1) The contents of the two tanks are a closed system. (2) There is no work. (3) The air behaves as an ideal gas. (4) Kinetic and potential energy effects can be neglected.

ANALYSIS: To find the heat transfer, begin with the energy balance.

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q - \cancelto{0}{W}$$

Since the final state is an equilibrium state

$$\Delta U = (m_{1A} + m_{1B})u_2 - (m_{1A}u_{1A} + m_{1B}u_{1B})$$

Thus

$$Q = (m_{1A} + m_{1B})u_2 - (m_{1A}u_{1A} + m_{1B}u_{1B})$$

Using data from Table A-22E

$$Q = (3\ \text{lb})(88.62\ \text{Btu/lb}) - [(1)(254.66) + (2)(154.57)]$$

$$= -297.9\ \text{Btu} \longleftarrow Q$$

To find the final pressure, first determine the volume of each tank

$$V_A = \frac{m_{1A} R T_{1A}}{P_{1A}} = \frac{(1\ \text{lb})\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(1440°\text{R})}{(50\ \text{lbf/in}^2)\left|144\ \text{in}^2/1\ \text{ft}^2\right|} = 10.66\ \text{ft}^3$$

$$V_B = \frac{(2)\left(\frac{1545}{28.97}\right)(900)}{(50)\left|144/1\right|} = 13.33\ \text{ft}^3$$

Thus, at the final state

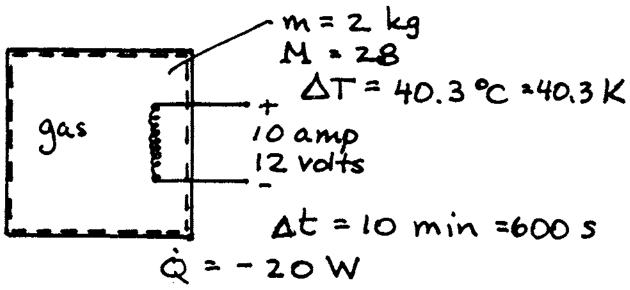
$$P_2 = \frac{mRT_2}{(V_A + V_B)} = \frac{(3)\left(\frac{1545}{28.97}\right)(520)}{(10.66 + 13.33)\left|144/1\right|} = 24.07\ \text{lbf/in}^2 \longleftarrow P_2$$

PROBLEM 3.101

KNOWN: A gas is contained in a closed rigid tank fitted with an electric resistor. The constant current and voltage and the constant heat transfer rate are known.

FIND: Determine an average value of specific heat c_p.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The gas is a closed system. (2) The gas behaves as an ideal gas. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: To determine the specific heat, begin with the energy balance

$$\overset{0}{\cancel{\Delta KE}} + \overset{0}{\cancel{\Delta PE}} + \Delta U = Q - W$$

with $\Delta U = m\,c_v\,\Delta T = m(c_p - R)\,\Delta T$

$$c_p = \frac{Q - W}{m\,\Delta T} + R$$

Next, evaluate the heat transfer and work, respectively

$$Q = \int_{t_1}^{t_2} \dot{Q}\,dt = \dot{Q}\,\Delta t = (-20\text{ W})(600\text{ s})\left|\frac{1\text{ kJ/s}}{10^3\text{ W}}\right|$$

$$= -12\text{ kJ}$$

$$W = \int_{t_1}^{t_2} \dot{W}\,dt = \dot{W}\,\Delta t$$

$$= -(10\text{ amp})(12\text{ volts})\left|\frac{1\text{ W/amp}}{1\text{ volt}}\right|(600\text{ s})\left|\frac{1\text{ kJ/s}}{10^3\text{ W}}\right|$$

$$= -72\text{ kJ}$$

Thus

$$c_p = \frac{(-12\text{ kJ}) - (-72\text{ kJ})}{(2\text{ kg})(40.3\text{ K})} + \left(\frac{8.314}{28}\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)$$

$$= 1.041\text{ kJ/kg}\cdot\text{K} \longleftarrow c_p$$

PROBLEM 3.102

KNOWN: CO_2 gas is compressed from a specified initial state in a process for which $pv^{1.2}$ constant. The work per unit of mass is also known.

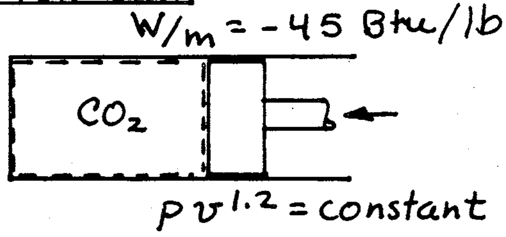
FIND: Determine the temperature at the final state and the heat transfer per unit of mass.

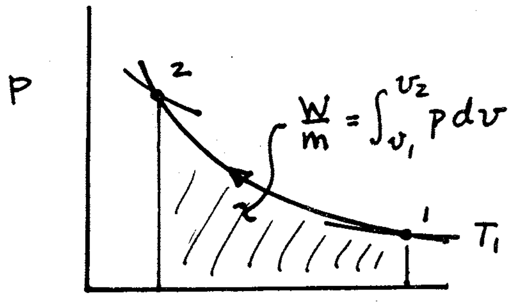
SCHEMATIC & GIVEN DATA:

$T_1 = 530°R$

$P_1 = 15\ lbf/in^2$

$V_1 = 1\ ft^3$





ASSUMPTIONS: (1) The gas is a closed system. (2) The carbon dioxide behaves as an ideal gas. (3) The process is $pv^{1.2}$ = constant. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The heat transfer is determined using an energy balance. First, the final temperature is found using $W = \int_1^2 pdV$ in the form of Eq. 3.57:

$$\frac{W}{m} = \frac{R(T_2 - T_1)}{1-n}$$

Solving for T_2

$$T_2 = \left(\frac{1-n}{R}\right)\frac{W}{m} + T_1$$

$$= \frac{(1-1.2)}{\left(\frac{1545}{44.01}\ \frac{ft \cdot lbf}{lb \cdot °R}\right)}\left(-45\ \frac{Btu}{lb}\right)\left|\frac{778\ ft \cdot lbf}{1\ Btu}\right| + 530°R$$

$$= 729.5°R \quad \blacktriangleleft \quad T_2$$

Now, the energy balance is

$$\Delta \overset{0}{\cancel{KE}} + \Delta \overset{0}{\cancel{PE}} + \Delta U = Q - W$$

or

$$\frac{Q}{m} = (\bar{u}_2 - \bar{u}_1)\frac{1}{M} + \frac{W}{m}$$

From Table A-23E, $\bar{u}_1 = 2916.1$ Btu/lbmol and $\bar{u}_2 = 4394.0$ Btu/lbmol.

Inserting values

$$\frac{Q}{m} = \frac{4394.0 - 2916.1}{44.01} + (-45)$$

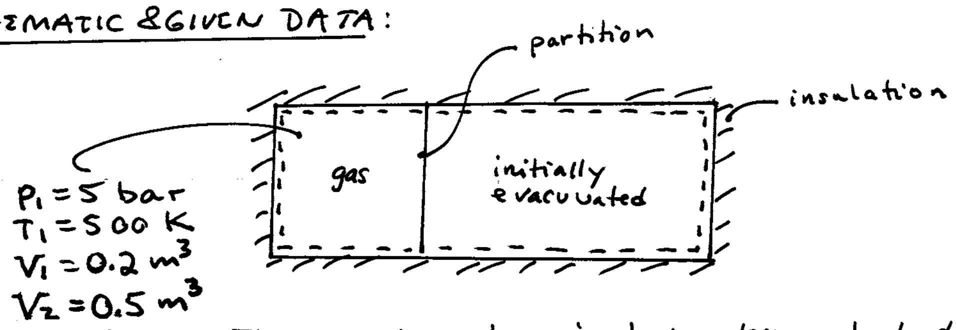
$$= -11.42\ Btu/lb \quad \blacktriangleleft \quad Q/m$$

PROBLEM 3.103*

KNOWN: A gas is confined to one side of a rigid, insulated container divided by a partition. The other side is initially evacuated. The partition is removed and the gas expands to fill the entire container.

FIND: Determine the final pressure.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The closed system is shown by a dashed line on the above schematic. 2. The gas behaves as an ideal gas. 3. For the process, $Q = W = 0$, and kinetic and potential energy changes are neglibible.

ANALYSIS: Applying the ideal gas equation of state

$$\left.\begin{array}{l} p_1 V_1 = mRT_1 \\ p_2 V_2 = mRT_2 \end{array}\right\rangle \quad \frac{p_2 V_2}{p_1 V_1} = \frac{T_2}{T_1} \Rightarrow p_2 = p_1\left[\frac{T_2}{T_1}\right]\left[\frac{V_1}{V_2}\right]$$

The final temperature is required. Applying an energy balance

$$\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = \cancel{Q} - \cancel{W} \Rightarrow \Delta U = 0$$

Since internal energy of an ideal gas depends on temperature alone, $T_2 = T_1$. Thus

$$p_2 = (5\text{ bar})[1]\left[\frac{0.2\ m^3}{0.5\ m^3}\right] = 2\text{ bar} \longleftarrow$$

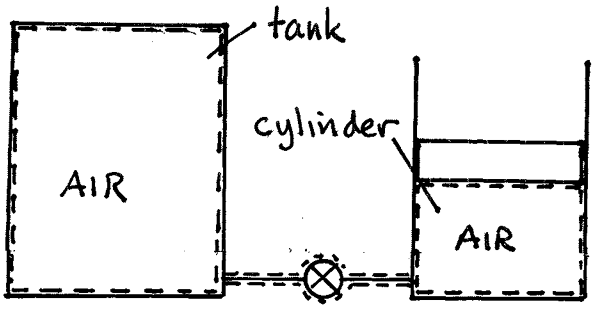
PROBLEM 3.104

KNOWN: Air flows through a valve from a rigid tank into a vertical piston-cylinder assembly.

FIND: Determine the total amount of energy transfer by work and heat.

SCHEMATIC & GIVEN DATA:

$m_{1t} = 3$ kg
$p_{1t} = 500$ kPa
$T_t = 290$K



$p_c = 200$ kPa
$T_c = 290$K
$V_{1c} = 0.05\ m^3$

ASSUMPTIONS: (1) The air in the tank and cylinder is a closed system. (2) The air behaves as an ideal gas. (3) The pressure in the cylinder remains constant. (4) Kinetic and potential energy effects are negligible. (5) The temperature of the air is constant.

ANALYSIS: Taking the air in both the tank and the cylinder as the system, the energy balance is

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \cancelto{0}{\Delta U} = Q - W$$

where the change in internal energy is zero because the temperature is constant. Thus

$$Q = W \qquad \longleftarrow Q = W$$

Using $W = \int p_c\, dV$, with constant pressure p_c

$$Q = p_c(V_2 - V_1)$$

① Now, with the ideal gas model

$$m_{1c} = \frac{p_c V_{1c}}{R T_c} = \frac{(200\ kPa)(0.05\ m^3)}{\left(\frac{8.314}{28.97}\ \frac{kJ}{kg\cdot K}\right)(290K)} \left|\frac{10^3\ N/m^2}{1\ kPa}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 0.12\ kg$$

$$V_1 = V_{1c} + V_t = V_{1c} + \frac{m_{1t} R T_t}{p_{1t}} = 0.05 + \frac{(3)\left(\frac{8.314}{28.97}\right)(290)}{(500)} = 0.549\ m^3$$

$$V_2 = \frac{(m_{tot}) R T_2}{p_2} = \frac{(3 + 0.12)\left(\frac{8.314}{28.97}\right)(290)}{(200)} = 1.298\ m^3$$

Finally

$$Q = (200\ kPa)(1.298 - 0.549)\ m^3 \left|\frac{10^3\ N/m}{1\ kPa}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 149.8\ kJ \longleftarrow Q$$

---

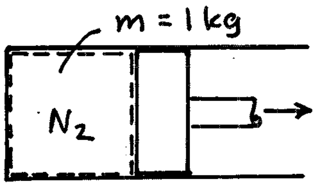
1. The applicability of the ideal gas model can be verified by referring to the compressibility chart.

PROBLEM 3.105*

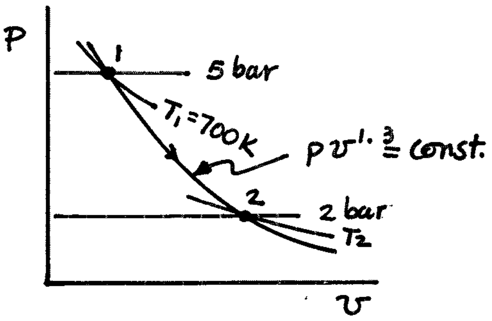
KNOWN: Nitrogen gas undergoes a polytropic expansion in a piston-cylinder assembly.

FIND: Determine the heat transfer using (a) constant specific heat evaluated at 300K, (b) constant specific heat evaluated at 700K, and (c) nitrogen tables.

SCHEMATIC & GIVEN DATA:



$T_1 = 700$ K
$P_1 = 5$ bar
$P_2 = 2$ bar



ASSUMPTIONS: (1) The nitrogen is a closed system. (2) The nitrogen behaves as an ideal gas. (3) The process is polytropic, with n = 1.3. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, the work is evaluated for the polytropic process. With assumption (2), Eqs. 3.56 and 3.57 apply

$$T_2 = \left(\frac{P_2}{P_1}\right)^{\left(\frac{n-1}{n}\right)} T_1 = \left(\frac{2}{5}\right)^{(.3/1.3)} (700\text{K}) = 566.6\text{ K}$$

and

$$W = \frac{mR(T_2 - T_1)}{1-n} = \frac{(1\text{ kg})\left(\frac{8.314}{28.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(566.6-700)\text{K}}{(1-1.3)} = 132\text{ kJ}$$

To determine the heat transfer, use an energy balance

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = Q - W$$

or

$$m(u_2 - u_1) = Q - W$$

$$Q = m(u_2 - u_1) + W$$

(a) From Table A-20 at 300K, $c_v = 0.743$ kJ/kg·K. Thus

$$Q = mc_v(T_2 - T_1) + W$$

$$= (1\text{ kg})(0.743\text{ kJ/kg}\cdot\text{K})(566.6-700)\text{K} + (132\text{ kJ})$$

$$= 32.88\text{ kJ} \longleftarrow \text{Q (a)}$$

(b) From Table A-20 at 700K, $c_v = 0.801$ kJ/kg·K. Thus

$$Q = (1)(0.801)(566.6 - 700) + (132) = 25.15\text{ kJ} \longleftarrow \text{Q (b)}$$

(c) Using values for $\bar{u}$ from Table A-23

$$Q = m\left(\frac{\bar{u}_2 - \bar{u}_1}{M}\right) + W$$

① $$= (1\text{ kg})\frac{(11{,}850 - 14{,}784)\text{ kJ/kmol}}{(28.01\text{ kg/kmol})} + 132\text{ kJ}$$

$$= 27.25\text{ kJ} \longleftarrow \text{Q (c)}$$

---

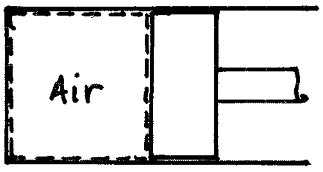
1. The approach of part (c) explicitly accounts for the variation of specific heat c_v with temperature.

PROBLEM 3.106

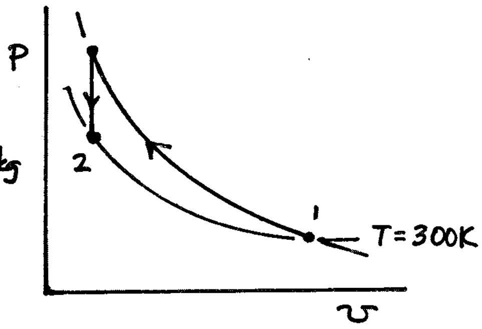
KNOWN: Air undergoes an adiabatic compression followed by constant volume cooling.

FIND: Determine the work for the first process and the heat transfer for the second one, using (a) air table data, (b) constant specific heats.

SCHEMATIC & GIVEN DATA:



Process 1-2: adiabatic
$p_1 = 1$ bar, $T_1 = 300$K
$p_2 = 15$ bar, $v_2 = .1227$ m³/kg
Process 2-3: Constant volume
$T_3 = 300$K



ASSUMPTIONS: (1) The air is a closed system. (2) There is no heat transfer for the first process and no work for the second one. (3) The air behaves as an ideal gas. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, using the ideal gas equation of state to determine T_2.

$$T_2 = \frac{p_2 v_2}{R} = \frac{(15\text{ bar})(.1227\text{ m}^3/\text{kg})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 641.3\text{K}$$

(a) Using the energy balance for each process

Process 1-2: $\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + m(u_2 - u_1) = \cancel{Q_{12}}^{0} - W_{12}$

$$\frac{W_{12}}{m} = u_1 - u_2$$

From Table A-22, $u_1 = 214.07$ kJ/kg and, interpolating, $u_2 \approx 466.5$ kJ/kg. Thus

$$\frac{W_{12}}{m} = 214.07 - 466.5 = -252.4\text{ kJ/kg} \longleftarrow W_{12}/m$$

Process 2-3: $\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{0}$

$$\frac{Q_{23}}{m} = u_3 - u_2$$

Since $T_3 = T_1$, $u_3 = u_1$ and $Q_{23}/m = W_{12}/m = -252.4$ kJ/kg $\longleftarrow$ Q_{23}/m

(b) Assuming a constant value for c_v

$$\frac{W_{12}}{m} = c_v(T_1 - T_2)$$

From Table A-20, at 300K, $c_v = 0.718$ kJ/kg·K. Thus

$$\frac{W_{12}}{m} = (0.718\text{ kJ/kg}\cdot\text{K})(300 - 641.3)\text{K}$$
$$= -245\text{ kJ/kg} \longleftarrow W_{12}/m$$

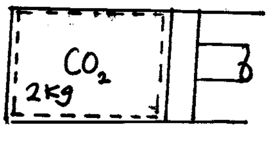
and $$\frac{Q_{23}}{m} = \frac{W_{12}}{m} = -245\text{ kJ/kg} \longleftarrow Q_{23}/m$$

PROBLEM 3.107

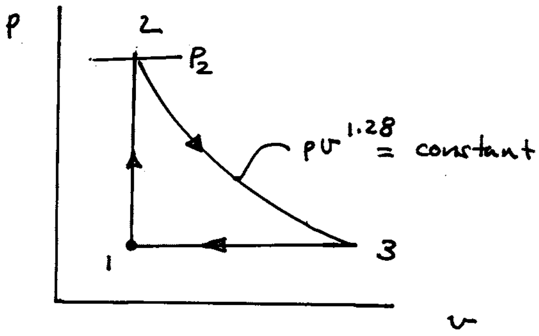
KNOWN: A system consisting of 2 kg of CO_2 undergoes a power cycle consisting of three processes.

FIND: Skech the cycle on a p-v diagram, and plot the thermal efficiency versus P_2/P_1 ranging from 1.05 to 4.

SCHEMATIC & GIVEN DATA:



P_1 = 1 bar
T_1 = 300K



ASSUMPTIONS: 1. The CO_2 is a closed system. 2. The CO_2 behaves as an ideal gas. 3. Process 2-3 is described by $pv^{1.28}$ = constant. 4. Kinetic and potential energy effects are negligible.

ANALYSIS: Begin by developing the key expressions describing the cycle. The thermal efficiency is given by $\eta = W_{net}/Q_{in}$, where $W_{net} = W_{12} + W_{23} + W_{31}$. Since volume is constant $W_{12} = 0$. For process 2-3 and process 3-1

$$W_{23} = m\int_2^3 p\,dv = m\frac{(P_3v_3 - P_2v_2)}{1-n} = mR\frac{(T_3-T_2)}{1-n} \qquad (1)$$

$$W_{31} = m\int_3^1 p\,dv = m(P_1 - P_3) = mR(T_1 - T_3) \qquad (2)$$

Applying an energy balance:

$$\text{Process 1-2:}\quad m(u_2-u_1) = Q_{12} - \cancel{W_{12}}^{\,0} \Rightarrow Q_{12} = m(u_2-u_1) \qquad (3)$$

$$\text{Process 2-3:}\quad m(u_3-u_2) = Q_{23} - W_{23} \Rightarrow Q_{23} = m(u_3-u_2) + W_{23} \qquad (4)$$

$$\text{Process 3-1:}\quad m(u_1-u_3) = Q_{31} - W_{31} \Rightarrow Q_{31} = m(u_1-u_3) + W_{31} \qquad (5)$$

Additionally, the ideal gas equation of state gives

$$\left.\begin{array}{l} P_1V = mRT_1 \\ P_2V = mRT_2 \end{array}\right\rangle \rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow T_2 = \left(\frac{P_2}{P_1}\right)T_1 \qquad (6)$$

and with Eq. 3.56 for process 2-3 $(P_3 = P_1)$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{(n-1)/n} \Rightarrow T_3 = T_2\left(\frac{P_1}{P_2}\right)^{(n-1)/n} \qquad (7)$$

SAMPLE CALCULATION: $P_2/P_1 = 4$

If $P_2/P_1 = 4$, Eq. (6) gives $T_2 = 4(300) = 1200\text{K}$, and Eq. (7) gives

$$T_3 = (1200\text{K})\left(\frac{1}{4}\right)^{.28/1.28} = 886\text{ K}$$

Eq. (1) then gives

$$W_{23} = \frac{(2\text{ kg})\left(\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg·K}}\right)(886-1200)\text{K}}{1-1.28} = 423.7\text{ kJ}$$

PROBLEM 3.107 (Contd.)

Eq.(2) gives

$$W_{31} = (2\,kg)\left(\frac{8.314}{44.01}\ \frac{kJ}{kg \cdot K}\right)(300-886)\,K = -221.4\ kJ$$

With data from Table A-23, Eq.(3) gives

$$Q_{12} = \left(\frac{2\ kg}{44.01\ kJ/kmol}\right)(43{,}871-6939)\ \frac{kJ}{kmol} = 1678.3\,kJ$$

Eq.(4) gives

$$Q_{23} = \left(\frac{2}{44.01}\right)(29{,}298-43{,}871) + 423.7 = -238.6\ kJ$$

Eq.(5) gives

$$Q_{31} = \left(\frac{2}{44.01}\right)(6939-29{,}298) - 221.4 = -1237.5\ kJ$$

Since Q_{23} and Q_{31} are both negative, we conclude that $Q_{in} = Q_{12}$. Thus, for the case $p_2/p_1 = 4$

$$\eta = \frac{423.7+(-221.4)}{1678.3} = 0.121\ (12.1\ \%) \longleftarrow \underset{(P_2/P_1=4)}{\eta}$$

Turning now to IT:

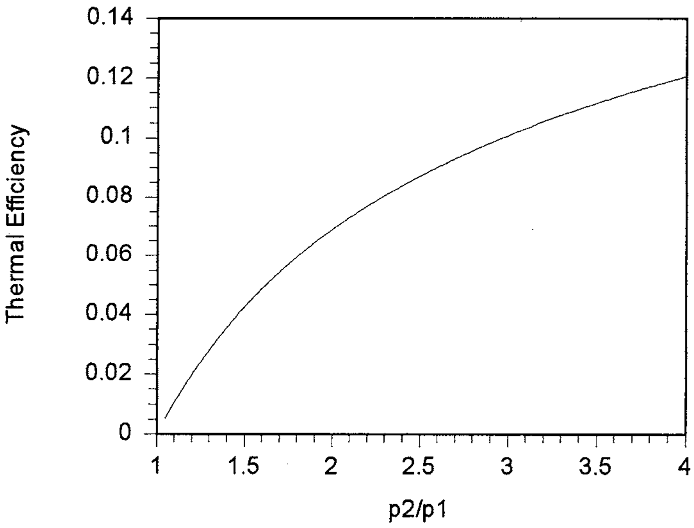
```
T1 = 300  // K
p1 = 1  // bar
m = 2  // kg
M = 44.01  // kg/kmol
R = 8.314 / M

W23 = m*R*(T3 - T2) / (1-1.28)
W31 = m*R*(T1 - T3)
m*(u2 - u1) = Q12
Wcycle = W23 + W31
eta = Wcycle / Q12
p2 / p1 = T2 / T1
p2 = r * p1
r = 4
p3 = p1
T3 / T2 = (p3 / p2)^((1.28-1) / 1.28)
u1 = u_T("CO2", T1)
u2 = u_T("CO2", T2)
u3 = u_T("CO2", T3)

/* Using the Explore button, sweep r from
   1.05 to 4 in steps of 0.01.  */
```

IT Results ($r = 4$)

Q_{in} = 1678 kJ
T_2 = 1200 K
T_3 = 886.1 K
W_{23} = 423.6 kJ
W_{31} = -221.4 kJ
η = 0.1205



PROBLEM 3.108

KNOWN: Air undergoes a power cycle consisting of three processes.

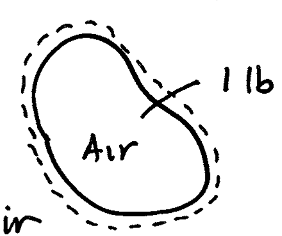
FIND: Sketch the cycle on a p-v diagram and determine (a) the pressure at state 2, (b) the temperature at state 3, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA: The following data are given for each process:

Process 1-2: constant volume from $p_1 = 20 \text{ lbf/in}^2$, $T_1 = 500°R$ to $T_2 = 820°R$

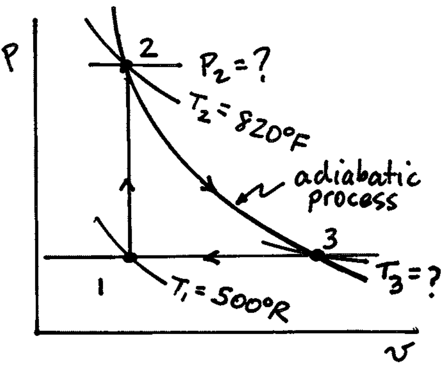
Process 2-3: adiabatic expansion to $v_3 = 1.4 v_2$

Process 3-1: constant-pressure compression



ASSUMPTIONS: (1) The air is the closed system. (2) The air behaves as an ideal gas. (3) Kinetic and potential energy effects are neglected.

ANALYSIS: (a) For process 1-2, the volume is constant. Thus, using the ideal gas equation of state



$$\left.\begin{array}{l} p_1 V = mRT_1 \\ p_2 V = mRT_2 \end{array}\right\} \Rightarrow \frac{p_1}{p_2} = \frac{T_1}{T_2} \Rightarrow p_2 = \frac{T_2}{T_1} \cdot p_1$$

$$= \frac{820}{500}\left(20 \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= \underline{\underline{32.8 \frac{\text{lbf}}{\text{in.}^2}}} \longleftarrow p_2$$

(b) Again, using the ideal equation of state

$$\frac{v_3}{v_2} = 1.4 = \frac{mRT_3/p_3}{mRT_2/p_2} = \frac{T_3}{T_2} \cdot \frac{p_2}{p_3} \Rightarrow T_3 = (1.4)\frac{p_3}{p_2} \cdot T_2 \overset{p_3 = p_1}{=} (1.4)\frac{p_1}{p_2} \cdot T_2$$

Thus $T_3 = (1.4)\left(\frac{20}{32.8}\right)(820) = 700°R \longleftarrow T_3$

(c) The thermal efficiency is $\eta = W_{cycle}/Q_{in}$. Analyze each process in turn.

Process 1-2: const. volume $\Rightarrow W_{12} = 0$.

Energy Balance: $\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q_{12} - \cancel{W_{12}} \Rightarrow Q_{12} = m(u_2 - u_1)$

with data from Table A-22

$$Q_{12} = (1 \text{ lb})(140.47 - 85.20) \text{Btu/lb} = 55.27 \text{ Btu}$$

Process 2-3: Adiabatic: $m(u_3 - u_2) = \cancel{Q_{23}} - W_{23}$

$$W_{23} = -m(u_3 - u_2) = -(1 \text{ lb})(119.58 - 140.47) \text{Btu/lb} = 20.89 \text{ Btu}$$

Process 3-1: p = constant. Using Eq. 2.17 $W_{31} = \int_{V_3}^{V_1} p\,dV = m\,p\,(v_1 - v_3) = mR(T_1 - T_3)$

$$W_{31} = (1 \text{ lb})\left(\frac{1545}{28.97}\right)\frac{\text{ft·lbf}}{\text{lb·°R}}\left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right|(500 - 700)°R = -13.71 \text{ Btu}$$

Energy Balance: $m(u_1 - u_3) = Q_{31} - W_{31} \Rightarrow Q_{31} = m(u_1 - u_3) + W_{31}$

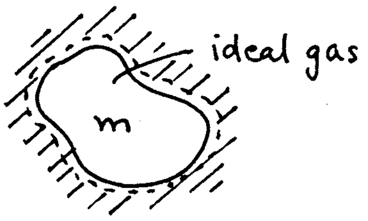
$$Q_{31} = (1 \text{ lb})(85.20 - 119.58) \text{Btu/lb} + (-13.71 \text{ Btu}) = -48.09 \text{ Btu}$$

Thus $W_{cycle} = \cancelto{0}{W_{12}} + W_{23} + W_{31} = 20.87 + (-13.71) = 7.18 \text{ Btu}$

$Q_{in} = Q_{12} = 55.27 \text{ Btu}$

Finally $\eta = W_{cycle}/Q_{in} = 7.18/55.27 = 0.1299 \; (12.99\%) \longleftarrow \eta$

PROBLEM 3.109



(a) For an adiabatic process with negligible effects of kinetic and potential energy

$$\cancelto{0}{\Delta KE} + \cancelto{0}{\Delta PE} + \Delta U = \cancelto{0}{Q} - W$$

Since the specific heats are constant

$$W = m c_v (T_1 - T_2)$$

or, using Eq. 3.47b, $c_v = R/(k-1)$, so

$$W = \frac{mR(T_2 - T_1)}{1-k} \longleftarrow \text{(a)}$$

(b) For a polytropic process

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{\text{const}}{V^n}\,dV = \frac{p_2V_2 - p_1V_1}{1-n}$$

Introducing $pV = mRT$ and letting n be the specific heat ratio k

$$W = \frac{mR(T_2 - T_1)}{1-k}$$

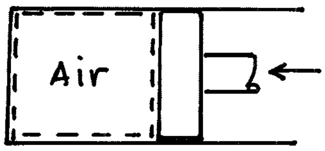
This result is identical to the result of part (a). Thus, for an ideal gas with constant specific heats, the polytropic process pV^k = constant corresponds to an adiabatic process. ⟵ (b)

PROBLEM 3.110

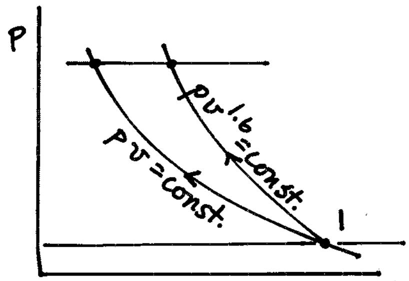
KNOWN: Air undergoes a polytropic process from a known initial state to a given final pressure.

FIND: Plot the heat transfer and work for $1.0 \leq n \leq 1.6$. Investigate the error in heat transfer introduced by assuming constant c_v evaluated at T_1.

SCHEMATIC & GIVEN DATA:



$p_1 = 14.7$ lbf/in.²
$T_1 = 70°F$
$p_2 = 100$ lbf/in.²
$pv^n = \text{constant}$



ASSUMPTIONS: (1) The air is a closed system. (2) The process is polytropic. (3) The air behaves as an ideal gas. (4) Neglect kinetic and potential energy effects.

Begin by fixing state 2. From the polytropic process relation

$$p_1 v_1^n = p_2 v_2^n \Rightarrow v_2 = (p_1/p_2)^{1/n} v_1$$

Thus, with $v_1 = RT_1/p_1$, and $T_2 = p_2 v_2/R$, state 2 is determined for any value of n. These expressions are evaluated using IT as follows:

```
p1 = 14.7  // lbf/in.²
T1 = 70  // °F
p2 = 100  // lbf/in.²
m = 1  // lb
n = 1.6

p1 * v1^n = p2 * v2^n
v1 = v_TP("Air",T1,p1)
v2 = v_TP("Air",T2,p2)
```

IT Results ($n = 1.6$)

```
T2 = 627.4 °F
v1 = 13.34 ft³/lb
v2 = 4.026 ft³/lb
```

The work is evaluated using Eq. 2.17 and Eq. 3.54

$$W = \int_{V_1}^{V_2} p\,dV = m\int_{v_1}^{v_2} p\,dv = \frac{m(p_2 v_2 - p_1 v_1)}{(1-n)} \qquad (n \neq 1)$$

Or, as expressed using IT

```
W/m =( (p2 * v2 - p1 * v1) / (1 - n)) * (144 / 778)  // Btu/lb
```

To find Q, use the energy balance: $\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$. or

$$m\,\Delta u = Q - W$$

Method 1

To evaluate Δu, use data from Table A-22

$$\Delta u = u(T_2) - u(T_1)$$

From IT:

```
m * delta_u_1 = Q_1 - W
u1 = u_T("Air", T1)
u2 = u_T("Air", T2)
delta_u_1 = u2 - u1
```

PROBLEM 3.110 (Contd.)

Method 2

An alternative method to evaluate Δu is to use a constant c_v value from Table A-20 determined at $T_1 = 70°F$. That is

$$\Delta u = c_v (T_2 - T_1)$$

From IT:

```
m * delta_u_2 = Q_2 - W
cv = cv_T("Air", T1)
delta_u_2 = cv * (T2 - T1)
```

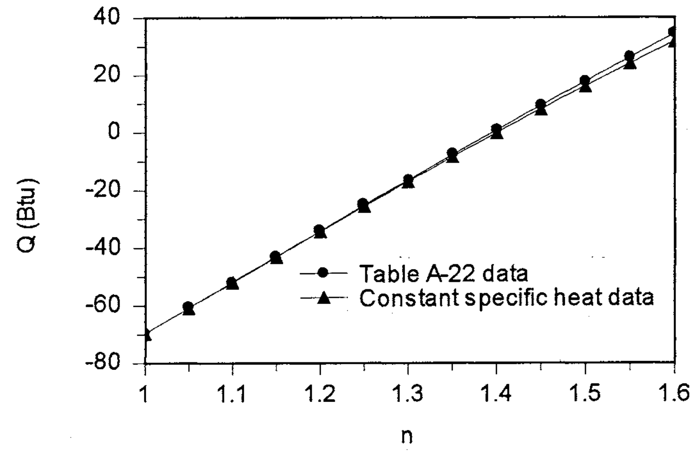
Now, solving gives

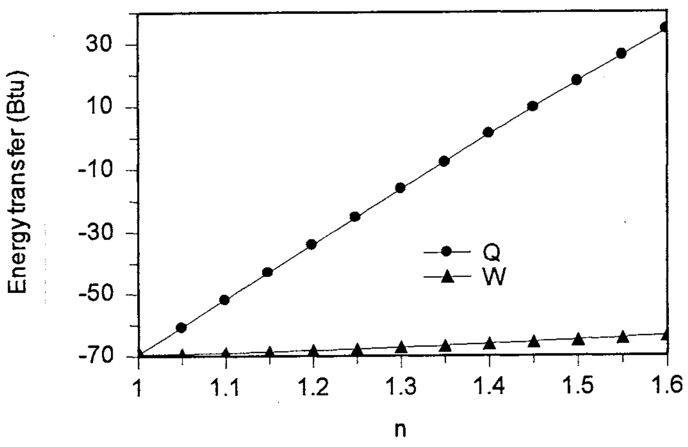
```
W = -63.68 Btu

u1 = 90.38 Btu/lb
u2 = 188.3 Btu/lb
delta_u_1 = 97.88 Btu
Q_1 = 34.19 Btu

cv = 0.1711 Btu/lb·°R
delta_u_2 = 95.38 Btu
Q_2 = 31.7 Btu
```

① Using the Explore button, sweep n from 1.001 to 1.6 in steps of 0.01. The data obtained give the following graphs:





From the graph of Q and W vs. n, we can see several effects:

- Q and W become equal as $n \to 1$, since for $n = 1$ the temperature is constant, and hence $\Delta u = 0$.
- Q changes sign, going from negative (heat out) to positive (heat in) at about $n = 1.4$. W is negative (work in) for all cases.
- W increases only slightly, whereas Q changes significantly.

From the graph of Q vs. n we see that the solutions become identical as $n \to 1$ (constant temperature) and diverge slightly as $n \to 1.6$. For $n = 1.6$, the percentage deviation in Q is about 7.3%.

---

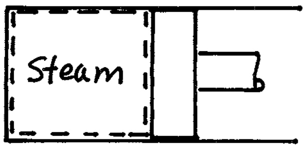
1. By using $n = 1.001$, we avoid the case of $n = 1$ for which Eq. 3.54 cannot be used. However, the slight deviation from $n = 1$ cannot be seen on the plots.

PROBLEM 3.111

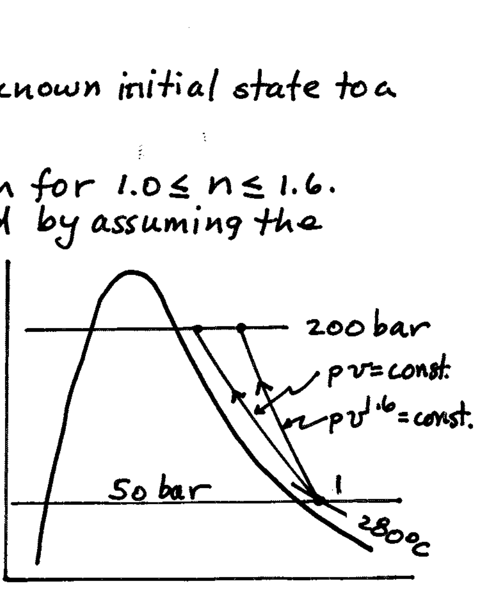
KNOWN: Steam undergoes a polytropic process from a known initial state to a given final pressure.

FIND: Plot the heat transfer per unit mass of steam for $1.0 \le n \le 1.6$. Investigate the error in heat transfer introduced by assuming the ideal gas model.

SCHEMATIC & GIVEN DATA:



$p_1 = 5\,\text{MPa} = 50\ \text{bar}$
$T_1 = 280°\text{C}$
$p_2 = 20\ \text{MPa} = 200\ \text{bar}$
$pv^n = \text{constant}$



ASSUMPTIONS: (1) The steam is a closed system. (2) The process is polytropic. (3) Kinetic and potential energy effects are neglected.

ANALYSIS: The following relations are used to determine the heat transfer, Q/m :

Process relation : $p_1 v_1^{\,n} = p_2 v_2^{\,n}$ (1)

Work expression (Eqs. 2.17 and 3.54) : $\frac{W}{m} = \frac{(p_2 v_2 - p_1 v_1)}{1-n}$ (2)

Energy balance : $\cancel{\Delta KE} + \cancel{\Delta PE} + \Delta U = Q - W$

$\Rightarrow \quad (u_2 - u_1) = \frac{Q}{m} - \frac{W}{m}$ (3)

When using steam table data (IT functions or Table A-4):

- $p_1 = 50\ \text{bar},\ T_1 = 280°\text{C} \Rightarrow v_1,\ u_1$
- with $p_2 = 200$ bar and (2), v_2 can be determined.
- $p_2, v_2 \Rightarrow u_2, T_2$

When using the ideal gas model (IT "H_2O" functions or Table A-23):

- $p_1 = 50\ \text{bar},\ T_1 = 280°\text{C} \Rightarrow v_1 = \frac{RT_1}{p_1}$
- $T_1 = 280°\text{C} \Rightarrow u(T_1)$
- with $p_2 = 200$ bar and (2), v_2 can be determined.
- $p_2, v_2 \Rightarrow T_2 = \frac{p_2 v_2}{R}$
- $T_2 \Rightarrow u(T_2)$

With either of these schemes, it is then possible to use (2) and (3) to solve for W/m and Q/m.

The corresponding IT code is given on the next page.

PROBLEM 3.111 (Contd.)

```
p1 = 50  // bar
T1 = 280  // °C
p2 = 200  // bar
m = 1 // kg
n = 1.6
p1 * v1^n = p2 * v2^n
W / m = ((p2 * v2 - p1 * v1) / (1 - n)) * 100
m * (u2 - u1) = Q - W
```

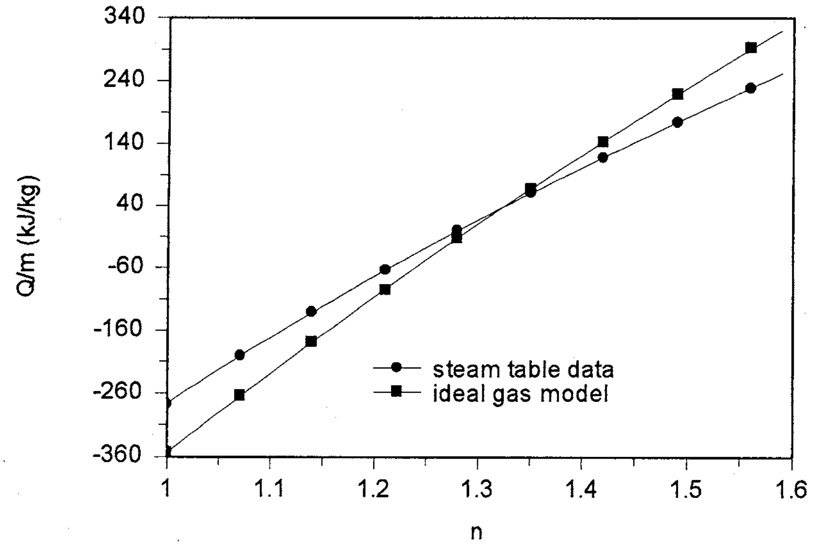
```
// Using steam table data:
v1 = v_PT("Water/Steam", p1, T1)
u1 = u_PT("Water/Steam", p1, T1)
v2 = v_PT("Water/Steam", p2, T2)
u2 = u_PT("Water/Steam", p2, T2)
```

```
// Using the ideal gas model:
v1 = v_TP("H2O",T1,p1)
u1 = u_T("H2O", T1)
v2 = v_TP("H2O",T2,p2)
u2 = u_T("H2O", T2)
```

IT Results for n = 1.6

quantity	steam table data	Ideal gas model
Q/m (kJ/kg)	251.9	320.8
T_2 (°C)	583.7	652.1
W/m (kJ/kg)	-240.8	-291
u_1 (kJ/kg)	2645	-1.319E4
u_2 (kJ/kg)	3138	-1.257E4
v_1 (m³/kg)	0.04224	0.05104
v_2 (m³/kg)	0.01766	0.02135

① Using the Explore button, sweep n from 1.001 to 1.6 in steps of 0.01. The data obtained give the following plot:



Discussion:

- Q/m varies significantly with n.
- Q/m changes sign, going from negative (heat out) to positive (heat in).
- The steam table and ideal gas solutions exhibit the same general trends. However, the values for Q/m differ significantly in some ranges of n.

---

1. By using n = 1.001, we avoid the case of n = 1 for which (2) above cannot be used. However, the slight deviation from n = 1 cannot be seen on the plot.

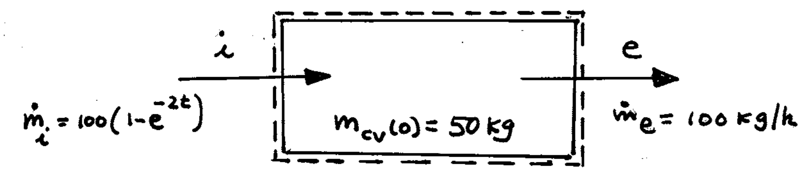
CHAPTER FOUR
CONTROL VOLUME ANALYSIS USING ENERGY

PROBLEM 4.1

KNOWN: The mass flow rates into and out of a control volume are known, and the initial mass is given.

FIND: (a) Plot the inlet and exit mass flow rates, the rate of change of mass, and the amount of mass in the control volume, each versus time. (b) Estimate the time, in h, when the tank is nearly empty.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (a) As shown in the schematic, a control volume having one inlet and one exit is under consideration.

ANALYSIS: The inlet and exit mass flow rates are given

$$\dot{m}_i = 100(1-e^{-2t})\ \text{kg/h} \quad , \quad \dot{m}_e = 100\ \text{kg/h} \qquad (1)$$

The instantaneous time rate of change of mass is obtained using Eq. 4.6, the mass rate balance

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e = 100(1-e^{-2t}) - 100$$

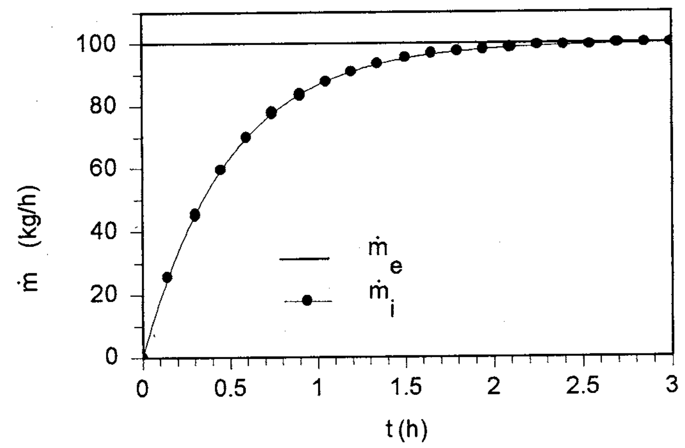
$$= -100\, e^{-2t} \qquad (2)$$

The mass contained in the control volume at time t is obtained by integrating Eq. (2)

$$m_{cv}(t) - m_{cv}(0) = -100\int_0^t e^{-2t}\,dt = 50e^{-2t}\Big]_0^t$$

$$\Rightarrow \quad m_{cv}(t) = 50\left[e^{-2t} - 1\right] + 50 = 50e^{-2t} \qquad (3)$$

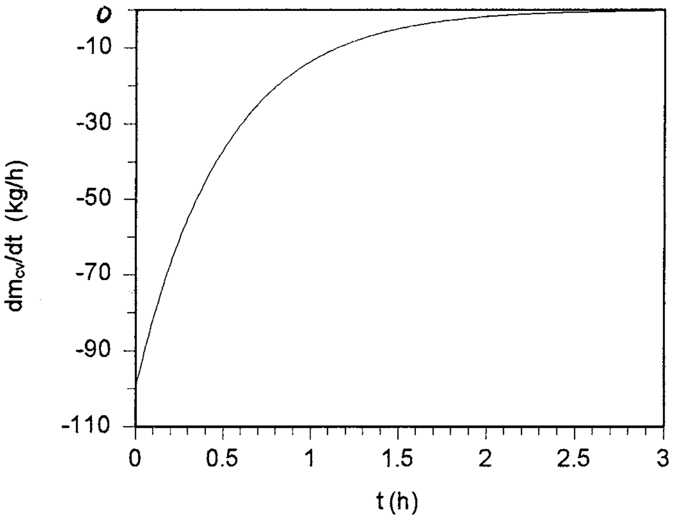
Plotting Eqs.(1) using IT gives the inlet and exit mass flow rates vs. time:



From this plot we see that initially mass flows out at a high rate, and no mass flows in. Eventually, the incoming flow rate equals the exit flow rate.

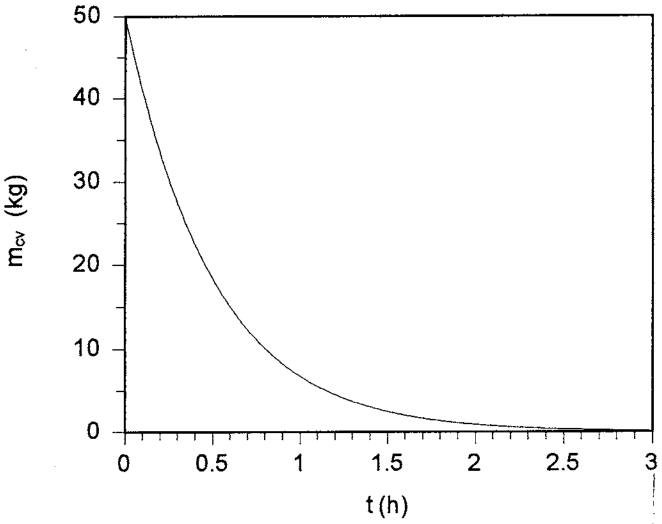
PROBLEM 4.1 (Contd.)

The following plot is obtained for the instantaneous rate of change of mass:



Initially, the amount of mass in the tank is decreasing at a rapid rate. Eventually, the rate of change (decrease) of mass slows to zero.

The following plot is obtained for the amount of mass in the tank as a function of time:



From this plot, we see that the tank initially contains 50kg and empties.

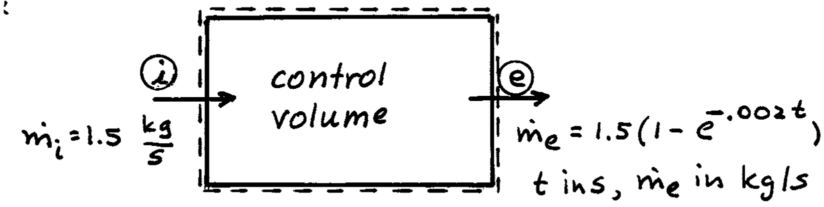
(b) From the plot, we see that the tank is nearly empty at $t \approx 2.5$ h.

PROBLEM 4.2

KNOWN: Mass flows into and out of a control volume with known flow rates.

FIND: Plot the time rate of change of mass and the net change in the amount of mass versus time.

SCHEMATIC AND GIVEN DATA :



i
control volume
e
$\dot{m}_i = 1.5\ \frac{kg}{s}$
$\dot{m}_e = 1.5(1-e^{-.002t})$
t in s, $\dot{m}_e$ in kg/s

ANALYSIS: The rate of change of mass at time t is

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e \Rightarrow \frac{dm_{cv}}{dt} = 1.5 - 1.5(1-e^{-.002t})\ kg/s \qquad (1)$$
$$= 1.5\,e^{-.002t}$$

Integrating the mass rate balance over time

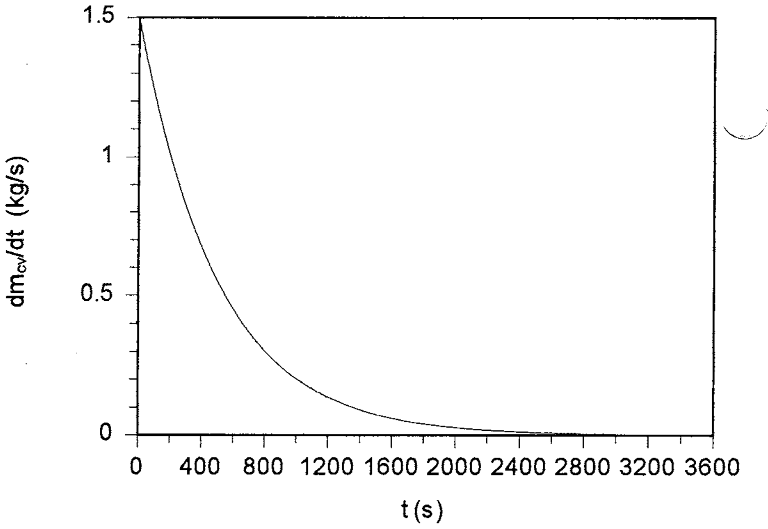
$$\int dm_{cv} = \int_0^t (\dot{m}_i - \dot{m}_e)dt \qquad (2)$$

or
$$\Delta m_{cv}(t) = \int_0^t [1.5\,e^{-.002t'}]dt' = \left[\frac{1.5\,e^{-.002t'}}{(-0.002)}\right]_0^t$$
$$= 750(1-e^{-.002t})$$

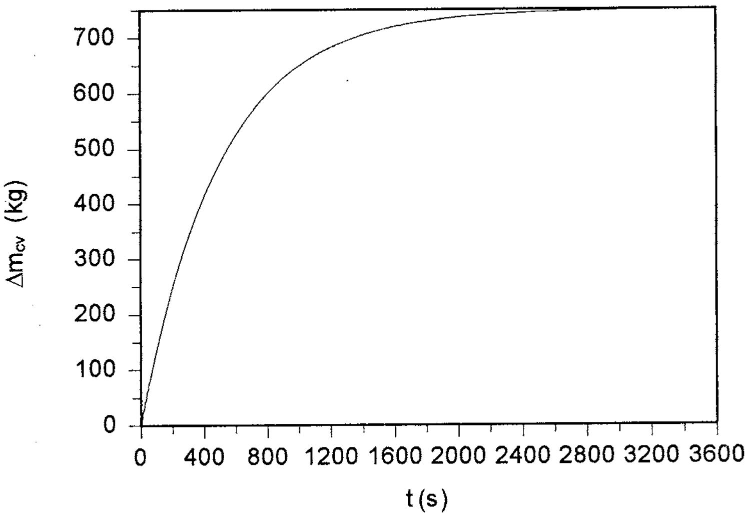
IT program

```
mdot_i = 1.5  // kg/s
mdot_e = 1.5 * (1 - exp(-.002*t))  // kg/s
dmcvdt = mdot_i - mdot_e
delta_m = Integral(dmcvdt,t)
```

Using the Explore button, sweep t from 0 to 3600 s in steps of 4.



From the upper graph and Eq. (1), we see that initially the inlet flow rate is high and the exit flow rate is zero. Thus, dm_{cv}/dt starts high. As t increases, the exit flow rate increases, and dm_{cv}/dt approaches zero as $t \rightarrow 2400$ s.



The bottom graph shows that the net change in mass increases until it approaches 750 kg. At that point there is little change in mass (t > 2400s).

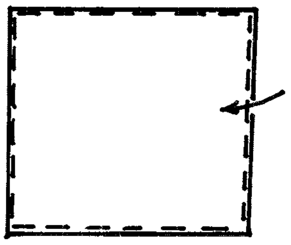
PROBLEM 4.3*

KNOWN: Air leaks into an initially evacuated tank at a constant mass flow rate.

FIND: Determine the pressure in the tank after 30 s.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) The control volume has one inlet and no exits. (2) The air behaves as an ideal gas.



$\dot{m}_i = 0.004$ lb/s

$m(t=0) = 0$

$V = 5$ ft³

$T_2 = 70°F = 530°R$

ANALYSIS: The mass rate balance, Eq. 4.6, reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i \Rightarrow dm_{cv} = \dot{m}_i\,dt$$

Integrating

$$\int_0^m dm_{cv} = \int_0^{30} \dot{m}_i\,dt = \int_0^{30} (0.004)\,dt$$

$$m(t=30) = (0.004)(30) = 0.12 \text{ lb}$$

Now, using the ideal gas model

$$P_2 = \frac{mRT_2}{V} = \frac{(0.12 \text{ lb})\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(530°R)}{(5 \text{ ft}^3)}\left|\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right|$$

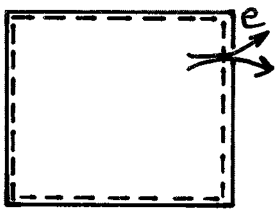
$$= 4.711 \text{ lbf/in}^2 \longleftarrow P_2$$

PROBLEM 4.4*

KNOWN: A tank containing Ammonia develops a leak. The ammonia escapes at a constant mass flow rate.

FIND: Determine the time at which half the mass has escaped and the pressure in the tank at that time.

SCHEMATIC & GIVEN DATA:



$\dot{m}_e = 0.04$ kg/s

$V = 0.5\ m^3$

$T_1 = 40°C$

$P_1 = 8$ bar

$m_2 = \frac{1}{2} m_1$

ASSUMPTIONS: (1) The control volume is shown on the accompanying diagram. (2) The temperature in the tank remains constant. (3) The volume is constant.

ANALYSIS: Applying the mass rate balance

$$\frac{dm_{cv}}{dt} = -\dot{m}_e = -0.04 \ \text{kg/s}$$

Integrating over time

$$\int_{m_{cv}(0)}^{m_{cv}(t)} dm_{cv} = -\int_0^t \dot{m}_e dt = -0.04 \ t$$

Solving for t

$$t = \frac{m_{cv}(t) - m_{cv}(0)}{(-0.04 \ \text{kg/s})}$$

From Table A-15, $v_1 = 0.17720\ m^3/kg$. Thus, the initial mass is

$$m_{cv}(0) = \frac{V}{v_1} = 2.822 \ \text{kg}$$

and $$m_{cv}(t) = \frac{1}{2} m_{cv}(0) = 1.411 \ \text{kg}$$

Thus

$$t = \frac{(1.411 - 2.822)\ \text{kg}}{(-0.04 \ \text{kg/s})} = 35.3 \ \text{s} \longleftarrow t$$

Now, to get P_2, determine v_2

$$v_2 = \frac{V}{m_{cv}(t)} = 0.3544 \ m^3/kg$$

Interpolating in Table A-15 at $T_2 = 40°C$, $v_2 = 0.3544\ m^3/kg$

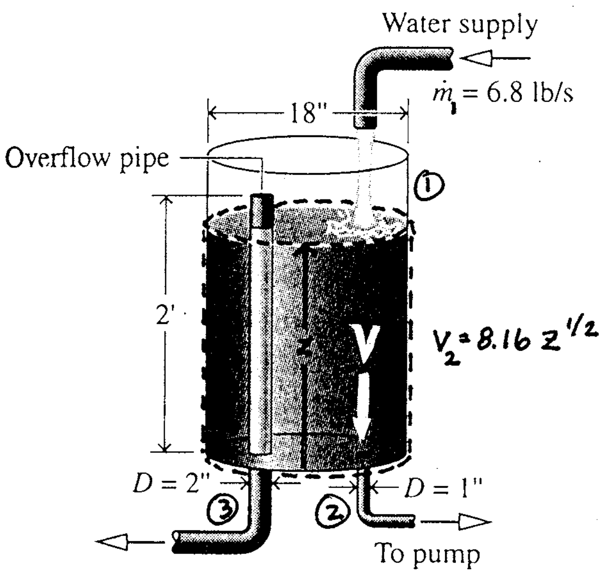
$$P_2 \approx 4.17 \ \text{bar} \longleftarrow P_2$$

PROBLEM 4.5

KNOWN: Water enters at the top of a tank through a supply pipe at a constant mass flow rate and exits to a pump through a pipe in the bottom of the tank. The velocity of the water exiting to the pump varies with height of the water surface. An overflow pipe stands in the tank.

FIND: Plot (a) the height of the water surface, (b) the rate mass exits to the pump, (c) the rate mass exit through the overflow pipe, each versus time. Determine the time when the water reaches the top of the overflow pipe.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume encloses the water in the tank. The control volume is at steady state when the water level reaches the top of the overflow pipe. (2) The water is incompressible, with $\rho = 1/v_{f\,@\,60°F} = 62.34$ lb/ft³. (3) Initially, the tank is empty.

ANALYSIS: (a) Before the water level reaches z = 2 ft, the mass rate balance is

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2 \Rightarrow \rho A_{tank} \frac{dz}{dt} = \dot{m}_1 - \rho A_2 V_2$$

Evaluating the areas:

$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi (1/12\ ft)^2}{4} = 5.454\times10^{-3}\ ft^2$$

$$A_{tank} = \frac{\pi D_{tank}^2}{4} = \frac{\pi (18/12\ ft)^2}{4} = 1.767\ ft^2$$

Thus

$$(62.34\ \tfrac{lb}{ft^3})(1.767\ ft^2)\frac{dz}{dt} = 6.8\ \tfrac{lb}{s} - (62.34\ \tfrac{lb}{ft^3})(5.454\times10^{-3}\ ft^2)\,8.16\, z^{1/2}$$

$$\Rightarrow \frac{dz}{dt} = 0.06173 - 0.02519\, z^{1/2} \quad (z \text{ in ft},\ t \text{ in s}) \qquad (*)$$

Equation (*) is a non-linear differential equation. By assumption (3), the initial condition is z = 0 at t = 0. The expression can be integrated using the integration feature of IT, as follows:

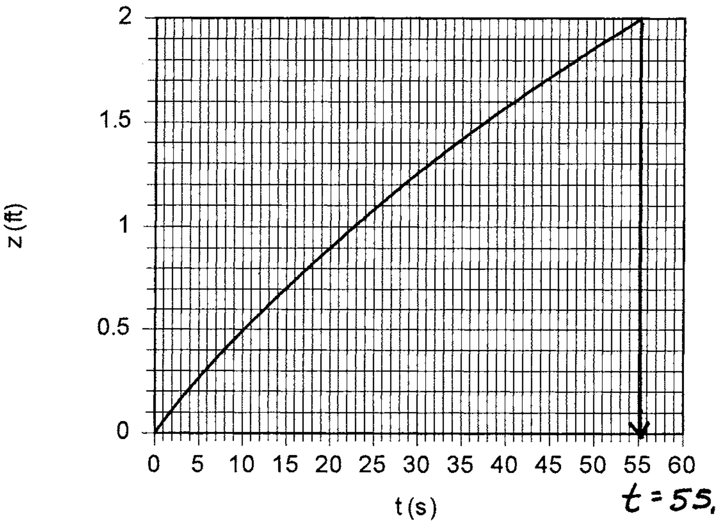
IT Program

```
f = 0.06173 - 0.02519 * z^0.5
der(z,t) = f
```

Using the Explore button, sweep t from 0 to 60 s in steps of 0.1.

The following graph can be obtained from the resulting data using IT:

PROBLEM 4.5 (Contd.)



$t = 55.2 \text{ s} \leftarrow$ time to overflow

$(z = 2 \text{ ft})$

(b) For $t < 55.2$ s, $\dot{m}_2$ varies with z as follows:

$$\dot{m}_2 = \rho A_2 V_2 = \rho A_2 (8.16 z^{1/2})$$

$$= (62.34)(5.454\times10^{-3})(8.16 z^{1/2}) = 2.7744 z^{1/2}$$

at $t = 55.2$ s; $z = 2$ ft and $\dot{m}_2 = 3.9236$ lb/s

For $t > 55.2$ s; $\dot{m}_2 = 3.9236$ lb/s (z remains constant at 2 ft) $\Big\} \leftarrow \dot{m}_2$

(c) For $t < 55.2$ s; $\dot{m}_3 = 0$ (flow hasn't reached the overflow pipe inlet)

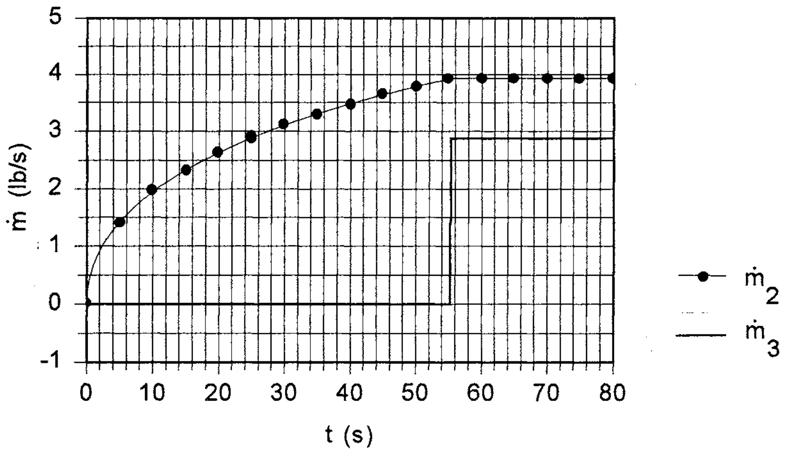
For $t > 55.2$ s; steady state. Thus

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2$$

$$= 6.8 \text{ lb/s} - 3.9236 \text{ lb/s}$$

$$= 2.876 \text{ lb/s} \leftarrow \dot{m}_3 \; (t > 55.2)$$

The following plot is constructed using IT:



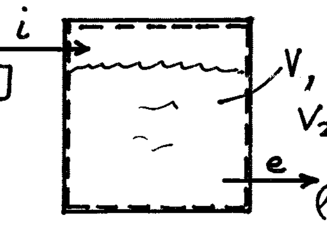
PROBLEM 4.6

KNOWN: A water storage tank contains a known volume of water. The volume flow rates in and out of the tank are given.

FIND: Determine how many days the tank will contain water.

SCHEMATIC & GIVEN DATA:

$(AV)_i = 5000[\exp(-t/20)]$
(t in days)



$V_1 = 100{,}000$ gal
$V_2 = 0$
$(AV)_e = 10{,}000$ gal/day

ASSUMPTIONS: (1) The control volume is as shown on the above sketch. (2) The water is incompressible.

ANALYSIS: The mass rate balance reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$

With $m_{cv} = \rho V$, and $\dot{m}_i = \rho(AV)_i$ and $\dot{m}_e = \rho(AV)_e$

$$\cancel{\rho}\frac{dV}{dt} = \cancel{\rho}(AV)_i - \cancel{\rho}(AV)_e$$

$$\Rightarrow \quad \frac{dV}{dt} = (AV)_i - (AV_e) = 5000\,[\exp(-t/20)] - 10{,}000$$

Integrating from $t=0$ to $t=t_f$

$$V_2 - V_1 = \int_0^{t_f} (5000\,[\exp(-t/20)] - 10{,}000)\,dt$$

$$= \left(\frac{5000}{(-1/20)}\exp(-t/20) - 10{,}000\,t\right)\Bigg|_0^{t_f}$$

① $$= [-100{,}000\exp(-t_f/20) - 1] - 10{,}000\,t_f \qquad (*)$$

With $V_1 = 100{,}000$ gal and for $V_2 = 0$

$$0 = 200{,}000 - 100{,}000\exp(-t_f/20) - 10{,}000\,t_f$$

Using IT to solve we get:

IT solution:

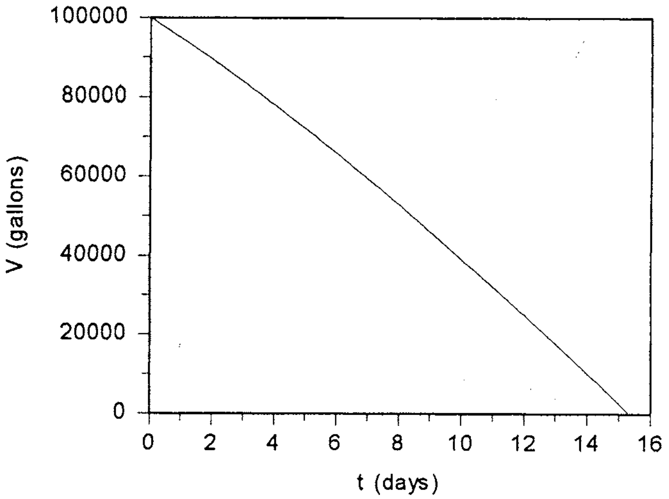
```
0 = 200000 - 100000 * exp(-tf/20) - 10000 * tf
// tf = 15.36 days
```

$\longleftarrow t_f$

---

1. To obtain V vs. t using *IT*:

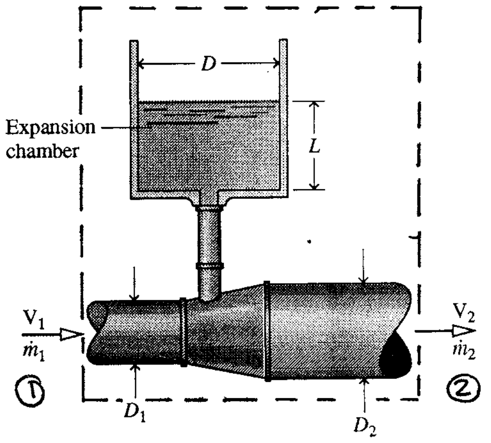
```
der(V,t) = AV1 - AV2
AV1 = 5000 * (exp(-t / 20))
AV2 = 10000
// Use the Explore button to sweep t from
// 0 to 15.36 days in steps of 0.05.
```



PROBLEM 4.7

KNOWN: A pipe carrying an incompressible liquid contains an expansion chamber.
FIND: (a) Develop an expression for the rate of change of liquid level in the chamber in terms of certain quantities. (b) Compare the relative magnitudes of the mass flow rates for specified values for dL/dt, the rate of change of liquid level.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume is shown on the accompanying schematic. 2. The liquid is modeled as incompressible. 3. Flow is one-dimensional at 1, 2.

ANALYSIS: (a) The mass rate balance for the control volume is

$$\frac{dm_{cv}}{dt} = \dot{m}_1 - \dot{m}_2$$

with

$$m_{cv} = \rho\left(\frac{\pi D^2}{4}\right)L$$

$$\dot{m}_1 = \rho\left(\frac{\pi D_1^2}{4}\right)V_1$$

$$\dot{m}_2 = \rho\left(\frac{\pi D_2^2}{4}\right)V_2$$

the mass rate balance becomes

$$\rho\left(\frac{\pi D^2}{4}\right)\frac{dL}{dt} = \rho\left(\frac{\pi D_1^2}{4}\right)V_1 - \rho\left(\frac{\pi D_2^2}{4}\right)V_2$$

or, solving for dL/dt and simpifying

$$\frac{dL}{dt} = \frac{D_1^2 V_1 - D_2^2 V_2}{D^2} \longleftarrow \text{dL/dt}$$

(b) The mass flow rate expressions indicate $\dot{m}_1 \sim D_1^2 V_1$, $\dot{m}_2 \sim D_2^2 V_2$.

Thus,

$$\frac{dL}{dt} > 0 \Rightarrow D_1^2 V_1 > D_2^2 V_2 \Rightarrow \dot{m}_1 > \dot{m}_2$$

$$\frac{dL}{dt} = 0 \Rightarrow D_1^2 V_1 = D_2^2 V_2 \Rightarrow \dot{m}_1 = \dot{m}_2$$

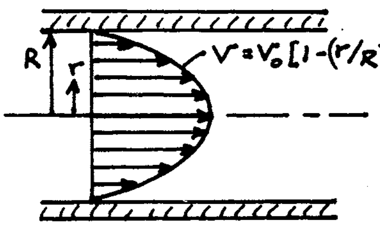
$$\frac{dL}{dt} < 0 \Rightarrow D_1^2 V_1 < D_2^2 V_2 \Rightarrow \dot{m}_1 < \dot{m}_2$$

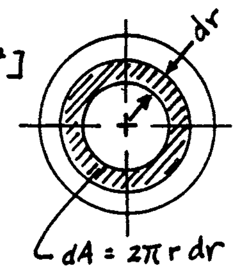
PROBLEM 4.8

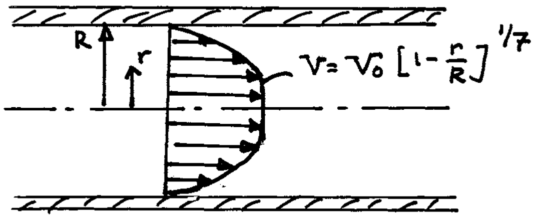
KNOWN: Velocity distributions are given for laminar and turbulent flow of an incompressible liquid in a circular pipe.

FIND: For each distribution, plot V/V_0 vs. r/R, derive expressions for the mass flow rate, average flow velocity, and specific kinetic energy. Determine the percent error if the specific kinetic energy is evaluated in terms of average velocity. Discuss.

SCHEMATIC & GIVEN DATA:





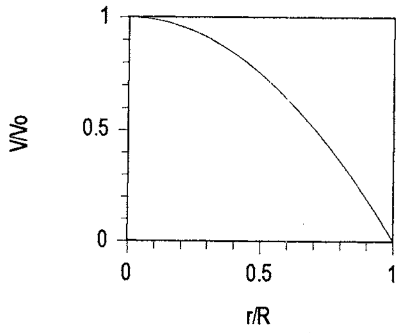


LAMINAR

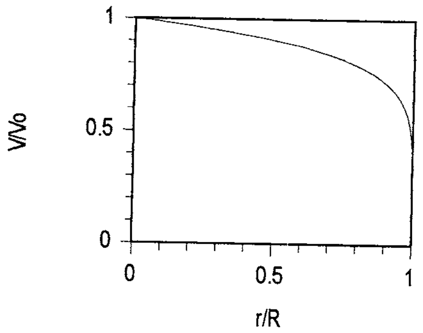
TURBULENT

ASSUMPTIONS: The liquid is modeled as incompressible.

ANALYSIS: (a)



Laminar



Turbulent

(b) Using Eq. 4.9, the mass flow rate for laminar flow is

$$\dot{m} = \int_A \rho V\, dA$$

$$= \int_0^R \rho V_0\left[1-\left(\frac{r}{R}\right)^2\right] 2\pi r\, dr$$

$$= \rho V_0 2\pi \int_0^R \left[r - \frac{r^3}{R^2}\right] dr$$

$$= \rho V_0 2\pi \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]\Bigg|_{r=0}^{r=R} = \rho V_0 2\pi\left[\frac{R^2}{2} - \frac{R^2}{4}\right]$$

$$= \rho V_0 \pi \frac{R^2}{2}$$

Using Eq. 4.11a, the average velocity is

$$V_{ave} = \frac{\dot{m}}{\rho A} = \frac{\rho V_0 \pi R^2/2}{\rho(\pi R^2)}$$

$$= \frac{V_0}{2}$$

PROBLEM 4.8 (Contd.)

Using Eq. 4.9, the mass flow rate for turbulent flow is

$$\dot{m} = \int_A \rho V \, dA$$

$$= \int_0^R \rho V_0 \left[1 - \frac{r}{R}\right]^{\frac{1}{7}} 2\pi r \, dr$$

$$= \rho V_0 2\pi \int_0^R r \left[1 - \frac{r}{R}\right]^{\frac{1}{7}} dr$$

To evaluate the integral, let $u = 1 - r/R$ and $dr = -R\,du$. Then

$$\dot{m} = \rho V_0 2\pi \int_1^0 R(1-u)\, u^{1/7} (-R\,du) = \rho V_0 2\pi \left[-R^2 \int_1^0 (u^{1/7} - u^{8/7})\, du\right]$$

$$= \rho V_0 2\pi \left[-R^2 \left(\frac{7}{8} u^{8/7} - \frac{7}{15} u^{15/7}\right)_{u=1}^{u=0}\right]$$

$$= \rho V_0 2\pi \left[R^2 \frac{49}{120}\right] = \rho V_0 \pi \left(\frac{49}{60}\right) R^2$$

Using Eq. 4.11a, the average velocity is

$$V_{ave} = \frac{\dot{m}}{\rho A} = \frac{\rho V_0 \pi \left(\frac{49}{60}\right) R^2}{\rho (\pi R^2)} = \frac{49}{60} V_0$$

(c) The specific kinetic energy (kinetic energy per unit of mass) carried through an area normal to the flow is

$$ke = \frac{\int_A \frac{V^2}{2} \rho V \, dA}{\dot{m}} = \frac{\rho \int_A \frac{V^3}{2} dA}{\rho \int_A V \, dA} = \frac{\int_A \frac{1}{2} V^3 dA}{\int_A V \, dA} = \frac{\int_A \frac{1}{2} V^3 dA}{V_{ave} A} \qquad (*)$$

Forming the ratio of Eq. (*) to the specific kinetic energy calculated as $V_{ave}^2/2$, we have the kinetic energy coefficient (or kinetic energy correction factor):

$$\alpha = \frac{\int_A \frac{1}{2} V^3 dA}{\frac{1}{2} V_{ave}^3 A} = \frac{\int_A V^3 dA}{V_{ave}^3 A}$$

For Laminar Flow: $\alpha = 2$ → % ERROR = 50

① For Turbulent Flow: $\alpha = 1.058$ → % ERROR = 5.5

The flatter turbulent velocity profile adheres most closely to the idealizations of one-dimensional flow.

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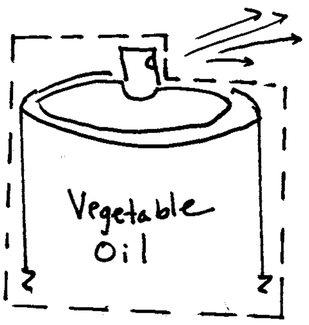
1. For further discussion, see R. W. Fox and A. T. McDonald, *Introduction to Fluid Mechanics*, 5th ed., J. Wiley & Sons, New York, pp 354-356.

PROBLEM 4.9

KNOWN: Data are provided for a vegetable oil-filled spray can.

FIND: Determine the mass flow rate per spray, and the mass remaining in the can after a specified number of sprays.

SCHEMATIC & GIVEN DATA:



- 560 sprays, each lasting 0.25 s and having a mass of 0.25 g
- initial mass of vegetable oil in the can is 170 g

ASSUMPTION: The control volume is shown in the accompanying schematic.

ANALYSIS: (a) Since each spray has a duration of 0.25 s and consists of 0.25 g

$$\dot{m}_e = \frac{0.25\,g}{0.25\,s} = 1\ g/s$$

(b) The mass rate balance, Eq. 4.3, reduces to

$$m_{cv}(t+\Delta t) - m_{cv}(0) = \overset{0}{\cancel{m_i}} - m_e$$

where $m_{cv}(0)$ is the initial amount of mass within the can and m_e is the amount of mass that exits. Thus, with $m_{cv}(0) = 170$ g and

$$m_e = (560\ \text{sprays})\left(\frac{0.25\,g}{\text{spray}}\right) = 140\,g$$

The mass of vegetable oil remaining in the can after 560 sprays is

$$m_{cv} = m_{cv}(0) - m_e$$
$$= 170\,g - 140\,g = 30\,g$$

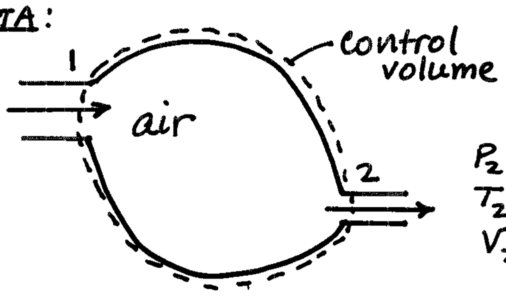
PROBLEM 4.10*

KNOWN: Air flows through a one-inlet, one-exit control volume. Data are known at the inlet and exit.

FIND: Determine (a) the mass flow rate, (b) the exit area.

SCHEMATIC & GIVEN DATA:

$P_1 = 8$ bar
$T_1 = 600$ K
$V_1 = 40$ m/s
$A_1 = 20$ cm^2
$= 0.002$ m^2



$P_2 = 2$ bar
$T_2 = 400$ K
$V_2 = 350$ m/s

ASSUMPTIONS: (1) The control volume is at steady state. (2) The air behaves as an ideal gas. (3) The flow is one-dimensional at the inlet and exit.

ANALYSIS: Beginning with the mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{\,0} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

Using data at the inlet and the ideal gas equation of state

(a) $$\dot{m} = \rho_1 A_1 V_1 = \left(\frac{P_1}{RT_1}\right) A_1 V_1$$

$$= \frac{(8 \text{ bar})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(600 \text{ K})}\left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|(0.002 \text{ m}^2)(40 \text{ m/s})$$

$$= 0.3717 \text{ kg/s} \longleftarrow \dot{m}$$

(b) From $\dot{m}_1 = \dot{m}_2$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Thus $$A_2 = \left(\frac{\rho_1}{\rho_2}\right)\left(\frac{V_1}{V_2}\right) A_1$$

With $\rho = P/RT$

$$A_2 = \left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right) A_1$$

$$= \left(\frac{8}{2}\right)\left(\frac{400}{600}\right)\left(\frac{40}{350}\right)(20 \text{ cm}^2)$$

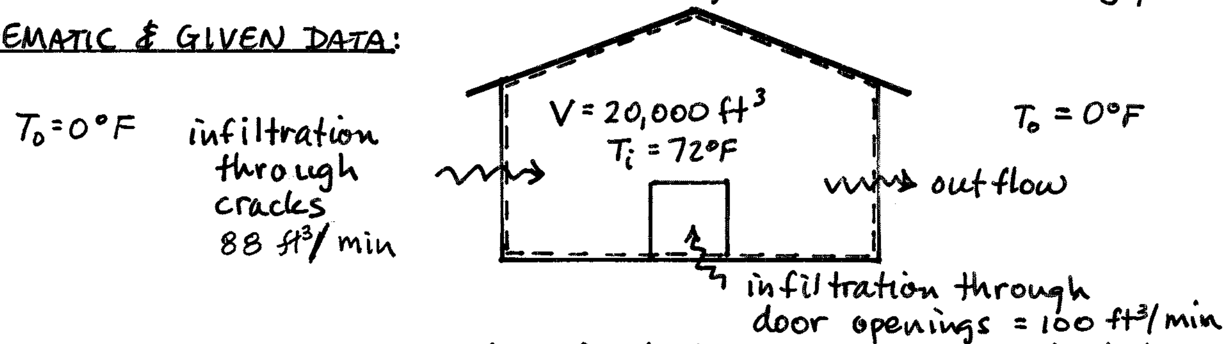
$$= 6.095 \text{ cm}^2 \longleftarrow A_2$$

PROBLEM 4.11

KNOWN: Air enters a building through cracks around doors and windows and due to door openings. The internal volume of the building is known.

FIND: Estimate the number of air changes within the building per hour.

SCHEMATIC & GIVEN DATA:


$T_o = 0°F$
infiltration through cracks 88 ft³/min
$V = 20{,}000\ ft^3$
$T_i = 72°F$
$T_o = 0°F$
outflow
infiltration through door openings = 100 ft³/min

ASSUMPTIONS: (1) The control volume is at steady state. (2) The air behaves as an ideal gas. (3) The inside and outside air pressures are nearly equal.

ANALYSIS: At steady state, the mass balance reduces to

$$\dot{m}_{outflow} = \dot{m}_{cracks} + \dot{m}_{doors}$$

$$\rho_i (AV)_{outflow} = \rho_o \left[(AV)_{cracks} + (AV)_{doors} \right]$$

where ρ_i and ρ_o are the inside and outside densities, respectively. Assuming ideal gas behavior

$$\frac{\rho_o}{\rho_i} = \frac{\cancel{p_o}/\cancel{R}T_o}{\cancel{p_i}/\cancel{R}T_i} = \frac{T_i}{T_o}$$

Thus

$$(AV)_{outflow} = \left(\frac{T_i}{T_o}\right) \left[(AV)_{cracks} + (AV)_{doors} \right]$$

Inserting values

$$(AV)_{outflow} = \left(\frac{532\ °R}{460\ °R}\right) \left[88\ ft^3/min + 100\ ft^3/min \right] = 217\ ft^3/min$$

$$= 217\ \frac{ft^3}{min} \left| \frac{60\ min}{1\ h} \right| = 13020\ \frac{ft^3}{h} \longleftarrow (AV)_{outflow}$$

$$\text{Air changes per hour} = \frac{13{,}020\ ft^3/h}{20{,}000\ ft^3/\text{air change}}$$

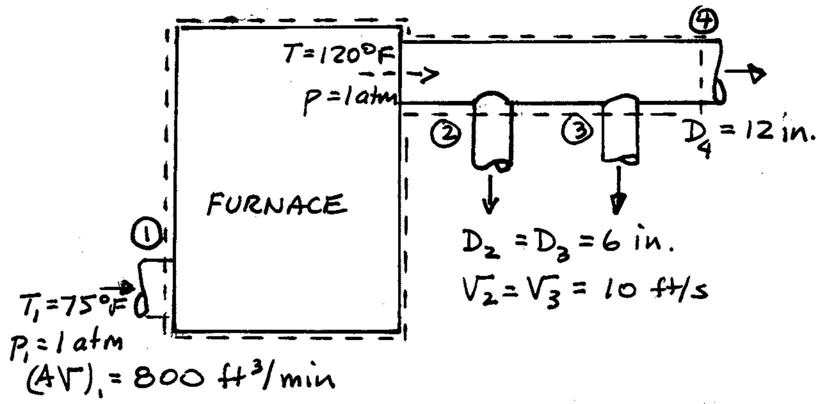
$$\approx 0.65\ \text{air changes/h} \longleftarrow \text{air changes}$$

PROBLEM 4.12

KNOWN: Air enters a furnace operating at steady state and is to a duct system consisting of three ducts. Data are known at the inlet and each of the discharge ducts.

FIND: Determine (a) the mass flow rate entering the furnace, (2) the volumetric flow rate in each of the 6-in. exit ducts, (c) the velocity in the 12-in. exit duct.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control shown on the accompanying sketch is at steady state. (2) The air behaves as an ideal gas. (3) The temperature and pressure in each duct are the same as the temperature and pressure of the air delivered to the duct system.

ANALYSIS: (a) Using Eq. 4.11b with the ideal gas equation

$$\dot m_1 = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{R T_1}$$

$$= \frac{(1\text{ atm})(800\text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot °\text{R}}\right)(535\,°\text{R})}\left|\frac{14.696\text{ lbf/in.}^2}{1\text{ atm}}\right|\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right|\left|\frac{1\text{ min}}{60\text{ s}}\right|$$

$= 0.989$ lb/s ← $\dot m_1$

(b) Since $D_2 = D_3$ and $V_2 = V_3$,

$$(AV)_2 = (AV)_3 = \left(\frac{\pi D^2}{4}\right)V = \left(\frac{\pi\left(\frac{6}{12}\right)^2\text{ft}^2}{4}\right)\left(10\frac{\text{ft}}{\text{s}}\right)\left|\frac{60\text{ s}}{1\text{ min}}\right|$$

$= 117.8$ ft³/min ← $(AV)_2, (AV)_3$

(c) Applying the mass balance to get $\dot m_4$

$$\overset{0}{\cancel{\frac{dm_{cv}}{dt}}} = \dot m_1 - \dot m_2 - \dot m_3 - \dot m_4 \Rightarrow \dot m_4 = \dot m_1 - \dot m_2 - \dot m_3 \qquad (*)$$

With Eq. 4.11b

$$\dot m_2 = \dot m_3 = \frac{(AV)}{v} = \frac{p(AV)}{RT} = \frac{(14.696)(117.8)}{\left(\frac{1545}{28.97}\right)(580)}\left|\frac{144}{60}\right| = 0.1343\text{ lb/s}$$

From (*)

$$\dot m_4 = 0.989 - 2(.1343) = 0.7204\text{ lb/s}$$

Finally, from $\dot m_4 = \frac{A_4 V_4}{v_4}$

$$V_4 = \frac{v_4 \dot m_4}{A_4} = \frac{(RT_4)(\dot m_4)}{(p_4)\left(\frac{\pi D_4^2}{4}\right)} = \frac{\left(\frac{1545}{28.97}\right)(580)(0.7204)}{(14.696)\frac{\pi(12)^2}{4}} = 13.41\text{ ft/s}$$ ← V_4

PROBLEM 4.13*

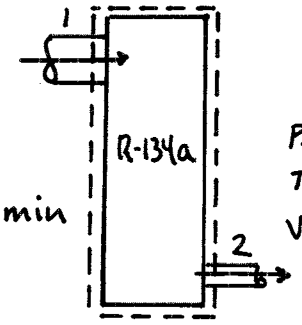
KNOWN: Refrigerant 134a flows through a refrigeration condenser. Data are known at the inlet and exit. The mass flow at the inlet is given.

FIND: Determine (a) the inlet velocity, (b) the diameter of the exit pipe.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) The control volume is at steady state. (2) The flow is one-dimensional at the inlet and exit.

$p_1 = 9$ bar
$T_1 = 50$ °C
$d_1 = 2.5$ cm
$\dot{m}_1 = 6$ kg/min



$p_2 = 9$ bar
$T_2 = 30$ °C
$V_2 = 2.5$ m/s

ANALYSIS: (a) Solving Eq. 4.11b for V_1

$$V_1 = \frac{\dot{m}_1 v_1}{A_1} = \frac{\dot{m}_1 v_1}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{4\dot{m}_1 v_1}{\pi d_1^2}$$

From Table A-12, $v_1 = 0.02472$ m³/kg. Thus

$$V_1 = \frac{(4)(6 \text{ kg/min})(0.02472 \text{ m}^3/\text{kg})}{\pi\ (2.5)^2 \text{cm}^2}\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\left|\frac{10^4 \text{ cm}^2}{1 \text{ m}^2}\right|$$

$= 5.04$ m/s ← V_1

(b) To find the diameter of the exit pipe, begin with mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_2 = \dot{m}_1$$

Thus, with $\dot{m}_2 = A_2 V_2 / v_2$

$$A_2 = \frac{\dot{m}_2 v_2}{V_2}$$

From Table A-11, $T_2 < T_{sat @ 9 \text{ bar}}$. Thus, the refrigerant is a sub-cooled liquid. From Table A-10,

$$v_2 \approx v_{f @ 30 °C} = 0.8417 \times 10^{-3} \text{ m}^3/\text{kg}$$

Inserting values

$$A_2 = \frac{(6 \text{ kg/min})(0.8417 \times 10^{-3} \text{ m}^3/\text{kg})}{(2.5 \text{ m/s})}\left|\frac{1 \text{ min}}{60 \text{ s}}\right| = 3.367 \times 10^{-5} \text{m}^2$$

Finally, with $A = \pi d^2/4$

$$d_2 = \left(\frac{4A_2}{\pi}\right)^{1/2} = 0.0065 \text{m} = 0.65 \text{ cm}$$ ← d_2

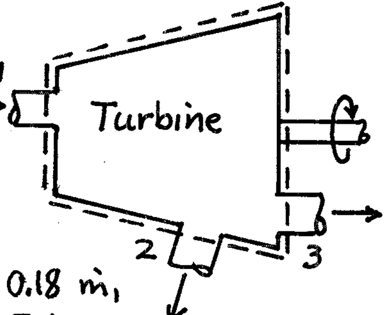
PROBLEM 4.14*

KNOWN: Data are given for steam flowing through a turbine with one inlet and two exits.

FIND: Determine the diameter of each exit duct.

SCHEMATIC & GIVEN DATA:

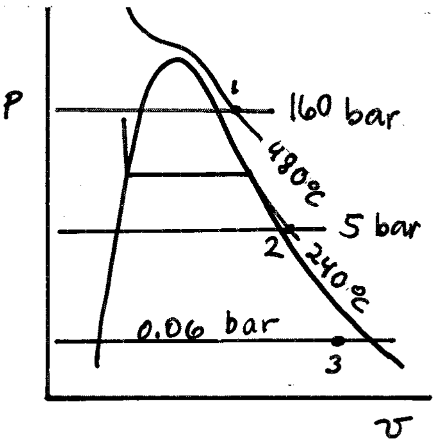
$P_1 = 160$ bar
$T_1 = 480$ °C
$(AV)_1 = 800\ \frac{m^3}{min}$



$P_3 = 0.06$ bar
$x_3 = 0.94$
$V_3 = 400$ m/s

$\dot{m}_2 = 0.18\ \dot{m}_1$
$P_2 = 5$ bar
$T_2 = 240$ °C
$V_2 = 25$ m/s

ASSUMPTIONS: (1) The control volume is at steady state. (2) The flow at the inlet and each exit is one-dimensional.



ANALYSIS: The mass flow rate at the inlet is

$$\dot{m}_1 = \frac{(AV)_1}{v_1}$$

From Table A-4, $v_1 = 0.01842$ m³/kg. Thus

$$\dot{m}_1 = \frac{(800\ m^3/min)}{(0.01842\ m^3/kg)}\left|\frac{1\ min}{60\ s}\right| = 723.9\ kg/s$$

and $\dot{m}_2 = 0.18\ \dot{m}_1 = 130.3$ kg/s

Applying the mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 593.6\ kg/s$$

Now, with $\dot{m} = AV/v$, and from Table A-4; $v_2 = 0.4646$ m³/kg

$$A_2 = \frac{\dot{m}_2 v_2}{V_2} = \frac{(130.3\ kg/s)(0.4646\ m^3/kg)}{(25\ m/s)} = 2.421\ m^2$$

Noting that $A = \pi d^2/4$

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = 1.756\ m \longleftarrow d_2$$

From Table A-3, at $P_3 = 0.06$ bar and $x_3 = 0.94$

$$v_3 = v_{f3} + x_3(v_{g3} - v_{f3})$$
$$= 1.0064\times10^{-3} + (0.94)(23.739 - 1.0064\times10^{-3}) = 22.315\ m^3/kg$$

Thus

$$A_3 = \frac{\dot{m}_3 v_3}{V_3} = \frac{(593.6)(22.315)}{(400)} = 33.115\ m^2$$

and

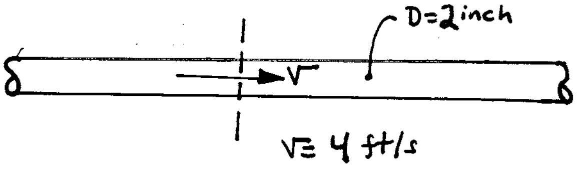
$$d_3 = \sqrt{\frac{4A_3}{\pi}} = 6.49\ m \longleftarrow d_3$$

PROBLEM 4.15*

KNOWN: Data are provided for substances flowing through a pipe of known diameter.

FIND: Determine the mass flow rate for each of three specified substances.

SCHEMATIC & GIVEN DATA:



ANALYSIS: With Eq. 4.11b

$$\dot{m} = \frac{A\mathrm{V}}{v} = \frac{(\pi D^2/4)\mathrm{V}}{v} = \frac{\pi\left[\left(\frac{1}{12}\,\mathrm{ft}\right)^2/4\right]\left[4\ \mathrm{ft/s}\right]}{v} = \frac{0.02182\ \mathrm{ft^3/s}}{v}$$

(a) water at 50 lbf/in², 80°F

with $v \simeq v_f(80°\mathrm{F}) = 0.01607\ \mathrm{ft^3/lb}$ (Table A-2E)

$$\therefore\ \dot{m} = \frac{0.02182\ \mathrm{ft^3/s}}{0.01607\ \mathrm{ft^3/lb}} = 1.35760\ \mathrm{lb/s} \longleftarrow \text{(a)}$$

(b) Nitrogen as an ideal gas at 50 lbf/in², 80°F = 540°R

$$v = \frac{RT}{P} = \frac{\left(\frac{1545}{28.01}\,\frac{\mathrm{ft \cdot lbf}}{\mathrm{lb \cdot °R}}\right)(540°\mathrm{R})}{(50\ \mathrm{lbf/in^2})}\left|\frac{1\ \mathrm{ft^2}}{144\ \mathrm{in^2}}\right| = 4.137\ \mathrm{ft^3/lb}$$

$$\therefore\ \dot{m} = \frac{0.02182\ \mathrm{ft^3/s}}{4.137\ \mathrm{ft^3/lb}} = 0.00527\ \mathrm{lb/s} \longleftarrow \text{(b)}$$

(c) R22 at 50 lbf/in², 80°F

$v = 1.2716\ \mathrm{ft^3/lb}$ from Table A-9E

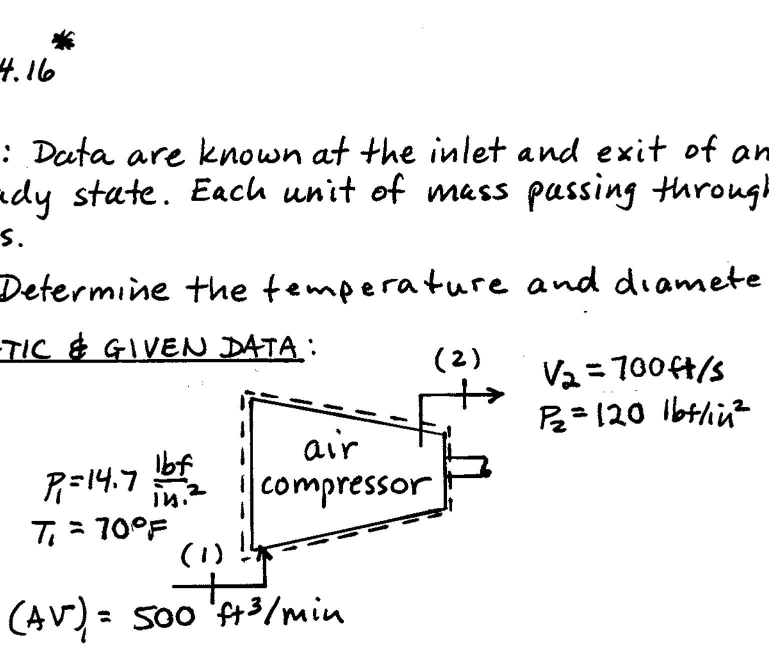
$$\therefore\ \dot{m} = \frac{0.02182\ \mathrm{ft^3/s}}{1.2716\ \mathrm{ft^3/lb}} = 0.01716\ \mathrm{lb/s} \longleftarrow \text{(c)}$$

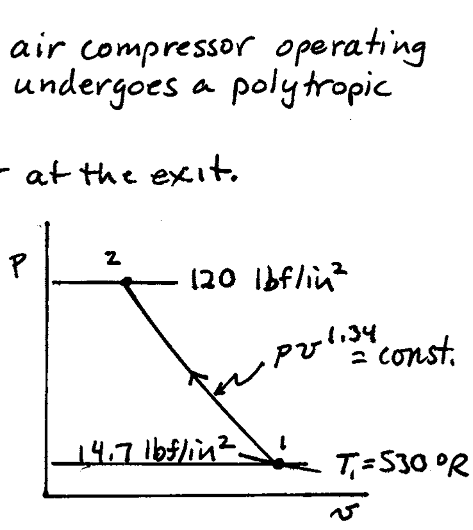
PROBLEM 4.16*

KNOWN: Data are known at the inlet and exit of an air compressor operating at steady state. Each unit of mass passing through undergoes a polytropic process.

FIND: Determine the temperature and diameter at the exit.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) Each unit of mass undergoes a polytropic process with n = 1.34. (3) The air behaves as an ideal gas, as can be verified by reference to the compressibility chart.

ANALYSIS: The exit velocity is determined by using the mass rate balance with Eq. 4.11 b, as follows:

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \Rightarrow \frac{(AV)_1}{v_1} = \frac{(AV)_2}{v_2} \qquad (*)$$

First, evaluate v_1 using the ideal gas equation

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(530\ °\text{R})}{(14.7\ \text{lbf/in}^2)}\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| = 13.35\ \text{ft}^3/\text{lb}$$

Now, with the polytropic process expression

$$v_2 = \left(\frac{P_1}{P_2}\right)^{\frac{1}{1.34}} v_1 = \left(\frac{14.7}{100}\right)^{\frac{1}{1.34}}\left(13.35\ \frac{\text{ft}^3}{\text{lb}}\right) = 3.192\ \text{ft}^3/\text{lb}$$

The exit temperature is determined using the ideal gas equation of state.

$$T_2 = \frac{P_2 v_2}{R} = \frac{(120\ \text{lbf/in}^2)(3.192\ \text{ft}^3/\text{lb})}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|$$

$$= 1034\,°\text{R} = 574\,°\text{F} \longleftarrow T_2$$

Solving (*) for A_2 and inserting values

$$A_2 = \frac{(AV)_1\, v_2}{V_2\, v_1}$$

$$= \frac{(500\ \text{ft}^3/\text{min})(3.192\ \text{ft}^3/\text{lb})}{(700\ \text{ft/s})(13.35\ \text{ft}^3/\text{lb})}\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 0.002846\ \text{ft}^2$$

$$= 0.410\ \text{in}^2$$

Using $A_2 = \pi d_2^2/4$

$$d_2 = \left(\frac{4A_2}{\pi}\right)^{1/2} = \left(\frac{(4)(0.410\ \text{in}^2)}{\pi}\right)^{1/2}$$

$$d_2 = 0.722\ \text{in} \longleftarrow d_2$$

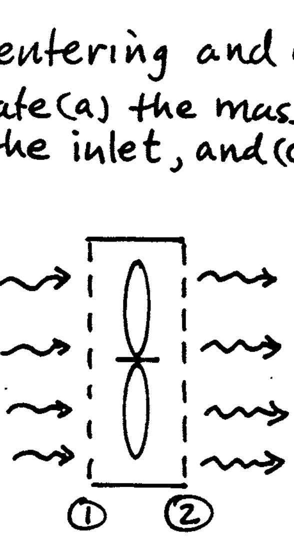
PROBLEM 4.17

KNOWN: Data are given for air entering and exiting from a fan.

FIND: Determine at steady state (a) the mass flow rate, (b) the volumetric flow rate at the inlet, and (c) the inlet and exit velocities.

SCHEMATIC & GIVEN DATA:

$T_1 = 16°C$
$P_1 = 101\ kPa$
$D = 0.6\ m$



$T_2 = 18\ °C$
$P_2 = 105\ kPa$
$(AV)_2 = 0.35\ m^3/s$

ASSUMPTIONS: (1) A control volume enclosing the fan is at steady state, (2) The air behaves as an ideal gas.

ANALYSIS:

(a) The mass balance reduces to: $\dot{m}_1 = \dot{m}_2$. Thus, using data at the exit of the fan

$$\dot{m} = \frac{(AV)_2}{v_2} = \frac{(AV)_2\, P_2}{R T_2}$$

$$= \frac{(0.35\ m^3/s)\ (105\ kPa)}{\left(\frac{8.314}{28.97}\ \frac{kJ}{kg\cdot K}\right)(291\ K)} \left|\frac{10^3\ N/m^2}{1\ kPa}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 0.44\ \ kg/s \longleftarrow$$

(b) At the inlet

$$(AV)_1 = \dot{m}\, v_1 = \dot{m}\left(\frac{RT_1}{P_1}\right) = 0.44\left[\frac{\left(\frac{8.314}{28.97}\right)(289)}{(101)}\right]$$

$$= 0.361\ \ m^3/s \longleftarrow$$

(c) The cross-sectional area is

$$A = \frac{\pi D^2}{4} = \frac{\pi(.6\ m)^2}{4} = 0.2827\, m^2$$

$$\Rightarrow \quad V_1 = \frac{(AV)_1}{A} = \frac{0.361\, m^3/s}{0.2827\ m^2} = 1.28\ m/s \longleftarrow$$

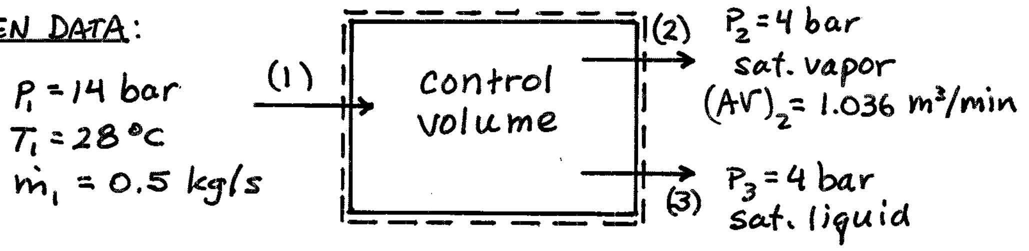
$$V_2 = \frac{(AV)_2}{A} = \frac{0.35}{0.2827} = 1.24\, m/s \longleftarrow$$

PROBLEM 4.18

KNOWN: Ammonia flows through a control volume at steady state. The control volume has one inlet and two exit, and data are known at each flow boundary.

FIND: Determine (a) the minimum inlet diameter so the ammonia velocity does not exceed 20 m/s. (b) the volumetric flow rate of the second exit stream.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) The flow at the inlet is one-dimensional.

ANALYSIS: (a) To relate velocity and pipe diameter at the inlet, use Eq. 4.11b

$$V_1 = \frac{\dot{m}_1 v_1}{A_1} = \frac{\dot{m}_1 v_1}{\left(\frac{\pi d_1^2}{4}\right)}$$

Thus, velocity varies inversely with diameter. The minimum diameter corresponds to $V_1 = 20$ m/s.

To get v_1, note from Table A-14 that $T_1 = 28$°c is less than T_{sat} at 14 bar. Hence, from Table A-13, $v_1 \approx v_{f@28°c} = 1.6714 \times 10^{-3}$ m³/kg, and

$$(d_1)_{min} = \sqrt{\frac{4\,\dot{m}_1 v_1}{\pi V_1}} = \sqrt{\frac{(4)(0.5\text{ kg/s})(1.6714\times10^{-3}\text{ m}^3/\text{kg})}{\pi\,(20\text{ m/s})}}$$

$$= 0.00729\text{ m} = 0.729\text{ cm} \longleftarrow \quad (d_1)_{min}$$

(b) To find $(AV)_3$, begin with the mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{\,0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2$$

With $\dot{m} = (AV)/v$

$$(AV)_3 = v_3\left[\dot{m}_1 - (AV)_2/v_2\right]$$

From Table A-14 at 4 bar ; $v_2 = 0.3094$ m³/kg and $v_3 = 1.5597\times10^{-3}$ m³/kg. Thus

$$(AV)_3 = \left(1.5597\times10^{-3}\,\frac{\text{m}^3}{\text{kg}}\right)\left[\left(0.5\,\frac{\text{kg}}{\text{s}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) - \frac{(1.036\text{ m}^3/\text{min})}{(0.3094\text{ m}^3/\text{kg})}\right]$$

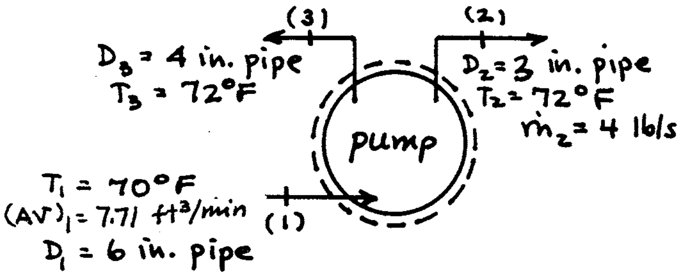
$$= 0.0416\text{ m}^3/\text{min} \longleftarrow \quad (AV)_3$$

PROBLEM 4.19

KNOWN: A pump operating at steady state provides water through two exit pipes. Data are known at the inlet and each exit.

FIND: Determine the water velocity in each exit pipe.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state.
(2) The water is incompressible and $v \approx v_f(T)$.

ANALYSIS: For the velocity in the 3 in. pipe, use Eq. 4.11b

$$V_2 = \frac{\dot{m}_2 v_2}{A_2} = \frac{4\dot{m}_2 v_2}{\pi D_2^2}$$

From Table A-2E at 72°F; $v_2 \approx v_f = 0.01606$ ft³/lb. Thus

$$V_2 = \frac{4(4 \text{ lb/s})(0.01606 \text{ ft}^3/\text{lb})}{\pi (3/12)^2 \text{ ft}^2} = 1.309 \text{ ft/s} \longleftarrow \underline{V_2}$$

To find the velocity in the 4 in. pipe, use the mass rate balance to determine the mass flow rate. That is

$$\cancel{\frac{dm_{cv}}{dt}}^{\,0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2$$

At the inlet

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{(7.71 \text{ ft}^3/\text{min})}{(0.01605 \text{ ft}^3/\text{lb})}\left|\frac{1 \text{ min}}{60 \text{ s}}\right| = 8.01 \text{ lb/s}$$

where v_1 is obtained from Table A-2E. Finally,

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 8.01 - 4 = 4.01 \text{ lb/s}$$

and

$$V_3 = \frac{4\dot{m}_3 v_3}{\pi D_3^2} = \frac{4(4.01 \text{ lb/s})(0.01606 \text{ ft}^3/\text{lb})}{\pi (4/12)^2 \text{ ft}^2}$$

$$= 0.738 \text{ ft/s} \longleftarrow \underline{V_3}$$

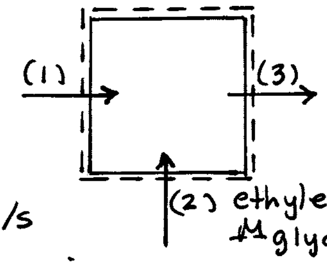
PROBLEM 4.20

KNOWN: Separate streams of water and ethylene glycol (glycol) mix to form a single mixture that is half glycol by mass. Data are given for the inlet flows.

FIND: Determine (a) the molar and volumetric flow rates of the entering glycol. (b) the diameters of the supply pipe.

SCHEMATIC & GIVEN DATA:

Water:
$T_1 = 20°C$
$p_1 = 1 \text{ bar}$
$\dot{n}_1 = 4.2 \frac{\text{kmol}}{\text{min}}$
$V_1 = V_2 = 2.5 \text{ m/s}$



Mixture: 50% glycol by mass

(2) ethylene glycol
$\mathcal{M}_{glycol} = 62.07 \text{ kg/kmol}$
$\rho_{glycol} = 1.115\, \rho_{H_2O}$

ASSUMPTIONS: (1) The control volume is at steady state. (2) The water and glycol are each incompressible substances.

ANALYSIS: (a) To find the glycol flow rates, begin with a mass rate balance. At steady state: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$. For the mixture

$$\left.\begin{array}{ll} 50\% \text{ glycol} \Rightarrow & \dfrac{\dot{m}_2}{\dot{m}_3} = 0.5 \\ 50\% \text{ water} \Rightarrow & \dfrac{\dot{m}_1}{\dot{m}_3} = 0.5 \end{array}\right\} \dot{m}_1 = \dot{m}_2 .$$

Now

$$\dot{m}_1 = \dot{n}_1 \mathcal{M}_{H_2O} = \left(4.2 \frac{\text{kmol}}{\text{min}}\right)\left(18.02 \frac{\text{kg}}{\text{kmol}}\right) = 75.68 \text{ kg/min}$$

$$\dot{n}_2 = \frac{\dot{m}_2}{\mathcal{M}_{glycol}} = \frac{75.68 \text{ kg/min}}{62.07 \text{ kg/kmol}} = 1.219 \text{ kmol/min} \longleftarrow \dot{n}_2$$

Also, with $\dot{m} = \rho(AV)$

$$(AV)_2 = \frac{\dot{m}_2}{\rho_{glycol}} = \frac{\dot{m}_2}{1.115\, \rho_{H_2O}}$$

From Table A-2, $v_{H_2O} \approx v_{f @ 20°C} = 1.0018 \times 10^{-3} \text{ m}^3/\text{kg}$ and $\rho_{H_2O} = 1/v_{H_2O} = 998.2 \text{ kg/m}^3$.
Thus

$$(AV)_2 = \frac{75.68 \text{ kg/min}}{1.115\,(998.2) \text{ kg/m}^3} = 0.068 \text{ m}^3/\text{min} \longleftarrow (AV)_2$$

(b) The area of the water supply pipe is

$$A_1 = \frac{\dot{m}_1}{\rho_{H_2O} V_1} = \frac{(75.68 \text{ kg/min})}{(998.2 \text{ kg/m}^3)(2.5 \text{ m/s})} \left|\frac{1 \text{ min}}{60 \text{ s}}\right| \left|\frac{10^4 \text{ cm}^2}{1 \text{ m}^2}\right| = 5.054 \text{ cm}^2$$

With $A = \pi d^2/4$

$$d_1 = \sqrt{\frac{4 A_1}{\pi}} = \sqrt{\frac{(4)(5.054 \text{ cm}^2)}{\pi}} = 2.54 \text{ cm} \longleftarrow d_1$$

Similarly for the glycol stream

$$A_2 = \frac{75.68}{(1.115)(998.2)(2.5)} \left|\frac{10^4}{60}\right| = 4.533 \text{ cm}^2$$

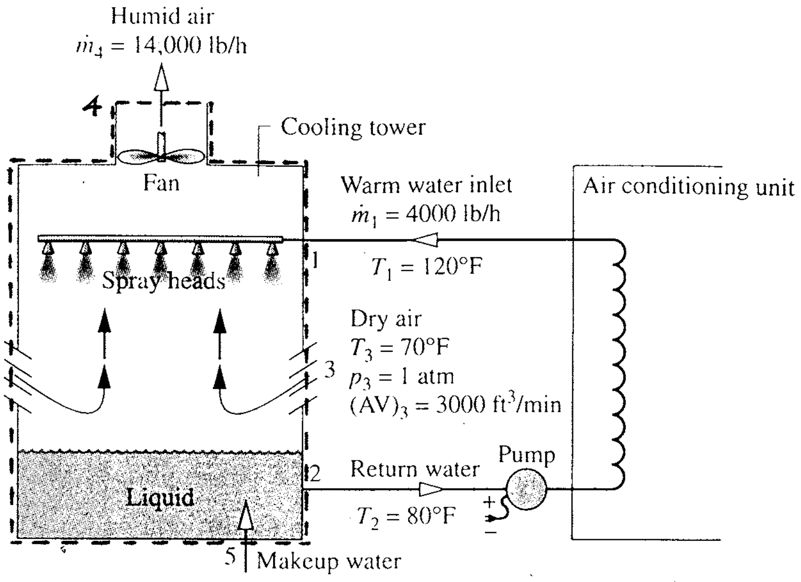
$$d_2 = \sqrt{\frac{(4)(4.533)}{\pi}} = 2.4 \text{ cm} \longleftarrow d_2$$

PROBLEM 4.21

KNOWN: Data are know for inlet and exit streams of an air conditioner cooling tower operating at steady state.

FIND: Determine the mass flow rate of the makeup water.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) The dry air stream at location 3 behaves as an ideal gas.

ANALYSIS: The mass rate balance for the control reduces as follows:

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 + \dot{m}_3 + \dot{m}_5 - \dot{m}_2 - \dot{m}_4$$

$$\therefore \quad \dot{m}_5 = \dot{m}_2 - \dot{m}_1 + \dot{m}_4 - \dot{m}_3$$

From the schematic, we see that circuit through the air conditioning unit is closed. Therefore, at steady state $\dot{m}_1 = \dot{m}_2$. Thus

$$\dot{m}_5 = \dot{m}_4 - \dot{m}_3 \qquad (*)$$

To get $\dot{m}_3$, use Eq. 4.11 b and the ideal gas equation

$$\dot{m}_3 = \frac{(AV)_3}{v_3} = \frac{p_3 (AV)_3}{R T_3}$$

$$= \frac{(1\ \text{atm})(3000\ \text{ft}^3/\text{min})}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(530\,{}^\circ\text{R})} \left|\frac{14.696\ \text{lbf/in}^2}{1\ \text{atm}}\right| \left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| \left|\frac{60\ \text{min}}{1\ \text{h}}\right|$$

$$= 13480\ \text{lb/h}$$

Finally, from (*)

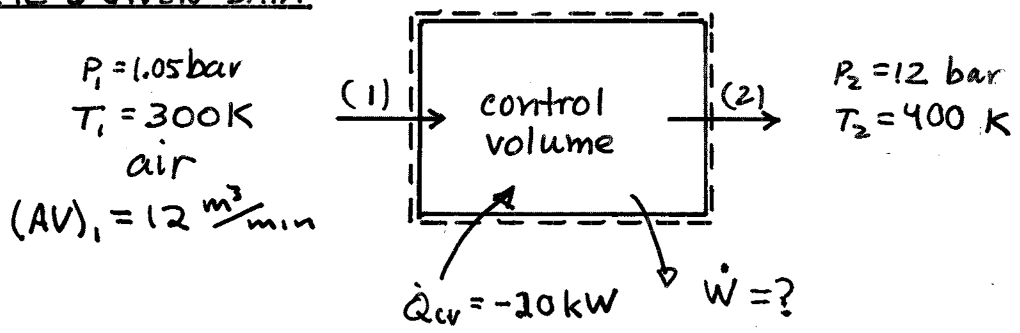
$$\dot{m}_5 = 14{,}000 - 13{,}480 = 520\ \text{lb/h} \longleftarrow \dot{m}_5$$

PROBLEM 4.22*

KNOWN: Air passes through a control volume operating at steady state. Data are given at the inlet and exit, and the heat transfer rate is known. The exit volumetric flow rate is known.

FIND: Determine the power.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) The air behaves as an ideal gas. (3) Kinetic and potential energy effects can be neglected.

ANALYSIS: To determine the heat transfer rate, begin with the energy rate balance

$$\cancel{\frac{dE_{cv}}{dt}}^{0} = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) - \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right)$$

With $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$

$$\dot{W}_{cv} = \dot{Q}_{cv} - \dot{m}(h_2 - h_1) + \cancel{\left(\frac{V_2^2 - V_1^2}{2}\right)}^{0} + g\cancel{(z_2 - z_1)}^{0}$$

The mass flow rate is

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{p_2 (AV)_1}{R T_1}$$

$$= \frac{(1.05\ \text{bar})(12\ m^3/\text{min})}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300\ K)} \left|\frac{1\ \text{min}}{60\ s}\right| \left|\frac{10^5\ N/m^2}{1\ \text{bar}}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 0.2439\ kg/s$$

Finally, using data from Table A-22

$$\dot{W}_{cv} = (-20\ kW) - (0.2439\ \tfrac{kg}{s})(400.98 - 300.19)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= -44.6\ kW \longleftarrow \dot{W}_{cv}$$

---

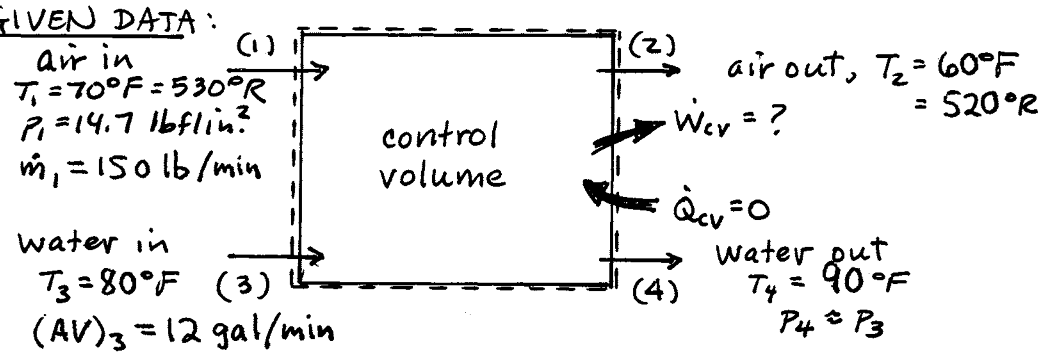
1. The negative sign for the power denotes energy transfer to the control volume from the surroundings.

PROBLEM 4.23*

KNOWN: Air and water pass through a control volume as separate streams. Data are known at the inlets and exits, and the control volume is at steady state.

FIND: Determine the power.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) There is no heat transfer: $Q_{cv} = 0$. (3) Kinetic and potential energy effects can be neglected. (4) The air behaves as an ideal gas. (5) The water behaves as an incompressible substance, with negligible change in pressure.

ANALYSIS: To find the power, we apply mass and energy rate balances to the control volume. First, since the air and water streams are separate, by assumption 1

$$\text{Air: } \cancelto{0}{\frac{dm_{cv}}{dt}} = \dot m_1 - \dot m_2 \Rightarrow \dot m_1 = \dot m_2 \equiv \dot m_a$$

$$\text{Water: } \cancelto{0}{\frac{dm_{cv}}{dt}} = \dot m_3 - \dot m_4 \Rightarrow \dot m_3 = \dot m_4 \equiv \dot m_w$$

and

$$\cancelto{0}{\frac{dE_{cv}}{dt}} = \cancelto{0}{\dot Q_{cv}} - \dot W_{cv} + \dot m_1\left(h_1 + \frac{V_1^2}{2} + g z_1\right) - \dot m_2\left(h_2 + \frac{V_2^2}{2} + g z_2\right)$$
$$+ \dot m_3\left(h_3 + \frac{V_3^2}{2} + g z_3\right) - \dot m_4\left(h_4 + \frac{V_4^2}{2} g z_4\right)$$

Combining

$$0 = -\dot W_{cv} + \dot m_a\left[(h_1 - h_2) + \cancelto{0}{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g\cancelto{0}{(z_1 - z_2)}\right]$$
$$+ \dot m_w\left[(h_3 - h_4) + \cancelto{0}{\left(\frac{V_3^2 - V_4^2}{2}\right)} + g\cancelto{0}{(z_3 - z_4)}\right]$$

or

$$\dot W_{cv} = \dot m_a (h_1 - h_2) + \dot m_w (h_3 - h_4)$$

The mass flow rate of water is found from $\dot m_w = (AV)_3 / v_3$
Using $v_3 = v_{f@80°F} = 0.01607$ ft^3/lb from Table A-2E

$$\dot m_w = \frac{(12 \text{ gal/min})}{(0.01607 \text{ ft}^3/\text{lb})}\left|\frac{0.13368 \text{ ft}^3}{1 \text{ gal}}\right| = 99.8 \text{ lb/min}$$

Now, from Table A-22E; $h_1 = 126.66$ Btu/lb and $h_2 = 124.27$ Btu/lb. Further, for the water; $h_3 \approx h_{f@80°F} = 48.09$ Btu/lb and $h_4 \approx h_{f@90°F} = 58.07$ Btu/lb. Thus

$$\dot W_{cv} = \left[\left(150 \frac{\text{lb}}{\text{min}}\right)(126.66 - 124.27)\frac{\text{Btu}}{\text{lb}} + \left(99.8 \frac{\text{lb}}{\text{min}}\right)(48.09 - 58.07)\frac{\text{Btu}}{\text{lb}}\right]\left|\frac{60 \text{ min}}{1 \text{ h}}\right|\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

① $= -15.0$ hp ← $\dot W_{cv}$

1. The negative sign denotes energy transfer into the control volume by work.

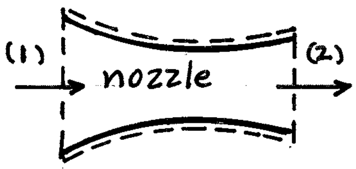
PROBLEM 4.24

KNOWN: Steam flows through a nozzle with known conditions at the inlet and exit. The mass flow rate is given.

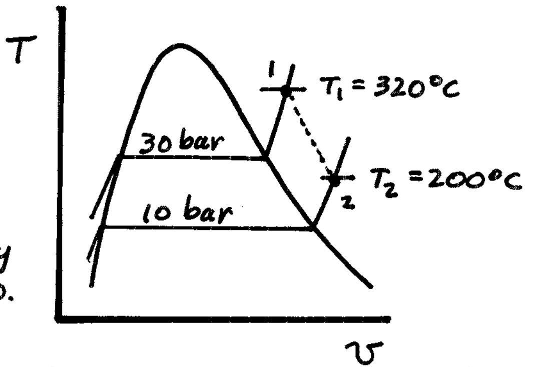
FIND: Determine (a) the exit velocity, (b) the inlet and exit flow areas.

SCHEMATIC & GIVEN DATA:

$p_1 = 30$ bar
$T_1 = 320°C$
$V_1 = 100$ m/s



$p_2 = 10$ bar
$T_2 = 200°C$
$\dot{m} = 2$ kg/s



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible.

ANALYSIS: (a) The velocity of steam at the exit is found from the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for V_2

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

From Table A-4, $h_1 = 3043.4$ kJ/kg and $h_2 = 2827.9$ kJ/kg. Thus

$$V_2 = \sqrt{2(3043.4 - 2827.9)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right| + (100^2)\ \text{m}^2/\text{s}^2}$$

$$= 664.1\ \text{m/s} \longleftarrow V_2$$

(b) To find the inlet and exit flow areas, use $\dot{m} = (AV)/v$. Solving

$$A_1 = \frac{\dot{m} v_1}{V_1} \quad \text{and} \quad A_2 = \frac{\dot{m} v_2}{V_2}$$

From Table A-4, $v_1 = 0.0850$ m³/kg and $v_2 = 0.2060$ m³/kg. Thus

$$A_1 = \frac{(2\ \text{kg/s})(0.0850\ \text{m}^3/\text{kg})}{(100\ \text{m/s})}\left|\frac{10^4\ \text{cm}^2}{1\ \text{m}^2}\right| = 17\ \text{cm}^2 \longleftarrow A_1$$

and

$$A_2 = \frac{(2)(0.2060)}{(664.1)}\left|\frac{10^4}{1}\right| = 6.2\ \text{cm}^2 \longleftarrow A_2$$

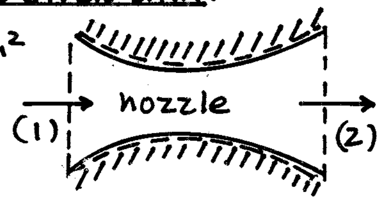
PROBLEM 4.25*

KNOWN: Steam flows through a well-insulated nozzle with known conditions at the inlet and exit.

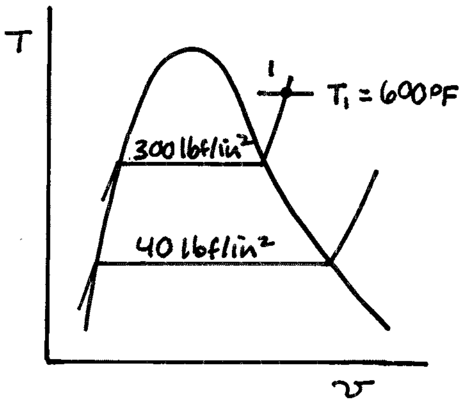
FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:

$p_1 = 300$ lbf/in²
$T_1 = 600$°F
$V_1 = 100$ ft/s



$p_2 = 40$ lbf/in²
$V_2 = 1800$ ft/s



ASSUMPTIONS: (1) The control volume is at steady state. (2) There is no heat transfer, and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible.

ANALYSIS: The pressure is known at the exit. The state is fixed by determining h_2 using the steady-state energy balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1-h_2)+\left(\frac{V_1^2-V_2^2}{2}\right)+g(\cancelto{0}{z_1-z_2})\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2

$$h_2 = h_1 + \left(\frac{V_1^2-V_2^2}{2}\right)$$

From Table A-4E, $h_1 = 1314.5$ Btu/lb. Thus

$$h_2 = (1314.5 \text{ Btu/lb}) + \left(\frac{100^2-1800^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\left|\frac{1 \text{ lbf}}{32.2 \text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$

$$= 1250.03 \text{ Btu/lb}$$

Interpolating in Table A-4E at $p_2 = 40$ lbf/in², $h_2 = 1250.03$ Btu/lb gives

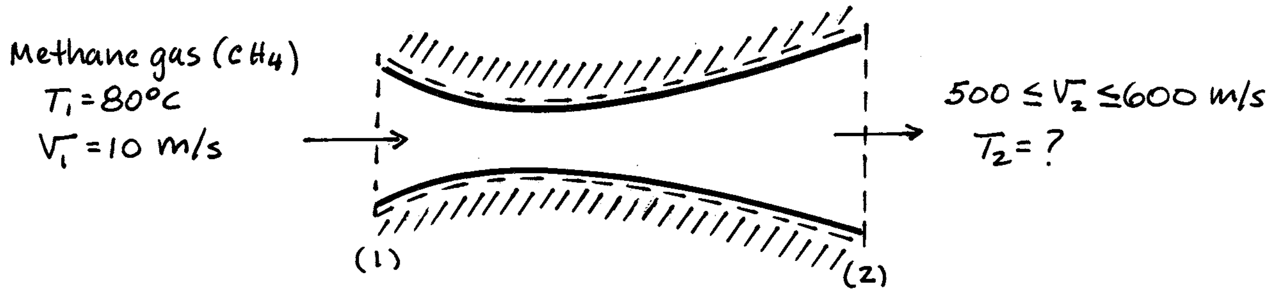
$$T_2 \approx 686.6 \text{ °F} \longleftarrow \underline{T_2}$$

PROBLEM 4.26

KNOWN: Methane gas flows through a nozzle with known inlet conditions and a specified range of exit velocities.

FIND: Plot exit temperature versus exit velocity.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (3) Potential energy effects can be ignored. (4) The methane behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for the ideal gas, the exit temperature is determined by evaluating h_2 using steady state mass and energy balances:

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\,\cancelto{0}{(z_1 - z_2)}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Thus, with $h = h(T)$

$$0 = h(T_1) - h(T_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| \qquad (*)$$

for h in kJ/kg and V in m/s.

Using IT, the functions $h(T)$ for methane (CH_4) as an ideal gas are accessed readily, and data for T_2 is obtained using the following code. The results are shown on the accompanying plot.

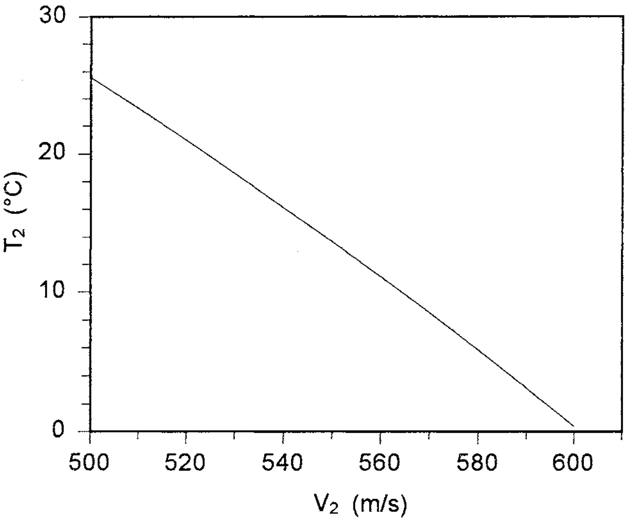
IT Code

```
T1 = 80  // °C
V1 = 10  // m/s
V2 = 550  // m/s

0 = (h1 - h2) + ((V1^2 - V2^2) / 2) * (1 / 10^3)  // kJ/kg
h1 = h_T("CH4", T1)
h2 = h_T("CH4", T2)

// Using the Explore button, sweep V2 from
// 500 to 600 in steps of 0.1.
```

① Result for V2 = 550: T2 = 13.63°C



From (*), $h(T_2) = h(T_1) + \left(\frac{V_1^2 - V_2^2}{2}\right)$. Thus, as V_2 increases, $h(T_2)$ decreases. Therefore, T_2 decreases as expected.

---

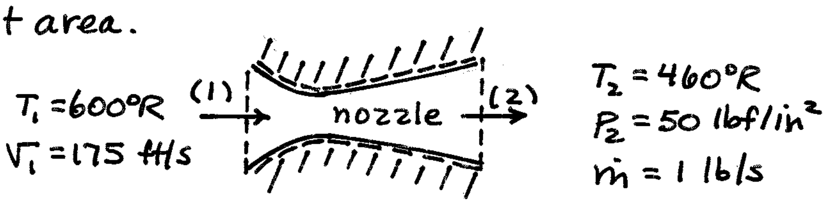
1. A sample calculation using the c_p function from Table A-21 to evaluate the enthalpy change in (*) confirms this result. With IT, this integration is not necessary.

PROBLEM 4.27

KNOWN: Helium flows through a well-insulated nozzle with known conditions at the inlet and exit. The mass flow rate is given.

FIND: Determine the exit area.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot W_{cv} = 0$. (3) The helium behaves as an ideal gas. (4) Potential energy effects are negligible.

ANALYSIS: From the mass rate balance, $\dot m_1 = \dot m_2 \equiv \dot m$. With $\dot m = (AV)/v$

$$A_2 = \frac{\dot m\, v_2}{V_2}$$

Introducing the ideal gas equation of state

$$A_2 = \frac{\dot m\, RT_2}{V_2\, p_2}$$

The exit velocity is found using the steady-state energy balance

$$0 = \cancel{\dot Q_{cv}}^{0} - \cancel{\dot W_{cv}}^{0} + \dot m\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\,\cancel{(z_1 - z_2)}^{0}\right]$$

From Table A-21, the specific heat c_p of helium is

$$c_p = (5/2)R = (5/2)\frac{(1.986\ \text{Btu/lbmol}\cdot°R)}{(4.003\ \text{lb/lbmol})} = 1.24\ \text{Btu/lb}\cdot°R$$

Since c_p is a constant $\Delta h = c_p \Delta T$, and

$$V_2 = \sqrt{2c_p(T_1 - T_2) + V_1^2}$$

$$= \sqrt{2\left(1.24\frac{\text{Btu}}{\text{lb}\cdot°R}\right)(600-460)°R \left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|\left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right| + (175^2)\frac{\text{ft}^2}{\text{s}^2}}$$

$$= 2954\ \text{ft/s}$$

Finally

$$A_2 = \frac{(1\ \text{lb/s})\left(\frac{1545}{4.003}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(460°R)}{(2954\ \text{ft/s})(50\ \text{lbf/in}^2)}\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right|$$

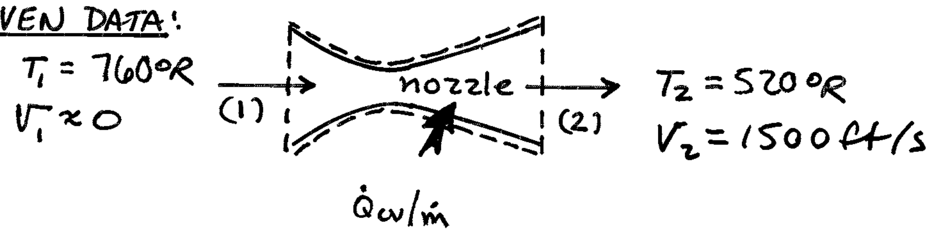
$$= 8.35\times10^{-3}\ \text{ft}^2 \longleftarrow A_2$$

PROBLEM 4.28*

KNOWN: Air flows through a nozzle with known conditions at the inlet and exit. Heat transfer occurs from the air to the surroundings.

FIND: Determine the heat transfer per lb of air flowing.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) For the control volume, $\dot{W}_{cv} = 0$. (3) The air behaves as an ideal gas. (4) The inlet kinetic energy and potential energy effects are negligible.

ANALYSIS: To determine the heat transfer, begin with the steady-state energy balance

$$0 = \dot{Q}_{cv} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\overset{0}{\cancel{V_1^2}} - V_2^2}{2}\right) + g\,\overset{0}{\cancel{(z_1 - z_2)}}\right]$$

Solving for $\dot{Q}_{cv}/\dot{m}$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = h_2 - h_1 + \frac{V_2^2}{2}$$

From Table A-22E, $h_1 = 182.08$ Btu/lb and $h_2 = 124.27$ Btu/lb. Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}} = (124.27 - 182.08)\frac{\text{Btu}}{\text{lb}} + \frac{\left(1500\frac{\text{ft}}{\text{s}}\right)^2}{2}\left|\frac{1\ \text{lbf}}{32.2\ \text{ft lb/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft lbf}}\right|$$

$$\frac{\dot{Q}_{cv}}{\dot{m}} = -12.9\ \text{Btu/lb} \longleftarrow \dot{Q}/\dot{m}$$

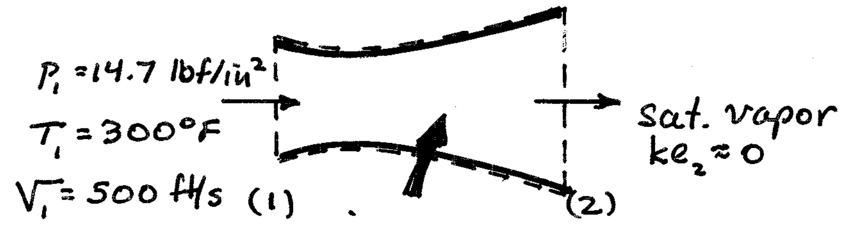
The negative sign indicates that the heat transfer is from the nozzle to the surroundings.

PROBLEM 4.29

KNOWN: Steam is decelerated in passing through a diffuser.

FIND: Determine the exit pressure.

SCHEMATIC & GIVEN DATA:



$$\frac{\dot{Q}_{cv}}{\dot{m}} = -19.59 \text{ Btu/lb}$$

ASSUMPTIONS: (1) The control volume is at steady state. (2) For the control volume, $\dot{W}_{cv} = 0$. (3) The exit kinetic energy is negligible. (4) Potential energy effects are negligible.

ANALYSIS: Applying the energy balance at steady state to get h_2

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - \cancel{V_2^2}^{\,0}}{2}\right) + \cancel{g(z_1 - z_2)}^{\,0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Thus

$$h_2 = \dot{Q}_{cv}/\dot{m} + h_1 + V_1^2/2$$

From Table A-4E at $p_1 = 14.7$ lbf/in², $T_1 = 300$°F ; $h_1 = 1192.6$ Btu/lb. Inserting values

$$h_2 = (-19.59 \text{ Btu/lb}) + (1192.6 \text{ Btu/lb}) + \left[\frac{(500^2)\text{ ft}^2/\text{s}^2}{2}\right]\left|\frac{1 \text{ lbf}}{32.2 \text{ ft}\cdot\text{lb/s}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$

$$= 1178 \text{ Btu/lb}$$

The pressure at the exit is the saturation pressure corresponding to $h_g = h_2 = 1178$ Btu/lb. From Table A-3E

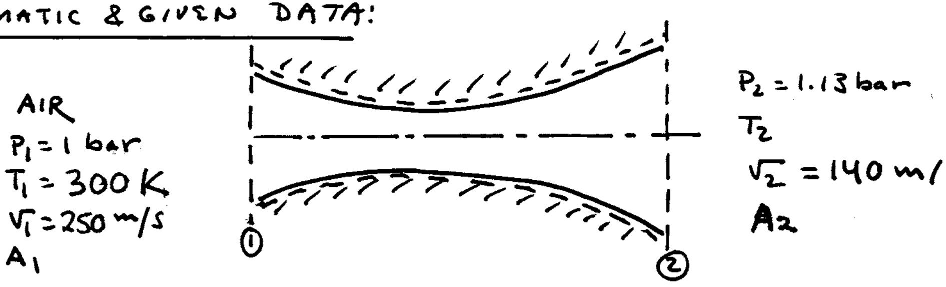
$$P_2 = 60 \text{ lbf/in}^2 \longleftarrow P_2$$

PROBLEM 4.30*

KNOWN: Data is provided for a diffuser at steady state, through which air is flowing.

FIND: Determine the ratio of the exit flow area to the inlet flow area, and the exit temperature.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and potential energy effects can be ignored. 3. Air is modeled as an ideal gas with constant c_p.

ANALYSIS: The mass rate balance reads $\dot{m}_2 = \dot{m}_1$, or

$$\frac{A_2 V_2}{v_2} = \frac{A_1 V_1}{v_1} \Rightarrow \frac{A_2 V_2}{(RT_2/p_2)} = \frac{A_1 V_1}{(RT_1/p_1)} \Rightarrow \frac{A_2}{A_1} = \left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right) \qquad (1)$$

The exit temperature, T_2, can be obtained using an energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g\cancel{(z_1 - z_2)}\right]$$

$$h_2 = h_1 + \frac{V_1^2}{2} - \frac{V_2^2}{2}$$

Using data from Table A-22

$$h_2 = \left(300.19\,\frac{\text{kJ}}{\text{kg}}\right) + \left(\frac{250^2 - 140^2}{2}\right)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1\text{ N}}{1\text{ kg m/s}^2}\right|\left|\frac{1\text{ kJ}}{1000\text{ Nm}}\right|$$

$$= 321.64\text{ kJ/kg}$$

Then, interpolating in Table A-22 for $h_2 = 321.64$ kJ/kg

$$T_2 = 321.3\text{ K} \longleftarrow T_2$$

Returning to Eq. (1)

$$\frac{A_2}{A_1} = \left(\frac{1\text{ bar}}{1.13\text{ bar}}\right)\left(\frac{321.3\text{ K}}{300\text{ K}}\right)\left(\frac{250\text{ m/s}}{140\text{ m/s}}\right)$$

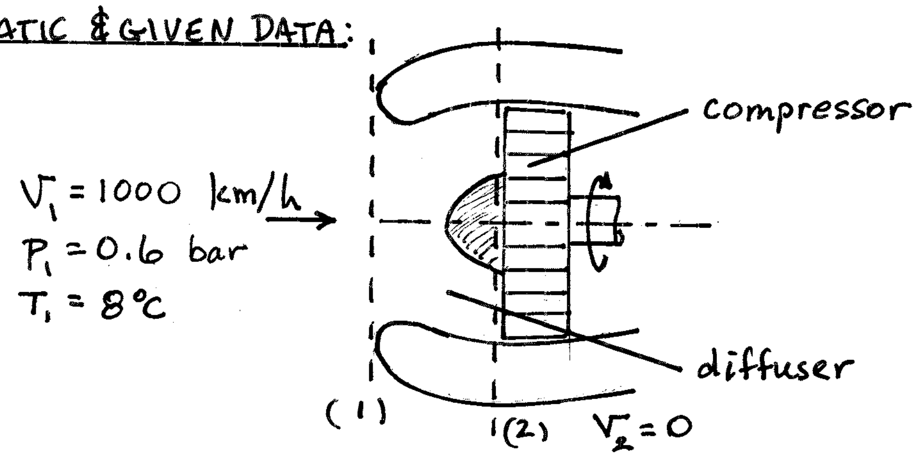
$$= 1.692 \longleftarrow A_2/A_1$$

PROBLEM 4.31

KNOWN: Air enters the diffuser at the inlet of a jet engine and decelerates to zero velocity before entering the jet engine's compressor.

FIND: Determine the temperature of the air entering the compressor.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Potential energy effects are ① negligible. (4) The air behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for an ideal gas, the exit temperature can be found by evaluating h_2. Beginning with the steady-state energy balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \frac{(V_1^2 - \cancelto{0}{V_2^2})}{2} + g(z_1 - z_3)\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and using assumption (3)

$$0 = (h_1 - h_2) + \frac{1}{2}V_1^2$$

or

$$h_2 = h_1 + \frac{1}{2}V_1^2$$

From Table A-22; $h_1 = 281.3$ kJ/kg, and

$$h_2 = 281.3\ \frac{\text{kJ}}{\text{kg}} + \frac{1}{2}(1\times10^6)^2\ \frac{\text{m}^2}{\text{h}^2}\left|\frac{1\ \text{h}^2}{3600^2\ \text{s}^2}\right|\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 319.9\ \text{kJ/kg}$$

Interpolating in Table A-22; $T_2 \approx 319.6$ K ← T_2

---

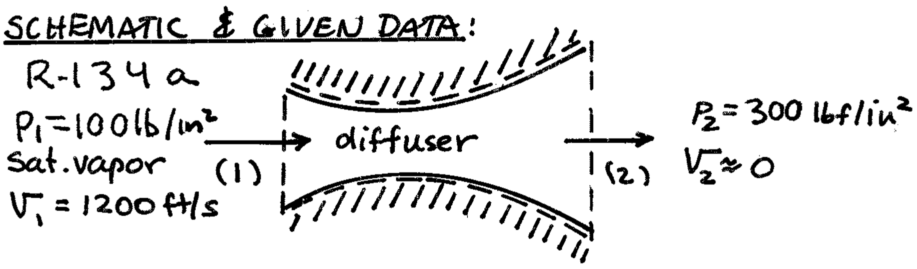
1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

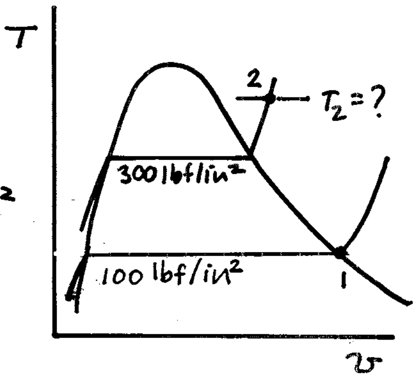
PROBLEM 4.32 *

KNOWN: Refrigerant 134a passes through an insulated diffuser. Data are known at the inlet and exit.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) The heat transfer is negligible and $W_{cv} = 0$. (3) Potential energy effects and kinetic energy at exit can be neglected.

ANALYSIS: The exit pressure is given. The state is fixed by determining h_2 using a steady-state energy balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - \cancelto{0}{V_2^2}}{2}\right) + g(\cancelto{0}{z_1 - z_2})\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2

$$h_2 = h_1 + \frac{V_1^2}{2}$$

From Table A-11E, at $p_1 = 100\,\frac{lbf}{in^2}$, $h_1 = h_g = 112.46$ Btu/lb. Thus

$$h_2 = 112.46\text{ Btu/lb} + \frac{1}{2}(1200^2\,ft^2/s^2)\left|\frac{1\text{ lbf}}{32.2\text{ lb}\cdot ft/s^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot lbf}\right|$$

$$= 141.20\text{ Btu/lb}$$

Interpolating in Table A-12E at $p_2 = 300$ lbf/in², $h_2 = 141.20$ Btu/lb gives

$$T_2 \approx 222.9\,°F \longleftarrow \quad T_2$$

PROBLEM 4.33

KNOWN: Carbon dioxide flows through a well-insulated diffuser with known conditions at the inlet and exit.

FIND: Determine the exit temperature and pressure and the mass flow rate.

SCHEMATIC & GIVEN DATA:

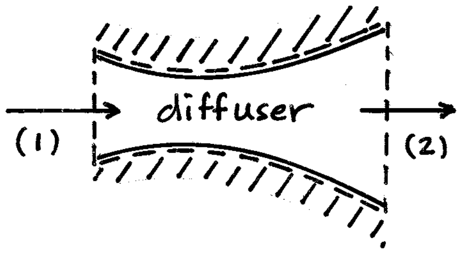
CO_2

$p_1 = 20\ \text{lbf/in}^2$

$T_1 = 500°R$

$V_1 = 800\ \text{ft/s}$

$A_1 = 1.4\ \text{in}^2$



$A_2/A_1 = 30$

$V_2 = 20\ \text{ft/s}$

ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible and $\dot{W}_{cv} = 0$. (3) Potential energy effects are negligible. (4) The carbon dioxide behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for an ideal gas, the exit temperature can be found by evaluating h_2. Beginning with the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2, and noting that $\bar{h} = h/M$

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right) \Rightarrow \bar{h}_2 = \bar{h}_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)M$$

where M is the molecular weight. From Table A-23E, $\bar{h}_1 = 3706.2\ \text{Btu/lbmol}$ and

$$\bar{h}_2 = 3706.2\ \frac{\text{Btu}}{\text{lbmol}} + \left(\frac{800^2 - 20^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{44.01\ \text{lb}}{1\ \text{lbmol}}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 4268\ \text{Btu/lbmol}$$

Interpolating in Table A-23E; $T_2 \approx 563.6°R \longleftarrow T_2$

To get p_2, begin with $\dot{m}_1 = \dot{m}_2$. Thus

$$\frac{A_1V_1}{v_1} = \frac{A_2V_2}{v_2}$$

with $pv = RT$

$$\frac{p_1}{RT_1}(A_1V_1) = \frac{p_2}{RT_2}(A_2V_2)$$

or

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)\left(\frac{A_1}{A_2}\right)\left(\frac{V_1}{V_2}\right) = \left(20\ \frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{563.6}{500}\right)\left(\frac{1}{30}\right)\left(\frac{800}{20}\right)$$

$$= 30.06\ \text{lbf/in}^2 \longleftarrow p_2$$

Finally, the mass flow rate is

$$\dot{m} = \frac{(A_1V_1)p_1}{RT_1} = \frac{(1.4\ \text{in}^2)(800\ \text{ft/s})(20\ \text{lbf/in}^2)}{\left(\frac{1545}{44.01}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(500°R)}$$

$$= 1.276\ \text{lb/s} \longleftarrow \dot{m}$$

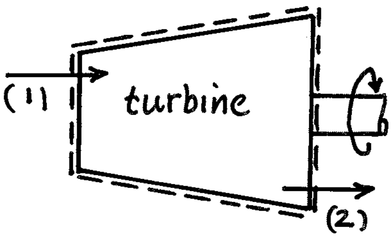
PROBLEM 4.34

KNOWN: Air expands through a turbine with known conditions at the inlet and exit. The power developed is known.

FIND: Determine the mass flow rate and the exit area.

SCHEMATIC & GIVEN DATA:

air
$p_1 = 10$ bar
$T_1 = 900$ K
$V_1 << V_2$



$\dot{W}_{cv} = 3200$ kW
$p_2 = 1$ bar
$T_2 = 500$ K
$V_2 = 100$ m/s

ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Potential energy effects and kinetic energy at the inlet can ① be neglected. (4) The air behaves as an ideal gas.

ANALYSIS: Begin with a steady-state energy balance

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{\overset{0}{\cancel{V_1^2}} - V_2^2}{2}\right) + g\overset{0}{\cancel{(z_1 - z_2)}}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cv}}{(h_1 - h_2) - \frac{V_2^2}{2}}$$

From Table A-22; $h_1 = 932.93$ kJ/kg and $h_2 = 503.02$ kJ/kg. Thus

$$\dot{m} = \frac{(3200 \text{ kW})\ 1 \text{ kJ/s}/1 \text{ kW}}{(932.93 - 503.02)\text{ kJ/kg} - \left(\frac{100^2 \text{ m}^2/\text{s}^2}{2}\right)\left|\frac{1 \text{ N}}{\text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|}$$

$$= 7.53 \text{ kg/s} \longleftarrow \dot{m}$$

The exit area is

$$A_2 = \frac{v_2 \dot{m}}{V_2} = \frac{R T_2 \dot{m}}{p_2 V_2}$$

$$= \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(500\text{K})(7.53 \text{ kg/s})}{(1 \text{ bar})(100 \text{ m/s})}\left|\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right|\left|\frac{10^3 \text{N}\cdot\text{m}}{1 \text{ kJ}}\right|$$

$$= 0.108 \text{ m}^2 \longleftarrow A_2$$

---

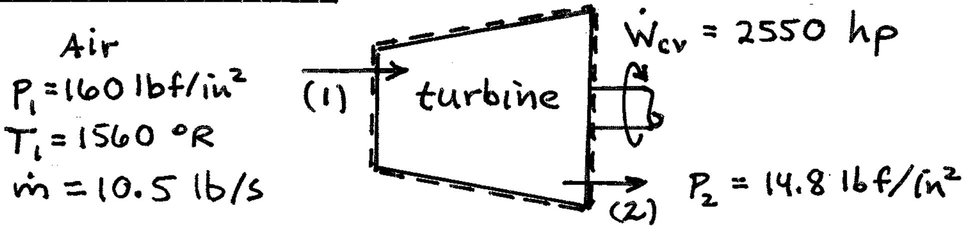
1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

PROBLEM 4.35*

KNOWN: Air expands through a turbine with known conditions at the inlet and exit. The inlet mass flow rate and the power developed are given.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:



Air
$p_1 = 160\ \text{lbf/in}^2$
$T_1 = 1560\ °R$
$\dot{m} = 10.5\ \text{lb/s}$
(1)
turbine
$\dot{W}_{cv} = 2550\ \text{hp}$
(2)
$p_2 = 14.8\ \text{lbf/in}^2$

ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Kinetic and potential energy effects are negligible. ① (4) The air behaves as an ideal gas.

ANALYSIS: Since $h = h(T)$ for an ideal gas, the exit temperature can be found by evaluating h_2. Beginning with the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and with assumption (3)

$$0 = -\dot{W}_{cv} + \dot{m}(h_1 - h_2)$$

or

$$h_2 = -\dot{W}_{cv}/\dot{m} + h_1 \qquad (1)$$

Using data from Table A-22E for h_1 and inserting values into (1)

$$h_2 = -\frac{(2550\ \text{hp})}{(10.5\ \text{lb/s})}\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right| + 385.08\ \frac{\text{Btu}}{\text{lb}}$$

$$= 213.39\ \text{Btu/lb}$$

Interpolating in Table A-22E

$$T_2 = 888.3\ °R \longleftarrow T_2$$

---

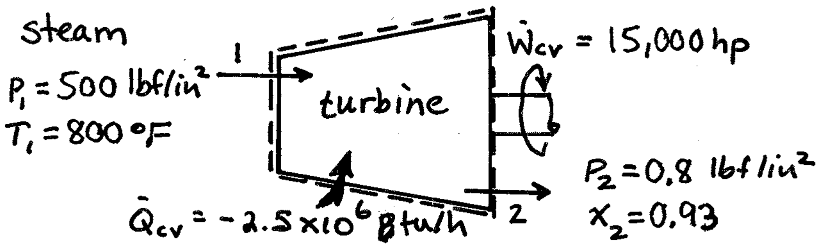
1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.

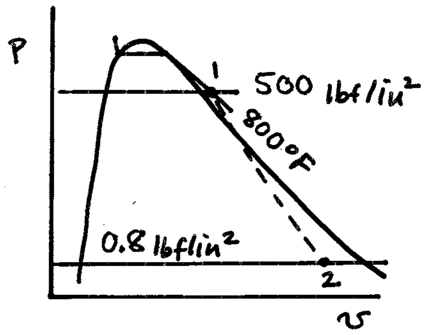
PROBLEM 4.36*

KNOWN: A steam turbine operates at steady state with known inlet and exit conditions. The power developed and the heat transfer rate between the turbine and its surroundings are specified.

FIND: Determine the volumetric flow rate of steam at the inlet.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: To calculate the volume flow rate, begin with mass and energy rate balances for the one-inlet, one-exit control volume at steady state

$$0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \overset{0}{\cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}} + g\overset{0}{\cancel{(z_1 - z_2)}}\right]$$

Solving for the mass flow rate

$$\dot{m} = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{(h_2 - h_1)}$$

From Table A-4E, at $p_1 = 500$ lbf/in², $T_1 = 800$°F; $h_1 = 1412.1$ Btu/lb. From Table A-3E, at $p_2 = 0.8$ lbf/in²

$$h_2 = h_{f2} + x_2 h_{fg2}$$

$$= (62.41) + (0.93)(1040.2) = 1029.80 \text{ Btu/lb}$$

Thus

$$\dot{m} = \frac{(-2.5 \times 10^6 \text{ Btu/h}) - (15{,}000 \text{ hp})\left|\frac{2545 \text{ Btu/h}}{1 \text{ hp}}\right|}{(1029.80 - 1412.1) \text{ Btu/lb}}$$

$$= 10.64 \times 10^4 \text{ lb/h}$$

Then, from Table A-4E at p_1 and T_1; $v_1 = 1.441$ ft³/lb. The volume flow rate at the inlet is then

$$(AV)_1 = \dot{m}\, v_1 = \left(10.64 \times 10^4 \frac{\text{lb}}{\text{h}}\right)\left(1.441 \frac{\text{ft}^3}{\text{lb}}\right)$$

$$= 1.53 \times 10^5 \text{ ft}^3/\text{h} \longleftarrow (AV)_1$$

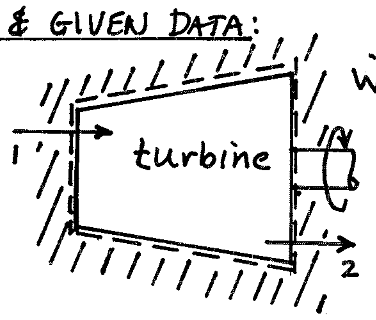
PROBLEM 4.37*

KNOWN: A well-insulated steam turbine operates at steady-state with known inlet and exit conditions. The power output and mass flow rate are specified.

FIND: Determine the inlet pressure.

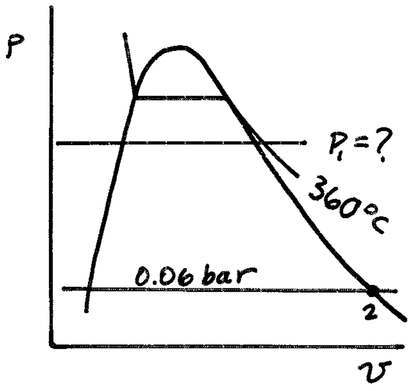
SCHEMATIC & GIVEN DATA:

Steam
$T_1 = 360$ °C
$V_1 = 35$ m/s



$\dot{W}_{cv} = 23$ MW

$p_2 = 0.06$ bar
sat. vapor
$V_2 = 120$ m/s
$\dot{m} = 40$ kg/s



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer can be neglected. (3) The potential energy change from inlet to exit can be neglected.

ANALYSIS: The temperature is known at the inlet. To fix the inlet state, h_1 is determined using mass and energy balances. For the control volume

$$\cancelto{0}{\frac{dm_{cv}}{dt}} = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

and

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\,\cancelto{0}{(z_1 - z_2)}\right]$$

Solving for h_1

$$h_1 = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 + \left(\frac{V_2^2 - V_1^2}{2}\right)$$

From Table A-3 for saturated vapor at $p_2 = 0.06$ bar; $h_2 = 2567.4$ kJ/kg. Thus

$$h_1 = \frac{(23\,\text{MW})}{(40\,\text{kg/s})}\left|\frac{10^3\,\text{kW}}{1\,\text{MW}}\right| + \left(2567.4\,\frac{\text{kJ}}{\text{kg}}\right) + \left(\frac{120^2 - 35^2}{2}\right)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= 3149.0\ \text{kJ/kg}$$

Interpolating in Table A-4, with $T_1 = 360$ °C and $h_1 = 3149.0$ kJ/kg gives

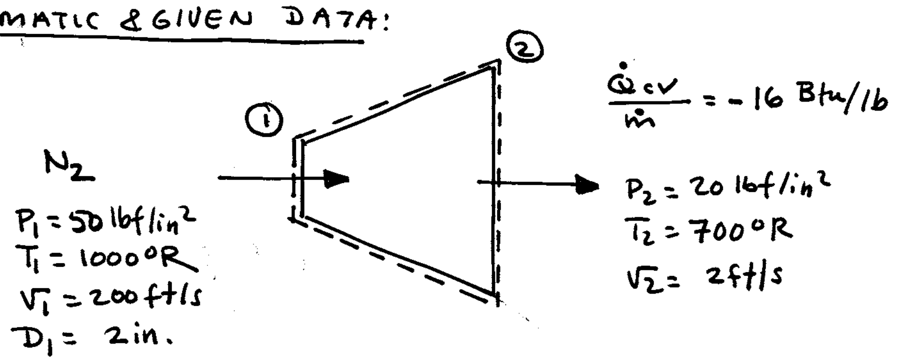
$$p_1 \approx 25.0\ \text{bar} \longleftarrow p_1$$

PROBLEM 4.38

KNOWN: Data are provided for a turbine at steady state, through which N_2 is expanding.

FIND: Determine the power developed.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, the change in potential energy from inlet to exit is negligible. 3. Nitrogen is modeled as an ideal gas.

ANALYSIS: Reducing the mass and energy rate balances,

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}^{\,0}\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right]$$

To find $\dot{m}$

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_1 V_1}{(RT_1/P_1)} = \frac{(\pi D_1^2/4) V_1 P_1}{RT_1} = \frac{[\pi(\frac{2}{12}\text{ ft})^2/4][200\text{ ft/s}][50 \times 144\text{ lbf/ft}^2]}{\left(\frac{1545}{28.01}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(1000^\circ\text{R})}$$

$$= 0.57\text{ lb/s}$$

Then, with enthalpy data from Table A-23E and $\dot{Q}_{cv} = (\dot{Q}_{cv}/\dot{m})(\dot{m})$

$$\dot{W}_{cv} = \left[-16\frac{\text{Btu}}{\text{lb}}\right]\left[0.57\frac{\text{lb}}{\text{s}}\right] + 0.57\frac{\text{lb}}{\text{s}}\left[\frac{(6977.9 - 4864.9)}{(28.01\text{ lb/lbmol})}\frac{\text{Btu}}{\text{lbmol}} + \left[\frac{(200\text{ ft/s})^2 - (2\text{ ft/s})^2}{2}\right]\left|\frac{1\text{ lbf}}{32.2\text{ lbf}\cdot\text{ft/s}^2}\right|\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|\right]$$

$$= 0.57\left[-16 + 75.44 + 0.8\right]\frac{\text{Btu}}{\text{s}}$$

$$= 34.34\frac{\text{Btu}}{\text{s}}\left|\frac{1\text{ hp}}{2545\text{ Btu/h}}\right|\left|\frac{3600\text{ s}}{1\text{ h}}\right| = 48.58\text{ hp} \quad \longleftarrow$$

PROBLEM 4.39

KNOWN: Steam expands through a well-insulated turbine. Inlet and exit conditions are known, and the exit diameter is given.

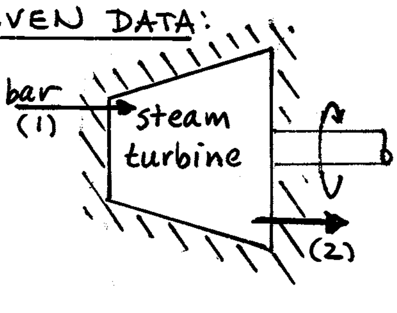
FIND: Plot the power developed by the turbine versus exit quality.

SCHEMATIC & GIVEN DATA:

$p_1 = 4\text{ MPa} = 40\text{ bar}$

$T_1 = 320°C$

$V_1 \approx 0$

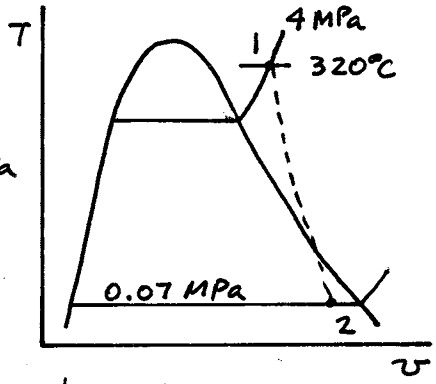


$p_2 = 0.07\text{ MPa}$

$V_2 = 90\text{ m/s}$

$d_2 = 0.6\text{ m}$

$0.9 \le x_2 \le 1.0$



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Kinetic energy at the inlet and potential energy effects are negligible.

ANALYSIS: To determine the power, use the energy balance at steady state

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_2 - z_1)\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. With assumption (3)

$$\dot{W}_{cv} = \dot{m}\left[(h_1 - h_2) - \frac{V_2^2}{2}\right] \qquad (*)$$

Evaluating the mass flow rate

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{\pi d_2^2 V_2}{4 v_2} \qquad (**)$$

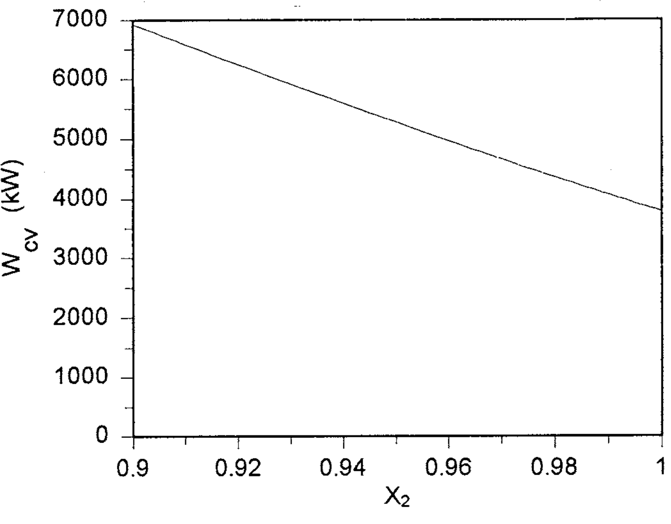
Sample Calculation: $\boxed{x_2 = 0.9}$ From Table A-4 at $p_1 = 4\text{MPa} = 40\text{ bar}$, $T_1 = 320°C$; we get $h_1 = 3015.4\text{ kJ/kg}$. From Table A-3; $v_2 = 1.036\times10^{-3} + (.9)(2.365 - 1.036\times10^{-3})$ or $v_2 = 2.1286\text{ m}^3/\text{kg}$. Similarly, $h_2 = 376.70 + (.9)(2283.3) = 2431.7\text{ kJ/kg}$. Now, from (**)

$$\dot{m} = \frac{\pi (.6)^2 m^2 (90\text{ m/s})}{4(2.1286\text{ m}^3/\text{kg})} = 11.95\text{ kg/s}$$

Now, from (*)

$$\dot{W}_{cv} = \left(11.95\frac{kg}{s}\right)\left[(3015.4 - 2431.7)\frac{kJ}{kg} - \left(\frac{90^2}{2}\right)\frac{m^2}{s^2}\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\right]\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 6927\text{ kW}$$



IT Code

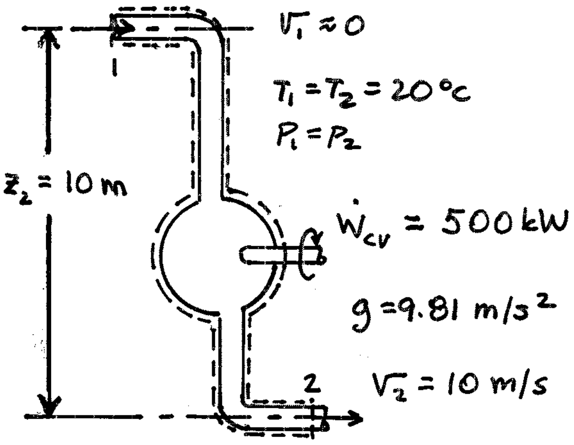
```
p1 = 40  // bar
T1 = 320  // °C
p2 = 0.7  // bar
V2 = 90  // m/s
d2 = 0.6  // m
x2 = 0.9
h1 = h_PT("Water/Steam", p1, T1)
h2 = hsat_Px("Water/Steam", p2, x2)
v2 = vsat_Px("Water/Steam", p2, x2)
mdot = pi * d2^2 * V2 / (4 * v2)
0 = -Wdot + mdot * ((h1 - h2) - V2^2 / (2 * 1000))
// Using the Explore button, sweep x2 from
// 0.9 to 1.0 in steps of 0.001
```

PROBLEM 4.40

KNOWN: Water flows through a hydraullic turbine with known conditions at the inlet and exit. The power output is specified.

FIND: Determine the mass flow rate.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Changes in temperature and pressure from inlet to exit are negligible. (4) Kinetic energy can be neglected at the inlet. (5) The acceleration of gravity is constant; $g = 9.81$ m/s².

ANALYSIS: To find the mass flow rate, begin with steady state mass and energy rate balances

$$0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

and

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \dot{W}_{cv} + \dot{m}\left[\overset{0}{(\cancel{h_1 - h_2})} + \left(\frac{\overset{0}{\cancel{V_1^2}} - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

where the enthalpy term is cancelled because of assumption (3). Solving

$$\dot{m} = \frac{\dot{W}_{cv}}{-\frac{V_2^2}{2} + g(z_1 - z_2)}$$

Inserting values

$$\dot{m} = \frac{(500\text{ kW})\left|\frac{1\text{ kJ/s}}{1\text{ kW}}\right|}{\left[\left(-\frac{10^2}{2}\right)\frac{\text{m}^2}{\text{s}^2} + \left(9.81\frac{\text{m}}{\text{s}^2}\right)(10\text{ m})\right]\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|}$$

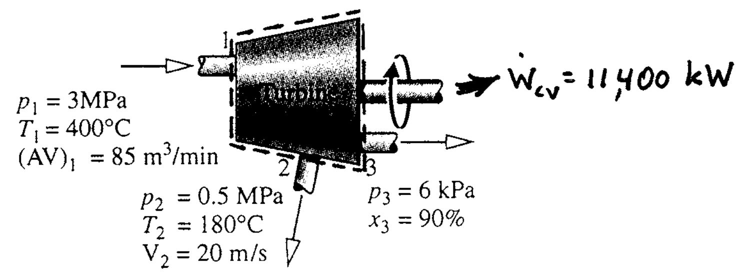
$= 10{,}400$ kg/s ← $\dot{m}$

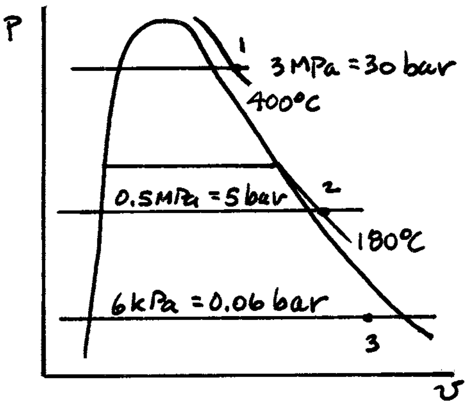
PROBLEM 4.41

KNOWN: Steam passes through a well-insulated extraction turbine with known conditions at the inlet and exits. The power output is given.

FIND: Determine (a) the mass flow rate at each exit, and (b) the diameter of the duct where steam is extracted.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) Kinetic and potential energy effects can be neglected.

ANALYSIS: (a) To determine the mass flow rates $\dot{m}_2$ and $\dot{m}_3$, begin with a mass rate balance

$$\cancel{\frac{dm_{cv}}{dt}}^{0} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 - \dot{m}_2 \qquad (*)$$

Using Eq. 4.4 b, with $v_1 = 0.0994$ m^3/kg from Table A-4

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{(85\ \text{m}^3/\text{min})}{(.0994\ \text{m}^3/\text{kg})}\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 14.25\ \text{kg/s}$$

With $\dot{m}_1$ known, (*) has two unknowns. Another relation is obtained using the energy rate balance.

$$\cancel{\frac{dE_{cv}}{dt}}^{0} = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right) - \dot{m}_2\left(h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right) - \dot{m}_3\left(h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right)$$

where the indicated terms are deleted based on the assumptions. Thus

$$0 = -\dot{W}_{cv} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \qquad (**)$$

With (*), we can reduce (**) as follows

$$0 = -\dot{W}_{cv} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - (\dot{m}_1 - \dot{m}_2) h_3$$

$$= -\dot{W}_{cv} + \dot{m}_1(h_1 - h_3) - \dot{m}_2(h_2 - h_3)$$

Solving for $\dot{m}_2$

$$\dot{m}_2 = \frac{-\dot{W}_{cv} + \dot{m}_1(h_1 - h_3)}{(h_2 - h_3)}$$

From Table A-4; $h_1 = 3230.9$ kJ/kg and $h_2 = 2812.0$ kJ/kg. The specific enthalpy at 3 is determined using data from Table A-3

$$h_3 = h_{f3} + x_3 h_{fg3} = 151.53 + (.9)2415.9 = 2325.8\ \text{kJ/kg}$$

PROBLEM 4.41 (Cont'd.)

Inserting values

$$\dot{m}_2 = \frac{-(11{,}400 \text{ kW})\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| + (14.25 \frac{\text{kg}}{\text{s}})(3230.9 - 2325.8)\frac{\text{kJ}}{\text{kg}}}{(2812.0 - 2325.8) \text{ kJ/kg}}$$

$$= 3.08 \text{ kg/s} = 11088 \text{ kg/h} \longleftarrow \dot{m}_2$$

Now, with $\dot{m}_3 = \dot{m}_1 - \dot{m}_2$

$$\dot{m}_3 = (14.25 \frac{\text{kg}}{\text{s}})\left|\frac{3600 \text{ s}}{1 \text{ h}}\right| - 11088 = 40{,}212 \frac{\text{kg}}{\text{h}} \longleftarrow \dot{m}_3$$

(b) At exit 2, with data from Table A-4

$$A_2 = \frac{\dot{m}_2 v_2}{V_2} = \frac{(3.08 \text{ kg/s})(0.4045 \text{ m}^3/\text{kg})}{(20 \text{ m/s})} = 0.0623 \text{ m}^2$$

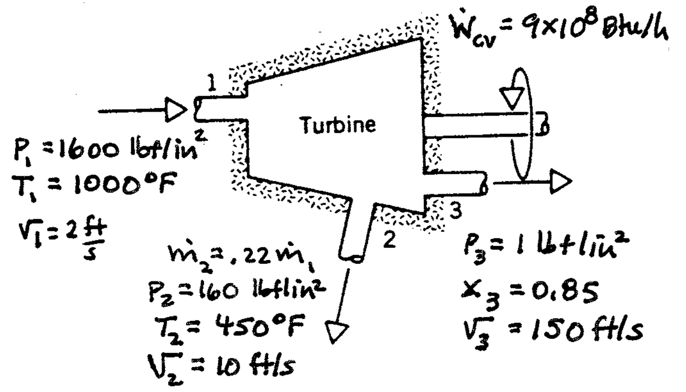
$$d_2 = \sqrt{\frac{4 A_2}{\pi}} = 0.282 \text{ m} \longleftarrow d_2$$

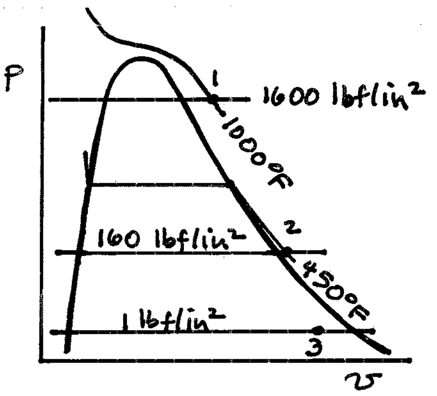
PROBLEM 4.42

KNOWN: Steam passes through an extraction turbine operating at steady state with know inlet and exit conditions. The power output is specified.

FIND: Determine (a) the inlet mass flow rate, (b) the diameter of the extraction duct.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. A control volume enclosing the turbine is at steady state. 2. For the control volume, heat transfer and potential energy effects are negligible.

ANALYSIS: (a) To find $\dot{m}_1$, apply mass rate balance: $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$. Then, since $\dot{m}_2/\dot{m}_1 = 0.22$, we have $\dot{m}_3/\dot{m}_1 = 0.78$. Next, apply an energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}_1\left[h_1 + \frac{V_1^2}{2}\right] - \dot{m}_2\left[h_2 + \frac{V_2^2}{2}\right] - \dot{m}_3\left[h_3 + \frac{V_3^2}{2}\right]$$

where the potential energy terms are omitted by assumption 2. Solving

$$\dot{m}_1 = \frac{\dot{W}_{cv}}{\left[h_1 + \frac{V_1^2}{2}\right] - \frac{\dot{m}_2}{\dot{m}_1}\left[h_2 + \frac{V_2^2}{2}\right] - \frac{\dot{m}_3}{\dot{m}}\left[h_3 + \frac{V_3^2}{2}\right]}$$

From Table A-4E, $h_1 = 1487$ Btu/lb, $h_2 = 1246.1$ Btu/lb. With Table A-3E data $h_3 = h_{f3} + x_3(h_{g3} - h_{f3}) = 69.74 + 0.85(1036) = 950.3$ Btu/lb. Thus

$$\dot{m}_1 = \frac{(9.\times10^8\,\text{Btu/h})}{\left[1487\frac{\text{Btu}}{\text{lb}} + \frac{(2\,\text{ft/s})^2}{2}\left|\frac{1\,\text{lbf}}{32.2\,\text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right|\right] - 0.22\left[1246.1 + \frac{(10)^2}{(2)|32.2||778|}\right] - 0.78\left[950.3 + \frac{(150)^2}{(2)|32.2||778|}\right]}$$

$$= 1.91\times10^6\ \text{lb/h} \longleftarrow \dot{m}_1$$

(b) Using $v_2 = 3.228$ from Table A-4E

$$A_2 = \frac{\dot{m}_2 v_2}{V_2} = \frac{(.22)(1.91\times10^6\,\text{lb/h})(3.228\,\text{ft}^3/\text{lb})}{(10\,\text{ft/s})}\left|\frac{1\,\text{h}}{3600\,\text{s}}\right| = 37.68\,\text{ft}^2$$

Since $A_2 = \pi d_2/4$

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{(4)(37.68\,\text{ft}^2)}{\pi}} = 6.93\,\text{ft} \longleftarrow d_2$$

PROBLEM 4.43

KNOWN: Air is compressed at steady state from a given initial state to a given final pressure. The mass flow is known, and each unit of mass undergoes a specified process in going from inlet to exit.

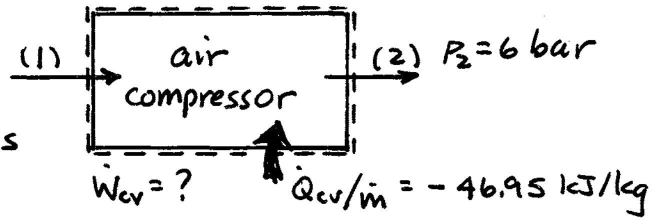
FIND: Determine the compressor power.

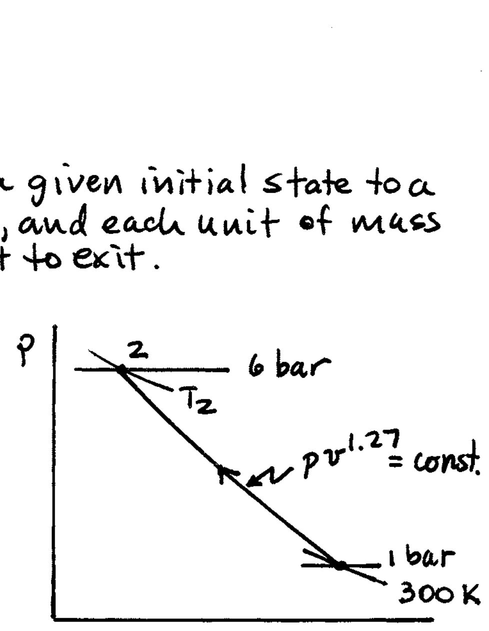
SCHEMATIC & GIVEN DATA:

$T_1 = 300K$

$P_1 = 1$ bar

$\dot{m} = 4$ kg/s





ASSUMPTIONS: (1) The control volume is at steady state. (2) Each unit of mass undergoes a process described by $pv^{1.27} =$ constant. (3) The air can be modeled as an ideal gas. (4) Kinetic and potential energy effects are negligible. ①

ANALYSIS: To determine the power, begin with mass and energy rate balances at steady state

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \overset{0}{\cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}} + \overset{0}{\cancel{g(z_1 - z_2)}}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and the indicated terms are deleted by assumption 4. Rearranging and solving for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \dot{m}\left[(\dot{Q}_{cv}/\dot{m}) + (h_1 - h_2)\right] \qquad (*)$$

Now, specific enthalpy h_1 is read from Table A-22 at 300K: $h_1 = 300.19 \frac{kJ}{kg}$.
To get T_2, we use Eq. 3.56 with $n = 1.27$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} T_1 = \left(\frac{6}{1}\right)^{\frac{1.27-1}{1.27}} (300K) = 439.1\ K$$

Interpolating in Table A-22; $h_2 = 440.7$ kJ/kg. Inserting values in (*)

$$\dot{W}_{cv} = \left(4\ \frac{kg}{s}\right)\left[\left(-46.95\ \frac{kJ}{kg}\right) + (300.19 - 440.7)\ \frac{kJ}{kg}\right]\left|\frac{1\ kW}{1\ kJ/s}\right|$$

② $= -750$ kW ◀———————— $\dot{W}_{cv}$

---

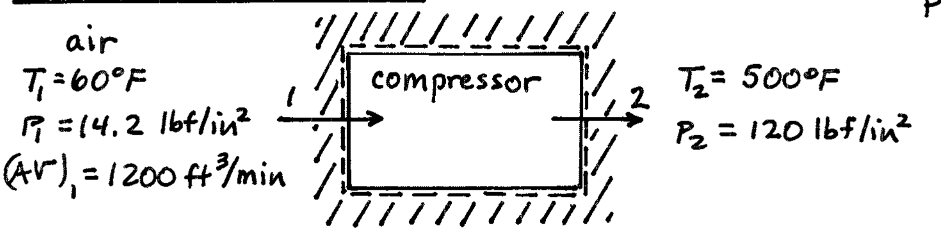
1. The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.
2. The negative sign for power denotes energy transfer by work into the control volume, as expected.

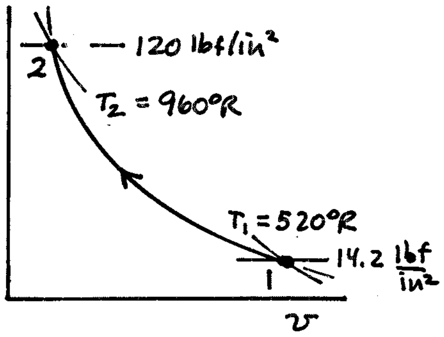
PROBLEM 4.44

KNOWN: A well-insulated air compressor operates with known inlet and exit states. The inlet volumetric flow rate is also known.

FIND: Determine the compressor power and the exit volumetric flow rate.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady
① state. (2) Heat transfer is negligible. (3) The air behaves as an ideal gas. (4) Kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: To find the power, begin with steady state mass and energy balances

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}^{0} + g\cancel{(z_1 - z_2)}^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for $\dot{W}_{cv}$

$$\dot{W}_{cv} = \dot{m}\left[h_1 - h_2\right]$$

To evaluate $\dot{m}$, use Eq. 4.11b and the ideal gas equation of state

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1}$$

$$= \frac{(14.2\ \text{lbf/in}^2)(1200\ \text{ft}^3/\text{min})}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(520°\text{R})} \left|\frac{60\ \text{min}}{1\ \text{h}}\right| \left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| = 5309\ \text{lb/h}$$

Thus, with $h_1 = 124.27$ Btu/lb and $h_2 = 231.06$ Btu/lb from Table A-22E

$$\dot{W}_{cv} = \left(5309\ \frac{\text{lb}}{\text{h}}\right)(124.27 - 231.06)\ \frac{\text{Btu}}{\text{lb}} \left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right|$$

② $$= -222.8\ \text{hp} \longleftarrow \dot{W}_{cv}$$

The exit volumetric flow rate is

$$(AV)_2 = \dot{m} v_2 = \dot{m}\left(\frac{RT_2}{p_2}\right)$$

$$= (5309\ \text{lb/h})\left|\frac{1\ \text{h}}{60\ \text{min}}\right| \frac{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(960°\text{R})}{(120\ \text{lbf/in}^2)} \left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right|$$

$$= 262\ \text{ft}^3/\text{min} \longleftarrow (AV)_2$$

---

1. The applicability of the ideal gas model can be checked by reference to the generalized compressibility chart.
2. The negative sign for power denotes energy transfer into the control volume, as expected.

PROBLEM 4.45*

KNOWN: An air compressor operates with known power, inlet conditions, and exit pressure. The inlet volumetric flow rate and the heat transfer rate per unit mass of air flow are also specified.

FIND: Determine the exit temperature.

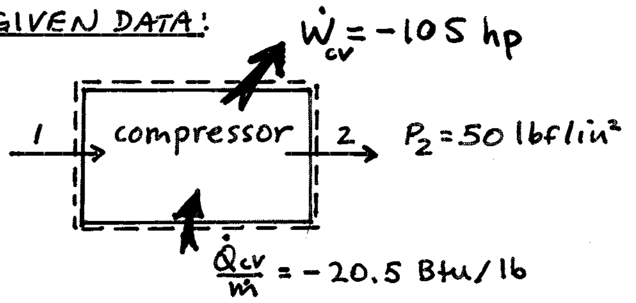
SCHEMATIC & GIVEN DATA:

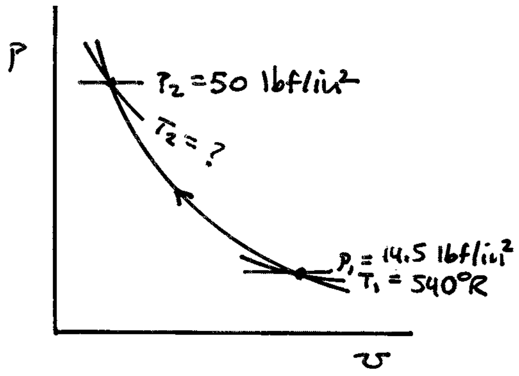
air

$T_1 = 80°F$

$P_1 = 14.5\ \text{lbf/in}^2$

$(AV)_1 = 20\ \text{ft}^3/\text{s}$





ASSUMPTIONS: (1) The control volume is at steady state. (2) Kinetic and ① potential energy changes from inlet to exit are negligible. (3) The air behaves as an ideal gas.

ANALYSIS: To find the compressor power, begin with steady state mass and energy rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancelto{0}{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g\cancelto{0}{(z_1 - z_2)}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving for h_2

$$h_2 = h_1 + \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}}$$

To evaluate $\dot{m}$, use Eq. 4.11b and the ideal gas equation of state

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1}$$

$$= \frac{(14.5\ \text{lbf/in}^2)(20\ \text{ft}^3/\text{s})}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(540°\text{R})}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| = 1.44\ \text{lb/s}$$

Using data from Table A-22E

$$h_2 = \left(129.06\frac{\text{Btu}}{\text{lb}}\right) + \left(-20.5\ \frac{\text{Btu}}{\text{lb}}\right) - \left(\frac{-105\ \text{hp}}{1.44\ \text{lb/s}}\right)\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right|$$

$$= 160.1\ \text{Btu/lb}$$

Interpolating in Table A-22E with $h_2 = 160.1$ Btu/lb

$$T_2 = 669°\text{R} = 209°\text{F} \longleftarrow T_2$$

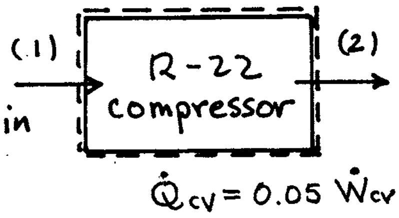
PROBLEM 4.46*

KNOWN: Data are provided for a R22 compressor operating at steady state.

FIND: Determine the power required by the compressor.

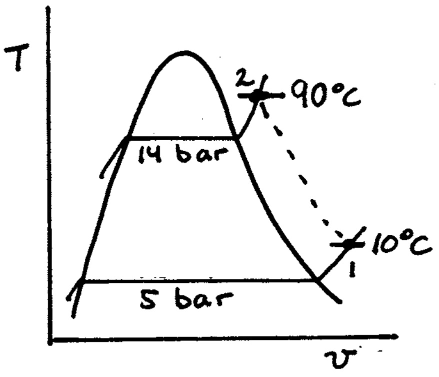
SCHEMATIC & GIVEN DATA:

$P_1 = 5$ bar
$T_1 = 10°C$
$(AV)_1 = 0.8$ m³/min
$D_1 = 4$ cm



$\dot{Q}_{cv} = 0.05\,\dot{W}_{cv}$

$P_2 = 14$ bar
$T_2 = 90$ °C
$D_2 = 2$ cm



ASSUMPTIONS: (1) The control volume is at steady state. (2) Potential energy effects can be neglected.

ANALYSIS: Noting that $\dot{W}_{cv}$ is negative for a compressor, the rate of heat transfer from the compressor is given by $\dot{Q}_{cv} = 0.05\,\dot{W}_{cv}$. For steady-state operation, the mass and energy rate balances reduce to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(\cancel{z_1 - z_2})\right]$$

$$\Rightarrow \dot{W}_{cv} = (0.05\,\dot{W}_{cv}) + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right] \Rightarrow \dot{W}_{cv} = \frac{\dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right]}{0.95}$$

From Table A-9, $h_1 = 257.22$ kJ/kg, $h_2 = 306.60$ kJ/kg, $v_1 = 0.04934$ m³/kg, $v_2 = 0.02217$ m³/kg. The mass flow rate is

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{0.8\ \text{m}^3/\text{min}}{0.04934\ \text{m}^3/\text{kg}} = 16.214\ \text{kg/min} = 0.2702\ \text{kg/s}$$

Further

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi(0.04\ \text{m})^2}{4} = 1.257 \times 10^{-3}\ \text{m}^2$$

$$A_2 = 0.3142 \times 10^{-3}\ \text{m}^2$$

Thus, the velocities at the inlet and exit are, respectively

$$V_1 = \frac{(AV)_1}{A_1} = \frac{(0.8\ \text{m}^3/\text{min})}{(1.257\times10^{-3}\ \text{m}^2)}\left|\frac{1\ \text{min}}{60\ \text{s}}\right| = 10.61\ \text{m/s}$$

$$V_2 = \frac{\dot{m}\,v_2}{A_2} = \frac{(0.2702\ \text{kg/s})(0.02217\ \text{m}^3/\text{kg})}{(0.3142\times10^{3}\ \text{m}^2)} = 19.07\ \text{m/s}$$

The required power is then

$$\dot{W}_{cv} = \frac{(0.2702\,\frac{\text{kg}}{\text{s}})(257.22 - 306.60)\frac{\text{kJ}}{\text{kg}} + \left(\frac{10.71^2 - 19.07^2}{2}\right)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1\ \text{N}}{1\ \text{kg m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{Nm}}\right|\,]}{0.95}$$

$$= \frac{(0.2702)[-49.38 + (-0.12)]\frac{\text{kJ}}{\text{s}}}{0.95} = -14.08\,\frac{\text{kJ}}{\text{s}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= -14.08\ \text{kW} \xleftarrow{\qquad\qquad\qquad \dot{W}_{cv}}$$

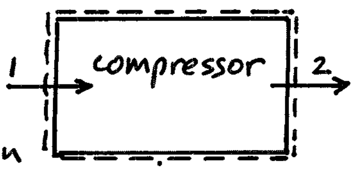
PROBLEM 4.47*

KNOWN: A Refrigerant 134a compressor has known conditions at the inlet and exit. The inlet volumetric flow rate and the compressor power per unit mass of refrigerant flowing are also specified.

FIND: Determine the heat transfer rate.

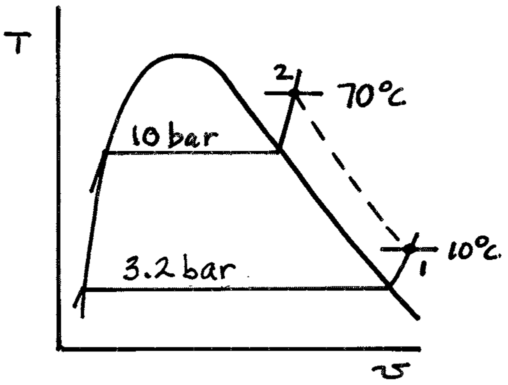
SCHEMATIC & GIVEN DATA:

Refrigerant 134a
$p_1 = 3.2\,\text{bar}$
$T_1 = 10°C$
$(AV)_1 = 3.0\ \text{m}^3/\text{min}$



$p_2 = 10\ \text{bar}$
$T_2 = 70\ °C$

$\frac{\dot{W}_{cv}}{\dot{m}} = -55.2\ \text{kJ/kg}$



ASSUMPTIONS: (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: To find the heat transfer rate, begin with steady state energy and mass rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \overset{0}{\cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)}} + g\overset{0}{\cancel{(z_1 - z_2)}}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving

$$\dot{Q}_{cv} = \dot{m}\left[\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) + (h_2 - h_1)\right]$$

To evaluate $\dot{m}$, use Eq. 4.11b and data for v_1 from Table A-12

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{3.0\ \text{m}^3/\text{min}}{0.06576\ \text{m}^3/\text{kg}} = 45.62\ \text{kg/min}$$

From Table A-12, $h_1 = 255.65$ kJ/kg, and $h_2 = 302.34$ kJ/kg
Thus

$$\dot{Q}_{cv} = \left(45.62\frac{\text{kg}}{\text{min}}\right)\left[(-55.2) + (302.34 - 255.65)\right]\frac{\text{kJ}}{\text{kg}}\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right)$$

$$= -0.14\ \text{kW} \longleftarrow \dot{Q}_{cv}$$

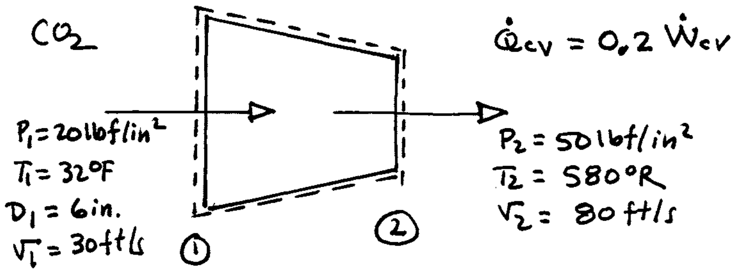
The negative sign indicates that the heat transfer is from the system.

PROBLEM 4.48

KNOWN: Data are provided for a CO_2 compressor operating at steady state.

FIND: Determine the power required by the compressor.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the figure is at steady state. 2. Potential energy effects can be ignored. 3. CO_2 is modeled as an ideal gas.

ANALYSIS: Since the compressor requires a power input, the heat transfer from the compressor is given by $\dot{Q}_{cv} = 0.2\,\dot{W}_{cv}$. To evaluate $\dot{W}_{cv}$, reduce the mass and energy rate balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \overset{0}{\cancel{g(z_1 - z_2)}}\right]$$

With $\dot{Q}_{cv} = 0.2\,\dot{W}_{cv}$

$$\dot{W}_{cv} = \frac{\dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right]}{0.8}$$

Using data at the inlet, the mass flow rate is

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_1 V_1 p_1}{RT_1} = \frac{\left[\frac{\pi}{4}(0.5\ ft)^2\right]\left[30\ ft/s\right]\left[(20)(144)\ lbf/ft^2\right]}{\left(\frac{1545}{44.01}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(492°R)} = 0.982\ lb/s$$

From Table A-23E, $\bar{h}_2 = 4417.2$ kJ/kmol, $\bar{h}_1 = 3637.7$ kJ/kmol. Thus

$$\dot{W}_{cv} = \frac{(0.982\ lb/s)\left[\left(\frac{3637.7 - 4417.2}{44.01}\right)\frac{Btu}{lb} + \left[\frac{(30\ ft/s)^2 - (80)^2}{2}\right]\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|\right]}{0.8}$$

$$= \frac{(0.982)}{0.8}\left[-17.71 + (-0.11)\right]\frac{Btu}{s} = -21.87\ \frac{Btu}{s}$$

$$= -21.8\ \frac{Btu}{s}\left|\frac{3600\ s}{1\ h}\right|\left|\frac{1\ hp}{2545\ Btu/h}\right|$$

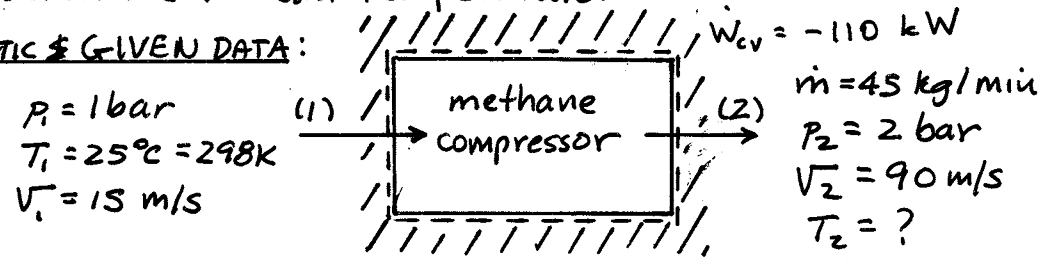
$$= -30.84\ hp \longleftarrow \dot{W}_{cv}$$

PROBLEM 4.49

KNOWN: Methane gas is compressed with negligible heat transfer. At the inlet, pressure, temperature, and velocity are given. Pressure and velocity are specified at the exit. The power input is known.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) For the control volume, $\dot Q_{cv} = 0$ and potential energy effects are negligible. (3) The methane ① gas can be modeled as an ideal gas.

ANALYSIS: By assumption (3), $h_2 = h(T_2)$. Thus, using energy and mass rate balances we can solve for h_2 and hence T_2. That is

$$0 = \cancelto{0}{\dot Q_{cv}} - \dot W_{cv} + \dot m\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(\cancelto{0}{z_1 - z_2})\right]$$

or

$$0 = -\dot W_{cv} + \dot m\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right] \qquad (*)$$

The term $h_1 - h_2$ can be expressed in terms of $c_p(T)$, as follows

$$h_1 - h_2 = \int_{T_2}^{T_1} c_p(T)\, dT \qquad (**)$$

One alternative to evaluate the integral is use the function for c_p in Table A-21. The result would be a polynomial in which T_2 would be unknown. Using the result in conjunction with Eq.(*), an iterative procedure or an equation-solving computer program would be needed.

A simpler alternative is to use the $h(T)$ function for methane included in IT, as follows:

IT Code

```
    T1 = 25 + 273  // K
    V1 = 15  // m/s
    V2 = 90  // m/s
    mdot = 45 / 60  // kg/s
    Wdot = -110  // kW
②   0 = - Wdot + mdot * ((h1 - h2) + (V1^2 - V2^2) / (2 * 1000))
    h1 = h_T("CH4", T1)
③   h2 = h_T("CH4", T2)
```

IT Result

T2 = 359.9 K ← T_2

---

1. The applicability of the ideal gas model can be checked by reference to the compressibility chart.
2. Carefully note the unit conversions required in this expression.
3. This result compares very favorably with the result obtaine using the alternative method involving Table A-21 data.

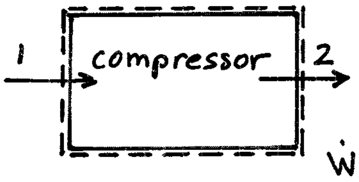
PROBLEM 4.50

KNOWN: An ammonia compressor has known conditions at the inlet and exit. The compressor power and the heat transfer rate are also specified.

FIND: Determine the inlet volumetric flow rate using ammonia tables and ideal gas relationships. Discuss.

SCHEMATIC & GIVEN DATA:

Ammonia
$P_1 = 20\ lbf/in^2$
$T_1 = 0°F$



$P_2 = 250\ lbf/in^2$
$T_2 = 300°F$
$\dot{W}_{cv} = -10\ hp$
$\dot{Q}_{cv} = -5000\ Btu/h$

ASSUMPTIONS: (1) The control volume is at steady state. (2) Kinetic and potential energy changes from inlet to exit are negligible. (3) For the second part, the ammonia behaves as an ideal gas.

ANALYSIS: To begin, determine the mass flow rate by using the steady-state mass and energy balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1-h_2) + \overset{0}{\cancel{\left(\frac{V_1^2-V_2^2}{2}\right)}} + g\overset{0}{\cancel{(z_1-z_2)}}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Solving

$$\dot{m} = \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{(h_2-h_1)} \qquad (*)$$

From Table A-15E; $h_1 = 614.84\ Btu/lb$ and $h_2 = 760.39\ Btu/lb$. Thus

$$\dot{m} = \frac{(-5000\ Btu/h) - (-10\ hp)\left|\frac{2545\ Btu/h}{1\ hp}\right|}{(760.39 - 614.84)\ Btu/lb}\left|\frac{1\ h}{60\ min}\right| = 2.342\ lb/min$$

Now, using $v_1 = 14.078\ ft^3/lb$ from Table A-15E and Eq. 4-11b

$$(AV)_1 = \dot{m}v_1 = (2.342)(14.078) = 32.97\ ft^3/min \longleftarrow \underset{\text{(Ammonia table)}}{(AV)_1}$$

To evaluate $h_2 - h_1$ for an ideal gas, integrate the specific heat function $c_p(T)$ for ammonia from Table A-21E

$$h_2 - h_1 = \frac{\bar{R}}{M}\int_{T_1}^{T_2}(\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)\,dT$$

$$= \frac{\bar{R}}{M}\left[\alpha(T_2-T_1) + \frac{\beta}{2}(T_2^2-T_1^2) + \frac{\gamma}{3}(T_2^3-T_1^3) + \frac{\delta}{4}(T_2^4-T_1^4) + \frac{\epsilon}{5}(T_2^5-T_1^5)\right]$$

With $T_1 = 460°R$, $T_2 = 760°R$, and coefficient values from Table A-21E

$$h_2 - h_1 = \frac{(1.986\ Btu/lbmol\cdot°R)}{(17.04\ lb/lbmol)}\left[1329\ °R\right]$$

$$= 154.9\ Btu/lb$$

PROBLEM 4.50 (Cont'd)

Using this result with (*)

$$\dot{m} = \frac{(-5000)-(-10)(2545)}{(154.9)}\left|\frac{1}{60}\right| = 2.2\ \text{lb/min}$$

Incorporating the ideal gas equation of state into Eq. 4-11b

$$(AV)_1 = \dot{m}v_1 = \dot{m}\left(\frac{RT_1}{p_1}\right)$$

$$= \left(2.2\ \frac{\text{lb}}{\text{min}}\right)\frac{\left(\frac{1545}{17.04}\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(460^\circ\text{R})}{(20\ \text{lbf/in}^2)}\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right|$$

$$= 31.86\ \text{ft}^3/\text{min} \longleftarrow \underset{(\text{ideal gas})}{(AV)_1}$$

Discussion: The % deviation in assuming ideal gas behavior is

$$\%\ \text{deviation} = \left[\frac{(AV)_{\text{tables}} - (AV)_{\text{ideal gas}}}{(AV)_{\text{tables}}}\right]\times 100$$

$$= \left[\frac{32.97 - 31.86}{32.97}\right]\times 100 = 3.4\%$$

Thus, the ideal gas model is reasonably accurate. To explore this further, consider

$$\%\ \text{deviation} = \left[\frac{\Delta h_{\text{ideal gas}} - \Delta h_{\text{tables}}}{\Delta h_{\text{tables}}}\right]\times 100$$

$$= \left[\frac{154.9 - 145.55}{145.55}\right]\times 100 = 6.4\%$$

The applicability of the ideal gas model can also be checked by determining the compressibility factor. For state 1

$$Z_1 = \frac{p_1 v_1}{RT_1} = \frac{(20)|144|(14.078)}{\left(\frac{1545}{17.04}\right)(460)} = 0.972$$

For state 2, $v_2 = 1.8191$ ft³/lb from Table A-15E. Thus

$$Z_2 = \frac{(250)|144|(1.8191)}{\left(\frac{1545}{17.04}\right)(760)} = 0.95$$

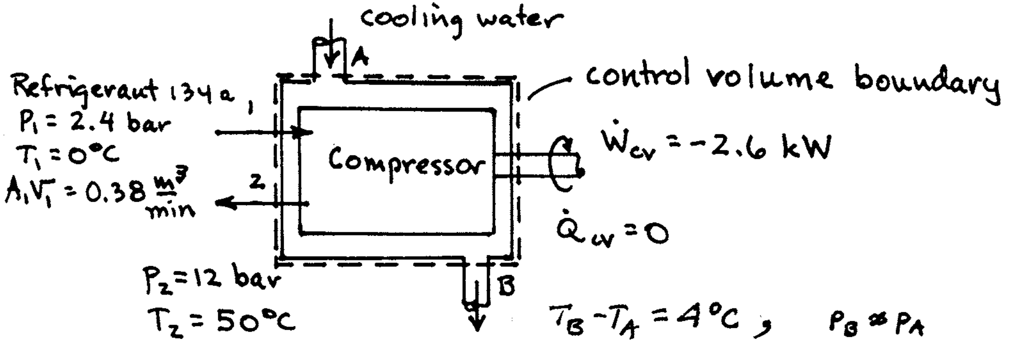
Both of these values are reasonably close to unity.

PROBLEM 4.51

KNOWN: Refrigerant 134a is compressed in a water-jacketed compressor between two known states. The inlet volumetric flow rate, input power, and cooling water temperature rise are given.

FIND: Determine the mass flow rate of the cooling water.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady-state. (2) There is no heat transfer with the surroundings. (3) Kinetic and potential energy effects can be neglected. (4) The cooling water is incompressible with c = 4.179 kJ/kg·K from Table A-19.

ANALYSIS: The energy rate balance applied to the overall compressor and water jacket at steady state reduces to

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \underline{\frac{V_1^2}{2} + gz_1}\right) - \dot{m}_2\left(h_2 + \underline{\frac{V_2^2}{2} + gz_2}\right)$$
$$+ \dot{m}_A\left(h_A + \underline{\frac{V_A^2}{2} + gz_A}\right) - \dot{m}_B\left(h_B + \underline{\frac{V_B^2}{2} + gz_B}\right)$$

where $\dot{Q}_{cv} = 0$ by assumption (2), and the underlined terms drop out by assumption (3). Since the water and refrigerant streams are separate

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_R$$
$$\dot{m}_A = \dot{m}_B \equiv \dot{m}_W$$

Thus

$$0 = -\dot{W}_{cv} + \dot{m}_R(h_1 - h_2) + \dot{m}_W(h_A - h_B)$$

Applying Eq. 3.20b for the water stream

$$h_A - h_B = c(T_A - T_B) + v\cancelto{0}{(p_A - p_B)}$$

Inserting this result and solving for $\dot{m}_W$

$$\dot{m}_W = \frac{-\dot{W}_{cv} + \dot{m}_R(h_1 - h_2)}{c(T_B - T_A)} \qquad (*)$$

From Table A-12; $h_1 = 248.89$ kJ/kg, $v_1 = 0.08574$ m³/kg and $h_2 = 275.52$ kJ/kg.

Evaluating $\dot{m}_R$

$$\dot{m}_R = \frac{(AV)_1}{v_1} = \frac{(0.38\ m^3/min)}{(0.08574\ m^3/kg)}\left|\frac{1\ min}{60\ s}\right| = 0.0739\ kg/s$$

Finally, inserting values in (*)

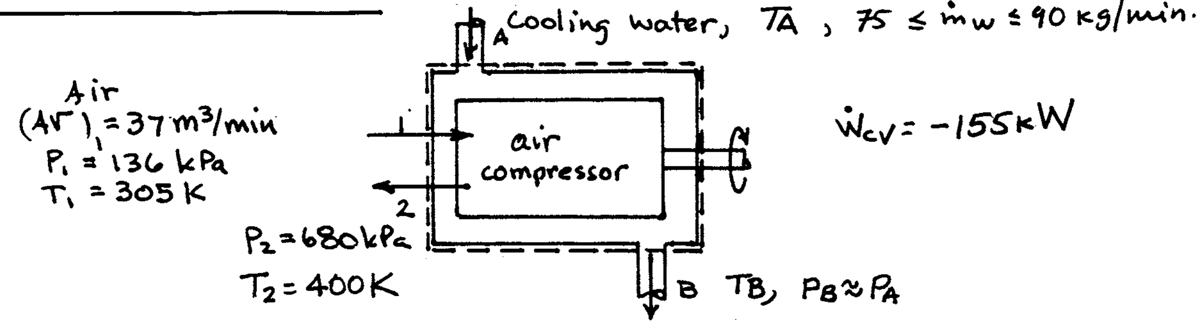
$$\dot{m}_W = \frac{-(-2.6\ kW)\left|\frac{1\ kJ/s}{1\ kW}\right| + (0.0739\ kg/s)(248.89 - 275.52)\frac{kJ}{kg}}{(4.179\ kJ/kg\cdot K)(4\ °C)} = 0.0378\ \frac{kg}{s} \longleftarrow \dot{m}_W$$

PROBLEM 4.52

KNOWN: Data are provided for a water-jacketed air compressor operating at steady state.

FIND: Determine the cooling water temperature increase for a specified cooling water mass flow rate. Plot the cooling water temperature increase versus cooling water mass flow rate.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer from the outside of the cooling water jacket is negligible. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas. (5) The cooling water is incompressible with $c_p = 4.179$ kJ/kg·K from Table A-19.

ANALYSIS: (a) The steady-state energy balance for the overall compressor is

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}_1\left(h_1 + \underline{\frac{V_1^2}{2}} + \underline{gz_1}\right) - \dot{m}_2\left(h_2 + \underline{\frac{V_2^2}{2}} + \underline{gz_2}\right)$$
$$+ \dot{m}_A\left(h_A + \underline{\frac{V_A^2}{2}} + \underline{gz_A}\right) - \dot{m}_B\left(h_B + \underline{\frac{V_B^2}{2}} + \underline{gz_B}\right)$$

where the indicated terms drop out by assumptions (2) and (3). Since the water and air streams are separate

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{air}$$
$$\dot{m}_A = \dot{m}_B \equiv \dot{m}_w$$

Thus
$$0 = -\dot{W}_{cv} + \dot{m}_{air}(h_1 - h_2) + \dot{m}_w(h_A - h_B) \qquad (1)$$

With assumption 5, the enthalpy change of the cooling water can be found using Eq. 3.20b:

$$h_B - h_A = c(T_B - T_A) + \cancel{v(P_B - P_A)} = c(T_B - T_A)$$

Accordingly, Eq. (1) gives

$$T_B - T_A = \frac{\left[-\dot{W}_{cv} + \dot{m}_{air}(h_1 - h_2)\right]}{c\,\dot{m}_w}$$

Using the data at location 1 to evaluate $\dot{m}_{air}$

$$\dot{m}_{air} = \frac{(AV)_1}{v_1} = \frac{(AV)_1 P_1}{RT_1} = \frac{(37\ \text{m}^3/\text{min})(136\ \text{kPa})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg·K}}\right)(305\ \text{K})}\left|\frac{10^3\ \text{N/m}^2}{1\ \text{kPa}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N·m}}\right|$$
$$= 57.49\ \text{kg/min}$$

Also, from Table A-22, $h_1 = 305.22$ kJ/kg, $h_2 = 400.48$ kJ/kg

Collecting results
$$T_B - T_A = \frac{\left[-(-155\ \text{kJ/s})(60\text{s/min}) + (57.49\ \text{kg/min})(305.22 - 400.48)\ \text{kJ/kg}\right]}{(4.179\ \text{kJ/kg·K})\ \dot{m}_w}$$

PROBLEM 4.52 (Contd.)

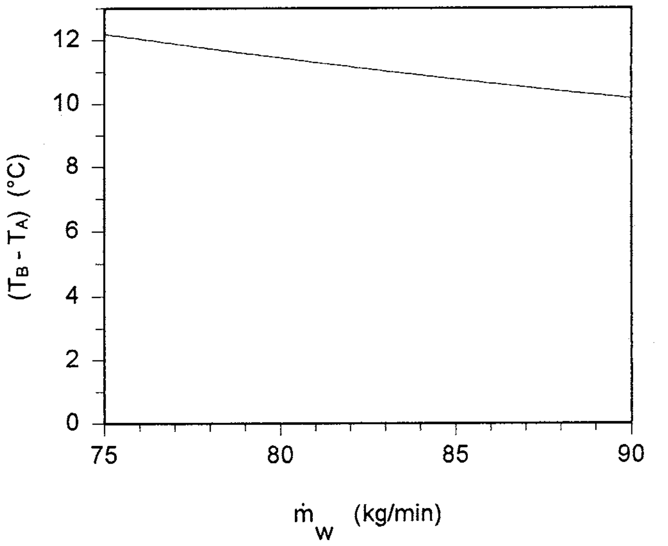
or

$$(T_B - T_A) = \frac{3823.5 \text{ kJ/min}}{(4.179 \text{ kJ/kg}\cdot\text{K})\ \dot{m}_w}$$

$$= \frac{914.93 \text{ kg}\cdot\text{K/min}}{\dot{m}_w} \qquad (*)$$

For the case of $\dot{m}_w = 82$ kg/min; $T_B - T_A = 11.2\text{ K} \longleftarrow \Delta T_w$ (part a)

(b) Equation (*) can be plotted readily using a spreadsheet, plotting program, or IT. The IT plot follows:

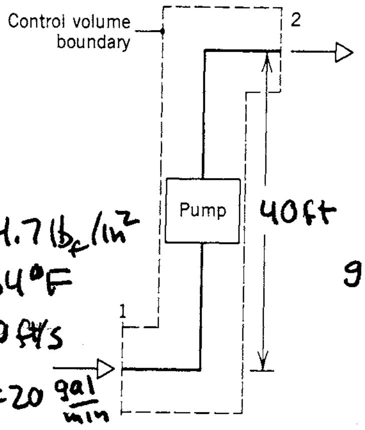


PROBLEM 4.53*

KNOWN: Water is steadily pumped through a piping arrangement to a higher elevation where it is discharged with a known mass flow rate.

FIND: Determine the power required by the pump.

SCHEMATIC & GIVEN DATA:



$P_2 = 35\ lb_f/in^2$
$T_2 = 64^\circ F$
$V_2 = 40 ft/s$

$P_1 = 14.7\ lb_f/in^2$
$T_1 = 64^\circ F$
$V_1 = 10\ ft/s$
$(AV)_1 = 20 \frac{gal}{min}$

$g = 32.0 \frac{ft}{s^2}$

ASSUMPTIONS:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} \sim 0$.
3. The water is modeled as incompressible.
4. $g = 32.0\ ft/s^2$

ANALYSIS: Reducing mass and energy rate balances

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$$

$$\Rightarrow \quad (-\dot{W}_{cv}) = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

With assumption 3 and Eq. 3.20b

$$(h_2 - h_1) = c(\cancel{T_2} - T_1) + v(P_2 - P_1) = v(P_2 - P_1)$$

$$= \left(0.01604 \frac{ft^3}{lb}\right)(35 - 14.7)\frac{lbf}{in^2}\left|\frac{144\ in^2}{ft^2}\right|\left|\frac{1\ Btu}{778\ ft\ lbf}\right| = 0.060\ \frac{Btu}{lb}$$

where $v = v_f(64^\circ F)$ from Table A-2E. The change in specific kinetic energy is

$$\frac{V_2^2 - V_1^2}{2} = \left[\frac{(40\ ft/s)^2 - (10\ ft/s)^2}{2}\right]\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft\ lbf}\right| = 0.0299\ \frac{Btu}{lb}$$

and

$$g(z_2 - z_1) = \left(32.0 \frac{ft}{s^2}\right)(40 ft)\left|\frac{1\ lbf}{32.2\ lb\ ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right| = 0.0511\ \frac{Btu}{lb}$$

The mass flow rate is calculated using

$$\dot{m} = \frac{(AV)_1}{v} = \frac{(20\ gal/min)}{(0.01604\ ft^3/lb)}\left|\frac{0.13368\ ft^3}{1\ gal}\right|\left|\frac{1\ min}{60\ sec}\right| = 2.78\ lb/s$$

Collecting results

$$(-\dot{W}_{cv}) = \left(2.78 \frac{lb}{s}\right)[0.060 + 0.0299 + 0.0511]\ \frac{Btu}{lb}$$

$$= 0.392\ Btu/s \longleftarrow$$

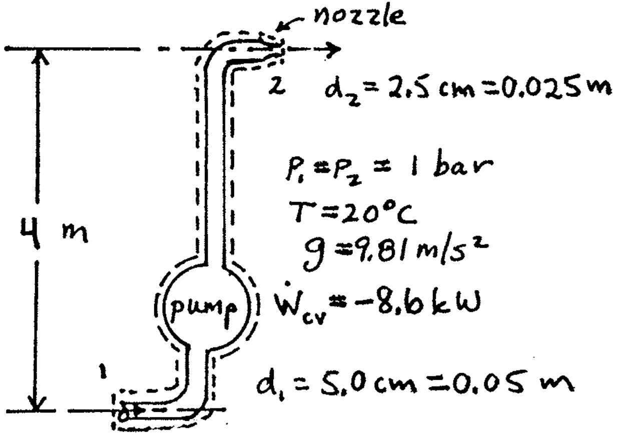
$$= \left(0.392 \frac{Btu}{s}\right)\left|\frac{3600 s}{1 h}\right|\left|\frac{1\ hp}{2545\ Btu/h}\right| = 0.55\ hp \xleftarrow{(-\dot{W}_{cv})}$$

PROBLEM 4.54*

KNOWN: Water flows through pumping system with known inlet and exit conditions. The power required by the pump is also specified.

FIND: Determine the mass flow rate.

SCHEMATIC & GIVEN DATA:


nozzle
2
$d_2 = 2.5\ \text{cm} = 0.025\ \text{m}$
$P_1 = P_2 = 1\ \text{bar}$
$T = 20°\text{C}$
$g = 9.81\ \text{m/s}^2$
4 m
pump
$\dot{W}_{cv} = -8.6\ \text{kW}$
1
$d_1 = 5.0\ \text{cm} = 0.05\ \text{m}$

ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible. (3) The water behaves as an incompessible liquid. (4) The acceleration of gravity is constant at $g = 9.81\ \text{m/s}^2$. (5) The temperature and pressure are nearly constant throughout.

ANALYSIS: To find $\dot{m}$, begin with steady-state mass and energy rate balances

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}\left[\cancel{(h_1 - h_2)}^{0} + \left(\frac{\mathrm{V}_1^2 - \mathrm{V}_2^2}{2}\right) + g(z_1 - z_2)\right]$$

$$= -\dot{W}_{cv} + \dot{m}\left[\left(\frac{\mathrm{V}_1^2 - \mathrm{V}_2^2}{2}\right) + g(z_1 - z_2)\right] \qquad (*)$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and the specific enthalpy term is eliminated based on assumption (3) and Eq. 3.20b

From Eq. 5.3a, $\mathrm{V} = \dot{m}v/A$, and (*) becomes

$$0 = -\dot{W}_{cv} + \dot{m}\left[\frac{(\dot{m}v/A_1)^2 - (\dot{m}v/A_2)^2}{2} + g(z_1 - z_2)\right]$$

$$= -\dot{W}_{cv} + \frac{\dot{m}^3 v^2}{2}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right) + \dot{m}g(z_1 - z_2) \qquad (**)$$

From $A = \pi d^2/4$; $A_1 = 0.000491\ \text{m}^2$ and $A_2 = 0.001964\ \text{m}^2$. Now, with $v \approx v_{f@20°C} = 1.0018 \times 10^{-3}\ \text{m}^3/\text{kg}$ from Table A-2, we can insert values in (**)

$$0 = -(-8.6\ \text{kW})\left|\frac{1\ \text{kJ/s}}{1\ \text{kW}}\right| + \dot{m}^3\frac{\left(1.0018\times10^{-3}\ \frac{\text{m}^3}{\text{kg}}\right)^2}{2}\left[\frac{1}{0.000491^2} - \frac{1}{0.001964^2}\right]\frac{1}{\text{m}^4} \times \left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| + \dot{m}\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(-4\ \text{m})\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

or

$$0 = 8.6 - 1.9513 \times 10^{-3}\ \dot{m}^3 - 0.03924\dot{m} \quad (\text{where } \dot{m} \text{ is in kg/s})$$

This equation is cubic in $\dot{m}$. The solution is

$$\dot{m} = 15.98\ \text{kg/s} \longleftarrow \dot{m}$$

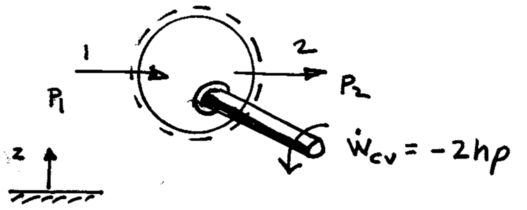
PROBLEM 4.55

KNOWN: Data are provided for a water pump operating at steady state

FIND: Plot the pressure rise from inlet to exit versus volumetric flow.

SCHEMATIC & GIVEN DATA:

$T = 70°F$
$D = 3\text{in.}$
$4 \le AV \le 5\ \text{gal/s}$
$z_1 = z_2$



ASSUMPTIONS: 1. The control volume shown in the schematic is at ① steady state. 2. For the control volume, $\dot Q_{cv} = 0$. 3. Water is modeled as incompressible.

ANALYSIS: The mass rate balance is $\dot m_2 = \dot m_1$. Since the inlet and exit pipe diameters are the same and the water is incompressible, $V_1 = V_2$; and so the kinetic term vanishes in the energy rate balance, which reads

$$0 = \cancel{\dot Q_{cv}} - \dot W_{cv} + \dot m \left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)} \right]$$

② With Eq. 3.20b,

$$h_2 - h_1 = c\cancelto{0}{(T_2 - T_1)} + v(p_2 - p_1)$$

Thus

$$(-\dot W_{cv}) = \dot m \left[v(p_2 - p_1) \right] \Rightarrow p_2 - p_1 = \frac{(-\dot W_{cv})}{\dot m\, v}$$

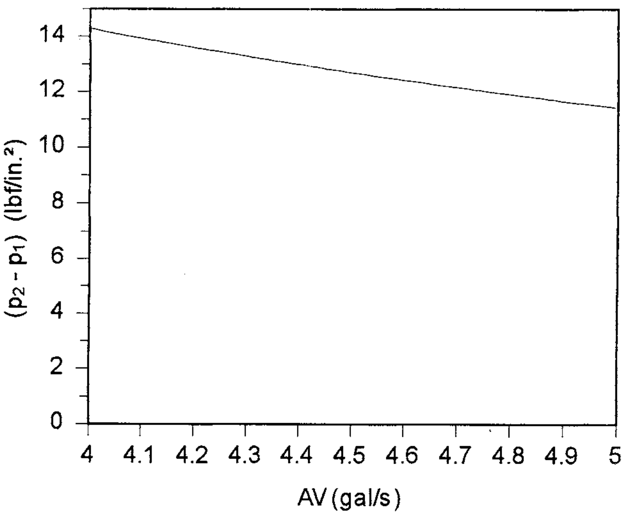
with $\dot m = \frac{AV}{v} \Rightarrow AV = \dot m v$,

$$(p_2 - p_1) = \frac{(-\dot W_{cv})}{(AV)}$$

$$= \frac{(2\ hp) \left| \frac{2545\ \text{Btu/h}}{1\ hp} \right| \left| \frac{1h}{3600s} \right| \left| \frac{778\ \text{ft·lbf}}{1\ \text{Btu}} \right| \left| \frac{1\ ft^2}{144\ in^2} \right|}{(AV)(\text{gal/s}) \left| \frac{0.13368\ ft^3}{1\ \text{gal}} \right|}$$

$$= \frac{57.14}{(AV)} \frac{\text{lbf}}{\text{in}^2} \qquad (*)$$

Eq. (*) can be plotted readily using a spreadsheet, plotting program, or IT. The IT plot is shown at the right.



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1. Since the water is at 70°F, only a small temperature difference with normal surroundings would be observed. So, heat transfer can be ignored.
2. Alternatively, Eq. 3.13 can be used to obtain the same result.

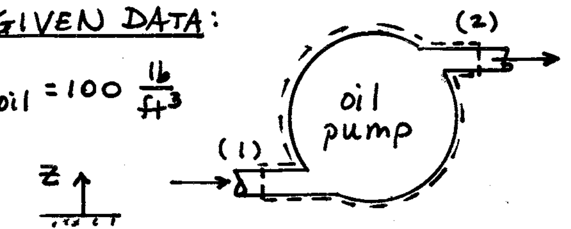
PROBLEM 4.56

KNOWN: An oil pump operating at steady state delivers oil with a known mass flow rate and pressure rise from inlet to exit.

FIND: If pumps are available in 1/4-horsepower increments, determine the horsepower rating of the pump needed for this application.

SCHEMATIC & GIVEN DATA:

$\rho_{oil} = 100 \frac{lb}{ft^3}$



$\dot m = 12$ lb/s
$d_2 = 1$ in. $= (1/12)$ ft
$p_2 - p_1 = 40$ lbf/in.^{2}
T is constant
$z_1 \approx z_2$

ASSUMPTIONS: (1) The control volume is at steady state. (2) For the control
① volume, $\dot Q_{cv} \approx 0$. (3) The oil is incompressible. (4) The potential energy change from inlet to exit and the inlet kinetic energy are negligible.

ANALYSIS: To determine the power required, begin with steady state forms of the mass and energy rate balances, as follows:

$$0 = \overset{0}{\cancel{\dot Q_{cv}}} - \dot W_{cv} + \dot m\left[(h_1 - h_2) + \left(\frac{\overset{0}{\cancel{V_1^2}} - V_2^2}{2}\right) + g\overset{0}{\cancel{(z_1 - z_2)}}\right]$$

where $\dot m_1 = \dot m_2 \equiv \dot m$ and assumptions (2) and (4) have been applied. Thus

$$\dot W_{cv} = \dot m\left[(h_1 - h_2) - \frac{V_2^2}{2}\right]$$

With Eq. 3.20b

② $$h_1 - h_2 = c\overset{0}{\cancel{(T_1 - T_2)}} + v(p_1 - p_2) = \frac{p_1 - p_2}{\rho}$$

Thus

$$\dot W_{cv} = \dot m\left[\frac{p_1 - p_2}{\rho} - \frac{V_2^2}{2}\right]$$

Using Eq. 4.11a with $A = \pi d^2/4$

$$V_2 = \frac{4\dot m}{\rho \pi d^2} = \frac{4\,(12\text{ lb/s})}{(100\text{ lb/ft}^3)\,\pi\,(1/12)^2\text{ ft}^2} = 22\text{ ft/s}$$

Inserting values and noting that $p_1 - p_2 = -(p_2 - p_1)$

$$\dot W_{cv} = \left(12\,\frac{lb}{s}\right)\left[\frac{-40\text{ lbf/in.}^2}{100\text{ lb/ft}^3}\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right| - \frac{(22\text{ ft/s})^2}{2}\left|\frac{1\text{ lbf}}{32.2\text{ lb}\cdot\text{ft/s}^2}\right|\right]$$

$$= \left(-781.4\,\frac{\text{ft}\cdot\text{lbf}}{s}\right)\left|\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lbf/s}}\right| = -1.42\text{ hp}$$

pump size = 1.5 hp ⟵ hp rating

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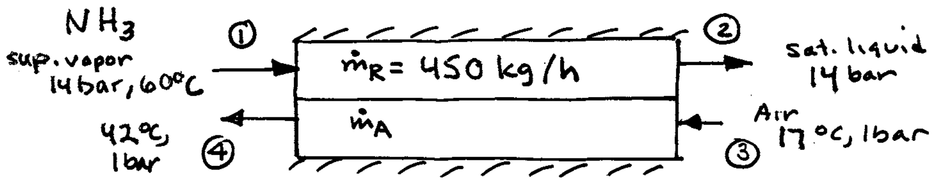
1. It is assumed that there is a small temperature difference between the pump and the surroundings, so heat transfer can be ignored.
2. Alternatively, Eq. 3.13 can be used to obtain the same result.

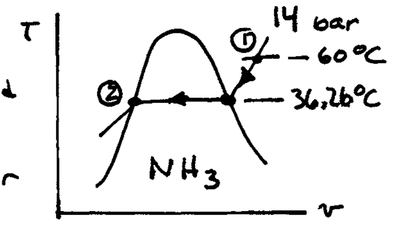
PROBLEM 4.57*

KNOWN: Ammonia and air pass in separate streams through a heat exchanger at steady state, for which data are provided.

FIND: Determine the mass flow rate of the air.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. A control volume enclosing the heat exchanger is at steady state. 2. For the control volume, $\dot W_{cv} = 0$, heat transfer can be ignored, and kinetic/potential energy effects are negligible.
①3. Air is modeled as an ideal gas.

ANALYSIS: Since the streams flow separately, the conservation of mass principle indicates at steady state: $\dot m_1 = \dot m_2 \equiv \dot m_R$ and $\dot m_3 = \dot m_4 \equiv \dot m_A$. An energy rate balance reads

$$0 = \cancel{\dot Q_{cv}} - \cancel{\dot W_{cv}} + \dot m_R\left[h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$
$$- \dot m_A\left[h_3 - h_4 + \cancel{\frac{V_3^2 - V_4^2}{2}} + \cancel{g(z_3 - z_4)}\right]$$

$\Rightarrow$

$$\dot m_A = \dot m_R\left[\frac{h_1 - h_2}{h_4 - h_3}\right]$$

From Table A-15, $h_1 = 1542.89$ kJ/kg. From Table A-14, $h_2 = 352.97$ kJ/kg. From Table A-22, $h_3 = 290.16$ kJ/kg, $h_4 = 315.27$ kJ/kg. Then

$$\dot m_A = (450)\frac{kg}{h}\left[\frac{1542.89 - 352.97}{315.27 - 290.16}\right]\left|\frac{1\ h}{60\ min}\right| = 355.4\ \frac{kg}{min} \longleftarrow \underline{\dot m_A}$$

---

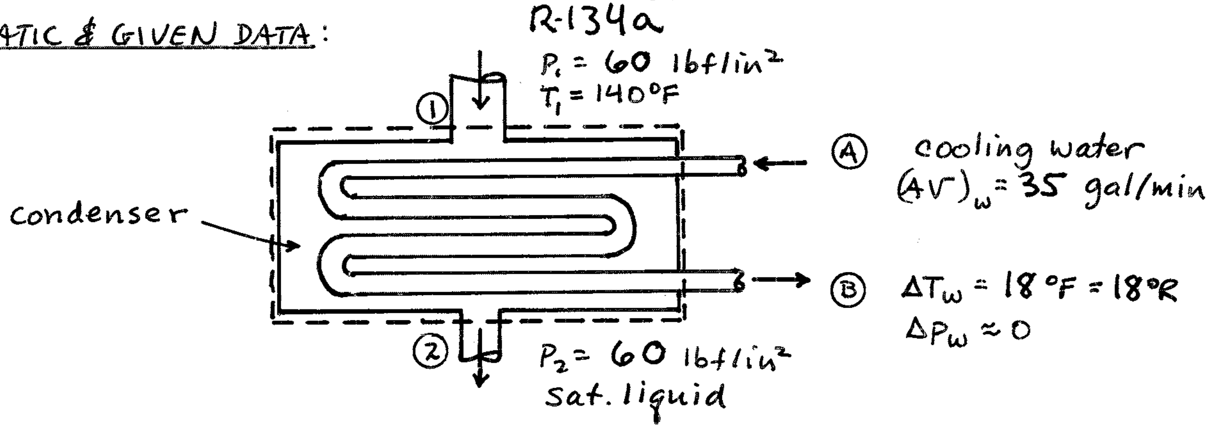
1. The validity of the ideal gas model is readily checked using the generalized compressibility chart.

PROBLEM 4.58*

KNOWN: Refrigerant 134a and cooling water pass in separate streams through a condenser (heat exchanger). The volumetric flow rate of cooling water and other data are given at the inlets and exits.

FIND: Determine (a) the mass flow rate of R-134a, and (b) the rate of energy transfer from the condensing R-134a to the cooling water.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer from the outside of the condenser is negligible. (3) Kinetic and potential energy changes from inlet to exit are negligible. (4) The cooling water is modeled as an incompressible liquid with constant specific heat.

ANALYSIS: (a) Since the R-134a and cooling water are separate streams

R-134a: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_R$

Water: $\dot{m}_A = \dot{m}_B \equiv \dot{m}_w$

The mass flow rate of R-134a is found from the steady-state energy balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1-h_2) + \cancelto{0}{\left(\frac{V_1^2-V_2^2}{2}\right)} + \cancelto{0}{g(z_1-z_2)}\right] + \dot{m}_w\left[(h_A-h_B) + \cancelto{0}{\left(\frac{V_A^2-V_B^2}{2}\right)} + \cancelto{0}{g(z_A-z_B)}\right]$$

Or

$$\dot{m}_R = \frac{\dot{m}_w(h_B-h_A)}{(h_1-h_2)}$$

For the water, using Eq. 3.20b

$$h_B - h_A = c\,(T_B - T_A) + v\,\cancelto{0}{(p_B - p_A)}$$

and

$$\dot{m}_w = \frac{(AV)_w}{v_w}$$

From Table A-19E; $c \approx 1$ Btu/lb·°R, and $v_w = 1/\rho = 0.0161$ ft³/lb.
For the R-134a, $h_1 = 129.53$ Btu/lb from Table A-12E and $h_2 = 27.24$ Btu/lb from Table A-11E. Inserting values

$$\dot{m}_R = \left(\frac{(35\,\text{gal/min})\left|\frac{0.13368\,\text{ft}^3}{1\,\text{gal}}\right|}{(0.0161\,\text{ft}^3/\text{lb})}\right)\frac{(1\,\text{Btu/lb}\cdot°\text{R})(18\,°\text{R})}{(129.53-27.24)\,\text{Btu/lb}}\left|\frac{60\,\text{min}}{1\,\text{h}}\right|$$

$= 3068.3$ lb/h ← $\dot{m}_R$

PROBLEM 4.58 (Cont'd)

(b) For a control volume enclosing only the R-134a

$$0 = \dot{Q}_R - \cancelto{0}{\dot{W}_R} + \dot{m}_R\left[(h_1-h_2) + \cancelto{0}{\left(\frac{V_1^2-V_2^2}{2}\right)} + g\cancelto{0}{(z_1-z_2)}\right]$$

where $\dot{Q}_R$ denotes the heat transfer rate for the R-134a only. Thus

$$\dot{Q}_R = \dot{m}_R(h_2-h_1)$$

$$= (3068.3 \text{ lb/h})(27.24-129.53)$$

① $$= -3.140\times10^5 \text{ Btu/h} \longleftarrow \dot{Q}_R$$

---

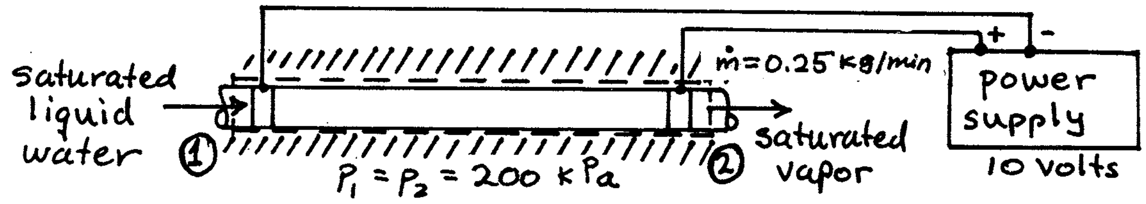
1. The negative value for $\dot{Q}_R$ denotes energy transfer heat from the R-134a to the cooling water, as expected.

PROBLEM 4.59

KNOWN: A steam boiler is constructed by passing a direct current through the stainless steel pipe through which the steam is flowing.

FIND: Determine the required size of the power supply and the expected current draw.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer can be neglected. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: To find the power required start with the steady-state energy balance

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. With assumption (3)

$$|\dot{W}_{cv}| = \dot{m}(h_2 - h_1)$$

From Table A-3 at 2 bar, $h_2 - h_1 = h_{fg} = 2201.9$ kJ/kg. Thus

$$|\dot{W}_{cv}| = \left(0.25\,\frac{\text{kg}}{\text{min}}\right)\left|\frac{1\text{ min}}{60\text{ s}}\right|\left(2201.9\,\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 9.175\text{ kW} \quad \longleftarrow$$

Then, with Eq. 2.21, the current is

$$i = \frac{|\dot{W}_{cv}|}{\mathcal{E}} = \left(\frac{9175\text{ W}}{10\text{ volts}}\right)\left(\frac{1\text{ Volt}}{1\text{ W/amp}}\right)$$

$$= 917.5\text{ amp} \quad \longleftarrow$$

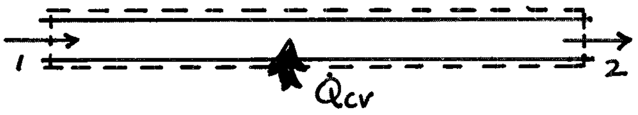
PROBLEM 4.60

KNOWN: Carbon dioxide is heated as it flows through a constant area pipe. Inlet and exit conditions are known.

FIND: Determine the rate of heat transfer.

SCHEMATIC & GIVEN DATA:

carbon dioxide
$p_1 = 2$ bar
$T_1 = 300$ K
$V_1 = 100$ m/s



$p_2 = 0.9413$ bar
$V_2 = 400$ m/s
$c_p = 0.94$ kJ/kg·K
$d = 2.5$ cm $= 0.025$ m

ASSUMPTIONS: (1) The control volume is at steady state. (2) $\dot{W}_{cv} = 0$. (3) The duct has constant area. (4) Potential energy change from inlet to exit can be neglected. (5) The carbon dioxide can be modeled as an ideal gas with constant specific heats.

ANALYSIS: To fix the exit state, begin with the steady-state mass balance

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$$

$$\frac{\cancel{A_1} V_1}{v_1} = \frac{\cancel{A_2} V_2}{v_2} \Rightarrow v_2 = \left(\frac{V_2}{V_1}\right) v_1$$

with the ideal gas equation of state

$$\frac{\cancel{R} T_2}{p_2} = \left(\frac{V_2}{V_1}\right)\frac{\cancel{R} T_1}{p_1} \Rightarrow T_2 = \left(\frac{V_2}{V_1}\right)\left(\frac{p_2}{p_1}\right) T_1 = \left(\frac{400}{100}\right)\left(\frac{0.9413}{2}\right)(300\text{K})$$
$$= 564.8\text{ K}$$

Evaluating the mass flow rate

$$\dot{m} = \frac{A V_1}{v_1} = \frac{\left(\frac{\pi d^2}{4}\right) V_1}{(R T_1 / p_1)}$$
$$= \frac{\left(\frac{\pi (.025^2\text{ m}^2)}{4}\right)(100\text{ m/s})(2\text{ bar})}{\left(\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg·K}}\right)(300\text{K})}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N·m}}\right|$$
$$= 0.1732\text{ kg/s}$$

Using the steady-state energy balance to find the heat transfer rate

$$0 = \dot{Q}_{cv} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(\cancelto{0}{z_1 - z_2})\right]$$

with $h_1 - h_2 = c_p(T_1 - T_2)$

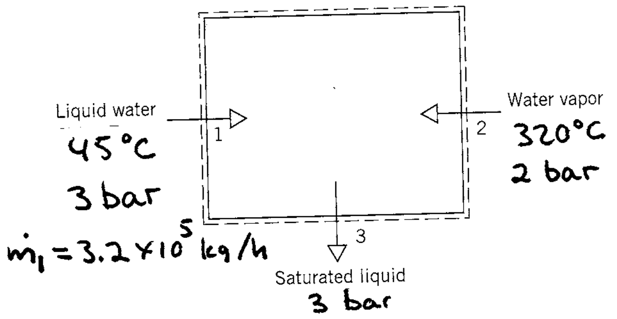
$$\dot{Q}_{cv} = \dot{m}\left[c_p(T_2 - T_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)\right]$$
$$= (0.1732\tfrac{\text{kg}}{\text{s}})\left[(0.94\tfrac{\text{kJ}}{\text{kg·K}})(564.8 - 300)\text{K} + \left(\frac{400^2 - 100^2}{2}\right)\frac{\text{m}^2}{\text{s}^2}\left|\frac{1\text{ N}}{1\text{ kg·m/s}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N·m}}\right|\right]\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$
$$= 56.1\text{ kW} \xleftarrow{\qquad\qquad\qquad\qquad} \dot{Q}_{cv}$$

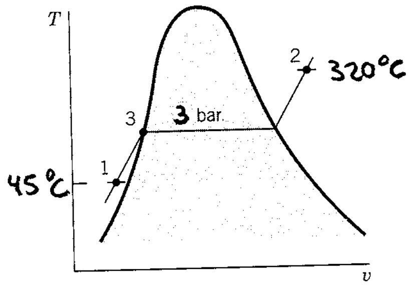
PROBLEM 4.61*

KNOWN: Data are provided for a feedwater heater at steady state.

FIND: Determine the mass flow rate of the incoming vapor stream, $\dot{m}_2$.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$, and heat transfer with the surroundings can be ignored. Kinetic and potential energy effects also can be neglected.

ANALYSIS: At steady state the mass rate balance reads

$$0 = \dot{m}_1 + \dot{m}_2 - \dot{m}_3 \Rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

The energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left(h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{g z_1}\right) + \dot{m}_2\left(h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{g z_2}\right) - \dot{m}_3\left(h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{g z_3}\right)$$

$$\Rightarrow 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

Introducing $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3 \Rightarrow \dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3}\right]$$

From Table A-4, $h_2 = 3110.1$ kJ/kg. From Table A-3, $h_3 = 561.47$ kJ/kg. With Eq. 3.14 and data from Table A-2

① $$h_1 \approx h_f(45°C) = 188.45 \text{ kJ/kg}$$

Thus $$\dot{m}_2 = \left(3.2 \times 10^5 \frac{\text{kg}}{\text{h}}\right)\left[\frac{561.47 - 188.45}{3110.1 - 561.47}\right] = 0.468 \times 10^5 \frac{\text{kg}}{\text{h}} \longleftarrow \dot{m}_2$$

---

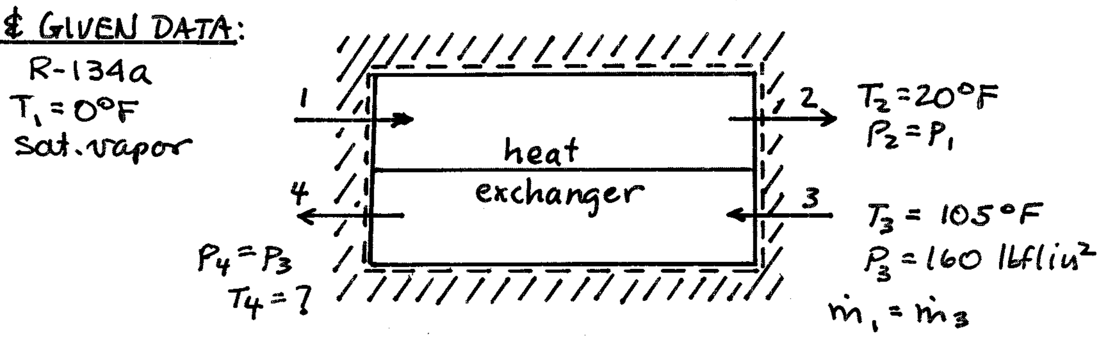
1. Using Eq. 3.13, $h_1 \approx 188.74$ kJ/kg and $\dot{m}_2 = 0.468 \times 10^5$ kg/h

PROBLEM 4.62

KNOWN: Separate vapor and liquid streams of Refrigerant 134a pass in counter flow trough a well-insulated heat exchanger. Data are known at the inlets and exits.

FIND: Determine the exit temperature of the liquid stream.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer between the heat exchanger and the surroundings can be neglected and $W_{cv} = 0$. (3) Kinetic and potential energy changes from inlet to exit are negligible.

ANALYSIS: To fix state 4, begin with steady state mass and energy balances to determine h_4

$$\dot{m}_1 = \dot{m}_2$$
$$\dot{m}_3 = \dot{m}_4$$

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_1\left[(h_1-h_2) + \cancelto{0}{\left(\frac{V_1^2-V_2^2}{2}\right)} + g\cancelto{0}{(z_1-z_2)}\right] + \dot{m}_3\left[(h_3-h_4) + \cancelto{0}{\left(\frac{V_3^2-V_4^2}{2}\right)} + g\cancelto{0}{(z_3-z_4)}\right]$$

with $\dot{m}_1 = \dot{m}_3$

$$h_4 = (h_1 - h_2) + h_3$$

From Table A-10E ; $h_1 = 101.75$ Btu/lb and $p_1 = 21.203$ lbf/in². Interpolating in Table A-12E ; $h_2 = 105.76$ Btu/lb.

States 3 and 4 are both subcooled liquid states. The following approximations are reasonable

$$h_3 \approx h_{f@T_3}$$
$$h_4 \approx h_{f@T_4}$$

with $h_3 = 46.01$ Btu/lb from Table A-10E

$$h_4 = (101.75 - 105.76) + 46.01 = 42 \text{ Btu/lb}$$

Interpolating in Table A-10E

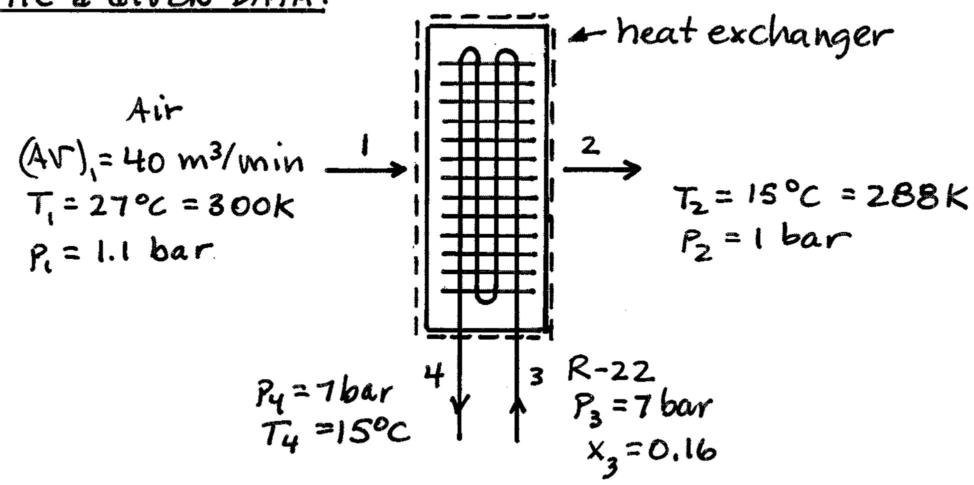
$$T_4 \approx 93.7\ °F \longleftarrow T_4$$

PROBLEM 4.63

KNOWN: Air and Refrigerant 22 pass in separate streams through a heat exchanger. Data are known at the inlet and exit of each stream.

FIND: Determine (a) the mass flow rate of refrigerant and (b) the rate of energy transfer from the air to the refrigerant.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer from the outside of the heat exchanger is negligible, and $\dot{W}_{cv} = 0$. (3) Kinetic and potential effects can be neglected. (4) The air behaves as an ideal gas, as can be verified by reference to the compressibility chart.

ANALYSIS: (a) The mass flow rate of refrigerant is determined using steady-state mass and energy balances. First, since the air and refrigerant flow as separate streams

$$\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{air}$$

$$\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_{R-22}$$

Thus, the energy rate balance reduces as follows

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}_{air}\left[(h_1-h_2) + \overset{0}{\cancel{\left(\frac{V_1^2-V_2^2}{2}\right)}} + g\overset{0}{\cancel{(z_1-z_2)}}\right] + \dot{m}_{R-22}\left[(h_3-h_4) + \overset{0}{\cancel{\left(\frac{V_3^2-V_4^2}{2}\right)}} + g\overset{0}{\cancel{(z_3-z_4)}}\right]$$

and

$$m_{R-22} = \dot{m}_{air}\left(\frac{h_1-h_2}{h_4-h_3}\right)$$

The mass flow rate of air is found using data at the inlet and the ideal gas equation of state

$$\dot{m}_{air} = \frac{(AV)_1}{v_1} = \frac{P_1(AV)_1}{RT_1}$$

$$= \frac{(1.1\text{ bars})(40\ m^3/min)}{\left(\frac{8.314}{28.97}\cdot\frac{kJ}{kg\cdot K}\right)(300\ K)}\left|\frac{10^5\ N/m^2}{1\text{ bar}}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 51.11\ kg/min$$

From Table A-22; $h_1 = 300.19$ kJ/kg and $h_2 = 288.15$ kJ/kg. Further, using data from Table A-8

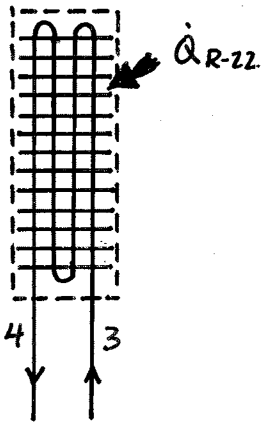
$$h_3 = h_{f3} + x_3 h_{fg3} = 58.04 + (.16)(195.60) = 89.34 \text{ kJ/kg}$$

And, from Table A-9; $h_4 = 256.86$ kJ/kg. Thus

$$\dot{m}_{R-22} = (51.11 \text{ kg/min})\left(\frac{300.19 - 288.15}{256.86 - 89.34}\right)$$

$$= 3.673 \text{ kg/min} \longleftarrow \underline{\dot{m}_{R-22}}$$

(b) Consider a control volume enclosing only the refrigerant stream



$$0 = \dot{Q}_{R-22} + \dot{m}_{R-22}(h_3 - h_4)$$

$$\dot{Q}_{R-22} = \dot{m}_{R-22}(h_4 - h_3)$$

$$= (3.673 \text{ kg/min})(256.86 - 89.34) \text{ kJ/kg}$$

$$= 615.3 \text{ kJ/min} \longleftarrow \underline{\dot{Q}_{R-22}}$$

COMMENT: For a control volume enclosing only the air stream

$$\dot{Q}_{air} = \dot{m}_{air}(h_2 - h_1)$$

$$= (51.11 \text{ kg/min})(288.15 - 300.19) \text{ kJ/kg}$$

$$= -615.3 \text{ kJ/min}$$

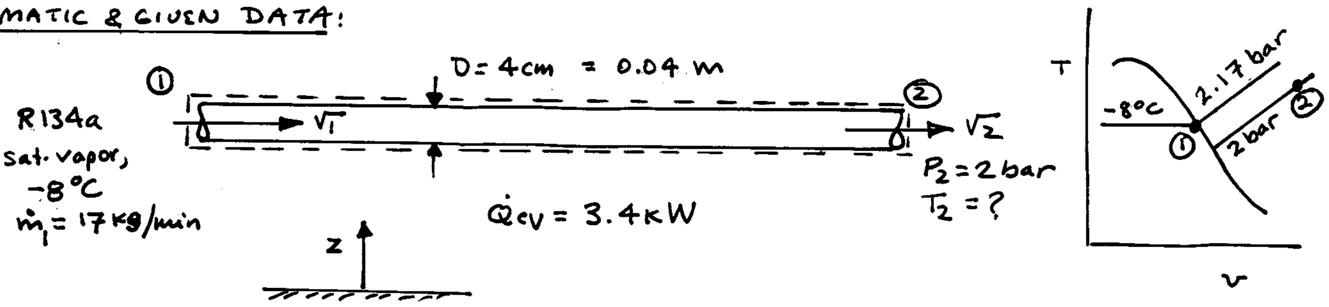
Thus, $\dot{Q}_{R-22} = -\dot{Q}_{air}$, as expected.

PROBLEM 4.64

KNOWN: Refrigerant 134a flows through a horizontal pipe at steady state, for which operating data are provided.

FIND: Determine the exit temperature and velocity, and the inlet velocity.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$ and there is no change in potential energy from inlet to exit.

ANALYSIS: At steady state, the mass rate balance reduces to $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. The inlet velocity is found as follows:

$$\dot{m} = \frac{A_1 V_1}{v_1} \Rightarrow V_1 = \frac{\dot{m} v_1}{A_1} = \frac{4 \dot{m} v_1}{\pi d^2}$$

with v_1 from Table A-10

$$V_1 = \frac{4(17 \text{ kg/min}) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| (0.0919 \text{ m}^3/\text{kg})}{\pi (0.04 \text{ m})^2} = 20.72 \text{ m/s} \longleftarrow V_1$$

Similarly, V_2 is found from $\dot{m}_1 = \dot{m}_2$ as follows:

$$\dot{m}_1 = \dot{m}_2 \Rightarrow \frac{\cancel{A_1} V_1}{v_1} = \frac{A_2 V_2}{v_2} \Rightarrow V_2 = \frac{v_2}{v_1} V_1 \qquad (*)$$

In this expression, v_1 and V_1 are known. However, $v_2 = v(T_2, P_2)$ is unknown.

Another relation is obtained using the energy rate balance

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{\,0}\right]$$

$$= \dot{Q}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right)\right]$$

Inserting known values, including h_1 from Table A-10

$$0 = (3.4 \text{ kW}) \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| + \left(17 \frac{\text{kg}}{\text{min}}\right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left[(242.54 - h_2) \frac{\text{kJ}}{\text{kg}} + \left(\frac{(20.72 \text{ m/s})^2 - V_2^2}{2} \right) \left| \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right| \right] \qquad (**)$$

where $h_2 = h(T_2, P_2)$.

Equations (*) and (**) can be solved simultaneously by referring to Table A-12 for v_2 and h_2 as functions of T_2 and P_2. The process is iterative. To avoid iteration, IT can be used effectively, as follows:

PROBLEM 4.64 (Contd.)

IT Code

```
// Data
T1 = -8  // °C
x1 = 1
p2  = 2  // bar
d = 4  // cm
mdot = 17  // kg/min
Qdot = 3.4  // kW

// Determine mdot
p1 = Psat_T("R134A", T1)
v1 = vsat_Px("R134A", p1, x1)
h1 = hsat_Px("R134A", p1, x1)
mdot  = ((pi * (d / 100)^2  / 4) * V1 / v1) * 60

// Find exit state
0 = Qdot + (mdot / 60) * ((h1 - h2) + (V1^2 - V2^2) / (2 * 1000))
V2 / v2 = V1 / v1
h2 = h_PT("R134A", p2, T2)
v2 = v_PT("R134A", p2, T2)
```

IT Results

① $T_2 = 4.976°C \approx 5°C \longleftarrow T_2$

$V_2 = 24.08$ m/s $\longleftarrow V_2$

$V_1 = 20.71$ m/s

---

1. These values can be verified using Eq.s (*) and (**) with data from Table A-12. The results compare very favorably.

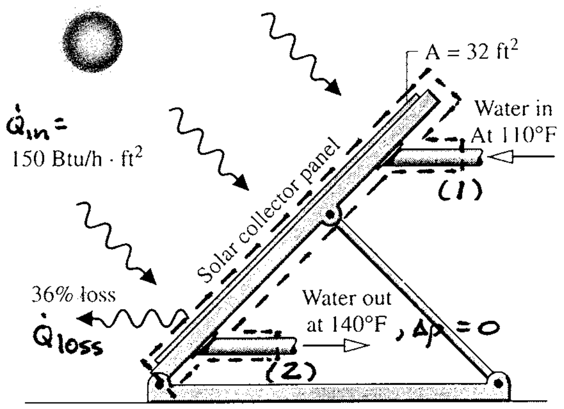
PROBLEM 4.65

KNOWN: Water is heated as it passes through a solar collector.

FIND: Determine the mass flow rate of water and the number of gallons of heated water that 8 collectors can provide in 30 min.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) The control volume is at steady state. (2) For the control volume, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects are negligible. (3) The water behaves as an incompressible substance with $c = 0.999$ Btu/lb·°R.



ANALYSIS: The mass flow rate is determined using the steady-state energy balance, as follows

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. With $\dot{Q}_{cv} = \dot{Q}_{in} - \dot{Q}_{loss}$ and assumption (3)

$$0 = \dot{Q}_{in} - \dot{Q}_{loss} + \dot{m}(h_1 - h_2)$$

① Using Eq. 3.20b, $h_1 - h_2 = c(T_1 - T_2) + v\,\cancel{(p_1 - p_2)}^{\,0}$. Thus

$$\dot{m} = \frac{\dot{Q}_{in} - \dot{Q}_{loss}}{c(T_2 - T_1)}$$

From the given data, $\dot{Q}_{in} = (150 \text{ Btu/h}\cdot\text{ft}^2)(32 \text{ ft}^2) = 4800$ Btu/h and $\dot{Q}_{loss} = (.36)\,\dot{Q}_{in} = 1728$ Btu/h. Inserting values

$$\dot{m} = \frac{(4800 - 1728)\text{ Btu/h}}{(1 \text{ Btu/lb}\cdot{}^\circ\text{R})(30\,{}^\circ\text{R})}\left|\frac{1\text{ h}}{60\text{ min}}\right| = 1.71 \text{ lb/min} \longleftarrow \dot{m}$$

In 30 min., one collector can provide the following amount of 140°F water:

$$m = \int_{t_1}^{t_2} \dot{m}\,dt = \dot{m}\,\Delta t = \left(1.71\,\frac{\text{lb}}{\text{min}}\right)(30\text{ min}) = 51.3 \text{ lb/collector}$$

Thus, the volume is

$$V = m v_2 = (51.3 \text{ lb/collector})\left(0.01629\,\frac{\text{ft}^3}{\text{lb}}\right)\left|\frac{1\text{ gal}}{0.13368\text{ ft}^3}\right| = 6.25\,\frac{\text{gal}}{\text{collector}}$$

where $v_2 \approx v_{f@\,140°F}$ from Table A-2E. Finally, for eight collectors

$$V_{tot} = (8\text{ collectors})\left(6.25\,\frac{\text{gal}}{\text{collector}}\right) = 50\text{ gal} \longleftarrow V_{tot}$$

---

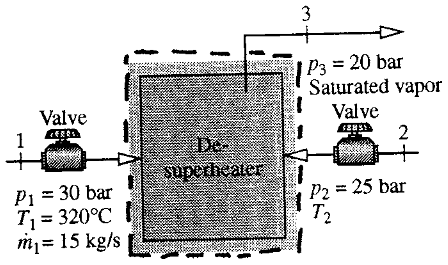
1. Alternatively, Eq. 3.13 could be invoked to obtain the same result.

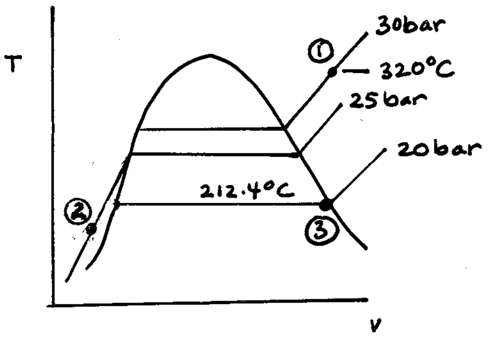
PROBLEM 4.66

KNOWN: Data are provided for a desuperheater operating at steady state.

FIND: (a) For a specified temperature for the entering liquid, determine the liquid mass flow rate. (b) Plot the liquid mass flow rate versus the liquid temperature.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. A control volume enclosing the desuperheater with inlets at 1 and 2 and an exit at 3 is at steady state. 2. For the control volume, $\dot{W}_{cv} = 0$ and heat transfer with the surroundings can be ignored. Kinetic and potential energy effects can be ignored.

ANALYSIS: The mass rate balance at steady state reads $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$.

The energy rate balance at steady state reduces as follows:

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_1\left[h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right] + \dot{m}_2\left[h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right] - \dot{m}_3\left[h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right]$$

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

Solving for $\dot{m}_2$

$$\dot{m}_2 = \dot{m}_1 \left[\frac{h_3 - h_1}{h_2 - h_3}\right] \qquad (1)$$

(a) From Table A-4; h_1 = 3043.4 kJ/kg. From Table A-3; h_3 = 2799.5 kJ/kg. Further, at T_2 = 200 °C, p_2 = 25 bar; Table A-5 gives h_2 = 852.8 kJ/kg. Thus

$$\dot{m}_2 = \left(15\ \frac{\text{kg}}{\text{s}}\right)\left[\frac{2799.5 - 3043.4}{852.8 - 2799.5}\right] = 1.88\ \text{kg/s} \longleftarrow \frac{\dot{m}_2}{\text{(part a)}}$$

(b) The following IT code is used to develop data to construct a plot of $\dot{m}_2$ vs. T_2:

IT Code

```
p1 = 30  // bar
T1 = 320  // °C
mdot1 = 15  // kg/s
p2 = 25  // bar
T2 = 20  // °C
p3 = 20  // bar

h1 = h_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
h3 = hsat_Px("Water/Steam", p3, 1)

mdot2 = mdot1 * ((h3 - h1) / (h2 - h3))
```

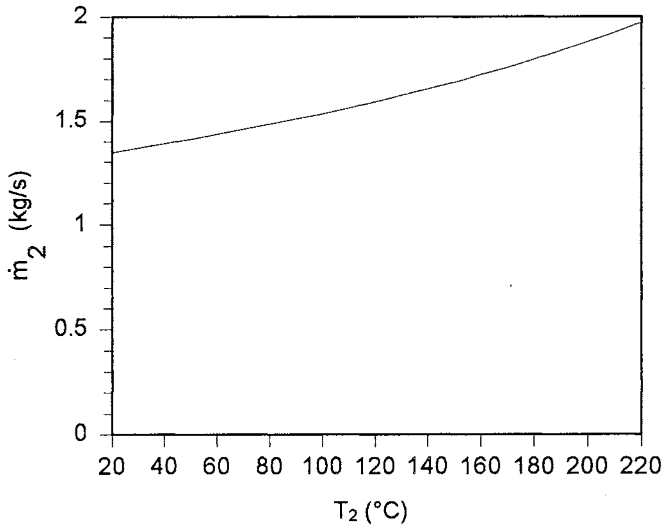
①

PROBLEM 4.66 (Cont'd.)

IT Result for T_2 = 200°C

h_1 = 3043 kJ/kg
h_2 = 852.4 kJ/kg
h_3 = 2799 kJ/kg
mdot2 = 1.879 kg/s

This result compares very favorably with the result of part (a). Thus, with the computer solution validated, use the Explore button and sweep T_2 from 20 to 220°C in steps of 10. The resulting data are used to construct the following plot:



---

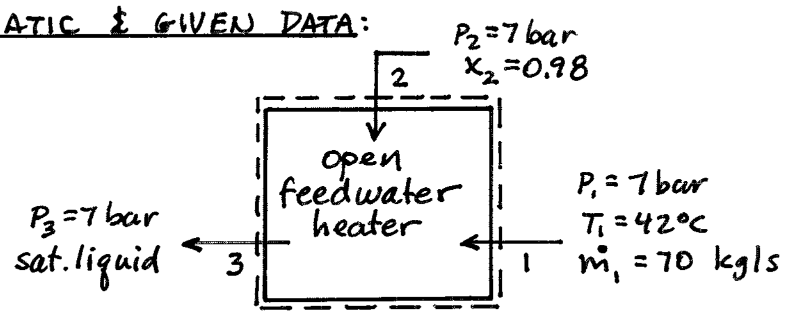
1. Note that IT uses the approximation of Eq. 3.14 for liquid enthalpies.

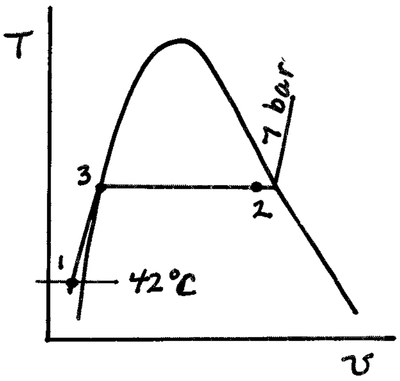
PROBLEM 4.67

KNOWN: An open feedwater heater operates with known inlet and exit conditions. The mass flow rate at one inlet is given.

FIND: Determine the mass flow rate at the second inlet.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer with the surroundings is negligible, and $\dot{W}_{cv} = 0$. (3) Kinetic and potential energy effects are negligible.

ANALYSIS: To find $\dot{m}_2$, begin with the steady-state mass and energy balances

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - \dot{m}_3\left(h_3 + \frac{V_3^2}{2} + gz_3\right)$$

With $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$ and assumption (3)

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

or

$$\dot{m}_2 = \dot{m}_1\left(\frac{h_1 - h_3}{h_3 - h_2}\right)$$

From Table A-3, $T_1 < T_{sat}$ at 7 bar. Hence, state 1 is compressed liquid. Using Eq. 3.14 and data from Table A-2 at 42°C

$$h_1 \approx h_f(T) + v_f(T)\left[p - p_{sat}(T)\right]$$
$$= 175.9 \text{ kJ/kg} + \left(1.0086 \times 10^{-3} \frac{m^3}{kg}\right)\left[7 - 0.08268\right] \text{bar} \left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N·m}}\right|$$
$$= 176.6 \text{ kJ/kg}$$

Further, from Table A-3

$$h_2 = h_{f2} + x_2 h_{fg2} = 697.22 + (.98)(2066.3) = 2722.2 \text{ kJ/kg}$$
$$h_3 = 697.22 \text{ kJ/kg}$$

Finally

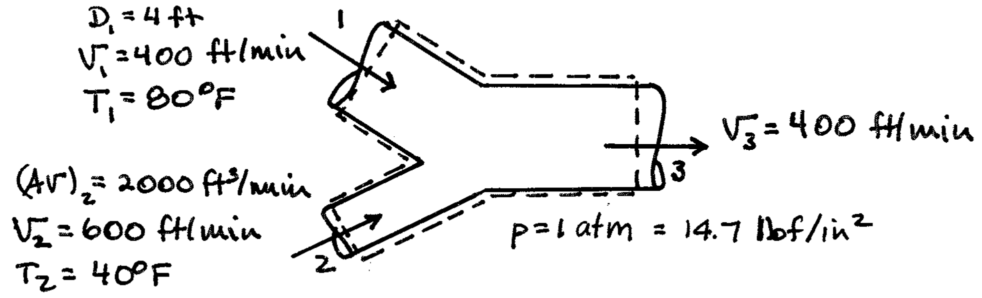
$$\dot{m}_2 = (70 \text{ kg/s})\left(\frac{176.6 - 697.22}{697.22 - 2722.2}\right)$$
$$= 18.0 \text{ kg/s} \longleftarrow \underline{\qquad\qquad\qquad\qquad} \dot{m}_2$$

PROBLEM 4.68

KNOWN: Two ducts carrying air in a ventilation system merge into one exit duct. Data are known at the inlets and exit.

FIND: Determine the exit temperature and the diameter of the exit duct.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer with the surroundings is negligible, and $\dot{W}_{cv} = 0$. (3) Potential energy effects can be neglected. (4) The air behaves as an ideal gas with constant specific heats.

ANALYSIS: To find T_3, begin with steady-state mass and energy balances

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_1 \left(h_1 + \frac{V_1^2}{2} + g z_1\right) + \dot{m}_2 \left(h_2 + \frac{V_2^2}{2} + g z_2\right) - \dot{m}_3 \left(h_3 + \frac{V_3^2}{2} + g z_3\right)$$

and $\dot{m}_1 + \dot{m}_3 - \dot{m}_3 = 0 \Rightarrow \dot{m}_3 = \dot{m}_1 + \dot{m}_2$

Combining and incorporating assumption (3)

$$0 = \dot{m}_1 \left[(h_1 - h_3) + \cancelto{0}{\left(\frac{V_1^2 - V_3^2}{2}\right)}\right] + \dot{m}_2 \left[(h_2 - h_3) + \left(\frac{V_2^2 - V_3^2}{2}\right)\right]$$

Referring to Table A-20E, $c_p = 0.24$ Btu/lb·°R for the temperature range in this problem. Using $\Delta h = c_p \Delta T$

$$0 = \dot{m}_1 \left[c_p (T_1 - T_3)\right] + \dot{m}_2 \left[c_p (T_2 - T_3) + \left(\frac{V_2^2 - V_3^2}{2}\right)\right] \qquad (*)$$

The mass flow rates are evaluated using Eq. 4-4b and the ideal gas equation of state

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{\left(\frac{\pi D_1^2}{4}\right) V_1 \, p_1}{R T_1} = \frac{\left(\frac{\pi (4^2) \text{ft}^2}{4}\right)(400 \text{ ft/min})(14.7 \text{ lbf/in}^2)}{\left(\frac{1545}{28.97} \frac{\text{ft·lbf}}{\text{lb·°R}}\right)(540°\text{R})} \left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|$$

$$= 369.5 \text{ lb/min}$$

$$\dot{m}_2 = \frac{(AV)_2 \, p_2}{R T_2} = \frac{(2000)(14.7)|144|}{\left(\frac{1545}{28.97}\right)(500)} = 158.8 \text{ lb/min}$$

Returning to (*)

$$T_3 = \frac{\dot{m}_1 c_p T_1 + \dot{m}_2 \left[c_p T_2 + \left(\frac{V_2^2 - V_3^2}{2}\right)\right]}{(\dot{m}_1 + \dot{m}_2) c_p}$$

PROBLEM 4.68 (Cont'd)

Evaluating the kinetic energy term

$$\frac{V_2^2 - V_3^2}{2} = \left(\frac{600^2 - 400^2}{2}\right)\frac{ft^2}{min^2}\left|\frac{1\ min^2}{3600\ s^2}\right|\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|$$

$$= 0.00111\ Btu/lb$$

Inserting values

① $$T_3 = \frac{(369.5\ lb/min)(.24\ \frac{Btu}{lb\cdot °R})(540°R) + (158.8)\left[(.24)(500) + (0.00111)\right]}{(369.5 + 158.8)(.24)}$$

$$= 528\ °R = 68\ °F \longleftarrow T_3$$

To get D_3, note that

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 528.3\ lb/min$$

Thus

$$A_3 = \frac{v_3 \dot{m}_3}{V_3} = \frac{R T_3 \dot{m}_3}{p_3 V_3} = \frac{\left(\frac{1545}{28.97}\right)(528)(528.3)}{(14.7)|144|(400)} = 17.57\ ft^2$$

and

$$D_3 = \sqrt{\frac{4 A_3}{\pi}} = 4.73\ ft \longleftarrow D_3$$

---

1. Note that if kinetic energy is neglected there is virtually no effect on the temperature at the exit.

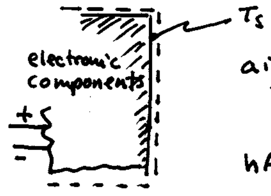
PROBLEM 4.69

KNOWN: The electronic components of Example 4.8 are cooled by air flowing through the electronics enclosure.

FIND: Determine the largest average surface temperature of the components for which specified limits are met.

SCHEMATIC & GIVEN DATA:

Also See Fig. E4.8



air temperature:

$$T_a = \frac{T_1 + T_2}{2}$$

$$hA = 5 \text{ W/K}$$

ASSUMPTIONS: 1. The electronic components form the system, which is at steady state.

ANALYSIS: The energy transfer from the electronic components to the air by convection is

$$\dot{Q}_c = hA[T_s - T_a]$$

① At steady state, the electric power provided to the electric components equals the energy removed by heat transfer: $\dot{Q}_c = 80$ W. Also, $hA = 5$ W/K. Solving for T_s, noting that $T_1 = 293$ K and $T_2 \leq 305$ K (22°C)

$$T_s = T_a + \frac{\dot{Q}_c}{hA}$$

$$T_s = \left(\frac{T_1 + T_2}{2}\right) + \frac{\dot{Q}_c}{hA}$$

② $$T_s \leq \left(\frac{293 + 305}{2}\right)\text{K} + \frac{80 \text{ W}}{5 \text{ W/K}} = 315 \text{ K} (42°\text{C}) \longleftarrow$$

---

1. This can be obtained formally by applying an energy rate balance to a system consisting of the electronic components.

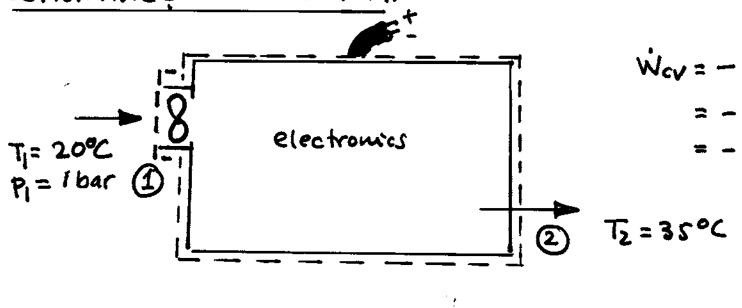
2. For reliability, excessive temperatures within the electronics are avoided by controlling the surface temperature T_s.

PROBLEM 4.70

KNOWN: Data are provided for an electronics enclosure cooled by an air flow induced by a fan.

FIND: Determine the volumetric flow rate of the air entering the fan.

SCHEMATIC & GIVEN DATA:



$$\dot{W}_{cv} = -[25W + 100W]$$
$$= -125W$$
$$= -0.125\,kW$$

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, heat transfer with the surroundings and kinetic/potential energy effects can be ignored. 3. Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. The energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

or

$$\dot{m} = \frac{(-\dot{W}_{cv})}{h_2 - h_1}$$

with $\dot{m} = (AV)_1/v_1$ and using the ideal gas equation of state

$$\dot{m} = \frac{(AV)_1 p_1}{RT_1}$$

Collecting results and inserting data, including enthalpy values from Table A-22

$$(AV)_1 = \left(\frac{RT_1}{p_1}\right)\left[\frac{(-\dot{W}_{cv})}{h_2 - h_1}\right]$$

$$= \frac{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(293\,K)}{(10^5\,N/m^2)}\left[\frac{0.125\ kJ/s}{(308.2 - 293.2)\frac{kJ}{kg}}\right]$$

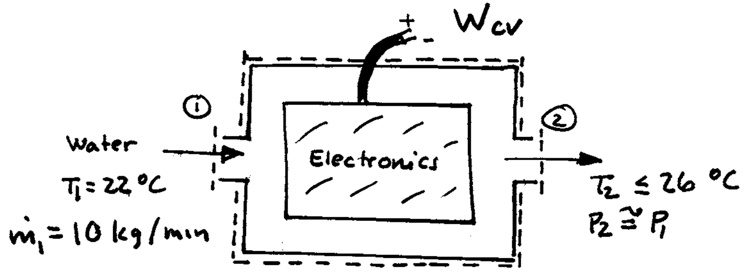
$$= 7 \times 10^{-3}\ \frac{m^3}{s} \longleftarrow$$

PROBLEM 4.71*

KNOWN: Data are provided for a water-jacketed housing filled with electronic components, which is at steady state.

FIND: Determine the maximum power input to satisfy a limit on the temperature of the water exiting the enclosure.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, heat transfer with the surroundings can be ignored, as can kinetic/potential energy effects. 3. For the water entering and exiting the housing; $h \approx h_f(T)$. ①

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. An energy rate balance reads

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancel{g(z_1 - z_2)}\right]$$

giving

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2)$$

$$= \dot{m}\left(h_f(T_1) - h_f(T_2)\right)$$

Since $T_2 \le 26°C$

$$\dot{W}_{cv} \ge \dot{m}\left(h_f(22°C) - h_f(26°C)\right)$$

$$\ge \left(10\ \frac{kg}{min}\right)(92.33 - 109.07)\frac{kJ}{kg}\left|\frac{1\ min}{60\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|$$

② $$\dot{W}_{cv} \ge -2.79\ kW \longleftarrow$$

---

1. Alternatively, Eq. 3.20b with c from Table A-19 can be used.

2. By the usual sign convention, $\dot{W}_{cv}$ represents power output. Hence, the constraint on power input can be expressed alternatively in terms of the magnitude of $\dot{W}_{cv}$ as

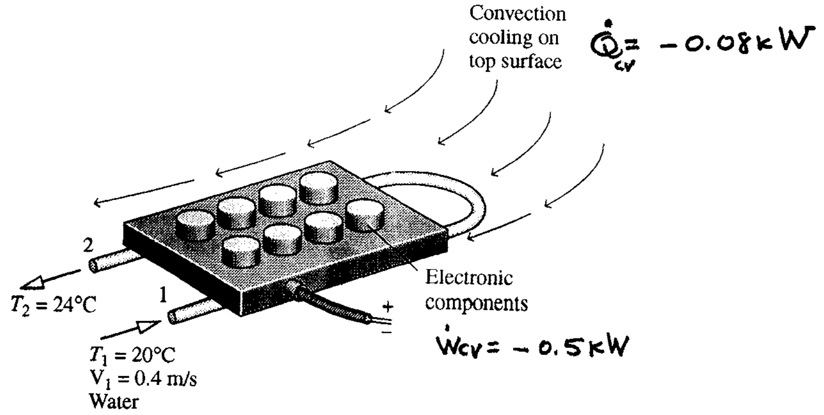
$$0 \le |\dot{W}_{cv}| \le 2.79\ kW$$

PROBLEM 4.72

KNOWN: Data are provided for electronic components mounted on a plate that are cooled by convection to the surroundings and water circulating through a tube bonded to the plate. Operation is at steady state.

FIND: Determine the tube diameter.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. A control volume encloses the plate-mounted electronic components with an inlet at 1 and an exit at 2. 2. The control volume ① is at steady state. 3. For the water entering and exiting, $h \approx h_f(T)$, $v \approx v_f(T)$. 4. Kinetic and potential energy effects can be ignored.

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Also,

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(\pi D^2/4) V_1}{v_f(T_1)}$$

An energy rate balance reads

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g\cancel{(z_1 - z_2)}\right]$$

or

$$\dot{m} = \frac{[\dot{Q}_{cv} - \dot{W}_{cv}]}{h_2 - h_1} = \frac{[\dot{Q}_{cv} - \dot{W}_{cv}]}{h_f(T_2) - h_f(T_1)}$$

Collecting results

$$D = \sqrt{\frac{4\, v_f(T_1)}{\pi V_1}\left[\frac{(\dot{Q}_{cv} - \dot{W}_{cv})}{h_f(T_2) - h_f(T_1)}\right]}$$

with data from Table A-2: $v_f(20°C) = (1.0018/10^3)\ \text{m}^3/\text{kg}$, $h_f(T_1) = 83.96$ kJ/kg, $h_f(T_2) = 100.7$ kJ/kg

$$D = \sqrt{\frac{(4)(1.0018/10^3)\ \text{m}^3/\text{kg}}{\pi\ (0.4\ \text{m/s})}\left[\frac{(-0.08 - (-0.5))\ \text{kJ/s}}{(100.7 - 83.96)\ \text{kJ/kg}}\right]}$$

$$= 0.0089\ \text{m}\ \left|\frac{10^2\ \text{cm}}{\text{m}}\right|$$

$$= 0.89\ \text{cm} \longleftarrow$$

---

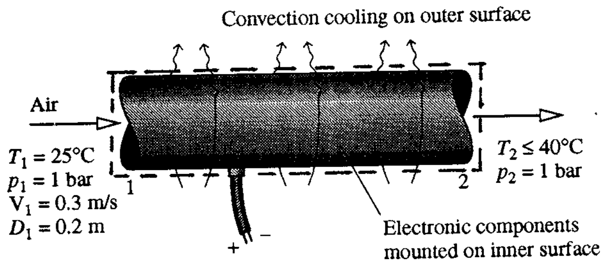
1. Alternatively, the incompressible model can be used, with Eq. 3.20b and c from Table A-19.

PROBLEM 4.73

KNOWN: Data are provided for electronic components mounted on the inner surface of a horizontal duct. The components are cooled by air flowing through the cylinder and by convection from the outer surface.

FIND: Determine the minimum heat transfer from the outer surface for which a limit on the exiting air temperature is satisfied.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cv} = -0.2\,kW$

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. The air is modeled as an ideal gas. 3. Kinetic and potential energy effects can be ignored.

ANALYSIS: At steady state, $\dot{m}_1 = \dot{m}_2 = \dot{m}$, where

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{(\pi D_1^2/4)V_1}{RT_1/P_1}$$

$$= \frac{\pi\,(0.2\,m)^2\,(0.3\,m/s)\,(10^5\,N/m^2)}{4\left(\frac{8314}{28.97}\,\frac{N\cdot m}{kg\cdot K}\right)(298\,K)} = 0.011\,\frac{kg}{s}$$

An energy rate balance is

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

or

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}\,(h_2 - h_1)$$
$$= \dot{W}_{cv} + \dot{m}\,(h(T_2) - h(T_1))$$

Since $T_2 \le 313\,K\,(40°C)$

$$\dot{Q}_{cv} \le (-0.2\,kW) + \left(0.011\,\frac{kg}{s}\right)\left[h(313\,K) - h(298\,K)\right]\left(\frac{kJ}{kg}\right)\left|\frac{1\,kW}{1\,kJ/s}\right|$$
$$\le (-0.2\,kW) + \left(0.011\,\frac{kg}{s}\right)\left[(313.3 - 298.2)\,\frac{kJ}{kg}\right]\left|\frac{1\,kW}{1\,kJ/s}\right|$$
$$\le -0.034\,kW \longleftarrow$$

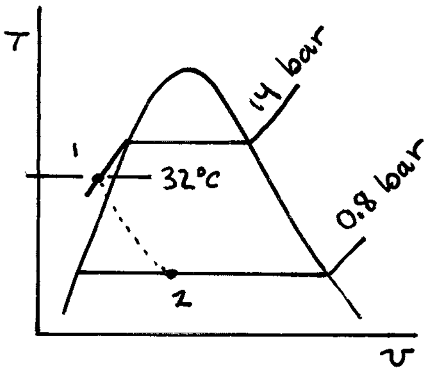
The energy removed by convection must be at least 0.034 kW.

PROBLEM 4.74*

KNOWN: Ammonia expands through a valve from a known pressure and temperature to a given final pressure.

FIND: Determine the exit quality.

SCHEMATIC & GIVEN DATA:



$p_1 = 1.4$ MPa
$= 14$ bar
$T_1 = 32$ °C

$p_2 = 0.08$ MPa
$= 0.8$ bar
$x_2 = ?$

ASSUMPTIONS: (1) A control volume enclosing the valve is at steady state. (2) The refrigerant undergoes a throttling process; $h_1 = h_2$.

ANALYSIS: According to data from Table A-14, at $p_1 = 14$ bar; $T_{sat} = 36.26$ °C. Since $T_1 < T_{sat}$, state 1 is in the compressed liquid region. For simplicity, ① we use Eq. 3.14 to evaluate h_1, as follows:

$$h_1 \approx h_f(T_1) = 332.17 \text{ kJ/kg}$$

where the value is obtained from Table A-13.

By assumption (2)

$$h_1 = h_2$$
$$= h_{f_2} + x_2 h_{fg2}$$

Solving and inserting data from Table A-14 at $p_2 = 0.8$ bar

$$x_2 = \frac{h_1 - h_{f2}}{h_{fg2}}$$
$$= \frac{332.17 \text{ kJ/kg} - 9.04 \text{ kJ/kg}}{1382.73 \text{ kJ/kg}}$$
$$= 0.2337 \ (23.37\%) \qquad x_2$$

---

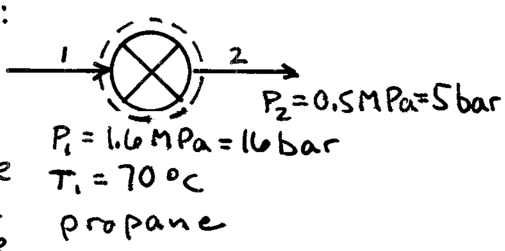
1. Here, we have ignored the effect of pressure on the specific enthalpy of liquid refrigerant. We could have used Eq. 3.13 to estimate the effect of pressure. In that case, h_1 would have been 332.44 kJ/kg, and the exit quality would have been $x_2 = 0.2339$ (23.39%). Thus, using Eq. 3.14 is very accurate in this case, and the approximation of Eq. 3.14 is commonly used when refrigeration systems are analyzed.

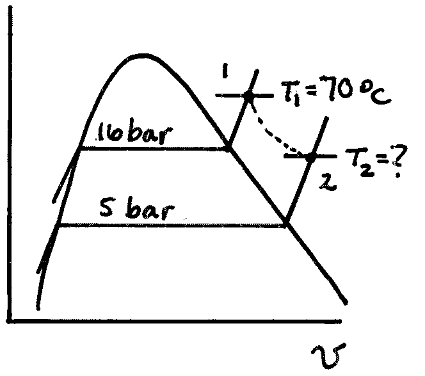
PROBLEM 4.75*

KNOWN: Propane expands through a valve from a known inlet state to a known exit pressure.

FIND: Determine the exit temperature.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible and $W_{cv} = 0$. (3) Kinetic and potential energy effects are negligible. (throttling process)

ANALYSIS: In accordance with the assumptions for a throttling process, $h_1 = h_2$. Using data from Table A-18

$$h_2 = h_1 = 568.5 \text{ kJ/kg}$$

Interpolating in Table A-18 at $p_2 = 5$ bar, $h_2 = 568.5$ kJ/kg

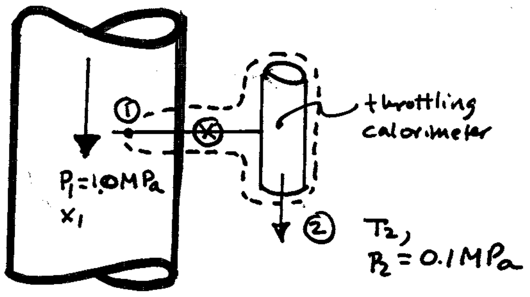
$$T_2 \approx 54.7\,°C \longleftarrow T_2$$

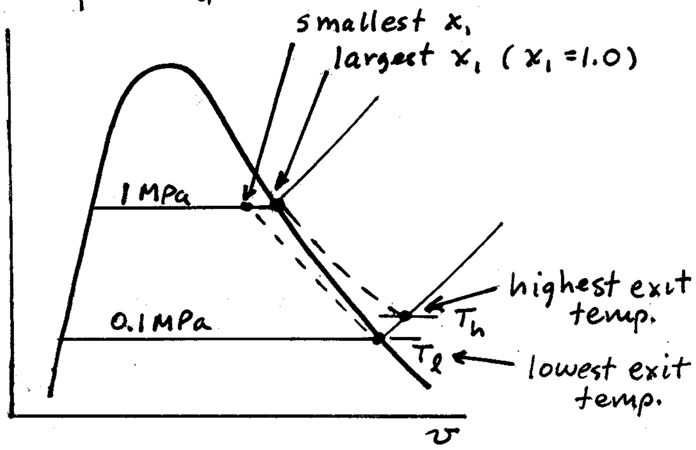
PROBLEM 4.76

KNOWN: Data are provided for a throttling calorimeter attached to a large pipe carrying a two-phase liquid–vapor mixture. Operation is at steady state. The substance is H_2O.

FIND: Determine the range of throttling calorimeter exit temperatures for which the device can determine the quality of steam in the pipe, and the corresponding range of steam quality values.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume shown in the figure is at steady state. 2. The expansion through the calorimeter adheres to the throttling process model: $h_2 \approx h_1$, where $h_1 = h_{f1} + x_1(h_{g1} - h_{f1})$.

ANALYSIS: With assumption 2

$$h(T_2, P_2) = h_{f1} + x_1(h_{g1} - h_{f1}) \tag{1}$$

where the state at the exit is fixed by T_2, P_2, and thus must be superheated vapor or, in the limit, saturated vapor. Also, the quality x_1 is required to be less than or, in the limit, equal to 1.0.

Referring to the T-v diagram, the highest exit temperature, T_h, corresponds to a steam quality of 1.0: That is, with data from Table A-3, Eq. (1) gives

$$h(T_h, P_2) = h_g(P_1)$$
$$= 2778.1 \text{ kJ/kg}$$

Interpolating in Table A-4 at 0.1 MPa gives $T_h = 151°C$.

The lowest exit temperature, T_ℓ, corresponds to saturated vapor exiting the calorimeter, for which $h_2 = 2675.5$ kJ/kg and $T_\ell = 99.6°C$. Eq. (1) gives

$$x_1 = \frac{h_2 - h_{f1}}{h_{g1} - h_{f1}} = \frac{2675.5 - 762.81}{2015.3} = 0.949$$

In summary

$$99.6 \le T_2 \le 151 \ °C$$
$$0.949 \le x_1 \le 1.0$$

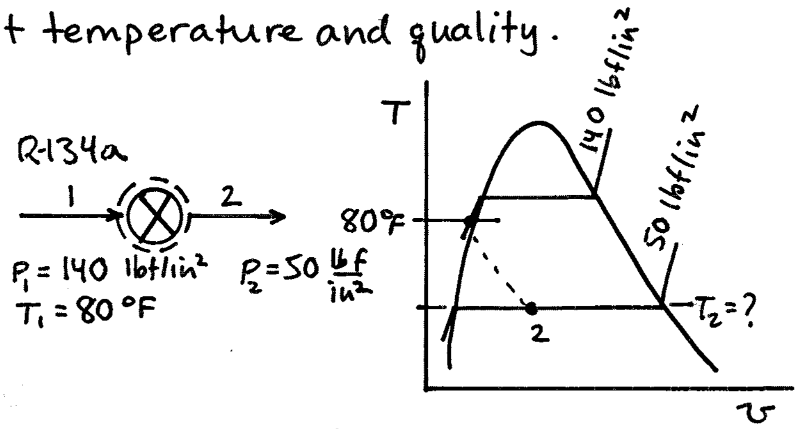
PROBLEM 4.77*

KNOWN: Refrigerant 134a expands through a valve from a known inlet state to a known final pressure.

FIND: Determine the exit temperature and quality.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible, and $W_{cv} = 0$. (3) Kinetic and potential energy effects are negligible. (throttling process)



ANALYSIS: In accordance with the assumptions for a throttling process, $h_1 = h_2$. From Table A-11E; $T_1 < T_{sat\,@\,140\ lbf/in^2}$. Thus, the inlet state is in the compressed liquid region. Ignoring the effect of pressure on the enthalpy of the liquid, Table A-10E gives

$$h_1 \approx h_f(80°F) = 37.27\ \text{Btu/lb}$$

Since $h_2 = h_1 = h_{f2} + x_2 h_{fg2}$, data from Table A-11E at 50 lbf/in² give

$$x_2 = \frac{h_2 - h_{f2}}{h_{fg2}}$$

$$= \frac{37.27 - 24.14}{83.29}$$

$$= 0.158 \longleftarrow x_2$$

and

$$T_2 = 40.27°F \longleftarrow T_2$$

PROBLEM 4.78

KNOWN: Steam flows through a well-insulated valve from specified inlet conditions to a known exit pressure.

FIND: (a) Determine the exit velocity and exit temperature for a given ratio of inlet-to-exit pipe diameters, d_1/d_2. (b) Plot the exit velocity, temperature, and specific enthalpy versus d_1/d_2 ranging from 0.25 to 4.

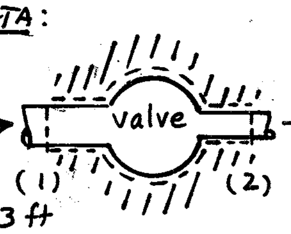
SCHEMATIC & GIVEN DATA:

Steam

$p_1 = 500 \ \frac{\text{lbf}}{\text{in}^2}$

$T_1 = 500°\text{F}$

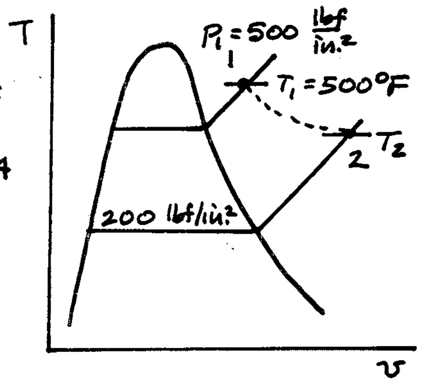
$d_1 = 1 \text{ in.} = 0.08333 \text{ ft}$



$p_2 = 200 \ \frac{\text{lbf}}{\text{in}^2}$

$0.25 \le \frac{d_1}{d_2} \le 4$

$\dot{m} = 0.11 \text{ lb/s}$



ASSUMPTIONS: (1) The control volume is at steady state.
(2) For the control volume, $\dot{Q}_{cv} = 0$ and $\dot{W}_{cv} = 0$.
(3) Potential energy effects are negligible.

ANALYSIS: (a) To determine the exit velocity, begin with the mass balance and Eq. 4.11b: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$, and

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2} \Rightarrow V_2 = \frac{A_1}{A_2} \cdot \frac{V_1}{v_1} \cdot v_2 = \left(\frac{d_1}{d_2}\right)^2 \cdot \frac{V_1}{v_1} \cdot v_2$$

Now, $V_1/v_1 = \dot{m}/A_1 = 4\dot{m}/\pi d_1^2$. Inserting values

$$\frac{V_1}{v_1} = \frac{(4)(.11 \text{ lb/s})}{\pi (.08333 \text{ ft})^2} = 20.17$$

Thus

$$V_2 = 20.17 \left(\frac{d_1}{d_2}\right)^2 \cdot v(T_2, p_2) \qquad (1)$$

and, with $v_1 = 0.992 \text{ ft}^3/\text{lb}$ from Table A-4E

$$V_1 = (20.17)(0.992) = 20.01 \text{ ft/s}$$

From (1) we see that it is necessary to fix state 2 to evaluate V_2. Another relation is obtained from the energy balance at steady state.

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\,\cancelto{0}{(z_1 - z_2)}\right] \qquad (2)$$

or

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

with h_1 from Table A-4E

$$V_2 = \sqrt{2\left[1231.5 - h(T_2, p_2)\right]\frac{\text{Btu}}{\text{lb}}\left|\frac{1 \text{ lbf}}{32.2 \text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right| + (20.01 \text{ ft/s})^2} \qquad (3)$$

Equations (1) and (3) can be solved simultaneously using data from Table A-4E and an iterative process. The results are, for $d_1/d_2 = 0.25$

$T_2 = 434.6 \ °\text{F}$ ◄

$V_2 = 3.140 \text{ ft/s}$ ◄

PROBLEM 4.78 (Contd.)

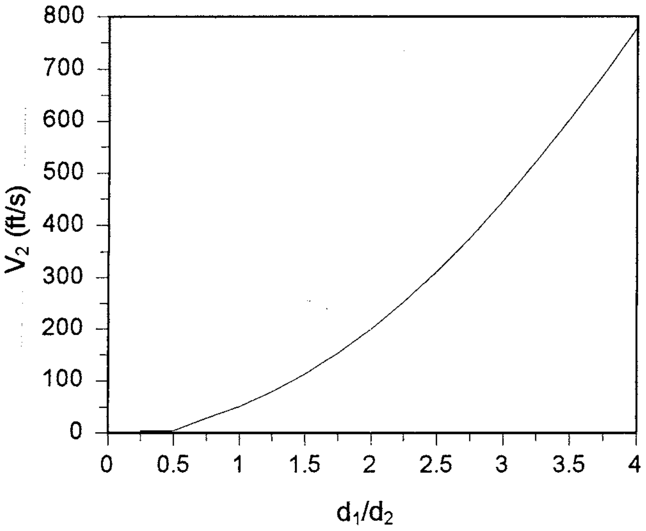
(b) The following *IT* code can be used to solve Eqs. (1) and (2) with the associated data for steam:

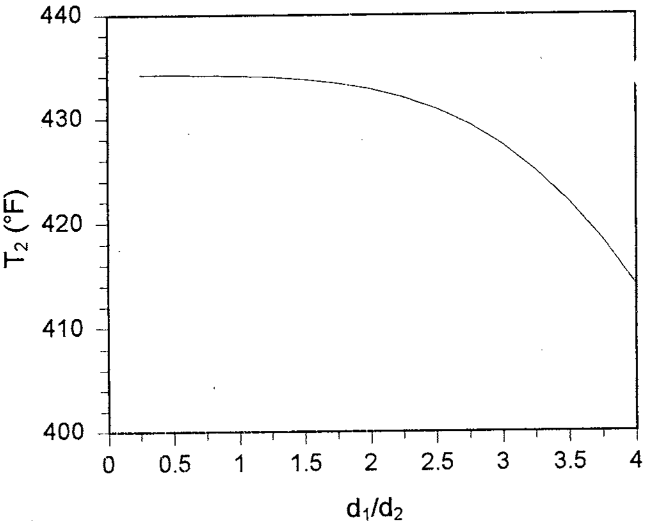
```
p1 = 500  // lbf/in.²
T1 = 500  // °F
mdot = 0.11  // lb/s
d1 = 1/12  // ft.
p2 = 200  // lbf/in.²
dratio = d1 / d2
dratio = .25

A1 = pi * d1^2 / 4
mdot = A1 * V1 / v1
V2 / V1 = (d1 / d2)^2 * (v2 / v1)

h1 = h_PT("Water/Steam", p1, T1)
v1 = v_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
v2 = v_PT("Water/Steam", p2, T2)

0 = (h1 - h2) + ((V1^2 - V2^2) / (2 * 32.2 * 778))
```

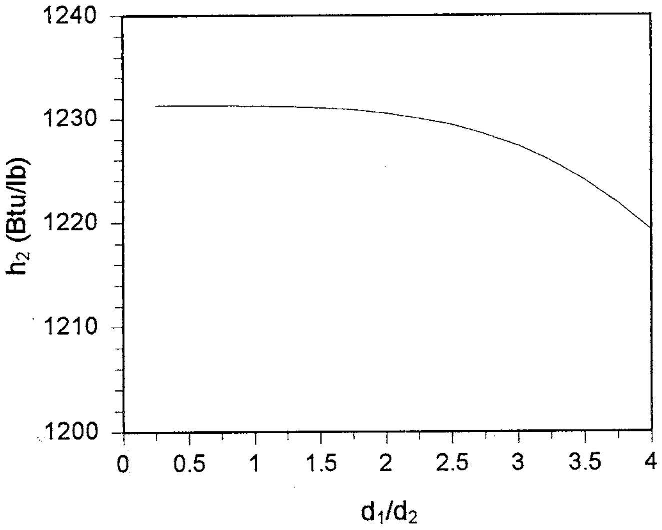
IT Results for the sample case of $d_1/d_2 = 0.25$

$T_2 = 434.2$ °F
$V_2 = 3.139$ ft/s
$h_2 = 1231$ Btu/lb
$V_1 = 20.01$ ft/s
$v_2 = 2.490$ ft³/lb

Using the Explore button, sweep dratio from 0.25 to 4 in steps of 0.25. Then, construct the following plots:







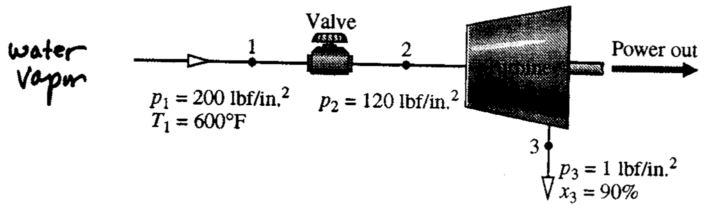
Note that as d_1/d_2 becomes large the throttling process assumptions (pp. 171-173) of negligible kinetic energy and $h_2 \approx h_1$ are not valid.

PROBLEM 4.79

KNOWN: Data are provided for a valve and turbine in series, each operating at steady state.

FIND: For the turbine, determine the temperature at the inlet and the power developed per unit mass of steam flowing.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. Control volumes at steady state enclose the valve and the turbine, respectively. 2. The expansion across the valve is a throttling process. 3. Heat transfer with the surroundings and kinetic/potential energy effects are negligible.

ANALYSIS: Considering the valve, since the expansion is a throttling process, we have $h_2 \approx h_1$. With data from Table A-4E, h_1 = 1322.1 Btu/lb. Then, interpolating at 120 lbf/in^2; $T_2 \approx 589$°F. ← T_2

Mass and energy rate balances applied to the turbine read

$\dot m_2 = \dot m_3 = \dot m$

$$0 = \cancel{\dot Q_{cv}} - \dot W_{cv} + \dot m\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot W_{cv} = \dot m[h_1 - h_2]$$

$$\frac{\dot W_{cv}}{\dot m} = h_3 - h_2$$

Then, with data from Table A-3E and $h_2 = h_1$

$$h_3 = h_f + x_3(h_g - h_f)$$
$$= 69.74 + 0.9[1036] = 1002.1 \text{ Btu/lb}$$

Finally

$$\frac{\dot W_{cv}}{\dot m} = 1322.1 - 1002.1$$
$$= 320 \text{ Btu/lb}$$

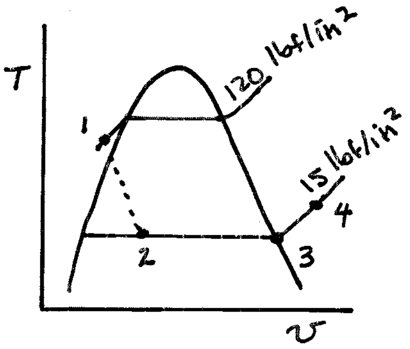
← $\dot W_{cv}/\dot m$

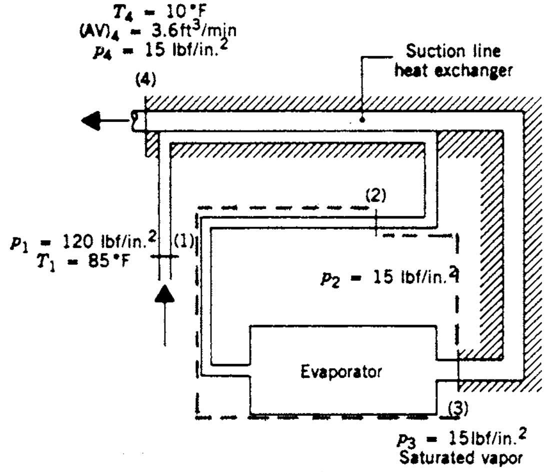
PROBLEM 4.80

KNOWN: Refrigerant 134a passes through a suction line heat exchanger and an evaporator. Data are given at various locations.

FIND: Determine the heat transfer rate for a control volume enclosing the evaporator.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) Heat transfer is negligible between the suction line heat exchanger and its surroundings. (3) $\dot{W}_{cv} = 0$. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: To find the evaporator heat transfer rate, begin with the steady-state energy and mass balances

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}\left[(h_2-h_3) + \left(\frac{\cancel{V_2^2-V_3^2}}{2}\right)^{0} + g(\cancel{z_2-z_3})^{0}\right]$$

where $\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4 \equiv \dot{m}$. Thus

$$\dot{Q}_{cv} = \dot{m}(h_3 - h_2)$$

From Table A-11E, $h_3 = h_{g@15\,lbf/in^2} = 99.66$ Btu/lb. To find h_2, write an energy balance for a control volume enclosing the heat exchanger

$$0 = \dot{m}(h_1 - h_2) + \dot{m}(h_3 - h_4)$$

or

$$h_2 = h_1 + h_3 - h_4$$

From Table A-12E, $h_4 = 104.38$ Btu/lb, and assuming $h_1 \approx h_f(T_1)$

$$h_2 = 38.99 + 99.66 - 104.38 = 34.27 \text{ Btu/lb}$$

Now, with $v_4 = 3.1680$ ft³/lb, the mass flow rate is

$$\dot{m} = \frac{(AV)_4}{v_4} = \frac{(3.6\ ft^3/min)}{(3.1680\ ft^3/lb)}\left|\frac{60\ min}{1\ h}\right| = 68.18\ lb/h$$

Finally, the evaporator heat transfer rate is

$$\dot{Q}_{cv} = \left(68.18\ \frac{lb}{h}\right)(99.66 - 34.27)\ Btu/lb$$

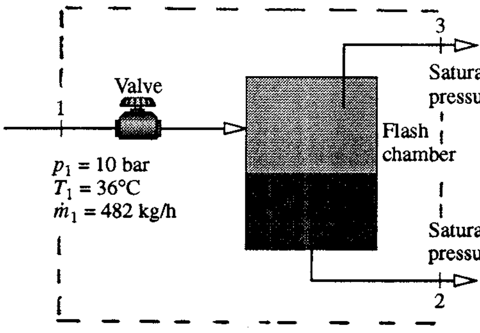
$$= 4458\ Btu/h \longleftarrow \dot{Q}_{cv}$$

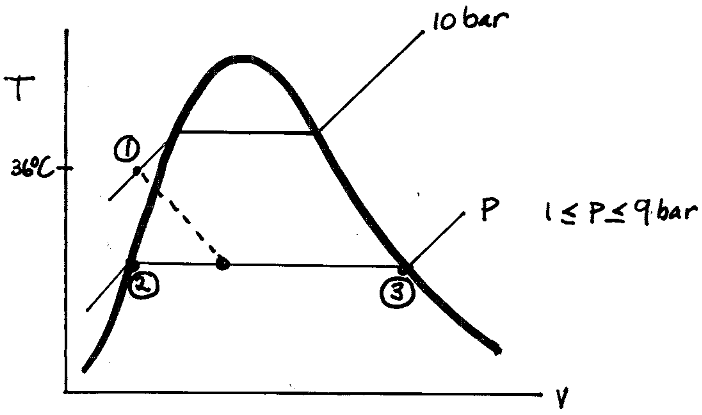
PROBLEM 4.81

KNOWN: Data are provided for a flash chamber operating at steady state. Saturated vapor and saturated liquid streams exit at pressure p.

FIND: If p = 4 bar, determine the mass flow rates of the exiting streams. Plot the mass flow rates of the exiting streams versus pressure p, $1 \leq p \leq 9$ bar.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume shown in the accompanying figure is at steady state. 2. For the control volume, $\dot W_{cv} = 0$, heat transfer with the surroundings is negligible, and kinetic/potential energy effects can be ignored. 3. For the liquid entering at 1, $h_1 \approx h_f(T_1)$.

ANALYSIS: For the control volume, the mass rate balance at steady state gives

$\dot m_1 = \dot m_2 + \dot m_3$, or $\dot m_3 = \dot m_1 - \dot m_2$.

An energy rate balance reduces to

$$0 = \cancel{\dot Q_{cv}} - \cancel{\dot W_{cv}} + \dot m_1\left[h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right] - \dot m_2\left[h_2 + \cancel{\frac{V_2^2}{2}} + \cancel{gz_2}\right] - \dot m_3\left[h_3 + \cancel{\frac{V_3^2}{2}} + \cancel{gz_3}\right]$$

$$\Rightarrow 0 = \dot m_1 h_1 - \dot m_2 h_2 - \dot m_3 h_3 \quad \text{or} \quad 0 = \dot m_1 h_1 - \dot m_2 h_2 - (\dot m_1 - \dot m_2) h_3$$

$$\Rightarrow \quad \dot m_2 = \dot m_1 \left[\frac{h_1 - h_3}{h_2 - h_3}\right]$$

(a) From Table A-10; $h_1 \approx h_f(36°C) = 100.25$ kJ/kg. Also, at $p = 4$ bar, Table A-11 gives; $h_2 = h_f(4\text{ bar}) = 62$ kJ/kg and $h_3 = h_g(4\text{ bar}) = 252.32$ kJ/kg. Thus

$$\dot m_2 = \left(482\ \frac{kg}{h}\right)\left[\frac{(100.25) - (252.32)}{(62) - (252.32)}\right] = 385.1\ \frac{kg}{h} \longleftarrow \dot m_2$$

From above

$$\dot m_3 = \dot m_1 - \dot m_2 = 482 - 385.1 = 96.9\ kg/h \longleftarrow \dot m_3$$

(b) The following IT code can be used to generate data for plots of $\dot m_2$ and $\dot m_3$ versus p:

IT Code

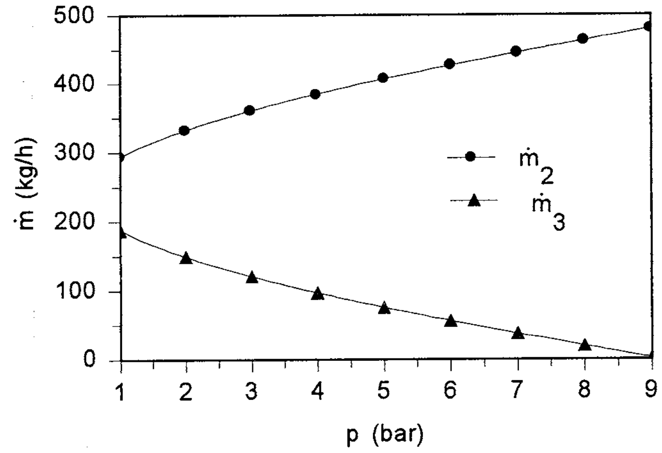
```
p1 = 10  // bar
T1 = 36  // °C
mdot1 = 482  // kg/h
p = 4  // bar
x2 = 0
x3 = 1
mdot1 = mdot2 + mdot3
0 = mdot1 * h1 - mdot2 * h2 - mdot3 * h3
h1 = h_PT("R134A", p1, T1)
h2 = hsat_Px("R134A", p, x2)
h3 = hsat_Px("R134A", p, x3)
```

PROBLEM 4.81 (Cont'd.)

Using $p = 4$ bar as a sample case

IT Results (p = 4 bar)
$\dot{m}_2 = 385.1$ kg/h
$\dot{m}_3 = 96.87$ kg/h
$h_1 = 100.3$ kJ/kg
$h_2 = 62$ kJ/kg
$h_3 = 252.3$ kJ/kg

These results compare very favorable with those of part(a). Now, using the Explore button, sweep p from 1 to 9 bar in steps of 0.1 bar. Then, the following plot can be constructed:



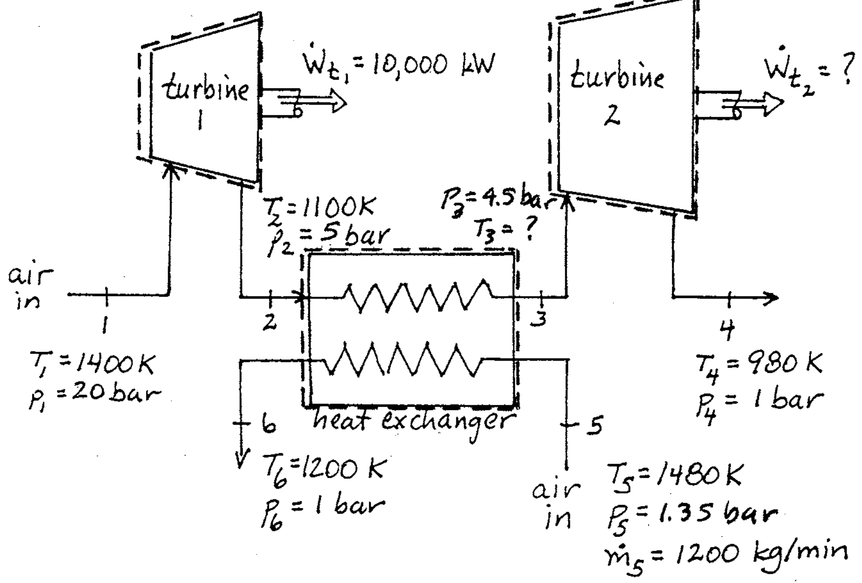
These plots imply that as the pressure in the flash chamber, p, is decreased a greater fraction of the incoming liquid flow "flashes" to saturated vapor.

PROBLEM 4.82

KNOWN: Air flows through two turbine stages and an inter-connecting heat exchanger. A separate hot air stream passes in counterflow through the heat exchanger. Data are known at various locations.

FIND: Determine the temperature of the main air stream exiting the heat exchanger and the power output of the second turbine.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volumes are at steady state. (2) Heat transfer to the surroundings can be neglected. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas. (5) For the heat exchanger, $\dot{W}_{cv} = 0$.

ANALYSIS: First, find the air flow rate at 1. Begin with steady-state energy and mass balances for turbine 1

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{t_1} + \dot{m}_1\left[(h_1-h_2) + \cancelto{0}{\left(\frac{V_1^2-V_2^2}{2}\right)} + g\cancelto{0}{(z_1-z_2)}\right]$$

and $0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2$

Solving for $\dot{m}_1$

$$\dot{m}_1 = \frac{\dot{W}_{t1}}{(h_1-h_2)}$$

From Table A-22; $h_1 = 1515.42$ kJ/kg and $h_2 = 1161.07$ kJ/kg. Thus

$$\dot{m}_1 = \frac{(10{,}000\ \text{kW})}{(1515.42-1161.07)\,\text{kJ/kg}}\left|\frac{1\ \text{kJ/s}}{1\ \text{kW}}\right| = 28.22\ \text{kg/s}$$

Turning next to the heat exchanger

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_4$$
$$\dot{m}_5 = \dot{m}_6$$

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_2\left[(h_2-h_3) + \cancelto{0}{\left(\frac{V_2^2-V_3^2}{2}\right)} + g\cancelto{0}{(z_2-z_3)}\right] + \dot{m}_5\left[(h_5-h_6) + \cancelto{0}{\left(\frac{V_5^2-V_6^2}{2}\right)} + g\cancelto{0}{(z_5-z_6)}\right]$$

$$0 = \dot{m}_2(h_2-h_3) + \dot{m}_5(h_5-h_6)$$

or

$$h_3 = h_2 + \frac{\dot{m}_5}{\dot{m}_2}(h_5-h_6)$$

Again, from Table A-22; $h_5 = 1611.79$ kJ/kg and $h_6 = 1277.79$ kJ/kg. Thus

PROBLEM 4.82 (Cont'd)

$$h_3 = 1161.07 + \left[\frac{(1200 \text{ kg/min})}{(28.22 \text{ kg/s})\left|60 \text{ s/1 min}\right|}\right](1611.79 - 1277.79)$$

$$= 1397.8 \text{ kJ/kg}$$

Interpolating in Table A-22

$$T_3 = 1301.5\text{K} \longleftarrow \underline{T_3}$$

Now, writing the steady-state energy balance for turbine 2

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{t_2} + \dot{m}_3\left[(h_3 - h_4) + \cancelto{0}{\left(\frac{V_3^2 - V_4^2}{2}\right)} + g\cancelto{0}{(z_3 - z_4)}\right]$$

or

$$\dot{W}_{t_2} = \dot{m}_3 (h_3 - h_4)$$

From Table A-22; $h_4 = 1023.25$ kJ/kg, and

$$\dot{W}_{t_2} = (28.22 \text{ kg/s})(1397.8 - 1023.25) \text{ kJ/kg}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

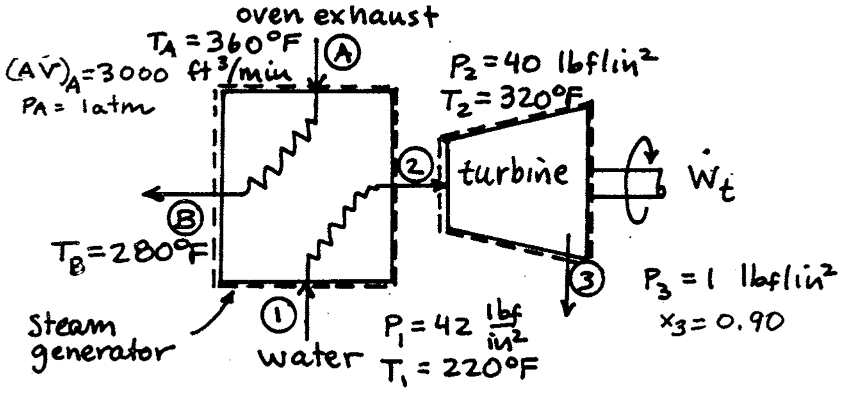
$$= 10{,}570 \text{ kW} \longleftarrow \underline{\dot{W}_{t_2}}$$

PROBLEM 4.83

KNOWN: Steam is developed from a water stream by energy transfer from oven exhaust in a waste heat recovery steam generator. The steam then expands through a turbine. Data are known at various locations.

FIND: Determine the power developed by the turbine, evaluate it economically on an annual basis, and comment.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Both control volumes are at steady state. (2) Heat transfer is negligible between each control volume and its surroundings, and $\dot{W}_{cv} = 0$ for the heat exchanger. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas, at a pressure of 14.7 lbf/in².

ANALYSIS: (a) The turbine power can be determined from an energy balance once the steam flow rate is determined. Begin with a steady-state energy balance on the steam generator

$$0 = \dot{m}_A - \dot{m}_B \Rightarrow \dot{m}_A = \dot{m}_B \equiv \dot{m}_{gas}$$

$$0 = \dot{m}_1 - \dot{m}_2 \Rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}_3 \equiv \dot{m}_{st}$$

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_{gas}\left[(h_A - h_B) + \cancelto{0}{\left(\frac{V_A^2 - V_B^2}{2}\right)} + \cancelto{0}{g(z_A - z_B)}\right] + \dot{m}_{st}\left[(h_1 - h_2) + \cancelto{0}{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancelto{0}{g(z_1 - z_2)}\right]$$

$$\dot{m}_{st} = \dot{m}_{gas}\frac{(h_A - h_B)}{(h_2 - h_1)}$$

The gas flow rate is

$$\dot{m}_{gas} = \frac{(A\bar{V})_A}{v_A} = \frac{P_A (A\bar{V})_A}{R T_A}$$

$$= \frac{(14.7\ lbf/in^2)(3000 ft^3/min)}{\left(\frac{1545}{28.97}\cdot\frac{ft\cdot lbf}{lb\cdot °R}\right)(820\ °R)}\left(\frac{144\ in^2}{1\ ft^2}\right) = 145.2\ lb/min$$

From Table A-22E; $h_A = 196.69$ Btu/lb and $h_B = 177.23$ Btu/lb. For the water, from Table A-2E; $P_1 > P_{sat@220°F} \Rightarrow$ compressed liquid. Thus, $h_1 \approx h_{f@220°F} = 188.2$ Btu/lb. At 2, using Table A-4E; $h_2 = 1196.8$ Btu/lb. The flow rate is

$$\dot{m}_{st} = (145.2\ lb/min)\frac{(196.69 - 177.23)}{(1196.8 - 188.2)} = 2.8\ lb/min$$

PROBLEM 4.83 (Contd.)

Turning now to the control volume enclosing the turbine

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_t + \dot{m}_{st}\left[(h_2 - h_3) + \cancel{\left(\frac{V_2^2 - V_3^2}{2}\right)}^{0} + g\cancel{(z_2 - z_3)}^{0}\right]$$

$$\dot{W}_t = \dot{m}_{st}(h_2 - h_3)$$

Using data from Table A-3E

$$h_3 = h_f + x_3(h_{fg}) = 69.7 + 0.9(1036) = 1002.1 \text{ Btu/lb}$$

Thus

$$\dot{W}_t = \left(2.8 \frac{\text{lb}}{\text{min}}\right)\left(1196.8 - 1002.1\right)\frac{\text{Btu}}{\text{lb}}\left|\frac{60 \text{ min}}{1 \text{ h}}\right|\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 12.85 \text{ hp} \qquad \longleftarrow$$

(b) Evaluating power at 8 cents per kW·h, the value for 8000h of operation annually is

$$\dot{\$} = (12.85 \text{ hp})\left|\frac{0.7457 \text{ kW}}{1 \text{ hp}}\right|\left(8000 \frac{\text{h}}{\text{year}}\right)\left(\frac{\$ 0.08}{\text{kW}\cdot\text{h}}\right)$$

$$= \$6133/\text{year}$$

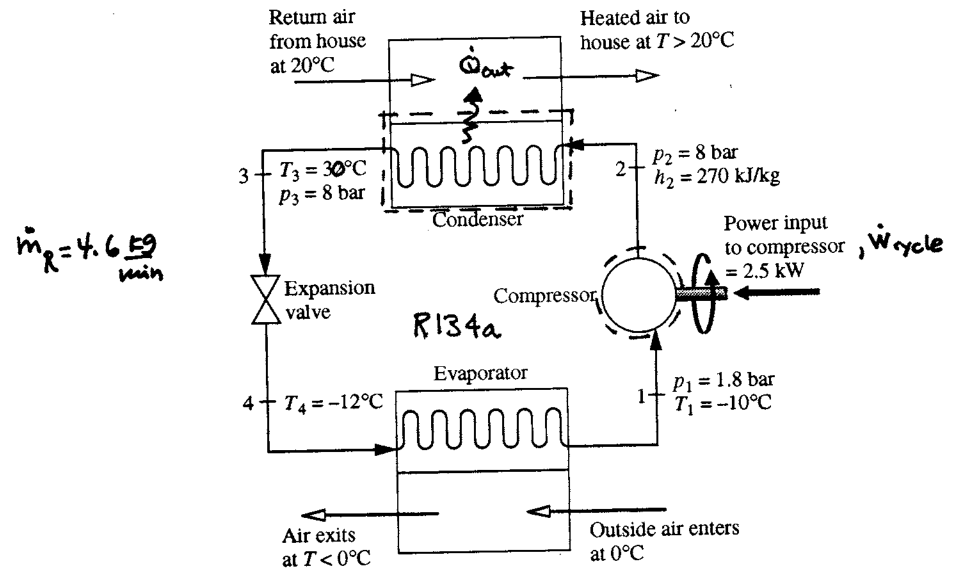
It is unlikely that this value would cover the capital and operating costs associated with the overall system. Accordingly, even though the oven exhaust has a thermodynamic potential for use, the potential is not likely to be exploited by the proposed means owing to economic considerations.

PROBLEM 4.84

KNOWN: Operating data are provided for a residential heat pump at steady state.

FIND: Determine the rate of heat transfer between the compressor and the surroundings. Also determine the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. Control volumes enclosing the compressor and the refrigerant side of the condenser are at steady state. 2. Kinetic and potential energy effects can be ignored.

ANALYSIS: (a) For the compressor, the mass rate balance reads $\dot{m}_2 = \dot{m}_1 \equiv \dot{m}_R$. The energy rate balance is

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_R\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}_R(h_2 - h_1)$$

From Table A-12, $h_1 = 242.06$ kJ/kg; $h_2 = 270$ kJ/kg (given). Thus

$$\dot{Q}_{cv} = \left(-2.5\ \frac{kJ}{s}\right)\left|\frac{60\ s}{min}\right| + \left(4.6\ \frac{kg}{min}\right)(270 - 242.06)\ \frac{kJ}{kg} = -21.48\ \frac{kJ}{min} \longleftarrow$$

(b) To determine the coefficient of performance requires the energy supplied by the condensing refrigerant to the return air, $\dot{Q}_{out}$. Thus, an energy rate balance for the refrigerant side of the condenser is required:

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}} + \dot{m}_R\left[(h_2 - h_3) + \cancel{\frac{V_2^2 - V_3^2}{2}} + \cancel{g(z_2 - z_3)}\right] \Rightarrow \dot{Q}_{cv} = \dot{m}_R(h_3 - h_2).$$

Then, with $h_3 \cong h_f(T_3) = 91.49$ kJ/kg, $\dot{Q}_{cv} = (4.6)(91.49 - 270) = -821.15$ kJ/min. Applying Eq. 2.47 with $\dot{W}_{cycle} = 2.5$ kW and $\dot{Q}_{out} = 821.15$ kJ/min

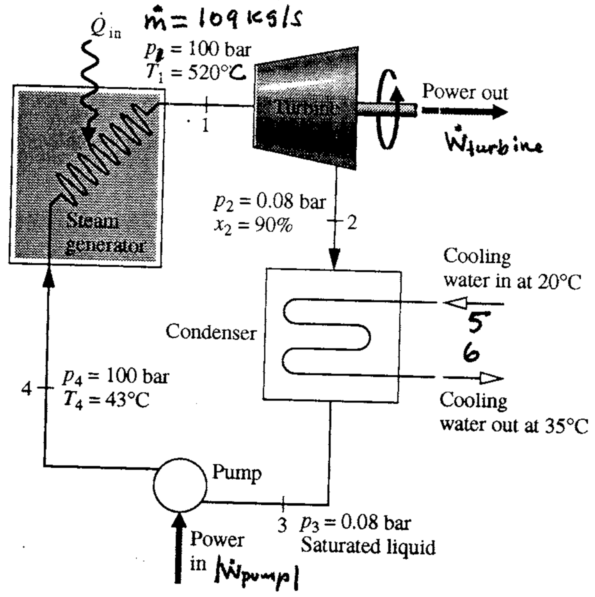
$$\gamma = \frac{821.15\ kJ/min}{\left(2.5\ \frac{kJ}{s}\right)\left|\frac{60\ s}{1\ min}\right|} = 5.47 \longleftarrow$$

PROBLEM 4.85

KNOWN: Data are provided for a simple vapor power plant at steady state.

FIND: Determine the thermal efficiency and the mass flow rate of the cooling water through the condenser.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. Control volumes enclosing each of the four components are at steady state. 2. For each control volume, kinetic/potential energy effects are negligible as are all stray heat transfers.

ANALYSIS: (a) For any power cycle, the thermal efficiency is the ratio of the net work developed to the heat added. In the present case

$$\eta = \frac{\dot{W}_{turbine} - |\dot{W}_{pump}|}{\dot{Q}_{in}}$$

Applying mass and energy rate balances to the turbine, pump, and steam generator, respectively, gives

$$\dot{W}_{turbine} = \dot{m}(h_1 - h_2).$$
$$|\dot{W}_{pump}| = \dot{m}(h_4 - h_3).$$
$$\dot{Q}_{in} = \dot{m}(h_1 - h_4).$$

Table A-4: $h_1 = 3425.1$ kJ/kg. Table A-3: $h_2 = h_f + x_2 h_{fg}$ or $h_2 = 173.88 + 0.9(2403.1) = 2336.7$ kJ/kg. Table A-3: $h_3 = 173.9$ kJ/kg. Table A-5: $h_4 = 188.9$ kJ/kg.

$$\Rightarrow \quad \eta = \frac{\dot{m}(h_1 - h_2) - \dot{m}(h_4 - h_3)}{\dot{m}(h_1 - h_4)} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} = \frac{(3425.1 - 2336.7) - (188.9 - 173.9)}{(3425.1 - 188.9)}$$

$$= 0.332 \; (33.2\%) \qquad \longleftarrow$$

(b) Applying mass rate balances to the condenser: $\dot{m}_2 = \dot{m}_3 = 109$ kg/s, $\dot{m}_5 = \dot{m}_6 \equiv \dot{m}_w$. An energy rate balance for the condenser reduces with assumption 4 to read

$$0 = \dot{m}_2[h_2 - h_3] + \dot{m}_{cw}[h_5 - h_6] \Rightarrow \dot{m}_{cw} = \dot{m}_2\left[\frac{h_2 - h_3}{h_6 - h_5}\right]$$

With $h_5 \sim h_f(T_5)$, $h_6 \sim h_f(T_6)$ from Table A-2

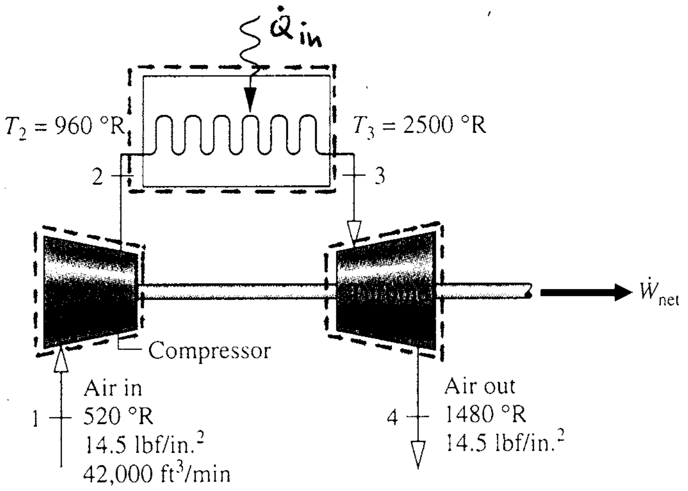
$$\dot{m}_{cw} = 109\,\frac{kg}{s}\left[\frac{2336.7 - 173.9}{146.68 - 83.96}\right] = 3759\,\frac{kg}{s} \qquad \longleftarrow$$

PROBLEM 4.86

KNOWN: A simple gas turbine power plant operates at steady state with a single stream of air as the working fluid. Data are known for the flow stream at various locations.

FIND: (a) Determine the power required by the compressor, and (b) define and evaluate a thermal efficiency for the gas turbine.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each control volume shown is at steady state. (2) Heat transfer is negligible for the control volumes enclosing the compressor and turbine. (3) For the heat exchanger, $\dot{W}_{cv} = 0$. (4) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas.

ANALYSIS: (a) The compressor power is determined by using energy and mass balances for the control volume enclosing the compressor. At steady state

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{comp} + \dot{m}\left[(h_1 - h_2) + \cancel{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g\cancel{(z_1 - z_2)}\right]$$

or $\quad \dot{W}_{comp} = \dot{m}(h_1 - h_2)$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. To get $\dot{m}$

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{R T_1}$$

$$= \frac{(14.5\ \text{lbf/in.}^2)(42{,}000\ \text{ft}^3/\text{min})}{\left(\frac{1545}{28.97}\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(520\ °\text{R})}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{60\ \text{min}}{1\ \text{h}}\right|$$

$$= 1.897\times10^5\ \text{lb/h}$$

Now, with specific enthalpies from Table A-22E; $h_1 = 124.27$ Btu/lb and $h_2 = 231.06$ Btu/lb. Thus

$$\dot{W}_{comp} = \left(1.897\times10^5\ \frac{\text{lb}}{\text{h}}\right)(124.27 - 231.06)\ \frac{\text{Btu}}{\text{lb}}\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right|$$

$$= -7960\ \text{hp} \longleftarrow \dot{W}_{comp}$$

(b) Comparing the operation of the gas turbine to Fig. 2.16, and Eq. 2.42, an appropriate thermal efficiency for the gas turbine is

$$\eta = \frac{\text{Net power developed}}{\text{Rate of heat transfer to the air passing through the heat exchanger}}$$

$$= \frac{\dot{W}_{net}}{\dot{Q}_{in}}$$

PROBLEM 4.86 (Cont'd)

To determine $\dot{W}_{net}$, consider the control volume enclosing the turbine. With assumptions (1), (2), and (4)

$$0 = -(\dot{W}_{net} + |\dot{W}_{comp}|) + \dot{m}(h_3 - h_4)$$

or

$$\dot{W}_{net} = -|\dot{W}_{comp}| + \dot{m}(h_3 - h_4)$$

With data from Table A-22E; $h_3 = 645.78$ Btu/lb and $h_4 = 363.89$ Btu/lb. Thus

$$\dot{W}_{net} = -(7960\ \text{hp}) + \left(1.897\times10^5\ \frac{\text{lb}}{\text{h}}\right)(645.78 - 363.89)\frac{\text{Btu}}{\text{lb}}\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right|$$

$$= 13{,}052\ \text{hp}$$

For $\dot{Q}_{in}$, consider the heat exchanger. With assumptions (1), (3), and (4)

$$0 = \dot{Q}_{in} - \cancel{\dot{W}_{cv}} + \dot{m}\left[(h_2 - h_3) + \cancel{\left(\frac{V_2^2 - V_3^2}{2}\right)} + \cancel{g(z_2 - z_3)}\right]$$

or

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2)$$

Inserting data

$$\dot{Q}_{in} = (1.897\times10^5)(645.78 - 231.06)\left|\frac{1}{2545}\right|$$

$$= 30{,}912\ \text{hp}$$

Finally

$$\eta = \frac{13{,}052}{30{,}912} = 0.422\ (42.2\%) \longleftarrow \eta$$

The following obsevations are made:

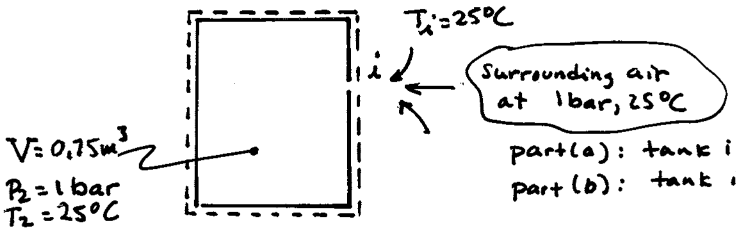
- In this case, 42.2% of the energy input by heat shows up as net power developed by the power plant. The second low of thermodynamics introduced in chapter 5 will allow us to compare this with ideal performance.
- Although the air doesn't undergo a cycle in passing through the gas turbine components, we could imagine that a cycle is completed in the surroundings as the air cools from 1480 °R back to 520°R.

PROBLEM 4.87*

KNOWN: A pinhole leak develops in the wall of a rigid tank of known volume, and air from the surroundings enters at a known temperature and pressure. The temperature of the air within the tank remains constant at the temperature of the surroundings due to heat transfer from the tank contents.

FIND: Determine the heat transfer between the tank contents and surroundings if (a) the tank is initially evacuated, (b) the tank initially contains air at a specified condition.

SCHEMATIC & GIVEN DATA:



part(a): tank is initially evacuated

part(b): tank initially contains air at 25 °C, 0.7 bar.

ASSUMPTIONS: 1. For the control volume shown in the accompanying figure, $\dot{W}_{cv} = 0$, and kinetic/potential energy effects are negligible. 2. The air is modeled as an ideal gas. 3. The condition of the air leaking into the tank at i remains constant at 1 bar, 25°C. 4. The temperature of the air within the tank is 25°C.

ANALYSIS: The mass rate balance for the control volume takes the form $dm_{cv}/dt = \dot{m}_i$. With indicated assumptions, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \dot{m}_i h_i \Rightarrow \frac{dU_{cv}}{dt} = \dot{Q}_{cv} + h_i \frac{dm_{cv}}{dt}$$

With assumption 3, h_i remains constant. Thus, on integration

$$\Delta U_{cv} = Q_{cv} + \int_1^2 h_i \, dm_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = Q_{cv} + h_i (m_2 - m_1)$$

Introducing $h_i = u_i + (pv)_i$, and collecting like terms

$$Q_{cv} = \underbrace{m_2(u_2 - u_i)}_{(1)} - \underbrace{m_1(u_1 - u_i)}_{(2)} - (m_2 - m_1)(pv)_i \qquad (1)$$

Since the specific internal energy of an ideal gas depends on temperature only, term (1) vanishes because $T_2 = T_i$. Term (2) vanishes in part (a) because $m_1 = 0$, and in part (b) because $T_1 = T_i$. Accordingly, Eq. (1) reduces in each part to the following working equation:

$$Q_{cv} = -(m_2 - m_1)(pv_i) \qquad (2a)$$

That is, the heat transfer from the tank removes the energy entering the tank at i by flow work (see Sec. 4.2.1). Since $pv_i = RT_i$, Eq. (2a) becomes alternatively

$$Q_{cv} = -(m_2 - m_1) RT_i \qquad (2b)$$

(a) $\underline{m_1 = 0}$. Using the ideal gas equation of state, $m_2 = P_2 V / RT_2 = P_2 V / RT_i$. So

$$Q_{cv} = -P_2 V = -(1 \text{ bar}) \left| \frac{10^5 N/m^2}{1 \text{ bar}} \right| (0.75 m^3) \left| \frac{1 kJ}{10^3 N \cdot m} \right| = -75 kJ \longleftarrow$$

(b) $\underline{m_1 \neq 0}$. Since $m_1 = P_1 V / RT_1 = P_1 V / RT_i$, $m_2 = P_2 V / RT_i$,

$$Q_{cv} = -(P_2 - P_1) V = -(1 \text{ bar} - 0.7 \text{ bar}) \left| \frac{10^5 N/m^2}{1 \text{ bar}} \right| (0.75 m^3) \left| \frac{1 kJ}{10^3 N \cdot m} \right|$$

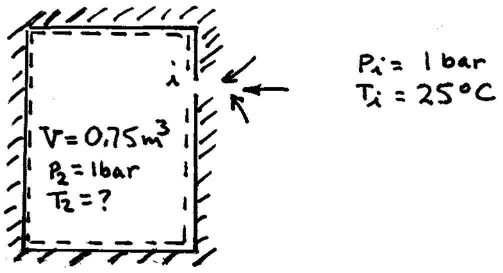
$$= -22.5 kJ \longleftarrow$$

PROBLEM 4.88*

KNOWN: A pinhole develops in the wall of an initially evacuate tank, and air enters from the surroundings until the pressure is 1 bar.

FIND: Determine the final temperature of the air in the tank in the absence of heat transfer between the tank contents and surroundings.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume is shown in the accompanying figure. 2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. 3. The air is modeled as an ideal gas. 4. The condition of the air entering the tank remain constant at 1 bar, 21°C.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$. The energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_i h_i$$

Combining the mass and energy rate balance, and integrating using assumption 4

$$\Delta U_{cv} = \int_1^2 \dot{m}_i h_i \, dt \Rightarrow \Delta U_{cv} = h_i \int_1^2 \dot{m}_i \, dt \Rightarrow \Delta U_{cv} = h_i(m_2 - m_1)$$

or since the tank is initially evacuated

$$m_2 u_2 - \cancel{m_1} u_1 = h_i(m_2 - \cancel{m_1}) \Rightarrow u_2 = h_i \qquad (1)$$

From Table A-22 at $T_i = 298 K$ (25°C), $h_i = 298.18$ kJ/kg. Then,

① interpolating with $u_2 = 298.18$ kJ/kg, $T_2 = 416.5 K$ (143.5°C) ←

---

1. To interpret the temperature increase of the air within the tank during the filling process, rewrite Eq. (1) with $h_i = u_i + (pv)_i$ to read

$$u_2 = u_i + \underline{(pv)_i}$$

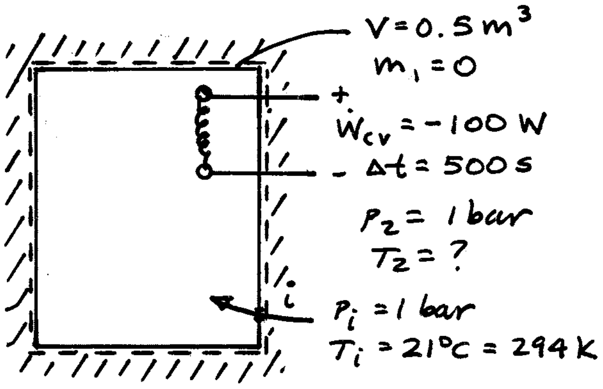
where the underlined term is the flow work discussed in Sec. 4.2.1. Thus, the temperature of air in the tank increases during the process because work is done by the surroundings -- flow work -- on the matter entering the tank.

PROBLEM 4.89

KNOWN: An electric resistor transfers energy to air from the surroundings entering a rigid, well-insulated tank. The final pressure is known.

FIND: Determine the final temperature of air in the tank.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is shown in the accompanying figure. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) The condition of the air entering the tank remains constant. (5) For the resistor, $\Delta U_{resistor} = 0$.

ANALYSIS: The mass rate balance reduces to $dm_{cv}/dt = \dot{m}_i$, and the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_i h_i$$

Combining and integrating using assumption (4)

$$\Delta U_{cv} = -\int_{t_1}^{t_2} \dot{W}_{cv}\, dt + \int_{t_1}^{t_2} \dot{m}_i h_i\, dt$$

$$= -\dot{W}_{cv}\Delta t + h_i \int_{t_1}^{t_2} \dot{m}_i\, dt = -\dot{W}_{cv}\Delta t + h_i(m_2 - m_1)$$

With $\Delta U_{cv} = m_2 u_2 - m_1 u_1$

$$m_2 u_2 - \cancel{m_1} u_1 = -\dot{W}_{cv}\Delta t + (m_2 - \cancel{m_1}) h_i$$

or

$$m_2 u_2 = -\dot{W}_{cv}\Delta t + m_2 h_i \Rightarrow u_2 = \frac{-\dot{W}_{cv}\Delta t}{m_2} + h_i$$

By assumption (3), $m_2 = p_2 V / R T_2$, and

$$u_2 = \left(\frac{R T_2}{p_2 V}\right)(-\dot{W}_{cv}\Delta t) + h_i \qquad (1)$$

Inserting values

$$-\dot{W}_{cv}\Delta t = -(-100\ W)(500\ s)\left|\frac{1\ kJ/s}{10^3\ W}\right| = 50\ kJ$$

$$\frac{R T_2}{p_2 V} = \frac{\left(\frac{8.314}{28.97}\ \frac{kJ}{kg\cdot K}\right)(T_2 \text{ in } K)}{(1\ bar)(0.5\ m^3)}\left|\frac{1\ bar}{10^5\ N/m^2}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|$$

$$= (5.74 \times 10^{-3})\, T_2$$

From Table A-22; $h_i = 294.17$ kJ/kg. Thus

$$u_2 = \left(5.74 \times 10^{-3}\ \frac{1}{kg\cdot K}\right)(50\ kJ)\, T_2 + 294.17\ kJ/kg \qquad (2)$$

Eq. (2) can be solved for T_2 using an iterative procedure with data for u_2 from Table A-22. The result is

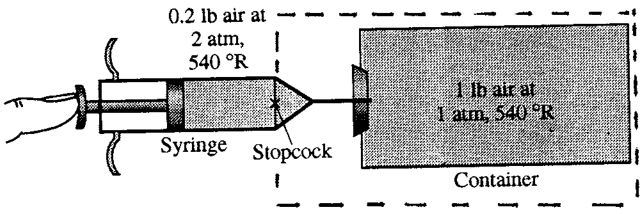
$$T_2 = 665.3\ K = 392.3\ °C$$

PROBLEM 4.90

KNOWN: A small amount of air at a given state is injected into a container initially holding air at a different state.

FIND: Determine the final temperature and pressure of the air in the container.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume is shown in the accompanying schematic. 2. The state of the air remains constant until it passes the stopcock. 3. For the control volume, $\dot{W}_{cv} = 0$, heat transfer with the surroundings can be ignored, and kinetic/potential energy effects are negligible. 4. The ideal gas model applies for the air.

ANALYSIS: The mass rate balance reads $dm_{cv}/dt = \dot{m}_i$. The energy rate balance reduces with given assumptions to

① $$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_i h_i \Rightarrow \frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt}$$

With assumption 2, h_i remains constant. Thus, integration over time gives

$$\Delta U_{cv} = \int_1^2 h_i \, dm_{cv} \Rightarrow \Delta U_{cv} = h_i \Delta m_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)$$

where 1 and 2 denote the initial and final states within the container, respectively. That is, $m_1 = 1$ lb and $m_2 = 1\text{ lb} + 0.2\text{ lb} = 1.2$ lb. Solving for u_2, and inserting data from Table A-22E for u_1 and h_i

$$u_2 = \frac{m_1 u_1 + (m_2 - m_1) h_i}{m_2} = \frac{(1\text{ lb})\left[92.04 \frac{\text{Btu}}{\text{lb}}\right] + (0.2\text{ lb})\left[129.06 \frac{\text{Btu}}{\text{lb}}\right]}{1.2\text{ lb}} = 98.21 \frac{\text{Btu}}{\text{lb}}$$

Interpolating in Table A-22E with u_2 gives $T_2 = 576°\text{R}$ (116°F).

Using the ideal gas equation of state, $P_2 = m_2 R T_2 / V$, where $V = m_1 R T_1 / P_1$, the final pressure is

$$P_2 = \left(\frac{m_2}{m_1}\right)\left(\frac{T_2}{T_1}\right) P_1 = \left(\frac{1.2\text{ lb}}{1\text{ lb}}\right)\left(\frac{576\,°\text{R}}{540\,°\text{R}}\right)\left(14.7 \frac{\text{lbf}}{\text{in}^2}\right) = 18.8\text{ lbf/in}^2$$

---

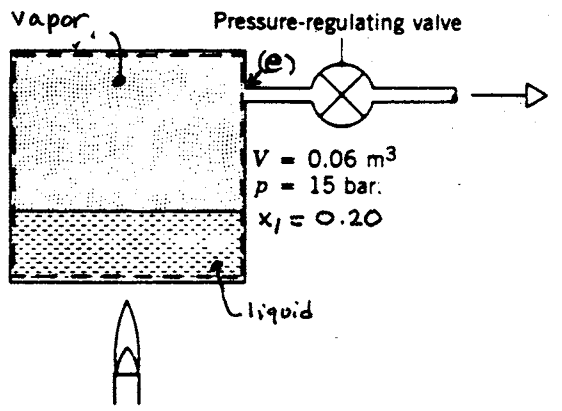
1. The plunger does not directly enter the analysis -- it is located in the surroundings of the control volume. However, there is flow work where air enters the control volume. This is incorporated in the $(pv)_i$ term of the specific enthalpy at the inlet, h_i. As shown by the following analysis, the final temperature of the air in the container is higher than the initial temperature. The higher temperature is due to the flow work effect.

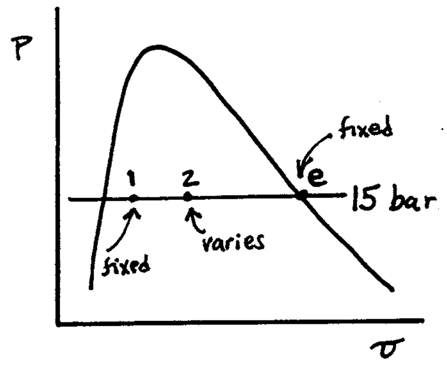
PROBLEM 4.91

KNOWN: A rigid tank of known volume initially containing a two-phase liquid-vapor mixture at a known state is heated, allowing saturated vapor to escape while maintaining constant pressure in the tank until a specified final quality is attained.

FIND: (a) If the final quality is 0.5, determine the mass in the tank and the amount of heat transfer. (b) Plot the mass in the tank and the heat transfer versus final quality ranging from 0.2 to 1.0.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume is shown in the schematic. 2. For the control volume, $\dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. 3. At exit e the state remains constant, as shown in the p-v diagram.

ANALYSIS: The mass rate balance reads $dm_{cv}/dt = -\dot{m}_e$. With the indicated assumptions, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e \Rightarrow \frac{dU_{cv}}{dt} = \dot{Q}_{cv} + h_e \frac{dm_{cv}}{dt}$$

Then, with assumption 3, integration over time results in

$$\Delta U_{cv} = Q_{cv} + h_e \int_1^2 dm_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = Q_{cv} + h_e [m_2 - m_1] \quad \text{, or}$$

$$Q_{cv} = m_2 u_2 - m_1 u_1 - h_e [m_2 - m_1] \qquad (1)$$

where $h_e = 2792.2$ kJ/kg, and

$$v_1 = v_f + x_1 (v_g - v_f)$$

$$= \left(\frac{1.1539}{10^3}\right) + 0.2\left[0.1318 - \frac{1.1539}{10^3}\right] = 0.02728 \frac{m^3}{kg} \Rightarrow m_1 = \frac{V}{v_1} = \frac{0.06 \, m^3}{0.02728 \, m^3/kg} = 2.199 \, kg$$

$$u_1 = u_f + x_1 (u_g - u_f) = 843.16 + 0.2[2594.5 - 843.16] = 1193.4 \, kJ/kg$$

With the same values for v_f, v_g, u_f and u_g, $v_2 = v_f + x_2 (v_g - v_f)$ and $u_2 = u_f + x_2 (u_g - u_f)$. Also, $m_2 = V/v_2$.

(a) $\underline{x = 0.5}$ Then,

$$v_2 = \left(\frac{1.1539}{10^3}\right) + 0.5\left[0.1318 - \frac{1.1539}{10^3}\right] = 0.06648 \frac{m^3}{kg} \Rightarrow m_2 = \frac{0.06 \, m^3}{0.06648 \, m^3/kg} = 0.903 \, kg. \longleftarrow m_2$$

$$u_2 = 843.16 - 0.5[2594.5 - 843.16] = 1718.8 \, kJ/kg$$

$$\Rightarrow Q_{cv} = (0.903)(1718.8) - (2.199)(1193.4) - 2792.2(0.903 - 2.199)$$

$$= 2546.5 \, kJ \longleftarrow Q_{cv}$$

PROBLEM 4.91 (Cont'd.)

(b) Data are obtained to make the required plots using *IT*, as follows:

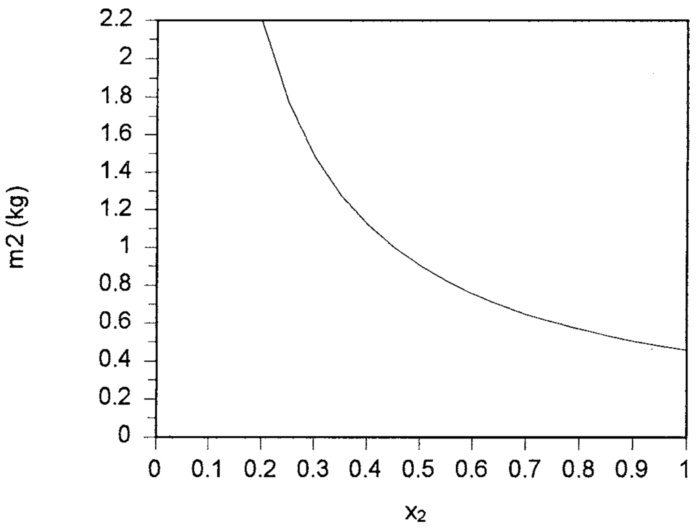
IT Code

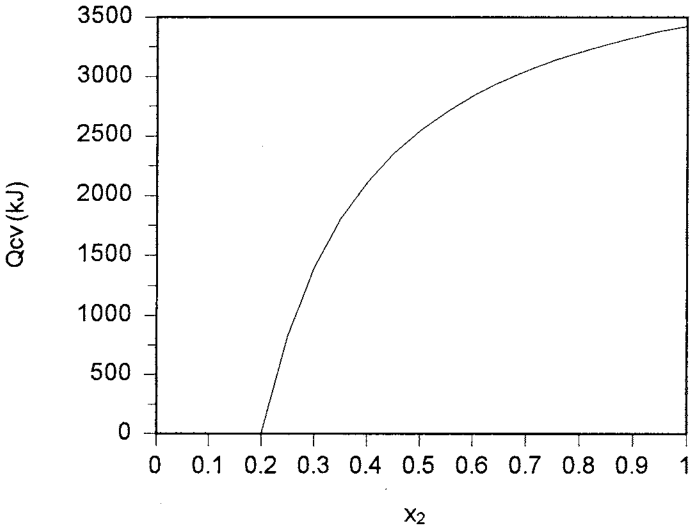
```
V = 0.06  // m³
p = 15  // bar
x1 = 0.2
x2 = 0.5

m1 = V / v1
m2 = V / v2
Qcv = m2 * u2 - m1 * u1 - he * (m2 - m1)

he = hsat_Px("Water/Steam", p, 1)
u1 = usat_Px("Water/Steam", p, x1)
u2 = usat_Px("Water/Steam", p, x2)
v1 = vsat_Px("Water/Steam", p, x1)
v2 = vsat_Px("Water/Steam", p, x2)
```

IT Result (x_2 = 0.5)

Q_{cv} = 2547 kJ

The result for $x_2 = 0.5$ compares very favorably with the result of part (a). Now, using the Exlope button, sweep x_2 from 0.2 to 1.0 in steps of 0.05. The following plots are constructed from the data:





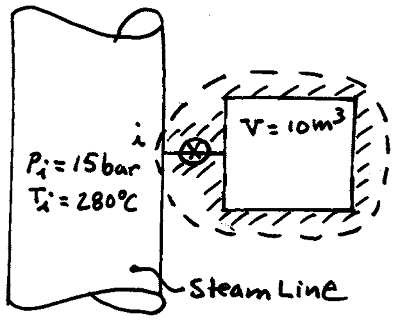
We see from the plots that the heat transfer increases rapidly with x_2 and that the mass in the tank drops rapidly as vapor escapes.

PROBLEM 4.92

KNOWN: An initially-evacuated, well-insulated tank is filled from a large steam line until the pressure in the tank attains a specified value, p.

FIND: (a) Determine the mass and temperature in the tank when p = 15 bar. (b) Plot the mass and temperature in the tank versus p ranging from 0.1 to 15 bar.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume is shown in the schematic. 2. For the control volume, $\dot{W}_{cv} = 0$, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. 3. At inlet i, the state remains constant.

ANALYSIS: The mass rate balance reads $dm_{cv}/dt = \dot{m}_i$. With the indicated assumptions, the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_i h_i \Rightarrow \frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt}$$

Since h_i is constant (assumption 3), integration over time gives

$$\Delta U_{cv} = h_i \Delta m_{cv} \Rightarrow m_2 u_2 - \overset{0}{\cancel{m_1}} u_1 = h_i (m_2 - \overset{0}{\cancel{m_1}}) \Rightarrow \quad u_2 = h_i \qquad (1)$$

(a) p = 15 bar. From Table A-4, h_i = 2992.7 kJ/kg. Then, interpolation at 15 bar with u_2 = 2992.7 kJ/kg ; T_2 = 425°C, v_2 = 0.211 m³/kg. Thus, the final mass is T_2

$$m_2 = \frac{V}{v_2} = \frac{10\ m^3}{0.211\ m^3/kg} = 47.39\ kg \qquad m_2$$

(b) IT is used as follows:

IT Code

```
V = 10  // m³
pline = 15  // bar
Tline = 280  // °C
p = 15  // bar
u2 = u_PT("Water/Steam", p, T2)
hi = h_PT("Water/Steam", pline, Tline)
u2 = hi
v2 = v_PT("Water/Steam", p, T2)
m2 = V / v2
```

IT Results (p = 15 bar)

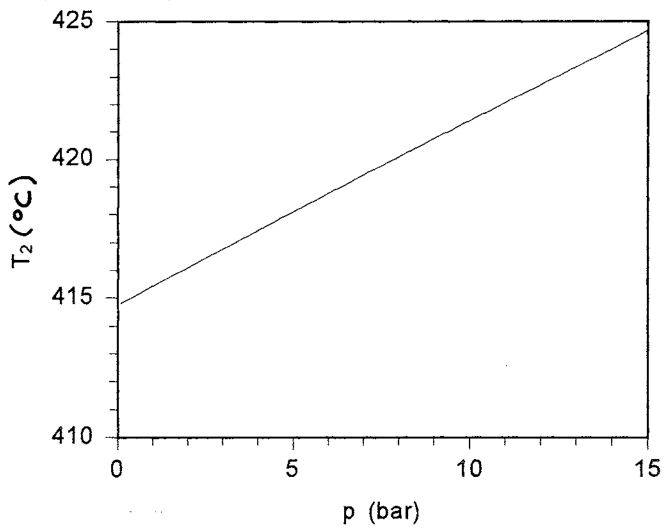
T_2 = 424.7 °C

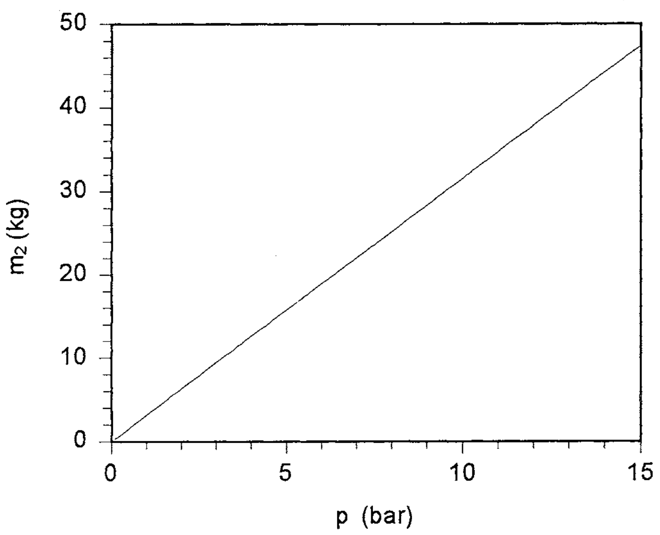
m_2 = 47.39 kg

v_2 = 0.211 m³/kg

These results compare very favorably with the results of part (a).

Now, using the Explore button, sweep p from 0.1 to 15 bar, in steps of 0.1. The following plots are constructed from the data:





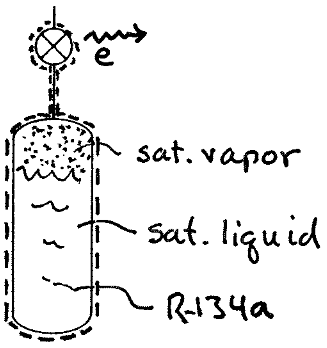
PROBLEM 4.93*

KNOWN: A storage tank containing Refrigerant 134a develops a leak allowing saturated vapor to escape until the level of saturated liquid in the tank has dropped to a known value.

FIND: Determine the mass of refrigerant that has escaped and the heat transfer.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) For the control volume, $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The pressure is constant.



$p = 100\ \text{lbf/in}^2$
$V = 2\ \text{ft}^3$
$V_{f_1} = 1.6\ \text{ft}^3$
$V_{f_2} = 0.8\ \text{ft}^3$

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. Integrating

$$\int_1^2 \dot{m}_e dt = m_1 - m_2 \quad \text{(mass escaped)}$$

To get the masses, use the given volumes and data from Table A-11E at a pressure of 100 lbf/in²

$$m_{f_1} = \frac{V_{f1}}{v_f} = \frac{(1.6\ \text{ft}^3)}{(0.01332\ \text{ft}^3/\text{lb})} = 120.12\ \text{lb}$$

$$m_{g_1} = \frac{V_{g_1}}{v_g} = \frac{V - V_{f1}}{v_g} = \frac{2 - 1.6}{0.4747} = 0.84\ \text{lb}$$

$$m_1 = m_{f_1} + m_{g_1} = 120.96\ \text{lb}$$

Similarly for state 2

$$m_{f_2} = 60.06\ \text{lb}\ ,\ m_{g2} = 2.53\ \text{lb}\ ;\ m_2 = 62.59\ \text{lb}$$

and

$$\int_1^2 \dot{m}_e dt = 120.96 - 62.59 = 58.37\ \text{lb} \longleftarrow \text{mass escaped}$$

With assumptions (1) and (2), the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Combining the mass and energy rate balances and integrating

$$Q_{cv} = m_2 u_2 - m_1 u_1 - h_e(m_2 - m_1)$$

From above, $x_1 = m_{g_1}/m_1 = 0.00694$ and $x_2 = m_{g2}/m_2 = 0.04042$. Thus, with data from Table A-11E

$$u_1 = u_f + x_1(u_g - u_f) = 37.21\ \text{Btu/lb}\ ;\ u_2 = 39.46\ \text{Btu/lb}$$

and $h_e = h_{g\,@\,100\ \text{lbf/in}^2} = 112.46\ \text{Btu/lb}$

Thus

$$Q_{cv} = (62.59)(39.46) - (120.96)(37.21) - (112.46)(62.59 - 120.96)$$

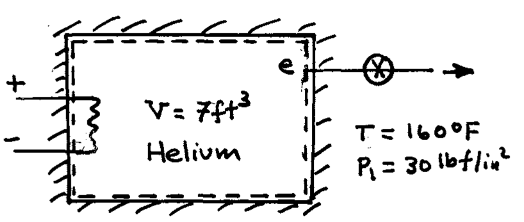
$$= 4533.2\ \text{Btu} \longleftarrow Q_{cv}$$

PROBLEM 4.94

KNOWN: Helium is withdrawn slowly from a well-insulated tank of known volume until the pressure within the tank is p. The temperature within the tank is kept constant by an electrical resistor.

FIND: (a) When p = 18 lbf/in.², determine the mass of helium withdrawn and the energy input to the resistor. (b) Plot the quantities of part (a) versus p ranging from 15 to 30 lbf/in.²

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume is shown in the schematic. 2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects can be ignore. 3. Helium is modeled as an ideal gas. 4. The mass of the resistor is small enough to be ignored.

ANALYSIS: The mass of helium withdrawn over a time interval equals the difference between the initial amount of mass in the tank and the mass in the tank at the later time: (Amount withdrawn) = $m_1 - m_2$. Since $T_1 = T_2$, the ideal gas equation of state can be used to rewrite this as follows, where $P_2 = p$:

$$\begin{pmatrix}\text{Amount}\\ \text{withdrawn}\end{pmatrix} = \frac{P_1 V}{RT} - \frac{p V}{RT} = (P_1 - p)\frac{V}{RT} \tag{1}$$

$$= (30-p)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\text{ in.}^2}{1\text{ ft}^2}\right| \frac{(7\text{ ft}^3)}{\left[\frac{1545}{4.003}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right](620^\circ\text{R})}$$

$$= (30-p)(4.21\times 10^{-3})\ \text{lb} \tag{2}$$

The mass rate balance is $dm_{cv}/dt = -\dot{m}_e$. The energy rate balance reduces to $dU_{cv}/dt = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} - \dot{m}_e h_e$. Introducing the mass rate balance, and noting that h_e remains constant because temperature is constant,

$$\frac{dU_{cv}}{dt} = -\dot{W}_{cv} + h_e\frac{dm_{cv}}{dt} \Rightarrow \Delta U_{cv} = -W_{cv} + h_e \Delta m_{cv} \Rightarrow (-W_{cv}) = m_2u_2 - m_1u_1 - (m_2 - m_1)h_e.$$

Since temperature is constant, $u_2 = u(T)$, $u_1 = u(T)$, $h_e = u(T) + (pv)_e$. Thus

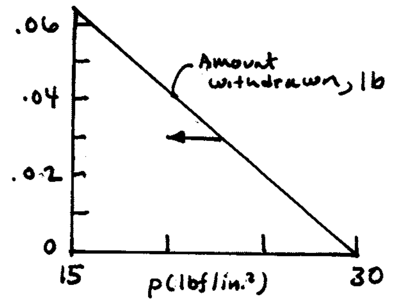
① $(-\dot{W}_{cv}) = (m_1 - m_2)(pv)_e$ $\Rightarrow$ with the ideal gas equation of state, $(-W_{cv}) = (m_1 - m_2)RT$.

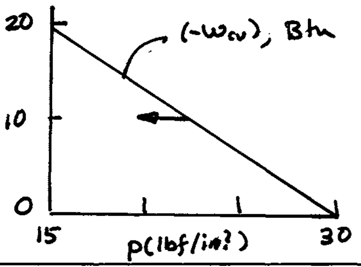
By Eq.(1) $(m_1 - m_2) = (P_1 - p)V/RT$, giving $(-W_{cv}) = (P_1 - p)V$. That is,

$$(-W_{cv}) = (30-p)\left(\frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\text{ in}^2}{1\text{ ft}^2}\right|(7\text{ ft}^3)\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right| = 1.296(30-p)\ \text{Btu} \tag{3}$$

(a) When p = 18 lbf/in², Eq.(2) gives (Amt. withdrawn) = 0.0505 kg, Eq.(3) gives $(-W_{cv})$ = 15.55 Btu.

(b) PLOTS





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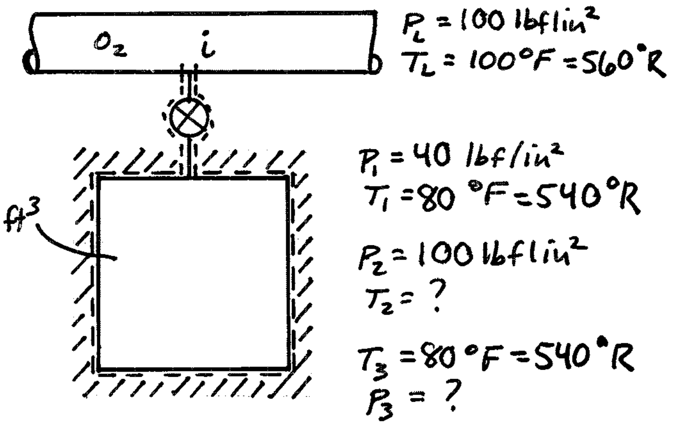
1. To maintain the temperature within the tank constant, the resistor must provide energy to the helium in the tank equal to the energy carried out by flow work at e.

PROBLEM 4.95*

KNOWN: An insulated tank containing oxygen is connected to a supply line carrying oxygen. A valve between the tank and the line is opened and gas flows into the tank until the pressure reaches that of the line. The valve is closed and eventually the tank contents cool back to their initial temperature.

FIND: Determine (a) the tank temperature when the valve closes and (b) the final pressure in the tank.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) For the control volume shown, $\dot{W}_{cv} = 0$. (2) There is no heat transfer during the filling process. (3) Kinetic and potential energy effects are negligible. (4) The state of the O_2 in the supply line remains constant. (5) The O_2 behaves as an ideal gas.

ANALYSIS: (a) Consider first the filling process. The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$ and with assumptions (1), (2), and (3), the energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_i h_i$$

The specific enthalpy in the supply line is constant. Combining the mass and energy rate balances, and integrating

$$m_2 u_2 - m_1 u_1 = h_i (m_2 - m_1)$$

Using the ideal gas equation of state

$$m_1 = \frac{P_1 V}{R T_1} \quad , \quad m_2 = \frac{P_2 V}{R T_2}$$

Combining and rearranging gives

$$u_2 = h_i + \frac{T_2}{T_1} \cdot \frac{P_1}{P_2} (u_1 - h_i)$$

From Table A-23E

$$h_i = \left(\frac{3886.6 \text{ Btu/lbmol}}{32.00 \text{ lb/lbmol}} \right) = 121.46 \text{ Btu/lb}$$

$$u_1 = 2673.8/32.00 = 83.56 \text{ Btu/lb}$$

Inserting values

$$u_2 = 121.46 - (0.02807) T_2$$

Since u_2 depends on T_2, the final temperature can be found by iteration using data from Table A-23E and this expression. The result is

$$T_2 \approx 661°R = 201°F \longleftarrow T_2$$

PROBLEM 4.95 (Cont'd)

(b) To determine the pressure after the contents of the tank cool off, first find the mass in the tank after the valve closes

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(100\ \text{lbf/in}^2)(5\ \text{ft}^3)}{\left(\frac{1545}{32.00}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(661\ {}^\circ\text{R})} \left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|$$

$$= 2.256\ \text{lb}$$

Thus, the final pressure is

$$P_3 = \frac{m_2 R T_3}{V}$$

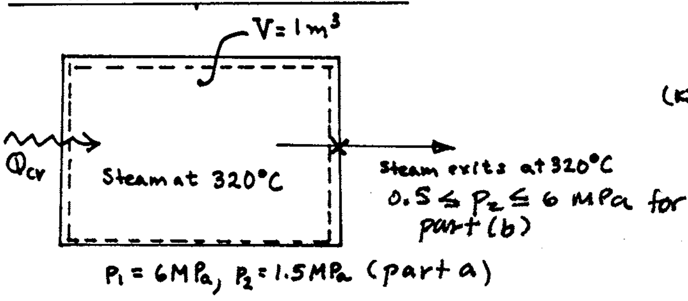
$$= \frac{(2.256)\left(\frac{1545}{32.00}\right)(540)}{(5)\,|144|} = 81.7\ \text{lbf/in}^2 \longleftarrow P_2$$

PROBLEM 4.96

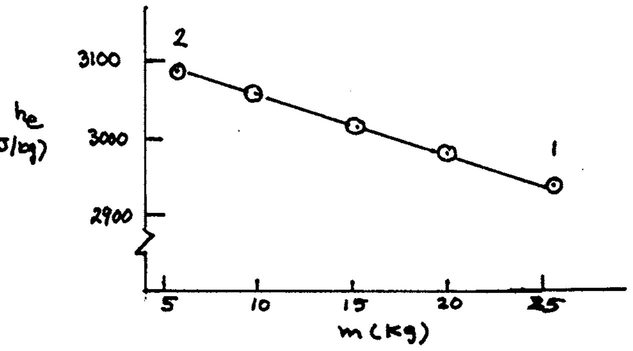
KNOWN: A tank of known volume initially contains steam at a specified state. Steam is withdrawn slowly until the pressure drops to pressure p. The temperature is kept constant by heat transfer to the tank contents.

FIND: (a) Determine the heat tranfer for $p = 1.5$ MPa. (b) Plot the heat transfer versus p ranging from 0.5 to 6 MPa.

SCHEMATIC & GIVEN DATA:



V = 1 m³
$\dot{Q}_{cv}$
Steam at 320°C
Steam exits at 320°C
0.5 ≤ p₂ ≤ 6 MPa for part (b)
P₁ = 6 MPa, P₂ = 1.5 MPa (part a)



ASSUMPTIONS: (1) For the control volume shown in the figure, $\dot{W}_{cv} = 0$ and kinetic and potential energy effects can be ignored. (2) At each instant, pressure is uniform throughout the steam.

ANALYSIS: (a) The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. Using the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e$$

Introducing the mass rate balance and integrating

$$\Delta U_{cv} = Q_{cv} + \int_1^2 h_e dm_{cv} \Rightarrow Q_{cv} = m_2 u_2 - m_1 u_1 - \int_1^2 h_e dm \qquad (1)$$

where m denotes the mass contained within the tank. At any instant, $m = V/v$ where v is specific volume at that instant determined by 320°C and the tank pressure. Initially, $p_1 = 6$ MPa, so Table A-4 gives $v_1 = 0.03876\ \text{m}^3/\text{kg}$, $u_1 = 2720$ kJ/kg. Finally, $p_2 = 1.5$ MPa, so $v_2 = 0.1765\ \text{m}^3/\text{kg}$, $u_2 = 2817.1$ kJ/kg. Thus

$$m_1 = \frac{V}{v_1} = \frac{1\ \text{m}^3}{0.03876\ \text{m}^3/\text{kg}} = 25.8\ \text{kg} \quad , \quad m_2 = \frac{V}{v_2} = \frac{1\ \text{m}^3}{0.1765\ \text{m}^3/\text{kg}} = 5.67\ \text{kg} \qquad (2)$$

For each of several pressures in the interval $1.5\ \text{MPa} < P < 6.0$ MPa, the mass within the tank can be calculated in like manner and the specific enthalpy h_e determined from Table A-4 at 320°C and the specified pressure. These values allow the plot of h_e vs m given above to be constructed. The area under the line from 1 to 2 equals the term $-\int_1^2 h_e dm$ of Eq. (1). As this variation is very nearly linear, the average value of h_e can be used to evaluate this term:

$$-\int_1^2 h_e dm = \left(\frac{(h_e)_1 + (h_e)_2}{2}\right)(m_1 - m_2) = \left(\frac{3081.9 + 2952.6}{2}\right)(25.8 - 5.67)$$

$$= \left(3017.25\ \frac{\text{kJ}}{\text{kg}}\right)(20.13\ \text{kg}) = 60737\ \text{kJ}$$

Finally, inserting values into Eq. (1)

$$Q_{cv} = (5.67\ \text{kg})\left(2817.1\ \frac{\text{kJ}}{\text{kg}}\right) - (25.8\ \text{kg})\left(2720\ \frac{\text{kJ}}{\text{kg}}\right) + 60737\ \text{kJ}$$

$$= 6534\ \text{kJ} \longleftarrow Q_{cv}$$

PROBLEM 4.96 (Cont'd.)

(b) Data are obtained to make the required plots using IT, as follows:

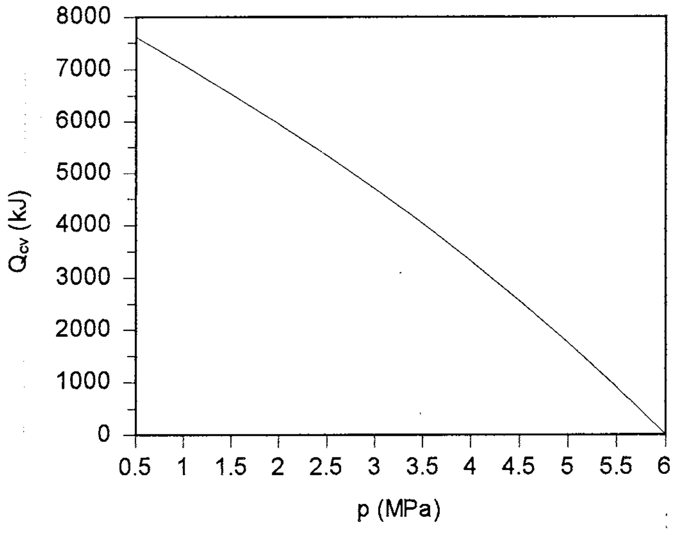
<u>*IT* Code</u>

```
V = 1  // m³
p1 = 60   // bar
T = 320  // °C
p = 15   // bar
pp = p / 10  // MPa

Qcv = m * u - m1 * u1 - ((he1 + he) / 2) * (m - m1)

he1 = h_PT("Water/Steam", p1, T)
u1 = u_PT("Water/Steam", p1, T)
v1 = v_PT("Water/Steam", p1, T)
m1 = V / v1
he = h_PT("Water/Steam", p,T)
u = u_PT("Water/Steam",p,T)
v = v_PT("Water/Steam", p, T)
m = V / v
```

<u>Key *IT* Results (p_2 = 1.5 MPa)</u>

Q_{cv} = 6534 kJ
h_{e2} = 3081 kJ/kg
h_{e1} = 2952 kJ/kg
m_2 = 5.666 kg
m_1 = 25.8 kg
u_2 = 2817 kJ/kg
u_1 = 2720 kJ/kg
v_2 = 0.1765 m³/kg
v1 = 0.03876 m³/kg

The results for p_2 = 1.5 MPa compare very favorably with the results of part (a). Now, using the Explore button, sweep p from 0.5 to 6 MPa in steps of 0.5. The following plot is constructed from the data:



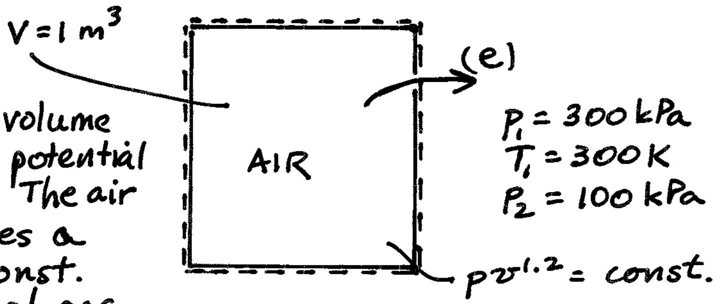
Note that the heat transfer increases as the pressure in the tank drops, as expected.

PROBLEM 4.97

KNOWN: Air escapes slowly from a tank. The initial state and final pressure are known. The that remains in the tank undergoes a polytropic process.

FIND: Determine the heat transfer for a control volume enclosing the tank.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) For the control volume shown, $\dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The air remaining in the tank undergoes a process described by $pv^{1.2} = \text{const.}$ (4) The air is modeled as an ideal gas with constant specific heats.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot{m}_e$. With the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} - \dot{m}_e h_e \qquad (1)$$

Since the process occurs slowly, the state of the air in the tank is uniform at any time. Thus, $U_{cv} = mu$, and $h_e = u + RT$. Inserting these expressions into (1) along with the mass rate balance

$$m\frac{du}{dt} + \cancel{u\frac{dm}{dt}} = \dot{Q}_{cv} + (\cancel{u} + RT)\frac{dm}{dt}$$

Thus

$$\dot{Q}_{cv}dt = m\,du - RT\,dm = m\,c_v\,dT - RT\,dm \qquad (2)$$

Each term on the right side of (2) can be expressed in terms of pressure, as follows. First, from $pv^{1.2} = \text{constant}$

$$m = \frac{V}{v} = \left(\frac{p}{\text{const}}\right)^{\frac{1}{1.2}} V = c\,p^{\frac{1}{1.2}}$$

and

$$dm = c\left(\frac{1}{1.2}\right) p^{\left(\frac{1}{1.2}-1\right)} dp \qquad (3)$$

Further

$$RT = \frac{pV}{m} = \frac{pV}{c\,p^{1/1.2}} = \frac{V}{c}\,p^{\left(1-\frac{1}{1.2}\right)} \qquad (4)$$

and

$$dT = \frac{V}{Rc}\left(1 - \frac{1}{1.2}\right) p^{(-1/1.2)} dp \qquad (5)$$

Substituting (3), (4) and (5) into (2) gives

$$\begin{aligned}\dot{Q}_{cv}dt &= \left[c\,p^{\frac{1}{1.2}}\right] c_v \left[\frac{V}{Rc}\left(1-\frac{1}{1.2}\right)p^{\left(-\frac{1}{1.2}\right)}dp\right] \\ &\quad - \left[\frac{V}{c}\,p^{\left(1-\frac{1}{1.2}\right)}\right]\left[c\left(\frac{1}{1.2}\right)p^{\left(\frac{1}{1.2}-1\right)}dp\right] \\ &= \left[\frac{c_v}{R}\left(1-\frac{1}{1.2}\right) - \left(\frac{1}{1.2}\right)\right] V\,dp\end{aligned}$$

Integrating

$$Q_{cv} = \left[\frac{c_v}{R}\left(1 - \frac{1}{1.2}\right) - \left(\frac{1}{1.2}\right)\right] V\,(p_2 - p_1)$$

PROBLEM 4.97 (Cont'd)

From Table A-20, $c_v \approx 0.717$ kJ/kg·K. Thus

$$Q_{cv} = \left[\frac{(0.717\ \text{kJ/kg}\cdot\text{K})}{\left(\frac{8.314}{28.97}\cdot\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\left(1-\frac{1}{1.2}\right)-\left(\frac{1}{1.2}\right)\right](1\,\text{m}^3)(100-300)\,\text{kPa}\left|\frac{10^3\,\text{N/m}^2}{1\,\text{kPa}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

① $= 83.39$ kJ ← Q_{cv}

---

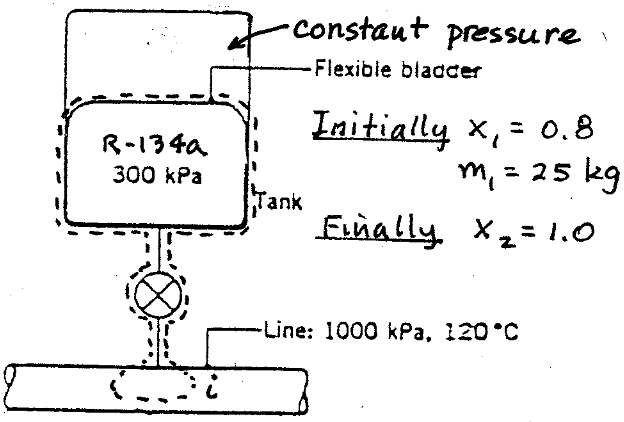
1. The positive sign denotes that the energy transfer is into the control volume. For a different value of polytropic exponent, the value would be different.

PROBLEM 4.98

KNOWN: A well-insulated tank containing R-134a is connected to a supply line. As refrigerant is allowed to flow into the tank, a flexible bladder in the tank expands to maintain the refrigerant in the tank at constant pressure.

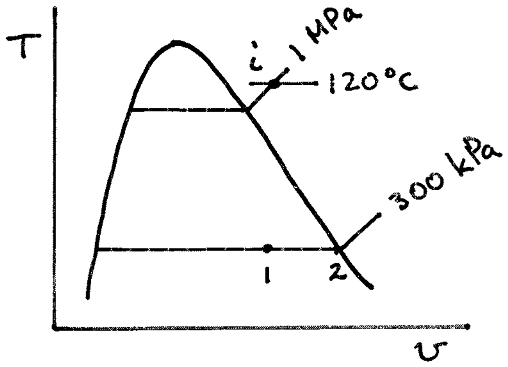
FIND: Determine the amount of mass admitted to the tank between the initial time and the instant when all the liquid in the tank is vaporized.

SCHEMATIC & GIVEN DATA:



Initially $x_1 = 0.8$
$m_1 = 25$ kg

Finally $x_2 = 1.0$



ASSUMPTIONS: (1) The control volume is shown with $\dot{Q}_{cv} = 0$. (2) Conditions in the supply line remain constant. (3) The pressure remains constant in the tank. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The mass rate balance takes the form; $dm_{cv}/dt = \dot{m}_i$. With the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}_i h_i$$

The specific enthalpy h_i is constant by assumption (2). Thus, combining the mass and energy rate balances and integrating

$$\Delta U_{cv} = -W_{cv} + \int_{t_1}^{t_2} \dot{m}_i h_i \, dt$$

Since h_i is constant

$$\Delta U_{cv} = -W_{cv} + h_i \int_{t_1}^{t_2} \dot{m}_i \, dt = -W_{cv} + h_i (m_2 - m_1) \qquad (1)$$

To evaluate the work, note that the pressure in the tank is constant. Thus

$$W_{cv} = \int p \, dV = p(V_2 - V_1) = p(m_2 v_2 - m_1 v_1) \qquad (2)$$

Combining (1) and (2), and noting that $\Delta U_{cv} = m_2 u_2 - m_1 u_1$

$$m_2 u_2 - m_1 u_1 = -p(m_2 v_2 - m_1 v_1) + h_i (m_2 - m_1)$$

or

$$m_2 [(u_2 + p v_2) - h_i] = m_1 [(u_1 + p v_1) - h_i]$$

$$m_2 [h_2 - h_i] = m_1 [h_1 - h_i]$$

PROBLEM 4.98 (Contd.)

Solving for m_2

$$m_2 = m_1\left(\frac{h_1 - h_i}{h_2 - h_i}\right)$$

Using data from Table A-11 at 3 bar: $h_f = 50.85$, $h_g = 247.59$ kJ/kg

$$h_1 = (50.85) + (0.81)[247.59 - 50.85] = 208.24 \text{ kJ/kg}$$

$$h_2 = 247.59 \text{ kJ/kg}$$

From Table A-12, $h_i = 356.52$ kJ/kg. Thus

$$m_2 = 25\text{ kg}\left(\frac{208.24 - 356.52}{247.59 - 356.52}\right) = 34.03 \text{ kg}$$

Finally,

$$\Delta m = m_2 - m_1$$

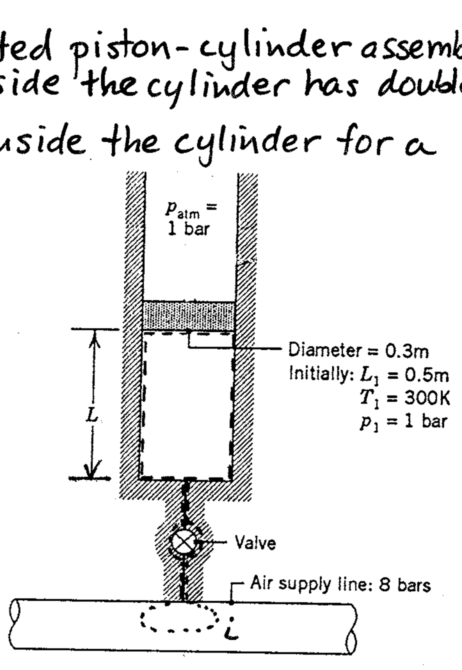
$$= 34.03 - 25 = 9.03 \text{ kg} \qquad \longleftarrow$$

PROBLEM 4.99

KNOWN: Air is admitted slowly into an insulated piston-cylinder assembly from a supply line until the volume inside the cylinder has doubled.

FIND: Plot the final temperature and mass inside the cylinder for a given range of supply temperatures.

SCHEMATIC & GIVEN DATA:



$300 \leq T_{supply} \leq 500K$

ASSUMPTIONS: (1) For the control volume shown, $\dot{Q}_{cv} = 0$. (2) Kinetic and potential energy effects can be neglected. (3) The weight of the piston and friction between the piston and cylinder wall can be neglected. (4) The air behaves as an ideal gas.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = \dot{m}_i$. With the assumptions listed, the energy balance is

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}_i h_i$$

The line conditions are constant, so h_i is constant. Combining the mass and energy rate balances and integrating

$$m_2 u_2 - m_1 u_1 = -W_{cv} + h_i (m_2 - m_1) \qquad (1)$$

To evaluate the work, note that the pressure in the cylinder is always atmospheric since the process is slow and since assumption (3) applies. Thus

$$W_{cv} = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1)$$

From the given data

$$V_1 = \left(\frac{\pi d_{pist}^2}{4}\right) L_1 = \left(\frac{\pi (.3^2) m^2}{4}\right)(0.5\,m) = 0.03534\,m^3$$

and $V_2 = 0.07068\ m^2$

$$W_{cv} = (1\,bar)(0.07068 - 0.03534) m^3 \left|\frac{10^5 N/m^2}{1\,bar}\right| \left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 3.534\ kJ$$

From Table A-22, at $T_1 = 300K$; $u_1 = 214.07$ kJ/kg. Also, $h_i = h_i(T_{supply})$. For $T_{supply} = 300K$, $h_i = 300.19$ K.

Using the ideal gas model

$$m_1 = \frac{P_1 V_1}{R T_1} = \frac{(1\,bar)(0.03534\,m^3)}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300K)} \left|\frac{10^5 N/m^2}{1\,bar}\right| \left|\frac{1\,kJ}{10^3 N\cdot m}\right| = 0.04105\ kg$$

and

$$m_2 = \frac{P_2 V_2}{R T_2} = \frac{(1)(0.07068)|100|}{\left(\frac{8.314}{28.97}\right)(T_2)} = \frac{24.628}{T_2} \qquad (2)$$

Incorporating these results into (1) and rearranging

PROBLEM 4.99 (Cont'd)

$$m_2 u_2 - h_i (m_2 - m_1) = m_1 u_1 - W_{cv}$$

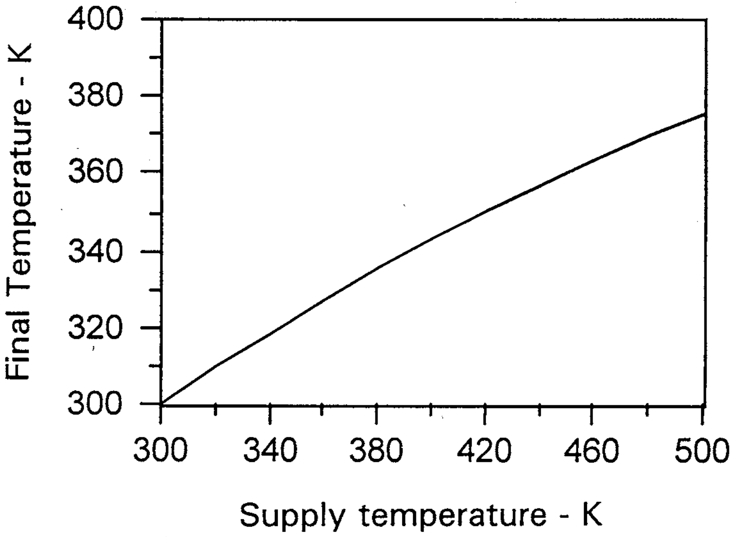
$$\frac{24.628\, u_2}{T_2} - h_i \left(\frac{24.628}{T_2} - 0.04105 \right) = 5.254 \qquad (3)$$

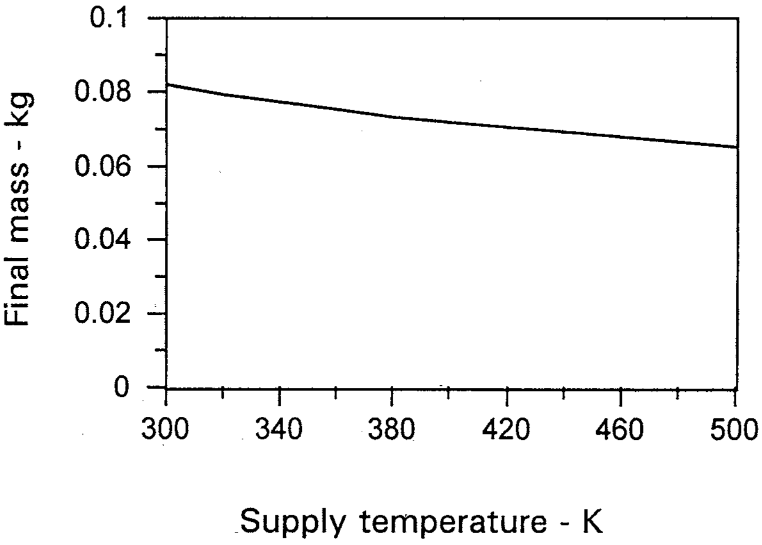
Equations (2) and (3) can be solved for given values of h_i (T_{supply}) by an iterative procedure and data from Table A-22. For $T_{supply} = 300K$,

$T_2 = 300K$

$m_2 = 0.0821$ kg

Plotting for the range of T_{supply} values





PROBLEM 4.100

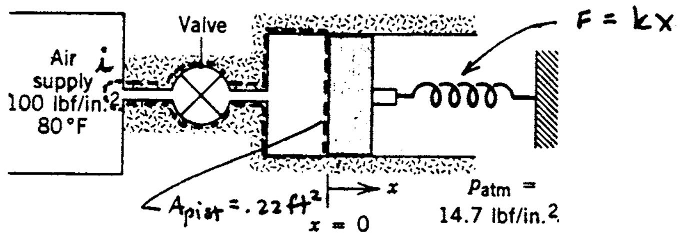
KNOWN: A well-insulated piston-cylinder assembly is connected by a valve to an air supply. The supply conditions and the initial state of air inside the cylinder. Air is admitted slowly causing the piston to compress a spring. The initial and final volumes within the cylinder are known.

FIND: Plot the final pressure and final temperature within the cylinder versus spring constant k varying from 650 to 750 lbf/ft.

SCHEMATIC & GIVEN DATA:

Initially: $p_1 = 14.7$ lbf/in^2
$T_1 = 80°F$
$V_1 = 0.1$ ft^3

Finally: $V_2 = 0.4$ ft^3



ASSUMPTIONS: (1) The control volume is shown on the accompanying diagram, with $\dot{Q}_{cv} = 0$. (2) Conditions in the air supply remain constant. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas. (5) There is no friction between the piston and cylinder wall.

ANALYSIS: The final pressure is found by applying Newton's Law to the piston. Since air is admitted slowly; $\Sigma F_x = 0$. Thus

$$p A_{pist} = p_{atm} A_{pist} + kx$$

with $V - V_1 = x A_{pist}$, the pressure is

$$p = p_{atm} + \frac{k(V-V_1)}{A_{pist}^2} \qquad (a)$$

At $V = V_2$

$$p_2 = 14.7 \frac{\text{lbf}}{\text{in}^2} + \frac{(k \text{ lbf/ft})(0.4 - 0.1)\text{ft}^3}{(.22^2)\text{ ft}^4}\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)$$

$$= \left[14.7 + 0.043\,k\right] \text{lbf/in}^2 \qquad (1)$$

The mass rate balance takes the form; $dm_{cv}/dt = \dot{m}_i$. With assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \dot{W}_{cv} + \dot{m}_i h_i$$

The specific enthalpy h_i is constant by assumption (3). Thus, combining the mass and energy rate balances and integrating

$$\Delta U_{cv} = -W_{cv} - \int_1^2 h_i\, dm_{cv} \Rightarrow m_2 u_2 - m_1 u_1 = -W_{cv} + h_i(m_2 - m_1) \qquad (2)$$

Since the process occurs slowly, $W_{cv} = \int p\,dV$. With Eq. (a) above

$$W_{cv} = \int_{V_1}^{V_2} \left[\left(p_{atm} - \frac{kV_1}{A_{pist}^2}\right) + \frac{kV}{A_{pist}^2}\right] dV$$

$$= \left(p_{atm} - \frac{kV_1}{A_{pist}^2}\right)(V_2 - V_1) + \frac{k(V_2^2 - V_1^2)}{2 A_{pist}^2}$$

PROBLEM 4.100 (Contd.)

$$\therefore W_{cv} = p_{atm}(V_2 - V_1) + \frac{k}{(A_{pist})^2}\left[\frac{V_2^2 - V_1^2}{2} - V_1(V_2 - V_1)\right]$$

$$= p_{atm}(V_2 - V_1) + \frac{k}{2(A_{pist})^2}\left[V_2 - V_1\right]^2$$

$$= \left[14.7\,\frac{\text{lbf}}{\text{in}^2}\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right|(0.3\,\text{ft}^3) + \frac{k\,(\text{lbf/ft})}{2(0.22\,\text{ft}^2)^2}\left[0.3\,\text{ft}^3\right]^2\right]\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right|$$

$$= \left[0.816 + (1.195\times10^{-3})k\right]\ \text{Btu} \qquad (3)$$

Also, with the ideal gas model equation of state

$$m_1 = \frac{p_1 V_1}{R T_1} = \frac{(14.7\times144\ \text{lbf/ft}^2)(0.1\,\text{ft}^3)}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(540^\circ\text{R})} = 7.35\times10^{-3}\ \text{lb}$$

$$m_2 = \frac{p_2 V_2}{R T_2}\ \text{, where } p_2 \text{ is given by Eq. (1)} \qquad (4)$$

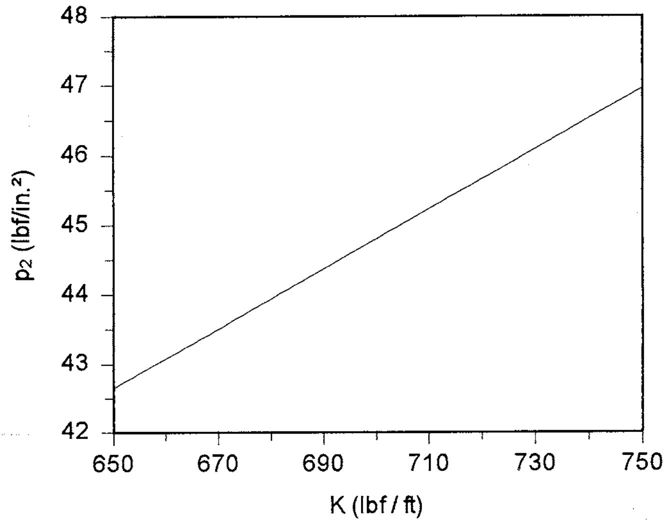
Since $T_1 = T_i = 540^\circ R\ (80^\circ F)$, $u_1 = u(540^\circ R)$, $h_i = h(540^\circ R)$.
Accordingly, T_2 can be determined by solving Eq. (2), together with Eqs. (1), (3), (4) and known values of V_2, u_1, and h_i.
This scheme is implemented via the following IT code:

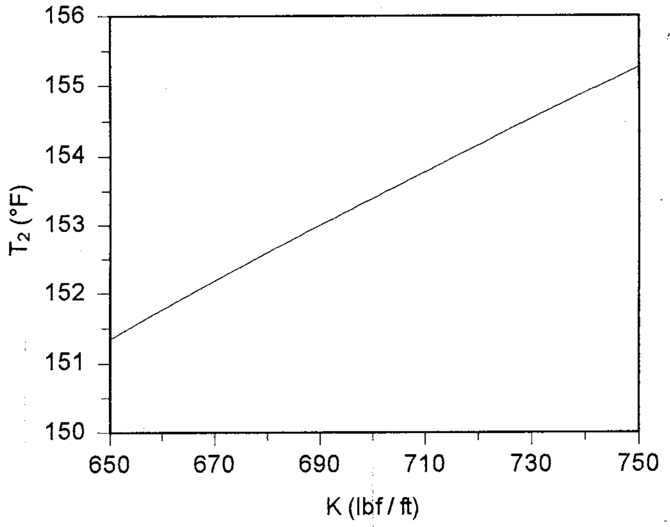
IT Code

```
p1 = 14.7  // lbf/in.²
T1 = 80  // °F
Ti = T1
V1 = 0.1  // ft²
V2 = 0.4  // ft²
K = 650 // lbf/ft

p2 = 14.7 + 0.043 * K
m2 * u2 - m1 * u1 = - Wcv + hi * (m2 - m1)
Wcv = 0.816 + 1.195E-3 * K
m1 = V1 / v1
m2 = V2 / v2

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1,p1)
u2 = u_T("Air", T2)
v2 = v_TP("Air",T2,p2)
hi = h_T("Air", Ti)
```

Now, using the Explore button, sweep K from 650 to 750 lbf/ft in steps of 1, to get the following plots:

PROBLEM 4.100 (Contd.)





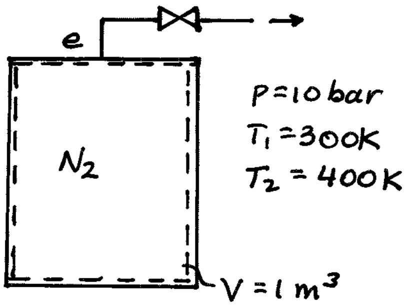
Note that the pressure plot is linear, as indicated by Eq.(1), and could have been readily plotted by hand. However, the temperature plot is non-linear and would be quite difficult to obtain without the use of IT or other software.

PROBLEM 4.101

KNOWN: Heat transfer occurs to nitrogen gas contained in a rigid tank. Gas escapes through a pressure relief valve, maintaining constant pressure in the tank. The initial and final temperatures are specified.

FIND: Determine the mass of nitrogen that escapes and the amount of energy transfer by heat.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) For the control volume shown, $\dot W_{cv} = 0$. (2) The state in the control can be assumed to be uniform at any time during the process. (3) Kinetic and potential energy effects can be neglected. (4) The nitrogen behaves as an ideal gas with constant specific heats evaluated at 350K.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = -\dot m_e$. Thus the mass that escapes is

$$\int_1^2 \dot m_e\, dt = -\int_{m_1}^{m_2} dm_{cv} = m_1 - m_2$$

With the ideal gas equation of state

$$\int_1^2 \dot m_e\, dt = \frac{PV}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$= \frac{(10\text{ bar})(1\text{ m}^3)}{\left(\frac{8.314}{28.97}\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)}\left[\frac{1}{300\text{K}} - \frac{1}{400\text{K}}\right]\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 2.904\text{ kg} \longleftarrow \text{mass escaped}$$

With the assumptions listed, the energy rate balance is

$$\frac{dU_{cv}}{dt} = \dot Q_{cv} - \dot m_e h_e$$

Noting that $U_{cv} = mu$ and $h_e = u + RT$, and with the mass rate balance

$$m\frac{du}{dt} + \cancel{u}\frac{dm}{dt} = \dot Q_{cv} + (\cancel{u} + RT)\frac{dm}{dt}$$

$$\dot Q_{cv} = m\frac{du}{dt} - RT\frac{dm}{dt}$$

For an ideal gas, $du = c_v\, dT$. Also, with $m = PV/RT$

$$dm = \left(\frac{PV}{R}\right)\frac{dT}{(-T^2)}$$

Thus

$$\dot Q_{cv} = \left(\frac{PV}{RT}\right)c_v\frac{dT}{dt} + \left(\frac{PV}{RT}\right)R\frac{dT}{dt} = \frac{PV}{R}(c_v + R)\frac{1}{T}\frac{dT}{dt}$$

Noting that $c_v + R = c_p$

$$\dot{Q}_{cv}\,dt = \left(\frac{PVc_p}{R}\right)\frac{dT}{T}$$

From Table A-20; $c_p = 1.041$ kJ/kg·K. Integrating and inserting values

$$\int_1^2 \dot{Q}_{cv}\,dt = \left(\frac{PVc_p}{R}\right)\int_{T_1}^{T_2}\frac{dT}{T}$$

$$Q_{cv} = \left(\frac{PVc_p}{R}\right)\ln\frac{T_2}{T_1}$$

$$= \frac{(10\text{ bar})(1\text{ m}^3)(1.041\text{ kJ/kg·K})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg·K}}\right)}\ln\left(\frac{400}{300}\right)\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N·m}}\right|$$

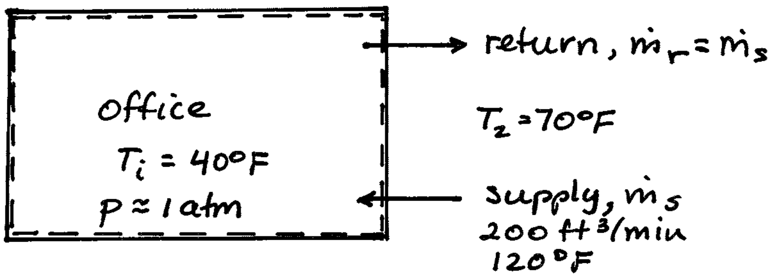
$$= 1043\text{ kJ} \longleftarrow Q_{cv}$$

PROBLEM 4.102

KNOWN: The air supply to an office is shut off overnight and the room temperature drops. In the morning, the thermostat is reset and a known volumetric flow rate of heated air is supplied.

FIND: Estimate the time it takes for the room to reach 70°F, and plot room temperature as a function of time.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) For the control volume shown, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air behaves as an ideal gas with constant specific heats. (3) The pressure is taken as 1atm everywhere. (4) Kinetic and potential energy effects can be neglected. (5) The room air is well-mixed.

ANALYSIS: The mass rate balance takes the form $dm/dt = \dot{m}_s - \dot{m}_r$. With $\dot{m}_s = \dot{m}_r$; $dm/dt = 0$. Thus, the mass in the room is constant with time. The energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}_s h_s - \dot{m}_r h_r$$

or, with $U_{cv} = mu$ and $h_r = h(T)$

$$m\frac{du}{dt} + u\cancel{\frac{dm}{dt}}^{\,0} = \dot{m}_s(h_s - h(T))$$

Further, $du = c_v\,dT$ and $h_s - h(T) = c_p(T_s - T)$. Thus

$$m c_v \frac{dT}{dt} = \dot{m}_s c_p (T_s - T)$$

With $dT = -d(T_s - T)$ and $c_p/c_v = k$

$$-\frac{m\,d(T_s-T)}{k\,(T_s-T)} = \dot{m}_s\,dt \qquad (1)$$

Evaluating $\dot{m}_s$

$$\dot{m}_s = \frac{(AV)_s}{v_s} = \frac{p\,(AV)_s}{RT_s}$$

$$= \frac{(14.7\ \text{lbf/in}^2)(200\ \text{ft}^3/\text{min})}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(580°\text{R})}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| = 13.69\ \text{lb/min}$$

and

$$m = \frac{pV}{RT} = \frac{(14.7)(20{,}00)\,|144|}{\left(\frac{1545}{28.97}\right)(515)} = 154\ \text{lb}$$

where the average of the initial and final temperatures is used to estimate the mass. Also, from A-20E ; $k = 1.4$.

PROBLEM 4.102 (Cont'd)

Integrating Eq. (1) from $t=0$ ($T=T_i=40°F$) to any time t

$$-\left(\frac{m}{k\cdot \dot{m}_s}\right)\ln\left(\frac{T_s-T}{T_s-T_i}\right)=t$$

or, solving for T

$$T=T_s-(T_s-T_i)\exp\left[\left(-\frac{\dot{m}_s k}{m}\right)t\right]$$

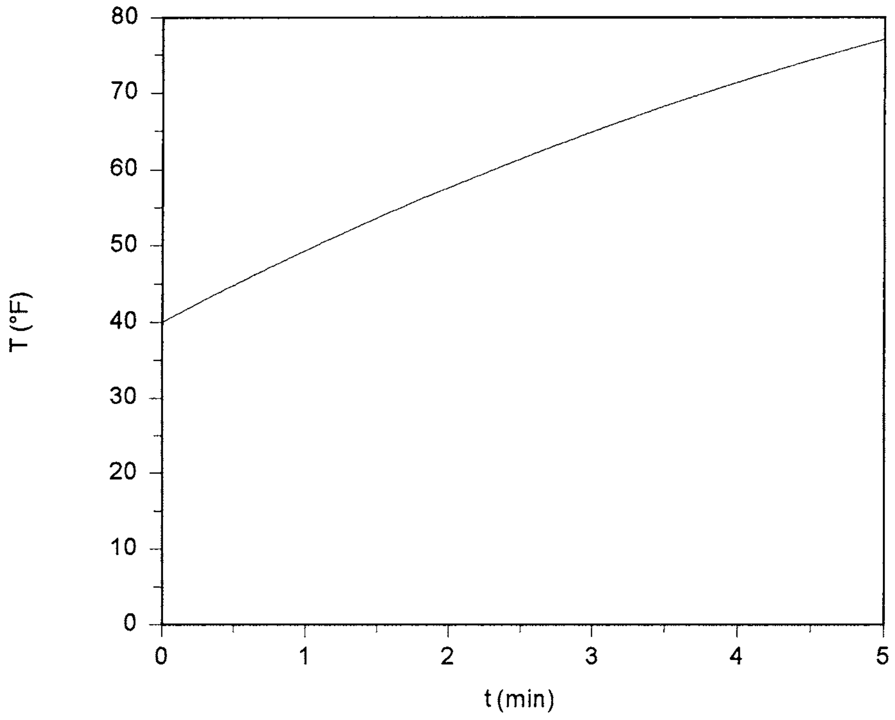
Inserting values

$$T=120-80\exp\left[(-0.1245)t\right] \qquad (2)$$

Solving for t_2 when $T_2=70°F$ gives

$$t_2=3.79\ \text{min} \longleftarrow \qquad t_2$$

Eq.(2) can readily be plotted using software. The following plot was constructed using IT:

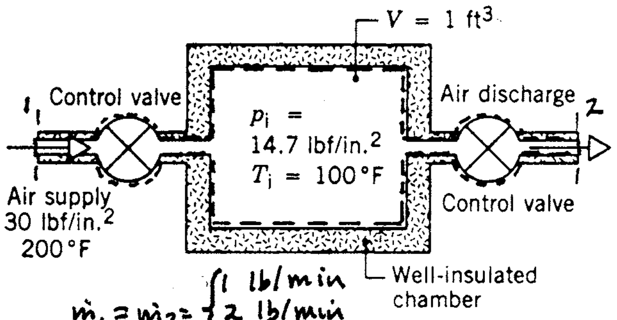


PROBLEM 4.103

KNOWN: Air flows through a well-insulated chamber. The initial conditions in the chamber, the supply conditions, and the inlet and exit mass flow rates are specified.

FIND: Determine the temperature and pressure of the air in the chamber as functions of time and plot for various mass flow rates.

SCHEMATIC & GIVEN DATA:



$\dot m_1 = \dot m_2 = \begin{cases} 1 \text{ lb/min} \\ 2 \text{ lb/min} \\ 5 \text{ lb/min} \end{cases}$

ASSUMPTIONS: (1) For the control volume shown, $\dot Q_{cv} = 0$ and $\dot W_{cv} = 0$. (2) The temperature and pressure in the chamber are uniform throughout at any time. (3) Kinetic and potential energy effects are negligible. (4) The air is modeled as an ideal gas with constant specific heats.

ANALYSIS: The mass rate balance takes the form $dm_{cv}/dt = \dot m_1 - \dot m_2$. With $\dot m_1 = \dot m_2 \equiv \dot m$, $dm_{cv}/dt = 0$. Thus, the amount of mass contained within the control volume is constant. Using assumptions listed, the energy balance is

$$\frac{dU_{cv}}{dt} = \cancelto{0}{\dot Q_{cv}} - \cancelto{0}{\dot W_{cv}} + \dot m (h_1 - h_2)$$

or, with $U_{cv} = m_{cv} u$

$$m_{cv}\frac{du}{dt} + u\cancelto{0}{\frac{dm_{cv}}{dt}} = \dot m [h_1 - h(t)]$$

where $h_2 = h(t)$ by assumption (2). Further, $du = c_v dT$ and $h(t) = c_p T(t)$, so

$$m_{cv} c_v \frac{dT}{dt} = \dot m [h_1 - c_p T(t)]$$

or

$$\frac{dT}{dt} + \left(\frac{\dot m c_p}{m_{cv} c_v}\right) T = \frac{\dot m c_p T_1}{m_{cv} c_v}$$

Introducing $k = c_p / c_v$

$$\frac{dT}{dt} + \left(\frac{\dot m k}{m_{cv}}\right) T = \left(\frac{\dot m k T_1}{m_{cv}}\right)$$

The solution of this differential equation takes the form

$$T(t) = c \exp\left[-\left(\frac{\dot m k}{m_{cv}}\right) t\right] + T_1$$

With $T(0) = T_i$; $c = T_i - T_1$, so

$$T(t) = (T_i - T_1) \exp\left[-\left(\frac{\dot m k}{m_{cv}}\right) t\right] + T_1 \tag{1}$$

PROBLEM 4.103 (Cont'd.)

From the given data

$$m_{cv} = \frac{p_i V_i}{R T_i} = \frac{(14.7\ \text{lbf/in.}^2)(1\ \text{ft}^3)}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(560^\circ\text{R})} \left| \frac{144\ \text{in.}^2}{1\ \text{ft}^2} \right| = 0.0709\ \text{lb}$$

From Table A-20E ; $k \approx 1.4$. Also, $T_i = 100°F$ and $T_1 = 200°F$. Thus, from (1)

$\underline{\dot{m} = 1\ \text{lb/s}}$ $\quad T(t) = -100\, e^{-0.3291 t} + 200$

$\underline{\dot{m} = 2\ \text{lb/s}}$ $\quad T(t) = -100\, e^{-0.6582 t} + 200$

$\underline{\dot{m} = 5\ \text{lb/s}}$ $\quad T(t) = -100\, e^{-1.6455 t} + 200$

The pressure is $p = \frac{m R T(t)}{V}$. Thus

$$p = \left[\frac{(0.0709\ \text{lb})\ \frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}}{(1\ \text{ft}^3) \left| \frac{144\ \text{in.}^2}{1\ \text{ft}^2} \right|} \right] [T(t) + 460]$$

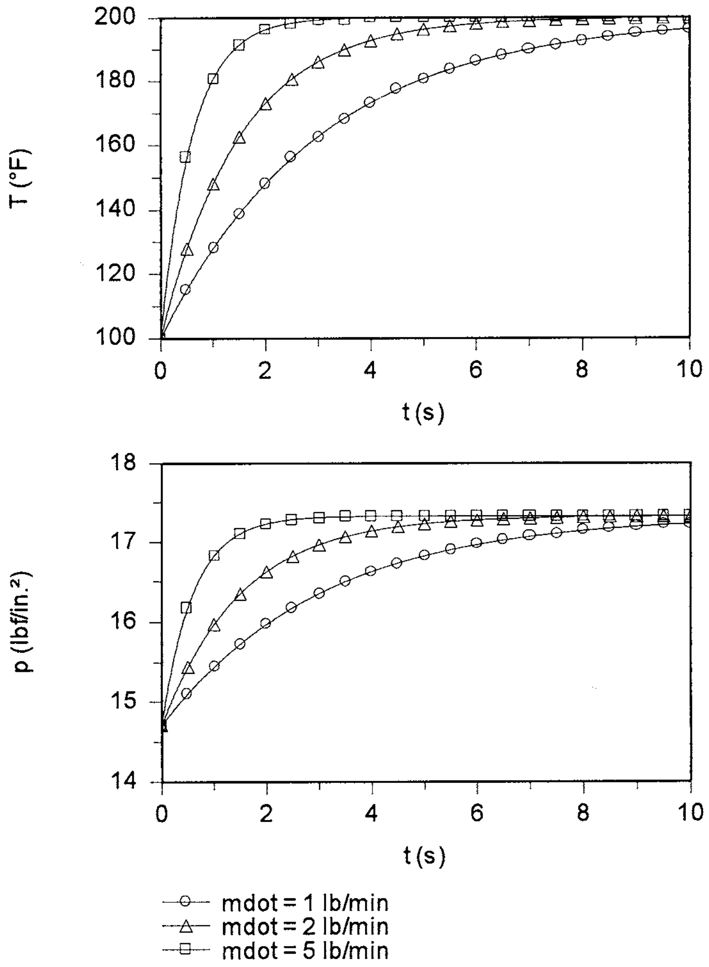
Finally

$\underline{\dot{m} = 1\ \text{lb/s}}$ $\quad p(t) = -2.6258\, e^{-0.3291 t} + 17.3304$

$\underline{\dot{m} = 2\ \text{lb/s}}$ $\quad p(t) = -2.6258\, e^{-0.6582 t} + 17.3304$

$\underline{\dot{m} = 5\ \text{lb/s}}$ $\quad p(t) = -2.6258\, e^{-1.6455 t} + 17.3304$

Using IT to plot, we get



CHAPTER FIVE

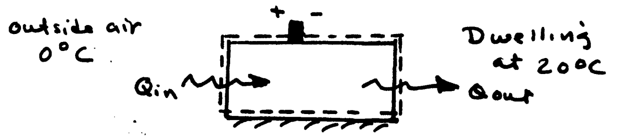
THE SECOND LAW OF THERMODYNAMICS

PROBLEM 5.1

KNOWN: Operating data are provided for a heat pump.

FIND: Explain whether the heat pump is in violation of the Clausius statement.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The heat pump is a conventional one intended for dwelling heating from an electric power input.

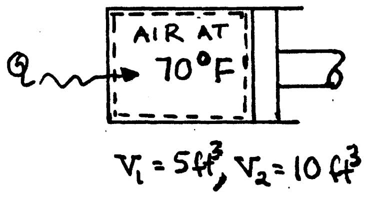
ANALYSIS: Since the heat pump is driven by an electric power input, there is no violation of the Clausius statement, which refers only to cases where the sole effect would be a heat transfer from a colder to a hotter body.

PROBLEM 5.2*

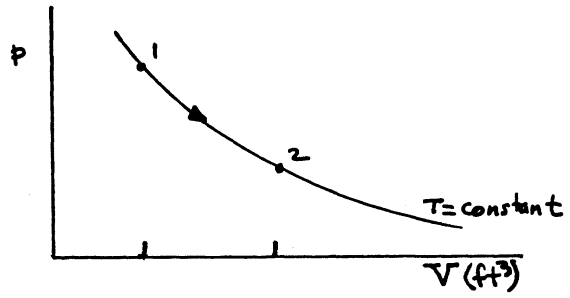
KNOWN: Air expands isothermally between specified states.

FIND: Evaluate the heat transfer and work for the process, and explain if a violation of the Kelvin-Planck statement occurs.

SCHEMATIC & GIVEN DATA:



$V_1 = 5\ ft^3$, $V_2 = 10\ ft^3$



ASSUMPTIONS: (1) The system is 1 lb of air modeled as an ideal gas. (2) Temperature remains constant. (3) Kinetic and potential energy changes are absent. (4) The surroundings play the role of a thermal reservoir. (5) Volume change is the only work mode.

ANALYSIS: An energy balance for the system reads $U_2 - U_1 = Q - W$. However, as the internal energy of an ideal gas depends on temperature only, $U_2 = U_1$. Thus, the energy balance reduces to $Q = W$. To find W

$$W = \int_1^2 p\,dV = \int_1^2 \frac{mRT}{V}\,dV = mRT \ln\frac{V_2}{V_1}$$

$$\Rightarrow \frac{W}{m} = \left(\frac{1545}{28.97}\right)\frac{ft{\cdot}lbf}{lb\,°R}(70+460)°R\left|\frac{1\ Btu}{778\ ft{\cdot}lbf}\right|\ln\left(\frac{10\ ft^3}{5\ ft^3}\right) = 25.2\ \frac{Btu}{lb} \longleftarrow W/m,\ Q/m$$

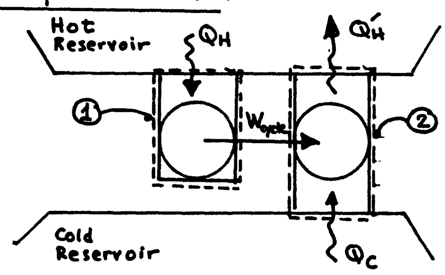
The Kelvin-Planck statement of the second law refers to systems that undergo thermodynamic cycles while communicating thermally with a single reservoir. Here, the system undergoes a process-not a cycle. Accordingly, the Kelvin-Planck statement is not applicable, so no violation can be claimed. Similarly, the Clausius statement, which is equivalent to the Kelvin-Planck statement, is not applicable in this case. With the information provided, there is no apparent violation of the second law.

PROBLEM 5.3

KNOWN: A system undergoes a cycle in violation of the Kelvin-Planck statement of the second law.

FIND: Show that a violation of the Clausius statement of the second law is a consequence.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) System 1 undergoes a cycle while receiving energy Q_H from the hot reservoir and developing work Wcycle. (This is in violation of the Kelvin-Planck statement of the second law.) (2) System 2 undergoes a cycle while removing energy Q_C from the cold reservoir and discharging energy Q_H' to the hot reservor. The work developed by System 1 is used to drive System 2.

ANALYSIS: An energy balance for system 1 reduces to

$$W_{cycle} = Q_H$$

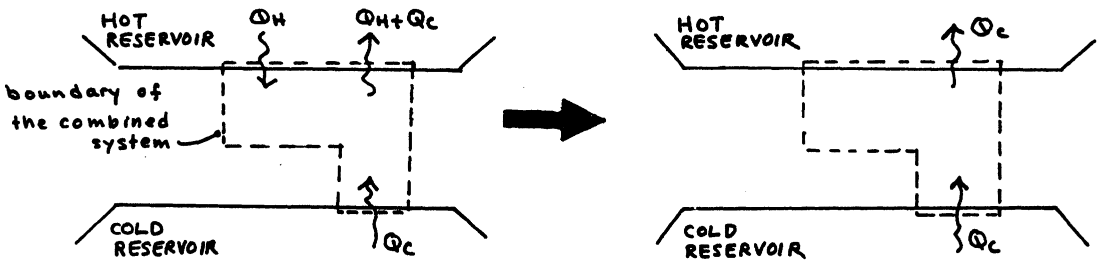
For System 2 an energy balance reads

$$W_{cycle} = Q_H' - Q_C$$

Combining these expressions

$$Q_H' = Q_H + Q_C$$

Next, consider a combined system consisting of Systems 1 and 2:



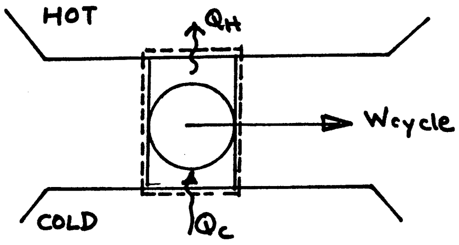
The sole result of the combined system is that a net amount of energy Q_C is transferred from the cold reservoir to the hot reservoir. This is a violation of the Clausius statement of the second law.

PROBLEM 5.4

KNOWN: A system undergoes a cycle while communicating with two reservoirs. The system develops work while receiving Q_C from the cold reservoir and discharging Q_H to the hot reservoir.

FIND: Evaluate the claimed performance using the second law.

SCHEMATIC & GIVEN DATA:



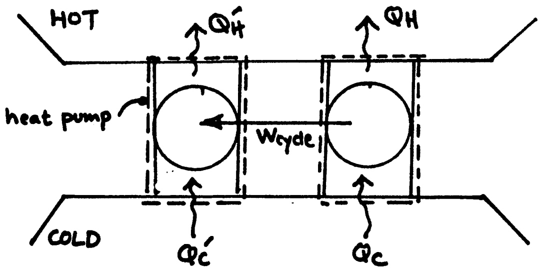
ASSUMPTION: The system shown in the accompanying figure undergoes a power cycle while communicating thermally with two reservoirs as shown.

ANALYSIS: An energy balance for the system reads

$$W_{cycle} = Q_C - Q_H$$

The claim can be evaluated using either the Clausius or the Kelvin-Planck statement:

(a) Clausius. As shown in the figure below, let the given cycle drive a heat pump cycle.



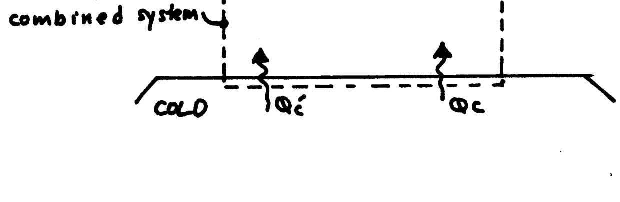
An energy balance for the heat pump reads

$$Q_H' = Q_C' + W_{cycle}$$

or, with $W_{cycle} = Q_C - Q_H$

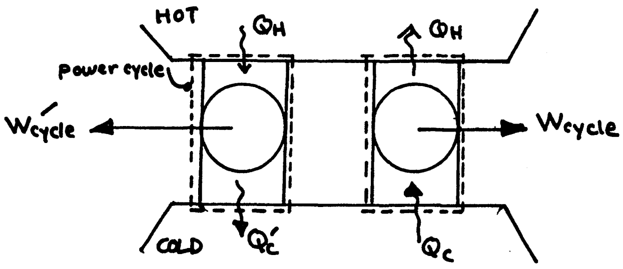
$$Q_H' = Q_C' + Q_C - Q_H$$

Next, consider the combined system of the two systems together. The sole result of the combined system is that a net amount of energy $Q_C' + Q_C$ is transferred from the cold reservoir to the hot reservoir. This is a violation of the Clausius statement of the second law, so the original system cannot operate as assumed. The inventor's claim is invalid.

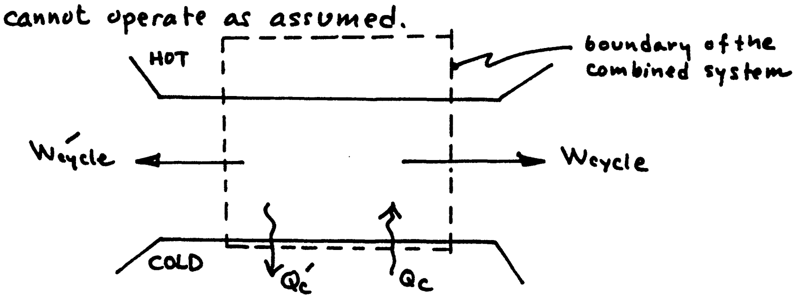


PROBLEM 5.4 (Contd.)

(b) Kelvin - Plank. As shown in the figure below, let Q_H be supplied to a system undergoing a power cycle.



Next, consider the combined system formed by the two systems and the hot reservoir. Observe that the hot reservoir experiences no net change as it receives Q_H from one cycle and supplies Q_H to the other. Thus, since the combined system is made up of parts that undergo cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the cold reservoir, while producing a net amount of work. However, this is a violation of the Kelvin-Planck statement of the second law, so the original system cannot operate as assumed.

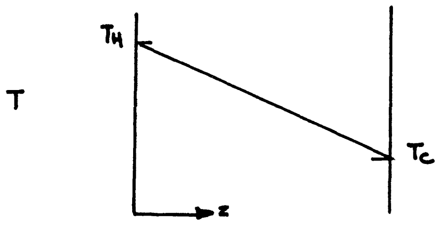


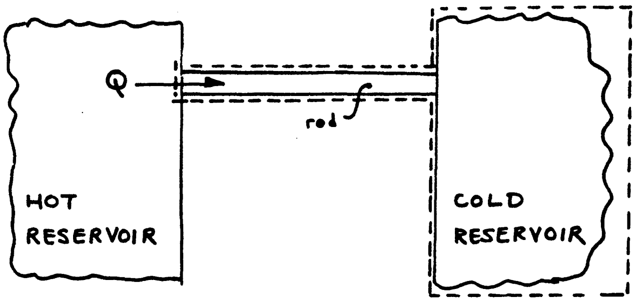
PROBLEM 5.5

KNOWN: Energy transfer by conduction from a hot reservoir to a cold reservoir takes place spontaneously.

FIND: Using the Kelvin - Planck statement of the second law, show that such a process is irreversible.

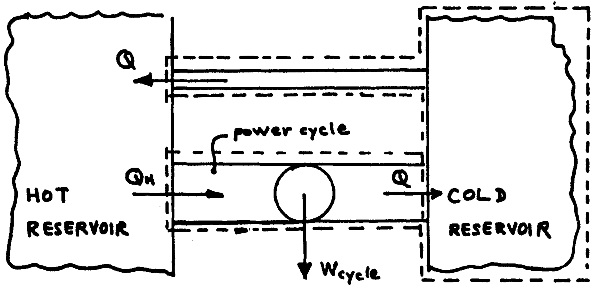
SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system shown in the accompanying figure receives energy Q from the hot reservoir which passes by conduction through a rod at steady state to the cold reservoir. (2) A power cycle is available for use in demonstrating that the process is irreversible.

ANALYSIS: The objective in this proof by contradiction is to devise a system that undergoes a cycle which develops work while the system communicates thermally with a single reservoir, thereby violating the Kelvin-Planck statement.



Note: Q represents the energy transferred from the cold reservoir to the hot reservoir without any other effect.

Note: Q represents the energy discharged to the cold reservoir from the power cycle.

The system shown above consists of the original system plus a system capable of undergoing a power cycle (assumption 2). This combined 'system undergoes a cycle as follows. (1) An amount of energy Q_H is transferred from the hot reservoir to the power cycle, producing work W_{cycle} and discharging energy Q to the cold reservoir. (2) An amount of energy Q is transferred from the cold reservoir to the hot reservoir without any other effects. (This would be possible only if the process described in assumption 1 were reversible.)

Since Q is added to the cold reservoir by the power cycle in the first

PROBLEM 5.5 (Contd.)

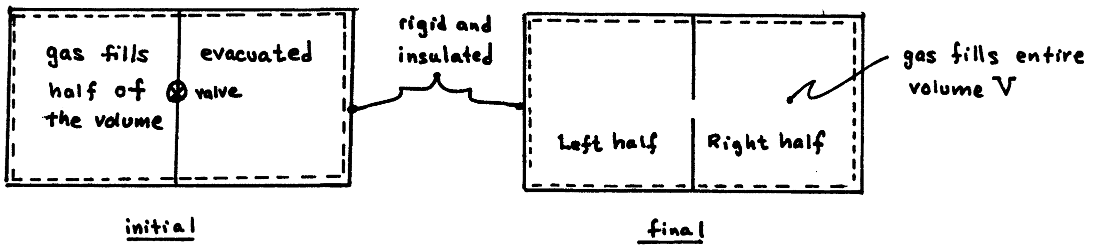
process of the cycle and the same amount of energy is removed from the cold reservoir in the second process of the cycle, this reservoir experiences no net change in its condition. The power cycle enclosed within the combined system also undergoes a cycle. Accordingly, the combined system undergoes a cycle in which work Wcycle is produced while exchanging energy by heat transfer with a single reservoir (the hot reservoir). Such a cycle violates the Kelvin-Planck statement of the second law, and thus is impossible. It follows that one, or both, of the processes making up the cycle executed by the combined system must be impossible. However, as the first process involving the power cycle can occur, it is the second process that must be impossible: energy Q cannot be transferred from the cold to the hot reservoir without other effects. By definition, then, the transfer of energy Q by conduction from the hot to the cold reservoir is irreversible.

PROBLEM 5.6

KNOWN: When an interconnecting valve is opened, a gas expands from one half of a tank to the other half which is initially evacuated.

FIND: Using the Kelvin-Planck statement of the second law, show that such a process is irreversible.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a process in which the gas expands spontaneously to fill the entire volume V. (2) During the process $Q = W = 0$. (3) The initial and final states are equilibrium states. There is no change in kinetic or potential energy between these states. (4) A turbine and a thermal reservoir are available for use in demonstrating that the process is irreversible.

ANALYSIS: The object in this proof by contradiction is to devise a system that undergoes a power cycle while the system communicates thermally with a single reservoir, thereby violating the Kelvin-Planck statement of the second law.

Before considering such a system, note that an energy balance for the spontaneous process is

$$U_{final} - U_{initial} = \overset{0}{\cancel{Q}} - \overset{0}{\cancel{W}}$$

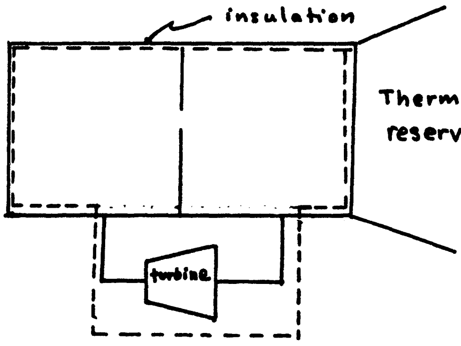
where assumptions 2 and 3 have been used. Accordingly,

$$U_{final} = U_{initial}$$

That is, the internal energy of the system does not change in the free expansion.

PROBLEM 5.6 (Contd.)

As shown in the figure below, modify the system to include a turbine and introduce a thermal reservoir in the surroundings (assumption 4).



Starting with the gas filling the entire volume V, let the modified system undergo a cycle consisting of three processes.

Process 1. Let the reverse of the free expansion occur without any other effects. That is, the gas passes spontaneously from the right half of the tank until it fills only the left half. (This would be possible only if the process described in assumption 1 were reversible.)

Process 2. Let part of the gas expand through the turbine into the right half of the tank until the pressure in both halves is the same. In expanding through the turbine the gas does work so that its internal energy is decreased: $U < U_{initial}$.

Process 3. Remove part of the tank insulation and add energy by heat transfer from the thermal reservoir until the internal energy of the gas is restored to its initial value. Thus, a cycle is completed.

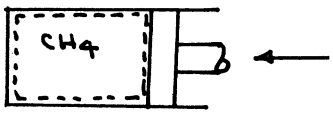
The net result of this cycle is to draw energy from a single reservoir by heat transfer and produce an equivalent amount of work. Such a cycle violates the Kelvin-Planck statement of the second law, and thus is impossible. Since both the heating of the gas by the reservoir (process 3) and the development of work as gas passes through the turbine (process 2) are possible, it can be concluded that it is process 1 that is impossible. Since process 1 is the reverse of the original free expansion, it follows that the original process is irreversible.

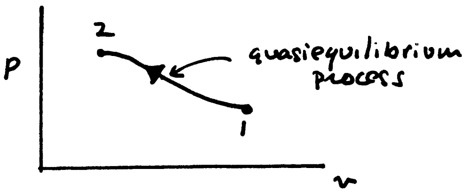
PROBLEM 5.7

KNOWN: A gas within a piston-cylinder assembly undergoes a quasiequilibrium process.

FIND: Determine if the process is internally reversible. If the process is reversible.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The gas is the system. 2. The process is a quasiequilibrium process.

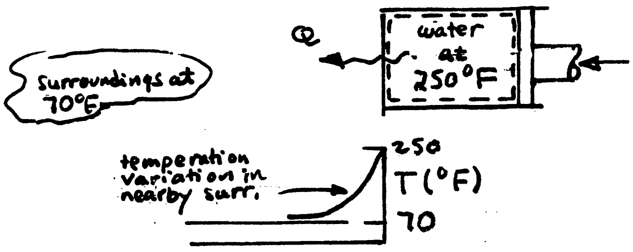
ANALYSIS: Referring to the discussion of Sec. 5.2, such a process is internally reversible. The process would be reversible only if the surroundings were also free of irreversibilities as the gas undergoes the indicated process. A reversible process is one in which there are no internal or external irreversibilities. Thus, reversibility in this case can be decided only if further information about the surroundings is provided.

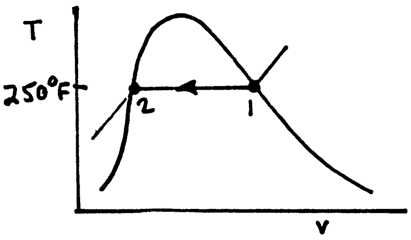
PROBLEM 5.8*

KNOWN: Water within a piston-cylinder assembly cools isothermally at 250°F from saturated vapor to saturated liquid.

FIND: Determine if the process is internally reversible, is reversible.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The system is the water in the piston-cylinder assembly. 2. The system undergoes a constant-temperature process from saturated vapor to saturated liquid.

ANALYSIS: Since temperature is constant during the process, the pressure also remains constant. As shown by the T-v diagram, the process is a sequence of equilibrium states, and thus is internally reversible.

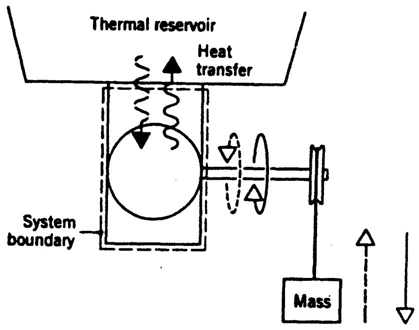
The process is not reversible because there is a significant irreversibility in the surroundings — namely, the spontaneous heat transfer taking place between the water at 250°F and the surroundings at 70°F.

PROBLEM 5.9

KNOWN: A system undergoes a cycle reversibly while communicating thermally with a single reservoir.

FIND: Show that $W_{cycle} = 0$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a reversible cycle. (2) During the cycle the system communicates thermally only with a single reservoir.

ANALYSIS: Let the system undergo one cycle. According to the Kelvin-Planck statement of the second law, the work for the cycle is zero or negative in value: $W_{cycle} \leq 0$.

As the cycle is reversible, it is possible to return both the system and its surroundings to their initial states. Accordingly, there would be no net change in the condition of the reservoir or the elevation of the mass used to store energy in the surroundings.

① The above statements are consistent only if the sign of equality is used: $W_{cycle} = 0$.

---

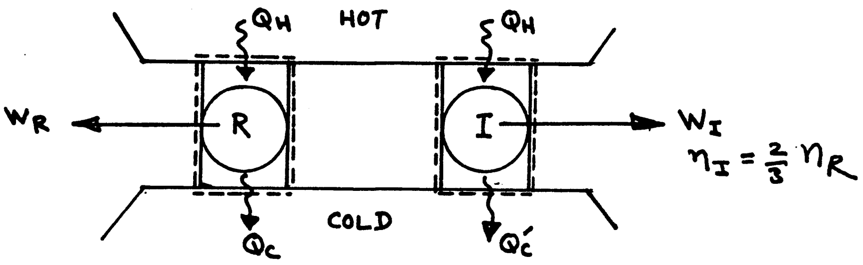
1. In Sec. 5.3.1, the converse is demonstrated: If the sign of equality applies: $W_{cycle} = 0$, then the cycle is reversible. Taken together, these two demonstrations establish the following proposition: If, and only if, the sign of equality applies in Eq. 5.1, the cycle is reversible.

PROBLEM 5.10

KNOWN: A power cycle I and a reversible power cycle R operate between the same two reservoirs. For these cycles, $\eta_I = \frac{2}{3}\eta_R$.

FIND: Show that cycle I must be irreversible.

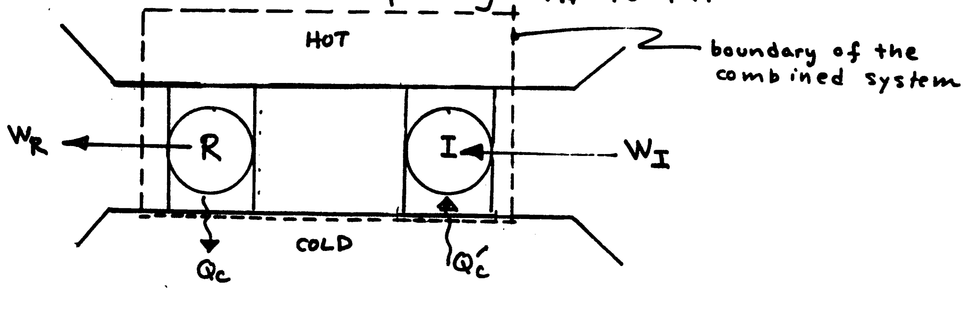
SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system denoted by R in the accompanying figure undergoes a reversible power cycle while system I undergoes a power cycle such that $\eta_I = \frac{2}{3}\eta_R$. (2) Both cycles receive the same energy Q_H from the hot reservoir.

ANALYSIS: If $\eta_I = 2/3\, R$ and both cycles receive Q_H from the hot reservoir, $W_I = 2/3\, W_R < W_R$.

In this proof by contradiction, assume I is reversible. Then, if I operates in the opposite direction as a refrigeration (or heat pump) cycle, the magnitudes of the energy transfers Q_H, Q_C', and W_I would remain the same but would be oppositely directed as shown in the figure below. With I operating in the opposite direction, the hot reservoir would experience no net change since it would receive Q_H from I while passing Q_H to R.



As the combined system shown in the figure above is made up of parts that execute cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the cold reservoir. Accordingly, the combined system must satisfy Eq. 5.1 expressed as $W_{cycle} = 0$, where the sign of equality is used because R is reversible and I has been assumed reversible. Evaluating W_{cycle} for the combined system in terms of the work amounts W_R and W_I, $W_{cycle} = W_R - W_I$, it follows that

$$W_I = W_R$$

As this conclusion is not in agreement with the requirement that $W_I < W_R$, it can be concluded that the hypothesis is false and I must be irreversible.

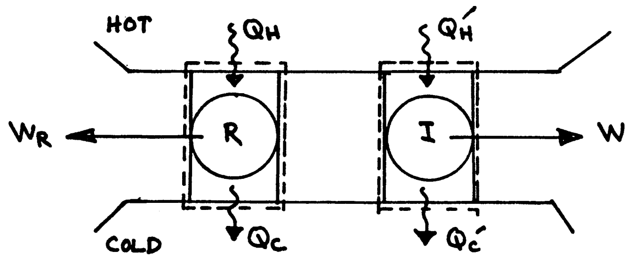
PROBLEM 5.11

KNOWN: A reversible power cycle R and an irreversible power cycle I operate between the same two reservoirs.

FIND: (a) If each cycle receives the same amount of energy from the hot reservoir, show that I discharges more energy to the cold reservoir.

(b) If each cycle develops the same net work, show that I receives more energy from the hot reservoir.

SCHEMATIC & GIVEN DATA:



To show:

(a) $Q_H = Q_H' \Rightarrow Q_C' > Q_C$

(b) $W_R = W_I \Rightarrow Q_H' > Q_H$

ASSUMPTION: The system denoted by R in the accompanying figure undergoes a reversible power cycle while system I undergoes an irreversible power cycle.

ANALYSIS: (a) By the first Carnot Corollary, $\eta_R > \eta_I$. Since both cycles receive the same amount of energy Q_H, it follows that $W_R > W_I$.

An energy balance for R and I read, respectively

$$W_R = Q_H - Q_C$$
$$W_I = Q_H - Q_C'$$

Collecting results

$$Q_H - Q_C > Q_H - Q_C'$$

Accordingly

$$Q_C' > Q_C \longleftarrow$$

as was to be demonstrated.

Thus, not only do actual cycles develop less work they also discharge more energy by heat transfer to their surroundings, thereby increasing the effect of thermal pollution.

(b) Since $\eta_R > \eta_I$ and $W_R = W_I \equiv W$,

$$\frac{W}{Q_H} > \frac{W}{Q_H'} \Rightarrow Q_H' > Q_H$$

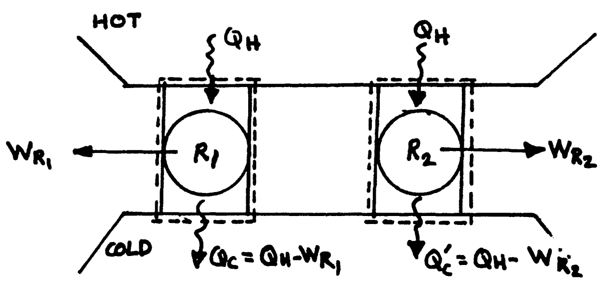
If the hot reservoir were maintained by, say, energy from the combustion of a fossil fuel, cycle I would have the greater fuel requirement. Also, note that cycle I also would have the greater energy discharge to the cold reservoir, so the comments of (a) would also be applicable.

PROBLEM 5.12

KNOWN: The Kelvin-Planck statement of the second law: $W_{cycle} \leq 0$ (single reservoir).

FIND: Show that all reversible power cycles operating between the same two reservoirs have the same thermal efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The systems shown in the accompanying figure each undergo reversible power cycles while receiving the same amount of energy Q_H from the hot reservoir.

ANALYSIS: Let cycle R_1 operate as a reversible refrigeration (or heat pump cycle). The magnitudes of W_{R1}, Q_H, and Q_C remain the same but are now oppositely directed. Further, with R_1 working in the opposite direction, the hot reservoir would experience no net change in its condition since it would receive Q_H from R_1 while passing Q_H to R_2.

The demonstration is completed by considering the combined system consisting of the two cycles and the hot reservoir. Since its parts execute cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the cold reservoir. Accordingly, the combined system must satisfy the Kelvin-Planck statement expressed as

$$W_{cycle} = 0$$

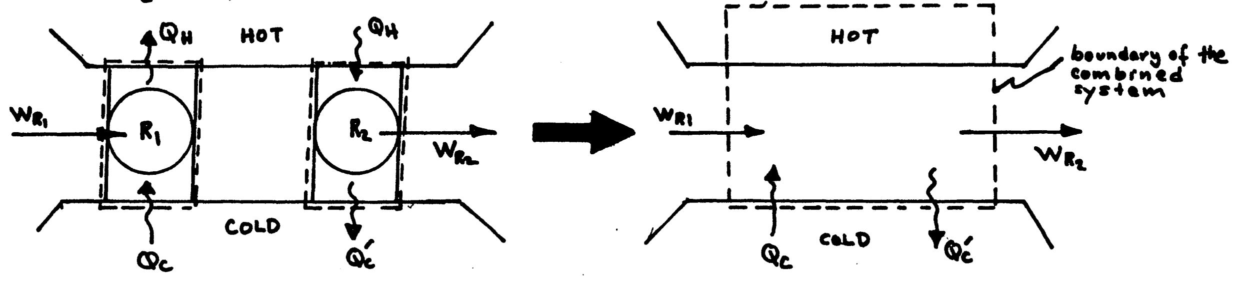
where the equality is used because all parts of the combined system are free of irreversibilities. Evaluating W_{cycle} in terms of the work amounts W_{R_1} and W_{R_2}

$$W_{R_2} - W_{R_1} = 0$$

or

$$W_{R_2} = W_{R1}$$

Since each of the power cycles receives the same energy input Q_H, it follows that $\eta_{R_2} = \eta_{R_1}$ and this completes the demonstration.

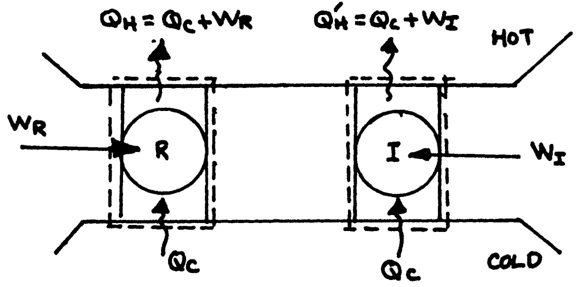


PROBLEM 5.13

KNOWN: The Kelvin-Planck statement of the second law: $W_{cycle} \leq 0$ (single reservoir).

FIND: Show that (a) the coefficient of performance of an irreversible refrigeration cycle is less than the coefficient of performance of a reversible refrigeration cycle when both exchange energy by heat transfer with the same two reservoirs, (b) all reversible refrigeration cycles operating between the same two reservoirs have the same coefficient of performance. For parts (c), (d), see next page.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: The systems shown in the accompanying figure undergo refrigeration cycles. Each cycle removes energy Q_C from the cold reservoir. R accomplishes this reversibly while I is irreversible.

ANALYSIS: (a) Let R operate as a reversible power cycle. The magnitudes of W_R, Q_C and Q_H remain the same but are now oppositely directed. Further, with R working in the opposite direction, the cold reservoir would experience no net change in its condition since it would receive Q_C from R while passing Q_C to I.

The demonstration is completed by considering the combined system consisting of the two cycles and the cold reservoir. Since its parts execute cycles or experience no net change, the combined system operates in a cycle. Moreover, it exchanges energy by heat transfer with a single reservoir: the hot reservoir. Accordingly, the combined system must satisfy the Kelvin-Planck statement expressed as

$$W_{cycle} < 0$$

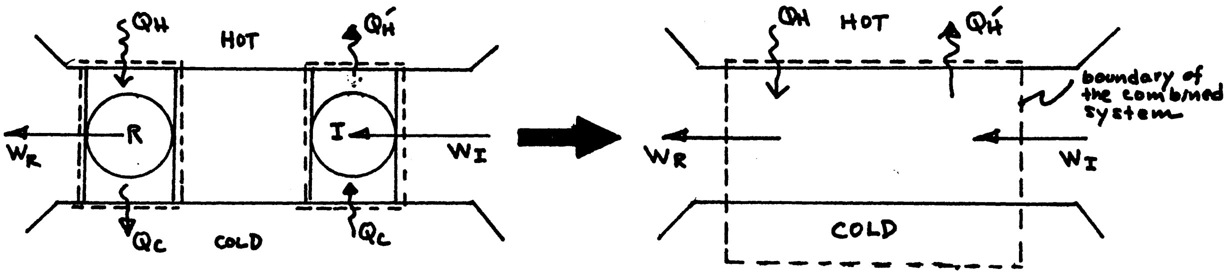
where the inequality is used because irreversible cycle I is included. Evaluating W_{cycle} in terms of the work amounts W_R and W_I

$$W_R - W_I < 0$$

or

$$W_R < W_I$$

Since each of the refrigeration cycles receives the same energy input Q_C, it follows that $\beta_R > \beta_I$ and this completes the demonstration.



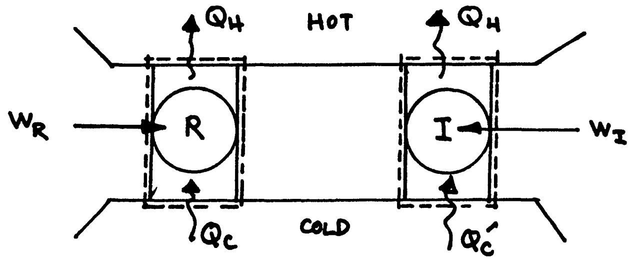
PROBLEM 5.13 (Contd.)

ANALYSIS (b) This proposition can be demonstrated in a parallel way by considering two reversible refrigeration cycles R_1 and R_2 operating between the same two reservoirs. Then, letting R_1 play the role of R and R_2 the role of I in the previous development, a combined system consisting of the two cycles and the hot reservoir may be formed which must satisfy $W_{cycle} = 0$, where the equality is used because all parts of the combined system are free of irreversibilities. Thus, it can be concluded that $W_{R_1} = W_{R_2}$, and therefore $\beta_{R_1} = \beta_{R_2}$.

Parts (c), (d):

FIND: Show that (c) the coefficient of performance of an irreversible heat pump cycle is less than the coefficient of performance a reversible heat pump cycle when both exchange energy by heat transfer with the same two reservoirs, (d) all reversible heat pump cycles operating between the same two reservoirs have the same coefficient of performance.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The systems shown in the accompanying figure undergo heat pump cycles. (2) Each cycle provides energy Q_H to the hot reservoir. R accomplishes this reversibly while I is irreversible. (3) Both cycles operate between the same two reservoirs.

ANALYSIS: (c) The demonstration parallels the approach used in part(a). However, in the present case the combined system would consist of the two cycles and the hot reservoir. (d). The demonstration parallels the approach used in part(b).

PROBLEM 5.14

KNOWN: The symbol Θ denotes temperature on Kelvin's logarithmic scale.

FIND: (a) Show that $\Theta = \ln T + C$, where T is temperature on the Kelvin scale. (b) Determine the range of temperature values on the logarithmic scale. (c) Obtain an expression for the thermal efficiency of a reversible power cycle operating between reservoirs at Θ_H and Θ_C on the logarithmic scale.

ANALYSIS: The two scales arise from different specifications of the function Ψ in Eq. 5.5:

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{\text{rev}\\ \text{cycle}}} = \Psi$$

That is

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{\text{rev}\\ \text{cycle}}} = \frac{T_C}{T_H} \qquad \underline{\text{Kelvin scale}}$$

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{\text{rev}\\ \text{cycle}}} = \frac{\exp \Theta_C}{\exp \Theta_H} \qquad \underline{\text{Logarithmic scale}}$$

(a) By comparison of the last two equations

$$\frac{T_C}{T_H} = \frac{\exp \Theta_C}{\exp \Theta_H} = \exp[\Theta_C - \Theta_H]$$

Thus

$$\ln T_C - \ln T_H = \Theta_C - \Theta_H$$

and therefore

$$\Theta = \ln T + C \longleftarrow\!\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!- \text{(a)}$$

where C is a constant determining the level of temperature corresponding to zero on the logarithmic scale.

(b) Temperatures on the Kelvin scale vary from 0 to $+\infty$. With the relationship of part (a), temperatures on the logarithmic scale vary from $-\infty$ to $+\infty$. $\longleftarrow$ (b)

(c) Use of

$$\left(\frac{Q_C}{Q_H}\right)_{\substack{\text{rev}\\ \text{cycle}}} = \frac{\exp \Theta_C}{\exp \Theta_H}$$

in Eq. 5.2 gives an expression for the thermal efficiency of a reversible power cycle while operating between reservoirs at temperatures Θ_H and Θ_C on the logarithmic scale

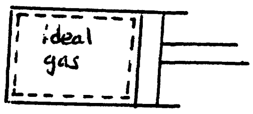
$$\eta = 1 - \frac{\exp \Theta_C}{\exp \Theta_H} \longleftarrow\!\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!- \text{(c)}$$

PROBLEM 5.15

KNOWN: The gas temperature scale introduced in Sec. 1.6.2 and the Kelvin scale defined in Sec. 5.4.

FIND: Demonstrate that the two scales are identical.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the figure consists of a gas obeying $pv = RT'$, where T' denotes temperature on the ① gas scale. (2) The system undergoes a reversible cycle while communicating thermally with reservoirs at T_H' and T_C'. (3) The cycle consists of four processes in series: 1-2 isothermal at T_H', 2-3 adiabatic, 3-4 isothermal at T_C', 4-1 adiabatic. (4) Kinetic and potential energy effects are absent.

ANALYSIS: With assumption (4), an energy balance in differential form reads

$$du = \delta(Q/m) - \delta(W/m)$$

Then, since the processes are internally reversible, $\delta(W/m) = pdv$. Also, for the ideal gas, $du = c_v dT'$. Collecting results

$$c_v dT' = \delta(Q/m) - pdv \Rightarrow c_v dT' = \delta(Q/m) - \frac{RT'}{v}dv \Rightarrow \underline{\underline{c_v dT' = \delta(Q/m) - RT' d\ln v}}$$

Process 1-2: The gas temperature is constant at T_H', so

$$\cancel{c_v d\overset{0}{T'}} = \delta(Q/m)_{1-2} - RT_H' d\ln v \Rightarrow (Q/m)_{1-2} = RT_H' \ln(v_2/v_1) \qquad (1)$$

Process 2-3: There is no heat transfer, and the gas temperature varies from T_H' to T_C', so

$$c_v dT' = \cancel{\delta(Q/m)}^{0} - RT' d\ln v \Rightarrow \frac{c_v dT'}{T'} = -R\, d\ln v \Rightarrow \int_{T_H'}^{T_C'} \frac{c_v dT'}{T'} = -R \ln\left(\frac{v_3}{v_2}\right) \qquad (2)$$

Process 3-4: The gas temperature is constant at T_C', and the heat rejected in the process is

$$(Q/m)_{3-4} = RT_C' \ln(v_4/v_3) \Rightarrow |(Q/m)_{3-4}| = RT_C' \ln\left(\frac{v_3}{v_4}\right) \qquad (3)$$

Process 4-1: There is no heat transfer, and the gas temperature varies from T_C' to T_H', so

$$\int_{T_C'}^{T_H'} \frac{c_v dT'}{T'} = -R \ln\left(\frac{v_1}{v_4}\right) \Rightarrow \int_{T_H'}^{T_C'} \frac{c_v dT'}{T'} = -R \ln\left(\frac{v_4}{v_1}\right) \qquad (4)$$

From Equations (2) and (4), $v_3/v_2 = v_4/v_1$ or $v_3/v_4 = v_2/v_1$. Using this result together with Equations (1) and (3) gives

$$\frac{|(Q/m)_{3-4}|}{(Q/m)_{1-2}} = \frac{T_C'}{T_H'}$$

which corresponds to Equation 5.6 underlying the Kelvin scale. This implies $T' \equiv T$, where T denotes temperature on the Kelvin scale.

---

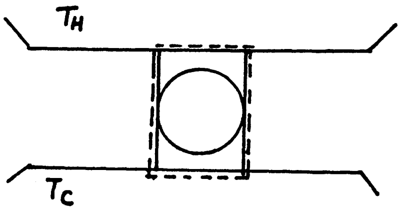
1. The reversible cycle considered here corresponds to the Carnot cycle discussed in Sec. 5.6.

PROBLEM 5.16

KNOWN: A system undergoes a reversible power cycle while receiving and discharging energy by heat transfer with two thermal reservoirs.

FIND: To increase the thermal efficiency, determine whether it would be better to increase T_H or decrease T_C.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system shown in the accompanying figure undergoes a reversible power cycle while receiving and discharging energy by heat transfer with two thermal reservoirs.

ANALYSIS: Trends can be determined by differentiation of Eq. 5.8, which is applicable in this case

$$\left(\frac{\partial\eta}{\partial T_H}\right)_{T_C} = \frac{T_C}{(T_H)^2} \Rightarrow \text{As } T_H \text{ increases, } \eta \text{ increases.}$$

$$\left(\frac{\partial\eta}{\partial T_C}\right)_{T_H} = -\frac{1}{T_H} \Rightarrow \text{As } T_C \text{ decreases, } \eta \text{ increases.}$$

Quantitative evaluations also can be had using Eq. 5.8

$$\eta = 1 - \frac{T_C}{T_H}$$

If T_C is decreased by ϵ degrees, the thermal efficiency is increased

$$\eta_{(-)} = 1 - \frac{(T_C - \epsilon)}{T_H} = \eta + \frac{\epsilon}{T_H} \qquad (1)$$

If T_H is increased by ϵ degrees, the thermal efficiency is increased

$$\eta_{(+)} = 1 - \frac{T_C}{T_H + \epsilon}$$

$$= \frac{T_H - T_C + \epsilon}{T_H + \epsilon} = \frac{\eta T_H + \epsilon}{T_H + \epsilon} = \frac{\eta + \epsilon/T_H}{1 + \epsilon/T_H} \qquad (2)$$

Combining Eqs. (1), (2)

$$\eta_{(+)} = \frac{\eta_{(-)}}{1 + \epsilon/T_H} \Rightarrow \eta_{(+)} < \eta_{(-)} \longleftarrow$$

Accordingly, it would be more beneficial to decrease ϵ than increase T_H. The possibility of increasing thermal efficiency by reducing T_C below that of ① the environment is not practical, however, for maintaining T_C below the ambient temperature would require a refrigerator that would have to be supplied work to operate. An increase in thermal efficiency by increasing T_H is limited by the properties of the materials used to fabricate the system undergoing the cycle.

---

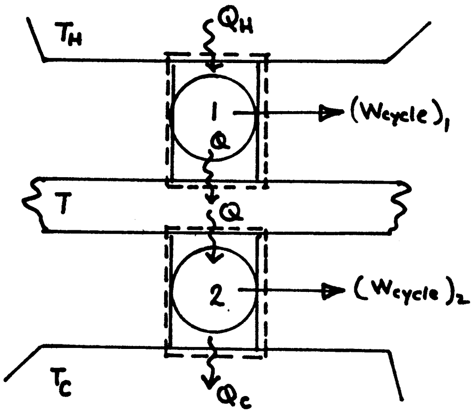
1. A discussion of the influence of the temperatures of heat addition and heat rejection on the thermal efficiency of practical power cycles is provided in Sec. 8.2.3.

PROBLEM 5.17

KNOWN: Two reversible power cycles are arranged in series. One cycle rejects energy by heat transfer to a reservoir at T while the other cycle receives energy by heat transfer from the reservoir at T.

FIND: Determine T when (a) the net work of the two cycles is equal, (b) the thermal efficiencies are equal.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The systems shown in the accompanying figure undergo reversible power cycles between thermal reservoirs.

ANALYSIS:

(a) Equal net work case. An energy balance for each system gives

$$(W_{cycle})_1 = Q_H - Q$$
$$(W_{cycle})_2 = Q - Q_C$$

In accord with the above assumption, Eq. 5.6 gives

$$\frac{Q}{Q_H} = \frac{T}{T_H}, \quad \frac{Q_C}{Q} = \frac{T_C}{T}$$

Setting the work expressions equal and eliminating Q_H and Q_C with the above expressions

$$Q_H - Q = Q - Q_C$$
$$\frac{T_H}{T}Q - Q = Q - \frac{T_C}{T}Q$$
$$T = \frac{T_H + T_C}{2} \longleftarrow T$$

(b) Equal thermal efficiency case. In accord with the above assumption, Eq. 5.8 applies, giving

$$\eta_1 = 1 - \frac{T}{T_H}$$
$$\eta_2 = 1 - \frac{T_C}{T}$$

Setting the efficiencies equal and solving

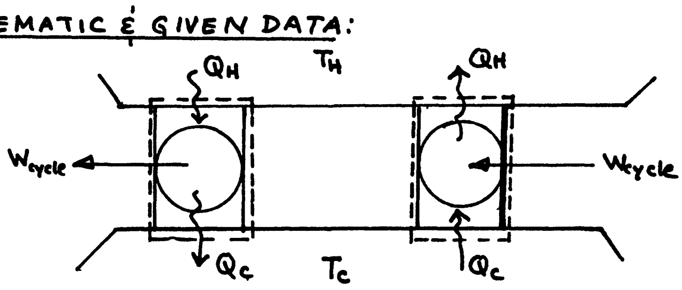
$$T = \sqrt{T_C T_H} \longleftarrow T$$

PROBLEM 5.18

KNOWN: Systems undergo reversible power, refrigeration, and heat pump cycles while operating between the same two thermal reservoirs.

FIND: Obtain expressions for the coefficients of performance of the refrigeration and heat pump cycles in terms of η_{max}, the thermal efficiency of the power cycle.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The systems shown in the accompanying figure undergo reversible cycles.

ANALYSIS: In accord with the above assumption, Eqs. 5.8, 5.9, and 5.10 are applicable. For the power cycle

$$\eta_{max} = 1 - \frac{T_C}{T_H}$$

Thus

$$\frac{T_C}{T_H} = 1 - \eta_{max}$$

For the refrigeration cycle

$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{T_C/T_H}{1 - T_C/T_H}$$

Accordingly

$$\beta_{max} = \frac{1 - \eta_{max}}{1 - (1 - \eta_{max})}$$

$$= \frac{1 - \eta_{max}}{\eta_{max}} \longleftarrow \beta_{max}$$

For the heat pump cycle

$$\gamma_{max} = \frac{T_H}{T_H - T_C} = \frac{1}{1 - T_C/T_H}$$

Accordingly

$$\gamma_{max} = \frac{1}{1 - (1 - \eta_{max})}$$

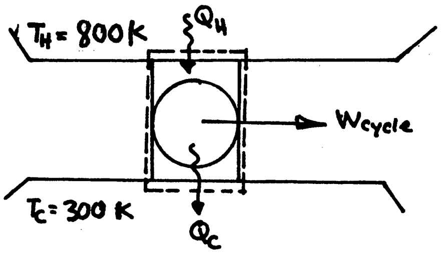
$$= \frac{1}{\eta_{max}} \longleftarrow \gamma_{max}$$

PROBLEM 5.19*

KNOWN: Operating data are provided for a system undergoing a power cycle while receiving and discharging energy by heat transfer with two thermal reservoirs at specified temperatures.

FIND: For each of three sets of data, determine if any principles of thermodynamics are violated.

SCHEMATIC & GIVEN DATA:



Energy balance:

$$W_{cycle} = Q_H - Q_C$$

Definition:

$$\eta = \frac{W_{cycle}}{Q_H}$$

ASSUMPTION: The system shown in the accompanying figure undergoes a power cycle.

ANALYSIS:

(a) $Q_H = 700\text{ kJ}$, $W_{cycle} = 400\text{ kJ}$, $Q_C = 300\text{ kJ}$. These values satisfy the energy balance, and give

$$\eta = \frac{W_{cycle}}{Q_H} = \frac{400\text{ kJ}}{700\text{ kJ}} = 0.571\ (57.1\%)$$

The maximum thermal efficiency for any power cycle under the stated conditions is given by Eq. 5.8

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{800} = 0.625\ (62.5\%)$$

Thus the second law is also satisfied. There is no apparent violation of the first and second laws.

(b) $Q_H = 640\text{ kJ}$, $W_{cycle} = 400\text{ kJ}$, $Q_C = 240\text{ kJ}$. These values satisfy the energy balance, and give

$$\eta = \frac{400\text{ kJ}}{640\text{ kJ}} = 0.625\ (62.5\%)$$

As $\eta = \eta_{MAX}$, this power cycle must be reversible. There is no apparent violation of the first and second laws.

(c) $Q_H = 640\text{ kJ}$, $W_{cycle} = 400\text{ kJ}$, $Q_C = 200\text{ kJ}$. With these values for Q_H and Q_C the energy balance gives

$$W_{cycle} = Q_H - Q_C = 640 - 200 = 440\text{ kJ}$$

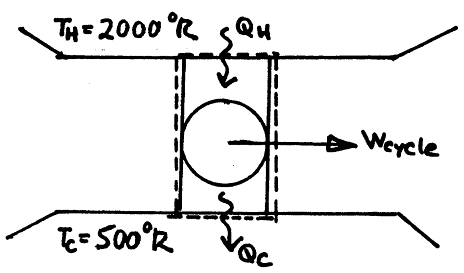
which does not agree with the given value. Accordingly, the conservation of energy principle is not satisfied. There is no need to consider the second law under such circumstances.

PROBLEM 5.20*

KNOWN: Operating data are provided for a system undergoing a power cycle while receiving and discharging energy by heat tranfer with two thermal reservoirs at specified temperatures.

FIND: For each of four sets of data, determine if the cycle is reversible, irreversible, or impossible.

SCHEMATIC & GIVEN DATA:



Energy balance:

$$W_{cycle} = Q_H - Q_C$$

Definition: $\eta = \frac{W_{cycle}}{Q_H}$

ASSUMPTION: The system shown in the accompanying figure undergoes a power cycle.

ANALYSIS:

(a) $Q_H = 800$ Btu, $W_{cycle} = 480$ Btu. The maximum thermal efficiency for any power cycle under the stated conditions is given by Eq. 5.8

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{500°R}{2000°R} = 0.75\ (75\%)$$

Using the given data

$$\eta = \frac{W_{cycle}}{Q_H} = \frac{480\text{ Btu}}{800\text{ Btu}} = 0.60\ (60\%)$$

Since $\eta < \eta_{MAX}$, this cycle is irreversible. ⟵ (a)

(b) $Q_H = 800$ Btu, $Q_C = 200$ Btu. In this case, it is convenient to use the thermal efficiency in the following form

$$\eta = 1 - \frac{Q_C}{Q_H} = 1 - \frac{200}{800} = 0.75\ (75\%)$$

Since $\eta = \eta_{MAX}$, this cycle is reversible. ⟵ (b)

(c) $W_{cycle} = 800$ Btu, $Q_C = 200$ Btu. Expressing the thermal efficiency in terms of given quantities

$$\eta = \frac{W_{cycle}}{W_{cycle} + Q_C} = \frac{800}{800 + 200} = 0.80\ (80\%)$$

Since $\eta > \eta_{MAX}$, this cycle is impossible. ⟵ (c)

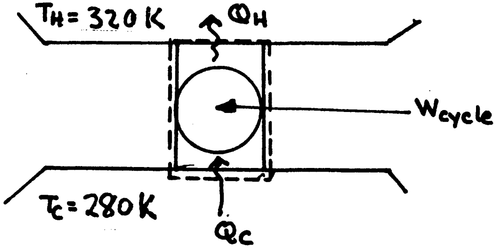
(d) $\eta = 50\%$. Since $\eta < \eta_{MAX}$, this cycle is irreversible. ⟵ (d)

PROBLEM 5.21*

KNOWN: Operating data are provided for a system undergoing a refrigeration cycle while receiving and discharging energy by heat transfer with two thermal reservoirs at specified temperatures.

FIND: For each of four sets of data determine if the cycle is reversible, irreversible, or impossible.

SCHEMATIC & GIVEN DATA:



Energy balance:

$$W_{cycle} = Q_H - Q_C$$

Definition:

$$\beta = \frac{Q_C}{W_{cycle}}$$

ASSUMPTION: The system shown in the accompanying figure undergoes a refrigeration cycle.

ANALYSIS:

(a) $Q_C = 1500\ kJ,\ W_{cycle} = 150\ kJ$. The coefficient of performance is

$$\beta = \frac{Q_C}{W_{cycle}} = \frac{1500\ kJ}{150\ kJ} = 10.0$$

The maximum coefficient of performance under the stated conditions is given by Eq. 5.9

$$\beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{280\ K}{(320-280)K} = 7.0$$

Accordingly, as $\beta > \beta_{max}$, the cycle is impossible. ⟵ (a)

(b) $Q_C = 1400\ kJ,\ Q_H = 1600\ kJ$. With the energy balance, the coefficient of performance can be written as

$$\beta = \frac{Q_C}{Q_H - Q_C} = \frac{1400\ kJ}{(1600-1400)kJ} = 7.0$$

As $\beta = \beta_{max}$, this cycle is reversible. ⟵ (b)

(c) $Q_H = 1600\ kJ,\ W_{cycle} = 400\ kJ$. With the energy balance, the coefficient of performance can be written as

$$\beta = \frac{Q_H - W_{cycle}}{W_{cycle}} = \frac{(1600-400)}{400} = 3.0$$

As $\beta < \beta_{max}$, this cycle is irreversible. ⟵ (c)

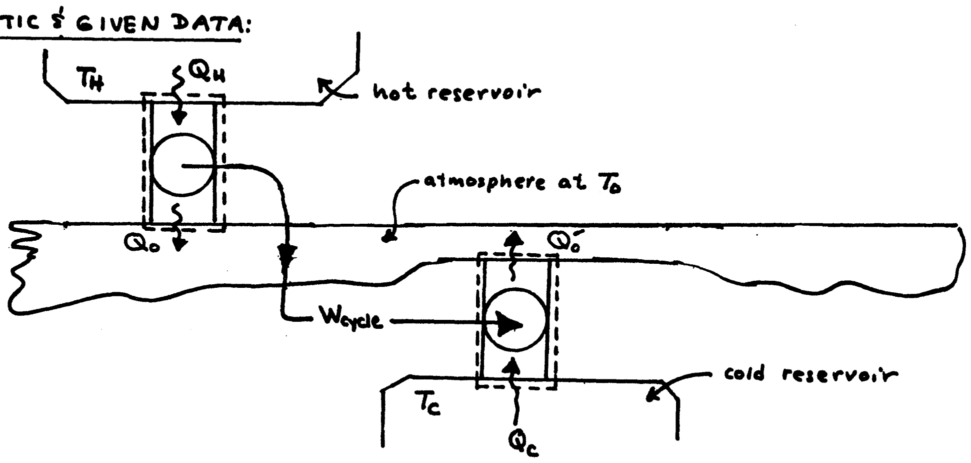
(d) $\beta = 5$. As $\beta < \beta_{max}$, this cycle is irreversible. ⟵ (d)

PROBLEM 5.22

KNOWN: A reversible power cycle receiving Q_H from a hot reservoir drives a refrigeration cycle that removes Q_C from a cold reservoir.

FIND: Develop an expression for Q_C/Q_H in terms of T_H/T_0, T_C/T_0. Plot Q_C/Q_H versus T_H/T_0 and T_C/T_0, each for specified values of the other temperature ratio.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each of the systems shown in the accompany figure undergo reversible cycles. (2) The atmosphere plays the role of a thermal reservoir.

ANALYSIS: (a) An energy balance for the power cycle gives

$$W_{cycle} = Q_H - Q_0$$

An energy balance for the refrigeration cycle gives

$$W_{cycle} = Q_0' - Q_C$$

Combining the last two expressions

$$Q_H - Q_0 = Q_0' - Q_C \qquad (1)$$

As each of the systems undergo reversible cycles between thermal reservoirs, Eq. 5.6 is applicable for each. That is

$$\frac{Q_0}{Q_H} = \frac{T_0}{T_H} \Rightarrow Q_0 = \frac{T_0}{T_H} Q_H \qquad (2)$$

$$\frac{Q_C}{Q_0'} = \frac{T_C}{T_0} \Rightarrow Q_0' = \frac{T_0}{T_C} Q_C \qquad (3)$$

Substituting Eqs. (2) and (3) into Eq. (1)

$$Q_H\left[1 - \frac{T_0}{T_H}\right] = Q_C\left[\frac{T_0}{T_C} - 1\right]$$

Solving for Q_C/Q_H

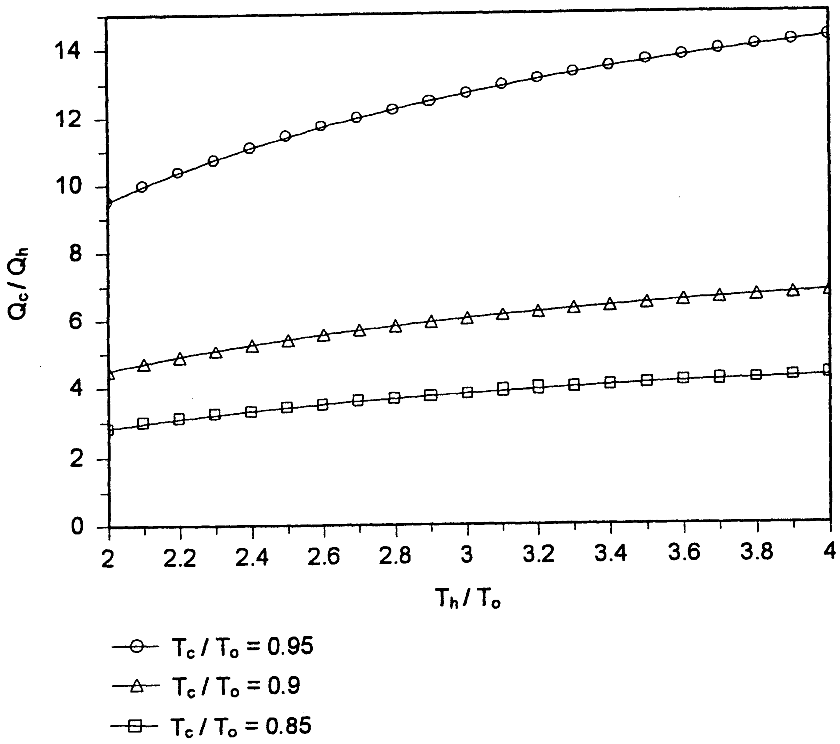
$$\frac{Q_C}{Q_H} = \frac{\left[1 - \frac{T_0}{T_H}\right]}{\left[\frac{T_0}{T_C} - 1\right]} = \frac{T_C}{T_H}\left[\frac{T_H - T_0}{T_0 - T_C}\right]$$

$$= \frac{(T_C/T_0)\left[\frac{T_H}{T_0} - 1\right]}{(T_H/T_0)\left[1 - \frac{T_C}{T_0}\right]} \qquad \longleftarrow$$

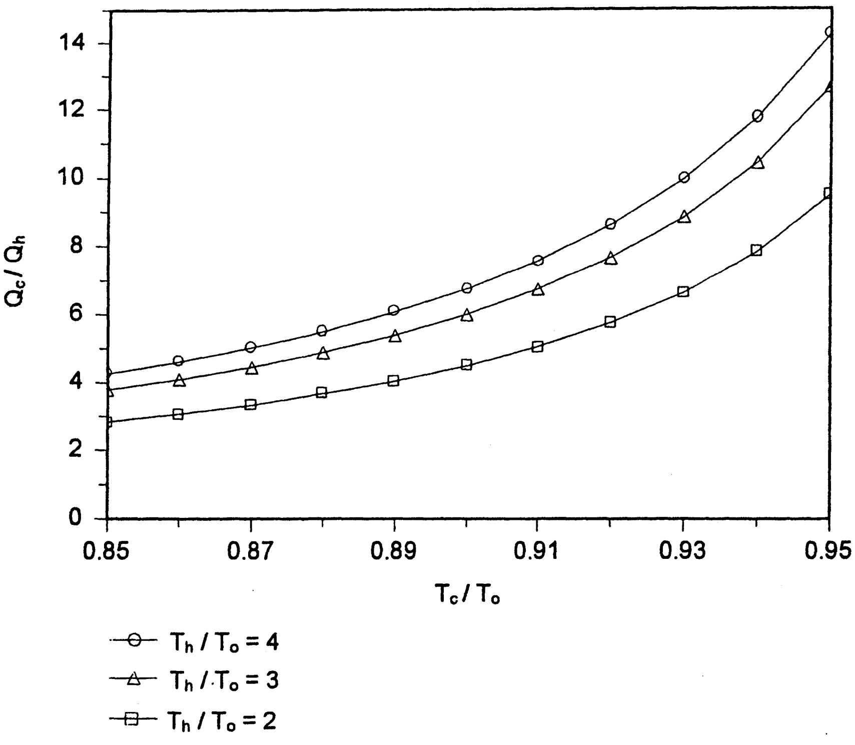
Sample calculations: When $T_H/T_0 = 2$, $T_C/T_0 = .9$, $Q_C/Q_H = 4.5$. When $T_H/T_0 = 4$, $T_C/T_0 = .9$, $Q_C/Q_H = 6.75$.

PROBLEM 5.22 (contd.)

(b) Plots:



Discussion:
When T_c/T_o is specified, larger values of the ratio Q_c/Q_h correspond to larger values of T_H/T_o. That is, $\frac{Q_c}{Q_h} \uparrow$ as $\frac{T_H}{T_o} \uparrow$ when T_c/T_o is fixed.



Discussion:
When T_H/T_o is specified, larger values of the ratio Q_c/Q_H correspond to larger values of T_c/T_o, which is limited by a value of unity.
That is, $\frac{Q_c}{Q_H} \uparrow$ as $\frac{T_c}{T_o} \to 1.0$ when T_H/T_o is fixed.

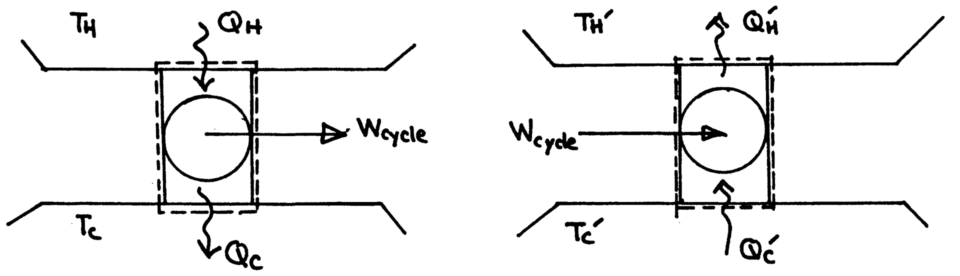
Also note that for a fixed value of the ratio Q_c/Q_H – e.g. a value of 5 – the required value of T_H increases as T_c decreases.

PROBLEM 5.23

KNOWN: A reversible power cycle receiving Q_H from a hot reservoir at T_H drives a heat pump that discharges Q'_H to a reservoir at T'_H.

FIND: Develop an expression for Q'_H/Q_H in terms of T_H, T_C, T'_H, T'_C and determine the relationship among the temperatures for the ratio to exceed unity. If $T'_H = T_C = T_0$, plot Q'_H/Q_H vesus T_H/T_0 and T'_C/T_0, each for selected values of the other temperature ratio.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: Each of the systems shown in the accompanying figure undergo reversible cycles.

ANALYSIS: (a) An energy balance on the power cycle gives

$$W_{cycle} = Q_H - Q_c$$

An energy balance for the heat pump cycle gives

$$W_{cycle} = Q'_H - Q'_c$$

Combining

$$Q_H - Q_c = Q'_H - Q'_c \qquad (1)$$

As each of the systems undergo reversible cycles between thermal reservoirs, Eq. 5.6 is applicable for each. That is

$$\frac{Q_c}{Q_H} = \frac{T_c}{T_H} \Rightarrow Q_c = \frac{T_c}{T_H} Q_H \qquad (2)$$

$$\frac{Q'_c}{Q'_H} = \frac{T'_c}{T'_H} \Rightarrow Q'_c = \frac{T'_c}{T'_H} Q'_H \qquad (3)$$

Substituting Eqs. (2) and (3) into Eq. (1)

$$Q_H\left[1 - \frac{T_c}{T_H}\right] = Q'_H\left[1 - \frac{T'_c}{T'_H}\right]$$

Solving for Q'_H/Q_H

$$\frac{Q'_H}{Q_H} = \frac{\left[1 - \frac{T_c}{T_H}\right]}{\left[1 - \frac{T'_c}{T'_H}\right]} = \frac{T'_H\left[T_H - T_c\right]}{T_H\left[T'_H - T'_c\right]} \longleftarrow \frac{Q'_H}{Q_H}$$

(b) $Q_H'/Q_H > 1$ when

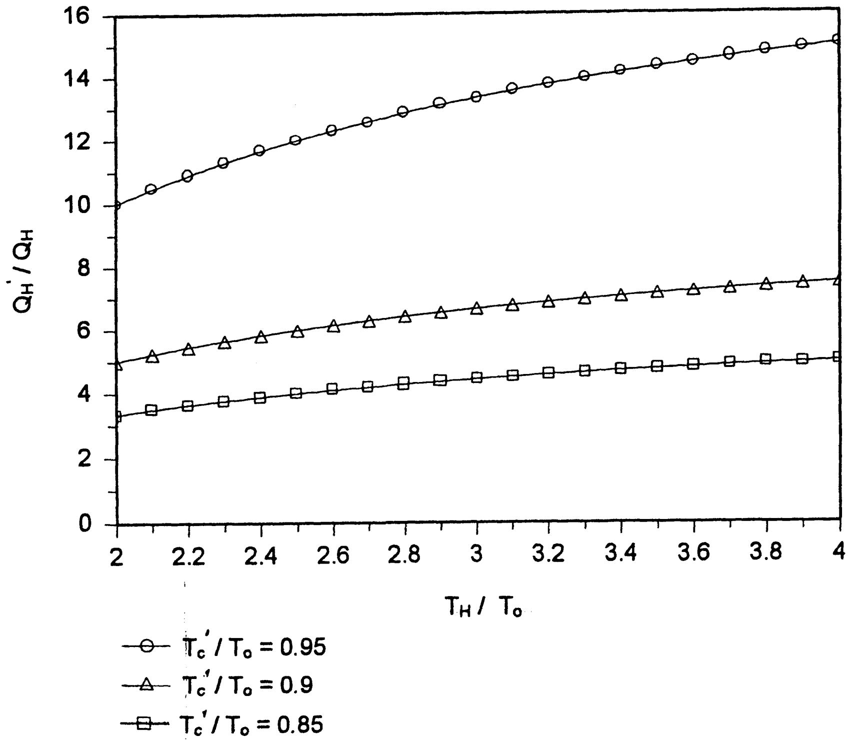
$$\frac{T_c}{T'_c} < \frac{T_H}{T'_H} \longleftarrow$$

(c) If $T_H' = T_c = T_0$,

$$\frac{Q'_H}{Q_H} = \frac{T_0}{T_H}\left[\frac{T_H - T_0}{T_0 - T_c'}\right] \Rightarrow \frac{Q'_H}{Q_H} = \frac{\left[\frac{T_H}{T_0} - 1\right]}{\frac{T_H}{T_0}\left[1 - \frac{T'_c}{T_0}\right]}$$

PROBLEM 5.23 (Contd.)

Sample Calculations : $T_H/T_0 = 2$, $T_c'/T_0 = .9$; $Q_H'/Q_H = 5$. $T_H/T_0 = 4$, $T_c'/T_0 = .9$; $Q_H'/Q_H = 7.5$.

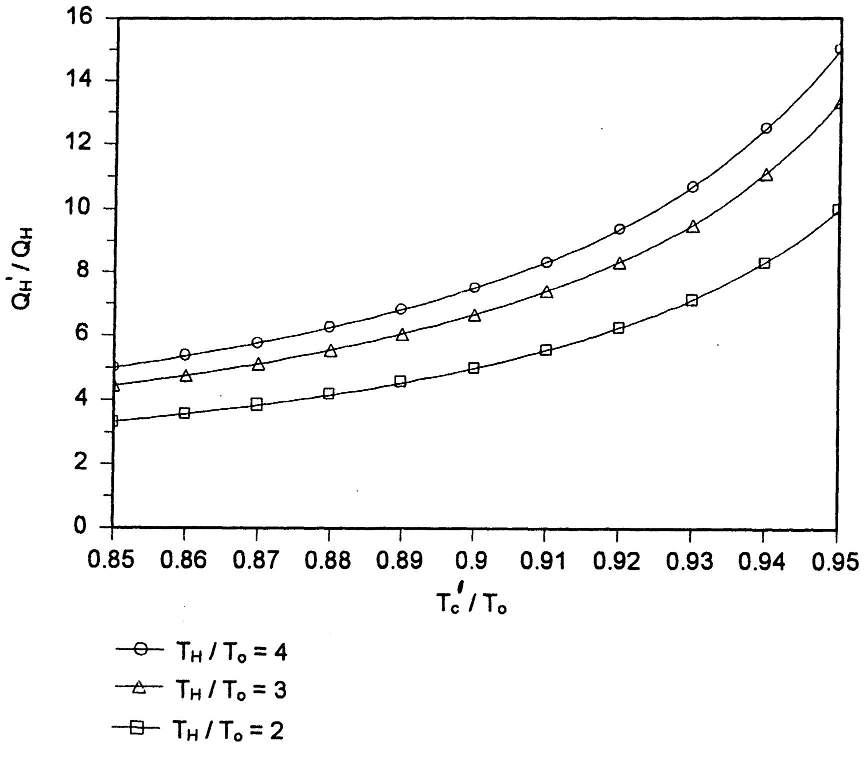


Discussion:

When T_c'/T_0 is fixed, the ratio Q_H'/Q_H increases as T_H/T_0 increases. That is,

$$\frac{Q_H'}{Q_H}\uparrow \text{ as } \frac{T_H}{T_0}\uparrow$$

when T_c'/T_0 is fixed.



Discussion:

When T_H/T_0 is fixed, the ratio Q_H'/Q_H increases as T_c'/T_0 increases, where T_c'/T_0 is limited by a value of unity. That is,

$$\frac{Q_H'}{Q_H}\uparrow \text{ as } \frac{T_c'}{T_0}\rightarrow 1.0$$

When T_H/T_0 is fixed.

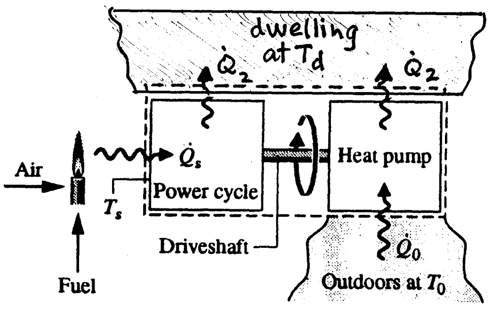
Also, note that for a fixed value of the ratio Q_H'/Q_H – e.g., a value of 5.0 – the required value of T_H increases as T_c decreases.

PROBLEM 5.24

KNOWN: Operating conditions specified for a system consisting of a power cycle driving a heat pump are shown on the schematic.

FIND: Obtain an expression for the maximum theoretical value of the ratio $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s$ in terms of T_s/T_d and T_0/T_d. Plot the ratio $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s$ versus (T_s/T_d) for specified values of T_0/T_d.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system shown in the schematic is at steady state.
2. The maximum theoretical value for $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s$ is achieved with the
① absence of internal irreversibilities.

ANALYSIS: (a) An energy rate balance for the system reduces to $\cancel{dE/dt}^{0} = \dot{Q} - \cancel{\dot{W}}^{0}$, or

$$0 = \dot{Q}_s + \dot{Q}_0 - \dot{Q}_1 - \dot{Q}_2 \Rightarrow \dot{Q}_0 = \dot{Q}_1 + \dot{Q}_2 - \dot{Q}_s \qquad (1)$$

With assumption 2, Eq. 5.6 can be invoked to write

$$\text{power cycle: } \frac{\dot{Q}_1}{T_d} = \frac{\dot{Q}_s}{T_s} \quad , \text{ heat pump: } \frac{\dot{Q}_2}{T_d} = \frac{\dot{Q}_0}{T_0} \qquad (2)$$

The ratio $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s$ can be expressed as follows, with Eqs. (1), (2):
From Eqs. (2), we have

$$\dot{Q}_1 = \frac{T_d}{T_s}\dot{Q}_s \quad \text{and} \quad \dot{Q}_0 = \frac{T_0}{T_d}\dot{Q}_2$$

Inserting these into Eq. (1) and solving for $\dot{Q}_2/\dot{Q}_s$

$$\frac{T_0}{T_d}\dot{Q}_2 = \frac{T_d}{T_s}\dot{Q}_s + \dot{Q}_2 - \dot{Q}_s \Rightarrow \left[\frac{T_0}{T_d} - 1\right]\dot{Q}_2 = \left[\frac{T_d}{T_s} - 1\right]\dot{Q}_s$$

$$\Rightarrow \dot{Q}_2 = \frac{T_d}{T_s}\left[\frac{T_d - T_s}{T_0 - T_d}\right]\dot{Q}_s$$

Then

$$\frac{\dot{Q}_1 + \dot{Q}_2}{\dot{Q}_s} = \frac{\frac{T_d}{T_s}\dot{Q}_s + \frac{T_d}{T_s}\left[\frac{T_d - T_s}{T_0 - T_d}\right]\dot{Q}_s}{\dot{Q}_s} = \frac{T_d}{T_s}\left[1 + \left[\frac{T_d - T_s}{T_0 - T_d}\right]\right]$$

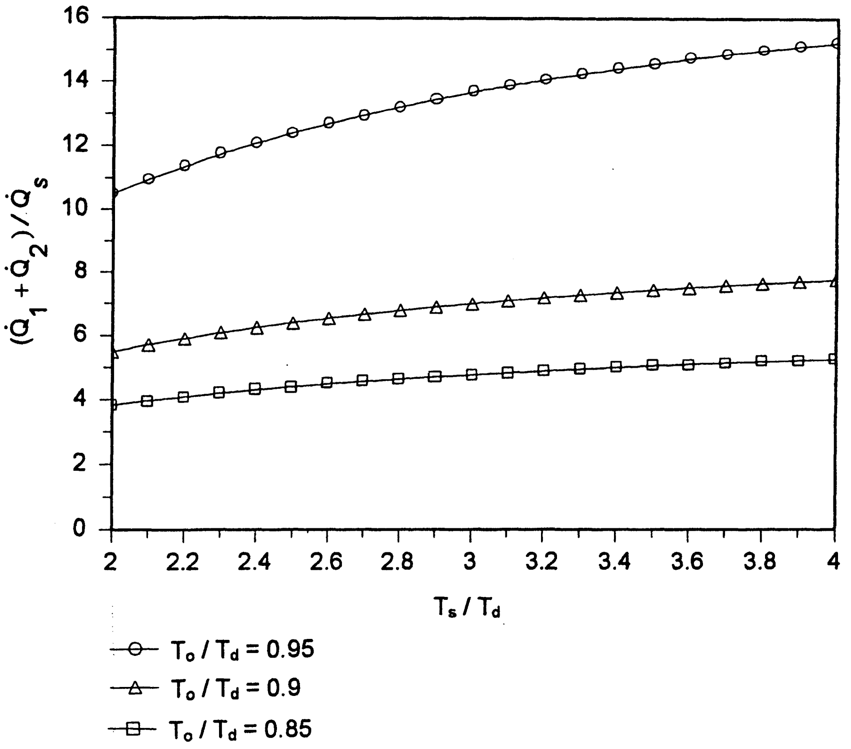
$$= \frac{T_d}{T_s}\left[\frac{T_0 - T_s}{T_0 - T_d}\right]$$

$$= \frac{1}{(T_s/T_d)}\left[\frac{(T_s/T_d) - (T_0/T_d)}{1 - (T_0/T_d)}\right] \quad \longleftarrow$$

Sample calculations:
$T_s/T_d = 2$, $T_0/T_d = 0.85$, $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s = 3.83$
$T_s/T_d = 2$, $T_0/T_d = 0.9$, $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s = 5.5$
$T_s/T_d = 2$, $T_0/T_d = 0.95$, $(\dot{Q}_1 + \dot{Q}_2)/\dot{Q}_s = 10.5$

PROBLEM 5.24 (Contd.)

(b) PLOT



The plot shows that for fixed T_o/T_d, the ratio $(\dot{Q}_1+\dot{Q}_2)/\dot{Q}_s$ increases as T_s/T_d increases. Note that T_o/T_d is limited by a value of unity: the dwelling temperature T_d would never be less than the outdoors temperature. For example, when T_d = 293 K (20°C), the three ratios shown in the plot correspond to outdoor temperatures T_o = 278 K (5°C), 264 K (-9°C), and 249 K (-24°C), respectively.

---

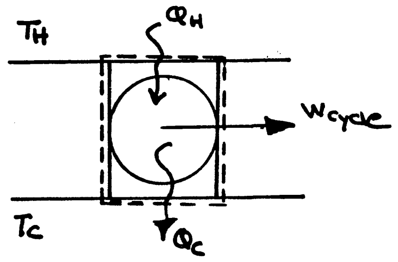
1. The presence of irreversibilities within the system is expected to exact a penalty. If two systems operating as shown in the schematic each receive $\dot{Q}_s$ from a fuel, and one system has internal irreversibilities and the other does not, it is in accord with intuition that the desirable outcome: $(\dot{Q}_1+\dot{Q}_2)$ would be greater in the ideal case than in the actual case. Using concepts from the present chapter, or from Chaps. 6 and 7 to follow, this conclusion can be demonstrated formally.

PROBLEM 5.25*

KNOWN: A system undergoes a reversible cycle while receiving and discharging energy by heat transfer with hot and cold reservoirs.

FIND: Determine the thermal efficiency and the net work developed.

SCHEMATIC & GIVEN DATA:



$Q_H = 500$ Btu
$T_H = 2120°R$ (1660°F)
$T_C = 530°R$ (70°F)

ASSUMPTIONS: (1) The system is shown on the accompanying figure. (2) The system undergoes a reversible cycle while communicating thermally with the hot and cold reservoirs only.

ANALYSIS: With assumption 2, Eq. 5.6 is applicable, giving

$$\left(\frac{Q_C}{500\text{ Btu}}\right)_{\substack{rev\\cycle}} = \frac{530\,°R}{2120\,°R} \Rightarrow Q_C = 125\text{ Btu}$$

For any thermodynamic cycle, the net work developed equals the net heat added:

$$W_{cycle} = Q_H - Q_C$$
$$= 500 - 125 = 375\text{ Btu} \longleftarrow W_{cycle}$$

For any power cycle the thermal efficiency is

① $$\eta = \frac{W_{cycle}}{Q_H} = \frac{375}{500} = 0.75 \;(75\%) \longleftarrow \eta$$

---

1. Alternatively, as the power cycle is reversible the thermal efficiency can be calculated using Eq. 5.8:

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{530}{2120} = 0.75 \;(75\%)$$

Then,

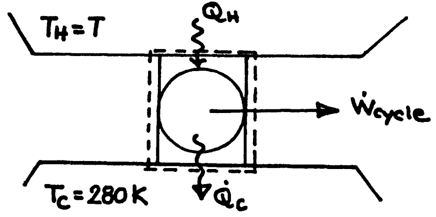
$$W_{cycle} = \eta\, Q_H = (0.75)(500\text{ Btu}) = 375\text{ Btu}$$

PROBLEM 5.26

KNOWN: Steady state operating data are provided for a system undergoing a power cycle while receiving and discharging energy by heat transfer with a hot and cold reservoir at temperatures T and 280 K, respectively

FIND: Determine the minimum theoretical value for T, in K.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cycle} = 40 kW$
$\dot{Q}_C = 1000\ kJ/min$

ASSUMPTIONS: (1) The system is shown on the accompanying figure. (2) The system undergoes a power cycle. (3) The data provided is for operation at steady state.

ANALYSIS: An energy rate balance gives

①
$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$$

or

$$\begin{aligned}\dot{Q}_H &= \dot{W}_{cycle} + \dot{Q}_C \\ &= 40 kW\left(\frac{1 kJ/s}{1 kW}\right) + 1000\frac{kJ}{min}\left(\frac{min}{60 s}\right) \\ &= 56.67\ kJ/s\end{aligned}$$

We know that $\eta \le \eta_{max}$; that is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} \le 1 - \frac{T_C}{T}$$

Inserting values

$$\frac{40\ kJ/s}{56.67\ kJ/s} \le 1 - \frac{280K}{T}$$

$$\Rightarrow \quad \frac{280}{T} \le 1 - \frac{40}{56.67} = 0.294$$

$$\Rightarrow \quad \underbrace{952K}_{T_{min}} \le T \qquad \xleftarrow{\qquad\qquad\qquad T_{min}}$$

---

1. At steady state, the cycle energy balance and thermal efficiency can be expressed in terms of rates:

$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$$

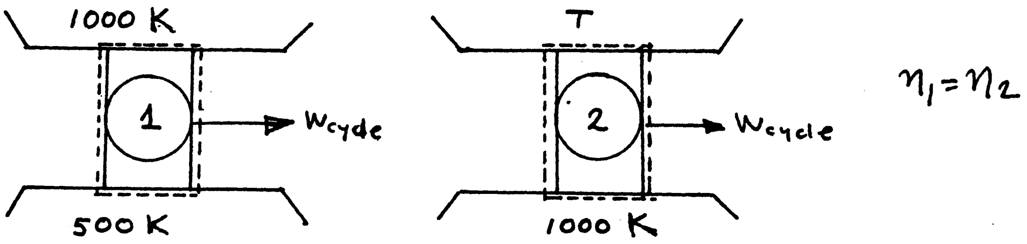
$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H}$$

PROBLEM 5.27

KNOWN: Systems undergo reversible power cycles while receiving and discharging energy with thermal reservoirs. The thermal efficiency is the same for hot and cold reservoirs at 1000 and 500 K, respectively, as for hot and cold reservoirs at T and 1000 K.

FIND: Determine the temperature T.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The systems shown in the accompanying figure undergo reversible power cycles having the same thermal efficiency.

ANALYSIS: In accord with the above assumption, Eq. 5.8 is applicable, giving

$$\eta_1 = 1 - \frac{500}{1000}$$

$$\eta_2 = 1 - \frac{1000}{T}$$

Since $\eta_1 = \eta_2$

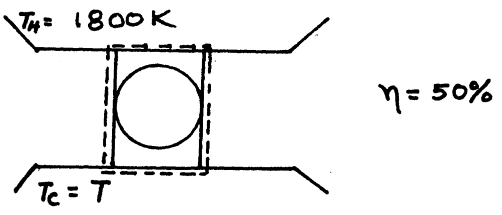
$$\frac{500}{1000} = \frac{1000}{T} \Rightarrow T = 2000\ K \longleftarrow T$$

PROBLEM 5.28

KNOWN: A system undergoes a reversible power cycle whose thermal efficiency is 50% while receiving and discharging energy by heat transfer with reservoirs at temperatures 1800 K (1527°C) and T, respectively.

FIND: Determine T.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system shown on the accompanying figure undergoes a reversible power cycle.

ANALYSIS: In accord with the above assumption, Eq. 5.8 is applicable, giving

$$0.5 = 1 - \frac{T}{1800}$$

Solving for T

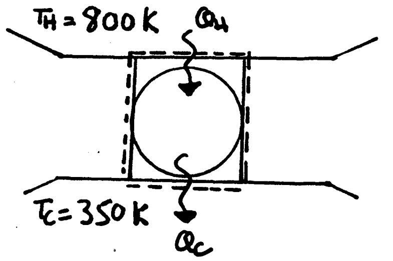
$$T = 900\ K \longleftarrow T$$

PROBLEM 5.29*

KNOWN: A system undergoes a cycle while receiving and discharging energy by heat transfer with two reservoirs of known temperatures.

FIND: For each of two specified values of thermal efficiency, determine if the claimed operation is feasible.

SCHEMATIC & GIVEN DATA:



(a) $\eta = 56\%$

(b) $\eta = 40\%$

ASSUMPTION: The system shown in the schematic undergoes a power cycle.

ANALYSIS: The maximum thermal efficiency for any power cycle under the stated conditions is given by Eq. 5.8:

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{350}{800} = 0.5625 \ (56.25\%)$$

(a) $\eta = 56\%$. Since this claimed value is nearly the maximum theoretical value, the cycle must be nearly ideal in operation. Although not ruled out by the second law, it is unlikely for an actual power cycle to perform at such a level.

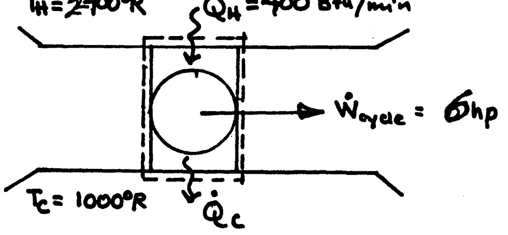
(b) $\eta = 40\%$. Since the claimed value is less than η_{max}, the cycle is feasible.

PROBLEM 5.30

KNOWN: A system undergoes a power cycle while receiving and discharging energy by heat transfer with two thermal reservoirs. Steady state operating data are provided.

FIND: Determine if the cycle can operate as claimed.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a power cycle. (2) The data provided is for operation at steady-state.

ANALYSIS: Using the given data, the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_H} = \frac{(6\,hp)\left|\frac{2545\ Btu/h}{1\ hp}\right|\left|\frac{1\ h}{60\ min}\right|}{400\ \frac{Btu}{min}} = 0.636 \ (63.6\%)$$

The maximum thermal efficiency for any power cycle under the stated conditions is given by Eq. 5.8:

$$\eta_{max} = 1 - \frac{1000}{2400} = 0.58 \ (58\%)$$

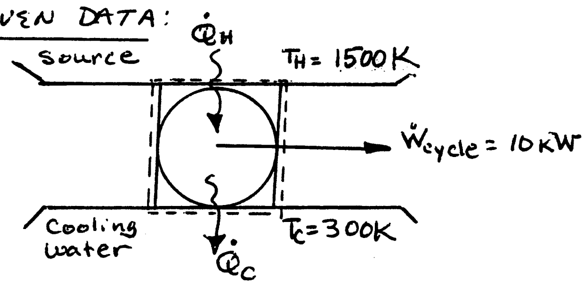
Since the claimed value of thermal efficiency is greater than η_{MAX}, the claim cannot be valid.

PROBLEM 5.31

KNOWN: A system undergoes a power cycle while receiving and discharging energy by heat transfer from a source at 1500K and to cooling water at 300K, respectively. Steady state operating data are provided.

FIND: Determine the minimum number of cycles/min.

SCHEMATIC & GIVEN DATA:



$$\frac{\dot{Q}_H}{n} = 10\ \frac{kJ}{cycle}$$

where n = cycles/min

ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a power cycle. (2) The data provided is for operation at steady state. (3) The source and cooling water play the roles of hot and cold reservoirs, respectively.

ANALYSIS: The power developed must be less than, or equal to, the power that would be developed by a reversible power cycle operating between thermal reservoirs at the specified temperatures:

$$10\,kW \le \left[1 - \frac{T_c}{T_H}\right]\dot{Q}_H$$

$$\le \left[1 - \frac{300K}{1500K}\right]\left[10\ \frac{kJ}{cycle}\right]\left[n\ \frac{cycles}{min}\right]\left[\frac{min}{60s}\right]\left[\frac{1\,kW}{1\,kJ/s}\right]$$

$$10\,kW \le \left[\frac{(0.8)(10)\,n}{60}\right]kW$$

$$75 \le n$$

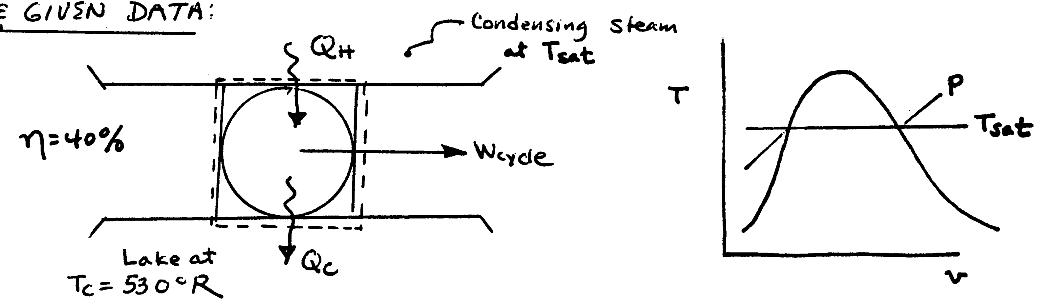
Accordingly, the actual device would require 75, or more, cycles per minute.

PROBLEM 5.32

KNOWN: A system undergoes a power cycle while receiving energy by heat transfer from condensing steam and discharging energy by heat transfer to a lake at 70°F. For the cycle, $\eta = 40\%$.

FIND: Determine the lowest possible condensing steam temperature and the corresponding pressure.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a power cycle. (2) The condensing steam and the lake play the roles of the hot and cold reservoirs, respectively.

ANALYSIS: Since P_{sat} decreases as T_{sat} decreases, the lowest possible condensing steam pressure corresponds to the lowest possible condensing steam temperature. Moreover, the thermal efficiency must be less than, or equal to, the thermal efficiency of a reversible power cycle operating between reservoirs at the specified temperatures:

$$0.40 \le \left[1 - \frac{T_C}{T_H}\right] = 1 - \frac{530}{T_{sat}}$$

or

$$0.60 \ge \frac{530}{T_{sat}} \Rightarrow T_{sat} \ge 883°R \ (423°F).$$

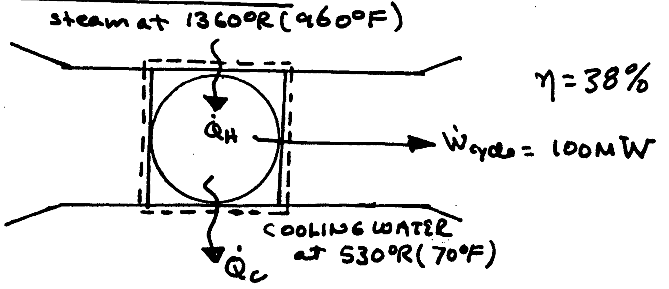
Thus, the lowest possible saturation temperature is 423 °F. Interpolating in Table A-2E, the corresponding saturation pressure is 319 lbf/in^2. ←

PROBLEM 5.33

KNOWN: Steady-state operating data are provided for a system undergoing a power cycle wh receiving and discharging energy by heat transfer at 900 and 70°F, respectively.

FIND: Determine the rate energy is discharged and compare with the minimum theoretical rate.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system shown in the schematic undergoes a power cycle. 2. Operation is at steady state. 3. The steam and cooling water play the roles of hot and cold reservoirs, respectively.

ANALYSIS: (a) For any cycle at steady state, $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C$, or $\dot{Q}_C = \dot{Q}_H - \dot{W}_{cycle}$. Also, $\eta = \dot{W}_{cycle}/\dot{Q}_H \Rightarrow \dot{Q}_H = \dot{W}_{cycle}/\eta$. Collecting results

$$\dot{Q}_C = \dot{W}_{cycle}\left[\frac{1}{\eta}-1\right] = 100\,MW\left[\frac{1}{0.38}-1\right] = 163.2\,MW\left|\frac{3.413\times10^6\,Btu/h}{1\,MW}\right| = 5.57\times10^8\,\frac{Btu}{h}$$

(b) The thermal efficiency must be less than, or equal to, the thermal efficiency of a reversible power cycle operating between reservoirs at $T_H = 1360°R$, $T_C = 530°R$:

$$\eta \le \left[1-\frac{T_C}{T_H}\right], \text{ where } \eta = \frac{\dot{W}_{cycle}}{\dot{W}_{cycle}+\dot{Q}_C}. \text{ That is}$$

① $$\left(\frac{100\,MW}{100\,MW+\dot{Q}_C}\right) \le 1-\frac{530}{1360} = 0.61 \Rightarrow \frac{100\,MW}{0.61} \le 100\,MW + \dot{Q}_C \Rightarrow$$

$$\dot{Q}_C \ge 63.9\,MW\left|\frac{3.413\times10^6\,Btu/h}{1\,MW}\right| = 2.18\times10^8\,\frac{Btu}{h}$$

Thus, the minimum theoretical rate at which energy could be discharged to the cooling water is 2.18×10^8 Btu/h.

Comparing the values for $\dot{Q}_C$

$$\frac{\dot{Q}_C}{(\dot{Q}_C)_{MIN}} = \frac{5.57\times10^8\,Btu/h}{2.18\times10^8\,Btu/h} = 2.56$$

Thus, the actual heat discharge rate is over 2½ times the minimum theoretical rate. This illustrates that irreversibilities within the system during each cycle of operation contribute significantly to thermal pollution.

---

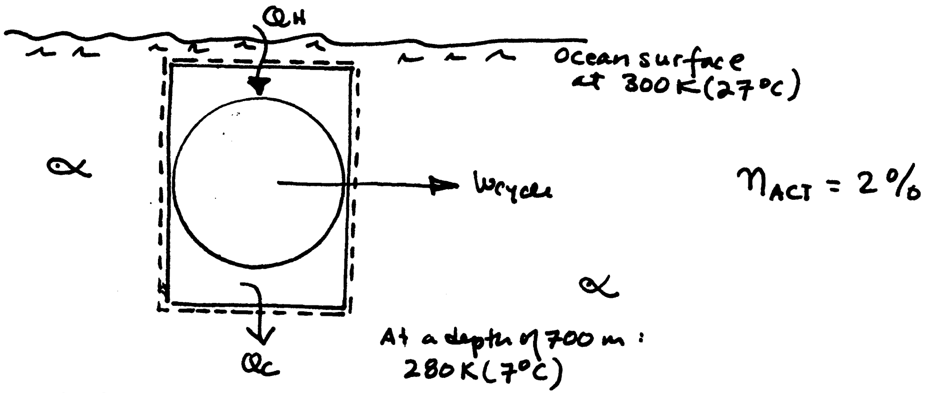
1. The actual thermal efficiency, 38%, is considerably less than the maximum value, 61%, owing to the effect of irreversibilities.

PROBLEM 5.34

KNOWN: A power cycle operates between surface ocean water at 27°C and water at a depth of 700 m where the temperature is 7°C.

FIND: Determine the maximum theoretical thermal efficiency for any such cycle and compare with the observed thermal efficiency of OTEC power plants

SCHEMATIC & GIVEN DATA:


Q_H
Ocean surface at 300 K (27°C)
W_{cycle}
$\eta_{ACT} = 2\%$
Q_C
At a depth of 700 m : 280 K (7°C)

ASSUMPTIONS: 1. The system shown in the schematic undergoes a power cycle. 2. The ocean surface water and water at a depth of 700 m play the roles of hot and cold reservoirs, respectively.

ANALYSIS: (a) The thermal efficiency of any power cycle under these conditions must be less than, or equal to, the thermal efficiency of a reversible power cycle operating between reservoirs at $T_H = 300\,K$, $T_C = 280\,K$, respectively. That is

$$\eta \le \left[1-\frac{T_C}{T_H}\right] = \left[1-\frac{280}{300}\right] = 0.067 \quad (6.7\%) \Rightarrow \eta_{MAX} = 6.7\% \longleftarrow$$

(b) If $\eta_{act} = 2\%$, then

$$\frac{\eta_{act}}{\eta_{MAX}} = \frac{2\%}{6.7\%} = 0.3$$

That is, for the assumed conditions, actual OTEC cycles operate at 30% of the maximum theoretical value. This indicates that there may be some scope for improving actual performance, by a few percentage points, via conventional engineering measures. Still, the characteristically low value of η_{max} for OTEC plants suggests that power development by such means would not be competitive economically with other types of power plants in use today.

PROBLEM 5.35*

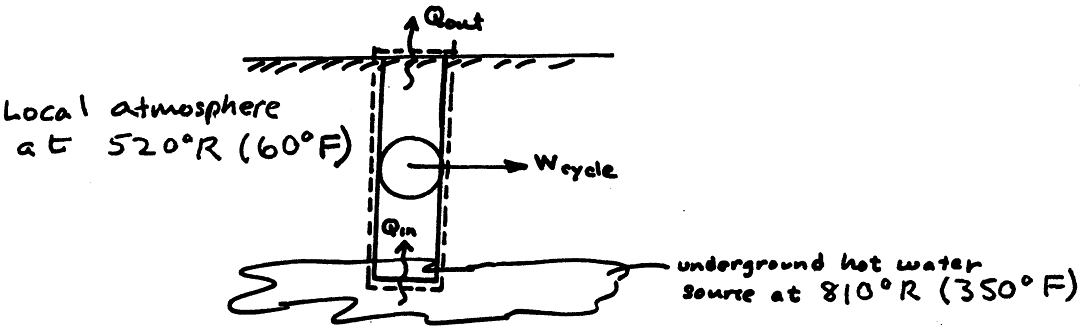
KNOWN: Hot water at 350°F is available from an underground source. The local atmosphere is at 60°F.

FIND: Determine the maximum possible thermal efficiency for any power cycle operating between these temperatures.

SCHEMATIC & GIVEN DATA:

(Continued)

PROBLEM 5.35 (Contd.)


Q_{out}
Local atmosphere at 520°R (60°F)
W_{cycle}
Q_{in}
underground hot water source at 810°R (350°F)

ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a power cycle. (2) The underground hot water source and the local atmosphere play the roles of the hot and cold reservoirs, respectively.

ANALYSIS: The maximum thermal efficiency corresponds to the case where $\dot{Q}_{in}$ is received from a thermal reservoir at 810°R and $\dot{Q}_{out}$ is discharged to a thermal reservoir at 520°R, and the system operates without irreversibilities. That is, applying Eq. 5.8

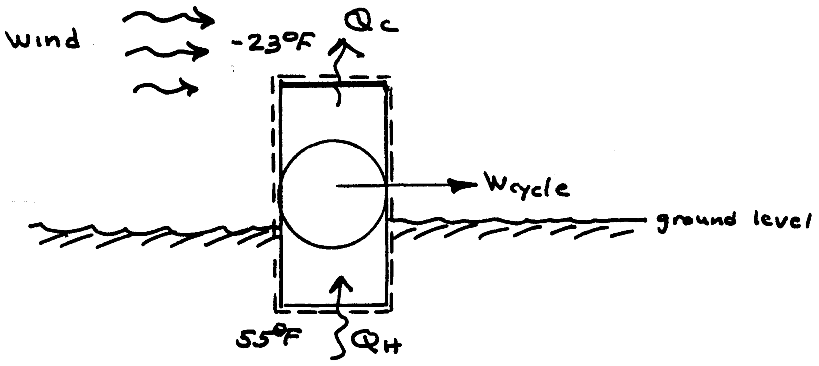
$$\eta_{MAX} = 1 - \frac{520}{810} = 0.358\ (35.8\%) \longleftarrow$$

PROBLEM 5.36 *

KNOWN: A power cycle operates between the ground at a temperature of 55°F and the atmosphere at a temperature of −23°F. A thermal efficiency of 10% is claimed.

FIND: Evaluate this claim.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the figure undergoes a power cycle. (2) The ground at 55°C and the atmosphere at −23°F play the roles of hot and cold reservoirs, respectively.

ANALYSIS: The maximum thermal efficiency any power cycle may have when operating between hot and cold reservoirs at T_H and T_C, respectively, is given by Eq. 5.8:

$$\eta_{MAX} = 1 - \frac{T_C}{T_H}$$

$$= 1 - \frac{(460-23)°R}{(460+55)°R} = 0.151 \quad (15.1\%)$$

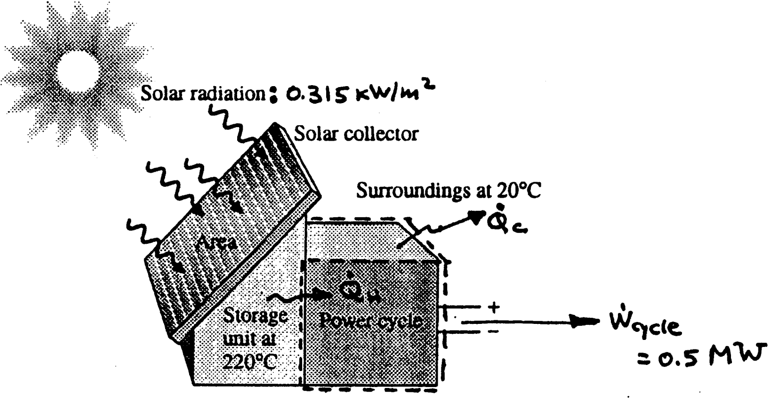
The claimed performance is in agreement with the requirements of the second law. Still, the efficiency value is so low as to raise concerns over economic viability.

PROBLEM 5.37

KNOWN: Steady-state operating data are provided for a solar-activated power plant.

FIND: Determine the minimum theoretical collector area required. Plot the collector area versus power plant thermal efficiency for specified values of the collector efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system shown in the schematic is at steady state. All other components are also at steady state. 2. Some of the incident solar energy is lost through unavoidable heat transfer from the solar collector. All other components experience negligible stray heat transfer. 3. The storage unit and surroundings play roles of hot and cold reservoirs, respectively.

ANALYSIS: The rate energy is collected and supplied to the storage unit is evaluated as (0.315 A e) where A is the collector area and e is the collector efficiency. Further, at steady state the rate energy is supplied to the system undergoing the power cycle equals the rate energy enters the storage unit: $\dot{Q}_H = (0.315\,A\,e)$. The thermal efficiency of the power cycle is $\eta = \dot{W}_{cycle}/\dot{Q}_H$. Combining these expressions and solving for A

$$A = \frac{\dot{W}_{cycle}}{0.315\,\eta\, e} = \frac{(0.5\,\text{MW})\left|\frac{10^3\,\text{kW}}{1\,\text{MW}}\right|}{\left(0.315\,\frac{\text{kW}}{\text{m}^2}\right)\eta\, e} = \frac{1587\,\text{m}^2}{\eta\, e} \qquad (1)$$

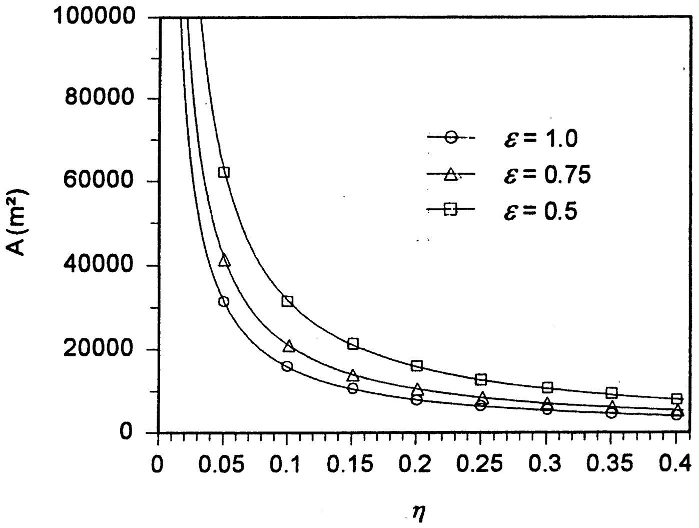
(a) By inspection of Eq. (1), the collector area A is a minimum when η and e take on their respective maximum values: e = 1.0 and

$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{293\,\text{K}}{493\,\text{K}} = 0.406.$$

Thus,

① $$A_{min} = \frac{1587\,\text{m}^2}{(0.406)(1)} = 3909\,\text{m}^2$$

(b) PLOT: For $0 < \eta \le \eta_{MAX} = 0.406$, the plot giving A versus η for e = 1.0, 0.75, and 0.5 is



Discussion:

1. The collector area required decreases as η approaches η_{max}, as expected.
2. For fixed η, the collector area decreases as ∈ increases, also as expected.

---

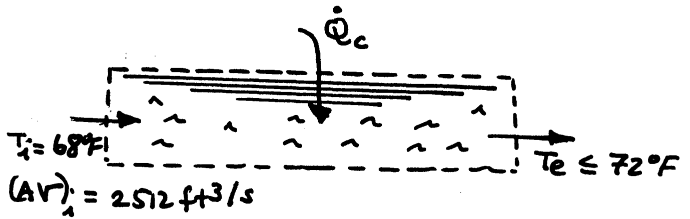
1. For comparison, the area of a football field is about 4350 m².

PROBLEM 5.38

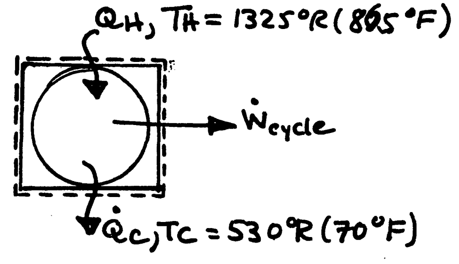
KNOWN: Operating data are provided for a power plant that discharges energy by heat transfer to a river. For environmental reasons, the temperature of the river downstream of the power plant is restricted.

FIND: Estimate the maximum theoretical power that can be developed subject to the restriction on river temperature.

SCHEMATIC & GIVEN DATA



(A) River


$\dot{Q}_H$, $T_H = 1325°R$ (865°F)
$\dot{W}_{cycle}$
$\dot{Q}_c$, $T_C = 530°R$ (70°F)

(B) Power Plant

ASSUMPTIONS: 1. The two systems shown in the schematic are at steady state. In each case, $\dot{Q}_c$ is in the direction of the arrow. 2. For system (A), effects of kinetic and potential energy can be ignored. 3. System (B) undergoes a power cycle that receives energy from a reservoir at T_H and discharges energy to a reservoir at T_C, which equals the average river temperature.

ANALYSIS: An energy rate balance for control volume (A) reduces to give

$$\dot{Q}_c = \dot{m}[h_e - h_i] = \frac{(AV)_i}{v_i}[h_e - h_i].$$

Then, with $v_i = v_f(T_i) = 0.01605\ ft^3/lb$, $h_i = h_f(T_i) = 36.09\ Btu/lb$, and $h_e \le h_f(72°F) = 40.09\ Btu/lb$ from Table A-2E

$$\dot{Q}_c \le \left(\frac{2512\ ft^3/s}{0.01605\ ft^3/lb}\right)(40.09 - 36.09)\left(\frac{Btu}{lb}\right) = 6.26 \times 10^5\ Btu/s$$

Thus, to meet the environmental constraint, the power cycle cannot discharge energy to the river at a rate greater than $(\dot{Q}_c)_{MAX} = 6.26 \times 10^5\ Btu/s$.

Turning to system (B), an energy balance reduces to $\dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_c$. Moreover, the thermal efficiency must be less than, or equal to, the thermal efficiency of a reversible power cycle operating between reservoirs at T_H, T_C: $\eta \le [1 - \frac{T_C}{T_H}]$, where $\eta = \dot{W}_{cycle}/\dot{Q}_H$. Collecting results

$$\frac{\dot{W}_{cycle}}{\dot{W}_{cycle} + \dot{Q}_c} \le \left[1 - \frac{T_C}{T_H}\right] \Rightarrow \dot{W}_{cycle} \le \left[1 - \frac{T_C}{T_H}\right](\dot{W}_{cycle} + \dot{Q}_c)$$

$$\Rightarrow \quad \dot{W}_{cycle} \le \left[\frac{T_H}{T_C} - 1\right](\dot{Q}_c) \rightarrow \dot{W}_{cycle} \le \left[\frac{T_H}{T_C} - 1\right](\dot{Q}_c)_{MAX} \qquad (1)$$

Inserting values, Eq. (1) gives

$$\dot{W}_{cycle} \le \left[\frac{1325°R}{530°R} - 1\right]\left(6.26 \times 10^5\ \frac{Btu}{s}\right) = (1.5)\left(6.26 \times 10^5\ \frac{Btu}{s}\right) = 9.39 \times 10^5\ \frac{Btu}{s}$$

$$\le \left(9.39 \times 10^5\ \frac{Btu}{s}\right)\left|\frac{3600\ s}{1\ h}\right|\left|\frac{1\ MW}{3.413 \times 10^6\ Btu/h}\right| = 990.4\ MW$$

or

① $$(\dot{W}_{cycle})_{\substack{MAX \\ THEO}} = 990.4\ MW \longleftarrow$$

---

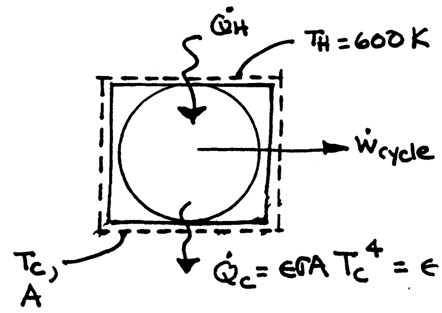
1. Owing to the effect of irreversibilities, an actual power plant would develop much less power than the maximum theoretical value determined here.

PROBLEM 5.39

KNOWN: A power cycle receives energy by heat transfer at a specified temperature and rejects energy by heat transfer at temperature T_c. The thermal efficiency is one-half that of a reversible cycle operating between reservoirs at the respective temperatures.

FIND: (a) For $T_c = 400K$, determine the net power developed per unit of radiator surface area and the thermal efficiency. (b) Plot these quantities versus T_c and determine the maximum value of the net power developed per unit area. (c) Determine the range of T_c for which the net power per unit area is within 2% of the maximum.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_c = \epsilon\sigma A T_c^4 = \epsilon\sigma A T_H^4 r^4$

$\epsilon = 0.6$

$\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$

$$\eta = 0.5\left[1 - \frac{T_c}{T_H}\right]$$

$$= 0.5[1 - r] \quad , \quad r = T_c/T_H$$

ASSUMPTIONS: (1) The system shown in the schematic undergoes a power cycle and operates at steady state. (2) Heat transfer at T_c is only by thermal radiation, as indicated on the schematic. The surface receives no radiation. (3) For the power cycle, $\eta = 0.5\ \eta_{max}$.

ANALYSIS: The thermal efficiency of a power cycle operating at steady state is $\eta = \dot{W}_{cycle}/\dot{Q}_H$. Introducing the energy rate balance, $\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_c$

$$\eta = \frac{\dot{W}_{cycle}}{\dot{W}_{cycle} + \dot{Q}_c} \Rightarrow \dot{W}_{cycle} = \left(\frac{\eta}{1-\eta}\right)\dot{Q}_c$$

Now, from above $\eta = 0.5(1-r)$ and $\dot{Q}_c = \epsilon\sigma A T_H^4 r^4$. Thus

$$\dot{W}_{cycle} = \left(\frac{0.5(1-r)}{1 - 0.5(1-r)}\right)\epsilon\sigma A T_H^4 r^4$$

Simplifying and solving for $\dot{W}_{cycle}/A$

$$\dot{W}_{cycle}/A = (\epsilon\sigma T_H^4)\left[\frac{(1-r)}{(1+r)} r^4\right] \qquad (1)$$

(a) $T_c = 400K$. In this case, $r = 400/600 = 0.6667$. Thus

$$\frac{\dot{W}_{cycle}}{A} = (0.6)(5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4})(600K)^4 \left[\frac{(1-.6667)}{(1+.6667)}(.6667)^4\right]\left|\frac{1 kW}{10^3 W}\right|$$

$$= 0.174\ \frac{kW}{m^2} \longleftarrow \dot{W}_{cycle}/A$$

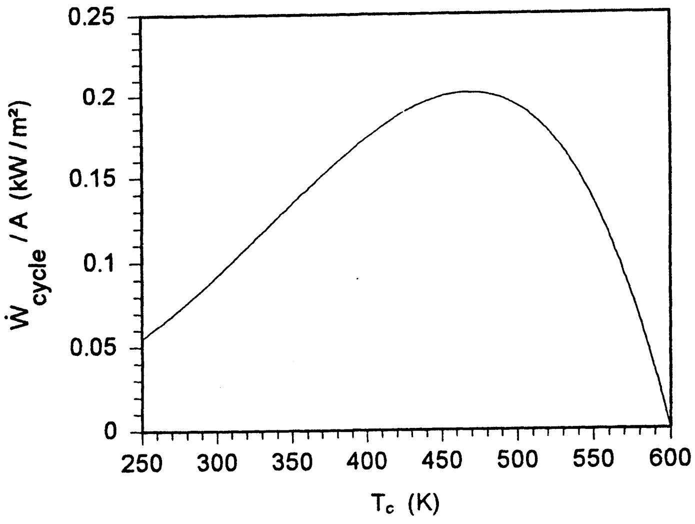
Also

$$\eta = 0.5(1-r) = 0.1667\ (16.67\%) \longleftarrow \eta$$

PROBLEM 5.39 (Cont'd)

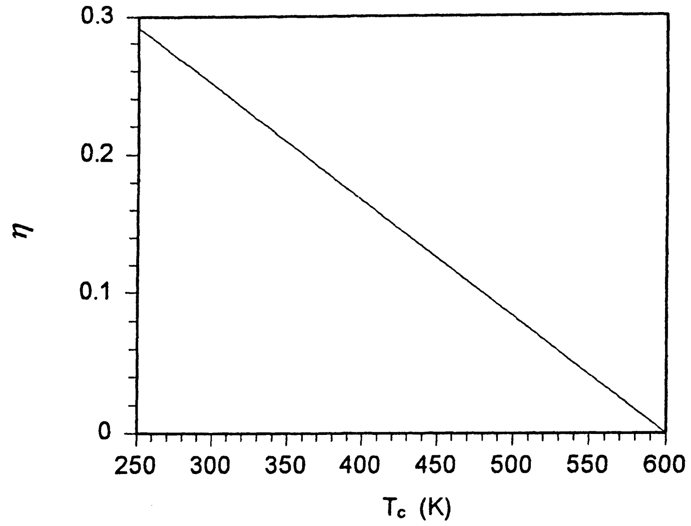
(b) Plots The following plots are constructed using IT:

①



From the computer data, the maximum value for $\dot{W}_{cycle}/A$ is 0.2017 kW/m², and occurs at T_c = 468 K. This is confirmed by inspection of the graph.

②



(c) For the net power per unit area within 2% of the maximum value,

$$0.98\,(\dot{W}_{cycle}/A)_{max} = (0.98)(0.2017) = 0.1977\ \text{kW/m}^2$$

By inspection of the computer data: 441 K < T_c < 491 K ← T_c range

Again, this is confirmed by inspection of the graph.

---

1. Alternatively, the value of r for which $\dot{W}_{cycle}/A$ is maximum could be obtained by differentiating (1) with respect to r, setting the resulting expression equal to zero, and solving for r. The result is r = 0.781, and $(\dot{W}_{cycle}/A)_{max} = 0.2017$. This confirms the computer solution.

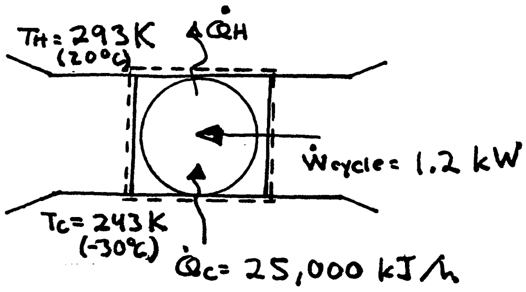
2. Note that the thermal efficiency corresponding to $(\dot{W}_{cycle}/A)_{max}$ is 0.1095 (10.95%). In this application, maximizing η may not be the objective. The size of the radiator surface may be a more significant consideration.

PROBLEM 5.40*

KNOWN: Operating data are claimed for a refrigeration cycle.

FIND: Evaluate the claim.

SCHEMATIC & GIVEN DATA:



$T_H = 293\text{K}$ (20°C)
$\dot{Q}_H$
$\dot{W}_{cycle} = 1.2\text{ kW}$
$T_C = 243\text{K}$ (−30°C)
$\dot{Q}_C = 25{,}000\text{ kJ/h}$

ASSUMPTION: The system shown in the schematic undergoes a refrigeration cycle while receiving energy by heat transfer from a reservoir at T_C and discharging energy by heat transfer to a reservoir at T_H.

ANALYSIS: Applying Eq. 2.45 together with claimed operating data, the coefficient of performance is

$$\beta = \frac{\dot{Q}_C}{\dot{W}_{cycle}} = \frac{(25{,}000\ \frac{kJ}{h})}{(1.2\text{ kW})}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|\left|\frac{1\text{ h}}{3600\text{ s}}\right| = 5.79$$

The maximum coefficient of performance any refrigeration cycle can have while operating between reservoirs at T_C, T_H is given by Eq. 5.9:

$$\beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{243\text{ K}}{(293-243)\text{K}} = 4.86$$

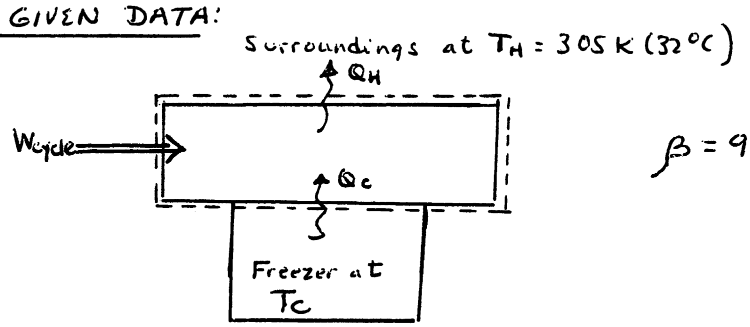
Since the claimed value for β exceeds β_{max}, the inventor's claim is invalid.

PROBLEM 5.41

KNOWN: A tray of ice cubes is placed in a freezer having a coefficient of performance of 9.0 operating in a room at 32 °C.

FIND: Determine if the cubes would remain frozen.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a refrigeration cycle. (2) The freezer compartment and the surroundings play the roles of the cold and hot reservoirs, respectively.

ANALYSIS: As discussed in Sec. 5.5.2, the actual coefficient of performance must be less than, or equal to, the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at the specified temperatures. Then, with Equation 5.9 and the given value of 9.0 for the coefficient of performance

$$9.0 \le \frac{T_C}{T_H - T_C} = \frac{1}{(305/T_C) - 1}$$

or

$$\frac{305}{T_C} \le 1 + \frac{1}{9}$$

giving

$$T_C \geq 275 \text{ K}$$

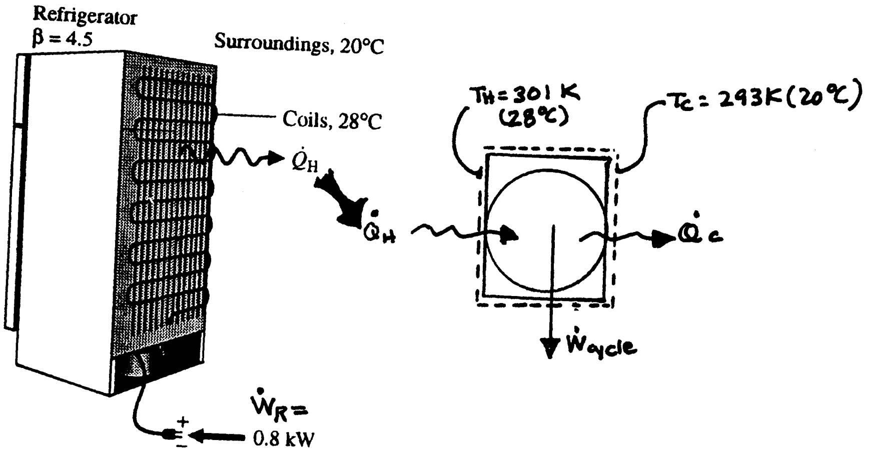
However, to maintain the frozen cubes the freezer must be at 273 K, or less. Accordingly, the cubes would not remain frozen. ◄

PROBLEM 5.42*

KNOWN: Steady-state operating data are provided for a refrigerator.

FIND: Determine (a) the rate energy is rejected from the refrigerator to the surroundings by heat transfer, (b) the lowest theoretical temperature inside the refrigerator, and (c) the maximum theoretical power that could be developed by a power cycle from the energy rejected. Discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Operation is at steady state. (2) In part (b) the interior of the refrigerator and the surroundings play the roles of cold and hot reservoirs, respectively. (3) In part (c), the coils and surroundings play the roles of the hot and cold bodies, respectively.

ANALYSIS: (a) With Eq. 2.45, $\beta = \dot Q_{in}/\dot W_R$. An energy balance for the refrigerator reads $\dot Q_H = \dot Q_{in} + \dot W_R$. In these expressions, $\dot Q_{in}$ represent the energy into the circulating refrigerant from the food and other contents inside the refrigerator. Collecting results

$$\beta = \frac{\dot Q_H - \dot W_R}{\dot W_R} \Rightarrow \dot Q_H = (1+\beta)\dot W_R = (1+4.5)(0.8\,\text{kW}) = 4.4\,\text{kW} \longleftarrow$$

(b) From Sec. 5.5.2, we know that $\beta \le \beta_{MAX}$, where β_{MAX} is given by Eq. 5.9. Thus, with $T_H = 293$ K (20°C) and T_C denoting the temperature inside the refrigerator,

① $$4.5 \le \frac{T_C}{T_H - T_C} \Rightarrow 4.5\,T_H \le 5.5\,T_C \Rightarrow \left(\frac{4.5}{5.5}\right)T_H \le T_C$$

Solving, $T_C \ge 239.7$ K. Thus, the lowest theoretical temperature inside the refrigerator is 239.7 K. ⟵

(c) For the power cycle shown in the sketch, the thermal efficiency is $\eta = \dot W_{cycle}/\dot Q_H$. From Sec. 5.5.1, we know $\eta \le \eta_{MAX}$, where η_{MAX} is given by Eq. 5.8. Then, with $T_C = 293$ K (20°C) and $T_H = 301$ K (28°C), we get

PROBLEM 5.42 (Continued)

$$\frac{\dot{W}_{cycle}}{\dot{Q}_H} \le 1 - \frac{T_C}{T_H} \Rightarrow \dot{W}_{cycle} \le \left[1 - \frac{T_C}{T_H}\right]\dot{Q}_H$$

$$\dot{W}_{cycle} \le \left[1 - \frac{293\,K}{301\,K}\right](4.4\,kW)$$

② $$\le 0.12\,kW$$

The maximum theoretical power that could be developed by the energy rejected is 0.12 kW. ←

---

1. Note that the surroundings is the natural hot reservoir for the refrigerator, and thus we use $T_H = 293\,K\,(20°C)$ and not the coil temperature, which would reduce to 293 K if the refrigerator were unplugged.

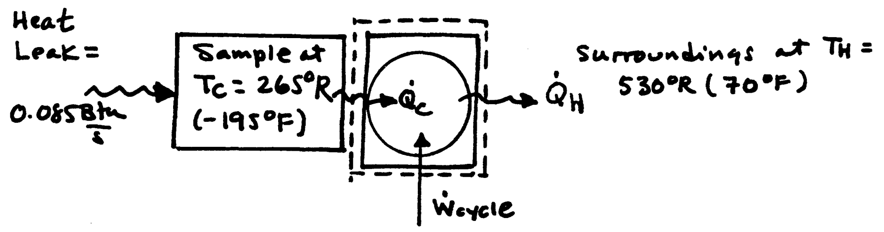
2. Although the refrigerator rejects energy at an appreciable rate, 4.4 kW, the thermodynamic value of this energy as measured by the power that might be developed from it is relatively small. Accordingly, there is *little* incentive for attempting to exploit this heat transfer, and $\dot{Q}_H$ is simply discharged to the surroundings without use.

PROBLEM 5.43

KNOWN: A cryogenic sample must be kept at −195°F by a refrigeration system.

FIND: Determine the minimum theoretical power required by the refrigerator.

SCHEMATIC & GIVEN DATA:


Heat Leak = 0.085 Btu/s
Sample at Tc = 265°R (−195°F)
Q̇c
Q̇H
Surroundings at TH = 530°R (70°F)
Ẇcycle

ASSUMPTIONS: 1. The system shown in the schematic undergoes a refrigeration cycle at steady state. 2. The sample is also at steady state. 3. The sample and surroundings play the role of cold and hot reservoirs, respectively.

ANALYSIS: The coefficient of performance of the refrigerator must be less than, or equal to, the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at Tc, TH. Thus, with Eq. 2.45 and Eq. 5.9

$$\beta \le \beta_{MAX} \Rightarrow \frac{\dot{Q}_C}{\dot{W}_{cycle}} \le \frac{T_C}{T_H - T_C} \Rightarrow \dot{Q}_C\left[\frac{T_H - T_C}{T_C}\right] \le \dot{W}_{cycle}$$

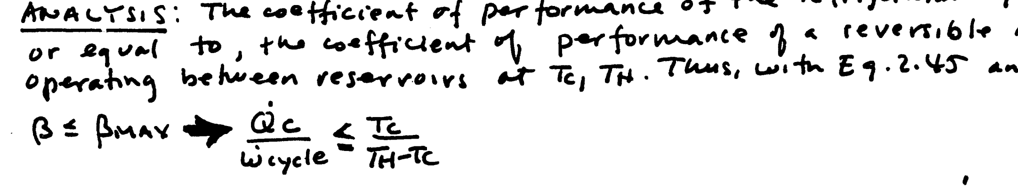
At steady state, the rate energy must be removed from the sample equals the rate energy leaks into the sample. Thus, $\dot{Q}_C = 0.085$ Btu/s, and

$$0.085\,\frac{\text{Btu}}{\text{s}}\underbrace{\left[\frac{530-265}{265}\right]}_{=1} \le \dot{W}_{cycle} \quad \text{or} \quad \dot{W}_{cycle} \ge 0.085\,\frac{\text{Btu}}{\text{s}} \longleftarrow \text{MIN. POWER}$$

PROBLEM 5.44

KNOWN: Steady-state operating data are provided for an ice maker.

FIND: Determine the maximum rate ice can be formed at 0°C, per kW of power input.


Water at Tc = 0°C (solid-liquid)
Q̇c
Q̇H
Ẇcycle
Surroundings at TH = 20°C
333 kJ/kg of energy must be removed to freeze water at 0°C

ASSUMPTIONS: 1. The system shown in the schematic undergoes a refrigeration cycle at steady state. 2. The water and the surroundings play the role of cold and hot reservoirs.

ANALYSIS: The coefficient of performance of the refrigeration cycle must be less than, or equal to, the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at Tc, TH. Thus, with Eq. 2.45 and Eq. 5.9

$$\beta \le \beta_{MAX} \Rightarrow \frac{\dot{Q}_C}{\dot{W}_{cycle}} \le \frac{T_C}{T_H - T_C}$$

At steady state, the energy removed while freezing water is $\dot{Q}_C = (333\ \text{kJ/kg})\dot{M}$, where $\dot{M}$ is the rate water freezes, in kg/h. Collecting results

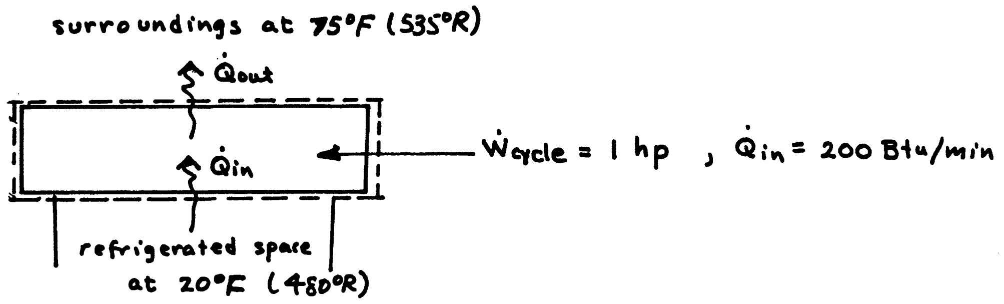
$$\frac{(333\ \text{kJ/kg})\dot{M}(\text{kg/h})}{\dot{W}_{cycle}(\text{kJ/s})}\left|\frac{1\ \text{h}}{3600\ \text{s}}\right| \le \frac{273\ \text{K}}{(293-273\ \text{K})} \Rightarrow \frac{\dot{M}}{\dot{W}_{cycle}} \le \left(\frac{273}{20}\right)\frac{|3600|}{333} = 147.6\ \frac{\text{kg/h}}{\text{kW}} \longleftarrow \text{MAX. RATE}$$

PROBLEM 5.45

KNOWN: A refrigerator maintains a refrigerated space at a specified temperature. Data for operation at steady state are provided.

FIND: Determine the coefficient of performance and the rate at which energy is discharged to the surroundings. Also, evaluate the minimum theoretical power input under the stated conditions.

SCHEMATIC & GIVEN DATA:


surroundings at 75°F (535°R)
$\dot{Q}_{out}$
$\dot{Q}_{in}$
$\dot{W}_{cycle}$ = 1 hp , $\dot{Q}_{in}$ = 200 Btu/min
refrigerated space at 20°F (480°R)

ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a refrigeration cycle. (2) The data are for operation at steady state. (3) The refrigerated space and the surroundings play the roles of cold and hot reservoirs, respectively.

ANALYSIS: (a) By definition of coefficient of performance

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{(200\ \text{Btu/min})\left|60\ \text{min/h}\right|}{(1\ \text{hp})\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right|} = 4.72 \longleftarrow \beta$$

From an energy balance

$$\dot{Q}_{out} = \dot{Q}_{in} + \dot{W}_{cycle}$$

$$= 200\ \frac{\text{Btu}}{\text{min}} + (1\ \text{hp})\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right|\left|\frac{1\ \text{h}}{60\ \text{min}}\right|$$

$$= 242.4\ \frac{\text{Btu}}{\text{min}} \longleftarrow \dot{Q}_{out}$$

(b) The coefficient of performance must be less than, or equal to, the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at $T_C = 480°R$, $T_H = 535°R$. With Eq. 2.45 and Eq. 5.9

$$\frac{\dot{Q}_{in}}{\dot{W}_{cycle}} \leq \frac{T_C}{T_H - T_C} \Rightarrow \dot{W}_{cycle} \geq \dot{Q}_{in}\left[\frac{T_H - T_C}{T_C}\right]$$

$$\geq \left(200\ \frac{\text{Btu}}{\text{min}}\right)\left|\frac{60\ \text{min}}{1\ \text{h}}\right|\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right|\left[\frac{535 - 480}{480}\right]$$

$$\geq 0.54\ \text{hp}$$

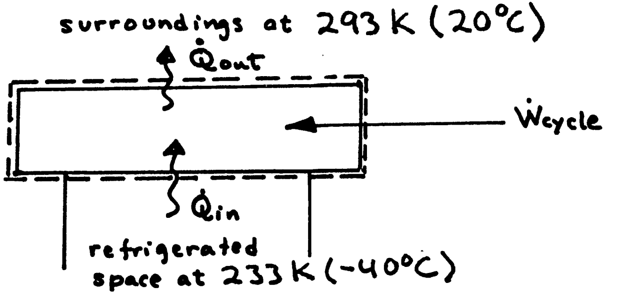
Thus, the minimum theoretical power required is 0.54 hp. ← min. power

PROBLEM 5.46*

KNOWN: A refrigerator maintains a refrigerated space at a specified temperature. Data for operation at steady state are provided.

FIND: Determine the power input required.

SCHEMATIC & GIVEN DATA:


surroundings at 293 K (20°C)
$\dot{Q}_{out}$
$\dot{W}_{cycle}$
$\dot{Q}_{in}$
refrigerated space at 233 K (−40°C)

$\dot{Q}_{in} = 18{,}000\ \mathrm{kJ/h}$

$\beta = 0.25\beta_{MAX}$

ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a refrigeration cycle. (2) The data are for operation at steady state. (3) The refrigerated space and the surroundings play the roles of the cold and hot reservoirs, respectively.

ANALYSIS: The coefficient of performance of a reversible refrigeration cycle operating between reservoirs at $T_H = 293\ \mathrm{K}$ and $T_C = 233\ \mathrm{K}$ is given by Eq. 5.9

$$\beta_{MAX} = \frac{T_C}{T_H - T_C} = \frac{233\ \mathrm{K}}{(293-233)\,\mathrm{K}} = 3.88$$

Then

$$\beta = (0.25)(3.88) = 0.97$$

Since $\beta = \dot{Q}_C / \dot{W}_{cycle}$

$$\dot{W}_{cycle} = \frac{\dot{Q}_C}{\beta} = \frac{(18{,}000\ \frac{\mathrm{kJ}}{\mathrm{h}})}{0.97}\left|\frac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right|\left|\frac{1\ \mathrm{kW}}{1\ \mathrm{kJ/s}}\right|$$

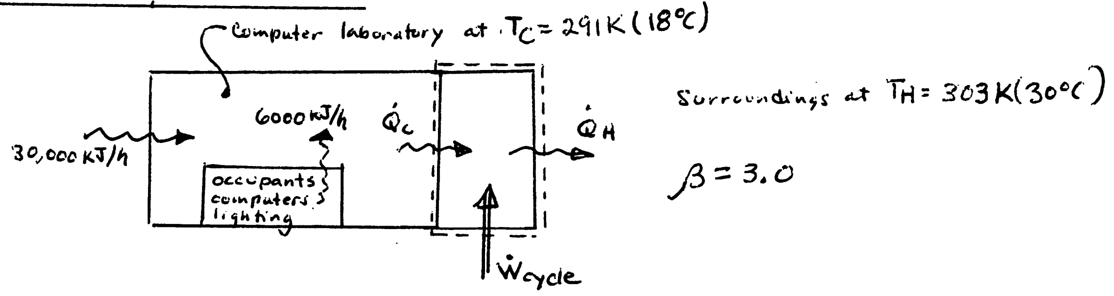
$$= 5.15\ \mathrm{kW} \longleftarrow \dot{W}_{cycle}$$

PROBLEM 5.47

KNOWN: A refrigeration cycle maintains a computer laboratory at 18°C on a day when the outside temperature is 30°C. Steady state operating data are provided.

FIND: Determine the power required, in kW, and compare with the minimum theoretical power required.

SCHEMATIC & GIVEN DATA



ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a refrigeration cycle. (2) All data provided are for operation at steady state. (3) The computer laboratory and surroundings play the roles of cold and hot reservoirs, respectively.

ANALYSIS: At steady state, the refrigeration cycle must remove energy from the computer laboratory at the same rate as energy enters from all sources: $\dot{Q}_C = (30{,}000 + 6000)\ kJ/h = 36{,}000\ kJ/h$. The coefficient of performance is given as 3.0. Thus

$$\beta = \frac{\dot{Q}_c}{\dot{W}_{cycle}} \Rightarrow 3.0 = \frac{36{,}000\ kJ/h}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = 12{,}000\ \frac{kJ}{h}$$

or

$$\dot{W}_{cycle} = \left(12000\ \frac{kJ}{h}\right)\left(\frac{h}{3600\ s}\right)\left(\frac{1\ kW}{1\ kJ/s}\right) = \mathbf{3.33}\ kW \longleftarrow \dot{W}_{cycle}$$

From Sec 5.5.2, we know that the actual coefficient of performance must be less than, or equal to, the coefficient of performance of a reversible refrigeration cycle operating between reservoirs at the specified temperatures. Then, with Equation 5.9

$$\frac{\dot{Q}_c}{\dot{W}_{cycle}} \le \frac{T_c}{T_H - T_c} = \frac{291}{303-291} = 24.25$$

Accordingly

$$\frac{36{,}000\ kJ/h}{24.25} \le \dot{W}_{cycle}$$

$$1484.5\ \frac{kJ}{h} \le \dot{W}_{cycle}$$

or

$$\dot{W}_{cycle} \ge \left(1484.5\ \frac{kJ}{h}\right)\left|\frac{1\ h}{3600\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right| = 0.41\ kW \longleftarrow (\dot{W}_{cycle})_{min}$$

Forming the ratio of the actual power required to the minimum theoretical value

$$\frac{3.33\ kW}{0.41\ kW} = 8.12$$

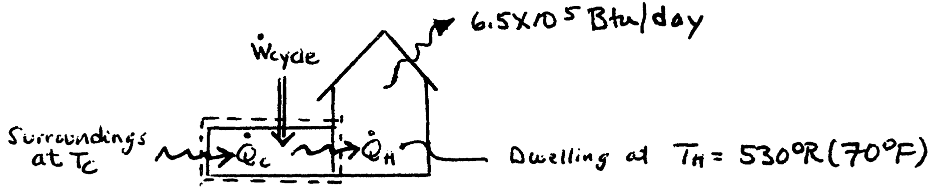
it can be concluded that the actual requirement is over 8 times greater than the minimum theoretical requirement.

PROBLEM 5.48

KNOWN: Operating data at steady state are provided for a heat pump working between a dwelling at 70°F and (a) outdoor air at 32°F, (b) a pond at 40°F, (c) the ground at 55°F.

FIND: For each case determine the minimum theoretical power required.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the cycle undergoes a heat pump cycle. (2) All data are for operation at steady state. (3) The surroundings and dwelling play the roles of cold and hot reservoirs, respectively.

ANALYSIS: At steady state, the heat pump cycle must provide energy to the dwelling equal to the energy leaking through the walls and roof:

$$\dot{Q}_H = \left(6.5\times10^5 \frac{\text{Btu}}{\text{day}}\right)\left|\frac{1\text{ day}}{24\text{ h}}\right| = 2.71\times10^4\ \frac{\text{Btu}}{\text{h}}$$

From Sec. 5.2.2, we know that the coefficient of performance of the heat pump must be less than, or equal to, the coefficient of performance of a reversible heat pump operating between reservoirs at $T_H = 293\text{K}$ and T_C. Then, with Eq. 5.10

$$\frac{\dot{Q}_H}{\dot{W}_{cycle}} \leq \frac{T_H}{T_H - T_C} \Rightarrow \dot{W}_{cycle} \geq \dot{Q}_H\left[\frac{T_H - T_C}{T_H}\right]$$

or

$$\dot{W}_{cycle} \geq \dot{Q}_H\left[1 - \frac{T_C}{T_H}\right]$$

Inserting values

$$\dot{W}_{cycle} \geq \left(2.71\times10^4 \frac{\text{Btu}}{\text{h}}\right)\left|\frac{1\text{ hp}}{2545\text{ Btu/h}}\right|\left[1 - \frac{T_C}{530}\right]$$

$$\geq (10.65\text{ hp})\left[1 - \frac{T_C}{530}\right]$$

(a) $T_C = 492°R$

$$\dot{W}_{cycle} \geq (10.65\text{ hp})\left[1 - \frac{492}{530}\right] = 0.76\text{ hp}$$

(b) $T_C = 500°R$

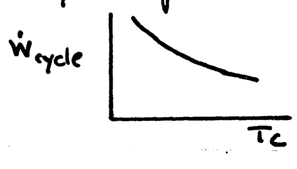
$$\dot{W}_{cycle} \geq (10.65\text{ hp})\left[1 - \frac{500}{530}\right] = 0.6\text{ hp}$$

① (c) $T_C = 515°R$

$$\dot{W}_{cycle} \geq (10.65\text{ hp})\left[1 - \frac{515}{530}\right] = 0.3\text{ hp}$$

---

1. The required power decreases as the outside source temperature increases:

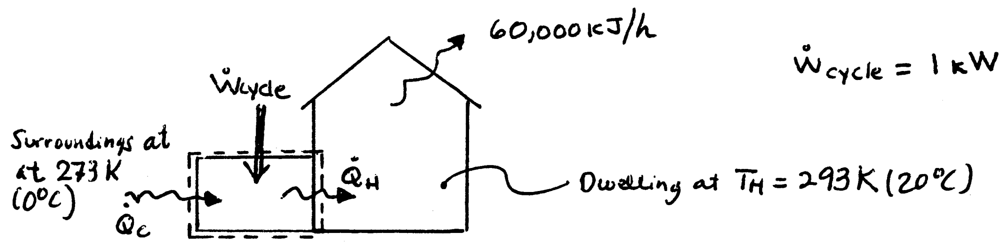


PROBLEM 5.49

KNOWN: Operating data are provided for a heat pump that provides heating for a building on a day when the outside temperature is 0°C and the building interior is to be kept at 20°C.

FIND: Determine if the heat pump would suffice.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cycle} = 1\ kW$

ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a heat pump cycle. (2) The data provided are for operation at steady state. (3) The surroundings and dwelling play the roles of cold and hot reservoirs, respectively.

ANALYSIS: At steady state, the heat pump cycle must provide energy to the dwelling equal to the energy leaking through the walls and roof: $\dot{Q}_H = 60{,}000\ kJ/h$.

From Sec. 5.2.2, we know that the coefficient of performance of the heat pump must be less than, or equal to, the coefficient of performance of a reversible heat pump operating between reservoirs at $T_C = 273\ K$, $T_H = 293\ K$. Then, with Eq. 5.10

$$\frac{\dot{Q}_H}{\dot{W}_{cycle}} \leq \frac{T_H}{T_H - T_C}$$

This equation can be used alternatively to determine if the heat pump would suffice: (a) Calculate $(\dot{Q}_H)_{MAX}$ from given values for $\dot{W}_{cycle}$, T_C, and T_H. (b) Calculate $(T_H)_{MAX}$ from known values of $\dot{Q}_H$, $\dot{W}_{cycle}$, T_C. (c) Calculate $(\dot{W}_{cycle})_{MIN}$ from known values of the remaining quantities. For the first alternative

$$\dot{Q}_H \leq \left(\frac{T_H}{T_H - T_C}\right)\dot{W}_{cycle} = \left(\frac{293}{20}\right) 1\ kW \left|\frac{1\ kJ/s}{1\ kW}\right|\left|\frac{3600\ s}{1\ h}\right| = 52{,}740\ \frac{kJ}{h}$$

$$\Rightarrow (\dot{Q}_H)_{MAX} = 52{,}740\ kJ/h$$

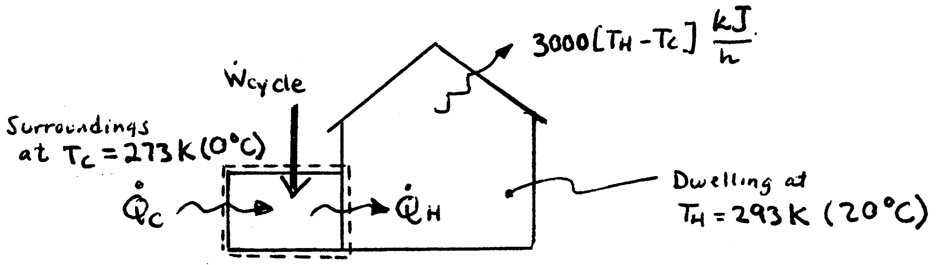
With alternative (b), $(T_H)_{MAX} = 290\ K\ (17°C)$. With alternative (c), $(\dot{W}_{cycle})_{MIN} = 1.14\ kW$. With each approach, we conclude the heat pump **would not** suffice.

PROBLEM 5.50*

KNOWN: Operating data are provided for a heat pump that maintains a dwelling at 20°C when the outside temperature is 0 °C.

FIND: Determine the minimum theoretical power required, in horsepower.

SCHEMATIC & GIVEN DATA:



3000[T_H - T_C] kJ/h
$\dot{W}_{cycle}$
Surroundings at T_C = 273 K (0°C)
$\dot{Q}_C$
$\dot{Q}_H$
Dwelling at T_H = 293 K (20°C)

ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a heat pump cycle. (2) The data provided are for operation at steady state, (3) The dwelling and surroundings play the roles of hot and cold reservoirs, respectively.

ANALYSIS: At steady state the heat pump must provide energy to the dwelling equal to the energy leaking through the walls and roof:

$$\dot{Q}_H = 3000\,[T_H - T_C] = 3000[293-273] = 60{,}000 \text{ kJ/h}$$

From Sec. 5.2.2, we know that the coefficient of performance of the heat pump must be less than, or equal to, the coefficient of performance of a reversible heat pump operating between reservoirs at T_H = 293 K and T_C = 273 K Then, with Eq. 5.10

$$\frac{\dot{Q}_H}{\dot{W}_{cycle}} \leq \frac{T_H}{T_H - T_C}$$

Rearranging

$$\dot{Q}_H\left[\frac{T_H - T_C}{T_H}\right] \leq \dot{W}_{cycle}$$

Inserting known values

$$\left(60{,}000\,\frac{\text{kJ}}{\text{h}}\right)\left[\frac{293-273}{293}\right] \leq \dot{W}_{cycle}$$

$$4096\,\frac{\text{kJ}}{\text{h}} \leq \dot{W}_{cycle}$$

Or

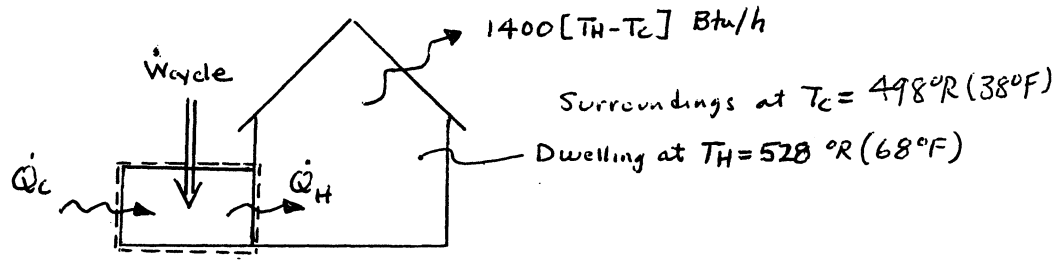
$$\dot{W}_{cycle} \geq \left(4096\,\frac{\text{kJ}}{\text{h}}\right)\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|\left|\frac{1\text{ h}}{3600\text{ s}}\right| = 1.14\text{ kW} \longleftarrow (\dot{W}_{cycle})_{min}$$

PROBLEM 5.51

KNOWN: Operating data are provided for a heat pump that maintains a dwelling at 68°F on a day when the outside temperature is 28°F

FIND: Determine the power required, in kW, to provide the heating by electrical resistance elements and compare with the minimum theoretical power required by a heat pump. Also repeat the comparison using typical coefficient of performance data for heat pumps.

SCHEMATIC & GIVEN DATA:



1400[T_H-T_C] Btu/h
$\dot{W}_{cycle}$
Surroundings at T_C= 498°R (38°F)
Dwelling at T_H=528 °R (68°F)
$\dot{Q}_C$
$\dot{Q}_H$

ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a heat pump cycle. (2) The data provided are for operation at steady state. (3) The dwelling and surroundings play the roles of hot and cold reservoirs, respectively.

ANALYSIS: At steady state, the electrical resistance elements, or the heat pump, must provide energy to the dwelling equal to the energy leaking through the walls and roof:

$$\dot{Q}_H = 1400[T_H - T_C] = 1400\,[528 - 498] = 42{,}000 \text{ Btu/h}$$

$$= (42{,}000 \text{ Btu/h})\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right| = 16.5 \text{ hp}$$

Accordingly, the resistors would be provided with electrical power = 16.5 hp.

Turning next to the heat pump option, we know from Sec. 5.2.2 that the coefficient of performance of a heat pump must be less than, or equal to, the coefficient of performance of a reversible heat pump operating between reservoirs at T_C and T_H. Then, with Eq. 5.10

$$\frac{\dot{Q}_H}{\dot{W}_{cycle}} \leq \frac{T_H}{T_H - T_C} \Rightarrow \dot{W}_{cycle} \geq \dot{Q}_H\left[\frac{T_H - T_C}{T_H}\right]$$

Substituting known values

$$\dot{W}_{cycle} \geq (16.5 \text{ hp})\left[1 - \frac{498}{528}\right] = 0.94 \text{ hp}$$

Accordingly, the minimum theoretical power required = 0.94 hp

From manufacturer's data, an actual heat pump would have a coefficient of performance in the range 2.0 - 3.0. Using the higher value

$$\beta = \frac{\dot{Q}_H}{\dot{W}_{cycle}} \Rightarrow \dot{W}_{cycle} = \frac{\dot{Q}_H}{\beta} = \frac{16.5 \text{ hp}}{3.0} = 5.5 \text{ hp}$$

Comparisons:

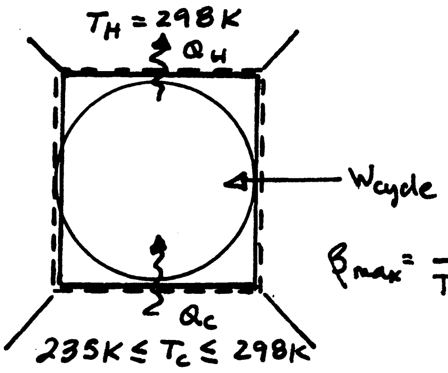
$$\frac{\text{All electric}}{\text{Ideal heat pump}} = \frac{16.5 \text{ hp}}{0.94 \text{ hp}} = 17.6, \quad \frac{\text{All electric}}{\text{Actual heat pump}} = \frac{16.5 \text{ hp}}{5.5 \text{ hp}} = 3$$

PROBLEM 5.52

KNOWN: Refrigeration and heat pump cycles operate reversibly between reservoirs at fixed T_H and a range of T_C values, respectively.

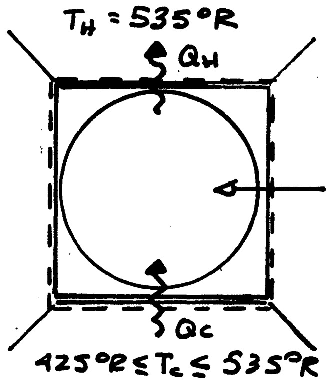
FIND: Plot (a) β_{max} given by Eq. 5.9 for the refrigeration cycle and (b) γ_{max} given by Eq. 5.10 for the heat pump cycle. Discuss practical implications in each case.

SCHEMATIC & GIVEN DATA:



$$\beta_{max} = \frac{T_C}{T_H - T_C}$$

(a)

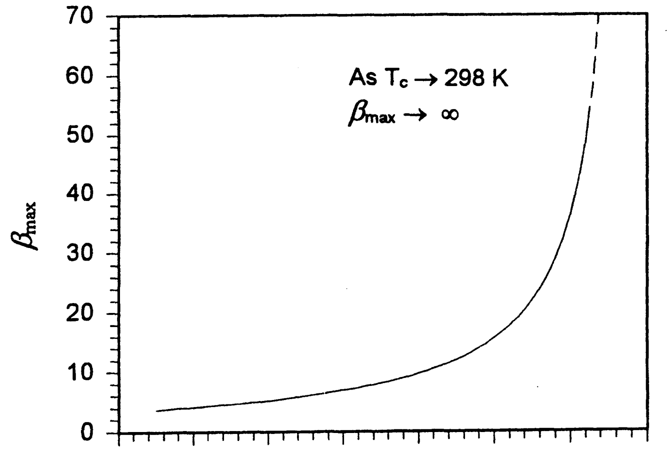


$$\gamma_{max} = \frac{T_H}{T_H - T_C}$$

(b)

ASSUMPTIONS: (1) Each system shown on the accompanying sketches undergoes a cycle. (2) The cycles are reversible.

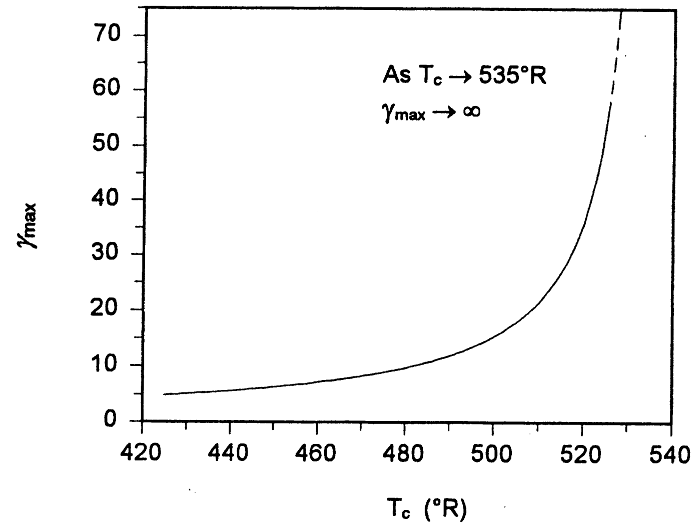
ANALYSIS: (a) The coefficient of performance for the reversible refrigeration cycle operating between reservoirs at T_H and T_C is given by Eq. 5.9. With $T_H = 298K$, we get $\beta_{max} = T_C/(298 - T_C)$. The following plot is obtained using IT:



For an actual refrigeration system operating at given values of T_H and T_C, the coefficient of performance β would be less than β_{max}: $\beta < \beta_{max}$. Still, the trend shown in the plot is qualitatively correct: β_{max} decreases rapidly as T_C decreases for fixed T_H. Accordingly, as T_C decreases, more work input is needed per unit of energy Q_C removed from the region at T_C (cold reservoir). When electricity is used to provide the work input, there would normally a higher cost of operation associated with the lowering of T_C.

PROBLEM 5.52 (Contd.)

(b) The coefficient of performance of a reversible heat pump cycle operating between reservoirs at T_H and T_C is given by Eq. 5.10. With $T_H = 535\,°R$, we get $\gamma_{max} = 535/(535 - T_C)$. The following plot is obtained using IT:



For an actual heat pump system operating given values of T_H and T_C, the coefficient of performance γ would be less than γ_{max} : $\gamma < \gamma_{max}$. Still, the trend shown on the plot is qualitatively correct: γ_{max} decreases rapidly as T_C decreases for fixed T_H. Accordingly, as T_C decreases, the energy delivered by heat transfer at T_H, Q_H, decreases for a given work input. Thus, when the heat pump is used to provede heat transfer to a dwelling, the system may not be able to provide enough Q_H when the outside temperture T_C gets very low.

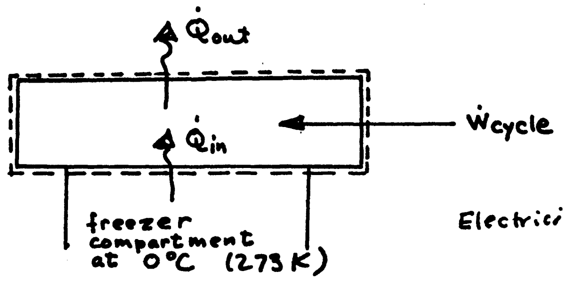
PROBLEM 5.53

KNOWN: A refrigerator maintains a refrigerated space at a specified temperature. Data for operation at steady state are provided.

FIND: Determine the power input and compare with the power input for a reversible refrigeration cycle operating between reservoirs at the specified temperatures. Evaluate the daily actual and minimum theoretical operating costs.

SCHEMATIC & GIVEN DATA:

surroundings at 20°C (293 K)



$\beta = 3$

$\dot{Q}_{in} = 6000$ kJ/h

Electricity : 8¢/kW·h

ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes a refrigeration cycle. (2) The data are for operation at steady state. (3) The freezer compartment and the surroundings play the roles of cold and hot reservoirs, respectively.

ANALYSIS: (a) Upon rearrangement, the defining expression for the coefficient of performance gives

$$\dot{W}_{cycle} = \frac{\dot{Q}_{in}}{\beta} = \frac{6000 \text{ kJ/h}}{3} = 2000 \frac{\text{kJ}}{\text{h}} \longleftarrow \dot{W}_{cycle}$$

The coefficient of performance for a reversible refrigeration cycle operating between reservoirs at $T_H = 293$ K (20°C) and $T_C = 273$ K (0°C) is given by Eq. 5.9

$$\beta_{max} = \frac{273}{293-273} = 13.65$$

The power input for such a cycle is then

$$(\dot{W}_{cycle})_{min} = \frac{6000 \text{ kJ/h}}{13.65} = 440 \frac{\text{kJ}}{\text{h}} \longleftarrow (\dot{W}_{cycle})_{min}$$

The actual power input is over 4½ times the minimum theoretical power requirement.

(b)

$$\text{Actual cost: } \dot{\$} = 2000 \frac{\text{kJ}}{\text{h}} \left|\frac{1 \text{ h}}{3600 \text{ s}}\right| \left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| \left|\frac{24 \text{ h}}{\text{day}}\right| \left(\$ \frac{0.08}{\text{kW}\cdot\text{h}}\right) = \$1.07/\text{day}$$

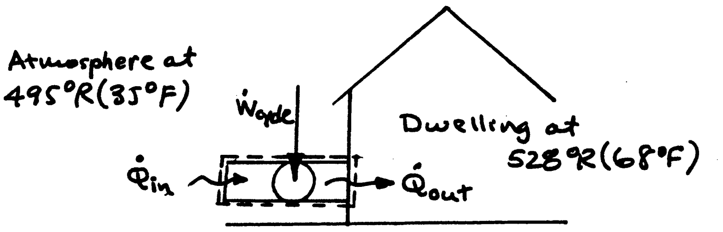
$$\text{Min. Theo. Cost: } \dot{\$} = 440 \frac{\text{kJ}}{\text{h}} \left|\frac{1 \text{ h}}{3600 \text{ s}}\right| \left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| \left|\frac{24 \text{ h}}{\text{day}}\right| \left(\$ \frac{0.08}{\text{kW}\cdot\text{h}}\right) = \$0.23/\text{day}$$

PROBLEM 5.54

KNOWN: A heat pump provides heating to a dwelling. Operating data are provided.

FIND: Determine the actual operating cost and compare with the minimum theoretical operating cost for each day of operation.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{out} = 30{,}000$ Btu/h

$\dot{W}_{cycle} = 5$ hp

ASSUMPTIONS: (1) The system shown in the figure undergoes a heat pump cycle. (2) All data are for operation at steady state. (3) The atmosphere and the dwelling play the roles of cold and hot reservoirs, respectively. (4) The cost of electricity is 8 cents per kW·h.

ANALYSIS: Using given data, the actual operating cost is

$$\left(\begin{matrix}\text{cost per}\\ \text{day}\end{matrix}\right) = (5\text{ hp})\left|\frac{1\text{ kW}}{1.341\text{ hp}}\right|\left|\frac{24\text{h}}{1\text{day}}\right|\left(\frac{0.08\$}{\text{kW}\cdot\text{h}}\right) = \$7.16/\text{day} \longleftarrow$$

The minimum theoretical operating cost corresponds to the minimum theoretical power requirement. Since $\beta < \beta_{MAX}$,

$$\Rightarrow \quad \frac{\dot{Q}_{out}}{\dot{W}_{cycle}} \le \frac{T_H}{T_H - T_C}$$

or

$$\frac{30{,}000\text{ Btu/h}}{\dot{W}_{cycle}} \le \frac{528\,°R}{(528 - 495)°R} = 16$$

$$\Rightarrow \quad \frac{30{,}000\text{ Btu/h}}{16} \le \dot{W}_{cycle}$$

$$1875\,\frac{\text{Btu}}{\text{h}} \le \dot{W}_{cycle}$$

The minimum cost is then

$$\left(\begin{matrix}\text{minimum cost}\\ \text{per day}\end{matrix}\right) = \left(1875\,\frac{\text{Btu}}{\text{h}}\right)\left(\frac{1\text{ kW}}{3413\text{ Btu/h}}\right)\left(\frac{24\text{h}}{1\text{day}}\right)\left(\frac{0.08\$}{\text{kW}\cdot\text{h}}\right)$$

$$= \$1.05/\text{day} \longleftarrow$$

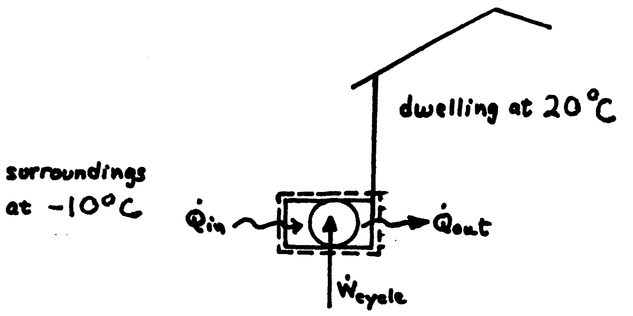
The actual operating cost is about 7 times higher than the minimum theoretical cost.

PROBLEM 5.55*

KNOWN: A heat pump maintains a dwelling at a specified temperature.

FIND: Determine the minimum theoretical cost.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{out} = 25{,}000$ kJ/h

cost of electricity = 0.08 \$/kW·h

ASSUMPTIONS: (1) The system shown on the accompanying figure undergoes a heat pump cycle. (2) The data are for operation at steady state. (3) The dwelling and the surroundings play the roles of hot and cold reservoirs, respectively.

ANALYSIS: The minimum theoretical cost for any heat pump cycle under the stated conditions is the cost for a reversible cycle operating between reservoirs at $T_H = 293$ K (20°C) and $T_C = 263$ K (−10°C). The power required by such a cycle can be obtained from

$$(\dot{W}_{cycle})_{min} = \frac{\dot{Q}_{out}}{\gamma_{max}}$$

where γ_{max} is

$$\gamma_{MAX} = \frac{293}{293-263} = 9.77$$

Accordingly

$$(\dot{W}_{cycle})_{Min} = \frac{(25{,}000 \frac{kJ}{h})}{(9.77)} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| \left| \frac{1 \text{ h}}{3600 \text{ s}} \right| = 0.71 \text{ kW}$$

Then, the minimum cost for one day of operation is

$$\text{Minimum theoretical cost: } \dot{\$} = (0.71 \text{kW}) \left| \frac{24 h}{day} \right| (\$ \, 0.08/day)$$

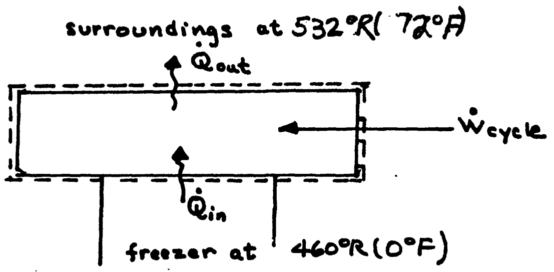
$$= \$ \, 1.36/day \longleftarrow \frac{MIN}{COST}$$

PROBLEM 5. 56

KNOWN: A refrigerator maintains a freezer compartment at a specified temperature. Data for operation at steady state are provided.

FIND: Determine the minimum theoretical operating cost per day.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{in} = 2000$ Btu/h

ASSUMPTIONS: (1) The system shown on the accompanying figure undergoes a refrigeration cycle. (2) The data provided are for operation at steady state. (3) The freezer compartment and the surroundings play the roles of cold and hot reservoirs, respectively. (4) The cost of electricity is 8 cents per kW·h.

ANALYSIS: The mimum cost of electricity corresponds to the minimum power input. Then, since $\beta \leq \beta_{max}$

$$\frac{\dot{Q}_{in}}{\dot{W}_{cycle}} \leq \frac{T_C}{T_H - T_C}$$

or

$$\frac{2000 \text{ Btu/h}}{\dot{W}_{cycle}} \leq \frac{460°R}{(532-460)°R} = 6.39$$

$$\Rightarrow \quad \frac{2000 \text{ Btu/h}}{6.39} \leq \dot{W}_{cycle}$$

$$313 \frac{\text{Btu}}{\text{h}} \leq \dot{W}_{cycle}$$

Then

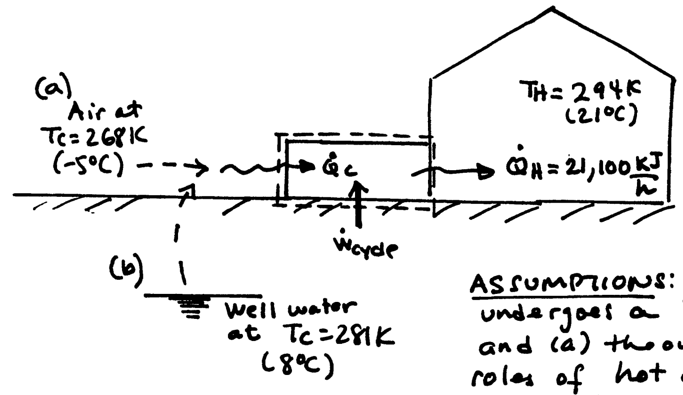
$$\begin{bmatrix}\text{Minimum cost} \\ \text{per day}\end{bmatrix} = \left[313 \frac{\text{Btu}}{\text{h}}\right]\left|\frac{24 \text{h}}{1 \text{ day}}\right|\left|\frac{1 \text{ kW}}{3413 \text{ Btu/h}}\right|\left[\frac{8 \text{ cents}}{\text{kW}\cdot\text{h}}\right]$$

$$= 17.6 \text{ cents/day} \longleftarrow$$

PROBLEM 5.57

KNOWN: Operating data are provided for a heat pump receiving energy by heat transfr from the outdoor air. From well water.

FIND: For each case, determine the minimum theoretical dailycost to operate.



ASSUMPTIONS: 1. The system shown in the schematic undergoes a heat pump cycle. 2. The dwelling and (a) the outside air, (b) the well water play the roles of hot and cold reservoirs, respectively. 3. Electricity costs 8cents per kW·h.

ANALYSIS: The coefficient of performance of a heat pump is less than, or equal to, the coefficient of performance of a reversible heat pump operating between reservoirs at T_C, T_H. That is, $\gamma \le \gamma_{MAX}$. With Eqs. 2.47and 5.10

$$\frac{\dot{Q}_H}{\dot{W}_{cycle}} \le \frac{T_H}{T_H - T_C} \Rightarrow \dot{Q}_H\left[1 - \frac{T_C}{T_H}\right] \le \dot{W}_{cycle}$$

Inserting values

$$\dot{W}_{cycle} \ge \left(21{,}100\ \frac{kJ}{h}\right)\left[1 - \frac{T_C}{294K}\right]\left|\frac{1h}{3600s}\right|\left|\frac{1kW}{1kJ/s}\right|$$

(a) Air at $T_C = 268K$

$$\dot{W}_{cycle} \ge (21{,}100)\left[1 - \frac{268}{294}\right]\left|\frac{1}{3600}\right| = 0.52kW$$

Costing: $\dot{\$} \ge (0.52kW)\left|\frac{24h}{1day}\right|\left(\frac{\$0.08}{kW\cdot h}\right) = \$1.00/day \longleftarrow$ minimum daily cost (a)

(b) Well water at $T_C = 281K$

$$\dot{W}_{cycle} = (21{,}100)\left[1 - \frac{281}{294}\right]\left|\frac{1}{3600}\right| = 0.26kW$$

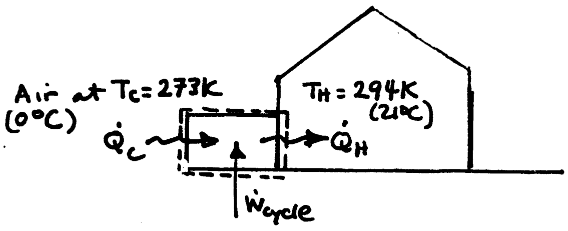
Costing: $\dot{\$} \ge (0.26kW)\left|\frac{24h}{day}\right|\left(\frac{\$0.08}{kW\cdot h}\right) = \$0.50/day \longleftarrow$ minimum daily cost (b)

PROBLEM 5.58

KNOWN: Operating data are provided for a heat pump that maintains the temperature within a building at a specified value.

FIND: Determine the actual and minimum theoretical daily operating costs, and compare with the daily cost for electrical resistance heating.

SCHEMATIC & GIVEN DATA:



$\gamma = 3.8$
$\dot{Q}_H = 75{,}000$ kJ/h

ASSUMPTIONS: 1. The system shown in the schematic undergoes a heat pump cycle. 2. Electricity costs 8 cents/kW·h. 3. The dwelling and outside air play the roles of the hot and cold reservoirs.

ANALYSIS: (a) Using Eq. 2.47: $\gamma = \dot{Q}_H / \dot{W}_{cycle}$

$$\dot{W}_{cycle} = \frac{\dot{Q}_H}{\gamma} = \frac{75{,}000 \text{ kJ/h}}{3.8} = 19{,}737 \frac{\text{kJ}}{\text{h}}$$

Costing: $$\dot{\$} = \left(19{,}737 \frac{\text{kJ}}{\text{h}}\right)\left|\frac{1\text{h}}{3600\text{s}}\right|\left|\frac{1\text{kW}}{1\text{kJ/s}}\right|\left(\frac{24\text{ h}}{\text{day}}\right)\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right) = \$10.53/\text{day}$$

For a reversible heat pump operating between reservoirs at T_C, T_H, Eq. 5.10 gives

$$\gamma_{MAX} = \frac{T_H}{T_H - T_C} = \frac{294}{294-273} = 14$$

Then

$$(\dot{W}_{cycle})_{MIN} = \frac{75000 \text{ kJ/h}}{14} = 5357 \frac{\text{kJ}}{\text{h}}$$

Costing: $$(\dot{\$})_{MIN} = (5357)\left|\frac{1}{3600}\right|(24)(0.08) = \$2.86/\text{day}$$

(b) For electrical-resistance heating, electricity would provide the full 75,000 kJ/h needed. Thus

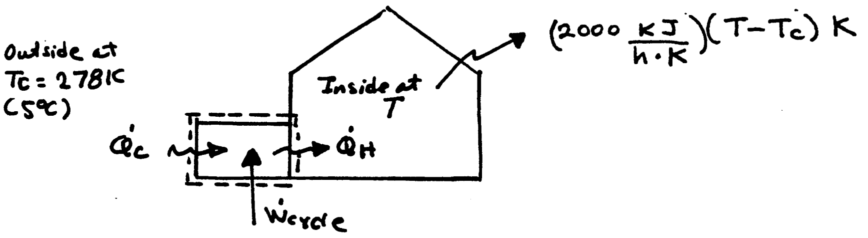
Costing: $$(\dot{\$})_{\substack{\text{electrical}\\ \text{resistance}}} = (75{,}000)\left|\frac{1}{3600}\right|(24)(0.08) = \$40/\text{day}$$

PROBLEM 5.59

KNOWN: Operating data are provided for a heat pump that maintains a dwelling at temperature T.

FIND: (a) When T = 20°C, determine the minimum theoretical daily operating cost. (b) Plot the minimum theoretical daily operating cost versus T.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system shown in the schematic undergoes a heat pump cycle. 2. The inside air and outside air play the roles of hot and cold reservoirs, respectively. 3. Electricity costs 8cents per kW·h.

ANALYSIS: To maintain the dwelling at temperature T, the heatpump must provide energy to the dwelling at the rate

$$\dot{Q}_H = \left(2000\ \frac{kJ}{h \cdot K}\right)(T - T_C) \quad , T_C = 278K \qquad (1)$$

The coefficient of performance must satisfy the following relationship, written using Eqs. 2.47 and 5.10

$$\frac{\dot{Q}_H}{\dot{W}_{cycle}} \le \frac{T}{T - T_C} \Longrightarrow \dot{W}_{cycle} \ge \left[\frac{T - T_C}{T}\right]\dot{Q}_H$$

Introducing Eq. (1)

$$\dot{W}_{cycle} \ge \left[\frac{T - T_C}{T}\right]\left[\left(2000\ \frac{kJ}{h \cdot K}\right)(T - T_C)K\right] = \left(2000\ \frac{kJ}{h \cdot K}\right)\frac{(T - T_C)^2}{T} \qquad (2)$$

where T, T_C are in K.

(a) When T = 293 K (20°C), Eq. (2) gives

$$\dot{W}_{cycle} \ge \left(2000\ \frac{kJ}{h \cdot K}\right)\frac{(293 - 278)^2}{293}K = 1536\ \frac{kJ}{h}$$

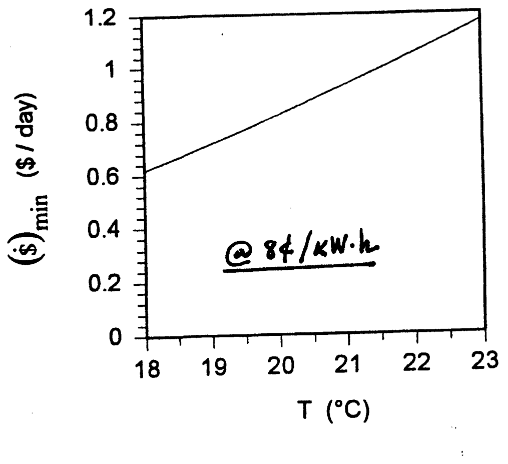
Thus, the minimum theoretical power required is 1536 kJ/h. The corresponding cost on a daily basis is

$$(\$)_{min} = \left(1536\ \frac{kJ}{h}\right)\left|\frac{1h}{3600s}\right|\left|\frac{1kW}{1kJ/s}\right|\left|\frac{24h}{day}\right|\left(\frac{\$0.08}{kW \cdot h}\right) = \$0.82/day \longleftarrow$$

(b) For T ranging from 18 to 23°C, the cost on a daily basis is

$$(\$)_{min} = \left[2000\ \frac{(T - T_C)^2}{T}\ \frac{kJ}{h}\right]\left|\frac{1h}{3600s}\right|\left|\frac{1kW}{1kJ/s}\right|\left|\frac{24h}{day}\right|\left(\frac{\$0.08}{kW \cdot h}\right) = 1.067\left[\frac{(T - T_C)^2}{T}\right](\$/day)$$

PLOT:



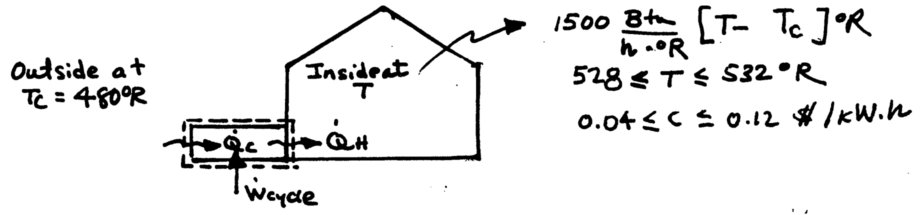
The minimum daily cost increases significantly as the inside temperature is required to be higher.

PROBLEM 5.60

KNOWN: Operating data are provided for a heat pump that maintains a dwelling at temperature T.

FIND: For a given electricity cost, plot the minimum theoretical operating cost versus T. For a given value of T, plot the minimum theoretical operating cost versus the cost of electricity in cents per kW·h.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system shown in the schematic undergoes a heat pump cycle. 2. The inside air and outside air play the roles of hot and cold reservoirs, respectively.

ANALYSIS: To maintain the dwelling at temperature T, the heat pump must provide energy to the dwelling at the rate

$$\dot{Q}_H = \left(1500 \frac{Btu}{h \cdot °R}\right)[T - T_C] , \quad T_C = 480°R \qquad (1)$$

The coefficient of performance must satisfy the following relationship, written using Eq. 2.47 and 5.10

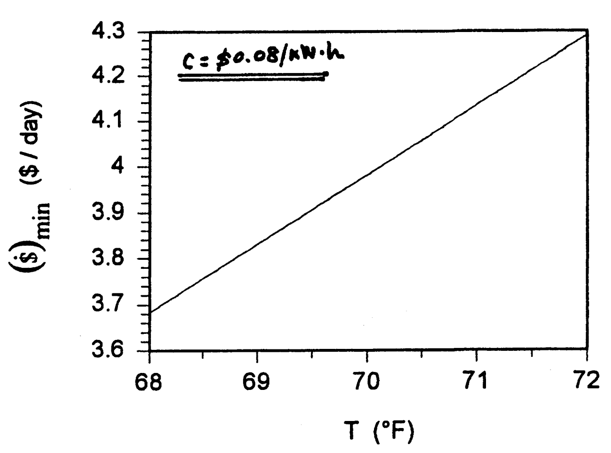
$$\frac{\dot{Q}_H}{\dot{W}_{cycle}} \le \frac{T}{T - T_C} \Rightarrow \dot{W}_{cycle} \ge \left[\frac{T - T_C}{T}\right]\dot{Q}_H \Rightarrow (\dot{W}_{cycle})_{MIN} = \left[\frac{T - T_C}{T}\right]\dot{Q}_H$$

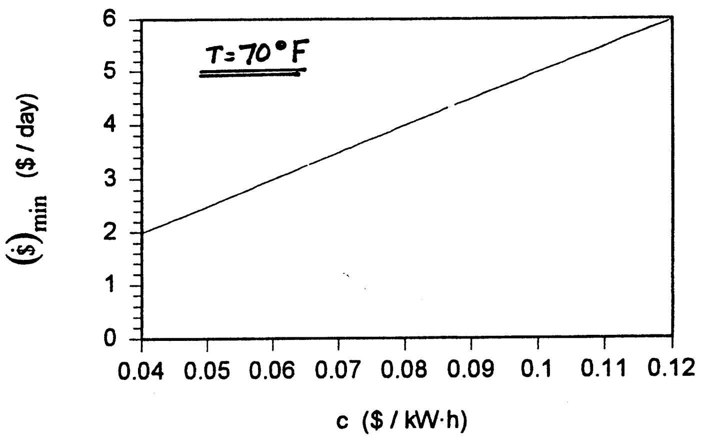
Introducing Eq. (1) and the unit cost of electricity, c, the minimum theoretical daily operating cost is

$$(\dot{\$})_{MIN} = \left[\frac{T - T_C}{T}\right]\left[\left(1500 \frac{Btu}{h \cdot °R}\right)[T - T_C]°R\right]\left(\frac{24 h}{day}\right)\left|\frac{1 \text{ kW} \cdot h}{3413 \text{ Btu}}\right| c \left(\frac{\$}{\text{kW} \cdot h}\right)$$

$$= 10.55 \left[\frac{(T - T_C)^2}{T}\right] c \left(\frac{\$}{day}\right) \qquad (2)$$

Sample calculation: T = 530°R (70°F), c = \$0.08/kW·h. Then, $(\dot{\$})_{MIN} = 10.55\left[\frac{(530 - 480)^2}{530}\right](.08) =$ \$3.98/day.





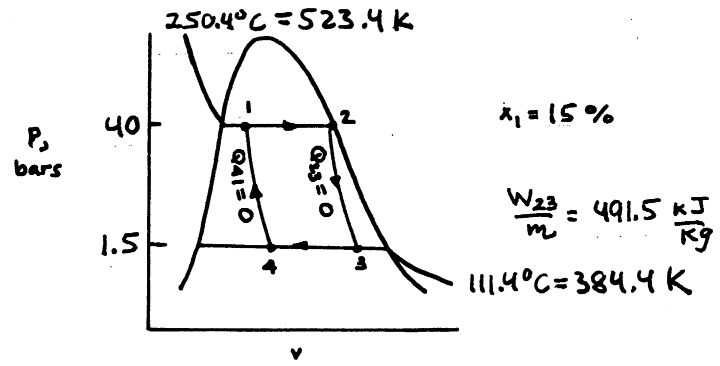
The minimum daily cost increases significantly as the inside temperature is required to be higher and/or as the unit cost of electricity increases.

PROBLEM 5.61*

KNOWN: 2 kg of water executes a Carnot cycle for which property data are provided.

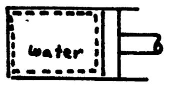
FIND: Sketch the cycle on p-v coordinates, evaluate the heat and work for each process, and evaluate the thermal efficiency.

SCHEMATIC AND GIVEN DATA:



$x_1 = 15\%$

$\frac{W_{23}}{m} = 491.5 \frac{kJ}{kg}$



Summary:

Process	Q	W
1-2	2914.0	330.0
2-3	0	983.0
3-4	-2140.2	-167.1
4-1	0	-373.0

ASSUMPTION: The system shown in the figure undergoes a Carnot cycle.

Process 1-2: $\frac{W_{12}}{m} = \int_1^2 p dv = p(v_2 - v_1)$. An energy balance gives $\frac{Q_{12}}{m} = u_2 - u_1 + \frac{W_{12}}{m}$. Thus

$$\frac{Q_{12}}{m} = (u_2 - u_1) + p(v_2 - v_1)$$
$$= h_2 - h_1$$

with data from Table A-3. $v_2 = 0.04978\ m^3/kg$, $h_2 = 2801.4\ kJ/kg$, $u_2 = 2602.3\ kJ/kg$

$$v_1 = v_f + x(v_g - v_f) = 1.2522 \times 10^{-3} + 0.15(0.04978 - 1.2522 \times 10^{-3}) = 8.531 \times 10^{-3}\ m^3/kg$$

$$h_1 = h_f + x(h_g - h_f) = 1087.3 + 0.15(1714.1) = 1344.42\ kJ/kg$$

With these values, $Q_{12} = (2\ kg)(2801.4 - 1344.42)\ kJ/kg = 2914.0\ kJ$, and $\longleftarrow Q_{12}$

$$W_{12} = (2.0\ kg)(40\ bar)\left(10^5 \frac{N/m^2}{bar}\right)\left[(49.78 - 8.531) \times 10^{-3} \frac{m^3}{kg}\right]\left(\frac{kJ}{10^3\ Nm}\right) = 330.0\ kJ \longleftarrow W_{12}$$

Process 2-3: $Q_{23} = 0$, $W_{23}/m = 491.5\ kJ/kg$. Thus $W_{23} = 983.0\ kJ$. $\longleftarrow Q_{23}, W_{23}$
Using an energy balance and data from Table A-3, $u_3 = 2110.8\ kJ/kg$, giving $x_3 = 0.801$, $h_3 = 2250.5\ kJ/kg$, $v_3 = 0.929\ m^3/kg$.

Process 3-4: As for process 1-2, $W_{34}/m = p(v_4 - v_3)$, $Q_{34}/m = (h_4 - h_3)$. Also, since the system undergoes a reversible cycle while communicating with reservoirs at $T_H = 523.4 K$, $T_C = 384.4 K$, Eq. 5.6 is applicable

$$\frac{|Q_{34}|}{Q_{12}} = \frac{384.4}{523.4} \Rightarrow h_3 - h_4 = \frac{384.4}{523.4}(1457.0) = 1070.1\ kJ/kg$$

Thus $h_4 = h_3 - 1070.1 = 2250.5 - 1070.1 = 1180.4\ kJ/kg$. Using this value,
$x_4 = (1180.4 - 467.11)/2226.5 = 0.320$ Then $v_4 = 0.0010528 + (0.320)(1.159 - 0.0010528) = 0.372\ m^3/kg$,
$u_4 = 466.94 + (0.320)(2519.7 - 466.94) = 1123.8\ kJ/kg$

Finally, $Q_{34} = -2140.2\ kJ$, $W_{34} = -167.1\ kJ$. $\longleftarrow Q_{34}, W_{34}$

Process 4-1: $Q_{41} = 0$. $W_{41}/m = u_4 - u_1 = 1123.8 - (1082.3 + 0.15(1520.0)) = -186.5\ kJ/kg$
So, $W_{41} = -373.0\ kJ$. $\longleftarrow Q_{41}, W_{41}$

Thermal Efficiency.

Method #1: $$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{330.0 + 983.0 - 167.1 - 373.0}{2914.0} = 0.265\ (26.5\%) \longleftarrow \eta$$

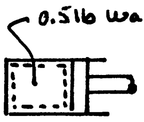
Method #2: Equation 5.8
$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{384.4}{523.4} = 0.266\ (26.6\%) \longleftarrow \eta_{max}$$

PROBLEM 5.62*

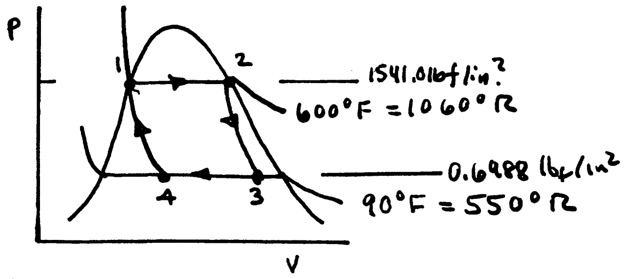
KNOWN: 0.5 lb of water executes a Carnot cycle for which property data are provided.

FIND: Sketch the cycle on p-v coordinates, evaluate the heat and work for each process, and evaluate the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$x_3 = 64.3\%$



SUMMARY:

Process	Q	W
1-2	274.85	34.81
2-3	0	200.21
3-4	−142.61	−8.29
4-1	0	−94.71
Cycle	132.24	132.02

ASSUMPTION: 1. The system is shown in the schematic. 2. Volume change is the only work mode.

ANALYSIS: (b) Process 1-2: $W_{12}/m = \int^{2} p\,dv = p(v_2 - v_1)$. Energy balance: $Q_{12}/m = (u_2 - u_1) + W_{12}/m$. Thus, $Q_{12}/m = (u_2 - u_1) + p(v_2 - v_1) = h_2 - h_1$ With data from Table A-3E, $Q_{12} = 0.5\,\text{lb}(549.7\ \text{Btu/lb}) = 274.85\ \text{Btu}$. $W_{12} = (0.5\,\text{lb})(1541 \times 144\ \text{lbf/ft}^2)(0.2677 - 0.02363)\text{ft}^3/\text{lb}\left|1\ \text{Btu}/778\ \text{ft·lbf}\right| = 34.81\ \text{Btu}$.

Process 2-3: $Q_{23} = 0$. An energy balance reduces to $W_{23} = m(u_2 - u_3)$. From Table A-2E $u_2 = 1090.0\ \text{Btu/lb}$. $u_3 = 58.07 + 0.643(1040.2 - 58.07) = 689.58\ \text{Btu/lb}$. Then $W_{23} = 0.5(1090.0 - 689.58) = 200.21\ \text{Btu}$.

Process 3-4: As for process 1-2: $W_{34} = mp(v_4 - v_3)$, $Q_{34} = m(h_4 - h_3)$. Since the system undergoes a reversible cycle while communicating with reservoirs at $T_H = 1060\,°R$ and $T_C = 550\,°R$, Eq. 5.6 is applicable: $\frac{|Q_{34}|}{550} = \frac{Q_{12}}{1060} \Rightarrow Q_{34} = -142.61\ \text{Btu}$

Thus, $h_4 = \frac{Q_{34}}{m} + h_3 = -285.22 + (58.07 + 0.643(1042.7)) = 443.31\ \text{Btu/lb}$. Using this, the quality at state 4 can be determined: $x_4 = (h_4 - h_f)/h_{fg} = (443.31 - 58.07)/1042.7 = 0.369$

Accordingly, $(v_4 - v_3) = (v_f + x_4 v_{fg}) - (v_f + x_3 v_{fg}) = v_{fg}(x_4 - x_3) = [467.7 - 0.01610](0.369 - 0.643) = -128.15\ \frac{\text{ft}^3}{\text{lb}}$

Finally, $W_{34} = (0.5\,\text{lb})(0.6988 \times 144\ \text{lbf/ft}^2)(-128.15\ \text{ft}^3/\text{lb})\left|1\ \text{Btu}/778\ \text{ft·lbf}\right| = -8.29\ \text{Btu}$

Process 4-1: $Q_{41} = 0$. Energy balance, $W_{41} = m(u_4 - u_1)$, $u_4 = 58.07 + 0.369(1040.2 - 58.07) = 420.48\ \text{Btu/lb}$, $u_1 = 609.9\ \text{Btu/lb}$. Then, $W_{41} = 0.5(420.48 - 609.9) = -94.71\ \text{Btu}$

(c) The thermal efficiency is found using

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{550}{1060} = 0.481\ (48.1\%)$$

or

① $$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{132.02}{274.85} = 0.480\ (48.0\%)$$

---

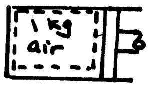
1. For any cycle, $W_{cycle} = Q_{cycle}$. From the process summary,
$132.24 \approx 132.02$
(difference due to round-off)

PROBLEM 5.63

KNOWN: One kg of air undergoes a Carnot cycle for which $\eta = 60\%$.

FIND: Determine the minimum and maximum temperatures, the pressure and volume at the beginning of the isothermal expansion, the work and heat transfer for each process, and sketch the cycle on p-v coordinates.

SCHEMATIC & GIVEN DATA:

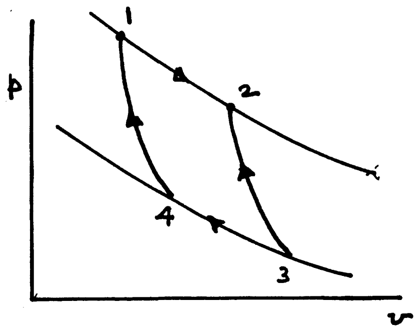


$\eta = 60\%$

$Q_{12} = 40\,kJ$

$P_2 = 5.6\,bar$

$V_2 = 0.3\,m^3$



Summary:

process	Q	W
1-2	40	40
2-3	0	256.7
3-4	-16	-16
4-1	0	-256.7
cycle	24	24

ASSUMPTIONS: 1. The system shown in the schematic consists of air modeled as an ideal gas. 2. Volume change is the only work mode.

ANALYSIS: (a) Using the ideal gas model equation of state, $T_2 = P_2 V_2 / mR$. Then

$$T_2 = \frac{(5.6 \times 10^5\,N/m^2)(0.3\,m^3)}{\left(\frac{8314}{28.97}\,\frac{N\cdot m}{kg\cdot K}\right)(1\,kg)} = 585.4\,K.$$

Then, since $\eta = 1 - \frac{T_C}{T_H} \Rightarrow T_C = T_H(1-\eta)$. With $T_H = T_2$,

$T_C = 585.4\,K(1-0.6) = 234.2\,K$, where $T_C = T_3 = T_4$.

(b) For process 1-2, $Q_{12} = 40\,kJ$ (given). An energy balance reads $m(u_2 - u_1) = Q_{12} - W_{12}$, but since internal energy of an ideal gas depends on temperature and $T_1 = T_2$, $W_{12} = Q_{12}$. Further, $W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT_H}{V}\,dV = mRT_H \ln V_2/V_1$. Solving and inserting values

$$\ln\frac{V_2}{V_1} = \frac{W_{12}}{mRT_H} \Rightarrow \ln\frac{V_2}{V_1} = \frac{40\,kJ}{(1\,kg)\left(\frac{8.314}{28.97}\,\frac{kJ}{kg\cdot K}\right)(585.4\,K)} = 0.2381 \Rightarrow V_1 = 0.24\,m^3$$

Since $T_1 = T_2$, $p_1 V_1 = mRT$, $P_2 V_2 = mRT \Rightarrow P_2 V_2 = P_1 V_1$, $P_1 = P_2(V_2/V_1) = 5.6\,bar(0.3/0.24) = 7\,bar$.

(c) For process 2-3: $Q_{23} = 0$. An energy balance reduces to give $W_{23} = m(u_2 - u_3)$. With data from Table A-22, $W_{23} = (1\,kg)(423.7 - 167.0) = 256.7\,kJ$. For process 3-4, $W_{34} = Q_{34}$ (as for process 1-2). Also, Eq. 5.6 is applicable:

$$\frac{|Q_{34}|}{T_C} = \frac{Q_{12}}{T_H} \Rightarrow |Q_{34}| = 0.4(40\,kJ) = 16\,kJ \Rightarrow Q_{34} = -16\,kJ,\ W_{34} = -16\,kJ.$$

① Process 4-1, $Q_{41} = 0$. An energy balance reduces to give, $W_{41} = m(u_4 - u_1)$. Since $u_1 = u_2$, $u_4 = u_3$, $W_{41} = -256.7\,kJ$. The work and heat transfers are summarized in the table.

---

1. As checks on the calculations, note that (1) $W_{cycle} = Q_{cycle}$ (see summary above), and (2)

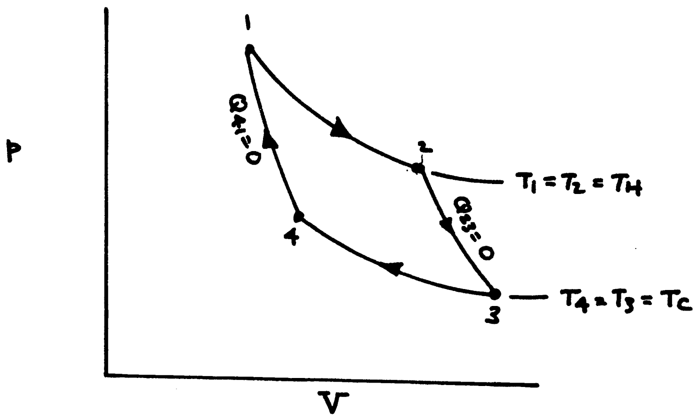
$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{24\,kJ}{40\,kJ} = 0.6$$

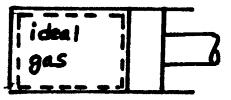
PROBLEM 5.64

KNOWN: A carnot cycle is executed by an ideal gas with constant specific heat ratio k.

FIND : Show that (a) $V_4 V_2 = V_1 V_3$, (b) $T_2/T_3 = (p_2/p_3)^{(k-1)/k}$, (c) $T_2/T_3 = (V_3/V_2)^{k-1}$

SCHEMATIC & GIVEN DATA.





ASSUMPTIONS: (1) The system shown in the figure consists of an ideal gas. (2) The specific heat ratio k is constant (required in part (b) only). (3) The system undergoes a Carnot cycle.

ANALYSIS: (a) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{W_{12}+W_{23}}{Q_{12}}$$

Since internal energy of an ideal gas depends on temperature only, an energy balance for process 1-2 reduces to $U_2 - U_1 = Q_{12} - W_{12}$, where $U_2 = U_1$. Thus, $Q_{12} = W_{12}$. Furthermore

$$W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT_H}{V}\,dV = mRT_H \ln\frac{V_2}{V_1}$$

Similarly, $W_{23} = mRT_c \ln V_4/V_3$. Collecting results

$$\eta = 1 - \frac{mRT_c \ln V_3/V_4}{mRT_H \ln V_2/V_1} = 1 - \left(\frac{\ln V_3/V_4}{\ln V_2/V_1}\right)\frac{T_c}{T_H}$$

However, for the Carnot cycle $\eta = 1 - T_c/T_H$, so it is necessary that

$$\frac{\ln(V_3/V_4)}{\ln(V_2/V_1)} = 1 \Rightarrow \ln\left(\frac{V_3}{V_4}\right) = \ln\left(\frac{V_2}{V_1}\right) \Rightarrow V_4 V_2 = V_3 V_1 \longleftarrow \quad (a)$$

(b) As process 2-3 is adiabatic, a energy balance in differential form reads

$$dU = \cancelto{0}{\delta Q} - \delta W$$

where $\delta W = p\,dV$ and with assumption 1 $dU = mc_v\,dT$. Collecting results and using $pV = mRT$ and $c_v = R/(k-1)$ (Eq. 3.47b)

$$\frac{1}{k-1}\,d\ln T = -\,d\ln V$$

Integration for constant k (assumption 2) gives

$$\ln\frac{T_3}{T_2} = -\ln\left(\frac{V_3}{V_2}\right)^{k-1} \Rightarrow \frac{T_3}{T_2} = \left(\frac{V_2}{V_3}\right)^{k-1} \longleftarrow \quad (c)$$

Finally, using $V = mRT/p$

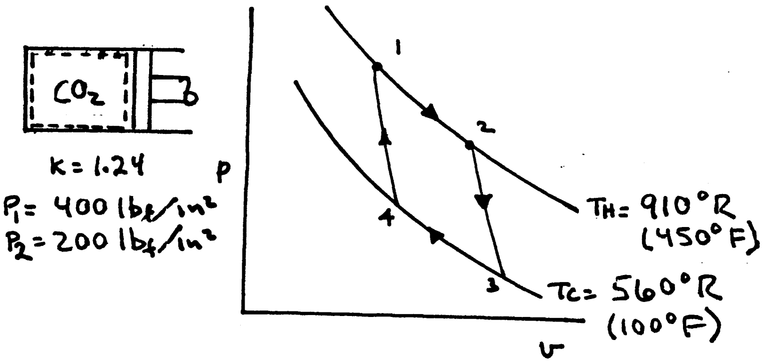
$$\frac{T_3}{T_2} = \left[\frac{T_2}{T_3}\,\frac{p_3}{p_2}\right]^{k-1} \Rightarrow \frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\frac{k-1}{k}} \longleftarrow \quad (b)$$

PROBLEM 5.65*

KNOWN: A quantity of CO_2 undergoes a Carnot cycle.

FIND: Determine the work and heat transfer for each process, the thermal efficiency, and the pressures at the initial and final states of the isothermal compression.

SCHEMATIC & GIVEN DATA:



Summary:

Process	Q/m	W/m
1-2	28.46	28.46
2-3	0	65.80
3-4	-17.52	-17.52
4-1	0	-65.80
Cycle	10.94	10.94

ASSUMPTIONS: 1. The system consists of a quantity of CO_2 modeled as an ideal gas with $k = 1.24$ 2. Volume change is the only work mode.

ANALYSIS: (a) Process 1-2: Since $U_2 = U_1$ for an ideal gas at fixed temperature, an energy balance reduces to give $Q_{12} = W_{12}$, where $W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT_H}{V}dV$, or $W_{12}/m = RT_H \ln(V_2/V_1)$. With $P_1V_1 = mRT_H$, $P_2V_2 = mRT_H$, we have

$$\frac{W_{12}}{m} = RT_H \ln\frac{P_1}{P_2} = \left(\frac{1545}{44.01}\right)\frac{\text{ft·lbf}}{\text{lb°R}}\left|\frac{1\ \text{Btu}}{778\ \text{ft·lbf}}\right|(910°\text{R})\ln\left(\frac{400}{200}\right) = 28.46\ \text{Btu/lb}$$

Process 2-3: $Q_{23} = 0$. An energy balance gives $W_{23} = m(u_2 - u_3)$. For an ideal gas with constant k, Eq. 3.47b gives $c_v = \frac{R}{k-1} = \left(\frac{1545}{44.01}\right)\left(\frac{1}{778}\right)\left(\frac{1}{1.24-1}\right) = 0.188\ \frac{\text{Btu}}{\text{lb°R}}$

$$\frac{W_{23}}{m} = c_v(T_2 - T_3) = \left(0.188\ \frac{\text{Btu}}{\text{lb°R}}\right)(910-560)°\text{R} = 65.8\ \text{Btu/lb}$$

Process 3-4: Since $U_3 = U_4$, an energy balance gives $Q_{34} = W_{34}$, where

$$\frac{W_{34}}{m} = \int_3^4 \frac{RT_C}{V}dV = RT_C \ln\frac{V_4}{V_3} = RT_C \ln\frac{P_3}{P_4} = \left(\frac{1545}{44.01}\right)\left(\frac{1}{778}\right)(560)\ln\left(\frac{16.28}{32.56}\right) = -17.52\ \frac{\text{Btu}}{\text{lb}}$$

where to find P_3, P_4, use part (b) of Problem 5.64:

$$\left.\begin{aligned}\frac{P_3}{P_2} &= \left(\frac{T_C}{T_H}\right)^{\frac{k}{k-1}} \Rightarrow P_3 = 200\ \frac{\text{lbf}}{\text{in}^2}\left[\frac{560}{910}\right]^{\frac{1.24}{.24}} = 16.28\ \frac{\text{lbf}}{\text{in}^2}\\ \frac{P_4}{P_1} &= \left(\frac{T_C}{T_H}\right)^{\frac{k}{k-1}} \Rightarrow \frac{P_4}{P_1} = \frac{P_3}{P_2} \Rightarrow P_4 = \frac{P_1}{P_2}P_3 = 32.56\ \frac{\text{lbf}}{\text{in}^2}\end{aligned}\right\} \leftarrow (c)$$

Process 4-1: $Q_{41} = 0$, and an energy balance gives $W_{41} = m(u_4 - u_1)$, or

$$\frac{W_{34}}{m} = c_v[T_4 - T_1] = \left(0.188\ \frac{\text{Btu}}{\text{lb°R}}\right)(560-910)°\text{R} = -65.8\ \text{Btu/lb}$$

(b) The thermal efficiency is given in this case by

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{10.94}{28.46} = 0.384\ (38.4\%)$$

or using Eq. 5.8

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{560}{910} = 0.384\ (38.4\%)$$

---

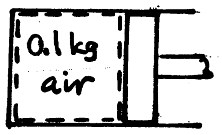
1. For any cycle, $Q_{cycle} = W_{cycle}$. See summary table above. This provides a check on the calculations.

PROBLEM 5.66*

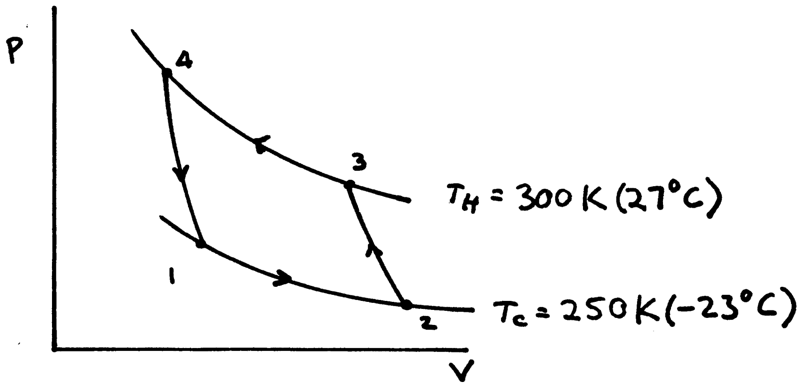
KNOWN: One-tenth kilogram of air executes a Carnot refrigeration cycle for which data are provided.

FIND: Determine (a) the pressure at each of the four principal states, (b) the work for each of the four processes, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



$V_4 = 0.01\ m^3$
$Q_{12} = 3.4\ kJ$
$k = 1.4$



ASSUMPTIONS: (1) The system consists of air modeled as an ideal gas with $k = 1.4$. (2) Volume change is the only work mode.

ANALYSIS: From the solution to Problem 5.64; $T_4/T_1 = (P_4/P_1)^{k-1/k}$ and $V_4 \cdot V_2 = V_3 \cdot V_1$.

(a) Using the ideal gas equation

$$P_4 = \frac{mRT_4}{V_4} = \frac{(0.1\,kg)\left(\frac{8.314}{28.97}\right)\frac{kJ}{kg\,K}(300\,K)}{(0.01\ m^3)}\left|\frac{1\ kN\cdot m}{1\ kJ}\right|\left|\frac{1\ kPa}{1\ kN/m^2}\right| = 861.0\ kPa \longleftarrow P_4$$

Now, for process 4-1; $T_4/T_1 = (P_4/P_1)^{k-1/k}$, or

$$P_1 = \left(\frac{T_1}{T_4}\right)^{k/k-1} P_4 = \left(\frac{250}{300}\right)^{1.4/1.4-1}\left(861\ \frac{lbf}{in^2}\right) = 454.9\ kPa \longleftarrow P_1$$

The volume at 1 is

$$V_1 = \frac{mRT_1}{P_1} = \frac{(0.1)\left(\frac{8.314}{28.97}\right)(250)}{(454.9)} = 0.01577\ m^3$$

For process 1-2: $m(u_2 - u_1) = Q_{12} - W_{12}$. Since $T_1 = T_2 = T_c$, $u_2 = u_1$, and

$$Q_{12} = W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{mRT}{V}\,dV = mRT_c \ln\frac{V_2}{V_1}$$

Solving for V_2/V_1 and inserting values

$$\frac{V_2}{V_1} = \exp\left[\frac{W_{12}}{mRT_c}\right] = \exp\left[\frac{(3.4\ kJ)}{(0.1\,kg)\left(\frac{8.314}{28.97}\right)\frac{kJ}{kg\,K}(250\,K)}\right]$$

$$= 1.6062$$

$$\therefore V_2 = (1.6062)(0.01577\ m^3) = 0.02533\ m^3$$

Thus

$$P_2 = \frac{(0.1)\left(\frac{8.314}{28.97}\right)(250)}{(0.02533)} = 283.2\ kPa \longleftarrow P_2$$

Next, from above

$$V_4V_2 = V_3V_1 \Rightarrow V_3 = \left(\frac{V_4}{V_1}\right)V_2 = \left(\frac{0.01}{0.01577}\right)(0.02533\ m^3) = 0.01606\ m^3$$

PROBLEM 5.66 (Cont'd.)

Finally

$$P_3 = \frac{(0.1)\left(\frac{8.314}{28.97}\right)(300)}{(0.01606)} = 536.1 \text{ kPa} \longleftarrow P_3$$

(b) Process 1-2: $Q_{12} = W_{12} = 3.4 \text{ kJ} \longleftarrow W_{12}$

process 2-3: $Q_{23} = 0 \Rightarrow W_{23} = -m(u_3 - u_2) = -m\,c_v(T_3 - T_2)$

With $c_v = R/(k-1) = \left(\frac{8.314}{28.97}\right)\left(\frac{1}{1.4-1}\right) = 0.717 \text{ kJ/kg K}$

$$W_{23} = -(0.1 \text{ kg})(0.717 \tfrac{\text{kJ}}{\text{kg K}})(300-250)\text{ K}$$

$$= -3.585 \text{ kJ} \longleftarrow W_{23}$$

Process 3-4: As for process 1-2 in part (a)

$$W_{34} = \int_3^4 p\,dV = mRT_H \ln\frac{V_4}{V_3}$$

$$= (0.1 \text{ kg})\left(\frac{8.314}{28.97}\right)\frac{\text{kJ}}{\text{kg K}}(300 \text{ K})\ln\left(\frac{0.01}{0.01606}\right) = -4.079 \text{ kJ} \longleftarrow W_{34}$$

Process 4-1: $Q_{41} = 0 \Rightarrow W_{41} = -m(u_1 - u_4) = -m\,c_v(T_1 - T_4)$. Again

$$W_{41} = -(0.1 \text{ kg})(0.717 \tfrac{\text{kJ}}{\text{kg K}})(250-300)\text{K} = 3.585 \text{ kJ} \longleftarrow W_{41}$$

(c) For the Carnot cycle

① $$\beta = \beta_{max} = \frac{T_C}{T_H - T_C} = \frac{250}{300-250} = 5.00 \longleftarrow \beta$$

---

1. Alternatively,

$$|W_{cycle}| = |W_{12} + W_{23} + W_{34} + W_{41}| = |-0.679 \text{ kJ}|$$

and, with $Q_{in} = Q_{12}$ for the cycle

$$\beta = \frac{Q_{in}}{|W_{cycle}|} = \frac{3.4}{0.679} = 5.01$$

The slight difference is due to round-off.

PROBLEM 5.67*

KNOWN: Systems execute Carnot refrigeration cycles for which data are provided.

FIND: For each such cycle, evaluate the coefficient of performance.

SCHEMATIC & GIVEN DATA: See the solutions to the respective problems.

ASSUMPTIONS: See the solutions to the respective problems.

ANALYSIS: For each case, Eq. 5.9 is applicable. Accordingly, $\beta_{MAX} =$
① $T_C/(T_H-T_C)$.

(a) PROBLEM 5.61

$$\beta_{MAX} = \frac{384.4}{523.4-384.4} = 2.765$$

(b) PROBLEM 5.62

$$\beta_{MAX} = \frac{550}{1060-550} = 1.078$$

(c) PROBLEM 5.63

$$\beta_{MAX} = \frac{234.2}{585.4-234.2} = 0.67$$

(d) PROBLEM 5.65

$$\beta_{MAX} = \frac{560}{910-560} = 1.60$$

---

1. Using the result of Problem 5.18(a),

$$\beta_{MAX} = \frac{1-\eta_{MAX}}{\eta_{max}}$$

ⓐ $\beta_{MAX} = \frac{1-.266}{.266} = 2.76$

ⓑ $\beta_{MAX} = \frac{1-.481}{.481} = 1.08$

ⓒ $\beta_{MAX} = \frac{1-0.6}{0.6} = 0.67$

ⓓ $\beta_{MAX} = \frac{1-0.384}{0.384} = 1.604$

PROBLEM 5.68*

KNOWN: Systems execute Carnot heat pump cycles for which data are provided.

FIND: For each cycle, evaluate the coefficient of performance.

SCHEMATIC & GIVEN DATA: See the solutions to the respective problems.

ASSUMPTIONS: See the solutions to the respective problems.

ANALYSIS: For each case, Eq. 5.10 is applicable. Accordingly, $\gamma_{MAX} =$
① $T_H/(T_H-T_C)$.

(a) PROBLEM 5.61

$$\gamma_{MAX} = \frac{523.4}{523.4-384.4} = 3.765$$

(b) PROBLEM 5.62

$$\gamma_{MAX} = \frac{1060}{1060-550} = 2.078$$

(c) PROBLEM 5.63

$$\gamma_{MAX} = \frac{585.4}{585.4-234.2} = 1.67$$

(d) PROBLEM 5.65

$$\gamma_{MAX} = \frac{910}{910-560} = 2.6$$

---

1. Using the result of Problem 5.18(b),

$$\gamma_{MAX} = \frac{1}{\eta_{MAX}}$$

(a) $\gamma_{MAX} = \frac{1}{0.266} = 3.76$

(b) $\gamma_{MAX} = \frac{1}{0.481} = 2.079$

(c) $\gamma_{MAX} = \frac{1}{0.6} = 1.67$

(d) $\gamma_{MAX} = \frac{1}{0.384} = 2.604$

CHAPTER SIX

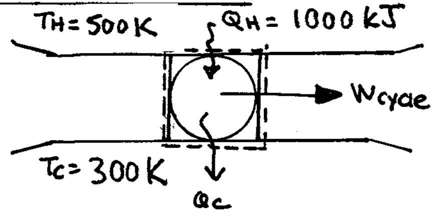
USING ENTROPY

PROBLEM 6.1*

KNOWN: A system undergoes a power cycle for which data are provided.

FIND: Using Eq. 6.2, evaluate σ_{cycle} if η is 60, 40, and 20%. Discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system shown in the accompanying figure undergoes a power cycle while receiving Q_H at T_H and discharging Q_C at T_C.

ANALYSIS: For any cycle, $\eta = 1 - Q_C/Q_H \Rightarrow Q_C = (1-\eta)Q_H$. Then, Eq. 6.2 gives

$$\sigma_{cycle} = -\oint\left(\frac{\delta Q}{T}\right)_b = -\left[\frac{Q_H}{T_H} - \frac{Q_C}{T_C}\right] = -Q_H\left[\frac{1}{T_H} - \frac{(1-\eta)}{T_C}\right]$$

(a) $\eta = 60\%$

$$\sigma_{cycle} = -(1000\,kJ)\left[\frac{1}{500\,K} - \frac{(1-0.6)}{300\,K}\right] = -0.667\,\frac{kJ}{K}$$

① Since σ_{cycle} must be positive or zero in value, this case is impossible.

(b) $\eta = 40\%$

$$\sigma_{cycle} = -(1000\,kJ)\left[\frac{1}{500\,K} - \frac{(1-0.4)}{300\,K}\right] = 0$$

Since $\sigma_{cycle} = 0$, this case corresponds to internally reversible operation.

(c) $\eta = 20\%$

$$\sigma_{cycle} = -(1000\,kJ)\left[\frac{1}{500\,K} - \frac{(1-0.2)}{300\,K}\right] = +0.667\,\frac{kJ}{K}$$

In this case, irreversibilities are present within the system.

---

1. Using Eq. 5.8, the maximum thermal efficiency any cycle can have while operating between reservoirs at T_H and T_C is

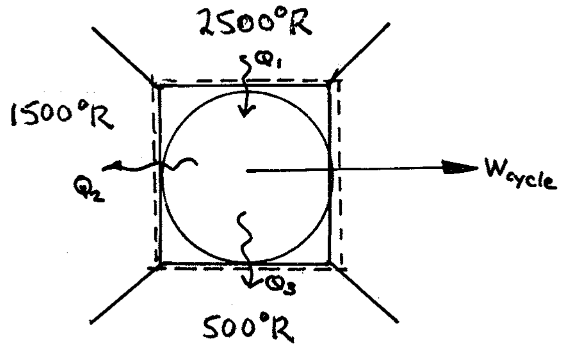
$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{500} = 0.4\ (40\%)$$

PROBLEM 6.2*

KNOWN: A system undergoes a reversible power cycle while communicating thermally with three reservoirs at specified temperatures. The values for the energy transfers by heat between two of the reservoirs and the system are also specified.

FIND: Determine the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$Q_1 = 1600$ Btu
$Q_3 = 200$ Btu

ASSUMPTION: 1. The system shown in the accompanying figure undergoes a power cycle with no internal irreversibilities. 2. All heat transfers take place at the indicated temperatures.

ANALYSIS: The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{W_{cycle}}{Q_1}$$

An energy balance gives

$$W_{cycle} = Q_1 - Q_2 - Q_3 = 1600 - Q_2 - 200$$
$$= 1400 - Q_2$$

Since the cycle involves no irreversibilities, and heat transfers are at T_1, T_2, T_3, Eq. 6.2 reads

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = -\overset{0}{\cancel{\sigma}}_{cycle}$$

$$\Rightarrow \quad 0 = \frac{1600}{2500} - \frac{Q_2}{1500} - \frac{200}{500} \Rightarrow Q_2 = 360 \text{ Btu}$$

Thus

$$W_{cycle} = 1400 - 360 = 1040 \text{ Btu}$$

and

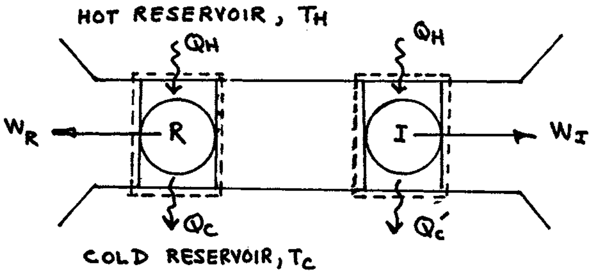
$$\eta = \frac{1040}{1600} = 0.65 \ (65\%)$$

PROBLEM 6.3

KNOWN: A reversible power cycle R and an irreversible cycle I operate between the same two reservoirs.

FIND: (A) Evaluate σ_{cycle} for I in terms of W_R, W_I, and the temperature T_C.
(b) Demonstrate that $W_I < W_R$, $Q_C' > Q_C$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The systems shown in the accompanying figure undergo power cycles. R is reversible and I is irreversible. (2) Each system receives Q_H at T_H from the hot reservoir and discharges energy at T_C to the cold reservoir.

ANALYSIS: (a) For cycle I, Equation 6.2 takes the form

$$\sigma_{cycle} = -\oint\left(\frac{\delta Q}{T}\right)_b = -\left[\frac{Q_H}{T_H} - \frac{Q_C'}{T_C}\right]$$

Energy balances for cycles R and I give, respectively

$$\begin{aligned} Q_H &= W_R + Q_C \\ Q_H &= W_I + Q_C' \end{aligned} \Longrightarrow Q_C' = Q_C + W_R - W_I$$

Inserting this result into the expression for σ_{cycle}

$$\sigma_{cycle} = -\left[\left(\frac{Q_H}{T_H} - \frac{Q_C}{T_C}\right) - \left(\frac{W_R - W_I}{T_C}\right)\right]$$

Since R is reversible, Eq. 5.6 applies and the first term on the right vanishes, leaving

$$\sigma_{cycle} = \frac{W_R - W_I}{T_C} \longleftarrow \text{(a)}$$

(b) When irreversibilities are present, $\sigma_{cycle} > 0$. Thus, Eq. (a) reads

$$\frac{W_R - W_I}{T_C} > 0 \Rightarrow W_R > W_I$$

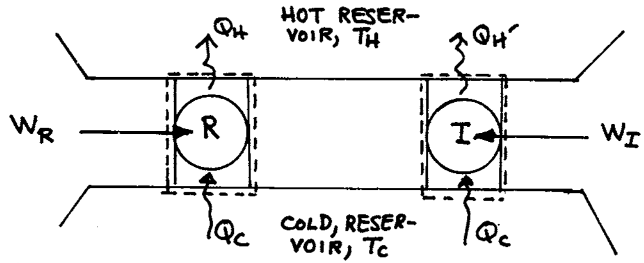
The energy balance result: $Q_C' = Q_C + W_R - W_I$ then indicates that $Q_C' > Q_C$. Accordingly, the effect of irreversibilities is to reduce the desired outcome: W_{cycle} and increase the heat rejected, which for actual power cycles increases the thermal pollution effect.

PROBLEM 6.4

KNOWN: A reversible refrigeration cycle R and an irreversible refrigeration cycle I operate between the same two reservoirs.

FIND: Show that $W_I > W_R$ and $Q_H' > Q_H$

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The systems shown in the accompanying figure undergo refrigeration cycles. R is reversible and I is irreversible. (2) Each system receives Q_C at T_C from the cold reservoir and discharges energy at T_H to the hot reservoir.

ANALYSIS: For cycle I, Eq. 6.2 takes the form

$$\sigma_{cycle} = -\oint\left(\frac{\delta Q}{T}\right)_b = -\left[\frac{Q_C}{T_C} - \frac{Q_H'}{T_H}\right]$$

Energy balances for cycles R and I give, respectively

$$\begin{aligned} Q_H &= W_R + Q_C \\ Q_H' &= W_I + Q_C \end{aligned} \Longrightarrow Q_H' = Q_H + W_I - W_R$$

Inserting this result into the expression for σ_{cycle}

$$\sigma_{cycle} = -\left[\left(\frac{Q_C}{T_C} - \frac{Q_H}{T_H}\right) - \left(\frac{W_I - W_R}{T_H}\right)\right]$$

Since R is reversible, Eq. 5.6 applies and the first term on the right vanishes leaving

$$\sigma_{cycle} = \frac{W_I - W_R}{T_H}$$

Since cycle I irreversible, $\sigma_{cycle} > 0$ (Sec. 6.1). Thus, $W_I - W_R > 0$, or $W_I > W_R$. The energy balance result then gives $Q_H' > Q_H$. ◄

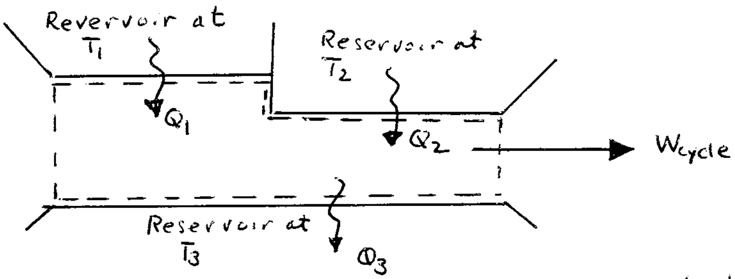
COMMENT: The current demonstration uses the Clausius inequality developed in Sec. 6.1. The conclusion that $W_I > W_R$ also can be demonstrated using the Kelvin-Planck statement of the second law, as illustrated in the solution to Problem 5.13a.

PROBLEM 6.5

KNOWN: A system undergoes a reversible power cycle while receiving Q_1, Q_2 from hot reservoirs at T_1, T_2 and discharging Q_3 to a cold reservoir at T_3.

FIND: Obtain $\eta = f(T_1/T_3, T_2/T_3, q)$, where $q = Q_2/Q_1$, and discuss the limiting behavior as $q \to 0$, $q \to \infty$, $T_1 \to \infty$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown by the dashed line undergoes a power cycle. (2) There are no internal irreversibilities or irreversibilities associated with heat transfer between the system and reservoirs. (3) The only energy transfers between system and surroundings are indicated on the sketch.

ANALYSIS: (a) An energy balance for the cycle reads $W_{cycle} = Q_1 + Q_2 - Q_3$. Equation 6.2 takes the form

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = -\cancelto{0}{\sigma_{cycle}} \Rightarrow Q_3 = \frac{T_3}{T_1} Q_1 + \frac{T_3}{T_2} Q_2$$

The thermal efficiency can be expressed, then, as

$$\eta = \frac{W_{cycle}}{Q_1 + Q_2} = \frac{Q_1 + Q_2 - Q_3}{Q_1 + Q_2} = 1 - \frac{Q_3}{(Q_1 + Q_2)} = 1 - \frac{\left[\frac{T_3}{T_1} Q_1 + \frac{T_3}{T_2} Q_2\right]}{(Q_1 + Q_2)}$$

or

$$\eta = 1 - \frac{1}{(T_1/T_3)}\left[\frac{1}{1+q}\right] - \frac{1}{(T_2/T_3)}\left[\frac{q}{1+q}\right], \quad q = Q_2/Q_1 \longleftarrow \eta$$

(b)

$$\lim_{q \to 0} \eta = 1 - \frac{1}{(T_1/T_3)} = 1 - \frac{T_3}{T_1}$$ (Corresponds to the case of Eq. 5.8)

$$\lim_{q \to \infty} \eta = 1 - \frac{1}{T_2/T_3} = 1 - \frac{T_3}{T_2}$$ (Corresponds to the case of Eq. 5.8)

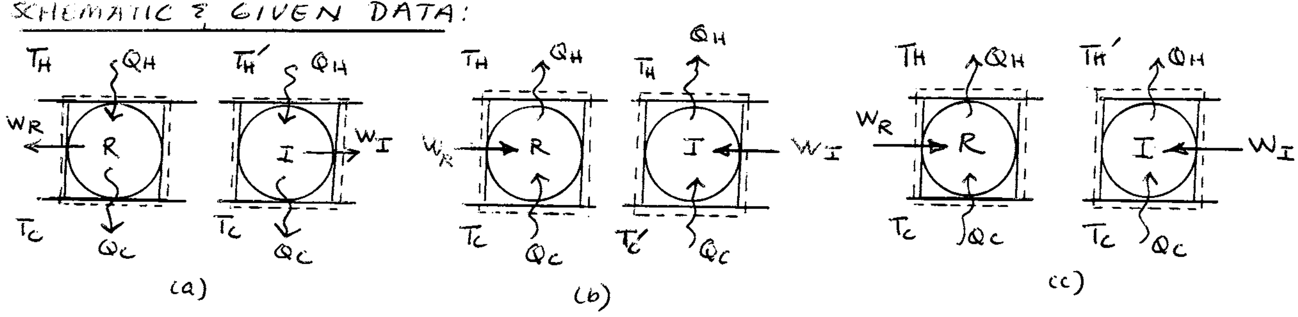
$$\lim_{T_1 \to \infty} \eta = 1 - \frac{1}{T_2/T_3}\left(\frac{q}{1+q}\right) = 1 - \frac{T_3}{T_2}\left[\frac{q}{1+q}\right]$$ (Although one reservoir might be at an extremely high temperature, the value of η would be strictly less than unity.)

PROBLEM 6.6

KNOWN: Operating details are provided for reversible and irreversible cycles.

FIND: (a) For power cycles, show $T_H' > T_H$, (b) For refrigeration cycles, show $T_C' > T_C$, (c) For heat pump cycles, show $T_H' < T_H$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The systems shown in the figure undergo cycles. R denotes a reversible cycle and I denotes an irreversible cycle, (2) The systems receive and discharge energy by heat transfer at the indicated temperatures. (3) The only energy transfers taking place are shown on the figures.

ANALYSIS: (a) Applying Eq. 6.2 to R and I, respectively

R: $\frac{Q_H}{T_H} - \frac{Q_C}{T_C} = -\overset{0}{\cancel{\sigma_{cycle}}} \Rightarrow Q_C = \frac{T_C}{T_H}Q_H$

I: $\frac{Q_H}{T_H'} - \frac{Q_C}{T_C} = -\sigma_{cycle}$

$$\frac{Q_H}{T_H'} - \left(\frac{T_C}{T_H}Q_H\right)\frac{1}{T_C} = -\sigma_{cycle}$$

$$Q_H\left[\frac{1}{T_H'} - \frac{1}{T_H}\right] = -\sigma_{cycle} \qquad \longleftarrow \text{(a)}$$

$$\Rightarrow \quad T_H' > T_H$$

(b) Similarly

R: $\frac{Q_C}{T_C} - \frac{Q_H}{T_H} = 0 \Rightarrow Q_H = \frac{T_H}{T_C}Q_C$

I: $\frac{Q_C}{T_C'} - \frac{Q_H}{T_H} = -\sigma_{cycle}$

$$\frac{Q_C}{T_C'} - \left(\frac{T_H}{T_C}Q_C\right)\frac{1}{T_H} = -\sigma_{cycle}$$

$$Q_C\left[\frac{1}{T_C'} - \frac{1}{T_C}\right] = -\sigma_{cycle} \qquad \longleftarrow \text{(b)}$$

$$\Rightarrow \quad T_C' > T_C$$

(c) Similarly

R: $\frac{Q_C}{T_C} - \frac{Q_H}{T_H} = 0 \Rightarrow Q_C = \frac{T_C}{T_H}Q_H$

I: $\frac{Q_C}{T_C} - \frac{Q_H}{T_H'} = -\sigma_{cycle}$

$$\left(\frac{T_C}{T_H}Q_H\right)\frac{1}{T_C} - \frac{Q_H}{T_H'} = -\sigma_{cycle} \qquad \longleftarrow \text{(c)}$$

$$Q_H\left[\frac{1}{T_H} - \frac{1}{T_H'}\right] = -\sigma_{cycle}$$

$$\Rightarrow \quad T_H' < T_H$$

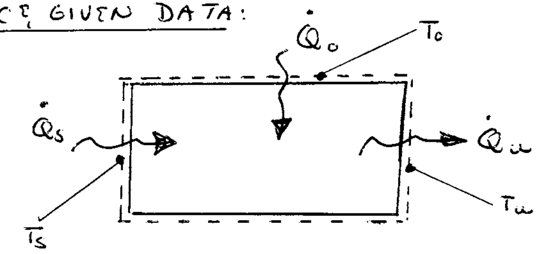
PROBLEM 6.7

KNOWN: A system undergoes a cycle while receiving $\dot{Q}_s, \dot{Q}_o$ at T_s, T_o, respectively, and delivering $\dot{Q}_u$ at T_u.

FIND: Obtain an expression for the maximum theoretical value of $\dot{Q}_u$ in terms of $\dot{Q}_s, T_s, T_u, T_o$.

SCHEMATIC & GIVEN DATA:

①



ASSUMPTIONS: (1) The system shown in the figure undergoes a cycle. (2) The only energy transfers experienced are shown on the figure. (3) $T_o < T_u < T_s$

② ANALYSIS: An energy balance reduces to $\dot{Q}_u = \dot{Q}_s + \dot{Q}_o$. Equation 6.2 takes the form

$$\frac{\dot{Q}_s}{T_s} + \frac{\dot{Q}_o}{T_o} - \frac{\dot{Q}_u}{T_u} = -\dot{\sigma}_{cycle}$$

Eliminating $\dot{Q}_o$ between these expressions

$$\frac{\dot{Q}_s}{T_s} + \frac{\dot{Q}_u - \dot{Q}_s}{T_o} - \frac{\dot{Q}_u}{T_u} = -\dot{\sigma}_{cycle}$$

$$\Rightarrow \quad \dot{Q}_s\left[\frac{1}{T_s} - \frac{1}{T_o}\right] + \dot{Q}_u\left[\frac{1}{T_o} - \frac{1}{T_u}\right] = -\dot{\sigma}_{cycle}$$

$$\Rightarrow \quad \dot{Q}_u = \frac{\dot{Q}_s\left[1 - \frac{T_o}{T_s}\right] - T_o\dot{\sigma}_{cycle}}{\left[1 - \frac{T_o}{T_u}\right]}$$

As $T_s > T_o$ and $T_u > 0$, the terms in brackets are positive. The maximum theoretical value of $\dot{Q}_u$ would be realized as $\dot{\sigma}_{cycle} \rightarrow 0$:

$$(\dot{Q}_u)_{MAX} = \dot{Q}_s\left[\frac{1 - \frac{T_o}{T_s}}{1 - \frac{T_o}{T_u}}\right] \longleftarrow$$

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1. This device corresponds to an absorption heat pump -- See Secs. 10.5, 6.
2. The energy balance shows that $\dot{Q}_u > \dot{Q}_s$

PROBLEM 6.8 No format – True/false

(a) TRUE. Entropy is a property. Thus, its change in value between two states is independent of the process -- see Sec. 1.3 for discussion.

(b) FALSE. When T is constant, Eq. 6.18 reduces to

$$s_2 - s_1 = \cancelto{0}{\int_{T_1}^{T_2} \frac{c_v(T)}{T}dT} + R\ln\frac{v_2}{v_1}$$. In a compression, $v_2 < v_1$; so, $s_2 - s_1 < 0$.

(c) TRUE. As shown by Eqs. 6.18, 6.19, the specific entropy of an ideal gas depends on T, v or T, p, respectively. Also, s can be expressed in terms of p, v.

(d) FALSE. See Eq. 6.12a.

(e) TRUE. See Eq. 6.24

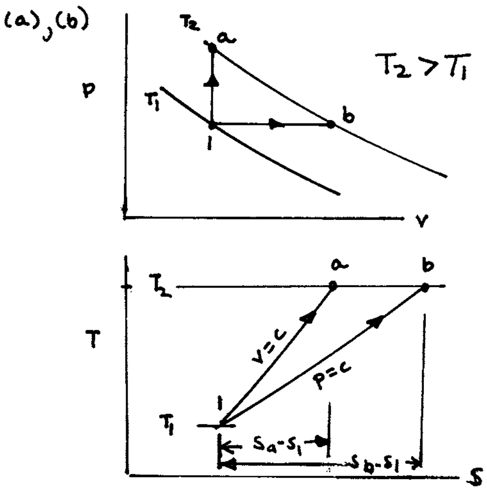
PROBLEM 6.9

KNOWN: A system consisting of an ideal gas with constant specific heat ratio k undergoes constant volume, constant pressure, and constant temperature processes from the same initial state.

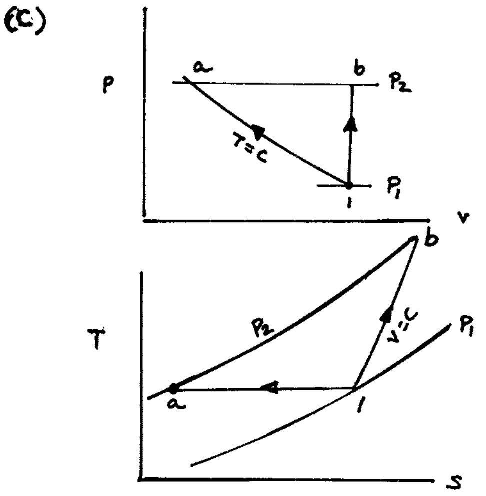
FIND: (a) Show that the entropy change is greater for the constant pressure process than for the constant volume process. Sketch the processes on p-v, T-s coordinates. (b) Using the T-s diagram, show that a line of constant v has a greater slope than a line of constant p. (c) Show that the ratio of the entropy change for a constant T process to the entropy change for a constant v process is (1-k). Sketch the processes on p-v and T-s coordinates.

SCHEMATIC & GIVEN DATA:

(a), (b)



(c)



ASSUMPTION: The system consists of an ideal gas with constant k.

ANALYSIS: (a) For process 1-a, specific volume is constant. For process 1-b, pressure is constant. Thus, Eqs. 6.22 and 6.23 reduce, respectively, to give

$$s_a - s_1 = c_v \ln \frac{T_2}{T_1}$$
$$s_b - s_1 = c_p \ln \frac{T_2}{T_1}$$
$$\Longrightarrow \quad \frac{s_b - s_1}{s_a - s_1} = \frac{c_p \ln T_2/T_1}{c_v \ln T_2/T_1} = \frac{c_p}{c_v} = k \quad \longleftarrow$$

As $k > 1$, $(s_b - s_1) > (s_a - s_1)$.

(b) Here, we compare at state 1 $(\partial T/\partial s)_v$ to $(\partial T/\partial s)_p$. Since $(\partial T/\partial s)$ at fixed v (or fixed p) $= \lim\limits_{\Delta s \to 0} \frac{\Delta T}{\Delta s}$, it is evident from the T-s diagram that a constant specific volume line passing through state 1 has a greater slope than a constant pressure line. $\longleftarrow$

PROBLEM 6.9 (Contd.)

For process 1-a, temperature is constant. For process 1-b, volume is constant. Thus, Eqs. 6.19 and 6.18 reduce, respectively, to give

$$s_a - s_1 = -R \ln \frac{p_2}{p_1} \quad \text{(entropy decreases)}$$

$$s_b - s_1 = c_v \ln \frac{T_b}{T_1} \quad \text{(entropy increases)}$$

Using Eq. 3.44, the first of these can be written as

$$s_a - s_1 = -(c_p - c_v) \ln \frac{p_2}{p_1}$$

Using the ideal gas equation of state with $v_b = v_1$, the second can be written as

$$s_b - s_1 = c_v \ln \frac{p_2}{p_1}$$

Forming the ratio

$$\frac{s_a - s_1}{s_b - s_1} = \frac{-(c_p - c_v) \ln p_2/p_1}{c_v \ln p_2/p_1} = 1 - k \quad \longleftarrow$$

PROBLEM 6.10 No Format. True/false

(a) FALSE. See discussion of Sec. 5.1.

(b) FALSE. For any process, internally reversible or otherwise, $\Delta E = Q - W$. Accordingly, the magnitude and direction of Q is not determined by W alone -- unless $\Delta E = 0$.

(c) FALSE. See Eq. 6.29 and the accompanying discussion.

(d) FALSE. See Eq. 6.27 and the accompanying discussion.

(e) FALSE. See Eq. 6.28 and the accompanying discussion.

(f) TRUE. See Sec. 6.4, p. 250 for discussion.

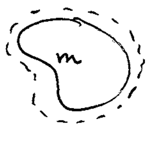
(g) FALSE. See Sec. 6.5.5

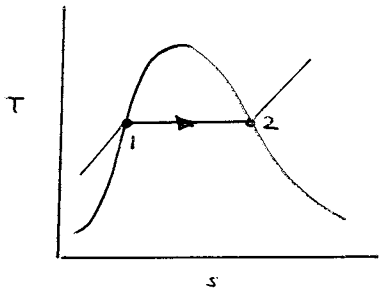
PROBLEM 6.11

KNOWN: A fixed mass of water, initially a saturated liquid, undergoes a constant temperature, constant pressure process to a saturated vapor condition.

FIND: (a) Derive expressions for W, Q in terms of m and steam table properties. (b) Show that the process is internally reversible.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system is a fixed mass of water. (2) Kinetic and potential energy effects are absent. (3) In the process, p = constant, T = constant (4) Volume change is the only work mode.

ANALYSIS: (a) With assumption (4) and since the pressure remains constant,

$$W = m\int_{1}^{2} p\,dv = m\,p\,(v_2 - v_1) = m\,p\,[v_g - v_f] \quad \longleftarrow \quad W$$

With assumption (2) and the above expression for W, an energy balance gives

$$m[u_2 - u_1] = Q - W \Rightarrow Q = m[u_g - u_f] + m\,p\,(v_g - v_f)$$

$$= m\,[(u_g + p v_g) - (u_f + p v_f)]$$

$$= m\,[h_g - h_f] \quad \longleftarrow \quad Q$$

(b) An entropy balance reduces to

$$m(s_2 - s_1) = \frac{Q}{T} + \sigma$$

Introducing the expression for Q

$$m(s_g - s_f) = \frac{m(h_g - h_f)}{T} + \sigma$$

From the discussion of Sec. 6.3.1, Eq. 6.14, $s_g - s_f = (h_g - h_f)/T$, and so

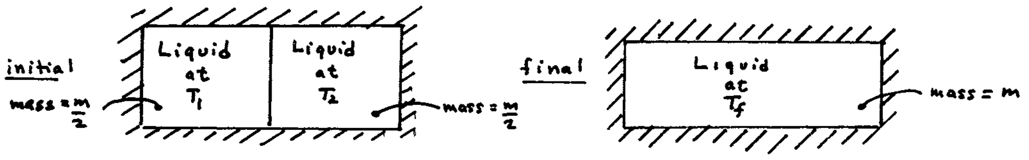
$\sigma = 0 \Rightarrow$ the process is internally reversible.

PROBLEM 6.14

KNOWN: An isolated system of total mass m is formed by mixing equal masses of the same liquid initially at temperatures T_1 and T_2.

FIND: (a) Show that the amount of entropy produced is $\sigma = mc \ln\left[(T_1+T_2)/2\,(T_1 \cdot T_2)^{1/2}\right]$ and (b) that σ must be positive.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system consists of the total mass of liquid. (2) The system is isolated. (3) The liquid is incompressible with constant specific heat c

ANALYSIS: (a) The final temperature T_f can be evaluated from an energy balance: $\Delta U = \cancel{Q}^0 - \cancel{W}^0$. Thus $\Delta U = 0$, or

$$m\,u(T_f) - \left[\frac{m}{2}u(T_1) + \frac{m}{2}u(T_2)\right] = 0$$

$$\Rightarrow \quad \frac{m}{2}\left[u(T_f) - u(T_1)\right] + \frac{m}{2}\left[u(T_f) - u(T_2)\right] = 0$$

Since the liquid is incompressible with constant specific heat c

$$\frac{m}{2}c\left[T_f - T_1\right] + \frac{m}{2}c\left[T_f - T_2\right] = 0$$

$$\Rightarrow \quad T_f = \frac{T_1+T_2}{2}$$

An entropy balance gives

$$\Delta S = \cancel{\int_1^2\left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma$$

or

$$\sigma = m s_f - \left[\frac{m}{2}s_1 + \frac{m}{2}s_2\right]$$
$$= \frac{m}{2}\left[(s_f - s_1) + (s_f - s_2)\right]$$

Using Eq. 6.28

$$\sigma = \frac{m}{2}c\left[\ln\frac{T_f}{T_1} + \ln\frac{T_f}{T_2}\right] = \frac{m}{2}c\ln\left[\frac{T_f^2}{T_1T_2}\right]$$
$$= mc\ln\left[\frac{T_f}{(T_1T_2)^{1/2}}\right]$$
$$= mc\ln\left[\frac{T_1+T_2}{2\,(T_1T_2)^{1/2}}\right] \longleftarrow \text{(a)}$$

(b) $\sigma \geq 0$ when $\ln\left[\frac{T_1+T_2}{2(T_1T_2)^{1/2}}\right] \geq 0$

$$\Rightarrow \quad \frac{T_1+T_2}{2(T_1T_2)^{1/2}} \geq 1 \quad \text{or} \quad T_1+T_2 \geq 2(T_1T_2)^{1/2}$$

Squaring both sides

$$(T_1+T_2)^2 \geq 4\,(T_1T_2) \quad \text{or} \quad T_1^2 + 2T_1T_2 + T_2^2 \geq 4T_1T_2$$
$$\Rightarrow \quad T_1^2 - 2T_1T_1 + T_2^2 \geq 0$$
$$\Rightarrow \quad (T_1 - T_2)^2 \geq 0$$

The inequality is satisfied for either $T_1 > T_2$ or $T_2 > T_1$. The equality applies only when $T_1 = T_2$ $\longleftarrow$ (b)

PROBLEM 6.13

KNOWN: A closed system and its surroundings make up an isolated system.

FIND: Four each of four cases answer true or false, and explain.

ANALYSIS: From the discussion of Sec. 6.5.5

(energy) $\Delta E]_{sys} + \Delta E]_{surr} = 0$ (1)

(entropy) $\Delta S]_{sys} + \Delta S]_{surr} = \sigma_{isol} (\geq 0)$ (2)

(a) False Equation (2) allows, but does not require, both entropy changes to be positive.

(b) True Equation (2) allows $\Delta S]_{system}$ to decrease while $\Delta S]_{surr}$ decreases, and conversely — provided the sum is nonnegative: provided $\sigma_{isol} \geq 0$.

(c) False If $\Delta S)_{sys} = \Delta S)_{surr} = 0$, it follows the σ_{isol} must be zero, and $\sigma_{isol} = 0$ is an allowed outcome.

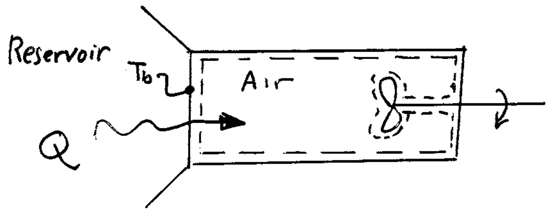
(d) False If both $\Delta S]_{sys}$ and $\Delta S]_{surr}$ decrease, it follows that $\sigma_{isol} < 0$, which is not an allowed outcome.

PROBLEM 6.12

KNOWN: A quantity of air undergoes a process in which the air is both stirred and heated from a reservoir.

FIND: Explain how the process can be conducted with (a) minimum entropy production, (b) maximum entropy production.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS (1) The system consists of the air only. (2) The temperature of the air at the place where energy is received from the reservoir by heat transfer is T_b (3) The air is modeled as an ideal gas. (4) The volume of the air is constant during the process and there is no significant change in kinetic or potential energy.

ANALYSIS: The change of state can be brought about by a combination of heat and work. Q and W are related by an energy balance:

$$\Delta U = Q - W \Rightarrow m\int_{T_1}^{T_2} c_v(T)\,dT = Q - W$$

where assumptions 3 and 4 have been used.

Since Q is received at T_b, the closed system entropy balance reads

$$\Delta S = \frac{Q}{T_b} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T_b}$$

Then, with Eq 6.18 and assumption 4

$$\Delta S = m\left[\int_{T_1}^{T_2} \frac{c_v(T)}{T}\,dT + \overset{0}{\cancel{R\ln\frac{V_2}{V_1}}}\right]$$

Collecting results

$$\sigma = \overbrace{m\int_{T_1}^{T_2}\frac{c_v(T)}{T}\,dT}^{\text{(strictly positive)}} - \overbrace{\frac{Q}{T_b}}^{\text{(positive or zero)}}$$

Accordingly, σ increases as Q decreases, and conversely. This suggests

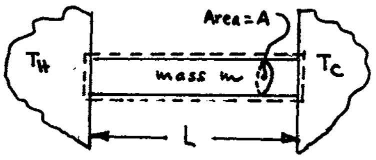
- Maximum σ corresponds to Q = 0: the process takes place by stirring only.
- Minimum σ corresponds to heating to the fullest extent, followed by stirring, as required. Observe ~~that~~ some stirring ~~would~~ be required if T_2 were greater than the reservoir temperature.

PROBLEM 6.15

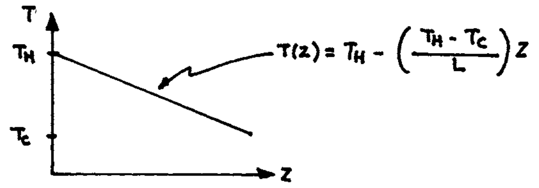
KNOWN: The temperature within a rod initially in contact with hot and cold walls at its ends is linear with position. The rod is insulated overall and eventually comes to a final equilibrium state where the temperature is T_f.

FIND: Evaluate the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



$T_C < T_H$



ASSUMPTIONS: (1) The system is the rod which is insulated on its lateral surface. (2) The rod is modeled as incompressible with constant specific heat c. (3) Initially, the temperature within the rod varies linearly from T_H to T_C.

ANALYSIS: The final temperature can be determined using an energy balance which reduces to give $\Delta U = \cancel{Q}^0 - \cancel{W}^0$ or $\Delta U = 0$. Each element of rod dz changes temperature from T(z) to the final temperature T_f, and thus contributes to the change in internal energy

$$dU = dm\, c\,(T_f - T(z))$$
$$= (\rho A\, dz)\, c\,(T_f - T(z))$$

Accordingly

$$\Delta U = \int_0^L (\rho A dz)c\,(T_f - T(z))$$
$$= \rho A c \int_0^L \left[T_f - T_H + \left(\frac{T_H - T_C}{L}\right)z\right]dz$$
$$= \rho A c \left[(T_f - T_H)z + \left(\frac{T_H - T_C}{L}\right)\frac{z^2}{2}\right]_0^L$$
$$= \rho A c L \left[(T_f - T_H) + \left(\frac{T_H - T_C}{2}\right)\right]$$

Since $\Delta U = 0$, $T_f = (T_H + T_C)/2$. ← T_f

To find the entropy production, an entropy balance reduces to give $\Delta S = \cancel{\int_1^2 \frac{\delta Q}{T}}^0 + \sigma$ or $\sigma = \Delta S$. With Eq. 6.24, the entropy change of an element of rod dz is

$$dS = dm\, c \ln \frac{T_f}{T(z)}$$
$$= (\rho A dz)\, c \ln \frac{T_f}{T(z)}$$

Accordingly

$$\sigma = \rho A c \int_0^L (\ln T_f - \ln T(z))dz = \rho A c \left[(\ln T_f)L - \int_0^L (\ln T(z))dz\right]$$

Using the given temperature distribution, the variable of integration can be changed from z to T:

$$dT = -\left(\frac{T_H - T_C}{L}\right)dz \Rightarrow dz = -\left(\frac{L}{T_H - T_C}\right)dT$$

With this, the integral can be expressed as

PROBLEM 6.15 (Contd.)

$$\int_0^L (\ln T(z))\,dz = \int_{T_H}^{T_c} (\ln T)\left(\frac{-L}{T_H-T_c}\right) dT$$

$$= \frac{L}{(T_H-T_c)} \int_{T_c}^{T_H} \ln T\, dT$$

$$= \frac{L}{(T_H-T_c)} \Big[T \ln T - T \Big]_{T_c}^{T_H}$$

$$= \frac{L}{T_H-T_c} \Big[(T_H \ln T_H - T_H) - (T_c \ln T_c - T_c) \Big]$$

$$= L \left[\frac{T_H \ln T_H}{T_H-T_c} - \frac{T_c \ln T_c}{T_H-T_c} - 1 \right]$$

Collecting results

$$\sigma = \rho A c \left[(\ln T_f) L - L \left[\frac{T_H \ln T_H}{T_H-T_c} - \frac{T_c \ln T_c}{T_H-T_c} - 1 \right] \right]$$

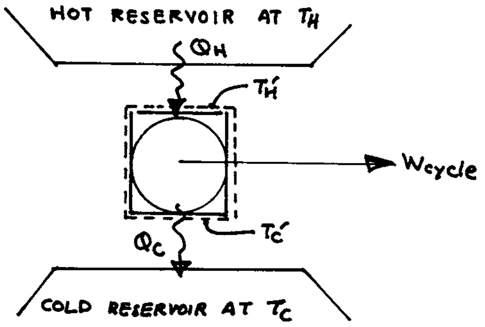
$$= mc \left[1 + \ln T_f + \frac{T_c \ln T_c}{T_H-T_c} - \frac{T_H \ln T_H}{T_H-T_c} \right] \xleftarrow{\qquad\qquad\qquad \sigma}$$

PROBLEM 6.16

KNOWN: A system undergoes a cycle while receiving Q_H at T_H' and discharging Q_C at T_C'. Q_H and Q_C are with hot and cold reservoirs at T_H and T_C, respectively.

FIND: (a) Determine an expression for W_{cycle} in terms of Q_H, T_C', T_H', and σ. (b) State the relationship of T_H' to T_H and T_C' to T_C. (c) Obtain an expression for W_{cycle} when there are (i) no internal irreversibilities, (ii) no irreversibilities.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: The system shown in the accompanying figure undergoes a power cycle while receiving Q_H at T_H' and discharging Q_C at T_C'. There are no other heat transfers.

ANALYSIS: (a) An energy balance gives

$$W_{cycle} = Q_H - Q_C \qquad (1)$$

An entropy balance gives

$$\cancelto{0}{\Delta S_{cycle}} = \frac{Q_H}{T_H'} - \frac{Q_C}{T_C'} + \sigma_{cycle} \qquad (2)$$

where σ is the amount of entropy produced within the system per cycle and $\Delta S = 0$ because the system undergoes a cycle. Solving Eq. (2) for Q_C and substituting the result into Eq. (1)

$$W_{cycle} = Q_H\left[1 - \frac{T_C'}{T_H'}\right] - T_C'\sigma_{cycle} \qquad (3) \qquad \xleftarrow{(a)}$$

(b) For heat transfer to occur from the hot reservoir to the system $T_H \geq T_H'$. For heat transfer to occur from the system to the cold reservoir $T_C' \geq T_C$. $\xleftarrow{(b)}$

(c) If there were no irreversibilities within the system during the cycle, the term σ_{cycle} in Eq. (3) would vanish leaving

$$W_{cycle} = Q_H\left[1 - \frac{T_C'}{T_H'}\right] \qquad (4) \qquad \xleftarrow{(c\text{-}i)}$$

External irreversibilities are associated with heat transfer between the reservoirs and the system. If these are also absent, $T_H' = T_H$ and $T_C' = T_C$, and Eq. (4) becomes

$$W_{cycle} = Q_H\left[1 - \frac{T_C}{T_H}\right] \qquad (5) \qquad \xleftarrow{(c\text{-}ii)}$$

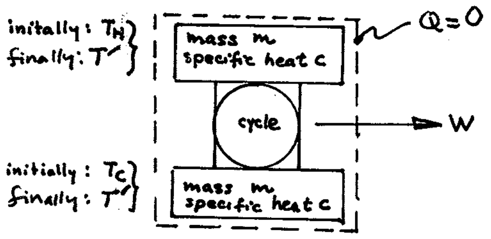
which is the maximum theoretical work that can be obtained (Sec. 5.6).

PROBLEM 6.17

KNOWN: A system undergoes a power cycle while receiving energy by heat transfer from a body initially at T_H and discharging energy by heat transfer to a similar body initially at T_C. Work is developed until the temperature of each body is T'.

FIND: Obtain expressions for the minimum theoretical final temperature T' and for the maximum theoretical work that can be developed. Also obtain an expression for the minimum theoretical work input that would be required by a refrigeration cycle to restore the two bodies to their respective initial temperatures.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the accompany figure is composed of three subsystems: An incompressible body of mass m and specific heat c initially at T_H. An incompressible body of mass m and specific heat c initially at T_C. A system that undergoes a power cycle. (2) For the overall system, $Q = 0$.

ANALYSIS: (a) An energy balance gives $\Delta U = \cancelto{0}{Q} - W \Rightarrow$

$$W = -[\Delta U]_{HOT} + \cancelto{0}{\Delta U]_{cycle}} + \Delta U]_{COLD}] = -[mc(T'-T_H) + mc(T'-T_C)]$$

or

$$W = mc[T_H + T_C - 2T'] \qquad (1)$$

An entropy balance gives $\Delta S = \cancelto{0}{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma$. Then, with Eq. 6.24

$$\sigma = [\Delta S]_{HOT} + \cancelto{0}{\Delta S]_{cycle}} + \Delta S]_{COLD}] = \left[mc \ln\frac{T'}{T_H} + mc \ln\frac{T'}{T_C}\right]$$

or

$$\sigma = mc \ln\left[\frac{(T')^2}{T_H T_C}\right]$$

Solving for T'

$$T' = [T_H T_C \exp(\sigma/mc)]^{1/2} \qquad (2)$$

From this, it can be concluded that the minimum value of T' corresponds to the case of no internal irreversibilities: $\sigma = 0$. Then

$$(T')_{min} = [T_H T_C]^{1/2} \qquad (3)$$

PROBLEM 6.17 (Continued)

(b) Substituting Eq. (2) into Eq. (1)

$$W = mc\left[T_H + T_C - 2\left[T_H T_C \exp(\sigma/mc)\right]^{1/2}\right]$$

Since $\sigma \geq 0$, it follows that W is maximum when $\sigma = 0$, giving

$$W_{MAX} = mc\left[T_H + T_C - 2(T_H T_C)^{1/2}\right] \longleftarrow$$

(c) In this part, the system shown in the schematic undergoes a refrigeration cycle that must be driven by a work input. The analysis very closely parallels that given above in parts (a), (b):

As $Q = 0$, the energy added by the work input results in an increase in the system internal energy: $-W = \Delta U$, or

$$-W = \Delta U]_{HOT} + \cancelto{0}{\Delta U]_{cycle}} + \Delta U]_{COLD} = \left[mc(T_H - T') + mc(T_C - T')\right]$$

$$\Rightarrow \quad -W \quad mc\left[T_H + T_C - 2T'\right] \qquad (4)$$

An entropy balance gives $\Delta S = \cancelto{0}{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma$. With Eq. 6.24

$$\sigma = \Delta S]_{HOT} + \cancelto{0}{\Delta S]_{cycle}} + \Delta S]_{COLD} = mc \ln \frac{T_H}{T'} + mc \ln \frac{T_C}{T'}$$

$$= mc \ln\left[\frac{T_H T_C}{(T')^2}\right]$$

Solving for T'

$$T' = \left[\frac{T_H T_C}{\exp(\sigma/mc)}\right]^{1/2} \qquad (5)$$

Substituting Eq. (5) into Eq. (4)

$$-W = mc\left[T_H + T_C - 2\left[\frac{T_H T_C}{\exp(\sigma/mc)}\right]^{1/2}\right]$$

As $\sigma \geq 0$, it follows that $(-W)$ is a minimum when $\sigma = 0$, giving

$$-W_{min} = mc\left[T_H + T_C - 2(T_H T_C)^{\frac{1}{2}}\right]$$

or

$$W_{min} = mc\left[2(T_H T_C)^{1/2} - T_H - T_C\right] \longleftarrow$$

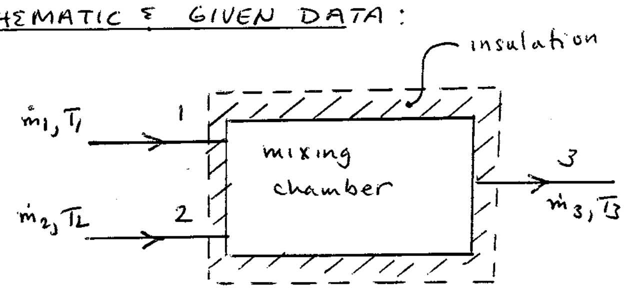
which corresponds to the result of part (b), except here a work input is evaluated.

PROBLEM 6.18

KNOWN: An insulated mixing chamber at steady state receives liquid streams of the same substance at $\dot{m}_1, T_1$ and $\dot{m}_2, T_2$. A single stream exits at $\dot{m}_3, T_3$.

FIND: (a) Evaluate T_3 in terms of $T_1, T_2, \dot{m}_1/\dot{m}_3$. (b) Evaluate $\dot{\sigma}/\dot{m}_3$ in terms of the specific heat c, T_1/T_2, $\dot{m}_1/\dot{m}_3$. (c) For fixed c and T_1/T_2, determine the value of $\dot{m}_1/\dot{m}_3$ for which $\dot{\sigma}/\dot{m}_3$ is a maximum.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and all kinetic and potential energy effects are negligible. (3) The liquid streams can be modeled as incompressible with constant specific heat c and negligible effects of pressure.

ANALYSIS: (a) At steady state, a mass balance reads $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$. An energy balance becomes, with assumptions(2), $0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$. Combining these equations gives

$$0 = \dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1) h_2 - \dot{m}_3 h_3$$

$$0 = \dot{m}_1 (h_1 - h_2) + \dot{m}_3 (h_2 - h_3)$$

① Or with assumption (3) and Eq. 3.20b

$$0 = \dot{m}_1 c [T_1 - T_2] + \dot{m}_3 c [T_2 - T_3] \Rightarrow T_3 = T_2 + \left[\frac{\dot{m}_1}{\dot{m}_3}\right](T_1 - T_2) \longleftarrow (a)$$

(b) An entropy balance reduces at steady state to give

$$0 = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma} \quad \text{or} \quad 0 = \dot{m}_1 s_1 + (\dot{m}_3 - \dot{m}_1) s_2 - \dot{m}_3 s_3 + \dot{\sigma}$$

Rearranging, and using Eq. 6.2

$$0 = \dot{m}_1 (s_1 - s_2) + \dot{m}_3 (s_2 - s_3) + \dot{\sigma} \quad \text{or} \quad 0 = \dot{m}_1 c \ln \frac{T_1}{T_2} + \dot{m}_3 c \ln \frac{T_2}{T_3} + \dot{\sigma}$$

Solving

② $$\frac{\dot{\sigma}}{\dot{m}_3} = c \left[\frac{\dot{m}_1}{\dot{m}_3} \ln \frac{T_2}{T_1} + \ln \frac{T_3}{T_2}\right]$$

Using the result of part(a) this becomes

$$\frac{\dot{\sigma}}{\dot{m}_3} = c \left[\frac{\dot{m}_1}{\dot{m}_3} \ln \frac{T_2}{T_1} + \ln\left[\frac{T_2 + \left(\frac{\dot{m}_1}{\dot{m}_3}\right)(T_1 - T_2)}{T_2}\right]\right] = c\left[\frac{\dot{m}_1}{\dot{m}_3} \ln \frac{T_2}{T_1} + \ln\left[1 + \left(\frac{\dot{m}_1}{\dot{m}_3}\right)\left(\frac{T_1}{T_2}\right) - \frac{\dot{m}_1}{\dot{m}_3}\right]\right]$$

Letting $x = \dot{m}_1/\dot{m}_3$, this can be expressed as

$$\frac{\dot{\sigma}}{\dot{m}_3} = c \ln\left[\left(\frac{T_2}{T_1}\right)^{x}\left[1 + x\left(\frac{T_1}{T_2}\right) - x\right]\right] = c \ln\left[\left(\frac{T_1}{T_2}\right)^{-x}\left[1 + x\left[\left(\frac{T_1}{T_2}\right) - 1\right]\right]\right] \longleftarrow (b)$$

(c) Taking the derivative of $\dot{\sigma}/\dot{m}_3$ with respect to x while holding c and T_2/T_1 constant,

$$\left.\frac{\partial(\dot{\sigma}/\dot{m}_3)}{\partial x}\right]_{c, T_1/T_2} = \frac{-\left(\frac{T_1}{T_2}\right)^{-x} \ln\left(\frac{T_1}{T_2}\right)\left[1 + x\left[\left(\frac{T_1}{T_2}\right) - 1\right]\right] + \left(\frac{T_1}{T_2}\right)^{-x}\left[\left(\frac{T_1}{T_2}\right) - 1\right]}{\left(\frac{T_1}{T_2}\right)^{-x}\left[1 + x\left[\left(\frac{T_1}{T_2}\right) - 1\right]\right]}$$

PROBLEM 6.18 (Contd.)

Setting this to zero

$$-\ln\left(\frac{T_1}{T_2}\right)\left[1 + x\left[\left(\frac{T_1}{T_2}\right)-1\right]\right] + \left[\left(\frac{T_1}{T_2}\right)-1\right] = 0$$

Solving

③ $$x = \frac{\ln\left(\frac{T_1}{T_2}\right) + \left[1-\left(\frac{T_1}{T_2}\right)\right]}{\ln\left(\frac{T_1}{T_2}\right)\left[1-\left(\frac{T_1}{T_2}\right)\right]} \quad \longleftarrow \quad (c)$$

1. In applying Eq. 3.20b the underlined term below has been omitted

$$h_2 - h_1 = c(T_2 - T_1) + \underline{v(p_2 - p_1)}$$

Since the specific volume of liquids are small, a substantial pressure difference (p_2-p_1) would be required before the underlined term becomes significant. Effects of pressure are neglected in assumption 3.

2 when $\dot{m}_1/\dot{m}_3 = 0$, Eq (a) gives $T_3 = T_2$, then

$$\frac{\dot{\sigma}}{\dot{m}_3} = c\left[\left(\frac{\dot{m}_1}{\dot{m}_3}\right)\ln\frac{T_2}{T_1} + \ln\frac{T_3}{T_2}\right] = 0$$

And when $\dot{m}_1/\dot{m}_3 = 1$, Eq. (a) gives $T_3 = T_1$, then

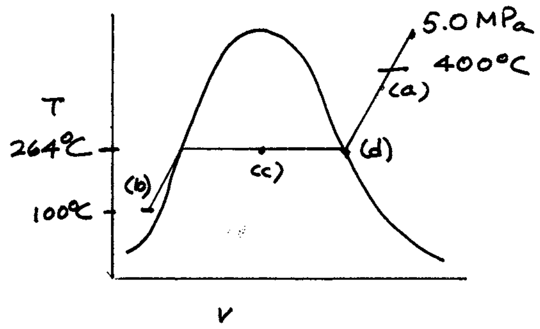
$$\frac{\dot{\sigma}}{\dot{m}_3} = c\left[\left(\frac{\dot{m}_1}{\dot{m}_3}\right)\ln\frac{T_2}{T_1} + \ln\frac{T_3}{T_2}\right] = c\left[\ln\frac{T_2}{T_1} + \ln\frac{T_1}{T_2}\right] = 0$$

In these cases there is no mixing, and thus no entropy production.

3. To verify that this locates a maximum, form the derivative $\partial^2(\dot{\sigma}/\dot{m}_3)/\partial x^2$ and evaluate the resulting expression with the expression for x here to show the second derivative is negative.

PROBLEM 6.19

FIND: Determine the specific entropy, in kJ/kg · K, of water at the following states:



(a) 5.0 MPa, 400°C. Interpolation in Table A-4: $s = 6.6549$ kJ/kg · K. $\longleftarrow$ (a)

(b) 5.0 MPa, 100°C. Table A-5: $s = 1.303$ kJ/kg · K. $\longleftarrow$ (b)

(c) 5.0 MPa, u = 1872.5 kJ/kg. Table A-3 gives $u_f = 1147.8$ kJ/kg, $u_g = 2597.1$ kJ/kg.

Thus

$$x = \frac{1872.5 - 1147.8}{2597.1 - 1147.8} = 0.5$$

Then

$$s = s_f + x(s_g - s_f) = 2.9202 + 0.5[5.9734 - 2.9202] = 4.4468 \text{ kJ/kg.K} \longleftarrow \text{(c)}$$

(d) 5.0 MPa, saturated vapor. Table A-3: $s_g = 5.9734$ kJ/kg · K $\longleftarrow$ (d)

IT results. Note at a liquid state, such as (b), IT returns entropy using Eq. 6.7. See Methodology Update — p. 244.

IT Code

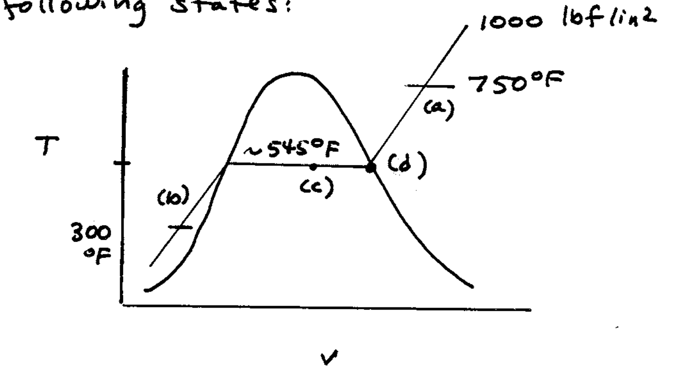
```
p = 50  // bar

Ta = 400  // °C
sa = s_PT("Water/Steam", p, Ta)

Tb = 100
sb = s_PT("Water/Steam", p, Tb)

uc = 1872.5  // kJ/kg
uc = usat_Px("Water/Steam", p, xc)
sc = ssat_Px("Water/Steam", p, xc)

xd = 1
sd = ssat_Px("Water/Steam", p, xd)
```

IT Results

```
(a)  sa = 6.645 kJ/kg·K
(b)  sb = 1.308 kJ/kg·K
(c)  xc = 0.5001
     sc = 4.447 kJ/kg·K
(d)  sd = 5.973 kJ/kg·K
```

PROBLEM 6.20

FIND: Determine the specific entropy, in Btu/lb·°R, of water at the following states:



(a) 1000 lbf/in², 750°F . Interpolation in Table A-4E : s = 1.54 Btu/lb·°R. ← (a)

(b) 1000 lbf/in², 300°F . Table A-5E : s = 0. 43552 Btu/lb·°R. ← (b)

(c) 1000 lbf/in², h = 932.4 Btu/lb . Table A-3E gives $h_f = 542.4$ Btu/lb, $h_{fg} = 650.0$, $h_g = 1192.4$ Btu/lb. Thus

$$x = \frac{932.4 - 542.4}{650} = 0.6$$

Then,

$$s = s_f + x(s_g - s_f)$$

$$= 0.7432 + 0.6(0.6471) = 1.1315 \text{ Btu/lb·°R}. \longleftarrow (c)$$

(d) 1000 lbf/in², saturated vapor. Table A-3E . $s_g = 1.3903$ Btu/lb·°R ← (d)

IT Results. Note that at liquid states, such as (b), IT returns entropy using Eq.6.7. See Methodology Update - p. 244.

IT Code

```
p = 1000  // lbf/in.²

Ta = 750  // °F
sa = s_PT("Water/Steam", p, Ta)

Tb = 300
sb = s_PT("Water/Steam", p, Tb)

hc = 932.4  // Btu/lb
hc = hsat_Px("Water/Steam", p, xc)
sc = ssat_Px("Water/Steam", p, xc)

xd = 1
sd = ssat_Px("Water/Steam", p, xd)
```

IT Results

```
(a)  sa = 1.541 Btu/lb·°R
(b)  sb = 0.4369 Btu/lb·°R
(c)  xc = 0.6001
     sc = 1.131 Btu/lb·°R
(d)  sd = 1.39 Btu/lb·°R
```

PROBLEM 6.21

FIND: Determine the change in specific entropy, in kJ/kg·K, between the specified states.

(a) Water. $p_1 = 10$ MPa, $T_1 = 400°C$, $p_2 = 10$ MPa, $T_2 = 100°C$.

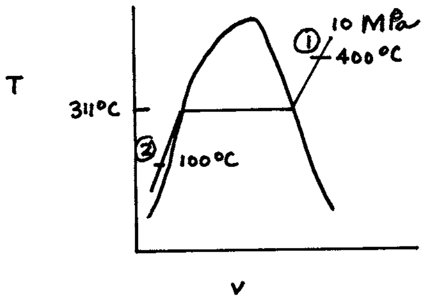


Table A-4, $s_1 = 6.2120$ kJ/kg·K

Table A-5, $s_2 = 1.2992$ kJ/kg·K

$s_2 - s_1 = 1.2992 - 6.2120$

$= -4.9128$ kJ/kg·K ⟵ (a)

(b) R134a. $h_1 = 111.44$ kJ/kg, $T_1 = -40°C$, saturated vapor at $P_2 = 5$ bar.

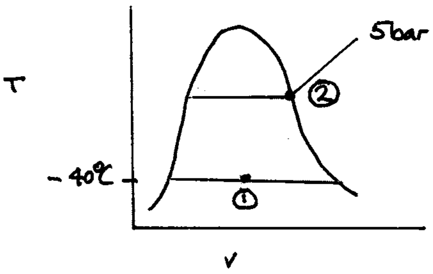


Table A-10 at −40°C

$h_f = 0$, $h_{fg} = 222.88$, $h_g = 222.88$ kJ/kg

Thus,

$$x_1 = \frac{111.44 - 0}{222.88} = 0.5$$

Then

$s_1 = s_f + x_1(s_g - s_f) = 0 + 0.5[0.9560 - 0]$

$= 0.478$ kJ/kg·K

Then, with s_2 from Table A-11

$s_2 - s_1 = 0.9117 - 0.478 = 0.4337$ kJ/kg·K ⟵ (b)

(c) Air as an ideal gas. $T_1 = 280$K (7°C), $P_1 = 2$bar, $T_2 = 600$K (327°C), $P_2 = 1$bar.

With Eq. 6.21a and $s°$ data from Table A-22

$$s_2 - s_1 = s°(T_2) - s°(T_1) - \frac{\bar{R}}{M} \ln \frac{P_2}{P_1}$$

$$= (2.40902 - 1.63279)\frac{\text{kJ}}{\text{kg·K}} - \left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg·K}}\right) \ln \frac{1}{2}$$

$$= 0.97515 \frac{\text{kJ}}{\text{kg·K}}$$ ⟵ (c)

(d) Hydrogen (H_2) as an ideal gas. $T_1 = 1000$ K (727°C), $P_1 = 1$ bar, $T_2 = 298$ K (25°C), $P_2 = 3$ bar

Table A-21 provides the following expression for $\bar{c}_p(T)$

$$\frac{\bar{c}_p}{\bar{R}} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4$$

Inserting this into Eq. 6.19, written on a molar basis, and integrating

$$\frac{\bar{s}_2 - \bar{s}_1}{\bar{R}} = \int_{T_1}^{T_2} \left(\frac{\alpha + \beta T + \gamma T^2 + \delta T^3 + \varepsilon T^4}{T}\right) dT - \ln \frac{p_2}{p_1}$$

$$= \alpha \ln \frac{T_2}{T_1} + \beta(T_2 - T_1) + \frac{\gamma}{2}(T_2^2 - T_1^2) + \frac{\delta}{3}(T_2^3 - T_1^3) + \frac{\varepsilon}{4}(T_2^4 - T_1^4) - \ln \frac{p_2}{p_1}$$

The values for the constants α, β, γ, δ, and ε for H_2 are obtained from Table A-21. Hence

PROBLEM 6.21 (Contd.)

$$\frac{\bar{s}_2-\bar{s}_1}{\bar{R}} = 3.057\ln\frac{298}{1000} + 2.677\frac{(298-1000)}{10^3} - \frac{5.810}{2}\left[\frac{\overline{298}^2-\overline{1000}^2}{10^6}\right]$$

$$+\frac{5.521}{3}\left[\frac{\overline{298}^3-\overline{1000}^3}{10^9}\right] - \frac{1.812}{4}\left[\frac{\overline{298}^4-\overline{1000}^4}{10^{12}}\right] - \ln\frac{3}{1}$$

$$\bar{s}_2-\bar{s}_1 = 8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\left[-3.701 - 1.8793 + 2.647 - 1.7916 + 0.4494 - 1.0986\right]$$

$$= -44.68\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$$

With the molecular weight for H_2 from Table A-1, $M = 2.016$

$$s_2-s_1 = \frac{\bar{s}_2-\bar{s}_1}{M} = -\frac{44.68\ \text{kJ/kmol}\cdot\text{K}}{2.016\ \text{kg/kmol}} = -22.163\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

IT SOLUTION:

IT Code

```
Ta1 = 400 // °C
pa1 = 100  // bar
Ta2 = 100
pa2 = 100
sa1 = s_PT("Water/Steam", pa1, Ta1)
sa2 = s_PT("Water/Steam", pa2, Ta2)
dels_a = sa2 - sa1

hb1 = 111.44  // kJ/kg
Tb1 = -40
xb2 = 1
pb2 = 5
pb1 = Psat_T("R134A", Tb1)
hb1 = hsat_Px("R134A", pb1, xb1)
sb1 = ssat_Px("R134A", pb1, xb1)
sb2 = ssat_Px("R134A", pb2, xb2)
dels_b = sb2 - sb1

Tc1 = 7
pc1 = 2
Tc2 = 327
pc2 = 1
dels_c = s_TP("Air", Tc2, pc2) -  s_TP("Air", Tc1, pc1)

Td1 = 727
pd1 = 1
Td2 = 25
pd2 = 3
dels_d = s_TP("H2", Td2, pd2) -  s_TP("H2", Td1, pd1)
```

IT Results

(a) Δs = -4.903 kJ/kg·K
(b) Δs = 0.4338 kJ/kg·K
(c) Δs = 0.9749 kJ/kg·K
(d) Δs = -22.16 kJ/kg·K

PROBLEM 6.22

FIND: Determine the change in specific entropy, in Btu/lb·°R.

(a) Water. $p_1 = 1000$ lbf/in², $T_1 = 800$°F, $p_2 = 1000$ lbf/in², $T_2 = 100$°F.

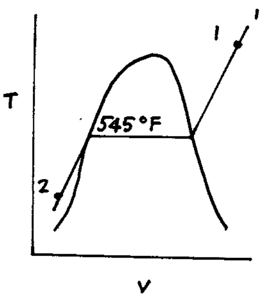


Table A-4E. $s_1 = 1.5665$ Btu/lb·°R

Table A-5E. $s_2 = 0.12901$ Btu/lb·°R

$s_2 - s_1 = -1.43749$ Btu/lb·°R ⟵ (a)

(b) R-134a. $h_1 = 47.91$ Btu/lb, $T_1 = -40$°F, saturated vapor at $p_2 = 40$ lbf/in².

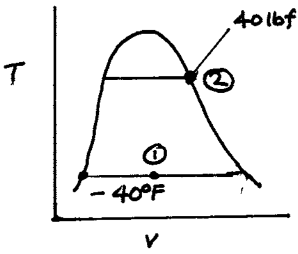


Table A-10E at -40°F: $h_f = 0$, $h_{fg} = 95.82$ Btu/lb

$$\therefore \; x_1 = \frac{49.91 - 0}{95.82} = 0.5$$

And $s_1 = s_f + x_1(s_g - s_f) = 0 + 0.5(0.2283 - 0) = 0.11415$ Btu/lb·°R.

Table A-11E, $s_2 = 0.2197$ Btu/lb·°R. Then

$s_2 - s_1 = 0.2197 - 0.11415 = 0.1056$ Btu/lb·°R ⟵ (b)

(c) Air as an ideal gas. $T_1 = 40$°F, $p_1 = 2$ atm, $T_2 = 420$°F, $p_2 = 1$ atm.
With Eq. 6.25a and $s°$ data from Table A-22E at 500°R and 880°R

$$s_2 - s_1 = s°(T_2) - s°(T_1) - \frac{\bar{R}}{M} \ln \frac{p_2}{p_1}$$

$$= (0.71886 - 0.58233)\left(\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) - \left(\frac{1.986}{28.97} \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) \ln \frac{1}{2}$$

$= 0.18405$ Btu/lb·°R ⟵ (c)

(d) Carbon dioxide as an ideal gas. $T_1 = 820$°F, $p_1 = 1$ atm, $T_2 = 77$°F, $p_2 = 3$ atm.
With Eq. 6.25b and $\bar{s}°$ data from Table A-23E at 1280°R and 537°R

$$\bar{s}_2 - \bar{s}_1 = \bar{s}°(T_2) - \bar{s}°(T_1) - \bar{R} \ln \frac{p_2}{p_1}$$

$$= (51.032 - 60.044)\left(\frac{\text{Btu}}{\text{lbmol}\cdot°\text{R}}\right) - \left(1.986 \frac{\text{Btu}}{\text{lbmol}\cdot°\text{R}}\right) \ln \frac{3}{1}$$

$$= -11.1938 \frac{\text{Btu}}{\text{lbmol}\cdot°\text{R}}$$

Then, with the molecular weight of CO_2 from Table A-1E

$$s_2 - s_1 = \frac{\bar{s}_2 - \bar{s}_1}{M}$$

$$= \frac{-11.1938 \frac{\text{Btu}}{\text{lbmol}\cdot°\text{R}}}{44.01 \frac{\text{lb}}{\text{lbmol}}}$$

$= -0.25435$ Btu/lb·°R ⟵ (d)

PROBLEM 6.22 (Contd). IT Solution ...

IT Code

```
 p = 1000  // lbf/in.²
Ta1 = 800  // °F
Ta2 = 100
sa1 = s_PT("Water/Steam", p, Ta1)
sa2 = s_PT("Water/Steam", p, Ta2)
dels_a = sa2 - sa1

hb1 = 47.91  // Btu/lb
Tb1 = -40
xb1 = 1
pb2 = 40
pb1 = Psat_T("R134A", Tb1)
hb1 = hsat_Px("R134A", pb1, xb1)
sb1 = ssat_Px("R134A", pb1, xb1)
sb2 = ssa_Px("R134A", pb2, xb2)
dels_b = sb2 - sb1

Tc1 = 40
pc1 = 2 * 14.696 // lbf/in.²
Tc2 = 420
pc2 = 1 * 14.696
sc1 = s_TP("Air", Tc1, pc1)
sc2 = s_TP("Air", Tc2, pc2)
dels_c = sc2 - sc1
```

```
Td1 = 820
pd1 = 1 * 14.696
Td2 = 77
pd2 = 3 * 14.696
sd1 = s_TP("CO2", Td1, pd1)
sd2 = s_TP("CO2", Td2, pd2)
dels_d = sd2 - sd1
```

IT Results

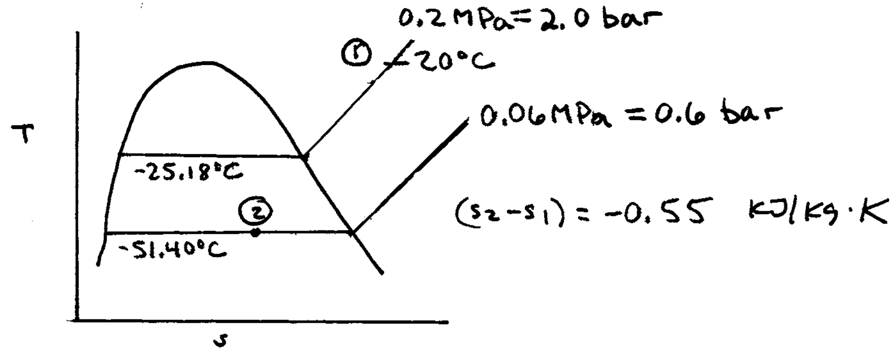
(a) Δs = -1.437 Btu/lb·°R
(b) Δs = 0.1056 Btu/lb·°R
(c) Δs = 0.184 Btu/lb·°R
(d) Δs = -0.2545 Btu/lb·°R

PROBLEM 6.23*

KNOWN: Refrigerant 22 undergoes a process between two specified states.

FIND: Determine the temperature at the final state, in °C, and the final specific enthalpy, in kJ/kg.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system is 1 kg of R-22

ANALYSIS: From Table A-9, $s_1 = 1.0786$ kJ/kg·K. Then

$$s_2 = s_1 + (s_2 - s_1) = 1.0786 + (-0.55) = 0.5286 \text{ kJ/kg·K}.$$

Since $s_f < s_2 < s_g$ at 0.6 bar, Table A-9, state 2 is in the two-phase liquid-vapor regime. Then, $T_2 = -51.40°C$ and $\xleftarrow{T_2}$

$$x_2 = \frac{0.5286 - (-0.0542)}{1.0294 - (-0.0542)} = 0.5378$$

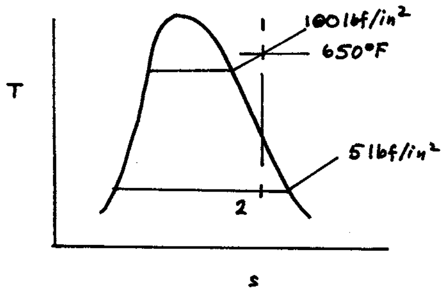
$$\Rightarrow \quad h_2 = h_f + x_2(h_{fg}) = -12.35 + (0.5378)(240.28) = 116.88 \text{ kJ/kg} \quad \xleftarrow{h_2}$$

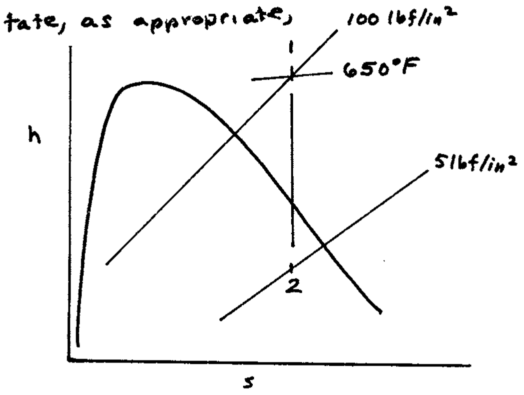
PROBLEM 6.24

KNOWN: One lb of water undergoes a process with no change in specific entropy between two specified states.

FIND: Determine x or T at the final state, as appropriate, using alternative methods.

SCHEMATIC & GIVEN DATA:





ASSUMPTION: The one lb of water is the system.

ANALYSIS: (a) From Table A-4E, $s_1 = 1.78075$ Btu/lb·°R. Then, with s_f an s_g at 5 lbf/in² from Table A-3E

$$x_2 = \frac{s_2 - s_f}{s_g - s_f} = \frac{1.78075 - 0.2349}{1.8441 - 0.2349} = 0.961$$

(b) By inspection of Figure A-8E at $p_1 = 100$ lbf/in², $T_1 = 650°F$, $s_1 = 1.781$ Btu/lb·°R. Then, with $s_2 = s_1$ and $p_2 = 5$ lbf/in², $x_2 = 0.96$,

(c) *IT* Code

```
T1 = 650  // °F
p1 = 100  // lbf/in.²
p2 = 5
s1 = s_PT("Water/Steam", p1, T1)
s2 = ssat_Px("Water/Steam", p2, x2)
s2 = s1
```

IT Results

$s_1 = s_2 = 1.781$ Btu/lb·°R
$x_2 = 0.9609$

PROBLEM 6.25

FIND: Determine the change in specific entropy, in kJ/kg·K, using the ideal gas model and three alternative approaches.

	Ideal Gas Table	Constant Specific Heat
① case	$\Delta s = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{P_2}{P_1}$ (6.21a)	$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$ (6.23)
(a) Air $P_1 = 100\text{ kPa}$ $P_2 = 100\text{ kPa}$ $T_1 = 293\text{ K}$ $T_2 = 373\text{ K}$	With s° data from Table A-22 $\Delta s = 1.92119 - 1.678298 - 0$ $= 0.24289$ kJ/kg·K	With Cp at 333 K from Table A-20 $\Delta s = 1.007 \ln \frac{373}{293} - 0$ $= 0.2431$ kJ/kg·K ← (a)
(b) Air $P_1 = 1\text{ bar}$ $P_2 = 3\text{ bar}$ $T_1 = 300\text{K}$ $T_2 = 650\text{K}$	With s° data from Table A-22 $\Delta s = 2.49364 - 1.70203 - \frac{8.314}{28.97} \ln \frac{3}{1}$ $= 0.47632$ kJ/kg·K	With Cp at 475K from Table A-20 $\Delta s = 1.0245 \ln \frac{650}{300} - \frac{8.314}{28.97} \ln \frac{3}{1}$ $= 0.47684$ kJ/kg·K ← (b)
(c) CO_2 $P_1 = 150\text{kPa}$ $P_2 = 300\text{kPa}$ $T_1 = 303\text{K}$ $T_2 = 573\text{K}$	With $\bar{s}^\circ$ data from Table A-23 and M from Table A-1 $\Delta s = \frac{241.033 - 214.284 - 8.314 \ln \frac{300}{150}}{44.01}$ $= 0.4769$ kJ/kg·K	With cp at 438K from Table A-20 $\Delta s = .9686 \ln \frac{573}{303} - \frac{8.314}{44.01} \ln \frac{300}{150}$ $= 0.4862$ kJ/kg·K ← (c)
(d) CO $T_1 = 300\text{K}$ $T_2 = 500\text{K}$ $v_1 = 1.1\text{m}^3/\text{kg}$ $v_2 = 0.75\text{m}^3/\text{kg}$	With $pv = RT$ $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{v_1}{v_2}\right)$ $= \left(\frac{500}{300}\right)\left(\frac{1.10}{0.75}\right) = 2.444$ With $\bar{s}^\circ$ data from Table A-23 and M from Table A-1 $\Delta s = \frac{212.719 - 197.723 - 8.314 \ln 2.444}{28.01}$ $= 0.2701$ kJ/kg·K	With cp at 400K from Table A-20 $\Delta s = 1.047 \ln \frac{500}{300} - \frac{8.314}{28.01} \ln 2.444$ $= 0.2696$ kJ/kg·K ← (d)
(e) N_2 $P_1 = 2\text{ MPa}$ $P_2 = 1\text{ MPa}$ $T_1 = 800\text{K}$ $T_2 = 300\text{K}$	With $\bar{s}^\circ$ data from Table A-23 and M from Table A-1 $\Delta s = \frac{191.682 - 220.907 - 8.314 \ln \frac{1}{2}}{28.02}$ $= -0.8373$ kJ/kg·K	With Cp at 550K from Table A-20 $\Delta s = 1.065 \ln \frac{300}{800} - \frac{8.314}{28.02} \ln \frac{1}{2}$ $= -0.8389$ kJ/kg·K ← (e)

IT Results

(a) Δs = 0.2429 kJ/kg·K
(b) Δs = 0.4762 kJ/kg·K
(c) Δs = 0.4764 kJ/kg·K
(d) Δs = 0.2702 kJ/kg·K
(e) Δs = -0.838 kJ/kg·K

---

1. The constant Cp value is evaluated from Table A-20 at the average temperature. Alternatively, the arithmetic average of the Cp values at the two end temperatures can be used.

PROBLEM 6.25 (Contd.)

IT Code

```
pa1 = 1  // bar
pa2 = 1
Ta1 = 293  // K
Ta2 = 373
sa1 = s_TP("Air", Ta1, pa1)
sa2 = s_TP("Air", Ta2, pa2)
dels_a = sa2 - sa1

pb1 = 1
pb2 = 3
Tb1 = 300
Tb2 = 650
sb1 = s_TP("Air", Tb1, pb1)
sb2 = s_TP("Air", Tb2, pb2)
dels_b = sb2 - sb1

pc1 = 1.5
pc2 = 3
Tc1 = 303
Tc2 = 573
sc1 = s_TP("CO2", Tc1, pc1)
sc2 = s_TP("CO2", Tc2, pc2)
dels_c = sc2 - sc1

Td1 = 300
Td2 = 500
vd1 = 1.1 // m³/kg
vd2 = 0.75
vd1 = v_TP("CO",Td1,pd1)
vd2 = v_TP("CO",Td2,pd2)
sd1 = s_TP("CO", Td1, pd1)
sd2 = s_TP("CO", Td2, pd2)
dels_d = sd2 - sd1

pe1 = 20
pe2 = 10
Te1 = 800
Te2 = 300
se1 = s_TP("N2", Te1, pe1)
se2 = s_TP("N2", Te2, pe2)
dels_e = se2 - se1
```

PROBLEM 6.26

FIND: Determine the change in specific entropy, in Btu/lbmol·°R, using the ideal gas model and three alternative approaches.

①

case	Ideal Gas Table	Constant Specific Heat
	$\Delta\bar{s} = \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln \frac{p_2}{p_1}$ (6.21b)	$\Delta s = \bar{c}_p \ln \frac{T_2}{T_1} - \bar{R} \ln \frac{p_2}{p_1}$ (6.23)
(a) Air $p_1 = 1$ atm $p_2 = 1$ atm $T_1 = 500°R$ $T_2 = 860°R$	With s° data from Table A-22E and M = 28.97 lb/lbmol $\Delta\bar{s} = 28.97(0.71323 - 0.58233) - \cancelto{0}{1.986 \ln 1}$ $= 3.7922$ Btu/lbmol·°R	With c_p at 220°F from Table A-20E and M = 28.97 lb/lbmol $\Delta\bar{s} = (28.97)(0.241) \ln \frac{860}{500} - \cancelto{0}{1.986 \ln 1}$ ← (a) $= 3.7864$ Btu/lbmol·°R
(b) Air $p_1 = 20$ lbf/in² $p_2 = 60$ lbf/in² $T_1 = 560°R$ $T_2 = 760°R$	with s° data from Table A-22E and M = 28.97 lb/lbmol $\Delta\bar{s} = 28.97(.68312 - .6095) - 1.986 \ln \frac{60}{20}$ $= -0.0491$ Btu/lbmol·°R	With c_p at 200°F from Table A-20E and M = 28.97 lb/lbmol $\Delta\bar{s} = (28.97)(0.241) \ln \frac{760}{560} - 1.986 \ln \frac{60}{20}$ ← (b) $= -0.0498$ Btu/lbmol·°R
(c) CO_2 $p_1 = 1$atm $p_2 = 3$atm $T_1 = 500°R$ $T_2 = 960°R$	With $\bar{s}^\circ$ data from Table A-23E $\Delta\bar{s} = 56.765 - 50.408 - 1.986 \ln \frac{3}{1}$ $= 4.1752$ Btu/lbmol·°R	With c_p at 270°F from Table A-20E and M = 44.01 lb/lbmol from Table A-1E $\Delta\bar{s} = (44.01)(0.225) \ln \frac{960}{500} - 1.986 \ln \frac{3}{1}$ ← (c) $= 4.2776$ Btu/lbmol·°R
(d) CO_2 $T_1 = 660°R$ $T_2 = 860°R$ $v_1 = 20$ ft³/lb $v_2 = 15$ ft³/lb	With pv = RT $\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)\left(\frac{v_1}{v_2}\right)$ $= \left(\frac{860}{660}\right)\left(\frac{20}{15}\right) = 1.7374$ With $\bar{s}^\circ$ data from Table A-23E $\Delta\bar{s} = 55.389 - 52.934 - 1.986 \ln 1.7374$ $= 1.558$ Btu/lbmol·°R	With c_p at 300°F from Table A-20E and M = 44.01 lb/lbmol from Table A-1E $\Delta\bar{s} = (44.01)(0.229) \ln \frac{860}{660} - 1.986 \ln 1.7374$ ← (d) $= 1.571$ Btu/lbmol·°R
(e) N_2 $p_1 = 2$atm $p_2 = 1$atm $T_1 = 1260°R$ $T_2 = 660°R$	With $\bar{s}^\circ$ data from Table A-23E $\Delta\bar{s} = 47.178 - 51.771 - 1.986 \ln \frac{1}{2}$ $= -3.2164$ Btu/lbmol·°R	With c_p at 500°F from Table A-20E and M = 28.02 lb/lbmol from Table A-1E $\Delta\bar{s} = (28.02)(0.254) \ln \frac{660}{1260} - 1.986 \ln \frac{1}{2}$ ← (e) $= -3.2255$ Btu/lbmol·°R

IT Results

(a) Δs = 3.79 Btu/lbmol·°R
(b) Δs = -0.04884 Btu/lbmol·°R
(c) Δs = 4.176 Btu/lbmol·°R
(d) Δs = 1.557 Btu/lbmol·°R
(e) Δs = -3.217 Btu/lbmol·°R

---

1. The constant c_p value is evaluated from Table A-20 at the average temperature. Alternatively, the arithmetic average of the c_p values at the two end states can be used.

PROBLEM 6.26 (Contd.)

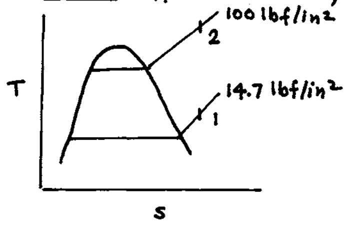
IT Code

```
pa1 = 1  // atm
pa2 = 1
Ta1 = 500  // °R
Ta2 = 860
sa1 = s_TP("Air", Ta1, pa1)
sa2 = s_TP("Air", Ta2, pa2)
dels_a = sa2 - sa1

pb1 = 20 / 14.696  // atm
pb2 = 60 / 14.696
Tb1 = 560
Tb2 = 760
sb1 = s_TP("Air", Tb1, pb1)
sb2 = s_TP("Air", Tb2, pb2)
dels_b = sb2 - sb1

pc1 = 1
pc2 = 3
Tc1 = 500
Tc2 = 960
sc1 = s_TP("CO2", Tc1, pc1)
sc2 = s_TP("CO2", Tc2, pc2)
dels_c = sc2 - sc1

Td1 = 660
Td2 = 860
vd1 = 20 * 44.01  // ft³/lbmol
vd2 = 15 * 44.01
vd1 = v_TP("CO2",Td1,pd1)
vd2 = v_TP("CO2",Td2,pd2)
sd1 = s_TP("CO2", Td1, pd1)
sd2 = s_TP("CO2", Td2, pd2)
dels_d = sd2 - sd1

pe1 = 2
pe2 = 1
Te1 = 1260
Te2 = 660
se1 = s_TP("N2", Te1, pe1)
se2 = s_TP("N2", Te2, pe2)
dels_e = se2 - se1
```

PROBLEM 6.27

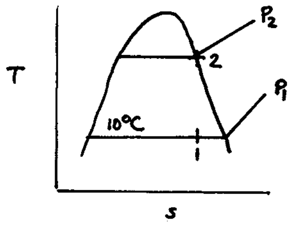
FIND: Determine the indicated property for a process where $s_2 = s_1$.

(a) Water. $P_1 = 14.7$ lbf/in², $T_1 = 500°F$, $P_2 = 100$ lbf/in². Find T_2.



From Table A-4E, $s_1 = 1.9263$ Btu/lb·°R.
Interpolating with $s_2 = s_1$ in Table A-4E at 100 lbf/in²,
$T_2 = 1017$ °F. ← (a)

(b) Water. $T_1 = 10°C$, $x_1 = 0.75$. Saturated vapor at state 2. Find P_2.



Using data from Table A-2

$$s_1 = s_f + x_1(s_g - s_f) = 0.1510 + 0.75(8.9008 - 0.1510)$$
$$= 6.71335 \text{ kJ/kg}\cdot\text{K}$$

Then, interpolating with $s_2 = s_1$ in Table A-3, $P_2 = 6.897$ bar. ← (b)

(c) Air as an ideal gas. $T_1 = 300$K (27°C), $P_1 = 1.5$ bar, $T_2 = 400$K (127°C). Find P_2.

Since $s_2 = s_1$, Eq. 6.25a gives

$$\ln \frac{P_2}{P_1} = \frac{s°(T_2) - s°(T_1)}{R}$$

with s° data from Table A-22

$$\ln \frac{P_2}{P_1} = \frac{1.99194 - 1.70203}{(8.314/28.97)}$$

$$\Rightarrow \quad \frac{P_2}{P_1} = 2.746 \quad \Rightarrow \quad P_2 = 4.119 \text{ bar}$$ ← (c)

(d) Air as an ideal gas. $T_1 = 560°R$ (100°F), $P_1 = 3$ atm, $P_2 = 2$ atm. Find T_2.

Since $s_2 = s_1$, Eq. 6.21a gives

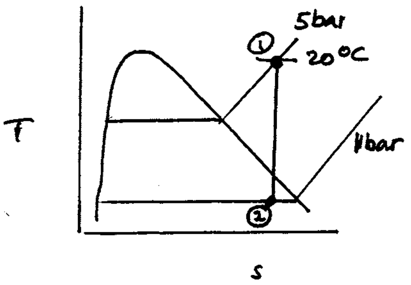
$$s°(T_2) = s°(T_1) + R \ln \frac{P_2}{P_1}$$

With s° at T_1 from Table A-22E

$$s°(T_2) = 0.60950 + \frac{1.986}{28.97} \ln \frac{2}{3} = 0.5817 \text{ Btu/lb}\cdot°\text{R}.$$

Interpolating with $s°(T_2)$ in Table A-22E, $T_2 = 498.7°R$ (39°F). ← (d)

(e) R134a. $T_1 = 20°C$, $P_1 = 5$bar, $P_2 = 1$ bar. Find v_2



From Table A-12, $s_1 = 0.9264$ kJ/kg·K.
Then, with data from Table A-11 and $s_2 = s_1$

$$x_2 = \frac{0.9264 - 0.0678}{0.9395 - 0.0678} = 0.985$$

And

$$v_2 = v_f + x_2(v_g - v_f)$$
$$= \frac{0.7258}{103} + 0.985\left[0.1917 - \frac{0.7258}{103}\right]$$
$$= 0.188 \text{ m}^3\text{/kg}$$ ← (e)

PROBLEM 6.27 (Contd.)

IT Code

```
/* (a)  Water
p1 = 14.7  // lbf/in.²
T1 = 500  // °F
p2 = 100
s1 = s_PT("Water/Steam", p1, T1)
s2 = s_PT("Water/Steam", p2, T2)
s2 = s1

(b)  Water
T1 = 10  // °C
x1 = 0.75
x2 = 1
p1 = Psat_T("Water/Steam", T1)
s1 = ssat_Px("Water/Steam", p1, x1)
s2 = ssat_Px("Water/Steam", p2, x2)
s2 = s1

(c)  Air
T1 = 27  // °C
p1 = 1.5  // bar
T2 = 127
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
s2 = s1

(d)  Air
T1 = 100  // °F
p1 = 3  // atm
p2 = 2
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
s2 = s1

(e)  R-134a
*/
T1 = 20  // °C
p1 = 5  // bar
p2 = 1
s1 = s_PT("R134A", p1, T1)
x2 = x_sP("R134A", s2, p2)
v2 = vsat_Px("R134A", p2, x2)
s2 = s1
```

IT Results

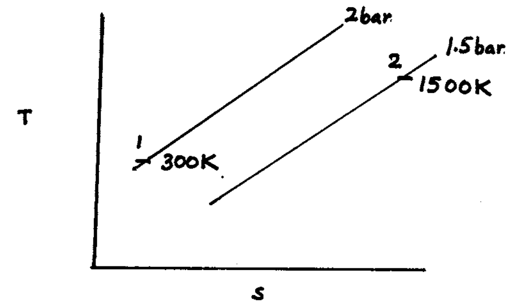
(a) s = 1.926Btu/lb·°R; T_2 = 1016°F
(b) s = 6.712 kJ/kg·K; p_2 = 6.903 bar
(c) p_2 = 4.117 bar
(d) T_2 = 38.75°F
(e) s = 0.9264 kJ/kg·K; x_2 = 0.985, v_2 = 0.1888 m³/kg

PROBLEM 6.28

KNOWN: One kg of O_2 undergoes a process between specified states.

FIND: Using three methods, determine the change in specific entropy: (a) Eq. 6.19 with $\bar{c}_p$ from Table A-21, (b) Eq. 6.21b with $\bar{s}°$ from Table A-23 (c) Eq. 6.23 with c_p at 900 K from Table A-20, (b) using IT.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system consists of one kg of O_2, which behaves as an ideal gas.

(a) Using Eq. 6.19

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p(T)}{T} dT - R \ln \frac{p_2}{p_1}$$

Table A-21 gives

$$\frac{\bar{c}_p}{\bar{R}} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4$$

where for O_2

$$\alpha = 3.626,\ \beta = -1.878 \times 10^{-3},\ \gamma = 7.056 \times 10^{-6},\ \delta = -6.764 \times 10^{-9},\ \epsilon = 2.156 \times 10^{-12}$$

Thus

$$\int_{T_1}^{T_2} \frac{c_p(T)}{T} dT = \frac{\bar{R}}{M} \int_{T_1}^{T_2} \left(\frac{\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4}{T}\right) dT = \frac{\bar{R}}{M}\left(\alpha \ln T + \beta T + \frac{\gamma T^2}{2} + \frac{\delta T^3}{3} + \frac{\epsilon T^4}{4}\right)\Bigg|_{T_1}^{T_2}$$

$$= \frac{8.314}{32}\left[3.626 \ln \frac{1500}{300} - \frac{1.878}{10^3}(1500-300) + \frac{7.056}{2(10^6)}(\overline{1500}^2 - \overline{300}^2) - \frac{6.764}{3(10^9)}(\overline{1500}^3 - \overline{300}^3) + \frac{2.156}{4(10^{12})}(\overline{1500}^4 - \overline{300}^4)\right]$$

$$= 1.657 \text{ kJ/kg}\cdot\text{K}$$

Finally

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p}{T} dT - R \ln \frac{p_2}{p_1} = 1.657 - \frac{8.314}{32} \ln \frac{1.5}{2} = 1.7317 \text{ kJ/kg}\cdot\text{K} \longleftarrow \text{(a)}$$

(b) Using Eq. 6.21b with $\bar{s}°$ from Table A-23

$$s_2 - s_1 = \frac{\bar{s}_2 - \bar{s}_1}{M} = \frac{\bar{s}°(T_2) - \bar{s}°(T_1) - \bar{R} \ln p_2/p_1}{M}$$

$$= \frac{257.965 - 205.213 - 8.314 \ln 1.5/2}{32} = 1.7235 \text{ kJ/kg}\cdot\text{K} \longleftarrow \text{(b)}$$

(c) Using Eq. 6.23 with c_p at 900 K from Table A-20

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$= 1.074 \ln \frac{1500}{300} - \frac{8.314}{32} \ln \frac{1.5}{2} = 1.8033 \text{ kJ/kg}\cdot\text{K} \longleftarrow \text{(c)}$$

(d)

IT Code

```
T1 = 300 // K                s1 = s_TP("O2", T1, p1)
p1 = 2  // bar               s2 = s_TP("O2", T2, p2)
T2 = 1500  // K              dels = s2 - s1
p2 = 1.5  // bar
```

IT Result

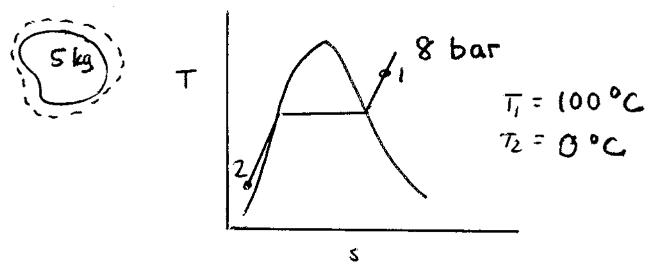
$\Delta s = 1.724$ kJ/kg·K

PROBLEM 6.29*

KNOWN: Five kilograms of ammonia undergo a specified process.

FIND: Determine the entropy change if the process is (a) irreversible, (b) internally reversible.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system consists of 5 kg of water.

ANALYSIS: Since entropy is a property, the change in entropy between the two states is the same regardless of the nature of the process.

s_1, Table A-15: $5.7545 \frac{kJ}{kg \cdot K}$

s_2, Table A-13: $0.7097 \frac{kJ}{kg \cdot K}$

Thus

$$\Delta s = s_2 - s_1 = (0.7097 - 5.7545)\frac{kJ}{kg \cdot K} = -5.0448 \frac{kJ}{kg \cdot K}$$

and for five kg

$$\Delta S = (5\,kg)\left(-5.0448 \frac{kJ}{kg \cdot K}\right) = -25.224 \frac{kJ}{K} \qquad \longleftarrow$$

PROBLEM 6.30

KNOWN: Water undergoes a process from a liquid state at 80°C, 5 MPa to sat. liq. at 40°C

FIND: Determine the change in specific entropy using four alternative methods.

ANALYSIS: (a) Tables A-2 and A-5. $s_1 = 1.0720$ kJ/kg·K, $s_2 = 0.5725$, $\Delta s = -0.4995$ kJ/kg·K.

(b) Sat. Liq. data from Table A-2. $s_1 \approx s_f(T_1) = 1.0753$ kJ/kg·K, s_2 (as above), $\Delta s = -0.5028$ kJ/kg·K.

(c) Incompressible liquid model with $c = 4.18 \frac{kJ}{kg \cdot K}$ from Table A-19 (at T_{ave}): with Eq. 6.24

$$\Delta s = c \ln \frac{T_2}{T_1} = 4.18 \frac{kJ}{kg \cdot K} \ln \frac{313}{353} = -0.5027 \text{ kJ/kg·K}$$

① (d) *IT* Code

```
T1 = 80  // °C
p1 = 50  // bar  = 5 MPa
T2 = 40  // °C
x2 = 0

s1 = s_PT("Water/Steam", p1, T1)
p2 = Psat_T("Water/Steam", T2)
s2 = ssat_Px("Water/Steam", p2, x2)
dels = s2 - s1
```

IT Results

s_1 = 1.077
s_2 = 0.5711
Δs = -0.506

---

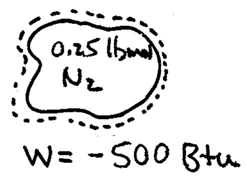
1. As noted in the Methodology Update on p. 244, IT returns liquid entropy data using Eq. 6.7.

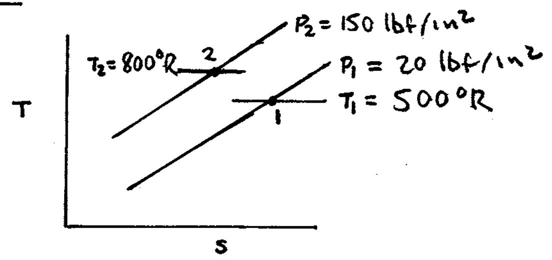
PROBLEM 6.31*

KNOWN: 0.25 lbmol of N_2 undergoes a process between known states.

FIND: Determine (a) the heat transfer and (b) the change in entropy.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system consists of the CO which behaves as an ideal gas. (2) There are no changes in kinetic and potential energy between the two end states.

ANALYSIS: (a) With assumption 2, an energy balance gives

$$Q = W + \Delta U$$
$$= W + n[\bar{u}_2 - \bar{u}_1]$$

With $\bar{u}$ data from Table A-23 E

$$Q = -500\ \text{Btu} + (0.25\ \text{lbmol})\{(3975.7 - 2479.3)\ \text{Btu/lbmol}\} = -125.9\ \text{Btu} \longleftarrow Q$$

(b) With Eq. 6.21b and $\bar{s}^\circ$ data from Table A-23 E

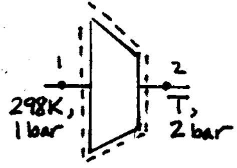
$$\Delta S = n[\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R}\ln(P_2/P_1)] = 0.25[48.522 - 45.246 - 1.986\ln(150/20)]$$
$$= -0.1814\ \text{Btu/°R} \longleftarrow \Delta S$$

PROBLEM 6.32

KNOWN: CH_4 is compressed from 298K, 1 bar to temperature T, 2 bar in a process for which $s_2 = s_1$

FIND: Determine T.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: CH_4 is modeled as an ideal gas.

① **ANALYSIS** Using IT

IT code

```
T1 = 298  // K
p1 = 1  // bar
p2 = 2  // bar

s1 = s_TP("CH4", T1, p1)
s2 = s_TP("CH4", T2, p2)
s2 = s1
```

IT Result

T_2 = 348.7 K $\longleftarrow$

---

1. Alternatively, using Eq. 6.19 with $\bar{c}_p(T)$ from Table A-21

$$\bar{s}_2 - \bar{s}_1 = \int_{T_1}^{T_2}\left(\frac{\bar{c}_p}{T}\right)dT - \bar{R}\ln\left(\frac{P_2}{P_1}\right)$$

$$0 = \cancel{\bar{R}}\left[\alpha\ln T + \beta T + \frac{\gamma}{2}T^2 + \frac{\delta}{3}T^3 + \frac{\epsilon}{4}T^4\right]_{298}^{T} - \cancel{\bar{R}}\ln\left(\frac{P_2}{P_1}\right)$$

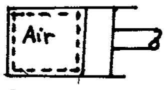
Using an equation solver, such as IT, this equation can be solved for T.

PROBLEM 6.33

KNOWN: A quantity of air undergoes a thermodynamic cycle consisting of three processes.

FIND: Evaluate the change in entropy for each process and sketch the cycle on p-v coordinates.

SCHEMATIC & GIVEN DATA:

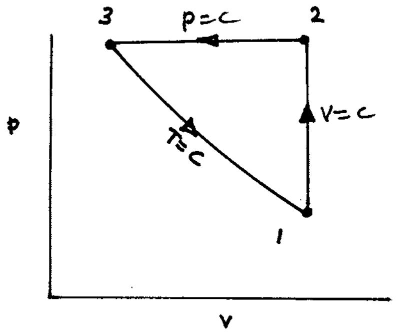


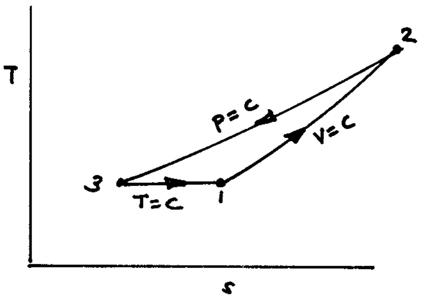
$p_1 = 0.1\ \text{MPa}$

$V_1 = 0.02\ m^3$

$p_2 = 0.42\ \text{MPa}$

$m = 2.42 \times 10^{-2}\ \text{kg}$





ASSUMPTIONS: (1) As shown in the accompanying figure, the system consists of the quantity of air. (2) The air behaves as an ideal gas with $c_p = 1\ \text{kJ/kg}\cdot\text{K}$.

ANALYSIS: The entropy changes can be evaluated using Eqs. 6.22 and 6.23. First, some preliminary results are calculated. Using the ideal gas equation of state

$$T_1 = \frac{p_1 V_1}{mR} = \frac{(0.1 \times 10^6\ N/m^2)(0.02 m^3)}{(2.42 \times 10^{-2}\ kg)\left(\frac{8314}{28.97}\ \frac{N\cdot m}{kg\,K}\right)} = 288\ K$$

Also, since $V_1 = V_2$

$$\left.\begin{array}{l} p_1 V = m R T_1 \\ p_2 V = m R T_2 \end{array}\right\} \Rightarrow \quad T_2 = \frac{p_2}{p_1} T_1 = \left(\frac{0.42}{0.10}\right)(288) = 1210\ K$$

With Eq. 3.44

$$c_v = c_p - R = 1\ \frac{kJ}{kg\cdot K} - \frac{8.314}{28.97}\ \frac{kJ}{kg\cdot K} = 0.713\ kJ/kg\cdot K$$

Process 1-2: With $V_2 = V_1$, Eq. 6.22 reduces so

$$S_2 - S_1 = m c_v \ln \frac{T_2}{T_1} = (0.024 kg)\left(0.713\ \frac{kJ}{kg\cdot K}\right) \ln \frac{1210}{288}$$

$$= 0.0246\ kJ/K \longleftarrow S_2 - S_1$$

Process 2-3: With $p_1 = p_2$, Eq. 6.23 reduces so

$$S_3 - S_2 = m c_p \ln \frac{T_3}{T_2} = m c_p \ln \frac{T_1}{T_2}$$

$$= (0.024 kg)\left(1.0\ \frac{kJ}{kg\cdot K}\right) \ln \frac{288}{1210}$$

$$= -0.0344\ kJ/K \longleftarrow S_3 - S_2$$

Process 3-1: Since $T_3 = T_1$, Eq. 6.23 reduces so

$$S_1 - S_3 = -mR \ln \frac{p_1}{p_3} = -mR \ln \frac{p_1}{p_2}$$

$$= -(0.024)\left(\frac{8.314}{28.97}\right) \ln \frac{0.1}{0.42}$$

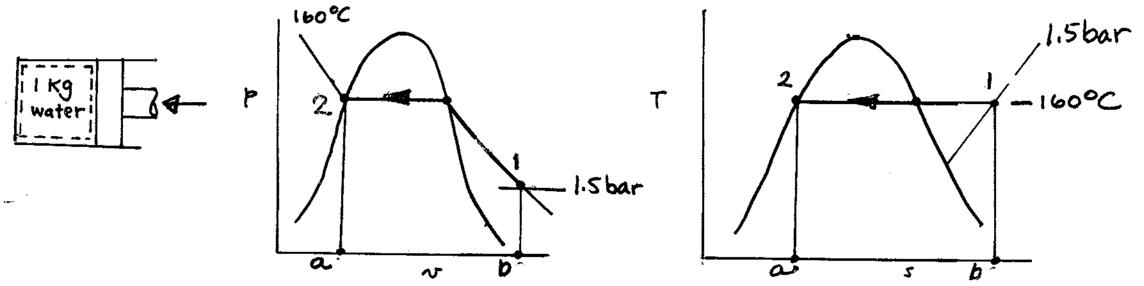
$$= 0.0099\ kJ/K \longleftarrow S_1 - S_3$$

PROBLEM 6.34

KNOWN: One kg of water undergoes an isothermal process between two specified states.

FIND: Determine the heat transfer and the work. Sketch the process on p-v and T-s coordinates

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) As shown in the accompanying figure, the system consists of one kg of water. (2) The compression takes place isothermally and without internal irreversibilities. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: Using assumption 2, Eq. 6.25 becomes

$$Q = \int_1^2 T dS = m T [s_2 - s_1]$$

Then, with data from Tables A-2 and A-4

$$Q = (1\,\text{kg})(433\,\text{K})(1.9427 - 7.4665)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = -2391.8\,\text{kJ} \longleftarrow Q$$

The magnitude of the heat transfer is represented by area 1-2-a-b-1 on the T-s diagram.

The energy balance reduces to give $W = Q - m(u_2 - u_1)$. With data from Tables A-2 and A-4

$$W = -2391.8 - (1\,\text{kg})[674.86 - 2595.2]\frac{\text{kJ}}{\text{kg}} = -471.5\,\text{kJ} \longleftarrow$$

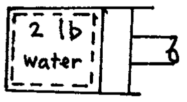
Alternatively, $W = \int_1^2 p dV$. The magnitude of the work is represented by area 1-2-a-b-1 on the p-v diagram.

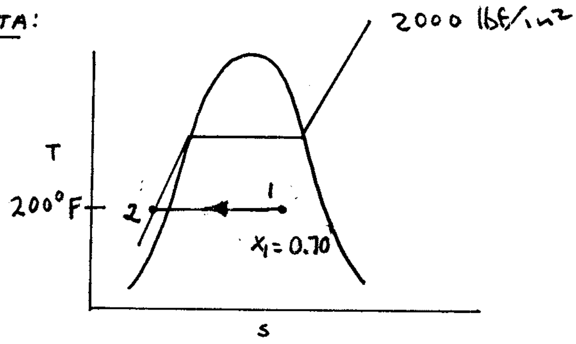
PROBLEM 6.35*

KNOWN: Two lb of water undergo an isothermal process.

FIND: Determine the heat transfer and the work, each in kJ.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) As shown in the accompanying figure, the system consists of 2 kg of water. (2) The process takes place isothermally and without internal irreversibilities. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: Using assumption 2, Eq. 6.25 becomes

$$Q = \int_1^2 T\,dS = mT(s_2 - s_1)$$

With data from Table A-2E at 200°F

$$s_1 = s_f + x_1(s_g - s_f) = 0.2940 + 0.70\,(1.7762 - 0.2940) = 1.33154\ \text{Btu/lb°R}$$

From Table A-5E, $s_2 = 0.29162$ Btu/lb°R

So $$Q = (2\,\text{lb})(660°\text{R})(0.29162 - 1.33154)\,\frac{\text{Btu}}{\text{lb°R}} = -1372.7\ \text{Btu} \longleftarrow Q$$

An energy balance reduces to give

$$W = Q - m(u_2 - u_1)$$

With data from Table A-2E at 200°F

$$u_1 = u_f + x_1(u_g - u_f) = 168.04 + 0.70\,(1074.2 - 168.04) = 802.35\ \text{Btu/lb}$$

and from Table A-5E, $u_2 = 166.49$ Btu/lb

So $$W = -1372.7 - 2\,(166.49 - 802.35)$$

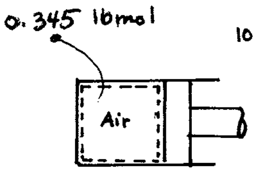
$$= -101.0\ \text{Btu} \longleftarrow W$$

PROBLEM 6.36

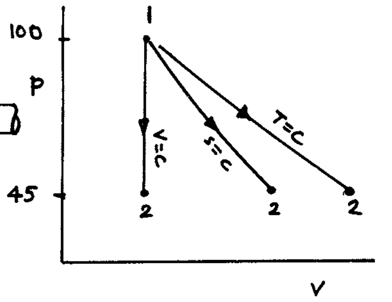
KNOWN: Air undergoes an internally reversible process between two specified states.

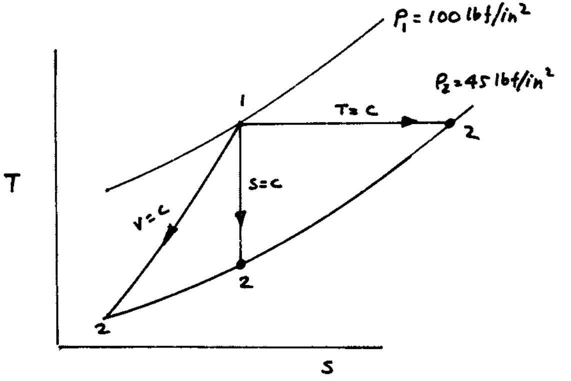
FIND: For each of three cases, determine the heat transfer and the work, per unit of mass, and show the process on p-v and T-s coordinates.

SCHEMATIC & GIVEN DATA:



$P_1 = 100$ lbf/in²
$P_2 = 45$ lbf/in²
$T_1 = 500°F$





ASSUMPTIONS: (1) The system consists of air which behaves as an ideal gas. (2) Each process is internally reversible. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: (a) Isothermal process. With assumption (2), Eq. 6.25 is applicable. Then, as temperature is constant

$$Q = \int_1^2 T\,dS = m\,T(s_2 - s_1)$$

With Eq. 6.21a and $m = nM$

$$Q = m\,T(s_2 - s_1) = mT\left[-R \ln \frac{P_2}{P_1}\right]$$

$$= \left((0.345)(28.97)\ \text{lb}\right)(960°\text{R})\left(-\frac{1.986}{28.97}\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) \ln \frac{45}{100} = +525.2\ \text{Btu} \longleftarrow Q$$

Since internal energy depends on temperature alone for an ideal gas, $\Delta U = 0$ because $T_1 = T_2$. Thus, an energy balance reduces with assumption 3 to give $W = Q = +525.2$ Btu. $\longleftarrow W$

(b) Adiabatic Process. Since adiabatic, $Q = 0$. $\longleftarrow Q$ Then, an energy balance gives

$$W = -m(u_2 - u_1)$$

This requires that state 2 be fixed. However, as the process is adiabatic and internally reversible, it is an isentropic process, so $s_2 = s_1$. Eq. 6.21a gives

$$s°(T_2) = s°(T_1) + R \ln \frac{P_2}{P_1}$$

With data from Table A-22E

$$s°(T_2) = 0.74030 + \frac{1.986}{28.97} \ln \frac{45}{100} = 0.68556 \Rightarrow u_2 = 131.36\ \text{Btu/lb}$$

At $T_1 = 960°R$, $u_1 = 165.26$ Btu/lb. Finally, with $m = nM = 9.99$ kg

$$W = -(9.99)(131.36 - 165.26) = +338.7\ \text{Btu} \longleftarrow W$$

(c) Constant volume process. Since volume is constant, $W = 0$. $\longleftarrow W$ Then, an energy balance gives $Q = m(u_2 - u_1)$. To find u_2, first determine T_2 using the ideal gas equation of state

$$\left.\begin{array}{l} P_1 V = RT_1 \\ P_2 V = RT_2 \end{array}\right\} \Rightarrow T_2 = T_1\left(\frac{P_2}{P_1}\right) = 960\left(\frac{45}{100}\right) = 432°\text{R}$$

Then, from Table A-22E $u_2 = 73.57$ Btu/lb, and

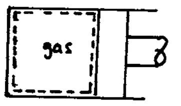
$$Q = (9.99)(73.57 - 165.26) = -916\ \text{Btu} \longleftarrow Q$$

PROBLEM 6.37

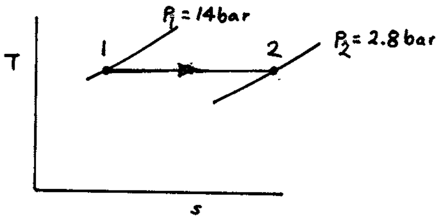
KNOWN: A gas expands from a specified initial state to a specified final pressure isothermally and without internal irreversibilities.

FIND: Determine the heat transfer and work, per unit of mass, for (a) R134a (b) Air as an ideal gas.

SCHEMATIC & GIVEN DATA:



T = 333 K (60°C)



ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the gas. (2) The expansion takes place isothermally and without internal irreversibilities. (3) There is no change in kinetic or potential energy between the end states. (4) Air is modeled as an ideal gas.

ANALYSIS: Using assumption 2, Eq. 6.25 becomes

$$Q = \int_1^2 T\,dS = mT(s_2 - s_1) \quad \Rightarrow \quad Q/m = T(s_2 - s_1)$$

With assumption 2, an energy balance gives

$$W = Q - \Delta U \quad \Rightarrow \quad W/m = Q/m - (u_2 - u_1)$$

(a) R134a. Table A-12: $u_1 = 262.17$ kJ/kg, $s_1 = 0.9297$ kJ/kg·K, $u_2 = 277.23$, $s_2 = 1.1079$ kJ/kg·K.

$$\Rightarrow \quad Q/m = 333\,\text{K}\,(1.1079 - 0.9297)\frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 59.34 \text{ kJ/kg} \quad \longleftarrow Q/m$$

$$W/m = 59.34 - (277.23 - 262.17) = 44.28 \text{ kJ/kg} \quad \longleftarrow W/m$$

(b) AIR. Since $T_1 = T_2$, Eq. 6.21a reduces to read

$$s_2 - s_1 = -R \ln p_2/p_1 = -\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\frac{2.8}{14} = +0.46188\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

and

$$Q/m = (333)(0.46188) = 153.81\,\frac{\text{kJ}}{\text{kg}} \quad \longleftarrow Q/m$$

Since internal energy depends on temperature alone for an ideal gas, $\Delta U = 0$ because $T_1 = T_2$. The energy balance then gives

$$W/m = Q/m - (\overset{0}{\cancel{u_2 - u_1}})$$

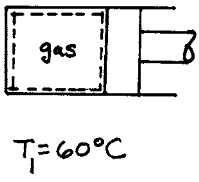
$$= Q/m = 153.81 \text{ kJ/kg} \quad \longleftarrow W/m$$

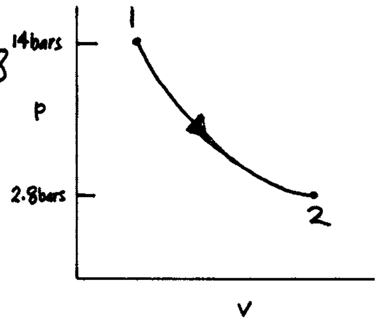
PROBLEM 6.38

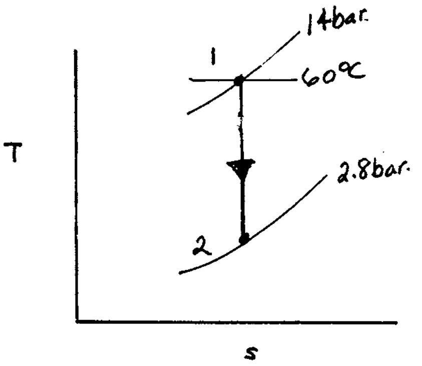
KNOWN: A gas expands from a specified initial state to a specified final pressure adiabatically and without internal irreversibilities.

FIND: Determine the work, per unit of mass, for (a) R134a, (b) air as an ideal gas.

SCHEMATIC & GIVEN DATA:







ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the gas. (2) The expansion takes place adiabatically and without internal irreversibilities. (3) There is no change in kinetic or potential energy between the end states. (4) Air is modeled as an ideal gas.

ANALYSIS: Since the process is both adiabatic and internally reversible, it is isentropic (Sec. 6.4). This together with the specified final pressure fixes state 2. With assumption 3, an energy balance for the adiabatic process reduces to give $W = -m(u_2 - u_1)$, or $W/m = -(u_2 - u_1)$.

(a) R134a. Table A-12: $u_1 = 262.17$ kJ/kg, $s_1 = 0.9297$ kJ/kg·K. Then, with $s_2 = s_1$ and data from Table A-12 at 2.8 bar, interpolation gives $u_2 = 228.82$ kJ/kg. Finally,

$$\frac{W}{m} = -(228.82 - 262.17)\frac{kJ}{kg} = 33,25\ \frac{kJ}{kg} \longleftarrow \text{(a)}$$

(b) AIR. With $s_2 = s_1$, Eq. 6.21a reduces to

$$s°(T_2) = s°(T_1) + R \ln p_2/p_1$$

with $s°(T_1)$ from Table A-22

$$s°(T_2) = 1.80685 + \frac{8.314}{28.97}\ln\frac{2.8}{14} = 1.345$$

Using this value, interpolation in Table A-22 gives $u_2 = 149.8$ kJ/kg. Also, $u_1 = 237.8$ kJ/kg. Finally

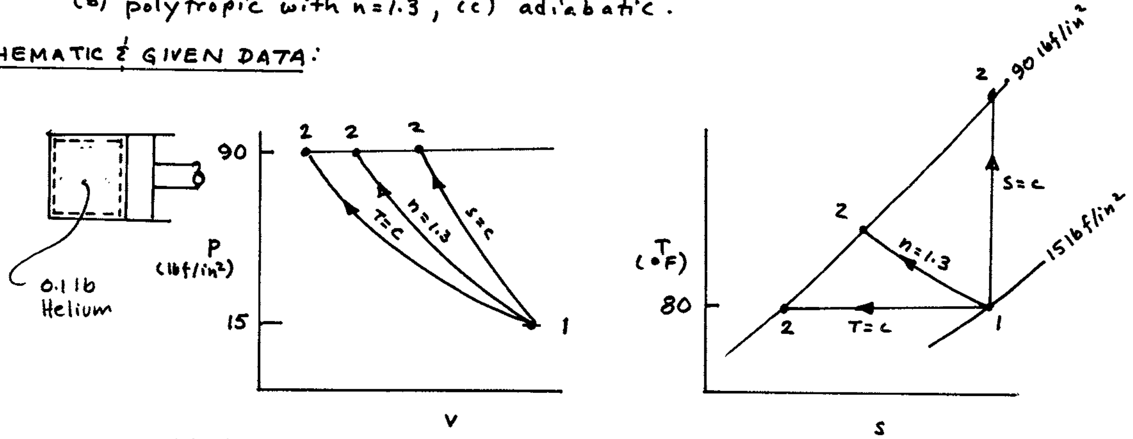
$$\frac{W}{m} = -(149.8 - 237.8) = 88\ kJ/kg \longleftarrow \text{(b)}$$

PROBLEM 6.39

KNOWN 0.1 lb of helium is compressed from a specified initial state to a specified final pressure without irreversibilities.

FIND: Determine the work and the change in entropy if the process is (a) isothermal, (b) polytropic with $n = 1.3$, (c) adiabatic.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the gas. (2) The gas is modeled as an ideal gas. (3) The compression is internally reversible.

ANALYSIS: (a) Isothermal. Using the ideal gas equation of state

$$W = \int_1^2 p\,dV = \int_1^2 \frac{mRT}{V}\,dV = mRT \ln \frac{V_2}{V_1}$$

Since $T_1 = T_2$, $V_2/V_1 = P_1/P_2$, so with the molecular weight from Table A-1E

$$W = mRT \ln \frac{P_1}{P_2} = (0.1\,\text{lb})\left(\frac{1.986}{4.003}\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)(540°\text{R}) \ln \frac{15}{90} = -48.0\ \text{Btu} \longleftarrow W$$

Since $T_1 = T_2$, Eq. 6.21a reduces to give

$$\Delta S = -mR \ln \frac{P_2}{P_1} = -(0.1\,\text{lb})\left(\frac{1.986}{4.003}\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) \ln \frac{90}{15} = -0.089\ \frac{\text{Btu}}{°\text{R}} \longleftarrow \Delta S$$

(b) Polytropic, $n = 1.3$. The process is described by $p = C/V^n$ where C is a constant (Sec. 2.2.3). Thus,

$$W = \int_1^2 p\,dV = \int_1^2 \frac{C}{V^n}\,dV = \frac{p_2V_2 - p_1V_1}{1-n} = \frac{mR(T_2 - T_1)}{1-n}$$

where in the last step the ideal gas equation of state has been used. Introducing Eq. 3.56

$$W = \frac{mRT_1}{1-n}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right] = \frac{(0.1\,\text{lb})\left(\frac{1.986}{4.003}\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)(540°\text{R})}{1-1.3}\left[\left(\frac{90}{15}\right)^{0.3/1.3} - 1\right]$$

$$= -45.73\ \text{Btu} \longleftarrow W$$

For a monatomic gas such as helium $c_p/R = 2.5$ (Table A-21), so Eq. 6.23 is appropriate for evaluating ΔS. The temperature at state 2 is obtained using Eq. 3.56 as 817°R. Thus

$$\Delta S = m\left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}\right] = (0.1\,\text{lb})\left(\frac{1.986}{4.003}\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\left[2.5 \ln \frac{817}{540} - \ln \frac{90}{15}\right]$$

$$= -0.0375\ \frac{\text{Btu}}{°\text{R}} \longleftarrow \Delta S$$

PROBLEM 6.39 (Contd.)

(c) Adiabatic. As the process is both adiabatic and internally reversible, it is isentropic (Sec. 6.4). Thus $\Delta S = 0$. ← ΔS

Since $Q=0$, an energy balance reduces with assumption 4 to give $W = -m(u_2-u_1)$. Using Eq. 3.44 with $c_p = 2.5\,R$, $c_v = 1.5R$. Thus

$$W = -m(1.5R)(T_2-T_1)$$

To find T_2, rewrite Eq. 6.23 using $s_2=s_1$ and $c_p = 2.5\,R$

$$0 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \Rightarrow 2.5\ln\frac{T_2}{T_1} = \ln\frac{P_2}{P_1} \Rightarrow T_2 = 1106\,^\circ R$$

Finally

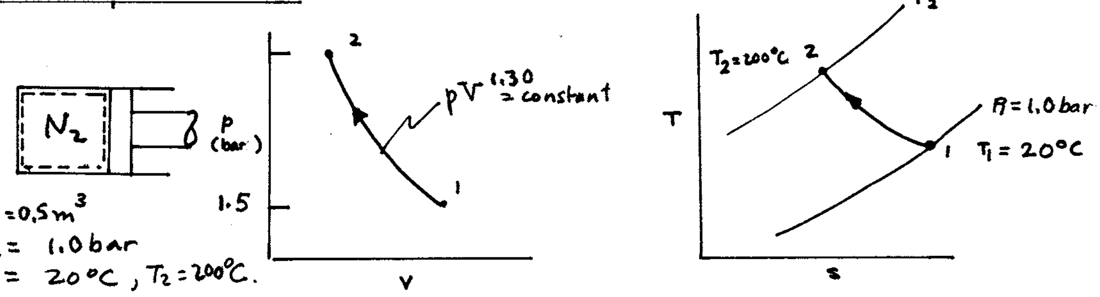
$$W = -(0.1\text{lb})(1.5)\left(\frac{1.986}{4.003}\ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ R}\right)(1106\,^\circ R - 540\,^\circ R) = -42.12\ \text{Btu} \longleftarrow W$$

PROBLEM 6.40*

KNOWN: N_2 undergoes an internally reversible compression between specified states during which $pV^n = $ constant, where $n = 1.30$.

FIND: Determine (a) P_2, (b) the work and heat transfer, (c) the entropy change.

SCHEMATIC & GIVEN DATA:



$V_1 = 0.5\,m^3$
$P_1 = 1.0$ bar
$T_1 = 20°C$, $T_2 = 200°C$.

ASSUMPTIONS: (1) As shown in the accompanying figure, the system consists of the N_2 which is modeled as an ideal gas. (2) The compression is internally reversible and described by $pV^n =$ constant. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: (a) With Eq. 3.56,

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{n/(1-n)} = (1.0\text{ bar})\left(\frac{473}{293}\right)^{\frac{1.30}{.30}} = 7.97\text{ bar} \longleftarrow P_2$$

(b) The mass of N_2 present can be found using the ideal gas equation of state

$$m = \frac{P_1V_1}{RT_1} = \frac{(1.0\times10^5\ N/m^2)(0.5\ m^3)}{\left(\frac{8314}{28.01}\right)\frac{N\,m}{kg\cdot K}(293\ K)} = 0.5749\ kg$$

The work is

$$W = \int_1^2 p\,dV = \int_1^2 \frac{C}{V^n}dV = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR(T_2-T_1)}{1-n} = \frac{(0.5749\,kg)\left(\frac{8.314}{28.01}\frac{kJ}{kg\,K}\right)(180K)}{1-1.3}$$

$$= -102.4\ kJ \longleftarrow W$$

With assumption 3, an energy balance reduces to $Q = m(u_2-u_1)+W$. Evaluating

$$Q = (0.5749\ kg)(9849-6083)\frac{kJ}{kmol}(1/28.01)\frac{kmol}{kg} + (-102.4\,kJ)$$

$$= -25.1\ kJ \longleftarrow Q$$

(c) The entropy change can be found with s° data from Table A-23 and Eq. 6.21a

$$\Delta S = m\left[s^\circ(T_2) - s^\circ(T_1) - R\ln\frac{P_2}{P_1}\right] = (0.5749)\left[(204.989-190.998)/28.01 - \left(\frac{8.314}{28.01}\right)\ln\left(\frac{7.97}{1.0}\right)\right]$$

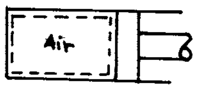
$$= -0.0670\ kJ/K \longleftarrow \Delta S$$

PROBLEM 6.41

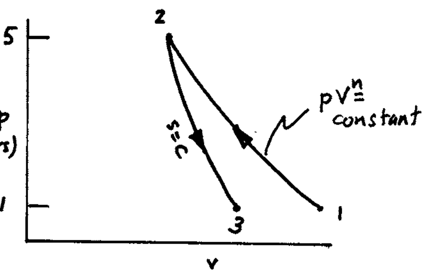
KNOWN: Air undergoes two internally reversible processes in series.

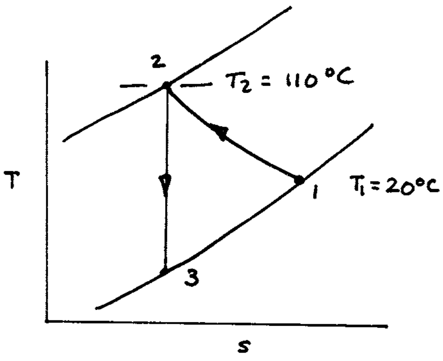
FIND: Sketch the processes on p-v and T-s diagrams. Determine the temperatures at states 2 and 3, and the net work.

SCHEMATIC & GIVEN DATA:



$V_1 = 1\ m^3$
$P_1 = 1$ bar
$T_1 = 20°C$
$T_2 = 110°C$
$P_2 = 5$ bar





ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the air. (2) The air is modeled as an ideal gas. (3) Each process is internally reversible. Process 1-2 is described by pV^n = constant and process 2-3 is adiabatic. (4) There is no change in kinetic or potential energy between the end states of either process.

ANALYSIS: (b) Using Eq. 3.56

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} \Rightarrow \frac{n-1}{n} = \frac{\ln(T_2/T_1)}{\ln(P_2/P_1)} = \frac{\ln(383/293)}{\ln(5/1)} \Longrightarrow n = 1.2 \longleftarrow n$$

(c) Since process 2-3 is adiabatic and internally reversible, it is isentropic (Sec. 6.4). Thus, with $s_3 = s_2$, Eq. 6.21a gives

$$s°(T_3) = s°(T_2) + R \ln\frac{P_3}{P_2} = 1.94791 + \frac{8.314}{28.97} \ln\frac{1}{5} = 1.48602$$

where $s°(T_2)$ is from Table A-22. Interpolating with $s°(T_3)$ in Table A-22, $T_3 = 242$ K $\longleftarrow T_3$

(d) The mass can be determined using the ideal gas equation of state.

$$m = \frac{P_1V_1}{RT_1} = \frac{(10^5 N/m^2)(1 m^3)}{\left(\frac{8314}{28.97}\frac{N \cdot m}{kg \cdot K}\right)(293 K)} = 1.189\ kg$$

The work for process 1-2 is

$$W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{C}{V^n} dV = \frac{P_2V_2 - P_1V_1}{1-n} = \frac{mR(T_2-T_1)}{1-n} = \frac{(1.189 kg)\left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(383-293)\ K}{1-1.2}$$

$$= -153.55\ kJ$$

For process 2-3 an energy balance reduces with assumption 4 to give, $W_{23} = -m(u_3 - u_2)$. With data from Table A-22

$$W_{23} = -\ 1.189\ kg\,(172.56 - 273.86)\,\frac{kJ}{kg} = +\ 120.45\ kJ$$

The net work is

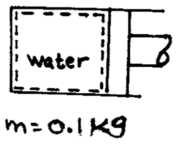
$$W = W_{12} + W_{23} = (-153.55) + (120.45) = -33.1\ kJ \longleftarrow W$$

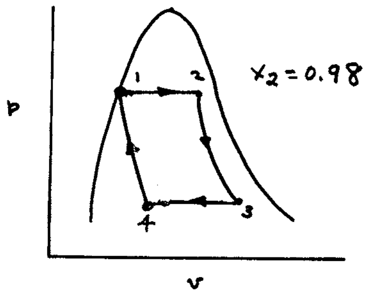
PROBLEM 6.42

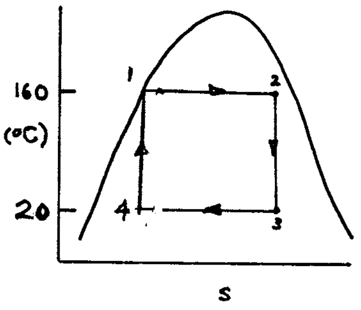
KNOWN: 0.1 kg of water executes a Carnot power cycle.

FIND: Sketch the cycle on p-v and T-s coordinates. Determine the heat added, net work, and thermal efficiency.

SCHEMATIC & GIVEN DATA:







ASSUMPTION: The system shown in the accompanying figure executes a Carnot cycle.

ANALYSIS: Since each process is internally reversible, Eq. 6.29 is applicable.

$$Q_{12} = \int_1^2 T\,dS = m\,T_1(s_2 - s_1) \qquad \text{(heat added)}$$

$$Q_{34} = \int_3^4 T\,dS = m\,T_3(s_4 - s_3) = -m\,T_3(s_2 - s_1)$$

$$Q_{23} = Q_{41} = 0$$

For any cycle $W_{cycle} = Q_{cycle}$. Thus $W_{cycle} = m[T_1(s_2 - s_1) - T_3(s_2 - s_1)]$, or

$$W_{cycle} = m(T_1 - T_3)(s_2 - s_1) \qquad \text{(net work)}$$

From Table A-2, $s_1 = 1.9427$ kJ/kg·K. To find s_2, use the known value for x_2 with data from Table A-2

$$s_2 = s_f + x_2(s_g - s_f) = 1.9427 + 0.98(6.7502 - 1.9427) = 6.654 \text{ kJ/kg·K}$$

Then

$$Q_{12} = (0.1\text{ kg})(433\text{ K})(6.654 - 1.9427)\frac{\text{kJ}}{\text{kg·K}} = 203.99 \text{ kJ}$$

$$W_{cycle} = (0.1)(140)(6.654 - 1.9427) = 65.96 \text{ kJ}$$

① $$\eta = \frac{W_{cycle}}{Q_{12}} = \frac{65.96}{203.99} = 0.323 \quad (32.3\%)$$

---

1. Alternatively, using Eq. 5.8

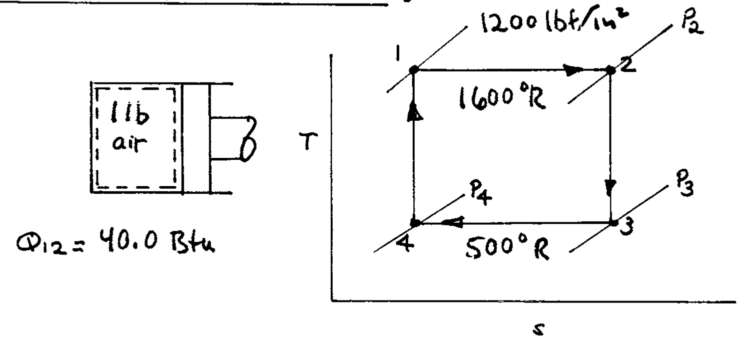
$$\eta_{MAX} = 1 - \frac{T_C}{T_H} = 1 - \frac{293}{433} = 0.323$$

PROBLEM 6.43*

KNOWN: Data is provided for 1 lb of air undergoing a Carnot power cycle.

FIND: Determine (a) the pressures at the end of the isothermal expansion, adiabatic expansion, and isothermal compression, (b) the net work, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is the 1 lb of air. (2) The air is modeled as an ideal gas. (3) The system undergoes a Carnot power cycle.

ANALYSIS: (a) As discussed in Sec. 6.4, for process 1-2 $Q_{12} = m T_H (s_2 - s_1)$. And for an ideal gas at fixed temperature, Eq. 6.25 reduces to read $s_2 - s_1 = -R \ln p_2/p_1$. Combining these $Q_{12} = -mRT_H \ln p_2/p_1$. Solving

$$\ln\frac{p_2}{p_1} = -\frac{Q_{12}}{mRT_H} = \frac{-40.0\ \text{Btu}}{(1.0\ \text{lb})\left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(1600^\circ\text{R})}$$

giving $p_2/p_1 = 0.6944$, $p_2 = 833.3$ lbf/in² ⟵ p_2

For process 2-3 and data from Table A-22E

$$0 = s^\circ(T_3) - s^\circ(T_2) - R\ln\frac{p_3}{p_2} \Rightarrow \ln\frac{p_3}{p_2} = \frac{s^\circ(T_3)-s^\circ(T_2)}{R} = \frac{0.58233-0.87130}{(1.986/28.97)}$$

giving $\ln p_3/p_2 = -4.2152$, $p_3/p_2 = 0.01477$, $p_3 = 12.31$ lbf/in² ⟵ p_3

Similarly, for process 4-1

$$\ln\frac{p_1}{p_4} = \frac{s^\circ(T_1)-s^\circ(T_4)}{R} = +4.2152,\quad \frac{p_1}{p_4} = 67.708,\quad p_4 = 17.72\ \text{lbf/in}^2$$ ⟵ p_4

(b) For any cycle, $W_{net} = \eta Q_{in}$. Here, η corresponds to η_{MAX} and $Q_{in} = Q_{12}$.

Thus
$$W_{net} = \underbrace{\left[1-\frac{500}{1600}\right]}_{0.6875\ (68.75\%)} 40.0\ \text{Btu} = 27.5\ \text{Btu}$$
⟵ W_{net}

⟵ η

Alternatively $W_{net} = Q_{net} = Q_{12} + Q_{34}$. From an energy balance, since T is constant, $Q_{34} = W_{34}$, where

$$W_{34} = m\int_3^4 p\,dv = m\int_3^4 \frac{RT}{v}dv = mRT\ln\overbrace{\frac{v_4}{v_3}}^{=p_3/p_4} = mRT\ln\frac{p_3}{p_4}$$

$$= (1.0)\left(\frac{1.986}{28.97}\right)(500)\ln\left(\frac{12.31}{17.72}\right) = -12.5\ \text{Btu}$$

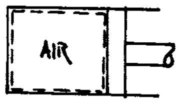
So $Q_{net} = 40.0 - 12.5 = 27.5$ Btu, and $W_{net} = 27.5$ Btu

PROBLEM 6.44

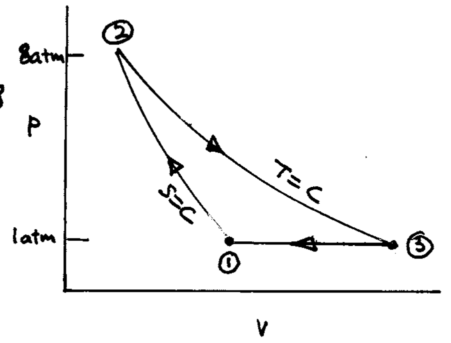
KNOWN: Air undergoes three internally reversible processes in series between specified states.

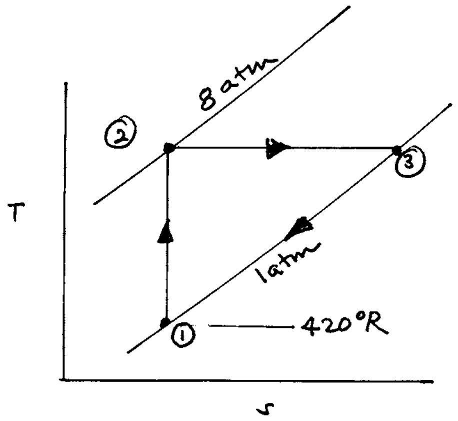
FIND: Sketch the processes on p-v and T-s coordinates. Determine the temperature at state 3. Also find the net work per unit of mass and the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$p_1 = 1$ atm
$T_1 = -40°F$





ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the air. (2) The air is modeled as an ideal gas. (3) Each process is internally reversible. (4) There is no change in kinetic or potential energy for either of the two processes.

ANALYSIS: As process 1-2 is adiabatic and internally reversible, it is an isentropic process (Sec. 6.4): $s_2 = s_1$. Thus, Eq. 6.21b reduces to give, with data from Table A-22E,

$$s^\circ(T_2) = s^\circ(T_1) + R \ln \frac{p_2}{p_1} = 0.54058 + \frac{1.986}{28.97} \ln 8 = 0.6831 \text{ Btu/lb·°R}$$

Thus, Table A-22E gives $T_2 = 760°R$, $u_2 = 129.99$ Btu/lb. Also, $u_1 = 71.52$ Btu/lb. Therefore, W_{12} can be evaluated using an energy balance:

$$\frac{W_{12}}{m} = \cancel{\frac{Q_{12}}{m}}^{\,0} - (u_2 - u_1) = -(129.99 - 71.52) \text{ Btu/lb} = -58.5 \text{ Btu/lb}$$

Since process 2-3 is isothermal, $T_3 = T_2 = 760°R$. ⟵ T_3

Also,

$$\frac{W_{23}}{m} = \int_2^3 p\,dv = \int_2^3 \frac{RT}{v}\,dv = RT \ln \frac{v_3}{v_2} = RT \ln \frac{p_2}{p_3}$$

$$\left[\begin{matrix} p_2 v_2 = RT_2 \\ p_3 v_3 = RT_3 \end{matrix} \Rightarrow \frac{v_3}{v_2} = \frac{p_2}{p_3}\right]$$

$$= \left(\frac{1.986}{28.97}\right)(760) \ln(8)$$

$$= +108.3 \text{ Btu/lb}$$

PROBLEM 6.44 (Contd.)

Since process 3-1 is at constant pressure

$$\frac{W_{31}}{m} = \int_3^1 p\,dv = p(v_1 - v_3) = R(T_1 - T_3)$$

$$= \left(\frac{1.986}{28.97}\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(420 - 760)^\circ\text{R} = -23.3 \text{ Btu/lb}$$

The net work is then,

① $$\frac{W_{cycle}}{m} = \frac{W_{12}}{m} + \frac{W_{23}}{m} + \frac{W_{31}}{m} = -58.5 + 108.3 - 23.3 = 26.5 \text{ Btu/lb}$$ ← W_{net}/m

By inspection of the T-s diagram, using Eq. 6-25, it can be concluded that $Q_{23} > 0$ (entropy increases) and $Q_{31} < 0$ (entropy decreases). Thus, the heat addition for the cycle occurs during process 2-3:

$$\frac{Q_{23}}{m} = \int_2^3 T\,ds = T(s_3 - s_2)$$

$$= T\Big(\underbrace{s^\circ(T_3) - s^\circ(T_2)}_{\substack{=0 \\ (T_2 = T_3)}} - R \ln p_3/p_2\Big)$$

$$= -760^\circ\text{R}\left(\frac{1.986}{28.97}\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\frac{1}{8}$$

$$= 108.3 \text{ Btu/lb}$$

Finally, the thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{26.5}{108.3} = 0.245 \quad (24.5\%)$$ ← η

---

1. Alternatively, $Q_{cycle} = W_{cycle}$, where

$$Q_{cycle} = \overset{0}{\cancel{Q_{12}}} + Q_{23} + Q_{31}$$

An energy balance reduces to give $Q_{31} = u_1 - u_3 + p(v_1 - v_3)$,
or

$$\frac{Q_{31}}{m} = h_1 - h_3 = 100.32 - 182.08 = -81.76 \text{ Btu/lb.}$$

Then, with Q_{23}/m determined below we get

$$\frac{Q_{cycle}}{m} = 108.3 - 81.76 = 26.5 \text{ Btu/lb.}$$

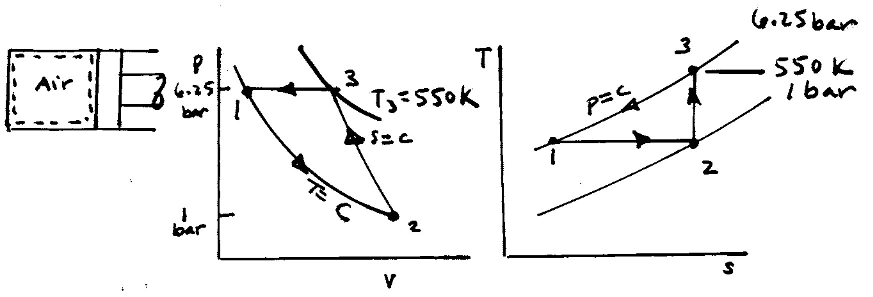
which checks the value given here for W_{cycle}/m.

PROBLEM 6.45

KNOWN: Air as an ideal gas undergoes a cycle consisting of three internally reversible processes.

FIND: Sketch the cycle on p-v and T-s coordinates. Determine temperature T_1 and the thermal efficiency or coefficient of performance, as appropriate.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. As shown in the schematic, the system is the air. 2. Air is modeled as an ideal gas. 3. All processes are internally reversible. 4. Kinetic & potential energy changes can be ignored.

ANALYSIS: (a) To find T_1, consider process 2-3, which has $s_2 = s_3$. Then, with Eq. 6.21a and data from Table A-22 and noting that $T_1 = T_2$

$$s^\circ(T_2) = s^\circ(T_3) - R\ln\frac{p_3}{p_2} = 2.31809 - \left(\frac{8.314}{28.97}\right)\ln\left(\frac{6.25}{1}\right) = 1.7922\,\frac{kJ}{kg\,K} \Rightarrow T_2 = 328\text{ K} = T_1 \longleftarrow \underline{T_1}$$

(b) Study of the areas under the processes on the p-v diagram indicates that there is a net work input requirement. Accordingly, the cycle can be a refrigeration cycle but not a power cycle. Study of the areas under the processes of the T-s diagram suggest that process 1-2 involves a heat addition (entropy increases) and process 3-1 involves a heat rejection (entropy decreases). Thus,

① $\beta = \dfrac{Q_{12}}{|W_{cycle}|}$, where $W_{cycle} = W_{12} + W_{23} + W_{31}$. Alternatively, $W_{cycle} = Q_{cycle} = Q_{12} + \cancel{Q_{23}} + Q_{31}$. Then, $\beta = \dfrac{Q_{12}}{|Q_{12} + Q_{31}|}$.

Using Eq. 6.25 with Eq. 6.21a

$$\frac{Q_{12}}{m} = \int_1^2 T\,ds = T(s_2 - s_1) = T[\underbrace{s^\circ(T_2) - s^\circ(T_1)}_{=0\text{ since }T\text{ is const.}} - R\ln\frac{p_2}{p_1}] = +RT\ln\frac{p_1}{p_2}$$

$$= \left(\frac{8.314}{28.97}\right)(328)\ln\left(\frac{6.25}{1.0}\right)$$

$$= 172.50\text{ kJ/kg}$$

An energy balance for process 3-1 reads: $(u_1 - u_3) = \frac{Q_{31}}{m} - \frac{W_{31}}{m}$, where $\frac{W_{31}}{m} = \int_3^1 p\,dv = p(v_1 - v_3)$. Thus, $\frac{Q_{31}}{m} = (u_1 - u_3) + p(v_1 - v_3)$, which can be rewritten concisely as $\frac{Q_{31}}{m} = h_1 - h_3$. With date from Table A-22

$\frac{Q_{31}}{m} = 328.33 - 554.74 = -226.41\,\frac{kJ}{kg}$. Finally, $Q_{cycle} = (172.50) + (-226.41) = -53.91\,\frac{kJ}{kg}$

$$\beta = \frac{172.50}{53.91} = 3.20 \longleftarrow \underline{\beta}$$

---

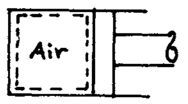
1. From the discussion of Sec. 2.6.3 recall that β requires the net work into the system to achieve the refrigeration effect. Thus, $|W_{cycle}|$ is used here.

PROBLEM 6.46

KNOWN: Air as an ideal gas undergoes a cycle consisting of three internally reversible processes.

FIND: Sketch the cycle on p-v, T-s coordinates, determine T_2, and evaluate the thermal efficiency or coefficient of performance, as appropriate.

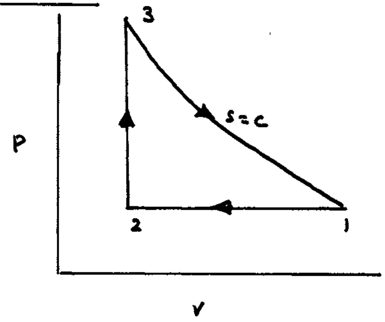
SCHEMATIC & GIVEN DATA:

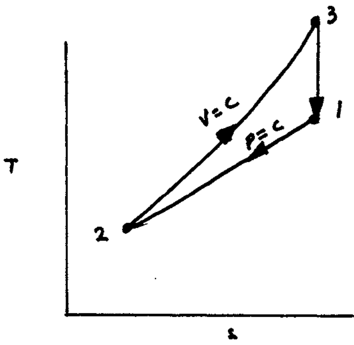


$P_1 = 12\ \frac{lbf}{in^2}$

$T_1 = 80°F$

$T_3 = 540°F$





ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the air. (2) The air is modeled as an ideal gas. (3) All processes are internally reversible. (4) There is no change in kinetic or potential energy between states 1, 2, 3.

ANALYSIS: As the processes occur in a clockwise sense, the cycle would be a power cycle. The heat added can be evaluated using an energy balance with assumption 4

$$\frac{Q_{23}}{m} = u_3 - u_2 + \cancel{\frac{W_{23}}{m}}^{0} = u_3 - u_2 \qquad (1)$$

Similarly the heat rejected is

$$\frac{Q_{12}}{m} = u_2 - u_1 + \frac{W_{12}}{m} = u_2 - u_1 + \int_1^2 p\,dv = u_2 - u_1 + p(v_2 - v_1) = h_2 - h_1 \qquad (2)$$

To evaluate these requires T_2. Since process 2-3 is at constant volume, the ideal gas equation of state gives $P_3/P_2 = T_3/T_2$. Also, with $s_3 = s_1$, Eq. 6.21a gives with $P_2 = P_1$

$$0 = s°(T_3) - s°(T_1) - R \ln P_3/P_1 \Rightarrow 0 = s°(T_3) - s°(T_1) - R \ln P_3/P_2$$

Combining these results, and using data from Table A-22E

$$\ln \frac{T_3}{T_2} = \frac{s°(T_3) - s°(T_1)}{R} = \frac{0.63395 - 0.60078}{(1.986/28.97)} \Rightarrow T_2 = 382.2\ °R \longleftarrow$$

Using Eqs. (1), (2) with data from Table A-22E

$$\frac{Q_{23}}{m} = u_3 - u_2 = 105.78 - 65.08 = 40.7\ Btu/lb, \quad \frac{Q_{12}}{m} = 91.28 - 129.06 = -37.78\ Btu/lb$$

The thermal efficiency is

① $$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{Q_{12} + Q_{23} + \cancel{Q_{31}}^{0}}{Q_{23}} = \frac{\overbrace{(-37.78) + (40.7)}^{2.92}}{40.7} = 0.072 \quad (7.2\%)$$

---

1. For the cycle, $W_{cycle} = Q_{cycle}$, where $W_{cycle} = W_{12} + \cancel{W_{23}}^{0} + W_{31}$. An energy balance gives $W_{31}/m = u_3 - u_1$. Also, $\frac{W_{12}}{m} = \int_1^2 p\,dv = p(v_2 - v_1)$, which can be expressed with the ideal gas equation of state as $W_{12}/m = R(T_2 - T_1)$. Then,

$$\frac{W_{31}}{m} = 105.78 - 92.04 = 13.74, \quad \frac{W_{12}}{m} = \frac{1.986}{28.97}(382.2 - 540) = -10.82\ Btu/lb$$

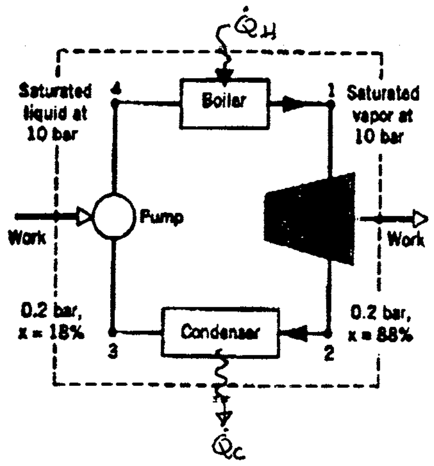
So, $(W_{cycle}/m) = 13.74 - 10.82 = 2.92\ Btu/lb$, which agrees with the value of Q_{cycle}/m, as expected.

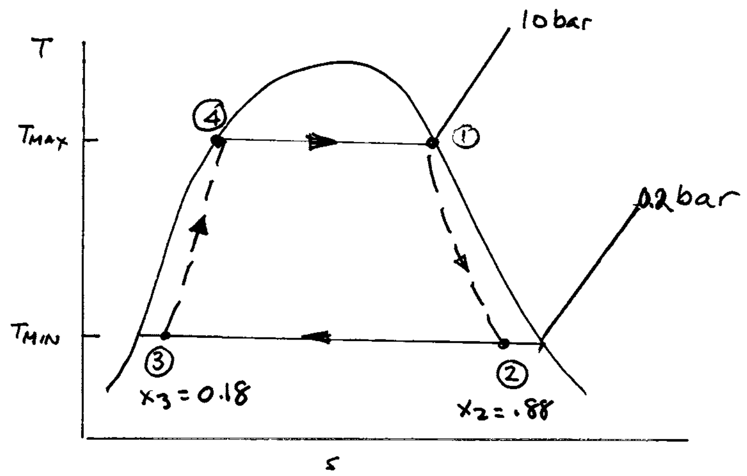
PROBLEM 6.47

KNOWN: The schematic and steady state operating data are provided for a vapor power plant.

FIND: (a) sketch the cycle on T-s coordinates, (b) determine the thermal efficiency and compare with the thermal efficiency of a Carnot cycle operating between the same maximum and minimum temperatures.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The overall system shown in the figure and thus each component is at steady state. (2) The only significant heat transfers occur with the two reservoirs. (3) Kinetic and potential energy effects can be ignored. (3) The maximum and minimum temperatures correspond, respectively, to the saturation temperatures at 10 bars and 1bar.

ANALYSIS: The thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_H} = 1 - \frac{\dot{Q}_C}{\dot{Q}_H}$$

The heat transfer rates are found by reducing mass and energy balances for the boiler and condenser: $\dot{Q}_H = \dot{m}(h_1 - h_4)$, $\dot{Q}_C = \dot{m}(h_2 - h_3)$. Thus

$$\eta = 1 - \left[\frac{h_2 - h_3}{h_1 - h_4}\right].$$

With data from Table A-3, $h_1 - h_4 = 2015.3$ kJ/kg. Also, $h_2 = h_f + x_2(h_g - h_f)$ and $h_3 = h_f + x_3(h_g - h_f)$. So, $h_2 - h_3 = (x_2 - x_3)(h_g - h_f)$. That is,

$$h_2 - h_3 = (0.88 - 0.18)(2358.3) = 1650.8 \text{ kJ/kg}$$

Accordingly,

$$\eta = 1 - \left[\frac{1650.8}{2015.3}\right] = 0.181 \quad (18.1\%) \longleftarrow \eta$$

For a Carnot Cycle

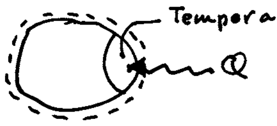
$$\eta_{MAX} = 1 - \frac{T_{MIN}}{T_{MAX}} = 1 - \frac{(60.06 + 273.15)}{(179.9 + 273.15)} = 0.265 \quad (26.5\%) \longleftarrow \eta_{MAX}$$

PROBLEM 6.48

KNOWN: A closed system undergoes a process in which Q occurs at T_b.

FIND: For each of several cases, determine whether the entropy change is positive, negative, zero, or indeterminate.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system shown in the figure interacts thermally with its surroundings only at a place on its boundary where temperature is T_b.

ANALYSIS: In this case, Eq. 6.27 takes the form $\Delta S = \frac{Q}{T_b} + \sigma$.

(a) $\sigma = 0$, $Q > 0$ $\Rightarrow$ $\Delta S = \frac{Q}{T_b} + \cancel{\sigma}^{0} > 0$

(b) $\sigma = 0$, $Q = 0$ $\Rightarrow$ $\Delta S = \cancel{\frac{Q}{T_b}}^{0} + \cancel{\sigma}^{0} = 0$

(c) $\sigma = 0$, $Q < 0$ $\Rightarrow$ $\Delta S = \frac{Q}{T_b} + \cancel{\sigma}^{0} < 0$

(d) $\sigma > 0$, $Q > 0$ $\Rightarrow$ $\Delta S = \frac{Q}{T_b} + \sigma > 0$

(e) $\sigma > 0$, $Q = 0$ $\Rightarrow$ $\Delta S = \cancel{\frac{Q}{T_b}}^{0} + \sigma > 0$

(f) $\sigma > 0$, $Q < 0$ $\Rightarrow$ $\Delta S = \underbrace{Q/T_b}_{\ominus} + \underbrace{\sigma}_{\oplus}$ Indeterminate: ΔS may be positive or negative depending on the relative magnitudes of the two terms.

PROBLEM 6.49

KNOWN: Closed systems undergo specified processes.

FIND: For each case, indicate whether the entropy change is $+$, $-$, 0, or indeterminate.

ANALYSIS: (a) Water undergoing an adiabatic process.

$\Delta S = \cancel{\int \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma \Rightarrow \Delta S = \sigma$. Since $\sigma \geq 0$, ΔS can be $+$ or 0 depending on the nature of the process. Indeterminate.

(b) Nitrogen heated internally reversibly.

$\Delta S = \int \left(\frac{\delta Q}{T}\right)_b + \cancel{\sigma}^{0}$. As entropy is transferred in the direction of heat transfer, the entropy would increase.

(c) R134a stirred adiabatically.

$\Delta S = \cancel{\int \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma$. Since stirring takes place, there would be entropy production. Thus, entropy would increase.

(d) CO_2 cooled isothermally.

$\Delta S = \int \left(\frac{\delta Q}{T}\right)_b + \sigma \Rightarrow \Delta S = \underbrace{\frac{Q}{T_b}}_{\ominus} + \underbrace{\sigma}_{\boxed{+ \text{ or } 0}}$. Indeterminate.

(e) Ideal gas undergoing a constant-pressure process, $T_2 > T_1$.
Using Eq. 6.21(a),
$\Delta S = s^\circ(T_2) - s^\circ(T_1) - \cancel{R \ln P_2/P_1}^{0}$. If $T_2 > T_1$, $s^\circ(T_2) > s^\circ(T_1)$. So, $\Delta S > 0$. Increase.

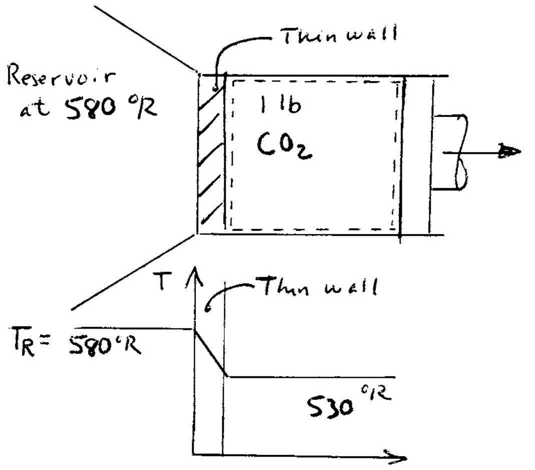
(f) Ideal gas undergoing a constant-temperature process, $P_2 < P_1$.
Using Eq. 6.21(a),
$\Delta S = \underbrace{s^\circ(T_2) - s^\circ(T_1)}_{=0} - R \ln P_2/P_1 = -R \ln P_2/P_1 > 0$ when $P_2 < P_1$ Increase

PROBLEM 6.50*

KNOWN: One lb of CO_2 expands isothermally between specified states while receiving energy through a thin intervening wall from a reservoir.

FIND: (a) For the CO_2 as the system, evaluate W, Q, σ. (b) For the CO_2 plus the thin wall as the system, evaluate σ and compare with the result of part (a).

SCHEMATIC & GIVEN DATA:



$p_1 = 180$ lbf/in^2
$T = 530$ °R
$p_2 = 15$ lbf/in^2

ASSUMPTIONS: (1) In part (a) the system is the CO_2 only. In part (b) the system is the CO_2 plus the thin wall. (2) CO_2 is modeled as an ideal gas. (3) There is no change in kinetic or potential energy. (4) The state of the thin wall does not change.

ANALYSIS: With assumptions (2), (3) an energy balance reduces to $\cancel{\Delta U}^{0} = Q - W$, where $\Delta U = 0$ because $u(T)$ for an ideal gas. Thus $Q = W$. To find W, use the ideal gas equation of state to write

$$W = m\int_1^2 p\,dv = m\int_1^2 \frac{RT}{v}\,dv = mRT\ln\frac{v_2}{v_1} = mRT\ln\frac{p_1}{p_2}$$

$$= (1\text{ lb})\left(\frac{1.986}{44.01}\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)(530\ °\text{R})\ln\left(\frac{180}{15}\right) = 59.43\text{ Btu} \quad \longleftarrow W, Q$$

An entropy balance reads

$$\Delta S = \frac{Q}{T} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T}$$

From Eq. 6.21(a), $\Delta S = -mR\ln p_2/p_1$. Also, from above $Q = mRT\ln p_1/p_2$. Thus

$$\sigma = -mR\ln\frac{p_2}{p_1} - \frac{mRT\ln\frac{p_1}{p_2}}{T} \equiv 0 \quad \text{(the process is internally reversible)} \longleftarrow \sigma$$

(b) For the enlarged system of O_2 plus thin wall an entropy balance reads

$$\Delta S = \frac{Q}{T_R} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T_R}$$

where by assumption (4) ΔS is the same as in part (a): $-mR\ln\frac{p_2}{p_1}$. Introducing $Q = mRT\ln\frac{p_1}{p_2}$ and simplifying

$$\sigma = -mR\ln\frac{p_2}{p_1} - \frac{mRT\ln\frac{p_1}{p_2}}{T_R} = mR\ln\frac{p_1}{p_2}\left[1 - \frac{T}{T_R}\right]$$

$$= (1\text{ lb})\left(\frac{1.986}{44.01}\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln\frac{180}{15}\left[1 - \frac{530}{580}\right] = 0.009667\text{ Btu/°R} \quad \longleftarrow \sigma$$

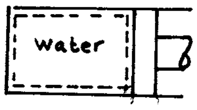
Discussion: The enlarged system includes an irreversibility not present in the CO_2 alone: heat transfer through a finite temperature difference, and so a nonzero value of σ is determined for the enlarged system.

PROBLEM 6.51

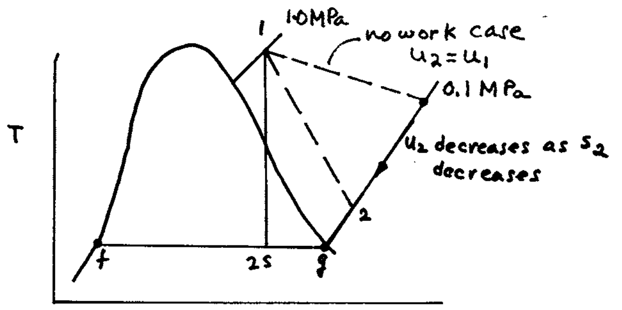
KNOWN: A quantity of water expands adiabatically from a specified state to a specified pressure.

FIND: Plot the work done versus the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



$V_1 = 0.04\,m^3$
$P_1 = 1.0\,MPa$
$T_1 = 320°C$
$P_2 = 0.1\,MPa$



ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the water. (2) The expansion occurs adiabatically. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: With assumptions 2 and 3 an energy balance reduces to

$$W = \overset{0}{\cancel{Q}} - \Delta U = m(u_1 - u_2)$$

With data from Table A-4, $u_1 = 2826.1$ kJ/kg, $v_1 = 0.2678\,m^3/kg$; $s_1 = 7.1962$ kJ/kg·K.

So,

$$W = \left(\frac{0.04\,m^3}{0.2678\,m^3/kg}\right)\left(2826.1 - u_2\right)\frac{kJ}{kg} = 0.1494\left(2826.1 - u_2\right)\,kJ \qquad (1)$$

An entropy balance reduces to read, $m(s_2 - s_1) = \sigma$. Thus

$$\sigma = (0.1494)(s_2 - 7.1962)\,\frac{kJ}{K} \qquad (2)$$

The T-s diagram shows that u_2 decreases as s_2 decreases. Thus W is a maximum and $\sigma = 0$, when $s_2 = s_1$.

Case $s_2 = s_1$: The specific internal energy at this state is the smallest allowed value: u_{2s}. To determine u_{2s}, first evaluate the quality with data from Tables A-3 and A-4

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{s_1 - s_f}{s_g - s_f} = \frac{7.1962 - 1.3026}{7.3594 - 1.3026} = 0.9731$$

$$\Rightarrow \quad u_{2s} = u_f + x_{2s}(u_g - u_f) = 417.36 + 0.9731(2506.1 - 417.36) = 2449.9\,kJ/kg$$

So

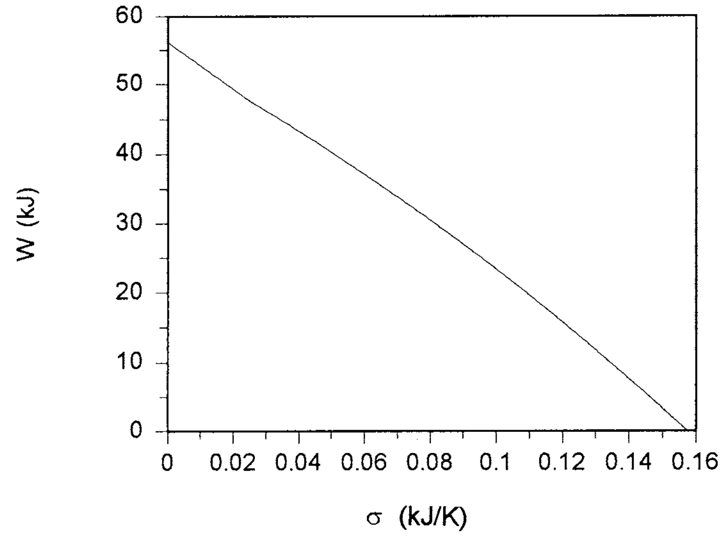
$$W_{MAX} = 0.1494(2826.1 - 2449.9) = 56.20\,kJ \quad \longleftarrow W_{MAX}$$

Case $u_2 = u_1$: In the limit as $u_2 \rightarrow u_1$, $W \rightarrow 0$, and the entropy production approaches a maximum. Interpolating in Table A-4 at 0.1MPa with $u_2 = u_1$ gives $s_2 = 8.25$ kJ/kg·K. Then

$$\sigma_{MAX} = (0.1494)(8.25 - 7.1962) = 0.1574\,\frac{kJ}{K} \quad \longleftarrow \sigma_{max}$$

PROBLEM 6.51 (Contd.)

PLOT: W vs. σ

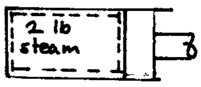


PROBLEM 6.52

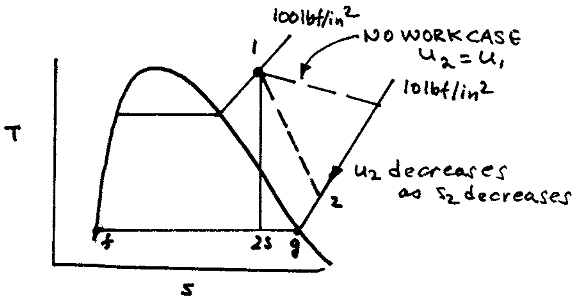
KNOWN: Two lb of steam expand adiabatically from a specified state to a specified pressure.

FIND: Plot the work done versus the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



$P_1 = 100$ lbf/in²
$T_1 = 500$°F
$P_2 = 10$ lbf/in²



ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the water. (2) The expansion occurs adiabatically. (3) There is no change in kinetic or potential energy between the end states.

ANALYSIS: (a) The work can be found from an energy balance using assumptions 2 and 3

$$W = \overset{0}{\cancel{Q}} - \Delta U = m(u_1 - u_2)$$

From Table A·4E at 100 lbf/in², 500°F, $u_1 = 1175.7$ Btu/lb, $s_1 = 1.7085$ Btu/lb·°R. Thus

$$W = (2\,\text{lb})(1175.7 - u_2)\,\frac{\text{Btu}}{\text{lb}} \qquad (1)$$

An entropy balance reduces to read $m(s_2 - s_1) = \sigma$. Thus

$$\sigma = (2\,\text{lb})(s_2 - 1.7085)\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} \qquad (2)$$

The T-s diagram shows that u_2 decreases as s_2 decreases. ~~Thus~~ W is a maximum and $\sigma = 0$ when $s_2 = s_1$,

PROBLEM 6.52 (contd.)

Case $s_2 = s_1$. To determine u_{2s}, first evaluate the quality x_{2s} with data from Tables A-3E and A-4E

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{s_1 - s_f}{s_g - s_f} = \frac{1.7085 - 0.2836}{1.5041} = 0.9473$$

Then

$$u_{2s} = u_f + x_{2s}(u_g - u_f) = 161.2 + 0.9473(1072.2 - 161.2) = 1024.19\ \text{Btu/lb}$$

Finally

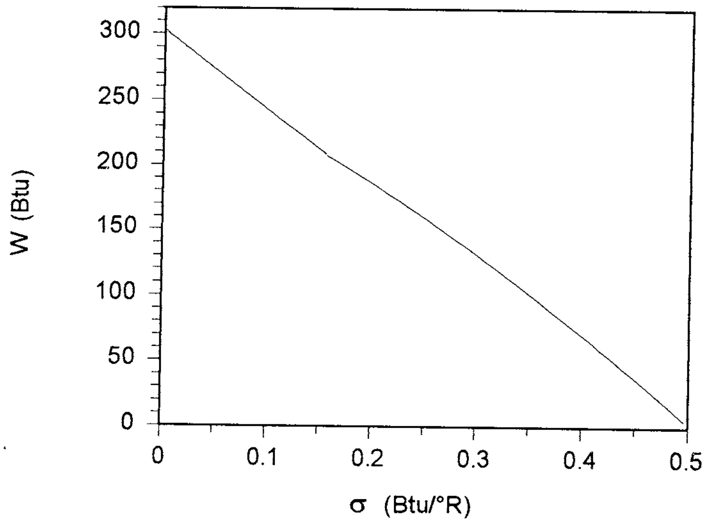
$$W_{max} = 2(1175.7 - 1024.19) = 303.02\ \text{Btu} \longleftarrow W_{max}$$

Case $u_2 = u_1$. In the limit as $u_2 \to u_1$, $W \to 0$, and the entropy production approaches a maximum.

Interpolation in Table A-4E at 10 lbf/in^2 with $u_2 = u_1$, gives $s_2 = 1.9597$ Btu/lb·°R. Then

$$\sigma_{MAX} = 2(1.9597 - 1.7085) = 0.5024\ \text{Btu/°R} \longleftarrow \sigma_{MAX}$$

PLOT: W vs σ:

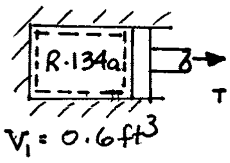


PROBLEM 6.53

KNOWN: R-134a expands adiabatically from a specified initial state to a specified pressure. The work developed is measured.

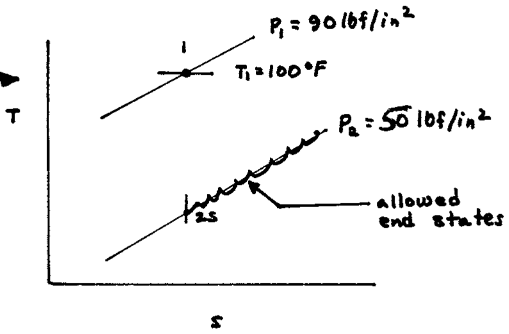
FIND: Determine if the value for work can be correct.

SCHEMATIC & GIVEN DATA:



$V_1 = 0.6\,ft^3$

$W = 5\,Btu$



ASSUMPTIONS: (1) As shown in the figure the R134a is the system. (2) The expansion occurs adiabatically and with no change in kinetic or potential energy between the end states.

ANALYSIS: The allowed end states can be determined using an entropy balance which reduces to give

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma$$

or

$$s_2 - s_1 = \sigma/m \geq 0 \Rightarrow \text{only allowed end states have } s_2 \geq s_1.$$

Next, use an energy balance to determine the end state that corresponds to the given value for W. With assumption 2

$$\Delta U = \cancel{Q}^{\,0} - W$$

or

$$u_2 - u_1 = -W/m$$

From Table A-12E, $u_1 = 108.82\,Btu/lb$, $v_1 = 0.5751\,ft^3/lb$, $s_1 = 0.2295\,Btu/lb\cdot°R$. Thus

$$m = \frac{V_1}{v_1} = \frac{0.6\,ft^3}{0.5751\,ft^3/lb} = 1.043\,lb$$

and

$$u_2 = 108.82 - \frac{5\,Btu}{1.043\,lb} = 104.03\,Btu/lb$$

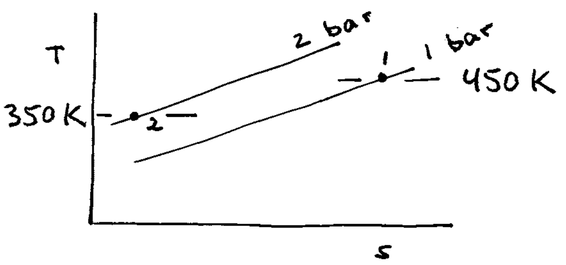
Turning to Table A-12E, interpolation at $50\,lbf/in^2$ with $u_2 = 104.03\,Btu/lb$ gives $s_2 = 0.2306\,Btu/lb\cdot°R$. Since $s_2 > s_1$, the measured value for W can be correct.

PROBLEM 6.54*

KNOWN: One kg of air undergoes a process between two specified states.

FIND: Determine if the process can occur adiabatically.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system is the one kg of air. 2. Air is modeled as an ideal gas.

ANALYSIS: Whether the process can occur adiabatically can be determined using an entropy balance together with s° data from Table A-22 and Eq. 6.21a. The entropy balance reads

$$\Delta S = \underline{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma$$

If the process takes place adiabatically, the underlined term vanishes, giving $\Delta S = \sigma$. Since σ cannot be negative, ΔS cannot be negative. Using Eq. 6.21a and s° from Table A-22

$$\Delta S = m\left[s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1}\right] = (1\,\text{kg})\left[1.85708 - 2.11161 - \frac{8.314}{28.97}\ln\frac{2}{1}\right]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$= -0.4535\ \text{kJ/K}$$

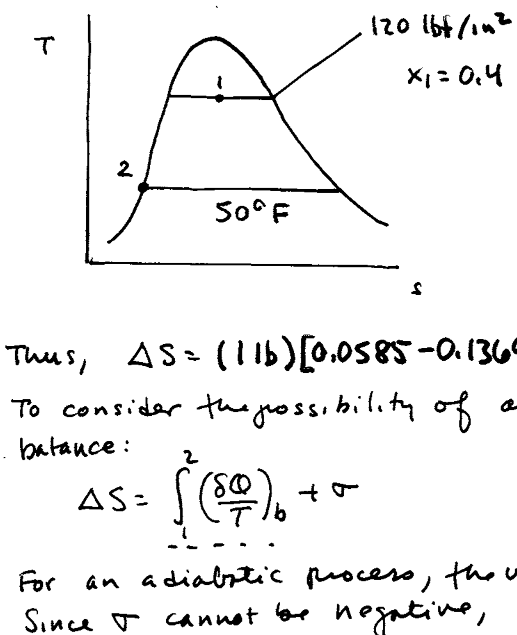
Since ΔS is negative, an adiabatic process between these states cannot occur.

PROBLEM 6.55*

KNOWN: One lb of R134a undergoes a process between specified states.

FIND: Determine the entropy change for the process. Can the process occur with $Q = 0$?

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system is the one lb of R134a.

ANALYSIS: The entropy change can be determined using data from Tables A-10E, 11E

$$s_1 = s_f + x_1(s_g - s_f) = 0.0839 + 0.4(0.2165 - 0.0839)$$

$$= 0.13694\ \text{Btu/lb}\cdot{}^\circ\text{R}$$

$$s_2 = 0.0585\ \text{Btu/lb}\cdot{}^\circ\text{R}$$

Thus, $\Delta S = (1\,\text{lb})[0.0585 - 0.13694]\,\text{Btu/lb}\cdot{}^\circ\text{R} = -0.07844\ \text{Btu/lb}\cdot{}^\circ\text{R}$

To consider the possibility of an adiabatic process, begin with an entropy balance:

$$\Delta S = \underline{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma$$

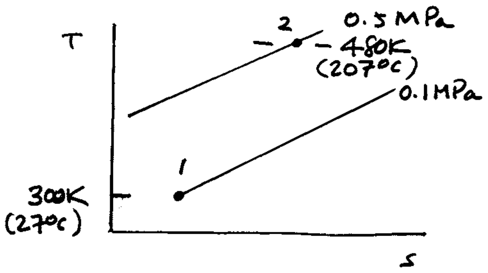
For an adiabatic process, the underlined term vanishes, giving $\Delta S = \sigma$. Since σ cannot be negative, ΔS also cannot be negative. Accordingly, the indicated process cannot occur adiabatically.

PROBLEM 6.56

KNOWN: Air is compressed between specified states.

FIND: Determine if the process can occur adiabatically. If so, evaluate the work. If not, determine the direction of the heat transfer.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system is the quantity of air under consideration. 2. Air is modeled as an ideal gas. 3. Ignore kinetic and potential energy.

ANALYSIS: Whether the process can occur adiabatically can be determined using an entropy balance together with Eq. 6.21a and s^o data from Table A-22:

The entropy balance reads, $\Delta S = \underline{\int \left(\frac{\delta Q}{T}\right)_b} + \sigma$. In an adiabatic process, the underlined term vanishes, giving: $\Delta S = \sigma$. Since σ cannot be negative, ΔS cannot be negative. Then, with Eq. 6.21a and s^o data,

$$\Delta s = s^o(T_2) - s^o(T_1) - R \ln \frac{P_2}{P_1} = 2.1776 - 1.70203 - \frac{8.314}{28.97} \ln 5 = +0.0137 \text{ kJ/kg}\cdot\text{K}$$

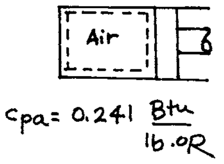
Accordingly, an adiabatic process between these states is allowed. For any such process, the energy balance reads, $\Delta U = \cancel{Q} - W$, or $W = -m(u_2 - u_1)$, $W/m = u_1 - u_2 = 214.07 - 344.7 = -130.6$ kJ/kg. ◄———

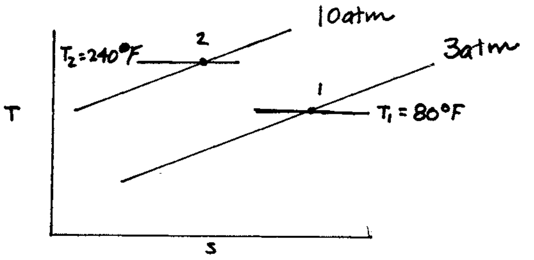
PROBLEM 6.57

KNOWN: Air is compressed in a process between two specified states.

FIND: Determine if the process can occur adiabatically. If adiabatic, evaluate the work. If not adiabatic, determine the direction of heat transfer.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the air. (2) Air is modeled as an ideal gas with $c_p = 0.241$ Btu/lb·°R.

ANALYSIS: Whether the process can occur adiabatically can be determined using an entropy balance together with Eq. 6.23 which gives

$$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 0.241 \ln \frac{700}{540} - \frac{1.986}{28.97} \ln \frac{10}{3} = -0.02 \text{ Btu/lb}\cdot°\text{R}$$

An entropy balance then reads

$$m[-0.02] = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma$$

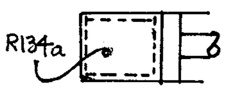
Since $\sigma \geq 0$ for all processes, the entropy transfer term must be negative which implies that energy is removed by heat transfer during the process. ◄———

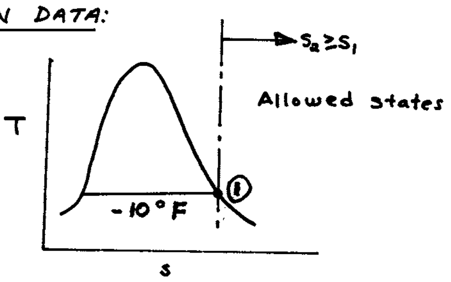
PROBLEM 6.58

KNOWN: R134a is compressed adiabatically from a specified initial state to a state where the volume is specified.

FIND: Determine if the pressure at the final state can be (a) 200 lbf/in², (b) 300 lbf/in²

SCHEMATIC & GIVEN DATA:





$m = 1.0$ lb
$V_2 = 0.8$ ft³

ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the R134a. (2) The compression occurs adiabatically.

ANALYSIS: The allowed end states can be determined using an entropy balance which reduces to give

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma$$

or

$$s_2 - s_1 = \sigma/m \geq 0 \Rightarrow \text{only allowed states have } s_2 \geq s_1 .$$

From Table A-10E at -10°F, $s_1 = 0.2236$ Btu/lb·°R. At state 2,

$$v_2 = \frac{V_2}{m} = \frac{0.8 \text{ ft}^3}{1 \text{ lb}} = 0.8 \text{ ft}^3/\text{lb}.$$

(a) $p_2 = 60$ lbf/in²:

From Table A-12E at $p_2 = 60$ lbf/in², $v_2 = 0.8$ ft³/lb, we see that $s_2 < s_1$, and so this state is not allowed. No. ◄———

(b) $p_2 = 70$ lbf/in²:

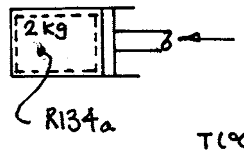
From Table A-12E at $p_2 = 70$ lbf/in², $v_2 = 0.8$ ft³/lb, we see that $s_2 > s_1$, and so this state is allowed. Yes. ◄———

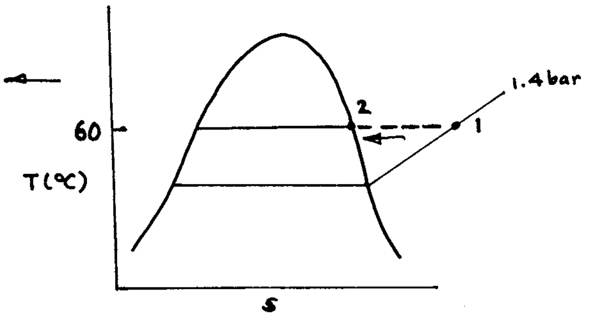
PROBLEM 6.59

KNOWN: Two kg of R12 is compressed between two specified states, during which the temperature of the R134a is 60°C ± 0.1 °C.

FIND: Determine the minimum theoretical heat transfer from the refrigerant.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The R134a is the closed system. (2) The temperature at which heat transfer occurs is $T_b = 60$°C.

ANALYSIS: An entropy balance for the process reads

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \sigma = \frac{Q}{T_b} + \sigma$$

PROBLEM 6.59 (Continued)

Thus

$$Q = T_b \Delta S - T_b \sigma$$

With data from Tables A-10 and A-12, $s_1 = 1.1690$ kJ/kg·K, $s_2 = 0.8973$ kJ/kg·K. Then

$$Q = (333K)(2kg)(0.8973 - 1.1690)\frac{kJ}{kg\,K} - T_b \sigma$$

$$= -180.95\,kJ - \underline{T_b \sigma}$$

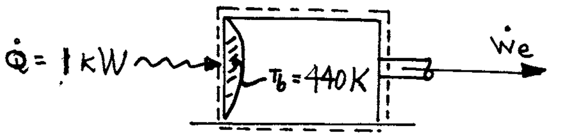
As $\sigma \geq 0$, the underlined term is positive (or zero). Thus, $Q_{min} = -180.95$ kJ ←

PROBLEM 6.60

KNOWN: A device operating at steady state receives 1 kW by heat transfer at 167°C and generates electricity. There are no other energy transfers.

FIND: Determine if the claimed performance of the device violates any principles of thermodynamics.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the accompanying figure operates at steady state. (2) The system receives energy by heat transfer at T_b = 440K (167°C).

ANALYSIS: An energy rate balance for the device reduces with assumption 1 to give

$$\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \dot{W}_e \Rightarrow \dot{W}_e = \dot{Q} = 1\ kW$$

This result is in accord with the conservation of energy principle. (If $\dot{W}_e$ had any other value, there would be a violation of this principle.)

With assumptions 1 and 2, entropy rate balance reduces to give

$$\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b}$$

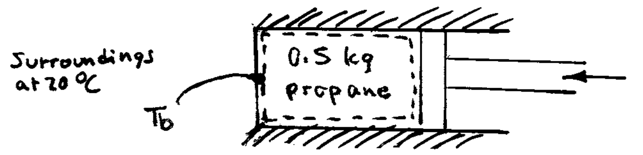
Thus

$$\dot{\sigma} = -\frac{(1\,kW)}{440K} = -2.27\times10^{-3}\ \frac{kW}{K}$$

However, as entropy must be produced the claimed performance violates the second law of thermodynamics.

PROBLEM 6.61*

KNOWN: Data is provided for a quantity of propane rapidly compressed.
FIND: Determine whether the reported value for the work is feasible.
SCHEMATIC & GIVEN DATA:



$W = -72.5$ kJ
$P_1 = 4$ bar , $T_1 = 30°C$
$P_2 = 14$ bar, $T_2 = 100°C$

ASSUMPTIONS: 1. The propane is the closed system. 2 Heat transfer with the surroundings occurs at T_b, which cannot be less than 293 K (20°C) and is most likely in the interval from 303 K (30°C) to 373 K (100°C). Kinetic and potential energy effects can be ignored.

ANALYSIS: An energy balance reduces to $\Delta U = Q - W$, or with data from Table A-15

$$Q = m(u_2 - u_1) + W = (0.5\,kg)[1480.79 - 1377.49]\frac{kJ}{kg} + (-72.5\,kJ) = -20.85\,kJ$$

An entropy balance takes the form,

$$\Delta S = \frac{Q}{T_b} + \sigma \Rightarrow \sigma = m[s_2 - s_1] - \frac{Q}{T_b}$$

With Eq. 6.21a and data from Table A-15

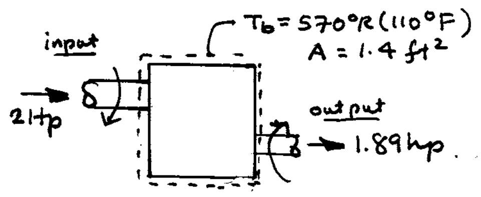
$$\sigma = (0.5\,kg)\left[(5.4433 - 5.6328)\frac{kJ}{kg\cdot K}\right] - \frac{(-20.85\,kJ)}{T_b}$$

$$\sigma = -0.09475\,\frac{kJ}{kg} + \frac{20.85\,kJ}{T_b}$$

For $T_b \geq 293$ K, $\sigma < 0$. Thus, the value for W cannot be correct.

PROBLEM 6.62

KNOWN: Operating data are provided for a gearbox at steady state.
FIND: Determine the rate of heat transfer and the rate of entropy production. Can the heat transfer be achieved by free convection?
SCHEMATIC & GIVEN DATA:



Surroundings at $T_f = 530°R (70°F)$

ASSUMPTIONS: 1. The system shown in the schematic is at steady state. 2. Heat transfer occurs at temperature T_b and is by convection.

ANALYSIS: At steady state an energy rate balance reduces to $\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \dot{W}$, or

$$\dot{Q} = \dot{W} \Rightarrow \dot{Q} = [1.89\,hp - 2\,hp]\left|\frac{2545\,Btu/h}{1\,hp}\right|\left|\frac{1\,h}{3600\,s}\right| = -7.78\times10^{-2}\,\frac{Btu}{s} \longleftarrow$$

At steady state an entropy rate balance reduces to $\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}}{T_b} + \dot{\sigma}$, or

$$\dot{\sigma} = -\frac{\dot{Q}}{T_b} = -\frac{(-7.78\times10^{-2}\,Btu/s)}{570°R} = 1.36\times10^{-4}\,\frac{Btu}{s\cdot°R} \longleftarrow$$

Using Eq. 2.34 written as $\dot{Q} = -hA[T_b - T_f]$, where the minus sign is needed for consistency with the sign for $\dot{Q}$ in the energy rate balance. Solving for the heat transfer coefficient,

$$h = \frac{-\dot{Q}}{A[T_b - T_f]} = -\frac{(-7.78\times10^{-2}\,Btu/s)\left|\frac{3600\,s}{h}\right|}{(1.4\,ft^2)(570-530)°R} = 5\,\frac{Btu}{h\cdot ft^2\cdot°R}$$

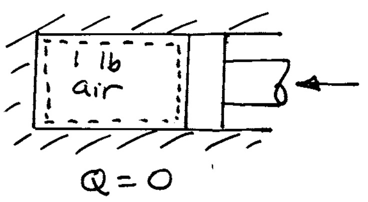
. Checking Table 2.1 with this value for h suggests that *forced* convection would be required.

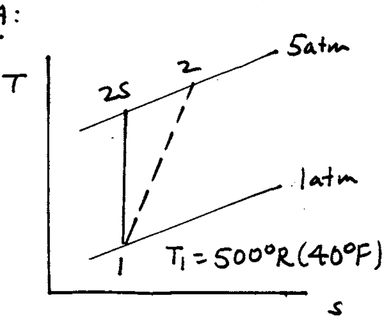
PROBLEM 6.63

KNOWN: One lb of air is compressed adiabatically from 40°F, 1atm to 5 atm.

FIND: If the air is compressed without internal irreversibilities, find the final temperature and the work. If the work is 20% greater than the value for no internal irreversibilities, find the final temperature and the amount of entropy produced. Show both processes on T-s coordinates.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The system is the air. 2. Air is modeled as an ideal gas. 3. There is no heat transfer nor significant kinetic/potential energy effects. 4. For part (a), $\sigma = 0$.

ANALYSIS: With assumption 3, the energy balance reduces to $\Delta U = \cancel{Q}^{\,0} - W$, which gives the following expression for the work input: $(-W) = m(u_2 - u_1)$. The entropy balance reduces to $\Delta S = \cancel{\int\left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma$, which gives $\sigma = m(s_2 - s_1)$.

(a) $\sigma = 0$. Reducing the above entropy balance gives $s_2 = s_1$. Thus, the process is 1-2s on the above T-s diagram. The corresponding expression for work is $(-W)_s = m(u_{2s} - u_1)$. From Table A-22E, $u_1 = 85.2$ Btu/lb. To find u_{2s}, first apply Eq. 6.21a with $s_2 = s_1$, obtaining

$$s^\circ_{2s} = s^\circ_1 + R \ln \frac{p_2}{p_1} \Rightarrow s^\circ_{2s} = 0.58233 + \frac{1.986}{28.97} \ln 5 = 0.69266$$

Then, interpolation in Table A-22E with s°_{2s} gives $u_{2s} = 135.3$ Btu/lb, $T_{2s} = 791°R\ (331°F)$. Finally

$$(-W)_s = (1\ \text{lb})(135.3 - 85.2)\frac{\text{Btu}}{\text{lb}} = 50.1\ \text{Btu}$$

(b) The actual work input is 20% greater than in part (a): $(-W) = 1.2(50.1) = 60.1$ Btu. Thus, the expression for $(-W)$ can be rearranged to give

$$u_2 = \frac{(-W)}{m} + u_1 = \left(\frac{60.1\ \text{Btu}}{1\ \text{lb}}\right) + 85.2\frac{\text{Btu}}{\text{lb}} = 145.3\frac{\text{Btu}}{\text{lb}}$$

Interpolating in Table A-22E, $T_2 = 848°R\ (388°F)$, $s^\circ_2 = 0.70963$. Finally,

$$\sigma = m\left[s^\circ_2 - s^\circ_1 - R \ln \frac{p_2}{p_1}\right] = (1\ \text{lb})\left[0.70963 - 0.58233 - \frac{1.986}{28.97} \ln 5\right]\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

$$= 0.017\ \frac{\text{Btu}}{°\text{R}}.$$

PROBLEM 6.64

KNOWN: Data are provided for a silicon chip at steady state.

FIND: Determine the rate of entropy production and identify the principal source of irreversibility.

SCHEMATIC & GIVEN DATA: See Fig. E 2.5.

ASSUMPTIONS: 1. The chip is a closed system at steady state. 2. There is no heat transfer between the chip and the substrate. All heat transfer is by convection from the top surface at T_b.

ANALYSIS: An entropy rate balance at steady state reads

$$\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} \quad \text{where} \quad \dot{Q} = -0.225\text{ W (from Example 2.5).}$$

Thus

$$\dot{\sigma} = \frac{-(-0.225)}{353\text{ K}} = 6.37 \times 10^{-4}\ \frac{\text{watt}}{\text{K}} \left| \frac{1\text{ kW}}{10^3\text{ W}} \right| = 6.37 \times 10^{-7}\ \frac{\text{kW}}{\text{K}} \longleftarrow$$

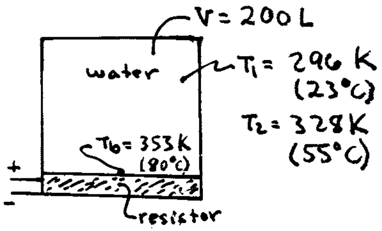
The principal source of internal irreversibility is electric current flow through a resistance.

PROBLEM 6.65*

KNOWN: Data are provided for an electric water heater.

FIND: For each of two systems determine the amount of entropy produced. Compare.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. In part (a) the system is the water alone. In part (b) the system is the water plus the resistor. 2. The water is modeled as incompressible. 3. Heat transfer from the outer surface of the water heater can be ignored. 4. The states of the tank wall and the resistor are constant.

(a) For the water as the system, the entropy balance reduces to $\Delta S = \frac{Q}{T_b} + \sigma$. The heat transfer to the water from the resistor is found from an energy balance on the water: $\Delta U = Q - \cancel{W}^{0}$, which with Eq. 3.20a takes the form

$Q = \Delta U \Rightarrow Q = m[c\Delta T]$. With Eq. 6.24, $\Delta S = mc \ln T_2/T_1$. Collecting results

$$\sigma = \Delta S - \frac{Q}{T_b} \Rightarrow \sigma = mc\left[\ln\frac{T_2}{T_1} - \frac{(T_2 - T_1)}{T_b}\right]$$

Using the known volume and $v = v_f(T_{ave}) = 1.008 \times 10^{-3}\ \text{m}^3/\text{kg}$ from Table A-2 and $c = 4.18\ \text{kJ/kg}\cdot\text{K}$ from Table A-19

$$m = \frac{V}{v} = \frac{(200\text{ L})\,|10^{-3}\text{ m}^3/\text{L}|}{1.008 \times 10^{-3}\text{ m}^3/\text{kg}} = 198.4\text{ kg}, \quad \sigma = (198.4\text{ kg})\left(4.18\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left[\ln\frac{328}{296} - \frac{(32)}{353}\right] = 9.95\ \frac{\text{kJ}}{\text{K}} \longleftarrow \text{(a)}$$

(b) For the water plus the resistor as the system, the entropy balance reduces to

$\Delta S = \cancel{\int\left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma \Rightarrow \sigma = \Delta S$. With assumption 4, ΔS can be identified with the water only: $\Delta S = mc \ln\frac{T_2}{T_1}$. Thus

$$\sigma = mc \ln\frac{T_2}{T_1} = (198.4\text{ kg})\left(4.18\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\frac{328}{296} = 85.13\ \frac{\text{kJ}}{\text{K}} \longleftarrow \text{(b)}$$

The entropy production in the enlarged system is greater because there is an additional source of irreversibility — namely, electric current flow through the resistor. For the water alone, the irreversibility is associated with internal heat transfer from the resistor through the water as it is heated.

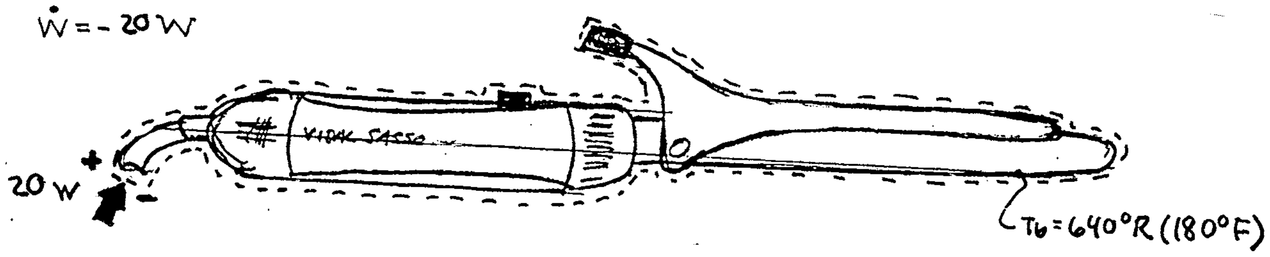
PROBLEM 6.66*

KNOWN: At steady state the outer surface of a 20-W curling iron is at 180°F

FIND: Determine the rate of heat transfer and the rate of entropy production.

SCHEMATIC & GIVEN DATA:

$\dot{W} = -20\ \text{W}$



ASSUMPTIONS: (1) The system shown in the figure is at steady state. (2) Heat transfer occurs at $T_b = 640°R\ (180°F)$ only.

ANALYSIS: An energy rate balance reduces to read

$$\overset{0}{\cancel{\frac{dE}{dt}}} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} = (-20\,\text{W})\left(3.413\,\frac{\text{Btu}}{\text{W}\cdot\text{h}}\right) = -68.3\ \text{Btu/h} \longleftarrow \dot{Q}$$

An entropy rate balance reduces to read

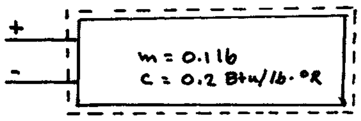
$$\overset{0}{\cancel{\frac{dS}{dt}}} = \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} = \frac{68.3\ \text{Btu/h}}{640°\text{R}} = 0.107\ \frac{\text{Btu}}{\text{h}\cdot°\text{R}} \longleftarrow \dot{\sigma}$$

PROBLEM 6.67

KNOWN: Data are provided for a thermally insulated resistor.

FIND: Plot the temperature and amount of entropy produced versus t for $0 \le t \le 3\,\text{s}$.

SCHEMATIC & GIVEN DATA:



R = 30 ohm
I = 6 amps
T_1 = 70°F

ASSUMPTIONS: (1) The system shown in the accompanying figure is modeled as an incompressible substance. (2) Q = 0.

ANALYSIS An energy balance reduces to give

$$\Delta U = \overset{0}{\cancel{Q}} - W \Rightarrow m(u_2 - u_1) = -W$$

With assumption 1

$$mc\,(T_2 - T_1) = -W \Rightarrow T_2 = T_1 - W/mc$$

Electrical work is done on the system. Using given data

$$W = -I^2 R \Delta t = -(6\,\text{amps})^2 (30\,\text{ohm})(\Delta t)\left(\frac{\text{W}}{(\text{amps})^2\cdot\text{ohm}}\right)$$

$$= -1080\,\Delta t\ (\text{W}\cdot\text{s})$$

Accordingly

$$T_2 = 530°\text{R} - \frac{(-1080\,\Delta t\ (\text{W}\cdot\text{s}))}{(0.1\,\text{lb})(0.2\,\text{Btu/lb}\cdot°\text{R})}\left(\frac{3.413\ \text{Btu/h}}{1\ \text{W}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right).$$

$$= (530 + 51.2\,\Delta t)\ °\text{R} \qquad (1)$$

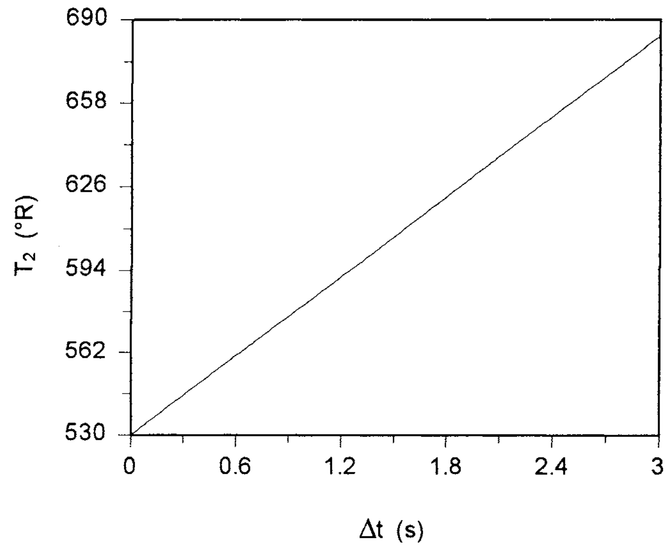
An entropy balance reduces to give

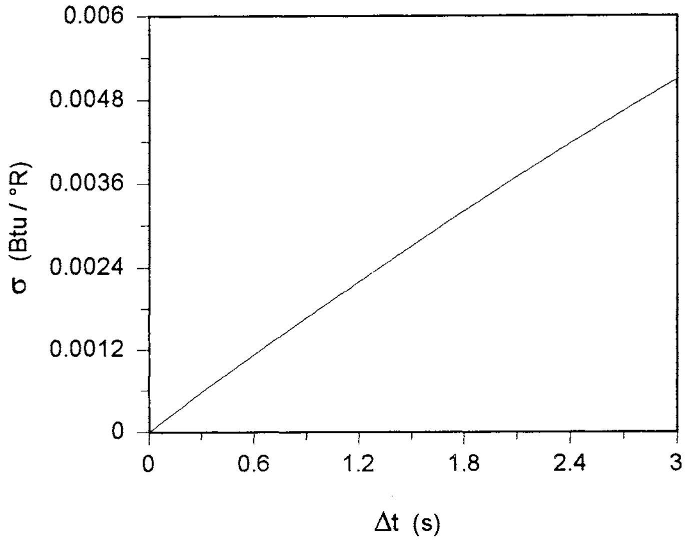
$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma \Longrightarrow \sigma = \Delta S = mc \ln \frac{T_2}{T_1} \qquad \text{(Eq. 6.24)}$$

PROBLEM 6.67 (Contd.)

Introducing Eq.(1) the expression for σ becomes

$$\sigma = (0.1\,\text{lb})\left(0.2\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln\left(1+\frac{51.2\,\Delta t}{530}\right) = 0.02\,\frac{\text{Btu}}{{}^\circ\text{R}}\ln\left(1+\frac{51.2\,\Delta t}{530}\right) \qquad (2)$$



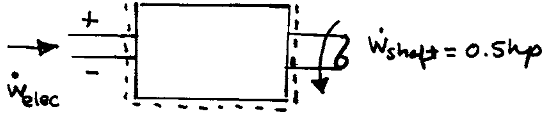


PROBLEM 6.68

KNOWN: Data are provided for an electric motor at steady state.

FIND: For the motor, determine the rates of heat transfer and entropy production.

SCHEMATIC & GIVEN DATA:



Motor draws 4 amps at 120 Volts

ASSUMPTIONS: 1. The system shown in the accompanying figure operates at steady state.
2. The power quantities indicated are in the directions of the respective arrows.

ANALYSIS: At steady state, the energy rate balance reduces to give

$$\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} \quad \text{where} \quad \dot{W} = \dot{W}_{shaft} - \dot{W}_{elec}. \text{ That is}$$

$$\dot{W} = 0.5\,\text{hp}\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right| - \left[(4\,\text{amps})(120\,\text{volts})\left|\frac{1\ \text{W}}{\text{amp}\cdot\text{volt}}\right|\left|\frac{3.413\ \text{Btu/h}}{1\ \text{W}}\right|\right] = -365.7\ \frac{\text{Btu}}{\text{h}}$$

$$\Rightarrow \dot{Q} = -365.7\ \text{Btu/h}.$$

At steady state, an entropy rate balance reduces to give

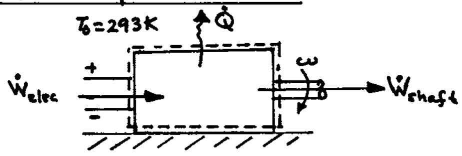
$$\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} = -\frac{(-365.7\ \text{Btu/h})}{580\ {}^\circ\text{R}} = 0.631\ \frac{\text{Btu}}{\text{h}\cdot{}^\circ\text{R}}$$

PROBLEM 6.69

KNOWN: Operating data are provided for an electric motor at steady state.

FIND: (a) Determine the motor outer surface temperature. Evaluate the rate of entropy production for (b) the motor as the system, (c) the motor plus the nearby surroundings as the system.

SCHEMATIC & GIVEN DATA:



$\dot Q = hA(T_b - T_0)$

$h = 100$ W/m²·K

$A = 0.195$ m²

$\omega = 1000$ RPM

$\mathcal{T} = 16$ N·m

$i = 10$ amp

$\mathcal{E} = 220$ Volts

ASSUMPTIONS: (1) Two systems at steady state are considered: The motor. The motor plus the nearby surroundings. (2) The temperature at the outer surface of the motor, T_b, is uniform with position. (3) Energy transfers are in the directions of the arrows.

ANALYSIS: (a) The temperature T_b can be evaluated from an energy rate balance applied to the motor. At steady-state, the rate energy enters equals the rate energy exits. Thus, for energy transfer in the directions indicated by the arrows

$$\dot W_{elect} = \dot W_{shaft} + \dot Q$$

where $\dot Q$ is given by the expression above and the other two terms are evaluated using Eq. 2.20 and 2.21:

$$\dot W_{shaft} = \mathcal{T}\omega = (16\ \text{N·m})\left(1000\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60\ \text{s}}\right)\left(\frac{1\ \text{kW}}{10^3\ \text{N·m}}\right) = 1.676\ \text{kW}$$

$$\dot W_{elec} = \mathcal{E}\,i = (220\ \text{volts})(10\ \text{amp})\left[\frac{1\ \text{watt/amp}}{1\ \text{volt}}\right]\left(\frac{\text{kW}}{10^3\ \text{W}}\right) = 2.2\ \text{kW}$$

Thus

$$2.2\ \text{kW} = 1.676\ \text{kW} + \left(100\ \frac{\text{W}}{\text{m}^2\text{K}}\right)(0.195\ \text{m}^2)\left(\frac{\text{kW}}{10^3}\right)(T_b - 293\ \text{K}) \Rightarrow T_b = 319.9\ \text{K} \longleftarrow T_b$$

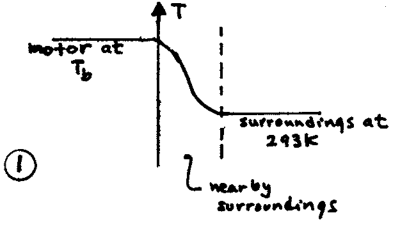
(b) With assumption 2, and taking in account that $\dot Q$ is positive in the direction of the arrow on the sketch above, an entropy rate balance for the motor as the system reduces at steady state to give

$$\cancel{\frac{dS}{dt}}^{\,0} = \frac{(-\dot Q)}{T_b} + \dot\sigma \quad\Rightarrow\quad \dot\sigma = \frac{\dot Q}{T_b}$$

From part (a), $\dot Q = \dot W_{elect} - \dot W_{shaft} = 0.524$ kW, so

$$\dot\sigma = \frac{0.524\ \text{kW}}{319.9\ \text{K}} = 1.64 \times 10^{-3}\ \frac{\text{kW}}{\text{K}} \longleftarrow \dot\sigma$$

(c) For a system consisting of the motor plus the nearby surroundings, the heat transfer takes place at $T_0 = 293$ K. At steady state an entropy rate balance reduces to give for this enlarged system



①

$$\dot\sigma = \frac{\dot Q}{T_0}$$

$$= \frac{0.524\ \text{kW}}{293\ \text{K}} = 1.79 \times 10^{-3}\ \frac{\text{kW}}{\text{K}} \longleftarrow \dot\sigma$$

---

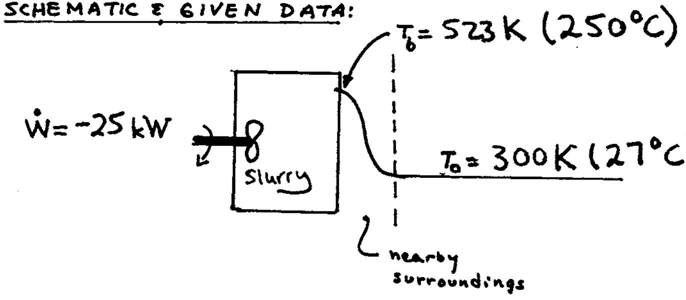
1. The rate of entropy production is greater for the enlarged system because it includes an additional source of irreversibility associated with heat transfer from the motor at T_b to the surroundings at T_0.

PROBLEM 6.70*

KNOWN: Operating data are provided for a closed tank fitted with a paddle wheel and containing a slurry.

FIND: Determine the rate of entropy production for systems consisting of (a) the tank and its contents, (b) the tank and its nearby surroundings.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Two systems are under consideration. One comprises the tank and the slurry within the tank. The other includes the tank, the slurry, and as shown in the sketch the nearby surroundings. (2) Each system operates at steady state, while receiving a power input from a paddle wheel.

ANALYSIS: An energy rate balance reduces at steady state to give

$$\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} = -25 \text{ kW}$$

(a) Tank and slurry as the system. An entropy rate balance reduces at steady state to give

$$\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}}{T_b} + \dot{\sigma}$$

$$\Rightarrow \quad \dot{\sigma} = -\frac{\dot{Q}}{T_b} = -\frac{(-25 \text{ kW})}{523 \text{ K}} = 0.0478 \text{ kW/K}$$

(b) Tank, slurry, and nearby surroundings as the system. An entropy rate balance reduces at steady state to give

① $$\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}}{T_0} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{(-25 \text{ kW})}{300 \text{ K}} = 0.0833 \frac{\text{kW}}{\text{K}}$$

---

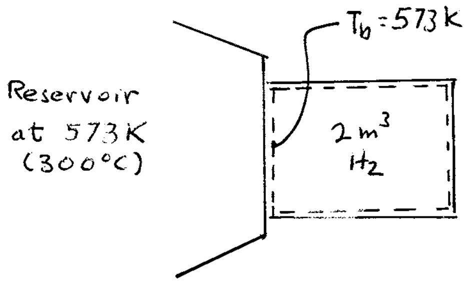
1. The entropy production rate is greater in part (b) than in part (a) because the enlarged system of part (b) has an additional source of irreversibility: the irreversibility associated with heat transfer from the tank to the surroundings.

PROBLEM 6.71

KNOWN: A system consisting of a fixed volume of hydrogen gas is brought from a specified initial state to a specified final temperature by heat transfer from a reservoir.

FIND: For the hydrogen, find Q, ΔS, and σ. For the reservoir, find ΔS. Compare ΔS values and discuss

SCHEMATIC & GIVEN DATA:



$P_1 = 215$ kPa
$T_1 = 308$K (35°C)
$T_2 = 433$ (160°C)

ASSUMPTIONS: (1) The hydrogen gas is the system. (2) The hydrogen gas is modeled as an ideal gas. (3) Heat transfer takes place at $T_b = 573$K. (4) Kinetic and potential energy effects are absent.

ANALYSIS: An energy balance reduces to give $\Delta U = Q - \cancel{W}^{\,0}$, or

$$Q = m(u_2 - u_1)$$

Inspection of Table A-20 indicates that c_v varies only slightly for H_2 over the range 308 to 433 K. Taking the value at $T_{ave} = 371$ K and evaluating m using the ideal gas equation of state

$$Q = \left(\frac{P_1 V}{R T_1}\right) c_v \left[T_2 - T_1\right] = \left[\frac{(215 \times 10^3\ N/m^2)(2\,m^3)}{\left(\frac{8314}{2.016}\ \frac{N\cdot m}{kg\cdot K}\right)(308K)}\right]\left(10.32\ \frac{kJ}{kg\cdot K}\right)\left[433 - 308\right]K = 436.7\ kJ \longleftarrow$$

To find ΔS, use Eq. 6.22

$$\Delta S = m\left[c_v \ln\frac{T_2}{T_1} + \cancel{R \ln\frac{v_2}{v_1}}^{\,0}\right] = \left[\frac{(215 \times 10^3)(2)}{\left(\frac{8314}{2.016}\right)(308)}\right](10.32)\ln\left(\frac{433}{308}\right) = 1.19\ \frac{kJ}{K} \longleftarrow$$

With assumption 3, an entropy balance reduces to

$$\Delta S = \frac{Q}{T_b} + \sigma \Rightarrow \quad \sigma = \Delta S - \frac{Q}{T_b} = \left(1.19 - \frac{436.7}{573}\right) = 0.428\ \frac{kJ}{K} \longleftarrow$$

Taking the reservoir as the system, the entropy balance reads

$$\Delta S)_{res} = \frac{Q)_{res}}{T_b} + \cancel{\sigma_{res}}^{\,0} = \frac{(-436.7)}{573} = -0.762\ \frac{kJ}{K}$$

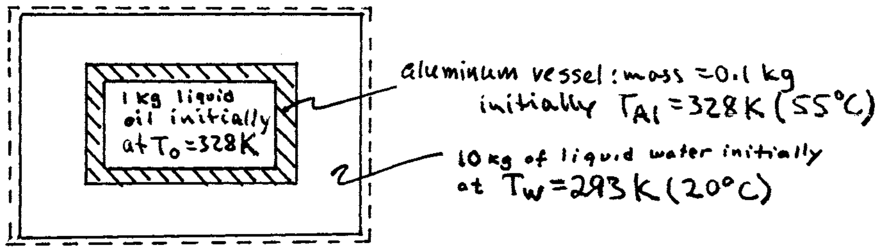
Entropy is transferred from the reservoir accompanying heat transfer, and so the entropy of the reservoir decreases by the amount carried out. For the hydrogen, entropy is carried in accompanying heat transfer. In addition, entropy is produced owing to irreversibilities. Accordingly, the entropy change for the hydrogen exceeds the amount carried in by the amount produced.

PROBLEM 6.72*

KNOWN: Data are provided on an isolated system consisting of an aluminum vessel, oil, and liquid water.

FIND: Determine (a) the final temperature when the system has come to equilibrium, (b) the entropy change for the aluminum vessel, the oil, and the water, (c) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:


1 kg liquid oil initially at T_0=328K
aluminum vessel: mass =0.1 kg initially T_{Al}=328 K (55°C)
10 kg of liquid water initially at T_W=293 K (20°C)

ASSUMPTIONS: (1) The system shown in the accompanying figure is isolated. (2) The aluminum vessel and each of the liquid masses can be modeled as incompressible with constant specific heat.

ANALYSIS: From Table A-19, the specific heat of aluminum is C_{Al}= 0.9 kJ/kg·K and of liquid water c_w = 4.18 kJ/kg·K, and of engine oil C_o = 1.91 kJ/kg·K.

(a) With assumption 1, Q = W = 0 and an energy balance reduces to

$$\Delta U = \cancelto{0}{Q} - \cancelto{0}{W}$$

$$(\Delta U)_w + (\Delta U)_0 + (\Delta U)_{Al} = 0$$

With assumption 2

$$m_w C_w[T_f - T_W] + m_o C_o[T_f - T_o] + m_{Al} C_{Al}[T_f - T_{Al}] = 0$$

where m_W and T_W are the mass and initial temperature of liquid water, m_o and T_o are the mass and initial temperature of the oil, and m_{Al} and T_{Al} are the mass and initial temperature of the vessel.

Solving for the final temperature, T_f

$$T_f = \frac{m_w C_w T_W + m_o C_o T_o + m_{Al} C_{Al} T_{Al}}{m_w C_w + m_o C_o + m_{Al} C_{Al}}$$

$$= \frac{(10)(4.18)(293) + (1)(1.91)(328) + (0.1)(0.9)(328)}{(10)(4.18) + (1)(1.91) + (0.1)(0.9)} = 294.6\,K\,(21.6°C) \xleftarrow{T_f}$$

(b) Entropy changes. Using Eq. 6.24 (assumption 2)

$$(\Delta S)_w = m_w C_w \ln\frac{T_f}{T_W} = (10)(4.18)\ln\frac{294.6}{293} = 0.22764\ \mathrm{kJ/K} \xleftarrow{\Delta S}$$

$$(\Delta S)_o = m_o C_o \ln\frac{T_f}{T_o} = (1)(1.91)\ln\frac{294.6}{328} = -0.20512\ \mathrm{kJ/K}$$

$$(\Delta S)_{Al} = m_{Al} C_{Al} \ln\frac{T_f}{T_{Al}} = (0.1)(0.9)\ln\frac{294.6}{328} = -0.00967\ \mathrm{kJ/K}$$

(c) Since there is no heat transfer to or from the system an entropy balance gives

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma \implies \sigma = (\Delta S)_W + (\Delta S)_0 + (\Delta S)_{Al}$$

$$= 0.01285\ \frac{kJ}{K} \xleftarrow{\sigma}$$

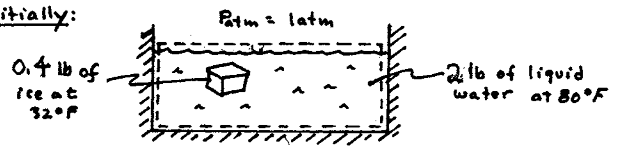
PROBLEM 6.73

KNOWN: A system initially consisting of 0.4 lb of ice at 32°F and 2 lb of liquid water at 80°F attains an equilibrium state adiabatically at a constant pressure of 1 atm.

FIND: Determine the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:

initially:



$h_{if} = 144$ Btu/lb

ASSUMPTIONS: (1) As shown in the accompanying figure, the system consists of the ice and liquid. (2) $Q = 0$ and pressure remains constant. (3) The liquid is incompressible with constant specific heat.

ANALYSIS: (a) The final temperature can be determined using an energy balance. Thus, $\Delta U = \cancel{Q}^{0} - W$, where $W = \int_1^2 p\,dV = p\Delta V$ since pressure is constant. So

$$\Delta U = -p\Delta V \Rightarrow \Delta H = 0 \quad \text{for the constant pressure process}$$

If all the ice does not melt, the final temperature would be 32°F. To investigate this, let T_f be the final temperature when all of the ice is regarded to have melted. Then with $c = 1.0$ Btu/lb·°R for liquid water from Table A-19E

$$\left\{ \Delta H\Big]_{ice} + \Delta H\Big]_{\substack{melt \\ 32°F \to T_f}} \right\} + \Delta H\Big]_{\substack{liquid \\ 80°F \to T_f}} = 0$$

$$\left\{(0.4\,\text{lb})\left(144\,\frac{\text{Btu}}{\text{lb}}\right) + (0.4\,\text{lb})\left(1\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)(T_f - 492)°\text{R}\right\} + (2\,\text{lb})\left(1\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)(T_f - 540)°\text{R} = 0$$

$$\Rightarrow T_f = 508°\text{R} \quad (48°\text{F}) \longleftarrow T_f$$

As $T_f > 32°F$, it can be concluded that all the ice melts.

(b) An entropy balance reduces to give

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma \Rightarrow \sigma = \Delta S$$

Thus

$$\sigma = \left\{ \Delta S\Big]_{\substack{ice \\ at\,32°F}} + \Delta S\Big]_{\substack{melt \\ 32°F \to T_f}} \right\} + \Delta S\Big]_{\substack{Liquid \\ 80°F \to T_f}}$$

The two liquid terms can be evaluated using Eq. 6.24. The entropy change for the melting ice can be found using the TdS equation, Eq 6.12b, which at constant pressure and temperature reduces to give

$$ds = \frac{dh}{T} \Rightarrow s_f - s_i = \frac{h_f - h_i}{T} = \frac{144\,\text{Btu/lb}}{492\,°\text{R}} = 0.2927\,\text{Btu/lb}\cdot°\text{R}$$

Thus

$$\sigma = (0.4\,\text{lb})\left(0.2927\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) + (0.4\,\text{lb})\left(1\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln\frac{508}{492} + (2\,\text{lb})\left(1\,\frac{\text{Btu}}{\text{lb}}\right)\ln\left(\frac{508}{540}\right)$$

$$= (0.1171) + (0.0128) + (-0.1222) = 0.0077\ \text{Btu/°R} \longleftarrow \sigma$$

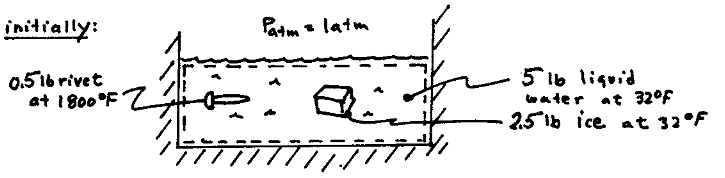
PROBLEM 6.74

KNOWN: A system initially consisting of a rivet and a two-phase solid-liquid mixture of water attains an equilibrium state adiabatically at constant pressure.

FIND: Determine the final temperature and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:

initially:



$h_{if} = 144$ Btu/lb

$c_r = 0.12$ Btu/lb·°R

ASSUMPTIONS: (1) As shown in the accompanying figure, the system consists of the ice, liquid, and rivet. (2) $Q = 0$ and pressure remains constant. (3) The rivet and liquid can be regarded as incompressible with constant specific heats.

ANALYSIS: (a) The final temperature can be determined using an energy balance. Thus, $\Delta U = \cancel{Q}^{0} - W$, where $W = \int_1^2 p\,dV = p\,\Delta V$ since pressure is constant. So

$$\Delta U = -p\Delta V \Rightarrow \Delta H = 0 \text{ for the constant pressure process.}$$

If all the ice does not melt, the final temperature would be 32°F. To investigate this, let T_f be the final temperature when all of the ice is regarded to have melted. Then, with $c = 1.0$ Btu/lb·°R for liquid water from Table A-19E

$$\left\{ \Delta H\Big]_{\substack{ice\\ at\ 32°F}} + \Delta H\Big]_{\substack{melt\\ 32°F \to T_f}} \right\} + \Delta H\Big]_{\substack{liquid\\ 32°F \to T_f}} + \Delta H\Big]_{\substack{rivet\\ 1800°F \to T_f}} = 0$$

$$0 = \left\{ (2.5\,lb)\left(144\,\frac{Btu}{lb}\right) + (2.5\,lb)\left(1\,\frac{Btu}{lb\cdot°R}\right)(T_f - 492) \right\} + (5\,lb)\left(\frac{1\,Btu}{lb\cdot°R}\right)(T_f - 492) + (0.5\,lb)\left(0.12\,\frac{Btu}{lb\cdot°R}\right)(T_f - 2260),$$

$$\Rightarrow T_f = 458.4\,°R$$

Since $T_f < 32°F$, it can be concluded that not all of the ice melts and the final temperature is 32°F. ◄——————— T_f

(b) An entropy balance reduces to give $\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{0} + \sigma$ or $\sigma = \Delta S$. Since not all of the ice melts it is necessary to determine the amount that does melt. Let x denote the amount of ice that melts. Then since the liquid remains at 32°F, the energy balance gives

$$\Delta H\Big]_{\substack{ice\\ at\ 32°F}} + \Delta H\Big]_{\substack{rivet\\ 1800°F \to 32°F}} = 0$$

or

$$(x\,lb)\left(144\,\frac{Btu}{lb}\right) + (0.5\,lb)\left(0.12\,\frac{Btu}{lb\cdot°R}\right)(492 - 2260)°R = 0 \Rightarrow x = 0.737\,lb$$

Returning to the calculation of the entropy production,

$$\sigma = \Delta S\Big]_{ice} + \Delta S\Big]_{rivet} + \cancel{\Delta S\Big]_{liq}}^{0}$$

The entropy change of the rivet can be found using Eq. 6.24. Similarly, $\Delta S = 0$ for the liquid by Eq. 6.24 since the temperature remains constant. The entropy change for the melting ice can be found using the TdS equation, Eq. 6.12b, which at constant pressure and temperature reduces to give

$$ds = \frac{dh}{T} \Rightarrow s_f - s_i = \frac{h_f - h_i}{T} = \frac{144\,Btu/lb}{492\,°R} = 0.2927\,Btu/lb\cdot°R$$

Thus

$$\sigma = (0.737\,lb)\left(0.2927\,\frac{Btu}{lb\cdot°R}\right) + (0.5\,lb)\left(0.12\,\frac{Btu}{lb\cdot°R}\right)\ln\left(\frac{492}{2260}\right)$$

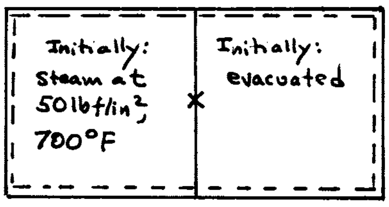
$$= (0.2157) + (-0.0915) = 0.1242\,Btu/°R$$ ◄——————— σ

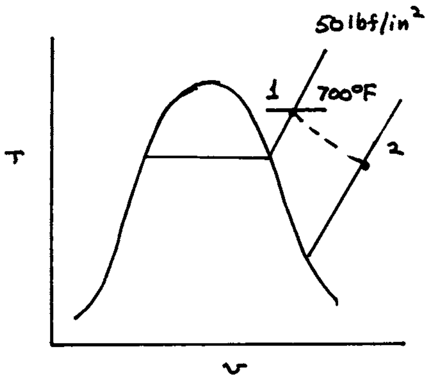
PROBLEM 6.75

KNOWN: An insulated vessel is divided into equal-sized compartments connected by a valve. Initially one compartment contains steam at a known state and the other is evacuated. The valve is opened and the steam fills the entire volume.

FIND: Determine (a) the final temperature and (b) the amount of entropy produced per unit mass of steam.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system is shown by the dashed line in the figure above. (2) For the system, $Q = 0$, $W = 0$, and kinetic/potential energy effects can be ignored.

ANALYSIS: To fix the final state requires the values of two independent intensive properties: u_2 and v_2. Thus, since the steam expands to fill twice the initial volume occupied by the steam, $V_2 = 2V_1$. And from an energy balance

$$\Delta U = \cancel{Q}^{\,0} - \cancel{W}^{\,0} \Rightarrow u_2 = u_1$$

An entropy balance reduces to

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma \Rightarrow \frac{\sigma}{m} = (s_2 - s_1) \qquad (1)$$

As the use of the pair of independent properties u_2, v_2 to fix the state with tabular steam table data is cumbersome, *IT* is employed to evaluate σ/m using Eq. (1):

IT Code

```
p1 = 50  // lbf/in.²
T1 = 700  // °F

u2 = u1
v2 = 2 * v1
u1 = u_PT("Water/Steam", p1, T1)
v1 = v_PT("Water/Steam", p1, T1)
u2 = u_PT("Water/Steam", p2, T2)
v2 = v_PT("Water/Steam", p2, T2)

sigma / m = s2 - s1
m = 1  // lb
s1 = s_PT("Water/Steam", p1, T1)
s2 = s_PT("Water/Steam", p2, T2)
```

IT Results

T_2 = 697.4 °F ← T_2

p_2 = 25.01 lbf/in.²

s_1 = 1.881 Btu/lb·°R

s_2 = 1.957 Btu/lb·°R

u_2 = 1255 Btu/lb

v_2 = 27.48 ft³/lb

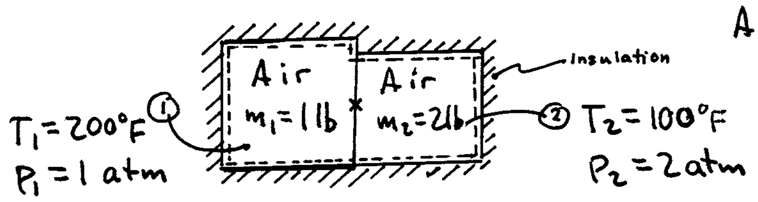
σ/m = 0.07611 Btu/lb·°R ← σ/m

PROBLEM 6.76*

KNOWN: Two insulated tanks containing air at known states are connected by a valve. The valve is opened and the two quantities of air mix.

FIND: Determine (a) the final temperature, (b) the final pressure, and (c) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



Air: $C_v = 0.17 \frac{\text{Btu}}{\text{lb}\,°\text{R}}$

ASSUMPTIONS: (1) As shown in the accompany figure, the system is the total quantity of air. (2) Air is modeled as an ideal gas. (3) $Q = W = 0$ and there is no change in kinetic or potential energy between the initial and final states.

ANALYSIS: (a) An energy balance reduces with assumption 3 to give $\Delta U = \cancelto{0}{Q} - \cancelto{0}{W}$ or $\Delta U = 0$. That is

$$\Delta U = (m_1 + m_2)u(T_f) - [m_1 u(T_1) + m_2 u(T_2)] = 0$$

or

$$0 = m_1[u(T_f) - u(T_1)] + m_2[u(T_f) - u(T_2)] \Rightarrow 0 = m_1 \cancel{c_v}[T_f - T_1] + m_2 \cancel{c_v}[T_f - T_2]$$

$$\Rightarrow \quad T_f = \frac{m_1 T_1 + m_2 T_2}{(m_1 + m_2)} = \frac{(1\text{ lb})(660°\text{R}) + (2\text{ lb})(560°\text{R})}{(3\text{ lb})} = 593°\text{R}\ (133°\text{F}) \xleftarrow{\quad (a) \quad}$$

(b) Using the ideal gas equation of state

$$P_f = \frac{(m_1 + m_2)\,R\,T_f}{(V_1 + V_2)}$$

where

$$V_1 = \frac{m_1 R T_1}{P_1} = \frac{(1)\left(\frac{1545}{28.97}\right)(660)}{(14.7)(144)} = 16.63\text{ ft}^3$$

$$V_2 = \frac{m_2 R T_2}{P_2} = \frac{(2)\left(\frac{1545}{28.97}\right)(560)}{(2)(14.7)(144)} = 14.11\text{ ft}^3$$

Thus

$$P_f = \frac{(3)\left(\frac{1545}{28.97}\right)(593)}{(16.63 + 14.11)(14.7)(144)} = 1.458\text{ atm} \xleftarrow{\qquad\qquad (b)}$$

(c) An entropy balance reduces to give $\Delta S = \cancelto{0}{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma$, or

$$\sigma = \Delta S = \{(m_1 + m_2)s_f - (m_1 s_1 + m_2 s_2)\} = m_1(s_f - s_1) + m_2(s_f - s_2)$$

$$= m_1\left[c_p \ln\frac{T_f}{T_1} - R \ln\frac{P_f}{P_1}\right] + m_2\left[c_p \ln\frac{T_f}{T_2} - R \ln\frac{P_f}{P_2}\right]$$

Using Eq. 3.44, $c_p = c_v + R = 0.17 + \left(\frac{1.986}{28.97}\right) = 0.24$ Btu/lb °R. Then

$$\sigma = (1)\left[(0.24)\ln\frac{593}{660} - \left(\frac{1.986}{28.97}\right)\ln\frac{1.458}{1}\right] + (2)\left[(0.24)\ln\frac{593}{560} - \left(\frac{1.986}{28.97}\right)\ln\frac{1.458}{2}\right]$$

$$= [-0.05154 + 0.07082]\ \text{Btu}/°\text{R}$$

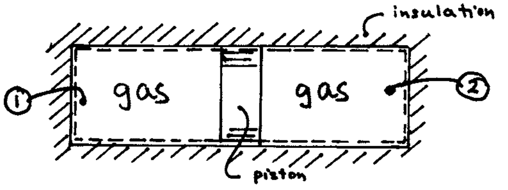
$$= 0.01928\ \text{Btu}/°\text{R} \xleftarrow{\qquad\qquad (c)}$$

PROBLEM 6.77*

KNOWN: An insulated cylinder is initially divided into halves by a piston. On either side of the piston is a gas at a known state. The piston is released and equilibrium is attained

FIND: Determine the final pressure, final temperature, and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



initially:

$T_1 = 300\,K \qquad T_2 = 300\,K$

$P_1 = 2\,bar \qquad P_2 = 1\,bar$

$V_1 = 1\,m^3 \qquad V_2 = 1\,m^3$

ASSUMPTIONS: (1) The system consists of both quantities of gas and the piston. But the piston experiences no net change in state. (2) The piston moves freely in the cylinder and is thermally conducting. (3) The gas is modeled as an ideal gas. (4) $Q = 0$.

ANALYSIS: (a) An energy balance reduces to give $\Delta U = \overset{0}{\cancel{Q}} - \overset{0}{\cancel{W}} \Rightarrow \Delta U = 0 \Rightarrow$

$$\Delta U]_1 + \Delta U]_2 + \overset{0}{\cancel{\Delta U}}]_{piston} = 0 \Rightarrow m_1[u(T_f) - u(300K)] + m_2[u(T_f) - u(300K)] = 0$$

$$\Rightarrow u(T_f) = u(300K) \Rightarrow T_f = 300\,K \longleftarrow T_f$$

(b) Using the ideal gas equation of state, and noting that the total mass of gas occupies the same total volume at the final state as initially: $V = 2\,m^3$,

$$P_f = \frac{(m_1 + m_2) R T_f}{V}$$

where

$$m_1 = \frac{P_1 (V/2)}{R T_1}, \quad m_2 = \frac{P_2 (V/2)}{R T_2}$$

Thus, since $T_1 = T_2 = T_f$

$$P_f = \frac{\left(\frac{P_1(V/2)}{RT} + \frac{P_2(V/2)}{RT}\right) RT}{V} = \frac{P_1 + P_2}{2} = 1.5\,bar \longleftarrow P_f$$

(c) An entropy balance reduces to give $\Delta S = \overset{0}{\cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}} + \sigma$ or

$$\sigma = \Delta S \Rightarrow \sigma = \Delta S]_1 + \Delta S]_2 + \overset{0}{\cancel{\Delta S}}]_{piston}$$

where

$$\Delta S]_1 = m_1 \Big[\underbrace{s°(T_f) - s°(T_1)}_{=0 \; (T_f = T_1)} - R \ln \frac{P_f}{P_1} \Big] = -R m_1 \ln \frac{P_f}{P_1}$$

$$\Delta S]_2 = m_2 \Big[\underbrace{s°(T_f) - s°(T_2)}_{=0 \; (T_f = T_2)} - R \ln \frac{P_f}{P_2} \Big] = -R m_2 \ln \frac{P_f}{P_2}$$

Thus

$$\sigma = -R \left[m_1 \ln \frac{P_f}{P_1} + m_2 \ln \frac{P_f}{P_2} \right] = -R \left[\frac{P_1 (V/2)}{RT} \ln \frac{P_f}{P_1} + \frac{P_2 (V/2)}{RT} \ln \frac{P_f}{P_2} \right]$$

$$= -\frac{V/2}{T} \left[P_1 \ln \frac{P_f}{P_1} + P_2 \ln \frac{P_f}{P_2} \right]$$

$$= -\frac{1\,m^3}{300K} \left[(2\,bar) \ln \frac{1.5}{2} + (1\,bar) \ln \frac{1.5}{1} \right] \left| \frac{100\,kN/m^2}{1\,bar} \right| \left| \frac{1\,kJ}{1\,kN \cdot m} \right|$$

① $$= 0.0566\,kJ/K \longleftarrow \sigma$$

---

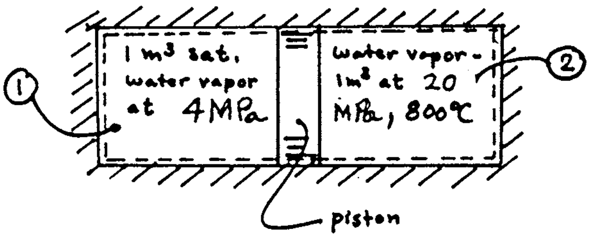
1. Provided the ideal gas model is assumed, the same numerical results are obtained for any gas.

PROBLEM 6.78

KNOWN: An insulated tank is divided into two compartments by a piston. On either side of the piston is water vapor at known states. The piston is released and equilibrium is attained.

FIND: Determine (a) the final pressure, (b) the final temperature, and (c) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:


1 m³ sat. water vapor at 4 MPa
water vapor - 1 m³ at 20 MPa, 800°C
①
②
piston

ASSUMPTIONS: (1) The closed system consists of both quantities of water vapor and the piston, but the piston experiences no change of state. (2) The piston moves freely and is thermally conducting. (3) For the system, $Q = 0$, $W = 0$, $\Delta KE = \Delta PE = 0$.

ANALYSIS: (a), (b) To fix the final state of the water vapor, we begin with an energy balance. That is, $\Delta U + \cancel{\Delta KE} + \cancel{\Delta PE} = \cancel{Q} - \cancel{W} \Rightarrow \Delta U = 0$. Thus

$$\Delta U\big]_1 + \Delta U\big]_2 + \cancel{\Delta U}\big]_{piston}^{\,0} = 0 \Rightarrow m_1(\Delta u)_1 + m_2(\Delta u)_2 = 0 \qquad (1)$$

For each side, $m = V/v$. With data from Table A-3 at the initial state

$$m_1 = \frac{V_1}{v_1} = \frac{1\ m^3}{0.04978\ m^3/kg} = 20.09\ kg$$

Similarly, for side 2 initially Table A-4 gives $v_2 = 0.02385\ m^3/kg$. Thus

$$m_2 = \frac{V_2}{v_2} = \frac{1\ m^3}{0.02385\ m^3/kg} = 41.93\ kg$$

It follows from assumption 2 that at equilibrium, the states of water vapor on either side of the piston are the same. Thus, from (1)

$$m_1(u - u_1) + m_2(u - u_2) = 0$$

From Table A-3, $u_1 = 2602.3$ kJ/kg and from Table A-4, $u_2 = 3592.7$ kJ/kg. Inserting values and solving for u we get

$$20.09(u - 2602.3) + 41.93(u - 3592.7) = 0 \Rightarrow u = 3271.9\ kJ/kg$$

The overall system volume is constant: 2 m³, and the total mass of water vapor is fixed. Thus, at equilibrium $v = (2\ m^3)/(20.09 + 41.93)\ kg = 0.03225\ m^3/kg$. The equilibrium state on both sides of the piston or fixed by $u = 3271.9$ kJ/kg and $v = 0.03225\ m^3/kg$. Interpolation in Table A-4 with u and v is very inconvenient. Instead, we use IT to get

$$P = 121.5\ bar \qquad \longleftarrow \qquad (a)\ (b)$$

$$T = 622.6\ °C$$

$$s = 6.861\ kJ/kg\cdot K$$

PROBLEM 6.78 (Contd.)

(c) The entropy produced can be determined from an entropy balance which reduces with assumptions (1) and (3) to give

$$\sigma = \Delta S \Rightarrow \sigma = \Delta S]_1 + \Delta S]_2 + \cancel{\Delta S}]_{piston}^{0} \Rightarrow \sigma = m_1(\Delta s)_1 + m_2(\Delta s)_2$$

From Table A-3, $s_1 = 6.0701$ kJ/kg·K, and from Table A-4, $s_2 = 7.0544$ kJ/kg·K. Thus

$$\sigma = 20.09\,[6.861 - 6.0701] + 41.93\,[6.861 - 7.0544] = 7.78 \text{ kJ/kg·K} \longleftarrow \sigma$$

ALTERNATIVE IT SOLUTION

*IT*Code

```
V1 = 1  // m³
x1 = 1
p1 = 40  // bar
V2 = 1  // m³
p2 = 200  // bar
T2 = 800  // °C

m1 = V1 / v1
v1 = vsat_Px("Water/Steam", p1, x1)
m2 = V2 / v2
v2 = v_PT("Water/Steam", p2, T2)


m1 * (u - u1) + m2 * (u - u2) = 0
u1 = usat_Px("Water/Steam", p1, x1)
u2 = u_PT("Water/Steam", p2, T2)
v = (V1 + V2) / (m1 + m2)

u = u_PT("Water/Steam", p, T)
v = v_PT("Water/Steam", p, T)
```

IT Results - Alternative Solution

```
sigma = m1 * (s - s1) + m2 * (s - s2)
s1 = ssat_Px("Water/Steam",00 p1, x1)
s2 = s_PT("Water/Steam", p2, T2)
s = s_PT("Water/Steam",p, T)
```

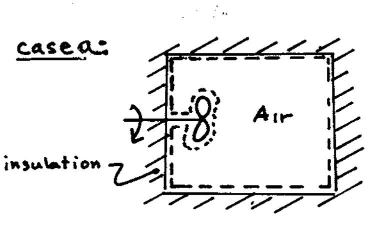
PROBLEM 6.79

KNOWN: A system consisting of air at a specified state undergoes constant volume processes in which the temperature increases in each of two ways.

FIND: (a) when the air is stirred adiabatically, determine the entropy produced. (b) When the air is heated by a reservoir at temperature T, plot the entropy produced versus T. Compare and discuss.

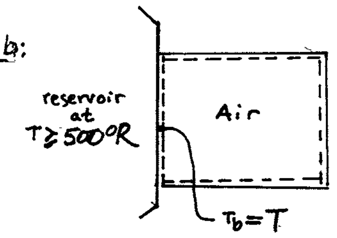
SCHEMATIC & GIVEN DATA:

case a:



$T_1 = 300\,K$
$p_1 = 1$ bar
$T_2 = 500K$

Case b:



ASSUMPTIONS: (1) The system consists of the air. (2) Air is modeled as an ideal gas. (3) In case (a), $Q = 0$, $W \neq 0$. In case (b), $Q \neq 0$, $W = 0$.

ANALYSIS: The change in entropy of the air is required in the evaluation of σ in each case. Thus, with data from Table A-22 and the ideal gas equation which gives $p_2/p_1 = T_2/T_1$

$$s_2 - s_1 = s°(T_2) - s°(T_1) - R \ln\frac{p_2}{p_1} = 2.21952 - 1.70203 - \frac{8.314}{28.97}\ln\frac{500}{300} = 0.3709 \text{ kJ/kg·K}$$

CASE a: An entropy balance reduces to give

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma_a \Rightarrow \frac{\sigma_a}{m} = s_2 - s_1 = 0.3709 \text{ kJ/kg·K} \longleftarrow \qquad (a)$$

CASE b: An entropy balance reduces to give

$$\Delta S = \frac{Q}{T_b} + \sigma_b \Rightarrow \frac{\sigma_b}{m} = (s_2 - s_1) - \frac{Q/m}{T_b}$$

To find Q, write an energy balance: $\Delta U = Q - \cancel{W}^{\,0}$. Thus, with data from Table A-22

$$\frac{Q}{m} = u(T_2) - u(T_1) = 359.49 - 214.07 = 145.42 \text{ kJ/kg}$$

Thus

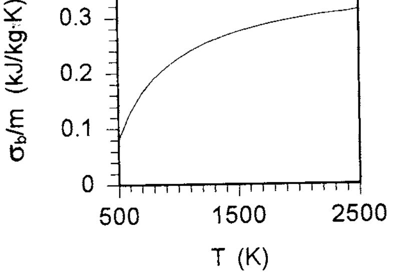
$$\frac{\sigma_b}{m} = 0.3709 - \frac{145.42}{T} \longleftarrow \qquad (b)$$

Sample calculation: when $T = 500K$, $\sigma_b/m = 0.08$ kJ/kg·K.

Equation (b) is plotted below. Comparing the two cases, we get

$$(\sigma_b/m) = (\sigma_a/m) - (145.42/T).$$

Thus, the entropy produced by heating is always less than the entropy produced by stirring. The entropy produced by heating approaches the entropy produced by stirring at higher reservoir temperatures (mathematically, as $T \to \infty$). See the accompanying plot.

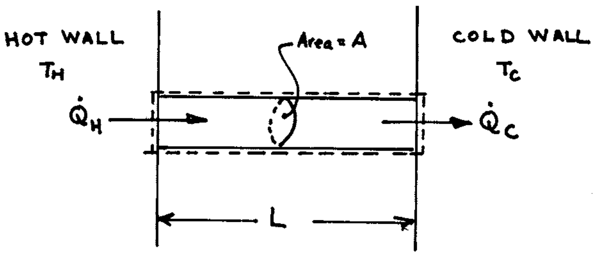


PROBLEM 6.80

KNOWN: Energy is conducted steadily through a copper rod from a hot wall to a cold wall. The rate of heat transfer, temperatures, and geometrical parameters are specified.

FIND: (a) Determine an expression for the rate of entropy production within the rod in terms of specified quantities, and (b) plot $\dot{Q}_H$ and $\dot{\sigma}$ versus L for a given set of numerical values for these quantities.

SCHEMATIC & GIVEN DATA:



$$\dot{Q}_H = \kappa A (T_H - T_c)/L \qquad (1)$$

$T_H = 600\ K\ (327°C)$
$T_c = 350\ K\ (77°C)$
$\kappa = 0.4\ kW/m\cdot K$
$A = 0.1\ m^2$

ASSUMPTIONS: (1) As shown in the accompanying figure, the system is the copper rod. (2) The system is at steady state. (3) The rod is insulated on its lateral surface. (4) An expression for evaluating $\dot{Q}_H$ is provided.

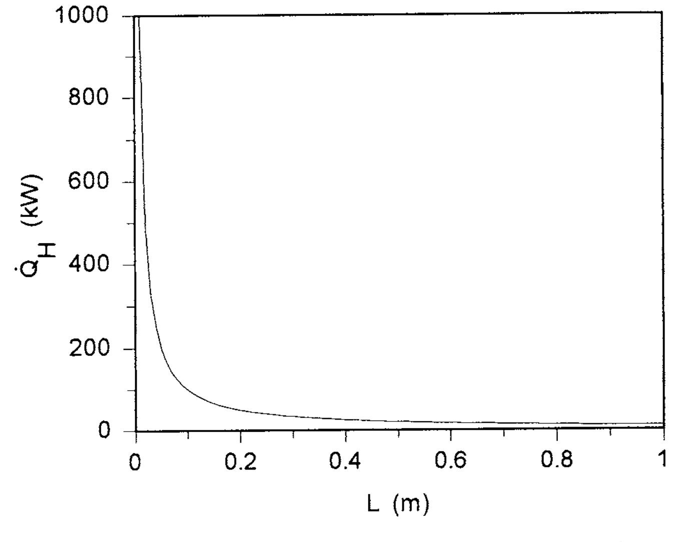
ANALYSIS: (a) At steady state an entropy rate balance reduces to give

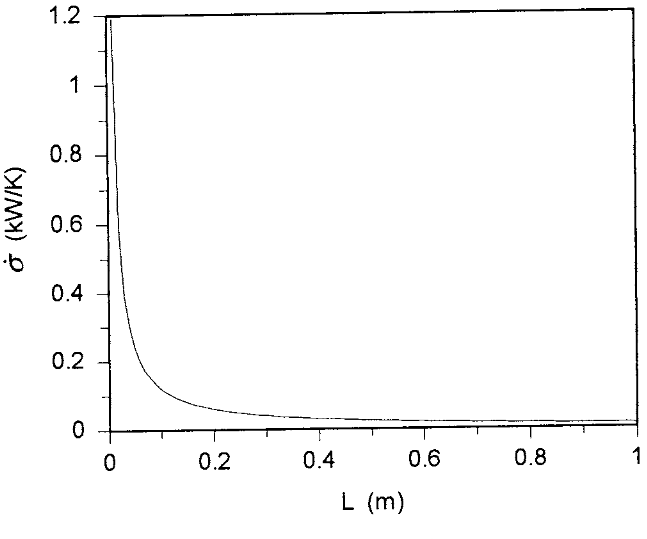
$$\frac{d\cancel{S}^{\,0}}{\cancel{dt}} = \frac{\dot{Q}_H}{T_H} - \frac{\dot{Q}_c}{T_c} + \dot{\sigma} \Rightarrow \dot{\sigma} = \frac{\dot{Q}_c}{T_c} - \frac{\dot{Q}_H}{T_H}$$

Noting that the energy transfers are positive in the directions of the arrows, an energy rate balance gives $\dot{Q}_c = \dot{Q}_H$. Collecting results

$$\dot{\sigma} = \dot{Q}_H \left[\frac{1}{T_c} - \frac{1}{T_H}\right] = \frac{\kappa A (T_H - T_c)^2}{L\, T_H T_c} \qquad (2)$$

(b) If $T_H = 600K\ (327°C)$, $T_c = 350K\ (77°C)$, $\kappa = 0.4\ kW/m\cdot K$, $A = 0.1\ m^2$, the variations of $\dot{Q}$ and $\dot{\sigma}$ with L are





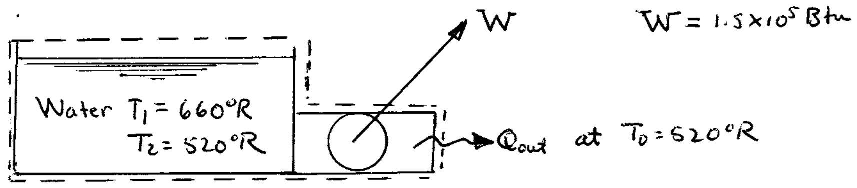
The plots show that both heat transfer and entropy production rate decrease rapidly with increasing L. $\dot{Q}_H$ decreases due to greater resistance to heat transfer as the length of the rod increases. The variation of $\dot{\sigma}$ is associated with the changing temperature gradient, dT/dx, in the rod. Higher temperature gradient ($L \to 0$; $dT/dx \to \infty$) corresponds to high rates of entropy production, and conversely. As a final point, Eq. (2) above indicates that $\dot{\sigma}$ is positive, illustrating the irreversible nature of heat transfer.

PROBLEM 6.81

KNOWN: A system undergoes a cycle while receiving energy by heat transfer from a tank of water.

FIND: Determine the minimum theoretical volume of the water for the cycle to produce net work of 1.5×10^5 Btu.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is composed of the tank of water plus a subsystem that undergoes a thermodynamic cycle. (2) The only energy transfers between system and cycle are shown on the figure. (3) Kinetic and potential energy effects are absent. (4) The liquid water is modeled as incompressible with constant specific heat c.

ANALYSIS: As the water temperature is reduced from T_1 to T_2 the system undergoes a multiplicity of thermodynamic cycles. The energy balance reads

$$\Delta U = -Q_{out} - W$$

where ΔU is the internal energy change for the water only because the subsystem undergoes cycles. Then with assumption (4)

$$mc[T_2 - T_1] = -Q_{out} - W \Rightarrow W = mc[T_1 - T_2] - Q_{out} \qquad (1)$$

With Eq. 6.24, an entropy balance yields

$$\Delta S = \frac{-Q_{out}}{T_0} + \sigma \Rightarrow mc\ln\frac{T_2}{T_1} = -\frac{Q_{out}}{T_0} + \sigma$$

$$\Rightarrow -Q_{out} = T_0 mc\ln\frac{T_2}{T_1} - T_0\sigma \qquad (2)$$

Eliminating $(-Q_{out})$ between Eqs. (1), (2) gives

$$W = mc[T_1 - T_2] + T_0 mc\ln\frac{T_2}{T_1} - T_0\sigma \qquad (3)$$

Solving Eq. (3) for m, and noting that $T_2 = T_0$, gives

$$m = \frac{W + T_0\sigma}{c\left[T_1 - T_0 + T_0\ln\frac{T_0}{T_1}\right]}$$

Since $\sigma \geq 0$, the minimum theoretical mass of water is

$$m_{MIN} = \frac{W}{c\left[T_1 - T_0 + T_0\ln\frac{T_0}{T_1}\right]} = \frac{(1.5\times10^5\ \text{Btu})}{\left(1\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\left(660 - 520 + 520\ln\frac{520}{660}\right)°\text{R}}$$

$$= 9359\ \text{lb}$$

where c is from Table A-19E. Then, with $v = v_f$ at 130°F from Table A-2E, the minimum volume of water is

$$V_{MIN} = m_{min}v = (9359\ \text{lb})\left(0.0163\ \frac{\text{ft}^3}{\text{lb}}\right)\left(\frac{1\ \text{gal}}{0.13368\ \text{ft}^3}\right)$$

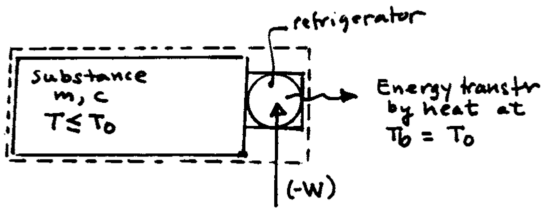
$$= 1141.2\ \text{gal} \longleftarrow$$

PROBLEM 6.82

KNOWN: The temperature of an incompressible substance of mass m and specific heat c is reduced from T_0 to T ($<T_0$) by a refrigeration cycle.

FIND: Plot (W_{min}/mcT_0) versus T/T_0, where W_{min} is the minimum theoretical work input required.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system shown in the accompanying figure is composed of an incompressible substance and a system that undergoes a refrigeration cycle. 2. The cycle discharges energy to the surroundings at T_0.

ANALYSIS: An energy balance gives $(-W) = \Delta U - Q$, where ΔU is the change in internal energy of the substance because $(\Delta U)_{cycle} = 0$. With Eq. 3.20a

$$(-W) = mc[T - T_0] - Q \qquad (1)$$

An entropy balance reads $\Delta S = (Q/T_0) + \sigma$, where ΔS is the change in entropy of the substance because $(\Delta S)_{cycle} = 0$. With Eq. 6.24

$$mc \ln\left(\frac{T}{T_0}\right) = \frac{Q}{T_0} + \sigma \Rightarrow Q = T_0 mc \ln\left(\frac{T}{T_0}\right) - T_0\sigma \qquad (2)$$

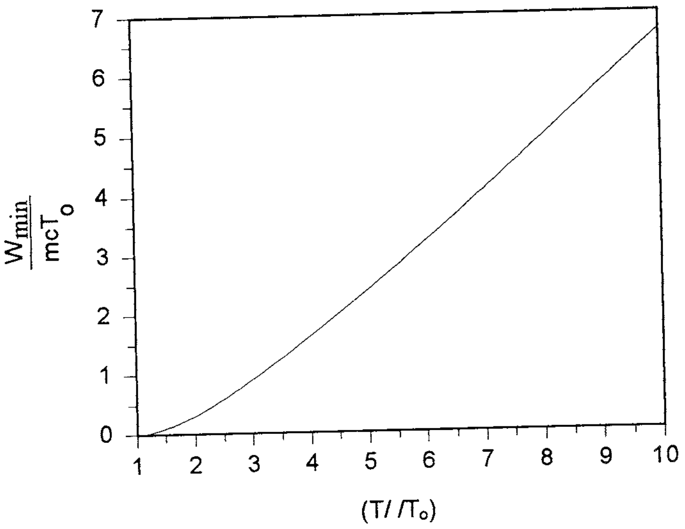
Collecting Eqs. (1), (2)

$$(-W) = mc[T - T_0] - T_0 mc \ln\left(\frac{T}{T_0}\right) + T_0\sigma$$

Since $\sigma \geq 0$, the minimum work input corresponds to $\sigma = 0$, giving

$$W_{min} = mc\left[T - T_0 - T_0 \ln\frac{T}{T_0}\right] \Rightarrow \left(\frac{W_{min}}{mcT_0}\right) = \frac{T}{T_0} - 1 - \ln\frac{T}{T_0} \qquad (3)$$

PLOT:

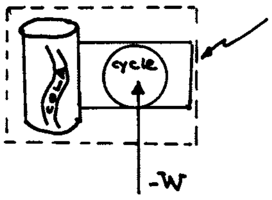


PROBLEM 6.83

__KNOWN__: A can of soft drink is cooled by a system undergoing a refrigeration cycle that receives energy by heat transfer from the soft drink and discharges energy by heat transfer to the surroundings.

__FIND__: Determine the minimum theoretical work required.

__SCHEMATIC & GIVEN DATA:__


soft drink:
0.354 ℓ
$T_i = 20°C$
$T_f = 5°C$
cycle
energy transfer by heat at $T_b = 293 K (20°C)$ to surroundings at 20°C
-W

__ASSUMPTIONS__: (1) The system shown in the accompanying figure is composed of two subsystems: The can of soft drink and a system that undergoes a refrigeration cycle. (2) The cycle discharges energy by heat transfer at $T_b = 293 K$. (3) The soft drink can be modeled as an incompressible liquid with the properties of water. The aluminum can itself can be ignored.

__ANALYSIS__: An energy balance gives $(-W) = -Q + \Delta U$ where ΔU is the change in internal energy of the liquid because $\Delta U]_{cycle} = 0$ and the can is ignored. Thus, with Eq. 3.20a the work input is

$$(-W) = -Q + mc(T_f - T_i) \qquad (1)$$

An entropy balance takes the form

$$\Delta S = \frac{Q}{T_b} + \sigma$$

where ΔS is the entropy change of the liquid because $\Delta S]_{cycle} = 0$ and the can is ignored. Thus, on solving for Q and using Eq. 6.24

$$Q = T_b \Delta S - T_b \sigma = T_b \left(mc \ln \frac{T_f}{T_i} \right) - T_b \sigma \qquad (2)$$

Substituting Eq. (2) into Eq. (1)

$$(-W) = mc \left[T_b \ln \frac{T_i}{T_f} + T_f - T_i \right] + T_b \sigma$$

Since $\sigma \geq 0$, the work input $(-W)$ is a minimum when $\sigma = 0$, giving

$$(-\dot{W})_{min} = mc \left[T_b \ln \frac{T_i}{T_f} + T_f - T_i \right]$$

With $c \approx 4.2\ kJ/kg \cdot K$ from Table A-19 and $v \approx 1\ cm^3/g$ from Table A-2

$$(-\dot{W})_{min} = \left[(0.354\, \ell) \left(\frac{10^{-3} m^3}{\ell} \right) \left(1 \frac{g}{cm^3} \right) \left(\frac{10^6 cm^3}{m^3} \right) \left(\frac{kg}{10^3 g} \right) \right] \left(4.2 \frac{kJ}{kg \cdot K} \right) \left[293 \ln \frac{293}{278} + 278 - 293 \right] K$$

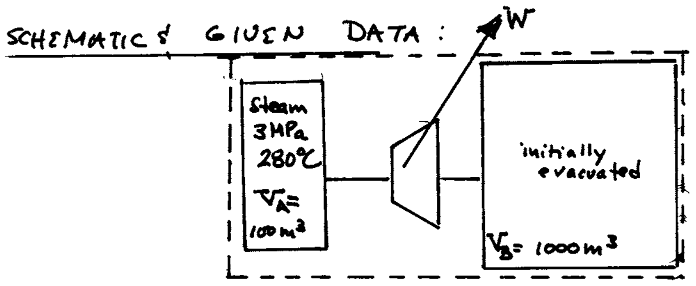
$$= 0.591\ kJ \longleftarrow$$

PROBLEM 6.84

KNOWN: A turbine is located between two tanks, initially one tank is filled with steam and the other is evacuated. Steam is allowed to flow through the turbine until equilibrium is established.

FIND: Determine the maximum theoretical work that can be developed.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is shown by the dashed line above. (2) Heat transfer can be ignored. (3) The initial and final states are equilibrium states. At the final state the mass of steam contained within the turbine and interconnecting piping can be ignored.

ANALYSIS: An energy balance reduces to $\Delta U = \cancel{Q} - W$, or $W = m(u_1 - u_2)$, where $m = \frac{V_A}{v_1}$. Thus, with steam table data, $u_1 = 2709.9$ kJ/kg, $v_1 = 0.0771$ m³/kg, $s_1 = 6.4462$ kJ/kg·K,

$$W = \left(\frac{100\,m^3}{0.0771\,m^3/kg}\right)(2709.9 - u_2)\frac{kJ}{kg} = 1297(2709.9 - u_2)\,kJ \qquad (1)$$

The final specific volume is

$$v_2 = \frac{V_A + V_B}{m} = v_1\left[\frac{V_A + V_B}{V_A}\right] = (0.0771)\left(\frac{1100}{100}\right) = 0.8481\,\frac{m^3}{kg} \qquad (2)$$

An entropy balance reduces to $\Delta S = \int_1^2 \left(\frac{\overset{0}{\cancel{\delta Q}}}{T}\right)_b + \sigma \Rightarrow m(s_2 - s_1) = \sigma$ (3)

By inspection of Eq.(1), the maximum value of W is attained when u_2 assumes the smallest allowed value at the value of v_2 given by Eq.(2). Since
① u and s change in the same direction at fixed v, the smallest allowed value for u_2 corresponds to the smallest allowed value for s_2. Since $\sigma \geq 0$, Eq.(3) indicates that the smallest value of s_2 corresponds to $\sigma = 0$: $s_2 = s_1$. The final state of the steam is then fixed by: $v_2 = .8481$ m³/kg, $s_2 = 6.4462$ kJ/kg·K.

As this pair of independent properties is cumbersome to use with tabular steam table data, IT is employed to obtain $u_2 = 2270$ kJ/kg. Returning to Eq.(1)

$$W_{MAX} = 1297(2709.9 - 2270) = 5.71 \times 10^5\,kJ \quad \longleftarrow$$

The final state is a two-phase liquid-vapor mixture with $T_2 = 117°C$, $P_2 = 1.815$ bar, $x_2 = 0.8741$.

When the entire problem is solved using IT, we get

PROBLEM 6.84 (Contd.)

IT Solution:

```
p1 = 30 bar
T1 = 280 °C
VA = 100 m³
VB = 1000 m³

v1 = v_PT("Water/Steam", p1, T1)
u1 = u_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
m = VA / v1

W = m * (u1 - u2)
v2 = (VA + VB) / m
s2 = s1

v2 = vsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
u2 = usat_Px("Water/Steam", p2, x2)
T2 = Tsat_P("Water/Steam", p2)

Note: These are the IT results when the entire problem is solved using IT
T2        117.2
W         5.707E5
s2        6.445
u2        2269
v2        0.8483
```

---

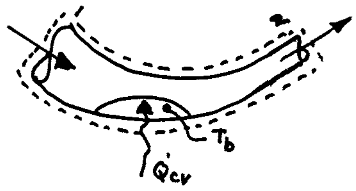
1. When v is fixed, the first Tds equation — Eq. 6.12a — reads $Tds = du$. Thus, changes in u and s occur in the same direction.

PROBLEM 6.85

KNOWN: A gas flows through a one inlet, one exit control volume operating at steady state. Heat transfer $\dot{Q}_{cv}$ takes place only at temperature T_b.

FIND: For each of several cases, determine if the change in specific entropy from inlet to exit is positive, negative, or indeterminate

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state. (2) Heat transfer occurs only where the temperature is T_b.

ANALYSIS: At steady-state Eq 6.42 reduces to Eq. 6.44. Then, since heat transfer takes place only at T_b and there is a single inlet and single exit, the entropy rate balance becomes

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

Solving for the change in specific entropy from inlet to exit

$$s_2 - s_1 = \frac{\dot{Q}_{cv}}{\dot{m}T_b} + \frac{\dot{\sigma}_{cv}}{\dot{m}}$$

(a) $\dot{\sigma}_{cv} = 0$, $\dot{Q}_{cv} = 0$

$$s_2 - s_1 = \overset{0}{\cancel{\frac{\dot{Q}_{cv}}{\dot{m}T_b}}} + \overset{0}{\cancel{\frac{\dot{\sigma}_{cv}}{\dot{m}}}} = 0$$

$s_2 = s_1$

(b) $\dot{\sigma}_{cv} = 0$, $\dot{Q}_{cv} < 0$:

$$s_2 - s_1 = \underbrace{\frac{\dot{Q}_{cv}}{\dot{m}T_b}}_{\text{negative}} + \overset{0}{\cancel{\frac{\dot{\sigma}_{cv}}{\dot{m}}}} < 0$$

s decreases

(c) $\dot{\sigma}_{cv} = 0$, $\dot{Q}_{cv} > 0$:

$$s_2 - s_1 = \underbrace{\frac{\dot{Q}_{cv}}{\dot{m}T_b}}_{\text{positive}} + \overset{0}{\cancel{\frac{\dot{\sigma}_{cv}}{\dot{m}}}} > 0$$

s increases

(d) $\dot{\sigma}_{cv} > 0$, $\dot{Q}_{cv} < 0$

$$s_2 - s_1 = \underbrace{\frac{\dot{Q}_{cv}}{\dot{m}T_b}}_{\text{negative}} + \underbrace{\frac{\dot{\sigma}_{cv}}{\dot{m}}}_{\text{positive}}$$

indeterminate without additional information about $\dot{Q}_{cv}/\dot{m}$, T_b, $\dot{\sigma}_{cv}/\dot{m}$

(e) $\dot{\sigma}_{cv} > 0$, $\dot{Q}_{cv} \geq 0$

$$s_2 - s_1 = \underbrace{\frac{\dot{Q}_{cv}}{\dot{m}T_b}}_{\substack{\text{positive}\\ \text{or zero}}} + \underbrace{\frac{\dot{\sigma}_{cv}}{\dot{m}}}_{\text{positive}} > 0$$

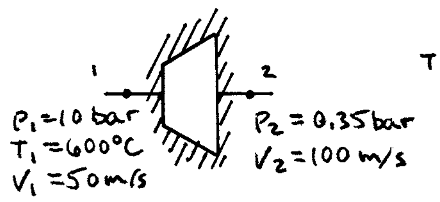
s increases

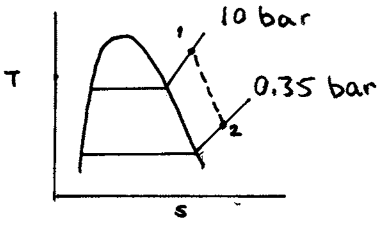
PROBLEM 6.86*

KNOWN: An insulated turbine at steady state has steam entering at 10 bar, 600°C, 50 m/s and exiting at 0.35 bar, 100 m/s. The work developed is claimed to be (a) 1000 kJ/kg, (b) 500 kJ/kg.

FIND: Determine if the claim can be correct.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The turbine operates at steady-state and is well-insulated. (2) Potential energy effects can be ignored.

ANALYSIS: At steady-state the entropy rate balance reduces with the mass rate balance: $\dot{m}_1 = \dot{m}_2 = \dot{m}$ to give

$$0 = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

Thus,

$$s_2 - s_1 = \frac{\dot{\sigma}_{cv}}{\dot{m}} \geq 0$$

Accordingly, the exiting specific entropy must be greater than, or equal to, the entering specific entropy.

From Table A-4 at 10 bar, 600°C, $s_1 = 8.0290$ kJ/kg·K. The exit specific entropy that corresponds to the claimed work value can be determined using an energy rate balance to fix state 2. Thus, at steady-state

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancelto{0}{g(z_1 - z_2)}\right]$$

Solving for h_2, and introducing h_1 from Table A-4: $h_1 = 3697.9$ kJ/kg

$$h_2 = -\frac{\dot{W}_{cv}}{\dot{m}} + h_1 + \frac{V_1^2 - V_2^2}{2} = -\frac{\dot{W}_{cv}}{\dot{m}} + 3697.9 + \left[\frac{(50)^2 - (100)^2}{2}\left(\frac{m}{s}\right)^2\right]\left|\frac{1\ N}{1\ kg \cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N \cdot m}\right|$$

$$= -\frac{\dot{W}_{cv}}{\dot{m}} + 3694.2\ \frac{kJ}{kg} \qquad (1)$$

(a) $\dot{W}_{cv}/\dot{m} = 1000$ kJ/kg. Eq.(1) gives $h_2 = 2694.2$ kJ/kg. Interpolating in Table A-4 at 0.35 bar with h_2 gives $s_2 = 7.8868$ kJ/kg·K. Since this value is less than s_1, the claimed value cannot be correct.

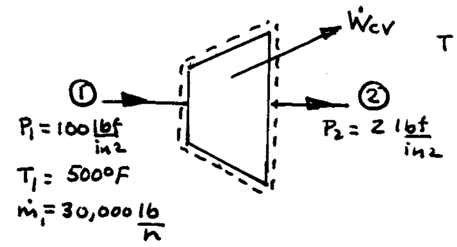
(b) $\dot{W}_{cv}/\dot{m} = 500$ kJ/kg. Eq.(1) gives $h_2 = 3194.2$ kJ/kg. Interpolating in Table A-4 at 0.35 bar with h_2 gives $s_2 = 8.8987$ kJ/kg·K. Since this value is greater than s_1, the claimed value could be correct.

PROBLEM 6.87

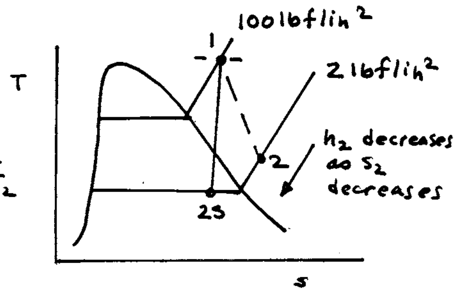
KNOWN: Steady state operating data are provided for a steam turbine.

FIND: Determine if either or both of two alternative values for the power developed by the turbine can be correct.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cv}$
①
$P_1 = 100\ \frac{lbf}{in^2}$
$T_1 = 500°F$
$\dot{m}_1 = 30{,}000\ \frac{lb}{h}$
②
$P_2 = 2\ \frac{lbf}{in^2}$



ASSUMPTIONS: 1. The control volume shown in the accompanying figure is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$ and changes in kinetic and potential energy can be ignored.

ANALYSIS: The mass rate balance reduces to $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Then, with assumption 2 the energy rate balance reduces to give

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) \qquad (1)$$

Since $\dot{Q}_{cv} = 0$, the entropy rate balance reduces at steady state to give

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow s_2 - s_1 = \dot{\sigma}_{cv}/\dot{m} \qquad (2)$$

Method 1. Eq. (1) indicates that $\dot{W}_{cv}$ increases as h_2 decreases. Inspection of Table A-4E shows that h_2 decreases as s_2 decreases. Thus, the maximum theoretical value of $\dot{W}_{cv}$ corresponds to the minimum theoretical value for h_2, which corresponds to the minimum allowed value for s_2. Since $\dot{\sigma}_{cv} \geq 0$, Eq. (2) shows that the minimum value for s_2 corresponds to $\dot{\sigma}_{cv} = 0$ — that is, $s_2 = s_1$.

From Table A-4E, $h_1 = 1279.1$ Btu/lb, $s_1 = 1.7085$ Btu/lb·°R. The state s2 falls in the two-phase region. With values from Table A-3E

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.7085 - 0.175}{1.7448} = 0.879 \Rightarrow h_{2s} = 94.02 + (0.879)(1022.1) = 992.4\ \text{Btu/lb}$$

Then, Eq. (1) gives

$$(\dot{W}_{cv})_{MAX} = \dot{m}(h_1 - h_{2s}) = \left(30{,}000\frac{lb}{h}\right)\left[(1279.1 - 992.4)\frac{Btu}{lb}\right]\left|\frac{1\ hp}{2545\ Btu/h}\right| = 3380\ hp$$

Of the two alternatives, only 3080 hp can be correct.

Method 2. The given power values can be used to determine $\dot{\sigma}_{cv}$. Thus, if $\dot{W}_{cv} = 3800$ hp, Eq. (1) gives on rearrangement

$$h_2 = h_1 - \frac{\dot{W}_{cv}}{\dot{m}} = 1279.1 - \left(\frac{3800\ hp}{30{,}000\ lb/h}\right)\left|\frac{2545\ Btu/h}{1\ hp}\right| = 956.7\ Btu/lb$$

Then, with data from Table A-3E

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{956.7 - 94.02}{1022.1} = 0.844 \Rightarrow s_2 = 0.175 + 0.844(1.7448) = 1.6476\ \frac{Btu}{lb\cdot°R}$$

With Eq. (2),

$$\dot{\sigma}_{cv}/\dot{m} = s_2 - s_1 = (1.6476 - 1.7085) = -0.061\ Btu/lb\cdot°R.$$

But since $\dot{\sigma}_{cv}/\dot{m}$ cannot be negative, the power value 3800 hp cannot be correct.

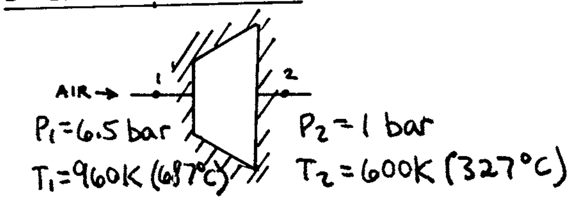
Using the value 3080 hp, the same approach gives $\dot{\sigma}_{cv}/\dot{m} > 0$, and thus this value is allowed.

PROBLEM 6.88*

KNOWN: Air enters and exits an insulated turbine at specified pressures and temperatures.

FIND: Determine the work developed per kg of air flowing, and whether the expansion is internally reversible, irreversible, or impossible.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The turbine operates at steady state and is well-insulated. (2) Changes in kinetic and potential energy from inlet to exit can be neglected. (3) Air is modeled ① as an ideal gas.

ANALYSIS: An energy rate balance at steady state for a control volume enclosing the turbine reduces with a mass rate balance: $\dot{m}_1 = \dot{m}_2 = \dot{m}$ to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \cancelto{0}{\frac{V_1^2 - V_2^2}{2}} + \cancelto{0}{g(z_1 - z_2)}\right)$$

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2$$

Then, with data from Table A-22

$$\frac{\dot{W}_{cv}}{\dot{m}} = 1000.55 - 607.02 = 393.53 \ \text{kJ/kg} \longleftarrow$$

An entropy rate balance at steady state gives

$$0 = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1$$

With Eq 6.25a and data from Table A-22

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s^\circ(T_2) - s^\circ(T_1) - R \ln P_2/P_1$$

$$= 2.40902 - 2.92128 - \frac{8.314}{28.97} \ln \frac{1}{6.5} = 0.0249 \ \text{kJ/kg·K}$$

Since $\dot{\sigma}_{cv}/\dot{m}$ is positive, the expansion is irreversible. However, the small value suggests that the expansion approaches ideality.

---

1. From Table A-1, the critical pressure of air is $P_c = 37.7$ bar and the critical temperature is $T_c = 133$ K. Thus, at the inlet and exit, respectively

$$P_{R1} = \frac{6.5}{37.7} = 0.17 \qquad P_{R2} = \frac{1}{37.7} = 0.027$$

$$T_{R1} = \frac{960}{133} = 7.22 \qquad T_{R2} = \frac{600}{133} = 4.51$$

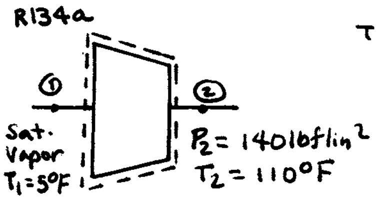
Referring to Fig A-1, these states fall into the region where the ideal gas model is appropriate.

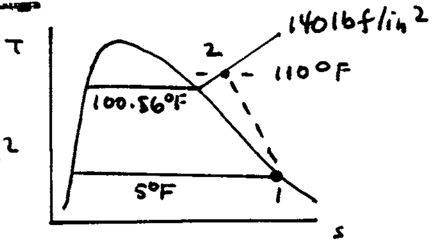
PROBLEM 6.89

KNOWN: Steady-state operating data are provided for an R134a compressor.

FIND What can be said about the direction of heat transfer between the compressor and its surroundings?

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume shown in the figure operates at steady state. 2. Kinetic and potential energy effects can be ignored.

ANALYSIS: Reducing mass and energy balances

$$\frac{\dot Q}{\dot m} = \frac{\dot W}{\dot m} + (h_2 - h_1)$$

With data from Tables A-10E, 12E

$$\frac{\dot Q}{\dot m} = \frac{\dot W}{\dot m} + (117.53 - 102.47) = \underbrace{\frac{\dot W}{\dot m}}_{\ominus} + 15.06 \frac{\text{Btu}}{\text{lb}}$$

Since $\frac{\dot W}{\dot m}$ must be minus, the direction of $\dot Q/\dot m$ is determined by the magnitude of $\dot W/\dot m$, which is unknown. Turning next to an entropy balance

$$0 = \sum \frac{\dot Q_j}{T_j} + \dot m (s_1 - s_2) + \dot\sigma_{cv} \Rightarrow \sum \frac{\dot Q_j}{T_j} = \dot m (s_2 - s_1) - \dot\sigma_{cv}$$

With table data

$$\frac{1}{\dot m} \sum \frac{\dot Q_j}{T_j} = s_2 - s_1 - (\dot\sigma_{cv}/\dot m)$$

$$= (0.2206 - 0.2219) - (\dot\sigma_{cv}/\dot m)$$

$$= (-0.0013 - \dot\sigma_{cv}/\dot m) \text{ Btu/lb·°R} \quad < 0$$

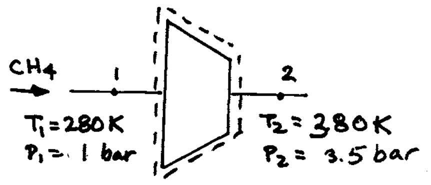
Thus, entropy is carried out of the control volume by heat transfer. The direction of heat transfer is from the control volume to the surroundings.

PROBLEM 6.90

KNOWN: Methane gas enters and exits a compressor operating at steady state at specified temperatures and pressures.

FIND: Determine the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume operates at steady state. (2) The methane is modeled as an ideal gas. (3) Heat transfer with the surroundings is negligible.

ANALYSIS: With assumption 3, the entropy rate balance reduces at steady state to

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) \qquad (1)$$

① The change in specific entropy is evaluated using Eq. 6.19 and $\bar{c}_p(T)$ from Table A-21

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p(T)}{T}\,dT - R \ln\frac{p_2}{p_1} = R\int_{T_1}^{T_2} \frac{(\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4)}{T}\,dT - R\ln\frac{p_2}{p_1}$$

$$= R\int_{T_1}^{T_2}\left(\frac{\alpha}{T} + \beta + \gamma T + \delta T^2 + \epsilon T^3\right)dT - R\ln\frac{p_2}{p_1}$$

$$= R\left[\alpha \ln T + \beta T + \frac{\gamma T^2}{2} + \frac{\delta T^3}{3} + \frac{\epsilon T^4}{4}\right]_{T_1}^{T_2} - R\ln\frac{p_2}{p_1}$$

$$= \left(\frac{8.314}{16.04}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left[\left[3.826 \ln\frac{380}{280} - \frac{3.979}{10^3}(380-280) + \frac{24.558}{2(10^6)}(\overline{380}^2 - \overline{280}^2) - \frac{22.733}{3(10^9)}(\overline{380}^3 - \overline{280}^3) + \frac{6.963}{4(10^{12})}(\overline{380}^4 - \overline{280}^4)\right] - \ln\frac{3.5}{1}\right]$$

$$= \left(\frac{8.314}{16.04}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left[1.357 - 1.253\right] = 0.0539\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

With Eq. (1)

$$\dot{\sigma}_{cv}/\dot{m} = 0.0539\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$ ←

---

1. Alternatively, using IT

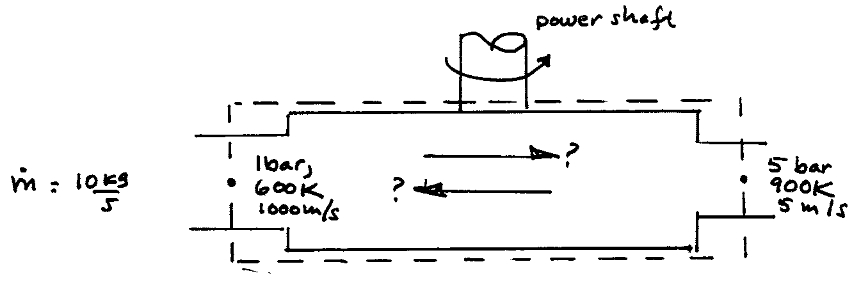
```
T1 = 280  // K
p1 = 1  // bar
T2 = 380
p2 = 3.5

dels = s2 - s1
s1 = s_TP("CH4", T1, p1)
s2 = s_TP("CH4", T2, p2)

/* Result
dels = 0.05407 kJ/kg·K

*/
```

PROBLEM 6.91

KNOWN: Steady-state operating data are provided for a well-insulated device carrying air at a known mass flow rate.

FIND: Determine the direction of flow and the power input or output.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and potential energy effects are negligible. (3) Air is modeled as an ideal gas.

ANALYSIS: Directionality is a second law aspect (Sec. 5.1). Thus, assume a flow direction and apply an entropy balance. Regarding the flow to be from left to right

$$0 = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}(s_i - s_e) + \dot{\sigma}_{cv}$$

With ideal gas relations and data from Table A-22

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_e - s_i = s^\circ(\underset{900K}{T_e}) - s^\circ(\underset{600K}{T_i}) - \frac{\bar{R}}{M} \ln \frac{\overset{5\,bar}{p_e}}{\underset{1\,bar}{p_i}}$$

$$= 2.84856 - 2.40902 - \frac{8.314}{28.97} \ln 5 = -0.0223 \frac{kJ}{kg \cdot K}$$

As $\dot{\sigma}_{cv}$ cannot be negative, the direction of flow must be from right to left. ←

Applying mass and energy rate balances with data from Table A-22

$$\dot{W}_{cv} = \dot{m}\left[\overset{900K}{h_i} - \overset{600K}{h_e} + \frac{\overset{5\,m/s}{V_i^2} - \overset{1000\,m/s}{V_e^2}}{2}\right]$$

$$= \left(10 \frac{kg}{s}\right)\left\{(932.93 - 607.02)\frac{kJ}{kg} + \left[\frac{(5)^2 - (1000)^2}{2}\right]\left(\frac{m^2}{s^2}\right)\left(\frac{1\,N}{1\,kg \cdot m/s^2}\right)\left[\frac{1\,kJ}{10^3\,N \cdot m}\right]\right\}$$

$$= \left(10 \frac{kg}{s}\right)\left\{325.91 \frac{kJ}{kg} - 499.99 \frac{kJ}{kg}\right\}$$

$$= -1740.8 \left(\frac{kJ}{s}\right)\left(\frac{1\,kW}{1\,kJ/s}\right)$$

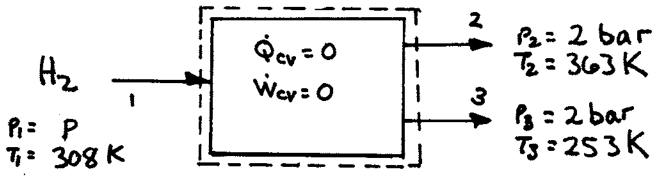
$$= -1740.8\,kW \longleftarrow$$

PROBLEM 6.92*

KNOWN: Hydrogen (H_2) gas enters an insulated control volume operating at steady state at 35°C and pressure p. Half exits at 90 °C, 2 bar and half exits at -20 °C, 2 bar. For the control volume, $\dot{W}_{cv} = 0$.

FIND: Determine the minimum allowed value for p.

SCHEMATIC & GIVEN DATA:



$\dot{m}_2 = \dot{m}_3 = \frac{1}{2}\dot{m}_1$

$c_p = 14.3$ kJ/kg·K

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state. (2) The effects of kinetic and potential energy are negligible. (3) Hydrogen is modeled as an ideal gas with constant c_p.

ANALYSIS: The given information must satisfy the conservation of energy principle. Thus, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and assumptions 1 and 2 an energy rate balance reduces to give

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

or with $\dot{m}_2 = \dot{m}_3 = \dot{m}_1/2$

$$0 = h_1 - \frac{1}{2}h_2 - \frac{1}{2}h_3$$

As c_p is constant, $h = c_p(T - T_{ref})$, where T_{ref} is an arbitrary reference temperature. With this for h_1, h_2, and h_3 the energy rate balance gives

$$0 = T_1 - \frac{1}{2}T_2 - \frac{1}{2}T_3$$

Introducing the given temperatures

$$0 = 308 - \frac{363}{2} - \frac{253}{2} \Rightarrow 0 = 308 - 181.5 - 126.5$$

Thus, the conservation of energy principle is satisfied.

Turning next to the second law, an entropy rate balance reduces to give

$$0 = \overset{0}{\cancel{\sum \frac{\dot{Q}_j}{T_j}}} + \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

With $\dot{m}_2 = \dot{m}_3 = \dot{m}_1/2$

$$0 = \frac{1}{2}(s_1 - s_2) + \frac{1}{2}(s_1 - s_3) + \frac{\dot{\sigma}_{cv}}{\dot{m}}$$

Introducing Eq 6.23 with $p_2 = p_3$ and $p_1 = p$

$$0 = \frac{1}{2}\left[c_p \ln\frac{T_1}{T_2} - R\ln\frac{p}{p_2}\right] + \frac{1}{2}\left[c_p \ln\frac{T_1}{T_3} - R\ln\frac{p}{p_2}\right] + \frac{\dot{\sigma}_{cv}}{\dot{m}}$$

$$\Rightarrow \ln\frac{p}{p_2} = \frac{c_p}{2R}\ln\left(\frac{T_1^2}{T_2 T_3}\right) + \frac{1}{R}\left(\frac{\dot{\sigma}_{cv}}{\dot{m}}\right) = \left[\frac{14.3\ \text{kJ/kg·K}}{2(8.314/2.016)\ \text{kJ/kg·K}}\right]\ln\left(\frac{(308)^2}{363 \times 253}\right) + \frac{1}{R}\left(\frac{\dot{\sigma}_{cv}}{\dot{m}}\right)$$

$$= 0.0562 + \frac{1}{R}\left(\frac{\dot{\sigma}_{cv}}{\dot{m}}\right)$$

Since $p_2 = 2$ bar

$$\Rightarrow \quad p = (2.12\ \text{bar})\exp\left(\frac{\dot{\sigma}_{cv}/\dot{m}}{R}\right)$$

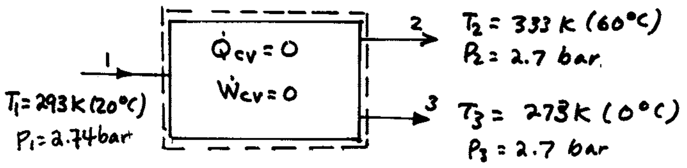
where $\dot{\sigma}_{cv}/\dot{m} \geq 0$. Thus, in the limit as $\dot{\sigma}_{cv}/\dot{m} \to 0$, $\to$ 2.12 bar ← min. pressure

PROBLEM 6.93

KNOWN: Air enters an insulated control volume operating at steady state at 20°C, 3 bar. A stream exits at 60°C, 2.7 bar, and another stream exits at 0°C, 2.7 bar. For the control volume $\dot{W}_{cv} = 0$.

FIND: Decide whether the device can operate as described.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state. (2) The effects of kinetic and potential energy are negligible. (3) Air is modeled as an ideal gas.

ANALYSIS: The conservation of mass and energy principles and the second law must be satisfied for the device to operate as described.

At steady state, the mass rate balance reduces to give

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \tag{1}$$

At steady state, the energy rate balance reduces with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and assumption 2 to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

or

$$0 = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \tag{2}$$

Letting $y = \dot{m}_2/\dot{m}_1$, Eqs. (1) and (2) combine to yield

$$y = \frac{h_1 - h_3}{h_2 - h_3}$$

with data from Table A-22

$$y = \frac{293.2 - 273.1}{333.3 - 273.1} = 0.334$$

Accordingly, the conservation of mass and energy principles require that the mass flow rates be in definite proportions: $\dot{m}_2/\dot{m}_1 = 0.334$, $\dot{m}_3/\dot{m}_1 = 0.666$.

An entropy rate balance at steady state reduces to give

$$0 = \cancelto{0}{\sum_j \frac{\dot{Q}_j}{T_j}} + \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

Thus

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_1} = \frac{\dot{m}_3}{\dot{m}_1} s_3 + \frac{\dot{m}_2}{\dot{m}_1} s_2 - s_1$$

$$= (1-y)s_3 + y s_2 - s_1$$

$$= (s_3 - s_1) + y(s_2 - s_3)$$

Then, with data from Table A-22, and using Eq. 6.21a

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \left(1.6073 - 1.6783 - \frac{8.314}{28.97} \ln\frac{2.7}{2.74}\right) + 0.334\left(1.8069 - 1.6073 - \cancelto{0}{\frac{8.314}{28.97} \ln\frac{2.7}{2.7}}\right)$$

$$= (-0.0668 + 0.0667) < 0$$

The device cannot operate as described.

PROBLEM 6.94*

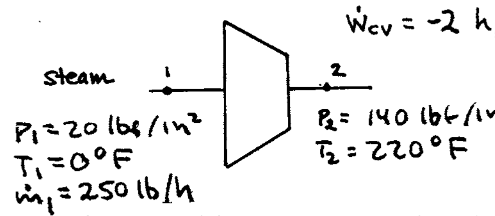
KNOWN: R134a enters and exits a compressor operating at steady state at specified temperatures and pressures. The ambient temperature is also specified. The required power input is claimed to be 2 hp.

FIND: Determine whether this claim can be correct.

SCHEMATIC & GIVEN DATA:

$T_0 = 530°R\ (70°F)$

$\dot{W}_{cv} = -2\ \text{hp}$ (claim)



steam
1
2
$p_1 = 20\ \text{lbf/in}^2$
$T_1 = 0°F$
$\dot{m}_1 = 250\ \text{lb/h}$
$p_2 = 140\ \text{lbf/in}^2$
$T_2 = 220°F$

ASSUMPTIONS: (1) Two control volumes at steady state are considered. One encloses the compressor. The other encloses the compressor plus a portion of the nearby surroundings so heat transfer occurs at the ambient temperature T_0. (2) Changes in kinetic and potential energy from inlet to exit can be ignored.

ANALYSIS: An entropy rate balance is used to determine if the claimed value for the power input can be correct. But first consider a control volume enclosing the compressor, and apply an energy rate balance to obtain the heat transfer. Thus with $\dot{m}_1 = \dot{m}_2 = \dot{m}$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right)$$

with assumption 2 and data from Table A-12E

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 - h_1 \Rightarrow \frac{\dot{Q}_{cv}}{\dot{m}} = \frac{(-2\ \text{hp})\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right|}{(250\ \text{lb/h})} + 146.18 - 101.88 = 23.94\ \text{Btu/lb}$$

As the variation of temperature over the surface of this control volume is not specified, the entropy transfer accompanying heat transfer cannot be evaluated without further information. Accordingly, consider an entropy rate balance for an enlarged control volume enclosing the compressor plus a portion of the nearby surroundings so heat transfer occurs at the ambient temperature T_0. That is

$$0 = \frac{\dot{Q}_{cv}}{T_0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

where $\dot{\sigma}_{cv}$ represents the rate of entropy production within the enlarged control volume. With data from Table A-4E

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_0} + s_2 - s_1 = \frac{-(23.94\ \text{Btu/lb})}{530°R} + 0.2667\ \frac{\text{Btu}}{\text{lb}°R} - 0.2238\ \frac{\text{Btu}}{\text{lb}°R}$$

① $$= -0.00227\ \text{Btu/lb}°R$$

As $\dot{\sigma}_{cv} \geq 0$, the claimed value for the work input cannot be correct. ←

---

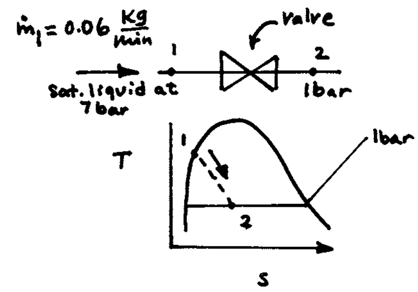
1. Note that the calculated value for $\dot{\sigma}_{cv}/\dot{m}$ is not for the compressor alone, but for an enlarged control volume of compressor plus nearby surroundings.

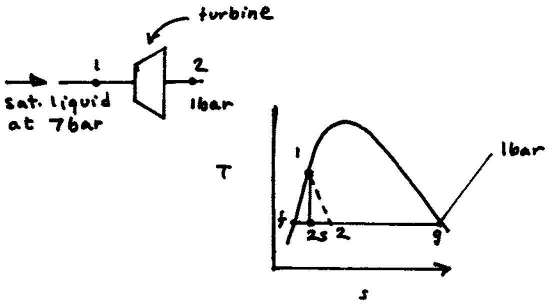
PROBLEM 6.95

KNOWN: Operating data are provided for ammonia throttled across a valve.

FIND: Determine the rate of entropy production. If the valve were relaced by a powery-recovery turbine, determine the maximum theoretical power that could be developed. Would you recommend the use of such a turbine?

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Control volumes enclosing the valve and enclosing the turbine operate at steady state. (2) $\dot Q_{cv} = 0$, and kinetic and potential energy changes from inlet to exit can be neglected.

ANALYSIS: Considering the valve first, the expansion is a throttling process for which $h_2 = h_1$ (Sec. 4.3). Thus, state 2 is fixed by $p_2 = 1$ bar and $h_2 = h_1$. The rate of entropy production can be evaluated from an entropy rate balance which a steady state reduces with $\dot m_1 = \dot m_2 = \dot m$ to

$$0 = \overset{0}{\cancel{\sum \frac{\dot Q_j}{T_j}}} + \dot m (s_1 - s_2) + \dot\sigma_{cv} \Rightarrow \dot\sigma_{cv} = \dot m (s_2 - s_1) \qquad (1)$$

From Table A-14 at 7bar, $s_1 = 0.9394$ kJ/kg·K. At state 2, $h_2 = h_1 = 244.69$ kJ/kg. Then

$$x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{244.69 - 28.18}{1370.23} = 0.158$$

Then

$$s_2 = s_f + x_2 (s_g - s_f) = 0.1191 + 0.158(5.8391 - .1191) = 1.0229 \text{ kJ/kg}\cdot\text{K}.$$

Thus, with Eq. (1)

$$\dot\sigma_{cv} = \left(0.06 \frac{kg}{min}\right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left((1.0229 - 0.9394) \frac{kJ}{kg \cdot K} \right) \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 8.35 \times 10^{-5} \frac{kW}{K} \longleftarrow \dot\sigma_{cv}$$

If the valve is replaced by a turbine operating at steady state, an energy rate balance reduces with assumption 2 to give

$$\dot W_{cv} = \dot m (h_1 - h_2)$$

An entropy rate balance gives Eq. (1) above. As $h_1 = 244.69$ kJ/kg, the work developed is

$$\dot W_{cv} = \left(\frac{0.06}{60} \frac{kg}{s}\right)\left(244.69 - h_2\right)\frac{kJ}{kg} \qquad (2)$$

Accordingly, the power developed increases as h_2 decreases. Since $s_2 \geq s_1$, the smallest allowed value is h_{2s}, corresponding to the state where $s_{2s} = s_1$, as shown on the sketch above. The quality at state 2s is

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{0.9394 - 0.1191}{5.8391 - 0.1191} = 0.1434 \Rightarrow h_{2s} = h_f + x_{2s}(h_{fg}) = 28.18 + (0.1434)(1370.23) = 224.67 \frac{kJ}{kg}$$

Then Eq. (2) gives

$$(\dot W_{cv})_{max} = \left(\frac{0.06}{60} \frac{kg}{s}\right)\left(244.69 - 224.67\right)\frac{kJ}{kg} \left| \frac{1 kW}{1 kJ/s} \right| = 0.02 \text{ kW} \longleftarrow (\dot W_{cv})_{max}$$

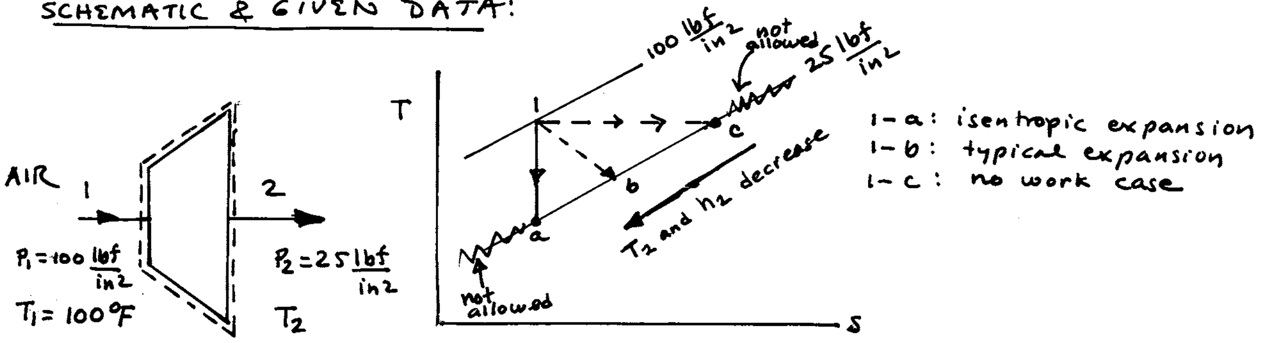
This is a small potential to develop power. Moreover, the affect of irreversibilities in an actual power-recovery turbine would be to reduce the actual power developed below the calculated value. Generally, for this type of application a power-recovery turbine would not be recommended.

PROBLEM 6.96

KNOWN: Steady-state operating data are provided for an air turbine.

FIND: Plot the work developed and entropy production, each per unit mass of air flowing, versus the turbine exit temperature.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the accompanying figure is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. 3. Air is modeled as an ideal gas.

ANALYSIS: At steady state, mass, energy and entropy rate balances reduce with stated assumptions to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad , \quad \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1$$

Since $\dot{\sigma}_{cv}/\dot{m} \geq 0$, we have $s_2 \geq s_1$. Inspection of the T-s diagram shows that the lowest allowed exit temperature corresponds to $s_2 = s_1$, for which $\dot{\sigma}_{cv} = 0$. Since specific enthalpy at the exit depends only on the exit temperature (ideal gas model), process 1-a also corresponds to the maximum theoretical value for $\dot{W}_{cv}/\dot{m}$.

The maximum exit temperature corresponds to the case of no work: $h_2 = h_1$ (a throttling process). Since $h(T)$ for an ideal gas, this case -- process 1-c -- has $T_2 = T_1$.

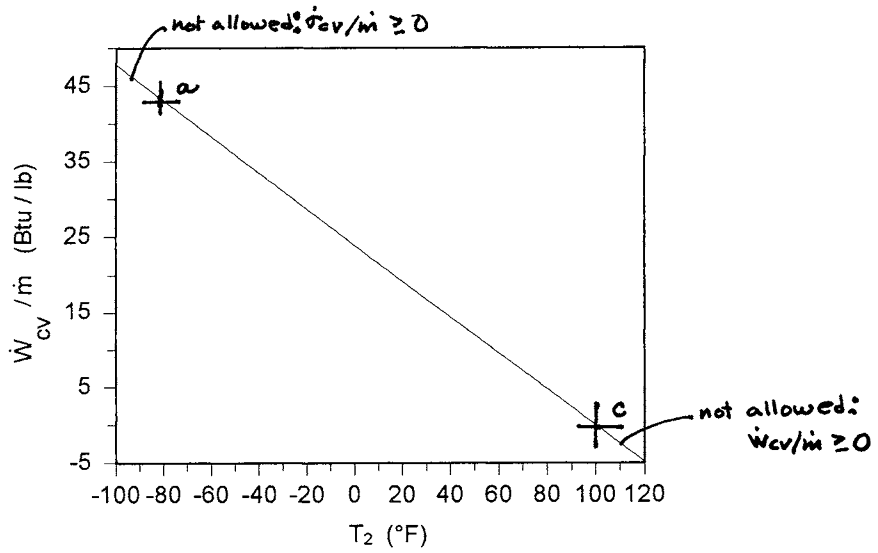
SAMPLE CALCULATIONS: Process 1-a: Eq. 6.21a reduces to give with Table A-22E data, $s^\circ(T_a) = s^\circ(T_1) + R \ln p_2/p_1 = 0.6095 + (1.986/28.97)\ln(25/100) = 0.51446\ \frac{Btu}{lb \cdot °R} \Rightarrow T_a = 377\ °R\ (-83°F)$, $h_a = 89.95$ Btu/lb. Then $(\dot{W}_{cv}/\dot{m})_a = 133.86 - 89.95 = 43.91$ Btu/lb. Process 1-c: $T_2 = T_1$. Thus, with Eq. 6.21a, $(\dot{\sigma}_{cv}/\dot{m})_c = \underbrace{s^\circ(T_2) - s^\circ(T_1)}_{=0} - R \ln p_2/p_1 = -\left(\frac{1.986}{28.97}\right)\ln\left(\frac{25}{100}\right) = 0.095\ \frac{Btu}{lb \cdot °R}$.

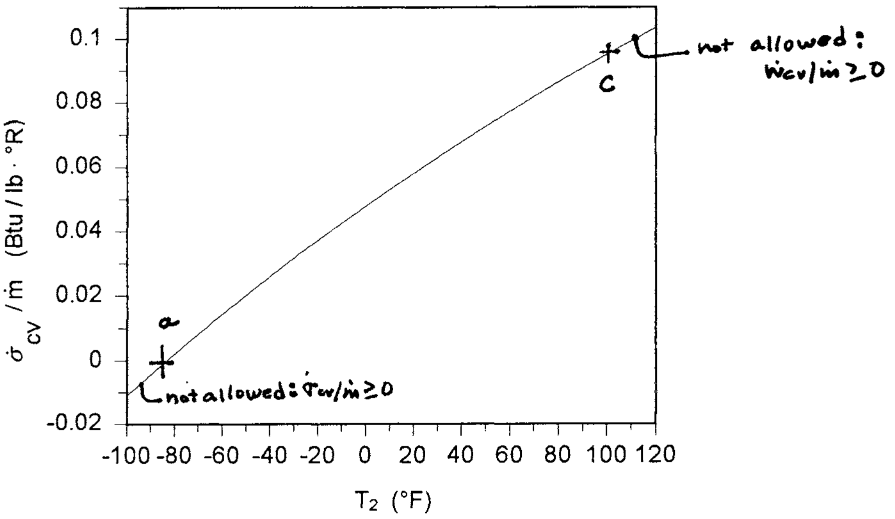
IT Code

```
T1 = 100  // °F
p1 = 100  // lbf/in.²
p2 = 25
T2 = 100

mdot = 1
Wdot / mdot = h1 - h2

h1 = h_T("Air", T1)
h2 = h_T("Air", T2)
sigmadot / mdot = s2 - s1
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
```

PROBLEM 6.96 (Contd.)

Data for the following plots are generated by sweeping T_2 from -100 to 120 in steps of 1 °F:



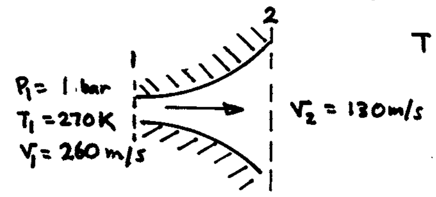


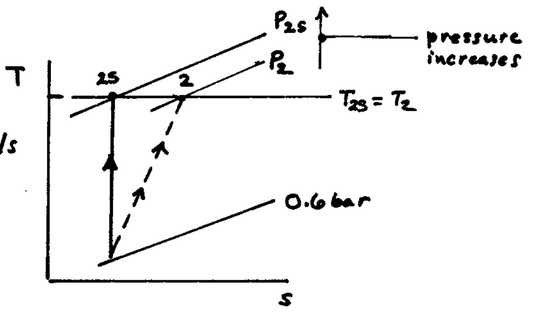
PROBLEM 6.97

KNOWN: Air enters an insulated diffuser operating at steady state at 1 bar, −3°C, 260 m/s and exits at 130 m/s.

FIND: Determine (a) the temperature of the air at the exit, (b) the maximum attainable exit pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The diffuser operates at steady state and is insulated. (2) The change in potential energy from inlet to exit can be neglected. (3) Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$ and with stated assumptions an energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}^{0}\right)$$

or

$$h_2 = h_1 + \frac{V_1^2 - V_2^2}{2}$$

With h_1 from Table A-22 at 270 K

$$h_2 = 270.11 + \left(\frac{260^2 - 130^2}{2}\right)\left(\frac{m^2}{s^2}\right)\left(\frac{1 N}{1 kg \cdot m/s^2}\right)\left(\frac{kJ}{10^3 N \cdot m}\right)$$
$$= 295.46 \text{ kJ/kg}$$

Table A-22 then gives T_2 = 295 K (22°C). ← T_2

An entropy rate balance reduces to read

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1$$

Since $\dot{\sigma}_{cv} \geq 0$, $s_2 \geq s_1$. For fixed exit temperature T_2 (and thus fixed exit velocity V_2), the accompanying T-s diagram shows that the maximum allowed exit pressure corresponds to the case $s_2 = s_1$. To determine this pressure, P_{2s}, use Eq. 6.21a to write

$$0 = s^\circ(T_{2s}) - s^\circ(T_1) - R \ln\frac{P_{2s}}{P_1} \Rightarrow \text{with } T_{2s} = T_2 \quad \ln\frac{P_{2s}}{P_1} = \frac{s^\circ(T_2) - s^\circ(T_1)}{R}$$

with data from Table A-22 at T_1 = 270 K, T_2 = 295 K

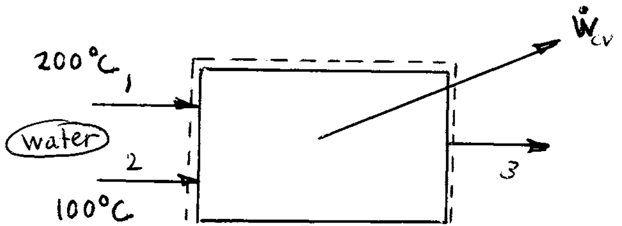
$$\ln\frac{P_{2s}}{P_1} = \frac{1.68515 - 1.59634}{(8.314/28.97)} \Rightarrow P_{2s} = 1.363\, P_1 = 1.363 \text{ bar}$$ ← $(P_2)_{MAX}$

PROBLEM 6.98*

KNOWN: Test data are provided for a new type of engine operating at steady state.

FIND: Determine the maximum theoretical rate power can be developed.

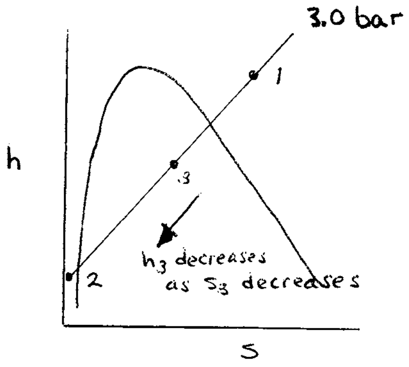
SCHEMATIC & GIVEN DATA:



$P_1 = P_2 = P_3 = 3$ bar

$\dot{m}_1 = 2\dot{m}_2$

$\dot{m}_3 = 5400 \frac{kg}{h}\left|\frac{1\,h}{3600\,s}\right| = 1.5\ kg/s$



ASSUMPTIONS: (1) The control volume shown above is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible.

ANALYSIS: At steady state, a mass rate balance gives $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$. Then, since $\dot{m}_3 = 1.5\ kg/s$ and $\dot{m}_1 = 2\dot{m}_2$, $\dot{m}_1 = 1\ kg/s$ and $\dot{m}_2 = 0.5\ kg/s$.

Using given assumptions, an energy rate balance reduces at steady state to give

$$\dot{W}_{cv} = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$$

With data from Tables A-2, 4, using $h_2 \approx h_f(T_2)$

$$\dot{W}_{cv} = \left(1\frac{kg}{s}\right)\left(2865.5\frac{kJ}{kg}\right) + \left(0.5\frac{kg}{s}\right)\left(419.04\frac{kJ}{kg}\right) - \left(1.5\frac{kg}{s}\right)(h_3)$$

$$= (3075.0 - 1.5\,h_3)\ kW \qquad (1)$$

From Eq.(1) it can be concluded that $\dot{W}_{cv}$ increases as h_3 decreases. The above Mollier diagram shows that the smallest allowed value for h_3 corresponds to the smallest allowed value for s_3.

To determine the smallest allowed value for s_3, apply an entropy rate balance at steady state: $0 = \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$

$$\Rightarrow s_3 = \frac{\dot{m}_1}{\dot{m}_3} s_1 + \frac{\dot{m}_2}{\dot{m}_3} s_2 + \frac{\dot{\sigma}_{cv}}{\dot{m}_3} \Rightarrow \quad s_3 = \left(\frac{2}{3}\right)s_1 + \left(\frac{1}{3}\right)s_2 + \frac{\dot{\sigma}_{cv}}{\dot{m}_3}$$

Since $\dot{\sigma}_{cv} \geq 0$, the smallest allowed value for s_3, and thus h_3, corresponds to $\dot{\sigma}_{cv} = 0$. With data from Tables A-2 and4, and using $s_2 = s_f(T_2)$

$$(s_3)_{min} = \left(\frac{2}{3}\right)s_1 + \left(\frac{1}{3}\right)s_2 = \left(\frac{2}{3}\right)(7.3115) + \left(\frac{1}{3}\right)(1.3069) = 5.3100\ kJ/kg\cdot K$$

Using this value, state 3 is located in the two-phase region with quality

$$(x_3)_{min} = \frac{5.3100 - 1.6718}{5.3201} = 0.684 \Rightarrow (h_3)_{min} = 561.47 + 0.684(2163.8) = 2041.5\ kJ/kg$$

① Finally, with Eq.(1), $(\dot{W}_{cv})_{MAX} = [3075.0 - 1.5(2041.5)] = 12.75\ kW \longleftarrow$

---

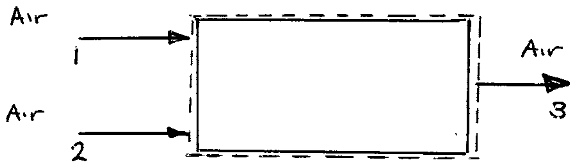
1. From Eq.(1), the case $\dot{W}_{cv} = 0$ corresponds to $h_3 = 2050\ kJ/kg$, or $x_3 = 0.688$, which differs only slightly from $(x_3)_{min}$. Accordingly, with such a small interval between maximum power and no power output, the effect of irreversibilities within the engine is expected to erode significantly the power that would be obtained.

PROBLEM 6.99

KNOWN: Steady state operating data are provided for a device.

FIND: Determine the maximum theoretical pressure achievable at the exit and discuss.

SCHEMATIC & GIVEN DATA:



$P_1 = P_2 = 1\ \text{atm}$

$T_1 = 600°R$, $\dot{m}_1 = 100\ \text{lb/h}$

$T_2 = 2000°R$, $\dot{m}_2 = 120\ \text{lb/h}$

ASSUMPTIONS: (1) The control volume shown above is at steady state. (2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and kinetic and potential energy effects are negligible. (3) The air is modeled as an ideal gas.

ANALYSIS: From a mass rate balance at steady state, $\dot{m}_3 = 220\ \text{lb/h}$. An energy rate balance reduces with assumptions (1) and (2) to give $0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$. Solving for h_3, and inserting data from Table **A-22E**

$$h_3 = \frac{\dot{m}_1 h_1 + \dot{m}_2 h_2}{\dot{m}_3} = \frac{(100)(143.47) + (120)(504.71)}{220} = 340.51\ \frac{\text{Btu}}{\text{lb}}$$

Using h_3, Table A-22E gives $s_3^\circ = 0.8343\ \text{Btu/lb}\cdot°R$. From the same table $s_1^\circ = 0.62607$, $s_2^\circ = 0.93205$,

Next, applying an entropy rate balance, $0 = \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}$. Or, since $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$, this gives

$$0 = \dot{m}_1 s_1 + \dot{m}_2 s_2 - (\dot{m}_1 + \dot{m}_2) s_3 + \dot{\sigma} \Rightarrow 0 = \dot{m}_1 (s_1 - s_3) + \dot{m}_2 (s_2 - s_3) + \dot{\sigma}$$

Then, using Eq 6.21a

$$0 = \dot{m}_1 \left[s_1^\circ - s_3^\circ - \frac{\bar{R}}{M} \ln \frac{P_1}{P_3} \right] + \dot{m}_2 \left[s_2^\circ - s_3^\circ - \frac{\bar{R}}{M} \ln \frac{P_2}{P_3} \right] + \dot{\sigma}$$

Solving

$$\dot{m}_1 \ln \frac{P_3}{P_1} + \dot{m}_2 \ln \frac{P_3}{P_2} = \frac{\dot{m}_1 [s_3^\circ - s_1^\circ] + \dot{m}_2 [s_3^\circ - s_2^\circ] - \dot{\sigma}}{\bar{R}/M}$$

Since $P_2 = P_1$, this becomes

$$\ln \frac{P_3}{P_1} = \frac{\dot{m}_1 (s_3^\circ - s_1^\circ) + \dot{m}_2 (s_3^\circ - s_2^\circ) - \dot{\sigma}}{(\dot{m}_1 + \dot{m}_2)\, \bar{R}/M}$$

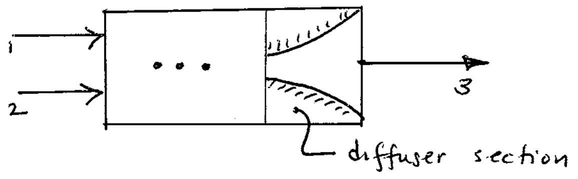
As $\dot{\sigma} \geq 0$, the maximum theoretical value for the ratio P_3/P_1 corresponds to $\dot{\sigma} = 0$. Thus, with known values

$$\ln \left(\frac{P_3}{P_1} \right)_{MAX} = \frac{(100)(0.8343 - 0.62607) + (120)(.8343 - 0.93205)}{(220)(1.986/28.97)} = 0.6029$$

or

$$\left(\frac{P_3}{P_1} \right)_{MAX} = 1.827 \Rightarrow (P_3)_{MAX} = 1.827\ \text{atm} \longleftarrow \text{MAX}$$

A pressure rise might be brought about by including a diffuser section as a component:

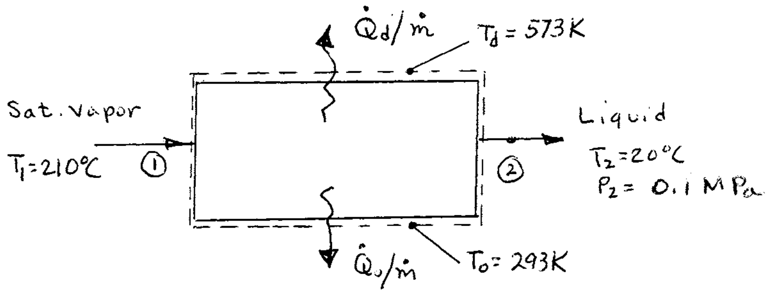


PROBLEM 6.100

KNOWN: Steady-state operating data are provided for a device to deliver energy by heat transfer at a specified temperature.

FIND: Determine the maximum theoretical amount of energy that could be delivered per unit mass of steam entering the device.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume $\dot{W}_{cv} = 0$, all heat transfers are in the directions indicated, and kinetic and potential energies are negligible.

ANALYSIS: From Table A-2, $h_1 = 2798.5\ kJ/kg$, $s_1 = 6.3585\ kJ/kg \cdot K$, $h_2 \approx h_f(T_2) = 83.96\ kJ/kg$, $s_2 \approx s_f(T_2) = 0.2966\ kJ/kg \cdot K$.

By combining mass and energy balances at steady state, using indicated assumptions, $0 = \dot{Q}_{cv} + \dot{m}(h_1 - h_2)$, where $\dot{Q}_{cv} = -[\dot{Q}_d + \dot{Q}_o]$. Thus

$$\frac{\dot{Q}_o}{\dot{m}} = (h_1 - h_2) - \frac{\dot{Q}_d}{\dot{m}} \tag{1}$$

At steady state, an entropy rate balance reduces to

$$0 = -\frac{\dot{Q}_o}{T_o} - \frac{\dot{Q}_d}{T_d} + \dot{m}(s_1 - s_2) + \dot{\sigma} \tag{2}$$

Next, eliminate $\dot{Q}_o$ between Eqs. (1), (2) to obtain

$$0 = -\left[\frac{(h_1 - h_2) - \dot{Q}_d/\dot{m}}{T_o}\right] - \frac{\dot{Q}_d/\dot{m}}{T_d} + (s_1 - s_2) + \frac{\dot{\sigma}}{\dot{m}} \Rightarrow \frac{\dot{Q}_d}{\dot{m}}\left[\frac{1}{T_o} - \frac{1}{T_d}\right] - \frac{(h_1 - h_2)}{T_o} + (s_1 - s_2) + \frac{\dot{\sigma}}{\dot{m}} = 0$$

Solving

$$\frac{\dot{Q}_d}{\dot{m}} = \frac{\left(\frac{h_1 - h_2}{T_o}\right) + (s_2 - s_1) - \dot{\sigma}/\dot{m}}{\left[\frac{1}{T_o} - \frac{1}{T_d}\right]}$$

As $\dot{\sigma} \geqq 0$, the maximum value corresponds to $\dot{\sigma} = 0$. That is

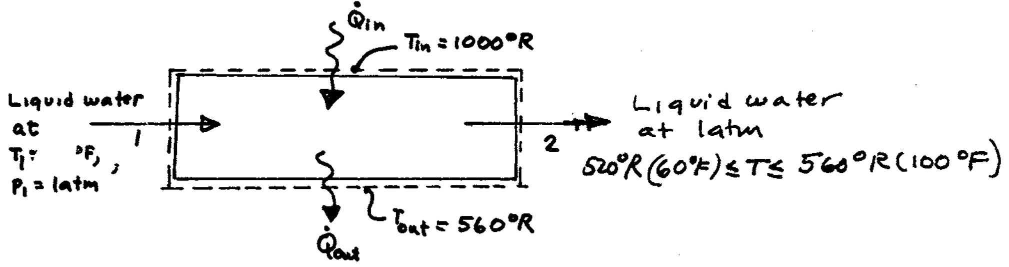
$$\left(\frac{\dot{Q}_d}{\dot{m}}\right)_{MAX} = \frac{\left(\frac{2798.5 - 83.96}{293}\right) + (0.2966 - 6.3585)}{\left[\frac{1}{293} - \frac{1}{573}\right]} = 1921\ kJ/kg \xleftarrow{\text{MAX}}$$

PROBLEM 6.101

KNOWN: A thermally activated device for chilling water requires no power input to operate. The chilled water exits at temperature T.

FIND: Plot the minimum theoretical heat addition required per lb of chilled water versus T.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$. (2) Changes in kinetic and potential energy from inlet to exit can be neglected. (3) The water can be modeled as incompressible with constant specific heat c.

ANALYSIS: At steadystate $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces with listed assumptions to give

$$0 = \dot{Q}_{in} - \dot{Q}_{out} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}(h_1 - h_2) \Rightarrow \frac{\dot{Q}_{in}}{\dot{m}} = \frac{\dot{Q}_{out}}{\dot{m}} + h_2 - h_1 \qquad (1)$$

At steady state an entropy rate balance reduces to give

$$0 = \frac{\dot{Q}_{in}}{T_{in}} - \frac{\dot{Q}_{out}}{T_{out}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

Solving for $\dot{Q}_{out}$

$$\frac{\dot{Q}_{out}}{\dot{m}} = \left(\frac{T_{out}}{T_{in}}\right)\left(\frac{\dot{Q}_{in}}{\dot{m}}\right) + T_{out}(s_1 - s_2) + T_{out}\frac{\dot{\sigma}_{cv}}{\dot{m}}$$

and substituting into Eq. (1)

$$\left[1 - \frac{T_{out}}{T_{in}}\right]\frac{\dot{Q}_{in}}{\dot{m}} = T_{out}(s_1 - s_2) + (h_2 - h_1) + T_{out}\dot{\sigma}_{cv}/\dot{m}$$

Or, upon rearrangement

$$\frac{\dot{Q}_{in}}{\dot{m}} = \frac{T_{out}(s_1 - s_2) + (h_2 - h_1)}{\left[1 - \frac{T_{out}}{T_{in}}\right]} + \frac{T_{out}\dot{\sigma}_{cv}/\dot{m}}{\left[1 - \frac{T_{out}}{T_{in}}\right]} \qquad (1)$$

Evaluating $(h_2 - h_1)$ using Eq. 3.20b and $(s_2 - s_1)$ using Eq. 6.24, the minimum heat addition corresponds to $\dot{\sigma}_{cv} = 0$ in Eq. (1), giving

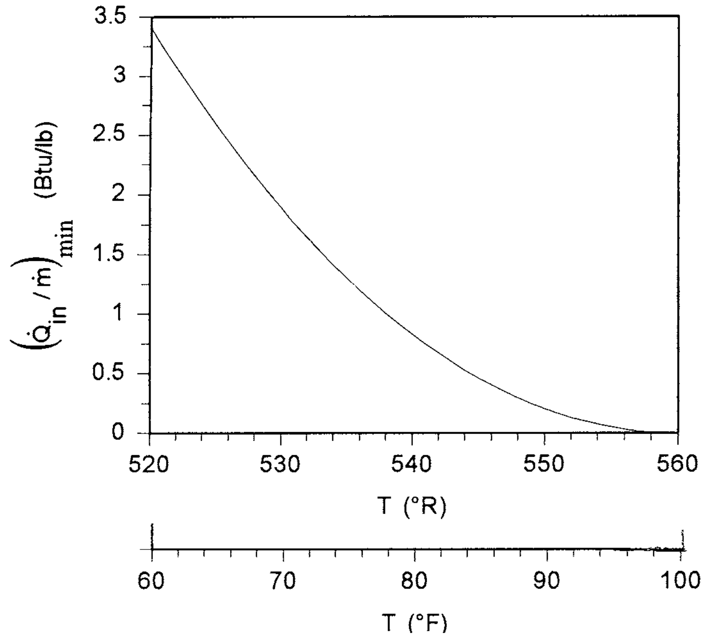
$$\left(\frac{\dot{Q}_{in}}{\dot{m}}\right)_{min} = \frac{c\left[T_{out}\ln\left(\frac{T_1}{T}\right) + (T - T_1)\right]}{\left[1 - \frac{T_{out}}{T_{in}}\right]}$$

, where c = 1 Btu/lb·°R (Table A-19E), $T_{in} = 1000°R$, $T_{out} = 560°R$, $T_1 = 560°R$, and $520 \le T \le 560°R$.

Sample calculation: $T = 520°R$, $(\dot{Q}_{in}/\dot{m})_{min} = 3.41$ Btu/lb.

Plot on next page

PROBLEM 6.101 (Contd.)

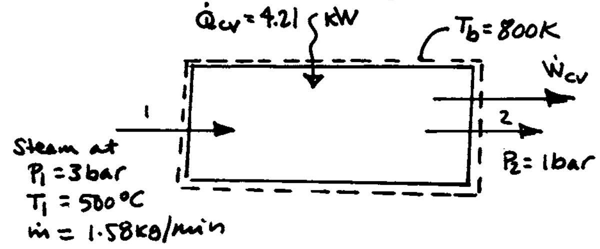


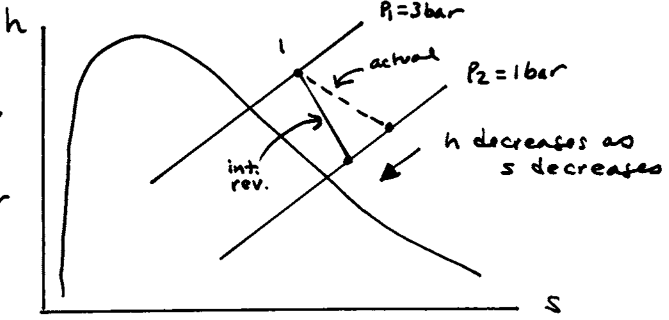
PROBLEM 6.102

KNOWN: A device is proposed for developing power using "waste heat."

FIND: Evaluate the maximum theoretical power that can be developed.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume shown in the accompanying schematic operates at steady state. 2. Kinetic/potential energy effects can be ignored. 3. Heat transfer occurs only at T_b.

ANALYSIS: At steady state, the mass and energy rate balances reduce to give

① $$\dot W_{cv} = \dot Q_{cv} + \dot m(h_1 - h_2) = (4.21\,kW) + \left(\frac{1.58}{60}\frac{kg}{s}\right)\left(3486\,\frac{kJ}{kg} - h_2\right)\left|\frac{1\,kW}{1\,kJ/s}\right| \qquad (1)$$

where h_1 is from Table A-4. Eq. (1) shows that $\dot W_{cv}$ increases as h_2 decreases. To determine the smallest allowed value for h_2, consider an entropy rate balance, which reduces at steady state to

$$0 = \frac{\dot Q_{cv}}{T_b} + \dot m(s_1 - s_2) + \dot\sigma_{cv} \Rightarrow s_2 = s_1 + \frac{\dot Q_{cv}}{\dot m T_b} + \frac{\dot\sigma_{cv}}{\dot m} \qquad (2)$$

The h-s diagram shows that h_2 decreases as s_2 decreases. The smallest s_2 value allowed corresponds to $(\dot\sigma_{cv}/\dot m) = 0$ in Eq. (2):

$$(s_2)_{min} = s_1 + \frac{\dot Q_{cv}}{\dot m T_b} = 8.3251 + \frac{4.21\,kJ/s}{\left(\frac{1.58}{60}\frac{kg}{s}\right)(800K)} = 8.5249\,\frac{kJ}{kg\cdot K}$$

where s_1 is from Table A-4. The corresponding value for specific enthalpy, $(h_2)_{min}$, is obtained by interpolation in Table A-4: $(h_2)_{min} = 3266$ kJ/kg. Then, Eq. (1) gives

$(\dot W_{cv})_{MAX} = (4.21) + (1.58/60)(3486 - 3266) = 10\,kW$ ←

---

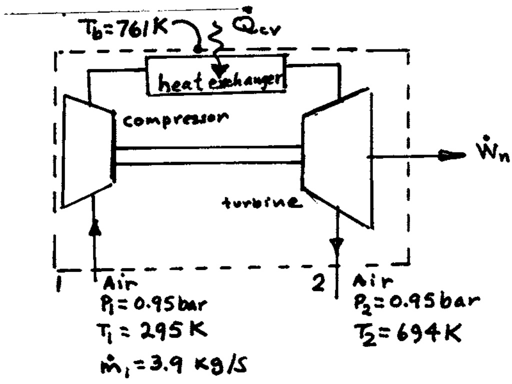
1. In principle, power is developed from two inputs: "waste" heat and steam. Note that $\dot W_{cv} = 0$ corresponds to $h_2 = 3645.9$ kJ/kg.

PROBLEM 6.103

KNOWN: Steady-state operating data are provided for a gas turbine power plant.

FIND: For the given conditions, determine the maximum theoretical value for the net power that can be developed.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the accompanying schematic operates at steady state. 2. Heat transfer to the control volume occurs only at T_b. 3. Kinetic/potential energy effects can be ignored. 4. Air is modeled as an ideal gas.

ANALYSIS: Mass and energy rate balances reduce at steady state to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\,[h_1 - h_2] \qquad (1)$$

An entropy rate balance at steady state gives

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}\,(s_1 - s_2) + \dot{\sigma}_{cv}$$

Solving for $\dot{Q}_{cv}$

$$\dot{Q}_{cv} = \dot{m}\,T_b(s_2 - s_1) - T_b\dot{\sigma}_{cv}$$

Inserting this into Eq. (1)

$$\dot{W}_{cv} = \underline{\dot{m}\left[(h_1 - h_2) + T_b(s_2 - s_1)\right]} - T_b\dot{\sigma}_{cv} \qquad (2)$$

The underlined term of Eq. (2) is fixed by conditions on the boundary of the control volume. The last term involving entropy production is greater than, or equal to, zero, depending on irreversibilities within the control volume. The maximum value of $\dot{W}_{cv}$ corresponds to $\dot{\sigma}_{cv} = 0$:

$$(\dot{W}_{cv})_{MAX} = \dot{m}\left[(h_1 - h_2) + T_b(s_2 - s_1)\right]$$

With Eq. 6.21a and data from Table A-22

$$(\dot{W}_{cv})_{MAX} = \dot{m}\left[h(T_1) - h(T_2) + T_b\left[s^\circ(T_2) - s^\circ(T_1) - \bar{R}\cancelto{0}{\ln\frac{p_2}{p_1}}\right]\right]$$

$$= \left(3.9\,\frac{kg}{s}\right)\left[(295.17 - 706.8)\frac{kJ}{kg} + 761\,K\,(2.5635 - 1.68515)\frac{kJ}{kg \cdot K}\right]$$

$$= 1001.5\,\frac{kJ}{s}\left|\frac{1\,MW}{10^3\,kJ/s}\right|$$

$$= 1\,MW \qquad \longleftarrow$$

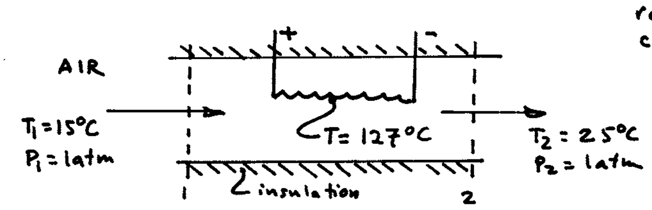
PROBLEM 6.104

KNOWN: Operating data are provided for an electrical resistor located in an insulated duct carrying air.

FIND: (a) For the resistor as the system, determine the rate of entropy production.
(b) For a control volume enclosing the air in the duct and the resistor, determine the volumetric flow rate and the rate of entropy production.

SCHEMATIC & GIVEN DATA:

resistor: 30 ohm
current: 15 amp



ASSUMPTIONS: (1) For part (a), the closed system consists of the resistor only. Heat transfer from this system takes place at $T_b = 400\,K (127°C)$. The system is at steady state. (2) For part (b), the control volume encloses the air in the duct and the resistor. $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. The control volume is at steady state. Air is modeled as an ideal gas.

ANALYSIS: (a) An energy rate balance for a closed system at steady state reduces to give $\dot{Q}_{cv} = \dot{W}_{cv}$. In this case, electrical work is done on the system. With given data

$$\dot{W}_{cv} = -(i)^2 R = -(15\,amp)^2(30\,ohm)\left[\frac{kW}{10^3 (amp)^2(ohm)}\right] = -6.75\,kW$$

At steady state an entropy rate balance becomes for heat transfer at T_b only

$$\cancelto{0}{\frac{dS}{dt}} = \frac{\dot{Q}_{cv}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}_{cv}}{T_b}$$

Finally

$$\dot{\sigma} = \frac{-(-6.75\,kW)}{400\,K} = 0.0169\,\frac{kW}{K} \longleftarrow \dot{\sigma}$$

(b) For the control volume at steady state the energy rate balance reduces with listed assumptions to

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \cancelto{0}{\left(\frac{V_1^2 - V_2^2}{2}\right)} + \cancelto{0}{g(z_1 - z_2)}\right]$$

Solving for $\dot{m}$ and using enthalpies from Table A-22

$$\dot{m} = \frac{(-\dot{W}_{cv})}{h_2 - h_1} = \frac{-(-6.75\,kW)(1\,kJ/s/kW)}{[298.2 - 288.2]\,kJ/kg} = 0.675\,\frac{kg}{s} \longleftarrow \dot{m}$$

The volumetric flow rate entering the duct is then

$$(AV)_1 = \dot{m} v_1 \Rightarrow (AV)_1 = \dot{m}\left(\frac{RT_1}{P_1}\right) = \left(0.675\,\frac{kg}{s}\right)\left(\frac{8314}{28.97}\,\frac{N \cdot m}{kg \cdot K}\right)(288\,K)\left(\frac{1}{1.01325 \times 10^5\,N/m^2}\right)$$

$$= 0.55\,m^3/s \longleftarrow (AV)_1$$

An entropy rate balance reduces at steady state to

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

With Eq. 6.21a

$$s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - R\,\cancelto{0}{\ln P_2/P_1}$$

Thus with data from Table A-22

$$\dot{\sigma}_{cv} = \dot{m}(s^\circ(T_2) - s^\circ(T_1)) = \left(0.695\,\frac{kg}{s}\right)(1.69528 - 1.66103)\,\frac{kJ}{kg \cdot K}$$

$$= 0.0238\,kW/K \longleftarrow \dot{\sigma}_{cv}$$

COMMENT: In part (a), $\dot{\sigma}$ accounts for the irreversibility of electrical current flow through a resistance. In part (b), $\dot{\sigma}_{cv}$ includes both the electrical irreversibility and the irreversibility of heat transfer from the resistor to the air.

PROBLEM 6.105

KNOWN: Operating data are provided for an electronics enclosure at steady state.

FIND: For the control volume of Example 4.8 determine the rate of entropy production when air exits at 32°C.

SCHEMATIC & GIVEN DATA: See Fig. E4.8

ASSUMPTIONS: 1. See Example 4.8. 2. Ignore the change in pressure from inlet to exit.

ANALYSIS: At steady state, mass and entropy rate balances reduce to give

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

Introducing $\dot{m}$ from Example 4.8 and evaluating $(s_2 - s_1)$ with Eq. 6.23

$$\dot{\sigma}_{cv} = \left[\frac{(-\dot{W}_{cv})}{c_p(T_2 - T_1)}\right]\left(c_p \ln\frac{T_2}{T_1} - \cancel{R \ln\frac{p_2}{p_1}}^{\,0}\right) \Rightarrow \dot{\sigma}_{cv} = \frac{(-\dot{W}_{cv})}{(T_2 - T_1)} \ln\frac{T_2}{T_1}$$

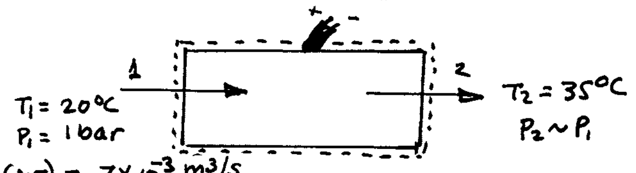
$$\Rightarrow \quad \dot{\sigma}_{cv} = \frac{(98\,\text{W})}{(305 - 293)\,\text{K}} \ln\frac{305}{293} = 0.328\,\frac{\text{W}}{\text{K}} \longleftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.106

KNOWN: Operating data are provided for an electronics enclosure at steady state.

FIND: For the control volume shown in the schematic determine the rate of entropy production.

SCHEMATIC & GIVEN DATA:



$(AV)_1 = 7 \times 10^{-3}\ \text{m}^3/\text{s}$

(from Solution to Prob. 4.70)

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$ and $p_2 \sim p_1$. 3. Air is modeled as an ideal gas.

ANALYSIS: Mass and entropy rate balances reduce at steady state to give

$$0 = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \quad \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

or with Eq. 6.21a and data from Table A-22

$$\dot{\sigma}_{cv} = \frac{(AV)_1}{v_1}\left[s^\circ(T_2) - s^\circ(T_1) - \cancel{R \ln\frac{p_2}{p_1}}^{\,0}\right]$$

$$= \frac{(AV)_1}{(RT_1/p_1)}\left[s^\circ(T_2) - s^\circ(T_1)\right] = \left[\frac{(7\times10^{-3}\,\text{m}^3/\text{s})(10^5\,\text{N/m}^2)}{\left(\frac{8314}{28.97}\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(293\,\text{K})}\right]\left[1.7284 - 1.6785\right]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$

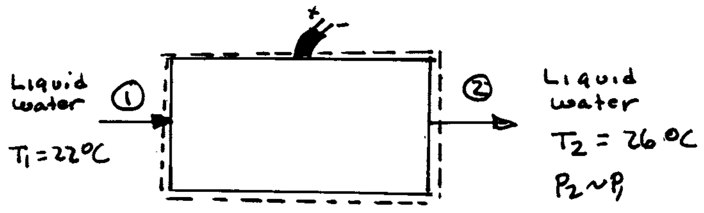
$$= 4.15 \times 10^{-4}\,\frac{\text{kW}}{\text{K}} \longleftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.107*

KNOWN: Steady-state operating data are provided for a water-jacketed electronics enclosure.

FIND: Determine the rate of entropy production when water exits at 26 °C.

SCHEMATIC & GIVEN DATA:



$\dot{m} = 10\ \text{kg/min} = 0.1667\ \text{kg/s}$
(from Problem 4.71)

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$ 3. For the liquid water, $s \approx s_f(T)$.

ANALYSIS: Mass and entropy rate balances reduce at steady state to give

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}\left[s_f(T_2) - s_f(T_1)\right]$$

With data from Table A-2

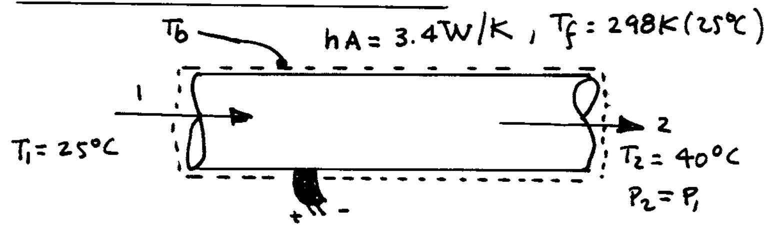
$$\dot{\sigma}_{cv} = \left(0.1667\ \frac{kg}{s}\right)\left[0.3814 - 0.3251\right]\frac{kJ}{kg\cdot K}\left|\frac{1\ kW}{1\ kJ/s}\right| = 9.4\times10^{-3}\ \frac{kW}{K} \longleftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.108

KNOWN: Steady-state operating data are provided for the electronics-laden cylinder of Problem 4.73.

FIND: Determine the rate of entropy production when air exits at 40°C.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{cv} = -34\ W$
$\dot{m} = 0.011\ kg/s$
] from Prob. 4.73

ASSUMPTIONS: 1. The control volume shown in the figure is at steady state. 2. Heat transfer by convection from the cylinder to the surroundings occurs at temperature T_b. 3. Air is modeled as an ideal gas.

ANALYSIS: Mass and energy rate balances reduce at steady state to give

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}[s_1 - s_2] + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_2 - s_1)$$

Using Eq. 6.21a

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}\left[s^\circ(T_2) - s^\circ(T_1) - \cancel{R\ln P_2/P_1}^{\,0}\right]$$

T_b can be evaluated using Eq. 2-34, written as $\dot{Q}_{cv} = -hA[T_b - T_f]$, where the minus sign is used to conform with the sign convention for heat transfer in the energy and entropy balances. Thus

$$T_b = \left(\frac{-\dot{Q}_{cv}}{hA}\right) + T_f = \left(\frac{34\ W}{3.4\ W/K}\right) + 298K = 308K$$

Then, with data from Table A-22

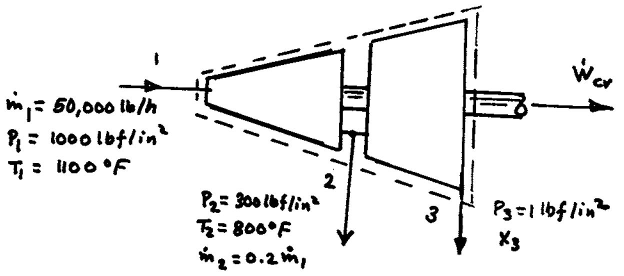
$$\dot{\sigma}_{cv} = -\frac{(-34W)}{308K} + \left(0.011\ \frac{kg}{s}\right)\left[1.7446 - 1.6953\right]\frac{kJ}{kg\cdot K}\left|\frac{1\ W}{1\ J/s}\right|\left|\frac{10^3 J}{1 kJ}\right| = 0.65\ \frac{W}{K} \longleftarrow \dot{\sigma}_{cv}$$

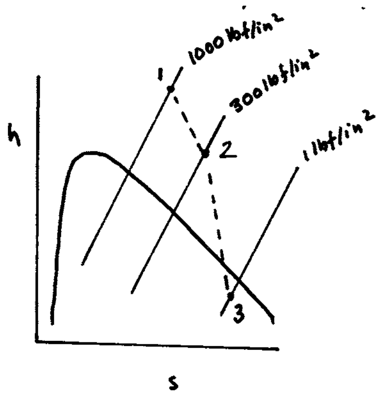
PROBLEM 6.109

KNOWN: Operating data are provided for a turbine operating at steady state from which steam is extracted at an intermediate pressure.

FIND: (a) For x=99% at the turbine exit, determine the power developed and the rate of entropy production. (b) Plot the quantities of part (a) versus x_3 ranging from 90 to 100%.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown on the accompanying figure operates at steady state. (2) $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible.

ANALYSIS: At steady state a mass rate balance reduces to $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$. Then, with $\dot{m}_2 = 0.1\,\dot{m}_1$, it follows that $\dot{m}_3 = 0.8\,\dot{m}_1$.

With assumption 2 an energy rate balance at steady state reduces to give

$$\dot{W}_{cv} = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

$$= \dot{m}_1 [h_1 - 0.2 h_2 - 0.8 h_3] \qquad (1)$$

An entropy rate balance at steady state takes the form

$$0 = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}_3 s_3 + \dot{m}_2 s_2 - \dot{m}_1 s_1$$

$$= \dot{m}_1 [0.8 s_3 + 0.2 s_2 - s_1] \qquad (2)$$

(a) From Table A-4E, $h_1 = 1562.9$ Btu/lb, $h_2 = 1421$ Btu/lb, $s_1 = 1.6908$ Btu/lb·°R, $s_2 = 1.7187$ Btu/lb·°R. For $x_3 = 0.99$, using data from Table A-3E at 1 lbf/in².

$$h_3 = h_f + x_3(h_g - h_f) = 69.74 + 0.99(1036) = 1095.38 \text{ Btu/lb}$$

$$s_3 = s_f + x_3(s_g - s_f) = 0.1327 + 0.99(1.8453) = 1.9595 \text{ Btu/lb·°R}$$

Substituting values in Eq. (1)

$$\dot{W}_{cv} = 50,000 \frac{\text{lb}}{\text{h}} \left[1562.9 - 0.2(1421) - 0.8(1095.38)\right] \frac{\text{Btu}}{\text{lb}}$$

$$= 2.012 \times 10^7 \text{ Btu/h} \qquad \longleftarrow \dot{W}_{cv}$$

Eq. (2) gives

$$\dot{\sigma}_{cv} = 50,000 \frac{\text{lb}}{\text{h}} \left[0.8(1.9595) + 0.2(1.7187) - 1.6908\right] \frac{\text{Btu}}{\text{lb·°R}} = 11,027 \frac{\text{Btu}}{\text{h·°R}} \longleftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.109 (cont'd.)

(b) Data for the required plots are generated using *IT*, as follows:

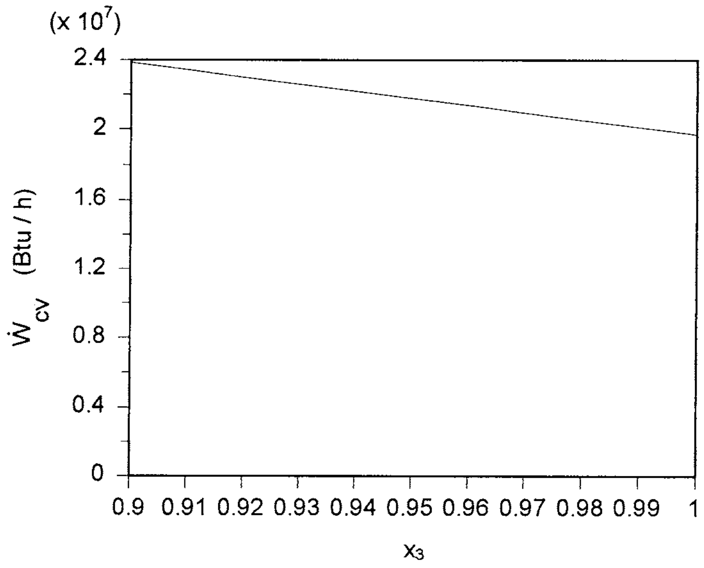
IT Code

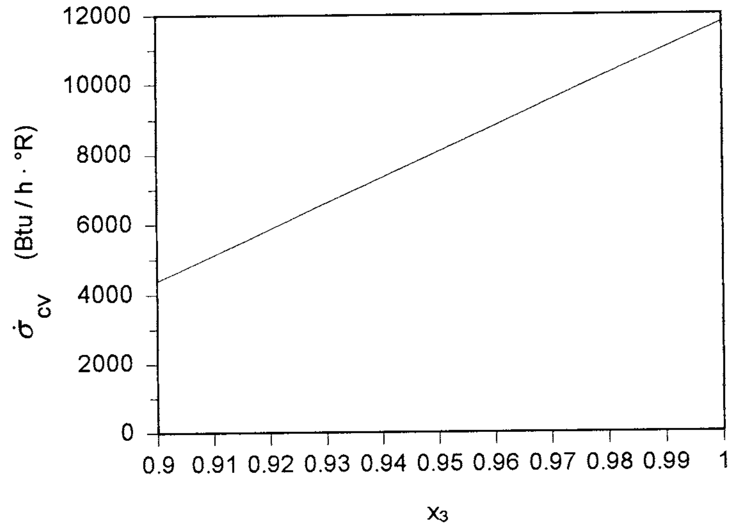
```
mdot1 = 50000  // lb/h
p1 = 1000  // lbf/in.²
T1 = 1100  // °F
p2 = 300
T2 = 800
mdot2 = 0.2 * mdot1
p3 = 1
x3 = 0.99

mdot1 = mdot2 + mdot3
Wdot = mdot1 * h1 - mdot2 * h2 - mdot3 * h3
sigmadot = mdot3 * s3 + mdot2 * s2 - mdot1 * s1

h1 = h_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
h3 = hsat_Px("Water/Steam", p3, x3)
s1 = s_PT("Water/Steam", p1, T1)
s2 = s_PT("Water/Steam", p2, T2)
s3 = ssat_Px("Water/Steam", p3, x3)
```

IT Results for $x_3 = 0.99$

h_1 = 1563 Btu/lb
h_2 = 1421 Btu/lb
h_3 = 1095 Btu/lb
s_1 = 1.691 Btu/lb·°R
s_2 = 1.719 Btu/lb·°R
s_3 = 1.959 Btu/lb·°R
$\dot{W}_{cv} = 2.012 \times 10^7$ Btu/h
$\dot{\sigma}_{cv} = 1.103 \times 10^4$ Btu/lb·°R

PLOTS:



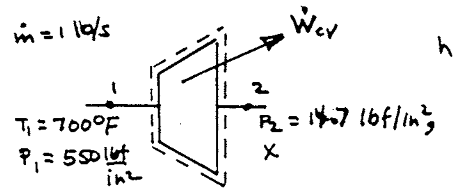


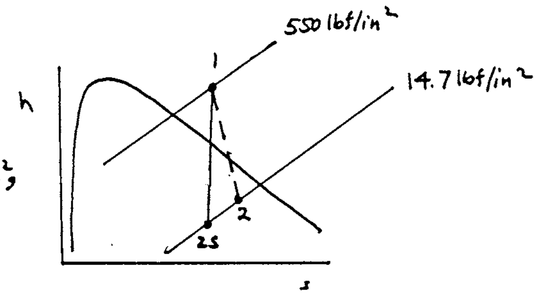
PROBLEM 6.110

KNOWN: Steady state operating data are provided for an insulated steam turbine. A two-phase liquid-vapor mixture with quality x exits the turbine.

FIND: Plot $\dot{W}_{cv}$ and $\dot{\sigma}_{cv}$ versus x

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) At the exit the steam is a two-phase liquid-vapor mixture with quality x. (3) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy changes can be ignored.

ANALYSIS: Reducing mass, energy, and entropy rate balances

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) \,, \quad \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

From steam table data, $h_1 = 1353.7$ Btu/lb, $s_1 = 1.5987$ Btu/lb·°R, $h_2 = h_f + x\, h_{fg} = 180.2 + x(970.4)$, $s_2 = s_f + x\, s_{fg} = 0.3121 + x(1.4446)$. Then

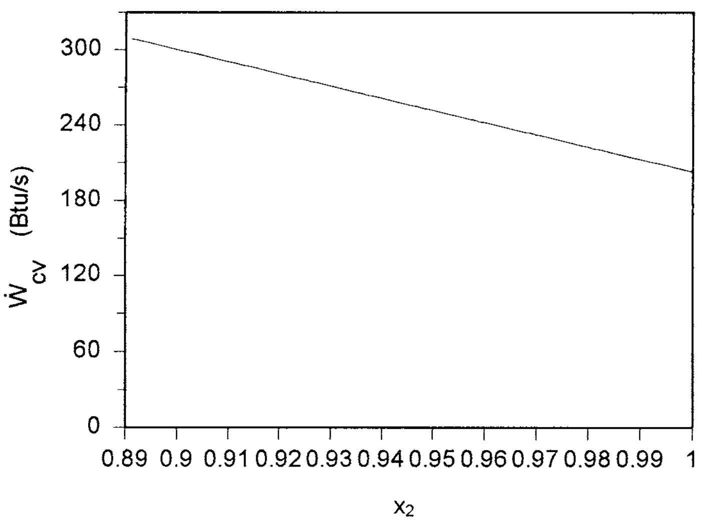
$$\dot{W}_{cv} = \left(1 \frac{lb}{s}\right)(1353.7 - 180.2 - 970.4\,x)\left(\frac{Btu}{lb}\right) = [1173.5 - 970.4\,x]\ \frac{Btu}{s} \qquad (3)$$

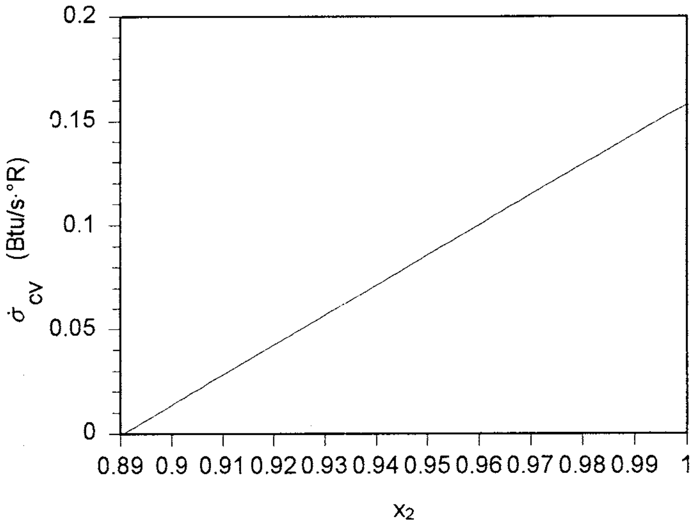
$$\dot{\sigma}_{cv} = \left(1 \frac{lb}{s}\right)(0.3121 + 1.4446\,x - 1.5987)\left(\frac{Btu}{lb \cdot °R}\right) = [1.4446\,x - 1.2866]\ \frac{Btu}{s \cdot °R} \qquad (4)$$

Sample calculation: Since $\dot{\sigma}_{cv} \geq 0$, the smallest allowed value for x is x_{2s} corresponding to $\dot{\sigma}_{cv} = 0$. Setting Eq. (4) to zero and solving; $x_{2s} = 0.8906$. The corresponding value of $\dot{W}_{cv}$ from Eq. (3) is 309.3 Btu/s.

The following plots of Eq. (3) and (4) are made using IT, although other plotting software such as a spreadsheet could be used.

Plots:





The plots show that greater entropy production (irreversibility) corresponds to less power output for the given operating conditions, as expected.

PROBLEM 6.110 (Contd.)

```
mdot = 1  // lb/s
T1 = 700  // °F
p1 = 550  // lbf/in.²
p2 = 14.7
x2 = 0.8906

Wdot = mdot * (h1 - h2)
sigmadot = mdot * (s2 - s1)

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = hsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
```

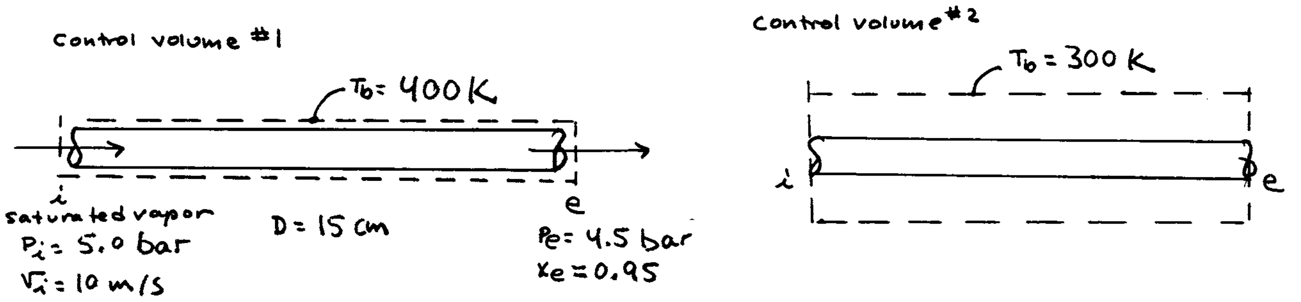
IT Results for $x_2 = 1$	*IT* Results for $x_2 = 0.8906$
$h_1 = 1353$ Btu/lb	$h_1 = 1353$ Btu/lb
$h_2 = 1150$ Btu/lb	$h_2 = 1044$ Btu/lb
$s_1 = 1.599$ Btu/lb·°R	$s_1 = 1.599$ Btu/lb·°R
$s_2 = 1.756$ Btu/lb·°R	$s_2 = 1.598$ Btu/lb·°R
$\dot{W}_{cv} = 203.2$ Btu/s	$\dot{W}_{cv} = 309.3$ Btu/s
$\dot{\sigma}_{cv} = 0.1579$ Btu/s·°R	$\dot{\sigma}_{cv} = 0$

PROBLEM 6.111*

KNOWN: Steady-state operating data are provided for steam flowing through a pipe.

FIND: Determine (a) the velocity of the steam at the pipe exit, (b) the rate of heat transfer from the pipe, (c) the rate of entropy production for each of two specifications of the control volume.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The two control volumes shown above are at steady state. (2) For the control volumes, heat transfer with the surroundings occurs at temperature T_b only; $\dot{W}_{cv} = 0$; and potential energy effects are absent.

ANALYSIS: Consider control volume #1. At steady state a mass rate balance reads $\dot{m}_i = \dot{m}_e$. Thus

$$\frac{\cancel{A_i} V_i}{v_i} = \frac{\cancel{A_e} V_e}{v_e} \Rightarrow V_e = \frac{v_e}{v_i} V_i = \left(\frac{0.3935\ \mathrm{m^3/kg}}{0.3749\ \mathrm{m^3/kg}}\right)\left(10\ \frac{\mathrm{m}}{\mathrm{s}}\right) = 10.50\ \frac{\mathrm{m}}{\mathrm{s}} \longleftarrow V_e$$

Steam table data: $v_i = 0.3749\ \mathrm{m^3/kg}$, $v_e = v_f + x_e v_{fg} = 0.0010882 + 0.95(0.4140 - 0.0010882) = 0.3935\ \mathrm{m^3/kg}$

With these data the mass flow rate is

$$\dot{m} = \frac{A_i V_i}{v_i} = \left(\frac{\pi D^2}{4}\right)\left(\frac{V_i}{v_i}\right) = \frac{\pi}{4}(0.15\,\mathrm{m})^2\left(\frac{10\ \mathrm{m/s}}{0.3749\ \frac{\mathrm{m^3}}{\mathrm{kg}}}\right) = 0.471\ \frac{\mathrm{kg}}{\mathrm{s}}$$

Mass and energy balances reduce to give

$$\dot{Q}_{cv} = \dot{m}\left(h_e - h_i + \frac{V_e^2 - V_i^2}{2}\right) = \left(0.471\ \frac{\mathrm{kg}}{\mathrm{s}}\right)\left[(2637.9 - 2748.7)\frac{\mathrm{kJ}}{\mathrm{kg}} + \frac{(10.50^2 - 10^2)}{2}\frac{\mathrm{m^2}}{\mathrm{s^2}}\left|\frac{1\ \mathrm{kN\,s^2}}{1000\ \mathrm{kg\,m}}\right|\left|\frac{1\ \mathrm{kJ}}{1\ \mathrm{kN\,m}}\right|\right]$$

$$= -52.18\ \mathrm{kW} \longleftarrow \dot{Q}_{cv}$$

Steam table data: $h_i = 2748.7\ \mathrm{kJ/kg}$, $h_e = h_f + x_e h_{fg} = 623.5 + 0.95(2120.7) = 2637.9\ \frac{\mathrm{kJ}}{\mathrm{kg}}$

Mass and entropy balances reduce to give

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_e - s_i)$$

Steam table data: $s_i = 6.8212\ \mathrm{kJ/kg\,K}$, $s_e = s_f + x_e s_{fg} = 1.8207 + 0.95(6.8565 - 1.8207) = 6.6047\ \mathrm{kJ/kg\,K}$

Control volume #1, $T_b = 400$ K:

$$\dot{\sigma}_{cv} = -\frac{(-52.18\ \mathrm{kW})}{400\ \mathrm{K}} + \left(0.471\ \frac{\mathrm{kg}}{\mathrm{s}}\right)(6.6047 - 6.8212)\frac{\mathrm{kJ}}{\mathrm{kg\,K}} = 0.02848\ \frac{\mathrm{kW}}{\mathrm{K}} \longleftarrow (\dot{\sigma}_{cv})_1$$

Control volume #2, $T_b = 300$ K:

$$\dot{\sigma}_{cv} = -\frac{(-52.18\ \mathrm{kW})}{300\ \mathrm{K}} + \left(0.471\ \frac{\mathrm{kg}}{\mathrm{s}}\right)(6.6047 - 6.8212)\frac{\mathrm{kJ}}{\mathrm{kg\,K}} = 0.07196\ \frac{\mathrm{kW}}{\mathrm{K}} \longleftarrow (\dot{\sigma}_{cv})_2$$

COMMENT: For control volume #1 there is a pressure decrease as the steam flows through the pipe. For this control volume the pressure drop is the most significant source of irreversibility. For control volume #2, in addition to the pressure drop irreversibility, there is the heat transfer from the outer surface of the pipe to the surroundings. Accordingly, the entropy production is greater for C.V. #2 than for C.V. #1. For control volume #1, the heat transfer effect is an *external* irreversibility.

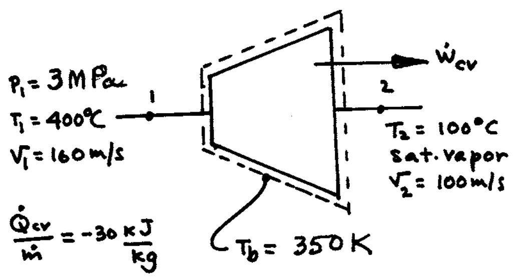
PROBLEM 6.112

KNOWN: Steady state operating data are provided for a steam turbine.

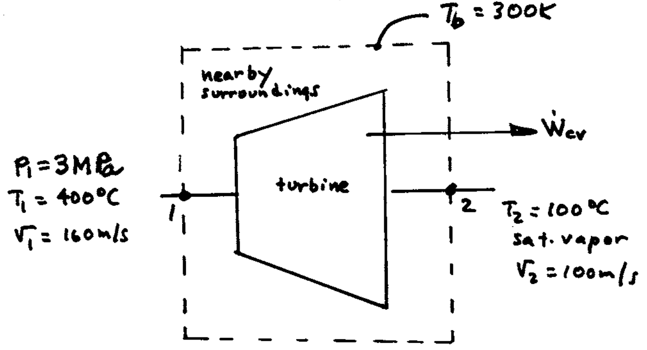
FIND: Determine the work developed, and for each of two choices of the control volume the entropy produced.

SCHEMATIC & GIVEN DATA:

CONTROL VOLUME #1:



CONTROL VOLUME #2:



ASSUMPTIONS: (1) The control volumes shown above are at steady state. (2) For the control volumes heat transfer takes place only at temperature T_b. (3) Potential energy changes can be ignored.

ANALYSIS: For control volume #1, mass and energy rate balances reduce to read

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

With data from Table A-4, $h_1 = 3230.9$ kJ/kg, $s_1 = 6.9212$ kJ/kg·K, and from Table A-2 $h_2 = 2676.1$ kJ/kg, $s_2 = 7.3549$ kJ/kg·K. Thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = -30\ \frac{kJ}{kg} + (3230.9 - 2676.1)\frac{kJ}{kg} + \frac{(160)^2 - (100)^2 (m^2/s^2)}{2}\left|\frac{1N}{1 kg\cdot m/s^2}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right|$$

$$= (-30 + 554.8 + 7.8)\frac{kJ}{kg} = +532.6\ \frac{kJ}{kg} \longleftarrow (\dot{W}_{cv}/\dot{m})$$

An entropy rate balance at steady state reduces with assumption 2

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \frac{(-\dot{Q}_{cv}/\dot{m})}{T_b} + (s_2 - s_1) \qquad (2)$$

$$= \frac{-(-30\ kJ/kg)}{350K} + (7.3549 - 6.9212)\frac{kJ}{kg\cdot K} = 0.5194\ \frac{kJ}{kg\cdot K} \longleftarrow (\dot{\sigma}_{cv}/\dot{m})_1$$

Equation (2) is applicable to control volume #2, except now $T_b = 300K$. Thus

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{(-30 kJ/kg)}{300K} + (7.3549 - 6.9212)\frac{kJ}{kg\cdot K} = 0.5337\ \frac{kJ}{kg\cdot K} \longleftarrow (\dot{\sigma}_{cv}/\dot{m})_2$$

COMMENT: The entropy production in control volume #2 is greater because this enlarged control volume includes an additional source of irreversibility: heat transfer from the outer surface of the turbine to the surroundings. For control volume #1 the heat transfer effect is an external irreversibility.

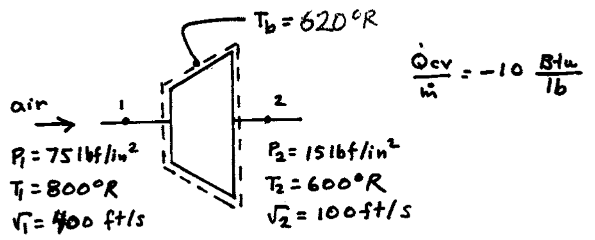
PROBLEM 6.113

KNOWN: Steady state operating data are provided for an air turbine.

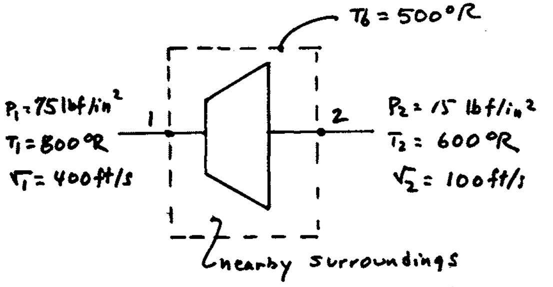
FIND: Determine the work developed, and for each of two choices of the control volume the entropy produced.

SCHEMATIC & GIVEN DATA:

Control Volume #1



Control Volume #2



ASSUMPTIONS: (1) The control volumes shown above are at steady state. (2) For the control volumes heat transfer takes place only at temperature T_b. (3) Potential ① energy changes can be ignored. (4) Air is modeled as an ideal gas.

ANALYSIS: For control volume #1 mass and energy balances reduce to read

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) \qquad (1)$$

With data from Table A-22E $h_1 = 191.81$ Btu/lb, $s_1^\circ = 0.69558$ Btu/lb·°R, $h_2 = 143.47$ Btu/lb, $s_2^\circ = 0.62607$ Btu/lb·°R. Thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = -10\,\frac{\text{Btu}}{\text{lb}} + (191.81 - 143.47)\frac{\text{Btu}}{\text{lb}} + \left(\frac{(400)^2 - (100)^2}{2}\right)\left(\frac{\text{ft}^2}{\text{s}^2}\right)\left|\frac{\text{lbf}}{32.2\,\text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right|$$

$$= -10 + 48.34 + 2.99 = +41.33\ \text{Btu/lb} \longleftarrow (\dot{W}_{cv}/\dot{m})$$

An entropy balance at steady state reduces with assumption 2

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \frac{(-\dot{Q}_{cv}/\dot{m})}{T_b} + (s_2 - s_1) \qquad (2)$$

$$= \left(-\frac{\dot{Q}_{cv}/\dot{m}}{T_b}\right) + (s_2^\circ - s_1^\circ - R \ln P_2/P_1)$$

$$= -\frac{(-10\ \text{Btu/lb})}{620°\text{R}} + \left(0.62607 - 0.69558 - \frac{1.986}{28.97}\ln\frac{15}{75}\right)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} = 0.057\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} \longleftarrow (\dot{\sigma}_{cv}/\dot{m})_1$$

Equation (2) is applicable to control volume #2, except now $T_b = 500°R$. Thus

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = -\frac{(-10)}{500} + \left(0.62607 - 0.69558 - \frac{1.986}{28.97}\ln\frac{15}{75}\right) = 0.061\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}} \longleftarrow (\dot{\sigma}_{cv}/\dot{m})_2$$

COMMENT:

The entropy production in control volume #2 is greater because this enlarged control volume includes an additional source of irreversibility: heat transfer from the outer surface of the turbine to the surroundings. For control volume #1 the heat transfer effect is an *external* irreversibility.

---

1. From Table A-1E for air $P_c = 37.2$ atm, $T_c = 239°R$. Thus

$$P_{R1} = \frac{75/14.7}{37.2} = 0.137, \quad T_{R1} = \frac{800}{239} = 3.35$$

$$P_{R2} = \frac{15/14.7}{37.2} = 0.027, \quad T_{R2} = \frac{600}{239} = 2.51$$

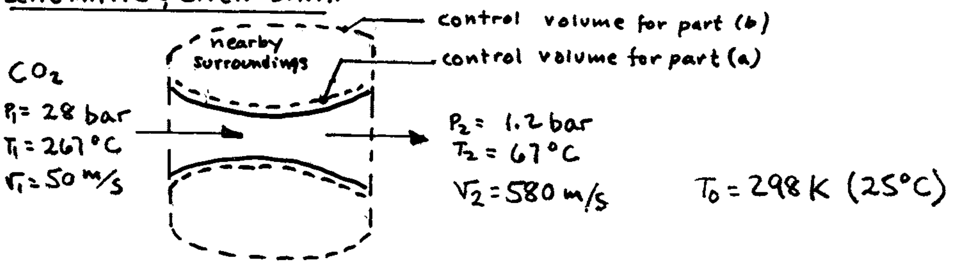
Inspection of Fig. A-1 indicates that the ideal gas model is appropriate at these states.

PROBLEM 6.114*

KNOWN: Operating data are provided for a nozzle at steady state through which CO_2 is passing.

FIND: (a) For a control volume enclosing the nozzle only, evaluate $\dot{Q}_{cv}/\dot{m}$, and the change in specific entropy from inlet to exit. Indicate the additional data required to evaluate $\dot{\sigma}_{cv}/\dot{m}$. (b) For an enlarged control volume including enough of the nearby surroundings that heat transfer occurs at the ambient temperature T_0, evaluate $\dot{\sigma}_{cv}/\dot{m}$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each of the control volumes shown in the accompanying figure operates at steady state. (2) The change in potential energy from inlet to exit ① is negligible. (3) CO_2 can be modeled as an ideal gas.

ANALYSIS: (a) At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}^{0}\right)$$

or

$$\frac{\dot{Q}_{cv}}{\dot{m}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

With M = 44.01 from Table A-1 and data from Table A-23, $\bar{h}_1 = 19{,}485$ kJ/kmol, $\bar{s}_1^\circ = 238.292$ kJ/kmol·K, $\bar{h}_2 = 10{,}959$ kJ/kmol, $\bar{s}_2^\circ = 218.694$ kJ/kmol·K. Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{(10{,}959 - 19{,}485)\,\text{kJ/kmol}}{44.01\,\text{kg/kmol}} + \left(\frac{580^2 - 50^2}{2}\right)\left(\frac{m^2}{s^2}\right)\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= -193.73\ \text{kJ/kg} + 166.95\ \text{kJ/kg} = -26.78\ \text{kJ/kg} \longleftarrow (\dot{Q}_{cv}/\dot{m})$$

With Eq. 6.2 b

$$s_2 - s_1 = \frac{1}{M}\left[\bar{s}_2^\circ - \bar{s}_1^\circ - \bar{R}\ln\frac{P_2}{P_1}\right] = \frac{\left[218.694 - 238.292 - 8.314\ln\frac{1.2}{28}\right]}{44.01}$$

$$= +0.1497\ \text{kJ/kg·K} \longleftarrow (s_2 - s_1)$$

To evaluate $\dot{\sigma}_{cv}/\dot{m}$ would require information about the temperature on the boundary of the control volume and the rate of heat transfer at each temperature.

(b) For the enlarged control volume the heat transfer takes place only at $T_0 = 298$ K, and the entropy rate balance reduces to

$$0 = \frac{\dot{Q}_{cv}}{T_0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv}/\dot{m} = -\frac{\dot{Q}_{cv}/\dot{m}}{T_0} + s_2 - s_1$$

Using previously calculated values

$$\dot{\sigma}_{cv}/\dot{m} = -\frac{(-26.78\ \text{kJ/kg})}{298\ \text{K}} + 0.1497\frac{\text{kJ}}{\text{kg·K}} = 0.2396\frac{\text{kJ}}{\text{kg·K}} \longleftarrow \dot{\sigma}_{cv}/\dot{m}$$

---

1. From Table A-1, for CO_2 $P_c = 73.9$ bar, $T_c = 304$ K. Then,

$$P_{R1} = \frac{28}{73.9} = 0.379 \qquad T_{R1} = \frac{540}{304} = 1.78$$

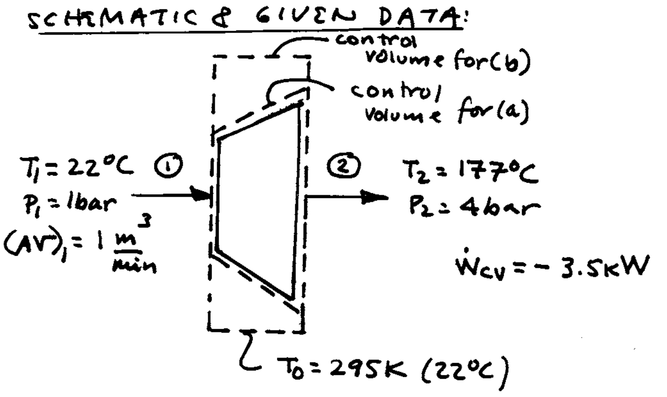
Inspection of Fig A-1 indicates that the ideal gas model is appropriate at this state. Similarly, the ideal gas model is appropriate at State 2, as can be verified.

PROBLEM 6.115

KNOWN: Steady-state operating data are provided for an aircompressor.

FIND: (a) Determine the heat transfer rate and change in entropy from inlet to exit for a control volume enclosing the compressor only. Discuss. (b) For an enlarged control volume, determine the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volumes shown in the accompanying figure are at steady state.
2. Kinetic and potential energy effects can be ignored.
3. Air is modeled as an ideal gas.

ANALYSIS: (a) At steady state, the mass rate balance reads, $\dot{m}_2 = \dot{m}_1 \equiv \dot{m}$, where

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(AV)_1}{(RT_1/P_1)} = \frac{(1\ m^3/min)(10^5 N/m^2)}{\left(\frac{8314}{28.97}\ \frac{N\cdot m}{kg\cdot K}\right)(295K)} = 1.18\ kg/min$$

The energy rate balance at steady state reduces to give on rearrangement

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1) = -3.5kW + \left(1.18\ \frac{kg}{min}\right)\left|\frac{1 min}{60 s}\right|\left[451.8 - 295.17\right]\frac{kJ}{kg}\left|\frac{1 kW}{1 kJ/s}\right|$$

$$= -0.42kW$$ $\dot{Q}_{cv}$

where enthalpy data is from Table A-22. The entropy change is obtained with Eq. 6.21a and s^o data from Table A-22:

$s_2 - s_1$

$$s_2 - s_1 = s^o(T_2) - s^o(T_1) - R\ln\frac{P_2}{P_1} = \left[2.11161 - 1.68515 - \frac{8.314}{28.97}\ln 4\right] = 0.0286\ \frac{kJ}{kg\cdot K}$$

To evaluate the entropy production for the control volume of part (a) would require information about the temperature at which heat transfer occurs on the boundary of the control volume.

(b) For the enlarged control volume of part (b), heat transfer takes place at $T_0 = 295K$. An entropy rate balance at steady state reads

$$0 = \frac{\dot{Q}_{cv}}{T_0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_0} + \dot{m}(s_2 - s_1)$$

with results from part (a)

$$\dot{\sigma}_{cv} = \frac{-(-0.42\ kW)}{295K} + \left(\frac{1.18}{60}\ \frac{kg}{s}\right)\left[0.0286\ \frac{kJ}{kg\cdot K}\right]\left|\frac{1 kW}{1 kJ/s}\right|$$

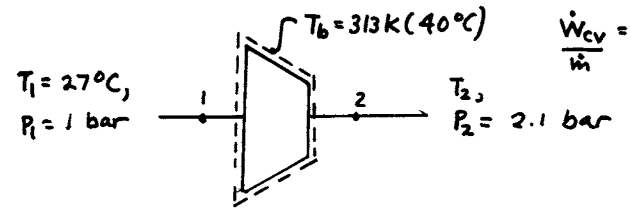
$$= 1.98 \times 10^{-3}\ \frac{kW}{K}$$ $\dot{\sigma}_{cv}$

PROBLEM 6.116

KNOWN: Operating data are provided for an air compressor at steady state.

FIND: Determine the temperature of the air exiting the compressor and the entropy production rate per kg of air flowing.

SCHEMATIC & GIVEN DATA:



$$\frac{\dot{W}_{cv}}{\dot{m}} = -94.6 \text{ kJ/kg}, \quad \frac{\dot{Q}_{cv}}{\dot{m}} = -14 \frac{\text{kJ}}{\text{kg}}$$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state. (2) Heat transfer takes place at T_b only. (3) Kinetic and potential energy changes from inlet to exit can be ignored. (4) Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces with assumption 3

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \cancelto{0}{\frac{V_1^2 - V_2^2}{2}} + \cancelto{0}{g(z_1 - z_2)}\right)$$

or

$$h_2 = h_1 + \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} = (300.19 - 14 + 94.6) \frac{\text{kJ}}{\text{kg}} = 380.79 \frac{\text{kJ}}{\text{kg}}$$

where h_1 is from Table A-22. From the same table with $h_2 = 380.79 \frac{\text{kJ}}{\text{kg}}$, $T_2 = 380 K$. ← T_2

With assumption 2, an entropy rate balance at steady state takes the form

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \frac{(-\dot{Q}_{cv}/\dot{m})}{T_b} + s_2 - s_1$$

$$= \left(\frac{-\dot{Q}_{cv}/\dot{m}}{T_b}\right) + \left(s_2^\circ - s_1^\circ - R \ln \frac{p_2}{p_1}\right)$$

$$= \left(\frac{14 \text{ kJ/kg}}{313 \text{ K}}\right) + \left(1.94001 - 1.70203 - \frac{8.314}{28.97} \ln \frac{2.1}{1}\right) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$= (0.0447 + 0.0251) \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

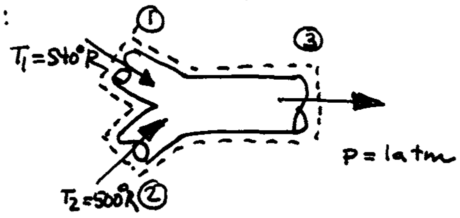
$$= 0.0698 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \longleftarrow \dot{\sigma}_{cv}/\dot{m}$$

PROBLEM 6.117

KNOWN: Operating data are provided for a duct system at steady state.

FIND: Determine the rate of entropy production.

SCHEMATIC & GIVEN DATA:



Data from the solution to Problem 4.68: $\dot{m}_1 = 369.5$ lb/min, $\dot{m}_2 = 158.8$ lb/min, $T_3 = 528°R$.

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. 3. Air is modeled as an ideal gas with constant $c_p = 0.24$ Btu/lb·°R.

ANALYSIS: An entropy rate balance at steady state reduces to read

$$0 = \cancelto{0}{\sum_j \frac{\dot{Q}_j}{T_j}} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

Since $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ at steady state

① $$\dot{\sigma}_{cv} = (\dot{m}_1 + \dot{m}_2) s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$= \dot{m}_1 (s_3 - s_1) + \dot{m}_2 (s_3 - s_2)$$

Introducing Eq. 6.23

$$\dot{\sigma}_{cv} = \dot{m}_1 \left[c_p \ln \frac{T_3}{T_1} - R \ln \cancelto{0}{\frac{p_3}{p_1}} \right] + \dot{m}_2 \left[c_p \ln \frac{T_3}{T_2} - R \ln \cancelto{0}{\frac{p_3}{p_2}} \right]$$

$$= c_p \left[\dot{m}_1 \ln \frac{T_3}{T_1} + \dot{m}_2 \ln \frac{T_3}{T_2} \right]$$

$$= 0.24 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}} \left[\left(369.5 \frac{\text{lb}}{\text{min}}\right) \ln \frac{528}{540} + \left(158.8 \frac{\text{lb}}{\text{min}}\right) \ln \frac{528}{500} \right]$$

$$= 0.084 \frac{\text{Btu}}{\text{min}\,°\text{R}} \qquad \longleftarrow \dot{\sigma}_{cv}$$

---

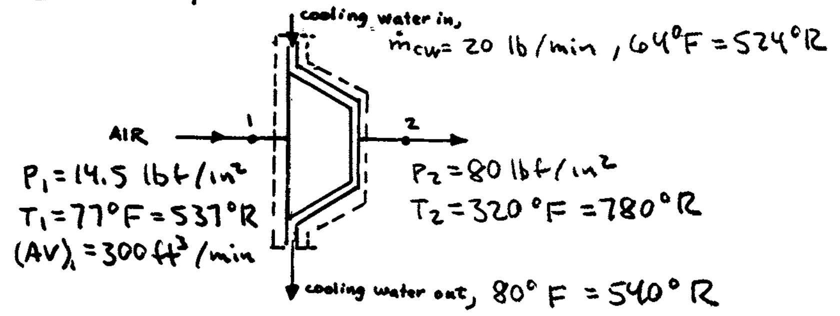
1. When using IT to perform this calculation, note that IT would return s_1, s_2 and s_3 directly for air as an ideal gas.

PROBLEM 6.118*

KNOWN: Operating data are provided for a water-jacketed air compressor.

FIND: Determine the power required and the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure operates at steady state with negligible kinetic and potential energy effects and $\dot Q_{cv} = 0$. (2) Air is modeled as an ideal gas. (3) The cooling water is modeled as incompressible with constant specific heat c and a negligible change in pressure. ①

ANALYSIS: At steady state $\dot m_1 = \dot m_2 = \dot m$ and the rates of cooling water entering and exiting are equal. An energy rate balance reduces with listed assumptions to give

$$\dot W_{cv} = \dot m[h_1 - h_2] + \dot m_{cw}[h_{cw,in} - h_{cw,out}]$$

With the ideal gas equation of state

$$\dot m = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{(14.5 \times 144)\frac{\text{lbf}}{\text{ft}^2}\left(300\frac{\text{ft}^3}{\text{min}}\right)}{\left(\frac{1545}{28.97}\right)\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}(537°\text{R})} = 21.87\,\frac{\text{lb}}{\text{min}}$$

Then, with specific enthalpies h_1 and h_2 from Table A-22E, and a specific heat value for liquid water from Table A-19E, $c = 1.00$ Btu/lb·°R, together with Eq. 3.20b: $\Delta h = c\Delta T + \overset{0}{\cancel{v\Delta p}}$, where Δp is dropped by assumption 3, the power is

$$\dot W_{cv} = (21.87)\frac{\text{lb}}{\text{min}}(128.34 - 186.94)\frac{\text{Btu}}{\text{lb}} + (20)\frac{\text{lb}}{\text{min}}\left(\left(1.00\frac{\text{Btu}}{\text{lb}°\text{R}}\right)(524 - 540)°\text{R}\right)$$

$$= (-1281.6 - 320.0)\frac{\text{Btu}}{\text{min}}\left|\frac{1\text{ min}}{60\text{ s}}\right|\left|\frac{1.055\text{ kJ}}{1\text{ Btu}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = -28.16\text{ kW} \longleftarrow \dot W_{cv}$$

At steady state an entropy rate balance reduces to give

$$0 = \overset{0}{\cancel{\sum \frac{\dot Q_j}{T_j}}} + \dot m(s_1 - s_2) + \dot m_w(s_{cw,in} - s_{cw,out}) + \dot\sigma_{cv}$$

Or, with Eq. 6.21a and Eq. 6.24

$$\dot\sigma_{cv} = \dot m\left(s_2^\circ - s_1^\circ - R\ln\frac{p_2}{p_1}\right) + \dot m_{cw}\, c\ln\frac{T_{cw,out}}{T_{cw,in}}$$

$$= \left(21.87\frac{\text{lb}}{\text{min}}\right)\left(0.68942 - 0.59945 - \frac{1.986}{28.97}\ln\left(\frac{80}{14.5}\right)\right)\frac{\text{Btu}}{\text{lb}°\text{R}} + \left(20\frac{\text{lb}}{\text{min}}\right)\left(1.00\frac{\text{Btu}}{\text{lb}°\text{R}}\right)\ln\left(\frac{540}{524}\right)$$

$$= -0.5929 + 0.6015$$

$$= 8.6 \times 10^{-3}\ \text{Btu/min}\cdot°\text{R} \longleftarrow \dot\sigma_{cv}$$

---

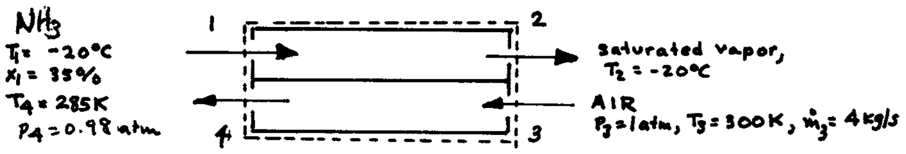
1. Alternatively, for the cooling water saturated liquid data could be used: $h \approx h_f(T)$, $s \approx s_f(T)$.

PROBLEM 6.119

KNOWN: Operating data are provided for a counterflow heat exchanger at steady state with NH_3 flowing on one side and air flowing on the other side.

FIND: Determine the mass flow rate of the ammonia and the rate of entropy production within the heat exchanger

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state. (2) $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. (3) Air is modeled as an ideal gas.

ANALYSIS: At steady state the inlet and exit mass flow rates of each side of the heat exchanger are equal: $\dot{m}_1 = \dot{m}_2 = \dot{m}_R$, $\dot{m}_3 = \dot{m}_4 = \dot{m}$, and the entropy rate balance reduces to give

$$0 = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}_R(s_1 - s_2) + \dot{m}(s_3 - s_4) + \dot{\sigma}_{cv} \qquad (1)$$

This requires the mass flow rate $\dot{m}_R$ which can be evaluated from an energy rate balance at steady state with assumption 2:

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_R(h_1 - h_2) + \dot{m}(h_3 - h_4)$$

or upon rearrangement

$$\dot{m}_R = \frac{\dot{m}(h_3 - h_4)}{(h_2 - h_1)}$$

The enthalpies h_3 and h_4 are obtained from Table A-22: $h_3 = 300.19$ kJ/kg, $h_4 = 285.14$ kJ/kg. The enthalpies h_1 and h_2 are obtained from Table A-13 at -20°C:

$$h_2 - h_1 = h_g - (h_f + x_1(h_g - h_f)) = (1 - x_1)(h_g - h_f) = 0.65\,(1329.1 \text{ kJ/kg}) = 863.9 \text{ kJ/kg}$$

Thus

$$\dot{m}_R = (4 \text{ kg/s})\frac{(300.19 - 285.14)\text{ kJ/kg}}{863.9 \text{ kJ/kg}} = 0.07 \text{ kg/s} \longleftarrow \dot{m}_R$$

Upon rearrangement Eq. (1) gives

$$\dot{\sigma}_{cv} = \dot{m}_R(s_2 - s_1) + \dot{m}(s_4 - s_3)$$

With Eq. 6.21a and data from Table A-22

$$s_4 - s_3 = s^\circ(T_4) - s^\circ(T_3) - R \ln p_4/p_3 = (1.65055 - 1.70203) - \frac{8.314}{28.97}\ln\frac{0.98}{1}$$
$$= -0.0457 \text{ kJ/kg·K}$$

With data from Table A-13 at -20°C

$$s_2 - s_1 = s_g - (s_f + x_1(s_g - s_f)) = (1 - x_1)(s_g - s_f) = 0.65\,(5.6144 - 0.3642) = 3.4126 \frac{\text{kJ}}{\text{kg·K}}$$

Finally,

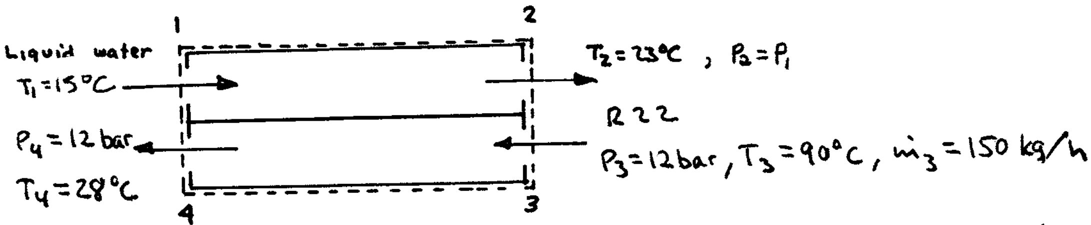
$$\dot{\sigma}_{cv} = \left(0.07 \frac{\text{kg}}{\text{s}}\right)\left(3.4126 \frac{\text{kJ}}{\text{kg·K}}\right) + \left(4 \frac{\text{kg}}{\text{s}}\right)\left(-0.0457 \frac{\text{kJ}}{\text{kg·K}}\right)$$
$$= 0.056 \text{ kW/K} \longleftarrow \dot{\sigma}_{cv}$$

PROBLEM 6.120*

KNOWN: Operating data are provided for a counterflow heat exchanger at steady state with liquid water flowing on one side and R22 flowing on the other side.

FIND: Determine the mass flow rate of the water stream and the rate of entropy production.

SCHEMATIC & GIVEN DATA:


1
2
Liquid water
$T_1 = 15°C$
$T_2 = 23°C$, $P_2 = P_1$
R22
$P_4 = 12$ bar
$T_4 = 28°C$
$P_3 = 12$ bar, $T_3 = 90°C$, $\dot{m}_3 = 150$ kg/h
4
3

ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state. (2) $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. (3) Liquid water is modeled as incompressible with constant specific ① heat c and negligible pressure drop.

ANALYSIS: At steady state the inlet and exit mass flow rates of each side of the heat exchanger are equal: $\dot{m}_1 = \dot{m}_2 = \dot{m}$, $\dot{m}_3 = \dot{m}_4 = \dot{m}_R$, and the energy rate balance reduces with assumption 2 to

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}(h_1 - h_2) + \dot{m}_R(h_3 - h_4)$$

Thus

$$\dot{m} = \frac{\dot{m}_R(h_3 - h_4)}{(h_2 - h_1)}$$

With assumption 3, Eq. 3.20 gives $(h_2 - h_1) = c(T_2 - T_1) + \overset{0}{\cancel{v(P_2 - P_1)}}$. With $c = 4.2$ kJ/kg·K from Table A-19, $h_2 - h_1 = (4.2 \text{ kJ/kg·K})(8 \text{ K}) = 33.6$ kJ/kg. Then, approximating $h_4 \approx h_f(T_4)$ and using data from Tables A-7, 9

$$\dot{m} = \frac{(150 \text{ kg/h})(308.70 - 79.05) \text{ kJ/kg}}{33.6 \text{ kJ/kg}} = 1025.2 \text{ kg/h} \longleftarrow \dot{m}$$

An entropy rate balance at steady state reduces to give

$$0 = \overset{0}{\cancel{\sum \frac{\dot{Q}_j}{T_j}}} + \dot{m}(s_1 - s_2) + \dot{m}_R(s_3 - s_4) + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) + \dot{m}_R(s_4 - s_3)$$

Introducing Eq. 6.24 and approximating $s_4 = s_f(T_4)$ and using data from Tables A-7, 9

$$\dot{\sigma}_{cv} = \dot{m} c \ln \frac{T_2}{T_1} + \dot{m}_R(s_4 - s_3)$$

$$= \left(1025.2 \frac{\text{kg}}{\text{h}}\right)\left(4.2 \frac{\text{kJ}}{\text{kg K}}\right) \ln \frac{296}{288} + \left(150 \frac{\text{kg}}{\text{h}}\right)(0.2936 - 1.0363) \frac{\text{kJ}}{\text{kg·K}}$$

$$= (117.98 - 111.41) \frac{\text{kJ/h}}{\text{K}}$$

$$= \left(6.57 \frac{\text{kJ/h}}{\text{K}}\right) \left| \frac{1 \text{ min}}{3600 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 1.83 \times 10^{-3} \text{ kW/K} \longleftarrow \dot{\sigma}_{cv}$$

---

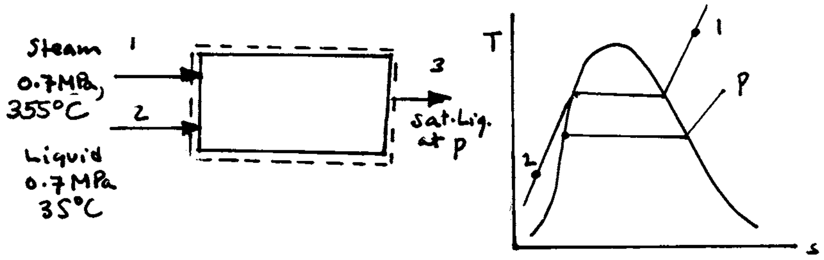
1. Alternatively, for the liquid water stream saturated liquid data can be used: $h \sim h_f(T)$, $s \sim s_f(T)$.

PROBLEM 6.121

KNOWN: Operating data are provided for an open feedwater heater at steady state. The exiting stream is at pressure p.

FIND: (a) If p = 0.7 MPa, determine the ratio of the mass flow rates of the two incoming streams and the rate of entropy production within the heater.
(b) Plot the quantities of part (a) versus p.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:
1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. Kinetic and potential energy effects can be ignored.

ANALYSIS: At steady state, the mass rate balance gives $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$, and the energy rate balance reduces as follows

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3 \Rightarrow \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_3 - h_2}{h_1 - h_3} \qquad (1)$$

An entropy rate balance reduces to

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}_1 s_1 + \dot{m}_2 - (\dot{m}_1 + \dot{m}_2) s_3 + \dot{\sigma}_{cv}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = s_3 - \left[\frac{\dot{m}_1}{\dot{m}_1 + \dot{m}_2}\right] s_1 - \left[\frac{\dot{m}_2}{\dot{m}_1 + \dot{m}_2}\right] s_2 \Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}_3} = s_3 - \left(\frac{r}{r+1}\right) s_1 - \left(\frac{1}{r+1}\right) s_2 \qquad (2)$$

where $r = \dot{m}_1 / \dot{m}_2$.

(a) p = 0.7 MPa. With $h_2 = h_f(T_2)$ and $s_2 = s_f(T_2)$; together with data from Tables A-2, 3, 4 Eq. (1) and Eq (2) give, respectively

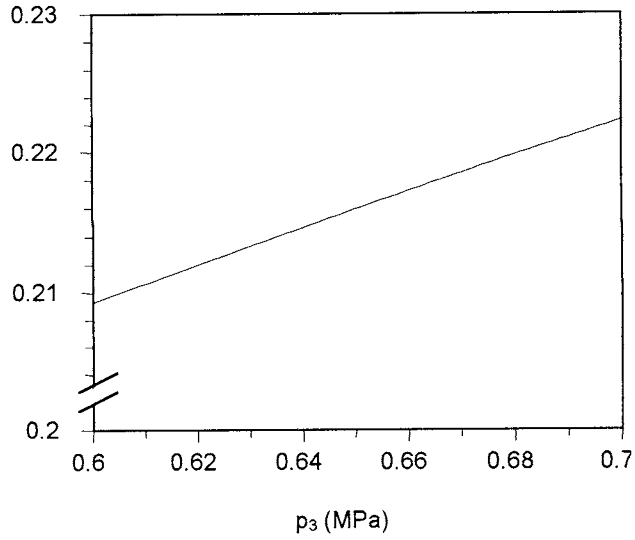
①

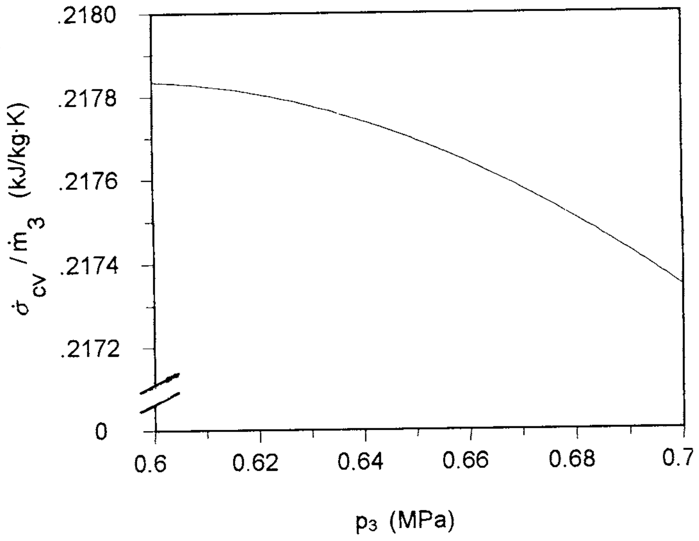
$$r = \frac{(697.22 - 146.68)\ \text{kJ/kg}}{(3174.2 - 697.22)\ \text{kJ/kg}} = 0.2223 \quad \longleftarrow \dot{m}_1/\dot{m}_2$$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = 1.9922 - \left(\frac{0.2223}{1.2223}\right)(7.4896) - \left(\frac{1}{1.2223}\right)(0.5053) = 0.217\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \quad \longleftarrow \dot{\sigma}_{cv}/\dot{m}_3$$

(b)

②





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1. Values determined using Eqs. (1), (2) are sensitive to roundoff.
2. The entropy production varies only slightly over this pressure range.

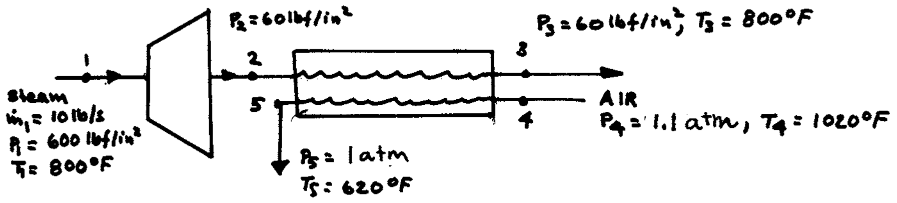
PROBLEM 6.122

KNOWN: Steam expands through a turbine and then passes through a counterflow heat exchanger in which the other stream is air. Operating data at steady state are provided.

FIND: Determine the mass flow rate of the air and the rates of entropy production in the turbine and heat exchanger, respectively.

SCHEMATIC & GIVEN DATA:

$\dot{W}_{cv} = 2852$ hp



ASSUMPTIONS: (1) The turbine and heat exchanger operate at steady state. (2) There is no significant heat transfer between either component and its surroundings. (3) Kinetic and potential energy effects can be ignored. (4) Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}$ and $\dot{m}_4 = \dot{m}_5 = \dot{m}_a$. Taking a control volume about the turbine and heat exchanger, an energy rate balance at steady state reduces to give with assumption 3

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \dot{W}_{cv} + \dot{m}(h_1 - h_3) + \dot{m}_a(h_4 - h_5)$$

Thus

$$\dot{m}_a = \frac{\dot{W}_{cv} + \dot{m}(h_3 - h_1)}{h_4 - h_5} = \frac{2852\,\text{hp}\left|\frac{2545\,\text{Btu/h}}{\text{hp}}\right|\left|\frac{\text{h}}{3600\,\text{s}}\right| + \left(10\frac{\text{lb}}{\text{s}}\right)(1431.2 - 1407.6)}{363.89 - 260.97}$$

$$= 21.88\ \text{lb/s} \longleftarrow\!\!\!\text{———————}\ \dot{m}_a$$

where enthalpies are from Tables A-4E and A-22E.

Taking a control volume about the turbine only, an entropy rate balance at steady state reduces to $\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$. To determine $\dot{\sigma}_{cv}$ requires state 2 to be fixed. This can be accomplished using an energy rate balance for the same control volume:

$$0 = \cancel{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow h_2 = h_1 - \dot{W}_{cv}/\dot{m}$$

Thus

$$h_2 = 1407.6\ \text{Btu/lb} - \frac{(2852)\left|2545/3600\right|\,\text{Btu/s}}{10\ \text{lb/s}} = 1205.98\ \text{Btu/lb}$$

State 2 is fixed by $p_2 = 60$ lbf/in², $h_2 = 1205.98$ Btu/lb. Interpolating in Table A-4E, $s_2 = 1.6802$ Btu/lb·°R. Thus, for the turbine

$$\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) = 10\frac{\text{lb}}{\text{s}}(1.6802 - 1.6343)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} = 0.459\ \frac{\text{Btu/s}}{°\text{R}} \longleftarrow\!\!\!\text{———————}\ \dot{\sigma}_{cv}$$

Taking a control volume around the heat exchanger only, an entropy rate balance reduces at steady state to

$$0 = \overset{0}{\cancel{\sum \frac{\dot{Q}_j}{T_j}}} + \dot{m}(s_2 - s_3) + \dot{m}_a(s_4 - s_5) + \dot{\sigma}_{cv}$$

or with Eq. 6.21a and data from Tables A-4E and A-22E

$$\dot{\sigma}_{cv} = \dot{m}(s_3 - s_2) + \dot{m}_a(s_5^\circ - s_4^\circ - R\ln p_5/p_4)$$

$$= \left(10\frac{\text{lb}}{\text{s}}\right)(1.9022 - 1.6802)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}} + \left(21.88\frac{\text{lb}}{\text{s}}\right)\left(0.76964 - 0.85062 - \frac{1.986}{28.97}\ln\frac{1}{1.1}\right)\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

$$= 2.22\ \frac{\text{Btu/s}}{°\text{R}} - 1.629\ \frac{\text{Btu/s}}{°\text{R}} = 0.591\ \frac{\text{Btu/s}}{°\text{R}} \qquad \text{———————}\ \dot{\sigma}_{cv}$$

PROBLEM 6.123

KNOWN: Steady state operating data are provided for two waste-heat recovery systems: (a) Example 4.10, (b) Problem 4.83.

FIND: For each system, determine the rates of entropy production for the steam generator and the turbine. Discuss.

SCHEMATIC & GIVEN DATA:

(a) See Fig. E 4.10

(b) See Fig. P 4.83. From the solution, $\dot{m}_A = 145.2$ lb/min, $\dot{m}_1 = 2.8$ lb/min.

ASSUMPTIONS: 1. Control volumes enclosing the steam generator and the turbine are at steady state. 2. For each control volume, $\dot{Q}_{cv} = 0$. 3. The combustion products of part (a) can be modeled as air as an ideal gas. The ideal gas model also applies to the air of part (b).

ANALYSIS: At steady state the entropy rate balance for the control volume enclosing the turbine reduces to give

(a) $\dot{\sigma}_t = \dot{m}(s_5 - s_4)$ (b) $\dot{\sigma}_t = \dot{m}(s_3 - s_2)$

An entropy rate balance for the steam generator gives

(a) $\dot{\sigma}_{HX} = \dot{m}_1(s_2 - s_1) + \dot{m}_3(s_4 - s_3)$ (b) $\dot{\sigma}_{HX} = \dot{m}_A(s_B - s_A) + \dot{m}_1(s_2 - s_1)$

(a) From the solution to Example 4.10, $h_4 = 1213.6$ Btu/lb, $p_4 = 40$ lbf/in²; thus Table A-4E gives $s_4 = 1.7334$ Btu/lb·°R. With data from Table A-3E

$s_5 = s_f + x_5(s_g - s_f) = 0.1327 + 0.93(1.8453) = 1.8488$ Btu/lb·°R. Thus,

$$\dot{\sigma}_t = \left(275 \frac{\text{lb}}{\text{min}}\right)(1.8488 - 1.7334) \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} \left|\frac{1 \text{ min}}{60 \text{ s}}\right| = 0.529 \frac{\text{Btu}}{\text{s}\cdot{}^\circ\text{R}}$$ ←

With s° data from Table A-22E and $s_3 = s_f(T_3)$ from Table A-2E

$$\dot{\sigma}_{HX} = \left[\left(9230.6 \frac{\text{lb}}{\text{min}}\right)\left[0.67002 - 0.71323 - R \overset{0}{\cancel{\ln p_2/p_1}}\right] \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} + 275(1.7334 - 0.1331)\right]\left|\frac{1 \text{ min}}{60 \text{ s}}\right|$$

$$= 0.687 \frac{\text{Btu}}{\text{s}\cdot{}^\circ\text{R}}$$ ← The steam generator contributes most to inefficient operation.

(b) With data from Table A-4E and A-3E, $s_2 = 1.7121$ Btu/lb·°R, $s_3 = s_f + x_3 s_{fg}$,
$s_3 = 0.1327 + 0.9(1.8453) = 1.7935$ Btu/lb·°R. Thus,

$$\dot{\sigma}_t = \left(2.8 \frac{\text{lb}}{\text{min}}\right)(1.7935 - 1.7121) \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} \left|\frac{1 \text{ min}}{60 \text{ s}}\right| = 3.8 \times 10^{-3} \frac{\text{Btu}}{\text{s}\cdot{}^\circ\text{R}}$$ ←

With s° data from Table A-22E, noting that $p_B = p_A$, and $s_1 = s_f(T_1)$ from Table A-2E

$$\dot{\sigma}_{HX} = \left[\left(145.2 \frac{\text{Btu}}{\text{min}}\right)\left[0.67665 - 0.70160\right] \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} + 2.8\left[1.7121 - 0.3241\right]\right]\left|\frac{1 \text{ min}}{60 \text{ s}}\right|$$

$$= 4.4 \times 10^{-3} \frac{\text{Btu}}{\text{s}\cdot{}^\circ\text{R}}$$ ←

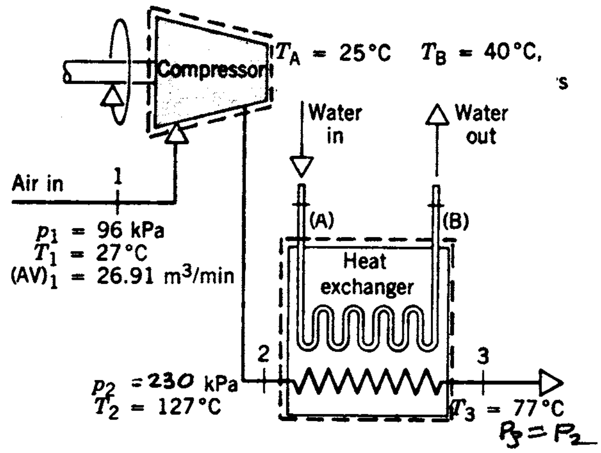
The steam generator contributes most to inefficient operation.

PROBLEM 6.124

KNOWN: Air passes through a compressor and heat exchanger. Data for the various flow streams are known.

FIND: Determine the compressor power and the mass flow rate of the cooling water. Also, evaluate the rates of entropy production in the compressor and heat exchanger.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Both control volumes shown are at steady state. (2) Heat transfer is negligible, and $\dot{W}_{cv} = 0$ for the heat exchanger. (3) Kinetic and potential energy effects are negligible. (4) The air behaves as an ideal gas.

ANALYSIS: (a) The compressor power is found from the steady-state energy balance for the compressor control volume

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_{air}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_{air}$. With assumption (3)

$$\dot{W}_{cv} = \dot{m}_{air}(h_1 - h_2)$$

The mass flow rate of air is

$$\dot{m}_{air} = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{(96\text{ kPa})(26.91\text{ m}^3/\text{min})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300\text{ K})}\left|\frac{10^3\text{ N/m}^2}{1\text{ kPa}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = 30\ \frac{\text{kg}}{\text{min}}$$

Thus

$$\dot{W}_{cv} = \left(30\ \frac{\text{kg}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)(300.19 - 400.98)\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right| = -50.4\text{ kW} \longleftarrow \dot{W}_{cv}$$

where the specific enthalpies are from Table A-22.

To find the cooling water flow rate, note that the air and water pass through the heat exchanger as separate streams. Thus

$$\dot{m}_2 = \dot{m}_3 \equiv \dot{m}_{air} \quad \text{and} \quad \dot{m}_A = \dot{m}_B \equiv \dot{m}_{cw}$$

With assumptions (2) and (3), the steady-state energy balance for the heat exchanger control volume reduces to

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_{air}(h_2 - h_3) + \dot{m}_{cw}(h_A - h_B)$$

or

$$\dot{m}_{cw} = \left(\frac{h_2 - h_3}{h_B - h_A}\right)\dot{m}_{air}$$

From Table A-22, $h_3 = 350.49$ kJ/kg. For the cooling water, $h = h_f(T)$:

$h_A = 104.89$ kJ/kg, $h_B = 167.57$ kJ/kg. Thus, the cooling water mass flow rate is

$$\dot{m}_{cw} = \left(\frac{400.98 - 350.49}{167.57 - 104.89}\right)\left(30\ \frac{\text{kg}}{\text{min}}\right)\left|\frac{1\text{ min}}{60\text{ s}}\right| = 0.403\text{ kg/s} \longleftarrow \dot{m}_{cw}$$

PROBLEM 6.1.24 (Contd.)

(b) Reducing mass and entropy rate balances for the compressor, and using Eq. 6.21a

$$\dot{\sigma}_C = \dot{m}_{Air}(s_2 - s_1)$$

$$= \dot{m}_{Air}\left(s^\circ(T_2) - s^\circ(T_1) - R \ln p_2/p_1 \right)$$

$$= \left(30 \frac{kg}{min}\right)\left|\frac{1 min}{60 s}\right|\left(1.99194 - 1.70203 - \frac{8.314}{28.97} \ln\left(\frac{230}{96}\right)\right]\frac{kJ}{kg \cdot K}\left|\frac{1 kW}{1 kJ/s}\right|$$

$$= 0.0196 \frac{kW}{K} \qquad \xleftarrow{\qquad \dot{\sigma}_C \qquad}$$

where s° values are from Table A-22.

Reducing mass and entropy rate balances for the heat exchanger

$$\dot{\sigma}_{HX} = \dot{m}_{Air}[s_3 - s_2] + \dot{m}_{cw}[s_B - s_A]$$

$$= \dot{m}_{Air}\left[s^\circ(T_3) - s^\circ(T_2) - R \cancel{\ln \frac{p_3}{p_2}}^{\,0} \right] + \dot{m}_{cw}\left[s_f(T_B) - s_f(T_A) \right]$$

With data from Tables A-2, 22

$$\dot{\sigma}_{HX} = \left[\left(0.5 \frac{kg}{s}\right)\left[1.85708 - 1.99194\right]\frac{kJ}{kg \cdot K} + 0.403\left(0.5725 - 0.3674\right)\right]\left|\frac{1 kW}{1 kJ/s}\right|$$

$$= 0.0152 \frac{kW}{K} \qquad \xleftarrow{\qquad \dot{\sigma}_{HX} \qquad}$$

PROBLEM 6.125

KNOWN: Operating data are provided for two turbine stages and an interconnecting heat exchanger.

FIND: Determine the rates of entropy production for the two turbines and the heat exchanger. Place in rank order.

SCHEMATIC & GIVEN DATA:

See Fig. P 4.82

From the solution to Problem 4.82, $T_3 = 1301.5\ \text{K}$, $\dot{m}_1 = 28.22\ \text{kg/s}$.

ASSUMPTIONS: 1. Control volumes enclosing each of the turbines and the heat exchanger are at steady state. 2. For each control volume, $\dot{Q}_{cv} = 0$. 3. Air is modeled as an ideal gas.

ANALYSIS: Mass and entropy rate balances reduce to give for the first turbine

$$\dot{\sigma}_{t1} = \dot{m}_1 (s_2 - s_1) = \dot{m}_1 \left(s^\circ(T_2) - s^\circ(T_1) - R \ln p_2/p_1 \right)$$

$$= \left(28.22\ \frac{\text{kg}}{\text{s}}\right) \left[3.07732 - 3.3620 - \frac{8.314}{28.97} \ln \frac{5}{20} \right] \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \left| \frac{1\ \text{kW}}{1\ \text{kJ/s}} \right|$$

$$= 3.1936\ \text{kW/K} \qquad \longleftarrow \dot{\sigma}_{t1}$$

where Eq. 6.21a and data from Table A-22 have been used. Likewise for turbine 2

$$\dot{\sigma}_{t2} = \dot{m}_1 (s_4 - s_3) = \dot{m}_1 \left[s^\circ(T_4) - s^\circ(T_3) - R \ln p_4/p_3 \right]$$

$$= (28.22) \left[2.94468 - 3.27481 - \frac{8.314}{28.97} \ln \frac{1}{4.5} \right] = 2.8649\ \text{kW/K} \qquad \longleftarrow \dot{\sigma}_{t2}$$

Mass and entropy rate balances for the interconnecting heat exchanger give

$$\dot{\sigma}_{HX} = \dot{m}_1 [s_3 - s_2] + \dot{m}_5 [s_6 - s_5]$$

$$= \dot{m}_1 \left[s^\circ(T_3) - s^\circ(T_2) - R \ln \frac{p_3}{p_2} \right] + \dot{m}_5 \left[s^\circ(T_6) - s^\circ(T_5) - R \ln \frac{p_6}{p_5} \right]$$

$$= 28.22 \left[3.27481 - 3.07732 - \frac{8.314}{28.97} \ln \frac{4.5}{5} \right] +$$

$$\left(\frac{1200}{60}\right) \left[3.17888 - 3.42892 - \frac{8.314}{28.97} \ln \frac{1}{1.35} \right]$$

$$= 6.4265 - 3.2783 = 3.1482\ \text{kW/K} \qquad \longleftarrow \dot{\sigma}_{HX}$$

In rank order: Turbine 1, heat exchanger, turbine 2.

PROBLEM 6.126

KNOWN: Steady-state operating data are provided for a simple vapor power plant.

FIND: Determine the rates of entropy production for the turbine, condenser, and pump. Place in rank order.

SCHEMATIC & GIVEN DATA:

- See Figure P 4.85
- From the solution to Problem 4.85, $\dot{m}_{cw} = 3759$ kg/s

ASSUMPTIONS: 1. Control volumes enclosing the turbine, condenser, and pump are at steady state. 2. For each control volume, $\dot{Q}_{cv} = 0$.

ANALYSIS: Mass and entropy rate balances for the turbine give with data from Tables A-3 and A-4

$$\dot{\sigma}_t = \dot{m}(s_2 - s_1) = \left(109 \frac{kg}{s}\right)[7.4651 - 6.6622] \frac{kJ}{kg \cdot K} \left|\frac{1\ kW}{1\ kJ/s}\right| = 87.51 \frac{kW}{K} \longleftarrow \dot{\sigma}_t$$

$$s_2 = s_f + x_2(s_g - s_f) = 0.5926 + 0.9(8.2287 - 0.5926) = 7.4651\ kJ/kg \cdot K.$$

For the pump, and data from Tables A-3, 5

$$\dot{\sigma}_p = \dot{m}[s_4 - s_3] = (109)[0.6061 - 0.5926] = 1.47 \frac{kW}{K} \longleftarrow \dot{\sigma}_p$$

Mass and entropy rate balances for the condenser give

$$\dot{\sigma}_{cond} = \dot{m}[s_3 - s_2] + \dot{m}_{cw}[s_6 - s_5]$$

Using $s = s_f(T)$ for the cooling water and data from Table A-2

$$\dot{\sigma}_{cond} = (109)[0.5926 - 7.4651] + (3759)[0.5053 - 0.2966]$$

$$= -749.1 + 784.5 = 35.4\ kW/K \longleftarrow \dot{\sigma}_{cond}$$

In rank order, turbine, condenser, pump.

PROBLEM 6.127

KNOWN: Steam contained in a large tank at a known state passes from the tank through a turbine into a small vessel until a specified final condition is attained in the vessel.

FIND: Determine the amount of entropy produced. Repeat for the case where no work is developed by the turbine.

SCHEMATIC & GIVEN DATA: See Figure E4.12.

ASSUMPTIONS: See assumptions listed in Example 4.12.

ANALYSIS: The mass rate balance reduces to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

An entropy rate balance takes the form

$$\frac{dS_{cv}}{dt} = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}_i s_i + \dot{\sigma}_{cv}$$

Combining the mass and entropy rate balances

$$\frac{dS_{cv}}{dt} = s_i \frac{dm_{cv}}{dt} + \dot{\sigma}_{cv}$$

Integrating

$$\Delta S_{cv} = \int_1^2 s_i \, dm_{cv} + \sigma_{cv} \Rightarrow \Delta S_{cv} = s_i \Delta m_{cv} + \sigma_{cv}$$

In accordance with assumption 3, the specific entropy of the steam entering the control volume is constant at the value corresponding to the state in the large tank.

Since the small vessel is initially evacuated, the terms ΔS_{cv} and Δm_{cv} reduce to the entropy and mass within the vessel at the end of the process. That is

$$\Delta S_{cv} = m_2 s_2 - \cancelto{0}{m_1 s_1} \quad , \quad \Delta m_{cv} = m_2 - \cancelto{0}{m_1}$$

Collecting results and solving for the amount of entropy produced

$$\sigma_{cv} = m_2 (s_2 - s_i) \qquad (1)$$

At 15 bar, 320°C, Table A-4 gives $s_i = 6.9938$ kJ/kg·K. At 15 bar, 400°C, $s_2 = 7.2690$ kJ/kg·K. The solution to Example 4.12 provides m_2. Thus, Eq. (1) gives

$$\sigma_{cv} = (2.96 \text{ kg})(7.2690 - 6.9938) \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 0.8146 \frac{\text{kJ}}{\text{K}} \longleftarrow \sigma_{cv}$$

In the case where no work is developed, the solution to Example 4.12 gives the final temperature of the steam in the vessel as 477°C. Thus, interpolation in Table A-4 at 15 bar gives $s_2 = 7.5024$ kJ/kg·K, $v_2 = 0.22784$ m³/kg. Finally, substituting into Eq. (1)

$$\sigma_{cv} = \left(\frac{0.6 \text{ m}^3}{0.22784 \text{ m}^3/\text{kg}}\right)(7.5024 - 6.9938) \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$= 1.3394 \text{ kJ/K} \longleftarrow \sigma_{cv}$$

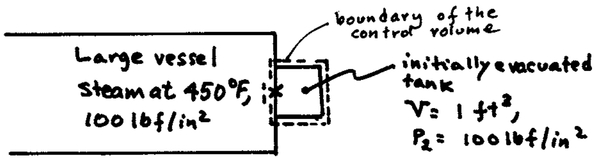
As there is less control of the entering stream in this case, the amount of entropy produced is greater.

PROBLEM 6.128

KNOWN: Steam contained in a large vessel at a specified state is allowed to flow into an initially evacuated tank until a specified pressure p is attained in the tank.

FIND: (a) For p = 100 lbf/in², determine the final temperature of the steam in the tank and the amount of entropy produced. (b) Plot the quantities of part (a) versus p ranging from 10 to 100 lbf/in².

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) For the control volume shown in the accompanying figure, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. (2) The state of the steam in the large vessel remains constant.

ANALYSIS: The mass rate balance and the energy rate balance reduce, respectively, to

$$\frac{dm_{cv}}{dt} = \dot{m}_i$$

$$\frac{dU_{cv}}{dt} = \dot{m}_i h_i$$

Combining these expressions and integrating

$$\frac{dU_{cv}}{dt} = h_i \frac{dm_{cv}}{dt} \Rightarrow \Delta U_{cv} = \int_1^2 h_i \, dm_{cv} \Rightarrow \Delta U_{cv} = h_i \Delta m_{cv}$$

In accordance with assumption 2, the specific enthalpy of the steam entering the control volume is constant at the value corresponding to the state in the large vessel. Since the tank is initially evacuated, the terms ΔU_{cv} and Δm_{cv} reduce to the internal energy and mass within the tank at the end of the process: $\Delta U_{cv} = m_2 u_2$, $\Delta m_{cv} = m_2$. Accordingly, the energy balance reduces simply to

$$m_2 u_2 = m_2 h_i \Rightarrow u_2 = h_i \qquad (1)$$

Thus, the final state of the steam in the tank is fixed by pressure p and $u_2 (= h_i)$.

An entropy rate balance for the control volume reduces to

$$\frac{dS_{cv}}{dt} = \overset{0}{\cancel{\sum \frac{\dot{Q}_j}{T_j}}} + \dot{m}_i s_i + \dot{\sigma}_{cv}$$

Introducing the above mass rate balance and integrating

$$\frac{dS_{cv}}{dt} = s_i \frac{dm_{cv}}{dt} + \dot{\sigma}_{cv} \Rightarrow \Delta S_{cv} = \int_1^2 s_i \, dm_{cv} + \sigma_{cv} \Rightarrow \Delta S_{cv} = s_i \Delta m_{cv} + \sigma_{cv}$$

As for h_i, s_i remains constant during the process. Then, since the tank is initially evacuated, $\Delta S_{cv} = m_2 s_2$, $\Delta m_{cv} = m_2$. Inserting these expressions and solving for the amount of entropy produced is

$$\sigma_{cv} = m_2 (s_2 - s_i) = \left(\frac{V}{v_2}\right)(s_2 - s_i) \qquad (2)$$

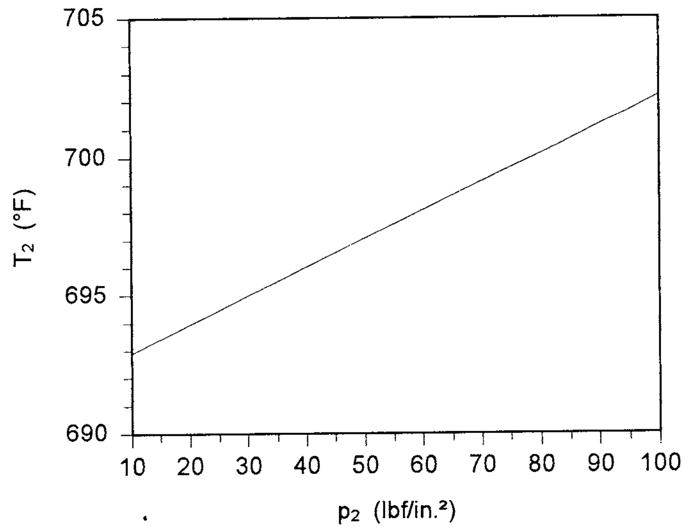
(a) p = 100 lbf/in² From Table A-4E at 100 lbf/in², 450°F, $h_i = 1253.6$ Btu/lb. Then, with $u_2 = 1253.6$ Btu/lb, interpolation in Table A-4E at 100 lbf/in² gives $T_2 = 702°F$, $s_2 = 1.8042$ Btu/lb·°R, $v_2 = 6.847$ ft³/lb $\longleftarrow T_2$

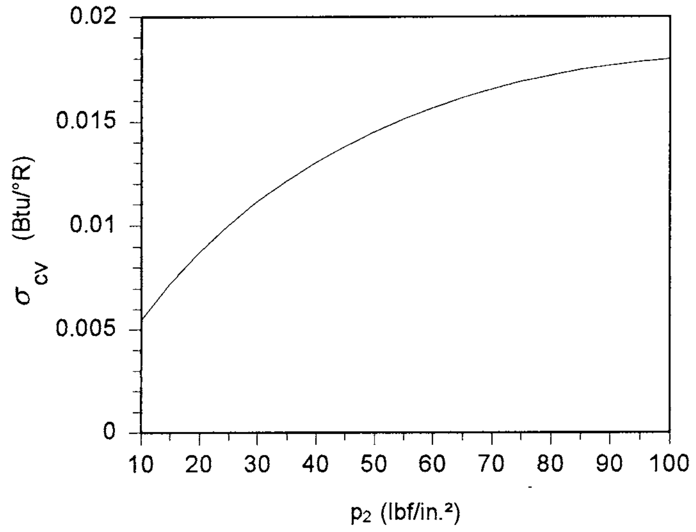
PROBLEM 6.128 (Contd.)

With Eq. (2) and s_i from Table A-4E at 100 lbf/in.², 450°F

$$\sigma_{cv} = \frac{V}{v_2}(s_2 - s_i) = \left(\frac{1\ \text{ft}^3}{6.847\ \text{ft}^3/\text{lb}}\right)(1.8042 - 1.6812)\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}} = 0.018\ \frac{\text{Btu}}{°\text{R}} \leftarrow \sigma_{cv}$$

(b) PLOTS



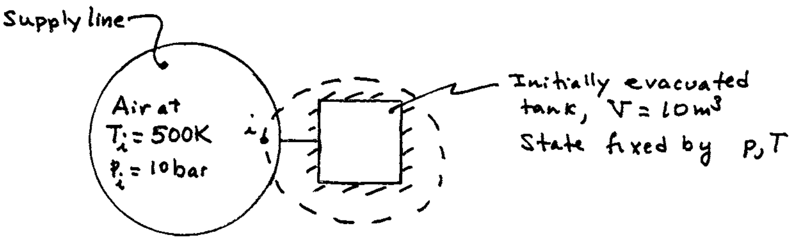


PROBLEM 6.129

KNOWN: Air flows from a large supply line into an initially evacuated, insulated tank until the tank pressure is p.

FIND: Plot the tank temperature, mass of air within the tank, and the amount of entropy produced versus p for p<10 bar.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is shown in the above figure. (2) For the control volume, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. (3) The state of the air within the supply line remains constant. (4) The mass of air within the piping connecting the supply line and tank can be ignored. (5) Air is modeled as an ideal gas with constant specific heat ratio k.

ANALYSIS: Paralleling the application of mass and energy balances in Example 4.12 but with $W_{cv} = 0$, the result is $u(T) = h(T_i)$, where T_i is the temperature of the air in the supply line and T is the temperature within the tank. For an ideal gas $h(T_i) = u(T_i) + RT_i$, so $u(T) = u(T_i) + RT_i$. Using ideal gas relations (assumption 5): $u(T) - u(T_i) = c_v[T - T_i]$, $R = c_p - c_v$, this gives

$$T = k T_i \tag{1}$$

That is, the temperature of the air within the tank is a constant independent of the pressure p.

Using the ideal gas equation of state, the mass of the air within the tank is

$$m = \frac{pV}{RT} = \frac{p}{p_i}\left[\frac{p_i V}{R(kT_i)}\right] = r\left[\frac{(10 \times 10^5 N/m^2)(10 m^3)}{\left(\frac{8314}{28.97}\frac{N \cdot m}{kg \cdot K}\right)((1.39)(500K))}\right] = (50.14 r) kg \tag{2}$$

where $r = p/p_i$ and $k = 1.39$ from Table A-20.

An entropy balance reads

$$\frac{dS_{cv}}{dt} = \dot{m}_i s_i + \dot{\sigma}_{cv} \xrightarrow{\text{(integration)}} [m s(T,p) - 0] = \int \dot{m}_i s_i + \sigma_{cv}$$
$$= m s(T_i, p_i) + \sigma_{cv}$$

That is, with ideal gas relations

$$\sigma_{cv} = m[s(T,p) - s(T_i,p_i)] = \underbrace{m}_{\text{Eq.(2)}}\Big[\overbrace{c_p}^{kR/k-1 \text{ (Eq.3.47a)}} \ln \underbrace{\frac{T}{T_i}}_{\text{Eq(1)}} - R \ln \underbrace{\frac{p}{p_i}}_{r}\Big]$$

$$\Rightarrow \dot{\sigma}_{cv} = (50.14r)R\left[\frac{k}{k-1}\ln k - \ln r\right] = (50.14 r)\left(\frac{8.314}{28.97}\right)\left[\frac{1.39}{0.39}\ln 1.39 - \ln r\right] \frac{kJ}{K}$$

$$\Rightarrow \sigma_{cv} = 14.39 r [1.174 - \ln r] \frac{kJ}{K} \tag{3}$$

SAMPLE CALCULATION: If p = 10 bar, r = 1. Eq.(1), T = 1.39 (500K) = 695K. Eq(2) gives, m = 50.14 kg. Eq (3) gives $\sigma_{cv} = 16.89 \frac{kJ}{K}$.

PROBLEM 6.129 (Cont'd.)

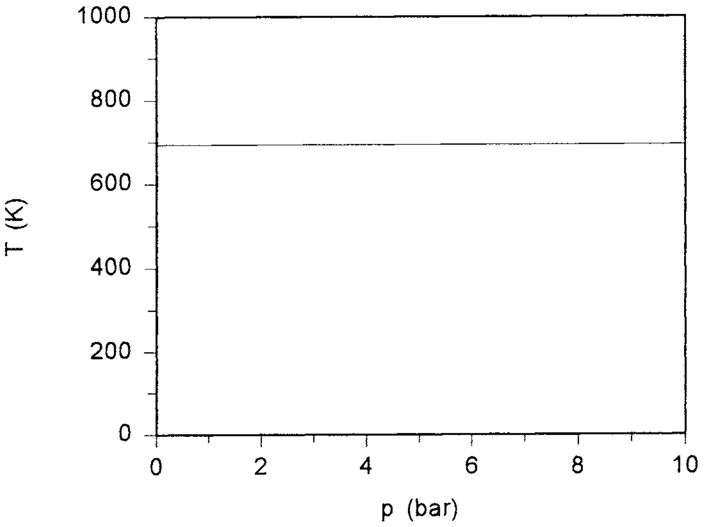
To generate data for the required plots, we use IT, as follows:

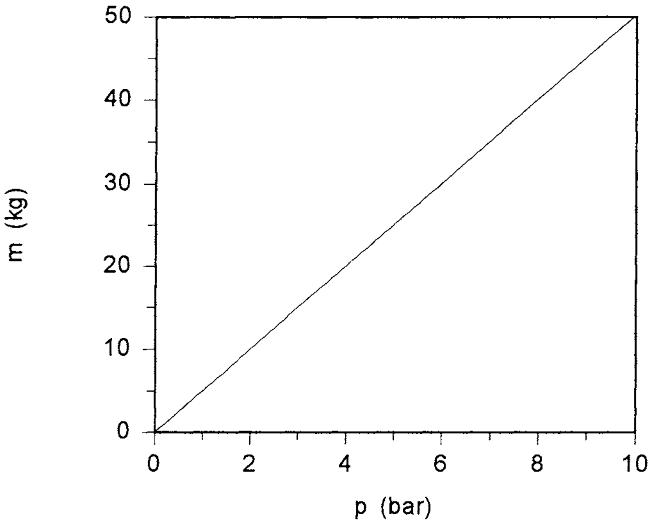
IT Code

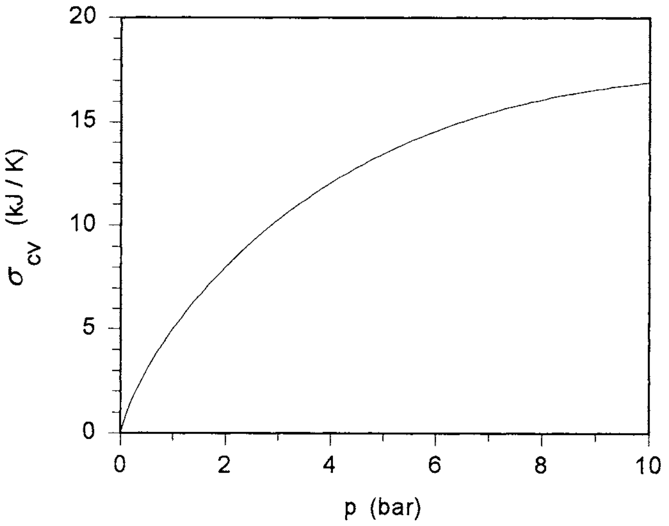
```
p_i = 10  // bar
Ti = 500  // K
V = 10  // m³

cp = cp_T("Air", Ti)
cv = cv_T("Air", Ti)
k = cp / cv

T = k * Ti

r = p / p_i
r = 1
v = v_TP("Air",T,p)
m = V / v

sigma_cv = m * (cp * ln(T / Ti) - R * ln(p / p_i))
R = 8.314 / 28.97
```

IT Results for p = 10 bar (r = 1)

```
c_p = 1.029 kJ/kg·K
k = 1.387
v = 0.199 m³/kg
m = 50.26 kg
T = 693.3 K
σ_cv = 16.91 kJ/kg·K
```







Observe:

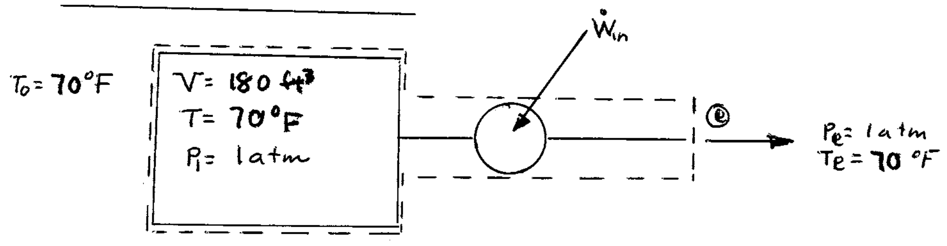
- The temperature in the tank is constant.
- The mass increases directly with tank pressure.
- There is more entropy produced as the pressure increases, as expected.

PROBLEM 6.130*

KNOWN: A tank initially filled with air is evacuated by pumping out the air.

FIND: Determine the minimum theoretical work required.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is shown in the figure. (2) Heat transfer with the surroundings maintains the temperature of the air in the tank and at the discharge of the pump at 70°F. (2) Kinetic and potential energy effects can be ignored. (3) The air is modeled as an ideal gas.

ANALYSIS: A mass balance reduces to $dm/dt = -\dot{m}_e$. An energy balace reads $dU/dt = \dot{Q} + \dot{W}_{in} - \dot{m}_e h_e$. Combinining these expressions

$$\frac{dU}{dt} = \dot{Q} + \dot{W}_{in} + h_e \frac{dm}{dt} \Rightarrow \Delta U = Q + W_{in} + h_e \Delta m \qquad (1)$$

where h_e is determined by T_e and thus is constant.

An entropy balance reads $dS/dt = \dot{Q}/T_0 - \dot{m}_e s_e + \dot{\sigma}$. With $-\dot{m}_e = dm/dt$

$$\frac{dS}{dt} = \frac{\dot{Q}}{T_0} + s_e \frac{dm}{dt} + \dot{\sigma} \Rightarrow \Delta S = \frac{Q}{T_0} + s_e \Delta m + \sigma \qquad (2)$$

where s_e is determined by T_e, P_e and thus is constant.

Eliminating Q between Eqs. (1), (2)

$$W_{in} = \Delta U - T_0 \Delta S + T_0 s_e \Delta m - h_e \Delta m + T_0 \sigma$$
$$= [\overset{0}{\cancel{m_2}} u_2 - m_1 u_1] - T_0(\overset{0}{\cancel{m_2}} s_2 - m_1 s_1) + T_0 s_e [\overset{0}{\cancel{m_2}} - m_1] - h_e(\overset{0}{\cancel{m_2}} - m_1) + T_0 \sigma$$
$$= m_1 T_0 (s_1 - s_e) + m_1 (h_e - u_1) + T_0 \sigma \qquad (3)$$

Since the states are the same initially and at the exit, $s_1 = s_e$. Also, with $h_e = u_e + P_e v_e$, $(h_e - u_1) = (u_e - u_1) + P_e v_e = P_e v_e$, since $T_1 = T_e$. With these, Eq. (3) reduces to

$$W_{in} = m_1 (P_e v_e) + T_0 \sigma = P_e V + T_0 \sigma \qquad (4)$$

The specific volume at the exit is the same as the initial specific volume in the tank, and so $V = m_1 v_e$.

Finally, since $\sigma \geq 0$, the minimum theoretical value corresponds to $\sigma = 0$:

$$(W_{in})_{MIN} = P_e V$$
$$= \left(14.7 \times 144 \frac{lbf}{ft^2}\right)(180\ ft^3)\left(\frac{1\ Btu}{778\ ft \cdot lbf}\right)$$
$$\cong 489.7\ Btu$$

$\xleftarrow{(W_{in})_{MIN}}$

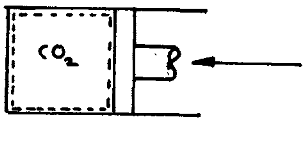
PROBLEM 6.131

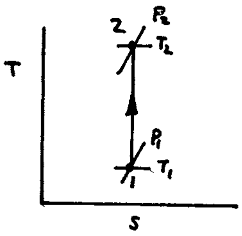
KNOWN: CO_2 contained in a piston cylinder is compressed isentropically from a specified initial state to a specified final pressure.

FIND: Using four alternative approaches, evaluate the final pressure and the work.

SCHEMATIC & GIVEN DATA:

$V_1 = 0.10\,m^3$
$p_1 = 0.3$ bar
$T_1 = 400$ K
$T_2 = 560$ K





ASSUMPTIONS: (1) As shown in the figure, the system is the CO_2 gas. (2) CO_2 is modeled as an ideal gas. (3) There are no significant kinetic or potential energy effects. 4 The process is isentropic, and thus $Q = 0$.

ANALYSIS: An energy balance reduces to $\Delta U = \overset{0}{\cancel{Q}} - W$, or $W = -\Delta U$.

$$W = -n(\bar{u}_2 - \bar{u}_1) \qquad (1)$$

With the ideal gas equation of state

$$n = \frac{p_1 V_1}{\bar{R} T_1} = \frac{(0.3 \times 10^5\ N/m^2)(0.1\ m^3)}{\left(8314\ \frac{N \cdot m}{kmol \cdot K}\right)(400\,K)} = 9.02 \times 10^{-4}\ kmol$$

(a) Table A-23. As $\Delta s = 0$, Eq. 6.21b gives

$$0 = \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln \frac{p_2}{p_1} \Rightarrow \ln \frac{p_2}{p_1} = \frac{\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1)}{\bar{R}} = \frac{239.962 - 225.225}{8.314} \Rightarrow p_2 = 1.766\ \text{bar} \longleftarrow p_2$$

where $\bar{s}^\circ$ values are from Table A-23, with $\bar{u}$ values from the same soure, Eq. (1) gives

$$W = -(9.02 \times 10^{-4}\ kmol)(15{,}751 - 10{,}046)\frac{kJ}{kmol} = -5.146\ kJ \longleftarrow W$$

(c) k from Table A-20 at 480 K: k = 1.233. The pressures and temperatures are related by Eq. 6.45. Thus

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/k-1} \Rightarrow \frac{p_2}{p_1} = \left(\frac{560}{400}\right)^{1.233/.233} \Rightarrow p_2 = 1.78\ \text{bar} \longleftarrow p_2$$

Using Eq. 3.47b on a molar basis, Eq. (1) becomes

$$W = -\frac{n\bar{R}}{k-1}(T_2 - T_1) = \frac{-(9.02 \times 10^{-4}\ kmol)(8.314\ kJ/kmol \cdot K)(560 - 400)K}{0.233} = -5.15\ kJ \longleftarrow W$$

(d) k from Table A-20 at 300 K: k = 1.288. Then, as in part (b)

① $$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/k-1} \Rightarrow \frac{p_2}{p_1} = \left(\frac{560}{400}\right)^{1.288/.288} \Rightarrow p_2 = 1.351\ \text{bar} \longleftarrow p_2$$

$$W = -\frac{n\bar{R}}{k-1}(T_2 - T_1) = \frac{-(9.02 \times 10^{-4}\ kmol)(8.314\ kJ/kmol \cdot K)(560 - 400)K}{0.288} = -4.166\ kJ \longleftarrow W$$

(b) *IT* Results

n = 0.0009021 kmol
v_1 = 110.9 m^3/kmol
p_2 = 1.766 bar
W = -5.144 kJ

IT Code

```
V1 = 0.1  // m³
p1 = 0.3  // bar
T1 = 400  // K
T2 = 560  // K

W = - n * (u2 - u1)
u1 = u_T("CO2", T1)   // kJ/kmol
u2= u_T("CO2", T2)
v1 = v_TP("CO2",T1,p1)   // m³/kmol
n = V1 / v1

s2 = s1
s1 = s_TP("CO2", T1, p1)
s2 = s_TP("CO2", T2, p2)
```

---

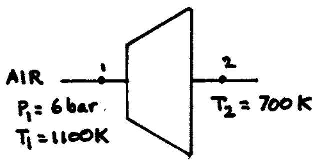
1. In the present case, the approach of part (d) gives results departing significantly from those obtained with the other approaches

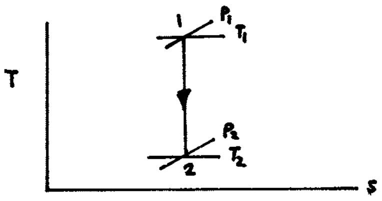
PROBLEM 6.132

KNOWN: Air expands isentropically through a turbine at steady state from a specified inlet state to a specified pressure.

FIND: Using three alternative approaches determine the exit pressure and the work developed per unit of air flowing.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The turbine is at steady state and the expansion is isentropic. (2) Kinetic and potential energy changes from inlet to exit can be neglected. (3) Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$ and an energy rate balance reduces to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \qquad (1)$$

(a) Table A-22. With h_1 and h_2 from the table, Eq. (1) gives

$$\frac{\dot{W}_{cv}}{\dot{m}} = (1161.07 - 713.27)\frac{\text{kJ}}{\text{kg}} = 447.8 \text{ kJ/kg} \longleftarrow (\dot{W}_{cv}/\dot{m})$$

The pressures and temperatures are related by Eq. 6.43, which with data from the table gives

$$p_2 = p_1\left(\frac{p_{r2}}{p_{r1}}\right) = 6 \text{ bar}\left(\frac{28.8}{167.1}\right) = 1.034 \text{ bar} \longleftarrow p_2$$

(c) k from Table A-20 at 900 K: k = 1.344. With Eq. 3.47a, Eq. (1) becomes

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{kR}{k-1}(T_1 - T_2)$$

$$= \frac{(1.344)(8.314/28.97)(\text{kJ/kg}\cdot\text{K})(1100-700)\text{K}}{0.344} = 448.5 \text{ kJ/kg} \longleftarrow (\dot{W}_{cv}/\dot{m})$$

The pressures and temperatures are related by Eq. 6.45. Thus

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/k-1} = \left(\frac{700}{1100}\right)^{1.344/.344} \Rightarrow p_2 = 1.026 \text{ bar} \longleftarrow p_2$$

(d) k from Table A-20 at 300 K: k = 1.4. Then, as in part (b)

① $$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{kR}{k-1}(T_1 - T_2) = \frac{(1.4)(8.314/28.97)(1100-700)}{0.4} = 401.8 \text{ kJ/kg} \longleftarrow (\dot{W}_{cv}/\dot{m})$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/k-1} = \left(\frac{700}{1100}\right)^{1.4/.4} \Rightarrow p_2 = 1.233 \text{ bar} \longleftarrow p_2$$

(b) *IT* Results

h1 = 1161 kJ/kg
h2 = 713 kJ/kg
$\dot{W}_{cv}$ = 447.6 kJ/kg
p2 = 1.034 bar

IT Code

```
p1 = 6  // bar
T1 = 1100  // K
T2 = 700  // K
```

```
Wdot_cv = mdot * (h1 - h2)
mdot = 1
h1 = h_T("Air", T1)
h2 = h_T("Air", T2)

s2 = s1
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
```

---

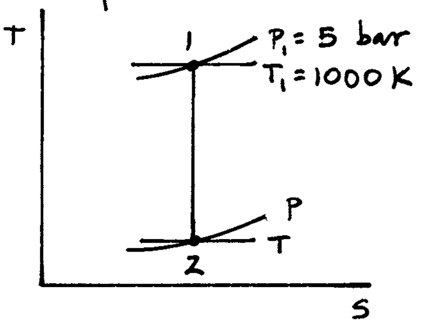
1. In the present case, the approach of part (d) gives results departing significantly from those obtained with the other approaches.

PROBLEM 6.133

KNOWN: Methane gas undergoes an isentropic expansion from a given initial state to a final state at T, p.

FIND: Using the ideal gas model, and $c_p(T)$ data from Table A-21, determine (a) p when T=500K and (b) T when p = 1 bar. Check using IT.

SCHEMATIC AND GIVEN DATA:



ASSUMPTIONS: (1) The methane can be modeled as an ideal gas. (2) T_1 and T_2 are in the range for which the $c_p(T)$ data in Table A-22 are applicable.

ANALYSIS: When expressed on a molar basis, Eq. 6.19 becomes for $\Delta s = 0$

$$0 = \int_{T_1}^{T} \frac{\bar{c}_p(T)}{T}\,dT - \bar{R}\ln\frac{p}{T_1}$$

where the integration is between state 1 (T_1, p_1) and a second state expressed a (T, p). With $\bar{c}_p$ from Table A-21: $\bar{c}_p = \bar{R}[\alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4]$ this expression becomes

$$\ln\frac{p}{p_1} = \int_{T_1}^{T} \left[\frac{\alpha}{T} + \beta + \gamma T + \delta T^2 + \epsilon T^3\right\} dT$$

or

$$\ln\frac{p}{p_1} = \alpha \ln\left(\frac{T}{T_1}\right) + \beta(T - T_1) + \frac{\gamma}{2}(T^2 - T_1^2) + \frac{\delta}{3}(T^3 - T_1^3) + \frac{\epsilon}{4}(T^4 - T_1^4) \qquad (1)$$

(a) and (b): Eq (1) can be solved for p when T is given or for T when p is given. The following IT code illustrates this, using data for the coefficients $\alpha, \beta, \ldots$ from Table A-21:

```
p1 = 5  // bar
T1 = 1000  // K

 ln (p / p1) = alpha*ln(T/T1) + beta*(T-T1) + (gamma/2)*(T^2-T1^2) +
(delta/3)*(T^3-T1^3) + (epsilon/4)*(T^4-T1^4)
alpha = 3.826
beta = -3.979E-03
gamma = 24.558E-06
delta = -22.733E-09
epsilon = 6.962E-12
```

Results: (a) T = 500 K
// p = 0.03825 bar ← (a) p

(b) p = 1 bar
// T = 821.1 K ← (b) T

(c) To check these results, we use the $s = s\_TP(\text{"CH4"}, T, p)$ functions of it, as follows:

```
s = s1
s1 = s_TP("CH4", T1, p1)
s = s_TP("CH4", T, p)
```

① Results: (c) T = 500 K; p = 0.03818 bar ← (c)
p = 1 bar; T = 821.2 K

---

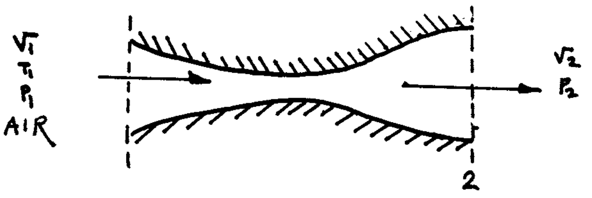
1. Note that the results using the internal functions of IT and using data from Table A-21 agree quite well in this case.

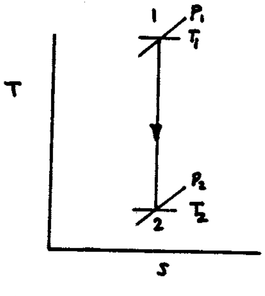
PROBLEM 6.134

KNOWN: Air as an ideal gas with constant specific heat ratio k expands isentropically from p_1, T_1, V_1 to a pressure p_2.

FIND: (a) Develop an expression for the exit velocity V_2 in terms of k, R, V_1, T_1, p_1, p_2.
(b) Plot V_2 versus p_2/p_1 for selected values of k, V_1, and T_1

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The expansion through the nozzle is isentropic. (2) The nozzle operates at steady state. (3) The gas is modeled as an ideal gas with constant specific heat ratio k. (4) Potential energy effects are negligible.

ANALYSIS: (a) At steady state $\dot m_1 = \dot m_2 = \dot m$, and an energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot m\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}^{0}\right)$$

or

$$0 = h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}$$

With Eq. 3.47a

$$\frac{V_2^2}{2} = \frac{V_1^2}{2} + \frac{kR}{k-1}(T_1 - T_2)$$

$$= \frac{V_1^2}{2} + \frac{kRT_1}{k-1}\left(1 - \frac{T_2}{T_1}\right)$$

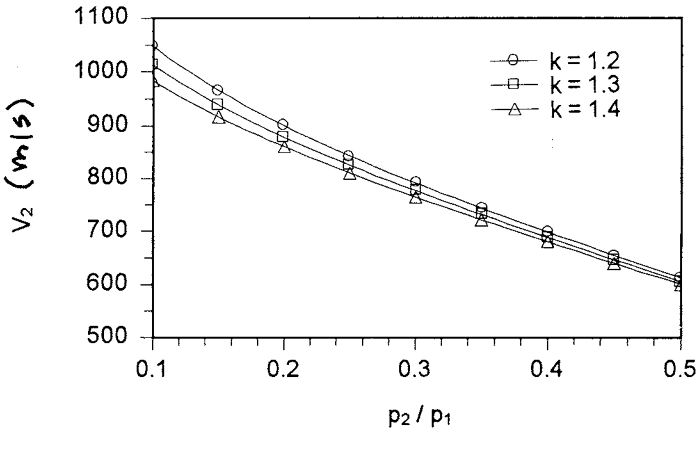
For constant k and no change in specific entropy, Eq. 6.45 relates the temperatures and pressures, giving

$$V_2 = \sqrt{V_1^2 + 2\frac{kRT_1}{k-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{(k-1)/k}\right]} \longleftarrow V_2$$

Sample calculation $V_1 = 0$, $T_1 = 1000K$, $k = 1.4$, $(p_2/p_1) = 0.1$

$$V_2 = \sqrt{\frac{(2)(1.4)}{1.4-1}\,\frac{8314}{28.97}\left(\frac{N\cdot m}{kg\cdot K}\right)(1000K)\left[\frac{1\,kg\cdot m/s^2}{1N}\right]\left[1 - (0.1)^{0.4/1.4}\right]} = 984\,m/s$$

(b) PLOT:



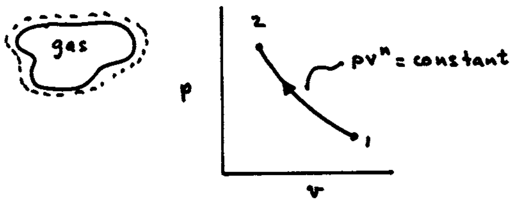
The plots show that the exit velocity increases as p_2/p_1 decreases. For a specified value of p_2/p_1, V_2 increases as k decreases.

PROBLEM 6.135

KNOWN: An ideal gas undergoes a polytropic process from T_1, p_1 to T_2.

FIND: (a) Derive an expression for $s_2 - s_1$ in terms of n, R, T_1, T_2 and s°. (b) Determine n when $\Delta s = 0$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system consists of a quantity of a gas that is modeled as an ideal gas. (2) The process is a polytropic process.

ANALYSIS: (a) With Eq. 6.21a

$$s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - R \ln \frac{p_2}{p_1}$$

For a polytropic process of an ideal gas (Eq. 3.56)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

Thus

$$s_2 - s_1 = s^\circ(T_2) - s^\circ(T_1) - R \ln \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} \longleftarrow (a)$$

(b) If $\Delta s = 0$, the expression of part (a) gives

$$\ln \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}} = \frac{s^\circ(T_2) - s^\circ(T_1)}{R}$$

$$\frac{n}{n-1} \ln \left(\frac{T_2}{T_1}\right) = \frac{s^\circ(T_2) - s^\circ(T_1)}{R}$$

or

$$\frac{n}{n-1} = \alpha \qquad \text{where} \qquad \alpha = \frac{s^\circ(T_2) - s^\circ(T_1)}{R \ln T_2/T_1}$$

①

$$\Rightarrow \quad n = \frac{\alpha}{\alpha - 1} = \frac{s^\circ(T_2) - s^\circ(T_1)}{s^\circ(T_2) - s^\circ(T_1) - R \ln T_2/T_1} \longleftarrow (b)$$

---

1. For the special case c_p = constant

$$s^\circ(T_2) - s^\circ(T_1) = \int_{T_1}^{T_2} \frac{c_p}{T} dT = c_p \ln \frac{T_2}{T_1}$$

Thus

$$\alpha = \frac{c_p \ln T_2/T_1}{R \ln T_2/T_1} = \frac{c_p}{R}$$

With Eq. 3.47a, $c_p = kR/k-1$, so

$$\alpha = \frac{k}{k-1}$$

and

$$n = \frac{k/(k-1)}{\left(\frac{k}{k-1}\right) - 1} = \frac{k}{k - (k-1)} = k$$

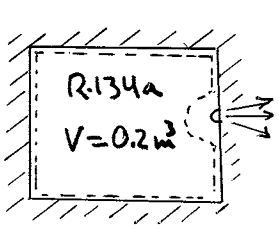
which is in accord with the discussion of Fig. 6.11.

PROBLEM 6.136*

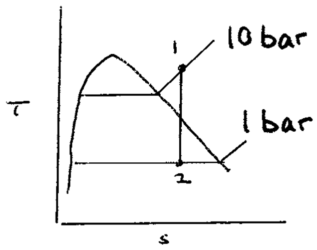
KNOWN: Data are provided for a rigid, insulated tank initially filled with R-134a vapor. A leak develops and R134a slowly escapes until the pressure becomes 1 bar.

FIND: Determine (a) the final temperature of the R134a within the tank and (b) the amount of mass that exits.

SCHEMATIC & GIVEN DATA:



Initially $P_1 = 10$ bar, $T_1 = 40°C$
$P_2 = 1$ bar



ASSUMPTIONS: (1) The system consists of the mass initially in the tank that remains in the tank. (2) For the system, $\dot{Q} = 0$ and the effects of kinetic and potential energy can be ignored. (3) Irreversibilities within the tank can be ignored as the air slowly escapes.

ANALYSIS: The solution closely follows that of Example 6.10, leading to the result that the mass within the system undergoes an isentropic expansion: $s_2 = s_1$.

With values from Table A-12, $s_1 = 0.9066$ kJ/kg·K, $v_1 = 0.02029$ m³/kg. Since $s_2 = s_1$, state 2 is located in the two phase liquid-vapor region with quality

$$x_2 = \frac{s_2 - s_f}{s_g - s_f} = \frac{0.9066 - 0.0678}{0.9395 - 0.0678} = 0.9623$$

(a) The final temperature is the saturation temperature corresponding to $P_2 = 1$ bar : $T_2 = -26.43°C$ ← (a)

(b) The mass that exits $= m_1 - m_2$, or

$$\Delta m = m_1 - m_2 = V\left[\frac{1}{v_1} - \frac{1}{v_2}\right]$$

where

$$v_2 = (0.7258 \times 10^{-3}) + 0.9623\left[0.1917 - 0.7258 \times 10^{-3}\right] = 0.1845 \ \frac{m^3}{kg}$$

So

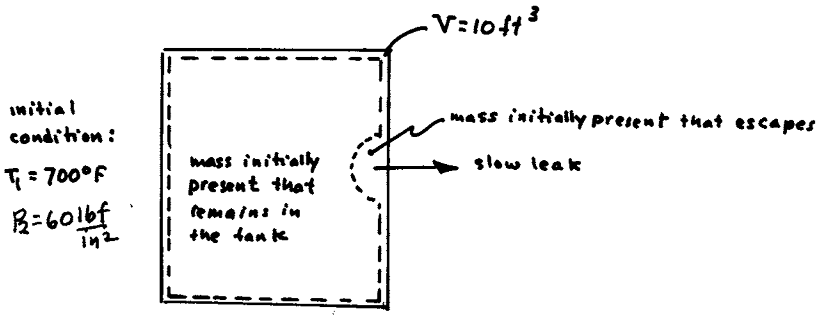
$$\Delta m = 0.2 m^3 \left[\frac{1}{0.02029} - \frac{1}{0.1845}\right] \frac{kg}{m^3} = 8.77 kg$$ ← (b)

PROBLEM 6.137

KNOWN: A rigid, insulated tank is initially filled with water vapor at a known state. Water vapor slowly escapes until vapor at pressure p remains.

FIND: (a) For p = 14.7 lbf/in², determine the final temperature and mass. (b) Plot final temperature and mass versus p ranging from 14.7 to 60 lbf/in.²

SCHEMATIC & GIVEN DATA:


$V = 10\ ft^3$
initial condition:
$T_1 = 700°F$
$P_2 = 60\ \frac{lbf}{in^2}$
mass initially present that escapes
slow leak
mass initially present that remains in the tank

ASSUMPTIONS: (1) As shown in the figure, the closed system is the mass initially in the tank that remains in the tank. (2) No significant heat transfer occurs between system and surroundings. (3) Irreversibilities within the tank can be ignored as the water vapor slowly escapes.

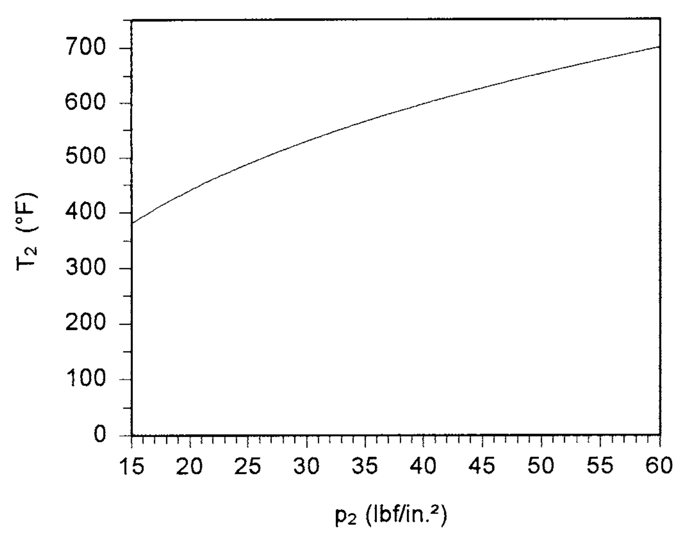
Analysis: For the closed system under consideration, there are no significant irreversibilities (assumption 3), and no heat transfer occurs (assumption 2). For this system an entropy balance reduces to

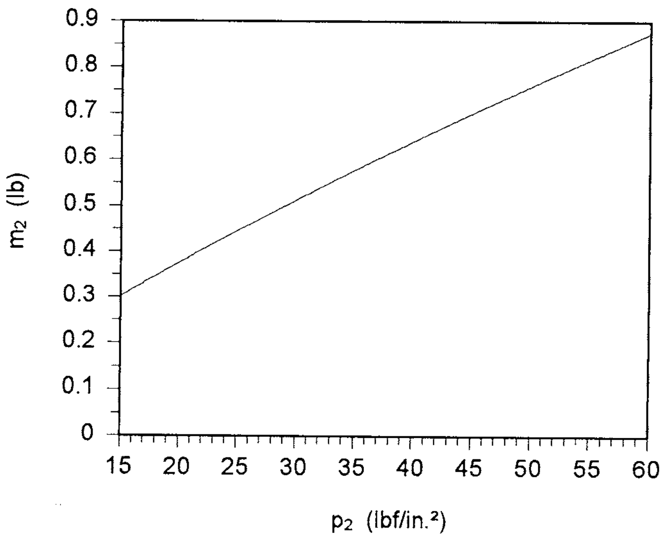
$$m\Delta s = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \cancel{\sigma}^{\,0} \Rightarrow s_2 = s_1$$

Thus, the final state within the tank is fixed by $p_2 = p$, $s_2 = s_1$. The final mass in the tank is $m_2 = V/v_2$.

(a) p = 14.7 lbf/in.² From Table A-4E, $s_1 = 1.8609$ Btu/lb · °R. Then, with $s_2 = s_1$ at 14.7 lbf/in.², interpolation in Table A-4E gives $T_2 = 377.4°F$ ← T_2
and $v_2 = 33.73$ ft³/lb, so $m_2 = 10\ ft^3/(33.73\ ft^3/lb) = 0.296$ lb. ← m_2
The initial mass is $m_1 = V/v_1 = (10/11.44) = 0.874$ lb.

(b) PLOTS



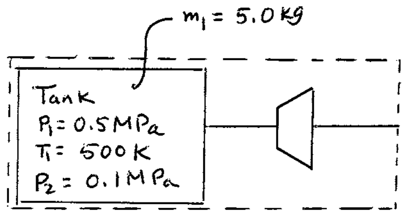


PROBLEM 6.138

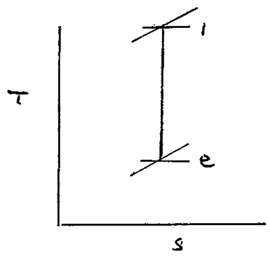
KNOWN: A tank initially filled with air is allowed to discharge through a turbine until the pressure in the tank becomes atmospheric

FIND: Determine the maximum theoretical work that could be developed.

SCHEMATIC & GIVEN DATA:



$P_e = 0.1$ MPa



ASSUMPTIONS: (1) The control volume is shown above. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) The control volume is free of irreversibilities.

ANALYSIS: A mass rate balance reduces to $dm/dt = -\dot{m}_e$. Using this, an energy rate balance reads

$$\frac{dU}{dt} = \overset{0}{\cancel{\dot{Q}_{cv}}} - \dot{W}_{cv} - \dot{m}_e h_e \quad , \quad \frac{dU}{dt} = -\dot{W}_{cv} + h_e \frac{dm}{dt}$$

or

$$\dot{W}_{cv} = -\frac{dU}{dt} + h_e \frac{dm}{dt} \Rightarrow W_{cv} = -\Delta U + \int h_e \, dm \qquad (1)$$

We expect that the maximum theoretical work would be developed in the absence of irreversibilities within the tank and the turbine (assumption (4)). The data given are recognized as corresponding to those for Example 6.10. The solution to Example 6.10 shows that a typical unit of mass remaining in the tank would undergo an isentropic expansion from T_1, P_1 to T_2, P_2. Moreover, each unit of mass passing through the turbine expands isentropically. Accordingly, if P, T denote the pressure and temperature within the tank at a particular instant

$$\frac{P_r(T)}{P_r(T_1)} = \frac{P}{P_1} \qquad (2)$$

But P, T would also correspond to the condition of the mass entering the turbine. So

$$\frac{P_r(T_e)}{P_r(T)} = \frac{P_e}{P} \qquad (3)$$

Combining Eqs. (2), (3)

$$\frac{P_r(T_e)}{P_r(T_1)} = \frac{P_e}{P_1} \rightarrow \frac{P_r(T_e)}{P_r(T_1)} = \frac{P_2}{P_1} \quad \text{since } P_e = P_2$$

Then, with the result of Example 6.10, $T_e = T_2 = 317$ K.

Returning to Eq. (1), h_e is fixed by T_e, and thus a constant:

$$(W_{cv})_{MAX} = -\Delta U + h_e \Delta m$$

$$= (m_1 u_1 - m_2 u_2) + h_e (m_2 - m_1)$$

From Example 6.10, $m_2 = 1.58$ kg. With data from Table A-22

$$(W_{cv})_{MAX} = (5\text{ kg})(359.49 \tfrac{kJ}{kg}) - (1.58\text{ kg})(226.28 \tfrac{kJ}{kg}) + (317.28 \tfrac{kJ}{kg})(1.58-5)\text{ kg}$$

$$= 1797.45\text{ kJ} - 357.52\text{ kJ} - 1085.1\text{ kJ}$$

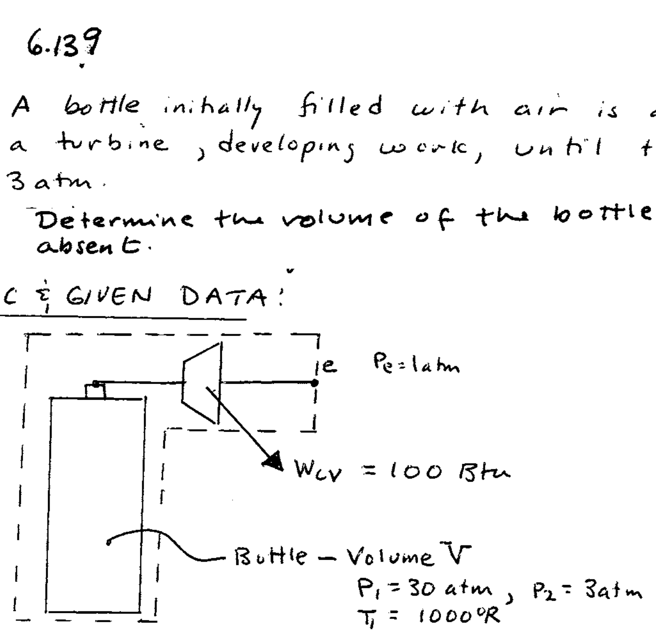
$$= 354.83\text{ kJ} \longleftarrow \text{MAX}$$

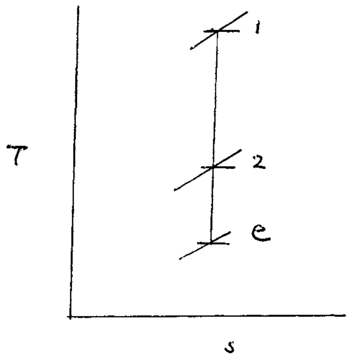
PROBLEM 6.139

KNOWN: A bottle initially filled with air is allowed to discharge through a turbine, developing work, until the pressure in the bottle becomes 3 atm.

FIND: Determine the volume of the bottle when irreversibilities are absent.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is shown in the figure. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) Irreversibilities are absent within the control volume.

①

ANALYSIS: A mass rate balance reduces to $dm/dt = -\dot{m}_e$. Using this an energy rate balance reads

$$\frac{dU}{dt} = \overset{0}{\cancel{\dot{Q}_{cv}}} - \dot{W}_{cv} - \dot{m}_e h_e \,, \quad \frac{dU}{dt} = -\dot{W}_{cv} + h_e \frac{dm}{dt} \Rightarrow W_{cv} = -\Delta U + \int h_e\, dm \qquad (1)$$

The plan is to evaluate the bottle volume in the absence of irreversibilities (assumption (4)). Following the reasoning of Example 6.10, a typical unit of mass remaining in the bottle would undergo an isentropic expansion from P_1, T_1 to P_2, T_2. Moreover, each unit of mass passing through the turbine expands isentropically. Accordingly, if P, T denote the pressure and temperature within the tank at a particular instant,

$$\frac{P_r(T)}{P_r(T_1)} = \frac{P}{P_1} \qquad (2)$$

But P, T would also correspond to the condition of the mass entering the turbine, so

$$\frac{P_r(T_e)}{P_r(T)} = \frac{P_e}{P} \qquad (3)$$

Combining Eqs. (2), (3), and using data from Table A-22E

$$\frac{P_r(T_e)}{P_r(T_1)} = \frac{P_e}{P_1} \Rightarrow P_r(T_e) = \left(\frac{1\text{atm}}{30\text{atm}}\right)(12.3) = 0.41 \Rightarrow h_e = 90.98 \text{ Btu/lb}.$$

Since h_e is fixed, Eq. (1) becomes, with the ideal gas equation of state ($m = pV/RT$),

$$\begin{aligned} W_{cv} = -\Delta U + h_e(\Delta m) \Rightarrow W_{cv} &= (m_1 u_1 - m_2 u_2) + h_e(m_2 - m_1) \\ &= \left(\frac{P_1 V}{RT_1}\right) u_1 - \left(\frac{P_2 V}{RT_2}\right) u_2 + h_e\left[\frac{P_2 V}{RT_2} - \frac{P_1 V}{RT_1}\right] \\ &= \frac{P_1 V}{R}\left[\frac{u(T_1)}{T_1} - \left(\frac{P_2}{P_1}\right)\frac{u(T_2)}{T_2} + h_e\left[\left(\frac{P_2}{P_1}\right)\frac{1}{T_2} - \frac{1}{T_1}\right]\right] \end{aligned} \qquad (4)$$

With data from Table A-22E, $u(T_1) = 172.43$ Btu/lb, $P_r(T_2) = P_2/P_1\, P_r(T_1) = (3/30)(12.3) = 1.23$, giving $T_2 = 522°R$, $u(T_2) = 88.93$ Btu/lb. Evaluating Eq. (4)

$$100 \text{ Btu} = \frac{V(30 \times 14.7 \times 144 \text{ lbf/ft}^2)}{(1545/28.97)(\text{ft·lbf/lb·°R})}\left[\frac{172.43}{1000} - \left(\frac{3}{30}\right)\frac{88.93}{522} + 90.98\left[\left(\frac{3}{30}\right)\frac{1}{522} - \frac{1}{1000}\right]\right]\frac{\text{Btu}}{\text{lb·°R}} \qquad (5)$$

Solving gives $V = 1.03 \text{ ft}^3$. ←

---

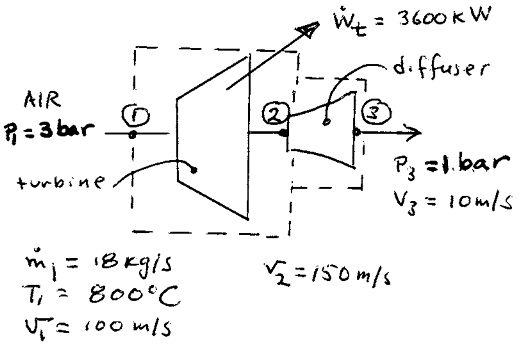
1. Checking the compressibility chart with $T_{R1} = 1000/239 = 4.18$, $P_{R2} = 3/37.2 = 0.081$. it can be concluded that the ideal gas model is appropriate at these states.

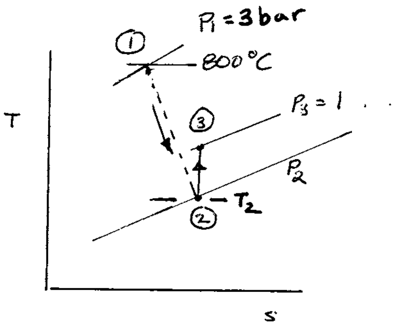
PROBLEM 6.140

KNOWN: Steady state operating data are provided for an air turbine fitted with a diffuser at its exit.

FIND: Determine the pressure and temperature at the turbine exit and the rate of entropy production in the turbine. Show the processes on a T-s diagram.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The overall control volume consists of two sub control volumes, each at steady state. (2) The turbine operates adiabatically (3) Flow through the diffuser is isentropic. (4) Potential energy effects can be ignored. (4) Air is modeled as an ideal gas.

ANALYSIS: (a) For the isentropic process of air from 2 to 3, $P_2/P_3 = P_r(T_2)/P_r(T_3)$. Accordingly, to find P_2 requires $P_r(T_2)$ and $P_r(T_3)$.

Mass and energy rate balances for the turbine reduce to give

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \dot{W}_t + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right) \Rightarrow h_2 = -\frac{\dot{W}_t}{\dot{m}} + h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

with h_1 from Table A-22

$$h_2 = -\frac{3600\ \text{kJ/s}}{18\ \text{kg/s}} + 1129.83\ \frac{\text{kJ}}{\text{kg}} + \frac{\left[(100)^2 - (150)^2\right](\text{m}^2/\text{s}^2)}{2}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = 923.6\ \frac{\text{kJ}}{\text{kg}}$$

Then, interpolation in Table A-22 gives $P_{r2} = 72.66$, $T_2 = 892\text{K}\ (619°\text{C})$. ← T_2

Mass and energy rate balances for the diffuser reduce to give

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left(h_2 - h_3 + \frac{V_2^2 - V_3^2}{2}\right) \Rightarrow h_3 = h_2 + \frac{V_2^2 - V_3^2}{2}$$

or

$$h_3 = 923.6 + \frac{(150)^2 - (10)^2}{2000} = 934.8\ \text{kJ/kg}.$$

Interpolating in Table A-22, $P_{r3} = 75.85$. Then

$$P_2 = P_3\frac{P_{r2}}{P_{r3}} = (1\ \text{bar})\left(\frac{72.66}{75.85}\right) = 0.958\ \text{bar}$$ ← P_2

(b) Mass and entropy rate balances reduce to give for the turbine

$$\dot{\sigma}_t = \dot{m}\,(s_2 - s_1)$$

With Eq. 6.21a this becomes

$$\dot{\sigma}_t = \dot{m}\left[s^\circ(T_2) - s^\circ(T_1) - R\ln P_2/P_1\right]$$

$$= (18\ \text{kg/s})\left[2.8381 - 3.0485 - \frac{8.314}{28.97}\ln\left(\frac{0.958}{3}\right)\right]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 2.11\ \frac{\text{kW}}{\text{K}}$$ ← $\dot{\sigma}_t$

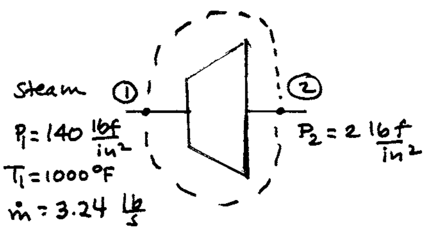
PROBLEM 6.141

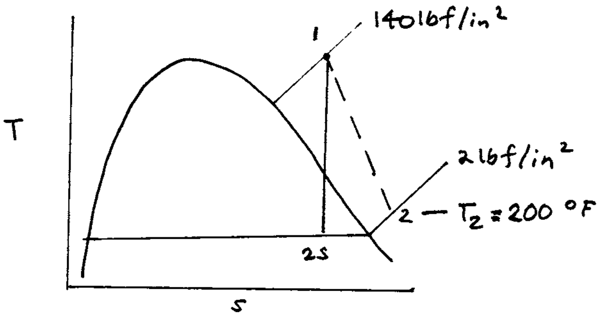
KNOWN: Steam at a specified state enters a turbine and expands adiabatically with a specified mass flow rate to a given exit pressure.

FIND: (a) Determine the maximum theoretical power that can be developed and the corresponding exit temperature.

(b) For a given exit temperature, determine the isentropic turbine efficiency.

SCHEMATIC & GIVEN DATA:


Steam ①
②
$P_1 = 140\ \frac{lbf}{in^2}$
$T_1 = 1000°F$
$\dot m = 3.24\ \frac{lb}{s}$
$P_2 = 2\ \frac{lbf}{in^2}$



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot Q_{cv} = 0$, and kinetic/potential energy effects are negligible.

ANALYSIS: Mass and energy rate balances reduce to give $\dot W_{cv} = \dot m(h_1 - h_2)$.

(a) As discussed in Sec. 6.8, the maximum theoretical power is obtained for an isentropic expansion: $s_{2s} = s_1$. With data from Table A-3E, 4E, $h_1 = 1531.0$ Btu/lb, $s_1 = 1.8827$ Btu/lb·°R,

$$x_{2s} = \frac{1.8827 - 0.1750}{1.7448} = 0.9787$$

$$h_{2s} = h_f + x_{2s} h_{fg} = 94.02 + 0.9787(1022.1) = 1094.3\ \text{Btu/lb}, \quad T_{2s} = 126°F$$

Then

$$(\dot W_{cv})_{MAX} = \left(3.24\ \frac{lb}{s}\right)(1531 - 1094.3)\left(\frac{Btu}{lb}\right)\left|\frac{3600\,s}{h}\right|\left|\frac{1\ hp}{2545\ Btu/h}\right| = 2000\ hp \longleftarrow (\dot W_{cv})_{max}$$

(b) The isentropic turbine efficiency, given by Eq. 6.48, takes the form

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{(1531 - 1149.7)\ \text{Btu/lb}}{(1531 - 1094.3)\ \text{Btu/lb}} = 0.873\ (87.3\%) \longleftarrow \eta_t$$

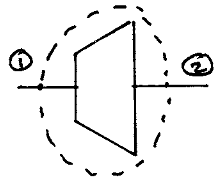
where h_2 is obtained from the steam tables at 2 lbf/in², 200°F.

PROBLEM 6.142

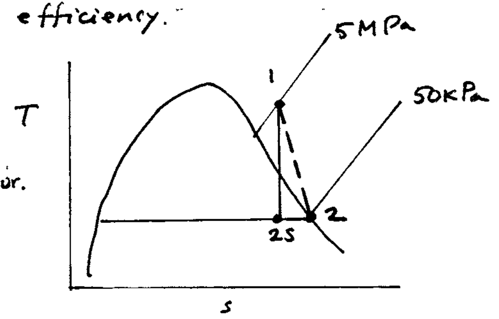
KNOWN: Steam at a specified state enters a turbine and expands adiabatically to a specified state.

FIND: Determine the work developed by the turbine per unit mass of steam flowing and the isentropic turbine efficiency.

SCHEMATIC & GIVEN DATA:



$P_1 = 5\ MPa$
$T_1 = 600°C$
$P_2 = 50\ kPa$, sat. vapor.



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot Q_{cv} = 0$, and kinetic/potential energy effects are negligible.

ANALYSIS: With the given assumptions, the isentropic turbine efficiency, Eq. 6.48, takes the form

PROBLEM 6.142 (Continued)

$$\eta_t = \frac{\dot{W}_t/\dot{m}}{h_1 - h_{2s}} \quad , \text{where } \frac{\dot{W}_t}{\dot{m}} = h_1 - h_2$$

To find h_{2s}, Table A-4 gives $s_1 = 7.2683$ kJ/kg·K. Then with data from Table A-3 at 50kPa

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{7.2683 - 1.0910}{7.5939 - 1.0910} = 0.95 \,,\; h_{2s} = h_f + x_{2s} h_{fg} = 340.49 + 0.95(2305.4) = 2530.6 \, \frac{kJ}{kg}$$

Then, with $h_1 = 3666.4$ kJ/kg, $h_2 = 2645.9$ kJ/kg, $\dot{W}_t/\dot{m} = 1020.5$ kJ/kg $\longleftarrow \dot{W}_t/\dot{m}$

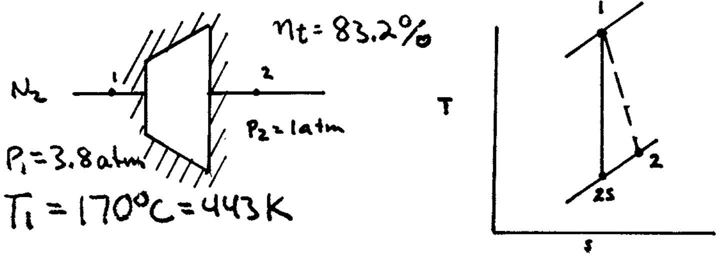
$$\eta_t = \frac{1020.5 \text{ kJ/kg}}{(3666.4 - 2530.6) \text{ kJ/kg}} = 0.898 \;(89.8\%) \longleftarrow \eta_t$$

PROBLEM 6.143*

KNOWN: Nitrogen (N_2) at a specified state expands through an insulated turbine at steady state to a specified pressure. The isentropic turbine efficiency is given.

FIND: Determine the temperature at the turbine exit.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The turbine is well insulated and at steady state. (2) Kinetic and potential energy effects are negligible. (3) N_2 is modeled as an ideal gas, with constant cp.

ANALYSIS: The isentropic turbine efficiency is

$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{(\dot{W}_{cv}/\dot{m})_s}$$

With the specified assumptions, mass and energy rate balances allow this to be expressed as

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{\cancel{c_p}[T_1 - T_2]}{\cancel{c_p}[T_1 - T_{2s}]} \qquad (1)$$

where T_{2s} can be determined using Eq. 6.45

$$T_{2s} = T_1 \left[\frac{P_2}{P_1}\right]^{(k-1)/k} \qquad (2)$$

As the N_2 expands through the turbine its temperature decreases -- see T-s diagram. Table A-20 shows that for $T \leq 450K$, the value of k is closely 1.4. Thus, Eq. (2) gives

$$T_{2s} = 443K \left[\frac{1}{3.8}\right]^{0.4/1.4} = 302.5K$$

Then, Eq. (1) gives on rearrangement

$$T_2 = T_1 - \eta_t (T_1 - T_{2s}) = 443K - 0.832(443 - 302.5)K = 326.1K$$

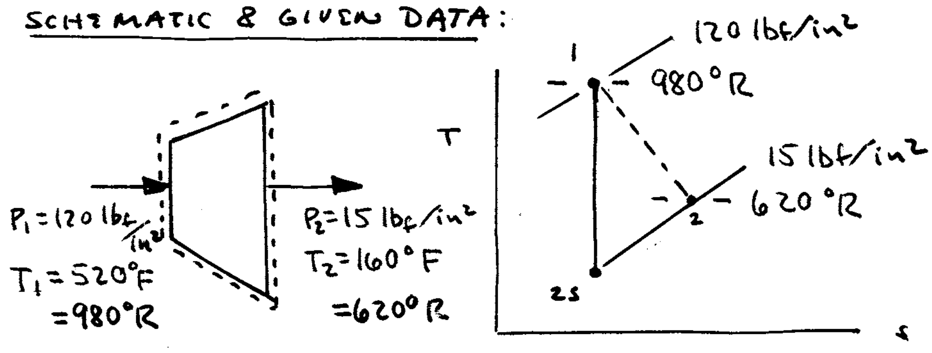
$$\Rightarrow T_2 = 53°C \longleftarrow T_2$$

PROBLEM 6.144*

KNOWN: Air at a specified temperature and pressure enters an insulated turbine operating at steady state and expands to a specified temperature and pressure.

FIND: Determine the work developed per unit mass flowing and the isentropic turbine efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The turbine is insulated and operates at steady state. 2. Kinetic and potential energy effects are negligible. 3. Air is modeled as an ideal gas.

ANALYSIS: Table A-22E gives $h_1 = 236.02$ Btu/lb $p_{r1} = 11.43$, and $h_2 = 148.28$ Btu/lb. To find h_{2s} use

$$\frac{p_{r2}}{p_{r1}} = \frac{p_2}{p_1} \Rightarrow p_{r2} = p_{r1}\left(\frac{p_2}{p_1}\right) = 11.43\left(\frac{15}{120}\right) = 1.429$$

Then, interpolating in Table A-22E with p_{r2}, $h_{2s} = 130.15$ Btu/lb

The work developed per unit of mass flowing is

$$\frac{\dot{W}_t}{\dot{m}} = h_1 - h_2 = 236.02 - 148.28 = 87.74 \text{ Btu/lb} \quad \longleftarrow \dot{W}_t/\dot{m}$$

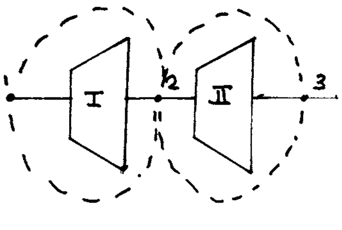
The isentropic turbine efficiency is

$$\eta_t = \frac{\dot{W}_t/\dot{m}}{h_1 - h_{2s}} = \frac{87.74}{236.02 - 130.15} = 0.829 \; (82.9\%) \quad \longleftarrow \eta_t$$

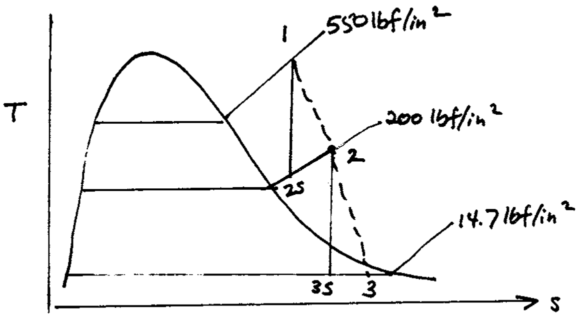
PROBLEM 6.145

KNOWN: Steady state operating data are provided for a well-insulated steam turbine having two stages in series.

FIND: Show the principal states on a T-s diagram. Determine the state at the exit of the second stage. Determine the work developed by each stage, per unit of mass of steam flowing.



$T_1 = 700°F$
$P_1 = 550\ \text{lbf/in}^2$
$P_2 = 200\ \text{lbf/in}^2$
$P_3 = 14.7\ \text{lbf/in}^2$
$\eta_I = 88\%$
$\eta_{II} = 92\%$



ASSUMPTIONS: (1) Each control volume shown in the figure is at steady state. (2) For each control volume, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects can be ignored.

ANALYSIS: Steam table data give $h_1 = 1353.7$ Btu/lb, $s_1 = 1.5987$ Btu/lb·°R. Interpolating at 200 lbf/in² with $s_{2s} = s_1$, $h_{2s} = 1245.2$ Btu/lb. Then, using the isentropic turbine efficiency of the first stage

$$\eta_I = 0.88 = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - 0.88(h_1 - h_{2s}) = 1353.7 - 0.88(1353.7 - 1245.2) = 1258.2\ \frac{\text{Btu}}{\text{lb}}$$

Interpolating at 200 lbf/in² with h_2 gives $s_2 = 1.6127$ Btu/lb·°R. Using $s_{3s} = s_2$,

$$x_{3s} = \frac{s_{3s} - s_f}{s_g - s_f} = \frac{1.6127 - 0.31212}{1.4446} = 0.9, \quad h_{3s} = h_f + x_{3s} h_{fg} = 180.15 + 0.9(970.4) = 1053.5\ \text{Btu/lb}.$$

Then, using the isentropic turbine efficiency of the second stage

$$\eta_{II} = 0.92 = \frac{h_2 - h_3}{h_2 - h_{3s}} \Rightarrow h_3 = h_2 - 0.92(h_2 - h_{3s}) = 1258.2 - 0.92(1258.2 - 1053.5) = 1069.9\ \frac{\text{Btu}}{\text{lb}}$$

Using h_3

$$x_3 = \frac{h_3 - h_f}{h_{fg}} = \frac{1069.9 - 180.15}{970.4} = 0.917 \quad (91.7\%)$$

The state at the exit of Stage I is fixed by two of P_2, h_2, s_2. The state at the exit of Stage II is fixed by two of P_3, h_3, x_3. ← States 2,3

The work developed for each of the stages is obtained by reducing mass and energy rate balances to give

$$(\dot{W}_{cv}/\dot{m})_I = h_1 - h_2 = 1353.7 - 1258.2 = 95.5\ \text{Btu/lb}$$ ← $(\dot{W}_{cv}/\dot{m})$

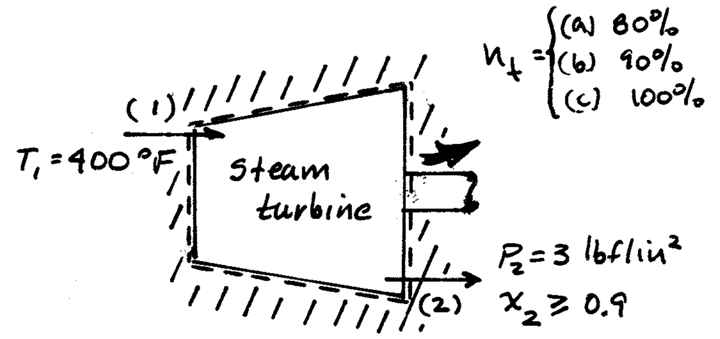
$$(\dot{W}_{cv}/\dot{m})_{II} = h_2 - h_3 = 1258.2 - 1069.9 = 188.3\ \text{Btu/lb}.$$

PROBLEM 6.146

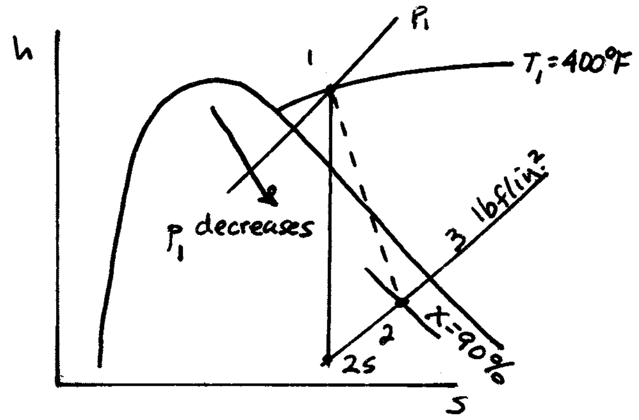
KNOWN: Steam at a known temperature enters an insulated turbine and expands to a known pressure. The isentropic efficiency is specified.

FIND: Determine the range of turbine inlet pressures that insures a turbine exit steam quality of at least 90% for isentropic turbine efficiencies of (a) 80%, (b) 90%, and (c) 100%.

SCHEMATIC & GIVEN DATA:



$\eta_t = \begin{cases} (a)\ 80\% \\ (b)\ 90\% \\ (c)\ 100\% \end{cases}$
(1)
$T_1 = 400°F$
Steam turbine
$p_2 = 3\ \text{lbf/in}^2$
$x_2 \geq 0.9$
(2)



ASSUMPTIONS: (1) The turbine is at steady state, and $\dot{Q}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible.

ANALYSIS: As shown on the h-s diagram, p_1 decreases as s_2 increases. Thus, the maximum allowed turbine inlet pressure corresponds to the case where $x_2 = 0.9$. For any pressure less than this, $x_2 > 0.9$.

Thus, with $x_2 = 0.9$ and data for h_f and h_{fg} at 3 lbf/in², h_2 is found from

$$h_2 = h_f + 0.9\, h_{fg} \tag{1}$$

and

$$h_{2s} = h_f + x_{2s}\, h_{fg} \tag{2}$$

thus

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{h_1 - h_f - 0.9\, h_{fg}}{h_1 - h_f - x_{2s}\, h_{fg}} \tag{3}$$

For a given value of p_1, there is a corresponding value of h_1 and x_{2s}. Thus, for that p_1 there is a value of η_t that can be calculated from (3). Using IT, an iterative numerical process is used to determine the unique combination of p_1 and x_{2s} and h_1 values to give a desired value of η_t:

IT Code

```
T1 = 400  // °F
p2 = 3  // lbf/in.²
x2 = 0.9

h2 = hsat_Px("Water/Steam", p2, x2)
h2s = hsat_Px("Water/Steam", p2, x2s)

eta = 80
h1 - h2 = (eta/100) * (h1 - h2s)

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = ssat_Px("Water/Steam", p2, x2s)
s1 = s2
```

IT Results

(a) η = 80%; $(p_1)_{max}$ = 116.1 lbf/in.² ← (a)

(b) η = 90%; $(p_1)_{max}$ = 80.04 lbf/in.² ← (b)

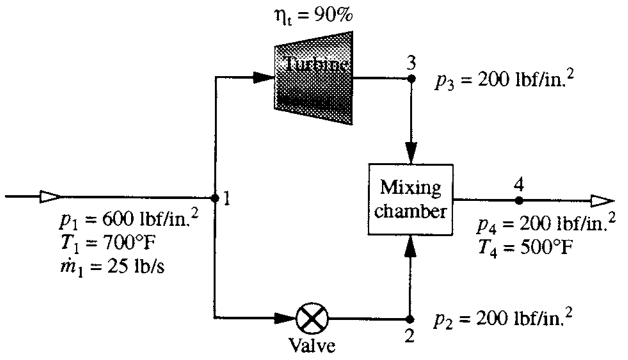
(c) η = 100%; $(p_1)_{max}$ = 57.86 lbf/in.² ← (c)

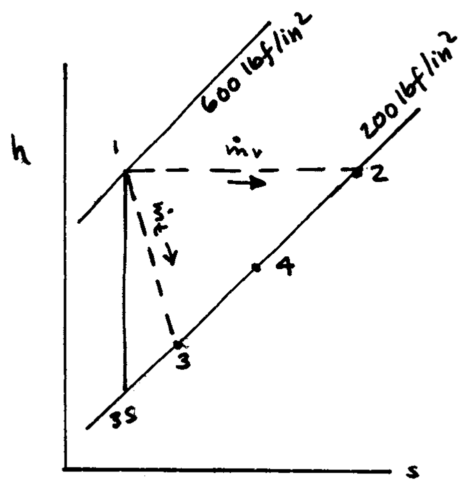
PROBLEM 6.147

KNOWN: Steady-state operating data are provided for a throttling valve in parallel with a steam turbine.

FIND: Determine the power developed by the turbine and the mass flow rate through the valve. Determine the rates of entropy production for the turbine, valve, and mixing chamber. Locate key states on an h-s diagram.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. Control volumes enclosing the turbine, valve, and mixing chamber are at steady state. For each, $\dot{Q}_{cv} = 0$. 2. The expansion across the valve is a throttling process. 3. Kinetic and potential energy effects can be ignored.

ANALYSIS: (a) Considering an overall control volume at steady state including the turbine, valve, and mixing chamber, mass and energy rate balances reduce to give $\dot{W}_t = \dot{m}[h_1 - h_4]$. With data from Table A-4E

$$\dot{W}_t = \left(25\ \frac{lb}{s}\right)[1350.6 - 1268.8]\ \frac{Btu}{lb}\left|\frac{3600\ s}{1\ h}\right|\left|\frac{1\ hp}{2545\ Btu/h}\right| = 2893\ hp \longleftarrow (a)$$

(b) Then mass and energy rate balances for a control volume enclosing only the turbine give $\dot{W}_t = \dot{m}_t[h_1 - h_3]$. Using the isentropic turbine efficiency: $\eta_t = (h_1 - h_3)/(h_1 - h_{3s}) \Rightarrow (h_1 - h_3) = \eta_t(h_1 - h_{3s})$. So

$$\dot{W}_t = \dot{m}_t \eta_t (h_1 - h_{3s}) \Rightarrow \dot{m}_t = \frac{\dot{W}_t}{\eta_t(h_1 - h_{3s})}$$

. With $s_1 = s_{3s} = 1.5872$ Btu/lb·°R,

Table A-4E gives $h_{2s} = 1234.86$ Btu/lb. Then

$$\dot{m}_t = \frac{(25)[1350.6 - 1268.8]}{(0.9)[1350.6 - 1234.86]} = 19.63\ lb/s$$

The mass flow rate through the valve is then $\dot{m} = \dot{m}_t + \dot{m}_v$ or $\longleftarrow$ (b)
$\dot{m}_v = \dot{m} - \dot{m}_t = 25 - 19.63 = 5.37$ lb/s

(c) Mass and entropy rate balances for the valve give

$$\dot{\sigma}_v = \dot{m}_v[s_2 - s_1]$$

Since $h_2 \sim h_1$, state 2 is fixed by $h_2 = 1350.6$ Btu/lb, $P_2 = 200$ lbf/in^2, giving $s_2 = 1.7024$ Btu/lb·°R. Also, $s_1 = 1.5872$ Btu/lb·°R. So

$$\dot{\sigma}_v = \left(5.37\ \frac{lb}{s}\right)[1.7024 - 1.5872]\ \frac{Btu}{lb\cdot°R} = 0.6186\ \frac{Btu}{s\cdot°R} \longleftarrow \dot{\sigma}_{valve}$$

PROBLEM 6.147 (Contd.)

Mass and entropy rate balances for the turbine give

$$\dot{\sigma}_t = \dot{m}_t [s_3 - s_1]$$

To find s_3, use the isentropic turbine efficiency to get

$$\eta_t = \frac{h_1 - h_3}{h_1 - h_{3s}} \Rightarrow h_3 = h_1 - \eta_t (h_1 - h_{3s}) = 1350.6 - .9(1350.6 - 1234.86)$$
$$= 1246.4 \text{ Btu/lb}$$

Interpolating at 200 lbf/in² in Table A-4E, $s_3 = 1.6$ Btu/lb·°R. Thus

$$\dot{\sigma}_t = 19.63[1.6 - 1.5872] = 0.2513 \frac{\text{Btu}}{\text{s}\cdot°\text{R}} \longleftarrow \dot{\sigma}_{turb}$$

Mass and entropy rate balances for the mixing chamber give

$$\dot{\sigma}_{mix} = \dot{m}_4 s_4 - \dot{m}_t s_3 - \dot{m}_v s_2$$, where $s_4 = 1.6239 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$ from Table A-4E.

Thus

$$\dot{\sigma}_{mix} = (25)(1.6239) - (19.63)(1.6) - (5.37)(1.7024)$$

① $$= 0.0476 \frac{\text{Btu}}{\text{s}\cdot°\text{R}} \longleftarrow \dot{\sigma}_{mix}$$

---

1. The total rate of entropy production: $\dot{\sigma}_v + \dot{\sigma}_v + \dot{\sigma}_{mix} = 0.9175$ Btu/s·°R. Alternatively, Mass and entropy rate balances for an overall control volume reduce to

$$\dot{\sigma}_{overall} = \dot{m}[s_4 - s_1] = 25[1.6239 - 1.5872] = 0.9175 \text{ Btu/s}\cdot°\text{R}$$

which is in agreement with the above-calculated sum, as expected.

Also, note that an energy balance on the mixing chamber reduces to $\dot{m}_2 h_4 = \dot{m}_3 h_3 + \dot{m}_2 h_2$. Inserting previously determined values, the equation is satisfied, providing a check on the data.

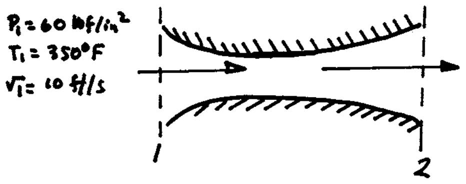
PROBLEM 6.148

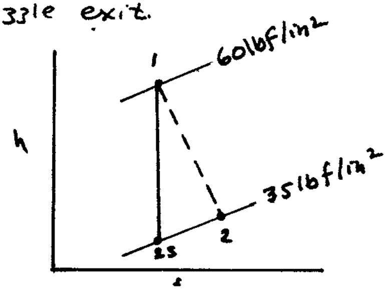
KNOWN: Steam at 60 lbf/in², 350°F, 10 ft/s enters an insulated nozzle at steady state and exits at 35 lbf/in². The isentropic nozzle efficiency is 94%.

FIND: Determine the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:

$\eta_{nozzle} = 94\%$





ASSUMPTIONS: (1) The nozzle is insulated and at steady state. (2) Potential energy effects are negligible.

ANALYSIS: By Eq. 6.49, the isentropic nozzle efficiency is

$$\eta_{nozzle} = \frac{V_2^2/2}{V_{2s}^2/2} \qquad (1)$$

where $V_{2s}^2/2$ is the exit kinetic energy per unit of mass for an isentropic expansion from the given inlet state to the specified exit pressure.

An energy rate balance for the nozzle in which the expansion is isentropic reduces at steady state to give

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}\left(h_1 - h_{2s} + \frac{V_1^2 - V_{2s}^2}{2} + \overset{0}{\cancel{g(z_1 - z_2)}}\right)$$

Thus

$$\frac{V_{2s}^2}{2} = \frac{V_1^2}{2} + h_1 - h_{2s} \qquad (2)$$

From Table A-4E $h_1 = 1208.2$ Btu/lb, $s_1 = 1.6830$ Btu/lb·°R. And with $s_{2s} = s_1$ Table A-3E gives

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.6830 - 0.3809}{1.6873 - 0.3809} = 0.9967$$

Accordingly

$$h_{2s} = h_f + x_{2s} h_{fg} = 228.04 + 0.9967(939.3) = 1164.24 \text{ Btu/lb}$$

Substituting values into Eq. (2)

$$\frac{V_{2s}^2}{2} = \frac{(10\text{ ft/s})^2}{2} + (1208.2 - 1164.24)\left(\frac{\text{Btu}}{\text{lb}}\right)\frac{778 \text{ ft·lbf}}{\text{Btu}}\;\frac{32.2 \text{ lb·ft/s}^2}{\text{lbf}}$$

$$= 1101318 (\text{ft/s})^2 \Rightarrow V_{2s} = 1484 \text{ ft/s}$$

Returning to Eq. (1)

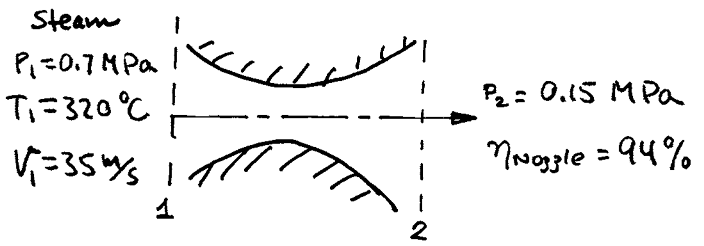
$$V_2 = \sqrt{\eta_{nozzle}}\,V_{2s} = \sqrt{0.94}\,(1484 \text{ ft/s}) = 1439 \text{ ft/s} \longleftarrow$$

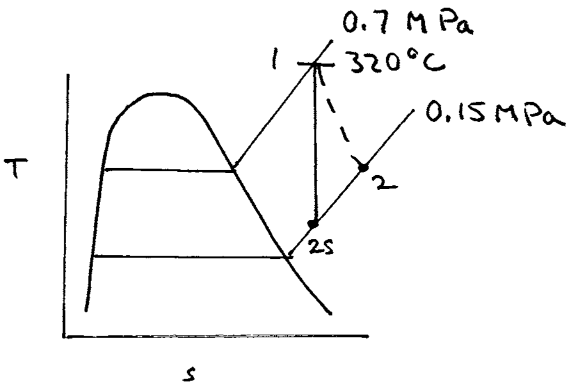
PROBLEM 6.149*

KNOWN: Steady state operating data are provided for an insulated nozzle.

FIND: Determine the exit velocity.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, and potential energy effects are negligible.

ANALYSIS: The isentropic nozzle efficiency, Eq. 6.49, yields

$$V_2 = \sqrt{\eta_{Nozzle}}\ V_{2s} \qquad (1)$$

where V_{2s} is the exit velocity for an isentropic expansion. Considering the isentropic expansion, reduction of mass and energy balances gives

$$\frac{V_{2s}^2}{2} = \frac{V_1^2}{2} + h_1 - h_{2s} \qquad (2)$$

With data from Table A-4, $h_1 = 3100.9$ kJ/kg, $s_1 = 7.3697$ kJ/kg·K. Then, interpolating at 0.15 MPa with $s_{2s} = s_1$, $h_{2s} = 2752.8$ kJ/kg.

Substituting values in Eq. (2)

$$\frac{V_{2s}^2}{2} = \frac{(35\ m/s)^2}{2} + (3100.9 - 2752.8)\frac{kJ}{kg}\left|\frac{1000\ N\ m}{kJ}\right|\left|\frac{1\ kg\ m}{1\ N\ s^2}\right|$$

giving

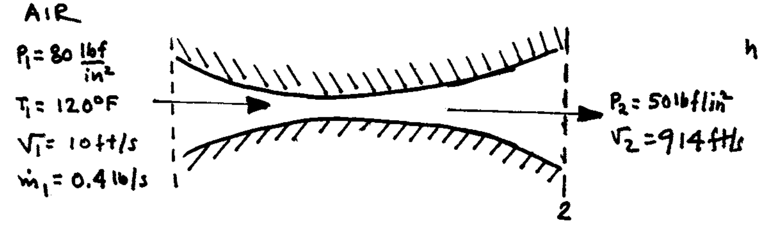
$$V_{2s} = 835.1\ m/s \Rightarrow V_2 = \sqrt{0.94}\,(835.1) = 809.7\ m/s \longleftarrow V_2$$

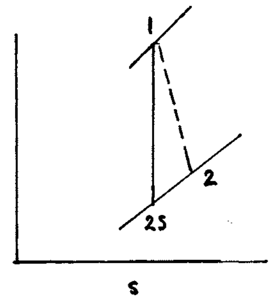
PROBLEM 6.150

KNOWN: Air enters an insulated nozzle at steady state at a specified state and mass flow rate and exits at a specified pressure with a known velocity.

FIND: Determine the exit area and the isentropic nozzle efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The nozzle is insulated and at steady state. (2) Potential energy effects are negligible. (3) Air is modeled as an ideal gas.

ANALYSIS: The isentropic nozzle efficiency is given by

$$\eta_{nozzle} = \frac{V_2^2}{V_{2s}^2} \qquad (1)$$

PROBLEM 6.150 (Continued)

where V_{2s} is the exit velocity for an isentropic expansion from the specified inlet state to the specified exit pressure. The velocity V_{2s} can be found from an energy rate balance a steady state

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left(h_1 - h_{2s} + \frac{V_1^2 - V_{2s}^2}{2} + \cancel{g(z_1 - z_2)}^{0}\right) \qquad (2)$$

Thus

$$V_{2s} = \sqrt{V_1^2 + 2(h_1 - h_{2s})}$$

From Table A-22E, $h_1 = 138.66$ Btu/lb, $p_{r1} = 1.78$. Using Eq. 6.43

$$\frac{p_{2s}}{p_1} = \frac{p_r(T_{2s})}{p_r(T_1)} \Rightarrow p_r(T_{2s}) = \left(\frac{50}{80}\right)(1.78) = 1.1125$$

Returning to Table A-13E, the corresponding value for h_{2s} is $h_{2s} = 121.13$ Btu/lb. Thus

$$V_{2s} = \sqrt{(10\,\text{ft/s})^2 + 2(138.66 - 121.13)(\text{Btu/lb})\left(778\,\frac{\text{ft}\cdot\text{lbf}}{\text{Btu}}\right)\left(32.2\,\frac{\text{lb}\cdot\text{ft/s}^2}{\text{lbf}}\right)}$$

$$= 937.2\ \text{ft/s}$$

Finally, Eq. (1) gives $\eta_{nozzle} = \dfrac{(914)^2/2}{(937.2)^2/2} = 0.951\ (95.1\%) \longleftarrow \eta_{nozzle}$

The exit area can be evaluated from the mass flow rate using the ideal gas equation of state: $\dot{m} = A_2 V_2 / v_2$ where $v_2 = RT_2/p_2$. The exit temperature, T_2, can be evaluated from an energy rate balance:

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}^{0}\right) \qquad (3)$$

Thus

① $$h_2 = h_1 + \frac{V_1^2 - V_2^2}{2} = 138.66 + \frac{(10)^2 - (914)^2}{2(32.2)(778)} = 121.99\ \text{Btu/lb}$$

Interpolating in Table A-22E, $T_2 = 510.5°R$. Accordingly

$$A_2 = \frac{\dot{m} v_2}{V_2} = \frac{\dot{m} R T_2}{p_2 V_2} = \frac{(0.4\,\text{lb/s})\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(510.5°\text{R})}{((50)(144)\,\text{lbf/ft}^2)(913.5\,\text{ft/s})} = 1.66 \times 10^{-3}\ \text{ft}^2 \longleftarrow A_2$$

---

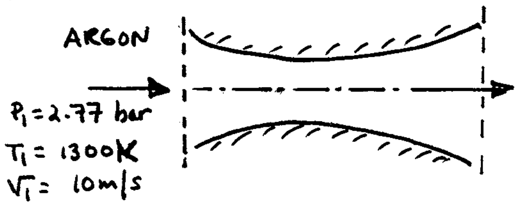
1. The energy rate balances Eqs.(2) and (3) are for the isentropic and actual expansions, respectively.

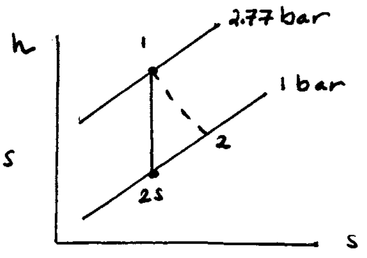
PROBLEM 6.151

KNOWN: Argon enters an insulated nozzle operating at steady state at a specified pressure, temperature, and velocity and expands to a known pressure and velocity.

FIND: Determine the exit temperature, the isentropic nozzle efficiency, and the rate of entropy production per unit mass flowing.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. A control volume enclosing the nozzle is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$ and potential energy effects ① can be ignored. 3. Argon is modeled as an ideal gas.

PROBLEM 6.151 (Contd.)

ANALYSIS: (a) Mass and energy rate balances reduce to give

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right] \Rightarrow 0 = h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}$$

Or since c_p is constant for Argon as an ideal gas: $c_p = 5/2\ R$ (Table A-21)

$$0 = c_p[T_1 - T_2] + \frac{V_1^2 - V_2^2}{2} \Rightarrow T_2 = T_1 + \frac{V_1^2 - V_2^2}{2c_p} = (1300\,\text{K}) + \frac{[(10)^2 - (645)^2]\ \text{m}^2/\text{s}^2}{2\left(\frac{5}{2}\right)\left(\frac{8314}{39.94}\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|$$

$$= 900\,\text{K} \longleftarrow T_2$$

(b) The isentropic nozzle efficiency is given by Eq. 6.49. To find V_{2s} requires the temperature at state 2s. Since $c_p = 5/2\ R$, $k = 1.666$ (Eq. 3.47a). Equation 6.45 gives

$$T_{2s} = T_1\left[\frac{P_2}{P_1}\right]^{(k-1)/k} = 1300\left[\frac{1}{2.77}\right]^{\frac{0.666}{1.666}} \Rightarrow T_{2s} = 865\ \text{K}.$$

Then, applying mass and energy rate balances to the isentropic expansion, we get

$$0 = c_p[T_1 - T_{2s}] + \frac{V_1^2 - V_{2s}^2}{2} \Rightarrow \frac{V_{2s}^2}{2} = \frac{V_1^2}{2} + c_p[T_1 - T_{2s}]$$

or

$$\frac{V_{2s}^2}{2} = \frac{(10\ \text{m/s})^2}{2} + \frac{5}{2}\left[\frac{8314}{39.94}\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right][1300 - 865]\text{K}\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|$$

$$= 226{,}426\,(\text{m/s})^2 \qquad (V_{2s} = 672.9\ \text{m/s})$$

Then

$$\eta_{\text{nozzle}} = \frac{(645)^2/2}{226{,}426} = 0.919 \quad (91.9\%) \qquad \longleftarrow \eta_{\text{Nozzle}}$$

(c) Reducing mass and entropy rate balances gives with Eq. 6.23

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}} = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}$$

with $c_p = 5/2\ R$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = R\left[\frac{5}{2}\ln\frac{T_2}{T_1} - \ln\frac{P_2}{P_1}\right] = \left(\frac{8.314}{39.94}\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left[\frac{5}{2}\ln\frac{900}{1300} - \ln\frac{1}{2.77}\right]$$

$$= 0.0207\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \qquad \longleftarrow \dot{\sigma}_{cv}/\dot{m}$$

---

1. Checking the generalized compressibility chart with

$$P_{R1} = \frac{P_1}{P_c} = \frac{2.77\ \text{bar}}{48.6\ \text{bar}} = 0.057 \qquad P_{R2} = \frac{P_2}{P_c} = \frac{1}{48.6} = 0.021$$

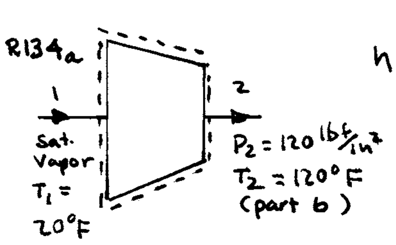
the ideal gas model is acceptable over a wide range of temperatures, including those here.

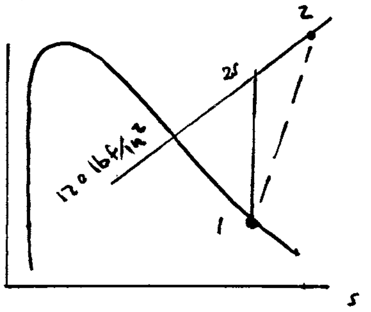
PROBLEM 6.152*

KNOWN: R134a enters a compressor operating adiabatically at steady state as a saturated vapor and exits at a known pressure.

FIND: (a) Determine the minimum theoretical work input per unit mass flowing and the corresponding exit temperature. (b) For a specified exit temperature, determine the isentropic compressor efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume shown in the schematic operates at steady state with no appreciable heat transfer with the surroundings.
2. Kinetic and potential energy effects can be ignored.

ANALYSIS: (a) The mass, energy, and entropy rate balances reduce to give

$$\left(\frac{-\dot{W}_{cv}}{\dot{m}}\right) = h_2 - h_1 \,, \quad \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1$$

The work input decreases as h_2 decreases. The entropy balance and h-s diagram show that the smallest allowed value for h_2 corresponds to state 2s, where $s_{2s} = s_1$. From Table A-10E, at 20°F, $h_1 = 104.61$ Btu/lb, $s_1 = 0.2205$ Btu/lb·°R. Then, interpolation in Table A-12E at 120 lbf/in² with $s_{2s} = s_1$ gives, $T_{2s} = 98.9°\text{F}$ ← and $h_{2s} = 116.04$ Btu/lb. Accordingly

$$\left(\frac{-\dot{W}_{cv}}{\dot{m}}\right)_{MIN} = h_{2s} - h_1 = (116.04 - 104.61) = 11.43\ \text{Btu/lb} \quad \longleftarrow$$

(b) The isentropic compressor efficiency is given by Eq. 6.50, which reduces to

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{11.43}{121.52 - 104.61} = \frac{11.43}{16.91} = 0.676\ (67.6\%) \quad \longleftarrow$$

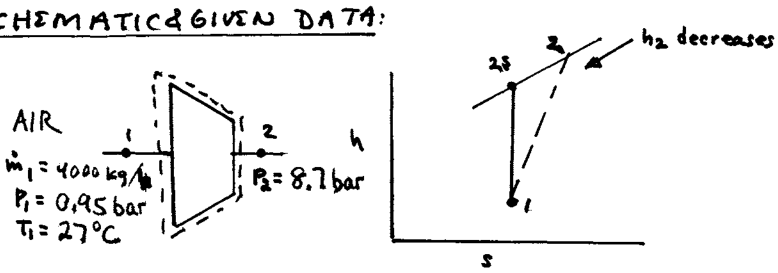
where h_2 is from Table A-12E at 120 lbf/in², 120°F.

PROBLEM 6.153*

KNOWN: Air enters an insulated compressor operating at steady state at a specified state and mass flow rate and exits at a known pressure.

FIND: (a) Determine the minimum theoretical power input and the corresponding exit temperature. (b) For a specified exit temperature, determine the power input and the isentropic compressor efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The compressor operates at steady state and $\dot{Q}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) Air is modeled as an ① ideal gas.

ANALYSIS: (a) At steady state $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and an energy rate balance reduces to

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right) = h_2 - h_1 \qquad (1)$$

An entropy rate balance at steady state gives, $s_2 - s_1 = \dot{\sigma}_{cv}/\dot{m} \geq 0$. The work input decreases as h_2 decreases. The h-s diagram and the entropy balance shows that the smallest allowed value for h_2 corresponds to state 2s, where $s_1 = s_{2s}$. That is

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right)_{min} = h_{2s} - h_1$$

To find h_{2s} use p_r data from Table A-22 and

$$\frac{p_r(T_{2s})}{p_r(T_1)} = \frac{p_{2s}}{p_1} \Rightarrow p_r(T_{2s}) = p_r(T_1)\left(\frac{p_{2s}}{p_1}\right) = 1.3860\left(\frac{8.7}{0.95}\right) = 12.69$$

Interpolating in Table A-22, $T_{2s} = 560.4$ K (287.4°C), $h_{2s} = 565.54$ kJ/kg. Thus, with h_1 from Table A-22 ← T_{2s}

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right)_{min} = 565.54 - 300.19 = 265.35 \text{ kJ/kg}$$

$$\Rightarrow (-\dot{W}_{cv})_{min} = 294.8 \text{ kW} \longleftarrow (-\dot{W}_{cv})_{min}$$

(b) $T_2 = 620$ K (347°C). With $h_2 = 628.07$ kJ/kg from Table A-22, Eq. (1) gives

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right) = h_2 - h_1 = 628.07 - 300.19 = 327.88 \text{ kJ/kg}$$

$$\Rightarrow (-\dot{W}_{cv}) = 364.3 \text{ kW} \longleftarrow (-\dot{W}_{cv})$$

With Eq. 6.50

$$\eta_c = \frac{294.8}{364.3} = 0.809 \; (80.9\%) \longleftarrow \eta_c$$

---

1. Checking the compressibility chart with

$$p_{r1} = \frac{0.95 \text{ bar}}{37.7 \text{ bar}} = 0.025, \quad p_{r2} = \frac{8.7}{37.7} = 0.231$$

indicates that the ideal gas model is valid over a wide range of temperature including those here.

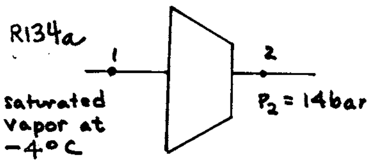
PROBLEM 6.154

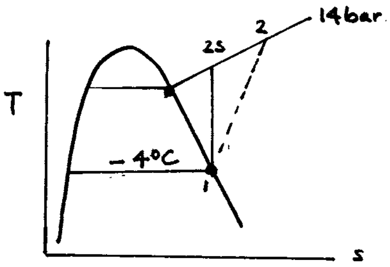
KNOWN: R134a enters a compressor at steady state at a specified state and exits at a specified pressure. The isentropic compressor efficiency is known.

FIND: Determine the exit temperature and the work input per unit mass of R134a flowing.

SCHEMATIC & GIVEN DATA:

$\eta_c = 0.75$





ASSUMPTIONS: (1) The compressor operates at steady state. (2) Heat transfer between the compressor and the surroundings can be ignored as can changes in kinetic and potential energy from inlet to exit.

ANALYSIS: At steady state $\dot m_1 = \dot m_2 = \dot m$ and an energy rate balance reduces to

$$\frac{-\dot W_{cv}}{\dot m} = h_2 - h_1 \qquad (1)$$

Similarly, for an isentropic compression

$$\left(\frac{-\dot W_{cv}}{\dot m}\right)_s = h_{2s} - h_1 \qquad (2)$$

State 2s is fixed by $P_2 = 14$ bar and $s_{2s} = s_1$. From Table A-10, at −4°C, $s_1 = 0.9213$ kJ/kg·K, $h_1 = 244.9$ kJ/kg. Interpolation in Table A-12 at 14 bar, $h_{2s} = 280.33$ kJ/kg. Thus

$$\left(\frac{-\dot W_{cv}}{\dot m}\right)_s = 280.33 - 244.9 = 35.43 \ \frac{\text{kJ}}{\text{kg}}$$

The isentropic compressor efficiency is given by Eq. 6.50. Thus

$$\left(\frac{-\dot W_{cv}}{\dot m}\right) = \frac{(-\dot W_{cv}/\dot m)_s}{\eta_c} = \left(\frac{35.43 \text{ kJ/kg}}{0.75}\right) = 47.24 \ \frac{\text{kJ}}{\text{kg}} \longleftarrow \frac{(-\dot W_{cv}/\dot m)}{}$$

Equation (1) allows h_2 to be evaluated:

$$h_2 = h_1 + (-\dot W_{cv}/\dot m) = 244.9 + 47.24 = 292.1 \text{ kJ/kg}$$

Interpolating in Table A-12 at 14 bar with h_2, $T_2 = 67.4$ °C $\longleftarrow \frac{T_2}{}$

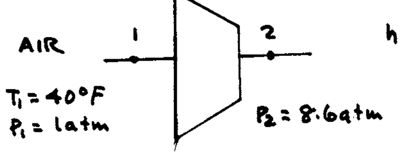
PROBLEM 6.155

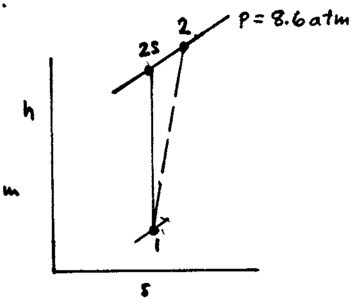
KNOWN: Steady-state operating data are provided for a compressor.

FIND: Determine the temperature of the air at the compressor exit and the rate of entropy production per unit of mass flowing.

SCHEMATIC & GIVEN DATA:

$\eta_c = 71.9\%$





ASSUMPTIONS: (1) The compressor operates at steady state. (2) Heat transfer between the compressor and the surroundings can be ignored as can changes in kinetic and potential energy from inlet to exit. (3) Air can be modeled as an
① ideal gas.

PROBLEM 6.155 (Contd.)

ANALYSIS: At steady state mass and energy rate balances give for the actual and isentropic compressions, respectively

$$\left(-\frac{\dot{W}_{cv}}{\dot{m}}\right) = h_2 - h_1 \quad , \quad \left(-\frac{\dot{W}_{cv}}{\dot{m}}\right)_s = h_{2s} - h_1$$

The isentropic compressor efficiency is given by Eq. 6.50:

$$\eta_c = \frac{(-\dot{W}_{cv}/\dot{m})_s}{(-\dot{W}_{cv}/\dot{m})} = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s}-h_1)}{\eta_c} \qquad (1)$$

From Table A-22E at 500°R, $h_1 = 119.48$ Btu/lb. Then, with Eq. 6.43

$(P_r)_{2s} = P_{r1}\left[\frac{P_2}{P_1}\right] = 1.0590\left[\frac{8.6}{1}\right] = 9.1074 \Rightarrow h_{2s} = 221.22$. Eq. (1) gives

$$h_2 = 119.48 + \frac{(221.22-119.48)}{0.719} = 260.98\ \frac{\text{Btu}}{\text{lb}} \Rightarrow T_2 = 1080°R\ (620°F) \longleftarrow T_2$$

Mass and entropy rate balances reduce to give with Eq. 6.21a

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = s°(T_2) - s°(T_1) - R \ln P_2/P_1 = 0.76964 - 0.58233 - \frac{1.986}{28.97}\ln\frac{8.6}{1}$$

$$= 0.0398\ \text{Btu/lb}\cdot°R \qquad \longleftarrow \dot{\sigma}_{cv}/\dot{m}$$

---

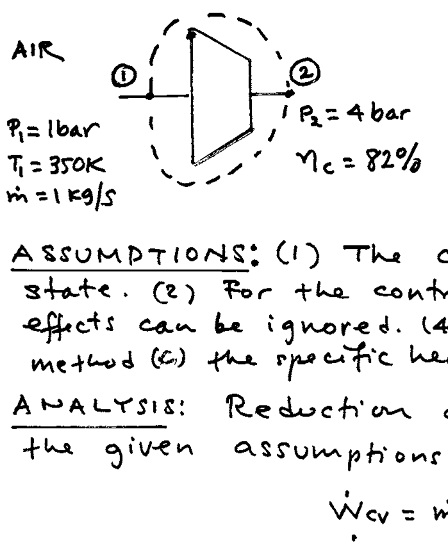
1. The validity of the ideal gas model can be checked using generalized compressibility data.

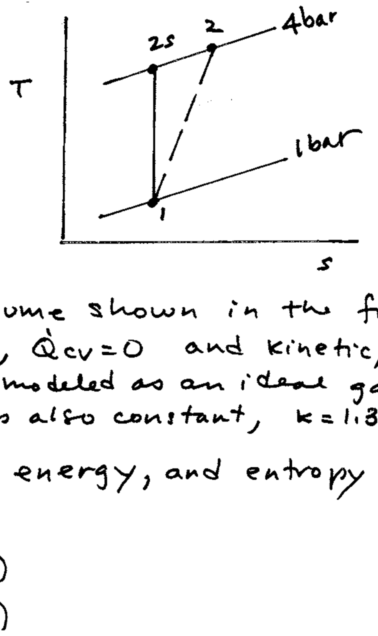
PROBLEM 6.156

KNOWN: Steady state operating data are provided for an air compressor.

FIND: Determine the power input and rate of entropy production using (a) Table A-22, (b) IT, (c) constant $k = 1.39$.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects can be ignored. (4) Air is modeled as an ideal gas. For solution method (c) the specific heat ratio is also constant, $k = 1.39$.

ANALYSIS: Reduction of mass, energy, and entropy balances with the given assumptions yields

$$\dot{W}_{cv} = \dot{m}\,(h_1 - h_2) \qquad (1)$$

$$\dot{\sigma}_{cv} = \dot{m}\,(s_2 - s_1) \qquad (2)$$

(a) Air Table Analysis. Consider process 1-2s. With data from Table A-22 $h_1 = 350.49$ kJ/kg, $s°(T_1) = 1.85708$ kJ/kg·K, and

$$P_{r,2s} = \frac{P_2}{P_1} P_{r,1} = \left(\frac{4\ \text{bars}}{1\ \text{bar}}\right)(2.379) = 9.516$$

Interpolation in Table A-22 gives $h_{2s} = 520.98$ kJ/kg. Then, using the isentropic compressor efficiency

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s}-h_1}{\eta_c} = 350.49 + \frac{(520.98-350.49)}{0.82} = 558.4\ \text{kJ/kg}$$

PROBLEM 6.156 (Continued)

Then, interpolating in Table A-22 with h_2 gives $s^\circ(T_2) = 2.3247$ kJ/kg·K.

Substituting values in Eqs. (1), (2)

$$\dot{W}_{cv} = \dot{m}\,(h_1 - h_2) = \left(1\,\frac{kg}{s}\right)(350.49 - 558.4)\left(\frac{kJ}{kg}\right)\left|\frac{1\,kW}{1\,kJ/s}\right| = -207.9\,kW \quad ◄$$

$$\dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) = \dot{m}\left(s^\circ(T_2) - s^\circ(T_1) - R\ln p_2/p_1\right) = \left(1\,\frac{kg}{s}\right)\left(2.3247 - 1.8571 - \frac{8.314}{28.97}\ln 4\right)\left(\frac{kJ}{kg\cdot K}\right)$$

$$= 0.0698\,\frac{kJ/s}{K} = 0.0698\,\frac{kW}{K} \quad ◄$$

(b) *IT* Code

```
p1 = 1  // bar
T1 = 350  // K
mdot = 1  // kg/s
p2 = 4  // bar
eta = 0.82

Wdotcv = mdot * (h1 - h2)
sigmadot = mdot * (s2 - s1)

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)

h2 = h1 + (h2s - h1) / eta
h2s = h_T("Air", T2s)
s2s = s_TP("Air", T2s, p2)
s2s = s1

h2 = h_T("Air", T2)
s2 = s_TP("Air", T2, p2)
```

IT Results

$\eta = 0.82$
$T_2 = 553.5$ K
$T_{2s} = 517.4$ K
$h_1 = 350.3$ kJ/kg
$h_2 = 558.3$ kJ/kg
$h_{2s} = 520.9$ kJ/kg

$\dot{W}_{cv} = -208$ kW ◄
$\dot{\sigma}_{cv} = 0.06993$ kW/K ◄

① (c) Ideal gas, k = 1.39 Analysis. Applying Eq. 6.45 to process 1-2s

$$T_{2s} = T_1\left[\frac{p_2}{p_1}\right]^{\frac{k-1}{k}} = 350\,K\left[\frac{4\,bars}{1\,bar}\right]^{\frac{1.39-1}{1.39}} = 516.4\,K$$

When k is constant, c_p is constant, and so the isentropic compressor efficiency gives

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{\cancel{c_p}\,[T_{2s} - T_1]}{\cancel{c_p}\,[T_2 - T_1]} \Rightarrow T_2 = T_1 + \frac{(T_{2s} - T_1)}{\eta_c} \Rightarrow T_2 = 350 + \frac{(516.4 - 350)}{.82}$$

$$T_2 = 552.9\,K$$

With Eq. 3.47a

$$c_p = \frac{kR}{k-1} = \frac{1.39\,(8.314/28.97)\,kJ/kg\cdot K}{1.39 - 1} = 1.02\,kJ/kg\cdot K$$

Accordingly, Eqs. (1), (2) give

$$\dot{W}_{cv} = \dot{m}\,(h_1 - h_2) = \dot{m}\,c_p\,(T_1 - T_2) = \left(1\,\frac{kg}{s}\right)\left(1.02\,\frac{kJ}{kg\cdot K}\right)[350 - 552.9]\,K$$

$$= -207\,\frac{kJ}{s} = -207\,kW \quad ◄$$

$$\dot{\sigma}_{cv} = \dot{m}\,(s_2 - s_1) = \dot{m}\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}\right] = \left(1\,\frac{kg}{s}\right)\left[\left(1.02\,\frac{kJ}{kg\cdot K}\right)\ln\frac{552.9}{350} - \left(\frac{8.314}{28.97}\,\frac{kJ}{kg\cdot K}\right)\ln 4\right]$$

$$= 0.0685\,\frac{kJ/s}{K} = 0.0685\,\frac{kW}{K} \quad ◄$$

---

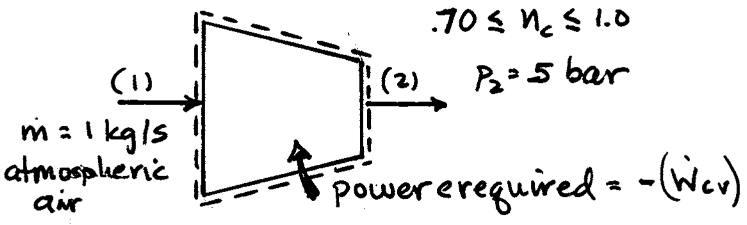
1. Inspection of Table A-20 indicates that k = 1.39 is a suitable constant value for the interval from T_1 to T_2. Thus, good agreement is realized between the results of this part and those of parts a) and b).

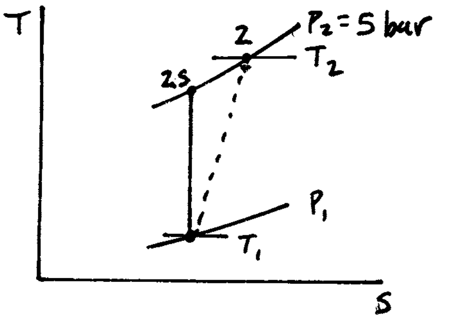
PROBLEM 6.157

KNOWN: A compressor operating at steady state takes in atmospheric air at a known mass flow rate and discharges air at a given pressure.

FIND: Plot the power required and the compressor exit temperature, each versus compressor isentropic efficiency ranging from 70% to 100%.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$. (3) Kinetic and potential energy changes from inlet to exit can be ignored. (4) Since the incoming air is atmospheric, we assume $p_1 = 1$ bar and $T_1 = 20°C$.

ANALYSIS: The mass and energy rate balances reduce to give

$$\text{power required} = -(\dot{W}_{cv}) = \dot{m}(h_2 - h_1) \qquad (1)$$

the enthalpy h_1 can be determined since p_1 and T_1 are known from assumption 4. The enthapy h_2 is found using compressor efficiency η_c, as follows:

$$h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} \qquad (2)$$

where h_{2s} corresponds to an isentropic compression from state 1 to p_2.

Sample Calculation

A sample calculation will be performed using data from Table A-22. From the table; $h_1 = 293.2$ kJ/kg and $p_{r_1} = 1.27652$. Then, from Eq. 6.43

$$p_r(T_{2s}) = 1.27652\left(\frac{5}{1}\right) = 6.3826$$

Interpolating in Table A-22; $h_{2s} = 464.8$ kJ/kg. Accordingly, for $\eta = 0.7$

$$h_2 = 293.2 + \frac{(464.8 - 293.2)}{0.7} = 538.3 \text{ kJ/kg}$$

and

$$\begin{pmatrix}\text{power}\\\text{required}\end{pmatrix} = \left(1 \frac{\text{kg}}{\text{s}}\right)(538.3 - 293.2)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 245.1 \text{ kW}$$

The software IT is used to generate data for the required plots, as follows:

PROBLEM 6.157 (Cont'd.)

IT Code

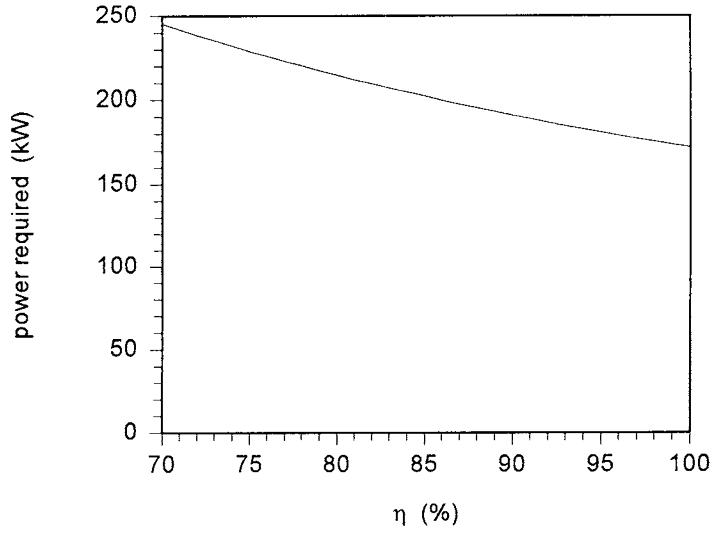
```
T1 = 20  // °C
p1 = 1  // bar
mdot = 1  // kg/s
p2 = 5  // bar
eta = 70  // %

Wdotcv = mdot * (h1 - h2)
power = - Wdotcv

h2 = h1 + (h2s - h1)*100 / eta
h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
h2s = h_T("Air", T2s)
s2s = s_TP("Air", T2s, p2)
s2s = s1
h2 = h_T("Air", T2)
```

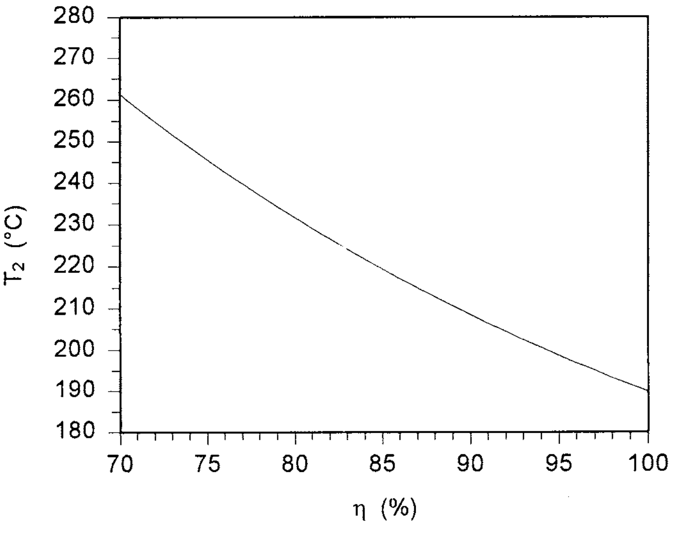
IT Results for $\eta = 70\%$

$h_1 = 293.2$ kJ/kg
$h_{2s} = 464.9$ kJ/kg
$h_2 = 538.6$ kJ/kg
$T_2 = 261.4$ °C
$T_{2s} = 189.6$ °C
$-(\dot{W}_{cv}) = 245.4$ kW

These results compare very favorably with those obtained in the sample calculation from table data.

PLOTS:





Note: As η_c decreases, the required power increases as does the exit temperature. This is expected.

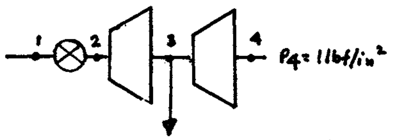
PROBLEM 6.158

KNOWN: Operating data are provided for a two stage turbine at steady state from which steam is extracted between the stages.

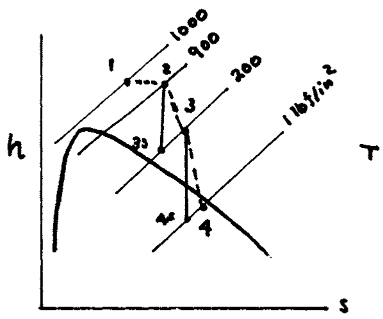
FIND: Determine the power output.

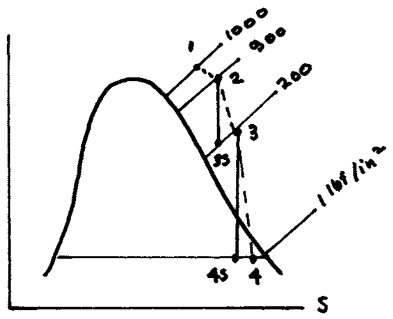
SCHEMATIC & GIVEN DATA:

$\eta_t = 85\%$ $\eta_t = 85\%$



$P_1 = 1000\ lbf/in^2$ $P_2 = 900\ lbf/in^2$
$T_1 = 900\ °F$ $P_3 = 200\ lbf/in^2$
$\dot{m}_1 = 30\ lb/s$ $\dot{m}_3 = 4.5\ lb/s$





ASSUMPTIONS: (1) The turbine operates at steady state. (2) The expansion through each stage occurs adiabatically. (3) The expansion through the valve is a throttling process. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: The total power developed is the sum of the power developed by the two stages. With the listed assumptions, mass and energy rate balances reduce to give

$$\dot{W}_t = \dot{m}_1 (h_2 - h_3) + (\dot{m}_1 - \dot{m}_2)(h_3 - h_4)$$

Introducing the isentropic turbine efficiency, $(h_2 - h_3) = \eta_t (h_2 - h_{3s})$, $(h_3 - h_4) = \eta_t (h_3 - h_{4s})$. Thus

$$\dot{W}_t = \eta_t \left[\dot{m}_1 (h_2 - h_{3s}) + (\dot{m}_1 - \dot{m}_2)(h_3 - h_{4s}) \right] \qquad (1)$$

With assumption 3, $h_2 = h_1$. So, state 2 is fixed by $900\ lbf/in^2$ and $h_2 = 1448.1\ Btu/lb$ (from Table A-4E). Interpolating in Table A-4E, $s_2 = 1.6228\ Btu/lb \cdot °R$. State 3s is fixed by $200\ lbf/in^2$ and $s_{3s} = s_2$. Interpolating, $h_{3s} = 1267.8\ Btu/lb$. Using the isentropic turbine efficiency

$$h_3 = h_2 - \eta_t (h_2 - h_{3s}) = 1448.1 - 0.85(1448.1 - 1267.8) = 1294.8\ Btu/lb$$

State 3 is fixed by $h_3 = 1294.8\ Btu/lb$, $200\ lbf/in^2$. Interpolating, $s_3 = 1.6503\ Btu/lb \cdot °R$. State 4s is fixed by $1\ lbf/in^2$, $s_3 = 1.6503\ Btu/lb \cdot °R$. Data from Table A-3E at $1\ lbf/in^2$ allows the quality to be determined

$$x_{4s} = \frac{s_{4s} - s_f}{s_g - s_f} = \frac{1.6503 - 0.1327}{1.8453} = 0.8224$$

Thus

$$h_{4s} = h_f + x_{4s} h_{fg} = 69.74 + 0.8224(1036) = 921.7\ Btu/lb$$

Inserting values into Eq. (1)

$$\dot{W}_t = 0.85 \left[(30\ lb/s)[1448.1 - 1267.8] \frac{Btu}{lb} + (25.5)[1294.8 - 921.7] \right] \left| \frac{3600 s}{h} \right| \left| \frac{1\ hp}{2545\ Btu/h} \right|$$

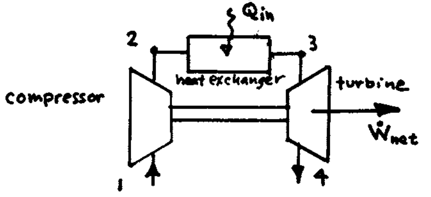
$$= 17,943\ hp \longleftarrow$$

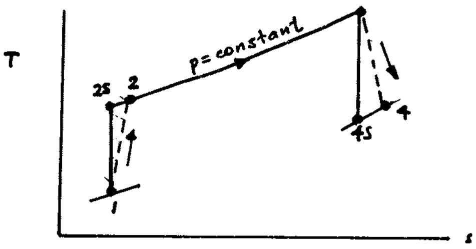
PROBLEM 6.159

KNOWN: Operating data are provided for a gas turbine power plant operating at steady state.

FIND: Determine the net power developed for each of two cases: (a) the turbine and compressor are without internal irreversibilities, (b) the compressor and turbine efficiencies are 82 and 85%, respectively.

SCHEMATIC & GIVEN DATA:





$p_1 = p_4 = 0.95$ bar, $p_2 = p_3 = 5.7$ bars
$T_1 = 22°C$, $T_3 = 1100$ K
$\dot{m}_1 = 5$ kg/s

ASSUMPTIONS: (1) The gas turbine power plant operates at steady state. (2) The compressor and turbine operate adiabatically. (3) Kinetic and potential energy effects are negligible. (4) Air is modeled as an ideal gas.

ANALYSIS: (a) $\eta_t = \eta_c = 100\%$. In this case, mass and energy rate balances at steady state give the following expression for the net work developed per unit mass flowing

$$\frac{\dot{W}_{net}}{\dot{m}} = \underbrace{(h_3 - h_{4s})}_{\left[\text{turbine output}\right]} - \underbrace{(h_{2s} - h_1)}_{\left[\text{compressor input}\right]}$$

From Table A-22, $h_1 = 295.17$ kJ/kg, $p_{r1} = 1.3068$, $h_3 = 1161.07$ kJ/kg, $p_{r3} = 167.1$. Then

$$p_{r2} = p_{r1}\left[\frac{p_2}{p_1}\right] = 1.3068\left[\frac{5.7}{0.95}\right] = 7.8408 \Rightarrow h_{2s} = 493.03 \text{ kJ/kg}$$

$$p_{r4} = p_{r3}\left[\frac{p_4}{p_3}\right] = 167.1\left[\frac{0.95}{5.7}\right] = 27.85 \Rightarrow h_{4s} = 706.51 \text{ kJ/kg}$$

Thus

$$\frac{\dot{W}_{net}}{\dot{m}} = (1161.07 - 706.51) - (493.03 - 295.17) = 454.56 - 197.86 = 256.7 \frac{\text{kJ}}{\text{kg}}$$

$$\Rightarrow \dot{W}_{net} = \left(5 \frac{\text{kg}}{\text{s}}\right)\left[256.7 \frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 1284 \text{ kW} \quad \longleftarrow$$

(b) $\eta_c = 0.82$, $\eta_t = 0.85$. In this case the net work developed per unit mass flowing is

$$\frac{\dot{W}_{net}}{\dot{m}} = \underbrace{(h_3 - h_4)}_{\left[\text{turbine output}\right]} - \underbrace{(h_2 - h_1)}_{\left[\text{compressor input}\right]}$$

Using the compressor and turbine efficiencies

$$(h_3 - h_4) = \eta_t(h_3 - h_{4s}), \quad h_2 - h_1 = \frac{(h_{2s} - h_1)}{\eta_c}$$

Thus

① $$\frac{\dot{W}_{net}}{\dot{m}} = \eta_t(h_3 - h_{4s}) - \frac{(h_{2s} - h_1)}{\eta_c}$$

$$= 0.85(454.56) - \frac{197.86}{.82} = 386.38 - 241.29 = 145.09 \frac{\text{kJ}}{\text{kg}}$$

$$\Rightarrow \dot{W}_{net} = (5 \text{ kg/s})\left[145.09 \frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 725 \text{ kW} \quad \longleftarrow$$

---

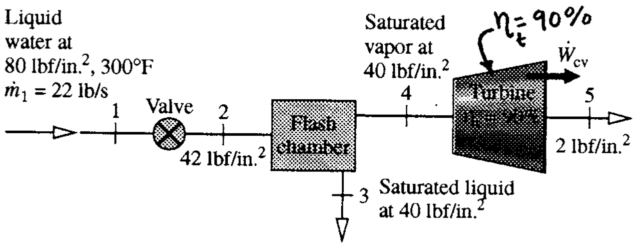
1. The effect of irreversibilities is to decrease the work developed by the turbine while increasing the work required by the compressor. Overall, the net work developed is significantly decreased.

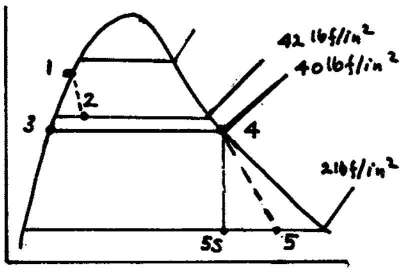
PROBLEM 6.160

KNOWN: Steady state operating data are provided for a valve, flash chamber, and turbine in series.

FIND: Determine the power developed by the turbine. Also, determine the rates of entropy production for each of the three components, and compare.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component operates at steady state with negligible heat transfer between each component and its surroundings. (2) Kinetic and potential energy effects can be ignored. (3) The expansion across the valve is a throttling process.

ANALYSIS At steady state, mass rate balances reduce to give $\dot{m}_2 = \dot{m}_1$, $\dot{m}_3 + \dot{m}_4 = \dot{m}_2$, $\dot{m}_4 = \dot{m}_5$. An entropy rate balance for a control volume enclosing the flash chamber takes the form

$$0 = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}_2 s_2 - \dot{m}_3 s_3 - \dot{m}_4 s_4 + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}_4 s_4 + \dot{m}_3 s_3 - \dot{m}_2 s_2 \qquad \text{[flash chamber]} \qquad (1)$$

Similarly, for the valve and turbine, the rates of entropy production are

$$\dot{\sigma}_{cv} = \dot{m}_1 (s_2 - s_1) \qquad \text{[valve]} \qquad (2)$$

$$\dot{\sigma}_{cv} = \dot{m}_4 (s_5 - s_4) \qquad \text{[turbine]} \qquad (3)$$

To evaluate Eqs. (1)-(3) requires $\dot{m}_3$, $\dot{m}_4$, s_1, s_2, s_3, s_4, and s_5. These will now be evaluated in turn.

Using the mass rate balance together with listed assumptions, an energy rate balance for a control volume enclosing the flash chamber reduces to

$$0 = \dot{m}_2 h_2 - \dot{m}_3 h_3 - \dot{m}_4 h_4$$

$$0 = \dot{m}_1 h_2 - \dot{m}_3 h_3 - (\dot{m}_1 - \dot{m}_3) h_4$$

Across the valve $h_2 = h_1$, so on solving the above equation

$$\dot{m}_3 = \dot{m}_1 \frac{[h_1 - h_4]}{[h_3 - h_4]} = \left(22 \frac{\text{lb}}{\text{s}}\right)\left[\frac{269.7 - 1170}{236.16 - 1170}\right] = 21.21 \text{ lb/s}$$

where $h_1 \approx h_f(T_1)$ from Table A-2E and h_3 and h_4 are from Table A-3E. Then, $\dot{m}_4 = \dot{m}_2 - \dot{m}_3 = 22 - 21.21 = 0.79$ lb/s.

From Table A-2E, $s_1 \approx s_f(T_1) = 0.4372$ Btu/lb·°R. State 2 is fixed by 42 lbf/in² and $h_2 = h_1 = 269.7$ Btu/lb. Thus, with data from Table A-3E

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{269.7 - 239.1}{931.8} = 0.0328$$

Thus

$$s_2 = s_f + x_2 s_{fg} = 0.3961 + 0.0328(1.2767) = 0.438 \text{ Btu/lb·°R}$$

From Table A-3E at 40 lbf/in², $s_3 = 0.3921$ Btu/lb·°R, $s_4 = 1.6767$ Btu/lb·°R. State 5 is fixed by 2 lbf/in² and h_5. Using the turbine isentropic efficiency,

$$\eta_t = \frac{h_4 - h_5}{h_4 - h_{5s}} \qquad (4)$$

where h_{5s} is the specific entropy at the turbine exit for an isentropic expansion from state 4 to 2 lbf/in². The value of h_{5s} is determined by 2 lbf/in² and $s_{5s} = s_4$.

PROBLEM 6.160 (Contd.)

Thus, with data from Table A-3E at 2 lbf/in²

$$x_{ss} = \frac{s_{ss}-s_f}{s_{fg}} = \frac{1.6767-0.175}{1.7448} = 0.861 \Rightarrow h_{ss} = h_f + x_{ss}h_{fg}$$
$$= 94.02 + (0.861)(1022.1) = 974.05\ \frac{\text{Btu}}{\text{lb}}$$

Then, with data from Table A-3E at 2 lbf/in²

$$x_{ss} = \frac{s_{ss}-s_f}{s_{fg}} = \frac{1.6767-0.175}{1.7448} = 0.861,\ h_{ss} = h_f + x_{ss}h_{fg} = 94.02 + 0.861(1022.1) = 974.05\ \text{Btu/lb}$$

Solving Eq. (4) for h_5 and inserting values

$$h_5 = h_4 - \eta_t(h_4 - h_{ss}) = 1170 - 0.9(1170 - 974.05) = 993.65\ \text{Btu/lb}$$

Using this together with data from Table A-3E at 2 lbf/in²

$$x_5 = \frac{h_5 - h_f}{h_{fg}} = \frac{993.65 - 94.02}{1022.1} = 0.88$$

$s_5 = s_f + x_5 s_{fg} = 0.175 + 0.88(1.7448) = 1.7104$ Btu/lb·°R.

Finally, substituting values into Eqs. (1)-(3)

- flash chamber: $\dot{\sigma}_{cv} = (0.79)[1.6767] + (21.21)[0.3921] - (22)[0.438] = 0.0054\ \frac{\text{Btu}}{\text{s}\cdot°\text{R}}$ ←
- valve: $\dot{\sigma}_{cv} = (22)[0.438 - 0.4372] = 0.0176\ \frac{\text{Btu}}{\text{s}\cdot°\text{R}}$ ←
- turbine: $\dot{\sigma}_{cv} = (0.79)[1.7104 - 1.6767] = 0.0266\ \frac{\text{Btu}}{\text{s}\cdot°\text{R}}$ ←

Mass and energy rate balances reduce for the turbine to give

$$\dot{W}_t = \dot{m}_4[h_4 - h_5] = (0.79\ \tfrac{\text{lb}}{\text{s}})[1170 - 993.65] = 139.3\ \frac{\text{Btu}}{\text{s}} \leftarrow$$

The overall rate of entropy production is obtained by summing the individual contributions:

$$\dot{\sigma}_{\text{overall}} = (0.0054 + 0.0176 + 0.0266) = 0.0496\ \frac{\text{Btu}}{\text{s}\cdot°\text{R}}$$

When expressed as percentages of the overall entropy production rate, the individual contributions are

①

Component	Contribution to overall, %
flash chamber	10.9
valve	35.5
turbine	53.6

---

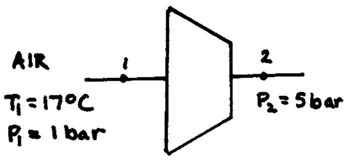
1. Although the turbine contributes most to the overall entropy production a significant reduction in the turbine's contribution might not be feasible because the turbine is operating near its practical limit: η_t = 90%. Attention would then turn to the next most important contributor — the valve. For example, the valve irreversibility could be reduced by replacing the valve with a power-recovery turbine. However, such a change would be governed by econonomic considerations.

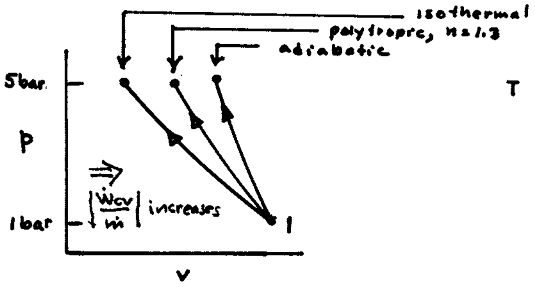
PROBLEM 6.161

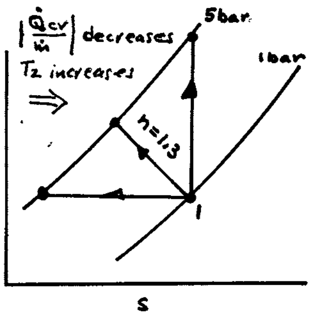
KNOWN: Data are provided for an air compressor operating at steady state.

FIND: For each of three cases, evaluate the work and heat transfer per unit mass of air flowing. Discuss.

SCHEMATIC & GIVEN DATA:







ASSUMPTIONS: (1) The air compressor operates at steady state without internal irreversibilities. (2) Kinetic and potential energy effects are negligible. (3) Air is modeled as an ideal gas.

ANALYSIS: Following the discussions of Eqs. 6.51 and 6.53 of Sec. 6.9, the areas below the process curves of the T-s diagram represent the magnitudes of the heat transfer per unit of mass flowing. The areas behind the process curves of the p-v diagram represent the magnitudes of the work per unit of mass flowing.

(a) Isothermal. An energy rate balance reduces at steady state to give

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}$$

As the specific enthalpy of an ideal gas depends on temperature alone, and temperature is constant, $h_2 = h_1$. The energy rate balance then becomes simply $\dot{Q}_{cv}/\dot{m} = \dot{W}_{cv}/\dot{m}$.

Since irreversibilities are absent, and $\Delta ke = \Delta pe \approx 0$, the work per unit of mass flowing is given by Eq. 6.53b. Introducing $v = RT/p$ and integrating

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\int_1^2 v\,dp = -\int_1^2 \frac{RT}{p}\,dp = -RT \ln P_2/P_1 = -\left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(290K) \ln\frac{5}{1} = -133.95 \frac{kJ}{kg} \longleftarrow \text{(a)}$$

(b) Polytropic with n = 1.3. As in part (a), the work per unit of mass flowing is given by Eq. 6.53b. Introducing $pv^n = \text{constant}$, and integrating, Eq. 6.55 results. Then, with the ideal gas equation of state, Eq. 6.57a is obtained:

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{nR}{(n-1)}(T_2 - T_1)$$

For a polytropic process of an ideal gas (Eq. 3.56)

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(n-1)/n} = (290K)\left(\frac{5}{1}\right)^{0.3/1.3} = 420.4\ K$$

Thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{(1.3)}{(0.3)}\left(\frac{8.314}{28.97}\right)(420.4 - 290) = -162.17 \frac{kJ}{kg}$$

(b)

An energy rate balance at steady state reduces to give

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 - h_1 = -162.17 + 421.67 - 290.16 = -30.66 \frac{kJ}{kg}$$

where enthalpy data is from Table A-22.

(c) Adiabatic. If the process is adiabatic and internally reversible, it is isentropic. An energy rate balance reduces with $\dot{Q}_{cv} = 0$ and assumption 2 to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2$$

To find h_2, use p_r data from Table A-22

$$p_{r2} = p_{r1}(P_2/P_1) = 1.2311\,(5/1) = 6.1555 \Rightarrow h_2 = 460.1\ kJ/kg$$

Finally

$$\frac{\dot{W}_{cv}}{\dot{m}} = 290.16 - 460.1 = -169.94\ kJ/kg \longleftarrow \text{(c)}$$

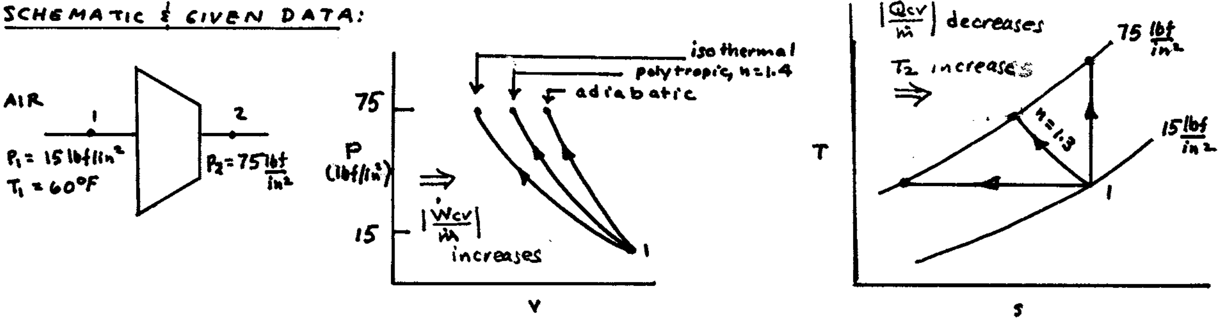
Only work and heat magnitudes are associated with areas on the p-v and T-s diagrams, respectively. Thus, the areas behind the curves on the p-v diagram increase from case (a) to case (b) to case (c) in accordance with the magnitudes calculated for $\dot{W}_{cv}/\dot{m}$. The areas below the curves on the T-s diagram decrease from case (a) to case (b) to case (c) in accordance with the magnitudes for $\dot{Q}_{cv}/\dot{m}$.

PROBLEM 6.162

KNOWN: Data are provided for an air compressor operating at steady state.

FIND: For each of three cases, evaluate the work and heat transfer per unit mass of air flowing.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The air compressor operates at steady state without internal irreversibilities. (2) Kinetic and potential energy effects are negligible. (3) Air is modeled as an ideal gas.

ANALYSIS: Following the discussions of Eqs. 6.51 and 6.53 of Sec. 6.9, the areas below the process curves of the T-s diagram represent the magnitudes of the heat transfer per unit of mass flowing. The areas behind the process curves of the p-v diagram represent the magnitudes of the work per unit of mass flowing.

(a) Isothermal. An energy rate balance reduces at steady state to give

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_1 - h_2) + \overset{0}{\cancel{\frac{V_1^2 - V_2^2}{2}}} + \overset{0}{\cancel{g(z_1 - z_2)}}$$

As the specific enthalpy of an ideal gas depends on temperature alone, and temperature is constant, $h_2 = h_1$. The energy rate balance then becomes simply $\dot{Q}_{cv}/\dot{m} = \dot{W}_{cv}/\dot{m}$.

Since irreversibilities are absent, and $\Delta ke = \Delta pe$, the work per unit of mass flowing is given by Eq. 6.53b. Introducing $v = RT/p$ and integrating

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\int_1^2 v\,dp = -\int_1^2 \frac{RT}{p}\,dp = -RT \ln\frac{p_2}{p_1} = -\left(\frac{1.986}{28.97}\,\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)(520°\text{R}) \ln\frac{75}{15} = -57.37\,\frac{\text{Btu}}{\text{lb}} \longleftarrow \text{(a)}$$

(b) Polytropic with n=1.3. As in part (a), the work per unit of mass flowing is given by Eq. 6.53b. Introducing pv^n = constant, and integrating, Eq. 6.55 results. Then, with the ideal gas equation of state, Eq. 6.57a, is obtained:

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{nR}{(n-1)}(T_2 - T_1)$$

For a polytropic process of an ideal gas (Eq. 3.56)

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 520\left(\frac{75}{15}\right)^{0.3/1.3} = 753.9\ °\text{R}$$

Thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{(1.3)}{(0.3)}\left(\frac{1.986}{28.97}\right)(753.9 - 520) = -69.48\,\frac{\text{Btu}}{\text{lb}}$$

An energy rate balance at steady state reduces to give

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + h_2 - h_1 = -69.48 + 180.61 - 124.27 = -13.14\ \text{Btu/lb}.$$

(b)

where enthalpy data is from Table A-22E.

(c) Adiabatic. If the process is adiabatic and internally reversible, it is isentropic. An energy balance reduces with $\dot{Q}_{cv}$ and assumption 2 to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2$$

To find h_2, use p_r data from Table A-22E

$$p_{r2} = p_{r1}(p_2/p_1) = 1.2147(75/15) = 6.0735 \Rightarrow h_2 = 197.06\ \text{Btu/lb}$$

Finally

$$\frac{\dot{W}_{cv}}{\dot{m}} = 124.27 - 197.06 = -72.79\ \text{Btu/lb} \longleftarrow \text{(c)}$$

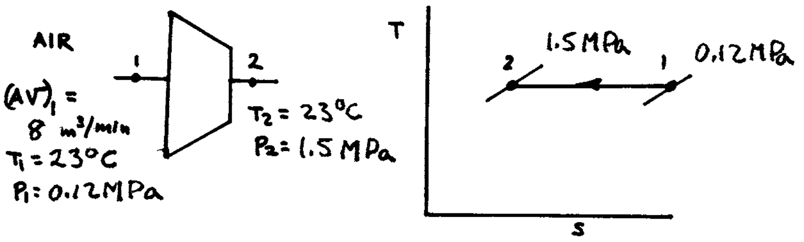
Only work and heat magnitudes are associated with areas on the p-v and T-s diagrams, respectively. Thus, the areas behind the curves on the p-v diagram increase from case (a) to case (b) to case (c) in accordance with the magnitudes calculated for $\dot{W}_{cv}/\dot{m}$. The areas below the curves on the T-s diagram decrease from case (a) to case (b) to case (c) in accordance with the magnitudes for $\dot{Q}_{cv}/\dot{m}$.

PROBLEM 6.163 *

KNOWN: Operating data are provided for an air compressor at steady state.

FIND: Determine the power required and the rate of heat transfer.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The compressor is at steady state. (2) The process is isothermal and internally reversible. (3) Kinetic and potential energy effects are negligible. (4) Air is modeled as an ideal gas.

ANALYSIS: At steady state $\dot m_1 = \dot m_2 = \dot m$, and an energy rate balance reduces to

$$0 = \frac{\dot Q_{cv}}{\dot m} - \frac{\dot W_{cv}}{\dot m} + (h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + \cancel{g(z_1 - z_2)}$$

As the enthalpy of an ideal gas depends on temperature only, and temperature remains constant, $h_2 = h_1$. Thus

$$\frac{\dot Q_{cv}}{\dot m} = \frac{\dot W_{cv}}{\dot m}$$

As the process is internally reversible, there are two solution approaches that can be taken: (1) Equation 6.51 can be integrated giving

$$\frac{\dot Q_{cv}}{\dot m} = T(s_2 - s_1)$$

$$= T\,(\underbrace{s°(T_2) - s°(T_1)}_{=0 \text{ since } T_1 = T_2} - R \ln P_2/P_1) = -RT \ln P_2/P_1$$

(2) Equation 6.53b can be integrated, giving

$$\frac{\dot W_{cv}}{\dot m} = -\int_1^2 v\,dp = -\int_1^2 \frac{RT}{p}\,dp = -RT \ln \frac{P_2}{P_1}$$

The mass flow rate is

$$\dot m = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{RT_1} = \frac{(1.2 \times 10^5 \frac{N}{m^2})(8 \frac{m^3}{min}) \left|\frac{min}{60 s}\right|}{\left(\frac{8314}{28.97} \frac{N \cdot m}{kg\,K}\right)(296\,K)} = 0.1884 \frac{kg}{s}$$

Thus

$$\dot Q_{cv} = \dot W_{cv} = -\dot m RT \ln \frac{P_2}{P_1} = -\left(0.1884 \frac{kg}{s}\right)\left(\frac{8.314}{28.97} \frac{kJ}{kg\,K}\right)(296\,K)\left(\ln \frac{1.5}{0.12}\right)\left|\frac{1\,kW}{1\,kJ/s}\right|$$

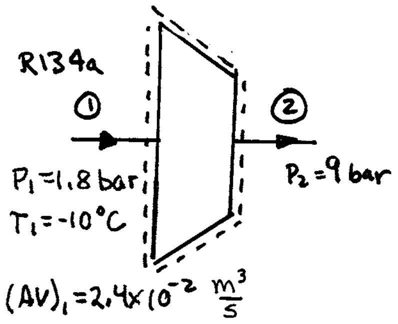
$$= -40.4\ kW \xleftarrow[\dot W_{cv}]{\dot Q_{cv}}$$

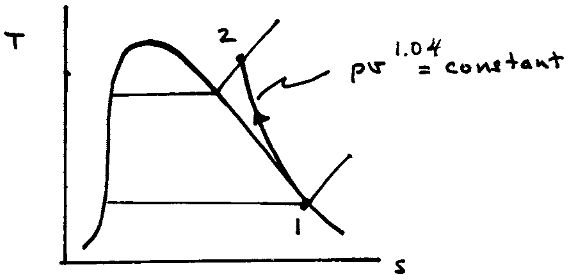
PROBLEM 6.164*

KNOWN: Steady state operating data are provided for a R134a compressor.

FIND: Determine the power required and rate of heat transfer.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume shown in the accompanying sketch is at steady state. 2. For the control volume, kinetic and potential energy effects can be ignored. 3. The compression is described by $pv^{1.04}$ = constant. The process is internally reversible.

ANALYSIS: (a) Using Eq. 6.55, the power is

$$(\dot{W}_{cv})_{\substack{int\\rev}} = -\dot{m}\left(\frac{n}{n-1}\right)\left[p_2v_2 - p_1v_1\right]$$

To find v_2, write $p_2 v_2^{1.04} = p_1 v_1^{1.04} \Rightarrow v_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.04}} v_1$, where $v_1 = 0.11135$ m³/kg from Table A-12. Thus

$$v_2 = \left(\frac{1.8}{9}\right)^{\frac{1}{1.04}}(0.11135) = 0.02369\ m^3/kg.$$

Also,

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{2.4\times10^{-2}\ m^3/s}{0.11135\ m^3/kg} = 0.2155\ kg/s$$

Finally

$$(\dot{W}_{cv})_{\substack{int\\rev}} = -\left(0.2155\frac{kg}{s}\right)\left(\frac{1.04}{1.04-1}\right)\left[\left(9\times10^5\frac{N}{m^2}\right)\left(0.02369\frac{m^3}{kg}\right) - (1.8\times10^5)(0.11135)\right]\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= -7.161\ kW \quad \longleftarrow$$

(b) Mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}} + g\cancel{(z_1 - z_2)}\right]$$

$$\Rightarrow \quad \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}\left[h_2 - h_1\right]$$

State 2 is fixed by 9 bar, $v_2 = 0.02369$ m³/kg. Thus, from Table A-12, $h_2 = 274.57$ kJ/kg and at state 1, $h_1 = 242.06$ kJ/kg.

$$\dot{Q}_{cv} = -7.161\ kW + \left(0.2155\frac{kg}{s}\right)\left[274.57 - 242.06\right]\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

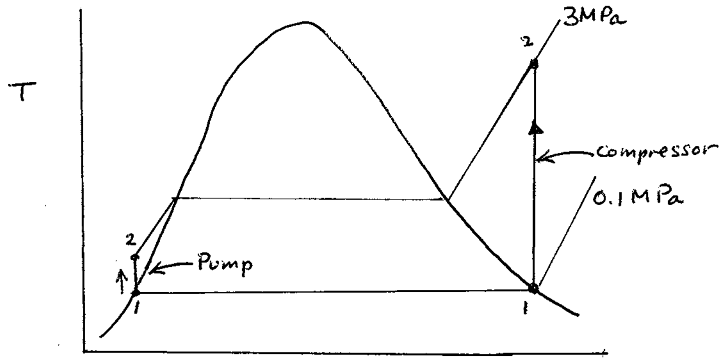
$$= -0.16\ kW \quad \longleftarrow$$

PROBLEM 6.165

KNOWN: At steady state, water vapor is compressed isentropically to 3 MPa from the saturated vapor state at 0.1 MPa and liquid water is pumped isentropically to 3 MPa from the saturated liquid state at 0.1 MPa.

① FIND: Compare the work required in each case.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) In each case, a control volume enclosing the device is under consideration. (2) For each control volume, $\dot{Q}_{cv} = 0$, kinetic/potential energy effects are negligible, and operation is at steady state.

ANALYSIS: Reducing mass and energy rate balances

$$\frac{-\dot{W}_{cv}}{\dot{m}} = h_2 - h_1$$

Compressor: From Table A-4, $h_1 = 2675.5$ kJ/kg, $s_1 = 7.3594$ kJ/kg·K, $h_2 = 3556.71$ kJ/kg. Thus

$$\frac{-\dot{W}_{cv}}{\dot{m}} = 3556.71 - 2675.5 = 881.21 \frac{kJ}{kg} \longleftarrow$$

PUMP: From Table A-3, $h_1 = 417.46$ Btu/lb, $s_1 = 1.3026$ kJ/kg·K. Then, interpolating in Table A-5 (double interpolation), $h_2 = 420.49$ kJ/kg. Thus

$$\frac{-\dot{W}_{cv}}{\dot{m}} = 420.49 - 417.46 = 3.03 \frac{kJ}{kg} \longleftarrow$$

Comparison:

$$\% = \left(\frac{\text{Pump Work}}{\text{Compressor Work}}\right)100 = \frac{(3.03)(100)}{881.21} = 0.34\% \longleftarrow$$

---

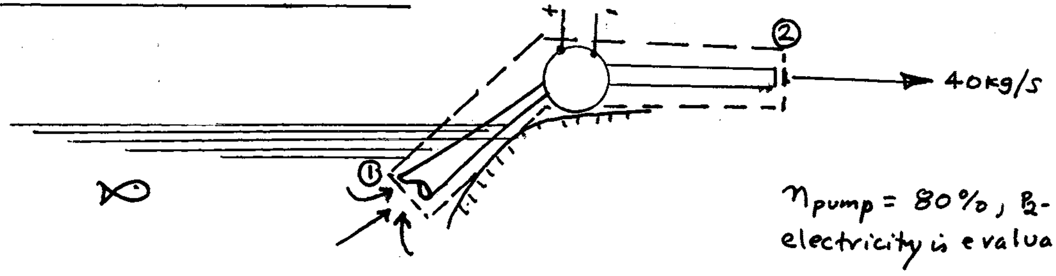
1. In chapters 8 and 9 the significance of pump work and compressor work and their relation to each other will be apparent in the study of Rankine and Brayton power cycles. Also, see the discussion of Eq. 6.53b in Sec. 6.9.

PROBLEM 6.166

KNOWN: Steady state operating data are provided for an electrically-driven pump that draws water from a pond.

FIND: Estimate the hourly cost of operating the pump.

SCHEMATIC & GIVEN DATA:



$\eta_{pump} = 80\%$, $p_2 - p_1 = 3$ bars
electricity is evaluated at 8¢/kW·h

ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, the changes in kinetic energy and potential energy from the inlet to the exit are ignored. (3) Liquid water is modeled as incompressible.

ANALYSIS: Using the pump efficiency, the power required to operate the pump is

$$\dot{W}_{pump} = \frac{\left(\dot{W}_{\substack{int\\rev}}\right)}{\eta_p}$$

where the numerator is provided by Eq. 6.53a or, with the assumptions listed, by Eq. 6.53c. Thus

$$\dot{W}_{pump} = -\frac{\dot{m}v(p_2 - p_1)}{\eta_p}$$

With v from Table A-2, $v \approx 10^{-3}\ \text{m}^3/\text{kg}$,

$$\dot{W}_{pump} = -\frac{\left(40\frac{\text{kg}}{\text{s}}\right)\left(10^{-3}\frac{\text{m}^3}{\text{kg}}\right)\left(3\times10^5\frac{\text{N}}{\text{m}^2}\right)}{0.80}\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= -15\text{ kW} \qquad \text{(minus denotes input)}$$

Then, the hourly cost of operating the pump is

$$\left(\begin{array}{c}\text{hourly}\\\text{cost}\end{array}\right) = (15\text{ kW})(1\cdot\text{h})\left(0.08\frac{\$}{\text{kW}\cdot\text{h}}\right)$$

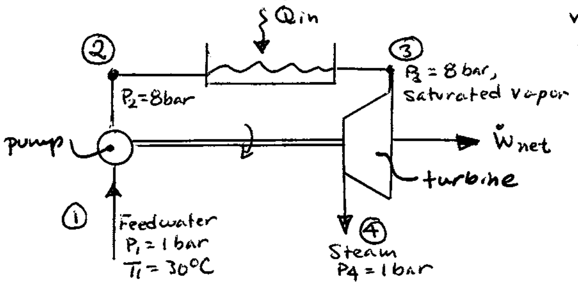
$$= \$1.20/\text{h}$$

PROBLEM 6.167

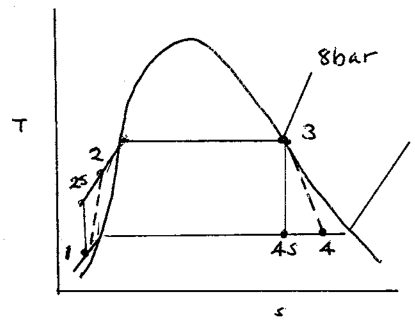
KNOWN: Steady state operating data are provided for a pump, a boiler, and a turbine in series.

FIND: Determine in kJ per kg of steam flowing (a) the pump work, (b) the net work developed by the turbine, and (c) the heat transfer to the boiler.

SCHEMATIC & GIVEN DATA:



$\eta_t = 90\%$
$\eta_p = 70\%$



ASSUMPTIONS: (1) Control volumes enclosing the pump, the boiler, and the turbine are at steady state. For the pump and turbine, $\dot{Q}_{cv} = 0$. (2) All kinetic and potential energy effects are negligible.

ANALYSIS: For the pump, **Eq. 6.53b** can be invoked to evaluate the work in the internally reversible process. With data from Table A-2, $h_1 \approx h_f(T_1) = 125.79$ kJ/kg, $v_1 \approx v_f(T_1) = 1.0043 \times 10^{-3}$ m³/kg. Then

$$\left(\frac{\dot{W}_p}{\dot{m}}\right)_{\substack{int\\rev}} \approx -v_1 \Delta p = \left(\frac{1.0043}{10^3} \frac{m^3}{kg}\right)(8-1)\times 10^5 \frac{N}{m^2} \left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = -0.7 \frac{kJ}{kg}$$

Then using the given isentropic pump efficiency

$$\eta_p = \frac{(-\dot{W}_p/\dot{m})_{s=c}}{(-\dot{W}_p/\dot{m})} \Rightarrow \frac{\dot{W}_p}{\dot{m}} = \frac{-0.7\ kJ/kg}{0.7} = -1\ \frac{kJ}{kg} \quad \longleftarrow \text{PUMP}$$

Mass and energy rate balances reduce to give, $0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_p + \dot{m}(h_1 - h_2)$. Thus $h_2 = (-\dot{W}_p/\dot{m}) + h_1 = 126.79$ kJ/kg.

For the boiler, mass and energy rate balances reduce to give

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = 2769.1 - 126.79 = 2642.3\ kJ/kg \quad \longleftarrow \text{BOILER}$$

where h_3 is from Table A-3.

For the turbine, mass and energy rate balances reduce to give

$$\frac{\dot{W}_t}{\dot{m}} = h_3 - h_4$$

Or on introducing the isentropic turbine efficiency

$$\frac{\dot{W}_t}{\dot{m}} = \eta_t (h_3 - h_{4s})$$

From Table A-3, $s_3 = s_{4s} = 6.6628$ kJ/kg·K. The quality at 4s is

$$x_{4s} = \frac{6.6628 - 1.3026}{7.3594 - 1.3026} = 0.885$$

Then, $h_{4s} = 417.46 + 0.885(2258) = 2415.8$ kJ/kg. Accordingly

$$\frac{\dot{W}_t}{\dot{m}} = 0.9\,(2769.1 - 2415.8) = 318\ kJ/kg$$

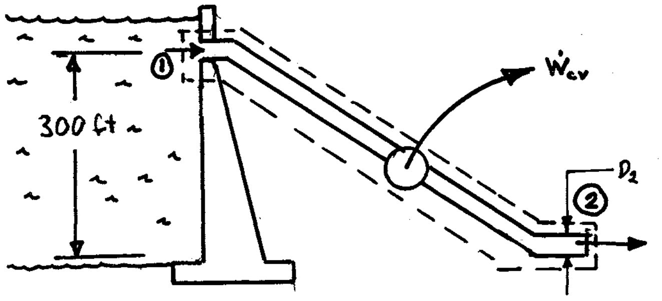
Of this amount, 1 kJ/kg is required by the pump, leaving $\frac{\dot{W}_{net}}{\dot{m}} = 317 \frac{kJ}{kg} \longleftarrow$ Net

PROBLEM 6.168*

KNOWN: Steady-state operating data are provided for a hydraulic turbine-generator.

FIND: In the absence of internal irreversibilities, determine the value of the power produced on a daily basis.

SCHEMATIC & GIVEN DATA:



$p_1 = 16\ \text{lbf/in}^2$, $V_1 = 4\ \text{ft/s}$

$p_2 = 14.7\ \text{lbf/in}^2$, $V_2 = 35\ \text{ft/s}$, $T_2 = 50°\text{F}$, $D_2 = 4\ \text{ft}$

$g = 32.2\ \text{ft/s}^2$

electricity is evaluated at 8¢/kW·h

ASSUMPTIONS: 1. The control volume shown in the figure is at steady state. 2. Internal irreversibilities are ignored. 3. Liquid water is modeled as incompressible.

ANALYSIS: Using assumption 2 indicating that internal irreversibilities are ignored, Eq. 6.53a can be used to evaluate the power developed. Since water is modeled as incompressible, we get

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -v(p_2 - p_1) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

With $v = v_f(50°\text{F}) = 0.01602\ \text{ft}^3/\text{lb}$ and given data

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -\left(0.01602\frac{\text{ft}^3}{\text{lb}}\right)\left[(14.7-16)(144)\frac{\text{lbf}}{\text{ft}^2}\right] + \left[\left(\frac{4^2-35^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\right]\left|\frac{1\ \text{lbf s}^2}{32.2\ \text{lb ft}}\right| + \left(32.2\frac{\text{ft}}{\text{s}^2}\right)(300\ \text{ft})\left|\frac{1\ \text{lbf s}^2}{32.2\ \text{lb ft}}\right|$$

$$= \left(2.999\frac{\text{ft lbf}}{\text{lb}}\right) + \left(-18.773\frac{\text{ft lbf}}{\text{lb}}\right) + \left(300\frac{\text{ft lbf}}{\text{lb}}\right)$$

$$= 284.23\frac{\text{ft lbf}}{\text{lb}}\left|\frac{1\ \text{Btu}}{778\ \text{ft lbf}}\right| = 0.365\ \text{Btu/lb}$$

The mass flow rate is

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{\pi D_2^2 V_2}{4 v_2} = \frac{\pi (4\ \text{ft})^2 (35\ \text{ft/s})}{4\ (0.01602\ \text{ft}^3/\text{lb})} = 27{,}455\frac{\text{lb}}{\text{s}}$$

so, $$(\dot{W}_{cv})_{\substack{int\\rev}} = \left(27{,}455\frac{\text{lb}}{\text{s}}\right)\left(0.365\frac{\text{Btu}}{\text{lb}}\right)\left|\frac{1\ \text{kW h}}{3413\ \text{Btu}}\right|\left|\frac{3600\ \text{s}}{1\ \text{h}}\right| = 10{,}570\ \text{kW}$$

① Daily Value = $(10{,}570\ \text{kW})\left(\frac{24\ \text{h}}{\text{day}}\right)\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right) = \$20{,}294/\text{day}$

---

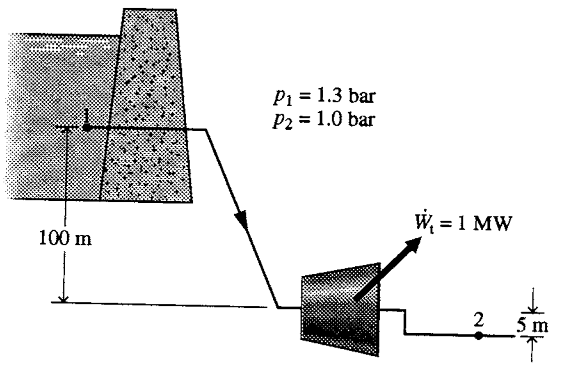
1. Owing to the effect of irreversibilities, the actual daily revenue would be **less** than the calculated value. Also, the daily value of the power developed is not the profit. The plant owners would have a considerable capital investment to recoup, and a variety of operating and maintenance costs to bear.

PROBLEM 6.169

KNOWN: Steady-state operating data are provided for a hydraulic turbine.

FIND: For a turbine power output of 1 MW, estimate the minimum required mass flow rate.

SCHEMATIC & GIVEN DATA:



$g = 9.8\ m/s^2$

$D_2 = D_1$

ASSUMPTIONS: 1. A one-inlet, one-exit control volume with inlet at 1 and exit at 2, and enclosing the turbine, is at steady state.

2. Liquid water is modeled as incompressible, with $v = 10^{-3}\ m^3/kg$ (Table A-19).

ANALYSIS: We expect that the effect of internal irreversibilities would be a lower power output in the actual case than in the ideal case — that is, we expect

$$(\dot{W}_t)_{act} \le (\dot{W})_{\substack{int\\rev}}$$

Introducing Eq. 6.53a with assumption 2

$$(\dot{W}_t)_{act} \le \dot{m}\left[-v(p_2-p_1) + g(z_1-z_2) + \cancel{\frac{V_1^2-V_2^2}{2}}\right] \qquad (1)$$

where the kinetic energy term drops out because $D_1 = D_2$, v is constant, and $\dot{m}_1 = \dot{m}_2$. From (1), we have

$$\dot{m} \ge \left[\frac{(\dot{W}_t)_{act}}{-v(p_2-p_1)+g(z_1-z_2)}\right] = \frac{(1\ MW)\left|\frac{10^6\ N\cdot m/s}{1\ MW}\right|}{\left[-\left(10^{-3}\frac{m^3}{kg}\right)\left(-0.3\times 10^5\frac{N}{m^2}\right)\right] + \left[\left(9.8\frac{m}{s^2}\right)(105\ m)\right]\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|}$$

$$\dot{m} \ge 944.3\ kg/s$$

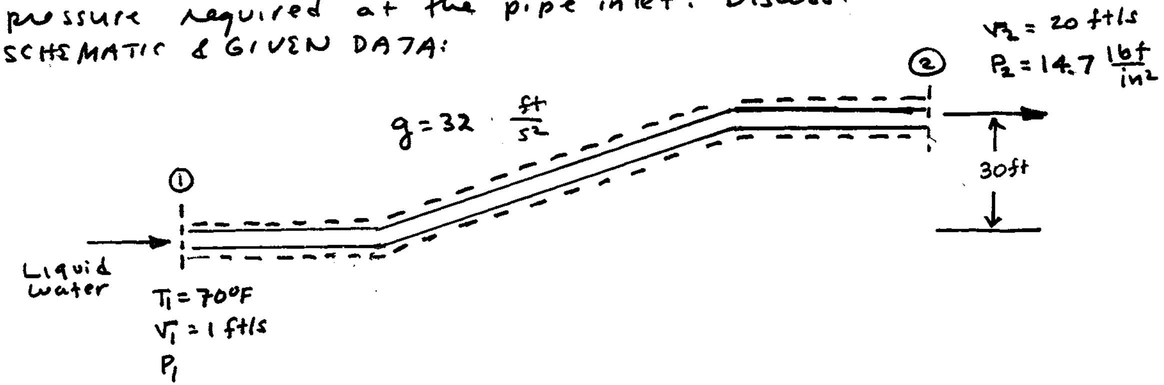
The minimum required mass flow rate is 944.3 kg/s. ←

PROBLEM 6.170

KNOWN: Operating data are provided for liquid water flowing through a pipe.

FIND: In the absence of internal irreversibilities, determine the pressure required at the pipe inlet. Discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. A one-inlet, one-exit control volume encloses the pipe. 2. The control volume is at steady state. 3. Liquid water is modeled as incompressible with $v = v_f(T_1) = 0.01605$ ft³/lb (Table A-2E). 4. Internal irreversibilities are ignored.

ANALYSIS: Since internal irreversibilities are ignored and there is no power input or output, Eq. 6.54 applies

① $$\int_1^2 v\,dp + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

$$\Rightarrow \quad v(p_2 - p_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

Solving for p_1

$$p_1 = p_2 + \frac{\left(\frac{V_2^2 - V_1^2}{2}\right) + g(z_2 - z_1)}{v}$$

$$= 14.7 \frac{lbf}{in^2} + \frac{\frac{1}{2}\left[(20)^2 - (1)^2\right]\left(\frac{ft}{s}\right)^2 + \left(32 \frac{ft}{s^2}\right)(30\,ft)}{\left(0.01605 \frac{ft^3}{lb}\right)} \left|\frac{1\,lbf}{32.2 \frac{lb\cdot ft}{s^2}}\right| \left|\frac{1\,ft^2}{144\,in^2}\right|$$

$$= 14.7 \frac{lbf}{in^2} + 15.58 \frac{lbf}{in^2} = 30.28 \frac{lbf}{in^2}$$

To overcome the effects of internal irreversibilities, such as friction between the flowing water and pipe wall, we expect the pressure at the inlet to be greater than calculated.

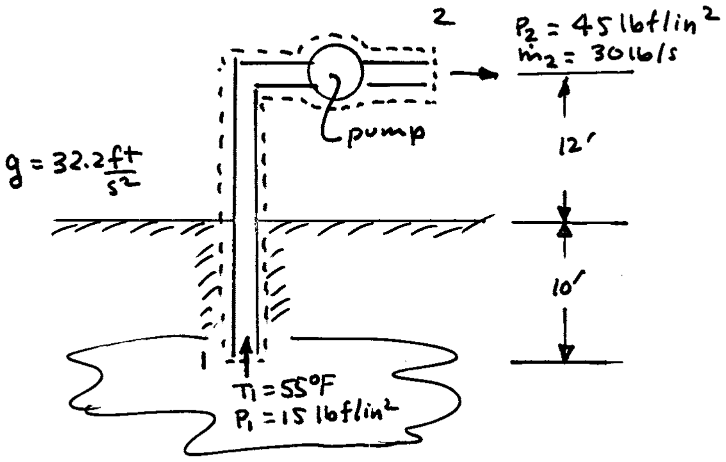
---

1. This is the Bernoulli equation.

PROBLEM 6.171

KNOWN: Steady-state operating data are provided for a pump drawing water from underground and delivering it above ground.

FIND: In the absence of internal irreversibilities, determine the power required by the pump. Discuss.


2
$p_2 = 45\ \text{lbf/in}^2$
$\dot m_2 = 30\ \text{lb/s}$
pump
12'
$g = 32.2\ \frac{\text{ft}}{\text{s}^2}$
10'
1
$T_1 = 55°\text{F}$
$p_1 = 15\ \text{lbf/in}^2$

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. Internal irreversibilities are absent. 3. Kinetic energy effects can be ignored. 4. Liquid water is modeled as incompressible with $v = v_f(T_1) = 0.01603\ \text{ft}^3/\text{lb}$ (Table A-2E).

ANALYSIS: In the absence of internal irreversibilities, Eq. 6.53a applies. Then, with assumptions 3, 4, we get

$$\left(\frac{\dot W_{cv}}{\dot m}\right)_{\substack{\text{int}\\ \text{rev}}} = -v(p_2-p_1) + g(z_1-z_2) + \cancelto{0}{\left(\frac{V_1^2-V_2^2}{2}\right)}$$

$$= -\left(0.01603\ \frac{\text{ft}^3}{\text{lb}}\right)(45-15)\frac{\text{lbf}}{\text{in}^2}\left|\frac{144\ \text{in}^2}{\text{ft}^2}\right| + \left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)(-22\ \text{ft})\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|$$

$$= -69.25\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}} - 22\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}} = -91.25\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}}$$

$$\Rightarrow \quad (\dot W_{cv})_{\substack{\text{int}\\ \text{rev}}} = \left(30\ \frac{\text{lb}}{\text{s}}\right)\left(-91.25\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}}\right)\left|\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lbf/s}}\right|$$

$$= -4.98\ \text{hp} \quad \Rightarrow \quad \text{Power input} = 4.98\ \text{hp} \longleftarrow$$

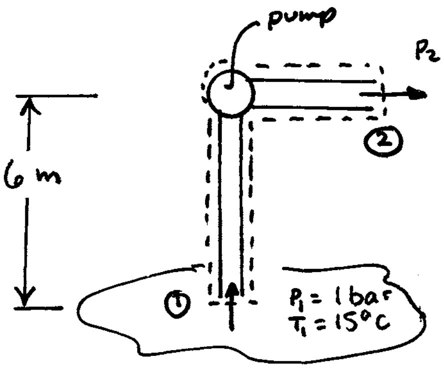
To overcome the effects of internal irreversibilities, such as friction between the flowing liquid and pipe wall, we expect the power input to the pump to be greater than determined here.

PROBLEM 6.172*

KNOWN: Steady state operating data are provided for a 5-kW pump.

FIND: Determine if it would be possible to pump 7.5 m³ in 10 min or less.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cv} = -5$ kW

$g = 9.8$ m/s²

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, kinetic energy effects can be ignored. 3. Liquid water is modeled as incompressible with $v = v_f(T_1) = 1.0009 \times 10^{-3}$ m³/kg (Table A-3).

ANALYSIS: To pump 7.5 m³ in 10 minutes, or less, would require a a volumetric flow rate (AV)

$$(AV) \geq \left(\frac{7.5 \text{ m}^3}{10 \text{ min}}\right)\left|\frac{1 \text{ min}}{60 \text{ s}}\right| = 0.0125 \text{ m}^3/\text{s}$$

which corresponds to a mass flow rate

$$\dot{m} \geq \frac{0.0125 \text{ m}^3/\text{s}}{1.0009 \times 10^{-3} \text{ m}^3/\text{kg}} = 12.49 \frac{\text{kg}}{\text{s}} \qquad (1)$$

In the absence of internal irreversibilities, the power input required would be given by the following expression obtained from Eq. 6.53a using assumptions 2, 3

$$(-\dot{W}_{cv})_{\substack{int\\rev}} = \dot{m}\left[v(P_2 - P_1) + g(z_2 - z_1) + \overset{0}{\cancel{\frac{(V_2^2 - V_1^2)}{2}}}\right]$$

Then, with (1)

$$(-\dot{W}_{cv})_{\substack{int\\rev}} \geq \left(12.49 \frac{\text{kg}}{\text{s}}\right)\left[\left(1.0009 \times 10^{-3} \frac{\text{m}^3}{\text{kg}}\right)\left(4 \times 10^2 \frac{\text{kN}}{\text{m}^2}\right)\left|\frac{1 \text{ kJ}}{1 \text{ kN m}}\right| + \left(9.8 \frac{\text{m}}{\text{s}^2}\right)(6 \text{ m})\left|\frac{1 \text{ N s}^2}{1 \text{ kg m}}\right|\left|\frac{1 \text{ kJ}}{1000 \text{ N m}}\right|\right]$$

$$\geq \left(12.49 \frac{\text{kg}}{\text{s}}\right)\left[0.400 \frac{\text{kJ}}{\text{kg}} + 0.059 \frac{\text{kJ}}{\text{kg}}\right]\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$\geq 5.73 \text{ kW}$$

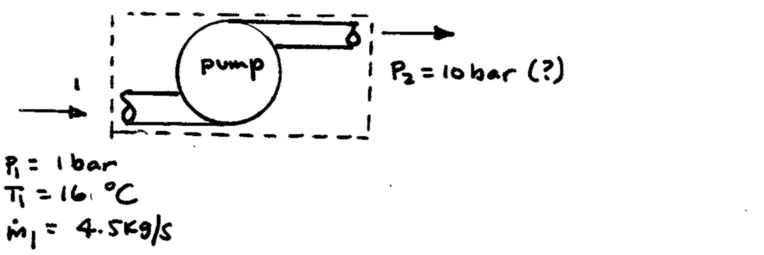
To overcome the effect of internal irreversibilities, such as friction between the flowing liquid and pipe wall, we expect the actual power input to the pump to be greater than calculated here. Since, only a 5-kW pump is available, the desired flow rate cannot be accomodated.

PROBLEM 6.173

KNOWN: Steady state operating data are provided for a 4-kW pump.

FIND: Determine if it would be possible for the pump to deliver water at a pressure of 10 bar.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. Kinetic and potential energy changes can be ignored. 3. Liquid water is modeled as incompressible with $v = v_f(T_1) = 1.0011 \times 10^{-3}$ m³/kg.

ANALYSIS: In the absence of internal irreversibilities, the power input required to deliver water at $P_2 = 10$ bar would be given by the following expression obtained from Eq. 6.53a using assumptions 3, 4

$$\left(-\dot{W}_{cv}\right)_{\substack{int\\rev}} = \dot{m}\, v (P_2 - P_1)$$

$$= \left(4.5 \frac{kg}{s}\right)\left(\frac{1.0011}{10^3}\right)\frac{m^3}{kg}\left(9 \times 10^5 \frac{N}{m^2}\right)\left|\frac{1\ kW}{10^3\ N\cdot m/s}\right|$$

$$= 4.05\ kW$$

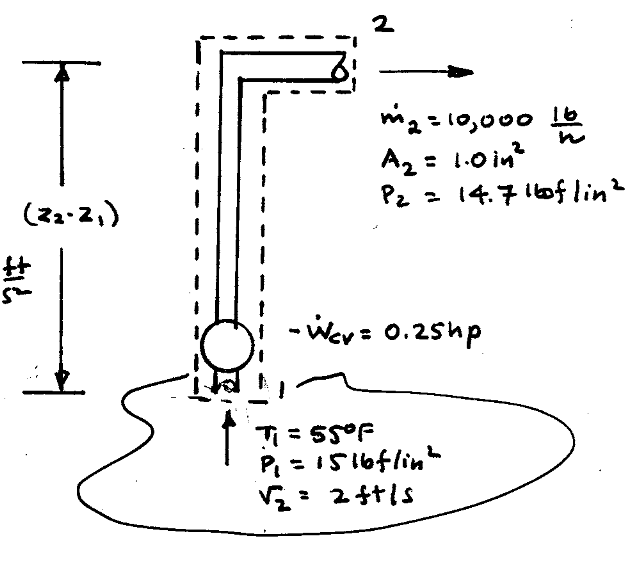
To overcome the effect of internal irreversibilities, such as friction between the flowing water and the pump walls, we expect the actual power required by the pump to be greater than calculated here. Since only a 4-kW pump is available, the desired exit pressure cannot be accomodated.

PROBLEM 6.174

KNOWN: Steady-state operating data are provided for a pump that delivers water at an elevation above the pump inlet.

FIND: Determine the maximum theoretical elevation at which the water can be delivered.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. Liquid water is modeled as incompressible with $v = v_f(T_1) = 0.01733\ \frac{ft^3}{lb}$ (Table A-2E).

ANALYSIS: Using data at 2, the velocity V_2 can be found:

$$\dot{m}_2 = \frac{A_2 V_2}{v}$$

$$\Rightarrow \quad V_2 = \frac{\dot{m}_2 v}{A} = \frac{\left(10{,}000\frac{lb}{h}\right)\left|\frac{1\,h}{3600\,s}\right|\left(0.01733\frac{ft^3}{lb}\right)}{(1.0\ in^2)\left|\frac{1\,ft^2}{144\,in^2}\right|}$$

$$= 6.9\ ft/s$$

Owing to the effect of internal irreversibilities, such as friction between the flowing water and pipe wall, the actual power input would be greater than in the ideal case — that is, we expect

$$(-\dot{W}_{cv})_{act} \geq (-\dot{W}_{cv})_{\substack{int\\rev}}$$

Introducing Eq. 6.53a with assumption 2

$$(-\dot{W}_{cv})_{act} \geq \dot{m}\left[v(p_2 - p_1) + g(z_2 - z_1) + \frac{(V_2^2 - V_1^2)}{2}\right]$$

$$\Rightarrow \quad (z_2 - z_1) \leq \frac{1}{g}\left[\frac{(-\dot{W}_{cv})_{act}}{\dot{m}} + v(p_1 - p_2) - \left(\frac{V_2^2 - V_1^2}{2}\right)\right]$$

$$\leq \left(\frac{1}{32.2\,ft/s^2}\right)\left[\frac{(0.25\,hp)\left|\frac{550\,ft\cdot lbf/s}{1\,hp}\right|}{(10^4\ lb/h)\left|\frac{1\,h}{3600\,s}\right|} + \left(0.01733\frac{ft^3}{lb}\right)\left(0.3 \times 144\ \frac{lbf}{ft^2}\right) - \frac{\left[(6.9)^2 - (2)^2\right]\frac{ft^2}{s^2}}{2}\left|\frac{1\,lbf}{32.2\,lb\cdot\frac{ft}{s^2}}\right|\right]$$

$$\leq \left(\frac{1}{32.2\,ft/s^2}\right)\left[(49.5 + 0.75 - 0.67)\ \frac{ft\cdot lbf}{lb}\right]\left|\frac{32.2\ lb\cdot ft/s^2}{1\,lbf}\right|$$

$$\leq 49.58\ ft$$

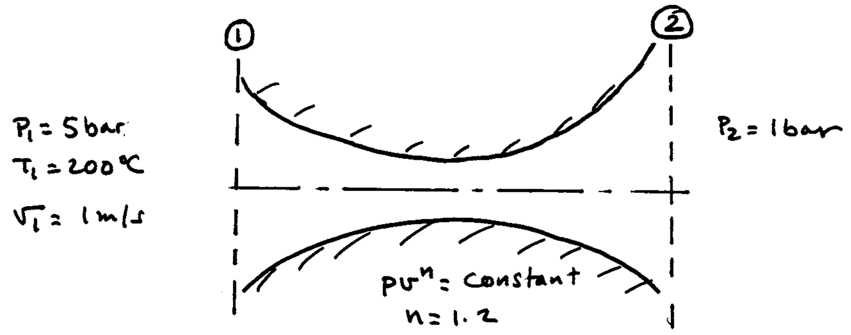
The maximum theoretical elevation at the exit is 49.58 ft.

PROBLEM 6.175

KNOWN: Data are provided for CO expanding through a nozzle operating at steady state.

FIND: Determine the velocity at the nozzle exit and the rate of heat transfer per kg of CO flowing.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The nozzle is at steady state. (2) The expansion is a polytropic process with $n = 1.2$. (3) CO is modeled as an ideal gas. (4) Potential energy effects are ignored.

ANALYSIS: (a) As a polytropic process is internally reversible, Eq. 6.53a is applicable. Then, with $\dot{W}_{cv} = 0$, the Bernoulli equation, Eq. 6.54 results:

$$\int_1^2 v\,dp + \frac{V_2^2 - V_1^2}{2} + \overset{0}{\cancel{g(z_2 - z_1)}} = 0 \qquad (1)$$

Using the relationship $pv^n = \text{constant}$, the integral can be performed (Sec. 6.9), giving

$$\int_1^2 v\,dp = \frac{n}{n-1}(p_2 v_2 - p_1 v_1)$$

$$= \frac{nR}{n-1}(T_2 - T_1) \qquad (2)$$

where the ideal gas equation of state has been used to obtain the last expression. For a polytropic process of an ideal gas (Eq. 3.56)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n} \qquad (3)$$

Collecting Eqs. (1) - (3)

$$V_2 = \sqrt{V_1^2 + 2\frac{nRT_1}{n-1}\left[1 - (p_2/p_1)^{(n-1)/n}\right]} \qquad (4)$$

$$= \sqrt{\left(1\frac{m}{s}\right)^2 + 2\left(\frac{1.2}{1.2-1}\right)\left(\frac{8314}{28}\frac{N\cdot m}{kg\cdot K}\right)(473K)\left|\frac{1\,kg\cdot m/s^2}{1N}\right|\left[1 - \left(\frac{1}{5}\right)^{0.2/1.2}\right]} = 629.7\,\frac{m}{s} \longleftarrow V_2$$

(b) An energy balance reduces to give

$$\frac{\dot{Q}_{cv}}{\dot{m}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

With Eq. (3), $T_2 = T_1 (p_2/p_1)^{\frac{n-1}{n}} = (473)\left(\frac{1}{5}\right)^{\frac{0.2}{1.2}} = 362K$

Then with data from Table A-23

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{(10531 - 13797)}{28}\frac{kJ}{kg} + \left[\frac{(629.7)^2 - (1)^2}{2}\left(\frac{m^2}{s^2}\right)\right]\left|\frac{1N}{1 kg\cdot m/s^2}\right|\left|\frac{1\,kJ}{10^3 N\cdot m}\right| = 81.7\,\frac{kJ}{kg} \longleftarrow \dot{Q}_{cv}/\dot{m}$$

CHAPTER SEVEN

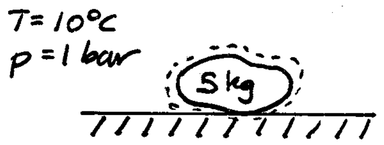
EXERGY ANALYSIS

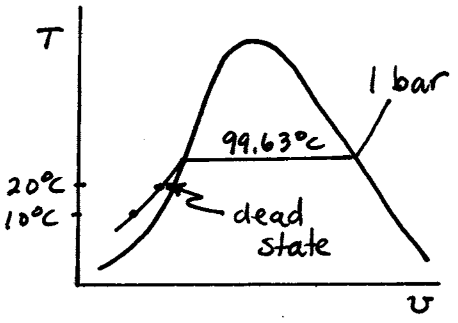
PROBLEM 7.1

KNOWN: A system consists of 5 kg of water at 10°C, 1 bar. The system is at rest and zero elevation relative to the exergy reference environment.

FIND: Determine the exergy.

SCHEMATIC & GIVEN DATA:





ASSUMPTION: The closed system shown in the figure is at rest and zero elevation relative to the environment for which $T_0 = 20°C$, $p_0 = 1$ bar.

ANALYSIS: Equation 7.2 gives

$$E = m\left[(u-u_0) + p_0(v-v_0) - T_0(s-s_0) + \cancelto{0}{\frac{V^2}{2}} + \cancelto{0}{gz}\right]$$

From Table A-2, using the approximations of sec. 3.3.6: $v \approx v_f(T)$ and $u \approx u_f(T)$ with the appoximation of Eq. 6.7: $s \approx s_f(T)$

$v_0 = 1.0018 \times 10^{-3}\ \text{m}^3/\text{kg}$ $\quad$ $v = 1.0004 \times 10^{-3}\ \text{m}^3/\text{kg}$

$u_0 = 83.95$ kJ/kg $\quad$ $u = 42.00$ kJ/kg

$s_0 = 0.2966$ kJ/kg·K $\quad$ $s = 0.1510$ kJ/kg

Thus

$$E = (5\ \text{kg})\left[(42.00-83.95)\frac{\text{kJ}}{\text{kg}} + (1\ \text{bar})\left(\frac{1.0004-1.0018}{10^3}\right)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| - (293\ \text{K})(0.1510-0.2966)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]$$

$$= (5\ \text{kg})\left[-41.95 - 1.4\times10^{-4} + 42.66\right]\frac{\text{kJ}}{\text{kg}}$$

$$= 3.55\ \text{kJ} \longleftarrow E$$

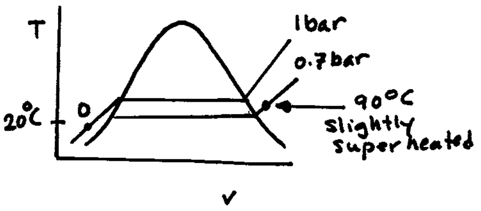
PROBLEM 7.2

KNOWN: A state is specified for each of three substances.

FIND: Determine the exergy.

ASSUMPTIONS: 1. Ignore the effects of motion and gravity. 2. $T_0 = 20°C$, $p_0 = 1\,bar$.

ANALYSIS: (a) Water at 0.7 bar, 90°C.



Tables A-2,3: $T_0 = 20°C$, $p_0 = 1\,bar$

$u_0 \approx u_f(T_0) = 83.95\ kJ/kg$

$v_0 \approx v_f(T_0) = \frac{1.0018}{10^3}\ m^3/kg$

$s_0 \approx s_f(T_0) = 0.2966\ kJ/kg\cdot K$

T 90°C, p = 0.7bar

$u \approx u_g(p) = 2494.5$

$v \approx v_g(p) = 2.365$

$s \approx s_g(p) = 7.4797$

Then, with Eq. 7.9, dropping $V^2/2$, gz terms

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$$

$$= (2494.5 - 83.95)\frac{kJ}{kg} + \left(10^5 \frac{N}{m^2}\right)\left(2.365 - \frac{1.0018}{10^3}\right)\frac{m^3}{kg}\left|\frac{1\,kJ}{10^3\,N\cdot m}\right| - (293K)\left(7.4797 - 0.2966\right)\frac{kJ}{kg\cdot K}$$

$$= \left[(2410.55) + (236.4) - (2104.65)\right] kJ/kg$$

$$= 542.27 \frac{kJ}{kg} \Rightarrow E = (1kg)\left(542.27 \frac{kJ}{kg}\right) = 542.27\ kJ \longleftarrow \mathbb{E}$$

(b) R134a at 0.7bar, 90°C

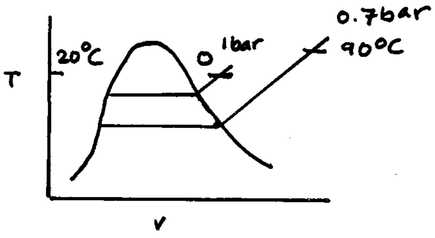


Table A-12:

$u_0 = 246.67\ kJ/kg$

$v_0 = 0.23349\ m^3/kg$

$s_0 = 1.0829\ kJ/kg\cdot K$

$u = 305.5$

$v = 0.441$

$s = 1.3118$

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$$

$$= (305.5 - 246.67) + 10^5(0.441 - 0.23349)\left|\frac{1}{10^3}\right| - 293(1.3118 - 1.0829)$$

$$= (58.83) + (20.75) - (67.07) = 12.51\ kJ/kg \Rightarrow E = 12.51\ kJ \longleftarrow \mathbb{E}$$

(c) Air as an ideal gas with c_p constant.

Constant $c_p \Rightarrow$ constant c_v. Evaluating c_v at Tave from Table A-20: $c_v = 0.72 \frac{kJ}{kg\cdot K}$

$c_p = 1.007 \frac{kJ}{kg\cdot K}$

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$$

$$= c_v[T - T_0] + p_0\left[\frac{RT}{p} - \frac{RT_0}{p_0}\right] - T_0\left[c_p \ln\frac{T}{T_0} - R\ln\frac{p}{p_0}\right]$$

$$= c_v[T - T_0] + R\left[T\left(\frac{p_0}{p}\right) - T_0\right] - T_0\left[c_p \ln\frac{T}{T_0} - R\ln\frac{p}{p_0}\right]$$

$$= \left(0.72\frac{kJ}{kg\cdot K}\right)\left[70K\right] + \frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\left[363K\left(\frac{1}{0.7}\right) - 293K\right]$$

$$- 293K\left[1.007\frac{kJ}{kg\cdot K}\ln\frac{363}{293} - \frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\ln\left(\frac{0.7}{1}\right)\right]$$

$$= \left[(50.4) + (64.74) - (93.2)\right]\frac{kJ}{kg} = 21.94\frac{kJ}{kg} \Rightarrow E = 21.94\ kJ \longleftarrow \mathbb{E}$$

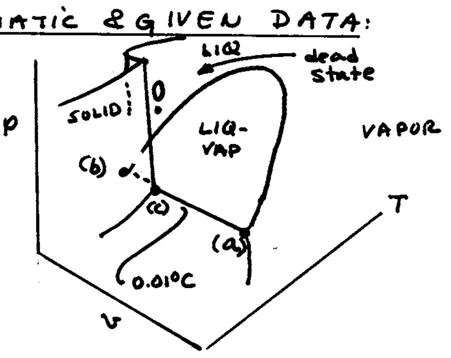
PROBLEM 7.3

KNOWN: Water at each of three alternative states.

FIND: Determine the specific exergy.

SCHEMATIC & GIVEN DATA:

①



dead state: $T_0 = 20°C$, $P_0 = 1$ bar

$u_0 \sim u_f(20°C) = 83.95$ kJ/kg

$v_0 \sim v_f(20°C) = (1.0018/10^3)$ m³/kg

$s_0 \sim s_f(20°C) = 0.2966$ kJ/kg·K

ASSUMPTION: 1. At the dead state $v \sim v_f(T_0)$, $u \sim u_f(T_0)$, $s \sim s_f(T_0)$. $T_0 = 20°C$, $P_0 = 1$ bar.
2. Ignore the effects of motion and gravity.

(a) Saturated vapor at 0.01 °C. With Eq. 7.9, dropping $V^2/2$, gz terms

$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$, with data from Table A-2:

$$= (2375.3 - 83.95)\frac{kJ}{kg} + \left(10^5 \frac{N}{m^2}\right)\left[206.136 - \frac{1.0018}{10^3}\right]\frac{m^3}{kg}\left|\frac{1\,kJ}{10^3\,\frac{N}{m^2}}\right| - 293\left[9.1562 - 0.2966\right]\frac{kJ}{kg\cdot K}$$

$$= \left[(2291.35) + 20{,}613.5 - (2595.86)\right]\frac{kJ}{kg} = 20{,}309 \text{ kJ/kg} \quad \longleftarrow \mathsf{e}$$

(b) Saturated liquid at 0.01 °C. With data from Table A-2:

$$e = (0 - 83.95) + (10^5)\left[\frac{1.002}{10^3} - \frac{1.0018}{10^3}\right]\left|\frac{1}{10^3}\right| - 293(0 - 0.2966)$$

$$= \left[(-83.95) + (\sim 0) + (86.90)\right]\frac{kJ}{kg} = 2.95 \text{ kJ/kg} \quad \longleftarrow \mathsf{e}$$

(c) Saturated Solid at 0.01 °C. With data from Table A-5:

$$e = ((-333.4) - 83.95) + (10^5)\left[\frac{1.0908}{10^3} - \frac{1.0018}{10^3}\right]\left|\frac{1}{10^3}\right| - 293((-1.221) - 0.2966)$$

$$= \left[(-417.35) + (0.01) + (444.66)\right]\frac{kJ}{kg} = 27.32 \text{ kJ/kg} \quad \longleftarrow \mathsf{e}$$

---

1. The "further" a state is from the dead state, the greater the value for exergy. Thus, state (a) is most distant, state (c) is closer, and state (b) is closest. The calculated exergy values confirm this rule.

PROBLEM 7.4*

KNOWN: Systems at specified states are defined.

FIND: Determine the specific exergy for 1 lb of each substance.

ASSUMPTIONS: The closed systems are at rest and zero elevation relative to the environment for which $T_0 = 70°F = 530°R$, $P_0 = 14.7\ lbf/in^2$.

ANALYSIS: Equation 7.2 gives

$$\mathsf{E} = m\left[(u-u_0) + p_0(v-v_0) - T_0(s-s_0) + \overset{0}{\cancel{\frac{V^2}{2}}} + \overset{0}{\cancel{gz}}\right]$$

(a) Saturated water vapor at 212°F. Using $u_0 \approx u_f(T_0)$, $v_0 \approx v_f(T_0)$, $s_0 \approx s_f(T_0)$, Table A-2E gives $u_0 = 38.09\ Btu/lb$, $v_0 = 0.01605\ ft^3/lb$, $s_0 = 0.07463\ Btu/lb\cdot°R$. Table A-2E also gives $u = 1077.6\ Btu/lb$, $v = 26.80\ ft^3/lb$, $s = 1.7567\ Btu/lb\cdot°R$.

Thus

$$\mathsf{E} = (1\,lb)\left[(1077.6-38.09)\frac{Btu}{lb} + (14.7)(144)\frac{lbf}{ft^2}(26.80-0.01605)\frac{ft^3}{lb}\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right| - (530°R)(1.7567-0.07463)\frac{Btu}{lb\cdot°R}\right]$$

$$= (1\,lb)\left[1039.51 + 72.87 - 891.50\right]\frac{Btu}{lb} = 220.88\ Btu \longleftarrow (a)$$

(b) Saturated liquid water at 40°F. The values of u_0, v_0, s_0 are the same as in part (a). From Table A-2E, $u = 8.02\ Btu/lb$, $v = 0.01602\ ft^3/lb$, $s = 0.01617\ Btu/lb°R$. Thus

$$\mathsf{E} = (1)\left[(8.02-38.09) + (14.7)(144)(0.01602-0.01605)\left|\frac{1}{778}\right| - (530)(0.01617-0.07463)\right]$$

① $$= (1\,lb)\left[-30.07 - 8.16\times10^{-5} + 30.98\right]\frac{Btu}{lb} = 0.914\ Btu \longleftarrow (b)$$

(c) Ammonia at −40°F, 6 lbf/in². From Table A-15

at 70°F, 14.7 lbf/in²	at −40°F, 6 lbf/in²
$v_0 = 22.55\ ft^3/lb$	$v = 43.533\ ft^3/lb$
$u_0 = 592.00\ Btu/lb$	$u = 551.03\ Btu/lb$
$s_0 = 1.5020\ Btu/lb\cdot°R$	$s = 1.4915\ Btu/lb\cdot°R$

$$\mathsf{E} = (1)\left[(551.03-592.00) + (14.7)(144)(43.533-22.55)\left|\frac{1}{778}\right| - (530)(1.4915-1.5020)\right]$$

$$= (1\,lb)\left[-40.97 + 57.09 + 5.57\right]\frac{Btu}{lb} = 21.69\ Btu \longleftarrow (c)$$

---

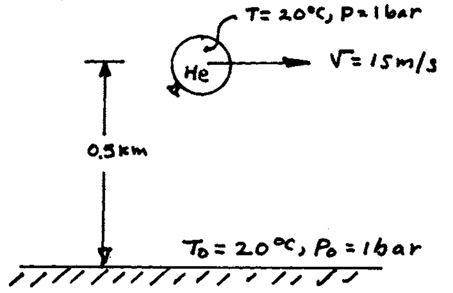
1. Even though $T < T_0$, the value of exergy is positive, as expected.

PROBLEM 7.5

KNOWN: A balloon filled with helium is at a specified state.

FIND: Determine the specific exergy.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The closed system is the helium within the baloon. (2) Helium is modeled as an ideal gas. (3) $g = 9.807\ m/s^2$.

ANALYSIS: The specific exergy is given by Eq. 7.9

$$e = u - u_0 + p_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

For a monatomic gas such as Helium, the specific heats c_p and c_v are constant*, so with ideal gas relations

$$e = \underline{c_v[T - T_0] + p_0\left[\frac{RT}{p} - \frac{RT_0}{p_0}\right] - T_0\left[c_p \ln\frac{T}{T_0} - R\ln\frac{p}{p_0}\right]} + \frac{V^2}{2} + gz$$

Since $T = T_0$ and $p = p_0$, the underlined term vanishes, leaving

$$e = \frac{V^2}{2} + gz$$

$$= \left[\frac{(15\ m/s)^2}{2} + (9.807\ m/s^2)(500\ m)\right]\left|\frac{1\ N}{1\ kg\cdot\frac{m}{s^2}}\right|\left|\frac{kJ}{10^3\ N\cdot m}\right|$$

$$= 5.016\ \frac{kJ}{kg} \longleftarrow e$$

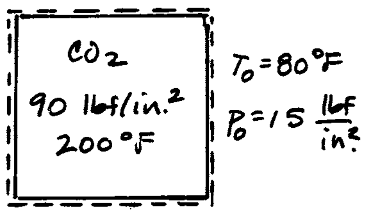
---

* See Tables A-21

PROBLEM 7.6

KNOWN: A vessel contains carbondioxide (CO_2).

FIND: (a) Determine specific exergy e at $p = 90$ lbf/in.², $T = 200°F$. (b) Plot e versus pressure for $T = 80°F$. (c) Plot e versus T for $p = 15$ lbf/in.²



ASSUMPTIONS: (1) The CO_2 is a closed system at rest and zero elevation relative to the reference environment at $T_0 = 80°F$, $p_0 = 15$ lbf/in.² (2) The CO_2 is modeled as an ideal gas.

ANALYSIS: Equation 7.9 reduces to give

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) \qquad (1)$$

(a) When using gas table data, $u - u_0 = \dfrac{\bar{u}(T) - \bar{u}(T_0)}{M}$, $s - s_0 = \dfrac{\bar{s}°(T) - \bar{s}°(T_0) - \bar{R}\ln p/p_0}{M}$

Thus, with ideal gas relations, (1) becomes

$$e = \frac{1}{M}\left\{[\bar{u}(T) - \bar{u}(T_0)] + p_0\left[\frac{\bar{R}T}{p} - \frac{\bar{R}T_0}{p_0}\right] - T_0\left[\bar{s}°(T) - \bar{s}°(T_0) - \bar{R}\ln\frac{p}{p_0}\right]\right\}$$

$$= \frac{1}{M}\left\{[\bar{u}(T) - \bar{u}(T_0)] + \bar{R}T\left[\frac{p_0}{p} - \frac{T_0}{T}\right] - T_0\left[\bar{s}°(T) - \bar{s}°(T_0) - \bar{R}\ln\frac{p}{p_0}\right]\right\}$$

Using data from Table A-23E

$$\bar{u}(T) - \bar{u}(T_0) = 3854.6 - 2984.4 = 870.2 \text{ Btu/lbmol}$$

$$\bar{R}T\left[\frac{p_0}{p} - \frac{T_0}{T}\right] = \left(1.986 \frac{\text{Btu}}{\text{lbmol}\cdot°R}\right)(660°R)\left[\frac{15}{90} - \frac{540}{660}\right] = -853.98 \text{ Btu/lbmol}$$

$$T_0\left[\bar{s}°(T) - \bar{s}°(T_0) - \bar{R}\ln p/p_0\right] = (540°R)\left[52.934 - 51.082 - (1.986)\ln\frac{90}{15}\right]\frac{\text{Btu}}{\text{lbmol}\cdot°R}$$
$$= -921.47 \text{ Btu/lbmol}$$

Finally

$$e = \left(\frac{1}{44.01 \frac{\text{lb}}{\text{lbmol}}}\right)\left\{(870.2 - 853.98 + 921.47)\right\}\frac{\text{Btu}}{\text{lbmol}} = 21.31 \frac{\text{Btu}}{\text{lb}} \longleftarrow e$$

(b)(c) The following IT code is used to generate data for the required plots. The evaluations are based on Eq.(1) above and internal functions in IT for u, v, and s of CO_2 as an ideal gas.

IT Code

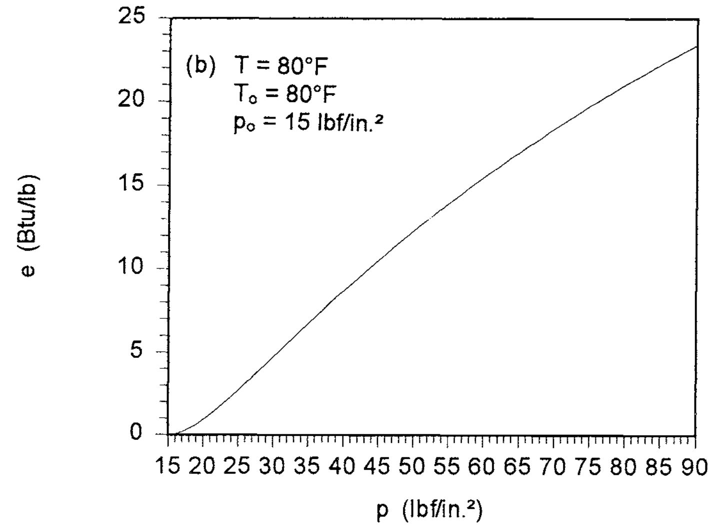
```
T = 200  // °F
p = 90  // lbf/in.²
To = 80  // °F
po = 15  // lbf/in.²

e = (u - uo) + (po * (144 / 778)) * (v - vo) - (To + 460) * (s - so)
u = u_T("CO2", T)
uo = u_T("CO2", To)
v = v_TP("CO2",T,p)
vo = v_TP("CO2",To,po)
s = s_TP("CO2", T, p)
so = s_TP("CO2", To, po)
```

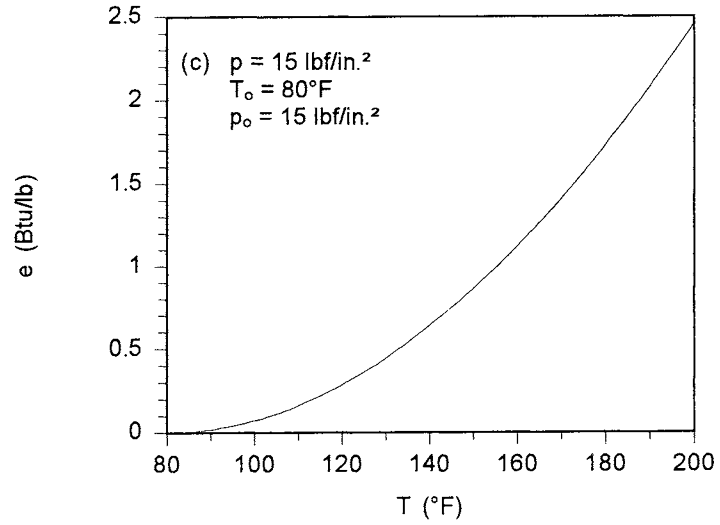
IT Result for p = 90 lbf/in.², T = 200 °F

① e = 21.32 Btu/lb

PROBLEM 7.6 (Cont'd.)

PLOTS:





---

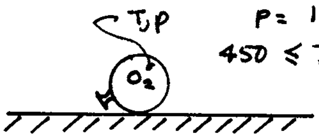
1. The IT result compares very favorably with the ideal gas table result in part (a).

PROBLEM 7.7

KNOWN: Oxygen (O_2) at T, 1atm fills a balloon at rest on the surface of the earth where the ambient temperature and pressure are known.

FIND: Plot specific exergy versus T.

SCHEMATIC & GIVEN DATA:



$p = 1\text{atm}$, $p_0 = 1\text{atm}$, $T_0 = 500°R$

$450 \leq T \leq 600°R$

ASSUMPTIONS: (1) The O_2 is at rest and zero elevation relative to the environ-
① ment for which $T_0 = 500°R$, $p_0 = 1\text{atm}$. (2) O_2 is modeled as an ideal gas with constant c_p.

ANALYSIS: Equation 7.9 reduces to

$$e = u - u_0 + p_0(v - v_0) - T_0(s - s_0)$$

With ideal gas relations, and noting that $p = p_0$

$$a = u(T) - u(T_0) + p_0\left(\frac{RT}{p} - \frac{RT_0}{p_0}\right) - T_0\left(s°(T) - s°(T_0) - \cancelto{0}{R\ln\frac{p}{p_0}}\right)$$

$$= u(T) - u(T_0) + R(T - T_0) - T_0(s°(T) - s°(T_0)) \qquad (1)$$

By inspection of Table A-20E, over the range $460 < T < 600K$, $c_p \cong 0.22\,\text{Btu/lb}\cdot°R$.
Thus, Eq.(1) can be expressed as

$$e = c_v[T - T_0] + R[T - T_0] - T_0 c_p \ln\frac{T}{T_0}$$

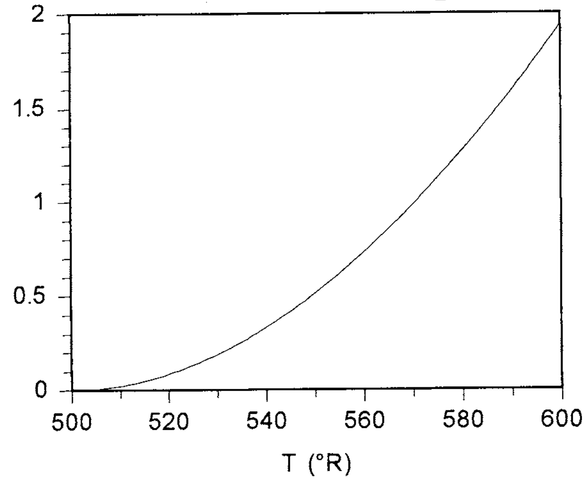
$$= \underbrace{(c_v + R)}_{c_p\ (\text{Eq. } 3.44)}[T - T_0] - T_0 c_p \ln\frac{T}{T_0}$$

or

$$e = T_0 c_p\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right]$$

$$= \underbrace{\left(0.22\,\frac{\text{Btu}}{\text{lb}\cdot°R}\right)(500°R)}_{110\,\text{Btu/lb}}\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right] = \left(110\,\frac{\text{Btu}}{\text{lb}}\right)\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right] \qquad (2)$$

SAMPLE CALCULATION: $T = 600°R$, $e = (110\,\text{Btu/lb})\left[\frac{600}{500} - 1 - \ln\frac{600}{500}\right] = 1.945\,\text{Btu/lb}$



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1. Alternatively, $(u - u_0)$ and $(s - s_0)$ can be evaluated using data from Table A-23E.

PROBLEM 7.8

KNOWN: A vessel contains a known amount of air at pressure p and 200°F.

FIND: Plot the specific exergy versus p ranging from 0.5 to 2 atm.

SCHEMATIC & GIVEN DATA:

$T = 200°F$
$0.5 \leq p \leq 2$ atm
$T_0 = 60°F$
$p_0 = 1$ atm

ASSUMPTIONS: (1) The air is a closed system, at rest and zero elevation relative to the reference environment which is at $T_0 = 60°F$, $p_0 = 1$ atm. (2) The air is modeled as an ideal gas.

ANALYSIS: Equation 7.9 reduces to $e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$ (1)

SAMPLE CALCULATION: This calculation will be done using gas table data. That is, $u - u_0 = u(T) - u(T_0)$, $s - s_0 = s°(T) - s°(T_0) - R \ln p/p_0$. With the ideal gas equation of state

$$e = u(T) - u(T_0) + p_0\left(\frac{RT}{p} - \frac{RT_0}{p_0}\right) - T_0\left[s°(T) - s°(T_0) - R \ln p/p_0\right]$$

Using data from Table A-22E, for the case $T = 200°F$, $p = 0.5$ atm

$$u(T) - u(T_0) = 112.67 - 88.62 = 24.05 \text{ Btu/lb}$$

$$p_0 R\left(\frac{T}{p} - \frac{T_0}{p_0}\right) = (1 \text{ atm})\left(\frac{1.986}{28.97}\frac{\text{Btu}}{\text{lb}\cdot°R}\right)\left(\frac{660°R}{0.5 \text{ atm}} - \frac{520°R}{1 \text{ atm}}\right) = 54.843 \text{ Btu/lb}$$

$$T_0\left[s°(T) - s°(T_0) - R \ln p/p_0\right] = (520°R)\left[(0.64902 - 0.59172) - \left(\frac{1.986}{28.97}\right)\ln\left(\frac{0.5}{1}\right)\right]\frac{\text{Btu}}{\text{lb}\cdot°R}$$
$$= 54.505 \text{ Btu/lb}$$

Finally

$$e = 24.05 + 54.843 - 54.505 = 24.388 \text{ Btu/lb}$$

Using *IT* to generate the data required, the evaluations are based on Eq. (1) above and internal *IT* functions for u, v, and s, as follows:

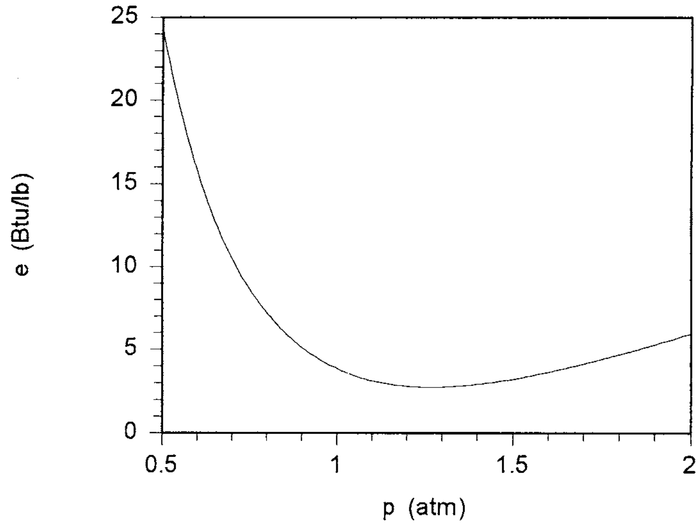
IT Code

```
T = 100  // °F
p = 0.5  // atm
To = 60 // °F
po = 1  // atm

e = (u - uo) + po * (v - vo) * (14.696*144/778) - (To + 460) * (s - so)
u = u_T("Air", T)
uo = u_T("Air", To)
v = v_TP("Air",T,p)
vo = v_TP("Air",To,po)
s = s_TP("Air", T, p)
so = s_TP("Air", To, po)
```

IT Result for p = 0.5 atm, T = 200°F

① e = 24.37 Btu/lb



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1. The *IT* result is in close agreement with the sample calculation based on Table A-22E, as expected.

PROBLEM 7.9

KNOWN: A state is specified for each of three substances.

FIND: Determine the specific exergy

ASSUMPTIONS: 1. Ignore the effects of motion and gravity. 2. $T_0 = 0°C$, $P_0 = 1\,bar$.

ANALYSIS: (a) Ammonia at 0.6 bar, −10°C.

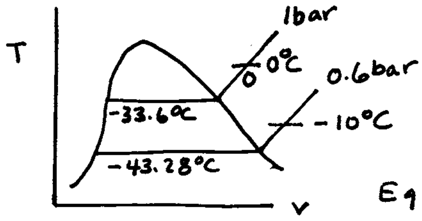


Table A-15

$u_0 = 1341\ kJ/kg$ $u = 1327.37$

$v_0 = 1.3136\ m^3/kg$ $v = 2.1188$

$s_0 = 6.1281\ kJ/kg\cdot K$ $s = 6.3077$

Equation 7.9 reduces with assumption 1 to give

$$e = (u-u_0) + P_0(v-v_0) - T_0(s-s_0)$$

$$= (1327.37-1341)\frac{kJ}{kg} + \left(10^5\frac{N}{m^2}\right)\left[2.1188-1.3136\right]\frac{m^3}{kg}\left|\frac{1\,kJ}{10^3 N\cdot m}\right| - 273K\left[6.3077-6.1281\right]\frac{kJ}{kg\cdot K}$$

$$= \left[(-13.63) + (80.52) - (49.03)\right]\frac{kJ}{kg} = 17.86\ kJ/kg \quad \longleftarrow \text{(a)}$$

(b) R22 at 0.6 bar, −10°C

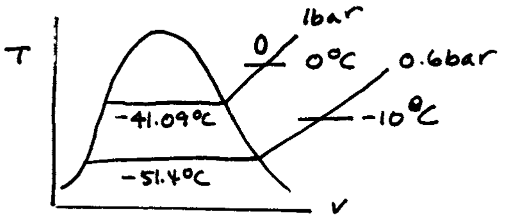


Table A-9

$u_0 = 232.36\ kJ/kg$ $u = 227.65$

$v_0 = 0.25747\ m^3/kg$ $v = 0.41608$

$s_0 = 1.1035\ kJ/kg\cdot K$ $s = 1.1314$

$$e = (u-u_0) + P_0(v-v_0) - T_0(s-s_0)$$

$$= (227.65-232.36) + (10^5)\left[0.41608-0.25747\right]\left|\frac{1}{10^3}\right| - 273(1.1314-1.1035)$$

$$= (-4.71) + (15.86) - (7.62) = 3.53\ kJ/kg \quad \longleftarrow \text{(b)}$$

(c) R134a at 0.6 bar, −10°C

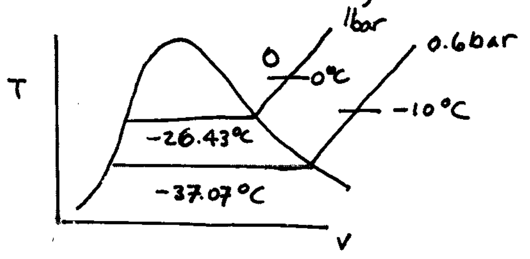


Table A-12

$u_0 = 231.41\ kJ/kg$ $u = 224.97$

$v_0 = 0.21587\ m^3/kg$ $v = 0.34992$

$s_0 = 1.0227\ m^3/kg$ $s = 1.0371$

$$e = (u-u_0) + P_0(v-v_0) - T_0(s-s_0)$$

$$= \left[224.97-231.41\right] + 10^5\left[0.34992-0.21587\right]\left|\frac{1}{10^3}\right| - 273(1.0371-1.0227)$$

$$= (-6.44) + (13.41) - (3.93) = 3.04\ kJ/kg \quad \longleftarrow \text{(c)}$$

PROBLEM 7.10

KNOWN: A two-phase solid-vapor mixture of H_2O is at a known temperature. The mass of each phase is the same.

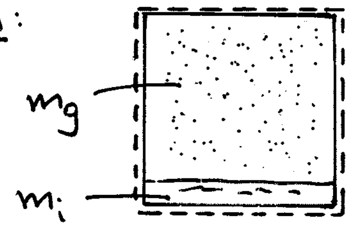
FIND: Determine the specific exergy.

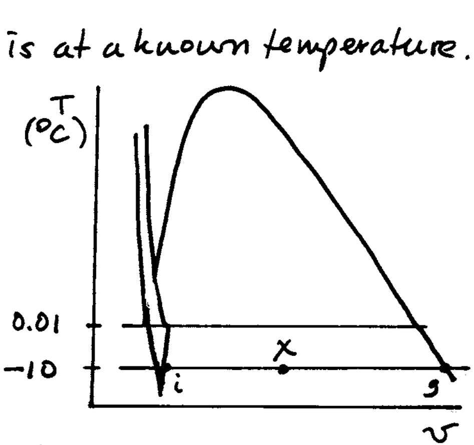
SCHEMATIC & GIVEN DATA:

$$x = \frac{m_g}{m_g + m_i} = 0.5$$

$T = -10°C$

$T_0 = 20°C$, $p_0 = 1\,atm$





ASSUMPTIONS: (1) The two-phase solid-vapor mixture is a closed system at rest and zero elevation relative to the reference environment at $T_0 = 20°C$, $p_0 = 1\,atm$.

ANALYSIS: Equation 7.9 reduces to

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) \qquad (1)$$

Using $u_0 \approx u_f(T_0)$, $v_0 \approx v_f(T_0)$, $s_0 \approx s_f(T_0)$ and data from Table A-2

$$u_0 = 83.95\ kJ/kg\ ,\ v_0 = 1.0018 \times 10^{-3}\ m^3/kg,\ s_0 = 0.2966\ kJ/kg\cdot K$$

Defining "quality" x as above, and using data from Table A-6

$$u = u_i + x(u_g - u_i) = -354.09 + (.5)(2715.5) = 1003.66\ kJ/kg$$

$$v = v_i + x(v_g - v_i) = 1.0891 \times 10^{-3}\ \frac{m^3}{kg} + (0.5)(466.7 - 1.0891 \times 10^{-3})\ \frac{m^3}{kg}$$

$$= 233.35\ m^3/kg$$

$$s = s_i + x(s_g - s_i) = -1.299 + (0.5)(10.781) = 4.0915\ kJ/kg\cdot K$$

Inserting values in (1)

$$e = (1003.66 - 83.95)\frac{kJ}{kg} + \left(101.325\ \frac{kN}{m^2}\right)(233.35 - 1.0018 \times 10^{-3})\ \frac{m^3}{kg}\left|\frac{1\ kJ}{1\ kN\cdot m}\right|$$

$$- (273 - 10)\,K\,(4.0915 - 0.2966)\ \frac{kJ}{kg\cdot K}$$

① $$= 919.71 + 23644 - 998.1 = 23{,}566\ kJ/kg \longleftarrow e$$

---

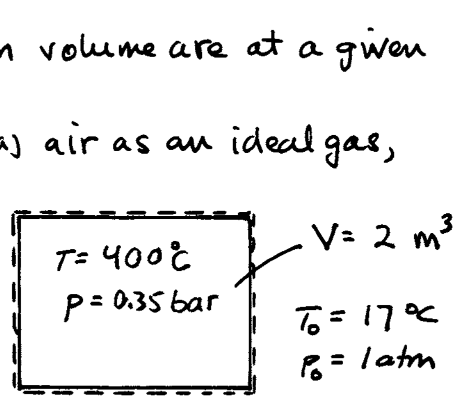
1. The large value indicates that a two-phase solid-vapor mixture of H_2O has a great deal of exergy relative to the dead state of 20°C, 1 atm.

PROBLEM 7.11*

KNOWN: The contents of a storage tank of known volume are at a given temperature and pressure.

FIND: Determine the exergy if the contents are (a) air as an ideal gas, (b) water vapor.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The tank contents are a closed system, and the effects of motion and gravity can be ignored. (2) The environment is at $T_0 = 17\,°C$, $p_0 = 1$ atm. (3) The air is modeled as an ideal gas.

ANALYSIS: With Eq. 7.9 and assumption (1)

$$E = m\left[(u-u_0) + p_0(v - v_0) - T_0(s - s_0)\right] \qquad (1)$$

(a) Introducing ideal gas relations, Eq. (1) becomes

$$E = m\left[u(T) - u(T_0) + RT\left(\frac{p_0}{p} - \frac{T_0}{T}\right) - T_0\left(s°(T) - s°(T_0) - R\ln p/p_0\right)\right]$$

Using the ideal gas equation of state

$$m = \frac{pV}{RT} = \frac{(0.35\,\text{bar})(2\,\text{m}^3)}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(673\,\text{K})}\left|\frac{10^5\,\text{N/m}^2}{1\,\text{bar}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| = 0.362\,\text{kg}$$

Now, with data from Table A-22

$$E = (0.362\,\text{kg})\left[(684.34 - 206.91)\frac{\text{kJ}}{\text{kg}} + \left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(673\,\text{K})\left(\frac{1.01325}{0.35} - \frac{290}{673}\right) - (290\,\text{K})\left\{(2.5306 - 1.66802) - \frac{8.314}{28.97}\ln\left(\frac{0.35}{1.01325}\right)\right\}\right]$$

$$= 617.7\,\text{kJ} \longleftarrow \text{(a)}$$

(b) For water, $u_0 \approx u_f(T_0)$, $v_0 \approx v_f(T_0)$, $s_0 \approx s_f(T_0)$. Thus, From Table A-2

$u_0 = 71.38$ kJ/kg, $v_0 = 1.0012\times10^{-3}$ m³/kg, $s_0 = 0.2535$ kJ/kg·K

Table A-4 gives

$u = 2968.6$ kJ/kg, $v = 8.872$ m³/kg, $s = 9.0291$ kJ/kg·K

The mass is: $m = V/v = (2\,\text{m}^3)/(8.872\,\text{m}^3/\text{kg}) = 0.225$ kg. Inserting values in Eq. (1)

$$E = (0.225\,\text{kg})\left[(2968.6 - 71.38)\frac{\text{kJ}}{\text{kg}} + \left(1.01325\times10^5\,\frac{\text{N}}{\text{m}^2}\right)(8.872 - 1.0012\times10^{-3})\frac{\text{m}^3}{\text{kg}}\left|\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| - (290\,\text{K})(9.0291 - 0.2535)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]$$

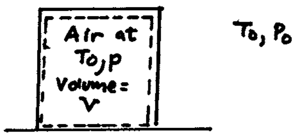
$$= 1251.2\,\text{kJ} \longleftarrow \text{(b)}$$

PROBLEM 7.12

KNOWN: Air as an ideal gas is stored in a closed vessel at pressure p, volume V, and temperature T_0.

FIND: (a) Obtain an expression for the exergy of the air in a specified form. (b) Use the result of (a) to plot V versus p/p_0.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The effects of motion and gravity can be ignored. (2) Air is modeled as an ideal gas.

ANALYSIS: (a) Ignoring motion and gravity, Eq. 7.2 takes the form

$$E = \overset{0}{\cancel{(U-U_0)}} + p_0(V-V_0) - T_0(S-S_0)$$

The first term drops out because the internal of an ideal gas depends on temperature only and $T = T_0$. Also,

$$S-S_0 = m\left[\overset{0}{\cancel{\int_{T_0}^{T}\frac{c_p}{T}dT}} - R\ln\frac{p}{p_0}\right] = -mR\ln\frac{p}{p_0}$$

Accordingly

$$E = p_0V\left[1-\frac{V_0}{V}+\frac{mRT_0}{p_0V}\ln\frac{p}{p_0}\right]$$

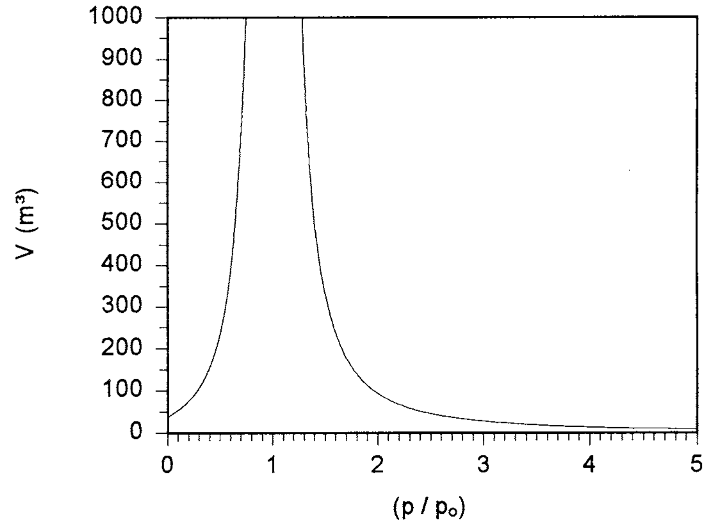
Since $T = T_0$, the ideal gas equation of state: $p = mRT_0/V$ gives $V_0/V = p/p_0$, so

$$E = p_0V\left[1-\frac{p}{p_0}+\frac{p}{p_0}\ln\frac{p}{p_0}\right] \longleftarrow E$$

(b) Using the above expression

$$V = \frac{E}{p_0\left[1-\frac{p}{p_0}+\frac{p}{p_0}\ln\frac{p}{p_0}\right]} = \frac{(1\ \text{kW}\cdot\text{h})\left|\frac{3600\ \text{kJ}}{1\ \text{kW}\cdot\text{h}}\right|\left|\frac{10^3\ \text{N}\cdot\text{m}}{\text{kJ}}\right|}{10^5\frac{\text{N}}{\text{m}^2}\left[1-\frac{p}{p_0}+\frac{p}{p_0}\ln\frac{p}{p_0}\right]}$$

$$= \frac{36\ \text{m}^3}{\left[1-\frac{p}{p_0}+\frac{p}{p_0}\ln\frac{p}{p_0}\right]}$$



$\lim_{p/p_0 \to 0} \quad V \to 36\ \text{m}^3$

$\lim_{p/p_0 \to 1} \quad V \to \infty$

$\lim_{p/p_0 \to \infty} \quad V \to 0$

PROBLEM 7.12 (Contd.)

For a fixed exergy value, the tank size depends on the pressure:

- As $p/p_0 \rightarrow \infty$, the required tank volume would become very small owing to the large difference between p and p_0. That is, with a large pressure difference not much air would be needed for a specified exergy value.
- As $p/p_0 \rightarrow 1$, the required tank volume would become very large owing to the small difference between p and p_0. That is, with a small pressure difference a large quantity of air would be needed for a specified exergy value.
- As $p/p_0 \rightarrow 0$, the tank volume approaches a constant as the tank holds less and less air. In the limit the tank would be evacuated, illustrating that an evacuated space has a nonzero value for exergy. We can think of this as the minimum theoretical work that would be required to create the evacuated space. See the 4th bulleted item in the discussion of Sec. 7.2.4.

PROBLEM 7.13

KNOWN: An ideal gas is stored in a closed vessel at p, T.

FIND: (a) If $T = T_0$, derive $e = f(p, p_0, T_0, R)$

(b) If $p = p_0$, derive $e = f(T, T_0, c_p)$

ASSUMPTIONS: (1) The gas obeys the ideal gas model. In part (b), c_p is constant as well. (2) The effects of motion and gravity can be ignored.

ANALYSIS:

(a) With assumptions (1), (2), Eq. 7.9 becomes

$$e = \cancelto{0}{(u-u_0)} + p_0(v-v_0) - T_0(s-s_0)$$

The first term vanishes because u of an ideal gas depends only on temperature, and $T = T_0$. Also,

$$s - s_0 = (\cancelto{0}{s^\circ(T) - s^\circ(T_0)}) - R \ln \frac{p}{p_0}$$

Collecting results

$$e = p_0(v-v_0) + RT_0 \ln \frac{p}{p_0}$$

With $v = RT_0/p$ and $v_0 = RT_0/p_0$

$$e = p_0\left[\frac{RT_0}{p} - \frac{RT_0}{p_0}\right] + RT_0 \ln \frac{p}{p_0}$$

$$= RT_0\left[\frac{p_0}{p} - 1 + \ln \frac{p}{p_0}\right] \quad \longleftarrow$$

(b) With assumption (2), Eq. 7.9 becomes

$$e = u - u_0 + p_0(v-v_0) - T_0(s-s_0)$$

Introducing ideal gas relations, with $p = p_0$

$$e = u(T) - u(T_0) + p_0\left[\frac{RT}{p_0} - \frac{RT_0}{p_0}\right] - T_0\left[\int_{T_0}^{T} \frac{c_p}{T}\,dT - R \cancelto{0}{\ln \frac{p_0}{p_0}}\right]$$

If c_p is constant, so is c_v and the two differ by the gas constant R. Thus

$$e = c_v[T - T_0] + R[T - T_0] - T_0\, c_p \ln \frac{T}{T_0}$$

$$= \underbrace{(c_v + R)}_{c_p}[T - T_0] - T_0\, c_p \ln \frac{T}{T_0}$$

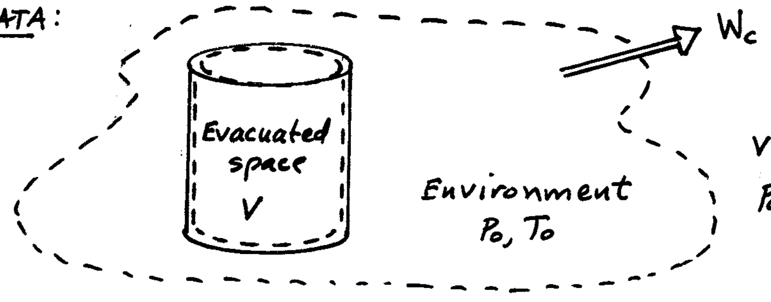
$$= c_p T_0\left[\frac{T}{T_0} - 1 - \ln \frac{T}{T_0}\right] \quad \longleftarrow$$

PROBLEM 7.14

KNOWN: An evacuated container of known volume is under consideration.

FIND: For the space inside the tank, determine the exergy.

SCHEMATIC & GIVEN DATA:



$V = 10 \text{ ft}^3$

$p_0 = 14.7 \text{ lbf/in.}^2$

ASSUMPTIONS: (1) A combined system is considered consisting of the evacuated space (a closed system) and the environment. (2) The volume of the combined system is constant. (3) For the combined system, $Q = 0$. (4) For the environment, p_0 and T_0 are constant. (5) The effects of gravity and motion are ignored.

ANALYSIS: Consider a process where the evacuated space collapses and work is developed by the combined system. An energy balance for the combined system reduces with assumption 3 to give

$$W_c = -\Delta E_c$$

Since $\Delta U = 0$ for the evacuated space, $\Delta E_c = \Delta U_e$. With Eq. 7.1

$$W_c = -[T_0 \Delta S_e - p_0 \Delta V_e]$$

By assumption 2, $\Delta V_e = -\Delta V = -(\cancel{V_0}^{\,0} - V) = V$. Further, an entropy balance for the combined system gives

$$\Delta S_c = \sigma_c \Rightarrow \cancel{\Delta S}^{\,0} + \Delta S_e = \sigma_c$$

Collecting results

$$W_c = p_0 V - T_0 \sigma_c$$

Since $\sigma_c \geq 0$, the maximum theoretical work, or exergy, is obtained when $\sigma_c = 0$. Thus

$$\mathbb{E} = p_0 V \tag{1}$$

Inserting values

$$\mathbb{E} = \left(14.7 \frac{\text{lbf}}{\text{in.}^2}\right)(10 \text{ ft}^3) \left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right| \left|\frac{1 \text{ Btu}}{778 \text{ ft} \cdot \text{lbf}}\right| = 27.21 \text{ Btu} \longleftarrow \mathbb{E}$$

PROBLEM 7.15

KNOWN: Equal molar amounts of CO_2 and He are at the same T, p.

FIND: Determine which gas has the greater exergy value, $\bar{e}$.

ASSUMPTIONS: (1) Each gas obeys the ideal gas model with constant $\bar{c}_v$.
(2) There are no significant effects of motion and gravity.

ANALYSIS: With assumption (2), Eq. 7.9 reduces to give on a molar basis

① $$\bar{e} = \bar{u} - \bar{u}_0 + p_0(\bar{v} - \bar{v}_0) - T_0(\bar{s} - \bar{s}_0)$$

Then, with assumption (1)

$$\bar{e} = \bar{c}_v(T - T_0) + \bar{R}T_0\left[\frac{T}{T_0}\cdot\frac{p_0}{p} - 1\right] - T_0\left[\bar{c}_p \ln\frac{T}{T_0} - \bar{R}\ln\frac{p}{p_0}\right]$$

$$= \bar{c}_v(T - T_0) - T_0\bar{c}_p \ln\frac{T}{T_0} + \bar{R}T_0\left\{\left[\frac{T}{T_0}\cdot\frac{p_0}{p} - 1\right] + \ln\frac{p}{p_0}\right\}$$

Applying this to each of CO_2 and He, and subtracting the resulting equations gives

$$\bar{e}_{CO_2} - \bar{e}_{He} \quad \underbrace{\left[\bar{c}_{v,CO_2} - \bar{c}_{v,He}\right]}_{\equiv(\bar{c}_{p,CO_2} - \bar{c}_{p,He})}(T - T_0) - \left[\bar{c}_{p,CO_2} - \bar{c}_{p,He}\right]T_0 \ln\frac{T}{T_0}$$

$$= \left[\bar{c}_{p,CO_2} - \bar{c}_{p,He}\right]\left[T - T_0 - T_0\ln\frac{T}{T_0}\right]$$

By inspection of Figure 3.13, $\bar{c}_{p,CO_2} > \bar{c}_{p,He}$, giving

② $$\bar{e}_{CO_2} > \bar{e}_{He} \qquad \longleftarrow$$

---

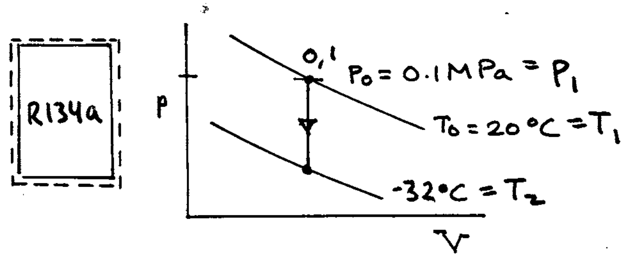
1. Here only the thermomechanical component is considered. The chemical contribution of Section 13.6 is not included in the present discussion.
2. But note that the molecular weights are much different: $M_{CO_2} = 44.01$, $M_{He} = 4.003$.

PROBLEM 7.16*

KNOWN: R134a vapor, initially at T_0, p_0, is cooled at fixed volume to a specified temperature.

FIND: For the R134a, evaluate Q/m and Δe.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is the R134a vapor, as shown in the figure. (2) For the system, $W = 0$ and there are no significant kinetic/potential energy effects. (3) For the exergy reference environment, $p_0 = 0.1$ MPa, $T_0 = 20°C$

ANALYSIS: An energy balance reduces to read $Q/m = u_2 - u_1$. With data from Table A-12 $v_1 = 0.23349$ m³/kg, $u_1 = 246.67$ kJ/kg, $s_1 = 1.0829$ kJ/kg·K. Then with $v_2 = v_1$ and data from Table A-10

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{0.23349 - (0.7172/10^3)}{0.2451 - (0.7172/10^3)} = 0.9525$$

and

$$u_2 = u_f + x_2 u_{fg} = 9.47 + 0.9525(199.54) = 199.53 \text{ kJ/kg}$$
$$s_2 = s_f + x_2 s_{fg} = 0.0401 + 0.9525(0.9055) = 0.9017 \text{ kJ/kg·K}$$

Then

$$\frac{Q}{m} = 199.53 - 246.67 - 47.14 \text{ kJ/kg} \longleftarrow Q/m$$

To find Δe, use Eq. 7.9 to obtain

$$\Delta e = \Delta u + \overset{0}{\cancel{p_0 \Delta v}} - T_0 \Delta s$$
$$= -47.14 \frac{\text{kJ}}{\text{kg}} - (293\text{K})(0.9017 - 1.0829)\frac{\text{kJ}}{\text{kg·K}}$$
$$= +5.95 \text{ kJ/kg} \longleftarrow \Delta a$$

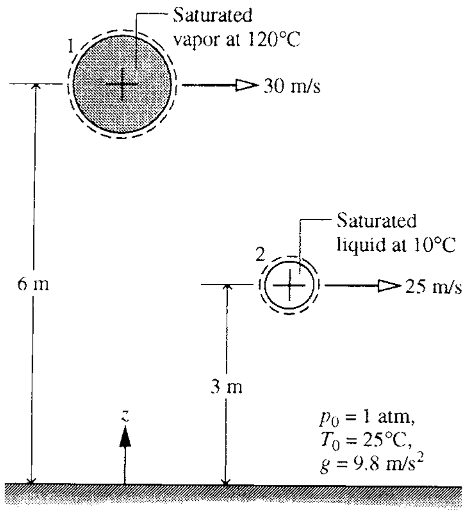
COMMENT: Here, despite the removal of energy by heat transfer the exergy increases. The exergy increases because the system is brought from the dead state to a state where T, p differ from T_0, p_0. The exergy at any state other than the dead state is strictly positive. For further comment, see the discussion of Figure 7.4.

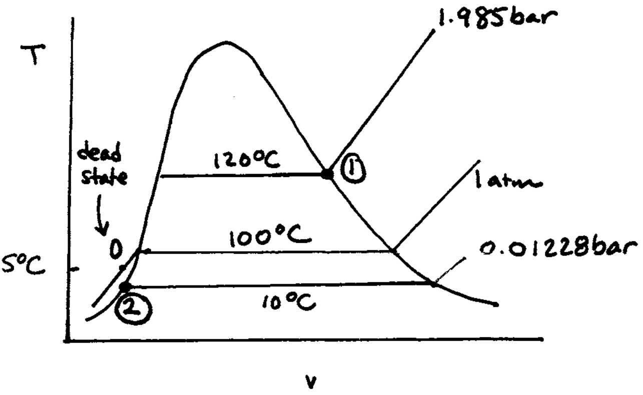
PROBLEM 7.17

KNOWN: Two kg of water undergo a process between specified states.

FIND: Determine the exergy at the initial and final states and the change in exergy.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The water is a closed system at equilibrium states initially and finally. 2. The velocities and elevations are measured relative to the environment. 3. $T_0 = 25°C$, $P_0 = 1atm$, $v_0 \sim v_f(T_0)$, $u_0 \sim u_f(T_0)$, $s_0 \sim s_f(T_0)$.

ANALYSIS: The exergy at the initial and final states can be calculated using Eq. 7.9:

① $$E = m\left[(u-u_0) + P_0(v-v_0) - T_0(s-s_0) + \frac{V^2}{2} + gz\right]$$

(a) For saturated vapor at 120°C, Table A-2 gives $v = 0.8919\ m^3/kg$, $u = 2529.3\ kJ/kg$, $s = 7.1296\ kJ/kg \cdot K$. Table A-2 gives $v_0 = 1.0029 \times 10^{-3}\ m^3/kg$, $u_0 = 104.88\ kJ/kg$, $S_0 = 0.3674\ kJ/kg \cdot K$. Thus,

$$E_1 = (2\,kg)\left\{\left[(2529.3-104.88)\frac{kJ}{kg} + \left(1.013\times10^5\frac{N}{m^2}\right)\left[0.8919 - \frac{1.0029}{10^3}\right]\frac{m^3}{kg}\left|\frac{1\,kJ}{10^3 N\cdot m}\right|\right.\right.$$
$$\left.\left. - (298\,K)[7.1296 - 0.3674]\frac{kJ}{kg\cdot K} + \left[\frac{[30\,m/s]^2}{2} + (9.8\,m/s^2)(6\,m)\right]\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|\left|\frac{1\,kJ}{10^3 N\cdot m}\right|\right]\right\}$$

$$= (2)[2424.42 + 90.25 - 2015.14 + 0.45 + 0.06]\,kJ = 1000.1\,kJ \quad \longleftarrow E_1$$

② (b) For saturated liquid at 10°C, $v = 1.0004\times10^{-3}\ m^3/kg$, $u = 42\ kJ/kg$, $s = 0.151\ kJ/kg\cdot K$. We get $E_2 = 3.9\ kJ$. $\longleftarrow E_2$

③ (c) The change in exergy is $\Delta E = E_2 - E_1 = 3.9 - 1000.1 = -996.2\ kJ \longleftarrow \Delta E$

---

1. The kinetic and potential energies measured relative to the environment contribute their full magnitudes to the value of exergy, for in principle each could be completely converted to work were the system brought to rest at zero elevation relative to the environment.

2. Exergy is a measure of the departure of the state of the system from that of the environment. At all states, $E \geq 0$. This applies when $T > T_0$, $P > P_0$, as in part (a), and when $T < T_0$, $P < P_0$, as in part (b).

3. Alternatively, Eq. 7.10 can be used. This requires dead state property values only for T_0, P_0. In parts (a), (b) u_0, v_0 and s_0 are also required; so more computation is needed with that approach.

PROBLEM 7.18

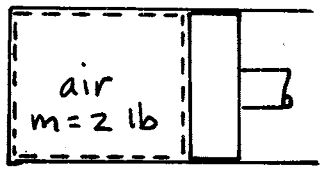
KNOWN: Air undergoes two specified processses in series.

FIND: (a) Represent each process on a p-v diagram and indicate the dead state. (b) Determine the change in exergy for each process.

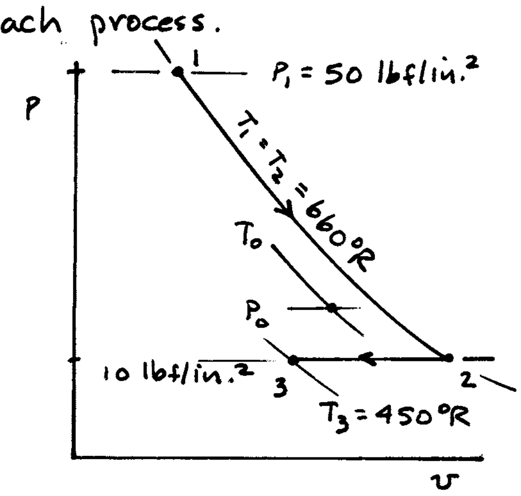
SCHEMATIC & GIVEN DATA:

Process 1-2: isothermal to $p_2 = 10$ lbf/in.²

Process 2-3: constant pressure to $T_3 = -10°F = 450°R$



$T_0 = 77°F = 537°R$
$p_0 = 14.7$ lbf/in.²



ASSUMPTIONS: (1) The air is the closed system. (2) The effects of motion and gravity can be ignored. (3) The air is modeled as an ideal gas. (4) For the environment, $T_0 = 77°F$, $p_0 = 14.7$ lbf/in.²

ANALYSIS: (a) From the given information, $p_1 = 50$ lbf/in.², $T_1 = 660°R$. Thus

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(660°R)}{(50\ \text{lbf/in.}^2)}\left|\frac{1\ \text{ft}^2}{144\ \text{in.}^2}\right| = 4.889\ \text{ft}^3/\text{lb}$$

Similarly, $p_2 = 10$ lbf/in.², $T_2 = T_1 = 660°R \Rightarrow v_2 = 24.44$ ft³/lb and $p_3 = p_2 = 10$ lbf/in.² and $T_3 = 450°R \Rightarrow v_3 = 16.67$ ft³/lb.

(b) Using Eq. 7.10 and ideal gas relations

$$\mathbb{E}_2 - \mathbb{E}_1 = m\left[\cancelto{0}{u(T_2) - u(T_1)} + p_0(v_2 - v_1) - T_0\left(\cancelto{0}{s°(T_2) - s°(T_1)} - R \ln p_2/p_1\right)\right]$$

$$\mathbb{E}_3 - \mathbb{E}_2 = m\left[u(T_3) - u(T_2) + p_0(v_3 - v_2) - T_0\left(s°(T_3) - s°(T_2) - \cancelto{0}{R \ln p_3/p_2}\right)\right]$$

With data from Table A-22E as needed

$$\mathbb{E}_2 - \mathbb{E}_1 = (2\ \text{lb})\left[\left(14.7\ \frac{\text{lbf}}{\text{in}^2}\right)(24.44 - 4.889)\frac{\text{ft}^3}{\text{lb}}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| - (537°R)\left(-\frac{1.986}{28.97}\ \frac{\text{Btu}}{\text{lb}\cdot°R}\ln\frac{10}{50}\right)\right]$$

$= -12.11$ Btu $\longleftarrow$ $\mathbb{E}_2 - \mathbb{E}_1$

$$\mathbb{E}_3 - \mathbb{E}_2 = (2)\left[(76.645 - 112.67) + (14.7)(16.67 - 24.44)\left|\frac{144}{778}\right| - (537)(.55704 - .64902)\right]$$

$= -15.55$ Btu $\longleftarrow$ $\mathbb{E}_3 - \mathbb{E}_2$

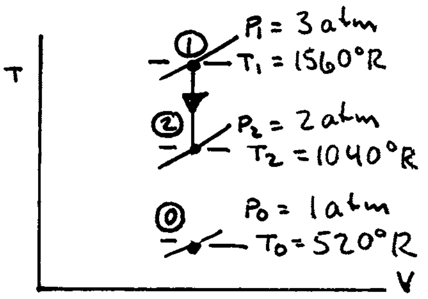
PROBLEM 7.19*

KNOWN: 1 lb of air fills a rigid tank. The air is cooled from a known initial state to a specified final state.

FIND: Locate the initial, final, and dead states on a T-v diagram. Evaluate the heat transfer and the exergy change, and interpret the sign of the exergy change using the p-v diagram.

SCHEMATIC & GIVEN DATA:

$T_1 = 1560°R$
$P_1 = 3\,atm$
$T_2 = 1040°R$
$P_2 = 2\,atm$



ASSUMPTIONS: 1. The air is the closed system. 2. For the system, $W = 0$ and the effects of motion and gravity can be ignored. 3. Air is modeled as an ideal gas. 4. $T_0 = 520°R$, $P_0 = 1\,atm$.

ANALYSIS: (a) Since volume and mass are fixed, the air undergoes a constant specific volume cooling process, as shown in the T-v diagram.

(b) Reducing an energy balance and using data from Table A-22E

$$m(u_2-u_1) = Q - \cancel{W}^{0} \Rightarrow Q = m(u_2-u_1) = (1\,lb)(179.66-278.13)\frac{Btu}{lb} = -98.47\,Btu \longleftarrow$$

(c) Using Eq. 7.10, the change in exergy is

$$E_2-E_1 = m\left[(u_2-u_1) + \underbrace{p_0(v_2-v_1)}_{=0 \text{ since } v_2=v_1} - T_0(s_2-s_1)\right] = m\left[(u_2-u_1) - T_0(s_2-s_1)\right]$$

where the kinetic/potential energy terms have been dropped by assumption 2. With Eq. 6.21a, this becomes

$$E_2-E_1 = m\left[(u_2-u_1) - T_0\left(s°(T_2) - s°(T_1) - R\ln\frac{p_2}{p_1}\right)\right]$$

$$= (1\,lb)\left[(179.66-278.13)\frac{Btu}{lb} - 520°R\left(0.76019 - 0.86456 - \frac{8.314}{28.97}\ln\frac{2}{3}\right)\frac{Btu}{lb°R}\right]$$

$$= -104.7\,Btu \longleftarrow$$

The T-v diagram shows that the state of the system is brought "closer" to the dead state as the air is cooled. Since the value of exergy is a measure of the departure of the state of the system from the state of the environment, exergy decreases in the process from 1 to 2.

---

1. Since $pv = RT$, $\frac{v}{R} = \frac{T}{p}$. Then, using given data we get

$$\frac{v_1}{R} = \frac{T_1}{P_1} = \frac{1560°R}{3\,atm} = 520, \quad \frac{v_2}{R} = \frac{T_2}{P_2} = \frac{1040°R}{2\,atm} = 520, \quad \frac{v_0}{R} = \frac{T_0}{T_0} = \frac{520°R}{1\,atm} = 520$$

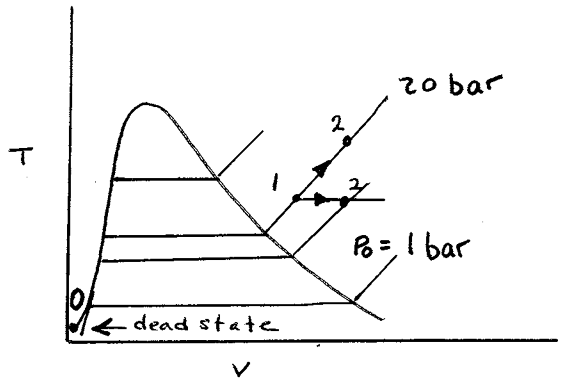
Thus, states 1, 2, and 0 fall as shown on the T-v diagram.

PROBLEM 7.20*

KNOWN: One kg of steam initially at 20 bar and 240°C undergoes two different processes.

FIND: For each process determine the change in exergy.

SCHEMATIC & GIVEN DATA:



$T_1 = 240°C$

1-2: constant pressure, $v_2 = 2\,v_1$

1-2': isothermal, $v_2 = 2\,v_1$

ASSUMPTIONS: (1) The system is the 1 kg of steam. (2) For the environment, $T_0 = 20°C$, $p_0 = 1$ bar. (3) Ignore the effects of motion and gravity.

ANALYSIS: Equation 7.10 reduces to give $E_2 - E_1 = m[u_2 - u_1 + p_0(v_2 - v_1) - T_0(s_2 - s_1)]$. From Table A-4, $u_1 = 2659.6$ kJ/kg, $v_1 = 0.1085$ m³/kg, $s_1 = 6.4952$ kJ/kg K. For each process $v_2 = 2\,v_1 = 0.217$ m³/kg.

(a) Constant pressure process: Interpolating in Table A-4 at 20 bar with $v_2 = 0.217$ m³/kg, $u_2 = 3423.1$ kJ/kg, $s_2 = 7.8849$ kJ/kg·K

$$E_2 - E_1 = (1\,\text{kg})\left[(3423.1 - 2659.6)\frac{\text{kJ}}{\text{kg}} + \left(100\frac{\text{kN}}{\text{m}^2}\right)(0.217 - 0.1085)\frac{\text{m}^3}{\text{kg}}\left|\frac{1\,\text{kJ}}{1\,\text{kN}\cdot\text{m}}\right| - 293\,\text{K}\,(7.8849 - 6.4952)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]$$

① $$= (1\,\text{kg})[763.5 + 10.9 - 407.2]\frac{\text{kJ}}{\text{kg}} = 367.2\ \text{kJ} \longleftarrow \text{(a)}$$

(b) Isothermal Process: Interpolating in Table A-4 at 240°C with $v_2 = 0.217$ m³/kg, $u_2 = 2690.8$ kJ/kg, $s_2 = 6.8527$ kJ/kg·K

$$E_2 - E_1 = (1)[(2690.8 - 2659.6) + (100)(0.1085) - 293\,(6.8527 - 6.4952)]$$

② $$= (1\,\text{kg})[31.2 + 10.9 - 104.7]\frac{\text{kJ}}{\text{kg}} = -62.6\ \text{kJ} \longleftarrow \text{(b)}$$

---

1. Here, pressure is kept constant and temperature increases relative to T_0. Exergy increases in the process.

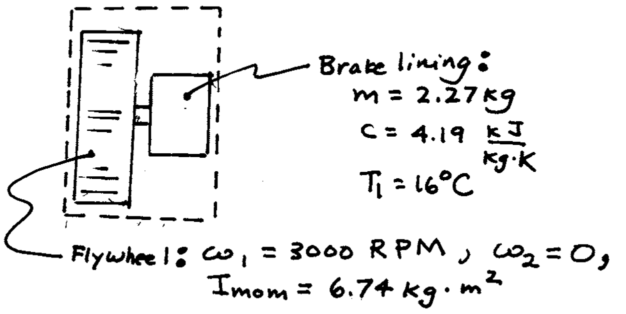
2. Here, temperature is kept constant and pressure decreases relative to P_0. Exergy decreases in the process.

PROBLEM 7.21

KNOWN: Data is provided for a flywheel braked to rest.

FIND: Determine the final temperature of the brake lining after braking to rest and the maximum theoretical rotational speed that could be attained by the flywheel using the energy stored in the flywheel.

SCHEMATIC & GIVEN DATA:


Brake lining:
$m = 2.27$ kg
$c = 4.19$ $\frac{kJ}{kg\cdot K}$
$T_1 = 16°C$
Flywheel: $\omega_1 = 3000$ RPM, $\omega_2 = 0$,
$I_{mom} = 6.74$ kg·m^2

Note:

$$\omega_1 = \left(3000\ \frac{\text{Rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)\left(\frac{1\text{min}}{60\text{s}}\right)$$

$$= 100\pi \ \text{rad/s}$$

ASSUMPTIONS: (1) For the system shown in the figure $W = Q = 0$ and $\Delta PE = 0$. (2) The brake lining is modeled as an incompressible substance with constant specific heat c. (3) For the environment, $T_0 = 16°C$.

ANALYSIS: (a) With $W = Q = \Delta PE = 0$, an energy balance reduces to read

$$\overset{\boxed{\text{brake lining}}}{\Delta U} + \overset{\boxed{\text{flywheel}}}{\Delta KE} = 0$$

or

$$mc[T_2 - T_1] + \left[0 - \tfrac{1}{2}\omega_1^2 I_{mom}\right] = 0$$

Thus

$$T_2 - T_1 = \frac{\frac{1}{2}\omega_1^2 I_{mom}}{mc} = \frac{\frac{1}{2}\left(100\pi\ \frac{\text{rad}}{\text{s}}\right)^2 (6.74\ \text{kg}\cdot\text{m}^2)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)}{(2.27\ \text{kg})\left(4.19\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left(\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right)}$$

$$= 35\text{K}$$

The final temperature of the brake lininng is then 324K (51°C). ← T_2

(b) As witnessed by the energy balance, energy is stored in the brake lining.

① However, in determining the maximum possible rotational speed achievable, it is the exergy stored, and not the energy stored, that is significant.

At the final state, the flywheel makes no contribution to the system exergy because the flywheel is at rest. The brake lining does contribute exergy because now $T_2 \neq T_0$. The final exergy is then found using incompressible substance relations as

$$E_2 = m[(u_2 - u_0) + \overset{0}{\cancel{p_0(v_2 - v_0)}} - T_0(s_2 - s_0)] = m[c(T_2 - T_0) - T_0 c \ln(T_2/T_0)]$$

$$= (2.27\text{kg})\left[4.19\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left[35\text{K} - 289 \ln\left(\frac{324}{289}\right)\right] = 18.67\ \text{kJ}$$

In principle, the exergy at the final state can be used to bring the flywheel into motion as the brake lining is restored to its initial temperature. Accordingly, the maximum theoretical rotational speed, ω_{MAX}, must satisfy $E_2 = \frac{1}{2}\omega_{MAX}^2 I_{mom}$, which is the maximum kinetic energy achievable. Then,

$$\omega_{MAX} = \sqrt{\frac{2E_2}{I_{mom}}} = \sqrt{\frac{(2)(18.67\ \text{kJ})}{6.74\ \text{kg}\cdot\text{m}^2}\left|\frac{10^3\ \text{N}\cdot\text{m}}{\text{kJ}}\right|\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|\left|\left(\frac{60\text{s}}{1\text{min}}\right)^2\right|}\ \left|\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right| = 711\ \text{RPM} \longleftarrow$$

---

1. Initially, the brake lining contributes nothing to the system exergy because the lining is at temperature $T_1 = T_0$. The initial system exergy equals the kinetic energy of the flywheel:

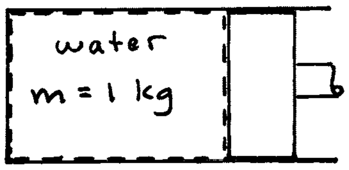
$$E_1 = \tfrac{1}{2}\omega_1^2 I_{mom} = \tfrac{1}{2}\left(100\pi\ \frac{\text{rad}}{\text{s}}\right)^2 (6.74\ \text{kg}\cdot\text{m}^2)\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right| = 332.61\ \text{kJ}.$$

PROBLEM 7.22*

KNOWN: One kilogram of water cools at constant pressure from an initial state to saturated liquid.

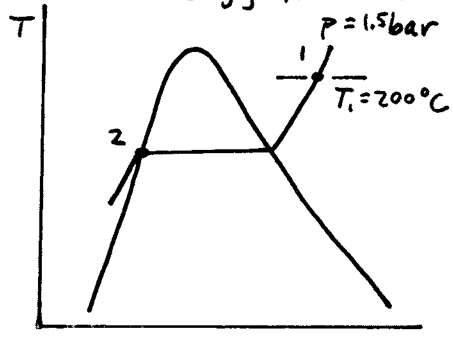
FIND: Determine the work, heat transfer, and amounts of exergy transfer accompanying heat and work.

SCHEMATIC & GIVEN DATA:



$p_1 = 1.5$ bar
$T_1 = 200°$C
p = const.
$x_2 = 0$

$T_0 = 20$ °C
$p_0 = 1$ bar



ASSUMPTIONS: (1) The water is the closed system. (2) The process occurs with no internal irreversibilities. (3) The effects of motion and gravity can be ignored. (4) $T_0 = 20°C$, $p_0 = 1$ bar.

ANALYSIS: To determine the work, we use Eq. 2.17

$$W = \int_1^2 p\,dV = m\,p(v_2 - v_1)$$

With data from Tables A-4 and A-3, respectively, $v_1 = 1.444$ m³/kg and $v_2 = 1.0528 \times 10^{-3}$ m³/kg

$$W = (1\text{ kg})(1.5\text{ bar})(1.0528\times10^{-3} - 1.444)\frac{\text{m}^3}{\text{kg}}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right| = -216.4\text{ kJ} \longleftarrow W$$

The heat transfer is determined by using the energy balance, which reduces to

$$Q = m(u_2 - u_1) + W$$

From Table A-4, $u_1 = 2656.2$ kJ/kg, and From Table A-3, $u_2 = 466.94$ kJ/kg. Thus

$$Q = (1\text{ kg})(466.94 - 2656.2)\text{ kJ/kg} + (-216.4\text{ kJ}) = -2405.7\text{ kJ} \longleftarrow Q$$

Now, the exergy transfer accompanying work, E_w, is

$$E_w = W - p_0\Delta V = W - m p_0(v_2 - v_1)$$
$$= (-216.4\text{ kJ}) - (1)(1)(1.0528\times10^{-3} - 1.444)\left|\frac{10^5}{10^3}\right| = -72.1\text{ kJ} \longleftarrow E_w$$

The exergy transfer accompanying heat transfer, is , is

$$E_Q = \int_1^2\left(1 - \frac{T_0}{T_b}\right)\delta Q = Q - T_0\int_1^2\frac{\delta Q}{T_b}$$

From an entropy balance: $m(s_2 - s_1) = \int_1^2 \frac{\delta Q}{T_b} + \cancel{\sigma}^{\,0}$. Thus

$$E_Q = Q - mT_0(s_2 - s_1)$$

With data from the appropriate tables

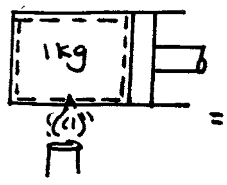
$$E_Q = (-2405.7\text{ kJ}) - (1\text{ kg})(293\text{ K})(1.4336 - 7.6433)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$
$$= -586.3\text{ kJ} \longleftarrow E_Q$$

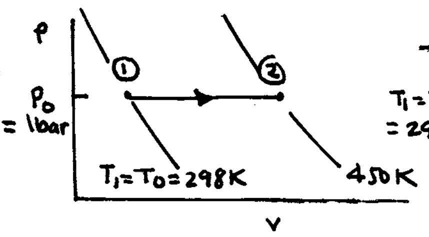
PROBLEM 7.23

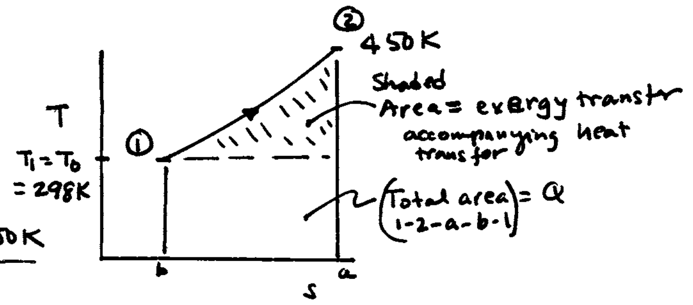
KNOWN: 1 kg of air is heated at constant pressure with no internal irreversibilities.

FIND: Determine the work, heat transfer, and the amounts of exergy transfer accompanying work and heat transfer.

SCHEMATIC & GIVEN DATA:






②
450K
T
①
$T_1 = T_0$ = 298K
Shaded Area = exergy transfer accompanying heat transfer
(Total area 1-2-a-b-1) = Q
b
a
s

ASSUMPTIONS: 1. The closed system is 1 kg of air, as shown in the schematic. 2. The process occurs at constant pressure without internal irreversibilities. 3. Air is modeled as an ideal gas. 4. $T_0 = 298K (25°C)$, $p_0 = 1$ bar 4. The effects of motion and gravity can be ignored.

ANALYSIS: Using the ideal gas equation of state, the work is

$$W = \int_1^2 p\,dV = m\,p(v_2 - v_1) = m R (T_2 - T_1) = (1\,kg)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(450 - 298)K = 43.62\,kJ \longleftarrow$$

Using Eq. 7.13, the accompanying exergy transfer is

$$\begin{bmatrix}\text{Exergy transfer} \\ \text{accompanying work}\end{bmatrix} = W - p_0 \Delta V = m p (v_2 - v_1) - m p_0 (v_2 - v_1)$$

$$= m(p - p_0)(v_2 - v_1) = 0, \text{ since } p = p_0. \longleftarrow$$

Reducing an energy balance for the process: $\Delta U = Q - W \Rightarrow Q = \Delta U + W$.
Or, with data from Table A-22

① $$Q = m(u_2 - u_1) + W = (1\,kg)[322.62 - 212.64]\frac{kJ}{kg} + 43.62$$
$$= 153.6\,kJ \longleftarrow$$

Using Eq. 7.12, the accompanying exergy transfer is

$$\begin{bmatrix}\text{Exergy transfer} \\ \text{accompanying heat}\end{bmatrix} = \int_1^2 \left[1 - \frac{T_0}{T_b}\right] \delta Q = Q - T_0 \underbrace{\int_1^2 \left(\frac{\delta Q}{T}\right)}_{\substack{= S_2 - S_1 \text{ since} \\ \text{the process is} \\ \text{without} \\ \text{internal irrevs.}}}$$

$$= Q - T_0[S_2 - S_1]$$

$$= Q - m T_0 [s°(T_2) - s°(T_1) - R \overset{\to 0}{\cancel{\ln}} p_2/p_1]$$

$$= 153.6\,kJ - (1\,kg)(298\,K)[2.11161 - 1.69528]\frac{kJ}{kg\cdot K}$$

$$= 29.53\,kJ \longleftarrow$$

---

1. Alternatively, for the constant pressure process

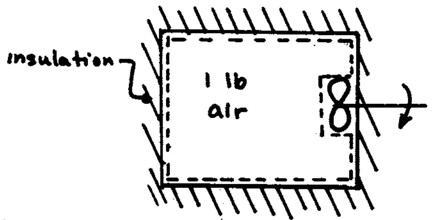
$$Q = m[u_2 - u_1] + m p (v_2 - v_1)$$
$$= m[h_2 - h_1] = 1\,kg(451.8 - 298.2)\frac{kJ}{kg} = 153.6\,kJ$$

PROBLEM 7.24

KNOWN: Air contained in a closed, rigid, insulated tank is stirred by a paddle wheel.

FIND: Determine the change in exergy, the transfer of exergy accompanying work, and the exergy destruction.

SCHEMATIC & GIVEN DATA:



$T_0 = 500°R$, $T_1 = 500°R$, $T_2 = 700°R$
$p_0 = 1 atm$, $p_1 = 1 atm$

ASSUMPTIONS: (1) For the system shown in the accompanying figure, $Q = 0$, and kinetic and potential energy effects can be ignored. (2) Air is modeled as an ideal gas. (3) For the environment $T_0 = 500°R$, $p_0 = 1 atm$.

ANALYSIS: With Eq. 7.10 the change in exergy is

$$E_2 - E_1 = m\left[(u_2 - u_1) + \overset{0}{\cancel{p_0(v_2 - v_1)}} - T_0(s_2 - s_1)\right]$$

$$= m\left[u(T_2) - u(T_1) - T_0\left(s°(T_2) - s°(T_1) - R \ln p_2/p_1\right)\right] \qquad (1)$$

This requires p_2. Using the ideal gas equation of state: $p_2/p_1 = T_2/T_1$. Then, with data from Table A-22E

$$E_2 - E_1 = (1\ lb)\left[119.58 - 85.2 - 500\left(0.66321 - 0.58233 - \frac{1.986}{28.97} \ln \frac{700}{500}\right)\right] = 5.47\ Btu \longleftarrow$$

To find W, reduce the energy balance to read $W = m(u_1 - u_2)$, or

$$W = (1\ lb)[85.2 - 119.58] = -34.38\ Btu$$

Then, since $\Delta V = 0$ in the process, Eq. 7.13 reads

$$\begin{bmatrix}\text{exergy transfer} \\ \text{accompanying work}\end{bmatrix} = W - \overset{0}{\cancel{p_0 \Delta V}} = -34.38\ Btu \longleftarrow$$

① Using the exergy balance, Eq. 7.11, with Eq. 7.14: $E_d = T_0 \sigma$, we get

$$E_d = \overset{0}{\cancel{\int_1^2 \left(1 - \frac{T_0}{T_b}\right)\delta Q}} - [W - \overset{0}{\cancel{p_0 \Delta V}}] - [E_2 - E_1]$$

$$= -W - [E_2 - E_1]$$

② $$= -(-34.38\ Btu) - [5.47\ Btu] = 28.91\ Btu \longleftarrow$$

---

1. Alternatively, $E_d = T_0 \sigma$, where σ is the amount of entropy produced, which is obtained from an entropy balance. Since the tank involves no heat transfer, the entropy balance reduces to give $\sigma = S_2 - S_1$. Thus, in this case, $E_d = T_0(S_2 - S_1)$.

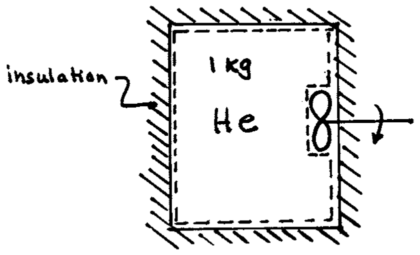
2. In this case, the destruction of exergy can be traced to the shearing action of the paddlewheel on the air in the tank.

PROBLEM 7.25

KNOWN: Helium contained in a closed, rigid, insulated tank is stirred by a paddle wheel.

FIND: Determine the work and the exergy destruction.

SCHEMATIC & GIVEN DATA:



$T_1 = 20°C$ $T_0 = 20°C$

$P_1 = 1 bar$ $P_0 = 1 bar$

$P_2 = 1.45 bar$

ASSUMPTIONS: (1) For the system shown in the accompanying figure, $Q = 0$, and kinetic and potential energy effects can be ignored. (2) Helium is modeled as an ideal gas. (3) For the environment, $T_0 = 293$ K, $P_0 = 1$ bar.

ANALYSIS: With assumption 1 an energy balance reduces to

$$\Delta U = \overset{0}{\cancel{Q}} - W \Rightarrow W = -m\left(u(T_2) - u(T_1)\right) \qquad (1)$$

To find T_2, use the ideal gas equation of state to write

$$\left.\begin{array}{l} P_1 V = m R T_1 \\ P_2 V = m R T_2 \end{array}\right\} : \quad T_2 = T_1\left(\frac{P_2}{P_1}\right) = 293\,K\left(\frac{1.45}{1}\right) = 425\,K$$

The value of $\bar{c}_p$ is constant for monatomic gases (see Table A-21): $\bar{c}_p = 5/2\,\bar{R}$.
Then, with $\bar{c}_p - \bar{c}_v = \bar{R}$, $\bar{c}_v = 3/2\,\bar{R}$. Equation (1) gives

$$W = -m\ c_v\,(T_2 - T_1)$$
$$= -(1\,kg)\left((1.5)\left(\frac{8.314}{4.003}\ \frac{kJ}{kg\cdot K}\right)\right)(425 - 293)\,K = -411.2\ kJ \longleftarrow W$$

The exergy destruction can be found from an exergy balance or using $E_d = T_0\,\sigma$, where σ is the amount of entropy produced, obtained from an entropy balance. Using the second of these approaches

$$\Delta S = \int_1^2 \overset{0}{\cancel{\left(\frac{\delta Q}{T}\right)_b}} + \sigma \Rightarrow \sigma = m(s_2 - s_1)$$

or

$$\sigma = m\left[\underbrace{c_p}_{\frac{5}{2}R} \ln \underbrace{\frac{T_2}{T_1}}_{= P_2/P_1} - R \ln \frac{P_2}{P_1}\right]$$

$$\Rightarrow \sigma = m\left(1.5\,\frac{\bar{R}}{M}\right) \ln \frac{P_2}{P_1}$$
$$= (1\,kg)(1.5)\left(\frac{8.314}{4.003}\ \frac{kJ}{kg\cdot K}\right) \ln 1.45 = 1.158\ \frac{kJ}{K}$$

So

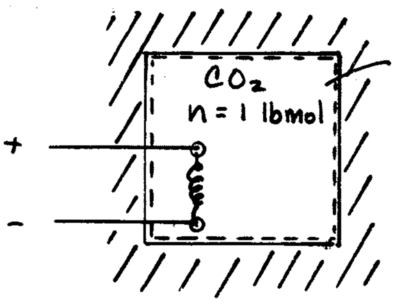
$$E_d = T_0\,\sigma = (293\,K)\left(1.158\ \frac{kJ}{K}\right) = 339.2\ kJ \longleftarrow E_d$$

PROBLEM 7.26

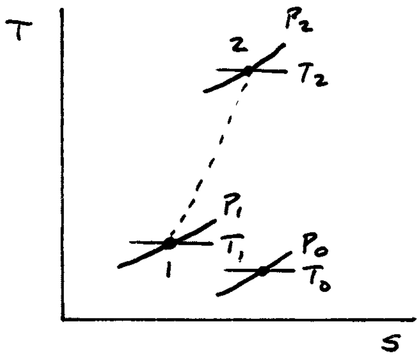
KNOWN: A known amount of carbon dioxide gas is contained in a rigid, insulated vessel of known volume and at a specified initial pressure. An electric resistor in the vessel transfers energy to the gas at a known rate for a specified period of time.

FIND: Determine the exergy change of the gas and, for a system consisting of the resistor and gas, the work and exergy destruction.

SCHEMATIC & GIVEN DATA:



$V = 100\ ft^3$
$P_1 = 4$ atm
$\dot{W}_{elec} = -12\ \frac{Btu}{s}$
$\Delta t = 1$ min
$T_0 = 70°F,\ P_0 = 1$ atm



ASSUMPTIONS: (1) The system consists of the gas and the resistor. (2) The volume is constant and kinetic and potential energy effects are neglected. (3) The resistor is of negligible mass and thus undergoes no change of state. (4) The carbon dioxide is modeled as an ideal gas. (5) For the system, Q=0.

ANALYSIS: First, fix both states, as follows:

$$T_1 = \frac{P_1 V}{n\bar{R}} = \frac{(4\ atm)(100\ ft^3)}{(1\ lbmol)(1545\ \frac{ft\cdot lbf}{lbmol\ °R})}\left|\frac{14.696\ lbf/in.^2}{1\ atm}\right|\left|\frac{144\ in.^2}{1 ft^2}\right| = 548\ °R$$

To determine T_2, begin with an energy balance: $\Delta U = Q - W$. With $\Delta U_{res} = 0$

$$n(\bar{u}_2 - \bar{u}_1) = \overset{0}{\cancel{Q}} - W$$

where $\bar{u}_2$ and $\bar{u}_1$ are obtained from Table A-23E. To get W, use Eq. 2.14

$$W = \int_1^2 \dot{W}dt = \dot{W}_{elec}\Delta t = (-12\ \tfrac{Btu}{s})(1\ min)\left|\frac{60\ s}{1\ min}\right| = -720\ Btu$$

Thus, solving for $\bar{u}_2$ and inserting data

$$\bar{u}_2 = -\frac{W}{n} + u_1 = -\frac{(-720\ Btu)}{(1\ lbmol)} + 3040.1\ Btu/lbmol = 3760.1\ Btu/lbmol$$

Interpolating in Table A-23E, $T_2 = 647.5\ °R$. Thus

$$P_2 = \frac{n\bar{R}T_2}{V} = \frac{(1)(1545)(647.5)}{(100)}\left|\frac{1}{144}\right|\left|\frac{1}{14.696}\right| = 4.727\ atm$$

Now, the change in exergy of the gas is obtained using Eq. 7.10 and ideal gas relations

$$E_2 - E_1 = n\left[\bar{u}(T_2) - \bar{u}(T_1) + p_0(\overset{0}{\cancel{V_2 - V_1}}) - T_0(\bar{s}°(T_2) - \bar{s}°(T_1) - \bar{R}\ln p_2/p_1)\right]$$

Inserting data interpolated from Table A-23E

$$E_2 - E_1 = (1\ lbmol)\left[(3760.1 - 3040.1)\frac{Btu}{lbmol} - (530°R)(52.750 - 51.2124 - 1.986\ln\frac{4.727}{4})\frac{Btu}{lbmol\ °R}\right]$$

$$= 80.85\ Btu \longleftarrow E_2 - E_1$$

The exergy transfer accompanying work, E_W, is

$$\mathsf{E}_W = W - p_0 \cancel{\Delta V}^{\,0} = -720 \text{ Btu} \longleftarrow \mathsf{E}_W$$

In this case, all of the energy transfer by work is "available" in the sense that the exergy transfer accompanying work equals the energy transfer.

The exergy destruction, E_d, is determined by solving the exergy balance, Eq. 7.11, for E_d

$$\mathsf{E}_2 - \mathsf{E}_1 = \int_1^2 \left(1 - \frac{T_0}{T_b}\right)\cancel{\delta Q}^{\,0} - (W - p_0 \cancel{\Delta V}^{\,0}) - \mathsf{E}_d$$

or

$$\mathsf{E}_d = -(\mathsf{E}_2 - \mathsf{E}_1) - (W) = -(80.85 \text{ Btu}) - (-720 \text{ Btu})$$

① $$= 639.15 \text{ Btu} \longleftarrow \mathsf{E}_d$$

---

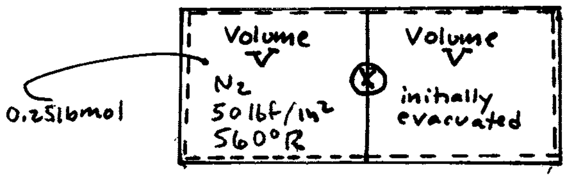
1. Note that the exergy input by work (720 Btu) results in only 80.85 Btu being stored as increased exergy of the gas. The rest, 639.15 Btu, is destroyed by irreversibilities.

PROBLEM 7.27*

KNOWN: A rigid insulated tank has two equal-volume compartments separated by a valve. Initially, one is evacuated and the other holds N_2 gas at a known condition. The valve is opened and the gas fills the entire volume.

FIND: Determine the final temperature and pressure, and evaluate the exergy destruction. Discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The closed system consists of both volumes, as shown in the schematic. 2. For the system, $Q = W = 0$ and effects of motion and gravity can be ignored. 3. N_2 is modeled as an ideal gas. 4. For the environment, $T_0 = 530°R$, $p_0 = 1$ atm.

ANALYSIS: With assumptions 1, 2 the energy balance reduces to $\Delta U = \overset{0}{\cancel{Q}} - \overset{0}{\cancel{W}} \Rightarrow \Delta U$. Since the specific internal energy of an ideal gas depends on temperature alone: $T_2 = T_1 = 560\ °R$

$T_2 = 100°F$ ◄——

Using the ideal gas equation of state

initially: $p_1 V = m R T_1$

finally: $p_2 (2V)\ m R T_2$

but $T_2 = T_1$, giving

$$p_2(2V) = p_1 V \Rightarrow p_2 = \tfrac{1}{2} p_1 = 25\ \text{lbf/in}^2$$ ◄——

The exergy destruction can be found from an exergy balance or using $E_d = T_0 \sigma$, where σ is obtained from an entropy balance. That is,

$$\Delta S = \cancelto{0}{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma$$

$$\Rightarrow \quad \sigma = \Delta S = n\left[\underbrace{\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1)}_{\substack{= 0 \text{ since} \\ T_2 = T_1}} - \bar{R}\ln p_2/p_1\right]$$

$$\sigma = -n\bar{R}\ln\frac{p_2}{p_1} = -(0.25\ \text{lbmol})\left(1.986\ \frac{\text{Btu}}{\text{lbmol}\cdot°R}\right)\ln\left(\tfrac{1}{2}\right)$$

$$= 0.344\ \text{Btu/°R}$$

Finally

$$E_d = T_0\sigma$$
$$= (530°R)\left(0.344\ \frac{\text{Btu}}{°R}\right)$$
$$= 182.3\ \text{Btu}$$ ◄——

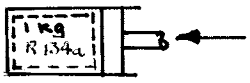
Exergy is destroyed in this case because the N_2 undergoes an unrestrained, and thus irreversible, expansion to a lower pressure.

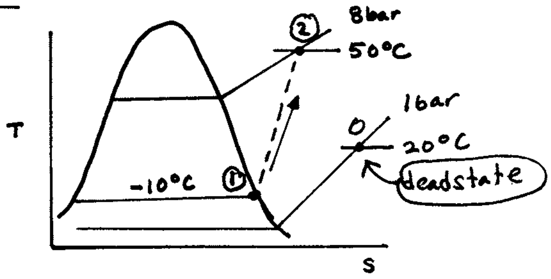
PROBLEM 7.28

KNOWN: One kg of R134a is compressed adiabatically from a specified initial state to a specified final pressure and temperature.

FIND: Determine the work and the exergy destruction.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system shown in the accompanying figure undergoes an adiabatic process between the indicated states. Changes in kinetic and potential energy can be ignored. (2) For the environment, $T_0 = 20°C$, $P_0 = 1 bar$.

ANALYSIS: An energy balance reduces with assumption 1 to

$$\Delta U = \overset{0}{\cancel{Q}} - W \Rightarrow W = -m(u_2 - u_1)$$

With data from Tables A-10 and A-12

$$W = -(1 kg)(261.62 - 221.48)\frac{kJ}{kg}$$

$$= -40.14 kJ \longleftarrow W$$

To find the exergy destruction, use $E_d = T_0\sigma$, where σ is the amount of entropy produced, obtained from an entropy balance:

$$\Delta S = \overset{0}{\cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}} + \sigma$$

With data from Tables A-10 and 12

$$\sigma = m(s_2 - s_1) = (1 kg)[0.9711 - 0.9253]\frac{kJ}{kg \cdot K}$$

$$= 0.0458 \frac{kJ}{K}$$

Finally

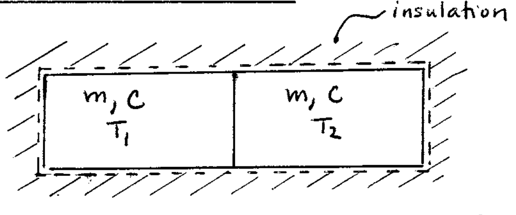
$$E_d = (293 K)(0.0458) = 13.42 kJ \longleftarrow I$$

PROBLEM 7.29

KNOWN: Two solid blocks of mass m and specific heat c are brought into contact and attain thermal equilibrium.

FIND: Derive an expression for the exergy destruction; demonstrate that the exergy destruction cannot be negative, and discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) As shown in the figure, the two solid blocks form a closed system. (2) The blocks are modeled as incompressible with constant specific heat c. (3) For the system, Q = W = 0 and there are no kinetic and potential energy effects.

ANALYSIS: (a) Using assumptions (3), the energy balance reduces to

$$\Delta U = \cancelto{0}{Q} - \cancelto{0}{W} \Rightarrow \Delta U = 0 \Rightarrow \left[(2m)u(T_f) - \left[m u(T_1) + m u(T_2)\right]\right] = 0$$

where T_f is the final temperature at equilibrium. Accordingly

$$\left[2u(T_f) - u(T_1) - u(T_2)\right] = 0 \Rightarrow \left[u(T_f) - u(T_1)\right] + \left[u(T_f) - u(T_2)\right] = 0$$

$$\Rightarrow c\left[T_f - T_1\right] + c\left[T_f - T_2\right] = 0 \Rightarrow T_f = \frac{T_1 + T_2}{2}.$$

The amount of exergy destroyed is $E_d = T_0 \sigma$, where σ is the amount of entropy produced, obtained from an entropy balance:

$$\Delta S = \cancelto{0}{\int \left(\frac{\delta Q}{T}\right)_b} + \sigma \Rightarrow \left[(2m)s(T_f) - \left[m s(T_1) + m s(T_2)\right]\right] = \sigma$$

or

$$\sigma = m\left[\left[s(T_f) - s(T_1)\right] + \left[s(T_f) - s(T_2)\right]\right]$$

$$= mc \ln \frac{T_f}{T_1} + mc \ln \frac{T_f}{T_2}$$

$$= mc \ln \left[\frac{T_f^2}{T_1 T_2}\right] = mc \ln \left[\frac{(T_1 + T_2)^2}{4 T_1 T_2}\right]$$

Finally

$$E_d = mc\, T_0 \ln \left[\frac{(T_1 + T_2)^2}{4 T_1 T_2}\right] \longleftarrow E_d$$

(b) The value of E_d would be negative only if $\left[\frac{(T_1 + T_2)^2}{4 T_1 T_2}\right]$ were less than unity. Considering this possibility ...

$$\frac{(T_1 + T_2)^2}{4 T_1 T_2} < 1 \Rightarrow (T_1 + T_2)^2 < 4 T_1 T_2 \Rightarrow T_1^2 + 2 T_1 T_2 + T_2^2 < 4 T_1 T_2$$

or

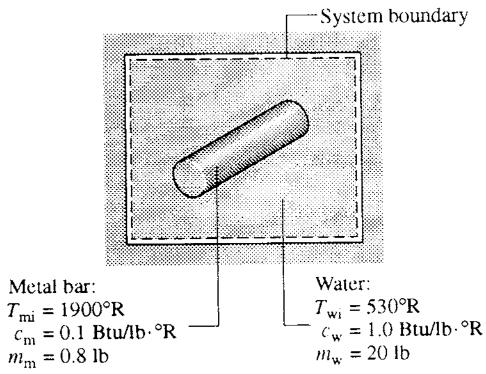
$$T_1^2 - 2 T_1 T_2 + T_2^2 < 0 \Rightarrow (T_1 - T_2)^2 < 0$$

Since this inequality cannot be satisfied, we can conclude that $E_d \geq 0$.

(c) The exergy destruction in this case can be traced to the spontaneous heat transfer that takes place within the two blocks as they come to thermal equilibrium.

PROBLEM 7.30

KNOWN: A hot metal bar is quenched by immersing it in a tank of water.
FIND: Determine the exergy destruction.
SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. As shown in the schematic, the metal bar and water form a ① closed system. 2. For the system, $Q = W = 0$ and there are no effect of motion or gravity. 3. The metal bar and water are each modeled as incompressible. 4. $T_0 = 537°R (77°F)$.

ANALYSIS: An energy balance for the system reduces to give

$$\Delta U)_{metal} + \Delta U)_{water} = 0 \qquad (1)$$

An exergy balance reduces to give

$$\Delta E = \overset{0}{\int_1^2 \left[1 - \cancel{\frac{T_0}{T_b}}\right]\delta Q} - [\overset{0}{\cancel{W}} - p_0 \overset{0}{\cancel{\Delta V}}] - E_d \Rightarrow E_d = -\Delta E$$

Since exergy is an extensive property, $\Delta E = \Delta E)_{metal} + \Delta E)_{water}$. Then, with Eq. 7.10

$$E_d = -\left\{[\Delta U + p_0 \overset{0}{\cancel{\Delta V}} - T_0 \Delta S]_{metal} + [\Delta U + p_0 \overset{0}{\cancel{\Delta V}} - T_0 \Delta S]_{water}\right\}$$

Using Eq. (1) this becomes

② $$E_d = T_0 [\Delta S)_{metal} + \Delta S)_{water}] \qquad (2)$$

The term in square brackets is the amount of entropy produced, which is evaluated in Example 6.5 as 0.0864 Btu/°R. Thus

$$E_d = 537°R (0.0864 \text{ Btu/°R}) = 46.4 \text{ Btu} \longleftarrow$$

---

1. With the indicated assumptions, the system is isolated. Eq. (1) indicates that the total energy of the system remains constant. Eq. (2) indicates that the exergy of the system does not remain constant because exergy is destroyed.

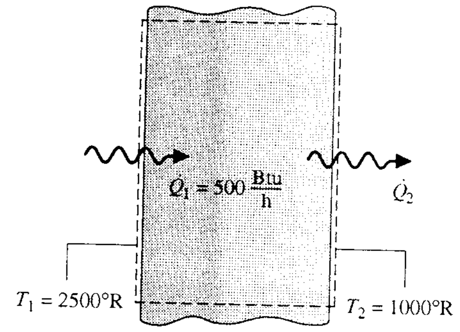
2. Alternatively, Eq. (2) can be expressed as $E_d = T_0 \sigma$, where σ is the amount of entropy produced within the system.

PROBLEM 7.31

KNOWN: There is heat transfer through a wall for which the temperatures at the inner and outer surfaces are known.

FIND: Determine the rates of exergy transfer accompanying heat transfer into and out of the wall and the rate of exergy destruction. Discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system shown in the schematic is at steady state with $\dot{W} = 0$ 2. $T_0 = 500°R$

ANALYSIS: At steady state an energy rate balance reduces to

$$\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \cancel{\dot{W}}^{0} \Rightarrow \dot{Q} = 0 \Rightarrow \dot{Q}_2 = \dot{Q}_1 = 500\ \text{Btu/h}.$$

The directions of $\dot{Q}_1$, $\dot{Q}_2$ are indicated by the arrows on the schematic.

Since T_1 and T_2 are are uniform over the inner and outer surfaces, the rates of exergy transfer accompanying heat transfer are, respectively

$$\begin{bmatrix}\text{rate of exergy transfer}\\ \text{in accompanying}\\ \text{heat transfer}\end{bmatrix} = \left[1 - \frac{T_0}{T_1}\right]\dot{Q}_1$$

$$= \left[1 - \frac{500}{2500}\right]\left(500\,\frac{\text{Btu}}{\text{h}}\right) = 400\ \frac{\text{Btu}}{\text{h}} \longleftarrow$$

$$\begin{bmatrix}\text{rate of exergy transfer}\\ \text{out accompanying}\\ \text{heat transfer}\end{bmatrix} = \left[1 - \frac{T_0}{T_2}\right]\dot{Q}_2$$

$$= \left[1 - \frac{500}{1000}\right]\left(500\,\frac{\text{Btu}}{\text{h}}\right) = 250\ \frac{\text{Btu}}{\text{h}} \longleftarrow$$

Reducing the exergy rate balance, Eq. 7.17, at steady state

$$\cancel{\frac{dE}{dt}}^{0} = \left[1 - \frac{T_0}{T_1}\right]\dot{Q}_1 - \left[1 - \frac{T_0}{T_1}\right]\dot{Q}_2 - \left[\cancel{\dot{W}}^{0} - p_0\cancel{\frac{dV}{dt}}^{0}\right] - \dot{E}_d$$

$$\Rightarrow \dot{E}_d = \left[1 - \frac{T_0}{T_1}\right]\dot{Q}_1 - \left[1 - \frac{T_0}{T_1}\right]\dot{Q}_2 = 400\ \frac{\text{Btu}}{\text{h}} - 250\ \frac{\text{Btu}}{\text{h}} = 150\ \frac{\text{Btu}}{\text{h}} \longleftarrow$$

① Exergy is destroyed because heat transfer occurs spontaneously through a finite temperature difference.

---

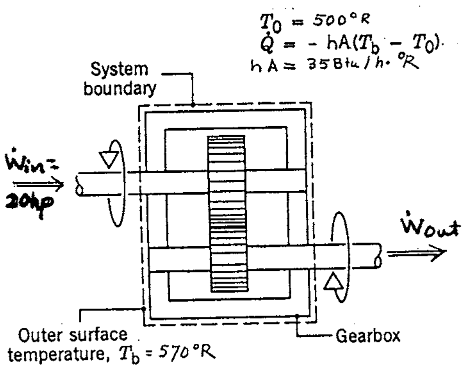
1. The term $\left[1 - \frac{T_0}{T_1}\right]\dot{Q}_1$ is the power that could be obtained in principle by supplying $\dot{Q}_1$ to a reversible power cycle operating between T_1 and T_0. Likewise, $\left[1 - \frac{T_0}{T_2}\right]\dot{Q}_2$ is the power that could be obtained in principle by supplying $\dot{Q}_2$ to a reversible power cycle operating between T_2 and T_0. The difference between these — $\dot{E}_d$ — represents the opportunity to develop power that is irrevocably lost in the spontaneous conduction process.

PROBLEM 7.32

KNOWN: A gear box operates at steady state. Performance data are provided.

FIND: Determine the power delivered along the output shaft, the rate of exergy transfer accompanying heat, and the rate of exergy destruction, each in Btu/h. Also express each as a percentage of the input power.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the figure is at steady state. (2) The temperature of the outer surface of the gearbox and the temperature of the surroundings are each uniform. (3) For the environment $T_0 = 500°R$.

ANALYSIS: An energy rate balance reduces at steady state to give: $\dot{W} = \dot{Q}$. Then, inserting given information

$$\dot{W}_{out} = \dot{Q} + \dot{W}_{in} = -hA[T_b - T_0] + \dot{W}_{in}$$
$$= -\left(35 \frac{Btu}{h\,°R}\right)(570-500)°R + (20hp)\left(\frac{2545\ Btu/h}{1\ hp}\right)$$
$$= -2450 \frac{Btu}{h} + 50{,}900 \frac{Btu}{h} = 48{,}450 \frac{Btu}{h} \longleftarrow$$

The rate of exergy transfer accompanying heat is

$$\left[1 - \frac{T_0}{T_b}\right]\dot{Q} = \left[1 - \frac{500}{570}\right]\left[-2450 \frac{Btu}{h}\right] = -300.9 \frac{Btu}{h} \longleftarrow$$

The rate of exergy destruction can be found by reducing the exergy rate balance at steady state to obtain

$$\dot{E}_d = \left[1 - \frac{T_0}{T_b}\right]\dot{Q} - \dot{W}$$
$$= -300.9 \frac{Btu}{h} - \left[48{,}450 - 50{,}900\right] \frac{Btu}{h} = 2149.1 \frac{Btu}{h} \longleftarrow$$

Expressing values as percentages of the input power

$$\frac{\dot{W}_{out}}{\dot{W}_{in}} = \left(\frac{48{,}450}{50{,}900}\right)(100) = 95.19\ \%$$

① $$\frac{\left|\left[1 - \frac{T_0}{T_b}\right]\dot{Q}\right|}{\dot{W}_{in}} = \left(\frac{300.9}{50{,}900}\right)(100) = 0.59\ \%$$

$$\frac{\dot{E}_d}{\dot{W}_{in}} = \left(\frac{2149.1}{50{,}900}\right)(100) = 4.22\ \%$$

$$= 100\ \%$$

---

1. This represents the true thermodynamic value of the heat loss. An energy analysis overstates its significance: $\frac{|\dot{Q}|}{\dot{W}_{in}} = \left(\frac{2450}{50{,}900}\right)(100) = 5\%$

PROBLEM 7.33

KNOWN: Steady-state data are provided for a gearbox.

FIND: Develop a full exergy accounting of the power input and compare with the results of Example 7.4.

SCHEMATIC & GIVEN DATA: See Fig. E 7.4

ASSUMPTIONS: 1. The gearbox and a portion of the surroundings are taken as the closed system. 2. The system is at steady state and heat transfer occurs on the boundary at temperature T_f. 3. $T_0 = 293\text{ K}$.

ANALYSIS: As in Example 7.4

$$\begin{bmatrix}\text{Rate exergy is transferred} \\ \text{in via the high-speed} \\ \text{shaft}\end{bmatrix} = 60\text{ kW}$$

$$\begin{bmatrix}\text{Rate exergy is transferred} \\ \text{out via the low-speed} \\ \text{shaft}\end{bmatrix} = 58.8\text{ kW}$$

Since heat transfer from the system occurs at $T_f = T_0 = 293\text{ K}$

$$\begin{bmatrix}\text{Rate exergy is transferred} \\ \text{out accompanying heat} \\ \text{transfer}\end{bmatrix} = \left[1 - \frac{T_0}{T_f}\right]\dot{Q} = 0$$

The rate of exergy destruction is then obtained from an exergy rate balance as

$$\dot{E}_d = (60 - 58.8)\text{ kW} = 1.2\text{ kW}$$

or, alternatively, from $\dot{E}_d = T_0\dot{\sigma}$, where $\dot{\sigma}$ is obtained from the solution to Example 6.4b:

$$\dot{E}_d = (293\text{ K})\left(4.1 \times 10^{-3}\frac{\text{kW}}{\text{K}}\right) = 1.2\text{ kW}$$

Exergy balance sheet in terms of exergy magnitudes on a rate basis:

• Rate of exergy in:	
– high-speed shaft	60 kW (100%)
• Disposition of the exergy:	
– Rate of exergy out	
low-speed shaft	58.8 kW (90%)
heat transfer	0 (0%)
– Rate of exergy destruction	1.2 (2%)
	60.0 kW (100%)

In the present case the rate of exergy destruction is greater than in Example 7.4 because an additional source of irreversibility is included in the system: the irreversibility associated with internal heat transfer from the gearbox to the surroundings.

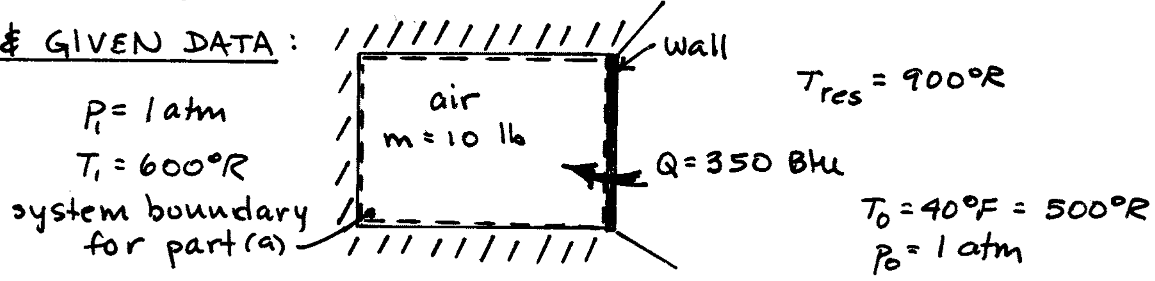
The analysis of Example 7.4 provides a sharper picture of gearbox performance because it distinguishes between the effects of exergy destruction within the gearbox and the exergy transferred from the gearbox associated with heat transfer. The present analysis lumps those effects.

PROBLEM 7.34

KNOWN: Air at a known initial state is contained in a closed, rigid tank. The air receives a specified energy transfer by heat through a wall separating the gas from a thermal reservoir at 900°R.

FIND: (a) For the air as the system, determine the exergy change, the exergy transfer accompanying heat transfer, and the exergy destruction. (b) Evaluate the exergy destruction for an enlarged system that includes the wall and discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Closed systems are under consideration for which the effects of motion and gravity can be ignored. (2) The volume is constant and there is no work. (3) The air behaves as an ideal gas. (4) The state of the wall remains unchanged. (5) The air undergoes an internally reversible process.

ANALYSIS: (a) First, we apply the energy balance in order fix state 2. That is

$$\cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0} + \Delta U = Q - \cancel{W}^{0} \Rightarrow m(u_2 - u_1) = Q$$

Solving for u_2 and inserting data from Table A-22E

$$u_2 = \frac{Q}{m} + u_1 = \frac{350 \text{ Btu}}{10 \text{ lb}} + 102.34 \text{ Btu/lb} = 137.34 \text{ Btu/lb}$$

Interpolating in Table A-22E; $T_2 = 802.1°R$, $s°(T_2) = 0.69622$ Btu/lb·°R.

The exergy change of the air is

$$\Delta E_{air} = m\left[u(T_2) - u(T_1) + p_0(\cancel{V_2 - V_1}^{0}) - T_0\left(s°(T_2) - s°(T_1) - R \ln p_2/p_1\right)\right]$$

Noting that $p_2/p_1 = T_2/T_1$ and with $s°(T_1)$ from Table A-22E

$$\Delta E_{air} = (10 \text{ lb})\left[(137.34 - 102.34)\frac{\text{Btu}}{\text{lb}} - (500°R)\left(0.69622 - 0.62607 - \left(\frac{1.986}{28.97}\right)\ln\frac{802.1}{600}\right)\frac{\text{Btu}}{\text{lb}\cdot°R}\right]$$

$$= 98.76 \text{ Btu} \longleftarrow \Delta E_{air}$$

Denoting the exergy transfer accompanying heat transfer as $E_{Q,air}$, the exergy balance gives

$$\Delta E_{air} = E_{Q,air} - (\cancel{W}^{0} - p_0 \cancel{\Delta V}^{0}) - \cancel{E_{d,air}}^{0}$$

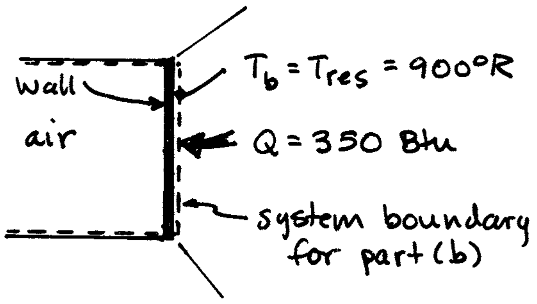
Thus

$$E_{d,air} = 0 \longleftarrow$$

$$E_{Q,air} = 98.76 \longleftarrow$$

PROBLEM 7.34 (Cont'd.)

(b) Applying the exergy balance to the enlarged system that includes the air and the wall



$$\cancelto{0}{\Delta E_{wall}} + \Delta E_{air} = \left(1 - \frac{T_0}{T_b}\right)Q - (\cancelto{0}{W} - p_0 \cancelto{0}{\Delta V}) - (E_d)_{\substack{\text{enlarged}\\ \text{system}}}$$

Thus

$$(E_d)_{\substack{\text{enlarged}\\ \text{system}}} = \left(1 - \frac{T_0}{T_b}\right)Q - \Delta E_{air}$$

$$= \left(1 - \frac{500}{900}\right)(350 \text{ Btu}) - (98.76 \text{ Btu})$$

$$= 155.56 - 98.76 = 56.80 \text{ Btu} \longleftarrow \text{(b)}$$

Discussion

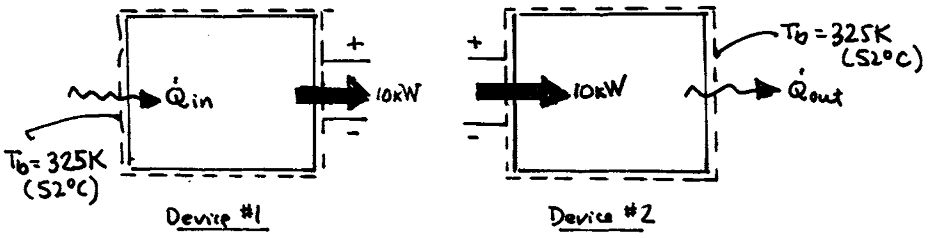
From the analysis of the enlarged system, the exergy transfer accompanying heat transfer to the wall is 155.56 Btu. This occurs at the reservoir temperature of 900°R. The exergy transfer from the wall to the air, $E_{Q,air}$, occurs at a much lower temperature, and amounts to only 98.76 Btu. The difference is accounted for by the exergy destruction associated with the temperature gradient across the wall.

PROBLEM 7.35

KNOWN: Steady-state operating data are provided for each of two systems.

FIND: For each system, evaluate the rates of exergy transfer accompanying heat and work, and the rate of exergy destruction. Discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. Each of the systems shown in the schematic is at steady state. 2. The only energy transfers are those occuring in the directions of the arrows. 3. $T_0 = 300K$.

ANALYSIS: Energy rate balances give

Device 1: $\cancel{\frac{dE}{dt}}^{0} = \dot{Q}_{in} - \dot{W}_{out} \Rightarrow \dot{Q}_{in} = \dot{W}_{out} = 10\,kW$

Device 2: $\cancel{\frac{dE}{dt}}^{0} = \dot{W}_{in} - \dot{Q}_{out} \Rightarrow \dot{Q}_{out} = \dot{W}_{in} = 10\,kW$

Exergy Analysis:

Device 1: $\begin{bmatrix} \text{rate of exergy transfer} \\ \text{in accompanying heat} \\ \text{transfer} \end{bmatrix} = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{in} = \left[1 - \frac{300}{325}\right](10\,kW) = 0.77\,kW$ ←

$\begin{bmatrix} \text{rate of exergy transfer} \\ \text{out accompanying power} \end{bmatrix} = 10\,kW$ ←

$\dot{E}_d = 0.77\,kW - 10\,kW = -9.23\,kW$ ←

Device 1 cannot operate as indicated because $\dot{E}_d$ cannot be negative. ←

Device 2: $\begin{bmatrix} \text{Rate of exergy transfer} \\ \text{in accompanying power} \end{bmatrix} = 10\,kW$ ←

$\begin{bmatrix} \text{Rate of exergy transfer} \\ \text{out accompanying heat transfer} \end{bmatrix} = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{out} = \left[1 - \frac{300}{325}\right](10\,kW) = 0.77\,kW$ ←

$\dot{E}_d = 10\,kW - 0.77\,kW = +9.23\,kW$ ←

Device 2 can operate as indicated. However, the very high exergy destruction rate, which is 92% of the exergy input, suggests that there is considerable scope for improving performance. Other uses for the exergy input also might be considered. ←

PROBLEM 7.36

KNOWN: Steady-state operating data are provided for a silicon chip.

FIND: Determine the rate of exergy destruction. Discuss.

SCHEMATIC & GIVEN DATA: See Fig. E2.5

ASSUMPTIONS: 1. The chip is a closed system at steady state. 2. No heat transfer occurs between the chip and substrate. 3. $T_0 = 293\,K$.

ANALYSIS: In this case it is convenient to evaluate the rate of exergy destruction using $\dot{E}_d = T_0\,\dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production.

PROBLEM 7.36 (Contd.)

An entropy rate balance for the chip reduces to give

$$\cancel{\frac{dS}{dt}}^{0} = \frac{\dot{Q}}{T_b} + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b}$$

Then, with data from Example 2.5

$$\dot{E}_d = -\frac{T_0}{T_b}\dot{Q} = -\left(\frac{293\,K}{353\,K}\right)(-0.225\,W)\left|\frac{1\,kW}{10^3\,W}\right|$$

$$= 1.87\times10^{-4}\ kW \quad \longleftarrow$$

Exergy destruction in this case is due to electrical resistance.

PROBLEM 7.37*

KNOWN: Steady-state operating data are provided for a curling iron.

FIND: Evaluate the rates of exergy destruction and exergy transfer accompanying heat transfer. Express each as a percentage of the electrical power supplied.

SCHEMATIC & GIVEN DATA: See solution to Problem 6.66.

ASSUMPTIONS: 1. The system shown in the figure is at steady state. 2. Heat transfer occurs at T_b ($= 640°R$ ($180°F$)) only. 3. $T_0 = 70°F = 530°R$.

ANALYSIS: At steady state, an energy rate balance reduces to give

$$\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \dot{W} \Rightarrow \dot{Q} = \dot{W} = -20\,W = -68.3\ Btu/h$$

Then

$$\left[\begin{matrix}\text{rate of exergy transfer}\\ \text{out accompanying heat}\end{matrix}\right] = \left[1-\frac{T_0}{T_b}\right]\dot{Q} = \left[1-\frac{530}{640}\right]\left(-68.3\,\frac{Btu}{h}\right) = -11.7\,\frac{Btu}{h} \quad \longleftarrow$$

The rate of exergy destruction can be found using $\dot{E}_d = T_0\dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production, or by reducing an exergy rate balance:

$$\cancel{\frac{dE}{dt}}^{0} = \left[1-\frac{T_0}{T_b}\right]\dot{Q} - \left[\dot{W} - \cancel{p_0\frac{dV}{dt}}^{0}\right] - \dot{E}_d$$

$$\Rightarrow \quad \dot{E}_d = \left[1-\frac{T_0}{T_b}\right]\dot{Q} - \dot{W}$$

$$= -11.7\,\frac{Btu}{h} - \left(-68.3\,\frac{Btu}{h}\right) = 56.6\ Btu/h \quad \longleftarrow$$

When expressed as percentages of the electrical power supplied

$$\left|\frac{\left[1-\frac{T_0}{T_b}\right]\dot{Q}}{\dot{W}}\right|(100) = \left(\frac{11.7}{68.3}\right)(100) = 17.1\,\%$$

① $$\frac{\dot{E}_d}{|\dot{W}|}(100) = \left(\frac{56.6}{68.3}\right)(100) = 82.9\,\%$$

---

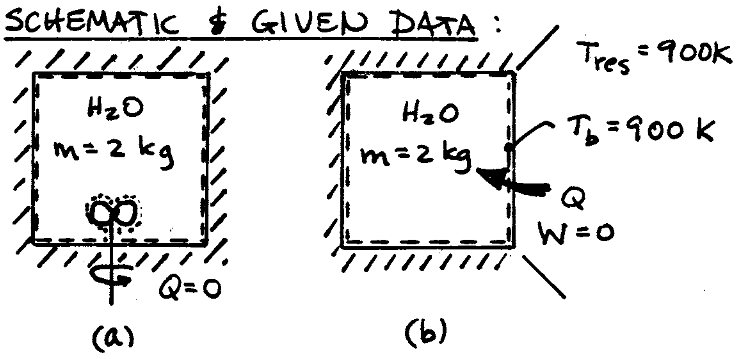
1. The significant exergy destruction in this case indicates that good use is not being made of the valuable electrical power input.

PROBLEM 7.38

KNOWN: Two kilograms of a two-phase liquid-vapor mixture of water undergo two different processes at constant volume between the same end states: (a) adiabatic process, stirring with a paddle wheel, (b) heat transfer from a thermal reservoir at 900K.

FIND: For each case, determine the change in exergy, the net amounts of exergy transfer by work and heat, and the amount of exergy destruction.

SCHEMATIC & GIVEN DATA:



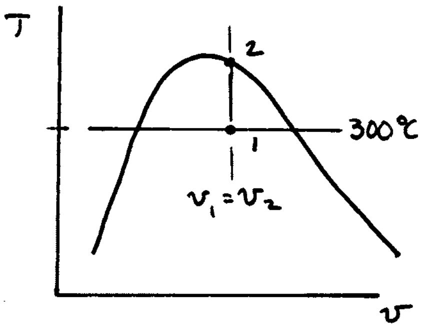
$T_1 = 300°C$

$x_1 = 0.5$

$x_2 = 1.0$

$T_0 = 300K$

$p_0 = 1$ bar



ASSUMPTIONS: (1) The water is a closed system. (2) The volume is constant. (3) In case (a); Q = 0, in case (b); W = 0. (4) Kinetic and potential energy effects are negligible. (5) For the environment, $T_0 = 300K$, $p_0 = 1$ bar.

ANALYSIS: For both cases, the initial and final states are the same. Let us begin by obtaining data at each state. From Table A-2 at 300°C

$$v_1 = v_{f1} + x_1(v_{g1} - v_{f1}) = 1.4036\times10^{-3} + (.5)(0.02167 - 1.4036\times10^{-3}) = 0.01154 \text{ m}^3/\text{kg}$$

$$u_1 = u_{f1} + x_1(u_{g1} - u_{f1}) = 1332.0 + (.5)(2563.0 - 1332.0) = 1947.5 \text{ kJ/kg}$$

$$s_1 = s_{f1} + x_1(s_{g1} - s_{f1}) = 3.2534 + (.5)(5.7045 - 3.2534) = 4.479 \text{ kJ/kg·K}$$

By assumptions (1) and (2), $v_2 = v_1 = v_g(T_2)$. From Table A-2, $T_2 = 336.8$ °C and

$$u_2 = 2474.2 \text{ kJ/kg}, \quad s_2 = 5.3673 \text{ kJ/kg·K}$$

(a) In this case, the volume is constant. Thus

$$\Delta\mathbb{E} = m\left[(u_2 - u_1) + p_0\overset{0}{\cancel{(v_2 - v_1)}} - T_0(s_2 - s_1)\right]$$

$$= (2 \text{ kg})\left[(2474.2 - 1947.5) - (300)(5.3673 - 4.479)\right]\frac{\text{kJ}}{\text{kg}} = 520.4 \text{ kJ} \longleftarrow \Delta\mathbb{E}$$

$\mathbb{E}_w = (W - p_0\overset{0}{\cancel{\Delta V}}) = W$. From an energy balance, $m(u_2 - u_1) = \overset{0}{\cancel{Q}} - W$. Thus

$$W = -m(u_2 - u_1) = -(2 \text{ kg})(2474.2 - 1947.5)\frac{\text{kJ}}{\text{kg}} = -1053.4 \text{ kJ} \longleftarrow \mathbb{E}_w$$

The exergy transfer accompanying heat transfer, denoted $\mathbb{E}_Q$, is

$$\mathbb{E}_Q = \int_1^2 \left(1 - \frac{T_0}{T_b}\right)\overset{0}{\cancel{\delta Q}} = 0 \longleftarrow \mathbb{E}_Q$$

Now, from an exergy balance

$$\Delta\mathbb{E} = \overset{0}{\cancel{\mathbb{E}_Q}} - \mathbb{E}_w - \mathbb{E}_d$$

$$\mathbb{E}_d = -\mathbb{E}_w - \Delta\mathbb{E} = -(-1053.4 \text{ kJ}) - (520.4 \text{ kJ})$$

$$= 533 \text{ kJ} \longleftarrow \mathbb{E}_d$$

PROBLEM 7.38 (Cont'd.)

(b) Since the change of state is the same, the change in exergy is the same as in part (a). That is

$$\Delta E = 520.4 \text{ kJ} \longleftarrow \Delta E$$

Since the work is zero, $E_W = 0 \longleftarrow E_W$

The exergy transfer accompanying heat transfer is evaluated at the boundary temperature, $T_b = 900\text{K}$. Thus

$$E_Q = \left(1 - \frac{T_0}{T_b}\right) Q$$

In this case, $m(u_2 - u_1) = Q - \cancel{W}^0 \Rightarrow Q = 1053.4 \text{ kJ}$ and

$$E_Q = \left(1 - \frac{300}{900}\right)(1053.4 \text{ kJ}) = 702.3 \text{ kJ} \longleftarrow E_Q$$

Now, from an exergy balance

$$\Delta E = E_Q - \cancel{E_W}^0 - E_d$$

$$E_d = E_Q - \Delta E = 702.3 - 520.4 = 181.9 \text{ kJ} \longleftarrow E_d$$

Discussion

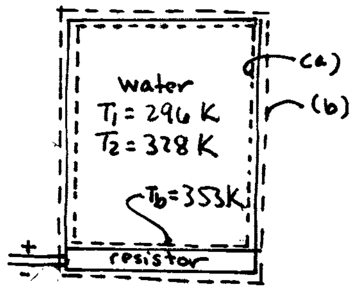
The exergy destruction for case (b) is significantly less than ~~for~~ case (a). Since the exergy change is the same in both cases, case (b) would allow the exergy change to be brought about by less exergy transfer from the surroundings.

PROBLEM 7.39*

KNOWN: Operating data are provided for a water heater.

FIND: Determine the exergy destruction and exergy transfer for (a) the water alone as the system (b) the water and resistor as the system. Compare and discuss.

SCHEMATIC & GIVEN DATA:



From solution to Problem 6.65...

(a) $\sigma = 9.95 \frac{kJ}{K}$

(b) $\sigma = 85.13 \frac{kJ}{kg}$

For water, m = 198.4 kg, c = 4.18 kJ/kg·K

ASSUMPTIONS: 1. In part (a) the system is the water only. In part (b) the system is the water plus the resistor. 2. The water is modeled as incompressible. 3. Heat transfer from the outer surface of the water heater can be ignored. 4. The states of the tank wall and the resistor are constant. 5. $T_0 = 293$ K.

ANALYSIS: (a) For system (a), the exergy is transfer is by heat transfer from the resistor to the water:

$$\begin{bmatrix}\text{exergy transfer}\\ \text{accompanying}\\ \text{heat transfer}\end{bmatrix} = \left[1 - \frac{T_0}{T_b}\right]Q = \left[1 - \frac{293}{353}\right](26{,}538\text{ kJ}) = 4511 \text{ kJ} \quad \leftarrow$$

where $Q = \Delta U$ from an energy balance, or $Q = mc(T_2 - T_1) = (198.4\text{ kg})(4.18 \frac{kJ}{kg\cdot k})(32\text{ K})$
$= 26{,}538$ kJ

The exergy destruction can be obtained conveniently here as $T_0\sigma$:

$$E_d = T_0\sigma = (293\text{ K})(9.95 \tfrac{kJ}{K}) = 2915 \text{ kJ} \quad \leftarrow$$

(b) For system (b), the exergy transfer is by electrical work. An overall energy balance reduces with assumptions 3, 4 to give $(-W) = \Delta U$, or $(-W) = 26{,}538$ kJ. As before the exergy destruction can be evaluated as

$$E_d = T_0\sigma = (293\text{ K})(85.13 \tfrac{kJ}{kg}) = 24{,}943 \text{ kJ}$$

DISCUSSION:

The exergy destruction in the enlarged system (b) is greater because there is an additional source of irreversibility — namely, electric current flow through the resistor. For the water alone, system (a), the irreversibility is associated with internal heat transfer from the resistor through the water as it is heated.

Observe that the increase in the exergy stored in the water equals, in each case, the difference between the exergy input and the exergy destroyed:

① System (a) $\Delta E = 4511 - 2915 = 1596$ kJ. System (b) $\Delta E = 26{,}538 - 24{,}943 = 1595$ kJ

Alternatively, ΔE for the water can be obtained using Eq. 7.10 with assumption 2

$$\Delta E = \Delta U + \overset{0}{\cancel{p_0\Delta V}} - T_0\Delta S = mc[T_2 - T_1] - T_0\, mc \ln\frac{T_2}{T_1} = mc\left[(T_2 - T_1) - T_0 \ln\frac{T_2}{T_1}\right]$$

$\Rightarrow \Delta E = (198.4\text{ kg})(4.18 \frac{kJ}{kg\cdot K})\left[32\text{ K} - 293\text{ K} \ln\frac{328}{296}\right] = 1594$ kJ which agrees to within roundoff.

---

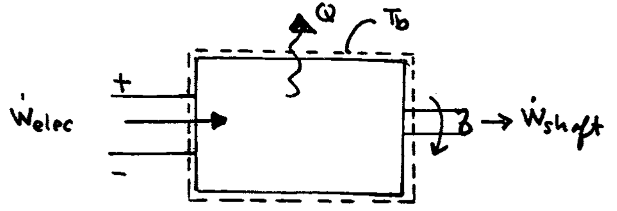
1. The percentage of the electrical input that is stored in the water: $\left(\frac{1595}{26{,}538}\right)(100) = 6.0\%$ suggests that such water heaters do not make effective use of the electrical power supplied to them.

PROBLEM 7.40

KNOWN: Steady-state operating data are provided for an electric motor.

FIND: Evaluate the rate of exergy destruction and the rate of exergy transfer accompanying heat. Express each as a percentage of the electrical power supplied.

SCHEMATIC & GIVEN DATA:



From the solution to Problem 6.69:

$\dot{W}_{elect} = 2.2\text{ kW}$

$\dot{W}_{shaft} = 1.676\text{ kW}$

$T_b = 319.9\text{ K}$

$\dot{Q} = 0.524\text{ kW}$

$\dot{\sigma} = 1.64 \times 10^{-3}\text{ kW/K}$

ASSUMPTIONS: 1. The system shown in the schematic is at steady state. 2. Energy transfers are in the directions of the arrows. 3. Heat transfer occurs at temperature T_b. 4. $T_0 = 293\text{ K}$.

ANALYSIS: With assumption 3

$$\begin{bmatrix}\text{rate of exergy transfer}\\ \text{accompanying heat transfer}\end{bmatrix} = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}$$

$$= \left[1 - \frac{293}{319.9}\right](0.524\text{ kW}) = 0.044\text{ kW} \longleftarrow$$

The rate of exergy destruction can be found by reducing an exergy rate balance or using $\dot{E}_d = T_0\dot{\sigma}$. Thus

$$\dot{E}_d = T_0\dot{\sigma} = (293\text{ K})\left(1.64 \times 10^{-3}\ \frac{\text{kW}}{\text{K}}\right) = 0.481\text{ kW} \qquad \longleftarrow$$

Exergy accounting:

• rate exergy enters electrically :	2.2 kW	(100%)
• disposition of the exergy :		
- exergy out via shaft	1.676 kW	(76.18 %)
- exergy out via heat	0.044 kW	(2.0 %)
- exergy destroyed	0.481 kW	(21.86%)

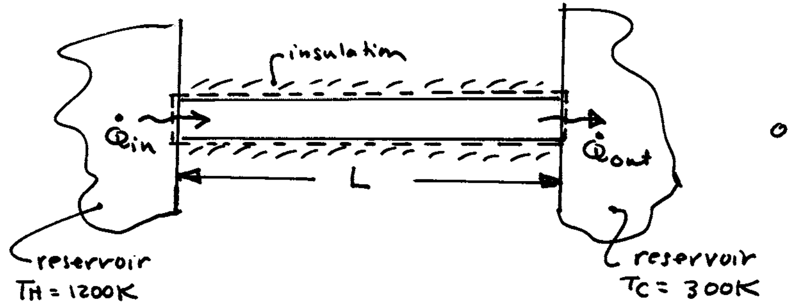
$\} \longleftarrow$

PROBLEM 7.41

KNOWN: Energy is conducted from a thermal reservior through a cylindrical rod at steady state to another thermal reservoir at a lower temperature.

FIND: Plot the rate of conduction through the rod, the rates of exergy transfer accompanying heat transfer into and out of the rod, and the rate of exergy destruction, each versus the rod length, L.

SCHEMATIC & GIVEN DATA:



$D = 2\,cm$

$k = 0.4\,kW/m\cdot K$

$0.01 \le L \le 1.0\,m$

ASSUMPTIONS: 1. The system shown in the schematic is at steady state. 2. Energy transfer is in the directions of the arrows only. 3. $T_0 = 300K$.

ANALYSIS: An energy rate balance reduces to give $\cancel{\frac{dE}{dt}}^{0} = \dot{Q} - \cancel{\dot{W}}^{0} \Rightarrow \dot{Q} = 0$, or $\dot{Q}_{in} = \dot{Q}_{out}$. Using Eq. 2.32, the common heat transfer rate by conduction is

$$\dot{Q}_{in} = \dot{Q}_{out} = kA\left[\frac{T_H - T_C}{L}\right] = \left(0.4\,\frac{kW}{m\cdot K}\right)\left(\frac{\pi}{4}(.02m)^2\right)\left(\frac{1200-300}{L}\right)K$$

$$= \left(\frac{0.1131}{L}\right)kW \qquad (1)$$

The rates of exergy transfer accompanying heat transfer are, respectively

$$\begin{bmatrix}\text{rate of exergy}\\ \text{transfer in}\end{bmatrix} = \left[1 - \frac{T_0}{T_H}\right]\dot{Q}_{in} = \left[1 - \frac{300}{1200}\right]\dot{Q}_{in} = 0.75\,\dot{Q}_{in} \qquad (2)$$

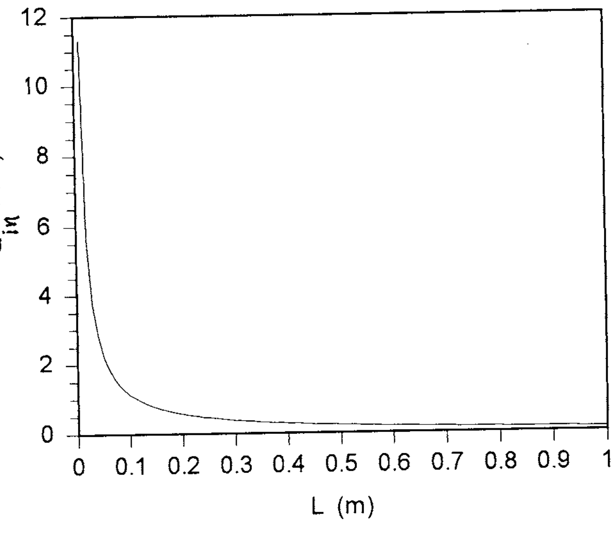
$$\begin{bmatrix}\text{rate of exergy}\\ \text{transfer out}\end{bmatrix} = \left[1 - \frac{T_0}{T_C}\right]\dot{Q}_{out} = \left[1 - \frac{300}{300}\right]\dot{Q}_{out} = 0 \qquad (3)$$

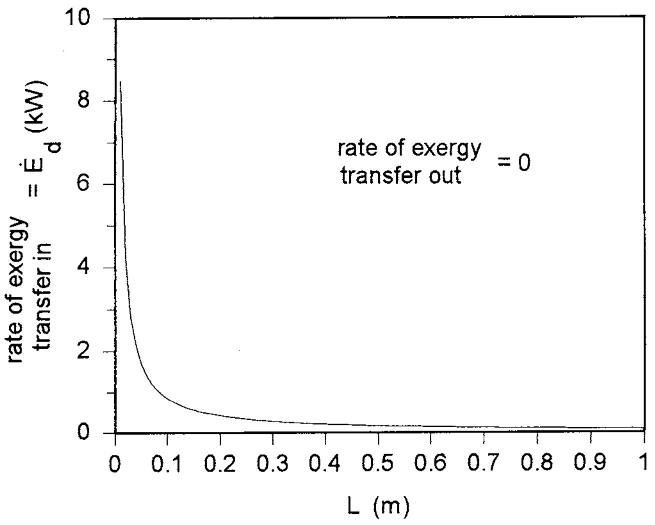
The rate of exergy destruction can be obtained by reducing an exergy rate balance, which in this case corresponds to finding the difference between the exergy entering and exiting the rod. Thus,

$$\dot{E}_d = \begin{bmatrix}\text{rate of exergy}\\ \text{transfer in}\end{bmatrix} - \begin{bmatrix}\text{rate of exergy}\\ \text{transfer out}\end{bmatrix} = 0.75\,\dot{Q}_{in} \qquad (4)$$

Sample Calculation: $L = 1\,m$, $\dot{Q}_{in} = 0.1131\,kW$, [Rate of exergy transfer in] = $\dot{E}_d = 0.0848\,kW$.

PLOTS:





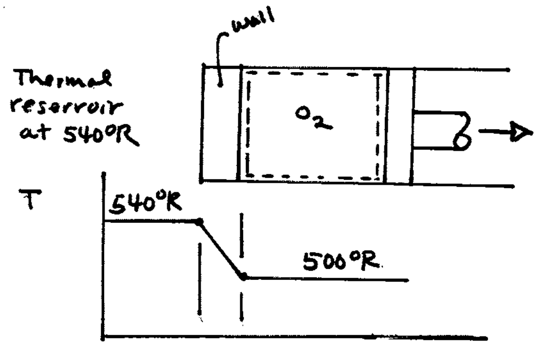
Discussion: As the length of the rod increases, the heat transfer rate and the exergy destruction rate decrease rapidly. Since exergy can be viewed as having economic value (secs. 7.6, 7.7), a reduction in exergy destruction can be viewed as a cost saving. However, the cost of the rod increases with length, so an economic tradeoff is inherent in selecting a value for L.

PROBLEM 7.42

KNOWN: One lb of O_2 expands isothermally while receiving energy by heat transfer through a wall separating the O_2 from a thermal reservoir.

FIND: (a) For the O_2 as the system, evaluate W, Q, the exergy transfers accompanying work and heat transfer, and the exergy destruction. (b) Evaluate the exergy destruction for a system consisting of the wall and O_2. Discuss.

SCHEMATIC & GIVEN DATA:



For O_2:
$T = 500°R$
$p_1 = 100\ \frac{lbf}{in^2}$
$p_2 = 10\ \frac{lbf}{in^2}$

ASSUMPTIONS: 1. In part(a), the system is the O_2 only. In part(b), the system is the O_2 plus the wall. 2. O_2 is modeled as an ideal gas. 3. The state of the wall does not change. 4. Effects of motion and gravity are ignored. 5. $T_0 = 540°R$, $p_0 = 15\ \frac{lbf}{in^2}$

ANALYSIS: (a) An energy balance reduces to $\Delta U = Q - W$. But $\Delta U = 0$ for an ideal gas at fixed temperature, giving $Q = W$. (1)

To find W, use the ideal gas model with

$$W = \int_1^2 p\,dV = \int_1^2 \frac{mRT}{V}\,dV = mRT \ln\frac{V_2}{V_1}. \quad \text{Since } p_1V_1 = mRT,\ p_2V_2 = mRT,$$

$$V_2/V_1 = \frac{p_1}{p_2} \quad \Rightarrow \quad W = mRT\ln\frac{p_1}{p_2}. \qquad (2)$$

An exergy balance takes the form

$$\Delta E = \underbrace{\left[1 - \frac{T_0}{T}\right]Q}_{\text{Exergy transfer with heat}} - \underbrace{\left[W - p_0\Delta V\right]}_{\text{Exergy transfer with work}} - \underbrace{E_d}_{\text{Exergy destruction}}$$

Alternatively

$$(\Delta U + p_0\Delta V - T_0\Delta S) = \left[1 - \frac{T_0}{T}\right]Q - \left[W - p_0\Delta V\right] - E_d$$

With $\Delta U = 0$, $Q = W$, and cancellation of $p_0\Delta V$ on both sides, we get

① $$E_d = T_0\left[\Delta S - \frac{Q}{T}\right]$$

$$= T_0\left[m[\underbrace{s°(T_2) - s°(T_1)}_{\substack{=0 \text{ since} \\ T=\text{constant}}} - R\ln\frac{p_2}{p_1}] - \frac{mRT\ln p_1/p_2}{T}\right] = 0 \quad \longleftarrow$$

(Q from Eqs. (1), (2))

Accordingly, the O_2 undergoes an internal reversible process.

With Eqs. (1), (2)

$$Q = W = (1\,lb)\left(\frac{1.986}{32}\frac{Btu}{lb \cdot °R}\right)(500°R)\ln\left(\frac{100}{10}\right) = 71.452\ Btu \quad \longleftarrow$$

Using the ideal gas equation of state and introducing Eq. 2

$$\begin{bmatrix}\text{Exergy transfer} \\ \text{associated with work}\end{bmatrix} = W - p_0\Delta V$$

$$= mRT\ln\frac{p_1}{p_2} - p_0\left[\frac{mRT}{p_2} - \frac{mRT}{p_1}\right] = mRT\left[\ln\frac{p_1}{p_2} - \frac{p_0}{p_2} + \frac{p_0}{p_1}\right]$$

$$= (1\,lb)\left(\frac{1.986}{32}\frac{Btu}{lb \cdot °R}\right)(500°R)\left[\ln 10 - \frac{15}{10} + \frac{15}{100}\right] = 29.56\ Btu \quad \longleftarrow$$

PROBLEM 7.42 (Contd.)

$$\begin{bmatrix}\text{Exergy transfer}\\ \text{associated with heat}\end{bmatrix} = \left[1-\frac{T_0}{T}\right]Q$$

② ③ $$= \left[1-\frac{540}{500}\right](71.452\,\text{Btu}) = -5.716\ \text{Btu}$$ ←

(b) For an enlarged system of O_2 plus wall, $E_d = T_0\sigma$, where σ is the amount of entropy produced within the enlarged system. Using assumption 3,

$$\Delta S = \frac{Q}{T_{res}} + \sigma \quad \text{or} \quad \cancelto{0}{(\Delta S)}_{wall} + (\Delta S)_{O_2} = \frac{Q}{T_{res}} + \sigma \Rightarrow$$

with Eq. (1), (2) and assumption 3

$$\sigma = (\Delta S)_{O_2} - \frac{Q}{T_{res}} = m\Big[\underbrace{s^\circ(T_2) - s^\circ(T_1)}_{=0} - R\ln\frac{P_2}{P_1}\Big] - \frac{mRT\ln(P_1/P_2)}{T_{res}}$$

or

④ ⑤ $$\sigma = mR\ln(P_1/P_2)\left[1-\frac{T}{T_{res}}\right] = (1\,\text{lb})\left(\frac{1.986}{32}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)\ln 10\left[1-\frac{500}{540}\right]$$

$$= 1.059\times10^{-2}\ \text{Btu}/^\circ\text{R}$$

Finally, $E_d = T_0\sigma = 540\,^\circ\text{R}(1.059\times10^{-2}\,\text{Btu}/^\circ\text{R}) = 5.72\ \text{Btu}$ ←

Exergy is destroyed within the enlarged system owing to spontaneous heat transfer through a finite temperature difference through the wall.

---

1. The term in brackets is the entropy production within the O_2, as can be seen by applying the entropy balance to the system of part (a). For this system, we get $\sigma = 0$.

2. Since heat transfer to the system occurs at $T < T_0$, the accompanying exergy transfer is oppositely directed. See the discussion of Sec. 7.3.2.

3. Inserting calculated values into exergy balance

   $\Delta E = (-5.716) - (29.56) - (0) = -35.28$ Btu

   This also can be determined using $\Delta E = \Delta U + p_0\Delta V - T_0\Delta S$, as can be readily checked.

4. $\sigma \to 0$ as $T_{res} \to T$.

5. Since the O_2 undergoes an internally reversible process, the entropy production (and exergy destruction) can be evaluated by study of the wall. That is, an entropy balance for the wall reduces with assumption 3 to give

$$\cancelto{0}{(\Delta S)}_{wall} = \frac{Q}{T_{res}} - \frac{Q}{T} + \sigma$$

$$\Rightarrow \quad \sigma = Q\left[\frac{1}{T} - \frac{1}{T_{res}}\right] = 71.452\,\text{Btu}\left[\frac{1}{500} - \frac{1}{540}\right]\left(\frac{1}{^\circ\text{R}}\right)$$

$$= 1.059\times10^{-2}\ \text{Btu}/^\circ\text{R}$$

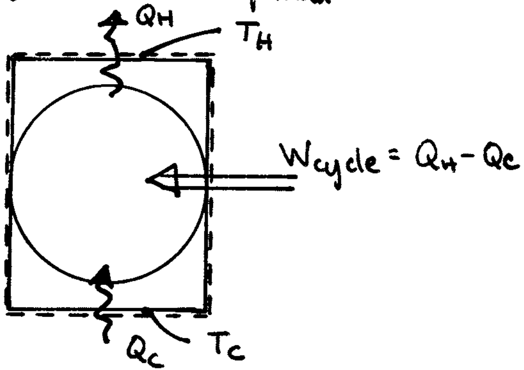
which agrees with the result reported here.

PROBLEM 7.43

KNOWN: A system undergoes a refrigeration cycle while receiving Q_c at temperature T_c and discharging Q_H at T_H, where $T_H > T_c$. Q_c and Q_H are the only heat transfers.

FIND: (a) Show that W_{cycle} cannot be zero. (b) Obtain a specified expression for coefficient of performance β. (c) Determine β_{max}.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown undergoes a refrigeration cycle. (2) Q_c and Q_H are the only heat transfers and are in the directions of the arrows. (3) T_c and T_H are constant and $T_H > T_c$. (4) The environment temperature is T_0.

ANALYSIS: (a) An exergy balance for the cycle reads

$$\cancel{\Delta E}\Big]_{cycle}^{\,0} = \left[1 - \frac{T_0}{T_c}\right]Q_c - \left[1 - \frac{T_0}{T_H}\right]Q_H - \left[W_{cycle} - p_0 \cancel{\Delta V}\Big]_{cycle}^{\,0}\right] - E_d$$

where $\Delta E = \Delta V = 0$ for a cycle. Introducing the energy balance, $Q_H = W_{cycle} + Q_c$ we get

$$0 = \left[1 - \frac{T_0}{T_c}\right]Q_c - \left[1 - \frac{T_0}{T_H}\right](W_{cycle} + Q_c) - W_{cycle} - E_d$$

$$= \left[(\cancel{1} - \frac{T_0}{T_c}) - (\cancel{1} - \frac{T_0}{T_H})\right]Q_c + \left[\cancel{1} - (\cancel{1} - \frac{T_0}{T_H})\right]W_{cycle} - E_d$$

$$= T_0\left[\frac{1}{T_H} - \frac{1}{T_c}\right]Q_c + \frac{T_0}{T_H}W_{cycle} - E_d \qquad (1)$$

Solving for E_d, and setting W_{cycle} to zero

$$E_d = \underbrace{T_0}_{pos.}\underbrace{\left[\frac{1}{T_H} - \frac{1}{T_c}\right]}_{neg.}\underbrace{Q_c}_{pos} + \frac{T_0}{T_H}\cancel{W_{cycle}}^{\,0} \Rightarrow E_d < 0 \quad \text{impossible!}$$

Thus W_{cycle} cannot be zero. ← (a)

(b) Solving (1) for $\beta = Q_c/W_{cycle}$

$$\left[\frac{T_H - T_c}{T_H T_c}\right]Q_c = \frac{W_{cycle}}{T_H} - E_d/T_0$$

$$\beta = \frac{Q_c}{W_{cycle}} = \left[\frac{T_c}{T_H - T_c}\right]\left[1 - \frac{T_H E_d}{T_0 W_{cycle}}\right] = \left[\frac{T_c}{T_H - T_c}\right]\left[1 - \frac{T_H E_d}{T_0(Q_H - Q_c)}\right] \leftarrow (b)$$

(c) From the result of part (b), β increases as $E_d \to 0$. Thus, when $E_d = 0$

$$\beta_{max} = \frac{T_c}{T_H - T_c}$$

as expected.

PROBLEM 7.44

KNOWN: Various substances are at specified states at the inlet to a control volume.

FIND: In each case, determine the specific exergy and specific flow exergy.

ASSUMPTIONS: (1) The velocity is relative to an exergy reference enviroment for which $T_0 = 20°C$, $p_0 = 1\ bar$. (2) The effects of gravity are negligible.

ANALYSIS: With assumption (2), the specific exergy is

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} \qquad (1)$$

and the specific flow exergy is

$$e_f = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} \qquad (2)$$

The relation between the two is

$$e_f = e + v(p - p_0) \qquad (3)$$

(a) Water vapor at 100 bar, 520°C, 100 m/s. With $h_0 \approx h_f(20°C) = 83.96\ kJ/kg$ and $s_0 \approx s_f(25°C) = 0.2966\ kJ/kg\cdot K$ from Table A-2, and $h = 3425.1\ kJ/kg$, $s = 6.6622\ kJ/kg\cdot K$, and $v = 0.03394\ m^3/kg$ from Table A-4, we get

$$e_f = (3425.1 - 83.96)\frac{kJ}{kg} - (293\ K)(6.6622 - 0.2966)\frac{kJ}{kg\cdot K} + \left(\frac{100^2}{2}\right)\frac{m^2}{s^2}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 1481\ kJ/kg \longleftarrow e_f$$

From (3) above

$$e = e_f - v(p - p_0) = 1481\frac{kJ}{kg} - \left(0.03394\frac{m^3}{kg}\right)(100 - 1)\ bar\left|\frac{10^5\ N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 1145\ kJ/kg \longleftarrow e$$

(b) Ammonia at 3 bar, 0°C, 5 m/s. With data from Table A-15, $h_0 = 1515.8\ kJ/kg$, $s_0 = 6.2816\ kJ/kg\cdot K$, $h = 1454.43\ kJ/kg$, $s = 5.5409\ kJ/kg\cdot K$, $v = 0.42382\ m^3/kg$. Thus

$$e_f = (1454.43 - 1515.8) - (293)(5.5409 - 6.2816) + \left(\frac{5^2}{2}\right)\left|\frac{1}{10^3}\right| = 155.67\ kJ/kg \longleftarrow e_f$$

and

$$e = e_f - V(p - p_0) = 155.67 - (0.42382)(3 - 1)\left|\frac{10^5}{10^3}\right| = 70.91\ kJ/kg \longleftarrow e$$

(c) Nitrogen as an ideal gas at 527°C (800 K), 50 bar, 200 m/s. Using ideal gas relations, Eq. (2) becomes

$$e_f = \left(\frac{1}{M}\right)\left[\bar{h}(T) - \bar{h}(T_0) - T_0\left\{\bar{s}^\circ(T) - \bar{s}^\circ(T_0) - \bar{R}\ln\frac{p}{p_0}\right\}\right] + \frac{V^2}{2}$$

With data from Table A-23

$$e_f = \frac{1}{28.01}\left[(23{,}714 - 8521) - (293)\left\{(220.907 - 190.805) - (8.314)\ln\left(\frac{50}{1}\right)\right\}\right] + \left(\frac{200^2}{2}\right)\left|\frac{1}{10^3}\right|$$

$$= 587.8\ kJ/kg \longleftarrow e_f$$

Now, from (3) above

$$e = e_f - V(p - p_0) = e_f - RT\left[1 - \frac{p_0}{p}\right]$$

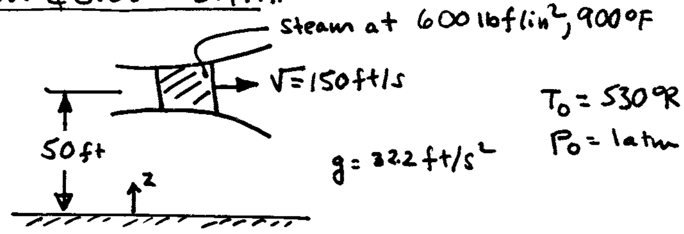
$$= 587.8\frac{kJ}{kg} - \left(\frac{8.314}{28.01}\frac{kJ}{kg\cdot K}\right)(800\ K)\left[1 - \frac{1}{50}\right] = 355.1\ kJ/kg \longleftarrow e$$

PROBLEM 7.45*

KNOWN: Steam at a specified state is under consideration.

FIND: Determine the specific exergy and the specific flow exergy.

SCHEMATIC & GIVEN DATA:



$g = 32.2\ \text{ft/s}^2$

$T_0 = 530°R\ (70°F)$

$p_0 = 1\ \text{atm}$

ASSUMPTIONS: The velocity and elevation are each relative to the exergy reference environment for which $T_0 = 530°R\ (70°F)$, $p_0 = 1\ \text{atm}$, $v_0 \approx v_f(T_0)$, $u_0 \approx u_f(T_0)$, $h_0 \approx h_f(T_0)$, $s_0 \approx s_f(T_0)$.

ANALYSIS: The specific exergy is found from Eq. 7.9:

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

With data from Tables A-2E, 4E

$$e = (1318.4 - 38.09)\,\frac{\text{Btu}}{\text{lb}} + \left(14.7 \times 144\ \frac{\text{lbf}}{\text{ft}^2}\right)(1.302 - 0.01605)\,\frac{\text{ft}^3}{\text{lb}}\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$- 530°R\,(1.6766 - 0.07463)\,\frac{\text{Btu}}{\text{lb}\cdot°R} + \left[\frac{(150\ \text{ft/s})^2}{2} + (32.2\ \text{ft/s}^2)(50\ \text{ft})\right]\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 1280.31 + 3.50 - 849.04 + 0.51 = 435.28\ \text{Btu/lb} \longleftarrow e$$

① The specific flow exergy is found from Eq. 7-20 and data from Tables A-2E, 4E:

$$e_f = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

$$= (1462.9 - 38.09) - 849.04 + 0.51 = 576.3\ \text{Btu/lb} \longleftarrow e_f$$

---

1. Alternatively, Eq. 7.30 can be used to write

$$e_f = e + v(p - p_0) = 435.28\,\frac{\text{Btu}}{\text{lb}} + \left(1.302\,\frac{\text{ft}^3}{\text{lb}}\right)(600 - 14.7)\,\frac{\text{lbf}}{\text{in}^2}\left|\frac{144\ \text{in}^2}{\text{ft}^2}\right|\left|\frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= (435.28 + 141.05)\,\frac{\text{Btu}}{\text{lb}} = 576.3\,\frac{\text{Btu}}{\text{lb}}$$

PROBLEM 7.46

KNOWN: An ideal gas with constant specific heat ratio k.

FIND: Show that in the absence of motion and gravity effects

$$\frac{e_f}{c_p T_0} = \frac{T}{T_0} - 1 - \ln\frac{T}{T_0} + \ln\left(\frac{p}{p_0}\right)^{(k-1)/k}$$

and develop plots for $k = 1.2, 1.3, 1.4$ of $e_f / c_p T_0$ vs. T/T_0 for $p/p_0 = 0.25, 0.5, 1, 2, 4$. Discuss the significance of $e_f < 0$.

ASSUMPTIONS: (1) The system consists of an ideal gas with constant specific heat ratio. (2) The effects of motion and gravity are not significant.

ANALYSIS: With assumption (2) and ideal gas relationships, Eq. 7.20 gives

$$e_f = h - h_0 - T_0(s - s_0)$$

$$= c_p[T - T_0] - T_0\left[c_p \ln\frac{T}{T_0} - \underbrace{R}_{= c_p\left[\frac{k-1}{k}\right]} \ln\frac{p}{p_0}\right] \quad (\text{Eq. } 3.47a)$$

Thus

$$\frac{e_f}{c_p T_0} = \frac{T}{T_0} - 1 - \ln\frac{T}{T_0} + \ln\left[\frac{p}{p_0}\right]^{(k-1)/k}$$

(a) See plots on the next page.

(b) Discussion.

The specific exergy is

$$e = (u - u_0) + p_0(v - v_0) - T_0(s - s_0)$$

and the specific flow exergy is

$$e_f = h - h_0 - T_0(s - s_0)$$

$$= (u + pv) - (u_0 + p_0 v_0) - T_0(s - s_0) = (u - u_0) + (pv - p_0 v_0) - T_0(s - s_0)$$

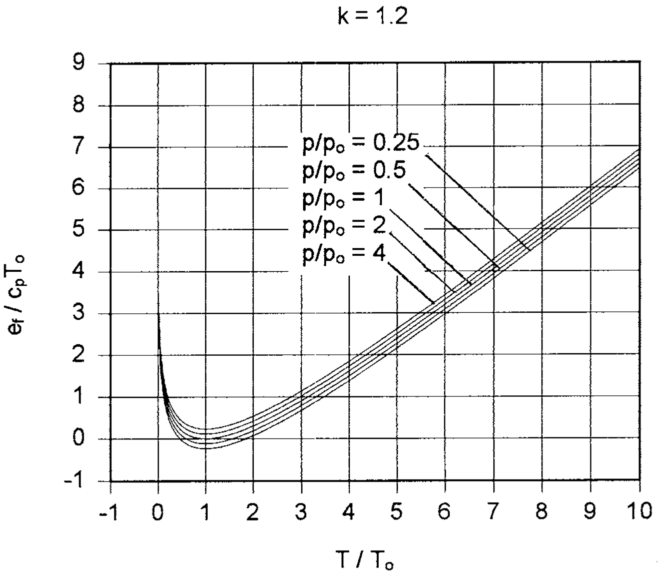
Subtracting e from e_f gives

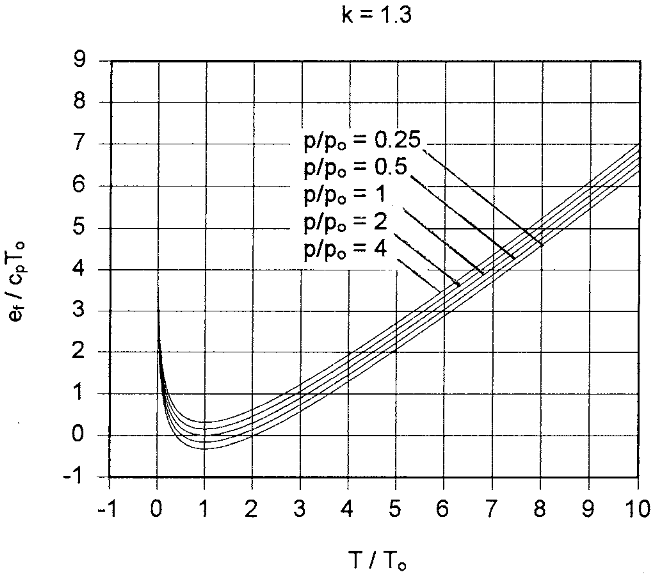
$$e_f = e + v(p - p_0)$$

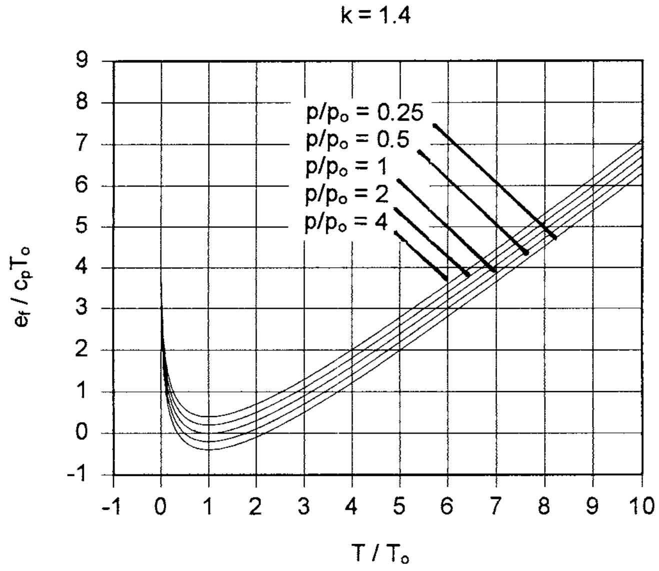
Thus, although the specific exergy e cannot be negative, e_f can take on negative values when $p < p_0$, as shown in the plot below. Physically, from the development of Sec 7.4, the flow exergy is the sum of the exergy transfers accompanying mass and accompanying flow work. The value of e_f can be negative, then, when these contributions have opposite signs: the flow work contribution is opposite to the direction of flow

PROBLEM 7.46 (Cont'd.)

PLOTS:







PROBLEM 7.47

KNOWN: A geothermal source provides a stream of liquid water at $T(\geq T_0)$ and pressure p.

FIND: Plot e_f/cT_0 versus T/T_0 for selected values of p/p_0.

ASSUMPTIONS: (1) The water can be modeled as incompressible with constant specific heat c. (2) The effects of motion and gravity can be neglected. (3) For the exergy reference environment, $T_0 = 60°F$, $p_0 = 1\,atm$.

ANALYSIS: With Eq. 7.20, using Eqs. 3.20b, 6.24 and assumption 2

$$e_f = (h-h_0) - T_0(s-s_0) = \left(c(T-T_0) + v(p-p_0)\right) - T_0 c \ln T/T_0$$

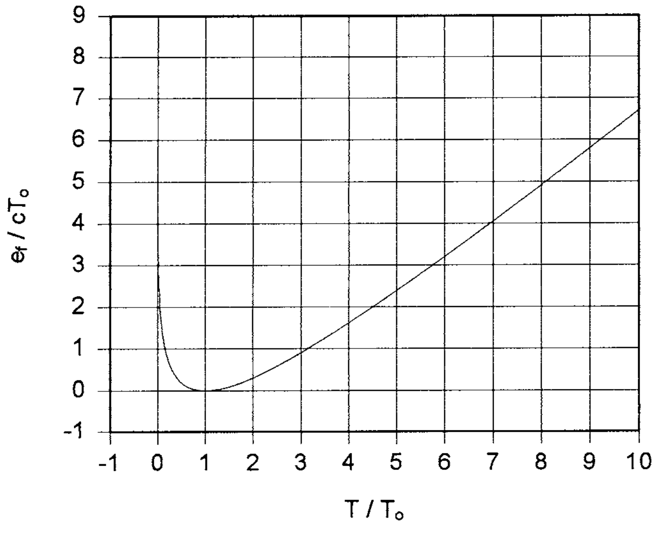
or

$$\frac{e_f}{cT_0} = \frac{T}{T_0} - 1 - \ln\frac{T}{T_0} + \underline{\frac{vp_0}{cT_0}}\left[\frac{p}{p_0} - 1\right] \qquad (1)$$

Selecting c and ρ from Table A-19E typical for geothermal applications, the underlined term of Eq. (1) is

$$\left[\frac{vp_0}{cT_0}\right] = \left[\frac{(14.7\times144)\ lbf/ft^2}{\left(1.01\ \frac{Btu}{lb\cdot°R}\right)\left(60\ \frac{lb}{ft^3}\right)(520°R)}\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|\right] = 8.63\times10^{-5}$$

The following plot is constructed using IT:



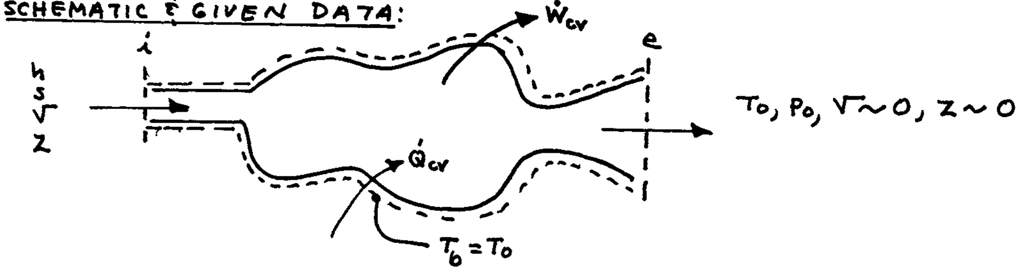
Note that the constant in Eq. (1) has a small value: $[vp_0/cT_0] = 8.63\times10^{-5}$, which indicates a weak dependence of (e_f/c_pT_0) on (p/p_0). The above plot reflects this, as the curves for various values of (p/p_0) fall on top of one another.

PROBLEM 7.48

KNOWN: The state of a flowing gas is defined by h, s, V, z.

FIND: Determine the maximum theoretical work, per unit of mass flowing, that could be developed as the stream is reduced to the dead state.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state. (2) Heat transfer occurs only at temperature $T_b = T_0$. (3) The velocity and elevation are relative to the environment, which is at T_0, p_0.

ANALYSIS: At steady state the exergy rate balance, Eq. 7.31, reduces to

$$\cancelto{0}{\frac{dE_{cv}}{dt}} = \underbrace{\left[1-\frac{T_0}{T_b}\right]}_{\substack{=0 \text{ since}\\ T_b = T_0}}\dot{Q}_{cv} - \left[\dot{W}_{cv} - p_0\cancelto{0}{\frac{dV}{dt}}\right] + \dot{m}\,(e_{fi} - \underbrace{e_{fe}}_{\substack{=0 \text{ since the stream is}\\ \text{reduced to the dead state}\\ \text{at the exit}}}) - \dot{E}_d$$

Thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = e_{fi} - \frac{\dot{E}_d}{\dot{m}}$$

The maximum theoretical value is obtained when $\dot{E}_d = 0$: when there are no internal irreversibilities. Thus

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{max} = (h-h_0) - T_0(s-s_0) + \frac{V^2}{2} + gZ \qquad \longleftarrow (\dot{W}_{cv}/\dot{m})_{max}$$

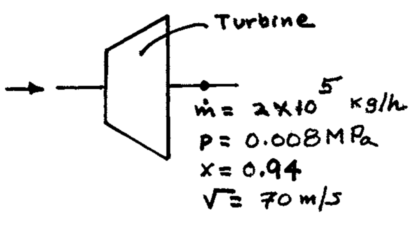
Comparing this expression to Eq. 7.20, it can be concluded that the specific flow exergy can be interpreted as the maximum theoretical work, per unit of mass flowing, that could be developed as the stream is brought to the dead state while allowing heat transfer only at T_0. This interpretation complements those given in Sec. 7.3.2.

PROBLEM 7.49

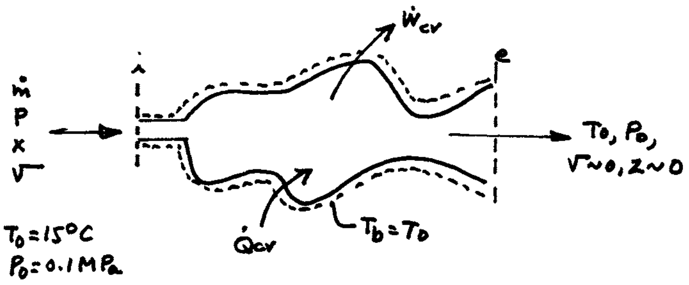
KNOWN: Steam exits a turbine at a specified state and mass flow rate.

FIND: Determine the maximum theoretical power that could be developed by reducing the exiting stream to the dead state while allowing heat transfer only at temperature T_0.

SCHEMATIC & GIVEN DATA:



ACTUAL



HYPOTHETICAL

ASSUMPTIONS: (1) The hypothetical control volume is at steady state and heat transfer occurs only at temperature $T_b = T_0$. (2) The velocity is relative to the environment for which temperature is T_0 and pressure is p_0. (3) The effect of gravity can be neglected.

ANALYSIS: The solution to Problem 7.48 shows that the maximum theoretical power that could be developed by any one-inlet, one-exit control volume at steady state that would reduce the stream to the dead state at the exit while allowing heat transfer only at T_0 is

$$(\dot{W}_{cv})_{max} = \dot{m}\, e_f = \dot{m}\left[(h - h_0) - T_0(s - s_0) + \frac{V^2}{2}\right]$$

Then, with $h_0 \sim h_f(T_0) = 62.99\ kJ/kg$, $s_0 \sim s_f(T_0) = 0.2245\ kJ/kg\cdot K$, and

$$h = h_f + x h_{fg} = 173.88 + 0.94(2403.1) = 2432.79\ kJ/kg$$

$$s = s_f + x s_{fg} = 0.5926 + 0.94(8.2287 - 0.5926) = 7.7705\ kJ/kg\cdot K$$

$$(\dot{W}_{cv})_{max} = \left(\frac{2\times10^5}{3600}\ \frac{kg}{s}\right)\left[(2432.79 - 62.99)\frac{kJ}{kg} - 288K(7.7705 - 0.2245)\frac{kJ}{kg\cdot K} + \frac{(70\frac{m}{s})^2}{2}\left|\frac{1N}{1kg\cdot m/s^2}\right|\left|\frac{1kJ}{10^3 N\cdot m}\right|\right]$$

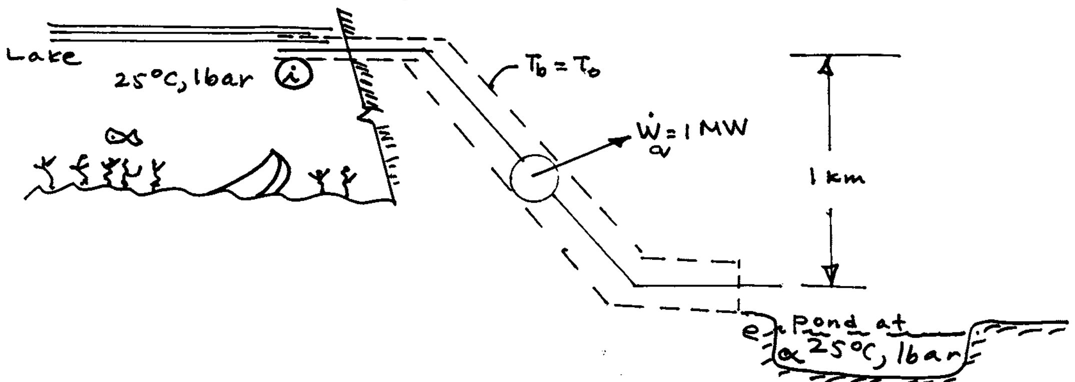
$$= \left(\frac{2\times10^5}{3600}\right)\left[2369.8 - 2173.25 + 2.45\right]\frac{kJ}{s}\left|\frac{1MW}{10^3 kJ/s}\right| = 11.1\ MW \quad \longleftarrow$$

PROBLEM 7.50

KNOWN: Water drawn from a lake flows through a hydraulic turbine-generator to a pond located 1 km below the lake.

FIND: Determine the minimum mass flow rate required to generate electricity at a rate of 1MW.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state, and heat transfer occurs only at $T_b = T_0$. 2. At the exit e, the state corresponds to the dead state: T_0, P_0, $V_0 \sim 0$, $z_0 \sim$, $h_e = h_0$, $s_e = s_0$. 3. At the inlet i, kinetic energy can be ignored. 4. For the environment, $T_0 = 25°C$, $P_0 = 1bar$, $g = 9.81 m/s^2$.

ANALYSIS: The actual power developed must satisfy, $\dot{W}_{cv} \le (\dot{W}_{cv})_{max}$. Using the result of Problem 7.48, the maximum power that could be developed by any control volume of the type under consideration is

① $$(\dot{W}_{cv})_{max} = \dot{m}\, e_{fi} = \dot{m}\left[(h_i - h_0) - T_0(s_i - s_0) + \frac{V_i^2}{2} + g z_i\right]$$

Accordingly

$$\dot{W}_{cv} \le \dot{m}\left[\underline{(h_i - h_0) - T_0(s_i - s_0) + \frac{V_i^2}{2}} + g z_i\right]$$

The underlined term vanishes because $P_i = P_0$, $T_i = T_0$ and the inlet kinetic energy is negligible by assumption 3. Thus,

$$\dot{W}_{cv} \le \dot{m}\,(g z_i) \quad \text{or} \quad \dot{m} \ge \frac{\dot{W}_{cv}}{g z_i} = \frac{(10^6\ J/s)}{(9.81\frac{m}{s^2})(1000m)}\left|\frac{1N\cdot m}{J}\right|\left|\frac{1kg\cdot m/s^2}{1N}\right|$$

$$\Rightarrow \dot{m} \ge 101.9\ kg/s \qquad \dot{m}_{min} = 101.9\ kg/s \quad \longleftarrow$$

---

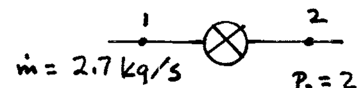
1. Alternatively, the analysis could invoke Eq. 6.53a.

PROBLEM 7.51*

KNOWN: Data are provided for the throttling of steam across a valve.

FIND: (a) Determine the flow exergy rates at the valve inlet and exit and the rate of exergy destruction. (b) Evaluate the annual cost associated with exergy destruction.

SCHEMATIC & GIVEN DATA:

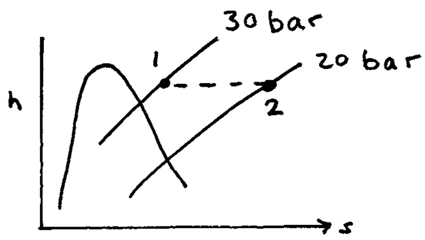


$\dot{m} = 2.7$ kg/s

$T_1 = 280°C$

$P_1 = 30$ bar

$P_2 = 20$ bar



ASSUMPTIONS: 1. The expansion across the valve is a throttling process: $h_2 \approx h_1$. 2. For the environment, $T_0 = 298\text{ K }(25°C)$, $P_0 = 1\text{ atm}$. 3. Exergy is valued at \$0.08/kW·h. 4. 7500 operating hours annually. 5. Ignore the effects of motion and gravity.

ANALYSIS: (a) With assumption 1, $h_2 \approx h_1$. Thus, state 2 is fixed by P_2, h_2. From Table A-4, $h_1 = 2941.3$ kJ/kg, $s_1 = 6.4462$ kJ/kg·K, $s_2 = 6.6169$ kJ/kg·K. Table A-2 gives $h_0 \sim h_f(T_0) = 104.89$ kJ/kg, $s_0 \sim s_f(T_0) = 0.3674$ kJ/kg·K. Then, with assumption 5, Eq. 7.20 gives

$$e_{f1} = h_1 - h_0 - T_0(s_1 - s_0) = (2941.3 - 104.89) - 298(6.4462 - 0.3674) = 1024.9\text{ kJ/kg}$$

$$e_{f2} = h_2 - h_0 - T_0(s_2 - s_0) = (2941.3 - 104.89) - 298(6.6169 - 0.3674) = 974.1\text{ kJ/kg}$$

The associated rates are

$$\dot{E}_{f1} = \dot{m}\, e_{f1} = (2.7\text{ kg/s})(1024.9\text{ kJ/kg}) = 2767.2\text{ kW} \quad \longleftarrow$$

$$\dot{E}_{f2} = \dot{m}\, e_{f2} = (2.7)(974.1) = 2630.1\text{ kW}$$

An exergy rate balance reduces to give

$$0 = \cancel{\sum_j \left(1 - \frac{T_0}{T_j}\right)\dot{Q}_j}^{\,0} - \cancel{\dot{W}}^{\,0} + (\dot{E}_{f1} - \dot{E}_{f2}) - \dot{E}_d$$

$$\Rightarrow \quad \dot{E}_d = \dot{E}_{f1} - \dot{E}_{f2} = (2767.2 - 2630.1) = 137.1\text{ kW} \quad \longleftarrow$$

(b) Evaluating exergy at \$0.08/kW·h, the cost associated with the exergy destroyed for 7500 hours of operation annually is

$$\$ = (137.1\text{ kW})\left(\frac{7500\text{ h}}{\text{year}}\right)\left(\frac{\$0.08}{\text{kW}\cdot\text{h}}\right)$$

① $$= \$79{,}260/\text{year} \quad \longleftarrow$$

---

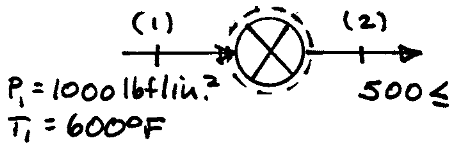
1. The irreversibility associated with the expansion across the valve can be reduced by replacing the valve with a power-recovery turbine. However, the costs of the turbine and its operation would have to considered carefully before such a replacement would be approved.

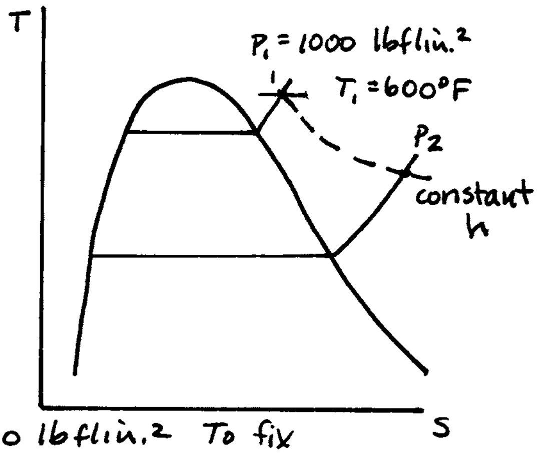
PROBLEM 7.52

KNOWN: Steam at a known state enters a valve operating at steady state and undergoes a throttling process to pressure p_2.

FIND: (a) For $p = 500$ lbf/in.², determine the exit temperature and the exergy destruction per unit of steam flowing. (b) Plot these quantities versus p ranging from 500 to 1000 lbf/in.²

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown is at steady state. (2) The steam undergoes a throttling process in passing through the valve. (3) For the reference environment, $T_0 = 70°F$, $p_0 = 14.7$ lbf/in.²

ANALYSIS: State 1 is fixed by $T_1 = 600°F$, $p_1 = 1000$ lbf/in.² To fix state 2, we use assumption (2) to get

$$h_1 = h_2 \qquad (1)$$

Thus, with p_2 and h_2, state 2 is fixed as well and T_2 can be determined.

To get $\dot{E}_d/\dot{m}$, we begin with an exergy balance at steady state

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right)\overset{0}{\cancel{\dot{Q}_j}} - \overset{0}{\cancel{\dot{W}}} + \dot{m}\left[e_{f_1} - e_{f_2}\right] - \dot{E}_d$$

or

$$\dot{E}_d/\dot{m} = e_{f_1} - e_{f_2} = \overset{0}{\cancel{(h_1 - h_2)}} - T_0(s_1 - s_2) \qquad (2)$$

① (a) From Table A-4E; $h_1 = 1248.8$ Btu/lb and $s_1 = 1.4450$ Btu/lb·°R. Interpolating in Table A-4E at $p_2 = 500$ lbf/in.², $h_2 = h_1 = 1248.8$ Btu/lb we get

$$T_2 = 524.6\,°F \qquad \text{(a) } T_2$$

$$s_2 = 1.5098 \text{ Btu/lb°R}$$

Now, from Eq. (2) above

$$\dot{E}_d/\dot{m} = -T_0(s_1 - s_2)$$

$$= -(530°R)(1.4450 - 1.5098)\frac{\text{Btu}}{\text{lb}\cdot°R}$$

$$= 34.34 \text{ Btu/lb} \qquad \text{(a) } \dot{E}_d/\dot{m}$$

(b) The data to construct the required plots are obtained using IT, as follows:

PROBLEM 7.52 (Cont'd.)

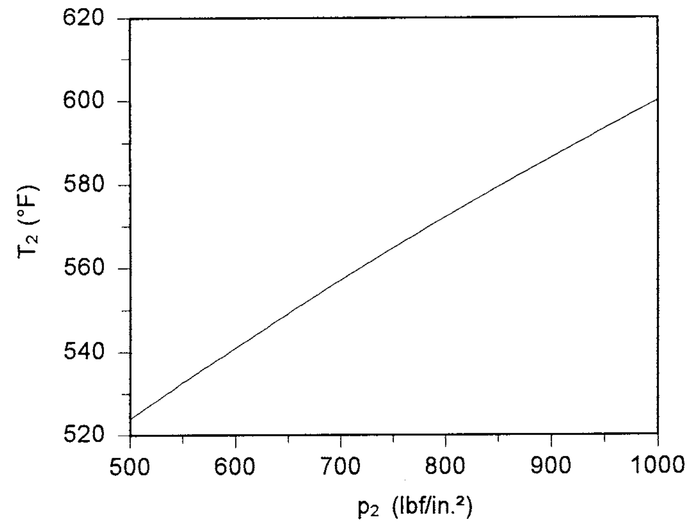
IT Code

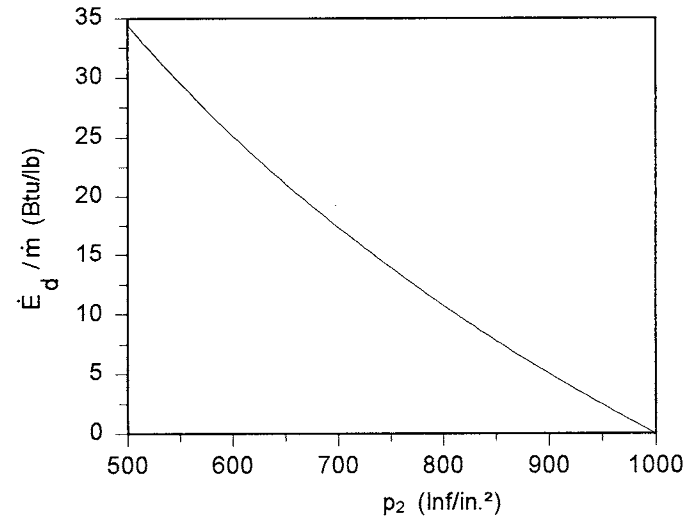
```
p1 = 1000  // lbf/in.²
T1 = 600  // °F
p2 = 500  // lbf/in.²
To = 530  // °R

h1 = h_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
h1 = h2
s1 = s_PT("Water/Steam", p1, T1)
s2 = s_PT("Water/Steam", p2, T2)

Ed = - To * (s1 - s2)
```

IT Results for p_2 = 500 lbf/in.²

h_1 = 1249 Btu/lb
s_1 = 1.445 Btu/lb·°R
s_2 = 1.51 Btu/lb·°R
$\dot{E}_d/\dot{m}$ = 34.43 Btu/lb
T_2 = 523.8 °F

PLOTS:





Discussion

- As seen on the accompanying T-s diagram, the exit temperature decreases as p_2 decreases for fixed h. Thus, the plot of exit temperature exhibits expected behavior.
- The rate of exergy destruction per unit mass of steam flowing increases with decreasing p_2. This is also expected based on greater entropy production and hence increasing s_2. (See the T-s diagram.)

---

1. In early printing of the 4th edition, h_1 is incorrectly given in Table A-4E as 1238.8 Btu/lb. The value used here (in the solution) is the correct value.

PROBLEM 7.53

KNOWN: Air at a given state enters a valve operating at steady state with a known volumetric flow rate. The air undergoes a throttling process to exit pressure p_2.

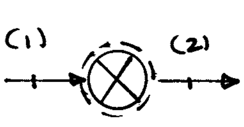
FIND: (a) Determine the rate of exergy destruction for $p_2 = 15\ \text{lbf/in}^2$. (b) Plot the exergy destruction rate versus p_2 ranging from 15 to 200 lbf/in.²

SCHEMATIC & GIVEN DATA:

$p_1 = 200\ \frac{\text{lbf}}{\text{in.}^2}$

$T_1 = 800°R$

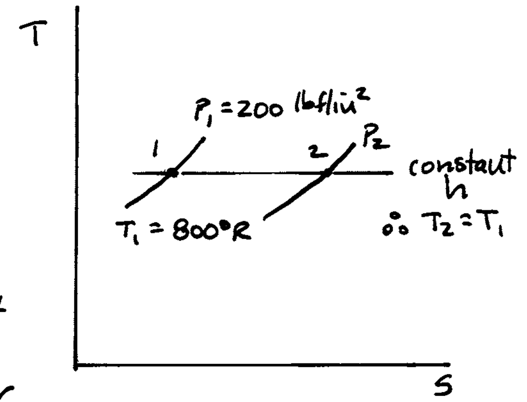
$(AV)_1 = 100\ \text{ft}^3/\text{min}$



$15 \le p_2 \le 200\ \text{lbf/in}^2$

$p_0 = 15\ \text{lbf/in}^2$

$T_0 = 530°R$



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) The air undergoes a throttling process in passing through the valve. (3) The air is modeled as an ideal gas. (4) For the reference environment, $T_0 = 530°R$, $p_0 = 15\ \text{lbf/in}^2$

ANALYSIS: State 1 is fixed by $T_1 = 800°R$, $p_1 = 200\ \text{lbf/in}^2$. To fix state 2, we use assumption (2) to get $h_1 = h_2$. By assumption (3)

$$T_2 = T_1 \tag{1}$$

To get $\dot{E}_d$, begin with an exergy balance at steady state. With $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$

$$0 = \sum_j \left[1 - \frac{T_0}{T_j}\right] \cancelto{0}{\dot{Q}_j} - \cancelto{0}{\dot{W}_j} + \dot{m}(e_{f_1} - e_{f_2}) - \dot{E}_d$$

or

$$\dot{E}_d = \dot{m}(e_{f_1} - e_{f_2}) = \dot{m}\left[\cancelto{0}{(h_1 - h_2)} - T_0(s_1 - s_2)\right] \tag{2}$$

To get $\dot{m}$, we use $\dot{m} = (AV)_1/v_1$ and the ideal gas equation for v_1.

(a) Evaluating $\dot{m}$

$$\dot{m} = \frac{p_1 (AV)_1}{RT_1} = \frac{(200\ \text{lbf/in}^2)(100\ \text{ft}^3/\text{min})}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(800°R)} \left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 67.5\ \text{lb/min}$$

Using ideal gas relations with Eq. (2) and inserting values

$$\dot{E}_d = -\dot{m}\, T_0 \left[\cancelto{0}{(s°(T_1) - s°(T_2))} - R \ln p_1/p_2\right]$$

$$= -\left(67.5\ \frac{\text{lb}}{\text{min}}\right)(530°R)\left[-\frac{1.986}{28.97}\ \frac{\text{Btu}}{\text{lb}\cdot°R} \ln\left(\frac{200}{15}\right)\right]$$

$$= 6353\ \text{Btu/min} \qquad (a)\ \dot{E}_d$$

(b) The data to construct the required plot are obtained using IT, as follows. For the IT solution, $v_1(T_1, p_1)$ and the entropy values $s_1(T_1, p_1)$ and $s_2(T_2, p_2)$ are evaluated directly using internal IT property functions.

PROBLEM 7.53 (Cont'd.)

IT Code

```
p1 = 200  // lbf/in.²
T1 = 800  // °R
p2 = 15  // lbf/in.²
AV1 = 100  // ft³/min
To = 530  // °R

T2 = T1
v1 = v_TP("Air",T1,p1)
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)

mdot = AV1 / v1
Edot = - mdot * To * (s1 - s2)
```

IT Results for p_2 = 15 lbf/in.²

v_1 = 1.481 ft³/lb

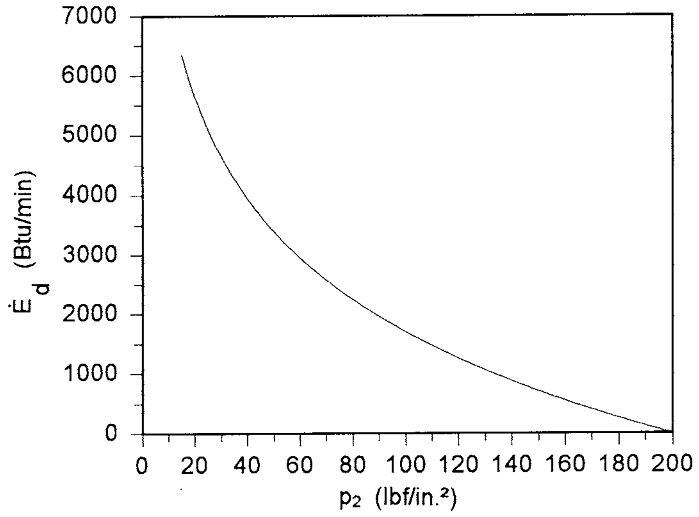
s_1 = 0.5167 Btu/lb·°R

s_2 = 0.6941 Btu/lb·°R

$\dot{m}$ = 67.5 lb/min

$\dot{E}_d$ = 6347 Btu/min

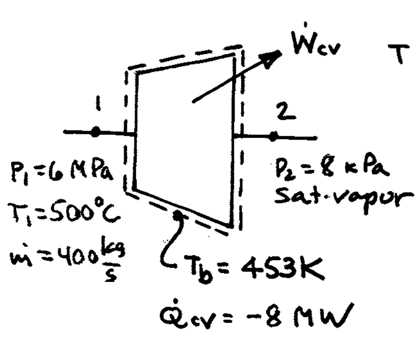
PLOT:



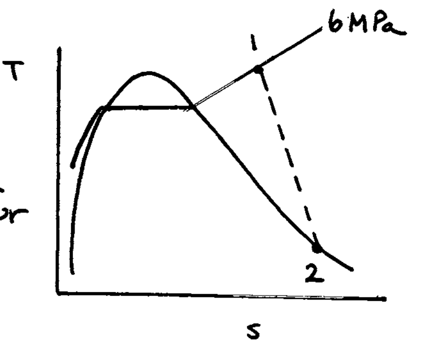
From the plot, we see that the rate of exergy destruction increases with decreasing p_2. This is expected based on greater entropy production and hence increasing s_2. (See the T-s diagram.)

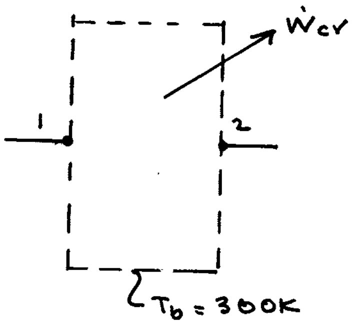
PROBLEM 7.54*

KNOWN: Steady-state operating data are provided for a steam turbine.
FIND: Determine the power developed. For each of two choices for the control volume, evaluate the rate of exergy destruction. Discuss.
SCHEMATIC & GIVEN DATA:



Control Volume (a)





Control Volume (b)

ASSUMPTIONS: 1. The control volumes shown in the schematic are at steady state. 2. In each case, heat transfer occurs at T_b. 3. Effects of motion and gravity can be ignored. 4. For the environment, $T_0 = 300K$, $p_0 = 100$ kPa.

ANALYSIS: (a) Mass and energy rate balances reduce at steady state to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}(h_1 - h_2).$$

Then, with data from Tables A-3, 4

$$\dot{W}_{cv} = (-8 \text{ MW}) + (400 \tfrac{kg}{s})[3422.2 - 2577.0]\tfrac{kJ}{kg}\left|\frac{1 MW}{10^3 kJ/s}\right| = 330.1 \text{ MW} \longleftarrow$$

The rate of exergy destruction can be obtained by reducing an exergy rate balance or using $\dot{E}_d = T_0\dot{\sigma}$. Selecting the second, an entropy rate balance reads

$$0 = \frac{\dot{Q}}{T_b} + \dot{m}(s_1 - s_2) + \dot{\sigma} \Rightarrow \dot{\sigma} = -\frac{\dot{Q}}{T_b} + \dot{m}(s_2 - s_1)$$

Then,

$$\dot{\sigma} = -\left(\frac{-8 MW}{453 K}\right) + (400 \tfrac{kg}{s})[8.2287 - 6.8803]\tfrac{kJ}{kg \cdot K}\left|\frac{1 MW}{10^3 kJ/s}\right|$$

$$= 0.55702 \frac{MW}{K} \qquad \Rightarrow \dot{E}_d = (300K)(0.55702 \tfrac{MW}{K}) = 167.1 \text{ MW}$$

(b) For the enlarged control volume, the entropy rate balance reads

$$\dot{\sigma} = -\frac{\dot{Q}}{T_b} + \dot{m}(s_2 - s_1)$$

$$= -\frac{(-8 MW)}{300 K} + (400)[8.2287 - 6.8803]\left|\frac{1}{10^3}\right| = 0.56603 \frac{MW}{K}$$

① $$\Rightarrow \dot{E}_d = (300K)(0.56603 \tfrac{MW}{K}) = 169.8 MW$$

The exergy destruction in control volume (b) is greater because there is an additional source of irreversibility — namely, the irreversibility related to heat transfer from the outer surface of the turbine to the ambient. For control volume (a), the heat transfer effect is an *external* irreversibility.

---

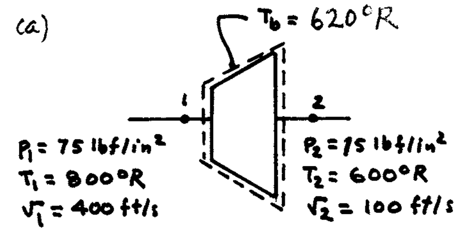
1. Using values obtained in the analysis, the net rate exergy is supplied to the turbine by the steam, $(\dot{E}_{f1} - \dot{E}_{f2})$, is 499.9 MW. Since $[\dot{E}_d / (\dot{E}_{f1} - \dot{E}_{f2})] = [169.8/499.9] = 0.34$ (34%), there is considerable scope in this case for improving thermodynamic performance.

PROBLEM 7.55

KNOWN: Steady state operating data are provided for an air turbine.

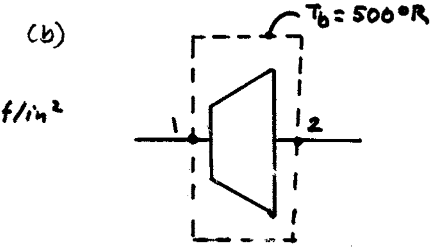
FIND: DETERMINE, per unit of mass flowing, the work developed and, for each of two choices of control volume, the exergy destruction. Discuss.

SCHEMATIC & GIVEN DATA:



$\frac{\dot{Q}_{cv}}{\dot{m}} = -2\ \frac{Btu}{lb}$

$T_0 = 40°F,\ P_0 = 15\ lbf/in^2$



ASSUMPTIONS: (1) The control volumes in the accompanying figure are at steady state. (2) Heat transfer takes place at temperature T_b. (3) There is no effect of gravity. (4) For the environment, $T_0 = 500°R$, $P_0 = 1\ atm$. (5) Air is modeled as an ideal gas.

ANALYSIS: At steady state an energy rate balance reduces to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + \overset{0}{\cancel{g(z_1 - z_2)}}\right]$$

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) \qquad (1)$$

With data from Table A-22E

$$\frac{\dot{W}_{cv}}{\dot{m}} = -2\ \frac{Btu}{lb} + (91.81 - 143.47)\ \frac{Btu}{lb} + \frac{[(400)^2 - (100)^2](ft^2/s^2)}{(2)\ 32.2\ \frac{lb \cdot ft}{lbf \cdot s^2}\ \ 778\ \frac{ft \cdot lbf}{Btu}}$$

$$= -2\ \frac{Btu}{lb} + 48.34\ \frac{Btu}{lb} + 2.99\ \frac{Btu}{lb} = 49.33\ \frac{Btu}{lb} \longleftarrow \dot{W}_{cv}/\dot{m}$$

Control Volume (a)

The exergy destruction can be evaluated by reducing an exergy rate balance. Thus

$$\frac{\dot{E}_d}{\dot{m}} = \left[1 - \frac{T_0}{T_b}\right]\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right) - \frac{\dot{W}_{cv}}{\dot{m}} + \left[(h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2}\right]$$

Introducing Eq. (1) and simplifying

$$\frac{\dot{E}_d}{\dot{m}} = T_0\left[(s_2 - s_1) - \frac{\dot{Q}_{cv}/\dot{m}}{T_b}\right] \qquad (2)$$

Or, for an ideal gas (assumption 5)

$$\frac{\dot{E}_d}{\dot{m}} = T_0\left[\left(s^\circ(T_2) - s^\circ(T_1) - \frac{\bar{R}}{M} \ln \frac{P_2}{P_1}\right) - \frac{\dot{Q}_{cv}/\dot{m}}{T_b}\right] \qquad (3)$$

With data from Table A-22E

$$\frac{\dot{E}_d}{\dot{m}} = 500°R\left[\left(0.62607 - 0.69558 - \frac{1.986}{28.97} \ln \frac{15}{75}\right)\frac{Btu}{lb \cdot °R} - \frac{(-2\ Btu/lb)}{620}\right]$$

$$= 500°R\left[0.04082 + 0.0032\right]\frac{Btu}{lb \cdot °R} = 22.01\ Btu/lb \longleftarrow \dot{I}_{cv}/\dot{m}$$

Control Volume (b). Eqs. (2)-(3) remain applicable. Thus

$$\frac{\dot{E}_d}{\dot{m}} = 500°R\left[0.04082 - \frac{(-2\ Btu/lb)}{500°R}\right]$$

$$= 22.41\ \frac{Btu}{lb} \longleftarrow (\dot{I}_{cv}/\dot{m})$$

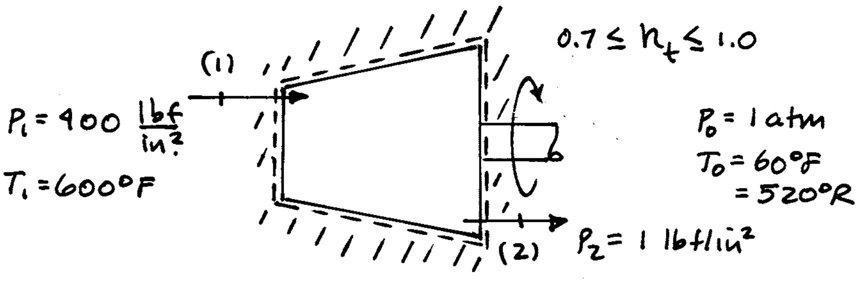
Discussion: The irreversibility is greater for control volume (b) because the enlarged control volume has an additional source of irreversibility: the irreversibility associated with heat transfer from the turbine's surface to the surroundings. For control volume (a), the heat transfer effect is an external irreversibility.

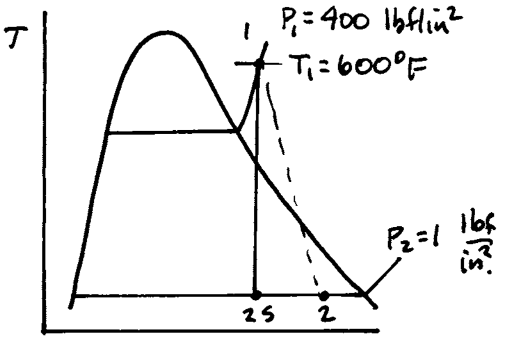
PROBLEM 7.56

KNOWN: An insulated steam turbine operating at steady state receives steam at a known state and exhausts at a given pressure.

FIND: Plot the exergy destruction rate per unit of steam flowing versus isentropic turbine efficiency ranging from 70 to 100%.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the turbine, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects can be neglected. (3) For the exergy reference environment, $T_0 = 520°R$, $p_0 = 1\ atm$.

ANALYSIS: To determine the exergy destruction rate, we begin with mass and entropy rate balances which reduce to give

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}_j}{T_j}\right)}^{0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv}/\dot{m} = (s_2 - s_1)$$

With $\dot{E}_d = T_0 \dot{\sigma}_{cv}$

$$\dot{E}_d/\dot{m} = T_0(s_2 - s_1) \tag{1}$$

Since $p_1 = 400\ lbf/in.^2$, $T_1 = 600°F$, state 1 is in the superheated vapor region and the state is fixed. To fix state 2, we use the isentropic turbine efficiency

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) \tag{2}$$

where h_{2s} is determined using $P_2 = 1\ lbf/in.^2$ and $s_{2s} = s_1$. Then, with h_2 known from Eq.(2) and p_2, s_2 can be determined.

Sample calculation: $\boxed{CASE: \eta_t = 0.7}$ From Table A-4E, $h_1 = 1306.6$ Btu/lb and $s_1 = 1.5892$ Btu/lb·°R. With $s_{2s} = s_1 = 1.5892$

$$x_{2s} = \frac{s_{2s} - s_{f2}}{s_{fg2}} = \frac{1.5892 - 0.1327}{1.8453} = 0.7893$$

and $h_{2s} = h_{f2} + x_{2s} h_{fg2} = 69.74 + (.7893)(1036.0) = 887.45$ Btu/lb

From (2)

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1306.6 - (.7)(1306.6 - 887.45) = 1013.2\ Btu/lb$$

Finally, with $p_2 = 1\ lbf/in.^2$; $x_2 = \frac{h_2 - h_{f2}}{h_{fg2}} = 0.9107$ and $s_2 = s_{f2} + x_2 s_{fg2} = 1.8132\ \frac{Btu}{lb°R}$

Thus

$$\dot{E}_d/\dot{m} = T_0(s_2 - s_1) = (520°R)(1.8132 - 1.5892)\frac{Btu}{lb \cdot °R}$$

$$= 116.5\ Btu/lb \longleftarrow$$ $\dot{E}_d/\dot{m}$ $(\eta_t = 70\%)$

PROBLEM 7.56 (Cont'd.)

The data to construct the required plot are obtained using IT, as follows:

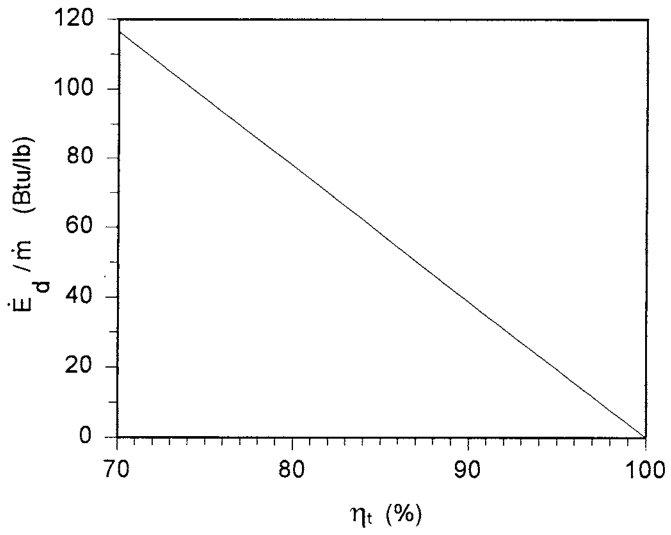
IT Code

```
p1 = 400  // lbf/in.²
T1 = 600  // °F
p2 = 1  // lbf/in.²
eff = 0.7
To = 60 + 460  // °R

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water/Steam", p2, s2s)

h2 = h1 - eff * (h1 - h2s)
Ed = To * (s2 - s1)
h2 = h_Ps("Water/Steam", p2, s2)
```

IT Results for $\eta_t = 70\%$

$h_1 = 1306$ Btu/lb
$h_2 = 1013$ Btu/lb
$h_{2s} = 887.3$ Btu/lb
$s_1 = 1.589$ Btu/lb·°R
$s_2 = 1.813$ Btu/lb·°R
$s_{2s} = 1.589$ Btu/lb·°R
$\dot{E}_d / \dot{m} = 116.5$ Btu/lb

PLOT:



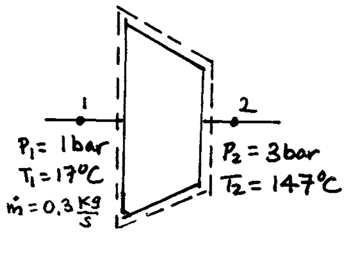
From the plot we see that lower values of isentropic turbine efficiency correspond to increased exergy destruction, as expected.

PROBLEM 7.57

KNOWN: Steady-state operating data are provided for an air compressor.

FIND: Determine the power required by the compressor and the rate of exergy destruction. Express the exergy destruction as a percentage of the power input.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume shown in the schematic is at steady state, $\dot{Q}_{cv} = 0$.
2. Effects of motion and gravity can be ignored.
3. ① Air is modeled as an ideal gas.
4. $T_0 = 290\,K$, $p_0 = 1\,bar$

ANALYSIS: Mass and energy rate balances reduce to give

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}\left[(h_1 - h_2) + \cancel{\frac{V_1^2 - V_2^2}{2}}^{0} + g\cancel{(z_1 - z_2)}^{0}\right]$$

with data from Table A-22

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) = \left(0.3\,\frac{kg}{s}\right)\left[290.16 - 421.26\right]\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right| = -39.33\,kW \leftarrow \dot{W}_{cv}$$

The rate of exergy destruction can be obtained by reducing an exergy rate balance or using $\dot{E}_d = T_0\dot{\sigma}_{cv}$, where $\dot{\sigma}$ is the rate of entropy production from an entropy balance. Using the entropy approach,

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) = \dot{m}\left[s^\circ(T_2) - s^\circ(T_1) - R\ln p_2/p_1\right]$$

$$= \left(0.3\,\frac{kg}{s}\right)\left[2.04142 - 1.66802 - \frac{8.314}{28.97}\ln\frac{3}{1}\right]\frac{kJ}{kg\cdot K}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 0.0174\,\frac{kW}{K}$$

Then,

$$\dot{E}_d = (290\,K)\left(0.0174\,\frac{kW}{K}\right) = 5.05\,kW \qquad \leftarrow \dot{E}_d$$

Forming a ratio

$$\left(\frac{\dot{E}_d}{-\dot{W}_{cv}}\right) = \frac{5.05\,kW}{39.33\,kW} = 0.128 \quad (12.8\%) \qquad \leftarrow$$

---

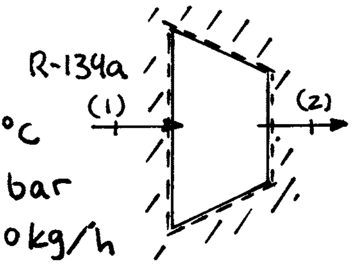
1. Can be checked using the generalized compressibility chart.

PROBLEM 7.58*

KNOWN: Operating data are provided for an insulated compressor at steady state.

FIND: Determine (a) the exit temperature, (b) the power input, and (c) the exergy destruction rate expressed as a percentage of the power input.

SCHEMATIC & GIVEN DATA:



$T_1 = -10°C$

$P_1 = 1.4$ bar

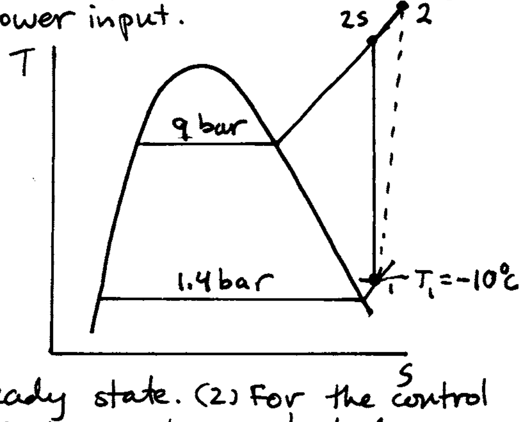
$\dot m = 280$ kg/h

$P_2 = 9$ bar

$\eta_c = 0.82$

$T_0 = 20°C = 293$ K

$P_0 = 1$ bar



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume, $\dot Q_{cv} = 0$ and kinetic and potential energy effects can be neglected. (3) For the exergy reference environment, $T_0 = 293$ K, $p_0 = 1$ bar.

ANALYSIS: (a) To find the exit temperature, we begin with the isentropic compressor efficiency

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s}-h_1}{\eta_c}$$

From Table A-12, $h_1 = 243.40$ kJ/kg, $s_1 = 0.9606$ kJ/kg·K. Interpolating in Table A-12 with $p_2 = 9$ bar and $s_{2s} = s_1 = 0.9606$; $h_{2s} = 283.65$ kJ/kg. Thus

$$h_2 = 243.40 + \frac{(283.65 - 243.40)}{0.82} = 292.49 \text{ kJ/kg}$$

Interpolating again in Table A-12 with $p_2 = 9$ bar, $h_2 = 292.49$ kJ/kg

$T_2 = 59.3$ °C ← T_2

$s_2 = 0.9875$ kJ/kg·K

(b) Applying energy and mass rate balances with assumptions (1) and (2)

$$\dot m_1 = \dot m_2 = \dot m$$

$$0 = \cancel{\dot Q_{cv}}^{0} - \dot W_{cv} + \dot m\left[(h_1-h_2) + \cancel{\left(\frac{V_1^2-V_2^2}{2}\right)}^{0} + g\cancel{(z_1-z_2)}^{0}\right]$$

Solving for the power input, $(-\dot W_{cv})$, and inserting values

$$-(\dot W_{cv}) = \dot m(h_2-h_1)$$

$$= \left(280\frac{kg}{h}\right)\left|\frac{1\,h}{3600\,s}\right|(292.49 - 243.40)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right| = 3.818 \text{ kW}$$ ← $(-\dot W_{cv})$

(c) The exergy destruction rate is conveniently evaluated using $\dot E_d = T_0\dot\sigma_{cv}$, where $\dot\sigma_{cv}$ is obtained from and entropy balance

$$0 = \cancel{\sum_j\left(\frac{\dot Q}{T}\right)_j}^{0} + \dot m(s_1-s_2) + \dot\sigma_{cv} \Rightarrow \dot E_d = T_0\dot m(s_2-s_1)$$

Thus

$$\dot E_d = (293\,K)\left(280\frac{kg}{h}\right)(0.9875 - 0.9606)\frac{kJ}{kg\cdot K}\left|\frac{1\,h}{3600\,s}\right|\left|\frac{1\,kW}{1\,kJ/s}\right| = 0.613 \text{ kW}$$

Finally

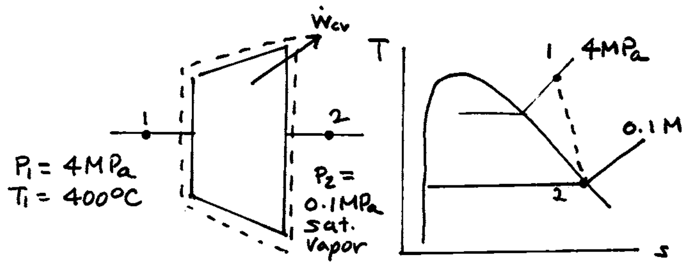
$$\% = \left(\frac{0.613}{3.818}\right)(100) = 16.1\,\%$$ ← %

PROBLEM 7.59

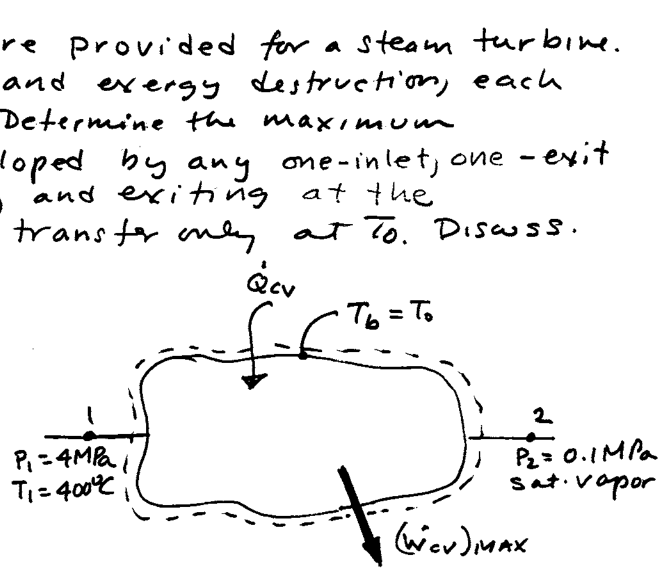
KNOWN: Steady-state operating data are provided for a steam turbine.

FIND: (a) Determine the work developed and exergy destruction, each per unit mass of steam flowing. (b) Determine the maximum theoretical work that could be developed by any one-inlet, one-exit control volume having steam entering and exiting at the specified states, while allowing heat transfer only at T_0. Discuss.

SCHEMATIC & GIVEN DATA:



ACTUAL



HYPOTHETICAL

ASSUMPTIONS: 1. The control volumes shown in the schematic are at steady state. 2. In the actual case, $\dot{Q}_{cv} = 0$. In the hypothetical case, heat transfer occurs only at $T_b = T_0$. 3. The effects of motion and gravity can be ignored. 4. $T_0 = 300K\,(27°C)$, $P_0 = 0.1\,MPa$.

ANALYSIS: (a) Reducing mass and energy rate balances, we get

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 = (3213.6 - 2675.5)\frac{kJ}{kg} = 538.1\ kJ/kg \qquad \longleftarrow$$

Reducing mass and entropy rate balances

$$\frac{\dot{\sigma}}{\dot{m}} = s_2 - s_1 = (7.3594 - 6.7690)\frac{kJ}{kg\cdot K} = 0.5904\frac{kJ}{kg\cdot K}$$

Then

$$\frac{\dot{E}_d}{\dot{m}} = T_0\left(\frac{\dot{\sigma}}{\dot{m}}\right) = 300K(.5904)\frac{kJ}{kg\cdot K} = 177.12\frac{kJ}{kg} \qquad \longleftarrow$$

(b) Mass and exergy rate balances reduce for the hypothetical case to give

$$0 = \underbrace{\left[1 - \frac{T_0}{T_b}\right]}_{\substack{=0 \text{ since} \\ T_b = T_0}}\dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

$$\Rightarrow \quad \frac{\dot{W}_{cv}}{\dot{m}} = (e_{f1} - e_{f2}) - \frac{\dot{E}_d}{\dot{m}} \implies \left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{MAX} = e_{f1} - e_{f2}$$

That is, the maximum power is obtained when there are no internal irreversibilities.

Then

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{MAX} = (h_1 - h_2) - T_0(s_1 - s_2) = (3213.6 - 2675.5) - 300\,(6.7690 - 7.3594)$$

$$= 538.1 + 177.12 = 715.22\ kJ/kg \qquad \longleftarrow$$

Combining the principal results for (a), (b)

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{MAX} = \left(\frac{\dot{W}_{cv}}{\dot{m}}\right) + \left(\frac{\dot{E}_d}{\dot{m}}\right)$$

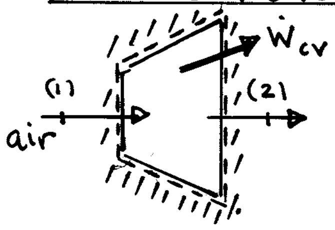
That is, the difference between the two work amounts is the exergy destruction for the actual turbine expansion.

PROBLEM 7.60

KNOWN: Operating data are provided for an insulated turbine at steady state.

FIND: Determine (a) the work developed and the exergy destruction, each per unit mass of air flowing, and (b) the maximum work that could be developed by any one-inlet, one-exit control volume at steady state with air entering and exiting at the specified states while allowing heat transfer only at temperature To.

SCHEMATIC & GIVEN DATA:

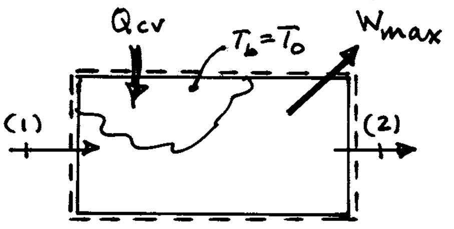


$P_1 = 8$ bar $\quad P_2 = 1$ bar

$T_1 = 500$K $\quad T_2 = 320$K

$V_1 = 150$ m/s $\quad V_2 = 10$ m/s

$T_0 = 300$K, $P_0 = 1$ bar



ASSUMPTIONS: (1) The control volumes shown are at steady state. (2) In part (a), $\dot{Q}_{cv} = 0$. In part (b), heat transfer occurs only at $T_b = T_0$. (3) Potential energy effects can be neglected. (4) The air is modeled as an ideal gas. (5) For the environment, $T_0 = 300$K, $P_0 = 1$atm.

ANALYSIS: The mass and energy rate balances reduce with assumptions (1)-(3) to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}\left[(h_1-h_2) + \frac{(V_1^2-V_2^2)}{2} + g(\cancelto{0}{z_1-z_2})\right] \Rightarrow \frac{\dot{W}_{cv}}{\dot{m}} = h_1-h_2+\frac{(V_1^2-V_2^2)}{2} \qquad (1)$$

With data from Table A-22

$$\frac{\dot{W}_{cv}}{\dot{m}} = (503.02-320.29)\frac{kJ}{kg} + \left(\frac{150^2-10^2}{2}\right)\frac{m^2}{s^2}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 193.93\ \frac{kJ}{kg} \longleftarrow (\dot{W}_{cv}/\dot{m})$$

The exergy destruction rate is found using $\dot{E}_d = T_0\dot{\sigma}_{cv}$ and an entropy balance.

$$0 = \cancelto{0}{\sum_j\left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}(s_1-s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{E}_d/\dot{m} = T_0(s_2-s_1) \qquad (2)$$

With ideal gas relations and data from Table A-22

$$\frac{\dot{E}_d}{\dot{m}} = T_0\left[s^\circ(T_2) - s^\circ(T_1) - R\ln P_2/P_1\right]$$

$$= (300K)\left[(1.76690-2.21952) - \left(\frac{8.314\ kJ}{28.97\ kg\cdot K}\right)\ln\left(\frac{1}{8}\right)\right] = 43.25\ kJ/kg \longleftarrow \dot{E}_d/\dot{m}$$

(b) In this case, the exergy rate balance reduces at steady state to give

$$0 = \cancelto{0}{\left[1-\frac{T_0}{T_0}\right]}\dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(e_{f1}-e_{f2}) - \dot{E}_d$$

The work is maximum when there are no internal irreversibilities: $\dot{E}_d = 0$. Thus

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{max} = e_{f1}-e_{f2} = (h_1-h_2) - T_0(s_1-s_2) + \frac{(V_1^2-V_2^2)}{2} + g(\cancelto{0}{z_1-z_2})$$

$$= (h_1-h_2) - T_0\left[s^\circ(T_2) - s^\circ(T_1) - R\ln P_2/P_1\right] + \frac{(V_1^2-V_2^2)}{2} \qquad (3)$$

$$= (503.02-320.29)-(300)\left[(2.21952-1.76690) - \frac{8.314}{28.97}\ln\left(\frac{8}{1}\right)\right] + \left(\frac{150^2-10^2}{2}\right)\left|\frac{1}{10^3}\right|$$

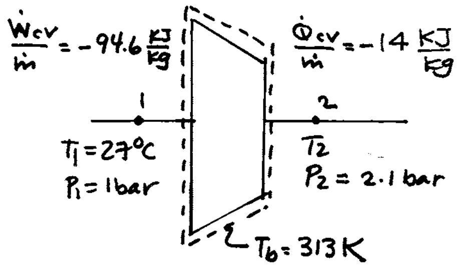
$$= 237.18\ kJ/kg \longleftarrow (\dot{W}_{cv}/\dot{m})_{max}$$

PROBLEM 7.61

KNOWN: Steady-state operating data are provided for an air compressor.

FIND: Determine the rate of exergy destruction and the rate of exergy transfer accompanying heat, each per unit mass of air flowing. Express each as a percentage of the compressor work input.

SCHEMATIC & GIVEN DATA:


$\frac{\dot{W}_{cv}}{\dot{m}} = -94.6 \frac{kJ}{kg}$
$\frac{\dot{Q}_{cv}}{\dot{m}} = -14 \frac{kJ}{kg}$
1
2
$T_1 = 27°C$
$p_1 = 1 bar$
T_2
$p_2 = 2.1 bar$
$T_b = 313 K$

Using energy and entropy balances, the solution of Problem 6.116 gives

$T_2 = 380K$, $\frac{\dot{\sigma}_{cv}}{\dot{m}} = 0.0698 \frac{kJ}{kg \cdot K}$

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. Heat transfer occurs only at T_b. 3. Air is modeled as an ideal gas. 4. The effects of motion and gravity can be ignored. 5. $T_0 = 293K$ (20°C), $p_0 = 1 atm$.

ANALYSIS: The exergy destruction can be found by reducing an exergy balance. Alternatively,

$$\frac{\dot{E}_d}{\dot{m}} = T_0 \left(\frac{\dot{\sigma}_{cv}}{\dot{m}}\right) = (293K)\left(0.0698 \frac{kJ}{kg \cdot K}\right) = 20.45 \frac{kJ}{kg} \quad \leftarrow$$

With assumption 2

$$\begin{bmatrix} \text{exergy transfer accompanying} \\ \text{heat transfer per unit of} \\ \text{mass flowing} \end{bmatrix} = \left[1 - \frac{T_0}{T_b}\right]\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)$$

$$= \left[1 - \frac{293}{313}\right]\left(-14 \frac{kJ}{kg}\right)$$

$$= -0.89 \frac{kJ}{kg}$$

Expressed as percentages of the work input:

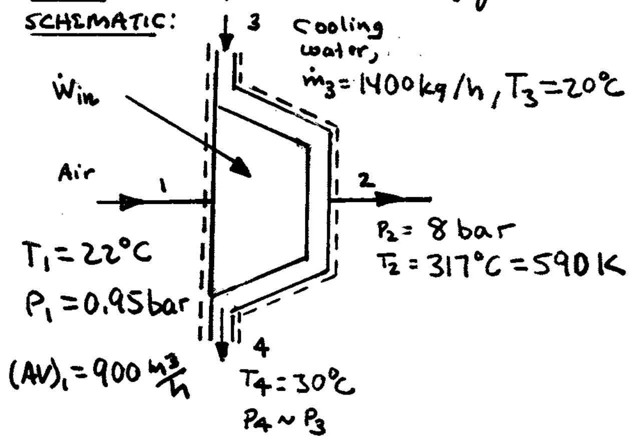
$$\left.\begin{aligned} &\text{exergy destruction:} \quad \left(\frac{20.45}{94.6}\right)(100) = 21.6\% \\ &\text{exergy transfer:} \quad \left(\frac{0.89}{94.6}\right)(100) = 0.9\% \end{aligned}\right\} \leftarrow$$

PROBLEM 7.62*

KNOWN: Steady-state operating data are provided for a water-jacketed air compressor.

FIND: Perform an exergy accounting of the power input

SCHEMATIC:



ASSUMPTIONS:

1. The control volume shown in the schematic operates at steady state with negligible kinetic and potential energy effects and $\dot{Q}_{cv} = 0$.
2. Air is modeled as an ideal gas.
3. Water is modeled as incompressible with $c = 4.19$ kJ/kg·K (Table A-19).
4. $T_0 = 293$K (20°C), $P_0 = 1$atm.

ANALYSIS: The exergy entering the control volume with the power input has the following disposition: (i) the exergy of the air stream is increased, (ii) the exergy of the water stream is increased, and (iii) exergy is destroyed by irreversibilities within the control volume. These quantities are now evaluated:

At steady state $\dot{m}_1 = \dot{m}_2$, $\dot{m}_3 = \dot{m}_4$. The energy rate balance reduces to give

$$\dot{W}_{cv} = \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4)$$

With the ideal gas model equation of state

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{(AV)_1}{(RT_1/P_1)} = \frac{(95{,}000 N/m^2)(900 m^3/h)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(295 K)} = 1009.9 \frac{kg}{h}$$

Then, with h_1, h_2 from Table A-22 and Eq. 3.20b $(P_4 \sim P_3)$

$$\dot{W}_{cv} = \left(1009.9 \frac{kg}{h}\right)\left[295.17 - 596.52\right]\frac{kJ}{kg} + \left(1400 \frac{kg}{h}\right)\left(4.19 \frac{kJ}{kg K}\right)(-10 K)$$

$$= \left[-304{,}333 + (-58{,}660)\right]\frac{kJ}{h}\left|\frac{1 kW}{3600 kJ/h}\right| = -100.83 kW$$

The rate of exergy destruction can be determined using an exergy rate balance or $\dot{E}_d = T_0\dot{\sigma}$ and an entropy rate balance. At steady state the entropy rate balance reduces to give

$$0 = \cancel{\sum_j \overset{0}{\dot{Q}_j}/T_j} + \dot{m}_1(s_1 - s_2) + \dot{m}_3(s_3 - s_4) + \dot{\sigma}_{cv}$$

Or with $s°$ data from Table A-22 and Eq. 6.24

$$\dot{\sigma}_{cv} = \dot{m}_1\left[s°(T_2) - s°(T_1) - R \ln P_2/P_1\right] + \dot{m}_3\, c \ln \frac{T_4}{T_3}$$

$$= \left(1009.9 \frac{kg}{h}\right)\left[2.39140 - 1.68515 - \frac{8.314}{28.97}\ln\frac{8}{1}\right]\frac{kJ}{kg K} + (1400)(4.19)\ln\frac{303}{293}$$

$$= \left[110.56 + 196.86\right]\frac{kJ}{h K}\left|\frac{1 kW}{3600 kJ/h}\right| = 0.08539 \frac{kW}{K}$$

$$\therefore \quad \dot{E}_d = T_0\dot{\sigma}_{cv} = (293K)\left(0.05839 \frac{kW}{K}\right) = 25.02 kW$$

PROBLEM 7.62 (Contd.)

For the air, the increase in flow exergy rate from inlet to exit is

$$\dot{E}_{f2} - \dot{E}_{f1} = \dot{m}_1(e_{f2} - e_{f1}) = \dot{m}_1\left[(h_2 - h_1) - T_0(s_2 - s_1)\right]$$

$$= \dot{m}_1(h_2 - h_1) - T_0\,\dot{m}(s_2 - s_1)$$

$$= \left(\frac{304{,}333}{3600}\right)\text{kW} - (293\,\text{K})\left[\left(\frac{110.56}{3600}\right)\right]\frac{\text{kW}}{\text{K}} = 75.54\ \text{kW}$$

For the water, the increase in flow exergy rate from inlet to exit is

$$\dot{E}_{f4} - \dot{E}_{f3} = \dot{m}_3(e_{f4} - e_{f3}) = \dot{m}_3\left[(h_4 - h_3) - T_0(s_4 - s_3)\right]$$

$$= \dot{m}_3(h_4 - h_3) - T_0\,\dot{m}_3(s_4 - s_3)$$

$$= \left(\frac{58{,}660}{3600}\right)\text{kW} - (293\,\text{K})\left(\frac{196.86}{3600}\right)\frac{\text{kW}}{\text{K}} \sim 0$$

Exergy Accounting:

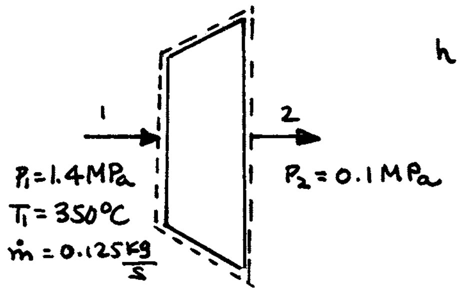
• Exergy input : power input	100.83 kW	
• Disposition :		
– Increase in exergy of the air stream	75.54 kW	(74.9 %)
– Increase in exergy of the water stream	0.27 kW	(0.3 %)
– Exergy destruction	25.02 kW	(24.8 %)
	100.83 kW	

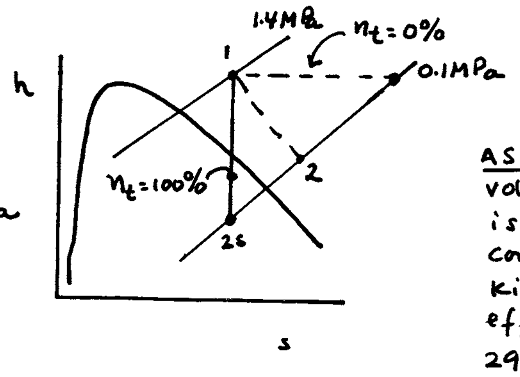
PROBLEM 7.63

KNOWN: Steady state operating data are provided for a steam turbine.

FIND: Plot versus the isentropic turbine efficiency ranging from 0 to 100%, the turbine exit temperature, power developed, and the rate of exergy destruction within the turbine.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are negligible. 3. $T_0 = 293 K (20°C)$, $P_0 = 1 atm$.

ANALYSIS: At steady state, mass and energy rate balances reduce to give $\dot{W}_{cv} = \dot{m}(h_1 - h_2)$. Introducing the isentropic turbine efficiency

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \quad (1) \Rightarrow \dot{W}_{cv} = \dot{m}\,\eta_t (h_1 - h_{2s}) \quad (2)$$

Note that Eq.(1) gives h_2, which together with P_2 fixes T_2. Mass and entropy rate balances reduce to give

$$\dot{E}_d = \dot{m}\, T_0 (s_2 - s_1) \quad (3)$$

Sample Calculation: $\eta_t = 80\%$. From Table A-4, $h_1 = 3149.5 \frac{kJ}{kg}$, $s_1 = 7.1406 \, kJ/kg \cdot K$ (double interpolation). Then, with Eq.(1) and h_{2s} determined with $s_{2s} = s_1$

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{7.1406 - 1.3026}{7.3594 - 1.3026} = 0.964, \quad h_{2s} = 417.6 + (.964)(2258) = 2594.3 \frac{kJ}{kg}$$

$$h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 3149.5 - .8(3149.5 - 2594.3) = 2705.3 \frac{kJ}{kg} \Rightarrow T_2 = 114.4°C,$$

$s_2 = 7.4373 \, kJ/kg \cdot K$. So, with Eqs.(2), (3)

$$\dot{W}_{cv} = \left(0.125 \frac{kg}{s}\right)(0.8)(3149.5 - 2594.3)\frac{kJ}{kg}\left|\frac{1 kW}{1 \frac{kJ}{s}}\right| = 55.52 kW, \quad \dot{E}_d = (0.125)(293)(7.4373 - 7.1406) = 10.87 kW$$

IT Code

```
p1 = 14  // bar
T1 = 350  // °C
p2 = 1  // bar
To = 293  // K
mdot = 0.125  // kg/s
eta = 0.8

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s1)
h2 = h1 - eta * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)
T2 = T_Ph("Water/Steam", p2, h2)
Wdotcv = mdot * (h1 - h2)
Edotcv = mdot * To * (s2 - s1)
```

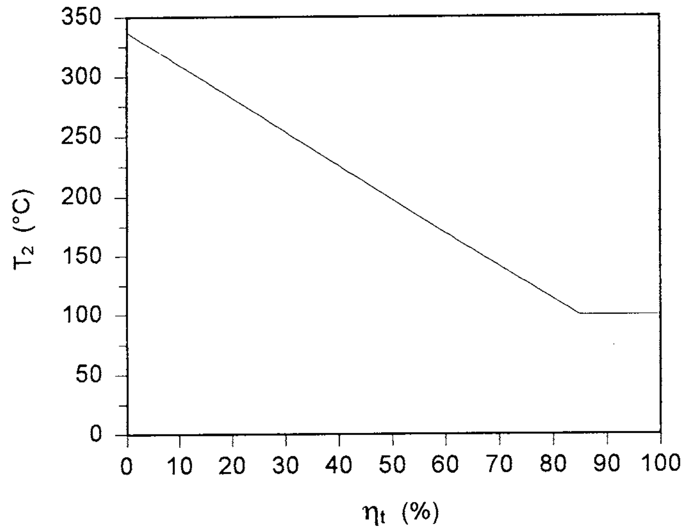
IT Results for $\eta_t = 80\%$

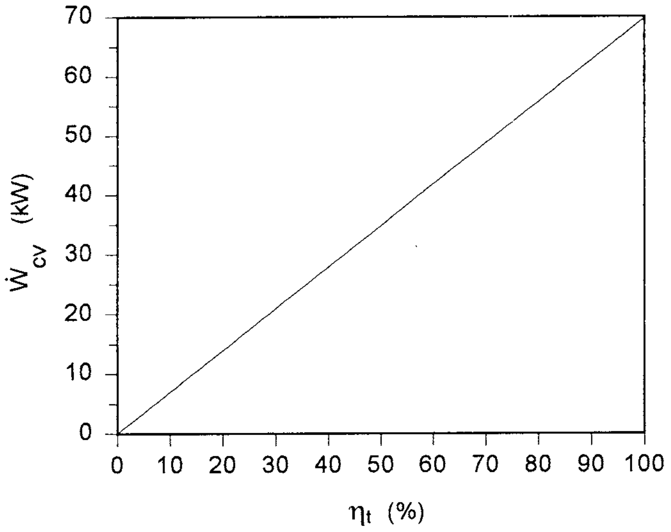
$h_1 = 3149$ kJ/kg
$s_1 = 7.135$ kJ/kg·K
$h_{2s} = 2592$ kJ/kg
$h_2 = 2703$ kJ/kg
$s_2 = 7.433$ kJ/kg·K
$T_2 = 113.6$ °C
$\dot{W}_{cv} = 55.73$ kW
$\dot{E}_d = 10.9$ kW

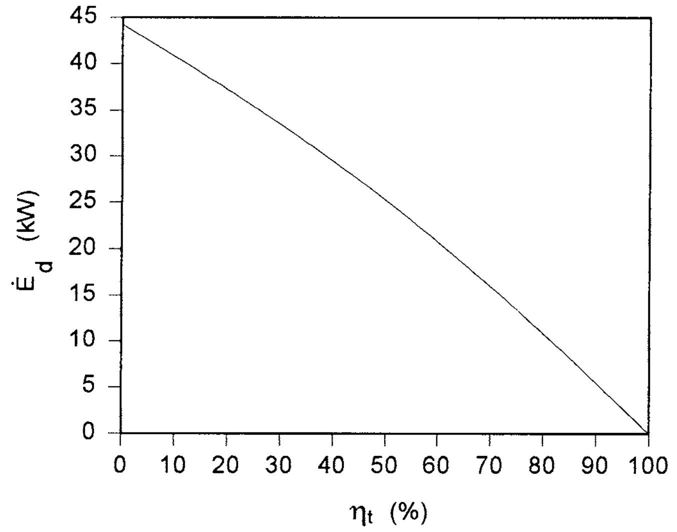
PROBLEM 7.63 (Cont'd.)

PLOTS:

①







Discussion:

1. The case of $\eta_t = 100\%$ corresponds to maximum power and zero exergy destruction. The case of $\eta_t = 0$ corresponds to zero power and maximum exergy destruction.
2. In the case of $\eta_t = 0$, there is no work, and the turbine acts as a throttling process, with $h_2 = h_1$.

---

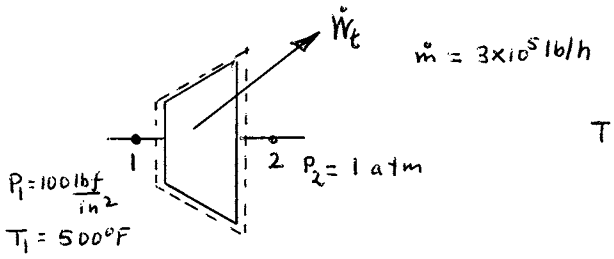
1. The flat portion of the plot corresponds to $x_2 \leq 1.0$. At these states the temperature corresponds to the saturation temperature at $p = 0.1$ MPa.

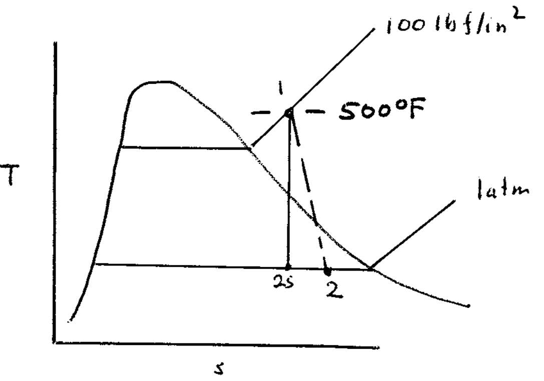
PROBLEM 7.64

KNOWN: Steady-state operating data are provided for a steam turbine.

FIND: (a),(b) If the isentropic efficiency is 80% and exergy is valued at 8 cents per kW·h, determine the value of the power produced and the cost of the exergy destroyed. (c) Plot the quantities of (a),(b) versus isentropic efficiency ranging from 80 to 100%

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume above is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects can be ignored. (3) For the environment $T_0 = 530°R$, $p_0 = 1$ atm (4) Exergy is valued at 8 cents per kW·h.

ANALYSIS: Reducing mass and energy rate balances and using the definition of isentropic turbine efficiency (Eq. 6.48)

$$\dot{W}_t = \dot{m}(h_1 - h_2) = \eta_t\, \dot{m}\,(h_1 - h_{2s}) \qquad (1)$$

Using $\dot{E}_d = T_0\,\dot{\sigma}_{cv}$ and $\dot{\sigma}_{cv}$ from the mass and entropy rate balances

$$\dot{E}_d = T_0\,\dot{m}\,(s_2 - s_1) \qquad (2)$$

(a) $\eta_t = 80\%$. From Table A-4E, $h_1 = 1279.1$ Btu/lb, $s_1 = 1.7085$ Btu/lb·°R. State 2s is fixed by $P_2 = 1$ atm and x_{2s}:

$$x_{2s} = \frac{1.7085 - 0.3121}{1.4446} = 0.967$$

Then

$$h_{2s} = h_f + x_{2s}(h_g - h_f) = 180.15 + 0.967(970.4) = 1118.5 \text{ Btu/lb}$$

Substituting into Eq. (1)

$$\dot{W}_t = (0.8)\left(3\times10^5 \frac{lb}{h}\right)(1279.1 - 1118.5)\frac{Btu}{lb} = 385.4\times10^5 \frac{Btu}{h}$$

The value is

$$\dot{\$} = \left(385.4\times10^5 \frac{Btu}{h}\right)\left|\frac{kW\cdot h}{3413\ Btu}\right|\left(\frac{0.08\ \$}{kW\cdot h}\right) = 903.4\ \frac{\$}{h} \longleftarrow$$

(b) Continuing the discussion of the case $\eta_t = 80\%$, to fix state 2 use Eq. (1) to write $h_2 = h_1 - (\dot{W}_t/\dot{m}) = 1279.1 - (385.4\times10^5/3\times10^5) = 1150.6$ Btu/lb. Then, at $P_2 = 1$ atm, state 2 is closely a saturated vapor for which $s_2 = 1.7567$ Btu/lb·°R. Substituting into Eq. (2)

$$\dot{E}_d = (530°R)\left(3\times10^5 \frac{lb}{h}\right)(1.7567 - 1.7085)\frac{Btu}{lb\cdot °R} = 76.6\times10^5 \frac{Btu}{h}$$

The cost rate is

$$\dot{\$} = \left(76.6 \frac{Btu}{h}\right)\left|\frac{1\ kW\cdot h}{3413\ Btu}\right|\left(\frac{0.08\ \$}{kW\cdot h}\right) = 179.5\ \frac{\$}{h} \longleftarrow$$

PROBLEM 7.64 (Cond'd.)

The data required for the plot are generated using *IT*, as follows:

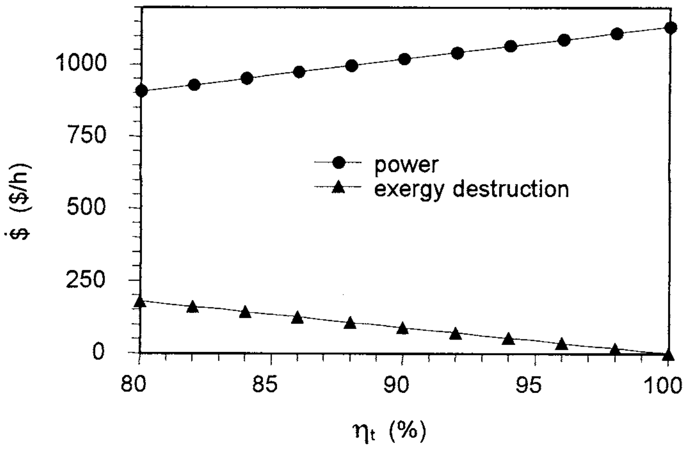
IT Code

```
p1 = 100  // lbf/in.²
T1 = 500  // °F
p2  = 1 * 14.696  // lbf/in.²
mdot = 300000  // lb/h
eta = 80  // %
To = 70 + 460  // °R

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
h2 = h1 - (eta / 100) * (h1 - h2s)
s2 = s_Ph("Water/Steam", p2, h2)

Wdot = mdot *  (h1 - h2)
CostW = Wdot * (0.08 / 3413)
Edot = mdot * To * (s2 - s1)
CostE = Edot * (0.08 / 3413)
```

IT Results for $\eta_t = 80\%$

$h_1 = 1279$ Btu/lb
$s_1 = 1.708$ Btu/lb·°R
$h_{2s} = 1118$ Btu/lb
$h_2 = 1150$ Btu/lb
$s_2 = 1.756$ Btu/lb
$\dot{W}_{cv} = 3.864 \times 10^7$ Btu/h
$\dot{\$}_{power} = 905.7$ \$/h
$\dot{E}_d = 7.623 \times 10^6$ Btu/h
$\dot{\$}_{exergy} = 178.7$ \$/h

PLOT:



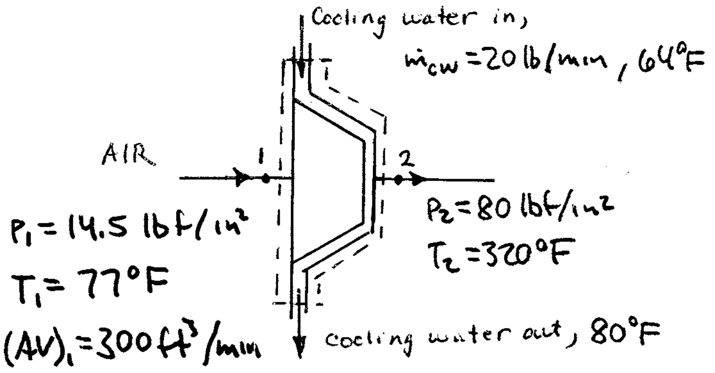
At higher values of η_t, there is less exergy destruction and greater power output, as expected.

PROBLEM 7.65*

KNOWN: Operating data are provided for a water-jacketed air compressor at steady state.

FIND: Determine rate of exergy destruction, in kW, and express it as a percentage of the power input.

SCHEMATIC & GIVEN DATA:



From solution to problem 6.118

$\dot{W}_{cv} = -28.16$ kW

$\dot{\sigma}_{cv} = 8.6\times10^{-3}$ Btu/min·°R

ASSUMPTIONS: (1) The control volume shown operates at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) Air is modeled as an ideal gas. (4) The cooling water is modeled as incompressible with constant specific heat c and a negligible change in pressure. (5) For the environment, $T_0 = 70$°F, $P_0 = 1$ atm.

ANALYSIS: The rate of exergy destruction is conveniently calculated in this case as

$$\dot{E}_d = T_0\dot{\sigma} = (530°R)(8.6\times10^{-3}\ \mathrm{Btu/min\cdot°R})\left|\frac{60\ \mathrm{min}}{1\ \mathrm{h}}\right|\left|\frac{1\ \mathrm{kWh}}{3413\ \mathrm{Btu}}\right| = 0.0801\ \mathrm{kW} \longleftarrow$$

When expressed as a percentage of the power input

$$\left(\frac{\dot{E}_d}{-\dot{W}_{cv}}\right) = \left(\frac{0.0801\ \mathrm{kW}}{28.16\ \mathrm{kW}}\right) = 0.285\% \qquad \longleftarrow$$

PROBLEM 7.66*

KNOWN: Steady-state operating data are provided for water-jacketed compressors — Problems 7.62, 7.65.

FIND: Determine for each case, the hourly costs of the power input and the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:

See solutions to Problems 7.62, 7.65

ASSUMPTIONS: 1. Refer to assumptions listed in the solutions for Problems 7.62, 7.65. 2. Exergy is valued at 8 cents per kW·h.

ANALYSIS:

(a) From the solution to Problem 7.62

$$-\dot{W}_{cv} = 100.83 \text{ kW}$$
$$\dot{E}_d = 25.02 \text{ kW}$$

$$\Rightarrow \quad \dot{\$} = (100.83\text{kW})\left(0.08 \frac{\$}{\text{kW}\cdot\text{h}}\right) = \$\,8.07/\text{h} \qquad \leftarrow \text{power}$$
$$\dot{\$} = (25.02\text{kW})\left(0.08 \frac{\$}{\text{kW}\cdot\text{h}}\right) = \$\,2.00/\text{h} \qquad \leftarrow \text{exergy destruction}$$

(b) From the solution to Problem 7.65

$$-\dot{W}_{cv} = 28.16 \text{ kW}$$
$$\dot{E}_d = 0.0801 \text{ kW}$$

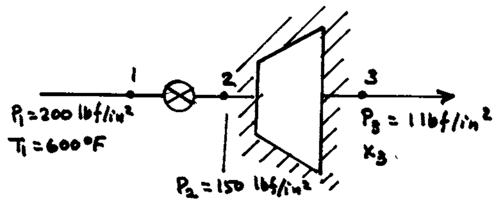
$$\Rightarrow \quad \dot{\$} = (28.16 \text{ kW})\left(0.08 \frac{\$}{\text{kW}\cdot\text{h}}\right) = \$\,2.25/\text{h} \qquad \leftarrow \text{power}$$
$$\dot{\$} = (0.0801 \text{ kW})\left(0.08 \frac{\$}{\text{kW}\cdot\text{h}}\right) = \$\,0.01/\text{h} \qquad \leftarrow \text{exergy destruction}$$

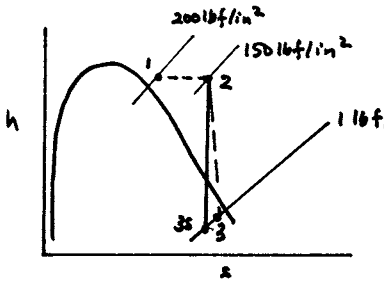
PROBLEM 7.67

KNOWN: Steady-state operating data are provided for a throttling valve and turbine in series.

FIND: Plot the rates of exergy destruction for the valve and the turbine, each versus the quality of the steam at the turbine exit.

SCHEMATIC & GIVEN DATA:





$T_0 = 60°F$, $P_0 = 1atm$

ASSUMPTIONS: (1) The turbine is insulated and at steady state. $\dot{Q}_{cv} = 0$. 2. The expansion across the valve is a throttling process. (3) For the environment, $T_0 = 60°F$, $P_0 = 1atm$.

ANALYSIS: For a 1-inlet, 1-exit control volume at steady state for which there is no heat transfer, an entropy rate balance reduces to give

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

Then, with $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, we get

$$\text{Valve: } \frac{\dot{E}_d}{\dot{m}} = T_0(s_2 - s_1) \quad (1) \qquad \text{Turbine: } \frac{\dot{E}_d}{\dot{m}} = T_0(s_3 - s_2) \quad (2)$$

Sample calculation: $x_3 = 0.98$. From Table A-4E, $h_1 = 1322.1\ Btu/lb$, $s_1 = 1.6767\ Btu/lb \cdot °R$. With assumption 2, $h_2 \sim h_1$. Interpolating in Table A-4E at $150\ lbf/in^2$ with $h_2 = 1322.1\ Btu/lb$, $s_2 = 1.7078\ Btu/lb \cdot °R$. With data from Table A-3E, $s_3 = s_f + x_3 s_{fg}$ or $s_3 = 0.1327 + 0.98(1.8453) = 1.9411\ Btu/lb \cdot °R$. Inserting values into Eqs. (1), (2)

$$(\dot{E}_d/\dot{m})_{valve} = 520°R\,(1.7078 - 1.6767)\frac{Btu}{lb \cdot °R} = 16.17\ \frac{Btu}{lb} \quad \leftarrow \text{valve}$$

$$(\dot{E}_d/\dot{m})_{turbine} = 520°R\,(1.9411 - 1.7078)\frac{Btu}{lb \cdot °R} = 121.32\ \frac{Btu}{lb}$$

IT Code

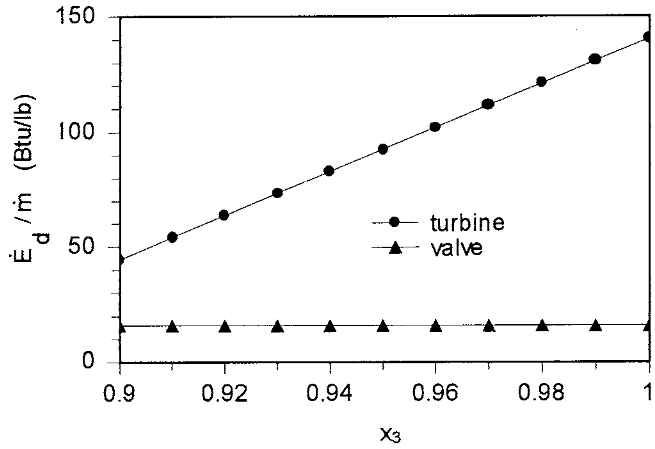
```
p1 = 200  // lbf/in.²
T1 = 600  // °F
p2 = 150  // lbf/in.²
p3 = 1  //  lbf/in.²
To = 60 + 460  // °R
x3 = 0.98

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = h_Ps("Water/Steam", p2, s2)
h2 = h1
s3 = ssat_Px("Water/Steam", p3, x3)
Edvalve= To * (s2 - s1)
Edturb= To * (s3 - s2)
```

IT Results for x_3 = 0.98

h1= 1322 Btu/lb
s1 = 1.676 Btu/lb·°R
s2 = 1.707 Btu/lb·°R
s3 = 1.941 Btu/lb·°R
$(\dot{E}_d/\dot{m})_{valve}$ = 121.4 Btu/lb
$(\dot{E}_d/\dot{m})_{turbine}$ = 16.04 Btu/lb

PROBLEM 7.67 (Cont'd.)

PLOT:



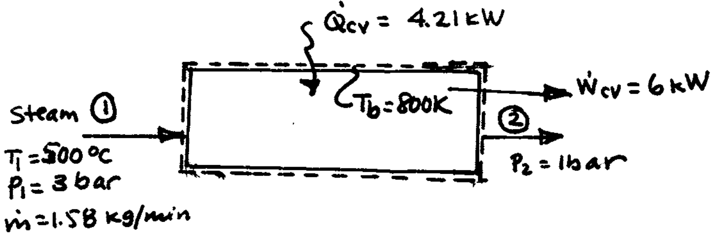
- Note that the exergy destruction in the valve is independent of x_3.
- The exergy destruction in the turbine increases as x_3 increases, corresponding to a decrease in isentropic turbine efficiency, as expected.

PROBLEM 7.68

KNOWN: A device for developing power using "waste heat" is described in Problem 6.102

FIND: If the device develops a net power output of 6kW, determine (a) the rate exergy enters accompanying heat transfer, (b) the net rate exergy is carried in by the steam, (c) the rate of exergy destruction within the device.

SCHEMATIC & GIVEN DATA



ASSUMPTIONS: 1. The control volume shown in the accompanying figure is at steady state. 2. Kinetic and potential energy effects can be ignored. 3. Heat transfer occurs only at T_b. 4. $T_0 = 293$ K, $p_0 = 1$ bar.

ANALYSIS: (a) The rate exergy enters accompanying heat transfer is obtained with Eq. 7.33:

$$\dot{E}_q = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{cv} = \left[1 - \frac{293}{800}\right](4.21\text{kW}) = 2.67\text{kW} \longleftarrow$$

(b) To find $(\dot{E}_{f1} - \dot{E}_{f2})$ requires state 2 to be fixed. Begin with a mass and energy rate balance to obtain with $h_1 = 3486$ kJ/kg from Table A-4

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}[h_1 - h_2] \Rightarrow h_2 = h_1 + \frac{\dot{Q}_{cv} - \dot{W}_{cv}}{\dot{m}} = 3486\frac{\text{kJ}}{\text{kg}} + \frac{(4.21 - 6)\text{ kJ/s}}{(1.58/60)\text{kg/s}} = 3418\frac{\text{kJ}}{\text{kg}}.$$

Then, from Table A-4, with $p_2 = 1$ bar and $h_2 = 3418$ kJ/kg, $s_2 = 8.7398$ kJ/kg·K. Also, Table A-4 gives $s_1 = 8.3251$ kJ/kg·K. With Eq. 7.36

$$(\dot{E}_{f1} - \dot{E}_{f2}) = \dot{m}\left[(h_1 - h_2) - T_0(s_1 - s_2)\right] = \left(\frac{1.58}{60}\frac{\text{kg}}{\text{s}}\right)\left[(3486 - 3418) - 293(8.3251 - 8.7398)\right]$$

$$= 4.99\text{ kW} \longleftarrow$$

① (c) The rate of exergy destruction can be obtained from Eq. 7.32b

$$0 = \dot{E}_q - \dot{W}_{cv} + (\dot{E}_{f1} - \dot{E}_{f2}) - \dot{E}_d$$

② $$\Rightarrow \dot{E}_d = \dot{E}_q - \dot{W}_{cv} + (\dot{E}_f - \dot{E}_{f2}) = (2.67 - 6 + 4.99)\text{kW} = 1.66\text{ kW} \longleftarrow$$

---

1. Alternatively, $\dot{E}_d = T_0\dot{\sigma}$, where

$$\dot{\sigma} = \left(-\frac{\dot{Q}_{cv}}{T_b}\right) + \dot{m}(s_2 - s_1) = -\frac{4.21\text{kW}}{800\text{K}} + \left(\frac{1.58}{60}\frac{\text{kg}}{\text{s}}\right)(8.7398 - 8.3251)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$\Rightarrow \dot{\sigma} = -0.00526 + 0.01092 = 0.00566\text{ kW/K}$$

$$\Rightarrow \dot{E}_d = T_0\dot{\sigma} = (293\text{K})(0.00566\text{ kW/K}) = 1.66\text{ kW}$$

2. Exergy accounting
   - Exergy supplied to the device
     - via heat transfer 2.67 kW
     - via steam flow* 4.99 kW $\Big\}$ 7.66 kW
   - Disposition of exergy supplied
     - power developed 6.00 kW (78.3%)
     - exergy destroyed 1.66 kW (21.7%)

* The steam provides nearly 2/3 of the exergy supplied to the device.

PROBLEM 7.69

KNOWN: Steady-state operating data are provided for a duct system in Problem 4.68.

FIND: Determine the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:

- See figure in solution of Problem 6.117
- From solution: $\dot{\sigma}_{cv} = 0.084 \frac{\text{Btu}}{\text{min}\cdot{}^\circ\text{R}}$

ASSUMPTIONS: 1. Assumptions from Problems 4.68, 6.117 apply. 2. Let $T_0 = 500^\circ\text{R}$, $p_0 = 1\,\text{bar}$.

ANALYSIS: Using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, we get

$$\dot{E}_d = 500^\circ\text{R}\left(0.084 \frac{\text{Btu}}{\text{min}\cdot{}^\circ\text{R}}\right) = 42 \frac{\text{Btu}}{\text{min}}$$

PROBLEM 7.70

KNOWN: Steady-state operating data are provided for a vortex tube.

FIND: Evaluate the rate of exergy destruction per unit mass of air entering.

SCHEMATIC & GIVEN DATA: ~~See~~ Figure E 6.7

ASSUMPTIONS: 1. The assumptions of Example 6.7 apply. 2. $T_0 = 530^\circ\text{R}$, $p_0 = 1\,\text{atm}$.

ANALYSIS: Using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ and $\dot{\sigma}_{cv}/\dot{m}_1 = 0.1086\ \text{Btu/lb}\cdot{}^\circ\text{R}$ from the solution to Example 6.7, we get

$$\frac{\dot{E}_d}{\dot{m}_1} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{m}} = (530\,^\circ\text{R})(0.1086)\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

$$= 57.6 \frac{\text{Btu}}{\text{lb}} \quad \longleftarrow$$

The inventor claims the device operates without work or heat transfer. However, the exergy destroyed within the vortex tube can be traced to a device, or devices, located upstream of the vortex tube that would require some combination of work, heat transfer, and fuel input.

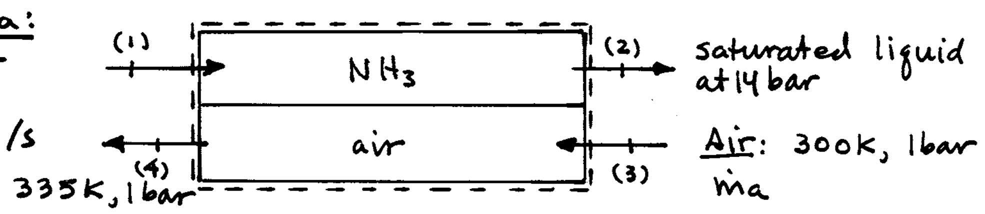
PROBLEM 7.71*

KNOWN: Ammonia and air flow on opposite sides of a counterflow heat exchanger operating at steady state. Operating data are given.

FIND: Determine (a) the change in flow exergy rate of each stream, and (b) the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:

Ammonia:
$p_1 = 14\text{ bar}$
$T_1 = 60°\text{C}$
$\dot m = 0.5\text{ kg/s}$



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume, $\dot W_{cv} = \dot Q_{cv} = 0$, and kinetic and potential effects are negligible. (3) The air is modeled as an ideal gas. (4) For the environment, $T_0 = 300\text{ K}$, $p_0 = 1\text{ bar}$.

ANALYSIS: First, we use mass and energy rate balances to determine $\dot m_a$. That is $\dot m_1 = \dot m_2 \equiv \dot m$ and $\dot m_3 = \dot m_4 \equiv \dot m_a$. Thus, with assumptions 1 and 2

$$0 = \cancelto{0}{\dot Q_{cv}} - \cancelto{0}{\dot W_{cv}} + \dot m(h_1 - h_2) + \dot m_a(h_3 - h_4) \qquad (1)$$

or, solving for $\dot m_a$ and inserting enthalpy data from Tables A-14, A-15, and A-22

$$\dot m_a = \left(\frac{h_1 - h_2}{h_4 - h_3}\right)\dot m = \frac{(1542.89 - 352.97)\text{kJ/kg}}{(335.38 - 300.19)\text{kJ/kg}}(0.5\text{kg/s}) = 16.91 \text{ kg/s}$$

(a) The change in flow exergy rate for the ammonia is

$$\dot E_{f_2} - \dot E_{f_1} = \dot m(e_2 - e_1) = \dot m\left[(h_2 - h_1) - T_0(s_2 - s_1)\right]$$

$$= (0.5\tfrac{\text{kg}}{\text{s}})\left[(352.97 - 1542.89)\tfrac{\text{kJ}}{\text{kg}} - (300\text{ K})(1.2987 - 5.1360)\tfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= -38.73\text{ kW} \longleftarrow (\Delta \dot E_f)_{water}$$

For the air stream, using data from Table A-22

$$\dot E_{f_4} - \dot E_{f_3} = \dot m_a\left[(h_4 - h_3) - T_0(s_4 - s_3)\right] = \dot m_a\left[(h(T_4) - h(T_3)) - T_0\left(s°(T_4) - s°(T_3) - R\cancelto{0}{\ln p_4/p_3}\right)\right]$$

① $$= (16.91)\left[(335.38 - 300.19) - (300)(1.8129 - 1.70203)\right] = 32.62\text{ kW} \longleftarrow (\Delta \dot E_f)_{air}$$

(b) At steady state, the exergy rate balance reduces to

$$0 = \sum_j\left[1 - \frac{T_0}{T_j}\right]\cancelto{0}{\dot Q_j} - \left[\cancelto{0}{\dot W_{cv}} - \cancelto{0}{p_0\frac{dV}{dt}}\right] + \dot m(e_{f_1} - e_2) + \dot m_a(e_{f_3} - e_{f_4}) - \dot E_d$$

Thus

$$\dot E_d = -\left[\dot E_{f_2} - \dot E_{f_1}\right] - \left[\dot E_{f_4} - \dot E_{f_3}\right] = -[-38.73] - [32.62] = 6.11\text{ kW} \longleftarrow \dot E_d$$

---

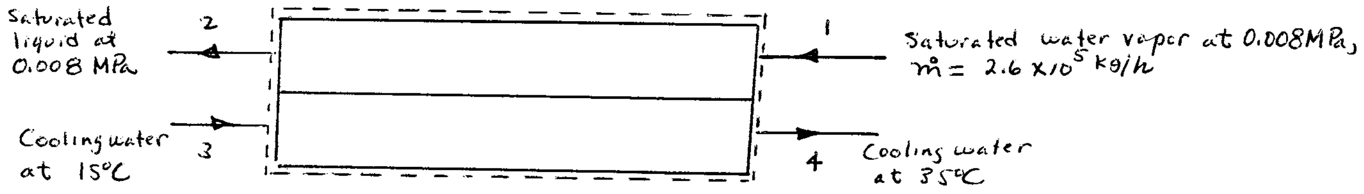
1. The decrease in energy of the ammonia exactly equals the increase in energy of the air stream, for energy is conserved — see Eq. (1). However, because exergy is destroyed, the decrease in exergy of the ammonia differs significantly from the increase of exergy of the air stream.

PROBLEM 7.72

KNOWN: Operating data are provided for a condenser at steady state.

FIND: For the cooling water stream determine the net rate, in MW, that (a) energy exits the plant, (b) exergy exits the plant. Discuss

SCHEMATIC & GIVEN DATA:


Saturated liquid at 0.008 MPa
2
Cooling water at 15°C
3
1
Saturated water vapor at 0.008 MPa, ṁ = 2.6 × 10^5 kg/h
4
Cooling water at 35°C

ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3). For the cooling water there is a negligible change in pressure from inlet to exit. (4) For the environment, $T_0 = 20°C$, $p_0 = 0.1$ MPa.

ANALYSIS: The mass flow rate of the cooling water is needed as a preliminary. Thus, reducing mass and energy rate balances with indicated assumptions

$$\dot{m}_{cw} = \frac{\dot{m}(h_1 - h_2)}{h_4 - h_3}$$

Using $h \approx h_f(T)$ for the cooling water and steam table data

$$\dot{m}_{cw} = \frac{(2.6 \times 10^5 \text{ kg/h})(2403.1 \text{ kJ/kg})}{(146.7 - 63) \text{ kJ/kg}} = 7.46 \times 10^6 \text{ kg/h}$$

(a) The net rate energy is carried from the power plant by the cooling water is given by

$$\dot{m}_{cw}[h_4 - h_3] = \left(7.46 \times 10^6 \frac{\text{kg}}{\text{h}}\right)(146.7 - 63)\frac{\text{kJ}}{\text{kg}} = 6.24 \times 10^8 \frac{\text{kJ}}{\text{h}}$$

$$= \left(6.24 \times 10^8 \frac{\text{kJ}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)\left(\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right) = 173.3 \text{ MW} \longleftarrow \text{(a)}$$

(b) The net rate exergy is carried from the power plant by the cooling water is given by

$$\dot{E}_{f4} - \dot{E}_{f3} = \dot{m}_{cw}[h_4 - h_3 - T_0(s_4 - s_3)] = 6.24 \times 10^8 \frac{\text{kJ}}{\text{h}} - \left[\left(7.46 \times 10^6 \frac{\text{kg}}{\text{h}}\right)(293 \text{ K})(0.5053 - 0.2245)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right]$$

$$= \left(0.1 \times 10^8 \frac{\text{kJ}}{\text{h}}\right)\left(\frac{\text{h}}{3600 \text{ s}}\right)\left(\frac{1 \text{ MW}}{10^3 \text{ kJ/s}}\right) = 2.8 \text{ MW} \longleftarrow \text{(b)}$$

where for the cooling water $s \approx s_f(T)$.

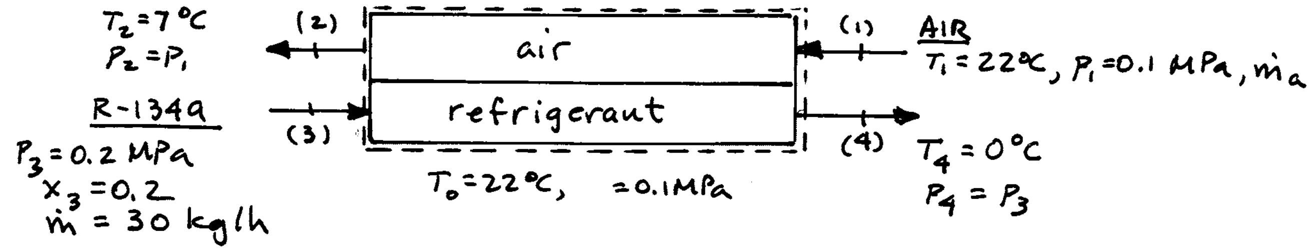
On an energy basis the quantity of energy exiting with the cooling water is significant. But when viewed in terms of exergy the potential for use of the effluent is seen to be limited: about 3% of the power developed. Many studies have shown that there are few practical uses for copious amounts of such low temperature condenser cooling water.

PROBLEM 7.73

KNOWN: Operating data are provider for a counterflow heat exchanger at steady state. One stream is R-134a and the other is air.

FIND: (a) For the R-134a, determine the rate of heat transfer, and (b) For each of the two streams, evaluate the change in flow exergy rate and compare.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state, with $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and negligible effects of motion and gravity. (2) For a control volume enclosing only the refrigerant stream, the foregoing applies, except $\dot{Q}_{cv} \neq 0$. (3) For each stream, pressure change is negligible. (4) The air is modeled as an ideal gas. (5) For the environment, $T_0 = 22°C = 295K$, $P_0 = 0.1$ MPa.

(a) For a control volume enclosing only the refrigerant stream, the mass and energy rate balances reduce to give

$$0 = \dot{Q} - \cancel{\dot{W}}^{0} + \dot{m}(h_3 - h_4) \Rightarrow \dot{Q} = \dot{m}(h_4 - h_3) \qquad (1)$$

From Table A-11 at 2 bar, $x_3 = 0.2$; $h_3 = h_{f3} + x_3 h_{fg3} = 36.84 + (.2)\,204.46 = 77.732 \frac{kJ}{kg}$
From Table A-12; $h_4 = 250.10$ kJ/kg. Thus

$$\dot{Q} = \left(30 \frac{kg}{h}\right)(250.10 - 77.732)\,kJ/kg = 5171 \text{ kJ/h} \longleftarrow \dot{Q}$$

(b) The change in flow exergy rate for the refrigerant stream is, with data from Tables A-11 and A-12

$$(\Delta \dot{E}_f)_{ref} = \dot{m}\left[(h_4 - h_3) - T_0(s_4 - s_3)\right] = (30)\left[(250.1 - 77.732) - (295)(.9582 - .30354)\right]$$

① $$= -622.7 \text{ kJ/h} \longleftarrow (\Delta \dot{E}_f)_{ref}$$

To determine the change in flow exergy rate for the air, we first evaluate $\dot{m}_a$ using mass and energy rate balances for the overall control volume.

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}_a[h_1 - h_2] + \dot{m}[h_3 - h_4] \qquad (1)$$

That is

$$\dot{m}_a = \dot{m}\left[\frac{h_4 - h_3}{h_1 - h_2}\right] = \left(30 \frac{kg}{h}\right)\left[\frac{250.1 - 77.732}{295.17 - 280.13}\right]\frac{kJ}{kg} = 343.8 \text{ kg/h}$$

where he specific enthalpies for air are from Table A-22.

Now, the change in exergy rate of the air is

$$(\Delta \dot{E}_f)_{air} = \dot{m}_a(e_{f2} - e_{f1}) = \dot{m}\left[(h_2 - h_1) - T_0(s_2 - s_0)\right]$$

$$= \dot{m}_a\left[(h_2 - h_1) - T_0\left(s^\circ(T_2) - s^\circ(T_1) - R \cancel{\ln P_2/P_1}^{0}\right)\right]$$

PROBLEM 7.73 (Cont'd.)

With $s^\circ(T)$ data from Table A-22

$$(\Delta \dot{E}_f)_{air} = (343.8)\left[(280.13 - 295.17) - (295)(1.63279 - 1.68515)\right]$$

$$= 139.7 \text{ kJ/h} \longleftarrow (\Delta \dot{E}_f)_{air}$$

Discussion. The change in energy of the air stream exactly equals the change in energy of the refrigerant stream in magnitude, for energy is conserved – see Eq. (1). However, because of exergy destruction, the decrease in exergy rate of the refrigerant stream differs significantly from the increase in flow exergy rate of the air.

---

1. Energy transfer by heat occurs from the air to the colder refrigerant. However, the exergy transfer accompanying this heat transfer occurs in the opposite direction; from the refrigerant to the air, because it takes place at temperatures below T_0. (See Sec. 7.3.2 for further discussion.) Even though its energy decreases, the flow exergy of the air increases, and conversely for the refrigerant.

PROBLEM 7.74

FIND: Determine the rate of exergy destruction for
(a) the computer of Example 4.8, when air exits at 32°C.
(b) the computer of Problem 4.70, ignoring the pressure change of the air.
(c) the water-jacketed electronics housing of Problem 4.71, when water exits at 24°C.

ANALYSIS: In each case, use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ where $T_0 = 293$ K and $\dot{\sigma}_{cv}$ is obtained from the counterpart Chap. 6 problem:

(a) From Problem 6.105, $\dot{\sigma}_{cv} = 0.328 \frac{W}{K}$

$$\therefore \dot{E}_d = (293 K)\left(0.328 \frac{W}{K}\right)\left|\frac{1 kW}{10^3 W}\right| = 0.096 \text{ kW} \quad \longleftarrow$$

(b) From Problem 6.106, $\dot{\sigma}_{cv} = 4.15 \times 10^{-4} \frac{kW}{K}$

$$\therefore \dot{E}_d = (293 K)(4.15 \times 10^{-4}) \frac{kW}{K} = 0.122 \text{ kW} \quad \longleftarrow$$

(c) From Problem 6.107, $\dot{\sigma}_{cv} = 8.5 \times 10^{-3} \frac{kW}{K}$

$$\therefore \dot{E}_d = (293 K)\left(8.5 \times 10^{-3} \frac{kW}{K}\right) = 2.491 \text{ kW} \quad \longleftarrow$$

PROBLEM 7.75

KNOWN: Steady-state operating data are provided for an electronics-laden cylinder.

FIND: Determine the rate of exergy destruction.

ANALYSIS: Using $T_0 = 293$ K and $\dot{\sigma}_{cv}$ from the solution to Problem 6.108

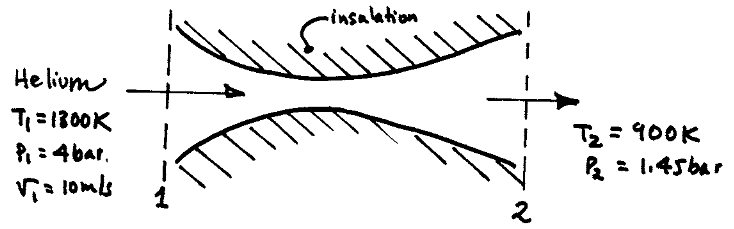
$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = (293 K)\left(0.65 \frac{W}{K}\right)\left|\frac{1 kW}{10^3 W}\right| = 0.19 \text{ kW} \quad \longleftarrow$$

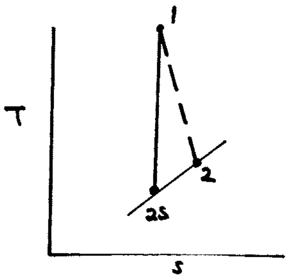
PROBLEM 7.76

KNOWN: Helium gas expands through an insulated nozzle at steady state.

FIND: Determine the exit velocity, the isentropic nozzle efficiency, and the exergy destruction per unit mass of gas flowing.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The nozzle is insulated and at steady state. (2) Potential energy effects can be ignored. (3) Argon is modeled as an ideal gas. (4) For the environment $T_0 = 20°C$, $P_0 = 1 atm$.

ANALYSIS: Mass and energy balances reduce at steady state with assumption 2 to give

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \overset{0}{\cancel{g(z_1 - z_2)}}\right)$$

Thus

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)} \qquad (1)$$

From Table A-21, for Helium $c_p = 5/2\,R$. Thus, $h_1 - h_2 = c_p(T_1 - T_2)$ and

$$V_2 = \sqrt{\left(10\frac{m}{s}\right)^2 + 2\left[(2.5)\frac{8314}{4.003}\frac{N\cdot m}{kg\cdot K}\right](1300K - 900K)\left|\frac{1\,kg\cdot m/s^2}{1N}\right|} = 2038\,\frac{m}{s}$$

$\longleftarrow$ V_2

To determine the isentropic nozzle efficiency requires the temperature at state 2s. With Eq. 6.45 and $k = 1.667$

$$T_{2s} = T_1\left[\frac{P_2}{P_1}\right]^{\frac{k-1}{k}} = 1300\left[\frac{1.45}{4}\right]^{.667/1.667} = 866.2\,K$$

Then, with Eq. (1)

$$V_{2s} = \sqrt{V_1^2 + 2(h_1 - h_{2s})} = \sqrt{V_1^2 + 2c_p(T_1 - T_{2s})}$$

$$= \sqrt{(10)^2 + 2\left[\frac{(2.5)(8314)}{4.003}\right](1300 - 866.2)} = 2122\,m/s$$

The isentropic nozzle efficiency is

$$\eta_{nozzle} = \frac{V_2^2/2}{V_{2s}^2/2} = \left(\frac{2038}{2122}\right)^2 = 0.922 \quad (92.2\%)$$

$\longleftarrow$ η_{nozzle}

The exergy destruction is given by $(\dot{E}_d/\dot{m}) = T_0(\sigma_{cv}/\dot{m})$, where $\sigma_{cv}/\dot{m}$ is the entropy production obtained from an entropy balance. Accordingly, as there is no heat transfer, we get using Eq. 6.23

$$\frac{\dot{E}_d}{\dot{m}} = T_0(s_2 - s_1) = T_0\left[c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}\right] = T_0 R\left[2.5\ln\frac{T_2}{T_1} - \ln\frac{P_2}{P_1}\right]$$

$$= (293K)\left(\frac{8.314}{4.003}\frac{kJ}{kg\cdot K}\right)\left[2.5\ln\frac{900}{1300} - \ln\frac{1.45}{4}\right]$$

$$= 58.1\,\frac{kJ}{kg}$$

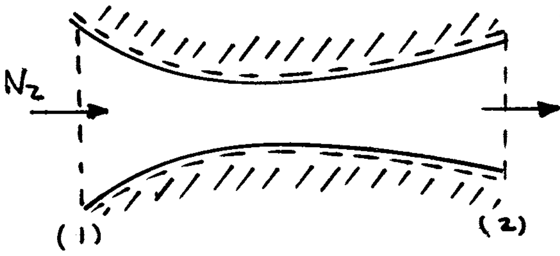
$\longleftarrow$

PROBLEM 7.77*

KNOWN: Nitrogen enters a well-insulated nozzle operating at steady state at a given state and expands to a specified pressure. The isentropic nozzle efficiency is known.

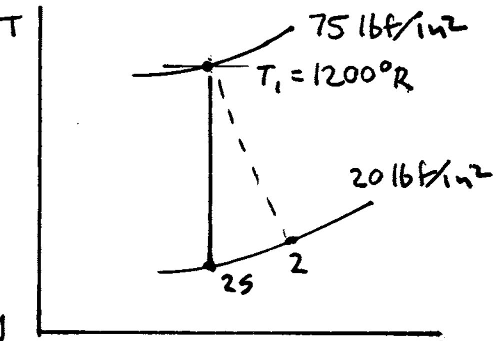
FIND: Determine the exergy destruction rate, per kg of oxygen flowing.

SCHEMATIC & GIVEN DATA:



Inlet: $P_1 = 75\text{ lbf/in}^2$
$T_1 = 1200°R$
$V_1 = 80\text{ ft/s}$

Exit: $P_2 = 20\text{ lbf/in}^2$
$\eta_N = 0.90$



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume, $\dot Q_{cv} = \dot W_{cv} = 0$ and potential energy effects are negligible. (3) The oxygen behaves as an ideal gas. (4) For the environment, $T_0 = 70°F = 530°R$, $P_0 = 14.7\text{ lbf/in}^2$

ANALYSIS: To determine the exergy destruction rate, we use mass and entropy rate balances and $\dot E_d = T_0\dot\sigma_{cv}$ to get

$$0 = \cancel{\sum_j \left(\frac{\dot Q}{T}\right)_j}^{\,0} + \dot m(s_1 - s_2) + \dot\sigma_{cv} \Rightarrow \dot E_d/\dot m = T_0(s_2 - s_1) \qquad (1)$$

To fix state 2, consider mass and energy balances which reduce as follows:

$$0 = \cancel{\dot Q_{cv}}^{\,0} - \cancel{\dot W_{cv}}^{\,0} + \dot m\left[\frac{(\bar h_1 - \bar h_2)}{M} + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\cancel{(z_1 - z_2)}^{\,0}\right]$$

or

$$\bar h_2 = \bar h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)M \qquad (2)$$

To get V_2, we use $\eta_N = (V_2^2/2)/(V_2^2/2)_s$. For the isentropic process from 1 to 2s

$$0 = \bar s^\circ(T_{2s}) - \bar s^\circ(T_1) - \bar R \ln P_2/P_1$$

or

$$\bar s^\circ(T_{2s}) = \bar s^\circ(T_1) + \bar R \ln P_2/P_1 = 51.413 + 1.986\ln\left(\frac{20}{75}\right) = 48.788\text{ Btu/lbmol·°R}$$

where the $\bar s^\circ$ values are from Table A-23E. Interpolating in the table with $\bar s^\circ(T_{2s})$ gives: $T_{2s} = 830.9°R$, $h_{2s} = 5781.2\text{ Btu/lbmol}$

Now

$$V_{2s} = \sqrt{\frac{2(\bar h_1 - \bar h_{2s})}{M} + V_1^2} = \sqrt{\frac{2(8420.0 - 5781.2)}{28.01}\frac{\text{Btu}}{\text{lb}}\left|\frac{778\text{ ft·lbf}}{1\text{ Btu}}\right|\left|\frac{32.2\text{ lb·ft}}{1\text{bf·s}^2}\right| + (80)^2\frac{\text{ft}^2}{\text{s}^2}}$$

$$= 1538.3\text{ ft/s}$$

Thus $V_2 = \sqrt{\eta_N}\,V_{2s} = 1459.4\text{ ft/s}$, and from (2)

$$\bar h_2 = 8420.0\frac{\text{Btu}}{\text{lbmol}} + \left(\frac{80^2 - 1459.4^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\left|\frac{\text{lbf·s}^2}{32.2\text{ lb·ft}}\right|\left|\frac{1\text{ Btu}}{778\text{ ft·lbf}}\right|\left|\frac{28.01\text{ lb}}{1\text{ lbmol}}\right| = 7232.9\frac{\text{Btu}}{\text{lbmol}}$$

From Table A-23E; $\bar s^\circ(T_2) = 50.350\text{ Btu/lbmol·°R}$. Finally, from (1)

$$\dot E_d/\dot m = T_0\left\{\bar s^\circ(T_2) - \bar s^\circ(T_1) - \bar R\ln P_2/P_1\right\}\frac{1}{M}$$

$$= (530°R)\left\{(50.350 - 51.413) - 1.986\ln(20/75)\right\}\frac{\text{Btu}}{\text{lbmol·°R}}\left|\frac{1\text{ lbmol}}{28.01\text{ lb}}\right|$$

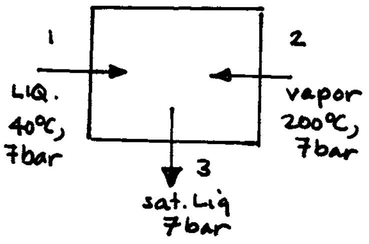
$$= 29.556\text{ Btu/lb} \longleftarrow \dot E_d/\dot m$$

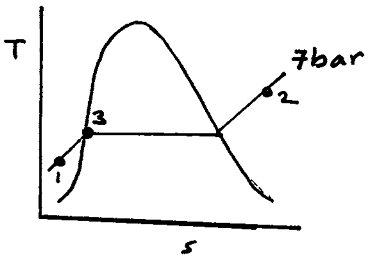
PROBLEM 7.78

KNOWN: Steady-state operating data are provided for an open feedwater heater.

FIND: Determine the ratio of the incoming mass flow rates, $\dot{m}_1/\dot{m}_2$, and the exergy destruction per unit mass exiting.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv}=0$, $\dot{W}_{cv}=0$. Kinetic and potential energy effects can be ignored.
3. $T_0 = 25°C$, $P_0 = 1\,atm$

ANALYSIS: (a) A mass rate balance reads, $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$. An energy rate balance reduces with assumptions 2 to give

$$0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3 \Rightarrow 0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$\Rightarrow \frac{\dot{m}_1}{\dot{m}_2} = \frac{h_2 - h_3}{h_3 - h_1} = \frac{2844.8 - 697.22}{697.22 - 167.57} = 4.05 \quad \longleftarrow$$

where data is from Tables A-2, 3, 4. At state 1, $h_1 \sim h_f(T_1)$.

(b) The exergy destruction can be found using $(\dot{E}_d/\dot{m}_3) = T_0(\dot{\sigma}_{cv}/\dot{m}_3)$, where $\dot{\sigma}_{cv}/\dot{m}_3$ is obtained from an entropy balance:

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

$$\Rightarrow \frac{\dot{\sigma}_{cv}}{\dot{m}_3} = s_3 - \frac{\dot{m}_1}{\dot{m}_3} s_1 - \frac{\dot{m}_2}{\dot{m}_3} s_3$$

From part (a) $\dot{m}_1 = 4.05\,\dot{m}_2$, $\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 5.05\,\dot{m}_2$. Thus, $\dot{m}_2/\dot{m}_3 = 0.198$, $\dot{m}_1/\dot{m}_3 = 0.802$.
with data from Tables A-2, 3, 4, At state 1, $s_1 \sim s_f(T_1)$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = [1.9922 - (0.802)(0.5725) - (0.198)(6.8865)]\,\frac{kJ}{kg \cdot K} = 0.1696\,\frac{kJ}{kg \cdot K}$$

$$\Rightarrow \dot{E}_d/\dot{m}_3 = T_0(\dot{\sigma}_{cv}/\dot{m}_3) = (298\,K)(0.1696\,kJ/kg \cdot K) = 50.5\,kJ/kg \quad \longleftarrow$$

PROBLEM 7.79

KNOWN: Steady-state operating data are provided in Problem 6.121 for an open feedwater heater.

FIND: Determine the cost of the exergy destroyed.

SCHEMATIC & GIVEN DATA: See solution to Problem 6.121

ASSUMPTIONS 1. The exiting mass flow rate is 1 kg/s. 2. 8000 hours of operation annually. 3. Exergy is valued 8 cents per kW·h. 4. $T_0 = 293\,K$ (20°C), $P_0 = 1\,atm$.

ANALYSIS: From the solution to Problem 6.121(a), $(\dot{\sigma}_{cv}/\dot{m}_3) = 0.218\,kJ/kg \cdot K$.
Then, with $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, the cost rate is

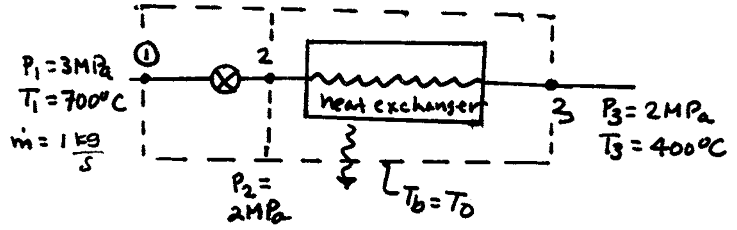
$$\dot{\$} = [(293\,K)(1\,kg/s)(0.218\,kJ/kg \cdot K)] \left| \frac{1\,kW}{1\,kJ/s} \right| \left[\frac{8000\,h}{year} \right] \left[\frac{\$0.08}{kW \cdot h} \right] = \$40,879/year$$

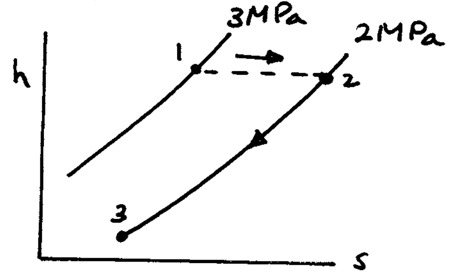
PROBLEM 7.80

KNOWN: A proposal is made for providing steam at 2MPa, 400°C from a source at 3MPa, 700°C.

FIND: (a) Determine the total rate of exergy destruction that would result from the implementation of the proposal. (b) Determine the annual cost of the exergy destruction found in part (a). Discuss.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volumes shown in the schematic are at steady state. 2. The expansion across the valve is a throttling process. 3. For the control volume enclosing the heat exchanger, heat transfer takes place at $T_b = T_0$ only. 4. The effects of motion and gravity can be ignored. 5. Exergy is evaluated at 8 cents per kW·h, and there are 8000 h of operation annually. 6. $T_0 = 293K$, $P_0 = 0.1MPa$

ANALYSIS: There are two important sources of exergy destruction: Exergy is destroyed in the expansion across the valve. Exergy is also destroyed in the spontaneous heat transfer that takes place between the heat exchanger and the ambient at a lower temperature. These exergy destructions can be evaluated individually by study of each control volume shown in the schematic. The total exergy destruction is the sum of the individual contributions.

The total exergy destruction also can be found by considering a single control volume enclosing both components. Since there is no heat transfer associated with the valve, and heat transfer for the portion of the control volume enclosing the heat exchanger occurs at $T_b = T_0$, an exergy rate balance for the overall control volume takes the form

$$0 = \overset{0}{\cancel{\left[1 - \frac{T_0}{T_b}\right]}}\dot{Q} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}\left[e_{f1} - e_{f3}\right] - \dot{E}_d \Rightarrow \dot{E}_d = \dot{m}\left[e_{f1} - e_{f3}\right]$$

or

$$\dot{E}_d = \dot{m}\left[(h_1 - h_3) - T_0(s_1 - s_3)\right]$$

With data from Table A-4

$$\dot{E}_d = (1\,kg/s)\left[(3911.7 - 3247.6)\frac{kJ}{kg} - (293\,K)\left[7.7571 - 7.1271\right]\frac{kJ}{kg\cdot K}\right] = 479.51\,\frac{kJ}{s} \longleftarrow$$

(b) The cost rate is

$$\dot{\$} = (479.51\,kW)\left(8000\frac{h}{year}\right)\left(\frac{\$0.08}{kW\cdot h}\right) = \$306,886/year \longleftarrow$$

Discussion: The result of part (a) shows that considerable exergy is destroyed in the proposed method for achieving the lower-pressure, lower-temperature steam condition. This amount of exergy destruction is not necessary, however. For example, a power-recovery turbine could replace the valve and achieve a lower pressure and temperature while generating power, and rather than cooling to the ambient, process heat could be developed for some purpose while the main steam flow is cooled. The cost rate determined in part (b) provides an indicator of cost savings that might result from reduced exergy destruction. Of course there would be a tradeoff with the costs to acquire and operate the turbine, process heat exchanger, etc. that would be specified for more exergy-thrifty approaches. In the end, the decision would be governed by economic considerations.

PROBLEM 7.81

KNOWN: Steam expands through a turbine and then passes through a counter flow heat exchanger in which the other stream is **air**. Steady state operating data are provided.

FIND: Determine the exergy destruction rates for the turbine and heat exchanger, in Btu/s. Evaluating exergy at 8¢/kW·h, determine the hourly cost of each.

SCHEMATIC & GIVEN DATA:

See the solution to Problem 6.122

ASSUMPTIONS: (1) See the solution to Problem 6.122. (2) For the environment $T_0 = 500°R$, $p_0 = 1\,atm$. (3) Exergy is evaluated at 8¢/kW·h.

ANALYSIS: From the solution to Problem 6.122

$$\text{turbine:}\quad \dot{\sigma}_{cv} = 0.459\ \frac{Btu/s}{°R}$$

$$\text{heat exchanger:}\quad \dot{\sigma}_{cv} = 0.591\ \frac{Btu/s}{°R}$$

Accordingly, with $\dot{E}_d = T_0\dot{\sigma}_{cv}$

① $$\text{turbine:}\quad \dot{E}_d = 500°R\left(0.459\ \frac{Btu/s}{°R}\right) = 229.5\ \frac{Btu}{s} \xleftarrow{\text{turbine}}$$

$$\text{heat exchanger:}\quad \dot{E}_d = 500°R\left(0.591\ \frac{Btu/s}{°R}\right) = 295.5\ \frac{Btu}{s} \xleftarrow{\text{HX}}$$

The corresponding cost rates are

$$\text{turbine:}\quad \dot{\$} = \left(229.5\ \frac{Btu}{s}\right)\left|\frac{3600\,s}{h}\right|\left|\frac{kW\cdot h}{3413\,Btu}\right|\left(0.08\ \frac{\$}{kW\cdot h}\right) = 19.37\ \frac{\$}{h} \xleftarrow{\text{turbine}}$$

$$\text{heat exchanger:}\quad \dot{\$} = (295.5)\,|3600|\left|\frac{1}{3413}\right|(0.08) = 24.94\ \frac{\$}{h} \xleftarrow{\text{HX}}$$

---

1. The power developed by the turbine is 2852 hp. The exergy destruction rates expressed in terms of the horsepower for comparison are, respectively

$$\text{turbine:}\quad \dot{E}_d = \left(229.5\ \frac{Btu}{s}\right)\left|\frac{3600\,s}{h}\right|\left|\frac{1\,hp}{2545\,Btu/h}\right| = 325\ hp$$

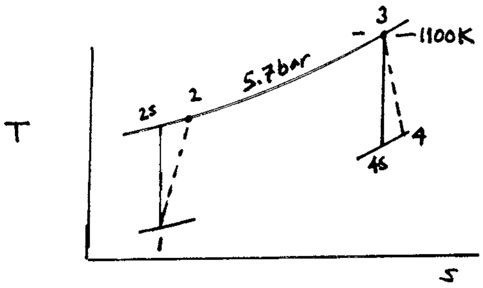
$$\text{HX}:\quad \dot{E}_d = (295.5)\,|3600|\left|\frac{1}{2545}\right| = 418\ hp$$

PROBLEM 7.82

KNOWN: Operating data are provided for a gas turbine power plant at steady state in Problem 6.159(b).

FIND: For the compressor and turbine, determine the rates of exergy destruction per kg of air flowing. Express each as a percentage of the net work developed.

SCHEMATIC & GIVEN DATA:



$p_1 = p_4 = 0.95$ bar
$p_2 = p_3 = 5.7$ bar
$T_1 = 22°C$, $T_3 = 1100K$
$\eta_c = 82\%$, $\eta_t = 85\%$

ASSUMPTIONS: (1) The gas turbine power plant operates at steady state. (2) The compressor and turbine operate adiabatically. (3) Kinetic and potential energy effects are negligible. (4) Air is modeled as an ideal gas. (4) For the environment, $T_0 = 22°C$, $p_0 = 0.95$ bar.

ANALYSIS: For the compressor and turbine the exergy destruction rates are conveniently determined using $\dot{E}_d/\dot{m} = T_0\, \dot{\sigma}_{cv}/\dot{m}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy balance. Thus

compressor: $\frac{\dot{E}_d}{\dot{m}} = T_0(s_2 - s_1)$, turbine: $\frac{\dot{E}_d}{\dot{m}} = T_0(s_4 - s_3)$

From the solution to Problem 6.159(b), $h_1 = 295.17$ kJ/kg, $h_3 = 1161.07$ kJ/kg, $h_{2s} = 493.03$ kJ/kg, $h_{4s} = 706.51$ kJ/kg, and

$$h_3 - h_4 = 386.38 \Rightarrow h_4 = 1161.07 - 386.38 = 774.69 \text{ kJ/kg}$$
$$h_2 - h_1 = 241.29 \Rightarrow h_2 = 295.17 + 241.29 = 536.46 \text{ kJ/kg}$$

Also, from the solution to Problem 6.159(b), $\dot{W}_{net}/\dot{m} = 145.09$ kJ/kg.

For the compressor,

$$\frac{\dot{E}_d}{\dot{m}} = T_0\left(s_2^\circ - s_1^\circ - R \ln p_2/p_1\right) = 295\left[2.2843 - 1.68515 - \frac{8.314}{28.97} \ln \frac{5.7}{0.95}\right] = 25.06 \frac{\text{kJ}}{\text{kg}} \longleftarrow$$ compressor

where s_1° and s_2° are obtained from Table A-22 at T_1 and h_2, respectively.

For the turbine,

$$\frac{\dot{E}_d}{\dot{m}} = T_0\left(s_4^\circ - s_3^\circ - R \ln p_4/p_3\right) = 295\left(2.6571 - 3.07732 - \frac{8.314}{28.97} \ln \frac{0.95}{5.7}\right) = 27.73 \frac{\text{kJ}}{\text{kg}} \longleftarrow$$ turbine

where s_3° and s_4° are obtained from Table A-22 at T_3 and h_4, respectively.

Expressed as a percentage of the net work developed

compressor: $\left(\frac{25.06}{145.09}\right)(100) = 17.3\%$ $\longleftarrow$ %

turbine: $\left(\frac{27.73}{145.09}\right)(100) = 19.1\%$

PROBLEM 7.83

KNOWN: Steady-state operating data are provided for a valve, flash chamber, and turbine in series.

FIND: For each component, determine the rate of exergy destruction and express it as a percentage of the turbine power.

SCHEMATIC & GIVEN DATA:

- See solution to Problem 6.160
- From the solution to Problem 6.160

$\dot{W}_t = 139.3\ \text{Btu/s}$

flash chamber: $\dot{\sigma}_{cv} = 0.0054\ \dfrac{\text{Btu/s}}{°R}$

valve: $\dot{\sigma}_{cv} = 0.0176\ \dfrac{\text{Btu/s}}{°R}$

turbine: $\dot{\sigma}_{cv} = 0.0266\ \dfrac{\text{Btu/s}}{°R}$

ASSUMPTIONS: 1. See solution to Problem 6.160. 2. $T_0 = 500°R$, $p_0 = 1\,\text{atm}$.

ANALYSIS: Since $\dot{\sigma}_{cv}$ values are known, it is convenient to find the exergy destruction rates using $\dot{E}_d = T_0\,\dot{\sigma}_{cv}$

– flash chamber: $\dot{E}_d = 500°R\left(0.0054\ \dfrac{\text{Btu/s}}{°R}\right)\left|\dfrac{3600\,s}{h}\right| = 9720\ \dfrac{\text{Btu}}{h}$

– valve: $\dot{E}_d = 500°R\left(0.0176\ \dfrac{\text{Btu/s}}{°R}\right)\left|\dfrac{3600\,s}{h}\right| = 31{,}680\ \dfrac{\text{Btu}}{h}$ ←

– turbine: $\dot{E}_d = 500°R\left(0.0266\ \dfrac{\text{Btu/s}}{°R}\right)\left|\dfrac{3600\,s}{h}\right| = 47{,}880\ \dfrac{\text{Btu}}{h}$

Expressing each as a percentage of the power output,

– flash chamber: $\% = \dfrac{9720\ \text{Btu/h}}{(139.3 \times 3600)\ \text{Btu/h}}(100) = 1.94$

– valve: $\% = \dfrac{(31680)(100)}{[139.3 \times 3600]} = 6.32$ ←

– turbine: $\% = \dfrac{(47{,}880)(100)}{[139.3 \times 3600]} = 9.55$

PROBLEM 7.84

KNOWN: Steady-state operating data are provided for two turbine stages and an interconnecting heat exchanger.

FIND: Determine the rates of exergy destruction. Place in rank order.

SCHEMATIC & GIVEN DATA:

- See Figure P4.82
- From the solution to Problem 6.125

$$\dot{\sigma}_{t1} = 3.1936\ \text{kW/K}\ ,\ \dot{\sigma}_{t2} = 2.8649\ \frac{\text{kW}}{\text{K}}\ ,\ \dot{\sigma}_{HX} = 3.1482\ \frac{\text{kW}}{\text{K}}$$

ASSUMPTIONS: 1. See solutions to Problems 4.82, 6.125. 2. $T_0 = 300\text{K}$, $p_0 = 1\text{bar}$.

ANALYSIS: Since the entropy production rates are known, it is convenient to determine the rates of exergy destruction using $\dot{E}_d = T_0\dot{\sigma}_{cv}$. Thus,

$$(\dot{E}_d)_{t1} = (300\text{K})(3.1936\text{kW/K}) = 958.1\text{kW}\ ,\ (\dot{E}_d)_{t2} = (300)(2.8649) = 859.5\text{kW},$$

$$(\dot{E}_d)_{HX} = 944.5\text{kW}$$

. In order, turbine 1, heat exchanger, turbine 2.

PROB 7.85

KNOWN: Steady-state operating data are provided for a simple vapor power plant.

FIND: Determine the rates of exergy destruction for the turbine, condenser, and pump. Place in rank order.

SCHEMATIC & GIVEN DATA:

- See Figure P4.85
- From the solution to Problem 6.126

$$\dot{\sigma}_{turb} = 87.51\ \frac{\text{kW}}{\text{K}}\ ,\ \dot{\sigma}_{pump} = 1.47\ \frac{\text{kW}}{\text{K}}\ ,\ \dot{\sigma}_{COND} = 35.4\ \frac{\text{kW}}{\text{K}}$$

ASSUMPTIONS: 1. See solutions to Problems 4.85, 6.126. 2. $T_0 = 293$, $p_0 = 1\text{bar}$.

ANALYSIS: Since the entropy production rates are known, it is convenient to determine the rates of exergy destruction using $\dot{E}_d = T_0\dot{\sigma}_{cv}$. Thus,

turbine: $\dot{E}_d = (293\text{K})(87.51\ \frac{\text{kW}}{\text{K}}) = 25{,}640\text{kW}$

① pump: $\dot{E}_d = (293)(1.47) = 431\text{kW}$

Condenser: $\dot{E}_d = (293)(35.4) = 10{,}372\text{kW}$

In order, turbine, condenser, pump.

---

1. Using results from the solution to Problem 4.85, the net power developed is

$$\dot{W}_{net} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)] = \left(109\ \frac{\text{kg}}{\text{s}}\right)[(3425.1 - 2336.7) - (188.9 - 173.9)]\ \frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{kW}}{1\text{kJ/s}}\right|$$

$$= 117{,}000\text{kW}$$

which provides a measure of the significance of each of the three exergy destructions determined here. Expressed as percentages of the net power output

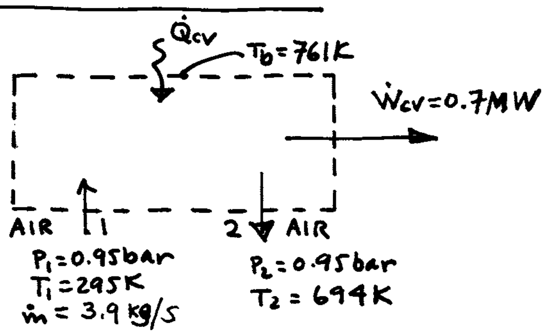
- turbine $\frac{25{,}640}{117{,}000}(100) \approx 22\%$, – pump $\frac{(431)(100)}{117{,}000} = 0.4\%$, – Condenser $\frac{(10{,}372)(100)}{117{,}000} = 9\%$

PROBLEM 7.86

KNOWN: Steady-state operating data are provided for a gas turbine power plant in Problem 6.103. The actual power output is known.

FIND: Determine (a) the rate of exergy transfer accompanying heat transfer, (b) the net rate exergy is carried out by the flowing air, (c) the total rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:
1. The control volume shown in the schematic is at steady state.
2. Heat transfer $\dot{Q}_{cv}$ takes place at temperature T_b only.
3. Air is modeled as an ideal gas.
4. $T_0 = 295K$, $P_0 = 0.95bar$. Effects of motion and gravity can be ignored.

ANALYSIS: (a) Mass and energy rate balances reduce to give

$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow \dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$. Then with data from Table A.22

$$\dot{Q}_{cv} = 0.7MW + \left(3.9\frac{kg}{s}\right)\left[706.8 - 295.17\right]\frac{kJ}{kg}\left|\frac{1MW}{10^3 kJ/s}\right| = 2.305 MW$$

Using Eq. 7.33

$$\dot{E}_q = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{cv} = \left[1 - \frac{295}{761}\right](2.305MW) = 1.411 MW \quad \leftarrow$$

(b) With Eqs. 7.34, 7.36

$$\dot{E}_{f2} - \dot{E}_{f1} = \dot{m}\left[(h_2 - h_1) - T_0(s_2 - s_1)\right] = \dot{m}\left[(h_2 - h_1) - T_0\left(s^\circ(T_2) - s^\circ(T_1) - R\overset{0}{\cancel{\ln\frac{P_2}{P_1}}}\right)\right]$$

$$= \left(3.9\frac{kg}{s}\right)\left[(706.8 - 295.17) - 295(2.5635 - 1.68515)\right]\frac{kJ}{kg \cdot K}\left|\frac{1MW}{10^3 kJ/s}\right|$$

$$= 0.595 MW \quad \leftarrow$$

(c) The total rate of exergy destruction can be obtained by use of the exergy rate balance, Eq. 7.32b

$$\dot{E}_d = \dot{E}_q - \dot{W}_{cv} + (\dot{E}_{f1} - \dot{E}_{f2})$$

$$= (1.411 - 0.7 - 0.595) MW$$

① $$= 0.116 MW \quad \leftarrow$$

---

1. Exergy balance sheet on a rate basis:

• Exergy rate in accompanying heat transfer:	1.411 MW
• Disposition of the exergy input	
— net power developed	0.7 MW (49.6%)
— net exergy carried out by airstream *	0.595 MW (42.2%)
— exergy destroyed	0.116 MW (8.2%)

* Accordingly, the exergy carried out by the air stream is significant, and means should be considered for utilizing it cost effectively.

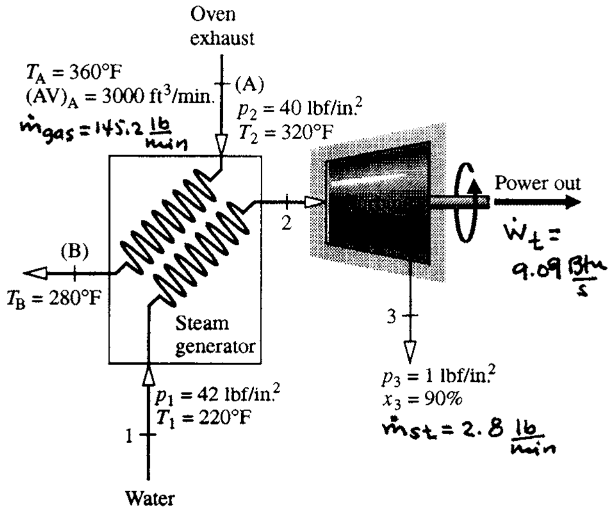
PROBLEM 7.87

KNOWN: Steady-state operating data are provided for a waste-heat-recovery steam generator and turbine in Problem 4.83.

FIND: Develop a full exergy accounting of the net exergy carried in the oven exhaust.

SCHEMATIC & GIVEN DATA:

Data from the solution to Problem 4.83 are shown on the schematic.



	State	h	s
	1	188.2	0.3241
Tables A-2,3,4:	2	1196.8	1.71206
	3	1002.1	1.79347
		h	s°
Table A-22:	A	196.69	0.70160
	B	177.23	0.67665

ASSUMPTIONS: 1. Control volumes enclosing the steam turbine and steam generator are at steady state. 2. $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects can be ignored. 3. The oven gas is modeled as air as an ideal gas. There is no pressure change for the air. 4. $T_0 = 540°R$, $p_0 = 1$ atm.

ANALYSIS: The net rate exergy is carried in by the oven gas is

$$\dot{E}_{fA} - \dot{E}_{fB} = \dot{m}_{gas}\left[h_A - h_B - T_0\left(s_A^\circ - s_B^\circ - \overset{0}{\cancel{R \ln \frac{p_A}{p_B}}}\right)\right]$$

$$= \left(\frac{145.2}{60} \frac{lb}{s}\right)\left[196.69 - 177.23 - 540\,(0.70160 - 0.67665)\right]\frac{Btu}{lb} = 14.49 \frac{Btu}{s}$$

The net rate exergy is carried out by the water stream is

$$\dot{E}_{f3} - \dot{E}_{f1} = \dot{m}_{st}\left[h_3 - h_1 - T_0(s_3 - s_1)\right] = \left(\frac{2.8}{60}\frac{lb}{s}\right)\left[(1002.1 - 188.2) - 540\,(1.79347 - 0.3241)\right]\frac{Btu}{lb}$$

$$= 0.95 \frac{Btu}{s}$$

The rate of exergy destruction in the turbine is $(\dot{E}_d)_t = T_0(\dot{\sigma}_{cv})_t \Rightarrow$

$$(\dot{E}_d)_t = T_0\,\dot{m}_{st}\left[s_3 - s_1\right] = (540\,°R)\left(\frac{2.8}{60}\frac{lb}{s}\right)\left[1.79347 - 1.71206\right]\frac{Btu}{lb\cdot°R} = 2.05 \text{ Btu/s}$$

The rate of the exergy destruction in the steam generator is $(\dot{E}_d)_{SG} = T_0(\dot{\sigma}_{cv})_{SG} \Rightarrow$

$$(\dot{E}_d)_{SG} = T_0\left[\dot{m}_{st}[s_2 - s_1] + \dot{m}_{gas}\left[s_B^\circ - s_A^\circ - \overset{0}{\cancel{R \ln p_B/p_A}}\right]\right]$$

$$= (540)\left[\left(\frac{2.8}{60}\right)(1.71206 - 0.3241) + \left(\frac{145.2}{60}\right)\left[0.67665 - 0.70160\right]\right] = 2.37 \text{ Btu/s}$$

Exergy Balance Sheet (rate basis):

- Net rate exergy is carried in by oven gas: 14.49 Btu/s
- Disposition of the exergy:

– power developed	9.09 Btu/s	(62.7%)
– net exergy carried out by water	0.95 Btu/s	(6.6%)
– exergy destroyed – turbine	2.05 Btu/s	(14.1%)
– exergy destroyed – steam generator	2.37 Btu/s	(16.4%)

PROBLEM 7.88

KNOWN: Steady-state operating data are provided in Problem 6.147 for a throttling valve in parallel with a steam turbine.

FIND: Determine a full exergy accounting of the net exergy carried in by the water stream.

SCHEMATIC & GIVEN DATA:

- See schematic in the solution to Problem 6.147.
- From the solution, $\dot{m}_1 = 25\,\text{lb/s}$

  $\dot{W}_t = 2045\ \text{Btu/s}$, $h_1 = 1350.6\ \frac{\text{Btu}}{\text{lb}}$, $s_1 = 1.5872\ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$, $h_4 = 1268.8\ \frac{\text{Btu}}{\text{lb}}$, $s_4 = 1.6239\ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$

  $\dot{\sigma}_{\text{valve}} = 0.6186\ \frac{\text{Btu/s}}{{}^\circ\text{R}}$, $\dot{\sigma}_{\text{turbine}} = 0.2513\ \frac{\text{Btu/s}}{{}^\circ\text{R}}$, $\dot{\sigma}_{\text{mix}} = 0.0476\ \frac{\text{Btu/s}}{{}^\circ\text{R}}$

ASSUMPTIONS: 1. See assumptions listed for Problem 6.147. 2. $T_0 = 500^\circ\text{R}$, $p_0 = 1\,\text{atm}$.

ANALYSIS: The net exergy carried in the water stream is

$$\dot{E}_{f1} - \dot{E}_{f4} = \dot{m}\left[(h_1 - h_4) - T_0(s_1 - s_4)\right] = 25\,\frac{\text{lb}}{\text{s}}\left[(1350.6 - 1268.8) - 500(1.5872 - 1.6239)\right]\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$
$$= 2503.75\ \text{Btu/s}$$

The rates of exergy destruction in the three components can be evaluated using $\dot{E}_d = T_0\dot{\sigma}_{cv}$. Thus

valve: $(\dot{E}_d)_v = (500^\circ\text{R})\left(0.6186\ \frac{\text{Btu/s}}{{}^\circ\text{R}}\right) = 309.3\ \text{Btu/s}$

turbine: $(\dot{E}_d)_t = (500^\circ\text{R})\left(0.2513\ \frac{\text{Btu/s}}{{}^\circ\text{R}}\right) = 125.65\ \text{Btu/s}$

mixer: $(\dot{E}_d)_m = (500^\circ\text{R})\left(0.0476\ \frac{\text{Btu/s}}{{}^\circ\text{R}}\right) = 23.8\ \text{Btu/s}$

Exergy Balance Sheet (rate basis):

• net rate exergy is carried in by the water stream	2503.75 Btu/s	
• Disposition of the exergy carried in		
– power developed:	2045 Btu/s	(82 %)
– exergy destroyed:		
valve	309.3 Btu/s	(12 %)
turbine	125.65 Btu/s	(5 %)
mixer	23.8 Btu/s	(1 %)
	2503.75 Btu/s	

PROBLEM 7.89

KNOWN: Steady-state operating data are provided for a compressor and heat exchanger in series.

FIND: Develop a full exergy accounting of the compressor power input.

SCHEMATIC & GIVEN DATA

- See the schematic in the solution to Problem 6.124.
- From the solution

$$(-\dot{W}_{cv}) = 50.4\,\text{kW},\quad \dot{m}_{air} = 0.5\,\text{kg/s},\quad \dot{m}_{cw} = 0.403\,\text{kg/s}$$

$$\dot{\sigma}_{comp} = 0.0196\,\frac{\text{kW}}{\text{K}},\quad \dot{\sigma}_{HX} = 0.0152\,\frac{\text{kW}}{\text{K}}$$

State	h	s°
1	300.19	1.70203
3	350.49	1.85708

State	h	s
A	104.89	0.3674
B	167.57	0.5725

ASSUMPTIONS: 1. See assumptions listed for Problem 6.124. 2. $T_0 = 300\,\text{K}$, $p_0 = 96\,\text{kPa}$.

ANALYSIS: The rates of exergy destruction can be found using $\dot{E}_d = T_0\dot{\sigma}_{cv}$.

compressor: $(\dot{E}_d)_c = 300\,\text{K}\left(0.0196\,\frac{\text{kW}}{\text{K}}\right) = 5.88\,\text{kW}$

heat exchanger: $(\dot{E}_d)_{HX} = 300\,(0.0152) = 4.56\,\text{kW}$

The net rate exergy exits in the air stream

$$\dot{E}_{f3} - \dot{E}_{f1} = \dot{m}_{air}\left(h_3 - h_1 - T_0(s_3^\circ - s_1^\circ - R\ln p_3/p_1)\right)$$

$$= \left(0.5\,\frac{\text{kg}}{\text{s}}\right)\left[(350.49 - 300.19) - 300\left(1.85708 - 1.70203 - \frac{8.314}{28.97}\ln\frac{230}{96}\right)\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$

$$= 39.5\,\text{kW}$$

The net rate exergy exits in the water stream

$$\dot{E}_{fB} - \dot{E}_{fA} = \dot{m}_{cw}\left[(h_B - h_A) - T_0(s_B - s_A)\right] = \left(0.403\,\frac{\text{kg}}{\text{s}}\right)\left[(167.57 - 104.89) - 300(.5725 - 0.3674)\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right|$$

$$= 0.46\,\text{kW}$$

Exergy Balance Sheet (rate basis)

• rate exergy is supplied by compressor power input	50.4	kW	
• disposition of the exergy			
– net exergy carried out by the air	39.5	kW	(78.4%)
– net exergy carried out by the water	0.46	kW	(0.9%)
– exergy destruction			
compressor	5.88	kW	(11.7%)
heat exchanger	4.56	kW	(9.0%)
	50.4	kW	

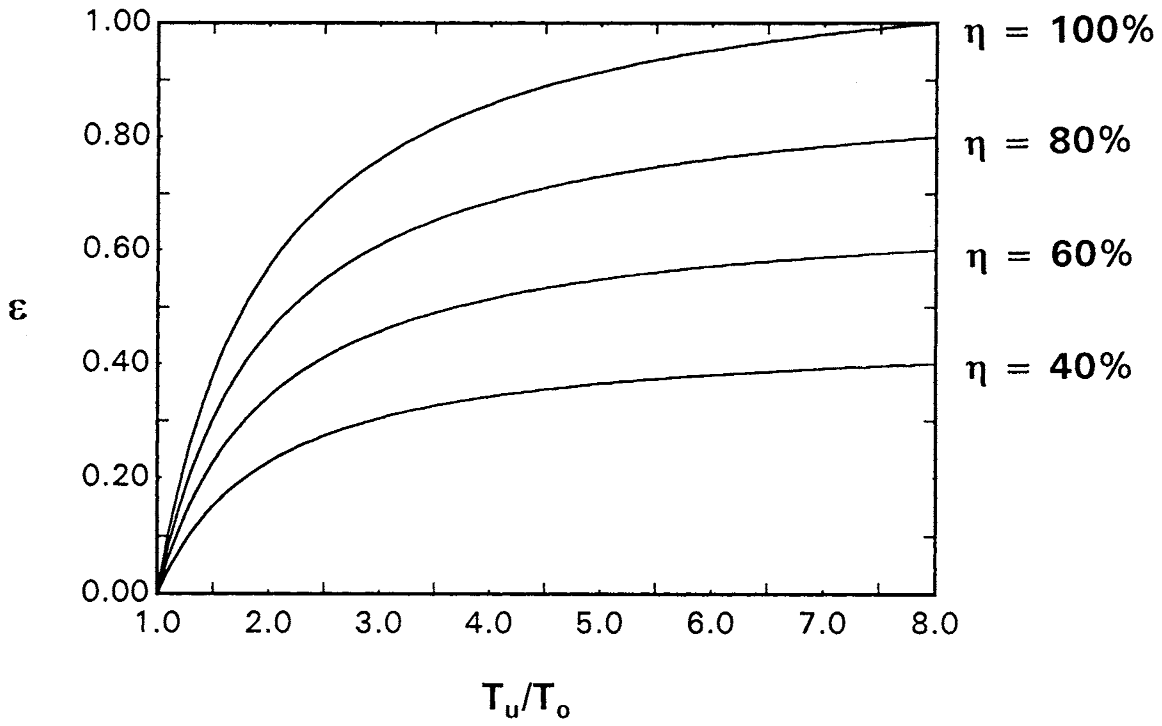
PROBLEM 7.90

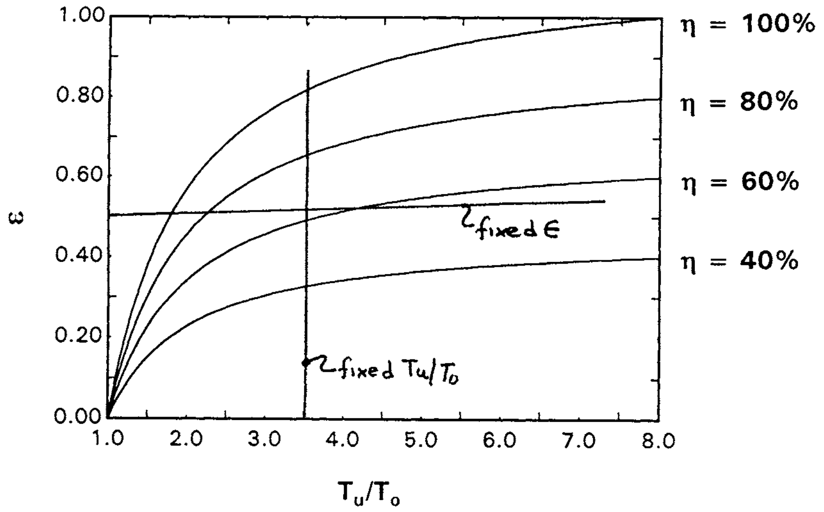
KNOWN: Equation 7.39b

FIND: Plot ϵ versus T_u/T_o for $T_s/T_o = 8$ and selected values of η.

ANALYSIS:

Eq. 7.39b $$\epsilon = \eta\left[\frac{1-T_o/T_u}{1-T_o/T_s}\right] = \eta\left[\frac{1-T_o/T_u}{1-\frac{1}{8}}\right] = \frac{8}{7}\eta\left[1-\frac{T_o}{T_u}\right]$$





- when ϵ is fixed, as η increases, lower use temperatures are allowed.
- when T_u/T_o is fixed, as η increases, ϵ increases.

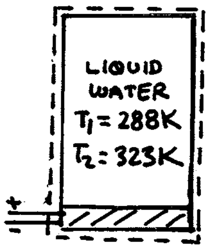
In sum, a high first-law efficiency, η, has practical benefit. So, both first-law and second-law considerations are important in proper energy resource utilization.

PROBLEM 7.91

KNOWN: Operating data are provided for an electrical water heater.

FIND: Devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The system is shown in the accompanying schematic. $Q = 0$.
2. The water can be modeled as incompressible with constant specific heat c.
3. The states of the resistor and tank do not change.
4. $T_0 = T_1 = 288K$.

ANALYSIS: Applying a closed system exergy balance

$$\Delta E = \int_1^2 \overset{0}{\cancel{\left[1-\frac{T_0}{T_b}\right]\delta Q}} - [W - p_0 \overset{0}{\cancel{\Delta V}}] - E_d$$

$$\Rightarrow \quad (-W) = \Delta E + E_d$$

That is, the exergy supplied to the system electrically, $(-W)$, is either stored, ΔE, or destroyed, E_d.

Thus, an exergetic efficiency in the form (exergy stored/exergy input) is

$$\epsilon = \frac{\Delta E}{(-W)} \qquad (1)$$

- The numerator can be evaluated using Eq. 7.10 and assumption 2 as

$$\Delta E = \Delta U + p_0 \overset{0}{\cancel{\Delta V}} - T_0 \Delta S = mc\,(T_2 - T_1) - T_0\, mc \ln\frac{T_2}{T_1} \qquad (2)$$

- The work input can be found from an energy balance

$$\Delta U = \overset{0}{\cancel{Q}} - W \Rightarrow (-W) = mc\,(T_2 - T_1) \qquad (3)$$

Collecting Eqs 1, 2, 3

$$\epsilon = \frac{mc\,[(T_2 - T_1) - T_0 \ln(T_2/T_1)]}{mc\,[T_2 - T_1]} = 1 - \frac{T_0 \ln(T_2/T_1)}{(T_2 - T_1)}$$

Inserting $T_1 = T_0 = 288\,K$, $T_2 = 323\,K$

① $$\epsilon = 1 - \frac{288 \ln(323/288)}{(323 - 288)} = 0.056 \; (5.6\%)$$

---

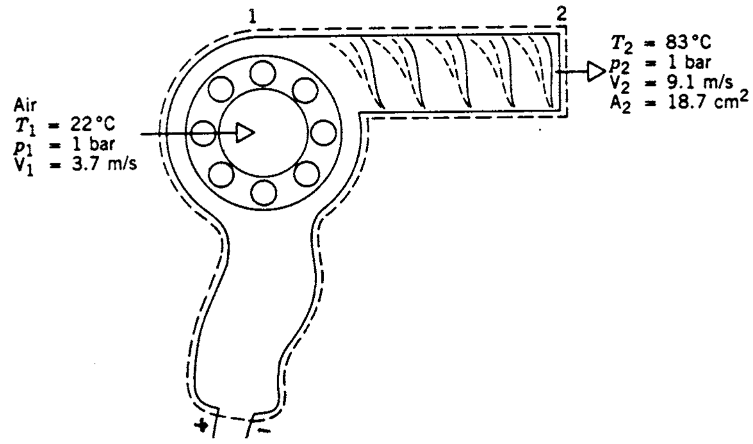
1. This calculation shows that ordinary water heating makes poor use of the exergy input: electricity. The electricity supplied has a high potential for use, but only relatively low temperature water is obtained in exchange. The largest portion of the input is destroyed by irreversibilities within the water heater, leaving only 5.6% stored for later use.

PROBLEM 7.92

KNOWN: Steady-state operating data are provided for a hand-held hair dryer.

FIND: (a) Evaluate the power input. (b) Devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume shown in the accompanying figure is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$ and potential energy effects can be ignored.
3. Air is modeled as air as an ideal gas.
4. $T_0 = 295K\ (22°C)$.

Analysis:

(a) The power $\dot{W}_{cv}$ for the control volume in the accompanying figure can be evaluated using mass and energy rate balances. At steady state, the mass rate balance reduces to give $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Using this relationship between the mass flow rates together with assumption 2, the energy rate balance reduces at steady state to give

$$\dot{W}_{cv} = \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right)$$

The mass flow rate $\dot{m}$ can be evaluated from the expression

$$\dot{m} = \frac{A_2V_2}{v_2} = \frac{A_2V_2}{RT_2/p_2}$$

$$\dot{m} = \frac{\left[\dfrac{18.7\ \text{cm}^2}{(100\ \text{cm/m})^2}\right]\left(9.1\ \dfrac{\text{m}}{\text{s}}\right)\left(10^5\ \dfrac{\text{N}}{\text{m}^2}\right)}{\left(\dfrac{8314}{28.97}\ \dfrac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(356\ \text{K})} = 0.0167\ \text{kg/s}$$

Then, with specific enthalpy values from Table A-22, h_1 = 295.2 kJ/kg and h_2 = 356.5 kJ/kg, the value of $\dot{W}_{cv}$ is obtained as follows:

$$\dot{W}_{cv} = 0.0167\ \frac{\text{kg}}{\text{s}}\left[295.2\ \frac{\text{kJ}}{\text{kg}} - 356.5\ \frac{\text{kJ}}{\text{kg}} + \frac{(3.7\ \text{m/s})^2 - (9.1\ \text{m/s})^2}{2}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|\right]$$

$$= 0.0167\ \frac{\text{kg}}{\text{s}}\left(-61.3\ \frac{\text{kJ}}{\text{kg}} - 0.03\ \frac{\text{kJ}}{\text{kg}}\right)\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = -1.02\ \text{kW} \longleftarrow$$

The minus sign reminds us that an electrical power *input* is required to operate the hair dryer. The change in kinetic energy between inlet and exit is seen to be negligible and is ignored in the remainder of the solution.

(b) Since $\dot{Q}_{cv} = 0$ and $\dot{m}_1 = \dot{m}_2$, the steady-state exergy rate balance reduces as follows:

$$0 = \sum_j \left[1 - \frac{T_0}{T_j}\right]\overset{0}{\dot{Q}_j} - \dot{W}_{cv} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d \Rightarrow (-\dot{W}_{cv}) = \dot{m}(e_{f2} - e_{f1}) + \dot{E}_d$$

This result shows that the exergy input, $(-\dot{W}_{cv})$, either goes into increasing the exergy of the air stream, $\dot{m}(e_{f2}-e_{f1})$, or is destroyed, $\dot{E}_d$. Thus, an exergetic efficiency can be defined as

$$\epsilon = \frac{\dot{m}(e_{f2} - e_{f1})}{(-\dot{W}_{cv})} = \frac{\dot{m}[(h_2 - h_1) - T_0(s_2 - s_1)]}{(-\dot{W}_{cv})} = \frac{\dot{m}[(h_2 - h_1) - T_0(s_2^\circ - s_1^\circ - \overset{0}{\cancel{R\ln p_2/p_1}})]}{(-\dot{W}_{cv})}$$

With data from Table A-22

① $$\epsilon = \frac{(0.0167\ \text{kg/s})[(356.5 - 295.2)\frac{\text{kJ}}{\text{kg}} - 295\text{K}(1.874 - 1.685)\frac{\text{kJ}}{\text{kg}\cdot\text{K}}]}{(1.02\ \text{kW})}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 0.091\ (9.1\%)$$

---

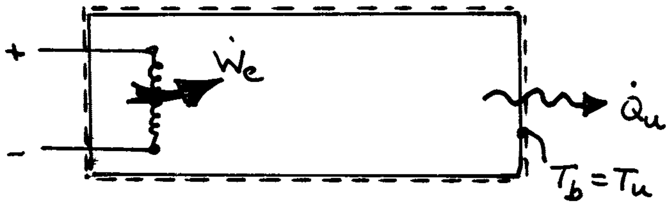
1. In this device, a relatively high potential for use electrical input is used to obtain a stream of slightly warm air. The low value of the efficiency ε emphasizes that source and end use are not well matched.

PROBLEM 7.93

KNOWN: From an input of electricity, a furnace delivers energy by heat transfer at a rate of $\dot{Q}_u$ at a use temperature of T_u. The furnace operates at steady state.

FIND: (a) Devise an exergetic efficiency. (b) Plot this efficiency versus T_u ranging from 300 to 900K.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The closed system is at steady state. (2) There are no significant energy transfers other than those shown. (3) For the environment, $T_0 = 20°C = 293K$.

ANALYSIS: (a) At steady state, the exergy balance with assumption 2 reads

$$0 = \dot{W}_e - \left[1 - \frac{T_0}{T_u}\right]\dot{Q}_u - \dot{E}_d$$

where the quantities $\dot{W}_e$ and $\dot{Q}_u$ have been taken as positive in the direction of the arrows. Rearranging

$$\underset{\text{(input)}}{\dot{W}_e} = \underset{\text{(transfer out)}}{\left[1 - \frac{T_0}{T_u}\right]\dot{Q}_u} + \underset{\text{(destroyed)}}{\dot{E}_d}$$

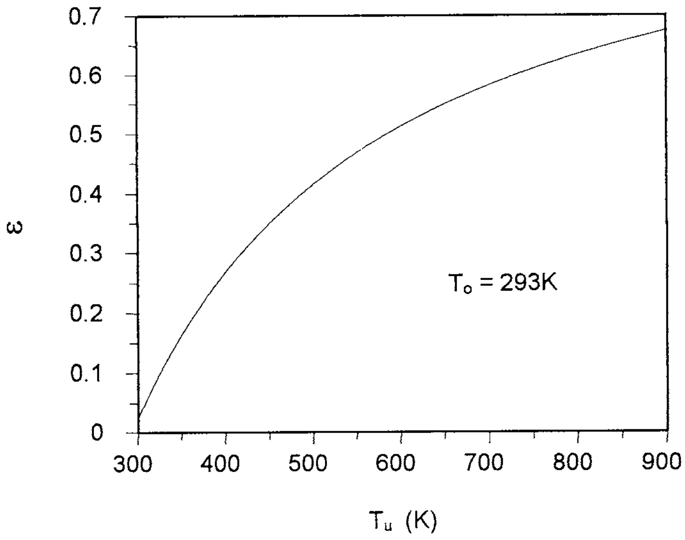
An exergetic efficiency in the form output/input is

$$\epsilon = \frac{\left[1 - \frac{T_0}{T_u}\right]\dot{Q}_u}{\dot{W}_e}$$

With $\dot{Q}_u = \dot{W}_e$ from an energy balance at steady state

$$\epsilon = \left[1 - \frac{T_0}{T_u}\right] \longleftarrow \epsilon$$

(b) The following plot is generated using IT with $T_0 = 293$:



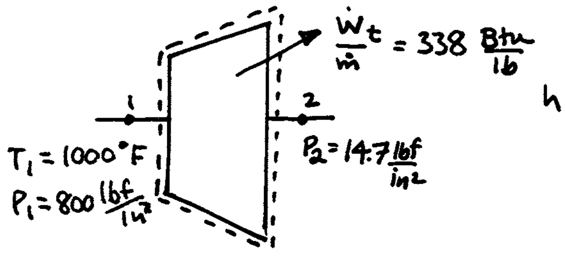
From the plot, we observe that as $T_u \to T_0$, $\epsilon \to 0$. Also, ϵ increases rapidly with T_u, since as T_u increases less exergy is destroyed, per unit of exergy input by electricity.

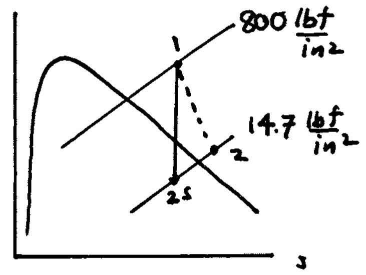
PROBLEM 7.94*

KNOWN: Steady-state operating data are provided for a steam turbine.

FIND: Evaluate the isentropic turbine efficiency and the exergetic turbine efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible. 3. $T_0 = 520°R$, $P_0 = 14.7$ lbf/in².

ANALYSIS: Using Eq. 6.48, the isentropic turbine efficiency is

$$\eta_t = \frac{\dot{W}_{cv}/\dot{m}}{h_1 - h_{2s}}$$

From Table A-4E $h_1 = 1511.9$ Btu/lb, $s_1 = 1.6807$ Btu/lb·°R. With $s_{2s} = s_1$ at 14.7 lbf/in², Table A-3E gives

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.6807 - 0.3121}{1.4446} = 0.947$$

$$\Rightarrow h_{2s} = h_f + x_{2s} h_{fg} = 180.15 + 0.947(970.4) = 1099.5 \text{ Btu/lb.}$$

Thus,

$$\eta_t = \frac{338 \text{ Btu/lb}}{[1511.9 - 1099.5] \text{ Btu/lb}} = \frac{338}{412.4} = 0.82 \; (82\%) \longleftarrow$$

Using Eq. 7.42, the exergetic turbine efficiency is

$$\epsilon = \frac{\dot{W}_{cv}/\dot{m}}{(h_1 - h_2) - T_0(s_1 - s_2)}$$

To fix the exit state, reduce the mass and energy rate balance to get

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \dot{W}_{cv} + \dot{m}(h_1 - h_2) \Rightarrow h_2 = h_1 - \dot{W}_{cv}/\dot{m} = 1511.9 - 338 = 1173.9 \text{ Btu/lb}$$

Then, at 14.7 lbf/in², with h_2 known, Table A-4E gives $s_2 = 1.7902$ Btu/lb·°R. Accordingly,

$$\epsilon = \frac{338 \text{ Btu/lb}}{[\underbrace{(1511.9 - 1173.9)}_{338} - 520(1.6807 - 1.7902)] \text{ Btu/lb}} = \frac{338}{394.9}$$

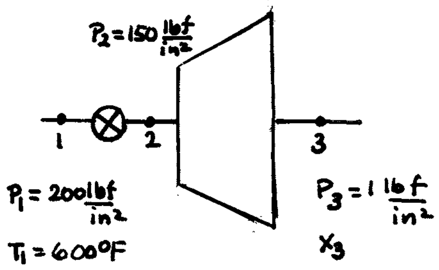
$$= 0.856 \; (85.6\%) \longleftarrow$$

PROBLEM 7.95

KNOWN: Steady-state operating data are provided in Problem 7.67 for a valve and turbine in series.

FIND: For the turbine, plot the exergetic efficiency versus the steam quality at the turbine exit ranging from 90 to 100 %.

SCHEMATIC & GIVEN DATA:


$P_2 = 150 \frac{lbf}{in^2}$
1
2
3
$P_1 = 200 \frac{lbf}{in^2}$
$T_1 = 600°F$
$P_3 = 1 \frac{lbf}{in^2}$
x_3

ASSUMPTIONS: 1. Control volumes enclosing the valve and turbine are at steady state.
2. For the control volumes, $\dot{Q}_{cv} = 0$, and the effects of motion and gravity can be ignored. The expansion across the valve is a throttling process.
3. $T_0 = 520°R$, $p_0 = 1 atm$.

ANALYSIS: The turbine exergetic efficiency is given by Eq. 7.42. With the indicated assumptions, this can be expressed as

$$\epsilon = \frac{h_2 - h_3}{(h_2 - h_3) - T_0(s_2 - s_3)}$$

. Since $h_2 = h_1$ for the throttling process, we get

$$\epsilon = \frac{h_1 - h_3}{(h_1 - h_3) - T_0(s_2 - s_3)} \quad (1).$$

In Eq. (1) h_1 is fixed by T_1, P_1, s_2 is fixed by $h_2 = h_1, P_2$, and h_3 and s_3 are fixed by P_3, x_3.

Sample Calculation: $x_3 = 0.98$. From Table A-4E, $h_1 = 1322.1$ Btu/lb. Interpolating in Table A-4E with $h_2 = h_1$ at 150 lbf/in.², $s_2 = 1.7078$ Btu/lb·°R. With data from Table A-3E at 1 lbf/in.², $h_3 = 69.74 + 0.98(1036) = 1085$ Btu/lb, $s_3 = 0.1327 + 0.98(1.8453) = 1.9411$ Btu/lb·°R. Inserting values, Eq. (1) gives

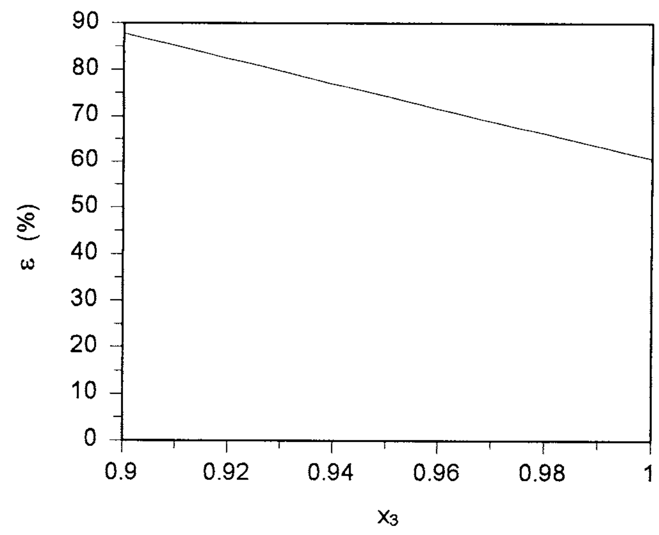
$$\epsilon = \frac{(1322.1 - 1085)}{(1322.1 - 1085) - 520(1.7078 - 1.9411)} = \frac{237.1}{358.4} = 0.662 \quad (66.2\%)$$

IT Code

```
p1 = 200  // lbf/in.²
T1 = 600  // °F
p2 = 150  // lbf/in.²
p3 = 1  // lbf/in.²
x3 = 0.98
To = 60 + 460  // °R

h1 = h_PT("Water/Steam", p1, T1)
s2 = s_Ph("Water/Steam", p2, h1)
s3 = ssat_Px("Water/Steam", p3, x3)
h3 = hsat_Px("Water/Steam", p3, x3)
eff = (h1 - h3) / ((h1 - h3) - To * (s2 - s3))
```

IT Results for $x_3 = 0.98$

```
eff   0.6613
h1    1322
h3    1085
s2    1.707
s3    1.941
```

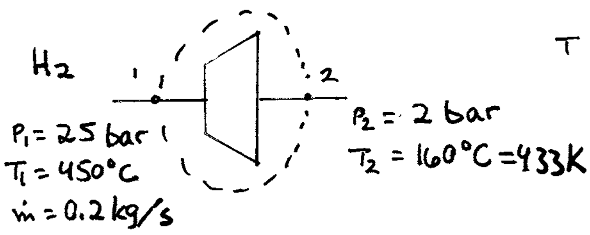


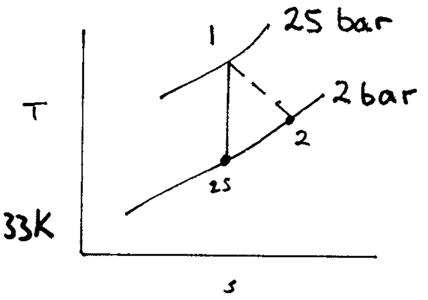
PROBLEM 7.96*

KNOWN: Steady state operating data are provided for a hydrogen turbine.

FIND: Determine (a) the isentropic turbine efficiency, (b) the exergetic turbine efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$, and kinetic/potential energy changes can be ignored. (3) Air is modeled as an ideal gas with $k = 1.37$ (4) The exergy reference environment is at $T_0 = 298$ K.

ANALYSIS: (a) The isentropic turbine efficiency is given by Eq. 6.48

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{c_p(T_1 - T_2)}{c_p(T_1 - T_{2s})} = \frac{T_1 - T_2}{T_1 - T_{2s}}$$ With Eq. 6.45, $T_{2s} = T_1\left(\frac{P_{2s}}{P_1}\right)^{\frac{k-1}{k}} = (723\ \text{K})\left(\frac{2}{25}\right)^{0.37/1.37} = 365.5\text{K}$

$$\therefore \eta_t = \frac{723 - 433}{723 - 365.5} = 0.81\ (81\%) \longleftarrow \eta_t$$

(b) The exergetic turbine efficiency is given by Eq. 7.42, which becomes

$$\epsilon = \frac{h_1 - h_2}{(h_1 - h_2) - T_0(s_1 - s_2)} = \frac{c_p(T_1 - T_2)}{c_p(T_1 - T_2) - T_0\left[c_p \ln\frac{T_1}{T_2} - R \ln\frac{P_1}{P_2}\right]}$$, Eq. 3.47a gives $R = c_p\left(\frac{k-1}{k}\right)$

$$\epsilon = \frac{T_1 - T_2}{T_1 - T_2 - T_0\left[\ln\frac{T_1}{T_2} - \left(\frac{k-1}{k}\right)\ln\frac{P_1}{P_2}\right]}$$

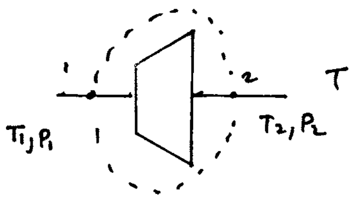
$$\epsilon = \frac{723 - 433}{723 - 433 - 298\left[\ln\left(\frac{723}{433}\right) - \left(\frac{1.37-1}{1.37}\right)\ln\left(\frac{25}{2}\right)\right]} = 0.852\ (85.2\%) \longleftarrow \epsilon$$

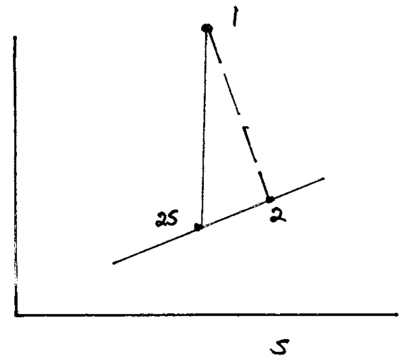
PROBLEM 7.97

KNOWN: An ideal gas with constant specific heat ratio k enters a turbine operating at steady state at T_1, P_1 and expands adiabatically to T_2, P_2.

FIND: Determine when the value of the exergetic turbine efficiency exceeds the value of the isentropic efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$, and the effects of kinetic/potential energy can be ignored. (3) The gas is modeled as an ideal gas with constant specific heat ratio k. (4) the exergy reference environment is at T_0.

ANALYSIS: The requirement is $\epsilon > \eta_t$. That is, with Eqs. 6.48 and 7.42

$$\frac{h_1 - h_2}{h_1 - h_2 - T_0(s_1 - s_2)} > \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow (h_1 - h_{2s}) > (h_1 - h_2) - T_0(s_1 - s_2)$$

$$\Rightarrow (h_2 - h_{2s}) > T_0(s_2 - \underbrace{s_1}_{= s_{2s}})$$

Thus, $\epsilon > \eta_t$ when

$$h_2 - h_{2s} > T_0(s_2 - s_{2s})$$

Since k is constant

$$c_p[T_2 - T_{2s}] > T_0\left[c_p \ln\frac{T_2}{T_{2s}} - \cancelto{0}{R \ln\frac{P_2}{P_{2s}}}\right]$$

$$\Rightarrow \quad (T_2 - T_{2s}) > T_0 \ln\frac{T_2}{T_{2s}}$$

or

$$\frac{T_2 - T_{2s}}{\ln(T_2/T_{2s})} > T_0 \qquad (1) \quad \longleftarrow$$

The left side is recognized as the logarithmic average of the temperature difference $T_2 - T_{2s}$.

Using the results of Problem 7.96 for illustration: $T_1 = 1273\text{K}$, $T_2 = 938\text{K}$, $T_{2s} = 854\text{K}$, $T_0 = 298$, Eq. (1) gives

$$\left[\frac{938 - 854}{\ln(938/854)}\right] = 895\text{K} > 298 \Rightarrow \epsilon > \eta_t$$

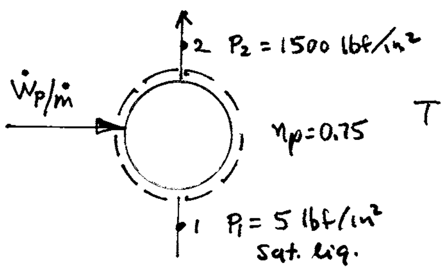
The solution to problem 7.96 gives $\epsilon = 85.2\%$, $\eta_t = 81\%$, which is in accord with the result elicited. [Note that the arithmetic average of T_2, T_{2s} is 896K, only slightly different in this case from the logarithmic average.]

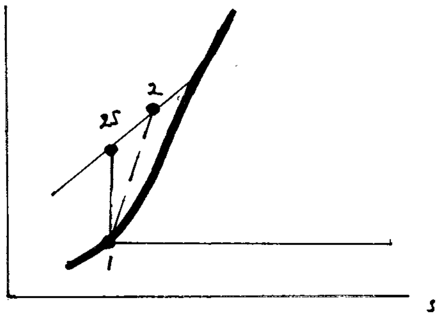
PROBLEM 7.98*

KNOWN: Steady state operating data are provided for a pump.

FIND: Determine (a) exergy destruction per unit of mass flowing, (b) the exergetic pump efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, and kinetic/potential energy effects are negligible. (3) $T_0 = 530°R$

ANALYSIS: (a) The exergy destruction per unit of mass flowing is conveniently found with

$$\frac{\dot{E}_d}{\dot{m}} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{m}} \Rightarrow \frac{\dot{E}_d}{\dot{m}} = T_0(s_2 - s_1)$$

State 1 is fixed. State 2 is fixed by P_2 and h_2 obtained as follows: using Eqs. 6.50, 6.53c together with an average value for the specific volume,

$$\eta_p = \frac{v_{ave}(P_2 - P_1)}{(-\dot{W}_{cv}/\dot{m})} = \frac{v_{ave}(P_2 - P_1)}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{v_{ave}(P_2 - P_1)}{\eta_p}$$

Here, v_{ave} is the average specific volume. Taking $v_{ave} \approx v_f(T_1)$, $h_1 = h_f(T_1)$, and data from Table A-3E

$$h_2 = 130.17 \frac{\text{Btu}}{\text{lb}} + \frac{(0.01641\ \text{ft}^3/\text{lb})[1500 - 5](144)\frac{\text{lbf}}{\text{ft}^2}\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|}{0.75} = 136.22 \frac{\text{Btu}}{\text{lb}}$$

① Interpolating in Table A-5E, $s_2 = 0.2367$ Btu/lb·°R, $v_2 = 0.01635$ ft³/lb. Also, $s_1 = s_f(T_1) = 0.2349$ Btu/lb·°R. Then

$$\frac{\dot{E}_d}{\dot{m}} = 530°R\,(0.2367 - 0.2349)\,\text{Btu/lb°R} = 0.954\ \text{Btu/lb} \quad \longleftarrow$$

(b) With Eq. 7.43

$$\epsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}_{cv}/\dot{m})} = \frac{(h_2 - h_1) - \overbrace{T_0(s_2 - s_1)}^{\dot{E}_d/\dot{m}}}{(h_2 - h_1)} = \frac{(136.22 - 130.17) - 0.954}{136.22 - 130.17}$$

$$= 0.842 \quad (84.2\%) \quad \longleftarrow$$

---

1. Comparing v_2 with $v_f(T_1)$, it is clear that specific volume does not vary appreciably in this case.

PROBLEM 7.99

KNOWN: Air enters an insulated turbine operating at steady state at a given state and with a known volumetric flow rate. The exit pressure and the isentropic turbine efficiency are specified.

FIND: Determine (a) the power developed and the exergy destruction rate. (b) the exergetic turbine efficiency.

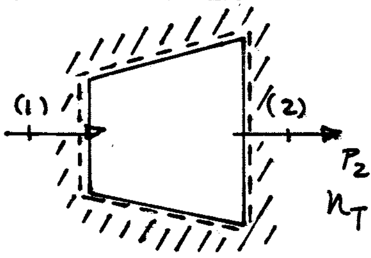
SCHEMATIC & GIVEN DATA:

air

$p_1 = 4$ bar

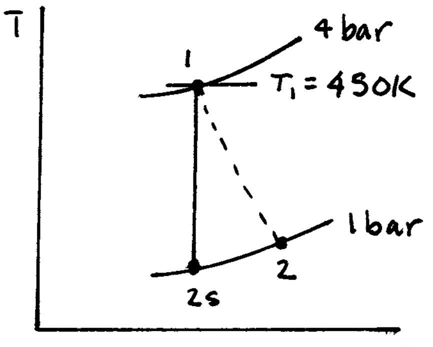
$T_1 = 450$K

$(AV)_1 = 5\ m^3/s$



$p_2 = 1$ bar

$\eta_T = 84\%$



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume, $\dot Q_{cv} = 0$ and the effects of motion and gravity are neglected. (3) The air is modeled as an ideal gas. (4) For the environment, $T_0 = 20°C = 293$ K, $p_0 = 1$ bar.

ANALYSIS: (a) We begin by using the isentropic turbine efficiency to fix state 2. With data from Table A-22

$$p_r(T_{2s}) = \left(\frac{p_2}{p_1}\right) p_r(T_1) = \left(\frac{1}{4}\right) .775 = 1.44375 \Rightarrow \begin{cases} T_{2s} = 303.5\ K \\ h_{2s} = 303.71\ \frac{kJ}{kg} \end{cases}$$

Now

$$\eta_T = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_T(h_1 - h_{2s}) = 451.8 - (.84)(451.8 - 303.71)$$
$$= 327.4\ kJ/kg$$

The mass flow rate of air is

$$\dot m = \frac{(AV)_1}{v_1} = \frac{p_1 (AV)_1}{RT_1} = \frac{(4\ bar)(5 m^3/s)}{\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(450 k)} \left|\frac{10^5\ N/m^2}{1\ bar}\right| \left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$
$$= 15.49\ kg/s$$

From mass and energy rate balances: $0 = \cancelto{0}{\dot Q_{cv}} - \dot W_{cv} + \dot m\left[(h_1 - h_2) + \cancelto{0}{\left(\frac{V_1^2 - V_2^2}{2}\right)} + g\cancelto{0}{(z_1 - z_2)}\right]$

$$\dot W_{cv} = \dot m\,[h_1 - h_2] = (15.49\ \tfrac{kg}{s})[451.8 - 327.4]\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right| = 1927\ kW \longleftarrow \dot W_{cv}$$

From mass and entropy balances: $0 = \cancelto{0}{\sum_j \left(\frac{\dot Q}{T}\right)_j} + \dot m(s_1 - s_2) + \dot\sigma_{cv} \Rightarrow \dot\sigma_{cv} = \dot m(s_2 - s_1)$

Thus

$$\dot E_d = T_0 \dot\sigma_{cv} = T_0 \dot m (s_2 - s_1) = T_0 \dot m\left[s°(T_2) - s°(T_1) - R \ln p_2/p_1\right]$$

Interpolating in Table A-22 with $h_2 = 327.4$; $s°(T_2) = 1.78886$.

$$\dot E_d = (293\ K)(15.49\ \tfrac{kg}{s})\left[(1.78886 - 2.11161) - \frac{8.314}{28.97}\ln\left(\frac{1}{4}\right)\right]\frac{kJ}{kg\cdot K}\left|\frac{1\ kW}{1\ kJ/s}\right|$$
$$= 340.8\ kW \longleftarrow \dot E_d$$

(b) The exergetic turbine efficiency is

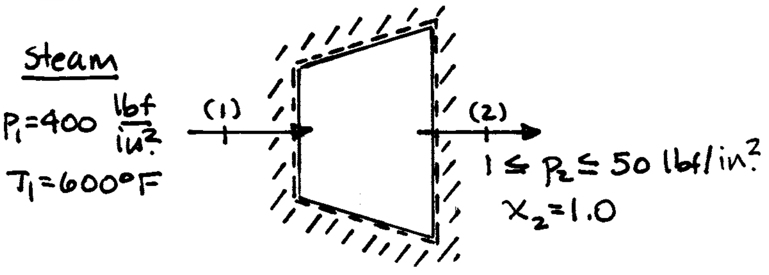
$$\epsilon = \frac{\dot W_{cv}}{[(h_1 - h_2) - T_0(s_1 - s_2)]\dot m} = \frac{\dot W_{cv}}{\dot W_{cv} + \dot E_d} = \frac{1927}{1927 + 340.8} = 0.8497\ (85\%) \longleftarrow \epsilon$$

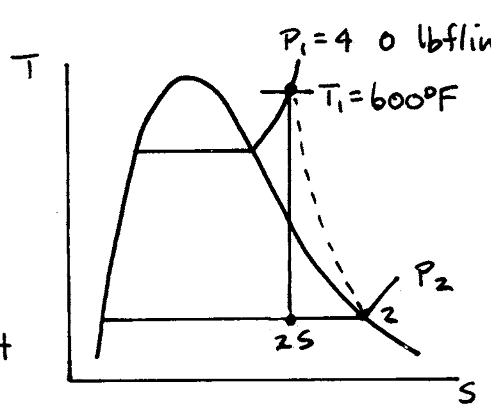
PROBLEM 7.100

KNOWN: Steam enters a well-insulated turbine operating at steady state at a specified state and exits at pressure p.

FIND: (a) For p = 50 lbf/in.^{2}, determine the exergy destruction rate per unit mass of steam flowing and the isentropic and exergetic turbine efficiencies. (b) Plot each of these quantities versus p ranging from 1 to 50 lbf/in.^{2}

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown operates at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are ignored. (3) For the environment, $T_0 = 60°F = 520°R$, $P_0 =$ 1atm.

ANALYSIS: With assumptions 1 and 2, the mass and energy rate balances reduce to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \qquad (1)$$

Further, mass and entropy rate balances reduce to give $\dot{\sigma}_{cv}/\dot{m} = s_2 - s_1$. The exergy destruction rate is $\dot{E}d/\dot{m} = T_0(\dot{\sigma}_{cv}/\dot{m})$, or

$$\dot{E}d/\dot{m} = T_0(s_2 - s_1) \qquad (2)$$

Furthermore, the isentropic and exergetic turbine efficiencies are, respectively

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \qquad (3)$$

$$\epsilon = \frac{\dot{W}_{cv}/\dot{m}}{e_{f1} - e_{f2}} = \frac{h_1 - h_2}{(h_1 - h_2) - T_0(s_1 - s_2)} \qquad (4)$$

(a) From Table A-4E, $h_1 = 1306.6$ Btu/lb, $s_1 = 1.5892$ Btu/lb·°R. Further, from Table A-3E at 50 lbf/in.^{2}, $h_2 = h_g = 1174.4$ Btu/lb, $s_2 = s_g = 1.6589$ Btu/lb·°R. Accordingly

$$\dot{E}d/\dot{m} = (520°R)(1.6589 - 1.5892)\frac{Btu}{lb \cdot °R} = 36.24 \text{ Btu/lb} \longleftarrow \dot{E}d/\dot{m}$$

State 2s is fixed by P_2 and $s_{2s} = s_1$: $x_{2s} = (s_{2s} - s_f)/s_{fg} = (1.5892 - 0.4113)/1.2476 = 0.9441$. Therefore $h_{2s} = h_f + x_{2s}h_{fg} = 250.24 + (.9441)(924.2) = 1122.8$ Btu/lb.

Thus

$$\eta_t = \frac{1306.6 - 1174.4}{1306.6 - 1122.8} = 0.719 \ (71.9\%) \longleftarrow \eta_t$$

Now, from Eq. (4)

$$\epsilon = \frac{(1306.6 - 1174.4)}{(1306.6 - 1174.4) - (520)(1.5892 - 1.6589)} = 0.785 \ (78.5\%) \longleftarrow \epsilon$$

PROBLEM 7.100 (Cont'd.)

(b) The data for the required plots are obtained using IT, as follows:

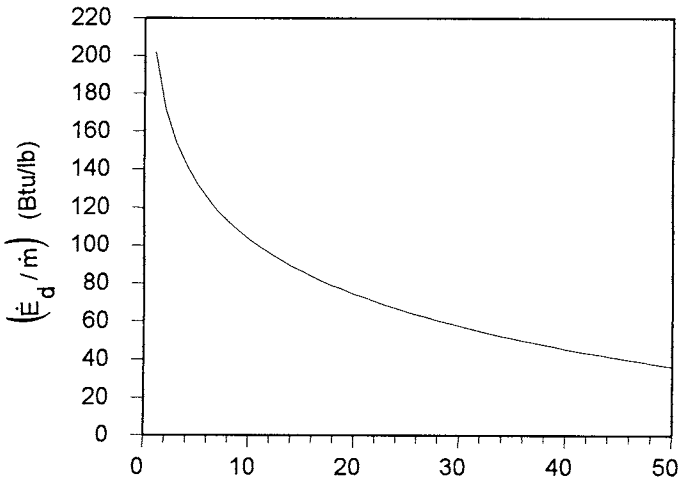
IT Code

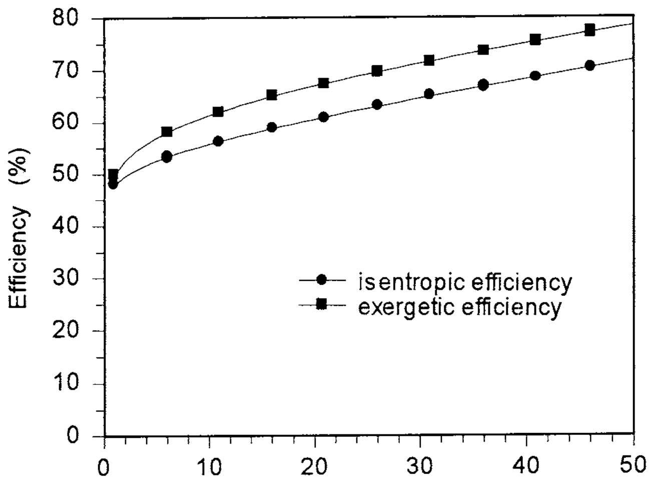
```
p1 = 400  // lbf/in.²
T1 = 600  // °F
x2 = 1.0
p2 = 50  // lbf/in.²
To = 60 + 460  // °R

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = hsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
s2s = s_Ph("Water/Steam", p2, h2s)
s2s = s1

Ed = To * (s2 - s1)
eta = (h1 - h2) / (h1 - h2s)
eff = (h1 - h2) / ((h1 - h2) - To * (s1  -s2))
```

IT Results for p_2 = 50 lbf/in.²

h_1 = 1306 Btu/lb
h_2 = 1174 Btu/lb
h_{2s} = 1123 Btu/lb
s_1 = 1.589 Btu/lb·°R
s_2 = 1.659 Btu/lb·°R
s_{2s} = 1.589 Btu/lb·°R
$\dot{E}_d/\dot{m}$ = 36.26
η_t = 0.719 (71.9%)
ε = 0.7847 (78.47%)

PLOTS:





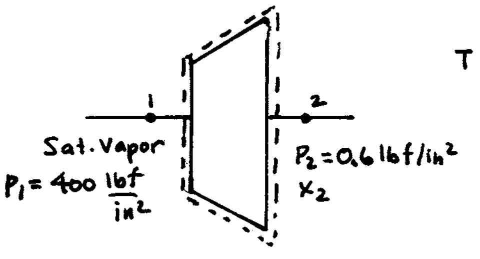
As expected, increased efficiency corresponds with decreased exergy destruction.

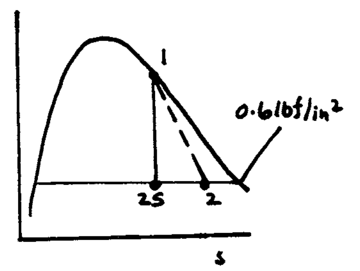
PROBLEM 7.101

KNOWN: Steady-state operating data are provided for a steam turbine.

FIND: Plot versus the steam quality at the turbine exit (a) the power developed and rate of exergy destruction, each per unit mass of steam flowing, (b) isentropic turbine efficiency, (c) exergetic turbine efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible.
3. $T_0 = 520°R$, $p_0 = 1\,atm$.

ANALYSIS: Reducing mass, energy, and entropy balances gives

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad (1) \quad \text{and} \quad \frac{\dot{E}_d}{\dot{m}} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{m}} = T_0(s_2 - s_1) \quad (2)$$

h_1 and s_1 are fixed by p_1, saturated vapor. h_2 and s_2 are fixed by p_2, x_2. Using Eq. 6.48,

$$(3) \quad \eta_t = \frac{(h_1 - h_2)}{(h_1 - h_{2s})}$$

where h_{2s} is fixed by p_2, $s_2 = s_1$. Then, with Eq. 7.42

$$(4) \quad \epsilon = \frac{\dot{W}_{cv}/\dot{m}}{e_{f1} - e_{f2}} = \frac{(h_1 - h_2)}{(h_1 - h_2) - T_0(s_1 - s_2)}$$

Sample calculation: $x_2 = 0.81$. From Table A-3E $h_1 = 1205.5\,Btu/lb$, $s_1 = 1.4856\,Btu/lb\cdot°R$,

$h_2 = h_f + x_2 h_{fg} = 53.27 + 0.81(1045.4) = 900\,Btu/lb$, $s_2 = s_f + x_2 s_{fg} = 0.1029 + 0.81(1.9184)$
$= 1.6568\,Btu/lb\cdot°R$. With $s_{2s} = s_1$,

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{1.4856 - 0.1029}{1.9184} = 0.721 \Rightarrow h_{2s} = 53.27 + (0.721)(1045.4) = 807\,Btu/lb$$

Then, we get from Eq. (1), (2), (3), and (4), respectively

$$\frac{\dot{W}_{cv}}{\dot{m}} = (1205.5 - 900) = 305.5\,Btu/lb, \quad \frac{\dot{E}_d}{\dot{m}} = 520(1.6568 - 1.4856) = 89\,Btu/lb$$

$$\eta_t = \left(\frac{305.5}{1205.5 - 807}\right) = \frac{305.5}{398.5} = 0.767\ (76.7\%), \quad \epsilon = \frac{303.5}{303.5 + 89} = \frac{303.5}{392.5} = .773\ (77.3\%)$$

Data required for the plot on the next page are obtained with the following IT code:

IT Code

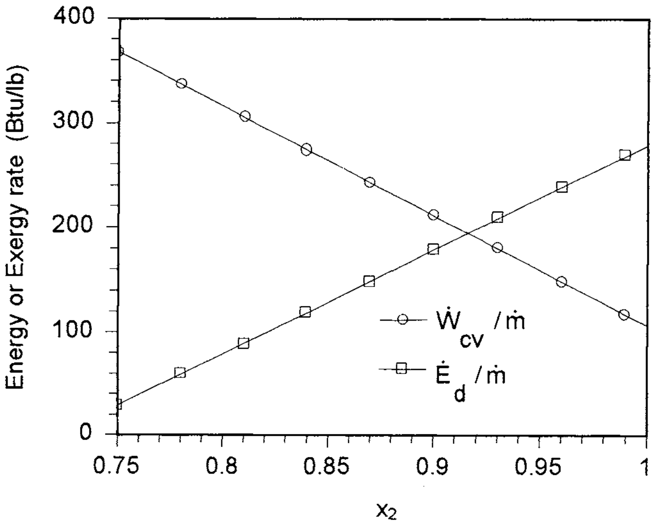
```
p1 = 400 // lbf/in.²
x1 = 1
p2 = 0.6  // lbf/in.²
x2 = 0.81
To = 60 + 460  // °R

h1 = hsat_Px("Water/Steam", p1, x1)
s1 = ssat_Px("Water/Steam", p1, x1)
h2 = hsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
s2s = s_Ph("Water/Steam", p2, h2s)
s2s = s1

Wcv = h1 - h2
Ed = To * (s2 - s1)
eta = Wcv / (h1 - h2s)
eff = Wcv / ((h1 - h2) - To * (s1 - s2))
```

IT Results for x_2 = 0.81

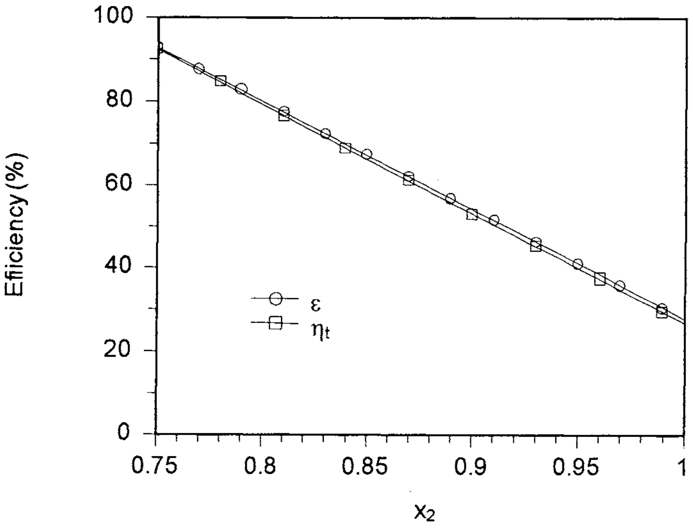
h_1 = 1205 Btu/lb
h_2 = 899.8 Btu/lb
h_{2s} = 806.6 Btu/lb
s_1 = 1.485 Btu/lb·°R
s_2 = 1.656 Btu/lb·°R
$\dot{W}_{cv}/\dot{m}$ = 305.5 Btu/lb
$\dot{E}_d/\dot{m}$ = 88.97 Btu/lb
ε = 0.7744 (77.44%)
η_t = 0.7662 (76.62%)

PROBLEM 7.101 (Cont'd.)

PLOTS:



①



As expected, increased exergy destruction corresponds with decreased power output and efficiency.

---

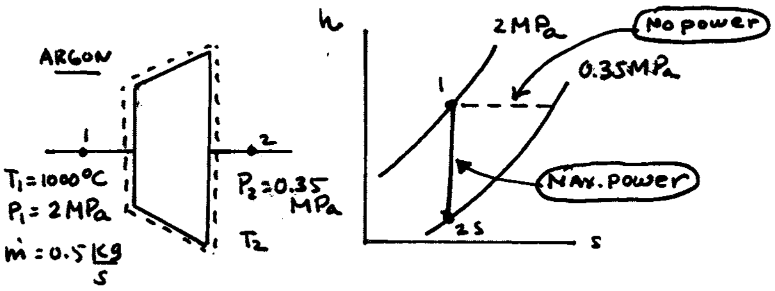
1. In this case the numerical values of ϵ and η_t for each x_2 are nearly the same, but this is not observed generally.

PROBLEM 7.102

KNOWN: Steady-state operating data is provided for a turbine through which argon is expanding.

FIND: Plot versus the turbine exit temperature, the power developed, the rate of exergy destruction, the exergetic turbine efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible.
3. Argon is modeled as an ideal gas with $c_p = \frac{5}{2} R$ (Table A-21).
4. $T_0 = 293\ K$, $p_0 = 1\ bar$.

ANALYSIS: Reducing mass, energy, and entropy balances together with ideal gas model relations for cp constant

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) = \dot{m} c_p (T_2 - T_1) = \dot{m}(2.5R)(T_2 - T_1) \quad (1)$$

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = \dot{m} T_0 (s_2 - s_1) = \dot{m} T_0 \left[c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \right] = \dot{m} T_0 R \left[2.5 \ln \frac{T_2}{T_1} - \ln \frac{P_2}{P_1} \right] \quad (2)$$

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{m}[e_{f1} - e_{f2}]} = \frac{\dot{W}_{cv}}{\dot{m}(h_1 - h_2 - T_0(s_1 - s_2))} = \frac{\dot{W}_{cv}}{\dot{W}_{cv} + \dot{E}_d} \quad (3)$$

Observe that T_2 can range from T_{2s}, corresponding to an isentropic expansion, to $T_2 = T_1$, corresponding to no power developed. With Eq. 6.45, $T_{2s} = T_1 (P_2/P_1)^{(k-1)/k}$. Using Eq. 3.47a, $\frac{k-1}{k} = \frac{R}{c_p} = \frac{R}{2.5R} = 0.4$. Thus, $T_{2s} = (1273 K)\left(\frac{0.35}{2}\right)^{0.4} = 634 K$. So, $634 K \leq T_2 \leq 1273 K$.

The data for the plots on the next page are obtained using IT as follows:

<u>*IT* Code</u>

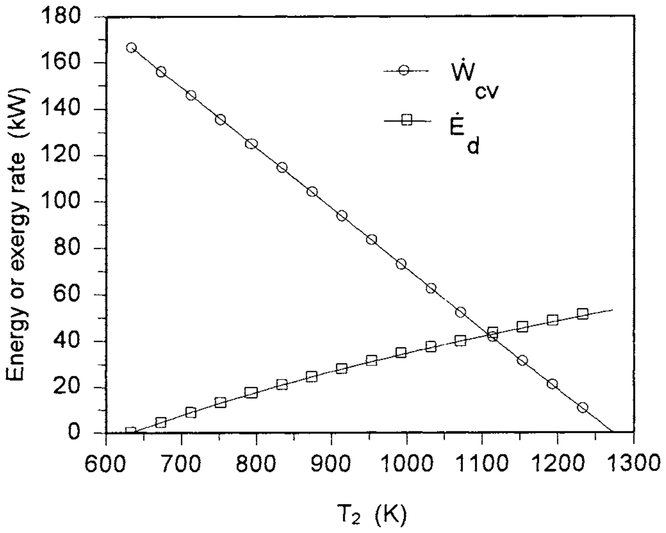
```
p1 = 2 // MPa
T1 = 1000 + 273  // K
p2 = 0.35  // MPa
T2 = 1273  // K
To = 20 + 273  // K
mdot = 0.5  // kg/s
cp = 2.5 * R
R = 8.314 / 39.94  // kJ/kg·K

Wdot = mdot * cp * (T1 - T2)
delta_s = cp * ln(T2 / T1) - R * ln(p2 / p1)
Edot = To * mdot * delta_s
eff = Wdot / (Wdot + Edot)
```

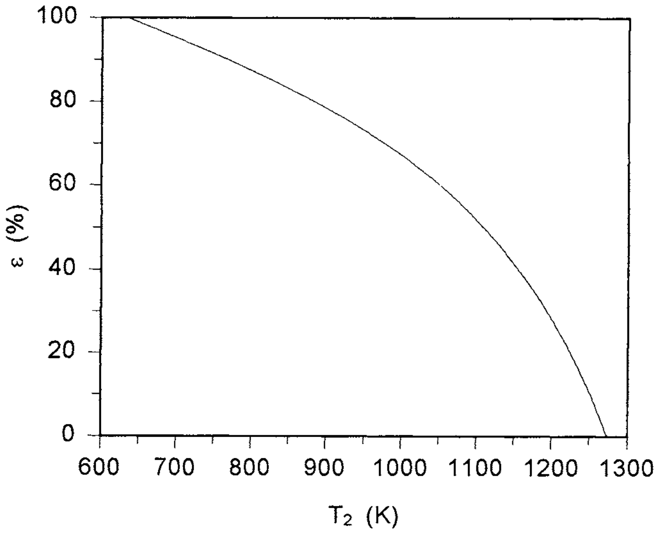
<u>*IT* Results</u>

for T_2 = 634 K:	For T_2 = 1273 K:
$\dot{W}_{cv}$ = 166.3 kW	$\dot{W}_{cv}$ = 0
$\dot{E}_d \approx 0$	$\dot{E}_d$ = 53.15 kW
ε = 1 (100%)	ε = 0

PROBLEM 7.102 (Cont'd.)

PLOTS:





PROBLEM 7.103

KNOWN: Operating data are provided for a steam turbine operating at steady state from which steam is extracted at a specified state and flow rate. Heat transfer between the turbine and its surroundings occurs at a given average surface temperature, and the power output is known.

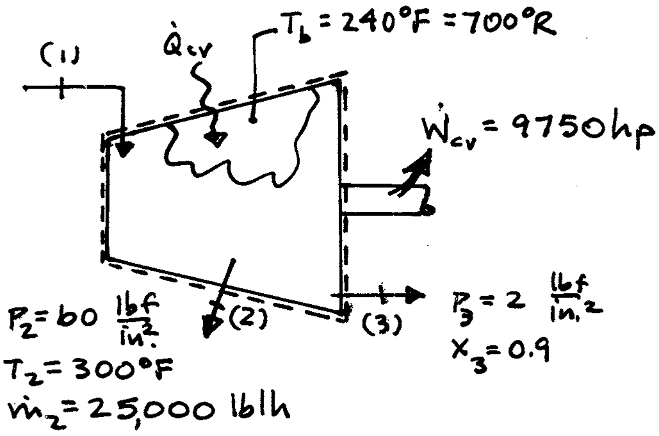
FIND: (a) Perform a full accounting of the net exergy supplied by the steam. (b) Devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:

$\dot{m}_1 = 10^5$ lb/h

$p_1 = 400$ lbf/in.^{2}

$T_1 = 600°F$



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) Kinetic and potential energy effects can be ignored. (3) For the environment, $T_0 = 77°F = 537°R$, $p_0 = 1$ atm.

ANALYSIS: At steady state, a mass rate balance reads: $0 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3$, or

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 10^5 - 25{,}000 = 75{,}000 \text{ lb/h}$$

With assumptions 1 and 2, an energy rate balance reduced to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 \Rightarrow \dot{Q}_{cv} = \dot{W}_{cv} - \dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{m}_3 h_3$$

From Table A-4E, $h_1 = 1306.6$ Btu/lb, $h_2 = 1181.9$ Btu/lb, $s_1 = 1.5892$ Btu/lb·°R, and $s_2 = 1.6496$ Btu/lb·°R. With data from Table A-3E

$$h_3 = h_{f_3} + x_3 h_{fg_3} = 94.02 + (.9)(1022.1) = 1013.9 \text{ Btu/lb}$$

$$s_3 = s_{f_3} + x_3 s_{fg_3} = 0.175 + (.9)(1.7448) = 1.7453 \text{ Btu/lb·°R}$$

Inserting values into the expression for $\dot{Q}_{cv}$

$$\dot{Q}_{cv} = (9750 \text{ hp})\left|\frac{2545 \text{ Btu/h}}{1 \text{ hp}}\right| - (10^5 \tfrac{\text{lb}}{\text{h}})(1306.6)\tfrac{\text{Btu}}{\text{lb}} + (25000)(1181.9) + (75000)(1013.9)$$

$$= -2.5625 \times 10^5 \text{ Btu/h}$$

Also, $\dot{W}_{cv} = (9750)(2545) = 2.4814 \times 10^7$ Btu/h

(a) With these preliminary values determined, we turn to the exergy accounting. The net exergy supplied is

$$\begin{pmatrix}\text{net exergy}\\ \text{supplied}\end{pmatrix} = \dot{E}_{f_1} - \dot{E}_{f_2} - \dot{E}_{f_3} = \dot{m}_1 e_{f_1} - \dot{m}_2 e_{f_2} - \dot{m}_3 e_{f_3}$$

$$= \dot{m}_1 [(h_1 - \cancel{h_0}) - T_0(s_1 - \cancel{s_0})] - \dot{m}_2 [(h_2 - \cancel{h_0}) - T_0(s_2 - \cancel{s_0})]$$

$$- \dot{m}_3 [(h_3 - \cancel{h_0}) - T_0(s_3 - \cancel{s_0})]$$

$$= \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 - T_0(\dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3)$$

where the indicated terms cancel when the mass balance is applied.

PROBLEM 7.103 (Cont'd.)

$$\begin{pmatrix}\text{net exergy}\\ \text{supplied}\end{pmatrix} = (10^5)\Big[(1)(1306.6)-(.25)(1181.9)-(.75)(1013.9)-(537)\big\{(1)(1.5892)$$
$$-(.25)(1.6496)-(.75)(1.7453)\big\}\Big]$$
$$= 3.217\times10^7 \text{ Btu/h}$$

The rate exergy is carried out with heat transfer is

$$\dot{E}_q = \left[1-\frac{T_0}{T_b}\right]|\dot{Q}_{cv}| = \left[1-\frac{537}{700}\right](2.5625\times10^5) = 5.97\times10^4 \text{ Btu/h}$$

The rate of exergy destruction is found from an exergy rate balance

$$0 = \left[1-\frac{T_0}{T_b}\right]\dot{Q}_{cv} - \left[\dot{W}_{cv} - \cancel{p_0\frac{dV}{dt}}\right] + \dot{m}_1 e_{f_1} - \dot{m}_2 e_{f_2} - \dot{m}_3 e_{f_3} - \dot{E}_d$$

or

$$\dot{E}_d = (\dot{m}_1 e_{f_1} - \dot{m}_2 e_{f_2} - \dot{m}_3 e_{f_3}) - \dot{W}_{cv} - \dot{E}_q$$
$$= 3.217\times10^7 - 2.4814\times10^7 - 5.97\times10^4 = 7.296\times10^6 \text{ Btu/h}$$

Exergy Accounting Summary

net exergy supplied	$= 3.217\times10^7$ Btu/h	(100%)
• Power output	$= 2.4814\times10^7$	(77.1%)
• tranfer out with heat transfer	$= 5.97\times10^4$	(0.2%)
• exergy destroyed	$= 7.296\times10^6$	(22.7%)
	3.217×10^7	(100%)

← exergy accounting

(b) An exergetic efficiency in the form (desired output / input) takes the form

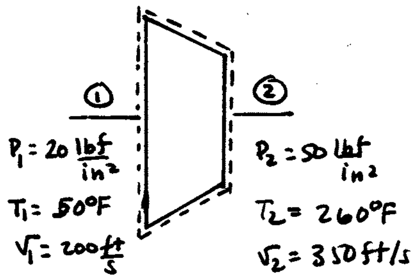
$$\epsilon = \frac{\dot{W}_{cv}}{\begin{pmatrix}\text{net exergy}\\ \text{supplied}\end{pmatrix}} = \frac{2.4814\times10^7}{3.217\times10^7} = 0.771\ (77.1\%) \longleftarrow \epsilon$$

PROBLEM 7.104

KNOWN: Steady-state operating data are provided for an air compressor.

FIND: (a) Perform a full exergy accounting of the power input, (b) evaluate the exergetic efficiency.

SCHEMATIC & GIVEN DATA:



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②
$P_1 = 20\ \frac{lbf}{in^2}$
$T_1 = 50°F$
$V_1 = 200\ \frac{ft}{s}$
$P_2 = 50\ \frac{lbf}{in^2}$
$T_2 = 260°F$
$V_2 = 350\ ft/s$

ASSUMPTIONS:

1. The control volume shown in the schematic is at steady state.
2. Neglect the effect of potential energy, $\dot{Q}_{cv} = 0$.
3. Air is modeled as an ideal gas.
4. $T_0 = 500°R$, $P_0 = 14.7\ lbf/in^2$

ANALYSIS: (a) At steady state, mass and energy rate balances reduce to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} = (121.875 - 172.39)\frac{Btu}{lb} + \frac{(200\,ft/s)^2 - (350\,ft/s)^2}{2}\left|\frac{1\,lbf}{32.2\,ft/s^2}\right|\left|\frac{1\,Btu}{778\,ft\cdot lbf}\right|$$
$$= -52.16\ Btu/lb$$

Mass and entropy balances reduce to give $\dot{E}_d = T_0 \dot{\sigma}_{cv} \Rightarrow \dot{E}_d/\dot{m} = T_0(s_2 - s_1)$.
or

$$\dot{E}_d/\dot{m} = T_0\left[s^\circ(T_2) - s^\circ(T_1) - R \ln\frac{P_2}{P_1}\right] = 500°R\left[0.67002 - 0.58703 - \frac{1.986}{28.97}\ln\frac{50}{20}\right] = 10.09\ \frac{Btu}{lb}$$

All h and s° data are from Table A-22E.

The increase in exergy of the air from 1 to 2 is given by Eq. 7.36

$$\frac{\dot{E}_{f2} - \dot{E}_{f1}}{\dot{m}} = h_2 - h_1 - T_0(s_2 - s_1) + (V_2^2 - V_1^2)/2 = \left(-\frac{\dot{W}_{cv}}{\dot{m}}\right) - \left(\frac{\dot{E}_d}{\dot{m}}\right)$$
$$= 52.16 - 10.09 = 42.07\ Btu/lb$$

Exergy Balance Sheet (unit mass basis)

• Exergy input — power	52.16 Btu/lb	
• Disposition of the exergy		
– Increase in exergy of the air	42.07 Btu/lb	(80.7%)
– Exergy destroyed	10.09 Btu/lb	(19.3%)

(b) With Eq. 7.43

$$\epsilon = \frac{e_{f2} - e_{f1}}{(-\dot{W}_{cv}/\dot{m})} = \frac{42.07}{52.16} = 0.807$$

This value can be seen by inspection of the exergy balance sheet of part (a).

PROBLEM 7.105*

KNOWN: The states at the inlet and exit of an air compressor operating at steady state are known. The compressor power and the average surface temperature for heat transfer from the compressor to its surrounding are also specified.

FIND: (a) Perform a full exergy accounting of the power input. (2) Devise and evaluate an exergetic efficiency. (3) Determine the hourly costs of each exergy quantity.

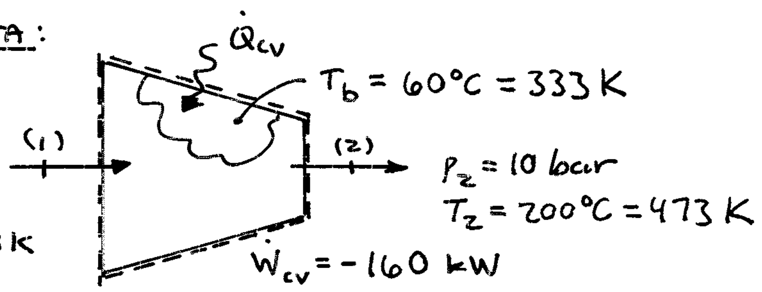
SCHEMATIC & GIVEN DATA:

air

$\dot{m} = 1980\ kg/h = 0.55\ kg/s$

$p_1 = 1$ bar

$T_1 = 25°C = 298 K$



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume, kinetic and potential energy effects are negligible. (3) The air is modeled as an ideal gas. (4) For the environment, $T_0 = 25°C = 298 K$, $p_0 = 1$ bar.

ANALYSIS: (a) First, we determine the heat transfer rate using mass and energy rate balances: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$ and $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(h_1 - h_2)$

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1) = -160\ kW + (0.55\tfrac{kg}{s})(475.32 - 298.18)\tfrac{kJ}{kg}\left|\tfrac{1\ kW}{1\ kJ/s}\right|$$
$$= -62.57\ kW$$

Exergy tranfer out with heat

$$\begin{pmatrix}\text{exergy out}\\\text{with heat}\end{pmatrix} = \left(1 - \frac{T_0}{T_b}\right)(-\dot{Q}_{cv}) = \left(1 - \frac{298}{333}\right)(62.57) = \underline{6.58\ kW}$$

Net flow exergy increase of air

$$\dot{E}_{f2} - \dot{E}_{f1} = \dot{m}\left[(h_2 - h_1) - T_0(s_2 - s_1)\right]$$
$$= \dot{m}\left[(h_2 - h_1) - T_0\left(s^\circ(T_2) - s^\circ(T_1) - R \ln p_2/p_1\right)\right]$$
$$= (0.55\tfrac{kg}{s})\left[(475.32 - 298.18) - (298\ K)\left(2.16251 - 1.69528 - \frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\ln\frac{10}{1}\right)\right] = \underline{129.15\ kW}$$

Exergy destroyed

Using an exergy rate balance

$$0 = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(e_{f1} - e_{f2}) - \dot{E}_d$$

$$\dot{E}_d = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}(e_{f1} - e_{f2})$$

$$= (-6.58) - (-160) + (-129.15) = \underline{24.27\ kW}$$

PROBLEM 7.105 (Cont'd.)

Exergy Accounting Summary

Input

power = 160 kW (100%)

Output and Destruction

flow exergy = 129.15 kW (80.72 %)

heat transfer = 6.58 kW (4.11 %) ← exergy accounting

Exergy destroyed = $\underline{24.27 \text{ kW} \quad (15.17\ \%)}$

160 kW (100 %)

(b) Defining an exergetic efficiency in terms of (desired output/ input)

$$\epsilon = \frac{\text{net flow exergy increase of air}}{\text{power input}} = \frac{129.15}{160} = 0.8072 \; (80.72\%) \longleftarrow \epsilon$$

(c) If the exergy cost rate is \$0.08 / kW·h

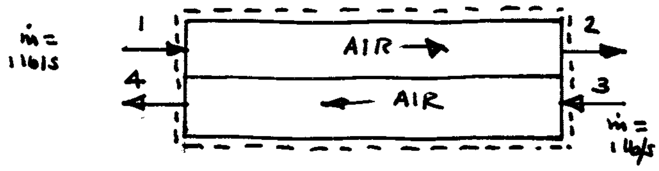
$$\left.\begin{aligned}
\begin{pmatrix}\text{hourly cost of}\\ \text{power input}\end{pmatrix} &= (160\text{ kW})(1\text{ h})\left(0.08\frac{\$}{\text{kW}\cdot\text{h}}\right) = \$\,12.80/\text{h}\\
\begin{pmatrix}\text{hourly cost of}\\ \text{exergy loss due}\\ \text{to heat transfer}\end{pmatrix} &= (6.58\text{ kW})(1\text{ h})\left(0.08\frac{\$}{\text{kW}\cdot\text{h}}\right) = \$0.53/\text{h}\\
\begin{pmatrix}\text{Cost of exergy}\\ \text{destruction}\end{pmatrix} &= (24.27\text{ kW})(1\text{ h})\left(0.08\frac{\$}{\text{kW}\cdot\text{h}}\right) = \$\,1.94/\text{h}
\end{aligned}\right\} \longleftarrow \$$$

PROBLEM 7.106

KNOWN: Steady-state operating data is provided for a counterflow heat exchanger having air flowing on both sides.

FIND: Determine (a) the rate of exergy destruction, (b) the exergetic efficiency given by Eq. 7.45.

SCHEMATIC & GIVEN DATA



State	T (°R)	p (lbf/in²)
1	1300	16
2	1155	14.7
3	850	60
4	1000	50

ASSUMPTIONS: 1. The control volume shown in the schematic is at steady state. 2. For the control volume, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and kinetic and potential energy effects are negligible. 3. Air is modeled as an ideal gas. 4. $T_0 = 520°R$, $p_0 = 14.7$ lbf/in².

ANALYSIS: Mass and energy rate balances reduce to give $0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}(h_1 - h_2) + \dot{m}(h_3 - h_4)$. $\Rightarrow 0 = (h_1 - h_2) + (h_3 - h_4)$. With data from Table A-22E, $h_1 = 316.95$ Btu/lb, $h_2 = 279.88$ Btu/lb, $h_3 = 204.01$ Btu/lb, $h_4 = 240.98$ Btu/lb. These values check the energy balance, indicating that the given state data are consistent with the conservation of mass and energy principles.

Reducing an exergy rate balance at steady state

$$0 = \sum_j \cancelto{0}{\left[1 - \frac{T_0}{T_j}\right]\dot{Q}_j} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}[e_{f1} - e_{f2}] + \dot{m}[e_{f3} - e_{f4}] - \dot{E}_d$$

$$\Rightarrow \frac{\dot{E}_d}{\dot{m}} = [h_1 - h_2 - T_0(s_1 - s_2)] + [h_3 - h_4 - T_0(s_3 - s_4)]$$

$$= [\underbrace{(h_1 - h_2) + (h_3 - h_4)}_{=0}] + T_0[(s_2 - s_1) + (s_4 - s_3)]$$

$$= T_0\left[\left(s°(T_2) - s°(T_1) - R \ln\frac{p_2}{p_1}\right) + \left(s°(T_4) - s°(T_3) - R \ln\frac{p_4}{p_3}\right)\right]$$

$$= 520°R\left[\left[0.78665 - 0.81678 - \frac{1.986}{28.97}\ln\frac{14.7}{16}\right]\frac{\text{Btu}}{\text{lb}\cdot°R} + \left[0.75042 - 0.71035 - \frac{1.986}{28.97}\ln\frac{50}{60}\right]\frac{\text{Btu}}{\text{lb}\cdot°R}\right]$$

$$= 520[-0.02432 + 0.05257] = 14.69 \text{ Btu/lb} \rightarrow \dot{E}_d = 14.69 \frac{\text{Btu}}{\text{s}}$$ ←

(b) The exergetic efficiency given by Eq. 7.45 is

① $$\epsilon = \frac{\dot{m}[e_{f4} - e_{f3}]}{\dot{m}[e_{f1} - e_{f2}]} = \frac{(h_4 - h_3) - T_0(s_4 - s_3)}{(h_1 - h_2) - T_0(s_1 - s_2)} = \frac{(240.98 - 204.01) - 520[0.05257]}{[316.95 - 279.88] - 520[0.02432]}$$

$$= \frac{9.634}{24.424} = 0.394 \quad (39.4\%)$$ ←

---

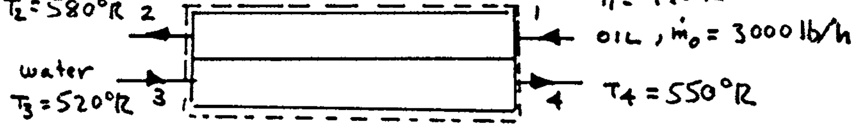
1. In accord with the discussion of Eq. 7.45, both streams are at temperatures above T_0.

PROBLEM 7.107*

KNOWN: Steady state operating data are provided for a counterflow heat exchanger through which oil and water flow in separate streams.

FIND: Determine (a) the mass flow rate of the water, (b) the exergetic efficiency given by Eq. 7.45, (c) the hourly cost of exergy destruction.

SCHEMATIC & GIVEN DATA:



oil : $c_o = 0.46$ Btu/lb·°R

water: $c_w = 1.00$ Btu/lb·°R

ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state. (2) $\dot{Q}_{cv} = 0$ and the effects of kinetic and potential energy can be ignored. (3) The oil and water can each be regarded as incompressible with constant specific heats c_o and c_w, respectively, and negligible change in pressure in flowing through the heat exchanger. (4) For the environment $T_0 = 60°F$, $p_0 = 1$ atm. (5) exergy is valued at 8¢ per kW·h.

ANALYSIS: (a) At steady state, mass rate balances reduce to give $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}_o$ and $\dot{m}_3 = \dot{m}_4 \equiv \dot{m}_w$. With assumptions 1 and 2, an energy balance gives

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}_o(h_1 - h_2) + \dot{m}_w(h_3 - h_4) \Rightarrow 0 = \dot{m}_o c_o (T_1 - T_2) + \dot{m}_w c_w (T_3 - T_4)$$

In writing the last expression, Eq. 3.20b was used together with assumption 3 to evaluate the enthalpy changes. Solving for $\dot{m}_w$

$$\dot{m}_w = \dot{m}_o \frac{c_o(T_1 - T_2)}{c_w(T_4 - T_3)} = 3000\frac{\text{lb}}{\text{h}} \frac{(0.46)(790-580)}{(1.00)(550-520)} = 9660 \text{ lb/h} \longleftarrow \dot{m}_w$$

(b) The exergetic efficiency is, with Eq. 6.24

① $$\epsilon = \frac{\dot{m}_w(e_{f4} - e_{f3})}{\dot{m}_o(e_{f1} - e_{f2})} = \frac{\dot{m}_w c_w [(T_4 - T_3) - T_0 \ln(T_4/T_3)]}{\dot{m}_o c_o [(T_1 - T_2) - T_0 \ln(T_1/T_2)]}$$

$$= \frac{(9660)(1.00)[(550-520) - (520)\ln(550/520)]}{(3000)(0.46)[(790-580) - (520)\ln(790/580)]}$$

$$= \frac{8{,}051.4 \text{ Btu/h}}{68{,}058.1 \text{ Btu/h}} = 0.1183 \;(11.83\%) \longleftarrow \epsilon$$

(c) The rate of exergy destruction is obtain using an exergy rate balance, as follows:

$$0 = \cancel{\sum_j \left[1 - \frac{T_0}{T_j}\right]\dot{Q}_j}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + \dot{m}_o(e_{f1} - e_{f2}) + \dot{m}_w(e_{f3} - e_{f4}) - \dot{E}_d$$

or

$$\dot{E}_d = \dot{m}_o(e_{f1} - e_{f2}) - \dot{m}_w(e_{f4} - e_{f3})$$

With values from above, $\dot{E}_d = 68{,}058.1 - 8{,}051.4 = 60{,}006.7 \frac{\text{Btu}}{\text{h}}$. Thus

$$\begin{pmatrix}\text{hourly}\\ \text{cost}\end{pmatrix} = \left(60{,}006.7\frac{\text{Btu}}{\text{h}}\right)\left|\frac{1 \text{ kW}}{3413 \text{ Btu/h}}\right|\left(0.08\frac{\$}{\text{kWh}}\right) = \$1.41/\text{h} \longleftarrow \$$$

---

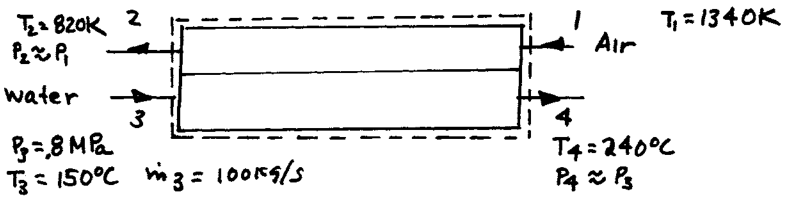
1. In accord with the discussion of Eq. 7.45, both streams are at temperatures $\geq T_0$.

PROBLEM 7.108

KNOWN: Steady-state operating data are provided for boiler tubes carrying water over which combustion gases flow.

FIND: Determine (a) the mass flow rate of the combustion gases, (b) the rate of exergy destruction, (c) the exergetic efficiency Eq. 7.45.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The combustion gases are modeled as air as an ideal gas. (3) There is no significant change in pressure for either stream as it passes from inlet to exit. (4) For the environment, $T_0 = 298K$, $p_0 = 1 atm$.

ANALYSIS: At steady state mass rate balances give $\dot{m}_1 = \dot{m}_2$ and $\dot{m}_3 = \dot{m}_4$. An energy rate balance reduces with assumption 1 to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4) \Rightarrow \frac{\dot{m}_1}{\dot{m}_3} = \frac{h_4 - h_3}{h_1 - h_2}$$

With data from Tables A-2 and A-4, $h_4 = 2928.3 kJ/kg$ and $h_3 \approx h_f(T_3) = 632.2 kJ/kg$. Table A-22 gives $h_1 = 1443.6 kJ/kg$, $h_2 = 843.98 kJ/kg$.

$$\Rightarrow \dot{m}_1 = \left(100 \frac{kg}{s}\right)\left[\frac{2928.3 - 632.2}{1443.6 - 843.98}\right] = (100)\left[\frac{2296.1}{599.62}\right] = 382.93 \ kg/s \quad \longleftarrow$$

The exergy rate balance reduces at steady state to give

$$\dot{E}_d = \dot{m}_1[e_{f1} - e_{f2}] + \dot{m}_3[e_{f3} - e_{f4}] = \dot{m}_1[(h_1 - h_2) - T_0(s_1 - s_2)] + \dot{m}_3[(h_3 - h_4) - T_0(s_3 - s_4)]$$

$$\dot{E}_d/\dot{m}_3 = \frac{\dot{m}_1}{\dot{m}_3}\left[h_1 - h_2 - T_0(s°(T_1) - s°(T_2)) - \cancelto{0}{R \ln p_1/p_2}\right] + [(h_3 - h_4) - T_0(s_3 - s_4)]$$

With s° data from Table A-22, $s_3 \approx s_f(T_3)$ from Table A-2 and s_4 from Table A-4

$$\frac{\dot{E}_d}{\dot{m}_3} \quad 3.8293[599.62 - 298(3.30959 - 2.74504)] + [-2296.1 - 298(1.8418 - 7.0033)]$$

$$= 3.8293[431.38] + [-757.97] = 893.91 \frac{kJ}{kg} \Rightarrow \dot{E}_d = 8.939 \times 10^4 \frac{kJ}{s} \quad \longleftarrow$$

① With Eq. 7.45 and previously calculated values

$$\epsilon = \frac{\dot{m}_3(e_{f4} - e_{f3})}{\dot{m}_1(e_{f1} - e_{f2})} = \frac{757.97}{3.8293(431.38)} = 0.459 \ (45.9\%) \quad \longleftarrow$$

---

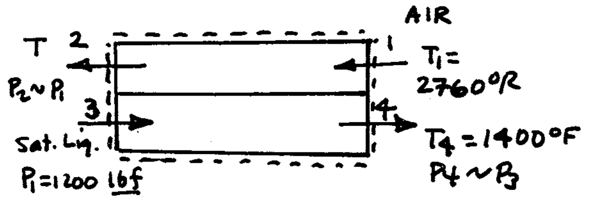
1. In accord with the discussion of Eq. 7.45, both streams are at temperatures above T_0.

PROBLEM 7.109

KNOWN: Steady-state operating data are provided for boiler tubes carrying water over which combustion gases flow.

FIND: Determine (a) the exit temperature of the combustion gases, (b) the rate of exergy destruction, (c) the exergetic efficiency given by Eq. 7.45.

SCHEMATIC & GIVEN DATA:



$\dot{m}_1 = 1.13 \times 10^7$ lb/h

$\dot{m}_3 = 1.4 \times 10^6$ lb/h

ASSUMPTIONS:

1. The control volume shown is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects are negligible.
3. The combustion gases are modeled as air as an ideal gas.
4. There is no significant change in pressure for either stream.
5. $T_0 = 530°R$, $P_0 = 1$ atm.

ANALYSIS: (a) Reducing mass and energy rate balances

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}_1(h_1 - h_2) + \dot{m}_3(h_3 - h_4) \Rightarrow h_2 = h_1 + \frac{\dot{m}_3}{\dot{m}_1}(h_3 - h_4)$$

with data from Table A-22E and Tables A-3 & 4E

$$h_2 = 720.73 + \left(\frac{1.4 \times 10^6}{1.13 \times 10^7}\right)(571.7 - 1730.7) = 577.14\ \text{Btu/lb}$$

Interpolating in Table A-22E,

$$T_2 = 2259°R. \longleftarrow$$

(b) The rate of exergy destruction is obtained using an exergy rate balance, which reduces to give

$$\dot{E}_d = \dot{m}_1[e_{f1} - e_{f2}] + \dot{m}_3[e_{f3} - e_{f4}]$$

$$= \dot{m}_1[(h_1 - h_2) - T_0(s_1 - s_2)] + \dot{m}_3[(h_3 - h_4) - T_0(s_3 - s_4)]$$

$$= \dot{m}_1\left[(h_1 - h_2) - T_0\left(s_1^o - s_2^o - \overset{0}{\cancel{R \ln\frac{p_1}{p_2}}}\right)\right] + \dot{m}_3[(h_3 - h_4) - T_0(s_3 - s_4)]$$

Inserting table data

$$\dot{E}_d = \left(1.13 \times 10^7 \frac{lb}{h}\right)\left[(720.73 - 577.14) - 530(1.02349 - 0.96609)\right]\frac{Btu}{lb} +$$

$$\left(1.4 \times 10^6 \frac{lb}{h}\right)\left[(571.7 - 1730.7) - 530(0.7712 - 1.7696)\right]\frac{Btu}{lb}$$

$$= (1.13 \times 10^7)[\underbrace{143.59 - 30.42}_{113.17}] + (1.4 \times 10^6)[\underbrace{-1159 - (-529.15)}_{-629.85}]$$

$$= 1.28 \times 10^9 + (-0.88 \times 10^9) = 4 \times 10^8\ \frac{Btu}{h}$$

① (c) With Eq. 7.45 and previously calculated values

$$\epsilon = \frac{\dot{m}_3(e_{f4} - e_{f3})}{\dot{m}_1(e_{f1} - e_{f2})} = \frac{0.88 \times 10^9}{1.28 \times 10^9} = 0.688\ (68.8\%). \longleftarrow$$

---

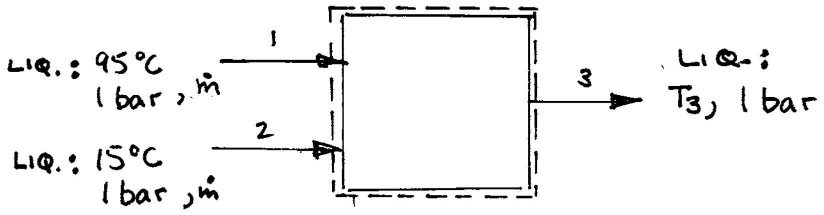
1. In accord with the discussion of Eq. 7.45, both streams are at temperatures above T_0.

PROBLEM 7.110*

KNOWN: Steady state operating data are provided for a direct-contact heat exchanger.

FIND: Determine (a) the rate of destruction per unit mass exiting, (b) the exergetic efficiency given by Eq. 7.47.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot Q_{cv} = 0$, $\dot W_{cv} = 0$, and kinetic/potential energy effects can be neglected. (3) For the exergy reference environment, $T_0 = 15°C$, $p_0 = 1$ bar.

ANALYSIS: Mass and energy rate balances reduce to give

$$0 = \cancel{\dot Q_{cv}}^{0} - \cancel{\dot W_{cv}}^{0} + \dot m h_1 + \dot m h_2 - 2\dot m h_3 \Rightarrow h_3 = \frac{h_1 + h_2}{2}.$$

Then, with saturated liquid data from Table A-2, $h \approx h_f(T)$

$$h_3 = \frac{397.96 + 62.99}{2} = 230.48 \ \frac{kJ}{kg}$$

Interpolation in Table A-2 gives $T_3 \approx 55°C$
$s_3 \approx 0.7679$ kJ/kg·K

The rate of exergy destruction is $\dot E_d = T_0 \dot\sigma_{cv}$, where $\dot\sigma_{cv} = \dot m_3 s_3 - (\frac{\dot m_3}{2}) s_1 - (\frac{\dot m_3}{2}) s_2$.
Then, with $s \approx s_f(T)$

$$\frac{\dot E_d}{\dot m_3} = T_0\left[s_3 - \left(\frac{s_1 + s_2}{2}\right)\right] = 288K\left[0.7679 - \left(\frac{1.2500 + 0.2245}{2}\right)\right]$$
$$= 8.827 \text{ kJ/kg} \longleftarrow$$

Applying Eq 7.47

$$\epsilon = \frac{\dot m\,[e_{f3} - e_{f2}]}{\dot m\,[e_{f1} - e_{f3}]} = \frac{(h_3 - h_2) - T_0(s_3 - s_2)}{(h_1 - h_3) - T_0(s_1 - s_3)} = \frac{(230.48 - 62.99) - 288(0.7679 - 0.2245)}{(397.96 - 230.48) - 288(1.2500 - 0.7679)}$$

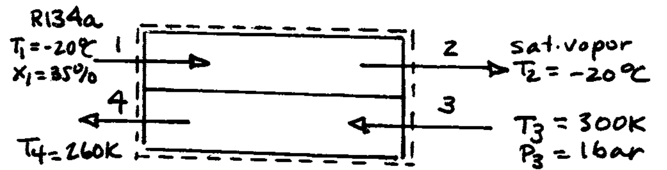
$$= \frac{10.99 \text{ kJ/kg}}{28.64 \text{ kJ/kg}} = 0.384 \ (38.4\%) \longleftarrow \epsilon$$

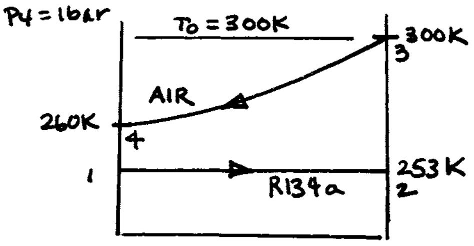
PROBLEM 7.111

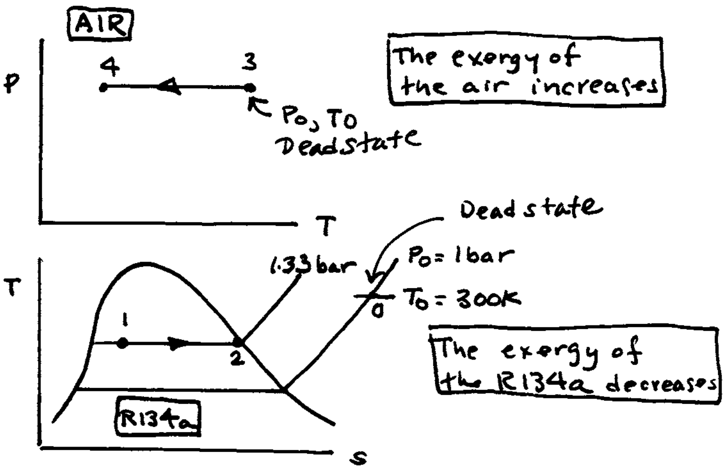
KNOWN: Steady-state operating data are provided for a counterflow heat exchanger.

FIND: (a) Sketch the variation of the temperature of each stream with with position. Locate To. (b) Determine the rate of exergy destruction. (c) Devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:







ASSUMPTIONS: 1. The control volume shown in the accompanying sketch is at steady state. 2. For the control volume, $\dot{Q}_{cv} = 0$, and kinetic and potential energy effects can be ignored. 3. Air is modeled as an ideal gas. 4. $T_0 = 300K$, $P_0 = 1 bar$.

ANALYSIS: The refrigerant mass flow rate is required for parts (b), (c). Reducing mass and energy rate balances,

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}_1[h_1 - h_2] + \dot{m}_3[h_3 - h_4] \Rightarrow \text{ with data from Tables A-10, A-22}$$

$$\dot{m}_1 = \frac{\dot{m}_3[h_3 - h_4]}{[h_2 - h_1]} = \frac{(4 kg/s)[300.19 - 260.09]kJ/kg}{[235.31 - \underset{\nwarrow = [24.26 + 0.35(211.05)]}{98.13}] kJ/kg} = 1.17 kg/s$$

(b) The rate of exergy destruction can be determined by reducing an exergy rate balance:

$$0 = \sum \overset{0}{\cancel{\left[1 - \frac{T_0}{T_j}\right]\dot{Q}_j}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}_1[e_{f1} - e_{f2}] + \dot{m}_3[e_{f3} - e_{f4}] - \dot{E}_d$$

$$\Rightarrow \dot{E}_d = \dot{m}_1[(h_1 - h_2) - T_0(s_1 - s_2)] + \dot{m}_3[h_3 - h_4 - T_0(s_3 - s_4)]$$

$$= \dot{m}_1[(h_1 - h_2) - T_0(s_1 - s_2)] + \dot{m}_3[h_3 - h_4 - T_0(s_3^\circ - s_4^\circ - R \overset{0}{\cancel{\ln P_3/P_4}})]$$

$$\swarrow 0.0996 + .35(0.9332 - 0.0996)]$$

$$= 1.17[(98.13 - 235.31) - 300(0.3914 - 0.9332)] + 4[300.19 - 260.09 - 300(1.70203 - 1.55848)]$$

$$= [(+29.67) + (-11.86)]\left(\frac{kJ}{s}\right)\left|\frac{1 kW}{1 kJ/s}\right| = 17.8 kW \quad \longleftarrow$$

(c) Since the air is brought from the dead state to a lower temperature, its exergy increases in the process. The state of the R134a is brought closer to the dead state, so its exergy decreases. This is confirmed by the calculations of part (b) that show

$$\dot{m}_3(e_{f4} - e_{f3}) = +11.86 kW, \quad \dot{m}_1(e_{f2} - e_{f1}) = -29.67 kW$$

Thus, the colder refrigerant provides the exergy that is either transferred to the air or destroyed by irreversibilities within the control volume.

① $$\Rightarrow \epsilon = \frac{\dot{m}_3(e_{f4} - e_{f3})}{\dot{m}_1(e_{f1} - e_{f2})} = \frac{11.86}{29.67} = 0.403 \quad (40.3\%)$$

---

1. Since the streams are each below T_0, this expression is formulated differently than Eq. 7.45, which regards the cold stream as receiving exergy from the hot stream. Here, the positions are reversed.

PROBLEM 7.112

KNOWN: Steady-state operating data are provided for two turbine stages and an interconnecting heat exchanger.

FIND: Determine the exergetic efficiencies of the two turbines and the heat exchanger.

SCHEMATIC & GIVEN DATA: See Fig. P4.82. Also, see Problem 7.84 solution for data.

ASSUMPTIONS: 1. See solutions to Probs. 4.82, 6.125, 7.84. 2 $T_0 = 300\ K$, $p_0 = 1\ bar$.

ANALYSIS: From the solution to Problem 7.84

$$(\dot{E}_d)_{t1} = 958.1\ kW,\ (\dot{E}_d)_{t2} = 859.5\ kW,\ (\dot{E}_d)_{HX} = 944.5\ kW$$

With Eq 7.42, and invoking the assumptions for the turbines in this case

$$\epsilon_t = \frac{\dot{W}_t}{\dot{m}(e_{f1}-e_{f2})} = \frac{\dot{W}_t}{\dot{m}[h_1-h_2-T_0(s_1-s_2)]} = \frac{\dot{W}_t}{\dot{W}_t+\dot{E}_d}$$

Then, with $\dot{W}_{t1}$ and $\dot{W}_{t2}$ from the solution to Problem 4.82

$$\epsilon_{t1} = \frac{10{,}000\ kW}{10{,}000\ kW + 958.1\ kW}\ 0.913\ (91.3\%)\ ,\ \epsilon_{t2} = \frac{10{,}570}{10{,}570+859.5} = .925\ (92.5\%)$$

ϵ_{t1} ϵ_{t2}

With Eqs 7.44, 7.45

$$\epsilon_{HX} = \frac{\dot{m}_2(e_{f3}-e_{f2})}{\dot{m}_5(e_{f5}-e_{f6})} = \frac{\dot{m}_2(e_{f3}-e_{f2})}{\dot{m}_2(e_{f3}-e_{f2})+\dot{E}_d}$$

$$= \frac{\dot{m}_2[h_3-h_2-T_0(s_3-s_2)]}{\dot{m}_2[h_3-h_2-T_0(s_3-s_2)]+\dot{E}_d} \quad (1)$$

With data from the solutions to Problems 4.82 and 6.125

$$\dot{m}_2[h_3-h_2-T_0(s_3-s_2)] = (28.22)\left[1397.8-1161.07\right]-300(6.4265)$$
$$= 4752.6\ kW$$

Then, Eq. (1) gives

$$\epsilon_{HX} = \frac{4752.6}{4752.6+944.5} = 0.834 \quad (83.4\%)$$ ←

PROBLEM 7.113

KNOWN: Steady-state operating data are provided for a steam generator and turbine.

FIND: Determine the exergetic efficiencies of the steam generator and turbine.

SCHEMATIC & GIVEN DATA: See solutions to Problems 4.83, 7.87.

ASSUMPTIONS: See assumptions listed for the solutions to Problems 4.83, 7.87.

ANALYSIS: With Eq. 7.42, and invoking the assumptions for the turbine in this case

$$\epsilon_t = \frac{\dot{W}_t}{\dot{m}(e_{f2}-e_{f3})} = \frac{\dot{W}_t}{\dot{m}[(h_2-h_3)-T_0(s_2-s_3)]} = \frac{\dot{W}_t}{\dot{W}_t+\dot{E}_d}$$

Inserting values from the solutions to Problems 4.83, 7.87

$$\epsilon_t = \frac{9.09 \text{ Btu/s}}{(9.09+2.05)\text{ Btu/s}} = 0.816 \quad (81.6\%) \qquad \longleftarrow \epsilon_t$$

With Eqs. 7.44, 7.45, and data from the solution to Problem 7.87

$$\epsilon_{SG} = \frac{\dot{E}_{f2}-\dot{E}_{f1}}{\dot{E}_{fB}-\dot{E}_{fA}} = \frac{(\dot{E}_{fB}-\dot{E}_{fA})-\dot{E}_d}{(\dot{E}_{fB}-\dot{E}_{fA})} = \frac{14.49-2.37}{14.49} = 0.836 \quad (83.6\%) \longleftarrow \epsilon_{SG}$$

PROBLEM 7.114

KNOWN: Steady-state operating data are provided for a heat pump system.

FIND: Determine the exergetic efficiencies of the compressor and condenser.

SCHEMATIC & GIVEN DATA: See Figs. E6.8, 14

ASSUMPTIONS: 1. See assumptions listed for Examples 6.8 and 6.14. 2. $T_0 = 273\text{ K}$, $p_0 = 1\text{ bar}$.

ANALYSIS: The exergetic efficiency of the compressor is obtained using Eq. 7.43 and data from the examples noted

① $$\epsilon = \frac{e_{f2}-e_{f1}}{(-\dot{W}_{cv}/\dot{m})} = \frac{h_2-h_1-T_0(s_2-s_1)}{h_2-h_1} = 1-\frac{T_0(s_2-s_1)}{(h_2-h_1)} = 1-\frac{(273)(0.98225-0.9572)}{(294.17-249.75)}$$

$$= 0.846 \quad (84.6\%) \qquad \longleftarrow$$

For condenser, use Eq. 7.45 and data from the examples noted

$$\epsilon = \frac{\dot{m}_{air}[h_6-h_5-T_0(s_6-s_5)]}{\dot{m}_{ref}[h_2-h_3-T_0(s_2-s_3)]} = \frac{0.5[30.15-273(0.098)]}{0.07[(294.17-79.05)-273(0.98225-0.2936)]}$$

② $$= 0.894 \quad (89.4\%) \qquad \longleftarrow$$

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1. Alternatively, with data from Example 7.9

$$\epsilon = \frac{(-\dot{W}_{cv})-\dot{E}_d}{(-\dot{W}_{cv})} = \frac{(3.11\text{ kW})-(0.478\text{ kW})}{3.11\text{ kW}} = 0.846$$

2. This calculation is sensitive to roundoff.

PROBLEM 7.115

KNOWN: Steady state operating data are provided for a compressor and heat exchanger in series.

FIND: Determine the exergetic efficiencies of the compressor and heat exchanger.

SCHEMATIC & GIVEN DATA: See the schematic from the solution to Problem 6.124. Also see Problem 7.89 solution for data.

ASSUMPTIONS: 1. See the assumptions listed for Problem 6.124.
2. $T_0 = 300\,K$, $P_0 = 96\,kPa$.

ANALYSIS: With Eq. 7.43 and invoking the assumptions listed

$$\epsilon_{comp} = \frac{\dot{m}(e_{f2} - e_{f1})}{(-\dot{W}_{cv})} = \frac{\dot{m}[h_2 - h_1 - T_0(s_2 - s_1)]}{(-\dot{W}_{cv})} = \frac{(-\dot{W}_{cv}) - \dot{E}_d}{(-\dot{W}_{cv})}$$

With values from the solution to Problem 7.89

$$\epsilon_{comp} = \frac{50.4 - 5.88}{50.4} = 0.883 \;(88.3\%) \longleftarrow$$

With Eqs. 7.44, 7.45, and data from the solution to Problem 7.89

$$\epsilon_{HX} = \frac{\dot{E}_{fB} - \dot{E}_{fA}}{(\dot{E}_{fB} - \dot{E}_{fA}) + \dot{E}_d} = \frac{(0.46)}{(0.46) + 4.56} = 0.092 \;(9.2\%) \longleftarrow$$

PROBLEM 7.116

KNOWN: Steady-state operating data are provided for a steam generator and turbine.

FIND: Determine the exergetic efficiencies of the steam generator and turbine.

Schematic & Given Data: See figs. E4.10, E7.8.

① ASSUMPTIONS: 1. See assumptions for Examples 4.10, 7.8 2. $T_0 = 537°R$, $P_0 = 1\,atm$.

ANALYSIS: With Eqs. 7.41, 42, and data from Example 7.8

$$\epsilon_t = \frac{\dot{W}_{cv}}{\dot{E}_{f4} - \dot{E}_{f5}} = \frac{\dot{W}_{cv}}{\dot{W}_{cv} + \dot{E}_d} = \frac{49{,}610\,Btu/min}{(49{,}610 + 17{,}070)\,Btu/min} = 0.744 \;(74.4\%) \longleftarrow$$

With Eqs. 7.44, 7.45 and data from Example 7.8

$$\epsilon_{SG} = \frac{\dot{E}_{f4} - \dot{E}_{f3}}{\dot{E}_{f1} - \dot{E}_{f2}} = \frac{(\dot{E}_{f1} - \dot{E}_{f2}) - \dot{E}_d}{(\dot{E}_{f1} - \dot{E}_{f2})} = \frac{100{,}300 - 22{,}110}{100{,}300} = 0.78 \;(78\%) \longleftarrow$$

PROBLEM 7.117

KNOWN: An expression is provided for the total cost rate $\dot{C}$ of a device as a function of pressure drop, $P_1 - P_2$.

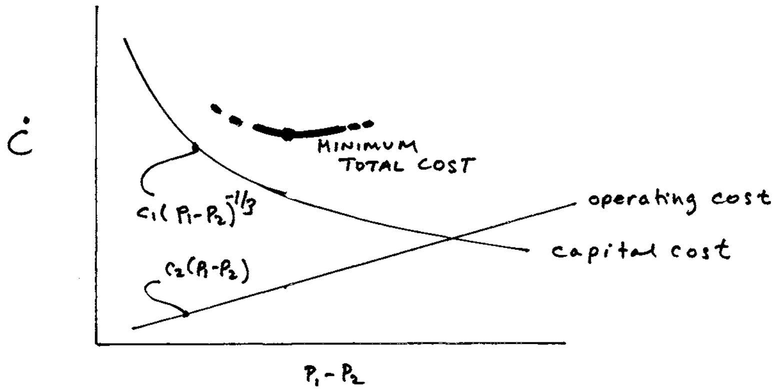
FIND: (a) Sketch $\dot{C}$ versus $(P_1 - P_2)$. (b) At the point of minimum total cost, determine the percent contributions of the capital and operating cost rates.

ASSUMPTION: The total cost rate is given by

$$\dot{C} = \underbrace{c_1 (P_1 - P_2)^{-1/3}}_{\text{Capital cost}} + \underbrace{c_2 (P_1 - P_2)}_{\text{operating Cost}} \qquad (1)$$

where c_1 and c_2 are constants.

ANALYSIS: (a) Sketch of Eq.(1)



(b) $$\frac{\partial \dot{C}}{\partial (P_1 - P_2)} = -\frac{1}{3} c_1 (P_1 - P_2)^{-4/3} + c_2 = 0 \Rightarrow \frac{1}{3} c_1 (P_1 - P_2)^{-4/3} = c_2$$

or, on rearrangement, at the point of minimum total cost

$$\underbrace{c_1 (P_1 - P_2)^{-1/3}}_{\text{Cap. cost}} = 3\, \underbrace{c_2 (P_1 - P_2)}_{\text{op. cost}}$$

In other words, at this point the capital cost is three times the

① operating cost. Thus, at the point of minimum total cost, the percent contributions to the total cost rate are

% operating cost = 25%

% capital cost = 75%

---

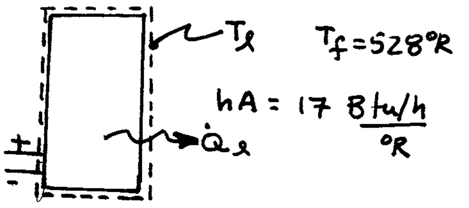
1. For further discussion, see D. Steinmeyer, "Optimum Δp and ΔT in Heat Exchange," Hydrocarbon Processing, April, 1992, 53-56.

PROBLEM 7.118

KNOWN: Heat transfer and cost data are provided for an electric water heater.

FIND: (a) Determine the cost of the heat loss when the outer surface temperature is 535 °R. (b) Plot cost of heat loss versus outer surface temperature ranging from 535 to 570 °R.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. For the system shown in the accompanying schematic, heat transfer occurs only at T_ℓ.
2. Electricity is evaluated at 8 cents/kW·h.
3. $T_0 = 528°R$.

ANALYSIS: Letting $\dot{Q}_\ell$ denote the rate of heat loss from the water heater, $\dot{Q}_\ell = hA[T_\ell - T_f]\ \frac{Btu}{h}$. Following the discussion of Sec. 7.6.1 concerning costing of heat loss, and letting electricity play the role of the fuel, we get

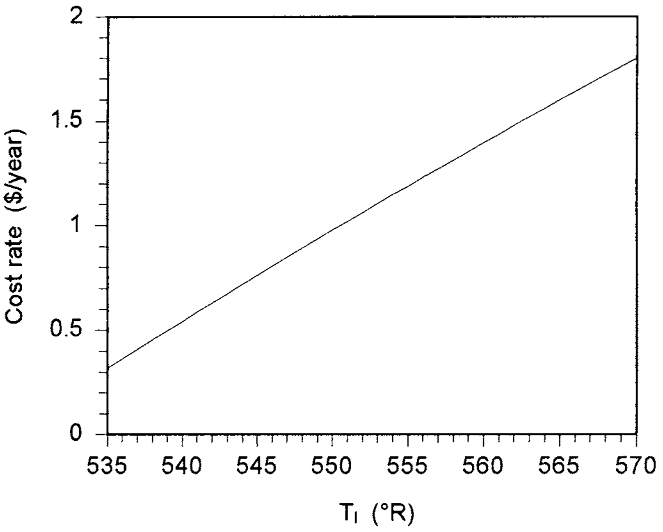
$$\dot{\$}_\ell = c_e\left[1 - \frac{T_0}{T_\ell}\right]\dot{Q}_\ell = c_e\left[1 - \frac{T_0}{T_\ell}\right](hA)(T_\ell - T_f) \qquad (1)$$

(a) When $T_\ell = 535°R$, the annual cost $(24 \times 365 = 8760\,h)$ is

$$\dot{\$}_\ell = \left(\frac{\$.08}{kW \cdot h}\right)\left[1 - \frac{528}{535}\right]\left(\frac{17\ Btu}{h \cdot °R}\right)(7\ °R)\left|\frac{8760h}{year}\right|\left|\frac{1\ kW}{3413\ Btu/h}\right| = \$0.32/year \longleftarrow$$

(b) Plot

①



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1. When considered over the several years life of such a water heater, the use of an insulating blanket to reduce the outer surface temperature is likely to be cost effective for the owner.

PROBLEM 7.119

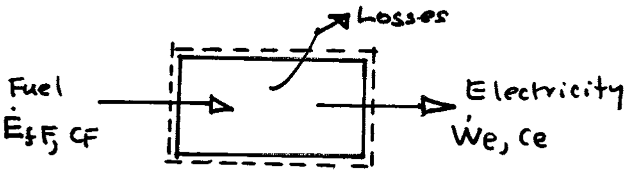
KNOWN: Cost rate data are provided for a system operating at steady state. The system generates electricity at the rate $\dot{W}_e$. The total cost rate is

$$\dot{C} = \underbrace{c_F \dot{E}_{fF}}_{\text{fuel cost}} + \underbrace{c\left(\frac{\epsilon}{1-\epsilon}\right)\dot{W}_e}_{\text{owning and operating cost}} \qquad (1)$$

where c_F is the unit cost based on exergy, $\epsilon = \dot{W}_e/\dot{E}_{fF}$, and c is a constant.

FIND: (a) Derive an expression for c_e, the unit cost of electricity based on $\dot{W}_e$, in terms of ϵ, c_e/c_F, and c/c_F. (b) For fixed c/c_F, derive an expression for ϵ corresponding to minimum c_e/c_F. (c) Plot c_e/c_F versus ϵ for selected values of c/c_F. For each c/c_F, evaluate the minimum value for c_e/c_F and the corresponding ϵ.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. The system shown in the figure is at steady state.
2. In Eq. (1), c and c_F are constants.
3. Eq. (1) accounts for all significant contributors to the total cost rate.

ANALYSIS: (a) Beginning with Eq. (1), use $\dot{E}_{fF} = \dot{W}_e/\epsilon$ to obtain

$$\dot{C} = c_F\left[\frac{\dot{W}_e}{\epsilon}\right] + c\left[\frac{\epsilon}{1-\epsilon}\right]\dot{W}_e = \left(\frac{c_F}{\epsilon} + c\left[\frac{\epsilon}{1-\epsilon}\right]\right)\dot{W}_e$$

Then, with $\dot{C} = c_e\dot{W}_e$, the unit cost of electricity is

$$c_e = \frac{c_F}{\epsilon} + c\left(\frac{\epsilon}{1-\epsilon}\right)$$

or

$$\frac{c_e}{c_F} = \frac{1}{\epsilon} + \underbrace{\left(\frac{c}{c_F}\right)}_{\equiv \gamma}\left(\frac{\epsilon}{1-\epsilon}\right) = \frac{1}{\epsilon} + \gamma\left(\frac{\epsilon}{1-\epsilon}\right) \longleftarrow \qquad (a)$$

(b) For fixed γ $(= c/c_F)$, differentiation of Eq. (a) with respect to ϵ gives

$$\frac{\partial(c_e/c_F)}{\partial\epsilon} = -\frac{1}{\epsilon^2} + \gamma\left[\frac{1(1-\epsilon) - \epsilon(-1)}{(1-\epsilon)^2}\right]$$

$$= -\frac{1}{\epsilon^2} + \gamma\left[\frac{1}{(1-\epsilon)^2}\right]$$

Setting $\frac{\partial(c_e/c_F)}{\partial\epsilon} = 0$ and solving

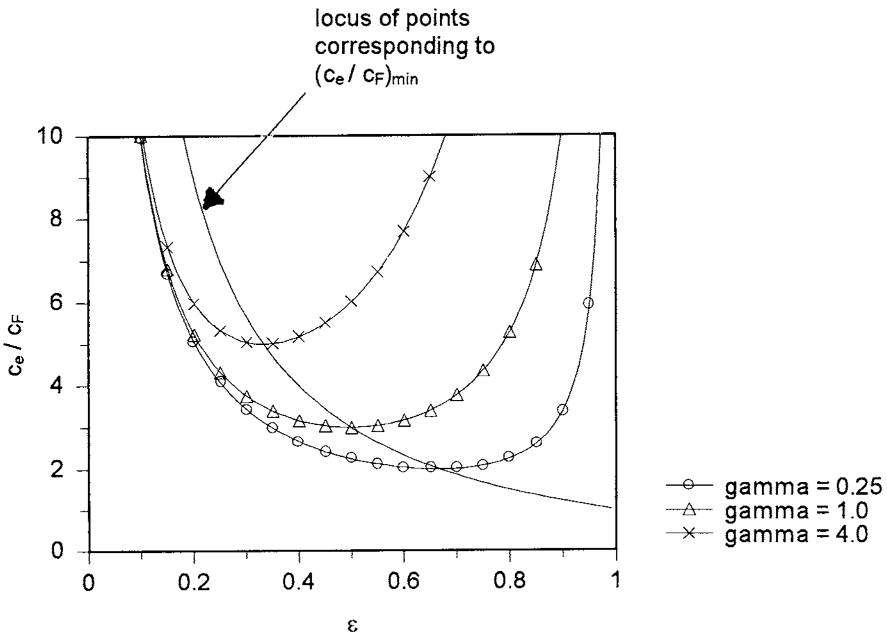
$$\frac{1}{\epsilon^2} = \frac{\gamma}{(1-\epsilon)^2} \Rightarrow \left(\frac{1-\epsilon}{\epsilon}\right)^2 = \gamma \Rightarrow \frac{1-\epsilon}{\epsilon} = \sqrt{\gamma} \Rightarrow \epsilon = \frac{1}{1+\sqrt{\gamma}} \longleftarrow \qquad (b)$$

which is the value of ϵ corresponding to a minimum in c_e/c_F when γ is fixed. Using this in the result of part (a)

$$\left(\frac{c_e}{c_F}\right)_{MIN} = 1 + 2\sqrt{\gamma}$$

PROBLEM 7.119 (Cont'd.)

PLOT:



The following chart lists values of $(c_e / c_F)_{min}$ and the corresponding values of ϵ for γ = 0.25, 1, and 4:

γ	ε	$(c_e / c_F)_{min}$
.25	2/3	2
1	.5	3
4	1/3	5

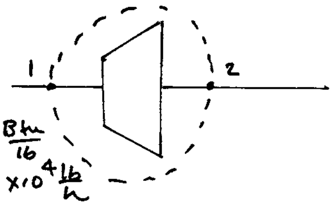
PROBLEM 7.120*

KNOWN: Operating and cost data are provided for a turbine operating at steady state.

FIND: (a) Evaluate the unit cost of the power developed.

(b) Evaluate the unit cost based on exergy of the steam entering and exiting the turbine, each in cents per lb of steam flowing.

SCHEMATIC & GIVEN DATA:



$e_{f1} = 645\ \frac{Btu}{lb}$

$\dot{m} = 32\times10^4\ \frac{lb}{h}$

$c_s = \$0.0182/kW\cdot h$

$\dot{W}_t = 18\times10^7\ kW\cdot h/yr$

$\dot{Z} = \$5.0\times10^5/yr$

$\epsilon = 85\%$

8000 h of operation/yr

ASSUMPTION: The control volume shown in the figure is at steady state.

ANALYSIS: (a) In this case, Eq. 7.52c is applicable:

$$c_e = \frac{c_s}{\epsilon} + \frac{\dot{Z}}{\dot{W}_e} = \left(\frac{0.0182\ \$/kW\cdot h}{0.85}\right) + \left(\frac{5.0\times10^5\ \$/yr}{18\times10^7\ kW\cdot h/yr}\right)$$

$$= (0.02141 + 0.00278)\ \$/kW\cdot h = 0.0242\ \$/kW\cdot h = 2.42\ \frac{cents}{kW\cdot h} \leftarrow (a)$$

(b) By assumption leading to Eq. 7.58c, the same unit cost based on exergy applies to the steam at the inlet and at the exit: $1.82\ \frac{cents}{kW\cdot h}$. Thus the unit cost of steam expressed on a unit of mass basis is

$$\begin{pmatrix}\text{unit cost of steam,}\\ \text{per lb, entering the}\\ \text{turbine}\end{pmatrix} = \left(1.82\ \frac{cents}{kW\cdot h}\right)\left[\left(645\ \frac{Btu}{lb}\right)\left|\frac{kW\cdot h}{3413\ Btu}\right|\right] = 0.344\ \frac{cents}{lb} \leftarrow$$

The same approach is used at the exit, but the value of e_{f2} is required. Using

$$\epsilon = \frac{\dot{W}_t}{\dot{m}(e_{f2}-e_{f1})} \Rightarrow e_{f2} = e_{f1} - \frac{\dot{W}_t}{\dot{m}\epsilon}$$

or

$$e_{f2} = 645\ \frac{Btu}{lb} - \frac{(18\times10^7 kW\cdot h/yr)\ |3413\ Btu/kW\cdot h|}{\left(32\times10^4\ \frac{lb}{h}\right)\left(\frac{8000 h}{yr}\right)(0.85)} = 363\ \frac{Btu}{lb}$$

Then

$$\begin{pmatrix}\text{unit cost of steam,}\\ \text{per lb, exiting the}\\ \text{turbine}\end{pmatrix} = \left(1.82\ \frac{cents}{kW\cdot h}\right)\left[\left(363\ \frac{Btu}{lb}\right)\left|\frac{kW\cdot h}{3413\ Btu}\right|\right] = 0.194\ \frac{cents}{lb} \leftarrow$$

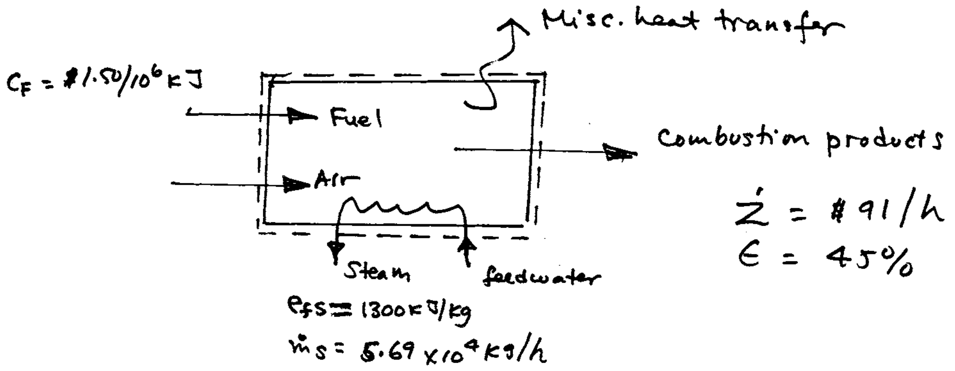
PROBLEM 7.121

KNOWN: Operating and cost data are provided for the boiler of Fig. P7.121.

FIND: (a) Develop an expression for the unit cost, based on exergy of the steam exiting the boiler.

(b) Using the result of (a), determine the unit cost of the exiting steam in cents per kg of steam exiting the boiler.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) The cost rates of the feedwater, combustion products, and heat transfer are ignored.

ANALYSIS: (a) With assumption (2), Equation 7.50b is applicable:

$$c_s = c_F\left(\frac{\dot E_{fF}}{\dot E_{fs}}\right) + \frac{\dot Z}{\dot E_{fs}}. \quad \text{With } \epsilon = \frac{\dot E_{fs}}{\dot E_{fF}}, \quad c_s = \frac{c_F}{\epsilon} + \frac{\dot Z}{\dot E_{fs}} \longleftarrow (a)$$

(b) Inserting values

$$c_s = \frac{(\$1.50/10^6\,\text{kJ})}{0.45} + \frac{91\,(\$/\text{h})}{\left(5.69\times10^4 \frac{\text{kg}}{\text{h}}\right)\left(1300\frac{\text{kJ}}{\text{kg}}\right)} = \left[\frac{(3.33 + 1.23)(\$/\text{kJ})}{10^6}\right]\left|\frac{3600\,\text{kJ}}{\text{kW}\cdot\text{h}}\right|$$

$$= 0.0164\ \$/\text{kW}\cdot\text{h}$$

This unit cost is on a per kW·h of exergy basis. It can be expressed on a per unit mass of steam basis as follows:

$$\left(\begin{matrix}\text{unit cost of steam}\\ \text{per kg}\end{matrix}\right) = \left(0.0164\frac{\$}{\text{kW}\cdot\text{h}}\right)\left(1300\frac{\text{kJ}}{\text{kg}}\right)\left|\frac{\text{kWh}}{3600\,\text{kJ}}\right| = 0.0059\ \$/\text{kg}$$

① $$= 0.59\ \frac{\text{cents}}{\text{kg}} \longleftarrow (b)$$

---

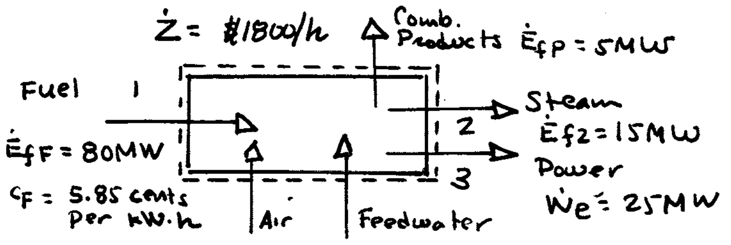
1. When expressed on a per lb of steam basis, the result of part (b) is 0.27 cents/lb, which corresponds to the unit cost of the steam entering the turbine of Problem 7.120, part(a).

PROBLEM 7.122

KNOWN: Steady-state operating and cost data are provided for a cogeneration system.

FIND: (a) Determine the rate of exergy destruction. (b) Devise and evaluate an exergetic efficiency. (c) Evaluate the unit cost, and the cost rates of the power and steam produced, each on an exergy basis.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume shown in the schematic is at steady state.
2. Feedwater and combustion air enter with negligible exergy and cost.
3. Combustion products exit with negligible cost.
4. Heat transfer with the surroundings can be ignored.
5. The exiting steam and power each have the same unit cost based on exergy.

ANALYSIS:

(a) At steady state, the exergy rate balance reduces with assumptions 2 and 4 to give

$$\dot{E}_d = \dot{E}_{fF} - \dot{E}_{fP} - \dot{E}_{f2} - \dot{W}_e$$

$$= 80 - 5 - 15 - 25 = 35 \text{ MW} \quad \leftarrow$$

(b) Regarding the exiting steam and power as the product of the system and the combustion products as a loss

$$\epsilon = \frac{\dot{E}_{f2} + \dot{W}_e}{\dot{E}_{fF}} = \frac{15 + 25}{80} = 0.5 \ (50\%) \quad \leftarrow$$

(c) With assumptions 2-4, a cost rate balance reads

$$\dot{C}_2 + \dot{C}_3 = \dot{C}_1 + \dot{Z}$$

Then with assumption 5, $c\dot{E}_{f2} + c\dot{W}_e = c_F \dot{E}_{fF} + \dot{Z}$

Solving for the unit cost of the steam and the power

$$c = \frac{c_F \dot{E}_{fF} + \dot{Z}}{\dot{E}_{f2} + \dot{W}_e} = c_F\left[\frac{\dot{E}_{fF}}{\dot{E}_{f2} + \dot{W}_e}\right] + \frac{\dot{Z}}{[\dot{E}_{f2} + \dot{W}_e]}$$

$$= \frac{c_F}{\epsilon} + \frac{\dot{Z}}{[\dot{E}_{f2} + \dot{W}_e]}$$

$$= \left[\frac{5.85 \text{ ¢/kW·h}}{0.5}\right] + \frac{\$ 1800/\text{h}}{[15 + 25] \text{ MW}} \left|\frac{1 \text{ MW}}{10^3 \text{ kW}}\right| \left|\frac{10^2 \text{ cents}}{1 \ \$}\right|$$

$$= [11.7 + 4.5] \frac{\text{cents}}{\text{kW·h}} = 16.2 \frac{\text{cents}}{\text{kW·h}} \quad \leftarrow$$

The cost rates are

$$\dot{C}_2 = c\dot{E}_{f2} = \left(16.2 \frac{\text{cents}}{\text{kW·h}}\right)(15 \text{ MW}) \left|\frac{10^3 \text{ kW}}{1 \text{ MW}}\right| \left|\frac{1 \ \$}{10^2 \text{ cents}}\right| = \$2430/\text{h} \quad \leftarrow$$

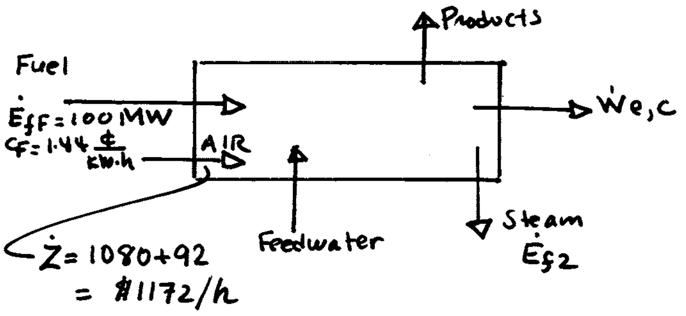
$$\dot{C}_3 = c\dot{W}_e = \left(16.2 \frac{\text{cents}}{\text{kW·h}}\right)(25 \text{ MW}) \left|\frac{10^3 \text{ kW}}{1 \text{ MW}}\right| \left|\frac{1 \ \$}{10^2 \text{ cents}}\right| = \$4050/\text{h} \quad \leftarrow$$

PROBLEM 7.123

KNOWN: Steady-state operating and cost data are provided in Example 7.10 for a cogeneration system.

FIND: Taking an overall control volume and regarding the unit cost based on exergy is the same for the power and steam produced, evaluate the unit cost and compare with the values obtained in Example 7.10

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume shown in the schematic is at steady state.
2. For the control volume, $\dot{Q}_{cv} = 0$.
3. The feedwater and combustion air enter with negligible exergy and cost.
4. The combustion products are discharged without cost.
5. The exiting steam and power each have the same unit cost based on exergy.

ANALYSIS: A cost rate balance reads

$$c\dot{W}_e + c\dot{E}_{f2} = c_F \dot{E}_{fF} + \dot{Z}$$

$$\Rightarrow \quad c = \frac{c_F \dot{E}_{fF} + \dot{Z}}{\dot{W}_e + \dot{E}_{f2}}$$

From the solution to Example 7.10, $\dot{W}_e = 12.75$ MW, $\dot{E}_{f2} = 20.67$ MW

$$c = \frac{(1.44¢/kW \cdot h)(100 \text{ MW})}{(12.75 + 20.67)\text{MW}} + \frac{\$1172/h}{(12.75+20.67)\text{MW}} \left| \frac{1 \text{ MW}}{10^3 \text{ kW}} \right| \left| \frac{10^2 ¢}{\$} \right|$$

$$= (4.309 + 3.507) = \quad 7.82 \text{ cents}/kW \cdot h \qquad \longleftarrow$$

From the solution to Example 7.10, the unit cost for power is \$8.81 cents/kW·h and for process steam 7.2 cents/kW·h. When the overall control volume is considered in the present case, the unit cost of the steam is higher because it now bears a part of the costs associated with the turbine in the analysis of Example 7.10. On the other hand, the unit cost of the power is lower because it is no longer required to bear the full burden of these costs. The approach of Example 7.10 is preferred because it takes into account important information related to the actual production processes.

PROBLEM 7.124

KNOWN: Steady-state operating and cost data are provided for the cogeneration system of Example 7.10.

FIND: Plot versus the pressure of the process steam (a) the power developed, (b) the unit costs of the power and process steam, per kW·h, (c) the unit cost of the process steam, per kg of process steam flowing.

SCHEMATIC & GIVEN DATA: See Figure E 7.10.

$\dot{Z}_t = 7.2\dot{W}_e \frac{\$}{h}$, (in MW)

p_2(bar)	40	30	20	9	5	2	1
T_2(°C)	436	398	349	262	205	128	sat.

Case considered in Example 7.10.

ASSUMPTIONS: See Example 7.10

ANALYSIS: Following the analysis of Example 7.10

$$\dot{W}_e = \dot{m}(h_1 - h_2)\left|\frac{1\,MW}{10^3\,kJ/s}\right| \qquad (1)$$

where $\dot{m} = 26.15$ kg/s, h_1 is fixed by $P_1 = 50$ bar, $T_1 = 466$°C, h_2 is fixed by P_2, T_2 from the table above.

The unit cost of the process steam is 7.2 cents/kW·h, as in Example 7.10. This value is determined by the boiler analysis together with the assumption underlying Eq. 7.52b. The unit cost of the power is determined by

$$c_e = c_2\left[\frac{\dot{E}_{f1} - \dot{E}_{f2}}{\dot{W}_e}\right] + \frac{\dot{Z}_t}{\dot{W}_e} \qquad (2)$$

where $c_2 = 7.2$ cents/kW·h, $\dot{W}_e$ is given by Eq.(1), $\dot{Z}_t = 7.2\dot{W}_e$, where $\dot{W}_e$ is in MW, and

$$\dot{E}_{f1} - \dot{E}_{f2} = \dot{m}[h_1 - h_2 - T_0(s_1 - s_2)]\left|\frac{1\,MW}{10^3\,kJ/s}\right| \qquad (3)$$

To determine the unit cost of the process steam on a unit mass basis

$$c_m = c_2\left[\frac{\dot{E}_{f2}}{\dot{m}}\right]\left|\frac{10^3\,kW}{1\,MW}\right|\left|\frac{1h}{3600s}\right| \qquad (4)$$

where $\dot{E}_{f2}$ is found using Eq.(3) and $\dot{E}_{f1} = 35$ MW.

The following IT code is used to generate data for each case in the above table. The data were imported to a spreadsheet program to obtain the plots on the next page.

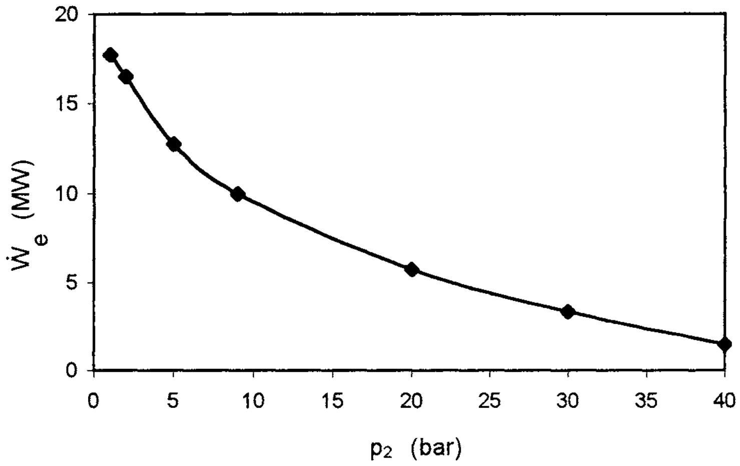
IT Code

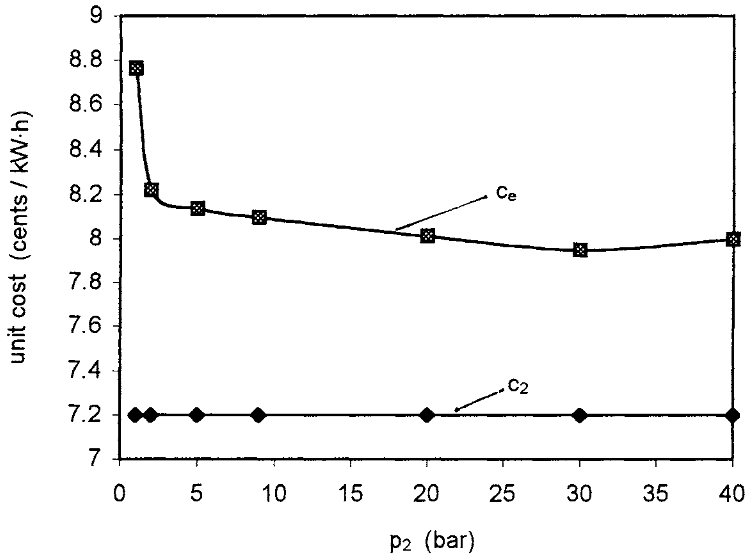
```
p1 = 50  // bar
T1 = 466  // °C
p2 = 40  // bar
T2 = 436  // °C
mdot = 26.15  // kg/s
To = 298  // K

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = h_PT("Water/Steam", p2, T2)
s2 = s_PT("Water/Steam", p2, T2)
s2s = s_Ph("Water/Steam", p2, h2s)
s2s = s1
```

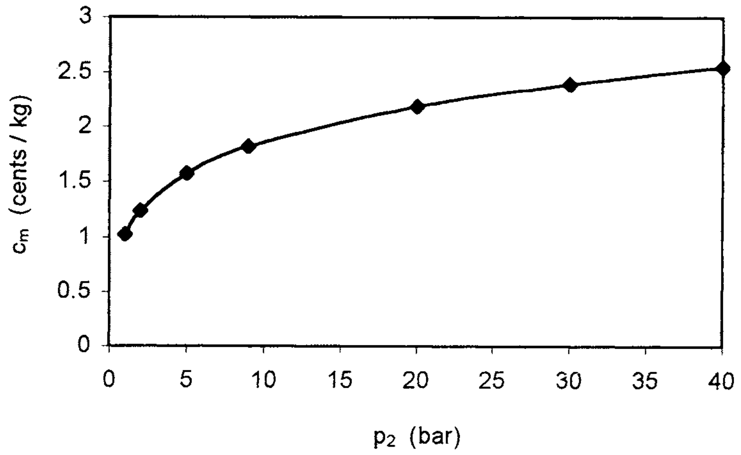
```
Wdote = mdot * (h1 - h2) / 1000
Edotf1 - Edotf2 = mdot * ((h1 - h2) - To * (s1 - s2)) / 1000
Edotf1 = 35
ce = c2 * ((Edotf1 - Edotf2) / Wdote) + (Zdott / Wdote) / 1000
Zdott = 7.2 * Wdote
c2 = 7.2
cm = c2 * (Edotf2 / mdot) * (1000 / 3600)
```

PROBLEM 7.124 (Cont'd.)

PLOTS:







CHAPTER EIGHT

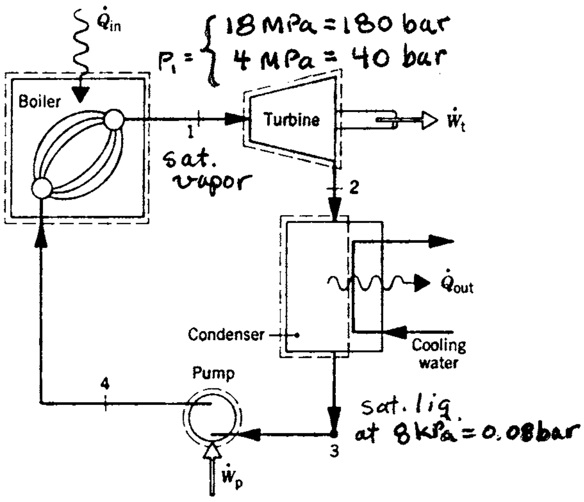
VAPOR POWER SYSTEMS

PROBLEM 8.1

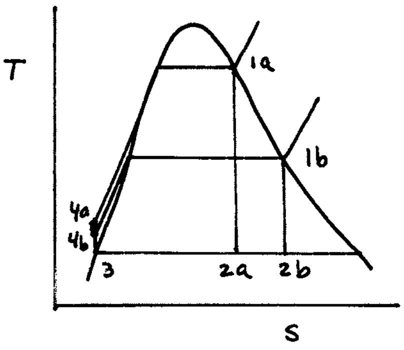
KNOWN: Water is the working fluid in an ideal Rankine cycle. The condenser pressure and turbine inlet state are specified. The net power output is given.

FIND: Determine for each of two boiler pressures, the mass flow rate of steam, the heat transfer rates for the working fluid passing through the condenser and boiler, and the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{in}$
$p_1 = \begin{cases} 18\ \text{MPa} = 180\ \text{bar} \\ 4\ \text{MPa} = 40\ \text{bar} \end{cases}$
Boiler
1
Turbine
$\dot{W}_t$
sat. vapor
2
$\dot{Q}_{out}$
Condenser
Cooling water
Pump
4
sat. liq. at 8 kPa = 0.08 bar
3
$\dot{W}_p$



ASSUMPTIONS: See Example 8.1

ANALYSIS: For the cycle, the net power output is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}\left[(h_1 - h_2) - (h_4 - h_3)\right]$$

thus

$$\dot{m} = \frac{\dot{W}_{cycle}}{(h_1 - h_2) - (h_4 - h_3)} \qquad (1)$$

The working fluid passing through the boiler experiences heat transfer at a rate

$$\dot{Q}_{in} = \dot{m}\,(h_1 - h_4) \qquad (2)$$

and in the condenser

$$\dot{Q}_{out} = \dot{m}\,(h_2 - h_3) \qquad (3)$$

The thermal efficiency of the cycle is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} \qquad (4)$$

Each of these expressions is evaluated for the two boiler pressures given.

(a) $p_1 = 180$ bar

State 1: $p_1 = 180$ bar, sat. vapor $\Rightarrow$ $h_1 = 2509.1$ kJ/kg, $s_1 = 5.1044$ kJ/kg·K

State 2: $p_2 = 0.08$ bar, $s_2 = s_1$ $\Rightarrow$ $x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.591$, $h_2 = 1594.1$ kJ/kg

State 3: $p_3 = 0.08$ bar, sat. liquid $\Rightarrow$ $h_3 = 173.88$ kJ/kg

PROBLEM 8.1 (Cont'd)

State 4: $h_4 \approx h_3 + v_3(p_4 - p_3)$; $h_3 = 173.88$ kJ/kg, $v_3 = 1.0084\times10^{-3}$ m³/kg

$$h_4 = 173.88\ \frac{kJ}{kg} + \left(1.0084\times10^{-3}\ \frac{m^3}{kg}\right)(180 - .08)\,bar \left|\frac{10^5 N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 173.88 + 18.14 = 192.02\ kJ/kg$$

Thus, from (1) above

$$\dot{m} = \frac{(100\times10^3\ kJ/s)}{\left[(2509.1 - 1594.1) - (18.14)\right]\frac{kJ}{kg}}\left|\frac{3600\,s}{1\,h}\right| = 4.014\times10^5\ kg/h \longleftarrow \dot{m}$$

From (2)

$$\dot{Q}_{in} = \left(4.014\times10^5\ \frac{kg}{h}\right)\left|\frac{1\,h}{3600\,s}\right|(2509.1 - 192.02)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 258.4\times10^3\ kW \longleftarrow \dot{Q}_{in}$$

From (3) $\dot{Q}_{out} = (4.014\times10^5)\left|\frac{1}{3600}\right|(1594.1 - 173.88) = 158.4\times10^3\ kW \longleftarrow \dot{Q}_{out}$

From (4) $\eta = \frac{100\times10^3\ kW}{258.4\times10^3\ kW} = 0.387\ (38.7\%) \longleftarrow \eta$

(b) $p_1 = 40$ bar:

State 1: $p_1 = 40$ bar, sat. vapor ⇒ $h_1 = 2801.4$ kJ/kg, $s_1 = 6.0701$ kJ/kg·K

State 2: $p_2 = 0.08$ bar, $s_2 = s_1$ ⇒ $x_2 = 0.7173$, $h_2 = 1897.6$ kJ/kg

State 3: $p_3 = 0.08$ bar, sat. liquid ⇒ $h_3 = 173.88$ kJ/kg

State 4: $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$= 173.88 + (1.0084\times10^{-3})(40 - 0.08)\left|\frac{10^5}{10^3}\right|$$

$$= 173.88 + 4.026 = 177.91\ kJ/kg$$

From (1)

$$\dot{m} = \frac{(100\times10^3)\ |3600|}{\left[(2801.4 - 1897.6) - 4.026\right]} = 4.001\times10^5\ kg/h \longleftarrow \dot{m}$$

① From (2) $\dot{Q}_{in} = (4.001\times10^5)\left|\frac{1}{3600}\right|(2801.4 - 177.91) = 291.6\times10^3\ kW \longleftarrow \dot{Q}_{in}$

From (3) $\dot{Q}_{out} = (4.001\times10^5)\left|\frac{1}{3600}\right|(1897.6 - 173.88) = 191.6\times10^3\ kW \longleftarrow \dot{Q}_{out}$

From (4) $\eta = \frac{100\times10^3}{291.6\times10^3} = 0.343\ (34.3\%) \longleftarrow \eta$

---

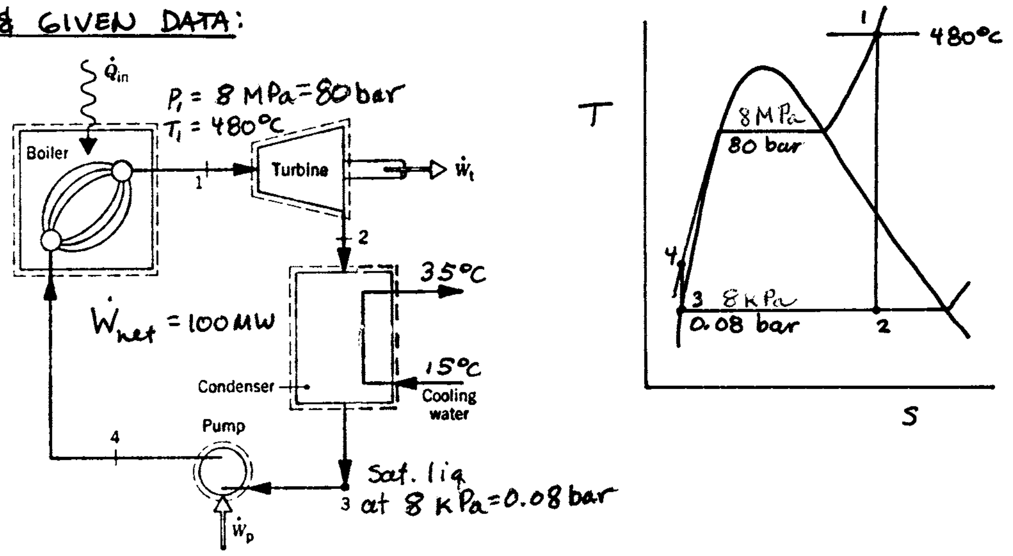
1. These results can be compared to results from Example 8.1 to see some effects of boiler pressure on the ideal Rankine cycle.

PROBLEM 8.2

KNOWN: Water is the working fluid in an ideal Rankine cycle with superheated vapor entering the turbine. The condenser pressure and the net power output are given.

FIND: Determine (a) the rate of heat transfer to the working fluid passing through the steam generator, (b) the thermal efficiency, and (c) the mass flow rate of condenser cooling water, which enters at 15°C and exits at 35°C.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 8.1, assumptions 1-4.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 80$ bar, $T_1 = 480°C \Rightarrow h_1 = 3348.4$ kJ/kg, $s_1 = 6.6586$ kJ/kg·K

State 2: $p_2 = 0.08$ bar, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.7944$, $h_2 = 2082.9$ kJ/kg

State 3: $p_3 = 0.08$ bar, Sat. liquid $\Rightarrow h_3 = 173.88$ kJ/kg, $v_3 = 1.0084 \times 10^{-3} \frac{m^3}{kg}$

State 4: $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$= 173.88 \frac{kJ}{kg} + (1.0084 \times 10^{-3} \frac{m^3}{kg})(80 - .08)\,bar \left|\frac{10^5 N/m^2}{1 bar}\right| \left|\frac{1 kJ}{10^3 N\cdot m}\right|$$

$$= 173.88 + 8.06 = 181.94 \text{ kJ/kg}$$

(a) To determine the heat transfer rate $\dot{Q}_{in}$, first determine the mass flow rate. The net power output of the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) - (h_4 - h_3)]$$

or

$$\dot{m} = \frac{\dot{W}_{cycle}}{(h_1 - h_2) - (h_4 - h_3)} = \frac{(100 \text{ MW})}{[(3348.4 - 2082.9) - 8.06] \frac{kJ}{kg}} \left|\frac{10^3 kJ/s}{1 MW}\right|$$

$$= 79.53 \text{ kg/s}$$

Thus

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = (79.53 \frac{kg}{s})(3348.4 - 181.94) \frac{kJ}{kg} \left|\frac{1 kW}{1 kJ/s}\right|$$

$$= 251.8 \times 10^3 \text{ kW} \longleftarrow \dot{Q}_{in}$$

PROBLEM 8.2 (Cont'd)

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{100 \times 10^3 \text{ kW}}{251.8 \times 10^3 \text{ kW}} = 0.397 \ (39.7\%) \longleftarrow \eta$$

(c) Taking a control volume around the condenser

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}(h_2 - h_3) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

where $\dot{m}_{cw}$ is the mass flow rate of the cooling water. Thus

① $$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{(h_{cw,out} - h_{cw,in})}$$

For the cooling water, $h \approx h_f(T)$. Thus, with data from Table A-2

$$\dot{m}_{cw} = \frac{(79.53 \text{ kg/s})(2082.9 - 173.88) \text{ kJ/kg}}{(146.68 - 62.99) \text{ kJ/kg}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right|$$

$$= 6.53 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}_{cw}$$

---

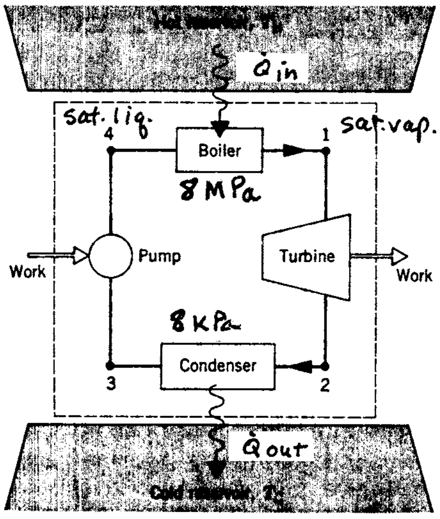
1. These results can be compared with results from Example 8.1 to see some effects of superheating the vapor at the turbine inlet on the ideal Rankine cycle.

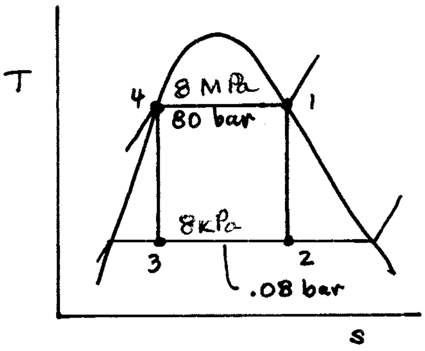
PROBLEM 8.3

KNOWN: Water is the working fluid in a Carnot vapor power cycle. The states at the boiler and turbine inlet are specified and the condenser pressure is known.

FIND: Determine (a) the thermal efficiency, (b) the back work ratio, (c) the heat transfer to the working fluid per unit mass passing through the boiler, and (d) the heat transfer from the working fluid per unit mass passing through the condenser. (e) Compare with results from Example 8.1.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible. (3) The turbine and the pump operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 80$ bar, sat. vapor $\Rightarrow h_1 = 2758.0$ kJ/kg, $s_1 = 5.7432$ kJ/kg·K

State 2: $p_2 = 0.08$ bar, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{g2} - s_{f2}} = 0.6745$, $h_2 = 1794.8$ kJ/kg

State 3: $p_3 = 0.08$ bar, $s_3 = s_4 = 3.2068$ kJ/kg·K $\Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{g3} - s_{f3}} = 0.3423$
$h_3 = 996.46$ kJ/kg

State 4: $p_4 = 80$ bar, sat. liquid $\Rightarrow h_4 = 1316.6$ kJ/kg

(a) The thermal efficiency of the Carnot cycle is given by Eq. 5.8. With data from Table A-3; $T_H = T_{sat\,@\,80\,bar} = 568.25$ K and $T_C = T_{sat\,@\,0.08\,bar} = 314.66$ K. Thus

①
$$\eta_{max} = 1 - \frac{T_C}{T_H} = 1 - \frac{314.66\text{ K}}{568.25\text{ K}} = 0.4463\ (44.63\%) \longleftarrow \eta$$

(b) The back work ratio is

$$bwr = \frac{(\dot{W}_p/\dot{m})}{(\dot{W}_t/\dot{m})} = \frac{h_4 - h_3}{h_1 - h_2} = \frac{1316.6 - 996.46}{2758.0 - 1794.8}$$

$$= \frac{320.14}{963.2} = 0.332\ (33.2\%) \longleftarrow bwr$$

PROBLEM 8.3 (Cont'd)

(c) From mass and energy rate balances for the boiler

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4 = 2758.0 \frac{kJ}{kg} - 1316.6 \frac{kJ}{kg} = 1441.4 \frac{kJ}{kg} \longleftarrow \underline{\dot{Q}_{in}/\dot{m}}$$

(d) From mass and energy rate balances for the condenser

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = 1794.8 \frac{kJ}{kg} - 996.46 \frac{kJ}{kg} = 798.34 \frac{kJ}{kg} \longleftarrow \underline{\dot{Q}_{out}/\dot{m}}$$

(e) Comparing these results with those of Example 8.1: In the present case,

- the thermal efficiency is greater
- the back work ratio is much greater.
- $\dot{Q}_{in}/\dot{m}$ is less
- $\dot{Q}_{out}/\dot{m}$ is less

The Rankine cycle of Example 8.1 avoids the work-intensive pumping process involving a two-phase liquid-vapor mixture of the present case: Process 3-4. See discussion on pp. 357-358 for further detail.

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1. Alternatively,

$$\eta = 1 - \frac{\dot{Q}_{out}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 1 - \frac{798.34}{1441.4} = 0.446$$

PROBLEM 8.4

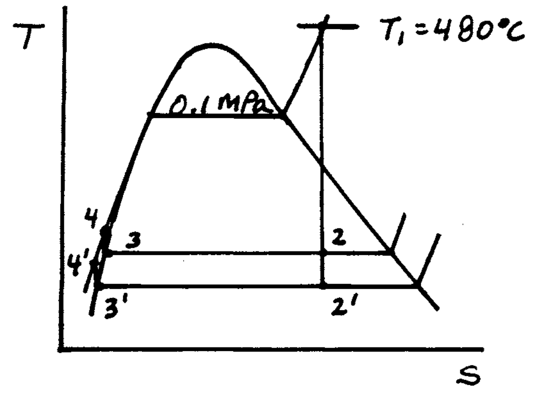
KNOWN: Water is the working fluid in an ideal Rankine cycle. with superheated vapor entering the turbine. Data are given in Problem 8.2.

FIND: Plot each of the quantities calculated in Problem 8.2 versus condenser pressure ranging from 6 kPa to 0.1 MPa.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.2.

ASSUMPTIONS: See Example 8.1, assumptions 1-4.

ANLYSIS: The procedure for fixing each of the principal states is the same as in Problem 8.2. State 1 is fixed, and states 2, 3, and 4 all vary with condenser pressure, as illustrated on the T-s diagram below



Sample results:

$P_{cond} = 6\ kPa = 0.06\ bar$

State	h (kJ/kg)	Comments
1	3348.4	$s_1 = 6.6586$ kJ/kg·K
2	2050.19	$s_2 = s_1$; $x_2 = 0.7859$
3	151.53	Sat. liquid at 0.06 bar
4	159.58	$h_4 \approx h_3 + v_3 (p_4 - p_3)$

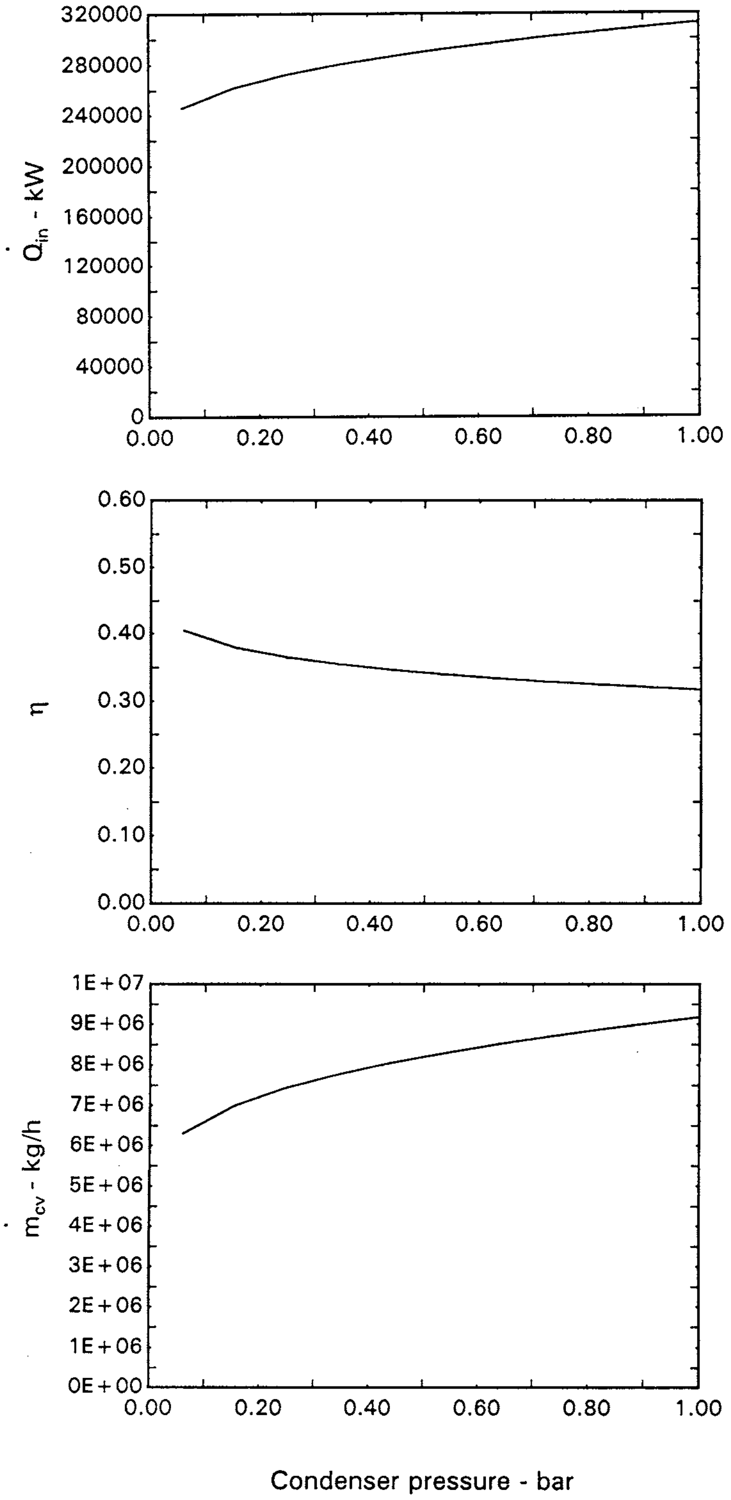
$P_{cond} = 0.1\ MPa = 1\ bar$

State	h (kJ/kg)	Comments
1	3348.4	$s_1 = 6.6586$ kJ/kg·K
2	2414.2	$s_2 = s_1$; $x_2 = 0.8843$
3	417.46	Sat. liquid at 1 bar
4	425.7	$h_4 \approx h_3 + v_3 (p_4 - p_3)$

P_{cond} (bar)	Result
0.06	$\dot{Q}_{in} = 247.2 \times 10^3$ kW, $\eta = 0.4045$, $\dot{m}_{cw} = 6.33 \times 10^6$ kg/h
1	$\dot{Q}_{in} = 315.7 \times 10^3$ kW, $\eta = 0.3168$, $\dot{m}_{cw} = 9.28 \times 10^6$ kg/h

PROBLEM 8.4 (Cont'd)

Plotting for the entire range of condenser pressures



The thermal efficiency decreases as condenser pressure increases. Since the power is constant, more heat input is required and the mass flow rate of cooling water must increase to accommodate the increased need for heat rejection.

PROBLEM 8.5

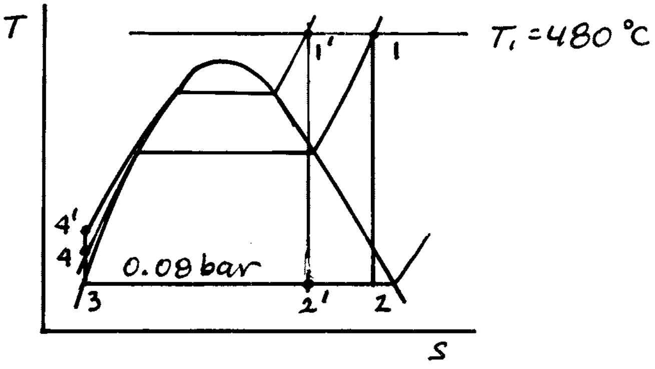
KNOWN: Water is the working fluid in an ideal Rankine cycle with superheated vapor at the turbine inlet. Data are given in Problem 8.2.

FIND: Plot each of the quantities calculated in Problem 8.2 versus steam generator pressure ranging from 4 to 24 MPa while maintaining constant turbine inlet temperature.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.2.

ASSUMPTIONS: See Example 8.1, assumptions 1-4.

ANALYSIS: The procedure for fixing each of the principal states is the same as in Problem 8.2. State 3 is fixed, and states 1, 2, and 4 all vary with steam generator pressure, as illustrated on the T-s diagram below



Sample results:

$P_1 = 4$ MPa $= 40$ bar

State	h(kJ/kg)	Comments
1	3399.2	$s_1 = 7.0281$
2	2199.1	$s_2 = s_1$; $x_2 = 0.84277$
3	173.88	$P_3 = 0.08$ bar , Sat. liquid
4	177.91	$h_4 \approx h_3 + v_3(p_4 - p_3)$

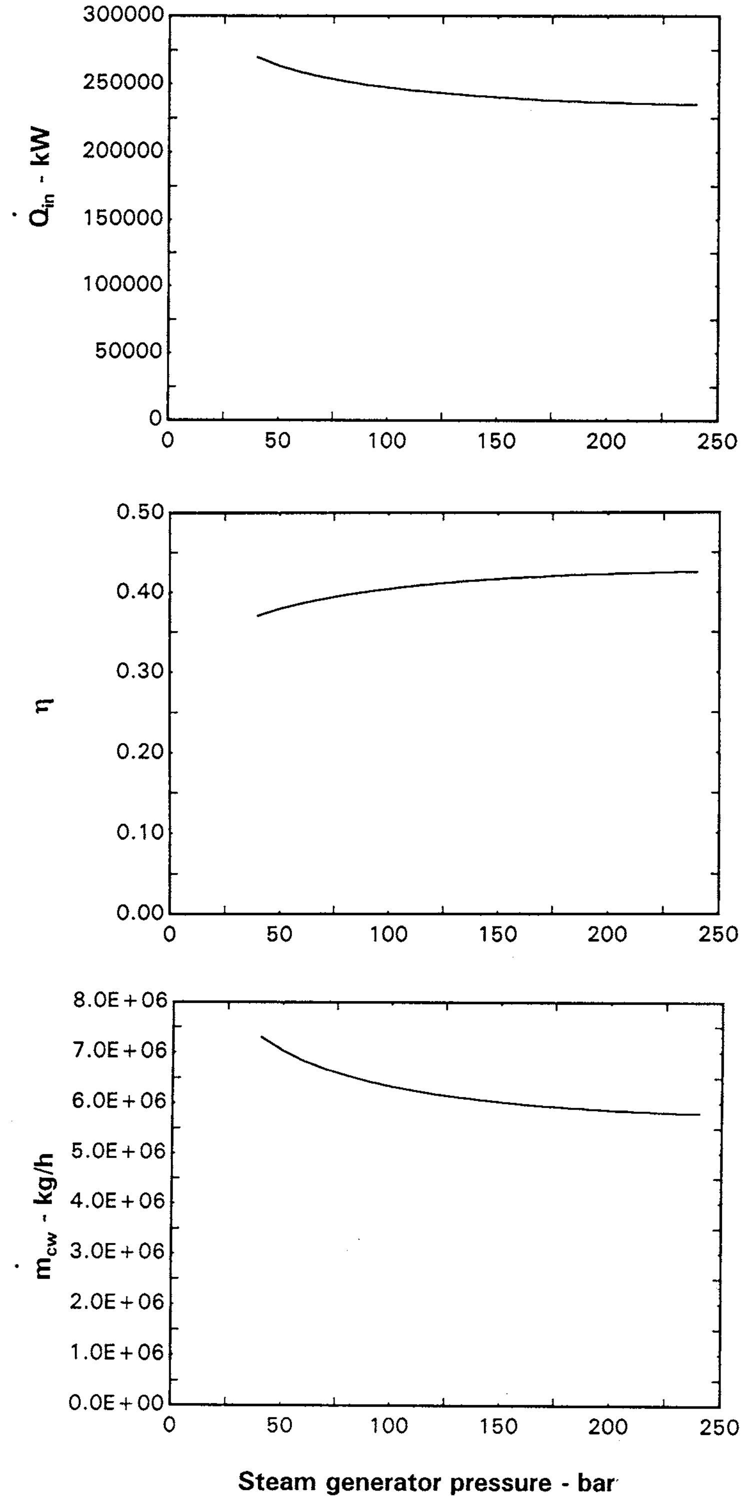
$P_1 = 24$ MPa $= 240$ bar

State	h(kJ/kg)	Comments
1	3102.3	$s_1 = 5.8950$
2	1842.6	$s_2 = s_1$; $x_2 = 0.6944$
3	173.88	$P_3 = 0.08$ bar, sat. liquid
4	198.07	$h_4 \approx h_3 + v_3(p_4 - p_3)$

P_1 (bar)	Result
40	$\dot{Q}_{in} = 269.3 \times 10^3$ kW, $\eta = 0.371$, $\dot{m}_{cw} = 7.28 \times 10^6$ kg/h
240	$\dot{Q}_{in} = 235.1 \times 10^3$ kW, $\eta = 0.425$, $\dot{m}_{cw} = 5.81 \times 10^6$ kg/h

PROBLEM 8.5 (Cont'd)

Plotting for the entire range of steam generator pressures



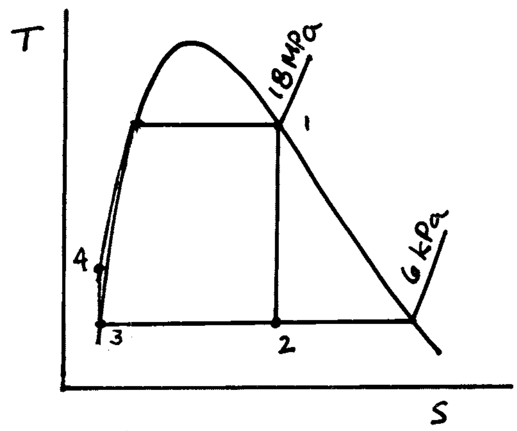
For fixed turbine inlet temperature, the thermal efficiency increases with increasing steam generator pressure. Since the power is constant, less heat input is required, and the mass flow rate of cooling water decreases since less heat rejection must be accommodated.

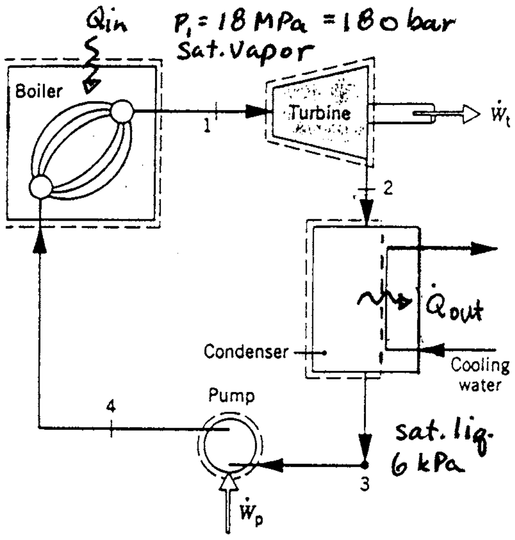
PROBLEM 8.6

KNOWN: Water is the working fluid in an ideal Rankine cycle. The condenser pressure and the turbine inlet state are specified.

FIND: Determine (a) the net work per unit mass of steam flow, (b) the heat transfer per unit mass of steam flow through the boiler, (c) the thermal efficiency, and (d) the heat transfer to cooling water passing through the condenser per unit mass of steam condensed.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 8-1.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 18\,\text{MPa}$, sat. vapor $\Rightarrow h_1 = 2509.1\ \text{kJ/kg}$, $s_1 = 5.1044\ \text{kJ/kg·K}$

State 2: $P_2 = 6\,\text{kPa}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = 0.5869$, $h_2 = 1569.4\ \text{kJ/kg}$

State 3: $P_3 = 6\,\text{kPa}$, sat. liquid $\Rightarrow h_3 = 151.53\ \text{kJ/kg}$

State 4: $h_4 \approx h_3 + v_3(P_4 - P_3)$

$$= 151.53\,\frac{\text{kJ}}{\text{kg}} + (1.0064\times10^{-3}\,\frac{\text{m}^3}{\text{kg}})(180 - 0.06)\,\text{bar}\left|\frac{10^5\,\text{N/m}^2}{1\,\text{bar}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N·m}}\right|$$

$$= 151.53 + 18.11 = 169.64\ \text{kJ/kg}$$

(a) The net work developed per unit mass of steam flow is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = (h_1 - h_2) - (h_4 - h_3) \longleftarrow \dot{W}_{cycle}/\dot{m}$$

$$= (2509.1 - 1569.4) - (169.64 - 151.53) = 921.6\ \text{kJ/kg}$$

(b) The rate of heat transfer per unit mass of steam flow through the boiler is

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_4) = 2509.1 - 169.64 = 2339.5\ \text{kJ/kg} \longleftarrow \dot{Q}_{in}/\dot{m}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{921.6}{2339.5} = 0.394\ (39.4\%) \longleftarrow \eta$$

(d) The rate of heat transfer to cooling water passing through the condenser is

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = 1569.4 - 151.53$$

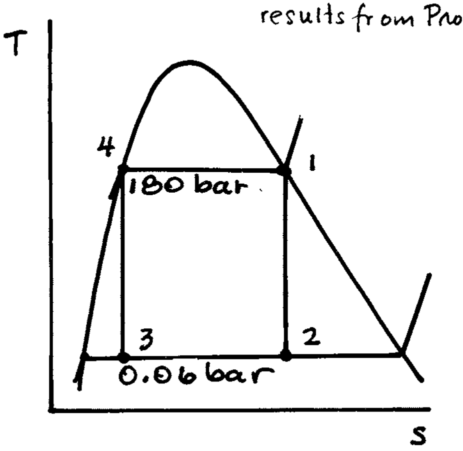
$$= 1417.9 \text{ kJ/kg} \longleftarrow \underline{\dot{Q}_{out}/\dot{m}}$$

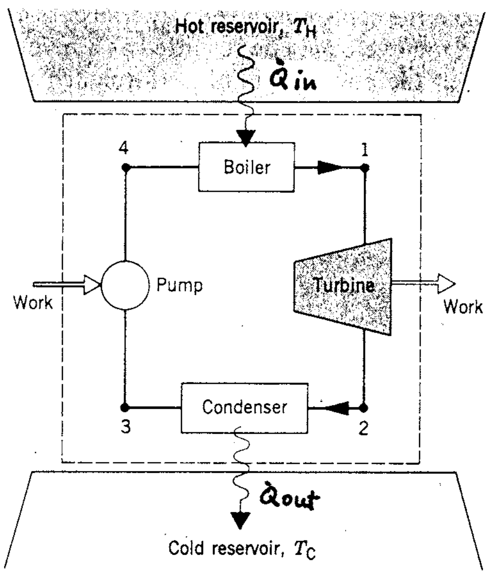
PROBLEM 8.7

KNOWN: Water is the working fluid in a Carnot vapor power cycle. The states at the boiler and turbine inlets are specified, and the condenser pressure is known.

FIND: Determine (a) the thermal efficiency, (b) the back-work ratio, (c) the net work per unit mass of water flow, (d) the heat transfer from the working fluid passing through the condenser per unit mass of steam flow.

FIND: Determine (a) the net work per unit mass of steam flow, (b) the heat transfer to the steam passing through the boiler per unit mass of steam flow, (c) the thermal efficiency, and (d) the heat transfer to cooling water passing through the condenser per unit mass of steam condensed. (e) Compare with results from Problem 8.6.





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The turbine and pump operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 180$ bar, sat. vapor $\Rightarrow h_1 = 2509.1$ kJ/kg, $s_1 = 5.1044$ kJ/kg·K

State 2: $p_2 = 0.06$ bar, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = 0.5869$, $h_2 = 1569.4$ kJ/kg

State 3: $p_3 = 0.06$ bar, $s_3 = s_4 = 3.8715$ kJ/kg·K $\Rightarrow x_3 = \frac{s_3 - s_{f3}}{s_{fg3}} = 0.429$
$h_3 = 1188.0$ kJ/kg

State 4: $p_4 = 180$ bar, sat. liquid $\Rightarrow h_4 = 1732.0$ kJ/kg

(a) The thermal efficiency of the Carnot cycle is given by Eq. 5.8. With data from Table A-3; $T_H = T_{sat @ 180 bar} = 630.25$ K and $T_c = T_{sat @ 0.06 bar} = 309.31$ K. Thus

① $$\eta_{max} = 1 - \frac{T_c}{T_H} = 1 - \frac{309.31}{630.25} = 0.509 \; (50.9\%) \longleftarrow \eta$$

(b) The back work ratio is

$$bwr = \frac{(\dot{W}_p/\dot{m})}{(\dot{W}_t/\dot{m})} = \frac{h_4 - h_3}{h_1 - h_2} = \frac{1732.0 - 1188.0}{2509.1 - 1569.4}$$

$$= \frac{544}{939.7} = 0.579 \; (57.9\%) \longleftarrow bwr$$

PROBLEM 8.7 (Cont'd)

(c) The net work per unit mass of water flow is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = (h_1 - h_2) - (h_4 - h_3)$$

$$= (2509.1 - 1569.4) - (1732.0 - 1188.0)$$

$$= 939.7 - 544 = 395.7 \text{ kJ/kg} \longleftarrow \dot{W}_{cycle}/\dot{m}$$

(d) From mass and energy rate balances for the working fluid passing through the condenser

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = 1569.4 - 1188.0 = 381.4 \text{ kJ/kg} \longleftarrow \dot{Q}_{out}/\dot{m}$$

(e) Compared to the results of Problem 8.6, in this case

- the net work per unit of mass flowing is less.
- the back work ratio is much greater
  Note: In Problem 8.6 the bwr = 0.019 (1.9%)
- $\dot{Q}_{out}/\dot{m}$ is greater

The Rankine cycle of Problem 8.6 avoids the work-intensive pumping process involving a two-phase liquid-vapor mixture of the present case: Process 3-4. See discussion on pp. 357-358 for further detail.

1· Alternatively

$$\eta = \left(\frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}}\right) = \frac{\dot{W}_{cycle}/\dot{m}}{h_1 - h_4} = \left(\frac{395.7}{2509.1 - 1732.0}\right) = 0.509$$

PROBLEM 8.8

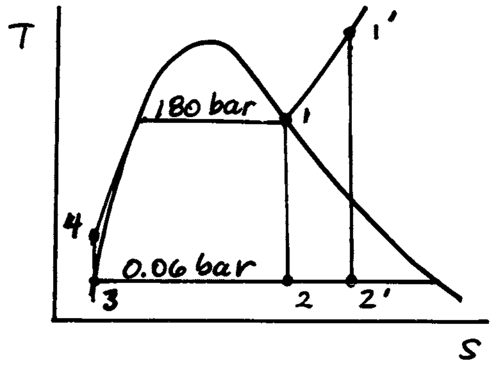
KNOWN: Water is the working fluid in an ideal Rankine cycle. The condenser and steam generator pressures are specified as in Problem 8.6.

FIND: Plot each of the quantities calculated in Problem 8.6 versus turbine inlet temperature ranging from saturation to superheated vapor.

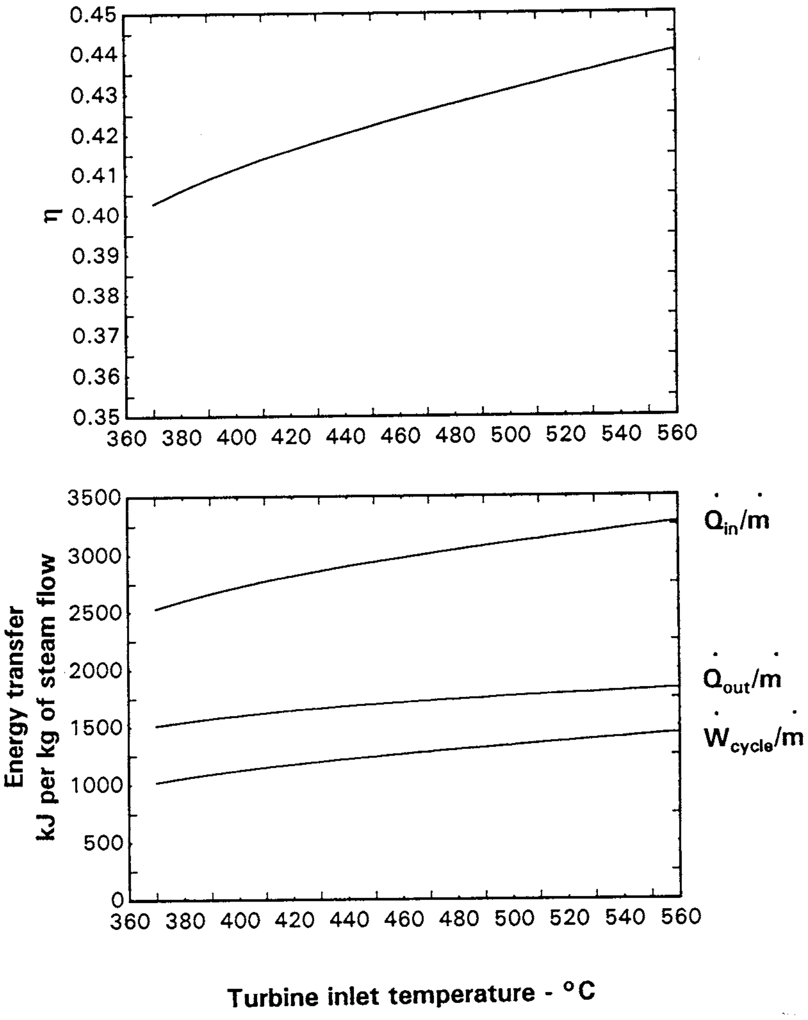
SCHEMATIC & GIVEN DATA: See Problem 8.6.

ASSUMPTIONS: See Problem 8.6.

ANALYSIS: The turbine inlet and exit states change as in the T-s diagram.



Plotting



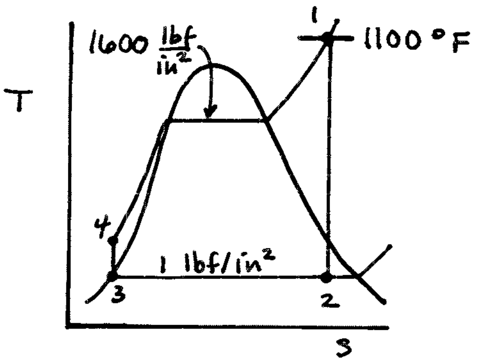
The cycle with superheating (1'-2'-3-4-1') has a higher temperature of heat addition than the cycle without superheating (1-2-3-4-1), so the thermal efficiency is higher.

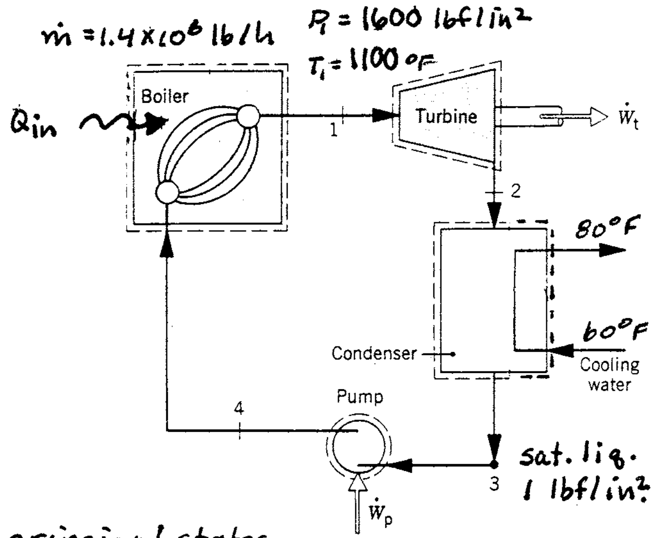
PROBLEM 8.9*

KNOWN: Water is the working fluid in an ideal Rankine cycle. The turbine inlet state and the condenser pressure are known. Also, the mass flow rate of steam entering the turbine is given.

FIND: Determine (a) the net power developed, (b) the thermal efficiency, and (c) the mass flow rate of cooling water.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 8.1, assumptions 1-4.

ANALYSIS: First, fix each of the principal states.

STATE 1: $p_1 = 1600$ lbf/in², $T_1 = 1100$ °F $\Rightarrow h_1 = 1547.7 \frac{\text{Btu}}{\text{lb}}$, $s_1 = 1.6315 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

STATE 2: $p_2 = 1$ lbf/in², $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = 0.81223$, $h_2 = 911.2$ Btu/lb

STATE 3: $p_3 = 1$ lbf/in², sat. liquid $\Rightarrow h_3 = 69.74$ Btu/lb

STATE 4: $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$= 69.74 \frac{\text{Btu}}{\text{lb}} + \left(0.01614 \frac{\text{ft}^3}{\text{lb}}\right)\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|(1600 - 1)\frac{\text{lbf}}{\text{in}^2}$$

$$= 69.74 + 4.78 = 74.52 \text{ Btu/lb}$$

(a) The net power developed by the cycle is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}\left[(h_1 - h_2) - (h_4 - h_3)\right]$$

$$= \left(1.4 \times 10^6 \frac{\text{lb}}{\text{h}}\right)\left[(1547.7 - 911.2) - (4.78)\right] \text{Btu/lb}$$

$$= 8.84 \times 10^8 \text{ Btu/h} \longleftarrow \dot{W}_{cycle}$$

(b) To find the cycle thermal efficiency, first find the heat transfer rate to the working fluid passing through the steam generator. Thus

$$\dot{Q}_{in} = \dot{m}\left[h_1 - h_4\right]$$

$$= \left(1.4 \times 10^6 \frac{\text{lb}}{\text{h}}\right)(1547.7 - 74.52)\frac{\text{Btu}}{\text{lb}}$$

$$= 2.06 \times 10^9 \text{ Btu/h}$$

PROBLEM 8.9 (Cont'd)

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{8.84 \times 10^8}{2.06 \times 10^9} = 0.429\ (42.9\%) \longleftarrow \underline{\qquad\qquad\qquad\qquad \eta}$$

(c) For the control volume enclosing the condenser

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}(h_2 - h_3) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

where $\dot{m}_{cw}$ is the mass flow rate of the cooling water. Thus

$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{(h_{cw,out} - h_{cw,in})}$$

For the cooling water, $h_{cw} \approx h_f(T)$. Thus, with data from Table A-2E

$$\dot{m}_{cw} = \frac{(1.4 \times 10^6\ \text{lb/h})(911.2 - 69.74)\ \text{Btu/lb}}{(48.09 - 28.08)\ \text{Btu/lb}}$$

$$= 5.89 \times 10^7\ \text{lb/h} \longleftarrow \underline{\qquad\qquad\qquad\qquad \dot{m}_{cw}}$$

PROBLEM 8.10*

KNOWN: Water is working fluid in an ideal Rankine cycle. Data are given in Prob. 8.9.

FIND: Plot each of the quantities calculated in Prob. 8.9 versus condenser pressure ranging from 0.4 to 14.7 lbf/in.^{2} Maintain constant steam mass flow rate.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.9.

ASSUMPTIONS: See Example 8.1, assumptions 1-4.

ANALYSIS: The following IT code is used to develop the data for the required plots. Results are presented for the two limiting cases.

IT Code

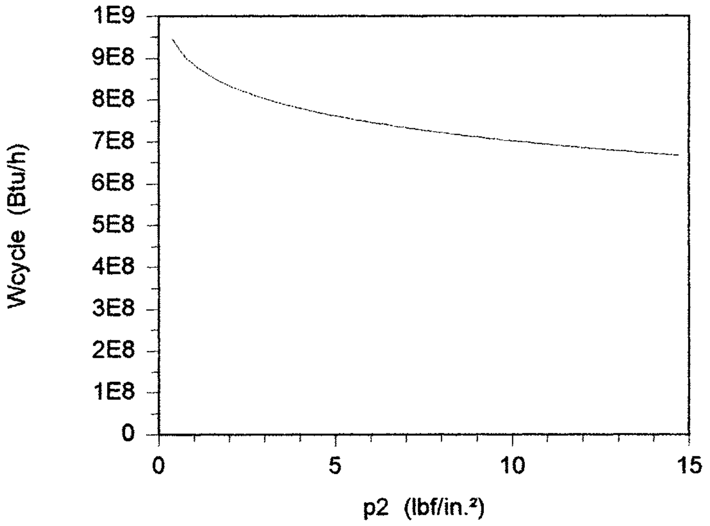
```
p1 = 1600  // lbf/in.²
T1 = 1100  // °F
p2 = 1
p3 = p2
x3 = 0
p4 = p1
mdot = 1.4E06
Twin = 60
Twout = 80

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
h3 = hsat_Px("Water/Steam", p3, x3)
v3 = vsat_Px("Water/Steam", p3, x3)
h4 = h3 + v3 * (p4 - p3) * (144 / 778)
pw1 = Psat_T("Water/Steam", Twin)
hwin = hsat_Px("Water/Steam", pw1, 0)
pw2 = Psat_T("Water/Steam", Twout)
hwout = hsat_Px("Water/Steam", pw2, 0)

Wcycle = mdot * ((h1 - h2s) - (h4 - h3))
Qin = mdot * (h1 - h4)
eta = Wcycle / Qin
0 = mdot * (h2s - h3) + mdotw * (hwin - hwout)
```

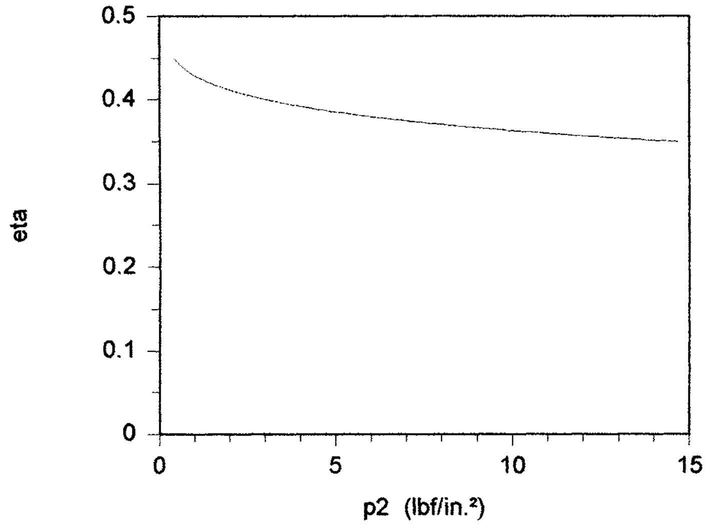
IT Results for p2 = 0.4 and 14.7 lbf/in^2

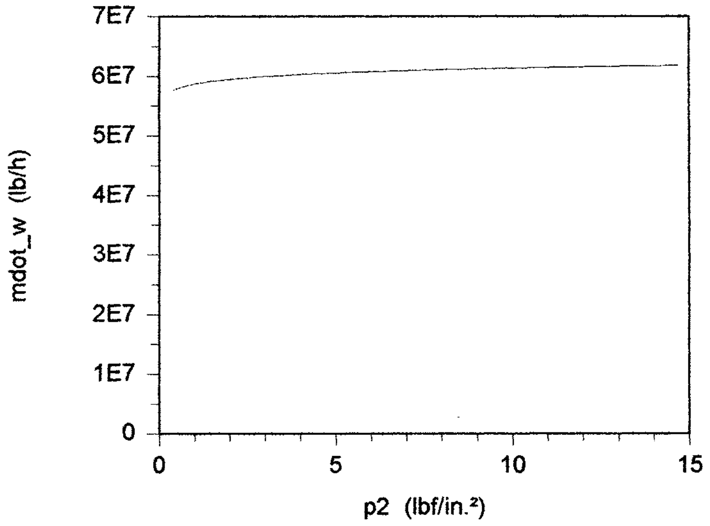
p2 (lbf/in2)	0.4	14.7
h1 (Btu/lb)	1547	1547
s1 (Btu/lb.R)	1.631	1.631
h2s	867.1	1066
h3	40.49	180.4
h4	45.25	185.3
hwin	27.63	27.63
hwout	47.7	47.7
v3 (ft3/lb)	0.01606	0.01671
Qin (Btu/h)	2.103×10^9	1.907×10^9
Wcycle (Btu/h)	9.46×10^8	6.669×10^8
eta	0.4498	0.3497
mdotw (lb/h)	5.765×10^7	6.178×10^7

PROBLEM 8.10 (Cont'd.)

PLOTS :







We see from the plots that as condenser pressure decreases, the power output increases as does the cycle thermal efficiency. Further, less energy is rejected in the conder, so the cooling water flow rate decreases slightly. These results are consistent with the discussion of Fig. 8.4(b) in Sec. 8.2.3.

PROBLEM 8.11*

KNOWN: Water is the working fluid in an ideal Rankine cycle. Data are given in Prob. 8.9.

FIND: Plot each of the quantities calculated in Prob. 8.9 versus steam generator pressure ranging from 250 to 4000 lbf/in.² Maintain constant steam mass flow rate and turbine inlet temperature.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.9.

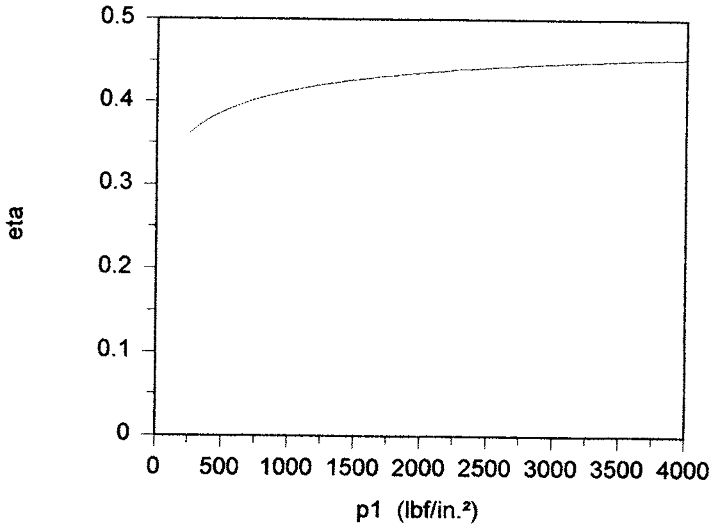
ASSUMPTIONS: See example 8.1, assumptions 1-4.

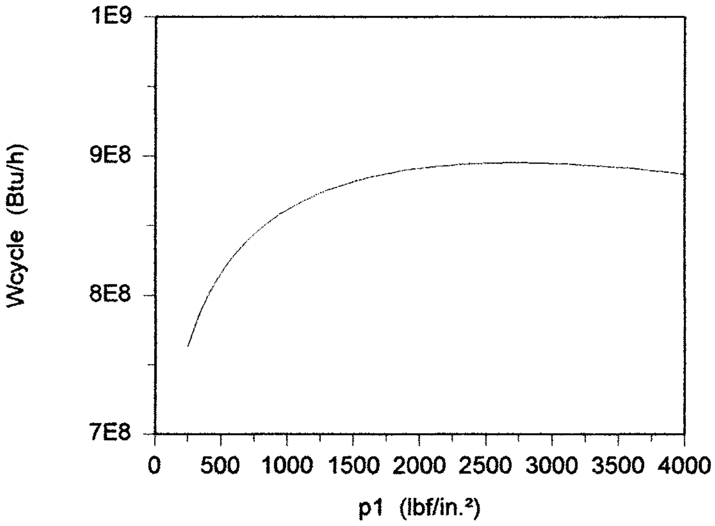
ANALYSIS: See solution to Prob. 8.10 for the IT code. Results are presented here for the two limiting cases.

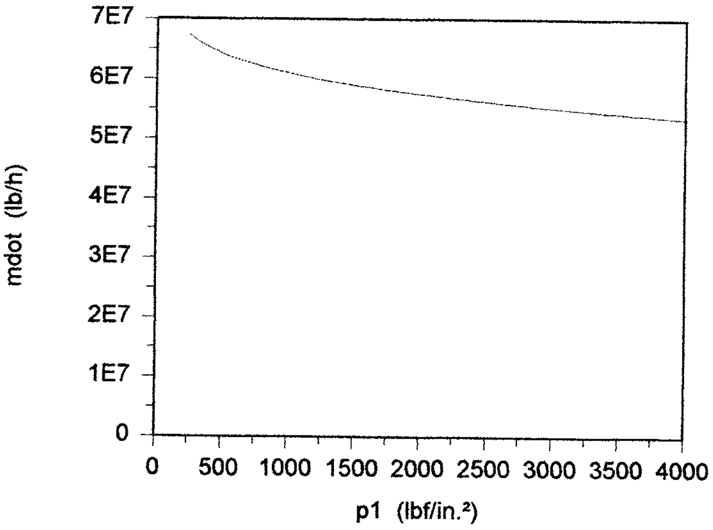
IT Results for p1 = 250 and 4000 lbf/in^2

p1 (lbf/in^2)	250	4000
h1 (Btu/lb)	1581	1481
s1 (Btu/lb.R)	1.852	1.497
h2s	1035	835.8
h3	69.58	69.58
h4	70.32	81.53
hwin	27.63	27.63
hwout	47.7	47.7
v3 (ft^3/lb)	0.01614	0.01614
Qin (Btu/h)	2.115×10^9	1.96×10^9
Wcycle (Btu/h)	7.629×10^8	8.872×10^8
eta	0.3608	0.4527
mdotw (lb/h)	6.734×10^7	5.343×10^7

PLOTS:







We see from the plots that the power output exhibits a maximum, but the thermal efficiency increases with increasing steam generator pressure. Further, less energy is rejected in the condenser, so the cooling water flow rate decreases slightly. For further discussion, see Sec. 8.2.3.

PROBLEM 8.12

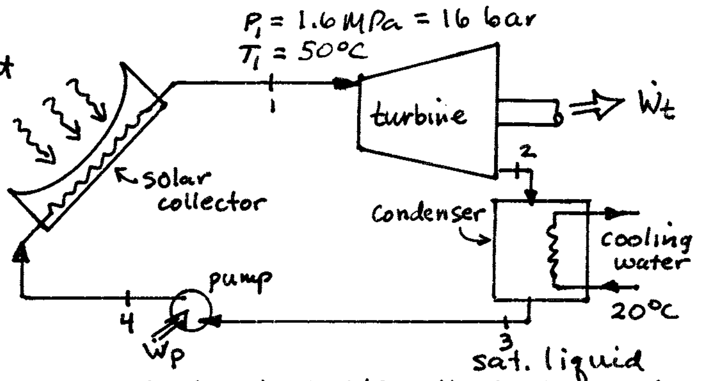
KNOWN: A power plant using R-22 as the working fluid is under development based on the Rankine cycle. The desired net power output, turbine inlet state, and available cooling water temperature are specified.

FIND: Specify the preliminary design of the cycle, estimate the thermal efficiency and steam and cooling water flow rates.

SCHEMATIC & GIVEN DATA:

ASSUMPTIONS: (1) Each component operates at steady state. (2) There are no stray heat transfers from any component. (3) Kinetic and potential energy effects are negligible. (4) Saturated liquid exits the condenser.

(Other assumptions discussed below in developing the preliminary design.)



ANALYSIS: To maximize performance, it is desirable that the condenser pressure be as low as possible. For $p_{cond} = 12$ bar; $T_{sat} = 30.25$ °C. With this choice, it is likely that the temperature rise of the cooling water passing through the condenser would be about 15°C.

Further, a characteristic value of $\eta_t \approx 85\%$ is reasonable for the preliminary design. Also, let $\eta_p \approx 70\%$. With these choices, each principal state can be fixed.

State 1: $p_1 = 16$ bar, $T_1 = 50$°C ⇒ $h_1 = 269.18$ kJ/kg, $s_1 = 0.8971$ kJ/kg·K

State 2: $p_2 = 12$ bar, $s_{2s} = s_1$ ⇒ $h_{2s} = 262.2$ kJ/kg. Thus, with the turbine efficiency

$$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 263.2 \text{ kJ/kg}$$

State 3: $p_3 = 12$ bar, sat. liquid ⇒ $h_3 = 81.90$ kJ/kg

State 4: $h_{4s} \approx h_3 + v_3(p_4 - p_3)$

$$= 81.90 \frac{\text{kJ}}{\text{kg}} + (.8546 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (16-12) \text{ bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}} \right| = 82.24 \frac{\text{kJ}}{\text{kg}}$$

Thus, with the pump efficiency

$$h_4 = h_3 + (h_{4s} - h_3)/\eta_p = 82.39 \text{ kJ/kg}$$

Using this data

$$\left.\begin{aligned} \dot{W}_t/\dot{m} &= h_1 - h_2 = 5.98 \text{ kJ/kg} \\ \dot{W}_p/\dot{m} &= h_4 - h_3 = 0.49 \text{ kJ/kg} \\ \dot{Q}_{in}/\dot{m} &= h_1 - h_4 = 186.8 \text{ kJ/kg} \end{aligned}\right\} \eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 0.0294 \; (2.94\%) \longleftarrow \eta$$

For a net power output of 10 MW = 10,000 kJ/s

$$\dot{m} = \frac{\dot{W}_{cycle}}{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}} = \frac{10{,}000 \text{ kJ/s}}{5.98 - 0.49 \text{ kJ/kg}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 6.56 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}$$

With $h_{cw} \approx h_f(T_{cw})$, the cooling water flow rate $\dot{m}_{cw}$ is

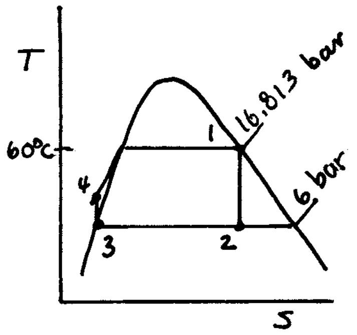
$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{(h_{cw,out} - h_{cw,in})} = \frac{(6.56 \times 10^6)(263.2 - 81.90)}{(117.43 - 83.96)} = 35.53 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}_{cw}$$

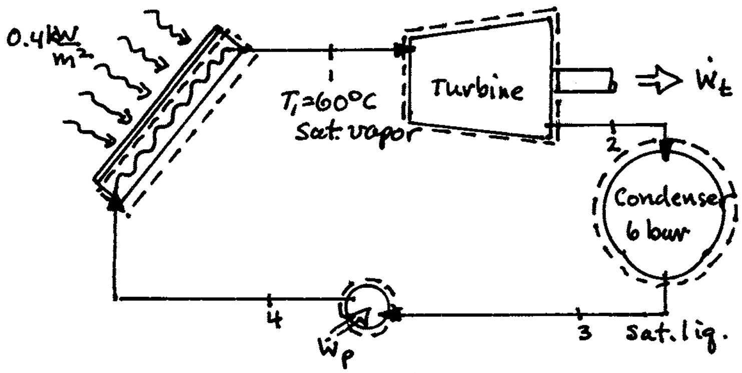
PROBLEM 8.13

KNOWN: Refrigerant 134a is the working fluid in a solar power plant operating on a Rankine cycle. The turbine inlet state and the condenser pressure are known.

FIND: Determine the minimum possible solar collector surface area per kW developed by the plant.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each control volume is at steady state. (2) All processes of the working fluid are internally reversible. (3) There are no stray heat transfers, and all solar radiation incident on the collector is absorbed by the working fluid. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the condenser as saturated liquid at 6 bars.

ANALYSIS: Fix each of the principal states.

State 1: $T_1 = 60°C$, sat. vapor $\Rightarrow h_1 = 275.99$ kJ/kg, $s_1 = 0.8973$ kJ/kg·K

State 2: $p_2 = 6$ bar, $s_2 = s_1 \Rightarrow x_2 = 0.9797$, $h_2 = 255.54$ kJ/kg

State 3: $p_3 = 6$ bar, sat. liquid $\Rightarrow h_3 = 79.48$ kJ/kg

State 4: $h_4 \approx h_3 + v_3(p_4 - p_3)$

$$= 79.48\ \frac{kJ}{kg} + (.8196\times10^{-3})\frac{m^3}{kg}(16.813-6)\,bar\left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 79.48 + 0.89 = 80.37\ kJ/kg$$

The ideal Rankine cycle described here requires the minimum $\dot{Q}_{in}$ to develop 1 kW under the prescribed constraints for turbine inlet state and condenser pressure. The mass flow rate of refrigerant is obtained from

$$\dot{W}_{cycle} = \dot{m}\left[\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}\right] = \dot{m}\left[(h_1-h_2)-(h_4-h_3)\right]$$

or

$$\dot{m} = \frac{(1\ kJ/s)}{[(275.99-255.54)-(.89)]\,kJ/kg} = 0.05112\ kg/s$$

Thus, the minimum value of $\dot{Q}_{in}$ is

$$\dot{Q}_{in} = \dot{m}(h_1-h_4) = (0.05112\ \frac{kg}{s})(275.99-80.37)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 10.00\ kW$$

$$\begin{pmatrix}\text{minimum}\\ \text{collector}\\ \text{area}\end{pmatrix} = \frac{10.00\ kW}{0.4\ kW/m^2} = 25\ m^2/kW \text{ power developed} \leftarrow \underline{\text{area}}$$

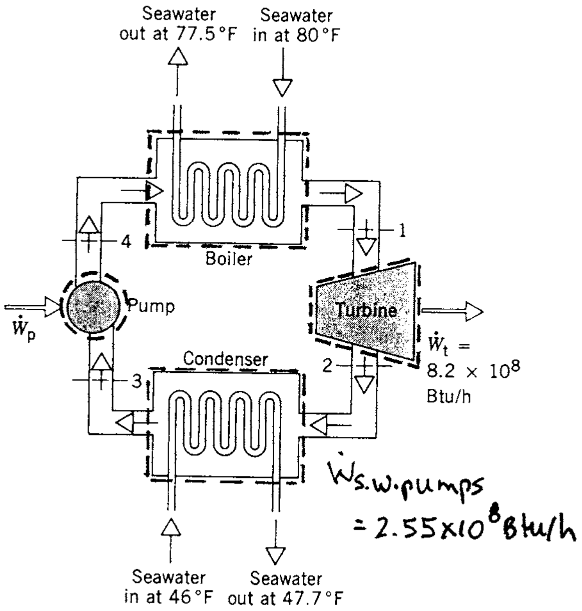
Irreversibilities or stray heat transfer from the collector will result in greater collector area being required.

PROBLEM 8.14

KNOWN: A Rankine cycle power plant, using ammonia as the working fluid, has been proposed to utilize the naturally-occurring temperature gradient within to ocean to produce electrical power.

FIND: Determine (a) the thermal efficiency and compare it to that of a reversible cycle operating between reservoirs at the seawater temperatures, (b) the net power output, and (c) the seawater flow rates through the boiler and condenser.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each control volume shown is at steady state, and $\dot{Q}_{cv} = 0$. (2) kinetic and potential energy effects can be neglected. (3) The seawater has properties of pure water, assumed to be an incompressible liquid with $c = 1.0$ Btu/lb·°R.

(Other assumptions are discussed below in developing the ammonia cycle.)

ANALYSIS: To maximize performance, it is desirable that the boiler pressure be as ① high a possible, and the condenser pressure be as low as possible. Based on the seawater temperatures in the boiler, let us assume an evaporating temperature of 15°F for the ammonia. Thus

State 1: $T_1 = 75°F, \; x_1 = 1.0 \Rightarrow p_1 = 140.60 \; lbf/in^2, \; h_1 = 629.20 \; Btu/lb$

$s_1 = 1.2048 \; Btu/lb \cdot °R$

Further, from the seawater temperatures in the condenser, assume a condensing temperature of the ammonia of 50°F. Thus, $p_2 = p_{sat\,@\,50°F} = 89.242 \frac{lbf}{in^2}$ and

State 2: $T_2 = 50°F, \; s_{2s} = s_1 \Rightarrow x_{2s} = 0.9625, \; h_{2s} = 604.8 \; Btu/lb$

With the turbine efficiency of 80% (assumed)

$$h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 609.7 \; Btu/lb$$

State 3: $p_3 = p_2$, sat. liquid $\Rightarrow h_3 = 97.55 \; Btu/lb$

State 4: $p_4 = p_1; \; h_{4s} \approx h_3 + v_3 (p_4 - p_3)$

$$= 97.55 \frac{Btu}{lb} + (0.02564 \; ft^3/lb)(140.6 - 89.242) \frac{lbf}{in^2} \left| \frac{144 \; in^2}{1 \; ft^2} \right| \left| \frac{1 \; Btu}{778 \; ft \cdot lbf} \right|$$

$$= 97.79 \; Btu/lb$$

Assuming a pump efficiency of 70%

$$h_4 \approx h_3 + (h_{4s} - h_3)/\eta_p = 97.893 \; Btu/lb$$

(a) For the ammonia passing through the boiler, the heat transfer from the seawater is

$$\dot{Q}_{in}/\dot{m} = h_1 - h_4 = (629.20 - 97.893) \; Btu/lb = 531.3 \; Btu/lb$$

PROBLEM 8.14 (Cont'd)

Similarly, for the ammonia passing through the condenser

$$\dot{Q}_{out}/\dot{m} = h_2 - h_3 = 609.7 - 97.55 = 512.2 \text{ Btu/lb}$$

Thus, the thermal efficiency is

$$\eta = 1 - \frac{\dot{Q}_{out}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 1 - \frac{512.2}{531.3} = 0.0359 \ (3.16\%) \longleftarrow \eta$$

For the reversible cycle, with $T_c = 506°R$ and $T_H = 540°R$

$$\eta_{max} = 1 - \frac{T_c}{T_H} = 0.0630 \ (6.3\%) \longleftarrow \eta_{max}$$

(b) The mass flow rate of ammonia is

$$\dot{m} = \dot{W}_t / (h_1 - h_2) = (8.2 \times 10^8 \text{ Btu/h}) / (629.2 - 609.7) \text{ Btu/lb} = 4.205 \times 10^7 \text{ lb/h}$$

Thus $\dot{W}_p = \dot{m}(h_4 - h_3) = 1.38 \times 10^7 \text{ Btu/h}$

and $\dot{W}_{net} = \dot{W}_t - \dot{W}_p - \dot{W}_{s.w.pumps} = 5.512 \times 10^8 \text{ Btu/h} \longleftarrow \dot{W}_{net}$

(c) Taking the boiler control volume overall

$$\dot{m}_{s.w.\,boiler} = \frac{\dot{m}(h_1 - h_4)}{c\,(T_{w,in} - T_{w,out})} = \frac{(4.205 \times 10^7)(531.3)}{(1)(80 - 77.5)} = 8.94 \times 10^9 \text{ lb/h} \longleftarrow$$

and for the condenser

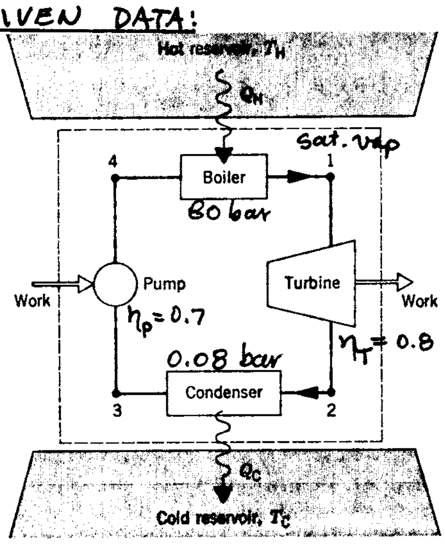
$$\dot{m}_{s.w.\,condenser} = \frac{(4.205 \times 10^7)(512.2)}{(1)(47.7 - 46)} = 1.267 \times 10^{10} \text{ lb/h} \longleftarrow$$

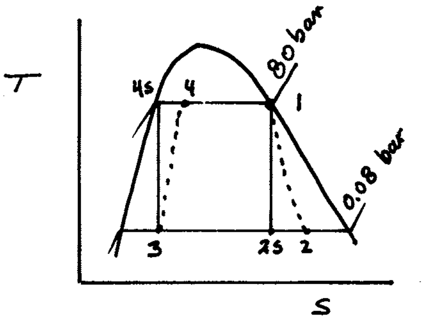
PROBLEM 8.15

KNOWN: A Carnot vapor power cycle is modified to include the effects of irreversibilities in the adiabatic expansion and compression processes.

FIND: Repeat the analysis of Problem 8.3 for the modified cycle.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The processes of the working fluid in the boiler and condenser are internally reversible. (3) The turbine and the pump operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: From Problem 8.3, $h_1 = 2758.0$ kJ/kg and $h_3 = 996.46$ kJ/kg. The specific enthalpy at state 2 is found using the turbine efficiency

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s})$$

With $h_{2s} = 1794.8$ kJ/kg from Problem 8.3, $h_2 = 1987.4$ kJ/kg. Similarly, the specific enthalpy at the pump exit is found using the pump efficiency.

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + \frac{h_{4s} - h_3}{\eta_p}$$

with $h_{4s} = 1316.6$ kJ/kg ; $h_4 = 1453.8$ kJ/kg.

(a) The thermal efficiency is

$$\eta = 1 - \frac{(\dot{Q}_{out}/\dot{m})}{(\dot{Q}_{in}/\dot{m})} = 1 - \frac{h_2 - h_3}{h_1 - h_4} = 0.24 \ (24\%) \longleftarrow \eta$$

(b) The back work ratio is

$$bwr = \frac{h_4 - h_3}{h_1 - h_2} = 0.597 \ (59.7\%) \longleftarrow bwr$$

(c) For the boiler

$$(\dot{Q}_{in}/\dot{m}) = h_1 - h_4 = 1304.2 \text{ kJ/kg} \longleftarrow \dot{Q}_{in}/\dot{m}$$

(d) For the condenser

$$(\dot{Q}_{out}/\dot{m}) = h_2 - h_3 = 990.9 \text{ kJ/kg} \longleftarrow \dot{Q}_{out}/\dot{m}$$

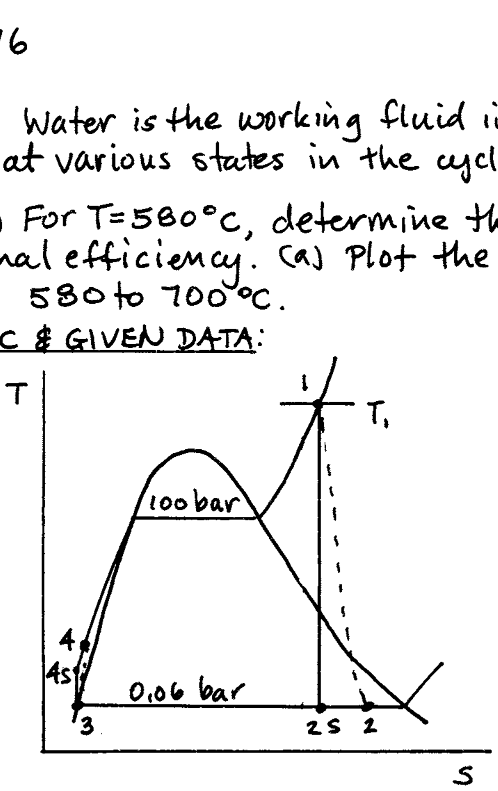
PROBLEM 8.16

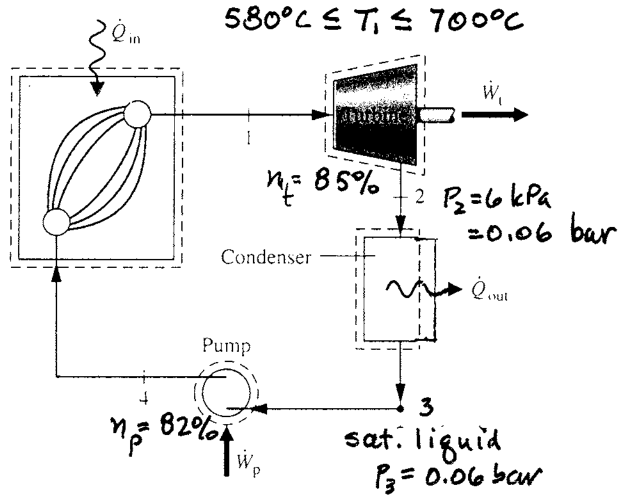
KNOWN: Water is the working fluid in a simple vapor power plant. Data are given at various states in the cycle. The turbine inlet temperature is T.

FIND: (a) For T=580°C, determine the turbine exit quality and the cycle thermal efficiency. (a) Plot the quantities of part (a) for T varying from 580 to 700°C.

SCHEMATIC & GIVEN DATA:

$P_1 = 10$ MPa $= 100$ bar
$580°C \le T_1 \le 700°C$





ASSUMPTIONS: See Example 8.1, assumptions 1-4.

ANALYSIS: First, fix each of the principal states.

State 1 $P_1 = 100$ bar, T_1 fix the state in the superheat region $\Rightarrow h_1, s_1$

State 2 P_2 and $s_{2s} = s_1$ gives x_{2s}, h_{2s}. Now, $h_2 = h_1 - (h_1 - h_{2s})/\eta_t \Rightarrow x_2$ (1)

State 3 $P_3 = 0.06$ bar, sat. liquid $\Rightarrow h_3, v_3$

State 4 $h_4 = h_3 + v_3(P_4 - P_3)/\eta_p$

Now, the net work of the cycle is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = (h_1 - h_2) - (h_4 - h_3) \qquad (2)$$

And, for the steam generator, $\dot{Q}_{in}/\dot{m} = h_1 - h_4$ (3)

Thus
$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1-h_2)-(h_4-h_3)}{(h_1 - h_4)} \qquad (4)$$

(a) For $T_1 = 580°C$, Table A-4 gives $h_1 = 3575.7$ kJ/kg, $s_1 = 6.8447$ kJ/kg·K. Thus $s_{2s} = s_1 = 6.8447 \Rightarrow x_{2s} = 0.80975 \Rightarrow h_{2s} = 2107.8$ kJ/kg. With $\eta_t = 0.85$

$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 3575.7 - (.85)(3575.7 - 2107.8) = 2328.0$ kJ/kg $\Rightarrow x_2 = 0.901 \leftarrow x_2$

Further, from Table A-3, $h_3 = 151.53$ kJ/kg, $v_3 = 1.0064 \times 10^{-3}$ m³/kg. Thus

$$h_4 = 151.53 \frac{kJ}{kg} + (1.0064\times10^{-3})\frac{m^3}{kg}(100 - 0.06)bar\left|\frac{10^5 N/m^2}{1 bar}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right| / (0.82)$$
$$= 163.8 \text{ kJ/kg}$$

Now
$$\eta = \frac{(h_1-h_2)-(h_4-h_3)}{(h_4 - h_4)} = \frac{(3575.7 - 2328) - (163.8 - 151.53)}{(3575.7 - 163.8)}$$

$$= \frac{1235.4}{3411.9} = 0.362 \longleftarrow \eta$$

PROBLEM 8.16 (Cont'd.)

(b) The data for the required plots are obtain using *IT*, as follows:

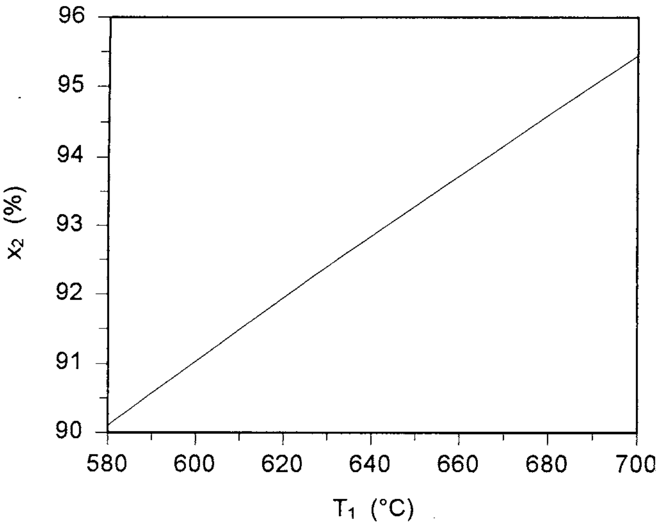
IT Code

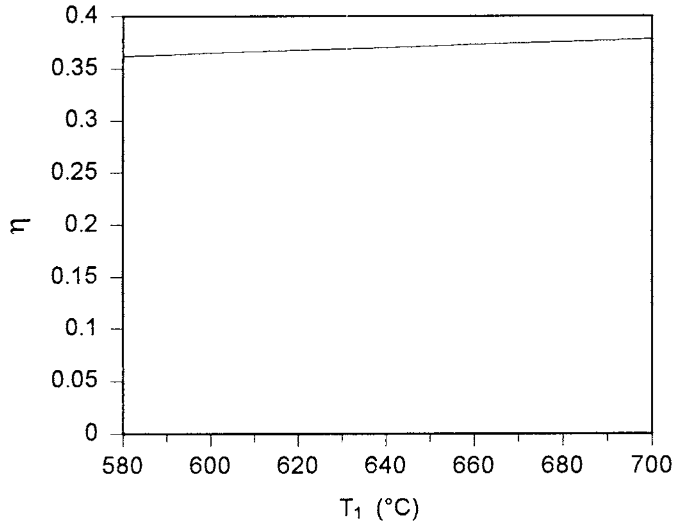
```
p1 = 100  // bar
T1 = 580  // °C
p2 = 0.06  // bar
eff_t = 0.85
eff_p = 0.82

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1

h2 = h1 - eff_t * (h1 - h2s)
h2 = hsat_Px("Water/Steam", p2, x2)
x2pct = x2 * 100
p3 = p2
h3 = hsat_Px("Water/Steam", p3, 0)
v3 = vsat_Px("Water/Steam", p3, 0)
p4 = p1
h4 = h3 + (v3 * (p4 - p3) * 100) / eff_p

Wcycle = (h1 - h2) - (h4 - h3)

Qin = h1 - h4
eta = Wcycle/Qin
```

IT Results for $T_1 = 580°C$

$h_1 = 3575$ kJ/kg
$s_1 = 6.845$ kJ/kg·K
$h_2 = 2328$ kJ/kg
$h_{2s} = 2108$ kJ/kg
$h_3 = 151$ kJ/kg
$v_3 = 0.001007$ m³/kg
$h_4 = 163.3$ kJ/kg
$\dot{Q}_{in}/\dot{m} = 3412$ kJ/kg
$\dot{W}_{cycle}/\dot{m} = 1235$ kJ/kg
$x_2 = 90.1$ %
$\eta = 0.362$

PLOTS:





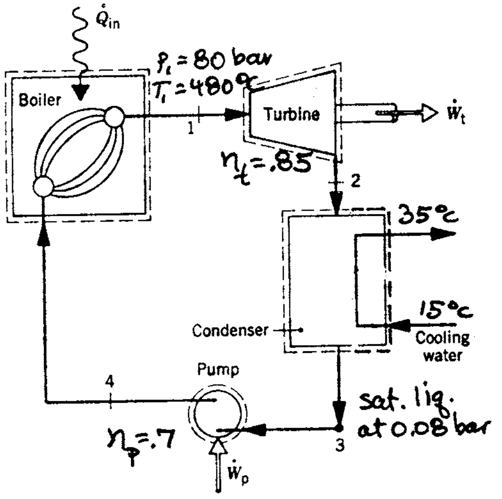
From the T-s diagram, we see that as T_1 increases, points 2s and 2 move to the right. Thus, x_2 increases. Further, as T_1 increases, the average temperature of heat addition increases. Thus, η increases slightly, as indicated in the above plot.

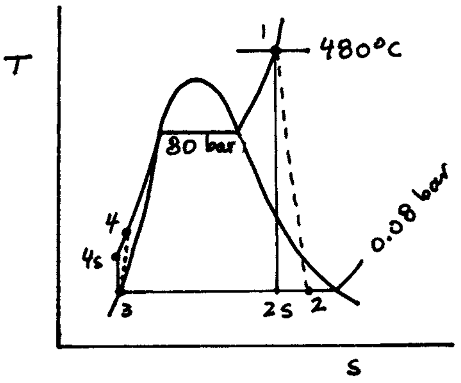
PROBLEM 8.17

KNOWN: The ideal Rankine cycle of Problem 8.2 is modified to include that the turbine and pump have isentropic efficiencies of 85 and 75 %, respectively.

FIND: Determine for the modified cycle (a) the thermal efficiency, (b) the mass flow rate of steam for a net power output of 100 MW, and (c) the mass flow rate of condenser cooling water.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Problem 8.2, except $\eta_T = 0.85$ and $\eta_P = 0.7$.

ANALYSIS: From Problem 8.2, $h_1 = 3348.4$ kJ/kg and $h_3 = 173.88$ kJ/kg. The specific enthalpy at state 2 is found using the turbine efficiency

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t (h_1 - h_{2s})$$

From Problem 8.2, $h_{2s} = 2082.9$ kJ/kg. Thus, $h_2 = 2272.7$ kJ/kg. Similarly, the specific enthalpy at state 4 is found using the pump efficiency

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + \frac{h_{4s} - h_3}{\eta_p}$$

From Problem 8.2, $h_{4s} = 181.94$ kJ/kg. Thus, $h_4 = 185.39$ kJ/kg.

(a) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

$$= \frac{(3348.4 - 2272.7) - (185.39 - 173.88)}{(3348.4 - 185.39)}$$

$$= 0.336 \; (33.6\%) \leftarrow \eta$$

(b) The mass flow rate of steam for a net power output of 100 MW is determined from

$$\dot{W}_{cycle} = \dot{m}\left[\dot{W}_t/\dot{m} - \dot{W}_P/\dot{m}\right] = \dot{m}\left[(h_1 - h_2) - (h_4 - h_3)\right]$$

PROBLEM 8.17 (Contd.)

Thus, solving for $\dot{m}$

$$\dot{m} = \frac{\dot{W}_{cycle}}{(h_1-h_2)-(h_4-h_3)}$$

$$= \frac{(100 \times 10^3 \text{ kJ/s})}{[(3348.4-2272.7)-(185.39-173.88)] \text{ kJ/kg}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right|$$

$$= 3.38 \times 10^5 \text{ kg/h} \longleftarrow \dot{m}$$

(c) The mass flow rate of cooling water is found from

$$0 = \dot{m}(h_2-h_3) + \dot{m}_{cw}(h_{in,cw} - h_{out,cw})$$

or

$$\dot{m}_{cw} = \frac{m(h_2-h_3)}{(h_{cw,out} - h_{cw,in})}$$

With specific enthalpies of the cooling water from Problem 8.2

$$\dot{m}_{cw} = \frac{(3.38 \times 10^5 \text{ kg/h})(2272.7 - 173.88) \text{ kJ/kg}}{(146.68 - 62.99) \text{ kJ/kg}}$$

$$= 8.48 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}_{cw}$$

These results can be compared with results from Problem 8.2 to see some effects of irreversibilities in the turbine and pump on the ideal Rankine cycle.

PROBLEM 8.18

KNOWN: Water is the working fluid in a Rankine cycle. The condenser and steam generator pressures are specified as in Problem 8.8.

FIND: Plot each of the quantities calculated in Problem 8.8, including in the analysis that the turbine and pump each have isentropic effiencies of (a) 90%, (b) 80%, (c) 70%.

SCHEMATIC & GIVEN DATA: See Problem 8.6.

ASSUMPTIONS: See Problem 8.6, except $\eta_t = \eta_p < 100\%$.

ANALYSIS:

For the turbine:

$$h_2 = h_1 - \eta_t (h_1 - h_{2s})$$

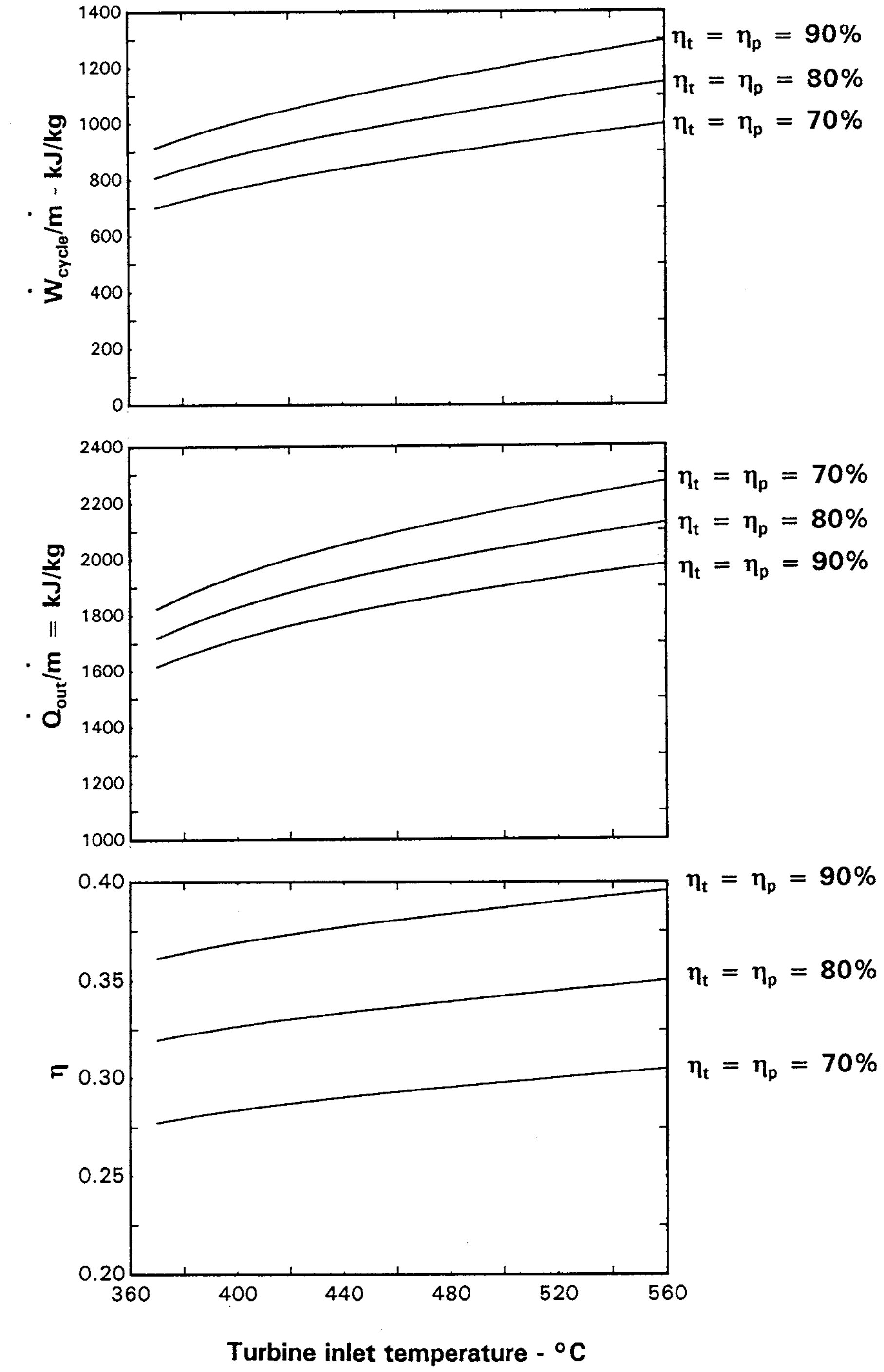
For the pump:

$$h_4 = h_3 + v_3 (p_4 - p_3)/\eta_p$$

Note that:

$$\dot{Q}_{in}/\dot{m} = h_1 - h_4$$

is only slightly affected by η_p, since h_4 varies minimally. Therefore, no plot of $\dot{Q}_{in}/\dot{m}$ vs. T_1 is included.

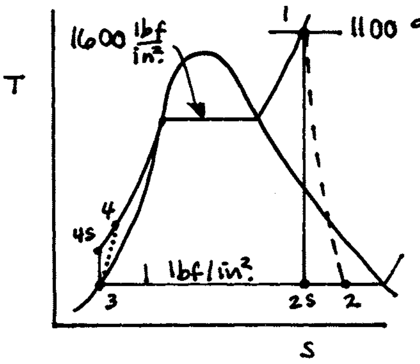


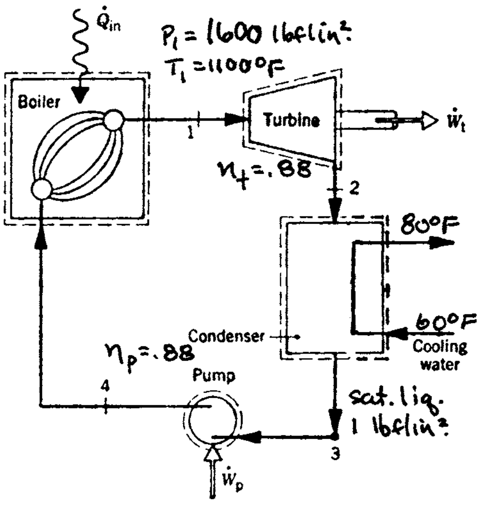
PROBLEM 8.19*

KNOWN: The ideal Rankine cycle of Problem 8.9 is modified to include that the turbine and pump each have an isentropic efficiency of 88%.

FIND: Determine (a) the net power developed, (b) the rate of heat transfer to the working fluid passing through the steam generator, (c) the thermal efficiency, and (d) the volumetric flow rate of the cooling water entering the condenser.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Problem 8.9, except $\eta_t = \eta_p = 0.88$.

ANALYSIS: From Problem 8.9, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R and $h_3 = 69.74$ Btu/lb. The specific enthalpy at state 2 is found using the turbine efficiency

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t (h_1 - h_{2s})$$

From Problem 8.9, $h_{2s} = 911.2$ Btu/lb. Thus, $h_2 = 987.6$ Btu/lb. Similarly, the specific enthalpy at state 4 is found using the pump efficiency

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + (h_{4s} - h_3)/\eta_p$$

From Problem 8.9, $h_{4s} = 74.52$ Btu/lb. Thus, $h_4 = 75.17$ Btu/lb.

(a) The net power developed is

$$\dot{W}_{cycle} = \dot{m}[(h_1 - h_2) - (h_4 - h_3)] = (1.4 \times 10^6 \tfrac{lb}{h})[(1547.7 - 987.6) - (75.17 - 69.74)] \tfrac{Btu}{lb}$$

$$= 7.77 \times 10^8 \text{ Btu/h} \longleftarrow \underline{\dot{W}_{cycle}}$$

(b) The heat transfer rate for the steam generator is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = (1.4 \times 10^6 \text{ lb/h})(1547.7 - 75.17) \text{ Btu/lb}$$

$$= 2.062 \times 10^9 \text{ Btu/h} \longleftarrow \underline{\dot{Q}_{in}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{7.77 \times 10^8}{2.062 \times 10^9} = 0.377\ (37.7\%) \longleftarrow \underline{\eta}$$

(d) The mass flow rate of cooling water through the condenser is

$$\dot{m}_{cw} = \frac{\dot{m}(h_2 - h_3)}{h_{cw,out} - h_{cw,in}} = \frac{(1.4 \times 10^6 \text{ lb/h})(987.6 - 69.74) \text{ Btu/lb}}{(48.09 - 28.08) \text{ Btu/lb}\ |60 \text{ min/h}|} = 1.071 \times 10^6 \text{ lb/min}$$

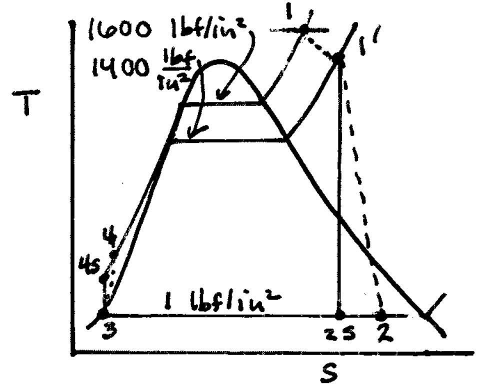
With $v_{cw} = v_f(60°F) = .01604$ ft³/lb, $(AV) = v_{cw}\dot{m}_{cw} = 17{,}176$ ft³/min $\longleftarrow \underline{(AV)_{cw}}$

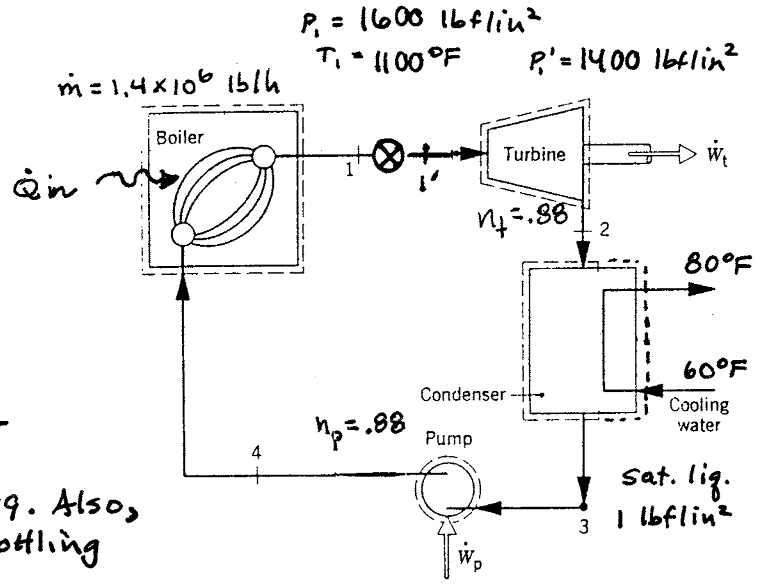
PROBLEM 8.20*

KNOWN: The cycle of Probem 8.19 is modified to include a throttling valve between the steam generator and the turbine inlet.

FIND: Repeat the analysis

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Problem 8.19. Also, process across the valve is a throttling process.

ANALYSIS: From Problem 8.19; $h_1 = 1547.7$ Btu/lb, $h_3 = 69.74$ Btu/lb, and $h_4 = 75.17$ Btu/lb. Further, $h_{1'} = h_1 = 1547.7$ Btu/lb. Interpolating in Table A-4E; $s_{1'} = 1.6453$ Btu/lb·°R. The specific enthalpy at State 2 is found using the turbine efficiency

$$\eta_t = \frac{(\dot W_t/\dot m)}{(\dot W_t/\dot m)_s} = \frac{h_{1'} - h_2}{h_{1'} - h_{2s}} \Rightarrow h_2 = h_{1'} - \eta_t(h_{1'} - h_{2s})$$

Using data from Table A-3E; $x_{2s} = (s_{2s} - s_{f2})/s_{fg2} = 0.8197$, and $h_{2s} = 918.96$ Btu/lb. Thus, $h_2 = 994.41$ Btu/lb.

(a) The net power developed is

$$\dot W_{cycle} = \dot m\left[(h_{1'} - h_2) - (h_4 - h_3)\right] = \left(1.4\times10^6\,\frac{\text{lb}}{\text{h}}\right)\left[(1547.7 - 994.41) - (75.17 - 69.74)\right]\frac{\text{Btu}}{\text{lb}}$$

$$= 7.670 \times 10^8 \text{ Btu/h} \longleftarrow \underline{\dot W_{cycle}}$$

(b) The heat transfer rate for the steam generator is

$$\dot Q_{in} = \dot m(h_1 - h_4) = (1.4\times10^6)(1547.7 - 75.17)\text{ Btu/lb}$$

$$= 2.062 \times 10^9 \text{ Btu/h} \longleftarrow \underline{\dot Q_{in}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot W_{cycle}}{\dot Q_{in}} = \frac{7.670\times10^8}{2.062\times10^9} = 0.372\ (37.2\%) \longleftarrow \underline{\eta}$$

(d) The mass flow rate of cooling water through the condenser is

$$\dot m_{cw} = \frac{\dot m(h_2 - h_3)}{h_{cw,out} - h_{cw,in}} = \frac{(1.4\times10^6\text{ lb/h})(994.41 - 69.74)\text{ Btu/lb}}{(48.09 - 28.08)\text{ Btu/lb}\ \left|60\text{ min/h}\right|} = 1.079\times10^6 \text{ lb/min}$$

With $v_{cw} = v_f(60°F) = 0.01604$ ft³/lb, $(AV) = v_{cw}\dot m_{cw} = 1.730\times10^4$ ft³/min $\longleftarrow \underline{(AV)_{cw}}$

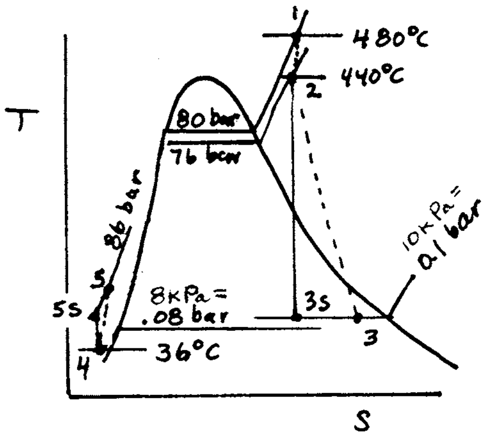
The primary effect of the throttling valve is to reduce the turbine work. This causes a decrease in thermal efficiency and an increase in the rate of cooling water flow required.

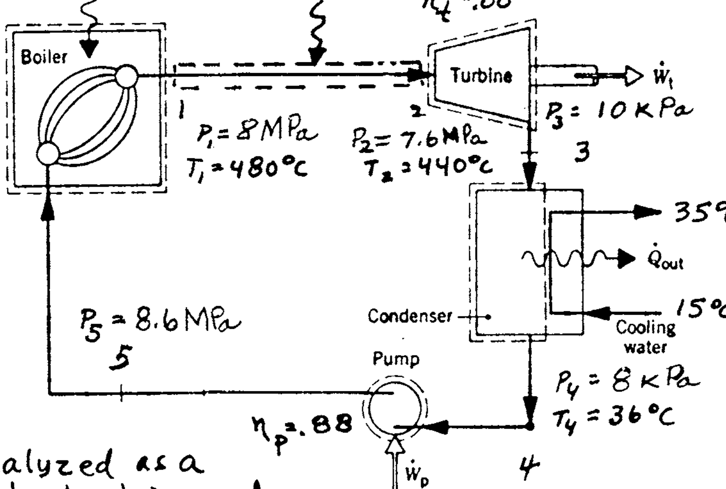
PROBLEM 8.21

KNOWN: Water is the working fluid in a vapor power plant. Data are known at various locations, and the mass flow rate is given.

FIND: Determine (a) the net power output, (b) thermal efficiency, (c) the rate of heat transfer from the line connecting the steam generator and the turbine, and (d) the mass flow rate of condenser cooling water.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The turbine and pump operate adiabatically (3) Kinetic & potential energy effects are negligible. (4) For the cooling water, assume $h_{cw} \approx h_f(T)$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 8\,\text{MPa}$, $T_1 = 480°\text{C} \Rightarrow h_1 = 3348.4\ \text{kJ/kg}$, $s_1 = 6.6586\ \text{kJ/kg·K}$

State 2: $p_2 = 7.6\,\text{MPa}$, $T_2 = 440°\text{C} \Rightarrow$ Interpolating in Table A-4;
$h_2 = 3252.3\ \text{kJ/kg}$, $s_2 \approx 6.5526\ \text{kJ/kg·K}$

State 3: State 3 is fixed using the turbine efficiency. First, at $p_3 = 10\,\text{kPa}$, $s_{3s} = s_2 = 6.5526 \Rightarrow x_{3s} = 0.787$; $h_{3s} = 2075.0\ \text{kJ/kg}$. Thus

$$\eta_t = \frac{(\dot{W}_t/\dot{m})}{(\dot{W}_t/\dot{m})_s} = \frac{h_2 - h_3}{h_2 - h_{3s}} \Rightarrow h_3 = h_2 - \eta_t(h_2 - h_{3s})$$
$$= 2216.3\ \text{kJ/kg}$$

Further, with $h_3 = 2216.3\ \text{kJ/kg}$; $x_3 = .8461 \Rightarrow s_3 = 6.9958\ \text{kJ/kg·K}$

State 4: $p_4 = 8\,\text{kPa}$, $T_4 = 36°\text{C} \Rightarrow h_4 \approx h_f(T_4) = 150.86\ \text{kJ/kg}$

State 5: $h_{5s} \approx h_4 + v_4(p_5 - p_4)$

$$= 150.86 + \left(1.0063 \times 10^{-3}\ \frac{\text{m}^3}{\text{kg}}\right)(86 - .08)\ \text{bars}\left(\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right)\left(\frac{1\ \text{kJ}}{10^3\ \text{N·m}}\right)$$
$$= 150.86 + 8.646 = 159.51\ \text{kJ/kg}$$

Thus, using the pump efficiency

$$\eta_p = \frac{(\dot{W}_p/\dot{m})_s}{(\dot{W}_p/\dot{m})} = \frac{h_{5s} - h_4}{h_5 - h_4} \Rightarrow h_5 = h_4 + \left(\frac{h_{5s} - h_4}{\eta_p}\right)$$
$$= 150.86 + \left(\frac{159.51 - 150.86}{.88}\right)$$
$$= 160.69\ \text{kJ/kg}$$

PROBLEM 8.21 (Cont'd.)

(a) To determine the net power output, we use energy and mass balances for the control volumes surrounding the turbine and pump to get

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}\left[(h_2 - h_3) - (h_5 - h_4)\right]$$

Inserting values

$$\dot{W}_{cycle} = \left(79.53\ \frac{kg}{s}\right)\left[(3252.3 - 2216.3) - (160.69 - 150.86)\right]\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 8.161 \times 10^4\ kW \longleftarrow \dot{W}_{cycle}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}(h_1 - h_5)} = \frac{8.161 \times 10^4}{(79.53)(3348.4 - 160.69)\left|\frac{1}{1}\right|}$$

$$= 0.322\ (32.2\%) \longleftarrow \eta$$

(c) For a control volume enclosing the line connecting the steam generator and the turbine

$$0 = \dot{Q}_{line} + \dot{m}(h_1 - h_2)$$

or

$$\dot{Q}_{line} = \dot{m}(h_2 - h_1) = \left(79.53\ \frac{kJ}{kg}\right)(3252.3 - 3348.4)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= -7643\ kW \longleftarrow \dot{Q}_{line}$$

(d) The mass flow rate of cooling water is found from

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}(h_3 - h_4) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

or

$$\dot{m}_{cw} = \dot{m}\frac{(h_3 - h_4)}{(h_{cw,out} - h_{cw,in})}$$

From Table A-2, $h_{cw,out} \approx h_f(35°C) = 146.68$ kJ/kg and $h_{cw,in} \approx h_f(15°C) = 62.99$ kJ/kg.
Inserting values

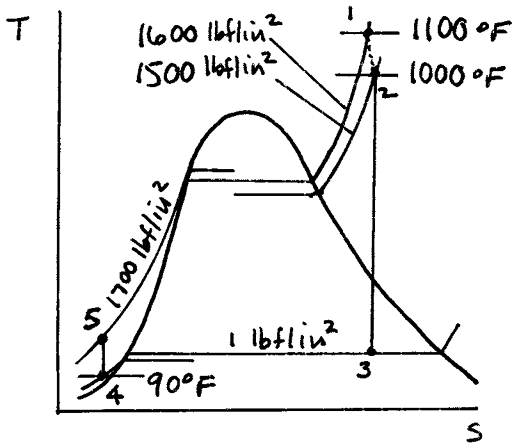
$$\dot{m}_{cw} = \left(79.53\ \frac{kg}{s}\right)\frac{(2216.3 - 150.86)}{(146.68 - 62.99)} = 1963\ kg/s \longleftarrow \dot{m}_{cw}$$

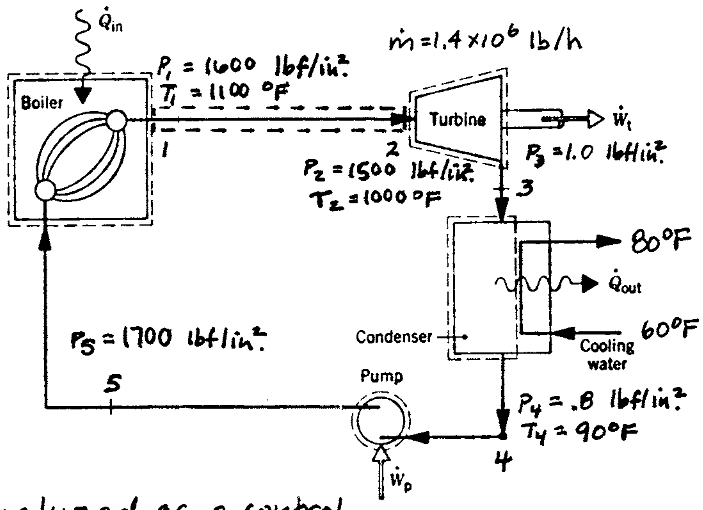
PROBLEM 8.22*

KNOWN: The ideal Rankine cycle of Prob. 8.9 is modified to include the effects of friction in the line connecting the steam generator to the turbine, the condenser, and the steam generator.

FIND: Determine for the modified cycle (a) the net power developed, (b) the thermal efficiency, (c) the heat rate, and (d) the mass flow rate of cooling water in the condenser.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The turbine and pump operate adiabatically and reversibly. (3) Kinetic and potential energy effects are negligible. (4) For the cooling water, assume $h_{cw} \approx h_f(T)$.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 1600\ \text{lbf/in}^2,\ T_1 = 1100\ °F \Rightarrow h_1 = 1547.7\ \text{Btu/lb},\ s_1 = 1.6315\ \text{Btu/lb}\cdot°R$

State 2: $P_2 = 1500\ \text{lbf/in}^2,\ T_2 = 1000\ °F \Rightarrow h_2 = 1490.3\ \text{Btu/lb},\ s_2 = 1.6004\ \text{Btu/lb}\cdot°R$

State 3: $P_3 = 1.0\ \text{lbf/in}^2,\ s_3 = s_2 \Rightarrow\ x_3 = \frac{s_3 - s_f}{s_{fg}} = \frac{1.6004 - 0.1327}{1.8453} = 0.795$

and $h_3 = h_f + x_3 h_{fg} = 69.74 + (0.795)(1036.0) = 893.4\ \text{Btu/lb}$

State 4: $P_4 = 0.8\ \text{lbf/in}^2,\ T_4 = 90°F \Rightarrow h_4 \approx h_f(90°F) = 58.07\ \text{Btu/lb}$

$s_4 \approx s_f(90°F) = 0.1117\ \text{Btu/lb}\cdot°R$

$v_4 \approx v_f(90°F) = 0.01610\ \text{ft}^3/\text{lb}$

State 5: $h_5 \approx h_4 + v_4(P_5 - P_4) = 58.07\ \frac{\text{Btu}}{\text{lb}} + \left(0.01610\ \frac{\text{ft}^3}{\text{lb}}\right)\left(1699.2\ \frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$

$= 63.13\ \text{Btu/lb}$

(a) The net power developed is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}\left[(h_2 - h_3) - (h_5 - h_4)\right]$$

$$= (1.4 \times 10^6\ \text{lb/h})\left[(1490.3 - 893.4) - (63.13 - 58.07)\right] \frac{\text{Btu}}{\text{lb}}$$

$$= 8.286 \times 10^8\ \text{Btu/h} \longleftarrow \dot{W}_{cycle}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}(h_1 - h_5)} = \frac{8.286 \times 10^8}{(1.4 \times 10^6)(1547.7 - 63.13)}$$

$$= \frac{8.286 \times 10^8}{2.078 \times 10^9} = 0.3987\ (39.87\%) \longleftarrow \eta$$

PROBLEM 8.22 (Cont'd.)

(c) Using the definition on Pg. 376, the heat rate is

$$\begin{pmatrix}\text{heat}\\ \text{rate}\end{pmatrix} = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{2.078\times10^{9}\ \text{Btu/h}}{8.286\times10^{8}\ \text{Btu/h}}\left|\frac{3413\ \text{Btu/h}}{1\ \text{kW}}\right|$$

$$= 8559\ \text{Btu/kW}\cdot\text{h} \longleftarrow \text{heat rate}$$

(d) Mass and energy balances for the control volume enclosing the condenser reduce as follows:

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}(h_3 - h_4) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

or

$$\dot{m}_{cw} = \dot{m}\left(\frac{h_3 - h_4}{h_{cw,out} - h_{cw,in}}\right)$$

From Table A-2E, $h_{cw,in} = h_f(60°F) = 28.08$ Btu/lb and $h_{cw,out} = h_f(80°F) = 48.09$ Btu/lb. Thus

$$\dot{m}_{cw} = \left(1.4\times10^{6}\ \frac{\text{lb}}{\text{h}}\right)\left(\frac{893.4 - 58.07}{48.09 - 28.08}\right)$$

$$= 5.844\times10^{7}\ \text{lb/h} \longleftarrow \dot{m}_{cw}$$

PROBLEM 8.23

KNOWN: Steady-state operating data are provided for a vapor power plant in which steam exits the turbine as a two-phase liquid-vapor mixture at temperature T, and condensate exits the condenser at 5°F lower than T.

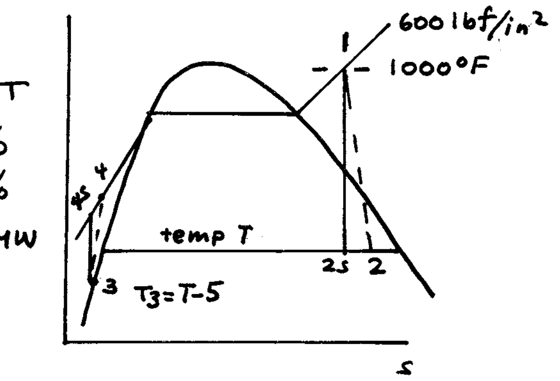
FIND: (a) For T = 80°F, determine the steam quality at the turbine exit, the mass flow rate, and the cycle thermal efficiency. (b) Plot the quantities of (a) versus T ranging from 80 to 105°F.

SCHEMATIC & GIVEN DATA:

$\eta_t = 90\%$

$\eta_p = 80\%$

$\dot{W}_{net} = 1 MW$



ASSUMPTIONS:

1. Control volumes enclosing each of the principal components is at steady state.
2. The pump and turbine operate adiabatically.
3. Kinetic/potential energy can be ignored.
4. Condensate exits the condenser and enters the pump as sat. liquid.

ANALYSIS: First fix each of the principal states for T = 80°F: From Table A-4E, $h_1 = 1517.8$ Btu/lb, $s_1 = 1.7155$ Btu/lb·°R. Then, with $s_{2s} = s_1$ and data from Table A-2E

$$x_{2s} = \frac{s_1 - s_f}{s_{g-f}} = \frac{1.7165 - 0.09332}{2.0356 - 0.09332} = 0.835 \Rightarrow h_{2s} = 48.09 + (0.835)(1048.3) = 923.4 \text{ Btu/lb}$$

Then, with

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1517.8 - 0.9(1517.8 - 923.4) = 982.8 \text{ Btu/lb}$$

$$\Rightarrow x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{982.8 - 48.09}{1048.3} = 0.892 \quad \longleftarrow$$

At state 3, $h_3 = h_f(75°F) = 43.1$ Btu/lb. Too, $h_{4s} \approx h_3 + v_3(P_4 - P_3)$, or

$$h_{4s} = 43.9 \frac{\text{Btu}}{\text{lb}} + \left(0.0161 \frac{\text{ft}^3}{\text{lb}}\right)(600 - 0.43) \frac{\text{lbf}}{\text{in}^2} \left| \frac{144 \text{ in}^2}{1 \text{ ft}^2} \right| \left| \frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}} \right| = 44.9 \text{ Btu/lb.}$$

Then, with

$$\eta_p = \frac{h_{4s} - h_3}{h_4 - h_3} \Rightarrow h_4 = h_3 + \frac{(h_{4s} - h_3)}{\eta_p} = 43.1 + \frac{(44.9 - 43.1)}{0.8} = 45.4 \text{ Btu/lb.}$$

The thermal efficiency is obtained as

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} = \frac{(1517.8 - 982.8) - (45.4 - 43.1)}{1517.8 - 45.4} = 0.362 \longleftarrow$$

The mass flow rate is

$$\dot{m} = \frac{\dot{W}_{net}}{(\dot{W}_t/\dot{m}) - (\dot{W}_p/\dot{m})} = \frac{(10^3 \text{ kW}) \left| \frac{3413 \text{ Btu/h}}{1 \text{ kW}} \right|}{(532.7 \text{ Btu/lb})} = 6.41 \times 10^3 \frac{\text{lb}}{\text{h}} \quad \longleftarrow$$

The data for the required plots are obtained using IT, as follows:

PROBLEM 8.23 (Cont'd.)

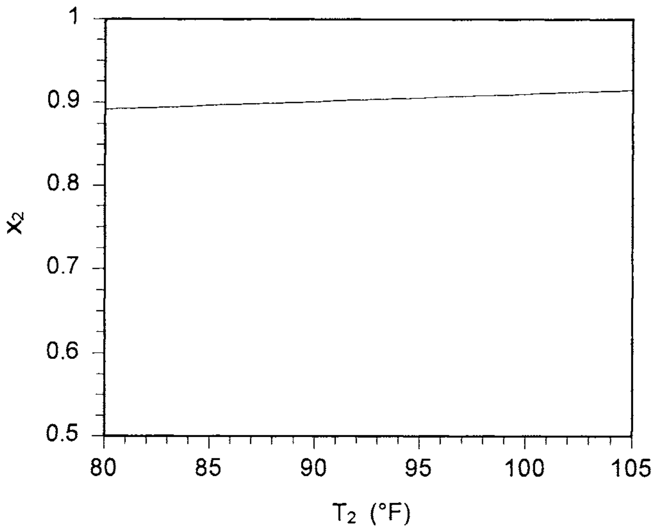
IT Code

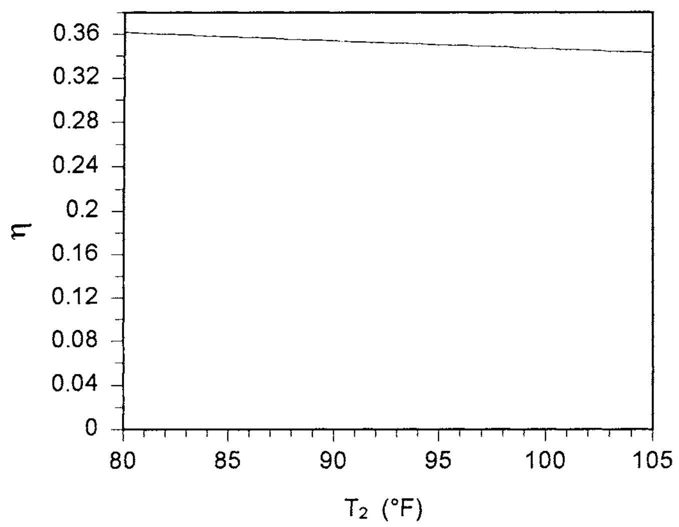
```
p1 = 600  // lbf/in.²
T1 = 1000  // °F
T2 = 80
T3 = T2 - 5
p3 = p2
p4 = p1
efft = 0.9
effp = 0.8
Wdotcycle = 1 * 1000 * 3413  // Btu/h

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
p2 = Psat_T("Water/Steam", T2)
h2s = h_Ps("Water/Steam", p2, s1)
h2 = h1 - (h1 - h2s) * efft
x2 = x_hP("Water/Steam", h2, p2)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 0)
v3 = vsat_Px("Water/Steam", psat, 0)
h4 = h3 + v3 * (p4 - p3) * (144 / 778) /
effp

Wcycle = (h1 - h2) - (h4 - h3)
eta = Wcycle / (h1 - h4)
Wdotcycle = mdot * (Wcycle)
```

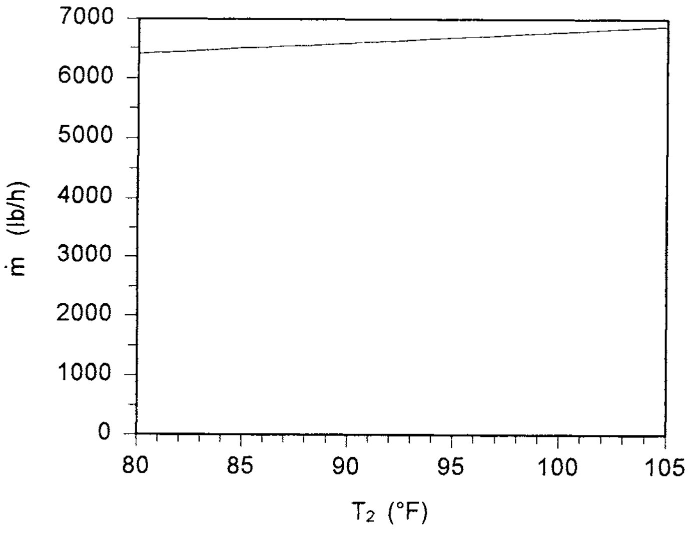
IT Results for T_1 = 80°F

```
h1 = 1518 Btu/lb
h2 = 982.9 Btu/lb
h3 = 42.67 Btu/lb
h4 = 44.9 Btu/lb
x2 = 0.8919
η = 0.3616
ṁ  = 6409 lb/h
```

PLOTS:







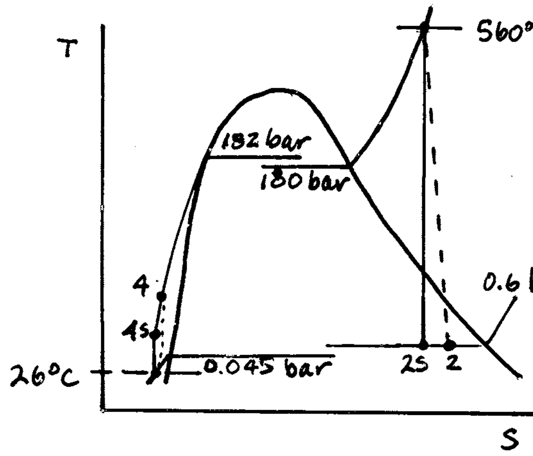
As the condenser temperature increases, the cycle thermal efficiency decreases, as expected. As a result, the turbine exit quality increases and the required mass flow rate increases.

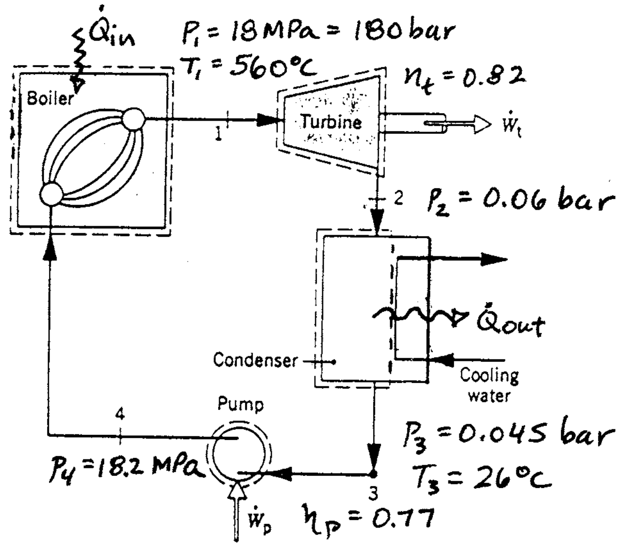
PROBLEM 8.24

KNOWN: Water is the working fluid in a simple vapor power plant. Data are known at varios locations.

FIND: Determine (a) the net work per unit mass of steam flow, (b) the heat transfer per unit mass of steam passing through the boiler, (c) the thermal efficiency, and (d) the heat transfer per unit mass of steam passing through the condenser.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each control volume shown is at steady state. (2) The turbine and the pump operate adiabatically. (3) Kinetic and potential energy effects can be neglected.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 180$ bar, $T_1 = 560°C \Rightarrow h_1 = 3444.4$ kJ/kg, $s_1 = 6.4392$ kJ/kg·K

State 2: First, $p_2 = 0.06$ bar, and $s_{2s} = s_1 \Rightarrow x_{2s} = 0.75783$, $h_{2s} = 1982.4$ kJ/kg
Using the turbine efficiency, $\eta_t = (h_1 - h_2)/(h_1 - h_{2s})$. Thus

$$h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 2245.6 \text{ kJ/kg}$$

State 3: From Table A-2, $p_3 > p_{sat @ 26°C}$. Thus, state 3 is a sub-cooled liquid state. Since the pressure is low, $h_3 \approx h_{f @ 26°C} = 109.07$ kJ/kg and $v_3 \approx v_{f @ 26°C} = 1.0032 \times 10^{-3}$ m³/kg.

State 4: First,

$$\begin{aligned} h_{4s} &\approx h_3 + v_3 (p_4 - p_3) \\ &= 109.07 \text{ kJ/kg} + (1.0032 \times 10^{-3} \tfrac{m^3}{kg})(182 - 0.045) \text{ bar} \left| \frac{10^5 N/m^2}{1 \text{ bar}} \right| \left| \frac{1 kJ}{10^3 N \cdot m} \right| \\ &= 127.32 \text{ kJ/kg} \end{aligned}$$

Using the pump efficiency, $\eta_p = (h_{4s} - h_3)/(h_4 - h_3)$. Thus

$$h_4 = h_3 + (h_{4s} - h_3)/\eta_p = 132.78 \text{ kJ/kg}$$

(a) The net work per unit mass of steam flow is

$$\begin{aligned} \frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} &= (h_1 - h_2) - (h_4 - h_3) \\ &= (3444.4 - 2245.6) - (132.78 - 109.07) \\ &= 1175.1 \text{ kJ/kg} \xleftarrow{\qquad \dot{W}_{cycle}/\dot{m} \qquad} \end{aligned}$$

(b) For control volume enclosing the steam passing through the boiler

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4$$

PROBLEM 8.24 (Cont'd)

Thus

$$\frac{\dot{Q}_{in}}{\dot{m}} = (3444.4 - 132.78)\ \text{kJ/kg}$$

$$= 3311.6\ \text{kJ/kg} \longleftarrow \underline{\qquad \dot{Q}_{in}/\dot{m}}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{1175.1}{3311.6} = 0.355\ (35.5\%) \longleftarrow \underline{\qquad \eta}$$

(d) The heat transfer to cooling water passing through the condenser per kg of steam condensed is

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_2 - h_3 = (2245.6 - 109.07)\ \text{kJ/kg}$$

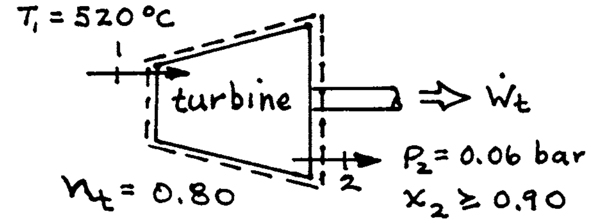
$$= 2136.5\ \text{kJ/kg} \longleftarrow \underline{\qquad \dot{Q}_{out}/\dot{m}}$$

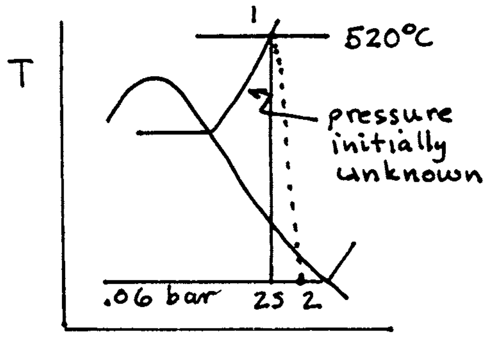
PROBLEM 8.25

KNOWN: The turbine inlet temperature and condenser pressure are specified in the preliminary design of a vapor power plant with water as the working fluid. The turbine efficiency is also known.

FIND: Determine the steam generator pressure required if the quality at the turbine exit must be at least 90%.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) The turbine operates adiabatically. (3) Kinetic and potential energy effects are negligible.

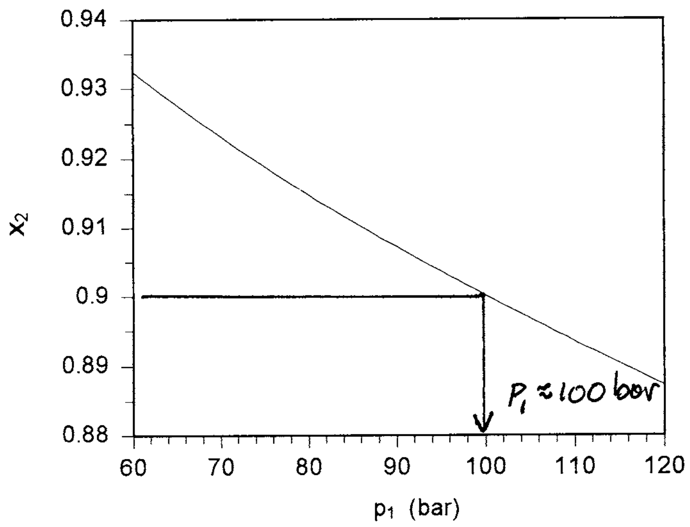
ANALYSIS: For fixed turbine inlet temperature, the value of turbine exit quality, x_2, decreases as turbine inlet pressure p_1 is increased. A trial and error procedure can ① be used to find p_1 corresponding to $x_2 = 90\%$ using table data. Alternatively, the following IT code can be used to sweep p_1 from 60 bar to 120 bar and determine the corresponding values of x_2.

IT Code

```
T1 = 520 // °C
p2 = 0.06 // bar
p1 = 60
efft = 0.8

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
h2 = h1 - efft * (h1 - h2s)
x2 = x_hP("Water/Steam", h2, p2)
```

IT Results

p_1 (bar)	60	80	**100**	120
h_1 (kJ/kg)	3469	3447	3425	3401
h_2 (kJ/kg)	2404	2361	2326	2295
h_{2s} (kJ/kg)	2137	2090	2051	2018
s_1 (kJ/kg·K)	6.94	6.786	6.661	6.555
s_{2s} (kJ/kg·K)	6.94	6.786	6.661	6.555
x_2	0.9324	0.9148	**0.9001**	0.8873



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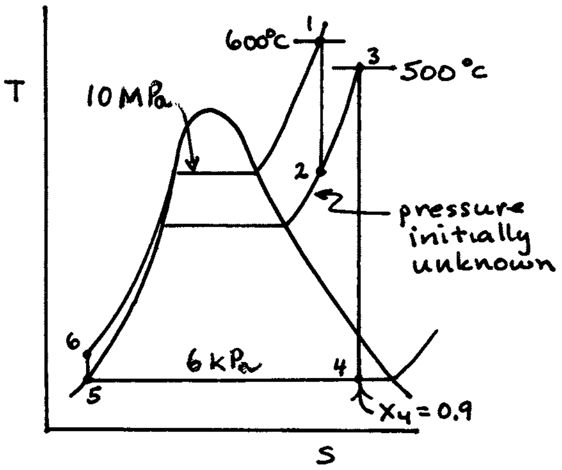
1. It is common practice to maintain at least 90% quality at the exit of a steam turbine to avoid undesirable effects.

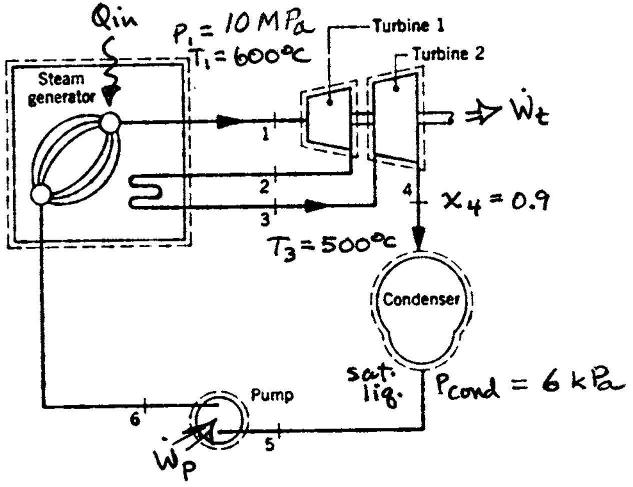
PROBLEM 8.26

KNOWN: Water is the working fluid in an ideal Rankine cycle with reheat. Data are known at various locations.

FIND: Determine the thermal efficiency of the cycle.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 8.3

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 10$ MPa, $T_1 = 600$°C ⇒ $h_1 = 3625.3$ kJ/kg, $s_1 = 6.9029$ kJ/kg·K

Next, skip to state 4: $P_4 = 6$ kPa, $x_4 = 0.9$ ⇒ $h_4 = 2325.8$ kJ/kg
$s_4 = 7.5495$ kJ/kg·K

State 3: $T_3 = 500$°C, $s_3 = s_4 = 7.5495$ kJ/kg·K ⇒ Interpolating in Table A-4; $P_3 \approx 15.735$ bar, $h_3 \approx 3472.3$ kJ/kg

Now, State 2: With $s_2 = s_1 = 6.9029$ kJ/kg, $P_2 = P_3 = 15.735$ ⇒ Double interpolation in Table A-4 gives $h_2 \approx 3040.5$ kJ/kg

State 5: $P_5 = 6$ kPa, sat. liquid ⇒ $h_5 = 151.53$ kJ/kg

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5)$

$$= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064\times10^{-3})\frac{\text{m}^3}{\text{kg}}(100-.06)\,\text{bar}\left|\frac{10^5\,\text{N/m}^2}{1\,\text{bar}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= 151.53 + 10.06 = 161.59\ \text{kJ/kg}$$

To determine the thermal efficiency, first evaluate the net work per unit mass of steam flow

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = (h_1-h_2)+(h_3-h_4)-(h_6-h_5)$$

$$= (3625.3-3040.5)+(3472.3-2325.8)-(10.06)$$

$$= 1721.2\ \text{kJ/kg}$$

Also, the total heat transfer for the boiling/superheating, per unit mass of steam flow is

$$\dot{Q}_{in}/\dot{m} = (h_1-h_6)+(h_3-h_2) = (3625.3-161.59)+(3472.3-3040.5) = 3895.5\ \frac{\text{kJ}}{\text{kg}}$$

Thus

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{1721.2}{3895.5} = 0.442\ (44.2\%) \longleftarrow \eta$$

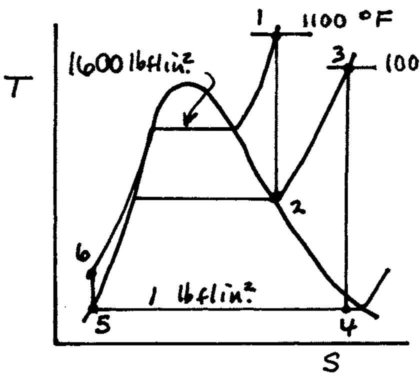
PROBLEM 8.27*

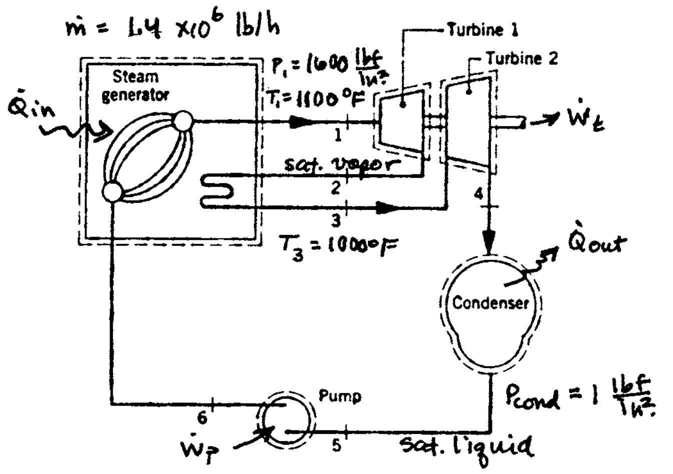
KNOWN: The ideal Rankine cycle of Problem 8.9 is modified to include reheat. The mass flow rate is the same as in Problem 8.9.

FIND: Determine for the modified cycle (a) the net power developed, (b) the rate of heat transfer to the working fluid in the reheat process, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:

$\dot{m} = 1.4 \times 10^6$ lb/h





ASSUMPTIONS: Same as Example 8.3

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 1600$ lbf/in², $T_1 = 1100$ °F $\Rightarrow$ $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R

State 2: $s_2 = s_1 = 1.6315$ Btu/lb·°R, saturated vapor $\Rightarrow$ Interpolating in Table A-3E; $P_2 \approx 70.5$ lbf/in², $h_2 \approx 1181.1$ Btu/lb

State 3: $P_3 = 70.5$ lbf/in², $T_3 = 1000$ °F $\Rightarrow$ $h_3 = 1532.9$ Btu/lb, $s_3 = 1.9605$ Btu/lb·°R

State 4: $P_4 = 1$ lbf/in², $s_4 = s_3$ $\Rightarrow$ $x_4 = 0.9905$, $h_4 = 1095.9$ Btu/lb

State 5: $P_5 = 1$ lbf/in², sat. liquid $\Rightarrow$ $h_5 = 69.74$ Btu/lb

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5) = 74.52$ Btu/lb (see Problem 8.9)

(a) The net power developed is

$$\dot{W}_{cycle} = \dot{m}\left[\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}\right] = \dot{m}\left[(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)\right]$$

$$= \left(1.4 \times 10^6 \frac{\text{lb}}{\text{h}}\right)\left[(1547.7 - 1181.1) + (1532.9 - 1095.9) - (74.52 - 69.74)\right]\frac{\text{Btu}}{\text{lb}}$$

$$= 1.118 \times 10^9 \text{ Btu/h} \longleftarrow \dot{W}_{cycle}$$

(b) For the reheat process

$$\dot{Q}_{reheat} = \dot{m}(h_3 - h_2) = \left(1.4 \times 10^6 \frac{\text{lb}}{\text{h}}\right)(1532.9 - 1181.1)\frac{\text{Btu}}{\text{lb}} = 4.93 \times 10^8 \frac{\text{Btu}}{\text{h}} \longleftarrow \dot{Q}_{reheat}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}\left[(h_1 - h_6) + (h_3 - h_2)\right]} = \frac{1.118 \times 10^9}{1.4 \times 10^6\left[(1547.7 - 74.52) + (1532.9 - 1181.1)\right]}$$

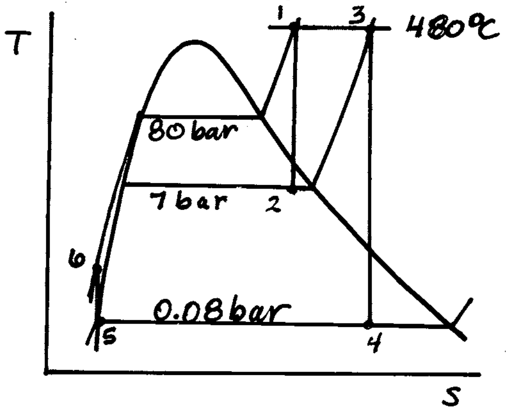
$$= 0.438\ (43.8\%) \longleftarrow \eta$$

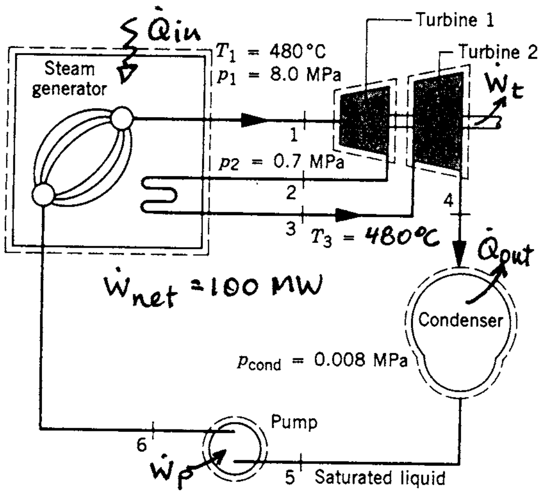
PROBLEM 8.28

KNOWN: The ideal Rankine cycle of Problem 8.2 is modified to include reheat to 480°C at a pressure of 0.7 MPa. The net power output is known.

FIND: Determine (a) the rate of heat transfer to the working fluid in the steam generator, (b) the thermal efficiency, and (c) the rate of heat transfer to cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 8.3.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 8\text{ MPa} = 80\text{ bar}$, $T_1 = 480°C \Rightarrow h_1 = 3348.4\text{ kJ/kg}$, $s_1 = 6.6586\text{ kJ/kg·K}$

State 2: $p_2 = 0.7\text{ MPa} = 7\text{ bar}$, $s_2 = s_1 \Rightarrow x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = 0.9895$, $h_2 = 2741.8\text{ kJ/kg}$

State 3: $p_3 = 0.7\text{ MPa}$, $T_3 = 480°C \Rightarrow h_3 = 3438.9\text{ kJ/kg}$, $s_3 = 7.8723\text{ kJ/kg}$

State 4: $p_4 = 0.08\text{ bar}$, $s_4 = s_3 \Rightarrow x_4 = \frac{s_4 - s_{f4}}{s_{fg4}} = 0.9533$, $h_4 = 2464.8\text{ kJ/kg}$

State 5: $p_5 = 0.08\text{ bar}$, sat. liquid $\Rightarrow h_5 = 173.88\text{ kJ/kg}$, $v_5 = 1.0084 \times 10^{-3}\text{ m}^3/\text{kg}$

State 6: $h_6 \approx h_5 + v_5(p_6 - p_5)$

$$= 173.88 + (1.0084\times10^{-3})\frac{\text{m}^3}{\text{kg}}(80 - 0.08)\text{bar}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N·m}}\right|$$

$$= 173.88 + 8.06 = 181.94\text{ kJ/kg}$$

(a) To determine the rate of heat transfer $\dot{Q}_{in}$, first find the mass flow rate. The net power output of the cycle is

$$\dot{W}_{net} = \dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_p = \dot{m}\left[(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)\right]$$

or

$$\dot{m} = \frac{\dot{W}_{net}}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}$$

$$= \frac{(100\text{ MW})\left|10^3\text{ kJ/s}/1\text{ MW}\right|}{\left[(3348.4 - 2741.8) + (3438.9 - 2464.8) - (8.06)\right]\text{kJ/kg}}$$

$$= 63.59\text{ kg/s}$$

The heat transfer rate is

$$\dot{Q}_{in} = \dot{m}\left[(h_1 - h_6) + (h_3 - h_2)\right]$$

PROBLEM 8.28 (Cont'd)

Inserting values

$$\dot{Q}_{in} = \left(63.59 \frac{kg}{s}\right)\left[(3348.4 - 181.94) + (3438.9 - 2741.8)\right]\frac{kJ}{kg}\left|\frac{1\ MW}{10^3\ kJ/s}\right|$$

$$= 245.7\ MW \longleftarrow \dot{Q}_{in}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{100}{245.7} = 0.407\ (40.7\%) \longleftarrow \eta$$

(c) The rate of heat transfer to the cooling water passing through the condenser is

$$\dot{Q}_{out} = \dot{m}[h_4 - h_5]$$

$$= \left(63.59 \frac{kg}{s}\right)\left[2464.8 - 173.88\right]\frac{kJ}{kg}\left|\frac{1\ MW}{10^3\ kJ/s}\right|$$

$$= 145.7\ MW \longleftarrow \dot{Q}_{out}$$

PROBLEM 8.29

KNOWN: The modified cycle of Problem 8.28 is reconsidered.

FIND: Investigate the effects on cycle performance as the reheat pressure and temperature take on other values. Construct suitable plots and discuss. Consider first the ideal cycle and then assume the turbine and pump have isentropic efficiencies less than unity.

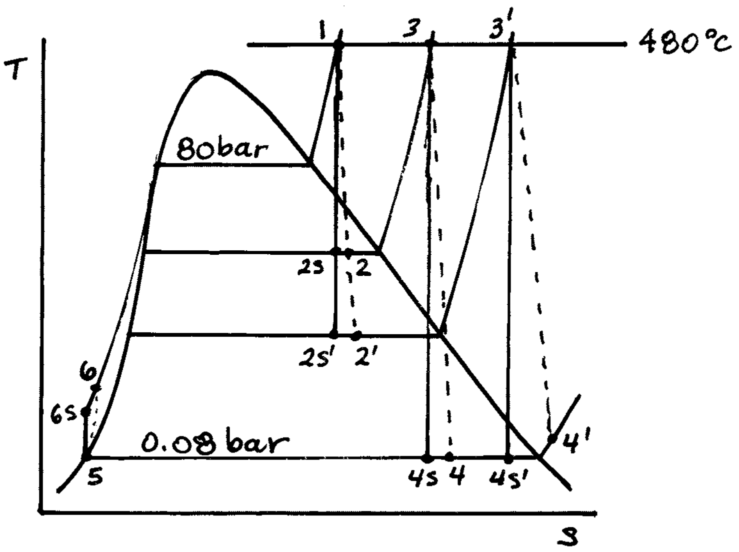
SCHEMATIC & GIVEN DATA: See solution to Problem 8.28.

ASSUMPTIONS: See Example 8.3, and $\eta_{t_1} = \eta_{t_2} = \eta_p = 0.8$ for the second part.

ANALYSIS: The performance of the cycle is investigated by determing the effects on thermal efficiency of varying the reheat pressure and the reheat temperature in turn.

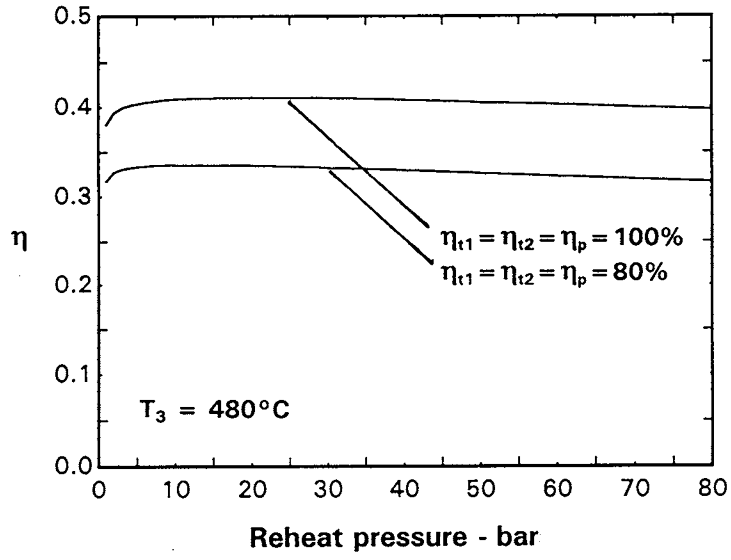
Reheat Pressure

The reheat pressure is varied while the reheat temperature is held constant. The effect on thermal efficiency is shown on the graph below.



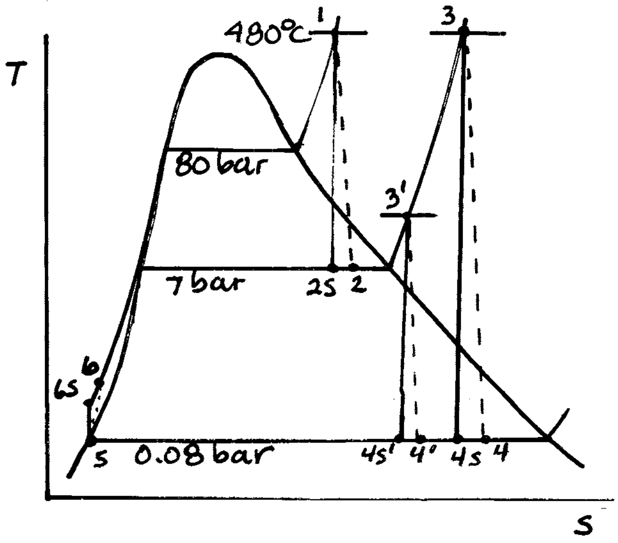
1. Note that the curves for both the reversible and irreversible cases exhibit maximums, indicating an optimum reheat pressure. However, the effect is slight.

2. Note that the problem of low quality at the turbine exit is reduced as the reheat pressure is lowered.

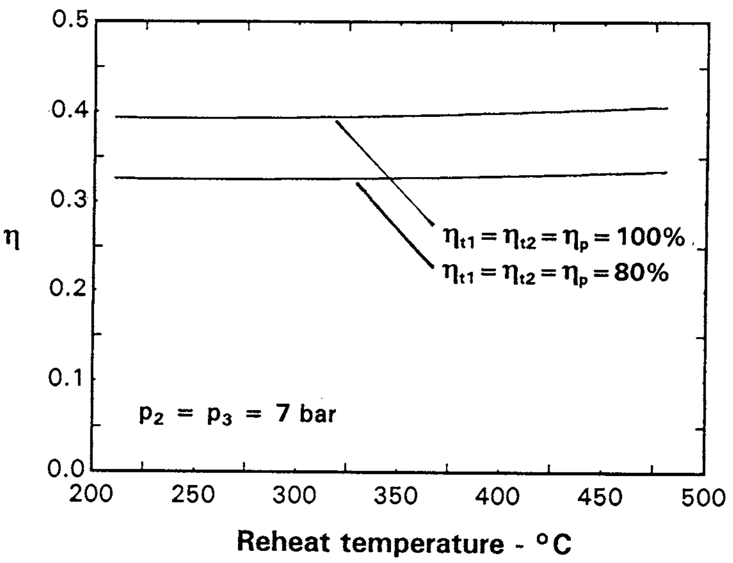


Reheat Temperature

The reheat temperature is varied while the reheat pressure is held constant. The effect on thermal efficiency is shown on the graph below.



1. Thermal efficiency is only slightly affected by reheat temperature in this case. Both the reversible and irreversible cases follow similar trends.
2. Note that the problem of low quality at the turbine exit is reduced as the reheat temperature increases.

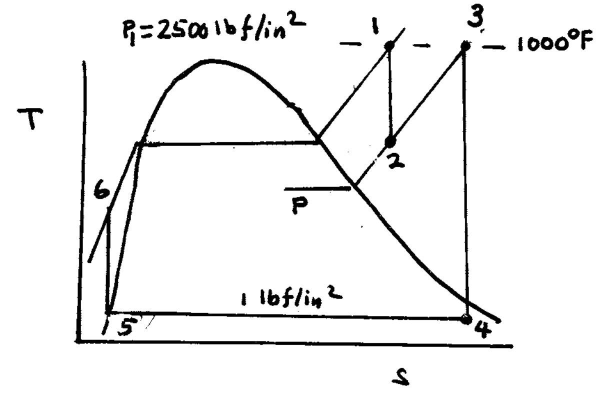


PROBLEM 8.30

KNOWN: Operating data are provided for an ideal Rankine cycle that has reheat at pressure p between the turbine stages.

FIND: (a) If $p/p_1 = 0.2$, where p_1 is the turbine inlet pressure, determine the cycle thermal efficiency and steam quality at the exit of the second turbine stage. (b) Plot the quantities of (a) versus p/p_1 ranging from 0.05 to 1.0.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. Control volumes enclosing each of the principal components are at steady state.
2. Each process is internally reversible, and the turbine stages and pump operate adiabatically.
3. Kinetic/potential energy can be ignored.

ANALYSIS: (a) For the case $p/p_1 = 0.2$, or $p = 500$ lbf/in², the principal states are fixed as follows: From Table A-4E, $h_1 = 1457.2$ Btu/lb, $s_1 = 1.5262$ Btu/lb·°R. With $s_2 = s_1$, interpolation in Table A-4E at 500 lbf/in² gives $h_2 = 1264.9$ Btu/lb.
① At state 3, $h_3 = 1520.7$ Btu/lb, $s_3 = 1.7371$ Btu/lb·°R. Then, with data from Table A-3E at 1 lbf/in², and $s_4 = s_3$ x_4

$$x_4 = \frac{s_3 - s_f}{s_g - s_f} = \frac{1.7371 - 0.1327}{1.8453} = 0.8695 \Rightarrow h_4 = 69.74 + .8695\,(1036.0) = 970.5 \text{ Btu/lb}$$

From Table A-3E, $h_5 = 69.7$ Btu/lb and $h_6 \approx h_5 + v_5(p_6 - p_5)$, or

$$h_6 = 69.7\,\frac{\text{Btu}}{\text{lb}} + 0.01614\,\frac{\text{ft}^3}{\text{lb}}\,(2500 - 1)\,\frac{\text{lbf}}{\text{in}^2}\left|\frac{144\,\text{in}^2}{1\,\text{ft}^2}\right|\left|\frac{1\,\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right| = 77.2\,\frac{\text{Btu}}{\text{lb}}$$

The thermal efficiency is given by

$$\eta = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)} = \frac{(1457.2 - 1264.9) + (1520.7 - 970.5) - (77.2 - 69.7)}{(1457.2 - 77.2) + (1520.7 - 1264.9)} = 0.449\ (44.9\%)$$

η

(b) The data for the required plots are obtained using IT, as follows:

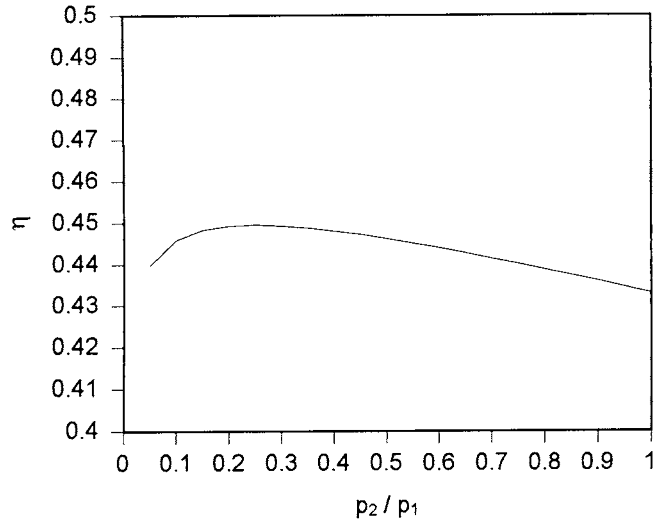
It Code

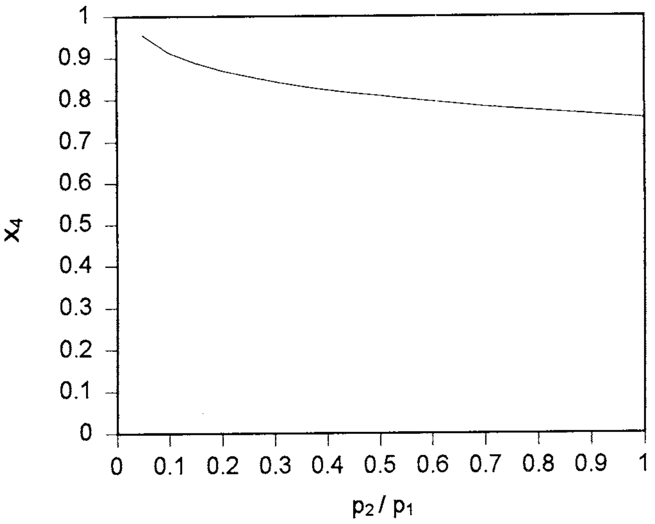
```
p1 = 2500  // lbf/in.²
T1 = 1000  // °F
pratio = p2 / p1
pratio = 0.2
p3 = p2
T3 = 1000  // °F
p4 = 1  // lbf/in.²
p5 = p4
p6 = p1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = h_Ps("Water/Steam", p2, s2)
s2 = s1
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4 = h_Ps("Water/Steam", p4, s4)
s4 = s3
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6 = h5 + v5 * (p6 - p5) * (144 / 778)

x4 = x_hP("Water/Steam", h4, p4)
eta = ((h1 - h2)  + (h3 - h4) - (h6 - h5)) / ((h1 - h6) + (h3 - h2))
```

IT Results for $p_2/p_1 = 0.2$ ($p_2 = 500$ lbf/in.²)

$h_1 = 1457$ Btu/lb
$s_1 = 1.526$ Btu/lb·°R
$h_2 = 1265$ Btu/lb
$h_3 = 1521$ Btu/lb
$s_3 = 1.737$ Btu/lb·°R
$h_4 = 970.4$ Btu/lb
$h_5 = 69.58$ Btu/lb
$h_6 = 77.05$ Btu/lb
$v_5 = 0.01614$ ft³/lb
$\eta = 0.4493$
$x_4 = 0.8695$

PLOTS:





The plots show the thermal efficiency is affected by reheat pressure, with a maximum at $p_2/p_1 \approx 0.2$ to 0.25. Also, as the reheat pressure increases the quality at the turbine exit decreases, which is consistent with point 4 on the T-s diagram moving to the left.

---

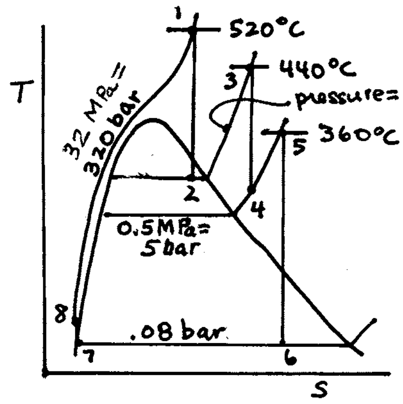
1. In early printings of the 4th edition, the value of s given in Table A-4E at p=500 lbf/in.², T = 1000°F was incorrectly listed as 1.7471 Btu/lb·°R. The corrected value of s = 1.7371 is used in this solution.

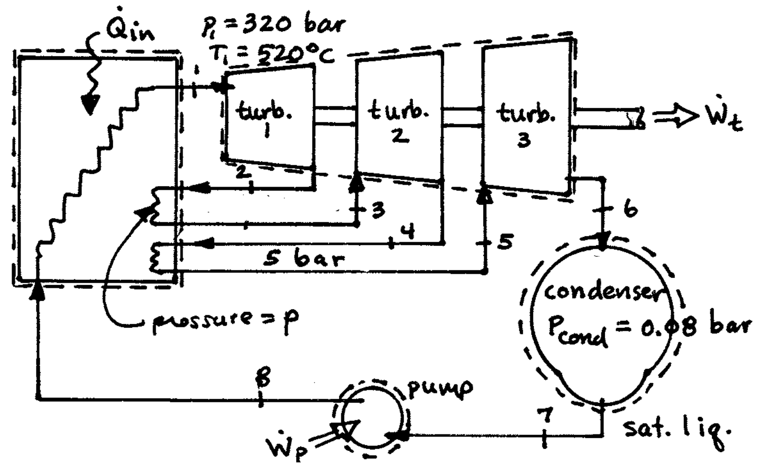
PROBLEM 8.31

KNOWN: Water is the working fluid in an ideal Rankine cycle modified to include three turbine stages with reheat between the stages. Data are known at various locations. The pressure of the steam exiting the first-stage turbine is p.

FIND: (a) If $p = 4$ MPa, determine the net work per unit of mass flowing and the thermal efficiency.
(b) Plot the quantities of part (a) versus p ranging from 0.5 to 1 MPa

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 8.3.

ANALYSIS: (a) $p = 4$ MPa. First, fix each of the principal states.

State 1: $P_1 = 320$ bar, $T_1 = 520$ °C ⇒ $h_1 = 3133.7$ kJ/kg, $s_1 = 5.8357$ kJ/kg·K

State 2: $P_2 = 40$ bar, $s_2 = s_1$ ⇒ $x_2 = 0.9284$, $h_2 = 2678.7$ kJ/kg

State 3: $P_3 = 40$ bar, $T_3 = 440$°C ⇒ $h_3 = 3307.1$ kJ/kg, $s_3 = 6.9041$ kJ/kg·K

State 4: $P_4 = 5$ bar, $s_4 = s_3$ ⇒ $h_4 = 2785.0$ kJ/kg

State 5: $P_5 = 5$ bar, $T_5 = 360$°C ⇒ $h_5 = 3188.4$ kJ/kg, $s_5 = 7.666$ kJ/kg·K

State 6: $P_6 = 0.08$ bar, $s_6 = s_5$ ⇒ $x_6 = .92631$, $h_6 = 2399.9$ kJ/kg

State 7: $P_7 = 0.08$ bar, sat. liquid ⇒ $h_7 = 173.88$ kJ/kg

State 8: $h_8 \approx h_7 + v_7 (p_8 - p_7)$

$$= 173.88 \frac{\text{kJ}}{\text{kg}} + (1.0084 \times 10^{-3}) \frac{\text{m}^3}{\text{kg}} (320 - .08)\,\text{bar} \left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right| \left|\frac{1\ \text{kJ}}{10^3\ \text{N·m}}\right|$$

$$= 173.88 + 32.26 = 206.14\ \text{kJ/kg}$$

(a) To determine the net work per unit mass of steam flowing, begin with the turbine stages. That is

$$\frac{\dot{W}_t}{\dot{m}} = (h_1 - h_2) + (h_3 - h_4) + (h_5 - h_6)$$

$$= (3133.7 - 2678.7) + (3307.1 - 2785.0) + (3188.4 - 2399.9)$$

$$= 1765.6\ \text{kJ/kg}$$

And, for the pump

$$\frac{\dot{W}_p}{\dot{m}} = h_8 - h_7 = 32.26\ \text{kJ/kg}$$

PROBLEM 8.31 (Contd.)

Thus

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_p}{\dot{m}} = 1765.6 - 32.26 = 1733.3 \text{ kJ/kg} \longleftarrow \underline{\dot{W}_{cycle}/\dot{m}}$$

For the working fluid passing through the steam generator, the total heat transfer including the reheat processes is

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_8) + (h_3 - h_2) + (h_5 - h_4)$$

$$= (3133.7 - 206.14) + (3307.1 - 2678.7) + (3188.4 - 2785.0)$$

$$= 3959.4 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{1733.3}{3959.4} = 0.438 \ (43.8\%) \longleftarrow \underline{\eta}$$

(b) The data for the required plots are obtained using IT, as follows:

IT Code

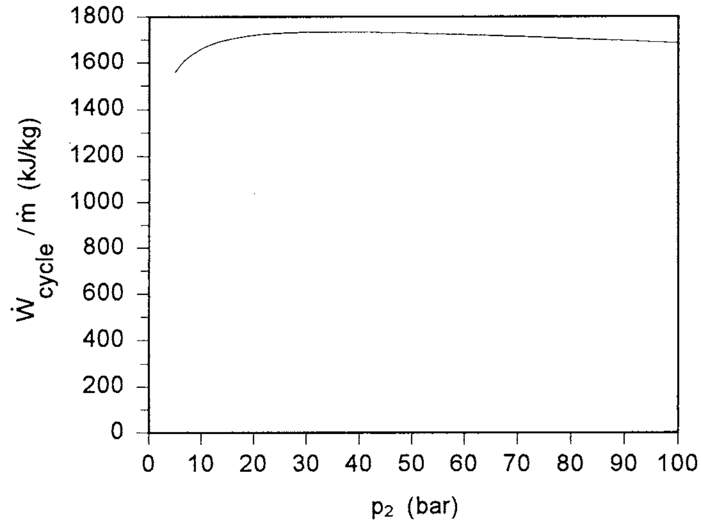
```
p1 = 320  // bar
T1 = 520  // °C
p2 = 40  // bar
p3 = p2
T3 = 440  // °C
p4 = 5  // bar
p5 = p4
T5 = 360  // °C
p6 = 0.08  // bar
p7 = p6
p8 = p1

// First turbine stage
h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1

// Second turbine stage
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4s = h_Ps("Water/Steam", p4, s4s)
s4s = s3
// Third turbine stage
h5 = h_PT("Water/Steam", p5, T5)
s5 = s_PT("Water/Steam", p5, T5)
h6s = h_Ps("Water/Steam", p6, s6s)
s6s = s5
// Pump
h7 = hsat_Px("Water/Steam", p7, 0)
v7 = vsat_Px("Water/Steam", p7, 0)
h8s = h7 + v7 * (p8 - p7) * 100

Wt = (h1 - h2s) + (h3 - h4s) + (h5 - h6s)
Wp = h8s - h7
Wcycle = Wt - Wp
Qin = (h1 - h8s) + (h3 - h2s) + (h5 - h4s)
eta = Wcycle / Qin
```

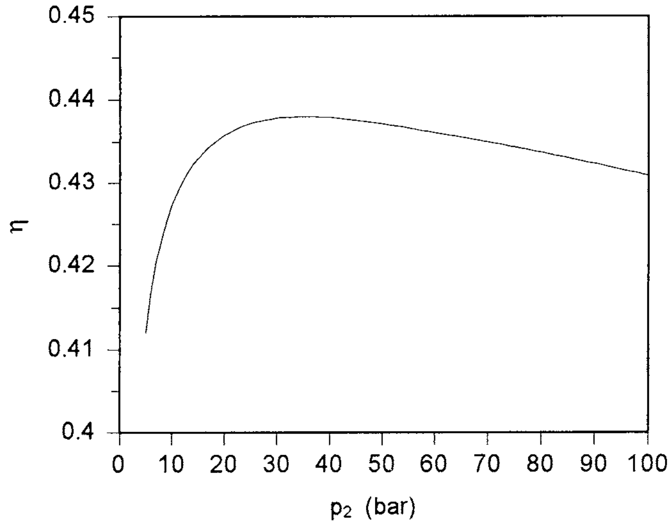
IT Results for p_2 = 4 MPa (40 bar)

h_1 = 3133 kJ/kg
s_1 = 5.835 kJ/kg·K
h_{2s} = 2678 kJ/kg
h_3 = 3307 kJ/kg
s_3 = 6.903 kJ/kg·K
h_{4s} = 2784 kJ/kg
h_5 = 3188 kJ/kg
s_5 = 7.665 kJ/kg·K
h_{6s} = 2399 kJ/kg
h_7 = 173.6 kJ/kg
h_{8s} = 205.9 kJ/kg
$\dot{W}_{cycle}/\dot{m}$ = 1734 kJ/kg
$\dot{Q}_{in}/\dot{m}$ = 3960 kJ/kg
η = 0.4379 (43.79%)

PROBLEM 8.31 (Cont'd.)

PLOTS:





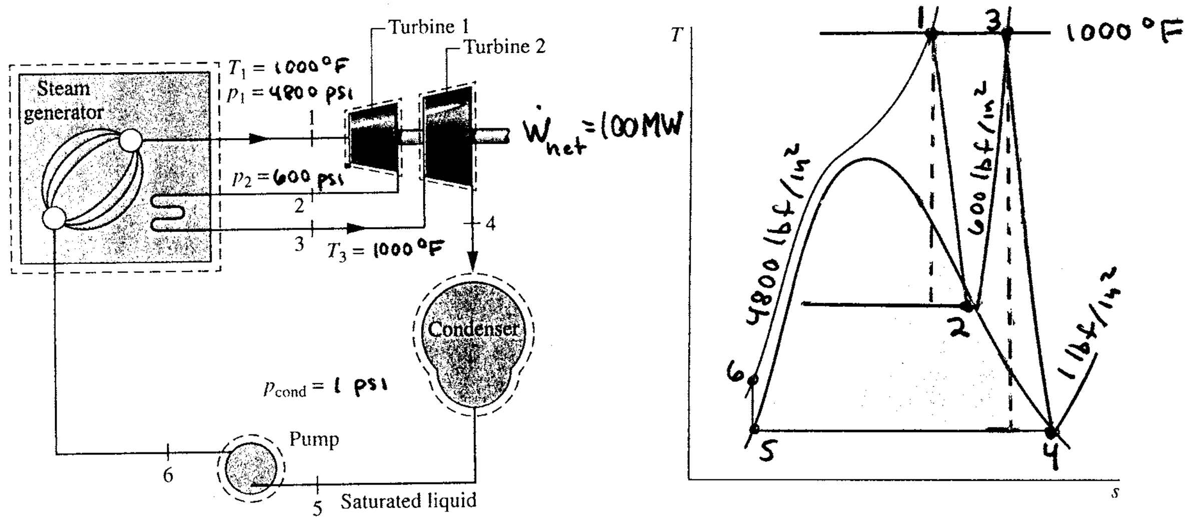
The plots show that the cycle net work and thermal efficiency are affected by the value of p_2. Each plot exhibits a maximum.

PROBLEM 8.32*

KNOWN: Water is the working fluid in an ideal Rankine cycle modified to include two turbine stages with reheat between the stages. The turbine and pump efficiencies are 85%.

FIND: (a) the rate of heat transfer to the working fluid passing through the steam generator, (b) the rate of heat transfer from the working fluid passing through the condenser, and (c) the cycle efficiency.

SCHEMATIC + GIVEN DATA:



ASSUMPTIONS: Same as in Example 8.3

ANALYSIS: First, fix each of the principle states.

state 1: $p_1 = 4800$ lbf/in², $T_1 = 1000$°F $\Rightarrow h_1 = 1317.4$ Btu/lb , $s_1 = 1.4078$ Btu/lb°R

state 2: $p_2 = 600$ lbf/in², $s_{2s} = s_1 \Rightarrow x_{2s} = 0.9500$, $h_{2s} = 1167.49$ Btu/lb

$$\eta_t = 0.85 = \frac{(h_1 - h_2)}{(h_1 - h_{2s})} \Rightarrow h_2 = 1189.98 \text{ Btu/lb}$$

state 3: $p_3 = 600$ lbf/in², $T_3 = 1000$°F $\Rightarrow h_3 = 1517.8$ Btu/lb , $s_3 = 1.7155$ Btu/lb°R

state 4: $p_4 = 1$ lbf/in², $s_{4s} = s_3 \Rightarrow x_{4s} = 0.8577$, $h_{4s} = 958.37$ Btu/lb

$$\eta_t = 0.85 = \frac{(h_3 - h_4)}{(h_3 - h_{4s})} \Rightarrow h_4 = 1042.28 \text{ Btu/lb}$$

state 5: $p_5 = 1$ lbf/in², sat. liq. $\Rightarrow h_5 = 69.74$ Btu/lb , $v_5 = 0.01614$ ft³/lb

state 6: $p_6 = 4800$ lbf/in², $h_6 \approx h_5 + v_5(p_6 - p_5)/\eta_p$

$$h_6 = 69.74 \frac{\text{Btu}}{\text{lb}} + (0.01614)\frac{\text{ft}^3}{\text{lb}}(4800-1)(144)\frac{\text{lbf}}{\text{in}^2}\left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right| = 69.74 + 14.34 = 84.08 \frac{\text{Btu}}{\text{lb}}$$

Next, determine the flow rate of the working fluid.

$$\dot{W}_{net} = \dot{W}_t - \dot{W}_p = \dot{m}[(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)]$$

$$\dot{m} = \frac{\dot{W}_{net}}{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)} = \frac{(100 \text{ MW})\left|\frac{1000 \text{ kJ/s}}{1 \text{ MW}}\right|\left|\frac{1 \text{ Btu}}{1.0551 \text{ kJ}}\right|}{[(1317.4 - 1189.98) + (1517.8 - 1042.28) - (84.08 - 69.74)] \frac{\text{Btu}}{\text{lb}}}$$

$$= 161.02 \text{ lb/s}$$

(a) The rate of heat transfer to the working fluid in the steam generator is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_6 + h_3 - h_2) = (161.02)\frac{\text{lb}}{\text{s}}(1317.4 - 84.08 + 1517.8 - 1189.98)\frac{\text{Btu}}{\text{lb}}$$

$$= 2.514 \times 10^5 \frac{\text{Btu}}{\text{s}}\left|\frac{1.0551 \text{ kJ}}{1 \text{ Btu}}\right|\left|\frac{1 \text{ MW}}{1000 \text{ kJ/s}}\right| = 265.23 \text{ MW} \longleftarrow \dot{Q}_{in}$$

PROBLEM 8.32 (cont'd)

(b) The rate of heat transfer from the working fluid in the condenser is

$$\dot{Q}_{out} = \dot{m}(h_4 - h_5) = (161.02)(1042.28 - 69.74)\left|\frac{1.0551}{1000}\right|$$

① $\dot{Q}_{out} = 165.23 \text{ MW} \longleftarrow$ $\dot{Q}_{out}$

(c) The cycle thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{sg}} = \frac{100 \text{ MW}}{265.23 \text{ MW}} = 0.377\ (37.7\%) \longleftarrow \eta$$

---

1. The overall energy balance holds for the cycle:

$$\dot{Q}_{in} = \dot{W}_{net} + \dot{Q}_{out}$$

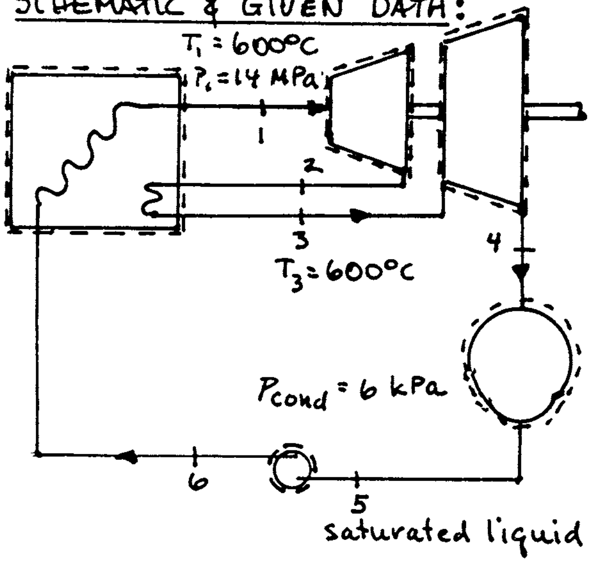
$$265.23 \text{ MW} = 100 \text{ MW} + 165.23 \text{ MW}$$

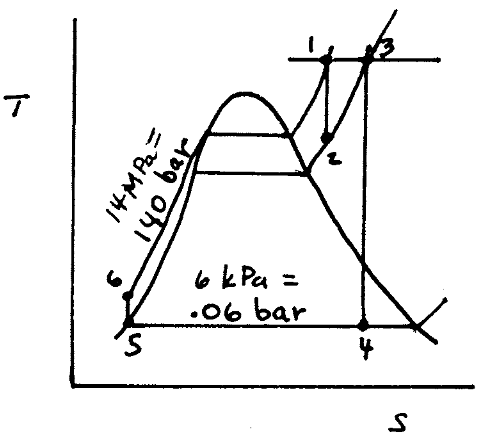
PROBLEM 8.33

KNOWN: An ideal reheat cycle operates with steam as the working fluid. Operating pressures and temperatures are given.

FIND: Plot thermal efficiency versus reheat pressure for pressurs ranging from 20 bars to 120 bars.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 8.3

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 140$ bar, $T_1 = 600°C \Rightarrow h_1, s_1$

State 2: For a given p_2, and with $s_2 = s_1$, we can get h_2.

State 3: $p_3 = p_2$, $T_3 = 600°C \Rightarrow h_3, s_3$

State 4: $p_4 = 0.06$ bar, and with $s_4 = s_3$, we can get x_4 and h_4.

State 5: $p_5 = 0.06$ bar, sat. liq. $\Rightarrow h_5, v_5$

State 6: $h_6 \approx h_5 + v_5(p_6 - p_5)$

Sample calculation: $p_2 = 2$ MPa $= 20$ bar. Using data from the steam tables, $h_1 = 3591.1$ kJ/kg, $s_1 = 6.7172$ kJ/kg·K. Now, with $p_2 = 20$ bar, $s_2 = s_1$; $h_2 = 2996.1$ kJ/kg. At state 3, $h_3 = 3690.1$ kJ/kg, $s_3 = 7.7024$ kJ/kg·K. Thus, $p_4 = 0.06$ bar, $s_4 = s_3$ gives

$$x_4 = \frac{s_4 - s_f}{s_g - s_f} = \frac{7.7024 - 0.5210}{8.3304 - 0.5210} = 0.9196 \Rightarrow h_4 = 151.53 + (.9196)(2415.9) = 2373.2 \text{ kJ/kg}.$$

Now, $p_5 = 0.06$ bar, sat. liq: $\Rightarrow h_5 = 151.53$ kJ/kg, $v_5 = 1.0064 \times 10^{-3}$ m³/kg. Finally,

$$h_6 \approx h_5 + v_5(p_6 - p_5) = 151.53 + (1.0064 \times 10^{-3})\frac{m^3}{kg}(140 - 0.06)\,bar\left|\frac{10^5 N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3 N\cdot m}\right|$$
$$= 165.61 \text{ kJ/kg}$$

Cycle Calculations

The work per unit mass of steam flow is obtained as follows:

$$\dot{W}_t/\dot{m} = (h_1 - h_2) + (h_3 - h_4) \tag{1}$$

$$\dot{W}_p/\dot{m} = h_6 - h_5 \tag{2}$$

and

$$\dot{W}_{cycle}/\dot{m} = \dot{W}_t/\dot{m} - \dot{W}_p/\dot{m} \tag{3}$$

For the heat input in the steam generator and reheat processes

$$\dot{Q}_{in}/\dot{m} = (h_1 - h_6) + (h_3 - h_2) \tag{4}$$

PROBLEM 8.33 (Cont'd.)

The thermal efficiency is $\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}}$ (5)

Inserting data from the sample calculation into Eqs. (1)-(5) gives, for $p_2 = 20$ bar

$\dot{W}_t/\dot{m} = 1911.9$ kJ/kg , $\dot{W}_p/\dot{m} = 14.08$ kJ/kg $\Rightarrow \dot{W}_{cycle}/\dot{m} = 1897.8$ kJ/kg

$\dot{Q}_{in}/\dot{m} = 4119.5$ kJ/kg $\Rightarrow \eta = 0.4607$ (46.07 %)

To obtain data for the required plot, we use IT, as follows

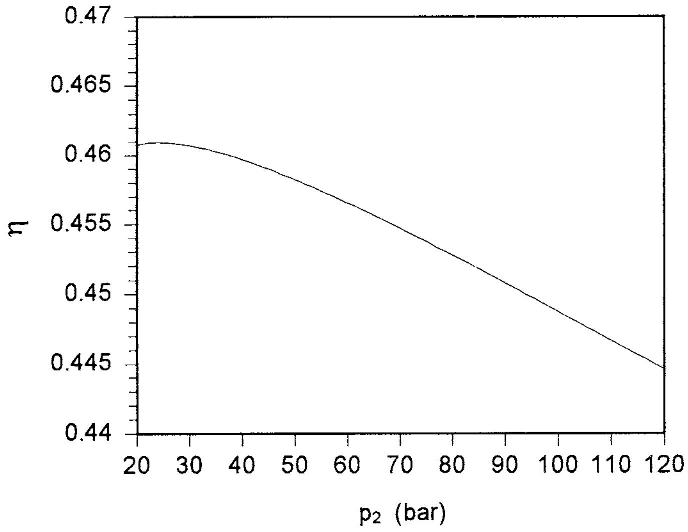
IT Code

```
p1 = 140  // bar
T1 = 600  // °C
p2 = 20  // bar
p3 = p2
T3 = 600  // °C
p4 = 0.06  // bar
p5 = p4
p6 = p1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4s = h_Ps("Water/Steam", p4, s4s)
s4s = s3
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6s = h5 + v5 * (p6 - p5) * 100

Wt = (h1 - h2s) + (h3 - h4s)
Wp = h6s - h5
Wcycle = Wt - Wp
Qin = (h1 - h6s) + (h3 - h2s)
eta = Wcycle / Qin
```

IT Results for p_2 = 2 MPa (20 bar)

h_1 = 3591 kJ/kg
s_1 = 6.716 kJ/kg·K
h_{2s} = 2995 kJ/kg
h_3 = 3690 kJ/kg
s_3 = 7.702 kJ/kg·K
h_{4s} = 2373 kJ/kg
h_5 = 151 kJ/kg
h_{6s} = 165.1 kJ/kg
$\dot{W}_t/\dot{m}$ = 1912 kJ/kg
$\dot{W}_p/\dot{m}$ = 14.09 kJ/kg
$\dot{W}_{cycle}/\dot{m}$ = 1898 kJ/kg
$\dot{Q}_{in}/\dot{m}$ = 4120 kJ/kg
η = 0.4608

PLOT:



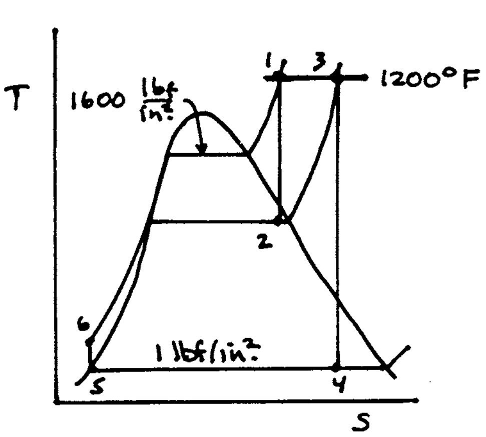
We see that the thermal efficiency exhibits a maximum value in the range of reheat pressures studied.

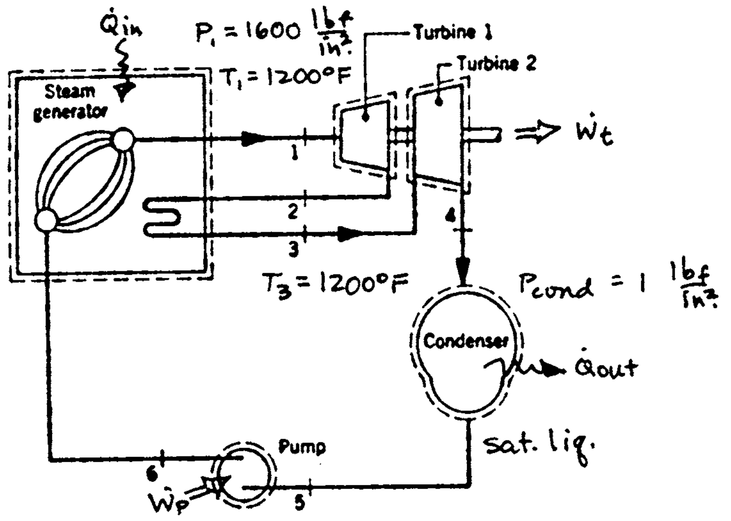
PROBLEM 8.34

KNOWN: An ideal reheat cycle uses steam as the working fluid. Operating pressures and temperatures are given.

FIND: Plot thermal efficiency versus reheat pressure for pressures ranging from 60 lbf/in² to 1200 lbf/in².

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 8.3

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 1600$ lbf/in², $T_1 = 1200°F \Rightarrow h_1, s_1$

State 2: For a given P_2, and with $s_2 = s_1$, we can get h_2.

State 3: $P_3 = P_2$, $T_3 = 1200°F \Rightarrow h_3, s_3$

State 4: $P_4 = 1$ lbf/in.², and with $s_4 = s_3$, we can get x_4 and h_4, or h_4 if superheated.

State 5: $P_5 = 1$ lbf/in.², sat. liq. $\Rightarrow h_5, v_5$

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5)$

Cycle calculations.

The work per unit mass of steam flow is

$$\dot{W}_t/\dot{m} = (h_1 - h_2) + (h_3 - h_4) \qquad (1)$$

$$\dot{W}_p/\dot{m} = h_6 - h_5 \qquad (2)$$

$$\dot{W}_{cycle}/\dot{m} = \dot{W}_t/\dot{m} - \dot{W}_p/\dot{m} \qquad (3)$$

The heat input to the steam generator and reheat process is

$$\dot{Q}_{in}/\dot{m} = (h_1 - h_6) + (h_3 - h_2) \qquad (4)$$

Finally, the thermal efficiency is $\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}}$ (5)

Sample calculation. $P_2 = 60$ lbf/in.² Using data from the steam tables, $h_1 = 1607.1$ Btu/lb, $s_1 = 1.6684$ Btu/lb·°R. Now, with $P_2 = 60$ lbf/in.², $s_2 = s_1$; $h_2 = 1196.7$ Btu/lb. At state 3, $h_3 = 1638.5$ Btu/lb, $s_3 = 2.0448$ Btu/lb.°R. Thus, $P_4 = 1$ lbf/in², $s_4 = s_3$, gives $h_4 = 1146.3$ Btu/lb. Now, $P_5 = 1$ lbf/in², sat. liq. $\Rightarrow$ $h_5 = 69.74$ Btu/lb, $v_4 = 0.01614$ ft³/lb. Finally

$$h_6 \approx h_5 + v_5(P_6 - P_5) = 69.74 + (0.01614)\frac{ft^3}{lb}(1600-1)\frac{lbf}{in^2}\left|\frac{144\ in^2}{1\ ft^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|$$

$$= 74.52\ lbf/in.^2$$

Inserting data into Eqs. (1) - (5) gives, for $P_2 = 60$ lbf/in²

PROBLEM 8.34 (Cont'd.)

$\dot{W}_t/\dot{m} = 902.6$ Btu/lb, $\dot{W}_p/\dot{m} = 4.78$ Btu/lb $\Rightarrow$ $\dot{W}_{cycle}/\dot{m} = 897.8$ Btu/lb

$\dot{Q}_{in}/\dot{m} = 1974.4$ Btu/lb $\Rightarrow$ $\eta = 0.4547$ (45.47%)

To obtain data for the required plots, we use *IT*, as follows:

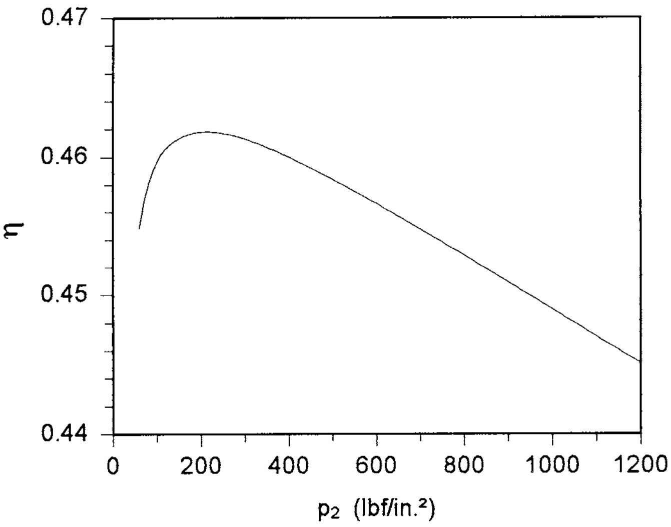
IT Code

```
p1 = 1600  // lbf/in.²
T1 = 1200  // °F
p2 = 60  // lbf/in.²
p3 = p2
T3 = 1200  // °F
p4 = 1  // lbf/in.²
p5 = p4
p6 = p1

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2s = h_Ps("Water/Steam", p2, s2s)
s2s = s1
h3 = h_PT("Water/Steam", p3, T3)
s3 = s_PT("Water/Steam", p3, T3)
h4s = h_Ps("Water/Steam", p4, s4s)
s4s = s3
h5 = hsat_Px("Water/Steam", p5, 0)
v5 = vsat_Px("Water/Steam", p5, 0)
h6s = h5 + v5 * (p6 - p5) * (144 / 778)

Wt = (h1 - h2s) + (h3 - h4s)
Wp = h6s - h5
Wcycle = Wt - Wp
Qin = (h1 - h6s) + (h3 - h2s)
eta = Wcycle / Qin
```

IT Results for p_2 = 60 lbf/in.²

h_1 = 1607 Btu/lb
s_1 = 1.668 Btu/lb·°R
h_{2s} = 1196 Btu/lb
h_3 = 1638 Btu/lb
s_3 = 2.045 Btu/lb·°R
h_{4s} = 1146 Btu/lb
h_5 = 69.58 Btu/lb
h_{6s} = 74.36 Btu/lb
$\dot{W}_t/\dot{m}$ = 902.8 Btu/lb
$\dot{W}_p/\dot{m}$ = 4.776 Btu/lb
$\dot{W}_{cycle}/\dot{m}$ = 898.1 Btu/lb
$\dot{Q}_{in}/\dot{m}$ = 1975 Btu/lb
η = 0.4548

PLOT:



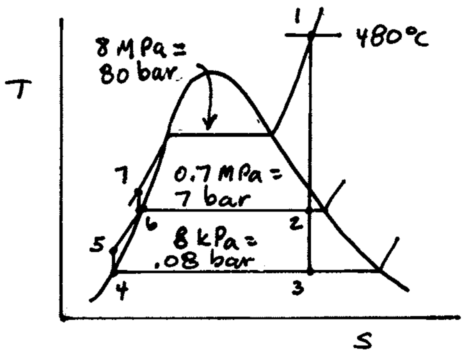
We see that the thermal efficiency exhibits a maximum in the range of reheat pressures studied.

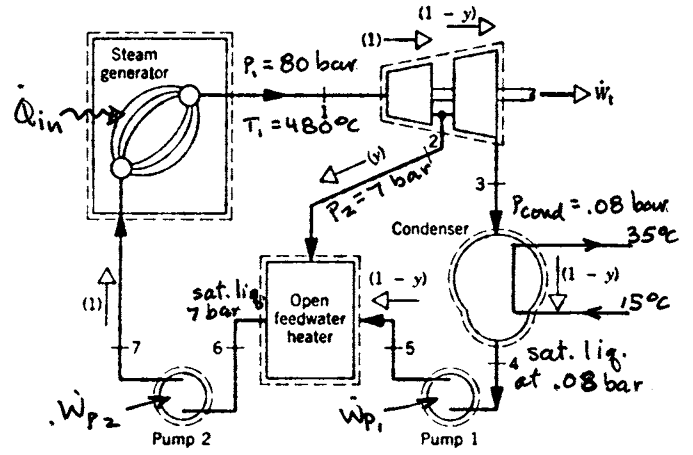
PROBLEM 8.35

KNOWN: The ideal Rankine cycle of Problem 8.2 is modified to include one open feedwater heater operating at 7 bar.

FIND: Answer the same questions about the modified cycle as in Problem 8.2, and discuss the results.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 8.5, except that the turbine stages operate in an internally reversible manner.

ANALYSIS: First, fix each of the principal states. From the solution to Problem 8.2; $h_1 = 3348.4$ kJ/kg, $s_1 = 6.6586$, $h_3 = 2082.9$ kJ/kg, and $h_4 = 173.88$ kJ/kg.

State 2: $p_2 = 7$ bar, $s_2 = s_1 \Rightarrow x_2 = .9895$, $h_2 = 2741.8$ kJ/kg

State 5: $h_5 \approx h_4 + v_4 (p_5 - p_4)$

$$= 173.88 + (1.0084\times10^{-3})\frac{m^3}{kg}(7-.08)\,\text{bars}\left|\frac{10^5\,N/m^2}{1\,\text{bar}}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 173.88 + .698 = 174.58\ \text{kJ/kg}$$

State 6: $p_6 = 7$ bar, sat. liquid $\Rightarrow h_6 = 697.22$ kJ/kg, $v_6 = 1.108\times10^{-3}\,m^3/kg$

State 7: $h_7 \approx h_6 + v_6 (p_7 - p_6)$

$$= 697.22 + (1.108\times10^{-3})(80-7)\left|\frac{10^5}{10^3}\right| = 697.22 + 8.09 = 705.31\ \text{kJ/kg}$$

Applying mass and energy rate balances to the control volume enclosing the open heater, the fraction y of flow extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{697.22 - 174.58}{2741.8 - 174.58} = 0.2036$$

(a) To find the heat transfer rate $\dot{Q}_{in}$, it is first necessary to determine the mass flow rate $\dot{m}_1$. To do so, begin with

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3)$$

$$= [(3348.4 - 2741.8) + (1 - .2036)(2741.8 - 2082.9)]\frac{kJ}{kg} = 1131.3\ \text{kJ/kg}$$

and

$$\frac{\dot{W}_p}{\dot{m}_1} = (1-y)(h_5 - h_4) + (h_7 - h_6)$$

$$= (1 - .2036)(.698) + (8.09) = 8.65\ \text{kJ/kg}$$

PROBLEM 8.35 (Cont'd)

Now, with

$$\dot{W}_{cycle} = \dot{m}_1 \left[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1 \right]$$

The mass flow rate entering the first turbine stage is

$$\dot{m}_1 = \frac{(100 \times 10^3 \text{ kW})}{(1131.3 - 8.65) \text{ kJ/kg}} \left| \frac{1 \text{ kJ/s}}{1 \text{ kW}} \right| = 89.07 \text{ kg/s}$$

Finally, the heat transfer rate $\dot{Q}_{in}$ is

$$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_7) = (89.07 \tfrac{\text{kg}}{\text{s}})(3348.4 - 705.31) \tfrac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 235 \times 10^3 \text{ kW} \longleftarrow \dot{Q}_{in}$$

(b) The thermal efficiency is

$$\eta = \frac{100 \times 10^3 \text{ kW}}{235 \times 10^3 \text{ kW}} = 0.425 \quad (42.5\%) \longleftarrow \eta$$

(c) The fraction of the flow $\dot{m}_1$ that passes through the condenser is $1-y = 0.7964$. Thus, the mass flow rate of cooling water can be determined from

$$0 = (1-y)\dot{m}_1 (h_3 - h_4) + \dot{m}_{cw} (h_{cw,in} - h_{cw,out})$$

or

$$\dot{m}_{cw} = \frac{(1-y)\dot{m}_1 (h_3 - h_4)}{h_{cw,out} - h_{cw,in}}$$

with $h_{cw} \approx h_f(T)$ from Table A-2

$$\dot{m}_{cw} = \frac{(0.7964)(89.07 \frac{\text{kg}}{\text{s}})(2082.9 - 173.88) \frac{\text{kJ}}{\text{kg}}}{(146.68 - 62.99) \text{ kJ/kg}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right|$$

$$= 5.825 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}_{cw}$$

Discussion. (1) The net work per unit mass entering the first turbine is less for this modified cycle than for the ideal cycle. Thus, for the same net power output more mass must flow through the steam generator. (2) The rate of heat input to the working fluid passing through the steam generator is less for the modified cycle because of regeneration in the open feedwater heater. As a result, the thermal efficiency is increased. (3) The rate of heat rejection to the cooling water is less for the modified cycle, leading to a smaller cooling water flow rate.

PROBLEM 8.36

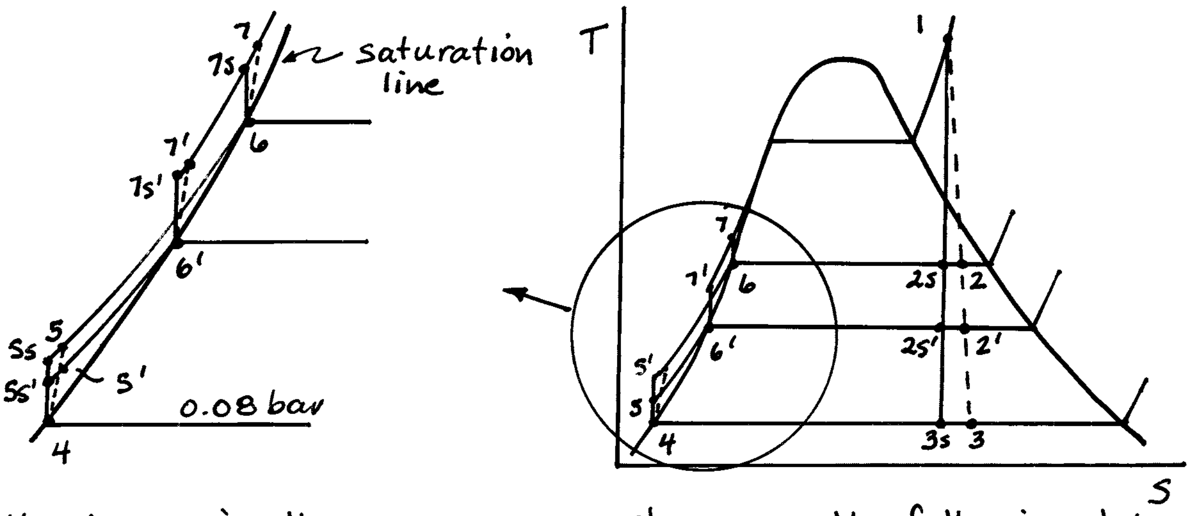
KNOWN: The modified cycle of Problem 8.35 is reconsidered.

FIND: Investigate the effects on cycle performance as the feedwater heater pressure takes on other values. Construct suitable graphs and discuss. Consider the ideal cycle as well as a cycle for which the turbine stages and pumps each have isentropic efficiencies less than unity.

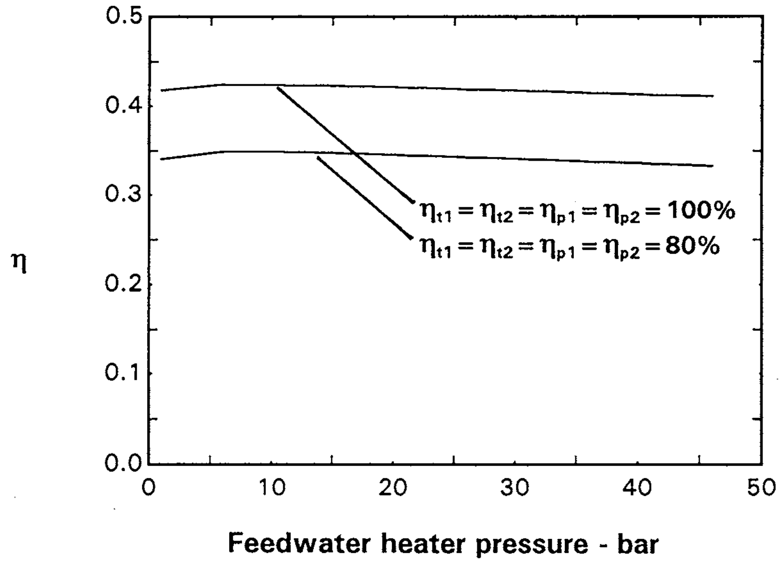
SCHEMATIC & GIVEN DATA: See solution to Problem 8.35.

ASSUMPTIONS: See Example 8.5, and $\eta_{t_1} = \eta_{t_2} = \eta_{p_1} = \eta_{p_2} = 80\%$ for the second part.

ANALYSIS: The performance of the cycle is investigated by determining the effects on thermal efficiency of varying the feedwater heater pressure.



The results of varying the pressure are shown on the following plot.



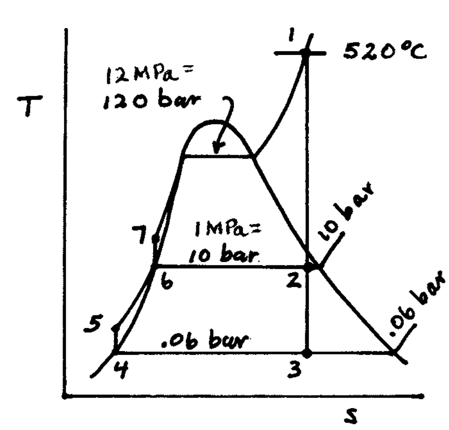
Note that the thermal efficiency does not vary greatly with feedwater heater pressure, but the curves do exhibit a maximum indicative of an optimum feedwater heater pressure.

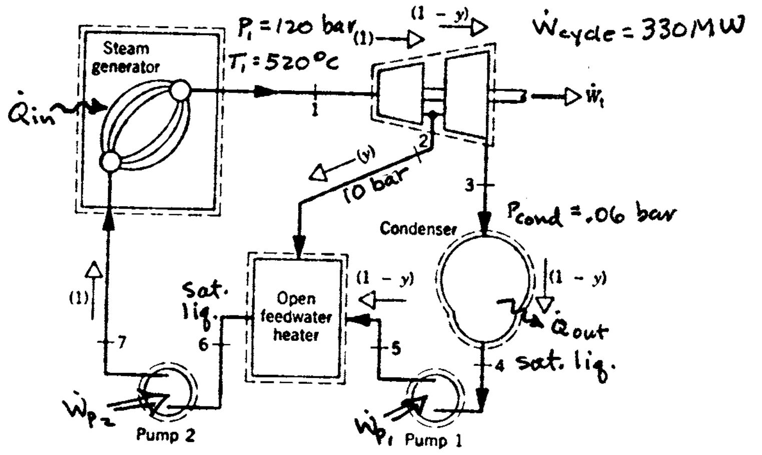
PROBLEM 8.37

KNOWN: Water is the working fluid in a regenerative power cycle with one open feedwater heater. Data at various locations are known and the net power output is given.

FIND: Determine the thermal efficiency and the mass flow rate into the first turbine stage.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 8.5, except that the turbine stages operate in an internally reversible manner.

ANALYSIS: First, fix each principal state.

State 1: $P_1 = 120$ bar, $T_1 = 520°C \Rightarrow h_1 = 3401.8$ kJ/kg, $s_1 = 6.5555$ kJ/kg·K

State 2: $P_2 = 10$ bar, $s_2 = s_1 \Rightarrow x_2 = 0.9931$, $h_2 = 2764.2$ kJ/kg

State 3: $P_3 = 0.06$ bar, $s_3 = s_1 \Rightarrow x_3 = 0.7727$, $h_3 = 2018.3$ kJ/kg

State 4: $P_4 = 0.06$ bar, sat. liquid $\Rightarrow h_4 = 151.53$ kJ/kg

State 5: $h_5 \approx h_4 + v_4(P_5 - P_4)$

$$= 151.53\ \frac{kJ}{kg} + \left(1.0064\times10^{-3}\ \frac{m^3}{kg}\right)(10-.06)\,\text{bar}\left|\frac{10^5\ N/m^2}{1\ \text{bar}}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

$$= 151.53 + 1.00 = 152.53\ kJ/kg$$

State 6: $P_6 = 10$ bar, sat. liquid $\Rightarrow h_6 = 762.81$ kJ/kg

State 7: $P_7 = 120$ bar, $h_7 \approx h_6 + v_6(P_7 - P_6)$

$$= 762.81 + (1.1273\times10^{-3})(120-10)\left|\frac{10^5}{10^3}\right| = 775.21\ kJ/kg$$

(a) Applying energy and mass balances to the control volume enclosing the open heater, the fraction of flow y extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{762.81 - 152.53}{2764.2 - 152.53} = 0.2337 \qquad (1)$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3) \qquad (2)$$

$$= (3401.8 - 2764.2) + (1 - .2337)(2764.2 - 2018.3) = 1209.2\ kJ/kg$$

And, for the pumps

$$\dot{W}_P/\dot{m}_1 = (h_7 - h_6) + (1-y)(h_5 - h_4) \qquad (3)$$

$$= (775.21 - 762.81) + (.7663)(152.53 - 151.53)$$

$$= 13.17\ kJ/kg$$

PROBLEM 8.37 (Contd.)

For the working fluid passing through the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_7 = 3401.8 - 775.21 = 2626.6 \text{ kJ/kg} \quad (4)$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.455 \ (45.5\%) \longleftarrow \eta \quad (6)$$

(b) The net power developed is

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1] \quad (7)$$

$$\Rightarrow \dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]}$$

$$= \frac{(330 \times 10^3 \text{ kJ/s})}{(1209.2 - 13.17) \text{ kJ/kg}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right|$$

$$= 9.93 \times 10^5 \text{ kg/h} \longleftarrow \dot{m}_1$$

PROBLEM 8.38

KNOWN: Steady-state operating data are provided for the regenerative cycle of Problem 8.37.

FIND: Plot the thermal efficiency of the cycle and the rate of exergy destruction within the feedwater heater versus feedwater heater pressure ranging from ① 0.5 to 10 MPa (5 to 100 bar).

SCHEMATIC & GIVEN DATA: See Problem 8.37

ASSUMPTIONS: See solution to Problem 8.37. Also, let $T_0 = 293\,\text{K}$.

ANALYSIS: The states are fixed as in the solution to Problem 8.37. Further, Eqs. (1) – (7) from the solution to Problem 8.37 are used.

The rate of exergy destruction in the feedwater heater can be obtained from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production in the control volume surrounding the feedwater heater. That is

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j}^{\,0} + \dot{m}_1 \left[(1-y)s_5 + y s_2 - s_6\right] + \dot{\sigma}_{cv}$$

or

$$\dot{E}_d = T_0 \dot{m}_1 \left[s_6 - y s_2 - (1-y) s_5\right] \tag{8}$$

Sample calculation: For $p_2 = 1.0\,\text{MPa} = 10\,\text{bar}$, see the solution to Problem 8.37. For the exergy destruction rate

$$\dot{E}_d = (293\,\text{K})\left(9.93\times10^5\,\frac{\text{kg}}{\text{h}}\right)\left[2.1387 - (0.2337)(6.5555) - (.7663)(.5210)\right]\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

$$= 6.0354\times10^7\,\text{kJ/h}\left|\frac{1\,\text{h}}{3600\,\text{s}}\right|\left|\frac{1\,\text{kW}}{1\,\text{kJ/s}}\right| = 1.677\times10^4\,\text{kW}$$

To obtain the data required for the plots, we use IT, as follows:

IT Code

```
p1 = 120  // bar
T1 = 520  // °C
p2 = 10   // bar
p3 = 0.06  // bar
p4 = p3
p5 = p2
p6 = p2
p7 = p1
Wdotcycle = 330  // MW

h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water", p2, s2s)
s3s = s2s
h3s= h_Ps("Water", p3, s3s)
h4 = hsat_Px("Water", p4, 0)
v4 = vsat_Px("Water", p4, 0)
s4 = ssat_Px("Water/Steam", p4, 0)
h5s = h4 + v4 * (p5 - p4) * (100)
s5s = s4
v6 = vsat_Px("Water", p6, 0)
h6 = hsat_Px("Water", p6, 0)
s6 = ssat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * (100)

0 = y * h2s + (1 - y) * h5s - h6
Wdott1 = mdot1 * (h1 - h2s)
Wdott2 = mdot1 * (1 - y) * (h2s - h3s)
Wdotp1 = mdot1 * (1 - y) * (h5s - h4)
Wdotp2 = mdot1 * (h7s - h6)
Wdotcycle * 10^3 = Wdott1 + Wdott2  - Wdotp1 - Wdotp2
mdot = mdot1 *3600  // kg/h
Qdotin = mdot1 * (h1 - h7s)
Qdotout = mdot1 * (1 - y) * (h3s - h4)
eta = Wdotcycle * 10^3 / Qdotin
Edotd = To * mdot1 * (s6 - y * s2s - (1 - y) * s5s)
To = 293  // K
```

PROBLEM 8.38 (Cont'd.)

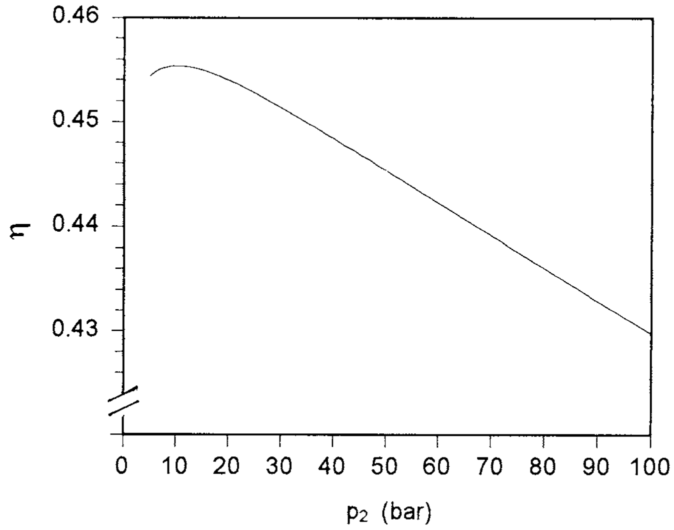
IT Results for p_2 = 1 MPa (10 bar)

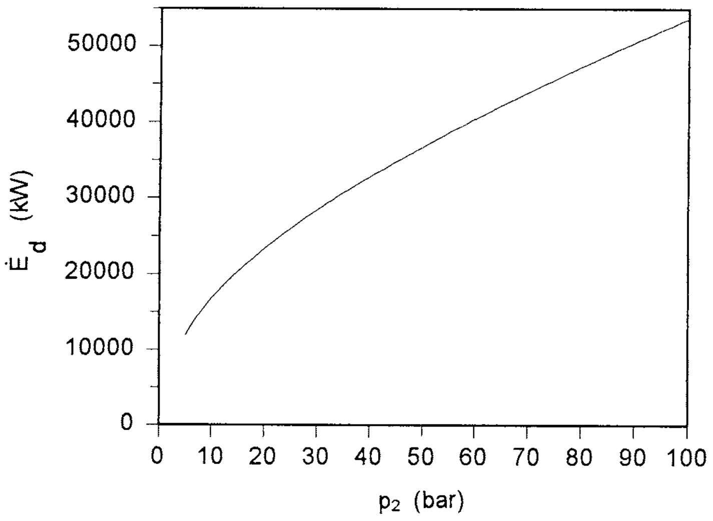
$h_1 = 3401$ kJ/kg
$h_{2s} = 2764$ kJ/kg
$h_{3s} = 2018$ kJ/kg
$h_4 = 151$ kJ/kg
$h_{5s} = 152$ kJ/kg
$h_6 = 762.5$ kJ/kg
$h_{7s} = 774.9$ kJ/kg
$s_1 = 6.555$ kJ/kg·K
$s_4 = 0.519$ kJ/kg·K
$s_6 = 2.138$ kJ/kg·K

$y = 0.2338$

$\dot{W}_{t1} = 1.76 \times 10^5$ kW
$\dot{W}_{t2} = 1.577 \times 10^5$ kW
$\dot{W}_{p1} = 211.6$ kW
$\dot{W}_{t2} = 3422$ kW
$\dot{Q}_{in} = 7.247 \times 10^5$ kW
$\eta = 0.4553$
$\dot{m}_1 = 9.934 \times 10^5$ kg/h
$\dot{E}_d = 1.681 \times 10^4$ kW

PLOTS:





From the plots, we see that the thermal efficiency exhibits a maximum in the range of feedwater heaters studied. In addition, the exergy destruction rate increases with increasing P_2, as the temperature difference between the extracted steam and the steam generator feedwater increases.

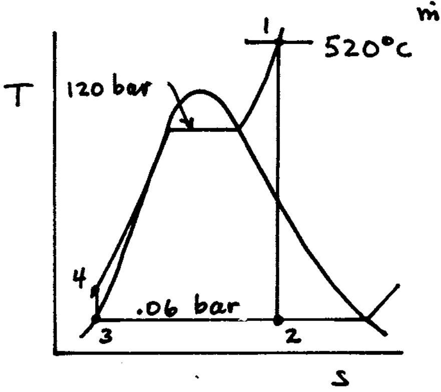
PROBLEM 8.39

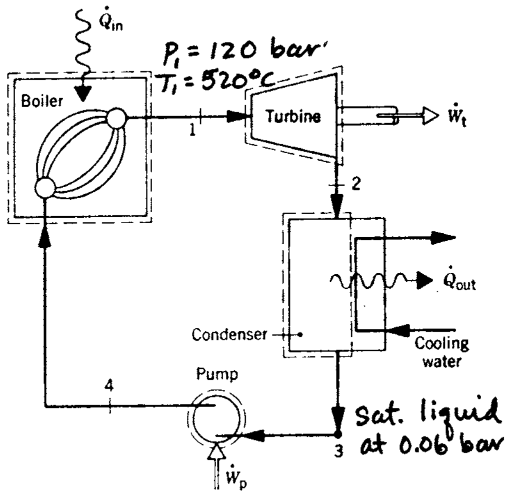
KNOWN: An ideal Rankine cycle having the same turbine inlet conditions and condenser pressure, but no regenerator, as the cycle of Problem 8.37 is considered.

FIND: Answer the same questions about the simple cycle as those in Problem 8.37.

SCHEMATIC & GIVEN DATA:

$\dot{m}_1 = 10^6\ \frac{kg}{h}$





ASSUMPTIONS: See Example 8.1, assumptions 1-4

ANALYSIS: Using appropriate data from the solution to Problem 8.37

$h_1 = 3401.8$ kJ/kg, $s_1 = 6.5555$ kJ/kg·K
$h_2 = 2018.3$ kJ/kg $h_3 = 151.53$ kJ/kg

State 4:
$$h_4 \approx h_3 + v_3(p_4 - p_3)$$
$$= 151.53\ \frac{kJ}{kg} + (1.0064 \times 10^{-3}\ \frac{m^3}{kg})(120 - .06)\ bars \left|\frac{10^5\ N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$
$$= 151.53 + 12.07 = 163.6\ kJ/kg$$

(a) For the turbine
$$\dot{W}_t/\dot{m} = h_1 - h_2 = 3401.8 - 2018.3 = 1383.5\ kJ/kg$$

And, for the pump
$$\dot{W}_p/\dot{m} = h_4 - h_3 = 12.07\ kJ/kg$$

For the working fluid passing through the steam generator
$$\dot{Q}_{in}/\dot{m} = h_1 - h_4 = 3401.8 - 163.6 = 3238.2\ kJ/kg$$

Thus, the thermal efficiency is
$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 0.424\ (42.4\%) \longleftarrow \eta$$

(b) The net power developed is
$$\dot{W}_{cycle} = \dot{m}\left[\dot{W}_t/\dot{m} - \dot{W}_p/\dot{m}\right]$$
$$= \left(10^6\ \frac{kg}{h}\right)\left|\frac{1\ h}{3600\ s}\right|\left[1383.5 - 12.07\right]\frac{kJ}{kg}\left|\frac{1\ MW}{10^3\ kJ/s}\right| = 381\ MW \longleftarrow \dot{W}_{cycle}$$

The introduction of the open feedwater heater into the cycle of Problem 8.37 results in an increase in thermal efficiency and a decrease in the net power developed.

PROBLEM 8.40

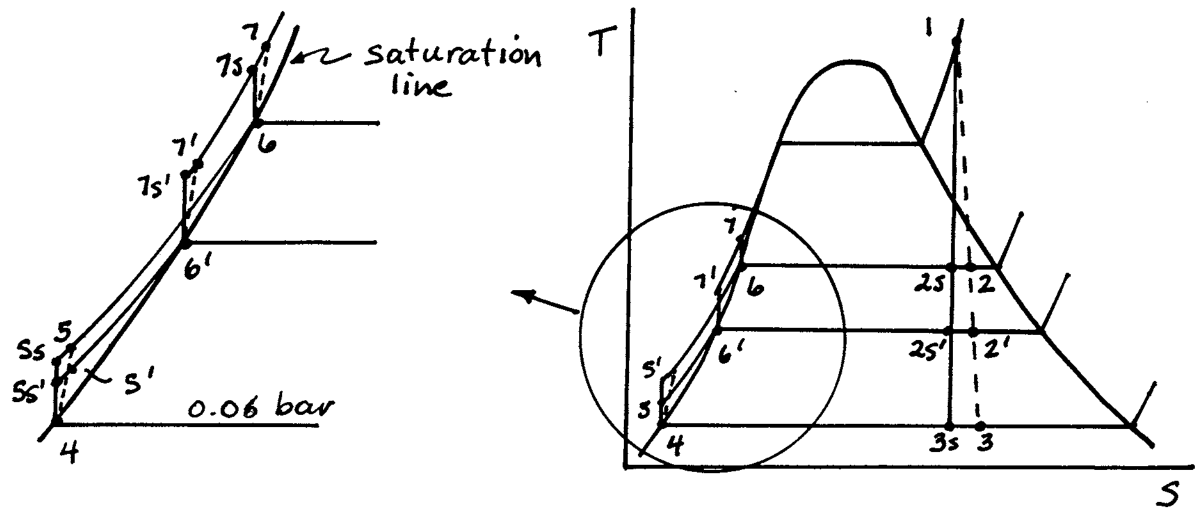
KNOWN: The regenerative vapor power cycle of Problem 8.37 is reconsidered.

FIND: Investigate the effects on cycle performance as the feedwater heater pressure takes on other values. Construct suitable plots and discuss. Consider the cycle for which each of the turbine stages and the pump have isentropic efficiencies less than unity.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.37.

ASSUMPTIONS: See Example 8.5, and $\eta_{t_1} = \eta_{t_2} = \eta_{p_1} = \eta_{p_2} = 80\%$.

ANALYSIS: The performance of the cycle is investigated by determining the effects on thermal efficiency of varying the feedwater heater pressure.



To obtain the data for the required plots, we use IT, as follows

```
IT Code

p1 = 120   // bar
T1 = 520   // °C
p2 = 10    // bar
p3 = 0.06  // bar
p4 = p3
p5 = p2
p6 = p2
p7 = p1
Wdotcycle = 330  // MW
etat1 = .8
etat2 = .8
etap1 = .8
etap2 = .8

h1 = h_PT("Steam", p1, T1)
s1 = s_PT("Steam", p1, T1)
s2s = s1
h2s = h_Ps("Water", p2,
s2s)
h2 = h1 - etat1 * (h1 - h2s)
s2 = s_Ph("Water/Steam",
p2, h2)
s3s = s2
h3s= h_Ps("Water", p3, s3s)
h3 = h2 - etat2 * (h2 - h3s)
```

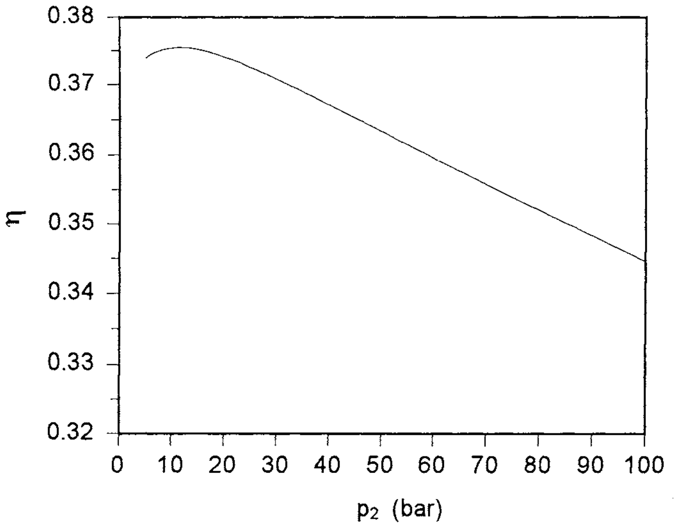
```
h4 = hsat_Px("Water", p4, 0)
v4 = vsat_Px("Water", p4, 0)
s4 = ssat_Px("Water/Steam", p4, 0)
h5s = h4 + v4 * (p5 - p4) * (100)
h5 = h4 + (h5s - h4) / etap1
s5s = s4
s5 = s_Ph("Water/Steam", p5, h5)
v6 = vsat_Px("Water", p6, 0)
h6 = hsat_Px("Water", p6, 0)
s6 = ssat_Px("Water/Steam", p6, 0)
h7s = h6 + v6 * (p7 - p6) * (100)
h7 = h6 + (h7s - h6) / etap2

0 = y * h2 + (1 - y) * h5 - h6

Wdott1 = mdot1 * (h1 - h2)
Wdott2 = mdot1 * (1 - y) * (h2 - h3)
Wdotp1 = mdot1 * (1 - y) * (h5 - h4)
Wdotp2 = mdot1 * (h7 - h6)
Wdotcycle * 10^3 = Wdott1 + Wdott2  - Wdotp1 - Wdotp2
Qdotin = mdot1 * (h1 - h7)
Qdotout = mdot1 * (1 - y) * (h3 - h4)
eta = Wdotcycle * 10^3 / Qdotin
mdot = mdot1 *3600

Edotd = To * mdot1 * (s6 - y * s2 - (1 - y) * s5)
To = 293
```

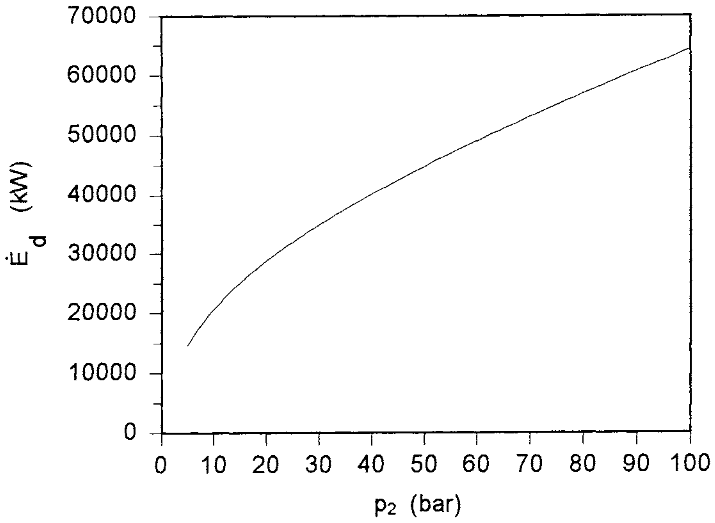
PROBLEM 8.40 (Cont'd.)

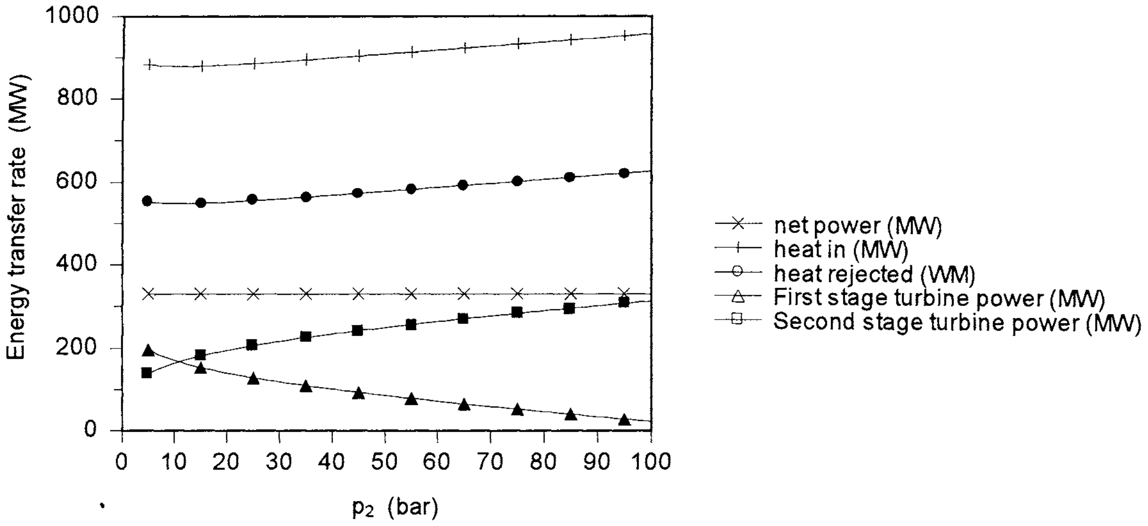
IT Results for p_2 = 1.0 MPa (10 bar)

h_1 = 3401 kJ/kg
h_2 = 2891 kJ/kg
h_{2s} = 2764 kJ/kg
h_3 = 2259 kJ/kg
h_{3s} = 2101 kJ/kg
h_4 = 151 kJ/kg
h_5 = 152.3 kJ/kg
h_{5s} = 152 kJ/kg
h_6 = 762.5 kJ/kg
h_7 = 778 kJ/kg
h_{7s} = 774.9 kJ/kg
s_1 = 6.555 kJ/kg·K
s_2 = 6.824 kJ/kg·K
s_4 = 0.519 kJ/kg·K
s_5 = 0.5231 kJ/kg·K
s_6 = 2.138 kJ/kg·K

$y = 0.2228$
$\dot{E}_d = 2.07 \times 10^4$ kW
$\dot{W}_{p1} = 325.7$ kW
$\dot{W}_{p2} = 5194$ kW
$\dot{W}_{t1} = 1.71 \times 10^5$ kW
$\dot{W}_{t2} = 1.646 \times 10^5$ kW
$\dot{m}_1 = 1.206 \times 10^6$ kg/h
$\dot{Q}_{in} = 8.791 \times 10^5$ kW
$\eta = 0.375$







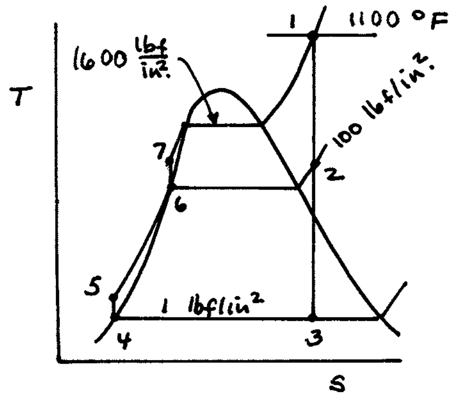
Even though turbine and pump isentropic efficiencies are less than unity, the trends for η and $\dot{E}_d$ are similar to those in Problem 8.38. Also, note that the power output of the first turbine stage decreases as $p_2 \rightarrow p_1$, and the output of stage two increases accordingly.

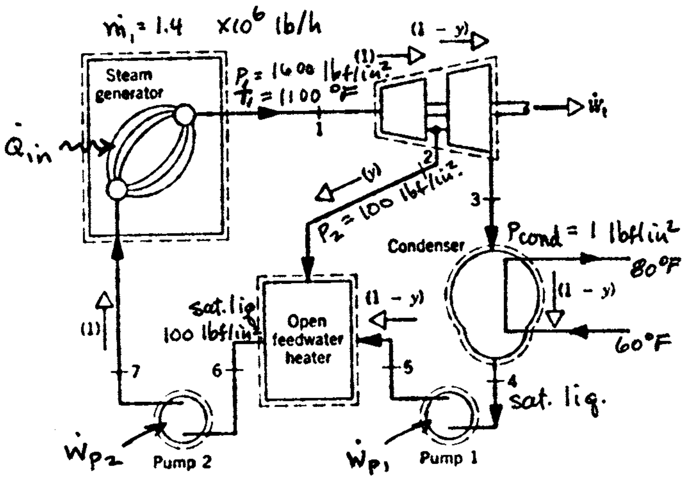
PROBLEM 8.41*

KNOWN: The ideal Rankine cycle of Problem 8.9 is modified to include one open feedwater heater operating at 100 lbf/in.²

FIND: Answer the same questions about the modified cycle as in Problem 8.9 and discuss the results.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 8.5, except that the turbine stages operate in an internally reversible manner.

ANALYSIS: First, fix each of the principal states. From the solution to Problem 8.9; h_1 = 1547.7 Btu/lb, s_1 = 1.6315 Btu/lb·°R, h_3 = 911.2 Btu/lb, and h_4 = 69.74 Btu/lb.

State 2: $p_2 = 100\ \text{lbf/in.}^2,\ s_2 = s_1 \Rightarrow h_2 = 1210.7\ \text{Btu/lb}$ (Table A-4E)

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$= 69.74\ \frac{\text{Btu}}{\text{lb}} + \left(.01614\ \frac{\text{ft}^3}{\text{lb}}\right)\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|(100-1)\ \frac{\text{lbf}}{\text{in.}^2}$$

$$= 69.74 + .296 = 70.04\ \text{Btu/lb}$$

State 6: $p_6 = 100\ \text{lbf/in.}^2$, sat. liquid $\Rightarrow h_6 = 298.6$ Btu/lb

State 7: $h_7 \approx h_6 + v_6(p_7 - p_6) = 298.6 + (.01774)\left|\frac{144}{778}\right|(1600-100) = 303.5$ Btu/lb

Applying energy and mass rate balances to the control volume enclosing the open heater, the fraction y of flow extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{298.6 - 70.04}{1209.2 - 70.04} = 0.2004$$

(a) To determine the net power developed by the cycle, begin with a control volume around the turbine

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3)$$

$$= (1547.7 - 1210.7) + (1 - 0.2004)(1210.7 - 911.2) = 576.48\ \text{Btu/lb}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_7 - h_6) + (1-y)(h_5 - h_4)$$

$$= (303.5 - 298.6) + (1 - 0.2004)(70.04 - 69.74) = 5.14\ \text{Btu/lb}$$

Thus, with $\dot{m}_1 = 1.4 \times 10^6$ lb/h (Problem 8.9)

$\dot{W}_{cycle}$

$$\dot{W}_{cycle} = \dot{m}_1\left[\dot{W}_t/\dot{m}_1 - \dot{W}_P/\dot{m}_1\right] = \left(1.4\times10^6\ \frac{\text{lb}}{\text{h}}\right)\left[576.48 - 5.14\right]\frac{\text{Btu}}{\text{lb}} = 8.00\times10^8\ \frac{\text{Btu}}{\text{h}}$$

PROBLEM 8.41 (Cont'd)

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}_1(h_1 - h_7)}$$

$$= \frac{(8.00 \times 10^8)}{(1.4 \times 10^6)(1547.7 - 303.5)} = 0.459\ (45.9\%) \longleftarrow \underline{\qquad\qquad \eta}$$

(c) The fraction of the flow $\dot{m}_1$ that passes through the condenser is $(1-y) = 0.7996$. Thus, the mass flow rate of cooling water can be determined from

$$0 = (1-y)\,\dot{m}_1(h_3 - h_4) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

or

$$\dot{m}_{cw} = \frac{(1-y)\,\dot{m}_1(h_3 - h_4)}{(h_{cw,out} - h_{cw,in})}$$

With $h_{cw} \approx h_f(T)$ from Table A-2E

$$\dot{m}_{cw} = \frac{(.7996)(1.4 \times 10^6\ \text{lb/h})(911.2 - 69.74)\,\text{Btu/lb}}{(48.09 - 28.08)\,\text{Btu/lb}}$$

$$= 4.71 \times 10^7\ \text{lb/h} \longleftarrow \underline{\qquad\qquad \dot{m}_{cw}}$$

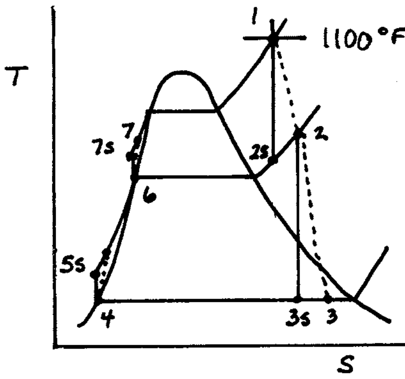
Discussion: (1) Since some mass is extracted from the turbine, the power developed in the modified cycle is less than for the cycle of Problem 8.9. (2) The rate of heat transfer to the working fluid passing through the steam generator is less for the modified cycle due to regeneration in the open feedwater heater. In this case, the thermal efficiency is increased due to regeneration. (3) In the modified cycle, less mass of working fluid passes through the condenser, leading to a reduced need for cooling water.

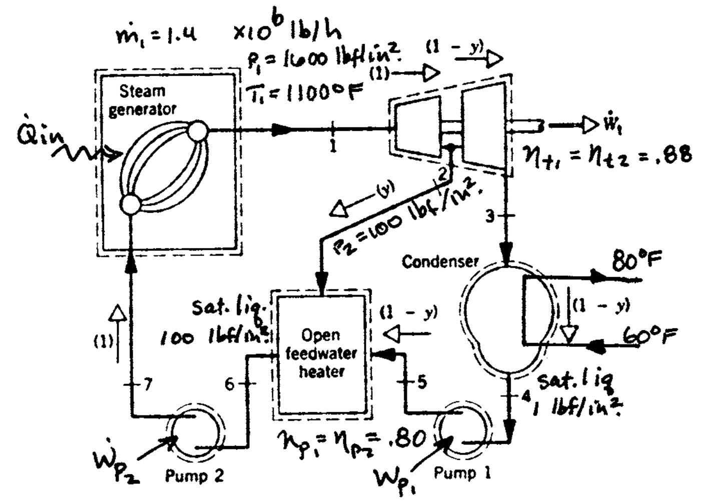
PROBLEM 8.42*

KNOWN: The regenerative power cycle of Problem 8.41 is modified to include that each turbine stage has an efficiency of 88% and each pump has an efficiency of 80%.

FIND: Answer the same questions for the modified cycle as in Problem 8.41

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Problem 8.41, except $\eta_{t1} = \eta_{t2} = .88$ and $\eta_{p1} = \eta_{p2} = 0.80$.

ANALYSIS: From Problem 8.41, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R, $h_4 = 69.74$ Btu/lb, and $h_6 = 298.6$ Btu/lb.

State 2: Use the turbine stage efficiency. First, $p_2 = 100$ lbf/in² and $s_{2s} = s_1$; $h_{2s} = 1210.7$ Btu/lb (Problem 8.41) Thus

$$\eta_{t1} = \frac{\dot{W}_{t1}/\dot{m}}{(\dot{W}_{t1}/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t1}(h_1 - h_{2s}) = 1251.1 \text{ Btu/lb}$$

From Table A-4E, $s_2 = 1.6784$ Btu/lb.

State 3: Similarly, $p_3 = 1$ lbf/in², $s_{3s} = s_2 \Rightarrow x_{3s} = 0.8376$, $h_{3s} = 937.52$ Btu/lb.
Thus $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 975.15$ Btu/lb

State 5: State 5 is determined using the pump efficiency. From Problem 8.41 $h_{5s} = 70.04$ Btu/lb. Thus

$$\eta_{p1} = \frac{(\dot{W}_{p1}/\dot{m})_s}{(\dot{W}_{p1}/\dot{m})} = \frac{h_{5s} - h_4}{h_5 - h_4} \Rightarrow h_5 = h_4 + (h_{5s} - h_4)/\eta_{p1} = 70.115 \text{ Btu/lb}$$

State 7: Similarly, $h_{7s} = 303.5$ Btu/lb and $h_7 = h_6 + (h_{7s} - h_6)/\eta_{p2}$ $= 304.7$ Btu/lb

The fraction y of the turbine flow extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{298.6 - 70.115}{1251.1 - 70.115} = 0.1935$$

(a) For the turbines

$$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$
$$= (1547.7 - 1251.1) + (1 - 0.1935)(1251.1 - 975.15) = 519.15 \text{ Btu/lb}$$

and the pumps

$$\dot{W}_p/\dot{m}_1 = (h_7 - h_6) + (1 - y)(h_5 - h_4)$$
$$= (304.7 - 298.6) + (1 - .1935)(70.115 - 69.74) = 6.402 \text{ Btu/lb}$$

PROBLEM 8.42 (Contd.)

Thus

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]$$

$$= (1.4 \times 10^6 \tfrac{lb}{h})[519.15 - 6.402]\tfrac{Btu}{lb} = 7.18 \times 10^8\ Btu/h \longleftarrow \underline{\quad \dot{W}_{cycle}}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{m}(h_1 - h_7)} = \frac{7.18 \times 10^8\ Btu/h}{(1.4 \times 10^6\ lb/h)(1547.7 - 304.7)\ Btu/lb}$$

$$= 0.413\ (41.3\%) \longleftarrow \underline{\quad \eta}$$

(c) The mass flow rate of cooling water passing through the condenser is

$$\dot{m}_{cw} = \frac{(1-y)\,\dot{m}_1\,(h_3 - h_4)}{h_{cw,out} - h_{cw,in}} = \frac{(1-.1935)(1.4 \times 10^6)(975.15 - 69.74)}{(48.09 - 28.08)}$$

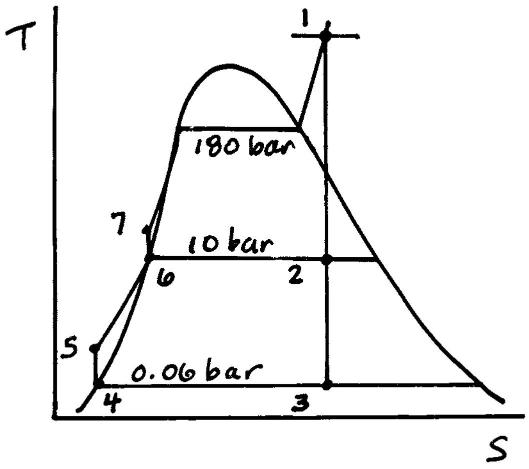
$$= 5.11 \times 10^7\ lb/h \longleftarrow \underline{\quad \dot{m}_{cw}}$$

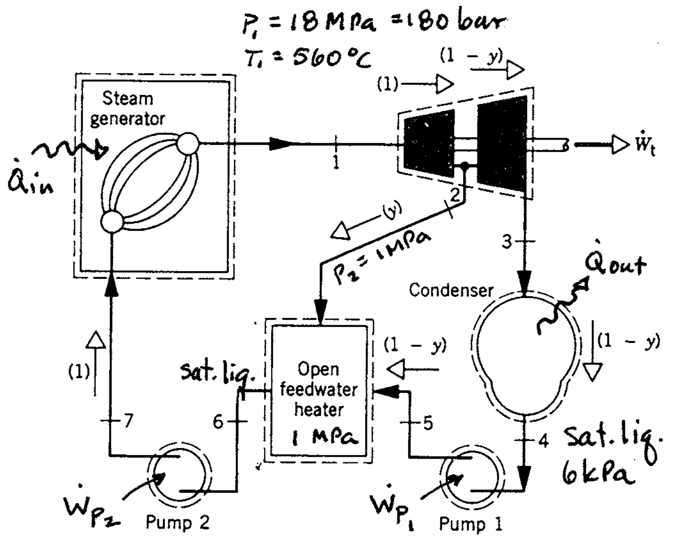
PROBLEM 8.43

KNOWN: The ideal Rankine cycle of Problem 8.6 is modified to include superheated vapor entering the first turbine stage and one open feedwater heater. Saturated liquid exits the feedwater heater and other data are known.

FIND: Determine for the modified cycle (a) the net work per kg of steam flow entering the first turbine stage, (b) the thermal efficiency, and (c) the heat transfer to cooling water in the condenser per kg of steam flow entering the first turbine stage.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 8.5, except the turbine stages operate in an internally reversible manner.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 18\,MPa = 180\,bar$, $T_1 = 560°C \Rightarrow h_1 = 3444.4\,kJ/kg$, $s_1 = 6.4392\,kJ/kg\cdot K$

State 2: $p_2 = 1\,MPa = 10\,bar$, $s_2 = s_1 \Rightarrow x_2 = 0.967$, $h_2 = 2711.6\,kJ/kg$

State 3: $p_3 = 6\,kPa = 0.06\,bar$, $s_3 = s_1 \Rightarrow x_3 = 0.7578$, $h_3 = 1982.3\,kJ/kg$

State 4: $p_4 = 0.06\,bar$, sat. liquid $\Rightarrow h_4 = 151.53\,kJ/kg$, $v_4 = 1.0064\times10^{-3}\,m^3/kg$

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$= 151.53 + (1.0064\times10^{-3}\,\frac{m^3}{kg})(10 - 0.06)\,bar\,\left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 151.53 + 1.00 = 152.53\,kJ/kg$$

State 6: $P_6 = 10\,bars$, sat. liquid $\Rightarrow h_6 = 762.81\,kJ/kg$, $v_6 = 1.1273\times10^{-3}\,m^3/kg$

State 7: $P_7 = P_1 = 180\,bar$

$$h_7 \approx h_6 + v_6(P_7 - P_6) = (762.81) + (1.1273\times10^{-3})(180-10)|100| = 781.97\,kJ/kg$$

(a) First, applying mass and energy rate balances to the control volume enclosing the open heater, the fraction y of flow extracted at 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_5} = \frac{762.81 - 152.53}{2711.6 - 152.53} = 0.2385$$

The total work developed by the turbine, per unit mass flow entering the first turbine stage is

PROBLEM 8.43 (Cont'd)

$$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1-y)(h_2 - h_3)$$
$$= (3444.4 - 2711.6) + (0.7615)(2711.6 - 1982.3)$$
$$= 1288.2 \text{ kJ/kg}$$

Similarly, for the pumps

$$\dot{W}_p/\dot{m}_1 = (1-y)(h_5 - h_4) + (h_7 - h_6)$$
$$= (0.7615)(152.53 - 151.53) + (781.97 - 762.81)$$
$$= 19.92 \text{ kJ/kg}$$

Thus, the net work is

$$\frac{\dot{W}_{cycle}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1268.3 \text{ kJ/kg} \longleftarrow \dot{W}_{cycle}/\dot{m}_1$$

(b) For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_7 = 3444.4 - 781.97 = 2662.4 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1268.3}{2662.4} = 0.476 \; (47.6\%) \longleftarrow \eta$$

(c) For the working fluid passing through the condenser

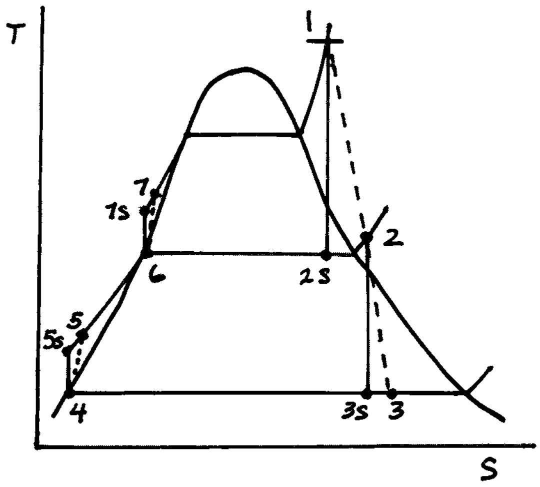
$$\frac{\dot{Q}_{out}}{\dot{m}_1} = (1-y)(h_3 - h_4)$$
$$= (0.7615)(1982.3 - 151.53)$$
$$= 1394.1 \text{ kJ/kg} \longleftarrow \dot{Q}_{out}/\dot{m}_1$$

PROBLEM 8.44.

KNOWN: The regenerative vapor power cycle of Problem 8.43 is modified to include that each turbine stage and pump has an isentropic efficiency of 85%.

FIND: See solution to Problem 8.43.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as Example 8.5, except $\eta_{t_1} = \eta_{t_2} = \eta_{p_1} = \eta_{p_2} = 0.85$.

ANALYSIS: First, fix each of the principal states. From Problem 8.43, $h_1 = 3444.4$ kJ/kg, $s_1 = 6.4392$ kJ/kg·K, $h_4 = 151.53$ kJ/kg, and $h_6 = 762.81$ kJ/kg.

State 2: Using the turbine efficiency, $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$. With $h_{2s} = 2711.6$ kJ/kg from Problem 8.43

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 2821.5 \text{ kJ/kg}$$

Interpolating in Table A-4, $s_2 = 6.6801$ kJ/kg·K

State 3: Similarly, $p_3 = 0.06$ bar, $s_{3s} = s_2 \Rightarrow x_{3s} = 0.7887$, $h_{3s} = 2057.0$ kJ/kg

Thus, $h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2171.7$ kJ/kg.

State 5: Using the pump efficiency, $\eta_{p_1} = (h_{5s} - h_4)/(h_5 - h_4)$. From Problem 8.43; $h_{5s} = 152.53$ kJ/kg. Thus

$$h_5 = h_4 + (h_{5s} - h_4)/\eta_{p_1} = 152.7 \text{ kJ/kg}$$

State 7: Similarly, with $h_{7s} = 781.97$ kJ/kg from Problem 8.43

$$h_7 = h_6 + (h_{7s} - h_6)/\eta_{p_2} = 785.4 \text{ kJ/kg}$$

(a) Using the modified data

$y = 0.2286$, $\dot{W}_t/\dot{m}_1 = 1124.2$ kJ/kg, $\dot{W}_p/\dot{m}_1 = 23.49$ kJ/kg

Thus $\quad \dfrac{\dot{W}_{cycle}}{\dot{m}_1} = 1100.7$ kJ/kg ← $\dot{W}_{cycle}/\dot{m}_1$

(b) $\dot{Q}_{in}/\dot{m}_1 = 2659$ kJ/kg and $\eta = 0.414$ (41.4 %) ← η

① (c) $\dot{Q}_{out}/\dot{m}_1 = 1558.3$ kJ/kg ← $\dot{Q}_{out}/\dot{m}_1$

---

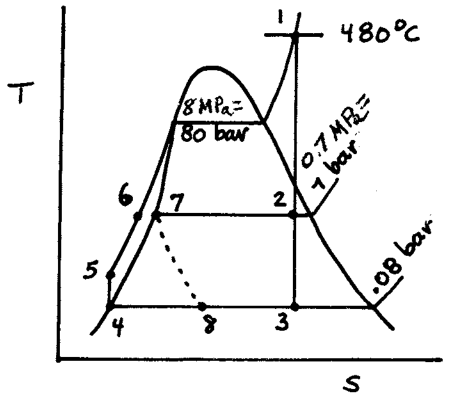
1. Comparing these results with those of Problem 8.43 indicates that turbine and pump irreversibilities affect the net work of the cycle significantly, thereby causing a significant drop in thermal efficiency.

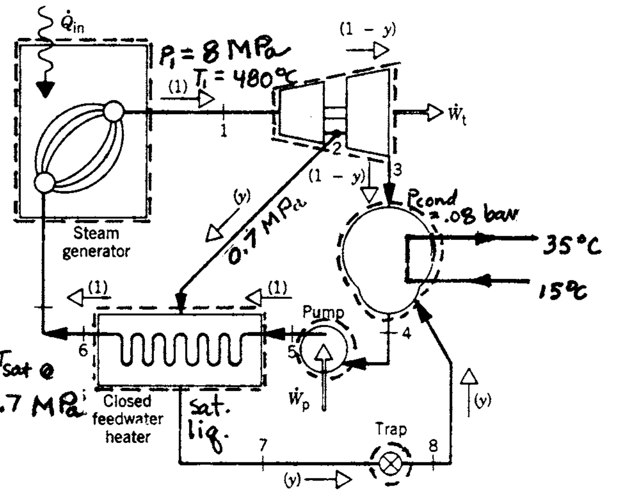
PROBLEM 8.45

KNOWN: The ideal Rankine cycle of Problem 8.2 is modified to include one closed feedwater heater operating at 0.7 MPa. The condensate from the heater is trapped into the condenser.

FIND: Answer the same questions about the modified cycle as in Problem 8.2, and discuss the results.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through the throttling valve (a throttling process) and in the closed feedwater heater. (3) The turbines, pump, and feedwater heater operate adiabatically. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the closed heater as saturated liquid at 7 bars and the condenser as saturated liquid at 0.08 bars.

ANALYSIS: First, fix each of the principal states. From the solution to Problem 8.2; $h_1 = 3348.4$ kJ/kg, $s_1 = 6.6586$ kJ/kg·K, $h_3 = 2082.9$ kJ/kg, $h_4 = 173.88$ kJ/kg, and $h_5 = 181.94$ kJ/kg.

State 2: $p_2 = 7$ bar, $s_2 = s_1 \Rightarrow x_2 = .9895$, $h_2 = 2741.8$ kJ/kg

State 6: $T_6 = T_{sat@7\,bar} = 165$°C, $p_6 = 80$ bar $\Rightarrow$ from Table A-5
$h_6 \approx 702$ kJ/kg (double interpolation)

State 7: $p_7 = 7$ bar, sat. liquid $\Rightarrow h_7 = 697.22$ kJ/kg

State 8: Throttling process $\Rightarrow h_8 = h_7 = 697.22$ kJ/kg

Applying mass and energy rate balances to the closed feedwater heater, the fraction of flow y extracted at 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{702 - 181.94}{2741.8 - 697.22} = 0.2544$$

(a) To find the heat transfer rate $\dot{Q}_{in}$, it is first necessary to determine the mass flow rate $\dot{m}_1$. To do so, begin with

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3)$$

PROBLEM 8.45 (Contd.)

Inserting values

$$\frac{\dot{W}_t}{\dot{m}_1} = (3348.4 - 2741.8) + (1 - .2544)(2741.8 - 2082.9) = 1097.9 \text{ kJ/kg}$$

Similarly

$$\frac{\dot{W}_p}{\dot{m}_1} = h_5 - h_4 = 181.94 - 173.88 = 8.06 \text{ kJ/kg}$$

Now, with

$$\dot{W}_{cycle} = \dot{m}_1 \left[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1 \right]$$

the mass flow rate $\dot{m}_1$ is

$$\dot{m}_1 = \frac{(100 \times 10^3 \text{ kW})}{(1097.9 - 8.06) \text{ kJ/kg}} \; \frac{1 \text{ kJ/s}}{1 \text{ kW}} = 91.76 \text{ kg/s}$$

Thus, the heat transfer rate $\dot{Q}_{in}$ is

$$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_6) = \left(91.76 \tfrac{\text{kg}}{\text{s}}\right)(3348.4 - 702) \tfrac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 243 \times 10^3 \text{ kW} \longleftarrow \dot{Q}_{in}$$

(b) The thermal efficiency is

$$\eta = \frac{100 \times 10^3 \text{ kW}}{243 \times 10^3 \text{ kW}} = 0.412 \; (41.2\%) \longleftarrow \eta$$

(c) For the condenser, the mass and energy rate balances reduce to

$$0 = \dot{m}_1 \left[(1-y) h_3 + y h_8 - h_4 \right] + \dot{m}_{cw} (h_{cw,in} - h_{cw,out})$$

Solving for the cooling water flow rate

$$\dot{m}_{cw} = \frac{\dot{m}_1 \left[(1-y) h_3 + y h_8 - h_4 \right]}{(h_{cw,out} - h_{cw,in})}$$

with $h_{cw} \approx h_f(T)$ from Table A-2

$$\dot{m}_{cw} = \frac{\left(91.76 \tfrac{\text{kg}}{\text{s}}\right)\left(3600 \tfrac{\text{s}}{\text{h}}\right) \left[(.7456)(2082.9) + (.2544)(697.22) - 173.88 \right] \tfrac{\text{kJ}}{\text{kg}}}{(146.68 - 62.99) \text{ kJ/kg}}$$

$$= 6.144 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}_{cw}$$

Discussion. (1) The net work per unit mass entering the first turbine is less for this modified cycle than for the ideal cycle. Thus, for the same net power output, more mass must flow through the steam generator.
(2) The rate of heat input to the working fluid passing through the steam generator is less for the modified cycle because of regeneration in the closed feedwater heater. As a result, the thermal efficiency is increased.
(3) The rate of heat rejection to cooling water is less for the modified cycle, leading to a smaller cooling water flow rate.

PROBLEM 8.46

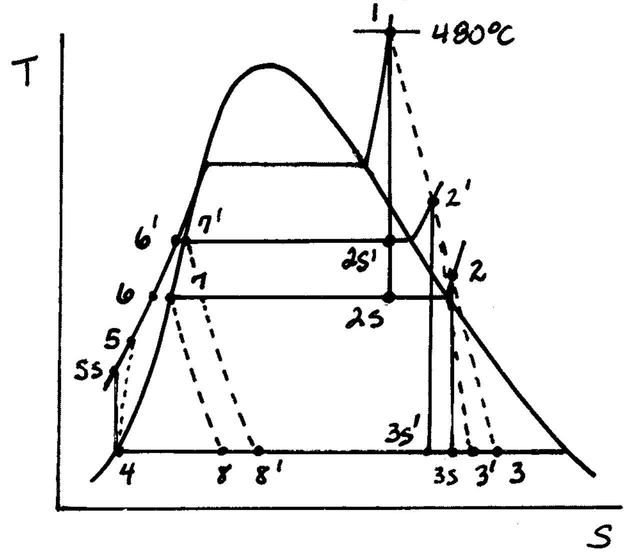
KNOWN: The regenerative vapor power cycle of Problem 8.45 is reconsidered.

FIND: Investigate the effects on performance as extraction pressure takes on other values. Construct suitable plots and discuss. Consider both reversible and irreversible operation of the turbine stages and pump.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.45.

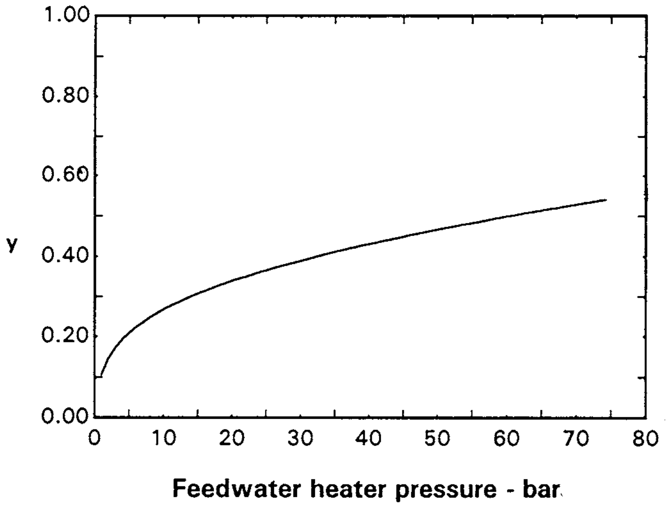
ASSUMPTIONS: See solution to Problem 8.45. Also, $\eta_{t_1} = \eta_{t_2} = \eta_p = 80\%$ for the second part.

ANALYSIS: The performance is investigated by determining the effects on thermal efficiency of varying the feedwater heater extraction pressure.

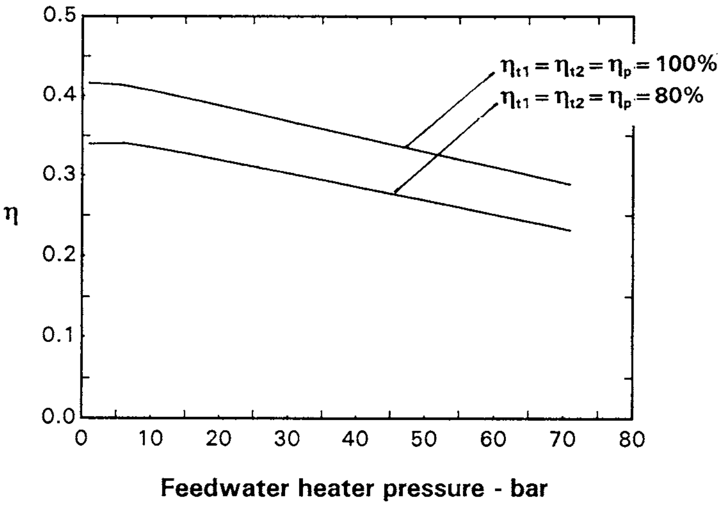


From the T-s diagram, we see that state 6' moves higher as the extraction pressure increases. This reduces the heat input to the cycle. However, from (a) we see that the fraction y of the total flow extracted at 2 also increases. This results in less mass flow through the second turbine stage, and hence, less power output. The net effect is a significant reduction in thermal efficiency, as illustrated in (b).

Another factor that reduces the power output is the irreversible expansion across the valve. This effect also worsens at higher extraction pressure.



(a)



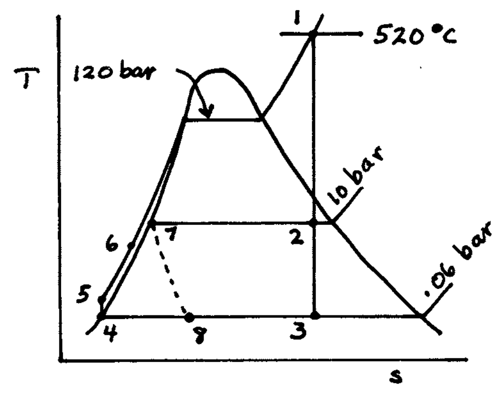
(b)

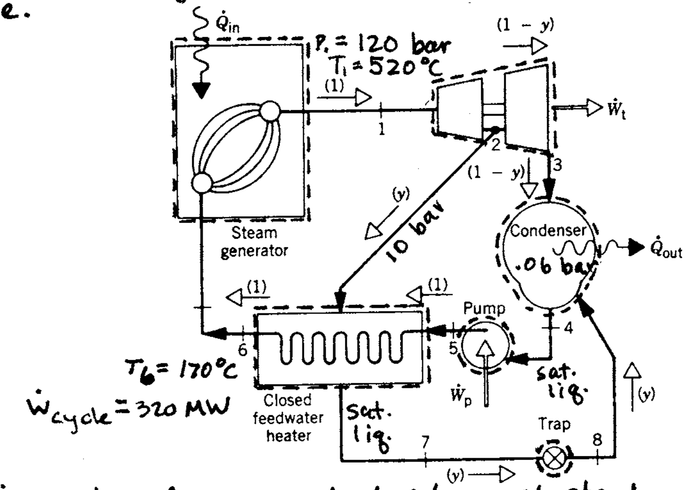
PROBLEM 8.47

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed feedwater heater. Data at various locations are known and the net power output is given.

FIND: Determine (a) the thermal efficiency and (b) the mass flow rate into the first turbine stage.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through the trap (a throttling process) and in the closed feedwater heater. (3) The turbines, pump, and feedwater heater operate adiabatically. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the closed heater and the condenser as saturated liquid at the respective pressures.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 120$ bar, $T_1 = 520°C \Rightarrow h_1 = 3401.8$ kJ/kg, $s_1 = 6.5555$ kJ/kg·K

State 2: $p_2 = 10$ bar, $s_2 = s_1 \Rightarrow x_2 = 0.9931$, $h_2 = 2764.2$ kJ/kg

State 3: $p_3 = 0.06$ bar, $s_3 = s_1 \Rightarrow x_3 = 0.7727$, $h_3 = 2018.3$ kJ/kg

State 4: $p_4 = 0.06$ bar, sat. liquid $\Rightarrow h_4 = 151.53$ kJ/kg

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$= 151.53\ \frac{\text{kJ}}{\text{kg}} + (1.0064\times10^{-3})\frac{\text{m}^3}{\text{kg}}(120 - 0.06)\ \text{bar}\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 151.53 + 12.07 = 163.6\ \text{kJ/kg}$$

State 6: $p_6 = 120$ bar, $T_6 = 170°C \Rightarrow$ Interpolating in Table A-5, $h_6 = 725.86\ \frac{\text{kJ}}{\text{kg}}$

State 7: $p_7 = 10$ bar, Sat. liquid $\Rightarrow h_7 = 762.81$ kJ/kg

State 8: Throttling process $\Rightarrow h_8 = h_7 = 762.81$ kJ/kg

(a) Applying mass and energy rate balances to the control volume surrounding the closed feedwater heater, the fraction of the flow y extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{725.86 - 163.6}{2764.2 - 762.81} = 0.2809$$

For the control volume surrounding the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$

$$= (3401.8 - 2764.2) + (1 - .2809)(2764.2 - 2018.3) = 1174.0 \text{ kJ/kg}$$

And, for the pump

$$\frac{\dot{W}_p}{\dot{m}_1} = h_5 - h_4 = 163.6 - 151.53 = 12.07 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_6 = (3401.8 - 725.86) = 2675.9 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1174.0 - 12.07}{2675.9} = 0.434 \; (43.4\%) \longleftarrow \underline{\qquad\qquad \eta}$$

(b) The mass flow rate into the first turbine stage is found from

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]$$

or

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]}$$

$$= \frac{(320 \text{ MW})}{[1174.0 - 12.07] \text{ kJ/kg}} \left| \frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right|$$

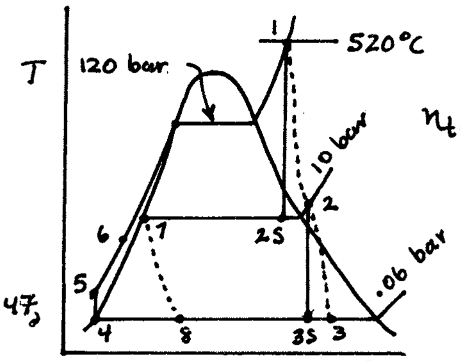
$$= 9.91 \times 10^5 \text{ kg/h} \longleftarrow \underline{\qquad\qquad \dot{m}_1}$$

PROBLEM 8.48

KNOWN: The regenerative vapor power cycle of Problem 8.47 is modified to include that each turbine stage has an efficiency of 82%.

FIND: Answer the same questions about the modified cycle as in Problem 8.47.

SCHEMATIC & GIVEN DATA:



$\eta_{t_1} = \eta_{t_2} = 0.82$

ASSUMPTIONS: Same as Problem 8.47, except $\eta_{t_1} = \eta_{t_2} = 0.82$.

ANALYSIS: From the solution of Problem 8.47

$h_1 = 3401.8$ kJ/kg , $s_1 = 6.5555$ kJ/kg·K
$h_4 = 151.53$ kJ/kg
$h_5 = 163.6$ kJ/kg
$h_6 = 725.86$ kJ/kg
$h_7 = h_8 = 762.81$ kJ/kg

State 2: Using the turbine efficiency, $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$. With $h_{2s} = 2764.2$ kJ/kg (Problem 8.47)

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 2879.0 \text{ kJ/kg}$$

From Table A-4, $s_2 = 6.7977$ kJ/kg·K

State 3: Similarly, with $p_3 = 0.06$ bar, $s_{3s} = s_2 \Rightarrow x_{3s} = .8037$ and $h_{3s} = 2093.2$ kJ/kg. Thus

$$h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2234.6 \text{ kJ/kg}$$

(a) The fraction of flow y is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = 0.2657$$

For the turbine stages

$$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1-y)(h_2 - h_3) = 996.0 \text{ kJ/kg}$$

And, for the pump

$$\dot{W}_p/\dot{m}_1 = h_5 - h_4 = 12.07 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\dot{Q}_{in}/\dot{m}_1 = h_1 - h_6 = 2675.9 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.368 \; (36.8\%) \longleftarrow \eta$$

(b) The mass flow rate into the first turbine stage is

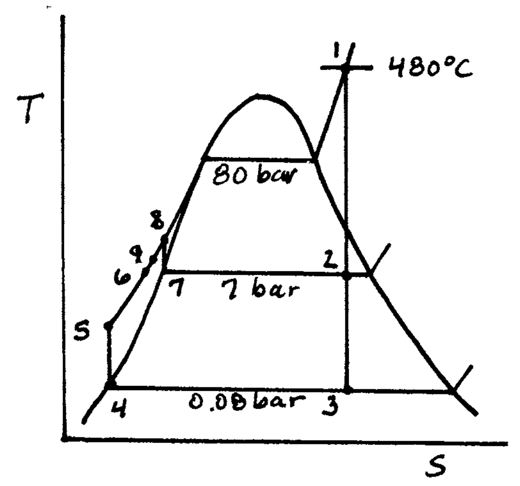
$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]} = \frac{(320 \text{ MW})}{[996.0 - 12.07] \text{ kJ/kg}} \left(\frac{10^3 \text{ kJ/s}}{1 \text{ MW}}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 1.17 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}_1$$

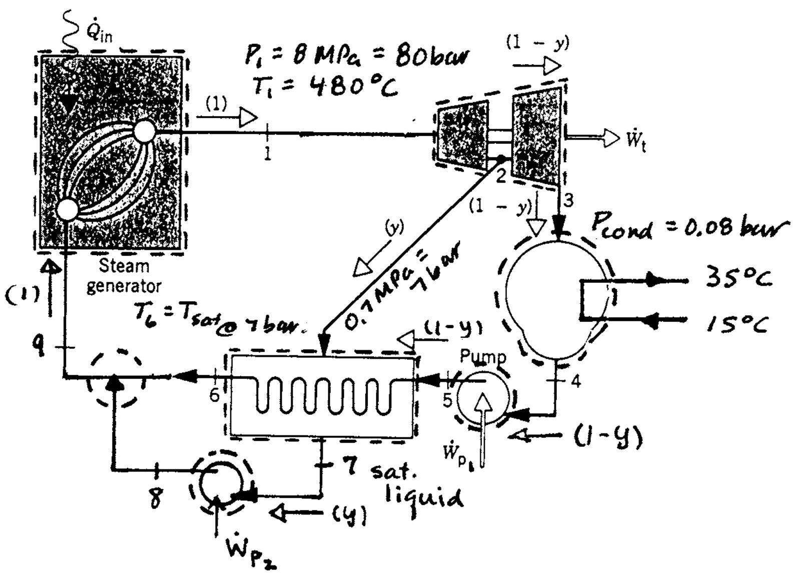
PROBLEM 8.49

KNOWN: The modified regenerative cycle in Problem 8.45 is reconsidered. The condensate is pumped into the boiler feedwater line.

FIND: Answer the same questions about the cycle with this further modification as in Problem 8.45. Compare the two condensate options.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the mixing of streams 8 and 6 to form 9. (3) Other assumptions are as in Problem 8.45.

ANALYSIS: First, fix each of the principal states. States 1-7 are the same as in Problem 8.45. Summarizing

h_1 = 3348.4 kJ/kg h_5 = 181.94
h_2 = 2741.8 h_6 = 702
h_3 = 2082.9 h_7 = 697.22
h_4 = 173.88

State 8: $h_8 \approx h_7 + v_7(p_8 - p_7)$

$$= 697.22\ \frac{kJ}{kg} + (1.108\times10^{-3})\frac{m^3}{kg}(80-7)\,bar\left|\frac{10^5\,N/m^2}{1\,bar}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 697.22 + 8.09 = 705.31\ kJ/kg$$

State 9: First, applying mass and energy rate balances to the closed feedwater heater, the fraction of flow extracted at 2 is obtained as follows

$$0 = (1-y)(h_5 - h_6) + y(h_2 - h_7)$$

or

$$y = \frac{(h_6 - h_5)}{(h_2-h_7)+(h_6-h_5)} = \frac{(702 - 181.94)}{(2741.8 - 697.22)+(702 - 181.94)} = 0.2028$$

Now, for the control volume enclosing the mixing process

$$0 = (1-y)\,h_6 + y\,h_8 - h_9$$

or

$$h_9 = (1-y)\,h_6 + y\,h_8 = (0.7972)(702) + (.2028)(705.31)$$
$$= 702.67\ kJ/kg$$

PROBLEM 8.49 (Cont'd)

(a) To find the heat transfer rate $\dot{Q}_{in}$, it is first necessary to determine the mass flow rate $\dot{m}_1$. To do so, begin with

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3)$$

$$= (3348.4 - 2741.8) + (1 - .2028)(2741.8 - 2082.9) = 1131.9 \text{ kJ/kg}$$

and

$$\frac{\dot{W}_p}{\dot{m}_1} = (1-y)(h_5 - h_4) + y(h_8 - h_7)$$

$$= (0.7972)(181.94 - 173.88) + (.2028)(705.31 - 697.22) = 8.07 \text{ kJ/kg}$$

Thus

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1} = \frac{(100 \text{ MW})}{(1131.9 - 8.07) \text{ kJ/kg}} \left| \frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right| = 88.98 \text{ kg/s}$$

The heat transfer rate $\dot{Q}_{in}$ is

$$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_9) = (88.96 \tfrac{\text{kg}}{\text{s}})(3348.4 - 702.67) \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right|$$

$$= 235 \times 10^3 \text{ kW} \longleftarrow \dot{Q}_{in}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{100 \times 10^3 \text{ kW}}{235 \times 10^3 \text{ kW}} = 0.426 \; (42.6\%)$$

(c) For the condenser, the mass and energy rate balances reduce to

$$0 = \dot{m}_1 (1-y)(h_3 - h_4) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

Solving for the mass flow rate of cooling water

$$\dot{m}_{cw} = \frac{\dot{m}_1 (1-y)(h_3 - h_4)}{(h_{cw,out} - h_{cw,in})}$$

with $h_{cw} \approx h_f(T)$ from Table A-2

$$\dot{m}_{cw} = \frac{(88.98 \text{ kg/s})(0.7972)(2082.9 - 173.88) \text{ kJ/kg}}{(146.68 - 62.99) \text{ kJ/kg}} \left| \frac{3600 \text{ s}}{1 \text{ h}} \right|$$

$$= 5.825 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}_{cw}$$

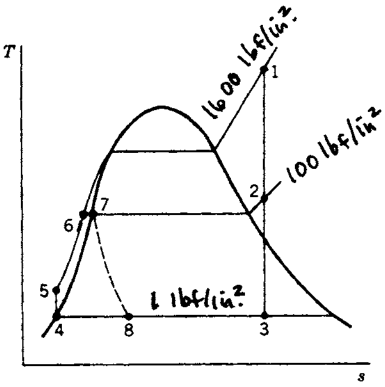
Discussion. Pumping the condensate into the boiler feedwater line has advantages over trapping it into the condenser. The fraction of steam y extrated at 2 is lower, resulting in greater work per unit mass flow through the turbine. Since the power is constant, less flow is required. This results in lower heat input and higher thermal efficiency. The pumping option also eliminates the irreversibility due to the trapping valve. Finally, the pumping option is somewhat more complex in terms of piping and requires an additional pump in the system with associated maintenance needs.

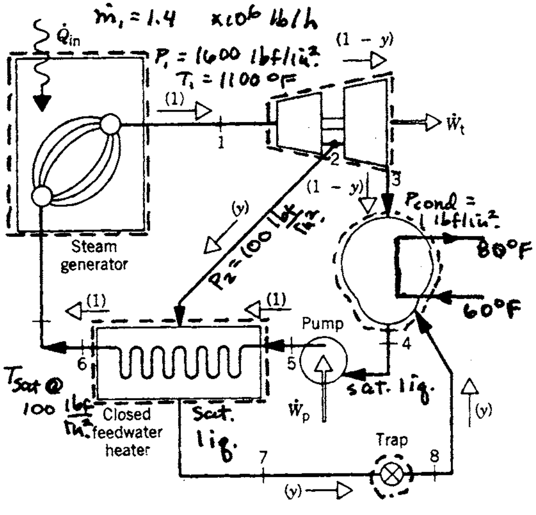
PROBLEM 8.50*

KNOWN: The ideal Rankine cycle of Problem 8.9 is modified to include one closed feedwater heater operating at 100 lbf/in². The condensate from the heater is trapped into the condenser.

FIND: Answer the same questions about the modified cycle as in Problem 8.9, and discuss the results.

SHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through the trap (a throttling process) and in the closed feedwater heater. (3) The turbines, pump, and feedwater heater operate adiabatically. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the closed heater as saturated liquid at 100 lbf/in² and the condenser as saturated liquid at 1 lbf/in².

ANALYSIS: First, fix each of the principal states. From the solution to Problem 8.9; $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R, $h_3 = 911.2$ Btu/lb, $h_4 = 69.74$ Btu/lb, and $h_5 = 74.52$ Btu/lb.

State 2: $p_2 = 100$ lbf/in², $s_2 = s_1 \Rightarrow h_2 = 1210.7$ Btu/lb (Table A-4E)

State 6: $p_6 = 1600$ lbf/in², $T_6 = T_{sat} @ 100$ lbf/in² $= 327.86$ °F $\Rightarrow$ from Table A-5E
$h_6 \approx 301.6$ Btu/lb (double interpolation)

State 7: $p_7 = 100$ lbf/in², sat. liquid $\Rightarrow h_7 = 298.6$ Btu/lb

State 8: Throttling process $\Rightarrow h_8 = h_7 = 298.6$ Btu/lb

Applying mass and energy rate balances to the closed feedwater heater, the fraction of flow y extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = 0.2490$$

(a) To determine the power developed by the cycle, begin with a control volume around the turbine

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y)(h_2 - h_3)$$
$$= [(1547.7 - 1210.7) + (1 - 0.2490)(1210.7 - 911.2)] \text{ Btu/lb}$$
$$= 561.2 \text{ Btu/lb}$$

PROBLEM 8.50 (Contd.)

For the pump

$$\frac{\dot{W}_p}{\dot{m}_1} = h_5 - h_4 = (74.52 - 69.74)\ \text{Btu/lb} = 4.78\ \text{Btu/lb}$$

Thus, with $\dot{m}_1 = 1.4 \times 10^6$ lb/h (Problem 8.9)

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]$$

$$= (1.4 \times 10^6\ \tfrac{\text{lb}}{\text{h}})[561.92 - 4.78]\ \tfrac{\text{Btu}}{\text{lb}} = 7.800 \times 10^8\ \text{Btu/h} \longleftarrow \underline{\dot{W}_{cycle}}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}(h_1 - h_6)}$$

$$= \frac{(7.800 \times 10^8)\ \text{Btu/h}}{(1.4 \times 10^6\ \text{lb/h})(1547.7 - 301.6)\ \text{Btu/lb}} = 0.447\ (44.7\%) \longleftarrow \underline{\eta}$$

(c) For the condenser, the mass and energy rate balances reduce to

$$0 = \dot{m}_1 [(1-y)h_3 + y h_8 - h_4] + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

where $\dot{m}_{cw}$ is the mass flow rate of cooling water. Solving for $\dot{m}_{cw}$

$$\dot{m}_{cw} = \frac{\dot{m}_1 [(1-y)h_3 + y h_8 - h_4]}{(h_{cw,out} - h_{cw,in})}$$

with $h_{cw} \approx h_f(T)$ from Table A-2E

$$\dot{m}_{cw} = \frac{(1.4 \times 10^6\ \text{lb/h})[(0.751)911.2 + (0.249)298.6 - 69.74]\ \text{Btu/lb}}{(48.09 - 28.08)\ \text{Btu/lb}}$$

$$= 4.820 \times 10^7\ \text{lb/h} \longleftarrow \underline{\dot{m}_{cw}}$$

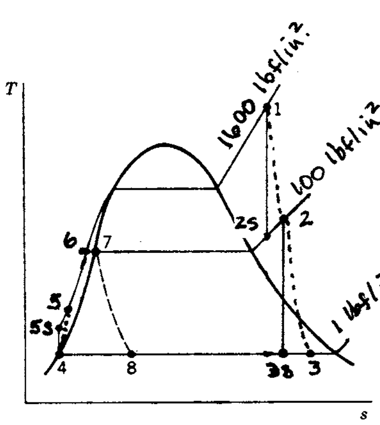
Discussion: (1) Since some mass is extracted from the turbine, the power developed in the modified cycle is less than for the cycle of Problem 8.9. (2) The rate of heat transfer to the working fluid passing through the steam generator is less for the modified cycle due to regeneration in the closed feed water heater. In this case, the thermal efficiency is increased due to regeneration.

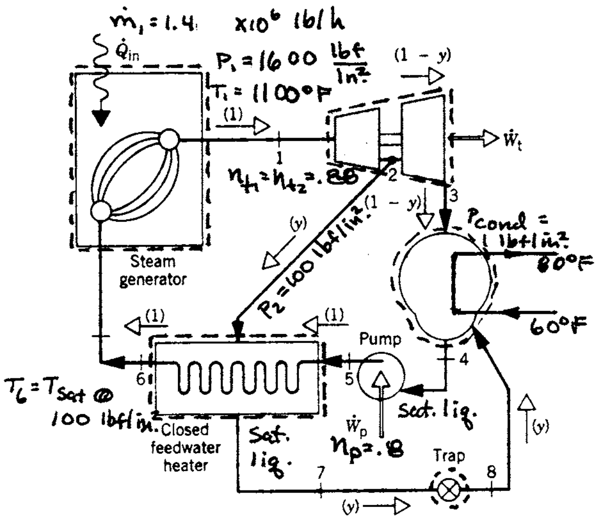
PROBLEM 8.51*

KNOWN: The regenerative power cycle of Problem 8.50 is modified to include that each turbine stage has an efficiency of 88% and the pump has an efficiency of 80%.

FIND: Answer the same questions for the modified cycle as in Problem 8.50

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Problem 8.50, except $\eta_{t_1} = \eta_{t_2} = .88$ and $\eta_p = .80$.

ANALYSIS: From Problem 8.50, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R, $h_4 = 69.74$ Btu/lb, $h_6 = 301.6$ Btu/lb, $h_7 = 298.6$ Btu/lb, $h_8 = h_7 = 298.6$ Btu/lb.

State 2: Use the turbine stage efficiency. First, $p_2 = 100$ lbf/in.² and $s_{2s} = s_1$; $h_{2s} = 1210.7$ Btu/lb (Problem 8.50). Thus

$$\eta_{t_1} = \frac{\dot{W}_{t_1}/\dot{m}}{(\dot{W}_{t_1}/\dot{m})_s} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 1251.1 \text{ Btu/lb}$$

From Table A-4E, $s_2 = 1.6784$ Btu/lb·°R.

State 3: Similarly, $P_3 = 1$ lbf/in.², $s_{3s} = s_2 \Rightarrow x_{3s} = 0.8377$, $h_{3s} = 937.55$ Btu/lb

Thus, $h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 975.2$ Btu/lb

State 5: State 5 is fixed using the pump efficiency. From Problem 8.50 $h_{5s} = 74.52$ Btu/lb. Thus

$$\eta_p = \frac{h_{5s} - h_4}{h_5 - h_4} \Rightarrow h_5 = h_4 + (h_{5s} - h_4)/\eta_p = 75.715 \text{ Btu/lb}$$

The fraction y of the turbine flow extracted at location 2 is

$$y = \frac{h_6 - h_5}{h_2 - h_7} = 0.2371$$

(a) For the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3)$$
$$= (1547.7 - 1251.1) + (1 - 0.2371)(1251.1 - 975.2)$$
$$= 507.08 \text{ Btu/lb}$$

For the pump

PROBLEM 8.51 (Cont'd)

$$\frac{\dot{W}_p}{\dot{m}} = h_5 - h_4 = 75.715 - 69.74 = 5.975 \text{ Btu/lb}$$

And the net power developed is

$$\dot{W}_{cycle} = \dot{m}_1 \left[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1 \right]$$

$$= (1.4 \times 10^6 \tfrac{\text{lb}}{\text{h}}) \left[507.08 - 5.975 \right] \tfrac{\text{Btu}}{\text{lb}} = 7.016 \times 10^8 \text{ Btu/h} \longleftarrow \underline{\dot{W}_{cycle}}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}_1 (h_1 - h_6)}$$

$$= \frac{(7.017 \times 10^8)}{(1.4 \times 10^6)(1547.7 - 301.6)} = 0.402 \ (40.2\%) \longleftarrow \underline{\eta}$$

(c) The mass flow rate of cooling water passing through the condenser is

$$\dot{m}_{cw} = \frac{\dot{m}_1 \left[(1-y) h_3 + y h_8 - h_4 \right]}{(h_{cw,out} - h_{cw,in})}$$

$$= \frac{(1.4 \times 10^6) \left[(0.7629) 975.2 + (.2371) 298.6 - 69.74 \right]}{(48.09 - 28.08)}$$

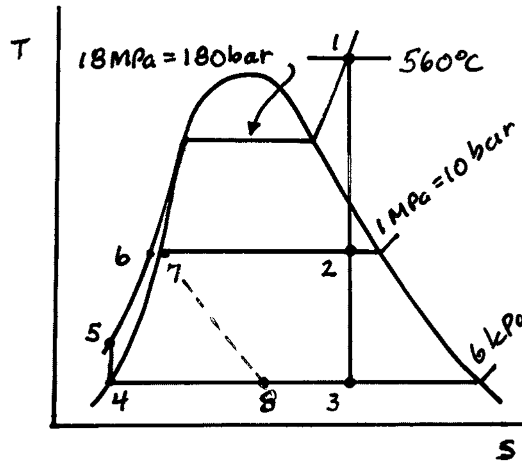
$$= 5.213 \times 10^7 \text{ lb/h} \longleftarrow \underline{\dot{m}_{cw}}$$

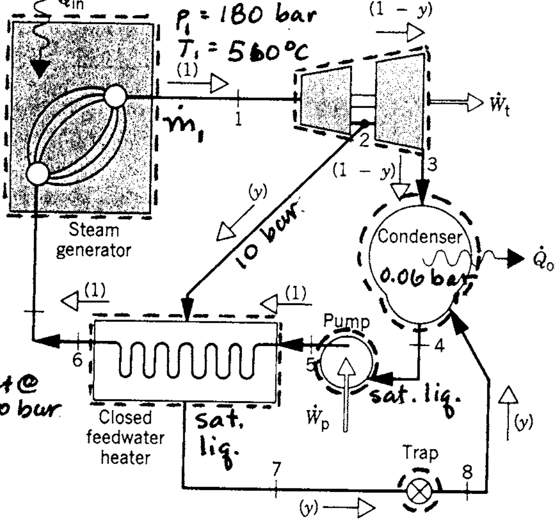
PROBLEM 8.52

KNOWN: The ideal Rankine cycle of Problem 8.6 is modified to include superheated vapor entering the first turbine stage and one closed feedwater heater using extracted steam. The condensate from the heater is trapped into the condenser.

FIND: Determine (a) the net work per unit of steam flow entering the first turbine stage, (b) the thermal efficiency, and (c) the heat transfer to cooling water passing through the condenser per unit mass of steam entering the first turbine stage.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through the trap, a throttling process, and in the heater. (3) The turbines, pump, and feedwater heater operate adiabatically. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the heater as saturated liquid at 10 bars.

ANALYSIS: First, fix each principal state.

State 1: $p_1 = 180$ bar, $T_1 = 560°C \Rightarrow h_1 = 3444.4$ kJ/kg, $s_1 = 6.4392$ kJ/kg·K

State 2: $p_2 = 10$ bar, $s_2 = s_1 \Rightarrow x_2 = 0.967$, $h_2 = 2711.6$ kJ/kg

State 3: $p_3 = 0.06$ bar, $s_3 = s_2 \Rightarrow x_3 = 0.7578$, $h_3 = 1982.3$ kJ/kg

State 4: $p_4 = 0.06$ bar, sat. liquid $\Rightarrow h_4 = 151.53$ kJ/kg, $v_4 = 1.0064 \times 10^{-3}$ m³/kg

State 5: $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$= 151.53 + \left(1.0064 \times 10^{-3}\,\frac{m^3}{kg}\right)(180 - 0.06)\,\text{bar}\left|\frac{10^5\,N/m^2}{1\,\text{bar}}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|$$

$$= 151.53 + 18.11 = 169.64\ \text{kJ/kg}$$

State 6: $\left.\begin{array}{l} T_6 = T_{sat}\,@\,10\text{ bar} = 179.9\,°F \\ p_6 = 180\text{ bar} \end{array}\right\} \Rightarrow$ Interpolating in Table A-5 $h_6 \approx 772.1$

State 7: $p_7 = 10$ bar, sat. liquid $\Rightarrow 762.81$ kJ/kg

State 8: Throttling process $\Rightarrow h_8 = h_7 = 762.81$ kJ/kg

(a) Begin by applying mass and energy rate balances to the closed feedwater heater

PROBLEM 8.52 (Cont'd)

$$0 = y(h_2 - h_7) + (h_5 - h_6)$$

or, solving for the fraction of flow y extracted at 2

$$y = \frac{h_6 - h_5}{h_2 - h_7} = \frac{772.1 - 169.64}{2711.6 - 762.81} = 0.3091$$

Now, for the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y)(h_2 - h_3)$$

$$= (3444.4 - 2711.6) + (0.6909)(2711.6 - 1982.3) = 1236.7 \text{ kJ/kg}$$

And, for the pump

$$\frac{\dot{W}_p}{\dot{m}_1} = h_5 - h_4 = 169.64 - 151.53 = 18.11 \text{ kJ/kg}$$

Thus, the net work is

$$\frac{\dot{W}_{cycle}}{\dot{m}_1} = \frac{\dot{W}_t}{\dot{m}_1} - \frac{\dot{W}_p}{\dot{m}_1} = 1218.6 \text{ kJ/kg} \longleftarrow \underline{\dot{W}_{cycle}/\dot{m}_1}$$

(b) To determine the thermal efficiency, first determine the heat input

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_6 = 3444.4 - 772.1 = 2672.3 \text{ kJ/kg}$$

and

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1218.6}{2672.3} = 0.456 \ (45.6\%) \longleftarrow \underline{\eta}$$

(c) For a control volume enclosing the working fluid passing through the condenser

$$\frac{\dot{Q}_{out}}{\dot{m}_1} = (1-y)h_3 + y h_8 - h_4$$

$$= (0.6909)(1982.3) + (.3091)(762.81) - 151.53$$

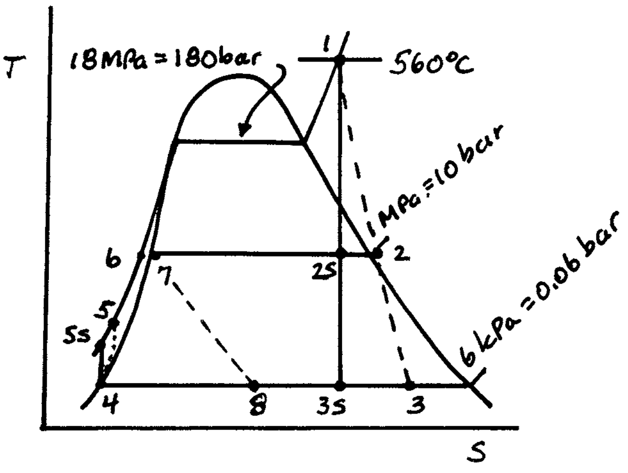
$$= 1453.8 \text{ kJ/kg} \longleftarrow \underline{\dot{Q}_{out}/\dot{m}_1}$$

PROBLEM 8.53

KNOWN: The regenerative power cycle of Problem 8.52 is modified to include that the turbine stages and pump each have efficiency of 85%.

FIND: Answer the same questions as in Problem 8.52.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as Problem 8.52, except $\eta_{t_1} = \eta_{t_2} = 0.85$ and $\eta_p = 0.85$.

ANALYSIS: First, fix each of the principal states. From Problem 8.52, $h_1 = 3444.4$ kJ/kg, $s_1 = 6.4392$ kJ/kg·K, $h_4 = 151.53$ kJ/kg, $h_6 = 772.1$ kJ/kg, and $h_7 = h_8 = 762.81$ kJ/kg.

State 2: Using the turbine efficiency, $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$. With $h_{2s} = 2711.6$ kJ/kg from Problem 8.52

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 2821.5 \text{ kJ/kg}$$

Interpolating in Table A-4; $s_2 = 6.6801$ kJ/kg

State 3: $P_3 = 0.06$ bar, $s_{3s} = s_2 \Rightarrow x_{3s} = 0.7887$, $h_{3s} = 2057.0$ kJ/kg

and $h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2171.7$ kJ/kg

State 5: Using the pump efficiency; $\eta_p = (h_{5s} - h_4)/(h_5 - h_4)$. With $h_{5s} = 169.64$ kJ/kg from Problem 8.52

$$h_5 = h_4 + (h_{5s} - h_4)/\eta_p = 172.84 \text{ kJ/kg}$$

(a) Using the modified data

$y = 0.2911$, $\dot W_t/\dot m_1 = 1083.5$ kJ/kg, $\dot W_p/\dot m_1 = 21.31$ kJ/kg

Thus $\dfrac{\dot W_{cycle}}{\dot m_1} = 1062.2$ kJ/kg ←—— $\dot W_{cycle}/\dot m_1$

(b) $\dot Q_{in}/\dot m_1 = 2672.3$ kJ/kg and $\eta = 0.397$ (39.7%) ←—— η

(c) $\dot Q_{out}/\dot m_1 = 1610.1$ kJ/kg ←—— $\dot Q_{out}/\dot m$

Discussion: Comparing the results with those of Problem 8.52 indicates that turbine and pump irreversibilities affect the net work of the cycle significantly, thereby causing a significant drop in thermal efficiency.

PROBLEM 8.54

KNOWN: The power plant layout of Fig. 8.12 is under consideration.

FIND: Determine expressions for the fractions of the total flow at states 8, 11, 17 in terms of the fractions y_2, y_3, y_6, and y_7.

SCHEMATIC & GIVEN DATA: See Fig. 8.12

ASSUMPTIONS: Control volumes at steady state enclose each of the principal components.

ANALYSIS: Letting $\dot{m}_1$ denote the total flow entering the first turbine stage, we have

State 2: $y_2 = \frac{\dot{m}_2}{\dot{m}_1}$, state 3: $y_3 = \frac{\dot{m}_3}{\dot{m}_1}$, state 6: $y_6 = \frac{\dot{m}_6}{\dot{m}_1}$, state 7: $y_7 = \frac{\dot{m}_7}{\dot{m}_1}$

Applying a mass rate balance to a control volume enclosing the overall turbine,

$$\dot{m}_8 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3 - \dot{m}_6 - \dot{m}_7 + \underbrace{\dot{m}_5 - \dot{m}_4}_{=0,\ \text{since } \dot{m}_4 = \dot{m}_5.}$$

$$\therefore \quad \frac{\dot{m}_8}{\dot{m}_1} = 1 - \frac{\dot{m}_2}{\dot{m}_1} - \frac{\dot{m}_3}{\dot{m}_1} - \frac{\dot{m}_6}{\dot{m}_1} - \frac{\dot{m}_7}{\dot{m}_1} = 1 - y_2 - y_3 - y_6 - y_7 \quad \longleftarrow y_8$$

A mass balance for a control volume enclosing the condenser reads

$$\dot{m}_9 = \dot{m}_8 + \dot{m}_{20}$$

For the closed feedwater heater and valve, $\dot{m}_7 = \dot{m}_{19} = \dot{m}_{20}$. And for the closed feedwater heater and pump, $\dot{m}_{11} = \dot{m}_{10} = \dot{m}_9$. Collecting results

$$\dot{m}_{11} = \dot{m}_8 + \dot{m}_7$$

$$\Rightarrow \quad y_{11} = y_8 + y_7 \quad \text{or} \quad y_{11} = (1 - y_2 - y_3 - y_6 - y_7) + y_7$$

$$= 1 - y_2 - y_3 - y_6 \quad \longleftarrow y_{11}$$

For the intermediate-pressure closed feedwater heater, $\dot{m}_{17} = \dot{m}_3 + \dot{m}_{16}$. And for the high-pressure closed feedwater heater and valve, $\dot{m}_2 = \dot{m}_{15} = \dot{m}_{16}$. Collecting results

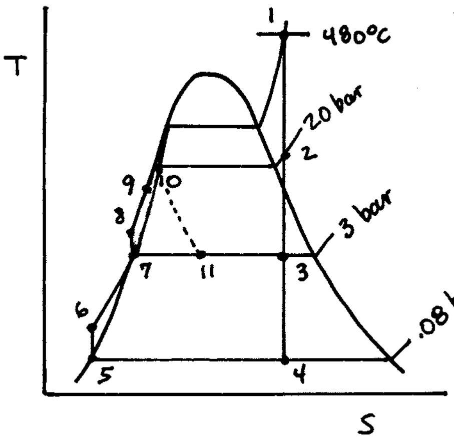
$$\dot{m}_{17} = \dot{m}_3 + \dot{m}_2 \Rightarrow y_{17} = y_3 + y_2 \quad \longleftarrow y_{17}$$

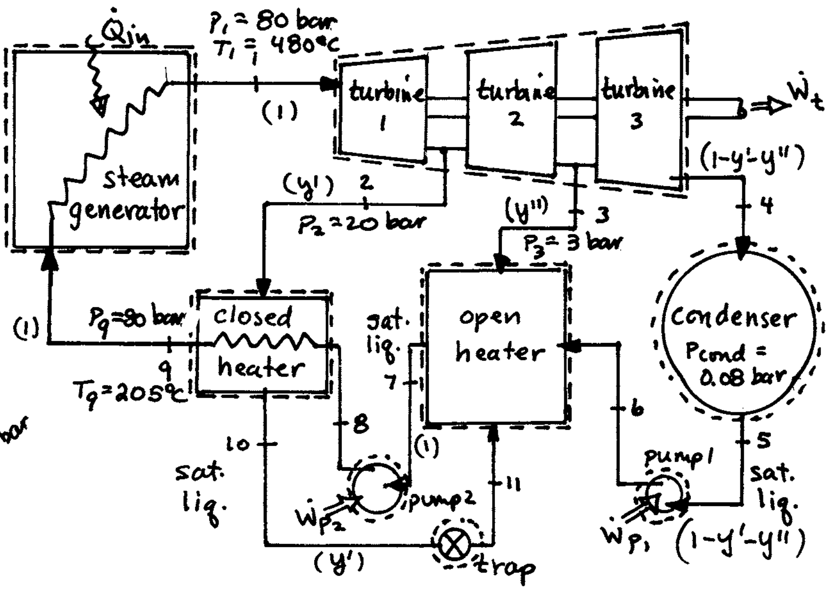
PROBLEM 8.55

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater. Data are known at various locations and the net power output is given.

FIND: Determine (a) the thermal efficiency and (b) the mass flow rate of steam entering the first turbine stage.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 8.6, except no reheater.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 80$ bar, $T_1 = 480°C \Rightarrow h_1 = 3348.4$ kJ/kg, $s_1 = 6.6586$ kJ/kg·K

State 2: $p_2 = 20$ bar, $s_2 = s_1 \Rightarrow h_2 = 2963.5$ kJ/kg

State 3: $p_3 = 3$ bar, $s_3 = s_1 \Rightarrow x_3 = 0.9373$, $h_3 = 2589.6$ kJ/kg

State 4: $p_4 = 0.08$ bar, $s_4 = s_1 \Rightarrow x_4 = .7944$, $h_4 = 2082.9$ kJ/kg

State 5: $p_5 = 0.08$ bar, sat. liq. $\Rightarrow h_5 = 173.88$ kJ/kg

State 6:
$$h_6 \approx h_5 + v_5(p_6 - p_5)$$
$$= 173.88\,\frac{\text{kJ}}{\text{kg}} + \left(1.0084\times10^{-3}\,\frac{\text{m}^3}{\text{kg}}\right)(3-.08)\,\text{bar}\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$
$$= 173.88 + 0.294 = 174.17\ \text{kJ/kg}$$

State 7: $p_7 = 3$ bar, sat. liq. $\Rightarrow h_7 = 561.47$ kJ/kg

State 8: $h_8 \approx h_7 + v_7(p_8 - p_7) = 561.47 + (1.0732\times10^{-3})(80-3)\left|\frac{10^5}{10^3}\right|$
$$= 561.47 + 8.26 = 569.73\ \text{kJ/kg}$$

State 9: $p_9 = 80$ bar, $T_9 = 205°C \Rightarrow$ (Interpolating in Table A-5) $h_9 = 878.34\,\frac{\text{kJ}}{\text{kg}}$

State 10: $p_{10} = 20$ bar, sat. liquid $\Rightarrow h_{10} = 908.79$ kJ/kg

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 908.79$ kJ/kg

PROBLEM 8.55 (Cont'd)

(a) Begin by determining the flow rate ratios y' and y''. Energy and mass rate balances for the closed feedwater heater reduce to

$$0 = y'(h_2 - h_{10}) + (h_8 - h_9)$$

or

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{878.34 - 569.73}{2963.5 - 908.79} = 0.1502$$

And, for the open feedwater heater

$$0 = y'' h_3 + (1 - y' - y'')h_6 + y' h_{11} - h_7$$

or

$$y'' = \frac{y'(h_6 - h_{11}) + (h_7 - h_6)}{(h_3 - h_6)} = \frac{(0.1502)(174.17 - 908.79) + (561.47 - 174.17)}{(2589.6 - 174.17)}$$

$$= 0.1147$$

For the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1 - y')(h_2 - h_3) + (1 - y' - y'')(h_3 - h_4)$$

$$= (3348.4 - 2963.5) + (.8498)(2963.5 - 2589.6) + (.7351)(2589.6 - 2082.9)$$

$$= 1075.1 \text{ kJ/kg}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_8 - h_7) + (1 - y' - y'')(h_6 - h_5)$$

$$= (569.73 - 561.47) + (.7351)(174.17 - 173.88) = 8.47 \text{ kJ/kg}$$

For the working fluid passing through the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_9 = 3348.4 - 878.34 = 2470.1 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.4318 \; (43.18\%) \longleftarrow \underline{\eta}$$

(b) To find the mass flow rate $\dot{m}_1$

$$\dot{W}_{cycle} = \dot{m}_1 \left[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1\right]$$

thus

$$\dot{m}_1 = \frac{(100 \times 10^3 \text{ kW})}{(1075.1 - 8.47) \text{ kJ/kg}} \left|\frac{3600 \text{ s}}{1 \text{ h}}\right| \left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| = 3.375 \times 10^5 \text{ kg/h} \longleftarrow \underline{\dot{m}_1}$$

PROBLEM 8.56

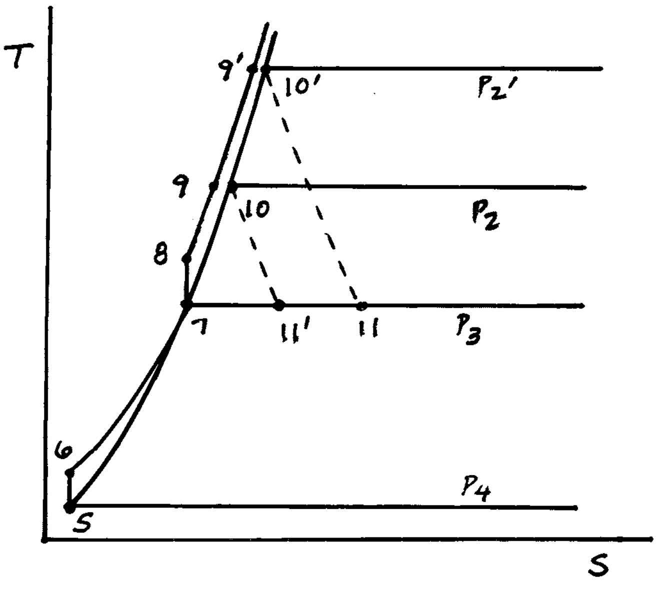
KNOWN: The regenerative vapor power cycle of Problem 8.55 is reconsidered.

FIND: Investigate the effects on performance as the higher extraction pressure takes on other values. Construct suitable plots and discuss. Consider both reversible and irreversible operation of the turbine stages and pumps.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.55.

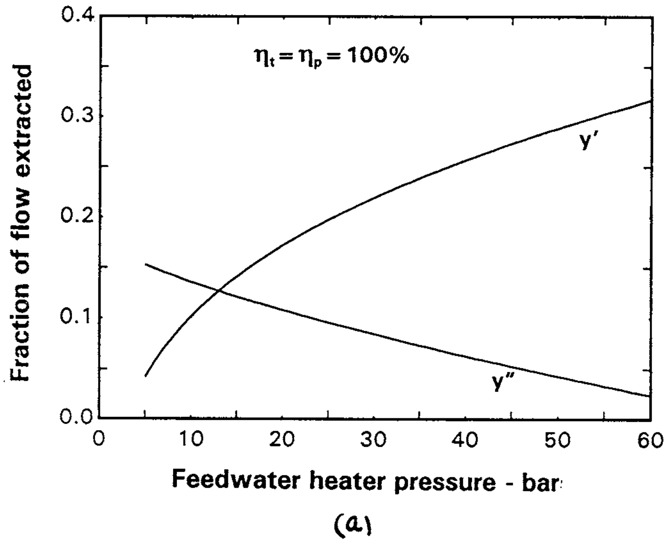
ASSUMPTIONS: See solution to Problem 8.55. Also, $\eta_{t_1} = \eta_{t_2} = \eta_{t_3} = 80\%$ and $\eta_{p_1} = \eta_{p_2} = 80\%$ for the second part.

ANALYSIS: The performance is investigated by determining the effects on thermal efficiency of varying the feedwater heater extraction pressure.

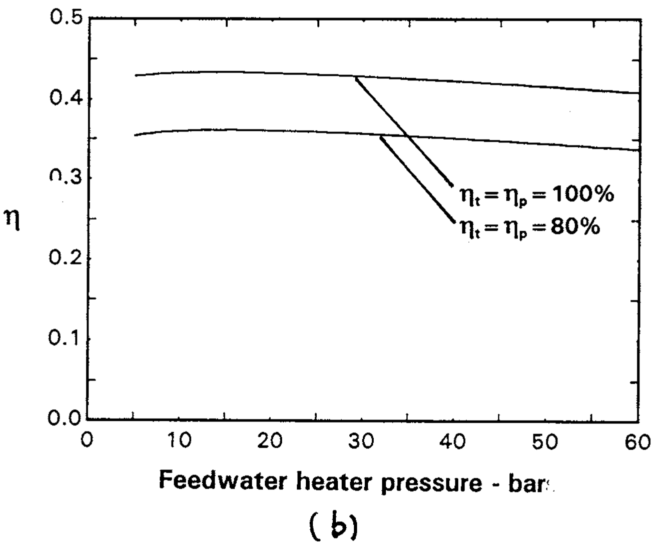


From the T-s diagram, we see that state 9' moves higher as the extraction pressure increases. This reduces the heat input to the cycle. However, from (a) we see that the fraction y' of the total flow extracted to the closed heater also increases. This results in less mass flow through the second and third turbine stages, and hence, less power output. The net effect is a reduction in thermal efficiency, as illustrated in (b).

Another factor that reduces the power output is the irreversible expansion across the valve. This affect also worsens at higher extraction pressure.



(a)



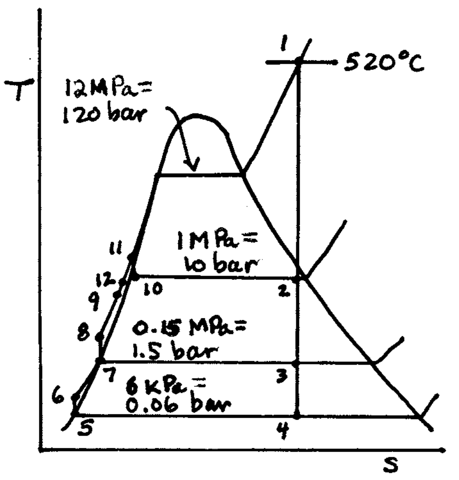
(b)

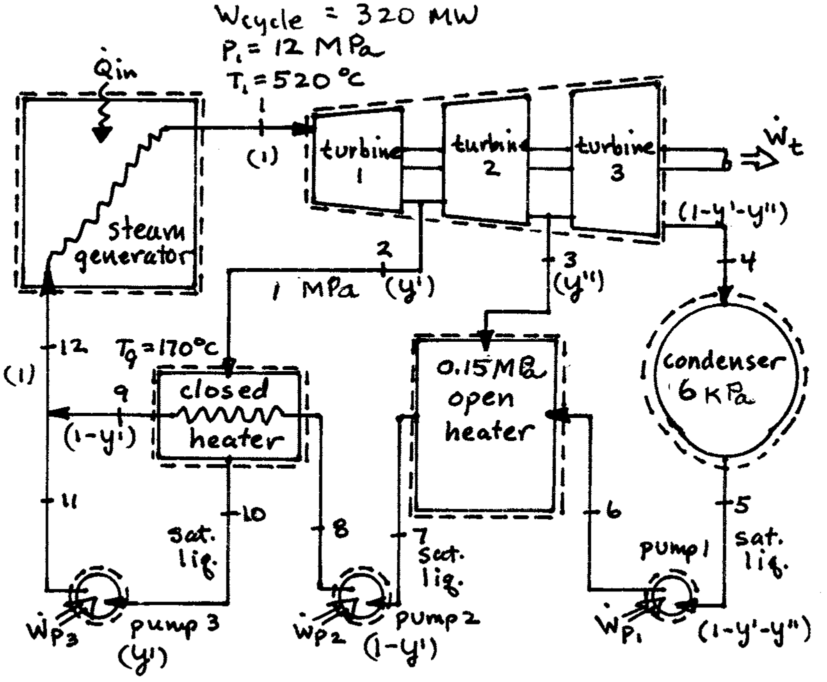
PROBLEM 8.57

KNOWN: Water is the working fluid in a regenerative vapor power cycle with one closed and one open feedwater heater. Data are known at various locations, and the mass flow rate entering the first turbine stage is given.

FIND: Determine (a) the thermal efficiency and (b) the net power developed.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) Each process of the working fluid is internally reversible, except in the feedwater heaters and the mixing of streams 9 and 11 to form stream 12. (3) The turbines, pumps, and feedwater heaters operate adiabatically. (4) Kinetic and potential energy effects are negligible. (5) Condensate exits the closed heater and condenser, and feedwater exits the open heater as saturated liquid the respective pressures.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 120$ bar , $T_1 = 520°C \Rightarrow h_1 = 3401.8$ kJ/kg , $s_1 = 6.5555$ kJ/kg·K

State 2: $P_2 = 10$ bar , $s_2 = s_1 \Rightarrow x_2 = 0.9931$, $h_2 = 2764.2$ kJ/kg

State 3: $P_3 = 1.5$ bar , $s_3 = s_1 \Rightarrow x_3 = 0.8847$, $h_3 = 2436.9$ kJ/kg

State 4: $P_4 = 0.06$ bar , $s_4 = s_1 \Rightarrow x_4 = 0.7727$, $h_4 = 2018.3$ kJ/kg

State 5: $P_5 = 0.06$ bar, sat. liquid $\Rightarrow h_5 = 151.53$ kJ/kg

State 6: $h_6 \approx h_5 + v_5(P_6 - P_5)$

$$= 151.53 \frac{kJ}{kg} + (1.0064 \times 10^{-3}) \frac{m^3}{kg} (1.5 - .06) \text{ bar} \left|\frac{10^5 N/m^2}{1 \text{ bar}}\right| \left|\frac{1 kJ}{10^3 N\cdot m}\right|$$

$$= 151.53 + 0.14 = 151.67 \text{ kJ/kg}$$

State 7: $P_7 = 1.5$ bar , sat. liquid $\Rightarrow h_7 = 467.11$ kJ/kg

State 8: $h_8 \approx h_7 + v_7(P_8 - P_7) = 467.11 + (1.0528 \times 10^{-3})(120 - 1.5)\left|\frac{10^5}{10^3}\right| = 479.59 \frac{kJ}{kg}$

State 9: $P_9 = 120$ bar , $T_9 = 170°C \Rightarrow$ Interpolating in Table A-5; $h_9 = 725.86$ kJ/kg

State 10: $P_{10} = 10$ bar , sat. liquid $\Rightarrow h_{10} = 762.81$ kJ/kg

PROBLEM 8.57 (Contd.)

State 11: $h_{11} \approx h_{10} + v_{10}(P_{11} - P_{10}) = 762.81 + (1.1273\times10^{-3})(120-10)\left|\frac{10^5}{10^3}\right| = 775.21$ kJ/kg

(a) Applying mass and energy rate balances to the control volume enclosing the closed heater, the fraction of flow y' extracted at location 2 is

$$y' = \frac{h_9 - h_8}{h_2 - h_{10} + h_9 - h_8} = \frac{725.86 - 479.59}{2764.2 - 762.81 + 725.86 - 479.59} = 0.1096$$

This allows the specific enthalpy at 12 to be determined by analyzing the mixing of streams 9 and 11:

$$0 = (1-y')h_9 + y'h_{11} - h_{12}$$

or

$$h_{12} = (1-y')h_9 + y'h_{11} = (0.8904)(725.86) + (0.1096)(775.21) = 731.27 \text{ kJ/kg}$$

Next, energy and mass balances for the control volume enclosing the open heater give the fraction y'' extracted at 3:

$$0 = y''h_3 + (1 - y' - y'')h_6 - (1-y')h_7$$

or

$$y'' = \frac{(1-y')(h_7 - h_6)}{(h_3 - h_6)} = \frac{(0.8904)(467.11 - 151.67)}{(2436.9 - 151.67)} = 0.1229$$

For the control volume enclosing the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y')(h_2 - h_3) + (1-y'-y'')(h_3 - h_4)$$

$$= (3401.8 - 2764.2) + (0.8904)(2764.2 - 2436.9) - (0.7675)(2436.9 - 2018.3) = 1250.3 \ \frac{\text{kJ}}{\text{kg}}$$

For the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = y'(h_{11} - h_{10}) + (1-y')(h_8 - h_7) + (1-y'-y'')(h_6 - h_5)$$

$$= (0.1096)(775.21 - 762.81) + (0.8904)(479.59 - 467.11) + (0.7675)(151.67 - 151.23)$$

$$= 12.58 \text{ kJ/kg}$$

For the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_{12} = (3401.8) - (731.27) = 2670.5 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1250.3 - 12.58}{2670.5} = 0.463 \ (46.3\%) \longleftarrow \eta$$

(b) The mass flow rate into the first turbine stage is

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}$$

$$= \frac{(320 \text{ MW})}{(1250.3 - 12.58) \text{ kJ/kg}} \left|\frac{10^3 \text{ kJ/s}}{1 \text{ MW}}\right| \left|\frac{3600 \text{ s}}{1 \text{ h}}\right|$$

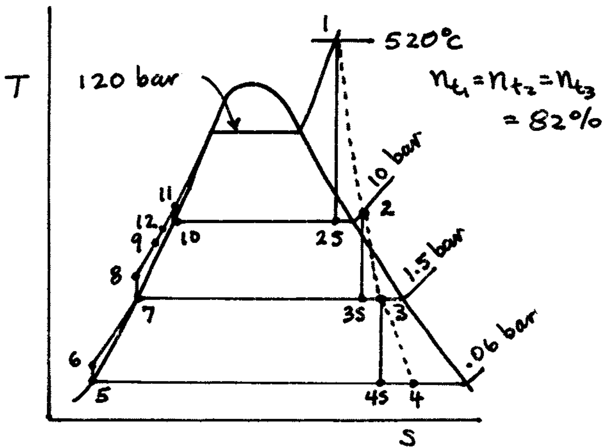
$$= 9.31 \times 10^5 \text{ kg/h} \longleftarrow \dot{m}_1$$

PROBLEM 8.58

KNOWN: The regenerative vapor power cycle of Problem 8.57 is modified to include that each turbine stage has an efficiency of 82%.

FIND: Answer the same questions about the modified cycle as in Problem 8.57.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as Problem 8.57, except $\eta_{t_1}=\eta_{t_2}=\eta_{t_3}=0.82$.

ANALYSIS: From the solution of Problem 8.57

$h_1 = 3401.8$ kJ/kg, $s_1 = 6.5555$ kJ/kg·K

$h_5 = 151.53$ kJ/kg $\qquad h_9 = 725.86$ kJ/kg

$h_6 = 151.67$ kJ/kg $\qquad h_{10} = 762.81$ kJ/kg

$h_7 = 467.11$ kJ/kg $\qquad h_{11} = 775.21$ kJ/kg

$h_8 = 479.59$ kJ/kg

State 2: Using the turbine efficiency, $\eta_{t_1} = (h_1-h_2)/(h_1-h_{2s})$. With $h_{2s} = 2764.2$ (Problem 8.57)

$$h_2 = h_1 - \eta_{t_1}(h_1-h_{2s}) = 2879.0 \text{ kJ/kg}$$

From Table A-4, $s_2 = 6.7977$ kJ/kg·K.

State 3: Similarly, $p_3 = 1.5$ bar, $s_{3s} = s_2 \Rightarrow x_{3s} = .9265$, $h_{3s} = 2530.0 \frac{\text{kJ}}{\text{kg}}$.

Thus $\quad h_3 = h_2 - \eta_{t2}(h_2 - h_{3s}) = 2592.8$ kJ/kg

and, using data from Table A-3; $x_3 = .9547$, $s_3 = 6.9610$ kJ/kg·K.

State 4: Again, $p_4 = 0.06$ bar, $s_{4s} = s_3 \Rightarrow x_{4s} = .82465$, $h_{4s} = 2143.8$ kJ/kg.

Thus $\quad h_4 = h_3 - \eta_{t3}(h_3 - h_{4s}) = 2224.6$ kJ/kg

(a) The flow fraction y' is

$$y' = \frac{h_9-h_8}{h_2-h_{10}} = \frac{725.86-479.59}{2879.0-762.81} = 0.1164$$

Thus, specific enthalpy h_{12} is

$$h_{12} = (1-y')h_9 + y'h_{11} = (.8836)(725.86) + (.1164)(775.21)$$
$$= 731.6 \text{ kJ/kg}$$

and the flow fraction y'' is

$$y'' = \frac{(1-y')(h_7-h_6)}{(h_3-h_6)} = \frac{(.8836)(467.11-151.67)}{(2592.8-151.67)} = 0.1142$$

For the control volume enclosing the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1-h_2) + (1-y')(h_2-h_3) + (1-y'-y'')(h_3-h_4)$$
$$= (3401.8-2879.0) + (.8836)(2879.0-2592.8) + (.7694)(2592.8-2224.6)$$
$$= 1059.0 \text{ kJ/kg}$$

PROBLEM 8.58 (Contd.)

And, for the pumps

$$\frac{\dot{W}_e}{\dot{m}_1} = y'(h_{11}-h_{10}) + (1-y')(h_8-h_7) + (1-y'-y'')(h_6-h_5)$$

$$= (.1164)(775.21-762.81) + (.8836)(479.59-467.11) + (.7694)(151.67-151.53)$$

$$= 12.58 \text{ kJ/kg}$$

For the steam generator

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = h_1 - h_{12} = 34018 - 731.6 = 2670.2 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1059 - 12.58}{2670.2} = 0.392 \ (39.2\%) \longleftarrow \eta$$

(b) The mass flow rate into the first turbine stage is

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]}$$

$$= \frac{(320 \text{ MW})}{[1059.0 - 12.58] \text{ kJ/kg}} \left|\frac{10^3 \text{ kJ/s}}{1 \text{ MW}}\right| \left|\frac{3600 \text{ s}}{1 \text{ h}}\right|$$

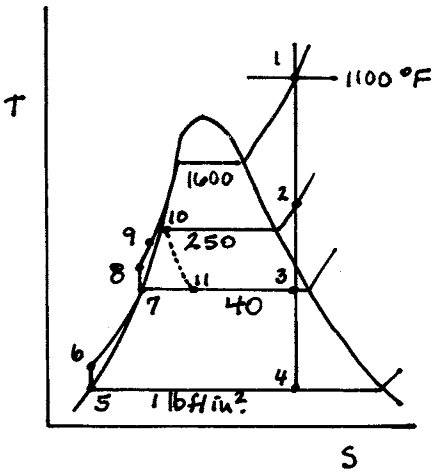
$$= 1.1 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}_1$$

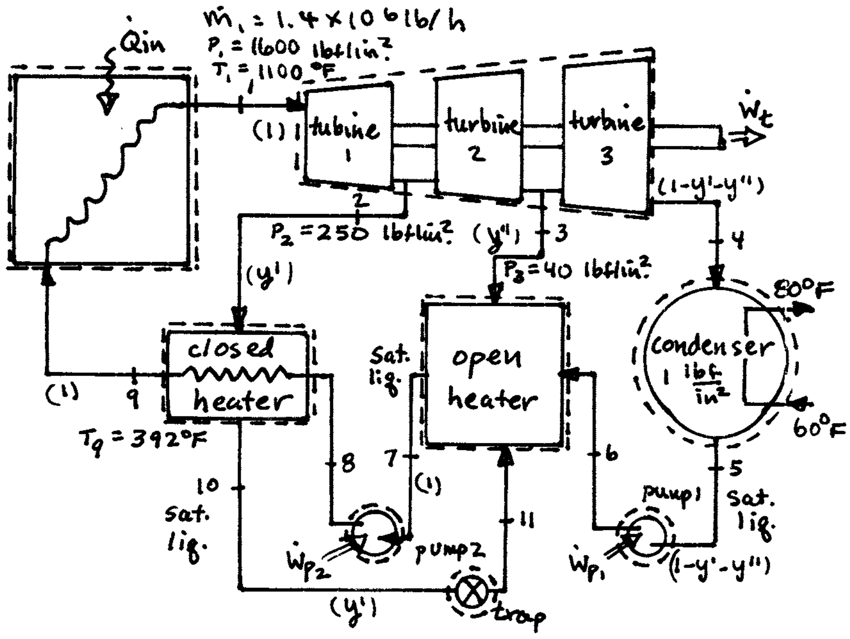
PROBLEM 8.59*

KNOWN: The ideal Rankine cycle of Probem 8.9 is modified to include one closed and one open feedwater heater. The condensate from the closed heater is trapped into the open heater.

FIND: Answer the same questions about the modified cycle as in Problem 8.9 and discuss the results.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 8.6, except no reheater

ANALYSIS: From the solution to Problem 8.9, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R, $h_4 = 911.2$ Btu/lb, and $h_5 = 69.74$ Btu/lb.

State 2: $p_2 = 250$ lbf/in², $s_2 = s_1 \Rightarrow h_2 = 1299.8$ Btu/lb

State 3: $p_3 = 40$ lbf/in², $s_3 = s_1 \Rightarrow x_3 = 0.9649$, $h_3 = 1137.2$ Btu/lb

State 6:
$$h_6 \approx h_5 + v_5(p_6 - p_5)$$
$$= 69.74\ \frac{\text{Btu}}{\text{lb}} + \left(.01614\ \frac{\text{ft}^3}{\text{lb}}\right)\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|(40-1)\ \frac{\text{lbf}}{\text{in}^2}$$
$$= 69.74 + 0.12 = 69.86\ \text{Btu/lb}$$

State 7: $p_7 = 40$ lbf/in², sat. liquid $\Rightarrow h_7 = 236.16$ Btu/lb

State 8:
$$h_8 \approx h_7 + v_7(p_8 - p_7) = 236.16 + (.01715)\frac{144}{778}(1600 - 40)$$
$$= 236.16 + 4.95 = 241.11\ \text{Btu/lb}$$

State 9: $p_9 = 1600$ lbf/in², $T_9 = 392$°F $\Rightarrow$ (Interpolating in Table A-5) $h_9 \approx 368.38$ Btu/lb

State 10: $p_{10} = 250$ lbf/in², sat. liquid $\Rightarrow h_{10} = 376.2$ Btu/lb

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 376.2$ Btu/lb

(a) Begin by determining the flow rate ratios y' and y''. Energy and mass rate balances for the control volume enclosing the closed feedwater heater reduce to

$$0 = y'(h_2 - h_{10}) + (h_8 - h_9)$$

or

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{368.38 - 241.11}{1299.8 - 376.2} = 0.1365$$

PROBLEM 8.59 (Contd.)

For the open heater

$$0 = y'' h_3 + (1-y'-y'') h_6 + y' h_{11} - h_7$$

or

$$y'' = \frac{y'(h_6-h_{11}) + (h_7-h_6)}{h_3-h_6} = \frac{(0.1365)(69.86-376.2)+(236.16-69.86)}{(1137.2-69.86)}$$

$$= 0.1166$$

For the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1-h_2) + (1-y')(h_2-h_3) + (1-y'-y'')(h_3-h_4)$$

$$= (1547.7-1299.8)+(.8635)(1299.8-1137.2)+(.7469)(1137.2-911.2)$$

$$= 557.10 \text{ Btu/lb}$$

and for the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_8-h_7) + (1-y'-y'')(h_6-h_5)$$

$$= (241.11-236.16)+(.7469)(69.86-69.74) = 5.04 \text{ Btu/lb}$$

With $\dot{m}_1 = 1.4 \times 10^6$ lb/h from Problem 8.9, the net power developed is

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]$$

$$= \left(1.4 \times 10^6 \frac{\text{lb}}{\text{h}}\right)[557.10-5.04]\frac{\text{Btu}}{\text{lb}} = 7.729 \times 10^8 \text{ Btu/h} \longleftarrow \underline{\dot{W}_{cycle}}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}_1(h_1-h_9)} = \frac{7.729 \times 10^8}{(1.4 \times 10^6)(1547.7-368.38)}$$

$$= 0.468 \; (46.8\%) \longleftarrow \underline{\eta}$$

(c) The mass flow rate of cooling water passing through the condenser is found from

$$\dot{m}_{cw} = \frac{\dot{m}_1(1-y'-y'')(h_4-h_5)}{h_{cw,out} - h_{cw,in}}$$

$$= \frac{(1.4 \times 10^6 \text{ lb/h})(.7469)(911.2-69.74) \text{ Btu/lb}}{(48.09-28.08) \text{ Btu/lb}}$$

$$= 4.40 \times 10^7 \text{ lb/h} \longleftarrow \underline{\dot{m}_{cw}}$$

Discussion: (1) The net power developed in the modified cycle is less than in Problem 8.9 due to some mass being extracted. (2) The rate of heat transfer to the working fluid passing through the steam generator is less for the modified cycle due to regeneration in the feedwater heaters. In this case, the thermal efficiency is increased due to regeneration. (3) The cooling water flow rate is less in the modified cycle because of the reduced flow of working fluid through the condenser in this case.

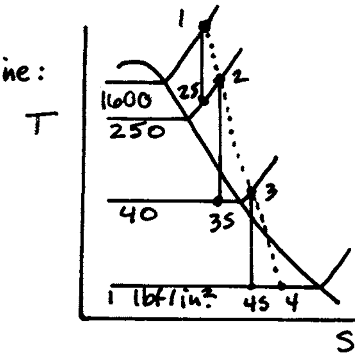
PROBLEM 8.60*

KNOWN: The regenerative vapor power cycle of Problem 8.59 is modified to include that each turbine stage has an efficiency of 88% and each pump has an efficiency of 80%.

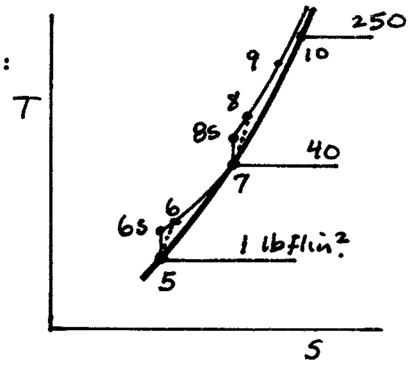
FIND: Repeat the analysis of Problem 8.59 for the modified cycle.

SCHEMATIC & GIVEN DATA:

For the turbine:



For the pumps:



ASSUMPTIONS: Same as Problem 8.59, except $\eta_{t_1} = \eta_{t_2} = \eta_{t_3} = 0.88$ and $\eta_{p_1} = \eta_{p_2} = 0.80$.

ANALYSIS: From Problem 8.59, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R, $h_5 = 69.74$ Btu/lb, $h_7 = 236.16$ Btu/lb, $h_9 = 368.38$ Btu/lb, and $h_{10} = h_{11} = 376.2$ Btu/lb.

State 2: Using the turbine efficiency, $\eta_{t_1} = (h_1 - h\ \ )/(h_1 - h_{2s})$. Thus, with h_{2s} from Problem 8.59

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 1329.5 \text{ Btu/lb}$$

and, interpolating in Table A-4E; $s_2 = 1.6595$ Btu/lb·°R.

State 3: First, $p_3 = 40$ lbf/in.², $s_{3s} = s_2 \Rightarrow x_{3s} = 0.9867$, $h_{3s} = 1157.5$ Btu/lb. Then

$$h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 1178.1 \text{ Btu/lb}$$

and, interpolating in Table A-4E; $s_3 = 1.6877$ Btu/lb·°R

State 4: With $p_4 = 1$ lbf/in.², $s_{4s} = s_3 \Rightarrow h_{4s} = 942.7$ Btu/lb

Thus

$$h_4 = h_3 - \eta_{t_3}(h_3 - h_{4s}) = 971.0 \text{ Btu/lb}$$

State 6: Using the pump efficiency, $\eta_{p_1} = (h_{6s} - h_5)/(h_6 - h_5)$. Thus, with h_{6s} from Problem 8.59

$$h_6 = h_5 + (h_{6s} - h_5)/\eta_{p_1} = 69.89 \text{ Btu/lb}$$

State 8: Similarly, with h_{8s} from Problem 8.59

$$h_8 = h_7 + (h_{8s} - h_7)/\eta_{p_2} = 242.35 \text{ Btu/lb}$$

The mass flow rate ratios y' and y'' are, respectively

$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{368.38 - 242.35}{1329.5 - 376.2} = 0.1322$$

and

$$y'' = \frac{y'(h_6 - h_{11}) + (h_7 - h_6)}{(h_3 - h_6)} = \frac{(.1322)(69.89 - 376.2) + (236.16 - 69.89)}{(1178.1 - 69.89)}$$

$$= 0.1135$$

PROBLEM 8.60 (Contd.)

(a) For the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y')(h_2 - h_3) + (1-y'-y'')(h_3 - h_4)$$
$$= 505.80 \text{ Btu/lb}$$

And for the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_8 - h_7) + (1-y'-y'')(h_6 - h_5) = 6.303 \text{ Btu/lb}$$

thus, with $\dot{m}_1 = 1.4 \times 10^6$ from Problem 8.59

$$\dot{W}_{cycle} = \dot{m}_1 \left[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1 \right] = 6.99 \times 10^8 \text{ Btu/h} \longleftarrow \underline{\dot{W}_{cycle}}$$

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}(h_1 - h_9)} = 0.424 \; (42.4\%) \longleftarrow \underline{\eta}$$

(c) The mass flow rate of cooling water through the condenser is

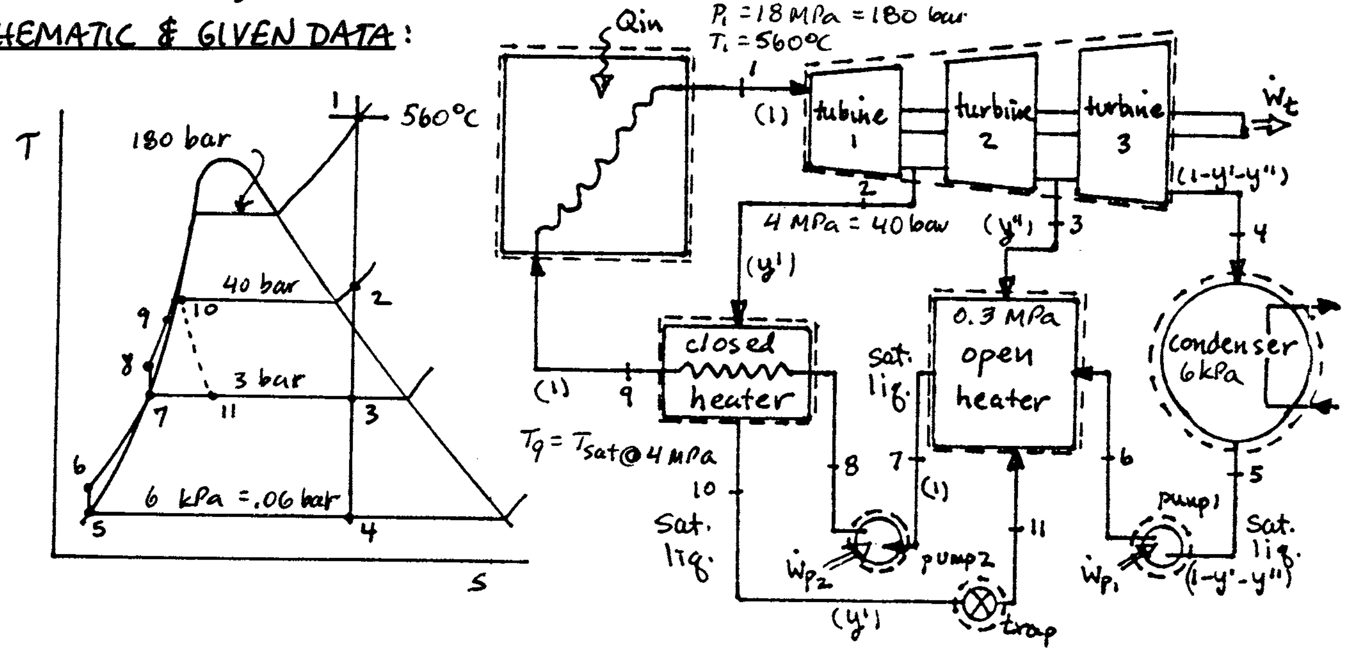
$$\dot{m}_{cw} = \frac{\dot{m}_1 (1-y'-y'')(h_4 - h_5)}{h_{cw,out} - h_{cw,in}} = 4.76 \times 10^7 \text{ lb/h} \longleftarrow \underline{\dot{m}_{cw}}$$

PROBLEM 8.61

KNOWN: The ideal Rankine cycle of Problem 8.6 is modified to include one closed and one open feedwater heater. Data are known at various locations.

FIND: Determine (a) the net work per kg of steam flow entering the first turbine stage, (b) the thermal efficiency, and (c) the heat transfer to condenser cooling water per kg of steam flow entering the first turbine stage.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as Example 8.6, except no reheater.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 180$ bar, $T_1 = 560°C \Rightarrow h_1 = 3444.4$ kJ/kg, $s_1 = 6.4392$ kJ/kg

State 2: $p_2 = 40$ bar, $s_2 = s_1 \Rightarrow h_2 = 3006.4$ kJ/kg

State 3: $p_3 = 3$ bar, $s_3 = s_2 \Rightarrow x_3 = 0.8961$, $h_3 = 2500.5$ kJ/kg

State 4: $p_4 = 0.06$ bar, $s_4 = s_3 \Rightarrow x_4 = 0.7578$, $h_4 = 1982.3$ kJ/kg

State 5: $p_5 = 0.06$ bar, sat. liquid $\Rightarrow h_5 = 151.53$ kJ/kg, $v_5 = 1.0064 \times 10^{-3}$ m³/kg

State 6: $h_6 \approx h_5 + v_5(p_6 - p_5)$

$$= 151.53 + (1.0064\times10^{-3}\ \tfrac{m^3}{kg})(3 - 0.06)\ bar \left|\frac{10^5 N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3 N\cdot m}\right|$$

$$= 151.53 + 0.296 = 151.83 \text{ kJ/kg}$$

State 7: $p_7 = 3$ bar, sat. liquid $\Rightarrow h_7 = 561.47$ kJ/kg, $v_7 = 1.0732\times10^{-3}$ m³/kg

State 8: $h_8 \approx h_7 + v_7(p_8 - p_7) = 561.47 + (1.0732\times10^{-3})(180 - 3)|10^2|$

$= 561.47 + 19 = 580.5$ kJ/kg

State 9: $T_9 = T_{sat@40\,bar} = 250.4\,°C$, $p_9 = 180$ bar $\Rightarrow h_9 \approx 1087$ kJ/kg (Table A-5)

State 10: $p_{10} = 40$ bar, sat. liquid $\Rightarrow h_{10} = 1087.3$ kJ/kg

State 11: Throttling process $\Rightarrow h_{11} = h_{10} = 1087.3$ kJ/kg

PROBLEM 8.61 (Cont'd)

(a) Begin by applying mass and energy rate balances to control volumes enclosing the feedwater heater in order to find the flow rate ratios y' and y''. For the closed heater

$$0 = y'(h_2 - h_{10}) + (h_8 - h_9)$$

or
$$y' = \frac{h_9 - h_8}{h_2 - h_{10}} = \frac{1087 - 580.5}{3006.4 - 1087.3} = 0.2639$$

And, for the open heater: $0 = y'' h_3 + (1 - y' - y'') h_6 + y' h_{11} - h_7$. Thus

$$y'' = \frac{y'(h_6 - h_{11}) + (h_7 - h_6)}{(h_3 - h_6)} = \frac{0.2639\ (151.83 - 1087.3) + (561.47 - 151.83)}{(2500.5 - 151.83)} = 0.0693$$

Now, for the turbine stages

$$\frac{\dot W_t}{\dot m_1} = (h_1 - h_2) + (1 - y')(h_2 - h_3) + (1 - y' - y'')(h_3 - h_4)$$
$$= (3444.4 - 3006.4) + (0.7361)(3006.4 - 2500.5) + (0.6668)(2500.5 - 1982.3)$$
$$= 1155.9\ \text{kJ/kg}$$

For the pumps

$$\frac{\dot W_p}{\dot m_1} = (h_8 - h_7) + (1 - y' - y'')(h_6 - h_5)$$
$$= (580.5 - 561.47) + (0.6668)(151.83 - 151.53) = 19.2\ \text{kJ/kg}$$

Thus, the net work is

$$\frac{\dot W_{cycle}}{\dot m_1} = \frac{\dot W_t}{\dot m_1} - \frac{\dot W_p}{\dot m_1} = 1136.7\ \text{kJ/kg} \longleftarrow \dot W_{cycle}/\dot m_1$$

(b) To determine the thermal efficiency, first find the heat input

$$\frac{\dot Q_{in}}{\dot m_1} = h_1 - h_9 = (3444.4 - 1087)\ \text{kJ/kg} = 2357.4\ \text{kJ/kg}$$

and
$$\eta = \frac{\dot W_{cycle}/\dot m_1}{\dot Q_{in}/\dot m_1} = \frac{1136.7}{2357.4} = 0.482\ (48.2\%) \longleftarrow \eta$$

(c) For a control volume enclosing the working fluid passing through the condenser

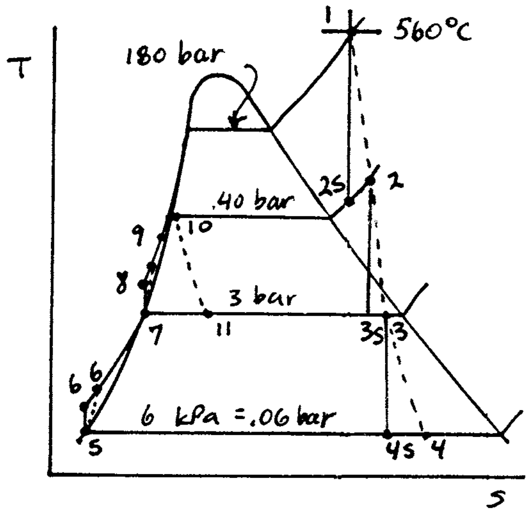
$$\frac{\dot Q_{out}}{\dot m_1} = (1 - y' - y'')(h_4 - h_5)$$
$$= (0.6668)(1982.3 - 151.53) = 1220.8\ \text{kJ/kg} \longleftarrow \dot Q_{out}/\dot m_1$$

PROBLEM 8.62

KNOWN: The regenerative power cycle of Problem 8.61 is modified to include that the turbine stages and pumps each have efficiency of 85%

FIND: Answer the same questions as in Problem 8.61.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as Problem 8.61, except $\eta_{t_1} = \eta_{t_2} = 0.85$ and $\eta_{p_1} = \eta_{p_2} = 0.85$.

ANALYSIS: First, fix each of the principal states. From Problem 8.61, $h_1 = 3444.4$ kJ/kg, $s_1 = 6.4392$ kJ/kg·K, $h_5 = 151.53$, $h_7 = 561.47$, $h_9 = 1087$, and $h_{11} = h_{10} = 1087.3$.

State 2: Using the turbine efficiency, $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$. With $h_{2s} = 3006.4 \frac{kJ}{kg}$ from Problem 8.61

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 3072.1 \text{ kJ/kg}$$

Interpolating in Table A-4; $s_2 = 6.5479$ kJ/kg·K

State 3: $P_3 = 3$ bar, $s_{3s} = s_2 \Rightarrow x_{3s} = 0.9165$, $h_{3s} = 2544.6$ kJ/kg

and $h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2623.7$ kJ/kg

Also, $x_3 = 0.9531$ and $s_3 = 6.7424$ kJ/kg·K

State 4: $P_4 = 0.06$ bar, $s_{4s} = s_3 \Rightarrow x_{4s} = 0.7967$, $h_{4s} = 2076.3$ kJ/kg

and $h_4 = h_3 - (h_3 - h_{4s})\eta_{t_3} = 2158.4$ kJ/kg

State 6: Using the pump efficiency, $\eta_{p_1} = (h_{6s} - h_5)/(h_6 - h_5)$. With $h_{6s} = 151.83$ from Problem 8.61

$$h_6 = h_5 + (h_{6s} - h_5)/\eta_{p_1} = 151.88 \text{ kJ/kg}$$

State 8: Similarly, with $h_{8s} = 580.5$ kJ/kg

$$h_8 = h_7 + (h_{8s} - h_7)/\eta_{p_2} = 583.9 \text{ kJ/kg}$$

(a) Using the modified data

$y' = 0.2622$, $y'' = 0.0665$, $\dot{W}_t/\dot{m}_1 = 1021.9$ kJ/kg, $\dot{W}_p/\dot{m}_1 = 22.67$ kJ/kg

Thus $\dfrac{\dot{W}_{cycle}}{\dot{m}_1} = 999.2$ kJ/kg ← $\dot{W}_{cycle}/\dot{m}_1$

(b) $\dot{Q}_{in}/\dot{m}_1 = 2357.4$ kJ/kg and $\eta = 0.424$ (42.4%) ← η

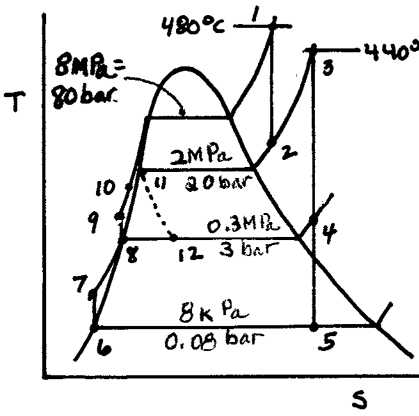
(c) $\dot{Q}_{out}/\dot{m}_1 = 1358.0$ kJ/kg ← $\dot{Q}_{out}/\dot{m}_1$

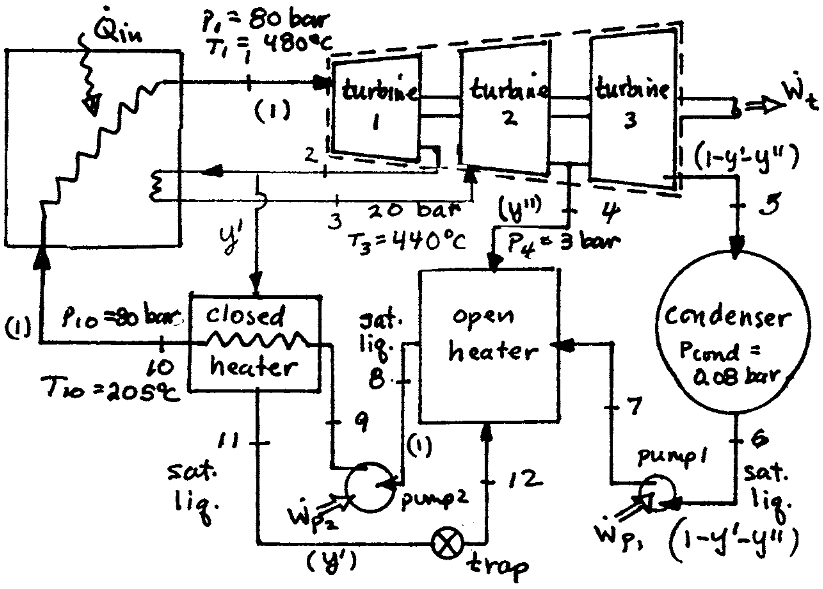
PROBLEM 8.63

KNOWN: The regenerative vapor power cycle in Problem 8.55 is modified to include reheat at 20 bar.

FIND: Determine the thermal efficiency of the modified cycle.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 8.6

ANALYSIS: Using appropriate data from Problem 8.55

$h_1 = 3348.4$ kJ/kg

$h_2 = 2963.5$ kJ/kg

$h_6 = 173.88$ kJ/kg

$h_7 = 174.17$ kJ/kg

$h_8 = 561.47$ kJ/kg

$h_9 = 569.73$ kJ/kg

$h_{10} = 878.34$ kJ/kg

$h_{11} = 908.79$ kJ/kg

$h_{12} = 908.79$ kJ/kg

State 3: $p_3 = 20$ bar, $T_3 = 440°C \Rightarrow h_3 = 3335.5$ kJ/kg, $s_3 = 7.2540$ kJ/kg·K

State 4: $p_4 = 3$ bar, $s_4 = s_3 \Rightarrow$ sup. vapor; $h_4 = 2839.5$ kJ/kg

State 5: $p_5 = 0.08$ bar, $s_5 = s_3 \Rightarrow x = 0.8723$, $h_5 = 2270.1$ kJ/kg

From the solution to Problem 8.55, $y' = 0.1502$. Regarding y''

$$y'' = \frac{y'(h_7 - h_{12}) + (h_8 - h_7)}{(h_4 - h_7)} = \frac{(0.1502)(174.17 - 908.79) + (561.47 - 174.17)}{(2839.5 - 174.17)}$$

$$= 0.1039$$

For the turbine stages

$$\dot{W}_t / \dot{m}_1 = (h_1 - h_2) + (1 - y')(h_3 - h_4) + (1 - y' - y'')(h_4 - h_5)$$

$$= (3348.4 - 2963.5) + (.8498)(3335.5 - 2839.5) + (.7459)(2839.5 - 2270.1)$$

$$= 1231.1 \text{ kJ/kg}$$

And, for the pumps

PROBLEM 8.63 (Contd.)

$$\dot{W}_p/\dot{m}_1 = (h_9 - h_8) + (1-y'-y'')(h_7 - h_6)$$
$$= (569.73 - 561.47) + (.7459)(174.17 - 173.88) = 8.48 \text{ kJ/kg}$$

The total heat added during boiling/superheating and reheating is expressed per unit mass entering the first turbine as

$$\dot{Q}_{in}/\dot{m}_1 = (h_1 - h_{10}) + (1-y')(h_3 - h_2)$$
$$= (3348.4 - 878.34) + (.8498)(3335.5 - 2963.5)$$
$$= 2786.2 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.4388 \ (43.88\%) \longleftarrow \underline{\eta}$$

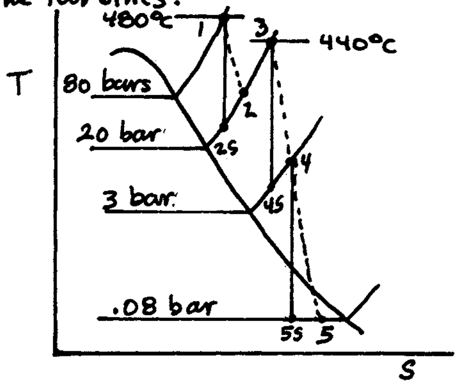
PROBLEM 8.64

KNOWN: The vapor power cycle of Probem 8.63 is modified to include that each turbine stage has an efficiency of 85% and each pump has an efficiency of 82%.

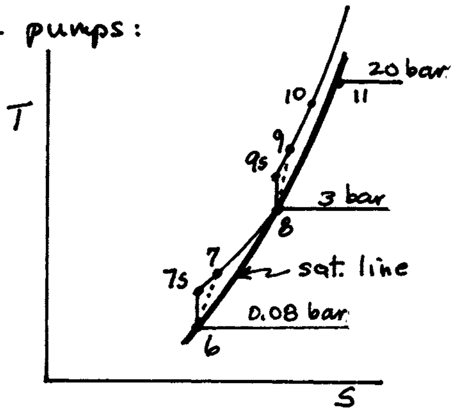
FIND: Repeat the analysis of Problem 8.63 for the modified cycle.

SCHEMATIC & GIVEN DATA:

For the turbines:



For the pumps:



ASSUMPTIONS: Same as Problem 8.63, except $\eta_{t_1} = \eta_{t_2} = \eta_{t_3} = 0.85$, and $\eta_{p_1} = \eta_{p_2} = 0.82$.

ANALYSIS: From Problem 8.63, $h_1 = 3348.4$ kJ/kg, $s_1 = 6.6586$ kJ/kg·K, $h_3 = 3335.8$ kJ/kg, $s_3 = 7.2540$ kJ/kg·K, $h_6 = 173.88$ kJ/kg, $h_8 = 561.47$ kJ/kg, $h_{10} = 878.34$ kJ/kg, $h_{11} = h_{12} = 908.79$ kJ/kg.

State 2: Using the turbine efficiency, $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$. Thus, with h_{2s} from Probem 8.63

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 3021.2 \text{ kJ/kg}$$

Also, interpolating in Table A-4; $s_2 = 6.7609$ kJ/kg.

State 4: $p_4 = 3$ bar, $s_{4s} = s_3 \Rightarrow h_{4s} = 2839.5$ kJ/kg (Problem 8.63). Then, with $\eta_{t_2} = 0.85$

$$h_4 = h_3 - \eta_{t_2}(h_3 - h_{4s}) = 2913.9 \text{ kJ/kg}$$

Further, interpolating in Table A-4; $s_4 = 7.3390$ kJ/kg·K.

State 5: $p_5 = .08$ bar, $s_{5s} = s_4 \Rightarrow h_{5s} = 2297.0$ kJ/kg. Then, with $\eta_{t_3} = .85$

$$h_5 = h_4 - \eta_{t_3}(h_4 - h_{5s}) = 2389.5 \text{ kJ/kg}$$

State 7: Using the pump efficiency, $\eta_{p_1} = (h_{7s} - h_6)/(h_7 - h_6)$. Thus, with h_{7s} from Problem 8.63

$$h_7 = h_6 + (h_{7s} - h_6)/\eta_{p_1} = 174.23 \text{ kJ/kg}$$

State 9: Similarly, with h_{9s} from Problem 8.63

$$h_9 = h_8 + (h_{9s} - h_8)/\eta_{p_2} = 571.54 \text{ kJ/kg}$$

The mass flow rate ratios y' and y'' are, respectively

$$y' = \frac{h_{10} - h_9}{h_2 - h_{11}} = \frac{878.34 - 571.54}{3021.2 - 908.79} = 0.1452$$

and

$$y'' = \frac{y'(h_7 - h_{12}) + (h_8 - h_7)}{(h_4 - h_7)} = \frac{(.1452)(174.23 - 908.79) + (561.47 - 174.23)}{(2913.9 - 174.23)}$$

$$= 0.1024$$

PROBLEM 8.64 (Contd.)

For the turbine stages

$$\dot{W}_t/\dot{m}_1 = (h_1-h_2) + (1-y')(h_3-h_4) + (1-y'-y'')(h_4-h_5)$$

$$= 1082.1 \text{ kJ/kg}$$

and, for the pumps

$$\dot{W}_p/\dot{m}_1 = (h_9-h_8) + (1-y'-y'')(h_7-h_6)$$

$$= 10.33 \text{ kJ/kg}$$

Finally, the total heat added per unit mass entering the first turbine stage is

$$\dot{Q}_{in}/\dot{m}_1 = (h_1-h_{10}) + (1-y')(h_3-h_2) = 2738.7 \text{ kJ/kg}$$

Thus, the thermal efficiency is

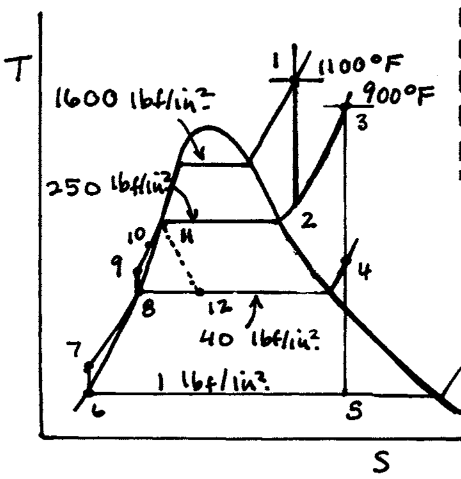
$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.391 \quad (39.1\%) \longleftarrow \underline{\quad \eta \quad}$$

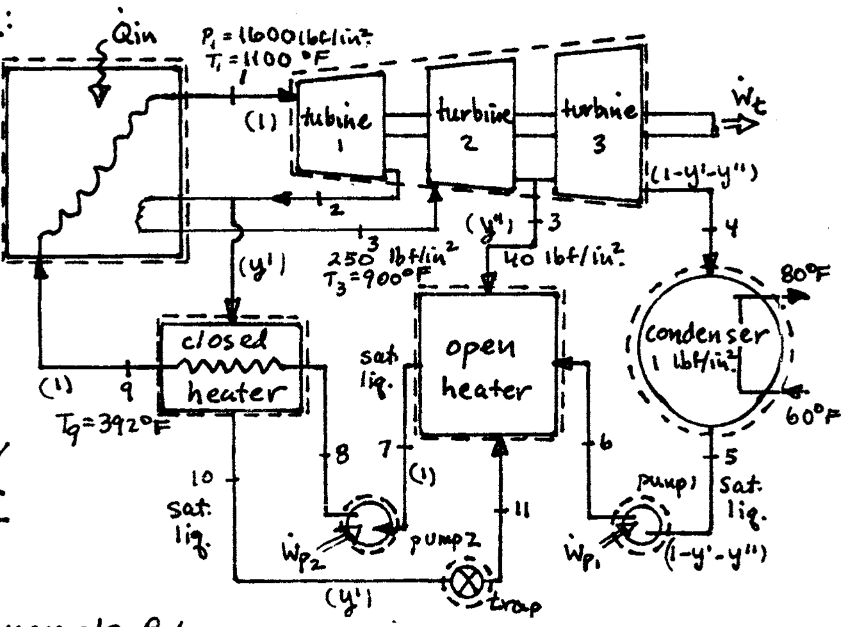
PROBLEM 8.65*

KNOWN: The regenerative power cycle in Problem 8.59 is modified to include reheat at 250 lbf/in².

FIND: Determine the thermal efficiency of the modified cycle.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 8.6

ANALYSIS: Using appropriate data from Problem 8.59

$h_1 = 1547.7$ Btu/lb
$h_2 = 1299.8$ Btu/lb
$h_6 = 69.74$ Btu/lb
$h_7 = 69.86$ Btu/lb
$h_8 = 236.16$ Btu/lb
$h_9 = 241.11$ Btu/lb
$h_{10} = 368.38$ Btu/lb
$h_{11} = 376.2$ Btu/lb
$h_{12} = 376.2$ Btu/lb

State 3: $p_3 = 250$ lbf/in², $T_3 = 900$°F $\Rightarrow h_3 = 1475.3$ Btu/lb, $s_3 = 1.7799 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

State 4: $p_4 = 40$ lbf/in², $s_4 = s_3 \Rightarrow$ sup. vapor ; $h_4 = 1253.9$ Btu/lb

State 5: $p_5 = 1$ lbf/in², $s_5 = s_3 \Rightarrow x_5 = 0.8926$, $h_5 = 994.5$ Btu/lb

From the solution to Problem 8.59, $y' = 0.1365$. Regarding y''

$$y'' = \frac{y'(h_7 - h_{12}) + (h_8 - h_7)}{(h_4 - h_7)} = \frac{(.1365)(69.86 - 376.2) + (236.16 - 69.86)}{(1253.9 - 69.86)}$$

$$= 0.1051$$

For the turbine stages

$$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1-y')(h_3 - h_4) + (1 - y' - y'')(h_4 - h_5)$$

$$= (1547.7 - 1299.8) + (.8635)(1475.3 - 1253.9) + (.7584)(1253.9 - 994.5)$$

$$= 635.8 \text{ Btu/lb}$$

And, for the pumps

$$\dot{W}_p/\dot{m}_1 = (h_9 - h_8) + (1-y'-y'')(h_7 - h_6)$$
$$= (241.11 - 236.16) + (.7584)(69.86 - 69.74) = 5.04 \text{ Btu/lb}$$

The total heat added during boiling/superheating and reheating is expressed per unit mass entering the first turbine as

$$\dot{Q}_{in}/\dot{m}_1 = (h_1 - h_{10}) + (1-y')(h_3 - h_2)$$
$$= (1547.7 - 368.38) + (.8635)(1475.3 - 1299.8)$$
$$= 1330.9 \text{ Btu/lb}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = 0.474 \; (47.4\%) \xleftarrow{\hspace{4cm}} \eta$$

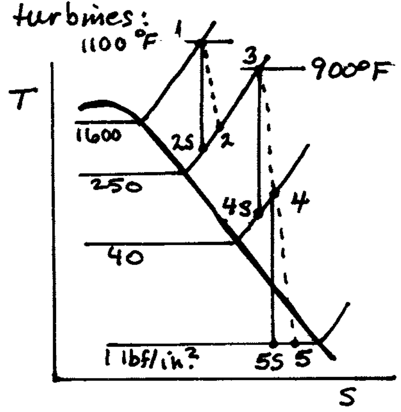
PROBLEM 8.66*

KNOWN: The vapor power cycle of Problem 8.65 is modified to include that each turbine stage has an efficiency of 88% and each pump has an efficiency of 80%.

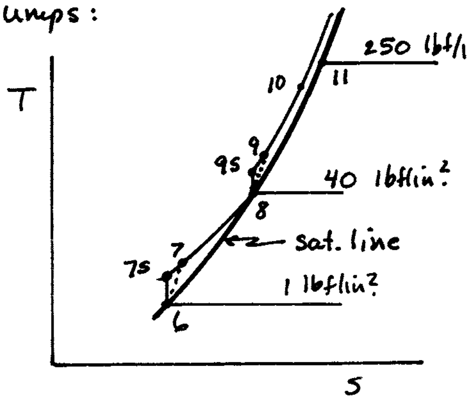
FIND: Repeat the analysis of Problem 8.65 for the modified cycle.

SCHEMATIC & GIVEN DATA:

For the turbines:



For the pumps:



ASSUMPTIONS: Same as Problem 8.65, except $\eta_{t_1} = \eta_{t_2} = \eta_{t_3} = 0.88$ and $\eta_{p_1} = \eta_{p_2} = 0.80$.

ANALYSIS: From Problem 8.65, $h_1 = 1547.7$ Btu/lb, $s_1 = 1.6315$ Btu/lb·°R, $h_3 = 1475.3$ Btu/lb, $s_3 = 1.7799$ Btu/lb·°R, $h_6 = 69.74$ Btu/lb, $h_8 = 236.16$ Btu/lb, $h_{10} = 368.38$ Btu/lb, and $h_{11} = h_{12} = 376.2$ Btu/lb.

State 2: Using the turbine efficiency, $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$. Thus, with h_{2s} from Problem 8.65

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 1329.5 \text{ Btu/lb}$$

State 4: $p_4 = 40$ lbf/in.², $s_{4s} = s_3 \Rightarrow h_{4s} = 1253.9$ Btu/lb (Problem 8.65). Then, with $\eta_{t_2} = 0.88$

$$h_4 = h_3 - \eta_{t_2}(h_3 - h_{4s}) = 1280.5 \text{ Btu/lb}$$

Further, interpolating in Table A-4E; $s_4 = 1.8092$ Btu/lb·°R

State 5: $p_5 = 1$ lbf/in.², $s_{5s} = s_4 \Rightarrow x_{5s} = 0.9085$, $h_{5s} = 1010.9$ Btu/lb. Then, with $\eta_{t_3} = 0.88$

$$h_5 = h_4 - \eta_{t_3}(h_4 - h_{5s}) = 1043.2 \text{ Btu/lb}$$

State 7: Using the pump efficiency, $\eta_{p_1} = (h_{7s} - h_6)/(h_7 - h_6)$. Thus, with h_{7s} from Problem 8.65

$$h_7 = h_6 + (h_{7s} - h_6)/\eta_{p_1} = 69.89 \text{ Btu/lb}$$

State 9: Similarly, with h_{9s} from Problem 8.65

$$h_9 = h_8 + (h_{9s} - h_8)/\eta_{p_2} = 242.35 \text{ Btu/lb}$$

The mass flow rate ratios y' and y'' are, respectively

$$y' = \frac{h_{10} - h_9}{h_2 - h_{11}} = \frac{368.38 - 242.35}{1329.5 - 376.2} = 0.1322$$

and

PROBLEM 8.66 (Contd.)

$$y'' = \frac{y'(h_7 - h_{12}) + (h_8 - h_7)}{(h_4 - h_7)}$$

$$= \frac{(.1322)(69.89 - 376.2) + (236.16 - 69.89)}{(1280.5 - 69.89)} = 0.1039$$

For the turbine stages

$$\dot{W}_t/\dot{m}_1 = (h_1 - h_2) + (1 - y')(h_3 - h_4) + (1 - y' - y'')(h_4 - h_5)$$

$$= (1547.7 - 1329.5) + (.8678)(1475.3 - 1280.5) + (.7639)(1280.5 - 1043.2)$$

$$= 568.5 \text{ Btu/lb}$$

And, for the pumps

$$\dot{W}_p/\dot{m}_1 = (h_9 - h_8) + (1 - y' - y'')(h_7 - h_6)$$

$$= (242.35 - 236.16) + (.7639)(69.89 - 69.74) = 6.30 \text{ Btu/lb}$$

The total heat added per unit mass entering the first turbine stage is

$$\dot{Q}_{in}/\dot{m}_1 = (h_1 - h_{10}) + (1 - y')(h_3 - h_2)$$

$$= (1547.7 - 368.38) + (.8678)(1475.3 - 1329.5)$$

$$= 1305.8 \text{ Btu/lb}$$

Thus, the thermal efficiency is

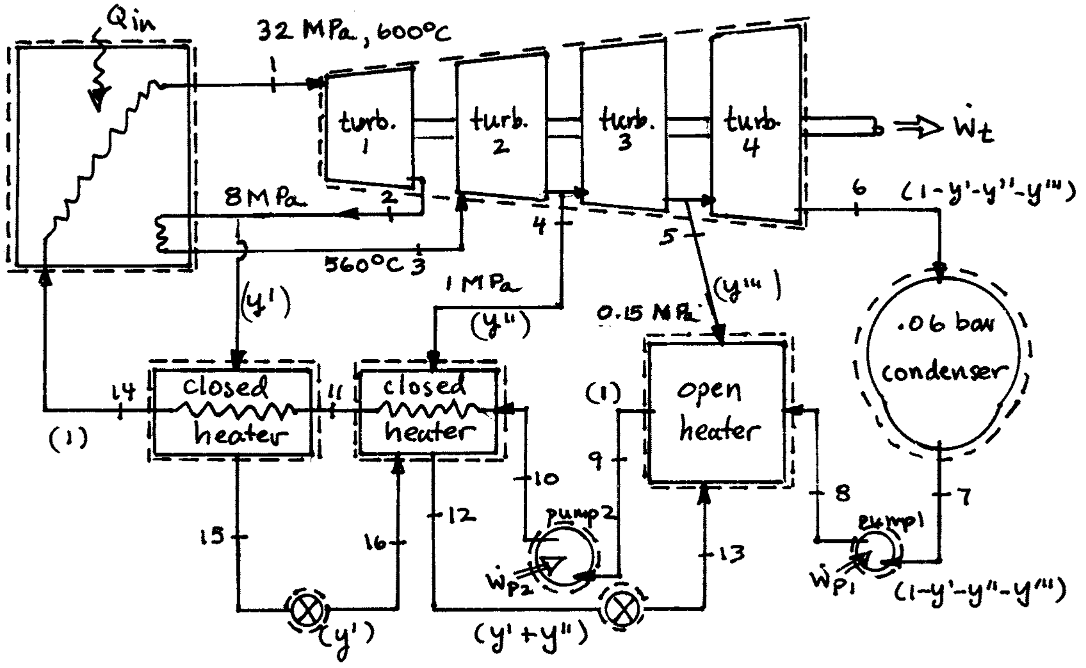
$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}} = \frac{568.5 - 6.30}{1305.8} = 0.431\ (43.1\%) \longleftarrow \underline{\eta}$$

PROBLEM 8.67

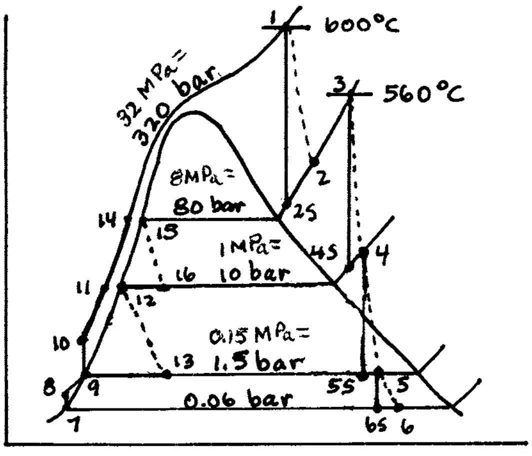
KNOWN: Water is the working fluid in a vapor power cycle with reheat and three regenerative feedwater heaters. Data are given at various locations and the net power output is known.

FIND: Sketch the layout of the cycle and determine the thermal efficiency and mass flow rate into the first turbine stage.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cycle} = 500$ MW



ASSUMPTIONS: Same as in Example 8.6, assumptions 1-5, (6) the feedwater leaving each closed heater is at the saturation temperature corresponding to the extraction pressure. Also, $\eta_{t_1} = \eta_{t_2} = \eta_{t_3} = \eta_{t_4} = 0.85$.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 320$ bar, $T_1 = 600°C \Rightarrow h_1 = 3424.6$ kJ/kg, $s_1 = 6.1858$ kJ/kg·K

State 2: $p_2 = 80$ bar, $s_{2s} = s_1 \Rightarrow h_{2s} = 3022.3$ kJ/kg

Using the turbine efficiency

$h_2 = h_1 - (h_1 - h_{2s})\eta_{t_1} = 3082.6$ kJ/kg and $s_2 = 6.2781$ kJ/kg·K

PROBLEM 8.67 (Contd. - Page 2)

State 3: $p_3 = 80$ bar, $T_3 = 560°C \Rightarrow h_3 = 3545.3$ kJ/kg, $s_3 = 6.9072$ kJ/kg·K

State 4: $p_4 = 10$ bar, $s_{4s} = s_3 \Rightarrow h_{4s} = 2934.0$ kJ/kg

$h_4 = h_3 - (h_3 - h_{4s})\eta_{t_2} = 3025.7$ kJ/kg, $s_4 = 7.0771$ kJ/kg

State 5: $p_5 = 1.5$ bar, $s_{5s} = s_4 \Rightarrow x_{5s} = 0.9747$, $h_{5s} = 2637.3$ kJ/kg

$h_5 = h_4 - (h_4 - h_{5s})\eta_{t_3} = 2695.6$ kJ/kg, $s_5 = 7.2284$ kJ/kg·K

State 6: $p_6 = 0.06$ bar, $s_{6s} = s_5 \Rightarrow x_{6s} = 0.8589$, $h_{6s} = 2226.5$ kJ/kg

$h_6 = h_5 - (h_5 - h_{6s})\eta_{t_4} = 2296.9$ kJ/kg

State 7: $p_7 = 0.06$ bar, sat. liquid $\Rightarrow h_7 = 151.53$ kJ/kg

State 8: $h_8 \approx h_7 + v_7(p_8 - p_7)$

$$= 151.53 \frac{\text{kJ}}{\text{kg}} + (1.0064 \times 10^{-3})\frac{\text{m}^3}{\text{kg}}(1.5 - 0.06)\text{bar} \left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kJ}}{10^3 \text{ N·m}}\right|$$

$$= 151.53 + 0.14 = 151.67 \text{ kJ/kg}$$

State 9: $p_9 = 1.5$ bar, sat. liquid $\Rightarrow h_9 = 467.11$ kJ/kg

State 10: $h_{10} \approx h_9 + v_9(p_{10} - p_9) = 467.11 + (1.0528 \times 10^{-3})(320 - 1.5)\left|\frac{10^5}{10^3}\right|$

$$= 467.11 + 33.53 = 500.64 \text{ kJ/kg}$$

State 11: $p_{11} = 320$ bar, $T_{11} = T_{sat\ @\ 10\ bar} = 180°C \Rightarrow$ Extrapolating using data from Table A-5 gives $h_{11} \approx 781.7$ kJ/kg

State 12: $p_{12} = 10$ bar, sat. liquid $\Rightarrow p_{12} = 762.81$ kJ/kg

State 13: Throttling process $\Rightarrow h_{13} = h_{12} = 762.81$ kJ/kg

State 14: $p_{14} = 320$ bar, $T_{14} = T_{sat\ @\ 80\ bar} = 295°C \Rightarrow$ Extrapolating using data from Table A-5 gives $h_{14} \approx 1303.8$ kJ/kg

State 15: $p_{15} = 80$ bar, sat. liquid $\Rightarrow h_{15} = 1316.6$ kJ/kg

State 16: Throttling process $\Rightarrow h_{16} = h_{15} = 1316.6$ kJ/kg

Applying mass and energy balances to the control volume enclosing the 80 bar closed heater, the fraction of the flow y' extracted at location 2 is

$$0 = y'(h_2 - h_{15}) + (1)(h_{11} - h_{14})$$

or

$$y' = \frac{h_{14} - h_{11}}{h_2 - h_{15}} = \frac{1303.8 - 781.7}{3082.6 - 1316.6} = 0.2956$$

Similarly, for the 10 bar closed heater

$$0 = y'' h_4 + y' h_{16} + (h_{10} - h_{11}) - (y' + y'') h_{12}$$

or

$$y'' = \frac{(h_{11} - h_{10}) + y'(h_{12} - h_{16})}{(h_4 - h_{12})}$$

$$= \frac{(781.7 - 500.64) + (.2956)(762.81 - 1316.6)}{(3025.7 - 762.81)} = 0.0519$$

PROBLEM 8.67 (Cont'd - Page 3)

And, for the open heater

$$0 = y''' h_5 + (y'+y'') h_{13} + (1-y'-y''-y''') h_8 - h_9$$

or

$$y''' = \frac{(h_9 - h_8) + (y'+y'')(h_8 - h_{13})}{(h_5 - h_8)}$$

$$= \frac{(467.11 - 151.67) + (.3475)(151.67 - 762.81)}{(2695.6 - 151.67)} = 0.0405$$

For the control volume enclosing the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1 - h_2) + (1-y')(h_3 - h_4) + (1-y'-y'')(h_4 - h_5) + (1-y'-y''-y''')(h_5 - h_6)$$

$$= (3424.6 - 3082.6) + (.7044)(3545.3 - 3025.7)$$

$$(.6525)(3025.7 - 2695.6) + (.6120)(2695.6 - 2296.9)$$

$$= 1167.4 \text{ kJ/kg}$$

And, for the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_{10} - h_9) + (1-y'-y''-y''')(h_8 - h_7)$$

$$= (500.64 - 467.11) + (.6120)(151.67 - 151.53) = 33.62 \text{ kJ/kg}$$

The total rate of heat transfer for the boiler/superheater and reheater, per unit mass of steam entering the first turbine stage, is

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = (h_1 - h_{14}) + (1-y')(h_3 - h_2)$$

$$= (3424.6 - 1303.8) + (.7044)(3545.3 - 3082.6)$$

$$= 2446.7 \text{ kJ/kg}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{1167.4 - 33.62}{2446.7} = 0.463 \, (46.3\%) \longleftarrow \eta$$

The net power developed by the cycle is

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]$$

or, with $\dot{W}_{cycle} = 500$ MW, the flow rate $\dot{m}_1$ is

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]}$$

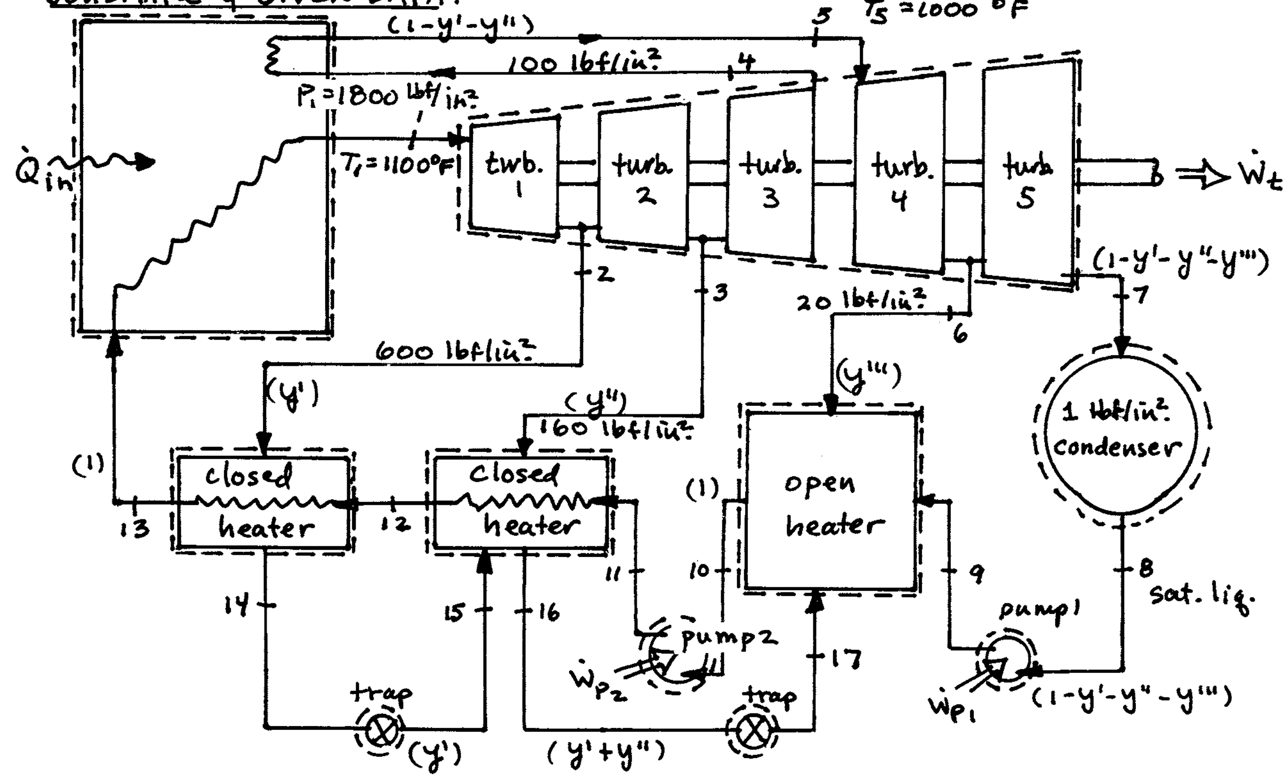
$$= \frac{(500 \text{ MW})}{(1167.4 - 33.62) \text{ kJ/kg}} \left| \frac{10^3 \text{ kJ/s}}{1 \text{ MW}} \right| \left| \frac{3600 \text{ s}}{1 \text{ h}} \right| = 1.588 \times 10^6 \text{ kg/h} \longleftarrow \dot{m}_1$$

PROBLEM 8.68

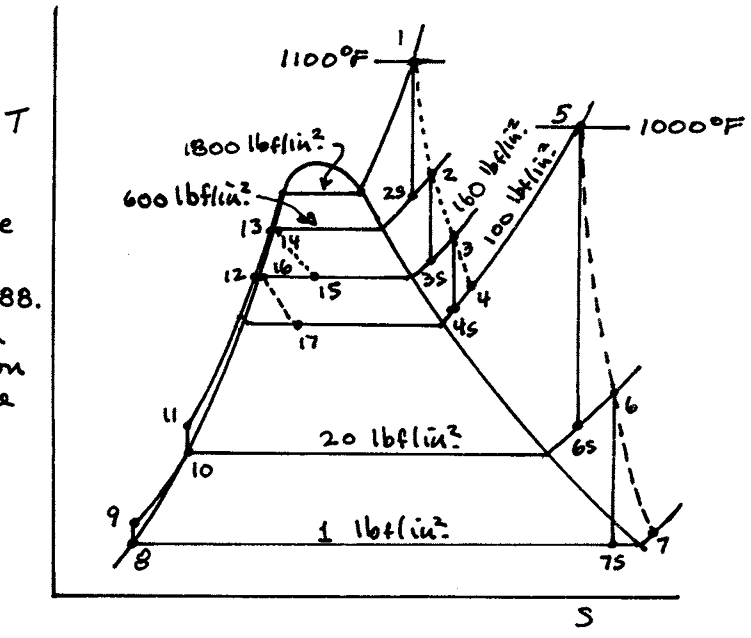
KNOWN: Water is the working fluid in a vapor power cycle with reheat and three regenerative feedwater heaters. Data are given at various locations, and the net power output is known

FIND: Sketch the layout of the cycle and determine the thermal efficiency, the mass flow rate into the first turbine stage, and the heat rate.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as in Example 8.6, assumptions 1-5, except $\eta_{t_1} = \eta_{t_2} = \eta_{t_3} = \eta_{t_4} = \eta_{t_5} = .88$. Also, the feedwater leaving each closed heater is at the saturation temperature corresponding to the extraction pressure.



ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 1800$ lbf/in.², $T_1 = 1100°F \Rightarrow h_1 = 1542.5$ Btu/lb, $s_1 = 1.6159$ Btu/lb°R

State 2: First, $P_2 = 600$ lbf/in.², $s_{2s} = s_1 \Rightarrow h_{2s} = 1385.3$ Btu/lb. With the turbine efficiency

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 1404.2 \text{ Btu/lb}$$

And, interpolating in Table A-4; $s_2 = 1.6315$ Btu/lb·°R

PROBLEM 8.68 (Contd.)

State 3: Similarly, $h_{3s} = 1254.0$ Btu/lb, and $h_3 = h_2 - \eta_{t2}(h_2 - h_{3s})$ $= 1272.0$ Btu/lb. From Table A-4; $s_3 = 1.6507$ Btu/lb·°R

State 4: Similarly, $h_{4s} = 1226.7$ Btu/lb, and $h_4 = h_3 - \eta_{t3}(h_3 - h_{4s})$ $= 1232.1$ Btu/lb. From Table A-4; $s_4 = 1.6569$ Btu/lb·°R

State 5: $p_5 = 100$ lbf/in.², $T_5 = 1000$°F $\Rightarrow h_5 = 1532.1$ Btu/lb, $s_5 = 1.9204 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

State 6: $p_6 = 20$ lbf/in.², $s_{6s} = s_5 \Rightarrow h_{6s} = 1315.5$ Btu/lb. With the turbine efficiency; $h_6 = h_5 - \eta_{t4}(h_5 - h_{6s}) = 1341.5$ Btu/lb. From Table A-4; $s_6 = 1.9455$ Btu/lb.

State 7: Similarly, $x_{7s} = .9824$, $h_{7s} = 1087.5$, and $h_7 = h_6 - \eta_{t5}(h_6 - h_{7s})$ $= 1118.0$ Btu/lb

State 8: $p_8 = 1$ lbf/in.², sat. liquid $\Rightarrow h_8 = 69.74$ Btu/lb

State 9:
$$h_9 \approx h_8 + v_8(p_9 - p_8)$$
$$= 69.74 \frac{\text{Btu}}{\text{lb}} + \left(.01614 \frac{\text{ft}^3}{\text{lb}}\right)\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|(20-1)\frac{\text{lbf}}{\text{in.}^2}$$
$$= 69.80 \text{ Btu/lb}$$

State 10: $p_{10} = 20$ lbf/in.², sat. liquid $\Rightarrow h_{10} = 196.26$ Btu/lb

State 11: $p_{11} = 1800$ lbf/in.², $s_{11} = s_{10} = 0.3358$ Btu/lb·°R. Double interpolation is Table A-5 gives $h_{11} = 203.28$ Btu/lb.

State 12: $p_{12} = 1800$ lbf/in.², $T_{12} = T_{sat\,@\,160\ \text{lbf/in.}^2} = 363.6$°F $\Rightarrow$ Double interpolation in Table A-5 gives $h_{12} = 339.10$ Btu/lb

State 13: $p_{13} = 1800$ lbf/in.², $T_{13} = T_{sat\,@\,600\ \text{lbf/in.}^2} = 486.3$°F $\Rightarrow$ Double interpolation in Table A-5 gives $h_{13} = 472.22$ Btu/lb

State 14: $p_{14} = 600$ lbf/in.², sat. liquid $\Rightarrow h_{14} = 471.7$ Btu/lb

State 15: Throttling process $\Rightarrow h_{15} = h_{14} = 471.7$ Btu/lb

State 16: $p_{16} = 160$ lbf/in.², sat. liquid $\Rightarrow h_{16} = 336.2$ Btu/lb

State 17: Throttling process $\Rightarrow h_{17} = h_{16} = 336.2$ Btu/lb

Applying mass and energy rate balances to the control volume enclosing the 600 lbf/in.² closed heater, the fraction of the flow y' extracted at location 2 is

$$0 = y'(h_2 - h_{14}) + (h_{12} - h_{13})$$

or

$$y' = \frac{h_{13} - h_{12}}{h_2 - h_{14}} = \frac{472.22 - 339.10}{1404.2 - 471.7} = 0.1428$$

Similarly, for the 160 lbf/in.² heater

$$0 = y'' h_3 + y' h_{15} - (y' + y'') h_{16} + (h_{11} - h_{12})$$

or

$$y'' = \frac{(h_{12} - h_{11}) + y'(h_{16} - h_{15})}{(h_3 - h_{16})}$$
$$= \frac{(339.10 - 203.28) + (.1428)(336.2 - 471.7)}{(1272.0 - 336.2)} = 0.1245$$

PROBLEM 8.68 (Contd.)

Also, for the open feedwater heater

$$0 = y''' h_6 + (y'+y'')\, h_{17} + (1-y'-y''-y''')\, h_9 - h_{10}$$

or

$$y''' = \frac{(h_{10}-h_9) + (y'+y'')(h_9 - h_{17})}{(h_6 - h_9)}$$

$$= \frac{(196.26 - 69.80) + (.2673)(69.80 - 336.2)}{(1341.5 - 69.80)} = 0.0434$$

For the control volume enclosing the turbine stages

$$\frac{\dot{W}_t}{\dot{m}_1} = (h_1-h_2) + (1-y')(h_2-h_3) + (1-y'-y'')[(h_3-h_4) + (h_5-h_6)] + (1-y'-y''-y''')(h_6-h_7)$$

$$= (1542.5 - 1404.2) + (.8572)(1404.2 - 1272.0) + (.7327)[(1272.0 - 1232.1) + (1532.1 - 1341.5)] + (.6893)(1341.5 - 1118.0) = 574.6 \text{ Btu/lb}$$

And, for the pumps

$$\frac{\dot{W}_p}{\dot{m}_1} = (h_{11} - h_{10}) + (1-y'-y''-y''')(h_9 - h_8)$$

$$= (203.28 - 196.26) + (.6893)(69.80 - 69.74) = 7.06 \text{ Btu/lb}$$

The total rate of heat transfer for the boiler/super heater and reheater, per unit mass of steam entering the first turbine stage, is

$$\frac{\dot{Q}_{in}}{\dot{m}_1} = (h_1 - h_{13}) + (1-y'-y'')(h_5 - h_4)$$

$$= (1542.5 - 472.22) + (.7327)(1532.1 - 1232.1) = 1290.1 \text{ Btu/lb}$$

Thus, the thermal efficiency is

$$\eta = \frac{\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1}{\dot{Q}_{in}/\dot{m}_1} = \frac{574.6 - 7.06}{1290.1} = 0.440\ (44.0\%) \longleftarrow \eta$$

The net power developed by the cycle is

$$\dot{W}_{cycle} = \dot{m}_1 [\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]$$

or, with $\dot{W}_{cycle} = 3 \times 10^9$ Btu/h, the flow rate $\dot{m}_1$ is

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1]} = \frac{3 \times 10^9 \text{ Btu/h}}{[574.6 - 7.06] \text{ Btu/lb}} = 5.286 \times 10^6 \text{ lb/h} \longleftarrow \dot{m}_1$$

The heat rate — the heat added to the cycle to produce a unit of net work output — is given here by

$$\text{heat rate} = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{(5.286 \times 10^6 \frac{\text{lb}}{\text{h}})(1290.1 \text{ Btu/lb})}{3 \times 10^9 \text{ Btu/h}} \left| \frac{3413 \text{ Btu/h}}{1 \text{ kW}} \right|$$

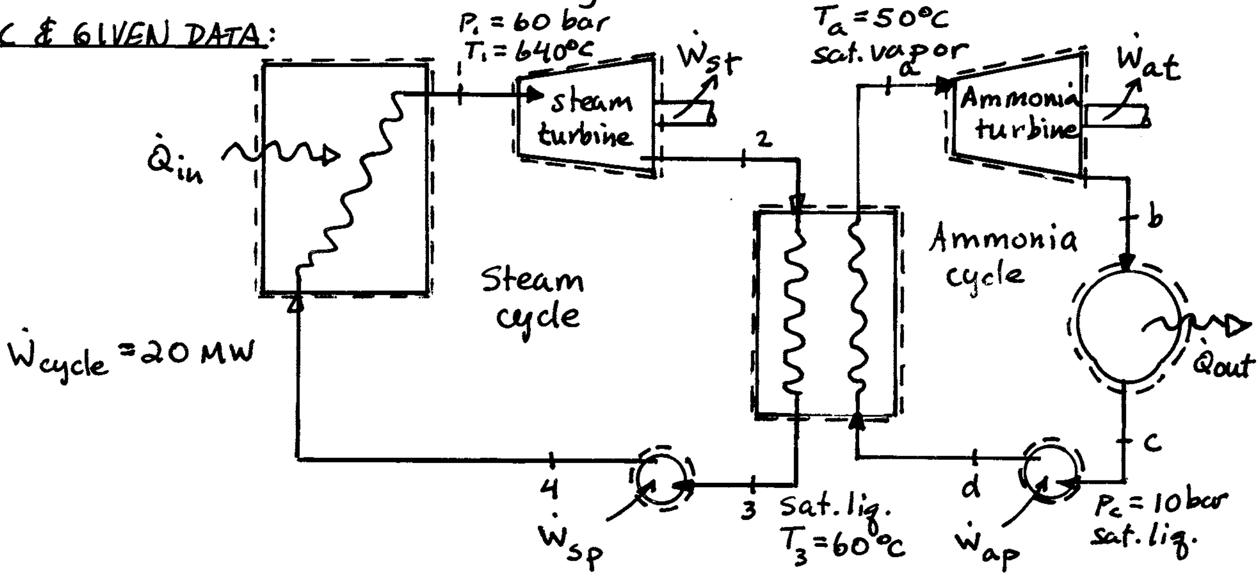
$$= 7758 \frac{\text{Btu}}{\text{kW}\cdot\text{h}} \longleftarrow \text{heat rate}$$

PROBLEM 8.69

KNOWN: Steam and ammonia are the working fluids in a binary vapor power cycle consisting of two ideal Rankine cycles. The heat rejected from the steam cycle is provided to the ammonia cycle. Data are known at various locations.

FIND: Determine (a) the power output of the steam and ammonia turbines, respectively, (b) the rate of heat addition to the cycle, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluids are internally reversible, except in the interconnecting heat exchanger. (3) There are no stray heat transfers from the turbines or the heat exchanger. (4) Kinetic and potential energy effects can be neglected.

ANALYSIS: First, fix each of principal states. For steam, use Tables A-2 and A-4.

State 1: $p_1 = 60$ bar, $T_1 = 640°C \Rightarrow h_1 = 3752.6$ kJ/kg, $s_1 = 7.2731$ kJ/kg

State 2: $T_2 = T_3 = 60°C$, $s_2 = s_1 \Rightarrow x_2 = 0.9101$, $h_2 = 2397.6$ kJ/kg

State 3: $T_3 = 60°C$, sat. liquid $\Rightarrow h_3 = 251.13$ kJ/kg, $v_3 = 1.0172 \times 10^{-3}$ m³/kg

State 4: $h_4 \approx h_3 + v_3(P_4 - P_3)$; $P_3 = 0.1994$ bar

$$h_4 = 251.13 + \left(1.0172 \times 10^{-3} \frac{m^3}{kg}\right)(60 - 0.1994)\text{ bar} \left|\frac{10^5 N/m^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 N \cdot m}\right|$$

$$= 251.13 + 6.083 = 257.21 \text{ kJ/kg}$$

Now, for Ammonia, use Tables A-13 and A-14.

State a: $T_a = 50°C$, sat. vapor $\Rightarrow h_a = 1471.26$ kJ/kg, $s_a = 4.7604$ kJ/kg·K

State b: $P_b = P_c = 10$ bar, $s_b = s_a \Rightarrow x_b = 0.9312$, $h_b = 1383.0$ kJ/kg

State c: $P_c = 10$ bar, sat. liquid $\Rightarrow h_c = 297.76$ kJ/kg, $v_c = 1.6584 \times 10^{-3}$ m³/kg

State d: $h_d \approx h_c + v_c(P_d - P_c)$; $P_d = P_a = 20.331$ bar

$$h_d = 297.76 + (1.6584 \times 10^{-3})(20.331 - 10)(100) = 299.47 \text{ kJ/kg}$$

(a) To determine the power output of each turbine, we must find the mass flow rates $\dot{m}_s$ and $\dot{m}_a$ for the steam and ammonia cycles, respectively. Beginning with mass and energy rate balances for the inter-connecting heat exchanger

PROBLEM 8.69 (Cont'd)

$$0 = \dot{m}_s(h_2-h_3) + \dot{m}_a(h_d-h_a)$$

or $$\frac{\dot{m}_a}{\dot{m}_s} = \frac{h_2-h_3}{h_a-h_d} = \frac{2397.6-251.13}{1471.26-288.47} = 1.832$$

For the steam cycle

$$\dot{W}_{cycle,s} = \dot{W}_{st} - \dot{W}_{sp} = \dot{m}_s[(h_1-h_2)-(h_4-h_3)]$$

And, for the ammonia cycle

$$\dot{W}_{cycle,a} = \dot{W}_{at} - \dot{W}_{ap} = \dot{m}_a[(h_a-h_b)-(h_d-h_c)]$$

Noting that $\dot{W}_{cycle} = \dot{W}_{cycle,s} + \dot{W}_{cycle,a}$ and combining

$$\dot{W}_{cycle} = \dot{m}_s\left\{[(h_1-h_2)-(h_4-h_3)]+\left(\frac{\dot{m}_a}{\dot{m}_s}\right)[(h_a-h_b)-(h_d-h_c)]\right\}$$

Solving

$$\dot{m}_s = \frac{\dot{W}_{cycle}}{[(h_1-h_2)-(h_4-h_3)]+\left(\frac{\dot{m}_a}{\dot{m}_s}\right)[(h_a-h_b)-(h_d-h_c)]}$$

$$= \frac{(20\text{ MW})}{[(1355)-(6.08)]+(1.832)[(88.26)-(1.71)]\text{ kJ/kg}}\left|\frac{10^3\text{ kJ/s}}{1\text{ MW}}\right|$$

$$= 13.27\text{ kg/s}$$

And $\dot{m}_a = 1.832\,\dot{m}_s = 24.31$ kg/s

Finally

$\dot{W}_{st} = \dot{m}_s(h_1-h_2) = 17.98$ MW ← $\dot{W}_{st}$

$\dot{W}_{at} = \dot{m}_a(h_a-h_b) = 2.15$ MW ← $\dot{W}_{at}$

(b) The rate of heat addition is

$$\dot{Q}_{in} = \dot{m}_s(h_1-h_4)$$

$$= \left(13.27\frac{\text{kg}}{\text{s}}\right)(3752.6-257.21)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right|$$

$= 46.38$ MW ← $\dot{Q}_{in}$

(c) The thermal efficiency is

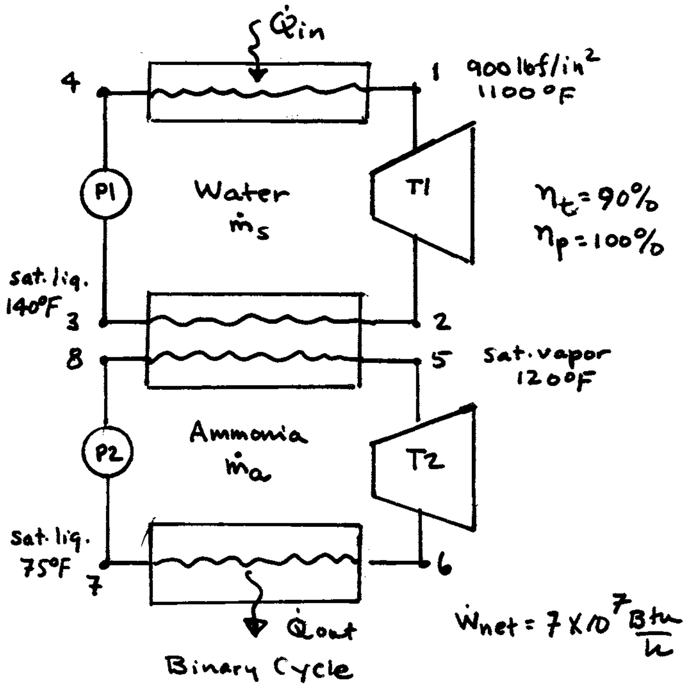
$\eta = \dfrac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \dfrac{20}{46.38} = 0.431$ (43.1%) ← η

PROBLEM 8.70

KNOWN: Operating data are provided for a binary vapor cycle consisting of two Rankine cycles with steam and ammonia as the working fluids.

FIND: (a) For the binary cycle, determine the quality at the exit of each turbine, the mass flow rate of each working fluid, and the overall thermal efficiency. (b) Compare the performance to a single Rankine cycle using water as the working fluid, condensing at 75°F.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. Control volumes enclosing the principal components are at steady state.
2. The turbines and pumps operate adiabatically. There are no stray heat transfers, and kinetic and potential energy effects can be ignored.
3. For the heat exchangers, the pressure of the working fluid remains constant.

ANALYSIS: (a) Begin by fixing the numbered states on the schematic. From Table A-4E, $h_1 = 1565.4$ Btu/lb, $s_1 = 1.7036$ Btu/lb. With $s_{2s} = s_1$ and data from Table A-2E at 140°F

$$x_{2s} = \frac{1.7036 - 0.1985}{1.8892 - 0.1985} = 0.89$$

$\Rightarrow h_{2s} = 107.96 + 0.89(1014) = 1010.42 \frac{Btu}{lb}$. Then, with $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$,

$h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1565.4 - 0.9(1565.4 - 1010.42) = 1065.92$ Btu/lb. And

$$x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{1065.92 - 107.96}{1014} = 0.945 \qquad \longleftarrow$$

For pump P1, Eq. 8.7b gives, $\frac{\dot{W}_{p1}}{\dot{m}_s} = v_3 \Delta p = \left(0.01629 \frac{ft^3}{lb}\right)(900 - 2.892) \frac{lbf}{in^2} \left| \frac{144\ in^2}{1\ ft^2} \right| \cdot \left| \frac{1\ Btu}{778\ ft \cdot lbf} \right|$
$= 2.7$ Btu/lb

$\Rightarrow h_4 = h_3 + \frac{\dot{W}_{p1}}{\dot{m}_s} = 107.96 + 2.7 = 110.66$ Btu/lb.

Considering the ammonia cycle, Table A-13E gives $h_5 = 632.95$ Btu/lb, $s_1 = 1.1405$ Btu/lb·°R. With $s_{6s} = s_5$ and data from Table A-13E at 75°F

$x_{6s} = \frac{1.1405 - 0.2636}{1.2048 - 0.2636} = 0.932 \Rightarrow h_{6s} = 126.02 + 0.932(503.18) = 594.98$ Btu/lb. And

$h_6 = h_5 - \eta_t(h_5 - h_{6s}) = 632.95 - 0.9(632.95 - 594.98) = 598.78$ Btu/lb

$$\Rightarrow x_6 = \frac{598.78 - 126.02}{503.18} = 0.94 \qquad \longleftarrow$$

For pump P2, $\frac{\dot{W}_{p2}}{\dot{m}_a} = v_7 \Delta p = (0.02650)(286.4 - 140.6) \left| \frac{144}{778} \right| = 0.72 \frac{Btu}{lb}$

$\Rightarrow h_8 = h_7 + \frac{\dot{W}_{p2}}{\dot{m}_a} = 126.02 + 0.72 = 126.74$ Btu/lb

PROBLEM 8.70 (Contd.)

Next, an energy rate balance on the interconnecting heat exchanger gives,

$$\frac{\dot m_s}{\dot m_a} = \frac{h_5-h_8}{h_2-h_3} = \frac{632.95-126.74}{1065.92-107.96} = 0.528$$

Then,

$$\dot W_{net} = \dot m_s\left[h_1-h_2-\frac{\dot W_{p1}}{\dot m_s}\right] + \dot m_a\left[h_5-h_6-\frac{\dot W_{p2}}{\dot m_a}\right]$$

$$\left(7\times10^7\ \frac{Btu}{h}\right) = 0.528\,\dot m_a\left[1565.4-1065.92-2.7\right] + \dot m_a\left[632.95-598.78-0.72\right]$$

$$\Rightarrow \dot m_a = 236{,}686\ \frac{lb}{h}\ ,\ \dot m_s = 124{,}970\ \frac{lb}{h} \quad \longleftarrow$$

The thermal efficiency is

$$\eta = \frac{\dot W_{net}}{\dot m_s(h_1-h_4)} = \frac{7\times10^7\ Btu/h}{\left(124{,}970\frac{lb}{h}\right)(1565.4-110.66)\frac{Btu}{lb}} = 0.385\ \ (38.5\%) \longleftarrow$$

(b) For a single steam cycle condensing at 75°F, the exit condition for the steam turbine is fixed using $s_{2s} = s_1$. Then, with data from Table A-2E

$$x_{2s} = \frac{1.7036-0.08402}{2.04975-0.08402} = 0.824 \Rightarrow h_{2s} = 43.09 + 0.824(1051.15) = 909.24\ Btu/lb$$

$$\Rightarrow h_2 = h_1 - \eta_t(h_1-h_{2s}) = 1565.4 - 0.9(1565.4-909.24) = 974.86\ Btu/lb$$

① $$\Rightarrow x_2 = \frac{974.86-43.09}{1051.15} = 0.886 \quad \longleftarrow$$

The pump work per unit of steam flowing is

$$\frac{\dot W_p}{\dot m_s} = v_3\,\Delta p = 0.01606(900-0.43)\left|\frac{144}{778}\right| = 2.67\ Btu/lb$$

$$\Rightarrow h_4 = h_3 + \frac{\dot W_p}{\dot m_s} = 43.09 + 2.67 = 45.76\ Btu/lb$$

The mass flow rate of the steam is

② $$\dot m_s = \frac{\dot W_{net}}{(h_1-h_2)-\dot W_p/\dot m_s} = \frac{7\times10^7\ Btu/h}{\left[(1565.4-974.86)-2.67\right]Btu/lb} = 119{,}074\ \frac{lb}{h} \quad \longleftarrow$$

The thermal efficiency is

③ $$\eta = \frac{\dot W_{net}}{\dot m_s(h_1-h_4)} = \frac{7\times10^7}{119{,}074(1565.4-45.76)} = 0.387\ \ (38.7\%) \longleftarrow$$

---

1. Since the quality at the exit of a steam turbine should be at least 90%, the performance of the single Rankine cycle is less satisfactory than the binary cycle in this respect.

2. The mass flow rate of the steam in the single Rankine cycle is about 5% less than in the binary cycle.

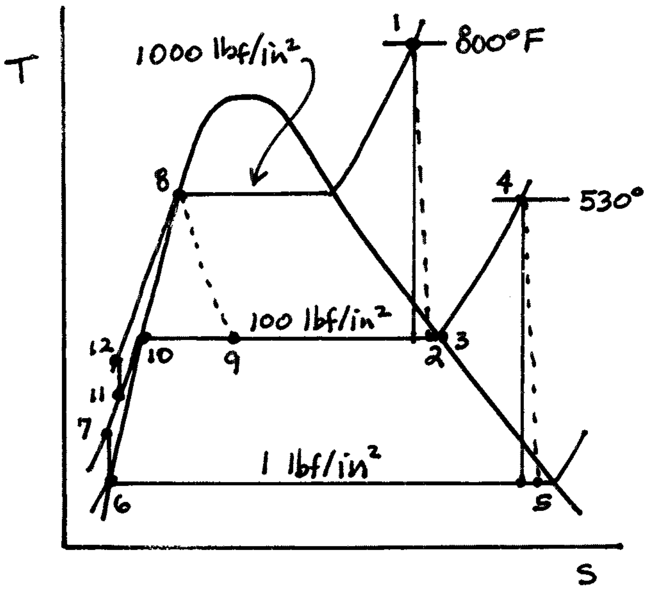
3. Since the average temperatures of heat addition and heat rejection are the same in the two cycles, the thermal efficiencies are the same when roundoff is taken into account.

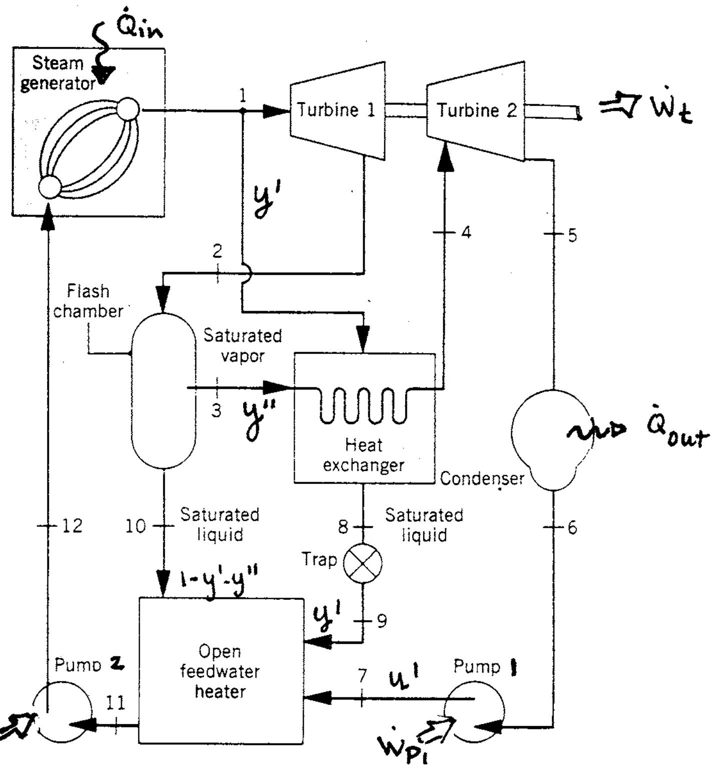
PROBLEM 8.71

KNOWN: Water is the working fluid in a vapor power cycle with reheat and regeneration. Data are known at various locations and the net power is given.

FIND: Determine (a) the mass flow rate through the steam generator, (b) the thermal efficiency, and (c) the rate of heat transfer to cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) There are no stray heat transfers. (3) The working fluid undergoes internally reversible processes in passing through the pumps, steam generator, flash chamber, and condenser. (4) The expansion through the trap is a throttling process. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 1000$ lbf/in², $T_1 = 800$°F ⇒ $h_1 = 1388.5$ Btu/lb, $s_1 = 1.5665$ Btu/lb·°R

State 2: $p_2 = 100$ lbf/in², $s_{2s} = s_1$ ⇒ $x_{2s} = 0.9673$, $h_{2s} = 1158.7$ Btu/lb

Then, with $\eta_{t_1} = (h_1 - h_2)/(h_1 - h_{2s})$

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 1186.3 \text{ Btu/lb}$$

State 3: $p_3 = 100$ lbf/in², sat. vapor ⇒ $h_3 = 1187.8$ Btu/lb

State 4: $p_4 = 100$ lbf/in², $T_4 = 530$°F ⇒ $h_4 = 1294.2$ Btu/lb, $s_4 = 1.7234$ Btu/lb·°R

State 5: First, $p_5 = 1$ lbf/in², $s_{5s} = s_5$ ⇒ $x_{5s} = .8620$, $h_{5s} = 962.77$ Btu/lb

Then, with $\eta_{t_2} = 0.88$; $h_5 = h_4 - \eta_{t_2}(h_4 - h_{5s}) = 1002.5$ Btu/lb

State 6: $p_6 = 1$ lbf/in², sat. vapor ⇒ $h_6 = 69.74$ Btu/lb

PROBLEM 8.71 (Cont'd - page 2)

State 7: $h_7 \approx h_6 + v_6(p_7 - p_6)$

$$= 69.74\ \frac{\text{Btu}}{\text{lb}} + \left(.01614\ \frac{\text{ft}^3}{\text{lb}}\right)(100-1)\ \frac{\text{lbf}}{\text{in}^2}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 69.74 + .296 = 70.04\ \text{Btu/lb}$$

State 8: $p_8 = 1000\ \text{lbf/in}^2$, sat. liquid $\Rightarrow h_8 = 542.4$ Btu/lb

State 9: throttling process $\Rightarrow h_9 = h_8 = 542.4$ Btu/lb

State 10: $p_{10} = 100\ \text{lbf/in}^2$, sat. liquid $\Rightarrow h_{10} = 298.6$ Btu/lb

State 11: Must be determined in the analysis.

The mass flow rate ratios y' and y'' are determined from energy and mass rate balances on the flash chamber and heat exchanger, respectively.

$$(1-y')h_2 = y''h_3 + (1-y'-y'')h_{10} \qquad (*)$$

$$y'(h_1 - h_8) + y''(h_3 - h_4) = 0 \qquad (**)$$

From (**)

$$y'' = \left(\frac{h_1 - h_8}{h_4 - h_3}\right)y' = \left(\frac{1388.5 - 542.4}{1294.2 - 1187.8}\right)y' = 7.952\,y'$$

Solving (*) and incorporating this result gives

$$y' = \frac{(h_2 - h_{10})}{(7.952\,h_3 - 8.952\,h_{10} + h_2)} = 0.1115$$

and $y'' = 7.952\,y' = 0.8866$

(a) For the turbine stages

$$\frac{\dot{W}_{t1}}{\dot{m}_1} = (1-y')(h_1 - h_2)$$

$$\frac{\dot{W}_{t2}}{\dot{m}_2} = y''(h_4 - h_5)$$

and for the pumps

$$\dot{W}_{p_1}/\dot{m}_1 = y''(h_7 - h_6)$$

$$\dot{W}_{p_2}/\dot{m}_1 = (h_{12} - h_{11})$$

To find h_{11}, use an energy balance for the open feedwater heater

$$(1-y'-y'')h_{10} + y'h_9 + y''h_7 = h_{11} = 123.1\ \text{Btu/lb}.$$

From Table A-2E, $T_{11} \approx 155°\text{F}$ and $v_{11} \approx 0.01637\ \text{ft}^3/\text{lb}$. Thus, for state 12

$$h_{12} \approx h_{11} + v_{11}(p_{12} - p_{11}) = 123.1 + 2.73 = 125.83\ \text{Btu/lb}$$

Finally

$$\dot{m}_1 = \frac{\dot{W}_{cycle}}{(1-y')(h_1-h_2) + y''(h_4-h_5) - y''(h_7-h_6) - (h_{12}-h_{11})}$$

$$= \frac{5\times10^9\ \text{Btu/h}}{[179.6 + 258.6 - 0.03 - 2.73]\ \text{Btu/lb}}$$

$$= 1.148\times10^7\ \text{lb/h} \longleftarrow \qquad \dot{m}_1$$

PROBLEM 8.71 (Cont'd - page 3)

(b) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{\dot{W}_{cycle}}{\dot{m}_1 (h_1 - h_{12})} = \frac{(5 \times 10^9)}{1.148 \times 10^7 (1388.5 - 125.83)}$$

$$= 0.345 \; (34.5\%) \longleftarrow \underline{\eta}$$

(c) The rate of heat transfer to cooling water passing through the condenser is

$$\dot{Q}_{out} = \dot{m}_1 y'' (h_5 - h_6)$$

$$= (1.148 \times 10^7 \text{ lb/h})(0.8866)(1002.5 - 69.74) \text{ Btu/lb}$$

$$= 9.494 \times 10^9 \text{ Btu/h} \longleftarrow \underline{\dot{Q}_{out}}$$

COMMENT: As a check on the calculations, note that

$$\dot{Q}_{out} = \dot{Q}_{in} - \dot{W}_{cycle} = 9.495 \times 10^9 \text{ Btu/h}$$

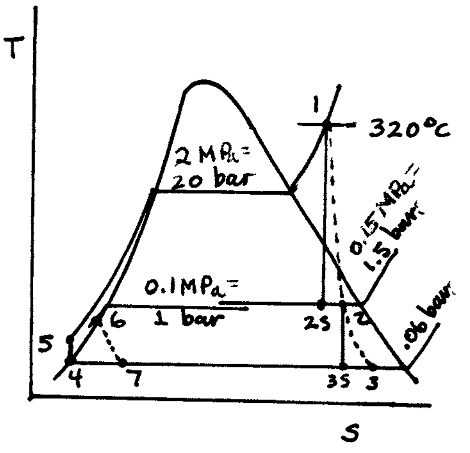
which is the same as the result determined in part (c), except for round-off errors.

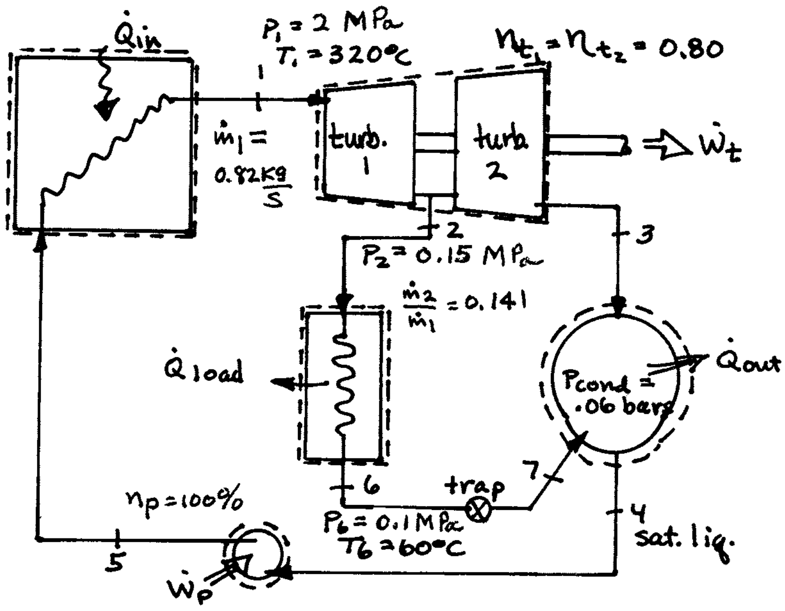
PROBLEM 8.72

KNOWN: Water is the working fluid in a cogeneration cycle that generates electricity and provides heating for buildings. Data are provided at several locations. The fraction of the total flow extracted is also known.

Determine: (a) the heat transfer rate to the working fluid passing through the steam generator, (b) the net power developed, (c) the rate of heat transfer for building heating, (c) the rate of heat transfer to cooling water.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each control volume shown operates at steady state. (2) The turbine stages operate adiabatically, with $\eta_{t_1} = \eta_{t_2} = 0.8$, and the pump operates isentropically. (3) The working fluid undergoes a throttling process in passing through the trap. (4) Kinetic and potential energy effects can be neglected. (5) Condensate leaves the condenser as saturated liquid at 0.06 bars.

ANALYSIS: First, fix each of the principal states.

State 1: $P_1 = 20$ bar, $T_1 = 320$°c $\Rightarrow h_1 = 3069.5$ kJ/kg, $s_1 = 6.8452$ kJ/kg·K

State 2: First, $P_2 = 1.5$ bar, $s_{2s} = s_1 \Rightarrow h_{2s} = 2548.2$. Using the turbine efficiency

$$\eta_{t_1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 2652.5 \text{ kJ/kg}$$

And, with data from Table A-3; $s_2 = 7.1163$ kJ/kg·K

State 3: $P_3 = 0.06$ bar, $s_{3s} = s_2 \Rightarrow h_{3s} = 2191.8$. Thus, $h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s})$ $= 2283.9$ kJ/kg. And, from Table A-3, $s_3 = 7.4139$ kJ/kg·K.

State 4: $P_4 = 0.06$ bar, Sat. liquid $\Rightarrow h_4 = 151.53$ kJ/kg

State 5: $h_5 \approx h_4 + v_4(P_5 - P_4)$

$$= 151.53 \frac{kJ}{kg} + (1.0064 \times 10^{-3})\frac{m^3}{kg}(20 - 0.06)\text{bar} \left|\frac{10^5 N/m^2}{1 \text{ bar}}\right| \left|\frac{1 kJ}{10^3 N\cdot m}\right|$$

$$= 153.5 \text{ kJ/kg}$$

State 6: $P_6 = 1$ bar, $T_6 = 60$°C, $h_6 \approx h_f(T_6) = 251.13$ kJ/kg

State 7: Throttling process $\Rightarrow h_7 = h_6 = 251.13$ kJ/kg

PROBLEM 8.72 (Contd)

(a) For a control volume enclosing the tubes carrying the working fluid through the steam generator

$$\dot{Q}_{in} = \dot{m}_1[h_1 - h_5] = \left(0.82\frac{kg}{s}\right)\left|\frac{3600\,s}{1\,h}\right|[3069.5 - 153.5]\frac{kJ}{kg} = 8.61\times10^6\frac{kJ}{h} \longleftarrow$$

(b) The net power developed is obtained as follows:

Turbines: $\dot{W}_t = \dot{m}_1(h_1 - h_2) + (\dot{m}_1 - \dot{m}_2)(h_2 - h_3)$

Pump: $\dot{W}_p = \dot{m}_1[h_5 - h_4]$

$$\therefore\ \dot{W}_{net} = \dot{m}_1(h_1 - h_2) + (\dot{m}_1 - \dot{m}_2)(h_2 - h_3) - \dot{m}_1(h_5 - h_4)$$

$$= \dot{m}_1[(h_1 - h_2) + (h_2 - h_3) - (h_5 - h_4)] - \dot{m}_2(h_2 - h_3)$$

$$= \dot{m}_1[h_1 - h_3 - h_5 + h_4] - \dot{m}_2[h_2 - h_3]$$

$$= (0.82\times3600)\frac{kg}{h}\left[3069.5 - 2283.9 - 153.5 + 151.5\right]\frac{kJ}{kg}$$

$$-0.141(0.82\times3600)[2652.5 - 2283.9] = 2.16\times10^6\frac{kJ}{h} \longleftarrow$$

(c) $\dot{Q}_{LOAD} = \dot{m}_2[h_2 - h_6] = 0.141(0.82\times3600)[2652.5 - 251.1]$

$$= 10^6\frac{kJ}{h}$$

(d) For a control volume enclosing the working fluid side of the condenser

$$\dot{Q}_{out} = (\dot{m}_1 - \dot{m}_2)h_3 + \dot{m}_2 h_7 - \dot{m}_1 h_4$$

$$= \dot{m}_1[h_3 - h_4] + \dot{m}_2[h_7 - h_3]$$

① $$= ((0.82)\times3600)[2283.9 - 151.5] + [(0.141)(0.82)(3600)][251.1 - 2283.9]$$

$$= 5.45\times10^6\frac{kJ}{h}$$

---

1. An overall energy rate balance reads:

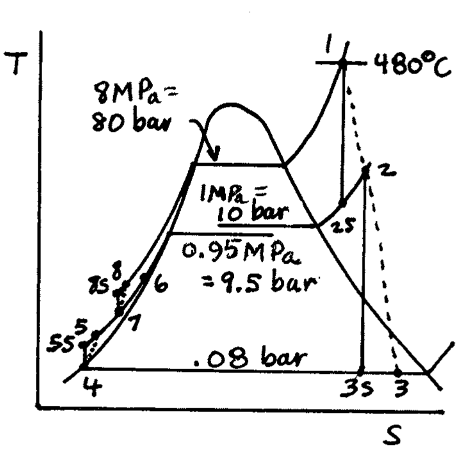
• Rate energy is added by heat transfer in the steam generator	8.61×10^6 kJ/h	
• Disposition:		
– net power developed	2.16×10^6 kJ/h	(25.1%)
– energy to campus buildings	1.00×10^6 kJ/h	(11.6%)
– energy to cooling water	5.45×10^6 kJ/h	(63.3%)
	8.61×10^6 kJ/h	

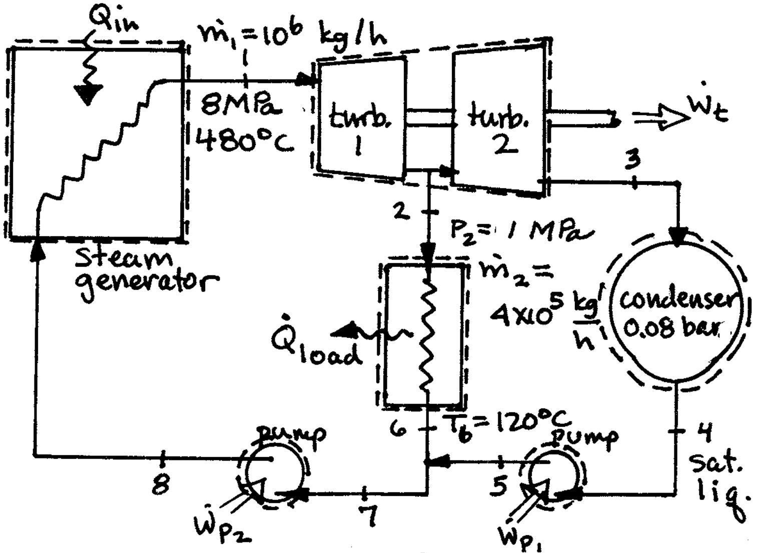
PROBLEM 8.73

KNOWN: Water is the working fluid in a cogeneration cycle producing power and providing steam for a process heating load. Data are known at various locations.

FIND: Determine (a) the heating load, (b) the power developed by the turbine, and (c) the rate of heat transfer to the working fluid passing through the steam generator.

SCHEMATIC & GIVEN DATA:





$P_5 = P_6 = 0.95$ MPa

ASSUMPTIONS: (1) Each control volume shown is at steady state. (2) The turbine stages and the pumps operate adiabatically, with $\eta_{t_1} = \eta_{t_2} = .86$ and $\eta_{p_1} = \eta_{p_2} = 0.8$. (3) Kinetic and potential energy effects are negligible. (4) The condenser operates at 0.08 bars, with the condensate leaving as saturated liquid.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 80$ bar, $T_1 = 480°C \Rightarrow h_1 = 3348.4$ kJ/kg, $s_1 = 6.6586 \frac{kJ}{kg \cdot K}$

State 2: First, $p_2 = 10$ bar, $s_{2s} = s_1 \Rightarrow h_{2s} = 2811.5$ kJ/kg. Using the turbine efficiency

$$\eta_{t_1} = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_{t_2}(h_1 - h_{2s}) = 2886.7 \text{ kJ/kg}$$

Also, from Table A-4; $s_2 = 6.8133$ kJ/kg·K

State 3: Similarly, $p_3 = 0.08$ bar, $s_{3s} = s_2 \Rightarrow h_{3s} = 2131.5$ kJ/kg.

Thus $h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 2237.2$ kJ/kg

State 4: $p_4 = 0.08$ bar, sat. liquid $\Rightarrow h_4 = 173.88$ kJ/kg

State 5: $h_{5s} \approx h_4 + v_4(p_5 - p_4)$

$$= 173.88 \frac{kJ}{kg} + (1.0084 \times 10^{-3}) \frac{m^3}{kg} (9.5 - .08) \text{bar} \left| \frac{10^5 \text{ N/m}^2}{1 \text{ bar}} \right| \left| \frac{1 \text{ kJ}}{10^3 \text{ N} \cdot \text{m}} \right|$$

$$= 173.88 + 0.95 = 174.83 \text{ kJ/kg}$$

Using the pump efficiency, $h_5 = h_4 + (h_{5s} - h_4)/\eta_{p_1} = 175.07$ kJ/kg

State 6: $p_6 = 9.5$ bar, $T_6 = 120°C \Rightarrow$ estimate h_6 using

$$h_6 \approx h_f(T_6) + v_f(T_6)[p - p_{sat}(T_6)]$$

With data from Table A-2

$$h_6 \approx 503.71 + (1.0603 \times 10^{-3})(9.5 - 1.985)\left|\frac{10^5}{10^3}\right| = 504.5 \text{ kJ/kg}$$

State 7: To fix state 7, consider the mixing of streams 5 and 6 to form stream 7. From the given data, $\dot{m}_1 = 10^6$ kg/h $= \dot{m}_7$ and $\dot{m}_2 = \dot{m}_6 = 4 \times 10^5$ kg/h. Thus, from the mass balance

$$\dot{m}_5 = \dot{m}_7 - \dot{m}_6 = 6 \times 10^5 \text{ kg/h}$$

thus, applying an energy balance

$$0 = \dot{m}_5 h_5 + \dot{m}_6 h_6 - \dot{m}_7 h_7$$

or

$$h_7 = \frac{\dot{m}_5 h_5 + \dot{m}_6 h_6}{\dot{m}_7} = 306.84 \text{ kJ/kg}$$

State 8: $h_{8s} \approx h_7 + v_7(p_8 - p_7)$. If $h_7 \approx 75°C$, $v_7 \approx (1.0259 \times 10^{-3}) \frac{m^3}{kg}$

thus $h_{8s} \approx 306.84 + (1.0259 \times 10^{-3})(80 - 9.5)\left|\frac{10^5}{10^3}\right| = 314.1$ kJ/kg

With the pump efficiency; $h_8 = h_7 + (h_{8s} - h_7)/\eta_{p2} = 315.88$ kJ/kg

(a) The heating load is

$$\dot{Q}_{load} = \dot{m}_2(h_2 - h_6) = \left(4 \times 10^5 \frac{kg}{h}\right)(2886.7 - 504.5)\frac{kJ}{kg}$$

$$= 9.529 \times 10^8 \text{ kJ/h} \longleftarrow \underline{\dot{Q}_{load}}$$

(b) For the control volume enclosing the turbine

$$\dot{W}_t = \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3$$

$$= (1 \times 10^6)(3348.4) - (4 \times 10^5)(2886.7) - (6 \times 10^5)(2237.2)$$

$$= 8.514 \times 10^8 \text{ kJ/h} = 236{,}500 \text{ kW} \longleftarrow \underline{\dot{W}_t}$$

(c) For the working fluid passing through the steam generator

$$\dot{Q}_{in} = \dot{m}_1(h_1 - h_8)$$

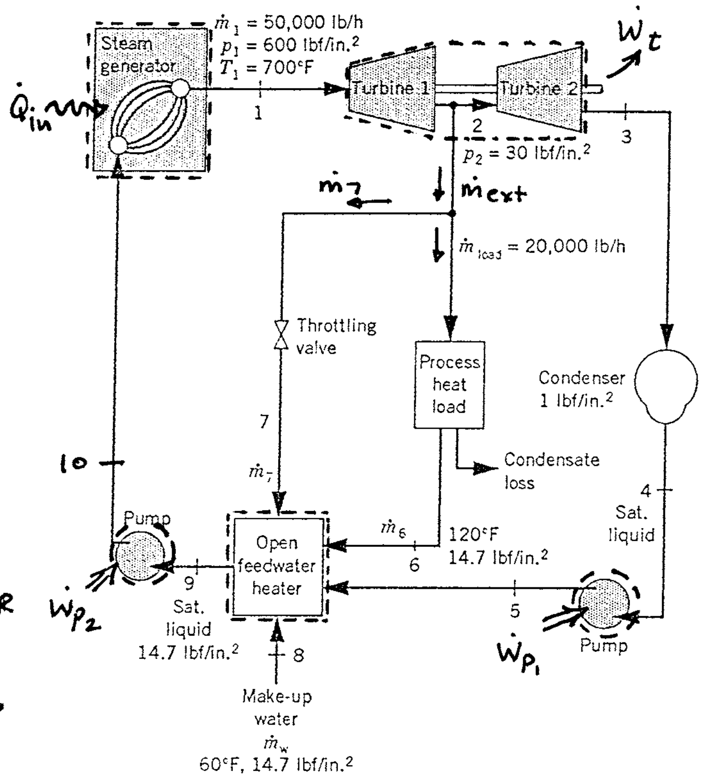
$$= (1 \times 10^6)(3348.4 - 315.88) = 3.032 \times 10^9 \text{ kJ/h} \longleftarrow \underline{\dot{Q}_{in}}$$

PROBLEM 8.74

KNOWN: Water is the working fluid in a cogeneration system providing power and process steam. Data are known at various locations.

FIND: Determine (a) the rate at which steam is extracted, (b) the net power developed by the cycle, (c) the rate of heat transfer to the working fluid passing through the steam generator, and (d) the heat rate.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each control volume shown is at steady state. (2) Each turbine stage, pump, and the open feedwater heater operate adiabatically. (3) For the turbine, $\eta_{t_1} = \eta_{t_2} = 0.88$ and the pumps operate isentropically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 600$ lbf/in^2, $T_1 = 700°F$
$\Rightarrow h_1 = 1350.6$ Btu/lb, $s_1 = 1.5872$ Btu/lb·°R

State 2: $p_2 = 30$ lbf/in^2, $s_{2s} = s_1$;
$x_{2s} = 0.9156$ and $h_{2s} = 1084.5$ Btu/lb

Using the turbine efficiency

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 1116.4 \text{ Btu/lb}$$

Further, $x_2 = 0.9493$ and $s_2 = 1.6321$ Btu/lb·°R

State 3: $p_3 = 1$ lbf/in^2, $s_{3s} = s_2 \Rightarrow x_{3s} = 0.81255$, $h_{3s} = 911.54$ Btu/lb

$$h_3 = h_2 - \eta_{t_2}(h_2 - h_{3s}) = 936.12 \text{ Btu/lb}$$

State 4: $p_4 = 1$ lbf/in^2, sat. liquid $\Rightarrow h_4 = 69.74$ Btu/lb, $v_4 = 0.01614$ ft^3/lb

State 5: $p_5 = 14.7$ lbf/in^2, $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$h_5 = 69.74 + (.01614 \tfrac{\text{ft}^3}{\text{lb}})(14.7 - 1)\tfrac{\text{lbf}}{\text{in}^2}\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft·lbf}}\right|$$

$$= 69.74 + 0.041 = 69.78 \text{ Btu/lb}$$

State 6: $h_6 \approx h_f(T_6) = 88.00$ Btu/lb

State 7: Throttling valve $\Rightarrow h_7 = h_2 = 1116.4$ Btu/lb

State 8: $h_8 \approx h_f(T_8) = 28.08$ Btu/lb

State 9: $p_9 = 14.7$ lbf/in^2, sat. liquid $\Rightarrow h_9 = 180.15$ Btu/lb, $v_9 = 0.01672$ ft^3/lb

State 10: $p_{10} = p_1 = 600$ lbf/in^2

$$h_{10} \approx h_9 + v_9(p_{10} - p_9)$$

$$= 180.15 + (0.01672)(600 - 14.7)\left|\frac{144}{778}\right|$$

$$= 180.15 + 1.81 = 182.96 \text{ Btu/lb}$$

(a) To find the rate at which steam is extracted, $\dot{m}_{ext}$, consider the control volume enclosing the open feedwater heater.

$$0 = \dot{m}_7 h_7 + \dot{m}_6 h_6 + \dot{m}_5 h_5 + \dot{m}_w h_8 - \dot{m}_1 h_9$$

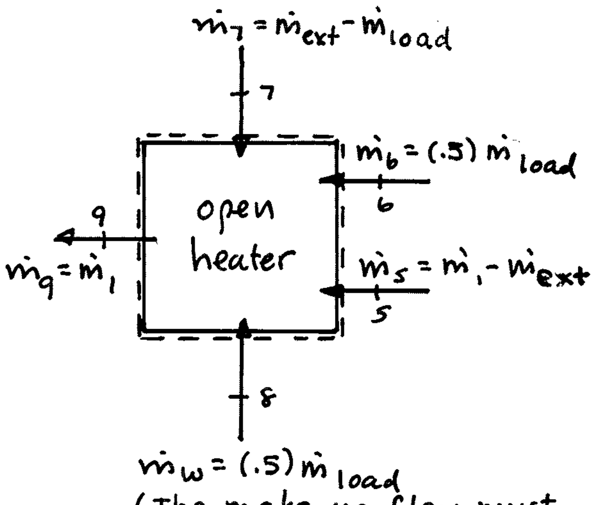
$$\dot{m}_9 = \dot{m}_1 = 50{,}000$$

$$\dot{m}_7 = \dot{m}_{ext} - \dot{m}_{load} = \dot{m}_{ext} - 20{,}000$$

$$\dot{m}_6 = (.5)\dot{m}_{load} = 10{,}000$$

$$\dot{m}_5 = \dot{m}_1 - \dot{m}_{ext} = 50{,}000 - \dot{m}_{ext}$$

$$\dot{m}_w = (.5)\dot{m}_{load} = 10{,}000$$



$\dot{m}_7 = \dot{m}_{ext} - \dot{m}_{load}$
7
$\dot{m}_6 = (.5)\dot{m}_{load}$
6
9
$\dot{m}_9 = \dot{m}_1$
open heater
$\dot{m}_5 = \dot{m}_1 - \dot{m}_{ext}$
5
8
$\dot{m}_w = (.5)\dot{m}_{load}$

(The make-up flow must equal the condensate loss.)

Inserting these relations, and solving for the extraction flow rate

$$\dot{m}_{ext} = \frac{50{,}000\,h_9 + 20{,}000\,h_7 - 10{,}000\,h_6 - 50{,}000\,h_5 - 10{,}000\,h_8}{(h_7 - h_5)}$$

$$= \frac{(50{,}000)(180.15) + (20{,}000)(1116.4) - (10{,}000)(88) - (50{,}000)(69.78) - (10{,}000)(28.08)}{(1116.4 - 69.78)}$$

$$= 25{,}500 \text{ lb/h} \longleftarrow \underline{\qquad\qquad \dot{m}_{ext}}$$

(b) For the turbine control volume

$$\dot{W}_t = \dot{m}_1 h_1 - \dot{m}_{ext} h_2 - (\dot{m}_1 - \dot{m}_{ext}) h_3$$

$$= \left(50{,}000 \tfrac{\text{lb}}{\text{h}}\right)\left(1350.6 \tfrac{\text{Btu}}{\text{lb}}\right) - (25{,}500)(1116.4) - (24{,}500)(936.12)$$

$$= 1.613 \times 10^7 \text{ Btu/h}$$

For the pumps

$$\dot{W}_p = \dot{W}_{p_1} + \dot{W}_{p_2} = (\dot{m}_1 - \dot{m}_{ext})(h_5 - h_4) + \dot{m}_1 (h_{10} - h_9)$$

$$= (24{,}500)(69.78 - 69.74) + (50{,}000)(182.96 - 180.15)$$

$$= 1.415 \times 10^5 \text{ Btu/h}$$

The net power developed is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = 1.599 \times 10^7 \text{ Btu/h} \longleftarrow \underline{\qquad\qquad \dot{W}_{cycle}}$$

(c) The heat input is

$$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_{10}) = (50{,}000)(1350.6 - 182.96)$$

$$= 5.84 \times 10^7 \text{ Btu/h} \longleftarrow \underline{\qquad\qquad \dot{Q}_{in}}$$

(d) The heat rate — the heat added to the cycle to produce a unit of net work output — is given here by

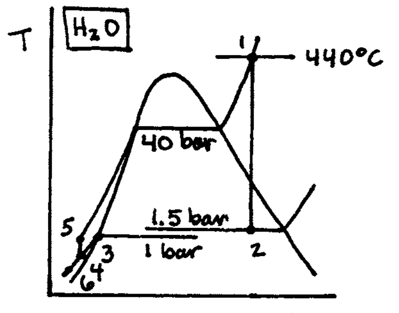
$$\text{heat rate} = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{5.84 \times 10^7 \text{ Btu/h}}{1.599 \times 10^7 \text{ Btu/h}} \left| \frac{3413 \text{ Btu/h}}{1 \text{ kW}} \right| = 12{,}465 \frac{\text{Btu}}{\text{kW} \cdot \text{h}} \longleftarrow \text{heat rate}$$

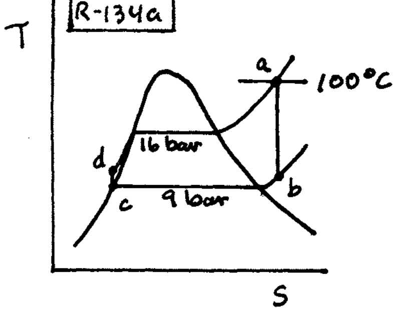
PROBLEM 8.75

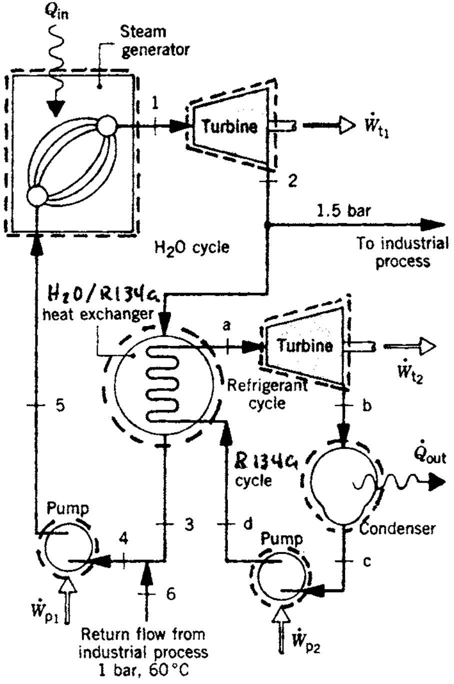
KNOWN: Water and Refrigerant 134a are the working fluids in a binary cycle used for cogeneration of power and process steam. Data are known at various locations.

FIND: Determine (a) the rate of heat transfer to the steam passing through the steam generator, (b) the net power output of the binary cycle, and (c) the rate of heat transfer to the industrial process.

SCHEMATIC & GIVEN DATA:







ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All turbines and pumps operate isentropically. (3) The control volume around the inter-connecting heat exchanger is adiabatic. (4) Kinetic and potential energy effects are negligible. (5) Saturated liquid exits each condenser at the respective pressure.

ANALYSIS: First, fix each of the principal states. For the H_2O cycle:

State 1: $p_1 = 40$ bar, $T_1 = 440°C \Rightarrow h_1 = 3307.1$ kJ/kg, $s_1 = 6.9041$ kJ/kg·K

State 2: $p_2 = 1.5$ bar, $s_2 = s_1 \Rightarrow h_2 = 2570.8$ kJ/kg

State 3: $p_1 = 1$ bar, sat. liquid $\Rightarrow h_3 = 417.46$ kJ/kg

State 6: $p_6 = 1$ bar, $T_6 = 60°C \Rightarrow$ compressed liquid;

$$h_6 \approx h_f(60) + v_f(60)\,(p - p_{sat@60°C})$$
$$= 251.13\ \frac{kJ}{kg} + (1.0172\times10^{-3})\frac{m^3}{kg}(1-.1994)\,bar\left|\frac{10^5\ N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$
$$= 251.2\ kJ/kg$$

State 4: For the mixing of streams 3 and 6 to form stream 4

$$0 = \dot{m}_3 h_3 + \dot{m}_6 h_6 - (\dot{m}_3 + \dot{m}_6) h_4$$

or, with $\dot{m}_3 = 2.5$ kg/s, $\dot{m}_6 = 2.5$ kg/s

$$h_4 = (.5)(417.46) + (.5)(251.2) = 334.33 \text{ kJ/kg}$$

State 5: $T_4 \approx 80°C$, $h_5 \approx h_4 + v_4(p_5 - p_4)$

$$= 334.33 + (1.0291 \times 10^{-3})(40-1)\left|\frac{10^5}{10^3}\right|$$

$$= 338.34 \text{ kJ/kg}$$

For the R-134a cycle ;

State a: $p_a = 16$ bar, $T_a = 100°C \Rightarrow h_a = 327.46$ kJ/kg, $s_a = 1.0467$ kJ/kg·K

State b: $p_b = 9$ bar, $s_b = s_a \Rightarrow h_b = 312.74$ kJ/kg

State c: $p_c = 9$ bar, sat. liquid $\Rightarrow h_c = 99.56$ kJ/kg

State d: $h_d \approx h_c + v_c(p_d - p_c) = 99.56 + (.8576 \times 10^{-3})(16-9)\left|\frac{10^5}{10^3}\right| = 100.16 \frac{\text{kJ}}{\text{kg}}$

(a) From mass and energy balances for the control volume enclosing steam passing through the steam generator

$$\dot{Q}_{in} = \dot{m}_1(h_1 - h_5)$$

$$= (5 \text{ kg/s})(3307.1 - 338.34)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 14{,}844 \text{ kW} \longleftarrow \dot{Q}_{in}$$

(b) For the H_2O cycle

$$\dot{W}_{H_2O} = \dot{W}_{t_1} - \dot{W}_{p_1} = (5 \text{ kg/s})\left[(3307.1 - 2570.8) - (338.34 - 334.33)\right]\frac{\text{kJ}}{\text{kg}}$$

$$= 3661 \text{ kW}$$

To determine the mass flow rate of R-134a, consider the inter-connecting heat exchanger

$$0 = \dot{m}_3(h_2 - h_3) + \dot{m}_R(h_d - h_a)$$

or

$$\dot{m}_R = \frac{\dot{m}_3(h_2 - h_3)}{(h_a - h_d)} = \frac{(2.5 \text{ kg/s})(2570.8 - 417.46)}{(327.46 - 100.16)}$$

$$= 23.68 \text{ kg/s}$$

thus

$$\dot{W}_R = \dot{m}_R\left[(h_a - h_b) - (h_d - h_c)\right]$$

$$= (23.68)\left[(327.46 - 312.74) - (100.16 - 99.56)\right] = 334.4 \text{ kW}$$

The total power developed is

$$\dot{W}_{cycle} = 3661 + 334.4 = 3995 \text{ kW} \longleftarrow \dot{W}_{cycle}$$

(c) For the industrial process

$$\dot{Q}_{process} = \dot{m}_{process}(h_2 - h_6)$$

$$= (2.5 \text{ kg/s})(2570.8 - 251.2)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

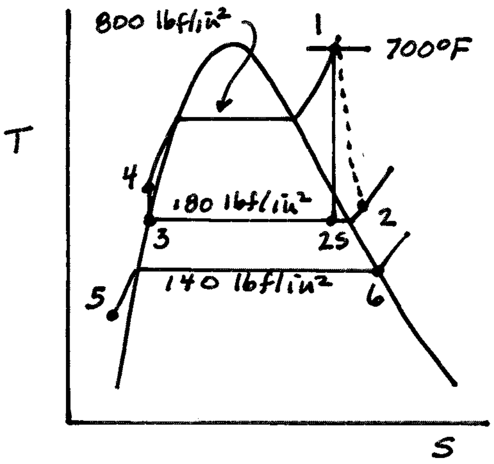
$$= 5799 \text{ kW} \longleftarrow \dot{Q}_{process}$$

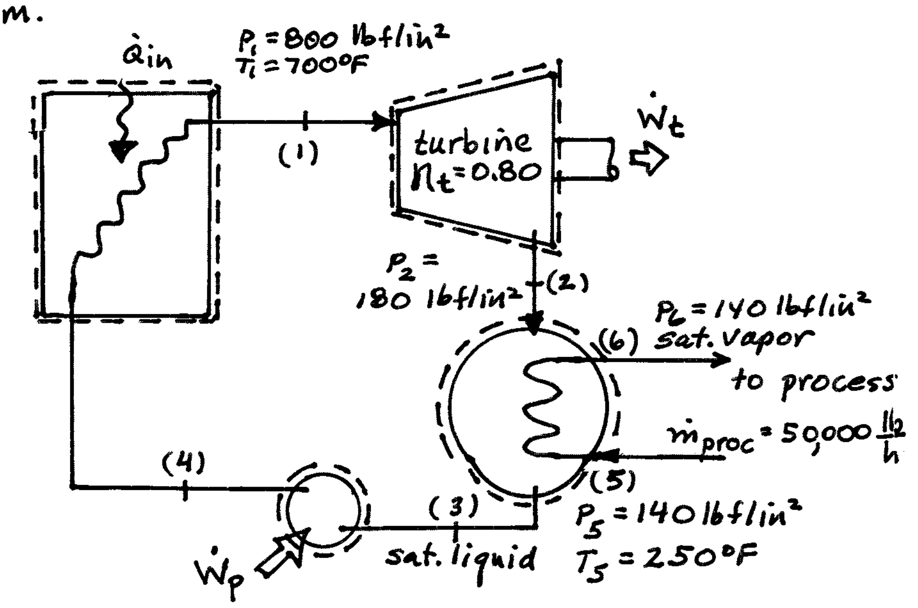
PROBLEM 8.76

KNOWN: Water is the working fluid in a Rankine cycle used for cogeneration. The cycle produces power, and energy rejected from the condensing steam is transferred to a separate process stream. Data are known at various locations.

FIND: Determine the mass flow rate for the Rankine cycle working fluid, and devise and evaluate an exergetic efficiency for the overall cogeneration system.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The turbine, pump, and condenser operate adiabatically. (3) For the turbine, $\eta_t = 0.80$, and the pump is assumed to operate isentropically. (4) Kinetic and potential energy effects are negligible. (5) Saturated liquid exits the condenser at the condenser pressure. (6) $T_0 = 530$°R, $p_0 = 14.7$ lbf/in².

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 800$ lbf/in², $T_1 = 700$°F $\Rightarrow h_1 = 1338.0$ Btu/lb, $s_1 = 1.5471$ Btu/lb·°R

State 2: First, $P_2 = 180$ lbf/in², $s_{2s} = s_1 \Rightarrow h_{2s} = 1191.1$ Btu/lb. With the turbine efficiency

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s}) = 1220.5 \text{ Btu/lb}$$

and, from Table A-4E; $s_2 \approx 1.5818$ Btu/lb·°R

State 3: $P_3 = 180$ lbf/in², sat. liquid $\Rightarrow h_3 = 346.3$ Btu/lb, $s_3 = 0.5329 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

State 4: $h_4 \approx h_3 + v_4(p_4 - p_3)$

$$= 346.3 \frac{\text{Btu}}{\text{lb}} + \left(0.01827 \frac{\text{ft}^3}{\text{lb}}\right)(800-180)\frac{\text{lbf}}{\text{in}^2}\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$

$= 346.3 + 2.10 = 348.4$ Btu/lb, $s_4 = s_3 = 0.5329$ Btu/lb·°R

State 5: $P_5 = 140$ lbf/in², $T_5 = 250$°F $\Rightarrow$ compressed liquid. Thus

$$h_5 \approx h_f(250) + v_f(250)\left[P - P_{sat@250°F}\right]$$

$$= 218.6 + (0.01700)(140 - 29.82)\left|\frac{144}{778}\right| = 218.9 \text{ Btu/lb}$$

and $s_5 \approx s_f(250) = 0.3677$ Btu/lb·°R

PROBLEM 8.76 (Cont'd)

State 6: $p_6 = 140$ lbf/in², sat. vapor $\Rightarrow h_6 = 1193.8$ Btu/lb, $s_6 = 1.5761$ Btu/lb·°R

To find the cycle mass flow rate, use an energy balance for the condenser

$$0 = \dot{m}_{cycle}(h_2 - h_3) + \dot{m}_{proc}(h_5 - h_6)$$

or

$$\dot{m}_{cycle} = \dot{m}_{proc}\left(\frac{h_5 - h_6}{h_3 - h_2}\right)$$

$$= (50{,}000 \text{ lb/h})\left(\frac{218.9 - 1193.8}{346.3 - 1220.5}\right) = 5.577 \times 10^4 \text{ lb/h} \leftarrow \dot{m}_{cycle}$$

For the cogeneration system, the cycle net work and the exergy transferred to the process stream are the outputs, and the exergy increase of the working fluid passing through the steam generator is the input. The difference between the input and output represents exergy destroyed due to irreversibilities. Thus, a reasonable exergetic efficiency is

$$\epsilon = \frac{\text{net rate of exergy output}}{\text{rate of exergy input}}$$

$$= \frac{\dot{W}_{cycle} + \dot{m}_{proc}(e_{f6} - e_{f5})}{\dot{m}_{cycle}(e_{f1} - e_{f4})} \leftarrow \epsilon \text{ (definition)}$$

Evaluating the various quantities in this expression

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}_{cycle}[(h_1 - h_2) - (h_4 - h_3)]$$

$$= (5.577 \times 10^4)[(1338.0 - 1220.5) - (2.10)] = 6.436 \times 10^6 \text{ Btu/h}$$

$$e_{f6} - e_{f5} = (h_6 - h_5) - T_0(s_6 - s_5)$$

$$= (1193.8 - 218.9) - (530)(1.5761 - 0.3677) = 334.4 \text{ Btu/lb}$$

$$e_{f1} - e_{f4} = (h_1 - h_4) - T_0(s_1 - s_4)$$

$$= (1338.0 - 348.4) - (530)(1.5471 - 0.5329) = 452.1 \text{ Btu/lb}$$

Thus

$$\epsilon = \frac{6.436 \times 10^6 \text{ Btu/h} + (5.577 \times 10^4 \text{ lb/h})(334.4 \text{ Btu/lb})}{(5.577 \times 10^4 \text{ lb/h})(452.1 \text{ Btu/lb})}$$

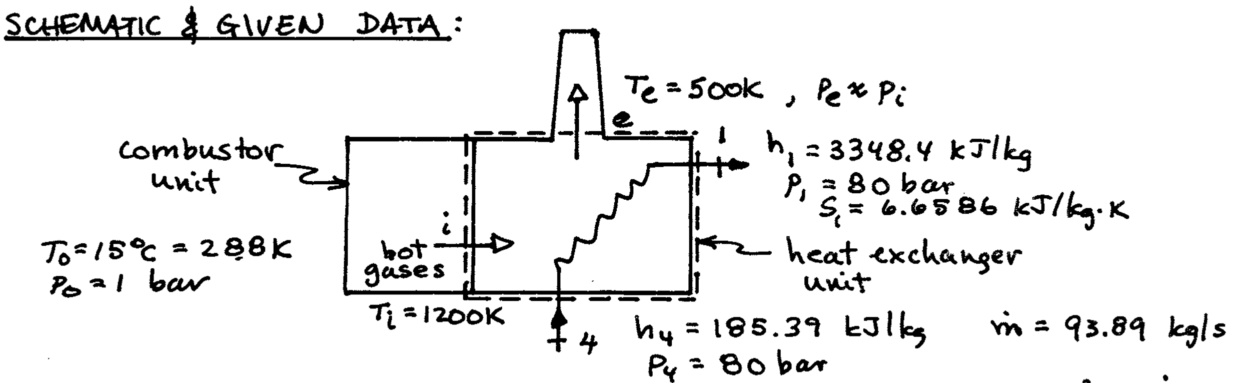
$$= \frac{6.436 \times 10^6 + 1.865 \times 10^6}{2.521 \times 10^7} = 0.329\ (32.9\%) \leftarrow \epsilon \text{ (value)}$$

PROBLEM 8.77

KNOWN: Hot combustion gases enter the heat exchanger unit of the steam generator of Problem 8.17. The inlet and exit temperatures of the gas stream are provided. Other data are from the solution to Problem 8.17.

FIND: Determine (a) the net rate exergy is carried in by the gas stream, (b) the net rate exergy is carried out by the water stream, (c) the rate of exergy destruction, and (d) the exergetic efficiency given by Eq. 7.45

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) For the steady-state control volume shown, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) Kinetic and potential energy effects are negligible. (3) The hot combustion gases are modeled as air as an ideal gas. (4) Each stream experiences negligible pressure drop. (5) $T_0 = 288K$, $p_0 = 0.1$ MPa $= 1$ bar

ANALYSIS: From steady-state mass balances on the water and air streams, respectively, $\dot{m}_4 = \dot{m}_1 \equiv \dot{m}$ and $\dot{m}_i = \dot{m}_e \equiv \dot{m}_a$. Thus, the energy rate balance can be written, with the above assumptions, as

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}(h_4 - h_1) + \dot{m}_a(h_i - h_e)$$

or

$$\dot{m}_a = \dot{m}\left(\frac{h_1 - h_4}{h_i - h_e}\right)$$

From Table A-22, $h_i = 1277.79$ kJ/kg and $h_e = 503.02$ kJ/kg. Thus

$$\dot{m}_a = \left(93.89\ \frac{kg}{s}\right)\left(\frac{3348.4 - 185.39}{1277.79 - 503.02}\right) = 383.3\ kg/s$$

(a) The net rate exergy is carried in by the gas stream is

$$\dot{m}_a(e_{fi} - e_{fe}) = \dot{m}_a[(h_i - h_e) - T_0(s_i - s_e)]$$

Since $p_i = p_e$, $s_i - s_e = s_i^\circ - s_e^\circ - R\,\cancelto{0}{\ln p_e/p_i} = s_i^\circ - s_e^\circ$. With values from Table A-22

$$\dot{m}_a(e_{fi} - e_{fe}) = \left(383.3\frac{kg}{s}\right)\left[(1277.79 - 503.2)\frac{kJ}{kg} - (288K)(3.17888 - 2.21952)\frac{kJ}{kg\cdot K}\right]\left|\frac{1\ kW}{1\ kJ/s}\right| =$$

$$= 191 \times 10^3\ kW \longleftarrow$$ net rate exergy carried in

(b) The net rate exergy is carried out by the water stream is

$$\dot{m}(e_{f1} - e_{f4}) = \dot{m}[h_1 - h_4 - T_0(s_1 - s_4)]$$

Double interpolation in Table A-5 at $p_4 = 80$ bar, $h_4 = 185.39$ kJ/kg gives $s_4 = 0.6018$ kJ/kg·K. Thus

PROBLEM 8.77 (Contd)

$$\dot{m}(e_{f1}-e_{f4}) = \left(93.89\,\frac{kg}{s}\right)\left[3348.4 - 185.39 - 288(6.6586 - 0.6018)\right]$$

$$= 133.2 \times 10^{3}\ kW \quad \leftarrow \text{net rate exergy is carried out}$$

(c) The exergy rate balance reduces to give

① $$\dot{E}_d = \dot{m}_a(e_{fi} - e_{fe}) + \dot{m}(e_{f4} - e_{f1})$$

$$= 191\times10^{3}\ kW - 133.2\times10^{3}\ kW = 57.8\times10^{3}\ kW \quad \leftarrow \text{rate of exergy destruction}$$

(d) Eq. 7.45 gives

$$\epsilon = \frac{\dot{m}(e_{f1} - e_{f4})}{\dot{m}_a(e_{fi} - e_{fe})} = \frac{133.2\times10^{3}}{191\times10^{3}} = 0.697 \quad (69.7\%) \quad \leftarrow$$

---

1. Exergy destruction in this case is due to heat transfer from the higher-temperature combustion gases to the vaporizing water.

PROBLEM 8.78

KNOWN: The vapor power cycle of Problem 8.17 is to be analyzed from the perspective of exergy accounting.

FIND: Determine the rate of input to the working fluid passing through the steam generator and account for all outputs, losses, and destructions of this exergy.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.17.

ASSUMPTIONS: Same as Problem 8.17. Also, $T_0 = 15°C = 288\ K$ and $p_0 = 0.1\ MPa$.

ANALYSIS: Specific enthalpies and entropies at the principal states are

State 1: $h_1 = 3348.4\ kJ/kg$, $s_1 = 6.6586\ kJ/kg\cdot K$

State 2: $h_2 = 2272.7\ kJ/kg$, $s_2 = 7.2620\ kJ/kg\cdot K$ $(x_2 = 0.8734)$

State 3: $h_3 = 173.88\ kJ/kg$, $s_3 = 0.5926\ kJ/kg\cdot K$

State 4: $h_4 = 185.39\ kJ/kg$, $s_4 = 0.6018\ kJ/kg\cdot K$ (double interpolation in Table A-5 is required)

Also, from Problem 8.17: $\dot{m} = 3.38\times10^5\ kg/h$ and $\dot{m}_{cw} = 8.48\times10^6\ kg/h$.

Input: For the working fluid passing through the steam generator

$$\dot{m}(e_{f1} - e_{f4}) = \dot{m}\left[(h_1 - h_4) - T_0(s_1 - s_4)\right]$$

$$= \left(3.38\times10^5\ \frac{kg}{h}\right)\left[(3348.4 - 185.39) - 288(6.6586 - 0.6018)\right]\frac{kJ}{kg}\left|\frac{1\ h}{3600\ s}\right|\left|\frac{1\ MW}{10^3\ kJ/s}\right|$$

$$= 133.2\ MW$$

input

Outputs

Net power: $\dot{W}_{cycle} = 100\ MW$

Condenser:

$$\dot{m}_{cw}(e_{f,out} - e_{f,in})_{cw} = \dot{m}_{cw}\left[(h_{cw,out} - h_{cw,in}) - T_0(s_{cw,out} - s_{cw,in})\right]$$

Approximating $s_{cw} \approx s_f(T_{cw})$

$$\dot{m}_{cw}(e_{f,out} - e_{f,in})_{cw}$$

$$= (8.48\times10^6\left|\frac{1}{3600}\right|\left|\frac{1}{10^3}\right|\left[(146.68 - 62.99) - 288(0.5053 - .2245)\right]$$

$$= 6.64\ MW$$

Destructions

For the turbine:

$$\dot{E}_{d,turb} = T_0\sigma_{turb} = T_0\dot{m}(s_2 - s_1)$$

$$= (288)(3.38\times10^5)(7.2620 - 6.6586)\left|\frac{1}{3600}\right|\left|\frac{1}{10^3}\right| = 16.32\ MW$$

and the pump:

$$\dot{E}_{d,pump} = T_0(\dot{m})(s_4 - s_3) = 0.25\ MW$$

PROBLEM 8.78 (Cont'd)

Finally, for the condenser:

$$0 = \sum_j \cancelto{0}{\left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}(s_2 - s_3) + \dot{m}_{cw}(s_{cw,in} - s_{cw,out}) + \dot{\sigma}_{cond}$$

With $\dot{E}_{d,cond} = T_0\, \dot{\sigma}_{cond}$

$$\dot{E}_{d,cond} = T_0\left[\dot{m}(s_3 - s_2) + \dot{m}_{cw}(s_{cw,out} - s_{cw,in})\right]$$

$$= \frac{288\left[(3.38\times10^5)(.5926 - 7.2620) + (8.48\times10^6)(.5053 - .2245)\right]}{|3600|\;|10^3|}$$

$$= 10.16 \text{ MW}$$

Checking

$$\left.\begin{array}{ll} \text{Outputs:} & 100\\ & \underline{6.64}\\ & 106.64\\ \text{Destructions:} & 16.32\\ & .25\\ & \underline{10.16}\\ & 133.37\ \text{MW}\\ \\ \text{Input} & 133.2\ \text{MW} \end{array}\right\} \longleftarrow \text{exergy summary}$$

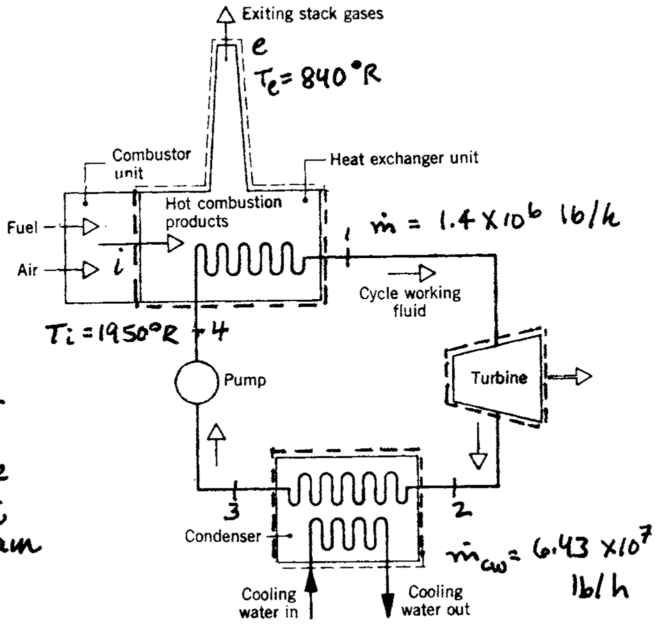
PROBLEM 8.79

KNOWN: Hot combustion gases enter the heat exchanger unit of the steam generator of Problem 8.19. The inlet and exit temperatures of the gas stream are known. Other data are from Problem 8.19.

FIND: Determine the exergy destruction rates in (a) the heat exchanger unit, (b) the turbine and pump, and (c) the condenser. Also, calculate the net rate exergy is carried away by the cooling water passing through the condenser.

SCHEMATIC & GIVEN DATA:

$T_o = 520°R$
$P_o = 14.7\ lbf/in^2$



ASSUMPTIONS: (1) Same as Problem 8.19. (2) $T_o = 520°R$, $P_o = 14.7\ lbf/in^2$ (3) The combustion gases are modeled as air as an ideal gas. (4) The gas stream, the working fluid passing through the heat exchanger unit and the condenser, and the condenser cooling water stream each experience no pressure drop.

ANALYSIS: (a) For the control volume surrounding the heat exchanger unit, Steady-state mass balances for the water and air stream reduce, respectively, to $\dot{m}_4 = \dot{m}_1 \equiv \dot{m}$ and $\dot{m}_i = \dot{m}_e \equiv \dot{m}_a$. Thus, the energy rate balance is

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}(h_4 - h_1) + \dot{m}_a(h_i - h_e)$$

or

$$\dot{m}_a = \frac{\dot{m}(h_1 - h_4)}{(h_i - h_e)}$$

From Table A-22E, $h_i = 490.88$ Btu/lb and $h_e = 201.56$ Btu/lb. Thus

$$\dot{m}_a = \frac{(1.4 \times 10^6\ lb/h)(1547.7\ Btu/lb - 75.17\ Btu/lb)}{(490.88 - 201.56)\ Btu/lb} = 7.13 \times 10^6\ lb/h$$

The rate of exergy destruction can be found from $\dot{E}_d = T_o \dot{\sigma}_{cv}$. The entropy rate balance for the control volume is

$$0 = \cancelto{0}{\sum_j \left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}(s_4 - s_1) + \dot{m}_a(s_i - s_e) + \dot{\sigma}_{cv}$$

Thus

$$\dot{E}_d = T_o\left[\dot{m}(s_1 - s_4) + \dot{m}_a(s_e - s_i)\right]$$

Since $P_i = P_e$, $s_e - s_i = s_e^\circ - s_i^\circ - \cancelto{0}{R\ln(P_e/P_i)} = s_e^\circ - s_i^\circ$. With values from Table A-22E, $s_e^\circ = 0.70747$ and $s_i^\circ = 0.92505$. Further, interpolating in

PROBLEM 8.79 (Contd.)

Table A-5E, $s_4 \approx 0.13364$ Btu/lb·°R, and with $s_1 = 1.6315$ Btu/lb·°R from Problem 8.19,

$$\dot{E}_d = (520\,°R)\left[(1.4 \times 10^6 \tfrac{lb}{h})(1.6315 - 0.13364)\ \text{Btu/lb} + (7.13 \times 10^6\ \text{lb/h})(0.70747 - 0.92505)\right]$$

$$= 2.84 \times 10^8\ \text{Btu/h} \longleftarrow \dot{E}_d\ \text{(heat exchange unit)}$$

(b) For the turbine, with $s_1 = 1.6315$ Btu/lb·°R and $s_2 = 1.7676$ Btu/lb·°R based on data from Problem 8.19

$$\dot{E}_d = T_0 \dot{m}(s_2 - s_1)$$

$$= (520)(1.4 \times 10^6)(1.7676 - 1.6315) = 9.91 \times 10^7\ \text{Btu/h} \longleftarrow \dot{E}_d\ \text{(turbine)}$$

Further, with $s_3 = 0.1327$ Btu/lb·°R (sat. liq. at 1 lbf/in.²), the rate of exergy destruction for the pump is

$$\dot{E}_d = T_0 \dot{m}(s_4 - s_3)$$

$$= (520)(1.4 \times 10^6)(0.13364 - 0.1327) = 6.84 \times 10^5\ \text{Btu/h} \longleftarrow \dot{E}_d\ \text{(pump)}$$

(c) Similarly, for the condenser

$$\dot{E}_d = T_0\left[\dot{m}(s_3 - s_2) + \dot{m}_{cw}(s_{cw,out} - s_{cw,in})\right]$$

$$= (520)\left[(1.4 \times 10^6)(0.1327 - 1.7676) + (6.43 \times 10^7)(.09332 - .05555)\right]$$

$$= 7.27 \times 10^7\ \text{Btu/h} \text{ ——— } \dot{E}_d\ \text{(condenser)}$$

where $s_{cw} \approx s_f(T)$.

Finally, the net rate exergy is carried out by the cooling water passing through the condenser is

$$\dot{m}_{cw}(e_{f,cw\,out} - e_{f,cw\,in}) = \dot{m}_{cw}\left[(h_{cw,out} - h_{cw,in}) - T_0(s_{cw,out} - s_{cw,in})\right]$$

$$= (6.43 \times 10^7)\left[(48.09 - 28.08) - (520)(.09332 - .05555)\right]$$

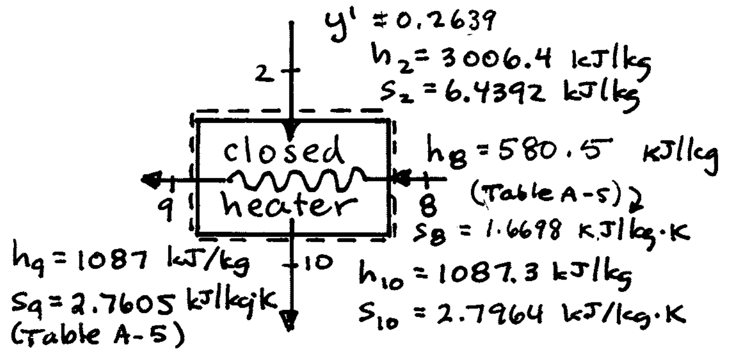
$$= 2.38 \times 10^7\ \text{Btu/h} \longleftarrow \text{net rate exergy is carried out by cooling water}$$

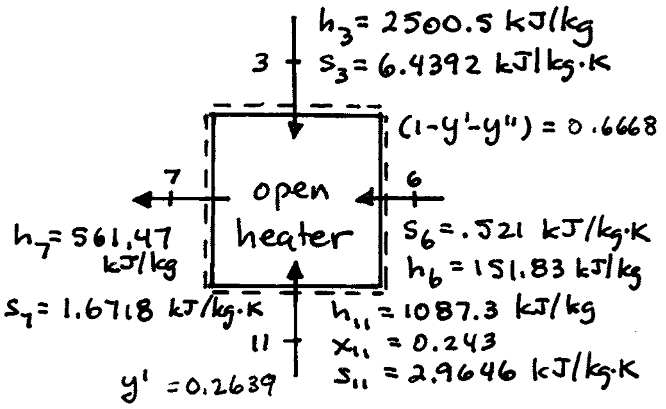
PROBLEM 8.80

KNOWN: Feedwater heaters operate as in the regenerative vapor power cycle of Problem 8.61.

FIND: Determine the exergy destruction rates of the feedwater heaters per unit mass of steam entering the first turbine stage, and express each as a fraction of the flow exergy increase of the working fluid passing through the steam generator. $y'' = 0.0693$

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See solution to Problem 8.61. Let $T_0 = 15°C = 288K$

ANALYSIS: For the closed heater, $\dot{E}_d/\dot{m}_1 = T_0\dot{\sigma}/\dot{m}_1$. Thus, using an entropy rate balance

$$0 = \sum_j \cancelto{0}{\left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}_1\left[y'(s_2 - s_{10}) + (s_8 - s_9) + \dot{\sigma}/\dot{m}_1\right]$$

Thus $\dot{E}_d/\dot{m}_1 = T_0\left[y'(s_{10} - s_2) + (s_9 - s_8)\right]$

$$= (288\,\text{K})\left[(0.2639)(2.7964 - 6.4392) + (2.7605 - 1.6698)\right]\,\text{kJ/kg·K}$$

$$= 37.3 \text{ kJ/kg} \longleftarrow \text{(closed heater)}$$

Similarly for the open heater

$$0 = \frac{1}{\dot{m}_1}\sum_j \cancelto{0}{\left(\frac{\dot{Q}}{T}\right)_j} + y''s_3 + y's_{11} + (1-y'-y'')s_6 - s_7 + \dot{\sigma}/\dot{m}_1$$

or $\dot{E}_d/\dot{m}_1 = T_0\left[s_7 - y''s_3 - y's_{11} - (1-y'-y'')s_6\right]$

$$= (288)\left[(1.6718) - (0.0693)(6.4392) - (0.2639)(2.9646) - (0.6668)(0.5210)\right]$$

$$= 27.6 \text{ kJ/kg} \longleftarrow \text{(open heater)}$$

For the working fluid passing through the steam generator

$$(e_{f1} - e_{f9}) = (h_1 - h_9) - T_0(s_1 - s_9)$$

$$= (3444.4 - 1087) - (288)(6.4392 - 2.7605) = 1297.9 \text{ kJ/kg}$$

Thus

$$\frac{(\dot{E}_d/\dot{m})_{\text{CLOSED HTR}}}{(e_{f1} - e_{f9})} = \frac{37.3}{1297.9} = 0.029 \quad (2.9\%) \longleftarrow \text{(closed heater)}$$

$$\frac{(\dot{E}_d/\dot{m})_{\text{OPEN HTR}}}{(e_{f1} - e_{f9})} = \frac{27.6}{1297.9} = 0.021 \quad (2.1\%) \longleftarrow \text{(open heater)}$$

PROBLEM 8.81

KNOWN: The regenerative vapor power cycle of Problem 8.67 is analyzed on the basis of exergy accounting.

FIND: Determine the rate of exergy input to the working fluid passing through the steam generator and account for the disposition of this exergy.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.67

ASSUMPTIONS: Same as in Problem 8.67. Also, let $T_0 = 15°C = 288\,K$ and $p_0 = 1$ bar.

ANALYSIS: Each principal state is fixed in the solution to problem 8.67. The specific enthalpies and entropies are:

State	h(kJ/kg)	s (kJ/kg·K)	
1	3424.6	6.1858	
2	3082.6	6.2781	
3	3545.3	6.9072	
4	3025.7	7.0771	
5	2695.6	7.2284	
6	2296.9	7.4559	
7	151.53	.5210	
8	151.67	.5210	
9	467.11	1.4336	
10	500.64	1.4336	
11	781.7	2.1396	(s_f @ 180°C)
12	762.81	2.1387	
13	762.81	2.2025	
14	1303.8	3.2064	(s_f @ 295°C)
15	1316.6	3.2068	
16	1316.6	3.3609	

From the solution to Problem 8.67 ; $\dot{m}_1 = 1.588 \times 10^6$ kg/h = 411.1 kg/s, $y' = 0.2956$, $y'' = 0.0519$, and $y''' = 0.0405$.

INPUTS

Boiler/superheater:

$$\dot{m}_1(e_{f1} - e_{f14}) = \dot{m}_1\left[(h_1 - h_{14}) - T_0(s_1 - s_{14})\right]$$

$$= \left(411.1\ \frac{kg}{s}\right)\left[(3424.6 - 1303.8) - 288(6.1858 - 3.2064)\right]\frac{kJ}{kg}$$

$$= 5.191 \times 10^5\ \frac{kJ}{s} = 519.1\ MW$$

Reheater:

$$\dot{m}_1(1-y')(e_{f3} - e_{f2}) = \dot{m}_1(1-y')\left[(h_3 - h_2) - T_0(s_3 - s_2)\right]$$

$$= 81.5\ MW$$

Total : 600.6 MW ← Input

OUTPUT

$$\text{net output of cycle} = \dot{m}_1\left[\dot{W}_t/\dot{m}_1 - \dot{W}_p/\dot{m}_1\right]$$

$$= (411.1)\left[1167.4 - 33.62\right]\frac{kJ}{s}\left(\frac{1\ MW}{10^3\ kJ/s}\right)$$

$$= 466.0\ MW$$ ← Output

PROBLEM 8.81 (Contd.)

LOSS

condenser loss $= \dot{m}_1(1-y'-y''-y''')(e_{f6}-e_{f7})$

$= \dot{m}_1(1-y'-y''-y''')\left[(h_6-h_7) - T_0(s_6-s_7)\right]$

$= 37.3\ \text{MW} \longleftarrow$ loss

DESTRUCTIONS

turbine:

$\dot{E}_d = \dot{m}_1 T_0(\dot{\sigma}/\dot{m}_1)$

$= \dot{m}_1 T_0\left[(s_2-s_1) + (1-y')(s_4-s_3) + (1-y'-y'')(s_5-s_4) + (1-y'-y''-y''')(s_6-s_5)\right]$

$= 53.27\ \text{MW}$

80-bar heater:

$\dot{E}_d = \dot{m}_1 T_0(\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0\left[y'(s_{15}-s_2) + (s_{14}-s_{11})\right]$

$= 18.82\ \text{MW}$

10-bar heater:

$\dot{E}_d = \dot{m}_1 T_0\left[s_{11} - s_{10} + (y'+y'')s_{12} - y's_{16} - y''s_v\right]$

$= 10.47\ \text{MW}$

Open heater:

$\dot{E}_d = \dot{m}_1 T_0\left[s_9 - y'''s_5 - (y'+y'')s_{13} - (1-y'-y''-y''')s_8\right]$

$= 6.7\ \text{MW}$

Traps:

$\dot{E}_d = \dot{m}_1 T_0\left[(s_{16}-s_{15})y'\right] = 5.39\ \text{MW}$

$\dot{E}_d = \dot{m}_1 T_0\left[(y'+y'')(s_{13}-s_{12})\right] = 2.62\ \text{MW}$

Total: $97.27\ \text{MW} \longleftarrow$ destroyed

SUMMARY

Input: 600.6 MW

Output: 466.0
Loss: 37.3
Destroyed: 97.27
600.57 MW

PROBLEM 8.82

KNOWN: The regenerative power cycle of Problem 8.68 is analyzed on the basis of exergy accounting.

FIND: Determine the rate of exergy input to the working fluid passing through the steam generator and account for its disposition.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.68.

ASSUMPTIONS: Same a Problem 8.68. Also, let $T_0 = 520°R$, $p_0 = 1\ lbf/in^2$.

ANALYSIS: Each principal state is fixed in the solution to Problem 8.68. The specific enthalpies and entropies are:

State	h (Btu/lb)	s (Btu/lb·°R)
1	1542.5	1.6159
2	1404.2	1.6315
3	1272.0	1.6507
4	1232.1	1.6569
5	1532.1	1.9204
6	1341.5	1.9455
7	1118.0	1.9988
8	69.74	.1327
9	69.80	.1327
10	196.26	.3358
11	203.28	.3358
12	339.10	.5159
13	472.22	.6674
14	471.7	.6723
15	471.7	.6854
16	336.2	.5208
17	336.2	.5221

From the solution to Problem 8.68; $\dot{m}_1 = 5.286\times10^6\ lb/h$, $y' = .1428$, $y'' = .1245$, $y''' = .0434$.

INPUTS

Boiler/superheater:

$$\dot{m}_1(e_{f1}-e_{f13}) = \dot{m}_1[(h_1-h_{13})-T_0(s_1-s_{13})]$$

$$=\left(5.286\times10^6\ \frac{lb}{h}\right)[(1542.5-472.22)-(520)(1.6159-.6674)]\frac{Btu}{lb}$$

$$= 3.05\times10^9\ Btu/h$$

Reheater:

$$(1-y'-y'')\dot{m}_1(e_{f5}-e_{f4}) = (1-y'-y'')\dot{m}_1[(h_5-h_4)-T_0(s_5-s_4)]$$

$$= 6.312\times10^8\ Btu/h$$

Total: 3.681×10^9 Btu/h $\longleftarrow$ Input

OUTPUT: net output of cycle $= \dot{W}_{cycle} = 3\times10^9$ Btu/h $\longleftarrow$ Output

Loss

Condenser loss

$$= \dot{m}_1(1-y'-y''-y''')(e_{f7}-e_{f8})$$

$$= \dot{m}_1(1-y'-y''-y''')[(h_7-h_8)-T_0(s_7-s_8)]$$

$$= 2.838\times10^8\ Btu/h$$ $\longleftarrow$ Loss

PROBLEM 8.82 (Contd.)

DESTRUCTIONS

Turbine:

$\dot{E}_d = T_0 \dot{m}_1 (\dot{\sigma}_{turb}/\dot{m}_1)$

$= T_0 \dot{m}_1 [(1-y'-y''-y''')s_7 + y'''s_6 - (1-y'-y'')(s_3-s_4) + y''s_3 + y's_2 - s_1]$

$= 2.521 \times 10^8$ Btu/h

600 lbf/in.² heater:

$\dot{E}_d = T_0 \dot{m}_1 [y'(s_{14}-s_2) + (s_{13}-s_{12})] = 3.99 \times 10^7$ Btu/h

160 lbf/in.² heater:

$\dot{E}_d = T_0 \dot{m}_1 [(y'+y'')s_{16} + s_{12} - s_{11} - y''s_3 - y's_{15}] = 4.377 \times 10^7$ Btu/h

Open heater:

$\dot{E}_d \quad T_0 \dot{m}_1 [s_{10} - y'''s_6 - (y'+y'')s_{17} - (1-y'-y''-y''')s_9]$

$= 5.59 \times 10^7$ Btu/h

Traps:

$\dot{E}_d = T_0 \dot{m}_1 [y'(s_{15}-s_{14}) + (y'+y'')(s_{17}-s_{16})] = 6.097 \times 10^6$ Btu/h

Total: 3.978×10^8 Btu/h ← destroyed

Finally, to summarize:

Input: 3.681×10^9 Btu/h

output: 3.00×10^9
Loss: 2.838×10^8
Destroyed: 3.978×10^8
3.682×10^8 Btu/h

PROBLEM 8.83

① KNOWN: The regenerative vapor power cycle of Problem 8.71 is analyzed from the perspective of exergy accounting.

FIND: Determine the rate of exergy input to the working fluid passing through the steam generator and perform calculations to account for this exergy.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.71.

ASSUMPTIONS: Same as in Problem 8.71. Also, let $T_0 = 520°R$ and $p_0 = 14.7\ lbf/in^2$.

ANALYSIS: Each principal state is fixed in the solution to Problem 8.71. The specific enthalpies and entropies are:

State	h (Btu/lb)	s (Btu/lb·°R)	
1	1388.5	1.5665	
2	1186.3	1.6015	$x_2 = 0.9983$
3	1187.8	1.6034	
4	1294.2	1.7234	
5	1002.5	1.7941	$x_5 = 0.9003$
6	69.74	0.1327	
7	70.04	0.1327	
8	542.4	0.6471	
9	542.4	0.7839	$x_9 = 0.2742$
10	298.6	0.4744	
11	123.1	0.22315	s_f @ 155°F
12	125.83	0.22315	

From the solution to Problem 8.71, $\dot{m}_1 = 1.148 \times 10^7$ lb/h, $y' = 0.1115$, $y'' = 0.8866$.

INPUT Boiler/superheater:

$$\dot{m}_1 (e_{f1} - e_{f12}) = \dot{m}_1 [(h_1 - h_{12}) - T_0(s_1 - s_{12})]$$

$$= \left(1.148 \times 10^7 \frac{lb}{h}\right)[(1388.5 - 125.83) - (520)(1.5665 - 0.22315)] \frac{Btu}{lb}$$

$$= 6.476 \times 10^9 \text{ Btu/h}$$ ← Input

OUTPUT

$$\begin{pmatrix}\text{net power}\\ \text{output of}\\ \text{cycle}\end{pmatrix} = 5 \times 10^9 \text{ Btu/h}$$ ← Output

LOSS

$$\begin{pmatrix}\text{condenser}\\ \text{loss}\end{pmatrix} = \dot{m}_1 y'' (e_{f5} - e_{f6})$$

$$= \dot{m}_1 y'' [(h_5 - h_6) - T_0(s_5 - s_6)]$$

$$= (1.148 \times 10^7)(0.8866)[(1002.5 - 69.74) - (520)(1.7941 - 0.1327)]$$

$$= 7.006 \times 10^8 \text{ Btu/h}$$ ← loss.

PROBLEM 8.83 (Cont'd)

DESTRUCTIONS

Turbine stages:

$$\dot{E}_d = \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 [(1-y')(s_2 - s_1) + y''(s_5 - s_4)]$$
$$= 5.598 \times 10^8 \text{ Btu/h}$$

Heat Exchanger:

$$\dot{E}_d = \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 [y''(s_4 - s_3) + y'(s_8 - s_1)]$$
$$= 2.316 \times 10^7 \text{ Btu/h}$$

Open heater:

$$\dot{E}_d = \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 [s_{11} - y'' s_7 - y' s_9 - (1 - y' - y'') s_{10}]$$
$$= 1.026 \times 10^8 \text{ Btu/h}$$

Trap:

$$\dot{E}_d = \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 [y'(s_9 - s_8)] = 9.106 \times 10^7 \text{ Btu/h}$$

Flash Chamber: $\dot{E}_d = 0$

Total: $7.7662 \times 10^8 \frac{\text{Btu}}{\text{h}}$ ← destroyed

Finally, to summarize:

Input: 6.476×10^9 Btu/h

Output: 5×10^9 (77.2%)
Loss: 7.006×10^8 (10.8%)
Destroyed: 7.7662×10^8 (12.0%)
6.477×10^9 Btu/h

PROBLEM 8.84

KNOWN: The regenerative vapor power cycle of Problem 8.45 is analyzed from the perspective of exergy accounting.

FIND: Determine the rate of exergy transfer to the working fluid passing through the steam generator and perform calculations to account for this exergy.

SCHEMATIC & GIVEN DATA: See solution to Problem 8.45.

ASSUMPTIONS: Same as in Probem 8.45. Also, let $T_0 = 15°\text{C} = 288\,\text{K}$ and $p_0 = 0.1\,\text{MPa}$.

ANALYSIS: From Problem 8.45, the principal states are fixed. Specific enthalpies and entropies are:

State	h(kJ/kg)	s(kJ/kg·K)
1	3348.4	6.6586
2	2741.8	6.6586
3	2082.9	6.6586
4	173.88	0.5926
5	181.94	0.5926
6	702	1.9805
7	697.22	1.9922
8	697.22	2.2557

Interpolating in Table A-5

$x_8 = 0.2178$

From the solution to Problem 8.45, $y = 0.2544$, $\dot{m}_1 = 91.76$ kg/s
$\dot{m}_{cw} = 1706.7$ kg/s, $\dot{W}_{cycle} = 100$ MW

INPUT Steam generator:

$$\dot{m}_1(e_{f1} - e_{f6}) = \dot{m}_1\left[(h_1 - h_6) - T_0(s_1 - s_6)\right]$$

$$= \left(91.76\,\frac{\text{kg}}{\text{s}}\right)\left[(3348.4 - 702) - 288(6.6586 - 1.9805)\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\,\text{MW}}{10^3\,\text{kJ/s}}\right|$$

$$= 119.2\ \text{MW} \longleftarrow \text{Input}$$

OUTPUT

$$\begin{pmatrix}\text{Net power output}\\ \text{of cycle}\end{pmatrix} = 100\ \text{MW} \longleftarrow \text{Output}$$

LOSS

$$\begin{pmatrix}\text{Condenser}\\ \text{loss}\end{pmatrix} = \dot{m}_1\left(e_{f,\,out} - e_{f,\,in}\right)_{cw}$$

$$= \dot{m}_1\left[(h_{cw,out} - h_{cw,in}) - T_0(s_{cw,out} - s_{cw,in})\right]$$

Using $h_{cw} = h_f(T_{cw})$ and $s_{cw} = s_f(T_{cw})$

$$\begin{pmatrix}\text{Condenser}\\ \text{loss}\end{pmatrix} = (91.76)\left[(146.68 - 62.99) - 288(.5053 - .2245)\right]\left|\frac{1}{10^3}\right|$$

$$= 0.259\ \text{MW} \longleftarrow \text{Loss}$$

PROBLEM 8.84 (Cont'd)

DESTRUCTIONS

Closed heater:

$$\dot{E}_d = T_0 \dot{m}_1 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 \left[y(s_7 - s_2) + (s_6 - s_5) \right]$$

$$= 5.3 \text{ MW}$$

Trap:

$$\dot{E}_d = \dot{m}_1 T_0 (\dot{\sigma}/\dot{m}_1) = \dot{m}_1 T_0 \left[y(s_8 - s_7) \right]$$

$$= 6.96 \text{ MW}$$

Condenser:

An entropy balance for a control volume enclosing the condenser gives

$$0 = \sum_j \cancelto{0}{\left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}_1 \left[(1-y)s_3 + y s_8 - s_4 \right] + \dot{m}_{cw}(s_{cw,in} - s_{cw,out}) + \dot{\sigma}$$

Thus

$$\dot{E}_d = T_0 \dot{\sigma} = \left\{ \dot{m}_1 \left[s_4 - (1-y)s_3 - y s_8 \right] + \dot{m}_{cw}(s_{cw,out} - s_{cw,in}) \right\} T_0$$

$$= 7.3 \text{ MW}$$

Total: 19.56 MW ← destroyed

SUMMARY

Input: 119.2 MW

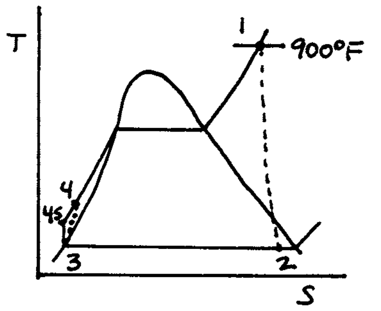
Output: 100
Loss: 0.26
Destroyed: 19.56
119.56 MW

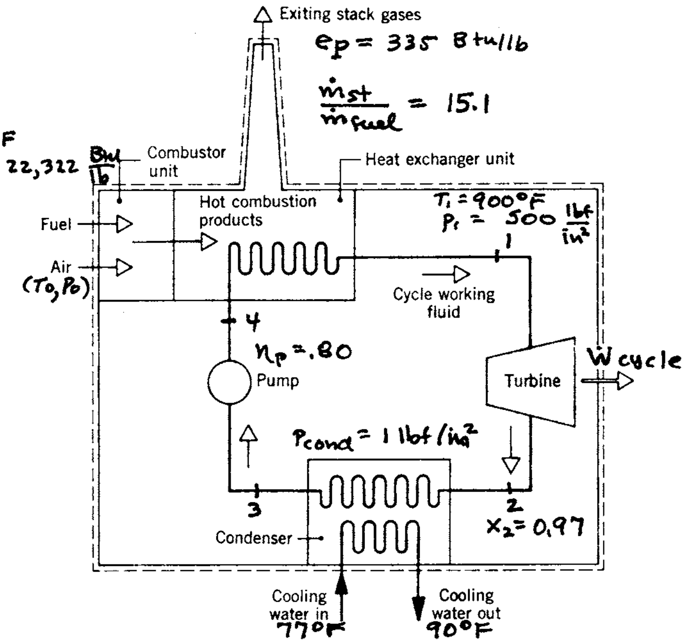
PROBLEM 8.85

KNOWN: Water is the working fluid in a simple vapor power plant. Data are known at various locations. For the steam generator, specific exergy values are provided for the fuel, air, and stack gases.

FIND: Determine, as percentages of the exergy entering with the fuel, (a) the exergy exiting with the stack gases, (b) exergy destroyed in the steam generator, (c) net power developed, (d) exergy destroyed in the turbine and pump, (e) exergy exiting with the cooling water, (f) exergy destroyed in the condenser.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) There are no stray heat transfers for any component. (3) Kinetic and potential energy effects are negligible. (4) Let $T_0 = 537°R$, $P_0 = 14.7$ lbf/in².

ANALYSIS: First, fix each of the principal states of the cycle.

State 1 $p_1 = 500$ lbf/in², $T_1 = 900°F \Rightarrow h_1 = 1466.5$ Btu/lb, $s_1 = 1.6987$ Btu/lb·°R

State 2 $p_2 = 1$ lbf/in², $x_2 = .97 \Rightarrow h_2 = 1074.66$ Btu/lb, $s_2 = 1.9226$ Btu/lb·°R

State 3 $p_3 = 1$ lbf/in², sat. liquid $\Rightarrow h_3 = 69.74$ Btu/lb, $s_3 = .1327$ Btu/lb·°R

State 4 $p_4 = 500$ lbf/in²; $s_{4s} = .1327 \Rightarrow$ Table A-5E; $h_{4s} = 71.34$ Btu/lb

With the pump efficiency; $h_4 = h_3 + \left(\frac{h_{4s} - h_3}{\eta_p}\right) = 71.74$ Btu/lb

$s_4 \approx 0.1334$ Btu/lb

ANALYSIS: (a) For the stack gases

$$\frac{\dot{E}_P}{\dot{E}_F} = \frac{335 \text{ Btu/lb(fuel)}}{22,322 \text{ Btu/lb(fuel)}} = 0.015 \quad (1.5\%). \quad \leftarrow \text{(a)}$$

(b) An exergy rate balance for the steam generator gives

$$0 = \cancel{\dot{E}_A}^{0} + \dot{E}_F - \dot{E}_P + \dot{m}_{st}[e_{f4} - e_{f1}] - \dot{E}_d$$

$$\Rightarrow \frac{\dot{E}_d}{\dot{m}_F} = e_F - e_P + \frac{\dot{m}_{st}}{\dot{m}_F}\left[(h_4 - h_1) - T_0(s_4 - s_1)\right]$$

$$= 22,322 - 335 + 15.1\left[(71.74 - 1466.5) - 537[0.1334 - 1.6987]\right]$$

$$= 13,619 \text{ Btu/lb(fuel)}$$

$$\Rightarrow \% \text{ destroyed} = \frac{13,619}{22,322} = 0.61 \quad (61\%) \quad \leftarrow \text{(b)}$$

PROBLEM 8.85 (Contd.)

(c) The net work developed per unit mass of fuel entering is

$$\frac{\dot{W}_{net}}{\dot{m}_F} = \frac{\dot{m}_{st}}{\dot{m}_F}\left[(h_1-h_2)-(h_4-h_3)\right] = 15.1\left[(1466.5-1074.66)-(71.74-69.74)\right]$$

$$= 5887 \text{ Btu/ lb(fuel)}$$

$$\%\ \underset{\text{as work}}{\text{output}} = \frac{5887}{22{,}322} = 0.264 \quad (26.4\%) \quad \longleftarrow (c)$$

(d) The exergy destroyed in the turbine and pump can be found using $\dot{E}_d = T_0\dot{\sigma}$. That is

$$\frac{\dot{E}_d}{\dot{m}_F} = \left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)T_0\left[(s_2-s_1)+(s_4-s_3)\right] = (15.1)(537)\left[(1.9226-1.6987)+(0.1334-0.1327)\right]$$

$$= 1821 \text{ Btu/lb(fuel)}$$

$$\%\ \text{destroyed} = \frac{1821}{22{,}322} = 0.082 \quad (8.2\%) \quad \longleftarrow (d)$$

(e) The exergy exiting with the cooling water per unit mass of fuel entering is given by

$$\left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)\left[\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}}\right)\left[e_{f,\underset{out}{cw}} - e_{f,\underset{in}{cw}}\right]\right] = \left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}}\right)\left[(h_{cw,out}-h_{cw,in}) - T_0(s_{cw,out}-s_{cw,in})\right]$$

An energy balance on the condenser using $h_{cw} \approx h_f(T)$ gives

$$\frac{\dot{m}_{cw}}{\dot{m}_{st}} = \frac{h_2-h_3}{h_f(90°F)-h_f(77°F)} = 77.42$$

Then, with $s_{cw} \approx s_f(T)$

$$\begin{bmatrix}\text{Exergy exiting with}\\ \text{the cooling water per}\\ \text{unit mass of fuel}\\ \text{entering}\end{bmatrix} = (15.1)(77.42)\left[(58.07-45.09)-(537)(0.1117-0.08775)\right]$$

$$= 139 \text{ Btu/lb(fuel)}$$

$$\%\ \underset{\text{the cooling water}}{\text{exiting with}} = \frac{139}{22{,}322} = 0.006 \quad (0.6\%) \quad \longleftarrow (e)$$

(f) Using $\dot{E}_d = T_0\dot{\sigma}$, the exergy destroyed in the condenser is

$$\frac{\dot{E}_d}{\dot{m}_F} = T_0\left[\frac{\dot{m}_{st}}{\dot{m}_F}(s_3-s_2) + \left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}}\right)\left[s_{cw,out}-s_{cw,in}\right]\right]$$

$$= (537)(15.1)\left[(0.1327-1.9226) + (77.42)\left[0.1117-0.08775\right]\right] = 521 \text{ Btu/lb(fuel)} \quad \longleftarrow f$$

$$\%\ \text{destroyed} = 521/22{,}322 = 0.023 \quad (2.3\%)$$

SUMMARY:

- **Exergy in with fuel** = 22,322 Btu/lb (fuel)

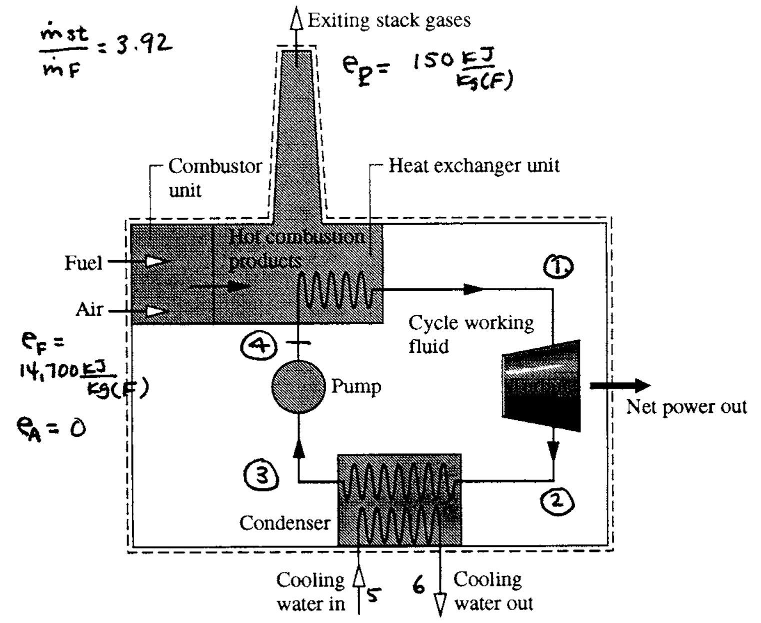
• **Exergy Out:**		
– Net work	5887	(26.4%)
– Cooling water	139	(0.6%)
– Stack gas	335	(1.5%)
• **Exergy Destroyed:**		
– Steam gen.	13,619	(61%)
– Turbine/pump	1821	(8.2%)
– Condenser	521	(2.3%)
	22,322	(100%)

PROBLEM 8.86

KNOWN: Water is the working fluid in a simple vapor powerplant. Data are known at various locations. For the steam generator, specific exergy values are provided for the fuel, air, and stack gases.

FIND: Develop an exergy accounting of the exergy entering the plant with the fuel.

SCHEMATIC & GIVEN DATA:



State	T(°C)	p(bar)	h(kJ/kg)	s(kJ/kg·K)
1	520	100	3425.1	6.6622
2	(x = .90)	0.08	2336.7	7.4651
3	(sat. Liq)	0.08	173.9	0.5926
4	43°C	100	188.9	0.6061
5	20	—	84	0.2966
6	35	—	146.7	0.5053

ASSUMPTIONS: 1. Control volumes enclosing each component are at steady state. 2. There are no stray heat transfers, and kinetic/potential energy effects can be ignored. 4. $T_0 = 20°C$, $p_0 = 1\,atm$

ANALYSIS: Begin by fixing each state and obtaining h, s data: Table A-4 gives $h_1 = 3425.1\ kJ/kg$, $s_1 = 6.6622\ kJ/kg \cdot K$. Then, with data from Table A-3

$h_2 = 173.88 + 0.9(2403.1) = 2336.7$, $s_2 = 0.5926 + 0.9(8.2287 - 0.5926) = 7.4651$.

$h_3 = h_f(0.08\ bar) = 173.9$, $s_3 = s_f = 0.5926$. Then, from Table A-5, $h_4 = 188.9$, $s_4 = 0.6061$. And with $h = h_f(T)$, $s = s_f(T)$ and data from Table A-2,

$h_5 = 84$, $s_5 = 0.2966$, $h_6 = 146.7$, $s_7 = 0.5053$

For the stack gases:

$$\frac{\dot{E}_P}{\dot{E}_F} = \frac{150\ kJ/kg(F)}{14{,}700\ kJ/kg(F)} = 0.01 \qquad (1\%)$$

Exergy Destruction – Steam Generator: An exergy rate balance reduces to read

$$0 = \overset{0}{\cancel{\dot{E}_{Air}}} + \dot{E}_F - \dot{E}_P + \dot{m}_{st}[e_{f4} - e_{f1}] - \dot{E}_d$$

$$\Rightarrow \frac{\dot{E}_d}{\dot{m}_F} = e_F - e_P + \frac{\dot{m}_{st}}{\dot{m}_F}\left[(h_4 - h_1) - T_0(s_4 - s_1)\right]$$

$$= 14{,}700 - 150 + 3.92\left[(188.9 - 3425.1) - 293\,(0.6061 - 6.6622)\right] = 8820\ kJ/kg(F)$$

Expressed as a percentage of the fuel exergy: $= \frac{8820}{14700} = 0.6 \quad (60\%)$

Net Work Developed:

$\dot{W}_{net}/\dot{m}_F = (\dot{m}_{st}/\dot{m}_F)\left[(h_1 - h_2) - (h_4 - h_3)\right] = 3.92\left[(3425.1 - 2336.7) - (188.9 - 173.9)\right] = 4208\ \frac{kJ}{kg(F)}$

Expressed as a percentage of the fuel exergy: $= \frac{4208}{14700} = 0.286 \quad (28.6\%)$

PROBLEM 8.86 (Contd)

Exergy Destruction in Turbine/Pump: Using $\dot{E}_d = T_0 \dot{\sigma}$

$$\frac{\dot{E}_d}{\dot{m}_F} = \frac{\dot{m}_{st}}{\dot{m}_F}\left[T_0[s_2 - s_1] - T_0[s_4 - s_3]\right] = (3.92)(293)\left[(7.4651 - 6.6622) + (0.6061 - 0.5926)\right]$$

$= 938 \; \frac{kJ}{kg(F)}$. When expressed as a percentage of the fuel exergy: $= \frac{938}{14,700} = 0.064$ (6.4%)

Exergy exiting with the cooling water:

$$= \left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}}\right)\left[e_{f,6} - e_{f,5}\right] = \left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}}\right)\left[(h_6 - h_5) - T_0(s_6 - s_5)\right]$$

An energy rate balance on the condenser gives

$$\frac{\dot{m}_{cw}}{\dot{m}_{st}} = \frac{h_2 - h_3}{h_6 - h_5} = \frac{2336.7 - 173.9}{146.7 - 84} = 34.49$$

Thus,

$$\left(\begin{array}{c}\text{Exergy exiting}\\ \text{with c.w.}\end{array}\right) = (3.92)(34.49)\left[(146.7 - 84) - 293(0.5053 - 0.2966)\right] = 210 \text{ kJ/kg(F)}$$

When expressed as a percentage of the fuel exergy: $= \frac{210}{14,700} = 0.014$ (1.4%)

Exergy Destruction in the Condenser. Using $\dot{E}_d = T_0 \dot{\sigma}$

$$\frac{\dot{E}_d}{\dot{m}_F} = T_0\left[\frac{\dot{m}_{st}}{\dot{m}_F}(s_3 - s_2) + \left(\frac{\dot{m}_{st}}{\dot{m}_F}\right)\left(\frac{\dot{m}_{cw}}{\dot{m}_{st}}\right)(s_6 - s_5)\right]$$

$$= (293)(3.92)\left[(0.5926 - 7.4651) + 34.49(0.5053 - 0.2966)\right] = 374 \text{ kJ/kg(F)}$$

When expressed as a percentage of the fuel exergy $= \frac{374}{14,700} = 0.025$ (2.5%)

SUMMARY:

• Exergy entering with the fuel, per unit mass of fuel:	14,700 kJ/kg(F)	(100%)
• Exergy Out		
– net work	4208	(28.6%)
– cooling water	210	(1.4%)
– stack gases	150	(1.0%)
• Exergy Destroyed		
– turbine/pump	938	(6.4%)
– condenser	374	(2.5%)
– steam generator	8820	(60%)
	14,700	(100%)

CHAPTER NINE

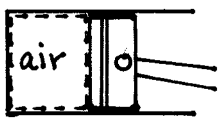
GAS POWER SYSTEMS

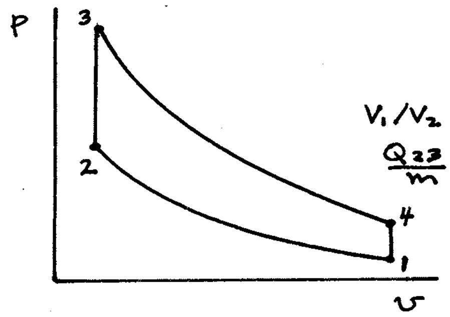
PROBLEM 9.1

KNOWN: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the net work per unit mass of air, (b) the thermal efficiency, (c) the mean effective pressure, and (d) the maximum cycle temperature. (e) Plot each of these quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:





$V_1/V_2 = 8.5$

$\frac{Q_{23}}{m} = 1400$ kJ/kg

$p_1 = 100$ kPa

$T_1 = 300$K

ASSUMPTIONS: See Example 9.1

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22).

State 1 $p_1 = 100$ kPa, $T_1 = 300$K $\Rightarrow$ $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$

State 2 For isentropic compression

$$v_{r2} = v_{r1}\frac{V_2}{V_1} = \frac{621.2}{8.5} = 73.082$$

Thus, $T_2 = 688.2$ K, $u_2 = 503.06$

State 3 The specific internal energy u_3 is found using the energy balance for process 2-3

$$m(u_3 - u_2) = Q_{23} - \cancelto{0}{W_{23}}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 1400\,\frac{kJ}{kg} + 503.06\,\frac{kJ}{kg} = 1903.06$$

Thus, $T_3 = 2231.3$ K, $v_{r3} = 1.9192$ (d) T_3

State 4 For the isentropic expansion

$$v_{r4} = v_{r3}\frac{V_4}{V_3} = v_{r3}\frac{V_1}{V_2} = (1.9192)(8.5) = 16.3132$$

Finally, $T_4 = 1154.3$ K, $u_4 = 892.95$ kJ/kg

(a) To find the net work, note that $W_{cycle} = Q_{cycle}$, so

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (u_4 - u_1)$$

$$= 1400 - (892.95 - 214.07) = 721.12\,\frac{kJ}{kg} \leftarrow \text{(a) } W_{cycle}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{721.12}{1400} = 0.515\ (51.5\%) \leftarrow \text{(b) } \eta$$

(c) The displacement volume is $V_1 - V_2 = m(v_1 - v_2)$, so the mean effective pressure is given by

PROBLEM 9.1 (Contd.)

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(300\,K)}{(100\,kPa)}\left|\frac{1\,kPa}{10^3\,N/m^2}\right|\left|\frac{10^3\,N \cdot m}{1\,kJ}\right|$$

$$= 0.861\,m^3/kg$$

Thus

$$mep = \frac{(721.12\,kJ/kg)}{(0.861\,\frac{m^3}{kg})(1 - \frac{1}{8.5})}\left|\frac{10^3\,N \cdot m}{1\,kJ}\right|\left|\frac{1\,kPa}{10^3\,N/m^2}\right|$$

$$= 949.2\,kPa \longleftarrow \text{(c) mep}$$

The data for the required plots are obtained using *IT*, as follows:

IT Code

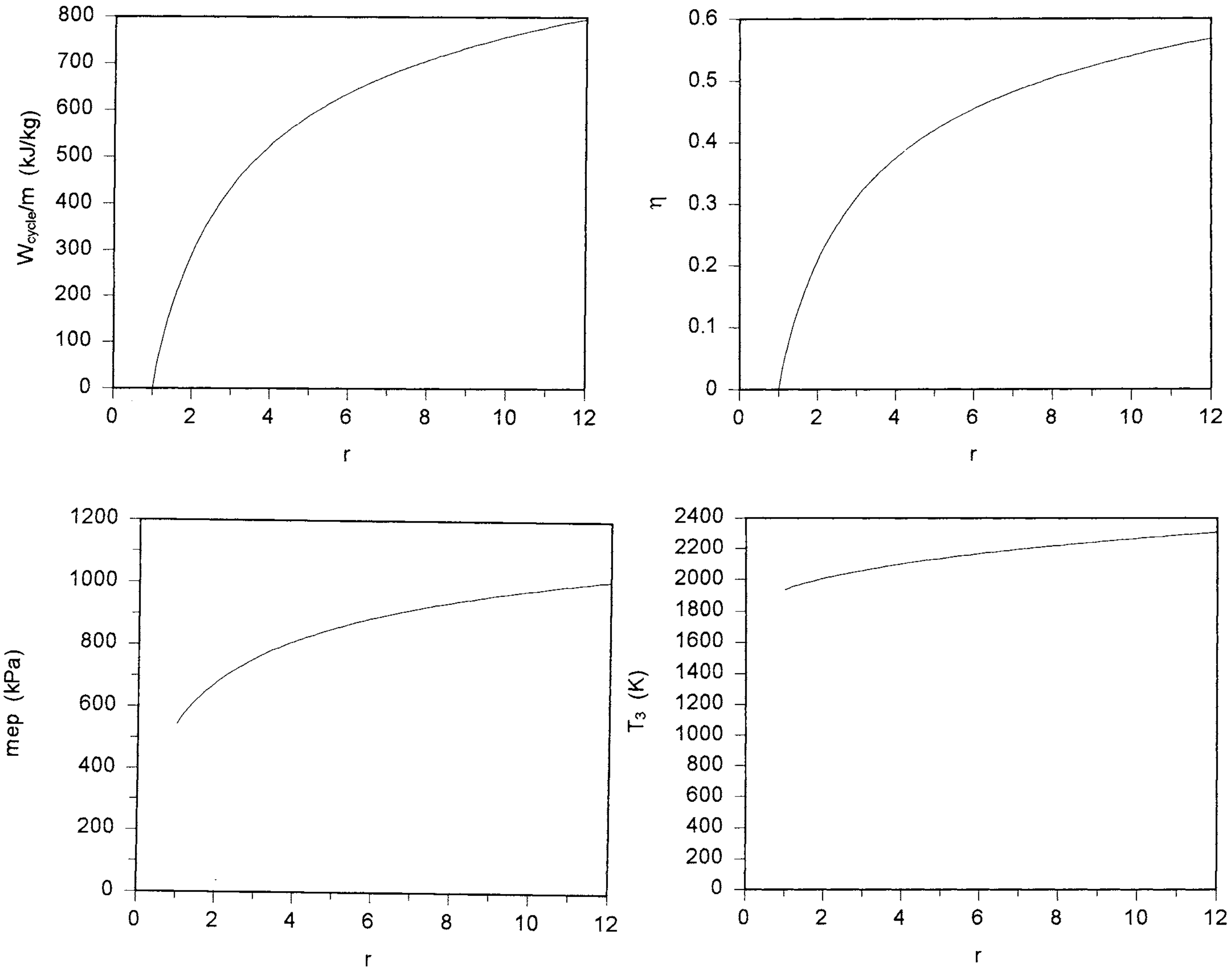
```
r = 8.5
p1 = 100   // kPa
T1 = 300   // K
Q23 / m = 1400   // kJ/kg
m = 1     // assume a unit mass of 1 kg.

v1 = v_TP("Air",T1,p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
s2 = s_TP("Air", T2, p2)
v2 = v_TP("Air",T2,p2)
u2 = u_T("Air", T2)
v3 = v2
m * (u3 - u2) = Q23 - W23
W23 = 0
u3 = u_T("Air", T3)
v3 = v_TP("Air",T3,p3)
s3 = s_TP("Air", T3, p3)
v4 = r * v3
s4 = s3
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)

Wcycle = Qcycle
Wcycle / m = Q23 / m - Q41 / m
Q41/ m = u4 - u1
eta = Wcycle / Q23
V1 = v1 * m
V2 = v2 * m
mep = Wcycle / (V1 - V2)
```

IT Results for r = 0.85

Q_{41}/m = 678.8 kJ/kg
T_4 = 1155 K
u_1 = 213.9 kJ/kg
u_2 = 502.9 kJ/kg
u_3 = 1903 kJ/kg
u_4 = 892.7 kJ/kg
v1 = 0.861 m^3/kg
W_{cycle}/m = 721.2 kJ/kg
η = 0.5151
T_3 = 2233 K
mep = 949.4 kPa

PROBLEM 9.1 (Cont'd.)

PLOTS:



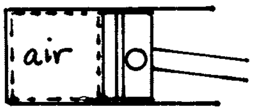
Thermal efficiency increases with increasing compression ratio. Since the heat addition is constant, the net work of the cycle increases, as expected. Also, the maximum cycle temperature increases, since the temperature at the end of compression, T_2, goes up with increasing r. The mep increases as well for similar reasons.

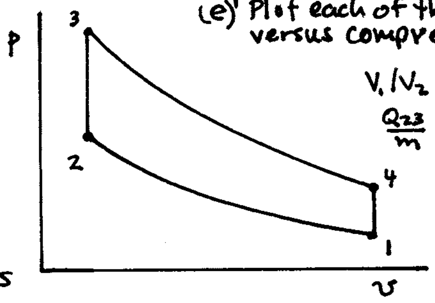
PROBLEM 9.2

KNOWN: A cold air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the net work per unit mass of air, (b) the thermal efficiency, and (c) the mean effective pressure, and (d) the maximum cycle temperature. (e) Plot each of these quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:





$V_1/V_2 = 8.5$

$\frac{Q_{23}}{m} = 1400\ \frac{kJ}{kg}$

$p_1 = 100$ kPa

$T_1 = 300K$

ASSUMPTIONS: See Example 9.1. Also, assume constant specific heats evaluated at 300K.

ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 300$ K is given.

State 2: For isentropic compression, $T_2 = (V_1/V_2)^{k-1} T_1 = 706.1$ K where k = 1.4 from Table A-20.

State 3: The temperature T_3 is found using the energy balance for process 2-3

$$m(u_3 - u_2) = Q_{23} - \cancelto{0}{W_{23}} \Rightarrow m c_v (T_3 - T_2) = Q_{23}$$

$$T_3 = \frac{(Q_{23}/m)}{c_v} + T_2 = \frac{1400\ kJ/kg}{0.718\ kJ/kg \cdot K} + 706.1 K = 2656 K \quad \longleftarrow T_3$$

State 4: For the isentropic expansion,

$$T_4 = \left(\frac{V_3}{V_4}\right)^{k-1} T_3 = \left(\frac{V_2}{V_1}\right)^{k-1} T_3 = \left(\frac{1}{8.5}\right)^{k-1}(2656) = 1128.4 K$$

(a) To find the net work, note that $W_{cycle} = Q_{cycle}$, so

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - c_v(T_4 - T_1)$$

$$= 1400 - (.718)(1128.4 - 300) = 805.2\ kJ/kg \longleftarrow W_{cycle}$$

(b) The thermal efficiency is $\eta = \frac{W_{cycle}/m}{Q_{23}/m} = 0.575\ (57.5\%) \longleftarrow \eta$

(c) The mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{m(v_1 - v_2)} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\ \frac{kJ}{kg \cdot K}\right)(300\ K)}{(100\ kPa)} \left|\frac{1\ kPa}{10^3\ N/m^2}\right| \left|\frac{10^3\ N \cdot m}{1\ kJ}\right|$$

$$= 0.861\ m^3/kg$$

Thus

$$mep = \frac{(805.2\ kJ/kg)}{(.861\ \frac{m^3}{kg})(1 - \frac{1}{8.5})} \left|\frac{10^3\ N \cdot m}{1\ kJ}\right| \left|\frac{1\ kPa}{10^3\ N/m^2}\right| = 1060\ kPa \longleftarrow mep$$

PROBLEM 9.2 (Cont'd)

The data for the required plots are obtained using IT, as follows:

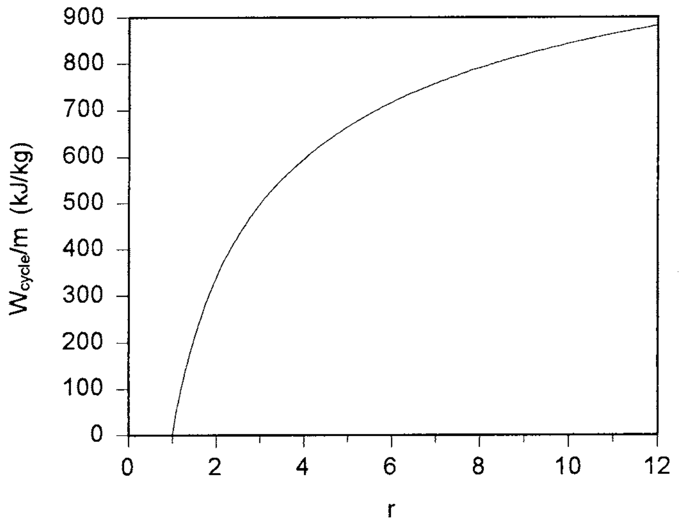
IT Code

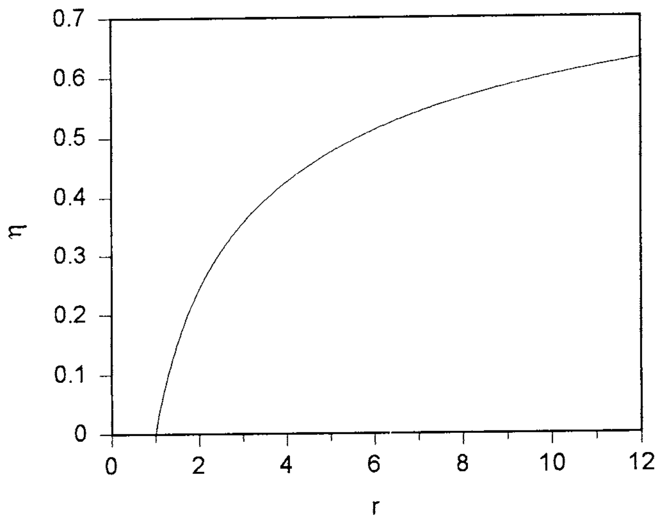
```
r = 8.5
p1 = 100    // kPa
T1 = 300   // K
Q23 / m = 1400   // kJ/kg
m = 1     // assume a unit mass of 1 kg.
cv = cv_T("Air", T1)
k = 1.4
Rbar = 8.314
M = 28.97

v1 = R * T1 / p1
R = Rbar / M
T2 = T1 * r^(k-1)
m * cv * (T3 - T2) = Q23 - W23
W23 = 0
T4 = T3 * (1 / r)^(k - 1)

Wcycle = Qcycle
Wcycle / m = Q23 / m - Q41 / m
Q41/ m = cv * (T4 - T1)
eta = Wcycle / Q23
V1 = v1 * m
mep = Wcycle / (V1 * ( 1 - 1 / r))
```

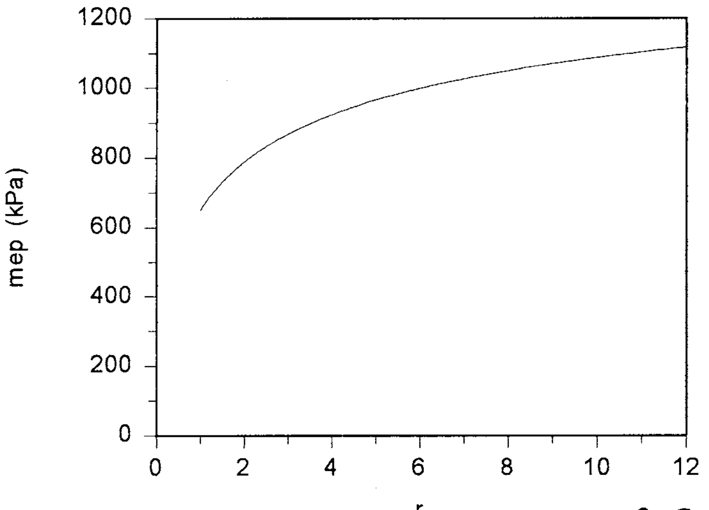
IT Results for r = 0.85

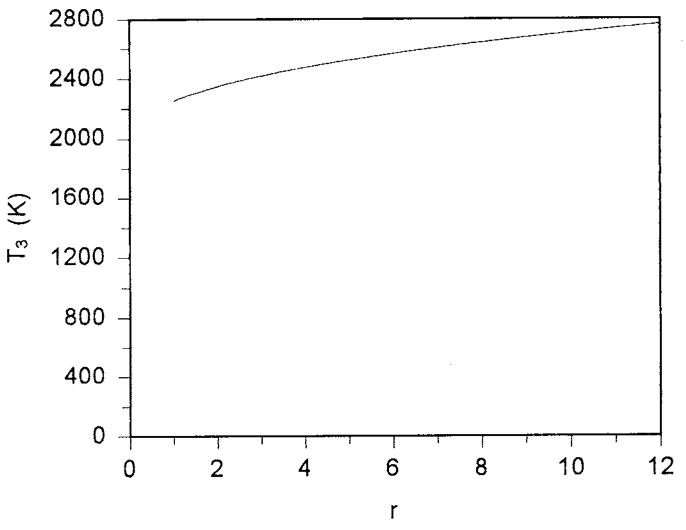
Q_{41}/m = 594.8 kJ/kg
T_2 = 706.1 K
T_4 = 1129 K
c_v = 0.7172 kJ/kg·K
v_1 = 0.861 m³/kg
k = 1.4
W_{cycle}/m = 805.2 kJ/kg
η = 0.5752
T_3 = 2658 K
mep = 1060 kPa

PLOTS:







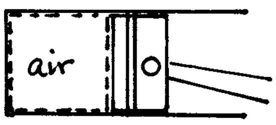


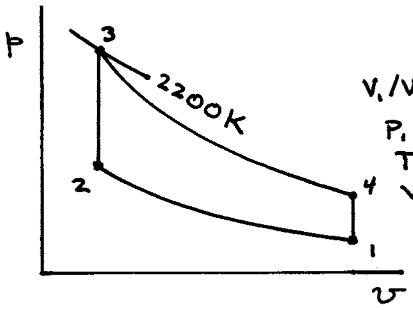
PROBLEM 9.3

KNOWN: An air-standard Otto cycle has a known compression ratio, a specified state at the beginning of compression, and a specified maximum temperature.

FIND: Determine (a) the heat added, (b) net work, (c) thermal efficiency, (d) mean effective pressure, Also, (e) develop a full accounting of the exergy transferred during heat addition and (f) devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:





$V_1/V_2 = 8$
$p_1 = 1$ bar
$T_1 = 290$K
$V_1 = 400$ cm^3

ASSUMPTIONS: See Example 9.1
Also, let $T_0 = 290$K; $p_0 = 1$ bar.

ANALYSIS: First, fix each of the principal states of the cycle (Table A-22).

State 1: $p_1 = 1$ bar, $T_1 = 290$K $\Rightarrow$ $u_1 = 206.91$ kJ/kg, $v_{r1} = 676.1$

State 2: For isentropic compression, $v_{r2} = v_{r1}(V_2/V_1) = \frac{676.1}{8} = 84.5125$
Thus, $T_2 = 652.4$ K, $u_2 = 475.11$ kJ/kg

State 3: $T_3 = 2200$K $\Rightarrow$ $u_3 = 1872.4$ kJ/kg, $v_{r3} = 2.012$

State 4: For isentropic expansion, $v_{r4} = v_{r3}(V_4/V_3) = v_{r3}(V_1/V_2) = 16.096$
Thus, $T_4 = 1159.3$ K, $u_4 = 897.3$ kJ/kg

(a) To find the heat addition, first determine the mass of air

$$m = \frac{p_1V_1}{RT_1} = \frac{(1\text{ bar})(400\text{ cm}^3)}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(290\text{K})}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\left|\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right|$$

$$= 4.806\times10^{-4}\text{ kg}$$

thus
$$m(u_3-u_2) = Q_{23} - \cancelto{0}{W_{23}}$$

$$Q_{23} = m(u_3-u_2) = (4.806\times10^{-4})(1872.4-475.11)$$

$$= 0.6715\text{ kJ} \longleftarrow Q_{23}$$

(b) To find the net work, note that $W_{cycle} = Q_{cycle}$, so

$$W_{cycle} = Q_{23} - Q_{41} = Q_{23} - m(u_4-u_1)$$

$$= 0.6715 - (4.806\times10^{-4})(897.3-206.91) = 0.3397\text{ kJ} \longleftarrow W_{cycle}$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{.3397}{.6715} = 0.506\ (50.6\%) \longleftarrow \eta$$

(d) The mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1-V_2} = \frac{W_{cycle}}{V_1(1-V_2/V_1)}$$

Inserting values

$$mep = \frac{(0.3397\text{ kJ})}{(400\text{ cm}^3)(1-\frac{1}{8})}\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right|\left|\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right|$$

$$= 9.71\text{ bar} \longleftarrow mep$$

PROBLEM 9.3(contd.)

(e) Since the heat addition process 2-3 involves no work or volume change and occurs without internal irreversibilities, an exergy balance reduces to read

① $$E_3 - E_2 = \underbrace{\int_2^3 \left[1 - \frac{T_0}{T_b}\right] \delta Q}_{E_Q} - \cancel{[W - p_0 \Delta V]} - \cancel{E_d} \Rightarrow E_Q = E_3 - E_2 .$$

Then, with Eq. 7.10, $E_3 - E_2 = \overbrace{(U_3 - U_2)}^{Q_{23}} + p_0 (\overset{0}{\cancel{V_3 - V_2}}) - T_0 (S_3 - S_2)$

$$\Rightarrow E_Q = Q_{23} - T_0 (S_3 - S_2) = Q_{23} - m T_0 \left[s^\circ(T_3) - s^\circ(T_2) - R \ln \underbrace{P_3/P_2}_{} \right]$$

Since $V_3 = V_2$, $P_3/P_2 = \frac{T_3}{T_2}$

With data from Table A-22

$$E_Q = 0.6715 - (4.806 \times 10^{-4})(290)\left[3.9191 - 2.49753 - \frac{8.314}{28.97} \ln \frac{2200}{652.4}\right] = 0.522 \text{ kJ}$$

The net exergy transfer accompanying work is obtained as follows:

Process 1-2: $E_W = W_{12} - \cancel{p_0 \Delta V}$, Process 2-3: $E_W = W_{23} - \overset{0}{\cancel{p_0 \Delta V}}$

$\Rightarrow$ $(E_W)_{cycle} = W_{cycle} = 0.3397$ kJ

The exergy transfer accompanying heat rejection is obtained as above:

$E_Q = (U_1 - U_4) + \overset{0}{\cancel{p_0 (V_1 - V_4)}} - T_0 (S_1 - S_4)$. Since $V_1 = V_4$, $P_1/P_4 = T_1/T_4$, and we get

$$= m\left[(u_1 - u_4) - T_0 \left(s_1^\circ - s_4^\circ - R \ln \frac{T_1}{T_4}\right)\right] = (4.806 \times 10^{-4})\left[(206.94 - 897.3)\right.$$

$$\left. - 290 \left(1.66802 - 3.13845 - \frac{8.314}{28.97} \ln \frac{290}{1159.3}\right)\right] = -0.1823 \text{ kJ}$$

In Summary:

Exergy In:	0.522 kJ (100%)	
Disposition of the Exergy:		
• Exergy Out		
– net work	0.3397	(65.1%)
– heat transfer	0.1823	(34.9%)
② • Exergy Destroyed	0	
	0.522	

f. An exergetic efficiency in this case can be written as the ratio of the desired exergy output (exergy transfer accompanying the net work developed) to the exergy added. By inspection of the above summary, we get $\epsilon = 65.1\%$.

---

1. Alternatively, for the internally reversible process we can write
$$E_Q = \int_2^3 \left[1 - \frac{T_0}{T_b}\right] \delta Q = Q_{23} - T_0 \int_2^3 \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev}} = Q_{23} - T_0 (S_3 - S_2)$$

2. Significant internal irreversibilities are present in actual internal combustion engines. Accordingly, significant exergy destruction occurs within such engines and the exergetic efficiency would be much less than determined here.

PROBLEM 9.4

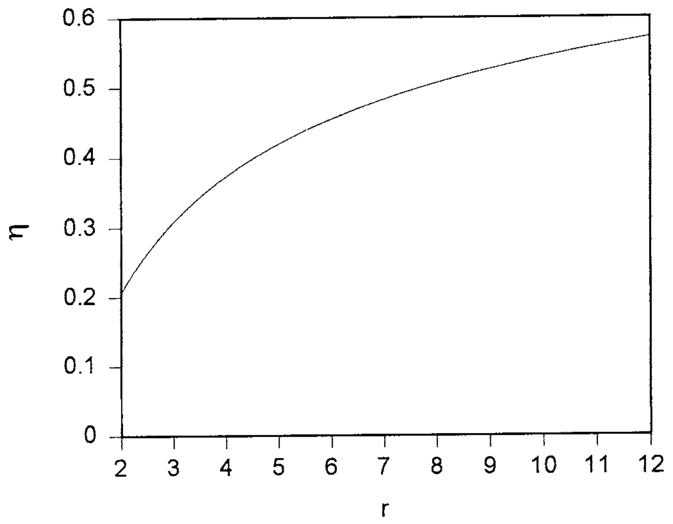
The data for the required plots are obtained using IT, as follows. See Problem 9.3 for sample calculations for the case of $r = 8$.

IT Code

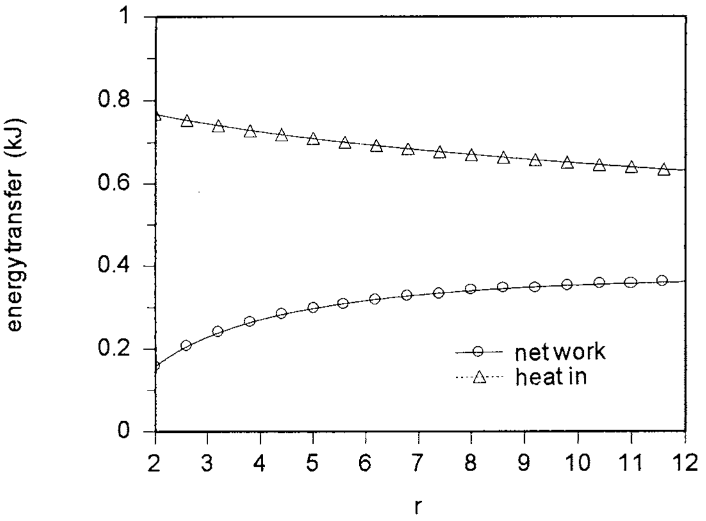
```
r = 8
p1 = 1    // bar
T1 = 290   // K
V1 = 400   // cm³
T3 = 2200    // K

v1 = v_TP("Air",T1,p1)
v1 = (V1 / 10^6) / m
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
s2 = s_TP("Air", T2, p2)
v2 = v_TP("Air",T2, p2)
u2 = u_T("Air", T2)
v3 = v2
u3 = u_T("Air", T3)
v3 = v_TP("Air",T3,p3)
s3 = s_TP("Air", T3, p3
v4 = r * v3
s4 = s3
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)

m * (u3 - u2) = Q23 - W23     // kJ
W23 = 0
Wcycle = Qcycle    // kJ
Wcycle / m = Q23 / m - Q41 / m
Q41/ m = u4 - u1
eta = Wcycle / Q23
mep = Wcycle * ( 10^3 / 10^5) / ((V1/ 10 ^6) * (1 - 1 / r))   // bar
```

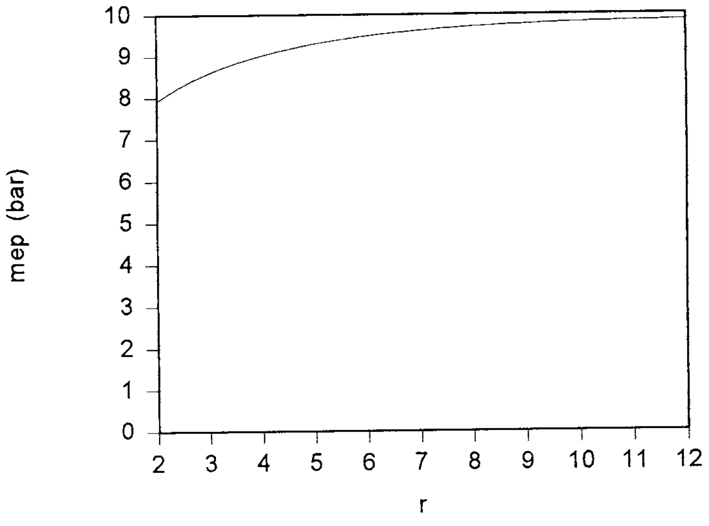
IT results for r = 8

$Q_4 = 0.3312$ kJ
$T_2 = 652.4$ K
$T_4 = 1158$ K
$u_1 = 206.8$ kJ
$u_2 = 474.9$ kJ
$u_3 = 1871$ kJ
$u4 = 896$ kJ
$T_3 = 2200$ K
$Q_{23} = 0.6707$ kJ
$W_{cycle} = 0.3395$ kJ
$\eta = 0.5062$
mep = 9.7 bar



PLOTS:





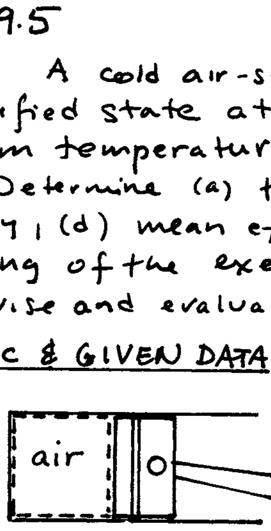
As seen in the accompanying plots, both η and mep increase with increasing compression ratio, r. Since T_3 is constant, the heat input decreases and the net work increases with r.

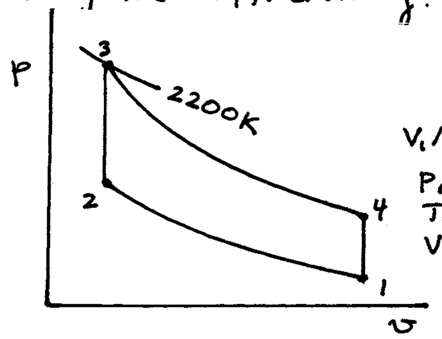
PROBLEM 9.5

KNOWN: A cold air-standard Otto cycle has a known compression ratio, a specified state at the beginning of compression, and a specified maximum temperature.

FIND: Determine (a) the heat added, (b) net work, (c) thermal efficiency, (d) mean effective pressure. Also, (e) develop a full exergy accounting of the exergy transferred during heat addition and (f) devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:





$V_1/V_2 = 8$
$p_1 = 1$ bar
$T_1 = 290$K
$V_1 = 400$ cm^3

ASSUMPTIONS: Same as Example 9.1 Also, assume constant specific heats evaluated at 300K and let $T_0 = 290$K, $p_0 = 1$ bar.

ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 290$K is given.

State 2: For isentropic compression, $T_2 = (V_1/V_2)^{k-1}T_1 = 666.2$ K where $k = 1.4$ from Table A-20

State 3: $T_3 = 2200$K is given.

State 4: For isentropic expansion,

$$T_4 = \left(\frac{V_3}{V_4}\right)^{k-1} T_3 = \left(\frac{V_2}{V_1}\right)^{k-1} T_3 = 957.6 \text{ K}$$

(a) To find the heat addition, first determine the mass of air

$$m = \frac{p_1 V_1}{RT_1} = \frac{(1\text{ bar})(400\text{ cm}^3)}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(290\text{K})}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{kJ}}{10^3\text{ N}\cdot\text{m}}\right|\left|\frac{1\text{ m}^3}{10^6\text{ cm}^3}\right| = 4.806\times10^{-4}\text{ kg}$$

Then

$$m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{\,0} \Rightarrow m\,c_v(T_3 - T_2) = Q_{23}$$

$$Q_{23} = (4.806\times10^{-4})(.718\ \tfrac{\text{kJ}}{\text{kg}\cdot\text{K}})(2200 - 666.2)\text{K}$$

$$= 0.5293 \text{ kJ} \longleftarrow Q_{23}$$

(b) To find the net work, note that $W_{cycle} = Q_{cycle}$, so

$$W_{cycle} = Q_{23} - Q_{41} = Q_{23} - m\,c_v(T_4 - T_1)$$

$$= 0.5293 - (4.806\times10^{-4})(.718)(957.6 - 290)$$

$$= 0.2989 \text{ kJ} \longleftarrow W_{cycle}$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{0.2989}{0.5293} = 0.565\ (56.5\%) \longleftarrow \eta$$

(d) The mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)}$$

Inserting values

$$mep = \frac{(0.2989\text{ kJ})}{(400\text{ cm}^3)(1 - 1/8)}\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right|\left|\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right|$$

$$= 8.54 \text{ bar} \longleftarrow mep$$

PROBLEM 9.5 (Contd.)

(e) Since the heat addition process involves no internal irreversibilities, the exergy transfer accompanying heat transfer can be expressed as

$$E_Q = \int_2^3 \left[1 - \frac{T_0}{T_b}\right]\delta Q = Q_{23} - T_0 \int_2^3 \left(\frac{\delta Q}{T}\right)_{\substack{int \\ rev}} = Q_{23} - T_0(S_3 - S_2)$$

The same result can be obtained by reducing an exergy balance:

$$E_3 - E_2 = \underbrace{\int_2^3 \left[1 - \frac{T_0}{T_b}\right]\delta Q}_{E_Q} - [\cancel{W} - \cancel{p_0 \Delta V}] - \cancel{E_d}^{\,0} \Rightarrow E_Q = E_3 - E_2. \text{ Then, with Eq. 7.10}$$

$$E_Q = \underbrace{(U_3 - U_2)}_{Q_{23}} + p_0(\cancel{V_3 - V_2})^{0} - T_0(S_3 - S_2) = Q_{23} - T_0\underbrace{(S_3 - S_2)}_{= m\left[c_p \ln\frac{T_3}{T_2} - R \ln\frac{p_3}{p_2}\right]}. \text{ with } c_p \text{ from Table A-20}$$

$$E_Q = 0.5293 - \left(\frac{4.806}{10^4}\right)(290)\left[1.005 \ln\frac{2200}{666.2} - \frac{8.314}{28.97} \ln\frac{2200}{666.2}\right] = 0.4097 \text{ kJ} \qquad \hookrightarrow = \frac{T_3}{T_2} \text{ since } V_3 = V_2$$

The net exergy transfer accompanying work is obtained as follows:

$$\text{Process 1-2: } E_W = W_{12} - p_0 \cancel{\Delta V}^{0}, \quad \text{Process 2-3: } E_W = W_{23} - p_0 \cancel{\Delta V}^{0}$$

$$\Rightarrow (E_W)_{cycle} = W_{cycle} = 0.2989 \text{ kJ}$$

The exergy transfer accompanying heat rejection is obtained as above:

$$E_Q = U_1 - U_4 + p_0(\cancel{V_1 - V_4}) - T_0(S_1 - S_4)$$

$$= m\left[c_v(T_1 - T_4) - T_0\left(c_p \ln\frac{T_1}{T_4} - R\,\frac{T_1}{T_4}\right)\right]$$

$$= (4.806 \times 10^{-4})\left[0.718(290 - 957.6) - 290\left(1.005 \ln\frac{290}{957.6} - \frac{8.314}{28.97} \ln\frac{290}{957.6}\right)\right]$$

$$= -0.1108 \text{ kJ}$$

In summary:

Exergy In:	0.4097 kJ	(100%)
Disposition of exergy:		
· Exergy out		
– net work	0.2989	(73%)
– heat transfer	0.1108	(27%)
① · Exergy Destroyed	0	

(f) An exergetic efficiency in this case can be written as the ratio of the desired exergy output (exergy transfer accompanying the net work developed) to the exergy added. By inspection of the above summary, we get $\epsilon = 73\%$.

---

1. Significant internal irreversibilities are present in actual internal combustion engines. Accordingly, significant exergy destruction occurs in such engines and the exergetic efficiency would be much less than determined here.

PROBLEM 9.6

KNOWN: The cycle in Problem 9.3 models the processes in a four-cylinder engine operating at 1200 cycles per minute.

FIND: Determine the net power output.

SHEMATIC & GIVEN DATA: See Problem 9.3.

ASSUMPTIONS: See Problem 9.3.

ANALYSIS: From the solution to Problem 9.3, the net work in one cylinder, per cycle, is

$$W_{cycle} = 0.3397 \text{ kJ/cycle}$$

Thus

$$\dot{W}_{engine} = \left(\frac{0.3397 \text{ kJ/cycle}}{1 \text{ cylinder}}\right)(4 \text{ cylinders})\left(1200 \frac{\text{cycles}}{\text{min}}\right)\left|\frac{\text{min}}{60 \text{ s}}\right|\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

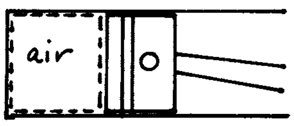
$$= 27.2 \text{ kW} \longleftarrow \dot{W}$$

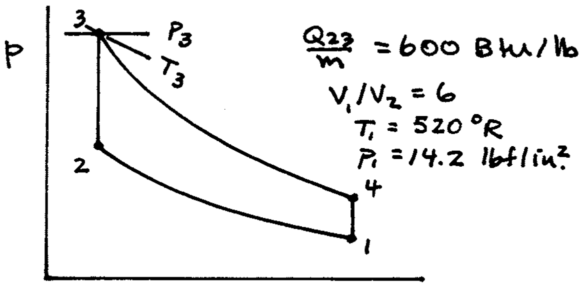
PROBLEM 9.7

KNOWN: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the maximum temperature, (b) the maximum pressure, and (c) the thermal efficiency. (d) Plot these quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:





$\frac{Q_{23}}{m} = 600$ Btu/lb

$V_1/V_2 = 6$

$T_1 = 520$ °R

$p_1 = 14.2$ lbf/in.²

ASSUMPTIONS: Same as Example 9.1

ANALYSIS: (a) To determine T_3, begin by fixing state 2. For the isentropic compression

$$v_{r2} = v_{r1}\left(\frac{V_2}{V_1}\right) = \frac{158.58}{6} = 26.43$$

Where data for air is obtained from Table A-22E. Thus, for $v_{r2} = 26.43$, $T_2 \approx 1051.4$ °R, $u_2 \approx 181.74$ Btu/lb. The energy balance for process 2-3 is

$$m(u_3 - u_2) = Q_{23} - \cancelto{0}{W_{23}}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 600\ \frac{\text{Btu}}{\text{lb}} + 181.74\ \frac{\text{Btu}}{\text{lb}} = 781.74\ \text{Btu/lb}$$

Thus, from Table A-22E; $T_3 \approx 3861$ °R ← T_3

(b) To find p_3, first determine p_2. For the isentropic compression

$$p_2 = p_1\,\frac{p_{r2}}{p_{r1}} = (14.2)\left(\frac{14.78}{1.2147}\right) = 172.78\ \text{lbf/in.}^2$$

Now, since $V_3 = V_2$; $p_3 = \frac{T_3}{T_2}\cdot p_2 = 634.5$ lbf/in.² ← p_3

(c) To determine the thermal efficiency, first find the net work. That is

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (u_4 - u_1)$$

To fix state 4, consider the expansion process

$$v_{r4} = v_{r3}\left(\frac{V_4}{V_3}\right) = v_{r3}\left(\frac{V_1}{V_2}\right) = (.5095)(6) = 3.057$$

Thus, $T_4 \approx 2228$ °R and $u_4 \approx 414.35$, and the net work is

$$\frac{W_{cycle}}{m} = 600 - (414.35 - 88.62) = 274.27\ \text{Btu/lb}$$

Finally, the thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{274.27}{600} = 0.457\ (45.7\%)$$ ← η

(d) The data for the required plots are obtained using IT, as follows:

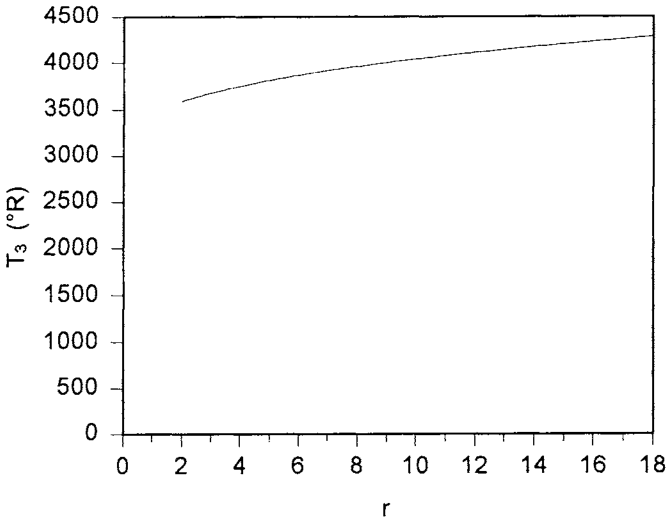
IT Code

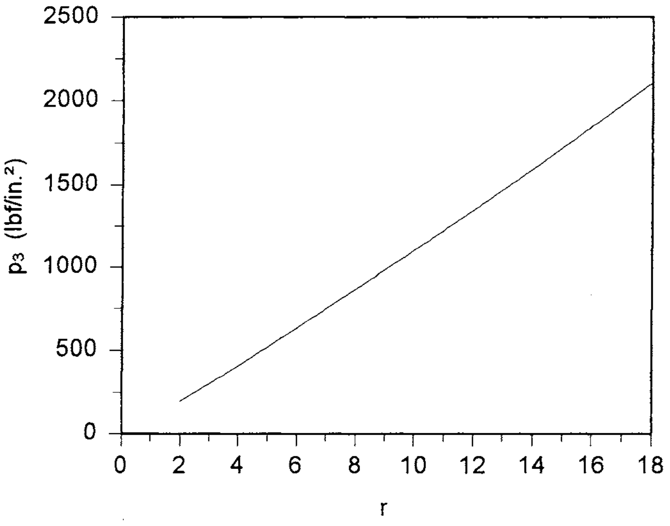
```
r = 6
p1 = 14.2    // lbf/in²
T1 = 520   // °R
Q23 / m = 600   // Btu/lb
m = 1      // Assume a unit mass of 1 lb.

v1 = v_TP("Air",T1,p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
s2 = s_TP("Air", T2, p2)
v2 = v_TP("Air",T2,p2)
u2 = u_T("Air", T2)
v3 = v2
m * (u3 - u2) = Q23 - W23
W23 = 0
u3 = u_T("Air", T3)
v3 = v_TP("Air",T3,p3)
s3 = s_TP("Air", T3, p3)
v4 = r * v3
s4 = s3
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)

Wcycle / m = Q23 / m - Q41 / m
Q41/ m = u4 - u1
eta = Wcycle / Q23
```

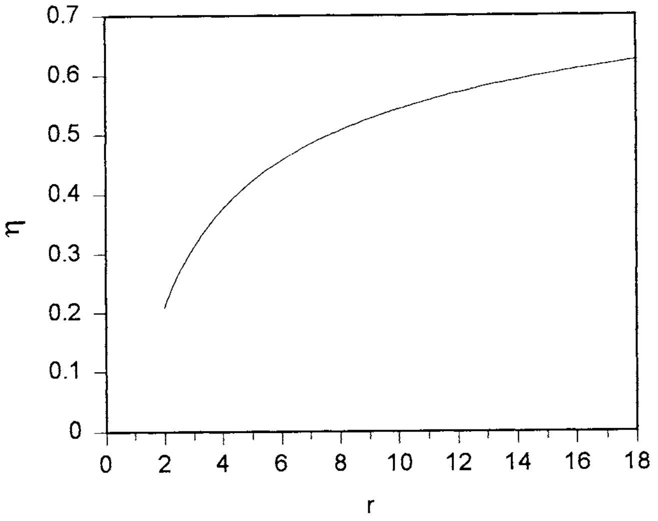
IT Results for r = 6

T_2 = 1051 °R
T_4 = 2229 °R
p_2 = 172.1 lbf/in.²
u_1 = 88.73 Btu/lb
u_2 = 181.6 Btu/lb
u_3 = 781.6 Btu/lb
u_4 = 415.4 Btu/lb
W_{cycle}/lb = 273.3 Btu/lb
Q_{41}/lb = 326.7 Btu/lb
T_3 = 3865 °R
p_3 = 633.2 lbf/in.²
η = 0.4555

PLOTS:





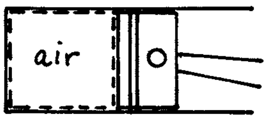


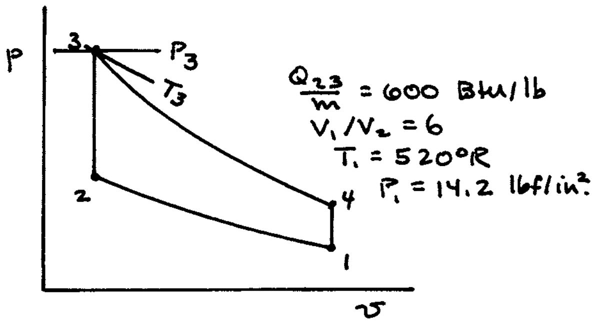
PROBLEM 9.8

KNOWN: A cold air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass of air is given.

FIND: Determine (a) the maximum temperature, (b) the maximum pressure, and (c) the thermal efficiency. (d) Plot those quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as Example 9.1. Also, assume constant specific heats evaluated at 520°R.

ANALYSIS: To determine T_3, begin by evaluating T_2. For the isentropic compression

$$T_2 = \left(\frac{V_1}{V_2}\right)^{k-1} T_1 = (6)^{.401}(520°R) = 1066.7\ °R$$

where k = 1.401 from Table A-20E. The energy balance for process 2-3 is

$$m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{0} \Rightarrow Q_{23} = m\,c_v(T_3 - T_2)$$

Thus

$$T_3 = \frac{Q_{23}}{m c_v} + T_2 = \frac{600\ \text{Btu/lb}}{.171\ \text{Btu/lb}\cdot°R} + 1066.7\ °R$$

$$= 4575.5\ °R \leftarrow \underline{\hspace{6cm} T_3}$$

(b) To find p_3, first determine p_2. For the isentropic compression

$$p_2 = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} p_1 = 174.8\ \text{lbf/in.}^2$$

Now, since $V_3 = V_2$; $p_3 = \frac{T_3}{T_2} p_2 = 749.8\ \text{lbf/in.}^2 \leftarrow \underline{\hspace{6cm} p_3}$

(c) The thermal efficiency can be determined using Eq. 9.8

$$\eta = 1 - \frac{1}{r^{k-1}} = 1 - \frac{1}{(V_1/V_2)^{k-1}}$$

$$= 1 - \frac{1}{6^{(1.401-1)}} = 0.5125\ (51.25\%) \leftarrow \underline{\hspace{6cm} \eta}$$

(d) The data for the required plots are obtained using IT, as follows:

IT Code

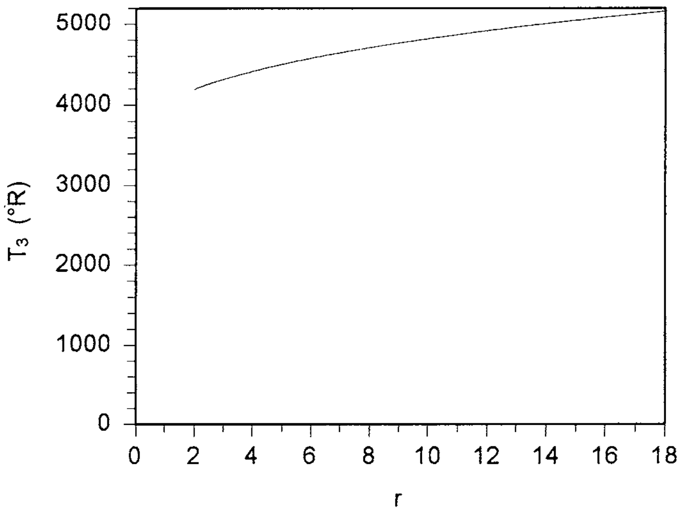
```
p1 = 14.2  // lbf/in.²
T1 = 520  // °R
r = 6
Q23 = 600  // Btu
m = 1  // Assume a unit mass of 1 lb.
cv = cv_T("Air", T1)
cp = cp_T("Air", T1)
k = cp / cv

(T2 / T1) = r^(k - 1)
Q23 = m * cv * (T3 - T2)
(p2 / p1) = (T2 / T1)^(k / (k - 1))
p3 / p2 = T3 / T2
eta = 1 - 1 / (r^(k - 1))
```

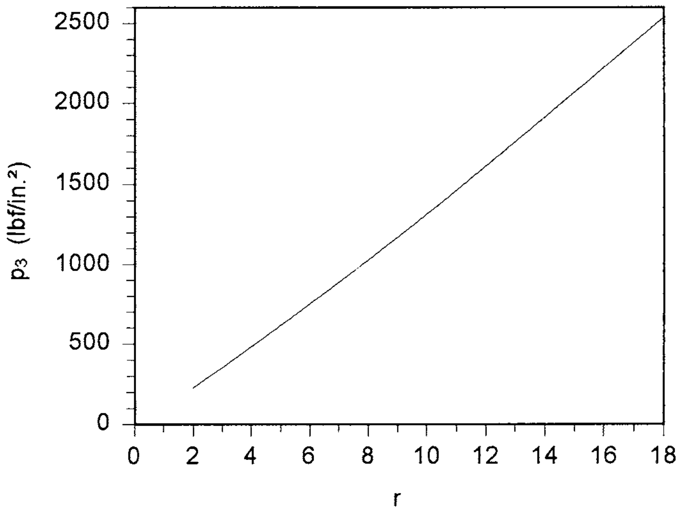
IT Results for r = 6

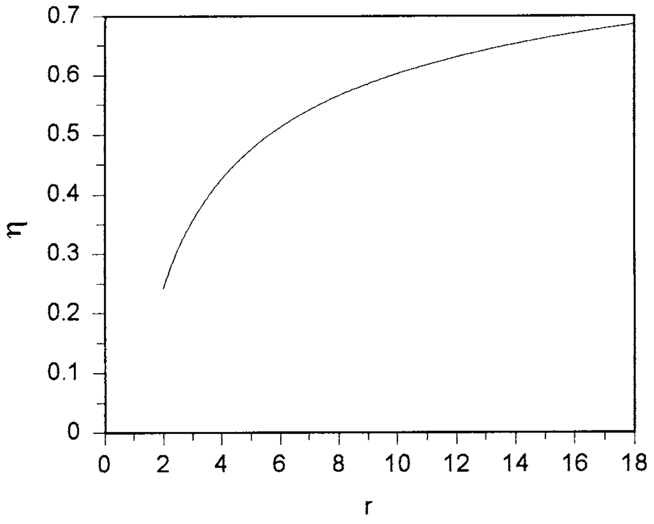
T2	1066
T3	4573
eta	0.512
k	1.4
p2	174.6
p3	749.3

PROBLEM 9.8 (Cont'd.)

PLOTS:





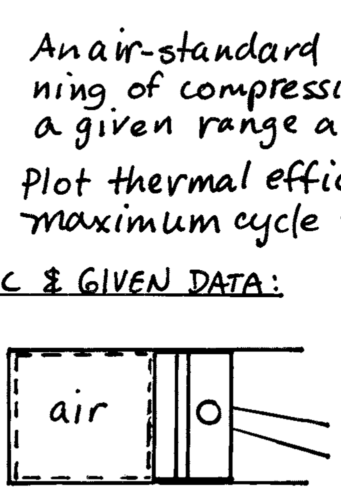


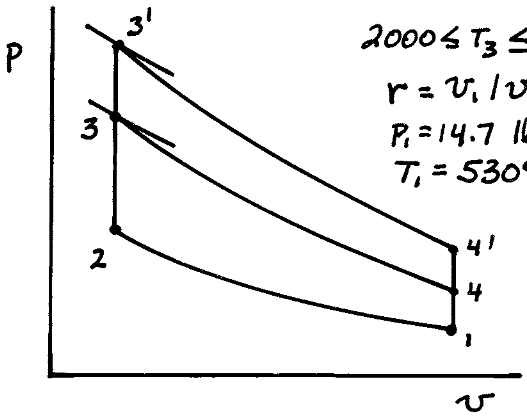
PROBLEM 9.9

KNOWN: An air-standard Otto cycle has known conditions at the beginning of compression. The maximum cycle temperature varies over a given range and the compression ratio takes on specified values.

FIND: Plot thermal efficiency and mean effective pressure versus maximum cycle temperature for each compression ratio.

SCHEMATIC & GIVEN DATA:





$2000 \le T_3 \le 5000°R$

$r = v_1/v_2 = 6, 8, 10$

$p_1 = 14.7\ lbf/in^2$

$T_1 = 530°R$

ASSUMPTIONS: Same as Example 9.1.

ANALYSIS: Sample calculation for $r = 8$, $T_3 = 2000°R$ using Table A-22E. From Table A-22E, $u_1 = 90.33$ Btu/lb, $v_{r_1} = 151.38$. For the compression process

$$v_{r_2} = \left(\frac{v_2}{v_1}\right) v_{r_1} = \left(\frac{1}{8}\right)(151.38) = 18.9225$$

Thus, $T_2 = 1190.7°R$ and $u_2 = 207.33$ Btu/lb.

For various values of T_3, u_3 and v_{r3} can be determined from Table A-22E. Then, state 4 is fixed using the relation for the isentropic expansion

$$v_{r4} = \left(\frac{v_4}{v_3}\right) v_{r3} = \left(\frac{v_1}{v_2}\right) v_{r3}$$

and u_4 can be evaluated from the tabular data. For $T_3 = 2000°R$

$$u_3 = 367.61\ Btu/lb,\ v_{r3} = 4.258,\ v_{r4} = 34.064,\ u_4 = 164.23\ Btu/lb$$

Continuing for this case, the heat added to the cycle is

$$\frac{Q_{23}}{m} = u_3 - u_2 = 160.28\ Btu/lb$$

and the heat rejected is

$$\frac{Q_{41}}{m} = u_4 - u_1 = 73.9$$

Thus, the thermal efficiency is

$$\eta = 1 - \frac{Q_{41}/m}{Q_{32}/m} = 0.539\ (53.9\%) \longleftarrow \eta$$

The mean effective pressure is

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)} = \frac{Q_{23}/m - Q_{41}/m}{v_1(1 - v_2/v_1)}$$

PROBLEM 9.9 (Cont'd)

Evaluating v_1

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(530°R)}{(14.7\ lbf/in^2)}\left|\frac{1\ ft^2}{144\ in^2}\right| = 13.35\ ft^3/lb$$

Thus

$$mep = \frac{(160.28 - 73.9)\ Btu/lb}{(13.35\ ft^3/lb)(1-\frac{1}{8})}\left|\frac{778\ ft\cdot lbf}{1\ Btu}\right|\left|\frac{1\ ft^2}{144\ in^2}\right| = 39.95\ lbf/in^2 \leftarrow \underline{mep}$$

The data for the required plots are obtained using *IT*, as follows:

IT Code

```
   r = 8
   p1 = 14.7    // lbf/in²
   T1 = 530  // °R
   T3 = 2000    // °R
   m = 1     // assume a unit mass of 1 lb.

   v1 = v_TP("Air",T1,p1)
   s1 = s_TP("Air", T1, p1)
   u1 = u_T("Air", T1)
①  s2 = s1
   v2 = v1 / r
   s2 = s_TP("Air", T2, p2)
   v2 = v_TP("Air",T2,p2)
   u2 = u_T("Air", T2)
   v3 = v2
   u3 = u_T("Air", T3)
   v3 = v_TP("Air",T3,p3)
   s3 = s_TP("Air", T3, p3)
   v4 = r * v3
   s4 = s3
   v4 = v_TP("Air",T4,p4)
   s4 = s_TP("Air", T4, p4)
   u4 = u_T("Air", T4)
```

```
Wcycle = Q23 - Q41
Q23 = m * (u3 - u2)
Q41 = m * (u4 - u1)
eta = Wcycle / Q23
V1 = v1 * m
V2 = v2 * m
mep = Wcycle * (778 / 144) / (V1 - V2)
```

IT Results for r = 8, T_3 = 2000°R

T_2 = 1191°R

T_4 = 953.6°R

p_2 = 264.2 lbf/in.²

p_3 = 443.8 lbf/in.²

v1 = 13.35 ft²/lb

Q_{23}/m = 160 Btu/lb

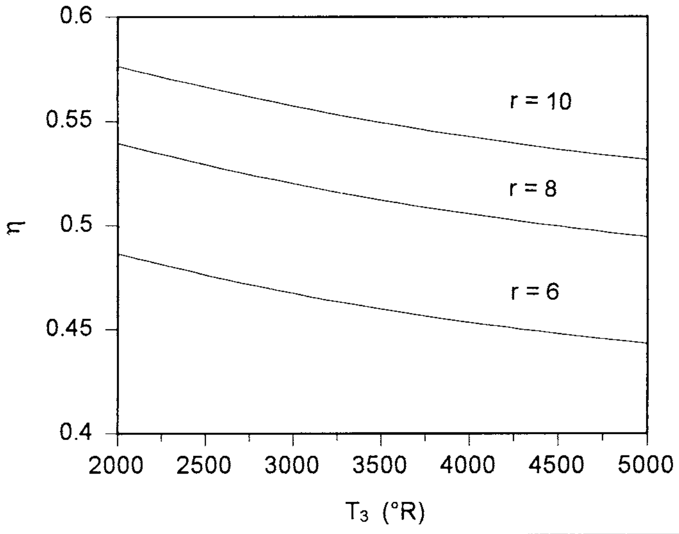
Q_{41}/m = 73.73 Btu/lb

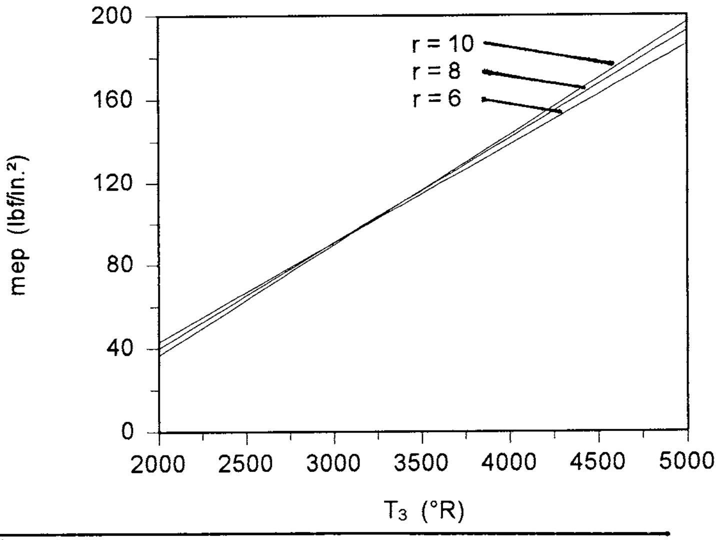
W_{cycle}/m = 86.29 Btu/lb

η = 0.5393

mep = 39.9 lbf/in.²

PLOTS:





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1. Note that IT evaluates s(T, p) directly, and thus does not utilize the special isentropic functions $v_r(T)$ and $p_r(T)$ as in Table A-22E.

PROBLEM 9.10

KNOWN: A cold air-standard Otto cycle has known conditions at the beginning of compression. The maximum cycle temperature varies over a given range and the compression ratio takes on specified values.

FIND: Plot thermal efficiency and mean effective pressure versus maximum cycle temperature for each compression ratio.

SCHEMATIC & GIVEN DATA: See solution to Problem 9.9.

ASSUMPTIONS: Same as Example 9.1 and $k = 1.4$.

ANALYSIS: First, determine the temperature at each principal state. T_1 is given, and $r = v_1/v_2$ is specified.

State 2: Isentropic compression; $T_2 = (v_1/v_2)^{k-1} T_1$

Also, for a particular value of T_3

State 4: $T_4 = (v_3/v_4)^{k-1} T_3 = (v_2/v_1)^{k-1} T_3$

Thus, with $k = 1.4$, c_v can be determined from Table A-20 ($c_v = 0.171$ Btu/lb·°R).

The thermal efficiency is

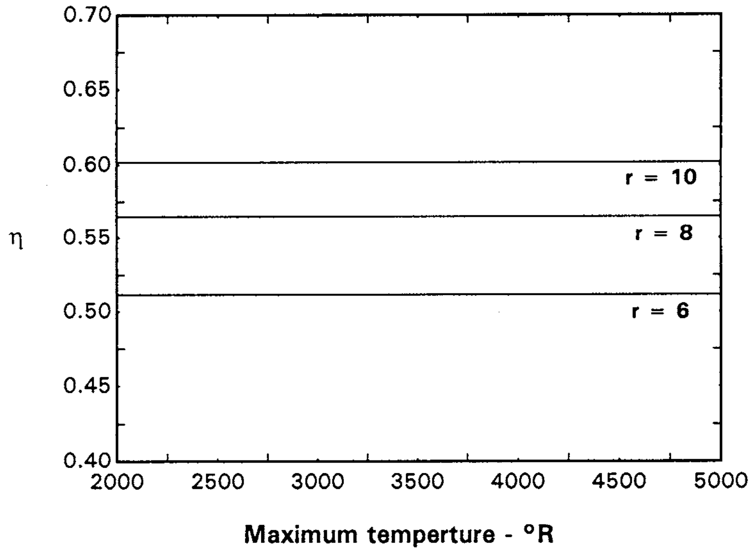
$$\eta = 1 - \frac{1}{r^{k-1}}$$

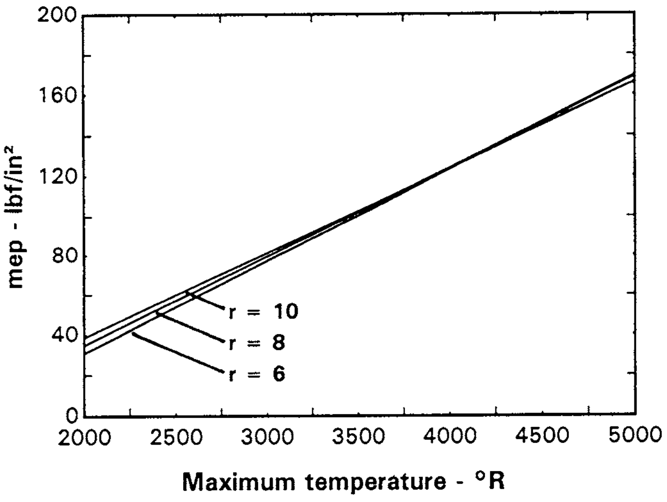
and

$$\frac{Q_{23}}{m} = c_v(T_3 - T_2) \;;\; \frac{Q_{41}}{m} = c_v(T_4 - T_1) \Rightarrow \frac{W_{cycle}}{m} = c_v\left[(T_3 - T_2) - (T_4 - T_1)\right]$$

$$\text{mep} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)} = \frac{c_v\left[(T_3 - T_2) - (T_4 - T_1)\right]}{v_1(1 - 1/r)}$$

Plotting for $r = 6, 8, 10$ and maximum cycle temperatures ranging from 2000 to 5000°R :



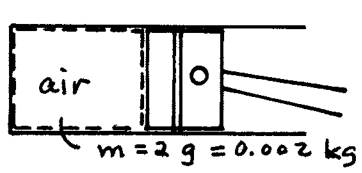


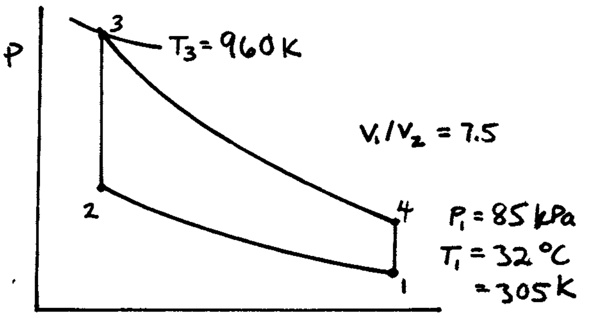
PROBLEM 9.11*

KNOWN: An air-standard Otto cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition and the maximum cycle temperature are given.

FIND: Determine (a) the heat rejection, (b) the net work, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 9.1

ANALYSIS: From the given data, $T_1 = 305$ K ; $u_1 = 217.67$ kJ/kg, $v_{r1} = 596.0$. Also, for $T_3 = 960$ K ; $u_3 = 725.02$ kJ/kg, $v_{r3} = 28.40$.
For the isentropic compression

$$v_{r2} = \frac{V_2}{V_1} v_{r1} = \left(\frac{1}{7.5}\right) 596.0 = 79.47$$

Thus, $T_2 = 367.4$ K, $u_2 = 486.77$ kJ/kg. Similarly, for the expansion

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = \frac{V_1}{V_2} v_{r3} = (7.5)\, 28.40 = 213.0$$

Thus, $T_4 = 458.7$ K, $u_4 = 329.01$ kJ/kg.

(a) Consider an energy balance for process 2-3

$$Q_{23} = m(u_3 - u_2) = (0.002 \text{ kg})(725.02 - 486.77) \text{ kJ/kg} = 0.4765$$

And, for process 4-1

$$Q_{41} = m(u_4 - u_1) = (0.002 \text{kg})(329.01 - 217.67) \text{kJ/kg}$$
$$= 0.2227 \text{ kJ} \longleftarrow \qquad Q_{41}$$

(b) The net work is

$$W_{cycle} = Q_{23} - Q_{41} = 0.4765 - 0.2227 = 0.2538 \text{ kJ} \longleftarrow \qquad W_{cycle}$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = 0.533 \; (53.3\%) \longleftarrow \qquad \eta$$

(d) To determine the mean effective pressure, first find V_1

$$V_1 = \frac{mRT_1}{p_1} = \frac{(0.002 \text{ kg})\left(\frac{8.314}{28.97} \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(305 \text{K})}{(85 \text{ kPa})} \left|\frac{1 \text{kPa}}{10^3 \text{N/m}^2}\right| \left|\frac{10^3 \text{N}\cdot\text{m}}{1 \text{kJ}}\right| = 2.06 \times 10^{-3} \text{m}^3$$

thus

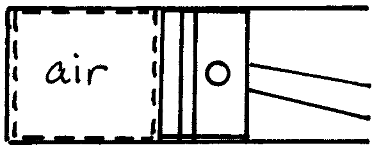
$$\text{mep} = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)} = \frac{(0.2538 \text{ kJ})}{(2.06 \times 10^{-3} \text{m}^3)(1 - 1/7.5)} \left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{kJ}}\right| \left|\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2}\right|$$
$$= 142.2 \text{ kPa} \longleftarrow \qquad \text{mep}$$

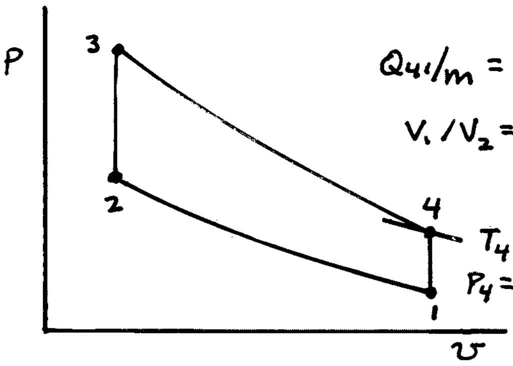
PROBLEM 9.12*

KNOWN: A cold air-standard Otto cycle has a known compression ratio and a specified state at the end of the expansion process. The heat rejection per unit mass of air is also given.

FIND: Determine (a) the net work per unit mass of air, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





$Q_{41}/m = 86$ Btu/lb

$V_1/V_2 = 9$

ASSUMPTIONS: Same as in Example 9.1 and $k = 1.4$.

ANALYSIS: Using the given heat rejection, $Q_{41}/m = u_4 - u_1$. Thus, with $\Delta u = c_v \Delta T$ and $c_v = 0.172$ Btu/lb·°R from Table A-20E

$$T_1 = T_4 - \frac{Q_{41}}{m \cdot c_v} = 1000 - \frac{86}{0.172} = 500 °R$$

For the isentropic compression $T_2/T_1 = (v_1/v_2)^{k-1}$. Therefore

$$T_2 = (9)^{1.4-1}(500) = 1204.1 °R$$

Also, for the isentropic expansion

$$T_3 = \left(\frac{v_4}{v_3}\right)^{k-1} T_4 = \left(\frac{v_1}{v_2}\right)^{k-1} T_4 = 2408.2 °R$$

(a) The net work per unit mass of air is

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = c_v(T_3 - T_2) - \frac{Q_{41}}{m} = 121.11\ \text{Btu/lb} \xleftarrow{\qquad W_{cycle}/m \qquad}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{W_{cycle}/m}{c_v(T_3 - T_2)} = \frac{121.11}{207.11} = 0.585\ (58.5\%) \xleftarrow{\qquad \eta \qquad}$$

(c) The mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - V_2/V_1)}$$

Evaluating the specific volume ($v_1 = v_4$)

$$v_4 = \frac{RT_4}{P_4} = \frac{\left(\frac{1545}{28.97} \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(1000\ °R)}{(95\ \text{lbf/in}^2)} \left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| = 3.898\ \text{ft}^3/\text{lb}$$

Thus

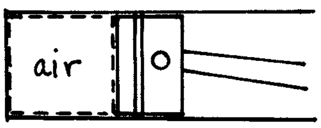
$$mep = \frac{(121.11\ \text{Btu/lb})}{(3.898\ \text{ft}^3/\text{lb})(1 - 1/9)} \left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| \left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right| = 188.8\ \text{lbf/in}^2 \xleftarrow{\qquad mep \qquad}$$

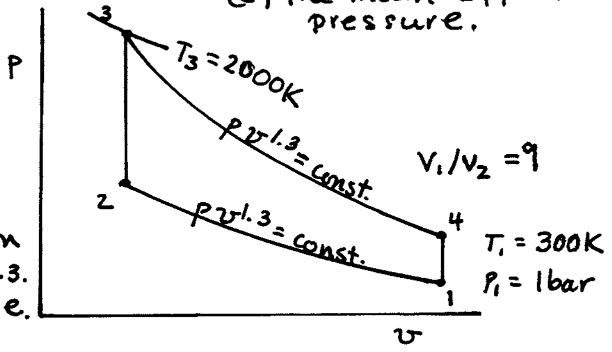
PROBLEM 9.13

KNOWN: An air-standard Otto cycle is modified by replacing the isentropic compression and expansion processes with polytropic processes having $n=1.3$. The compression ratio, state at the beginning of compression, and the maximum cycle temperature are all known.

FIND: Determine (a) the heat transfer and work, per unit mass, for each process in the cycle, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The air is the closed system. (2) The compression and expansion processes are polytropic, each with $n=1.3$. (3) All processes are internally reversible. (4) The air behaves as an ideal gas. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: Begin by fixing each of the four principal states. With $T_1 = 300K$, Table A-22 gives, $u_1 = 214.07$ kJ/kg. Next, using Eq. 3.56 for the polytropic compression

$$T_2 = T_1\left[\frac{V_1}{V_2}\right]^{n-1} = (300K)\left[9\right]^{0.3} = 580\,K$$. With Table A-22, $u_2 = 419.55$ kJ/kg.

Also, since $T_3 = 2000K$, $u_3 = 1678.7$ kJ/kg. For the polytropic expansion

$$T_4 = T_3\left[\frac{V_3}{V_4}\right]^{n-1} = 2000\left[\frac{1}{9}\right]^{0.3} = 1035\,K$$, and $u_4 = 789.05$ kJ/kg.

(a) Consider each process in series, beginning with process 1-2. With Eq. 3.57

$$\frac{W_{12}}{m} = \int_1^2 p\,dv = \frac{R(T_2-T_1)}{1-n} = \frac{(8.314/28.97\ kJ/kg\cdot K)(580-300)K}{(1-1.3)} = -267.9\ kJ/kg$$ ←

An energy balance gives

$$\frac{Q_{12}}{m} = (u_2-u_1) + \frac{W_{12}}{m} = (419.55 - 214.07) - 267.9 = -62.42\ kJ/kg$$ ←

Process 2-3: $W_{23} = 0$ and an energy balance gives ←

$$\frac{Q_{23}}{m} = (u_3-u_2) = (1678.7 - 419.55) = 1259.15\ kJ/kg$$ ←

Process 3-4: Using Eq. 3.57

$$\frac{W_{34}}{m} = \int_3^4 p\,dv = \frac{R(T_4-T_3)}{1-n} = \frac{(8.314/28.97)(1035-2000)}{-(0.3)} = 923.1\ kJ/kg$$ ←

An energy balance gives $Q_{34}/m = (u_4-u_3) + W_{34}/m = (789.05 - 1678.7) + 923.1 = 33.45\ \frac{kJ}{kg}$ ←

Process 4-1: $W_{41} = 0$ and an energy balance gives ←

① $$\frac{Q_{41}}{m} = u_1 - u_4 = 214.07 - 789.05 = -574.98\ kJ/kg$$ ←

PROBLEM 9.13 (Contd.)

(b) The thermal efficiency is $\eta = (W_{cycle}/m)/(Q_{in}/m)$, where

$$\frac{W_{cycle}}{m} = \frac{W_{12}}{m} + \frac{W_{34}}{m} = -267.9 + 923.1 = 655.2 \text{ kJ/kg}$$

$$\frac{Q_{in}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} = 1259.15 + 33.45 = 1292.6 \text{ kJ/kg}$$

Thus

$$\eta = \frac{655.2}{1292.6} = 0.507 \quad (50.7\%)$$ ← η

(c) The mean effective pressure is

$$mep = \frac{W_{cycle}/m}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1\left[1 - \frac{v_2}{v_1}\right]}$$

where

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8314}{28.97}\ \frac{N\cdot m}{kg\cdot K}\right)(300K)}{10^5\ N/m^2} = 0.861\ m^3/kg$$

Then

$$mep = \frac{655.2\ kJ/kg}{(0.861\frac{m^3}{kg})\left[1 - \frac{1}{9}\right]}\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ bar}{10^5\ N/m^2}\right| = 8.56\ bar$$ ← mep

---

1. For any cycle $Q_{cycle} = W_{cycle}$. In this case, $W_{cycle}/m = 655.2$ kJ/kg and

$$\frac{Q_{cycle}}{m} = -62.42 + 1259.15 + 33.45 - 574.98 = 655.2 \text{ kJ/kg}$$

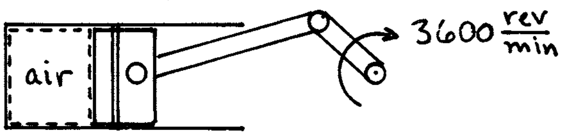
which serves to check the results obtained in part (a).

PROBLEM 9.14*

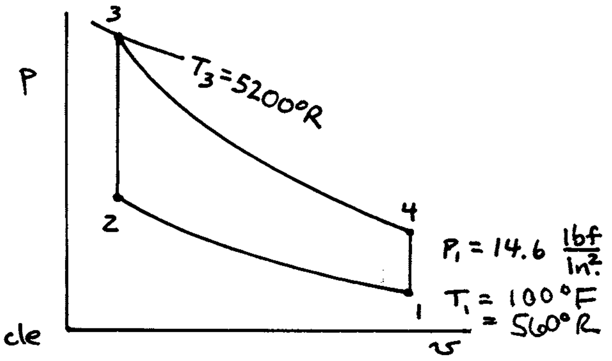
KNOWN: The bore, stroke, clearance volume, and speed of crankshaft rotation of a four-stroke, four-cylinder internal combustion engine are known. The processes within the cylinders are modeled by an Otto cycle with a known state at the beginning of compression and maximum cycle temperature.

FIND: Determine the net work per cycle and the power developed by the engine.

SCHEMATIC & GIVEN DATA:



bore = 2.55 in.
stroke = 2.10 in.
$V_2 = 0.12 V_1$



ASSUMPTIONS: Assumptions of Otto cycle are the same as in Example 9.1

ANALYSIS: Using the given data,

$$V_1 - V_2 = \frac{\pi\,(\text{bore})^2}{4} \times (\text{stroke}) = \frac{\pi (2.55)^2 (2.10)}{4}\ in^3 \left| \frac{1\ ft^3}{12^3\ in^3} \right|$$
$$= 0.006207\ ft^3$$

With $V_2 = 0.12\,V_1$, $V_1 = 0.007053\ ft^3$. Thus, the mass of air is

$$m = \frac{P_1 V_1}{R T_1} = \frac{(14.6\ lbf/in^2)(0.007053\ ft^3)}{\left(\frac{1545}{28.97}\ \frac{ft \cdot lbf}{lb \cdot °R}\right)(560\ °R)} \left| \frac{144\ in^2}{1\ ft^2} \right| = 0.000497\ lb$$

Next, fix each of the principal states of the cycle. From Table A-22E, at $T_1 = 560\ °R$; $u_1 = 95.47$ Btu/lb, $v_{r1} = 131.78$.
Thus, for the isentropic compression.

$$v_{r2} = \frac{V_2}{V_1} v_{r1} = (0.12)\,131.78 = 15.81$$

Thus, we get $u_2 = 222.48$ Btu/lb. At $T_3 = 5200°R$; $u_3 = 1098.0$ Btu/lb and $v_{r3} = 0.1828$. For the isentropic expansion

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = \frac{V_1}{V_2} v_{r3} = 1.523$$

Using this, we get $u_4 = 534.69$ Btu/lb.

The net work per cycle is

$$W_{cycle} = Q_{23} - Q_{41} = m\left[(u_3 - u_2) - (u_4 - u_1)\right]$$
$$= (0.000497\ lb)\left[(1098.0 - 222.48) - (534.69 - 95.47)\right]\ Btu/lb$$
$$= 0.217\ Btu \longleftarrow W_{cycle}$$

For a four-stroke engine, the cycle is completed once for every two revolutions of the crankshaft. For four cylinders, the net power is

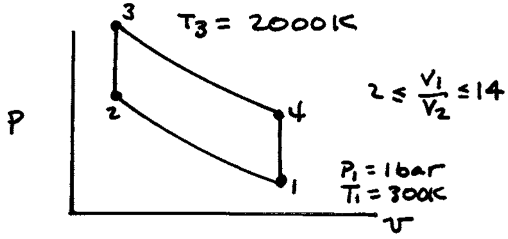
$$\dot{W}_{net} = (4)\,(1800\ cycles/min)(0.217\ Btu/cycle)\left|\frac{1\ hp}{42.42\ Btu/min}\right| = 36.8\ hp \longleftarrow \dot{W}_{net}$$

PROBLEM 9.15

KNOWN: For an air-standard Otto cycle, the state at the beginning of compression and the maximum cycle temperature are known.

FIND: Plot the net work per unit of mass, the thermal efficiency, and the mean effective pressure versus compression ratio ranging from 2 to 14.

SCHEMATIC & GIVEN DATA: ASSUMPTIONS: See Example 9.1.



ANALYSIS: The following reasoning is used to solve the problem:

1. Given values of p_1, T_1, r, and T_3.
2. Fix state 2 using ideal gas relations and $s(T_1, p_1) = s(T_2, p_2)$. When using Table A-22, this is expressed in terms of $v_{r_1}/v_{r_2} = v_1/v_2$.
3. Use $v_2 = v_3$ and the ideal gas model to get p_3.
4. Fix state 4 as in step 2; use $s(T_3, p_3) = s(T_4, p_4)$, or $v_{r4}/v_{r3} = v_4/v_3$.

Now, using energy balances, and the fact that $W_{23} = W_{41} = 0$

$$Q_{23} = m(u_3 - u_2) \qquad (1)$$

$$Q_{41} = m(u_1 - u_4) \qquad (2)$$

Further, for the cycle

$$W_{cycle}/m = Q_{23}/m + Q_{41}/m \qquad (3)$$

and

$$\eta = W_{cycle}/Q_{23} \qquad (4)$$

Finally

$$v_1 = \frac{RT_1}{p_1} \; ; \; mep = \frac{W_{cycle}}{m v_1 (1 - \frac{1}{r})} \qquad (5)$$

Sample calculation: $\boxed{r = 5}$ From Table A-22, at $p_1 = 1bar$, $T_1 = 300K$; $u_1 = 214.07$ kJ/kg, $v_{r_1} = 621.2$. Thus

$$v_{r_2} = \frac{v_2}{v_1} v_{r_1} = \left(\frac{1}{r}\right) v_{r_1} = \left(\frac{1}{5}\right)(621.2) = 124.24 \Rightarrow \begin{cases} T_2 = 564.8K \\ u_2 = 408.0K \end{cases}$$

Now, $T_3 = 2000$ K $\Rightarrow u_3 = 1678.7$ kJ/kg, $v_{r_3} = 2.776$. Thus, for state 4

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = \frac{v_1}{v_2} = 5 \Rightarrow v_{r4} = 13.88 \Rightarrow T_4 = 1216.32\text{ K}, \; u_4 = 947.82\text{ kJ/kg}.$$

Thus, with (1) – (5)

$$W_{cycle}/m = (u_3 - u_2) + (u_1 - u_4) = (1678.7 - 408.0) + (214.07 - 947.82)$$

$$= 1270.7 - 733.75 = 536.95\text{ kJ/kg} \longleftarrow W_{cycle}/m$$

and

$$\eta = \frac{536.95}{1270.7} = 0.4226\ (42.26\%) \longleftarrow \eta$$

Finally

$$v_1 = \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(300K)}{(1\text{ bar})} \left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right| \left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right| = 0.861\text{ m}^3\text{/kg}$$

$$mep = \frac{536.95\text{ kJ/kg}}{(0.861\text{ m}^3\text{/kg})(1 - \frac{1}{5})} \left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right| \left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right| = 7.795\text{ bar} \longleftarrow mep$$

PROBLEM 9.15 (Cont'd.)

IT solution. The data for the required plots are obtained using IT, as follows:

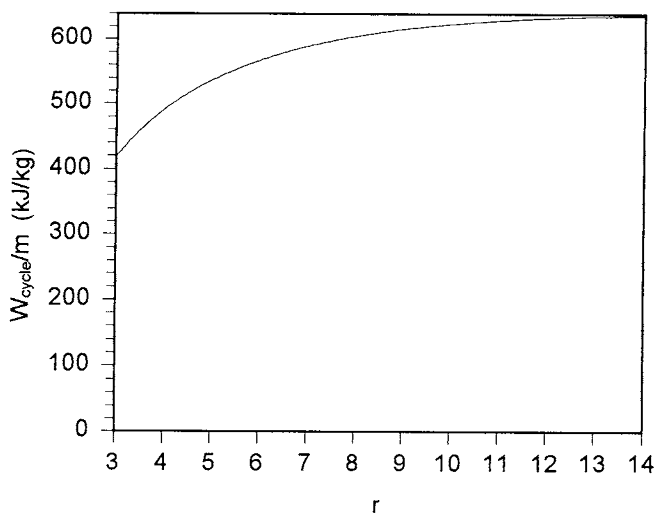
IT Code

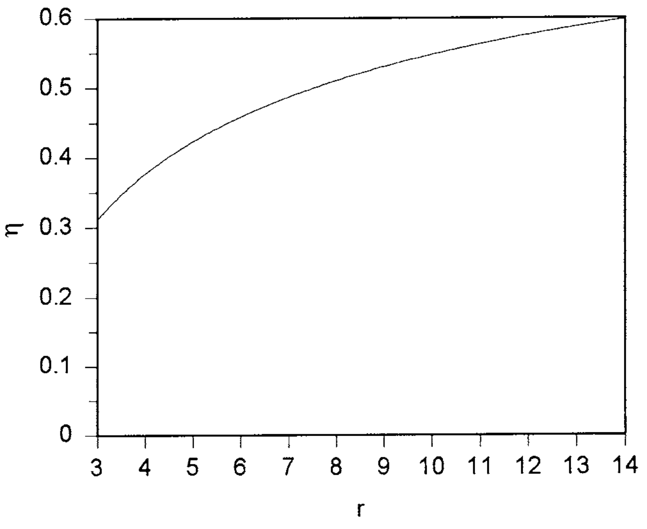
```
p1 = 1  // bar
T1 = 300  // K
r = 5
T3 = 2000  // K
m = 1  //  Assume a unit mass of 1 kg.

v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
v2 = v1 / r
v2 = v_TP("Air",T2,p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
v3 = v_TP("Air",T3, p3)
v3 = v2
u3 = u_T("Air", T3)
s3 = s_TP("Air", T3, p3)
v4 = v1
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
u4 = u_T("Air", T4)

Q23 = m * (u3 - u2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / Q23
mep = (Wcycle / (m * v1 * (1 - 1/r))) / 100   // bar
```

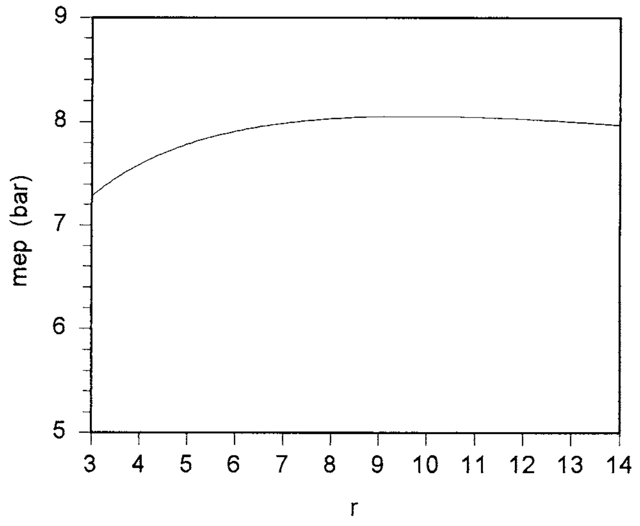
IT Results for r = 5

T_2 = 564.6 K
T_4 = 1215 K
p_3 = 33.33 bar
v1 = 0.861 m³/kg
Q_{23}/m = 1269 kJ/kg
Q_{41}/m = -732.3 kJ/kg
W_{cycle}/m = 536.3 kJ/kg
η = 0.4227
mep = 7.787 bar

PLOTS:







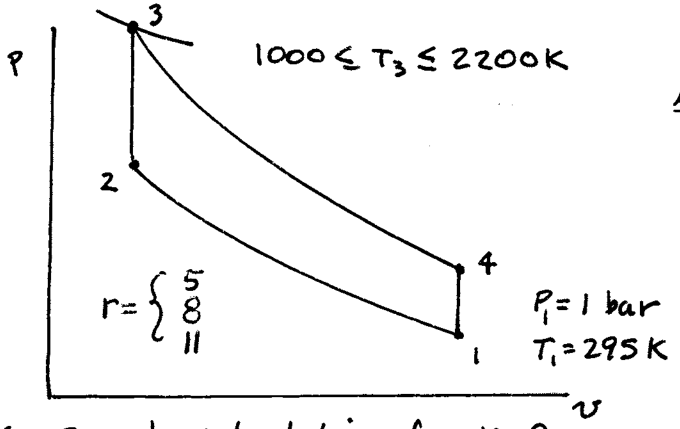
PROBLEM 9.16

KNOWN: An air-standard Otto cycle has known conditions at the beginning of compression. The maximum cycle temperature varies over a given range and the compression ratio takes on specified values.

FIND: Investigate the effect of maximum cycle temperature on the net work per unit mass of air.

SCHEMATIC & GIVEN DATA:

①



ASSUMPTIONS: Same as Example 9.1.

ANALYSIS: Sample calculation for $r = 8$, $T_3 = 1000$K using Table A-22.

From Table A-22, $u_1 = 210.49$ kJ/kg, $v_{r_1} = 647.9$. For the compression process

$$v_{r_2} = \left(\frac{v_2}{v_1}\right) v_{r_1} = \left(\frac{1}{8}\right)(647.9) = 80.99 \Rightarrow T_2 = 662.74 \text{ K}, u_2 = 483.15 \text{ kJ/kg}$$

Now, at $T_3 = 1000$ K; $u_3 = 758.94$ kJ/kg, $v_{r_3} = 25.17$. For the expansion

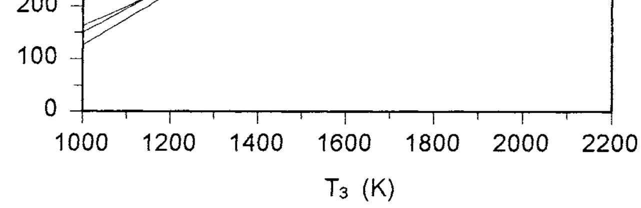
$$v_{r_4} = \left(\frac{v_4}{v_3}\right) v_{r_3} = \left(\frac{v_1}{v_2}\right) v_{r_3} = (8)(25.17) = 201.36 \Rightarrow u_4 = 336.50 \text{ kJ/kg}$$

Since $W_{23} = W_{41} = 0$; $Q_{23}/m = (u_3 - u_2)$ and $Q_{41}/m = (u_1 - u_4)$. For the cycle, $W_{cycle} = Q_{cycle}$. Thus

$$W_{cycle}/m = (u_3 - u_2) + (u_1 - u_4) = (758.94 - 483.15) + (210.49 - 336.50)$$
$$= 149.78 \text{ kJ/kg}$$

```
IT Code
p1 = 1  // bar
T1 = 295  // K
T3 = 1000  // K
r = 8
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
v2 = v1 / r
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
v3 = v_TP("Air",T3, p3)
s3 = s_TP("Air", T3, p3)
u3 = u_T("Air", T3)
v3 = v2
v4 = v1
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
u4 = u_T("Air", T4)
Wcycle = (u3 - u2) + (u1 - u4)
```

IT Results for r = 8 and T3 = 1000K
u_1 = 210.4 kJ/kg
T_2 = 662.8 K
u_2 = 483 kJ/kg
u_3 = 758.6 kJ/kg
T_4 = 468.6 K
u_4 = 336.2 kJ/kg
W_{cycle}/m = 149.8 kJ/kg



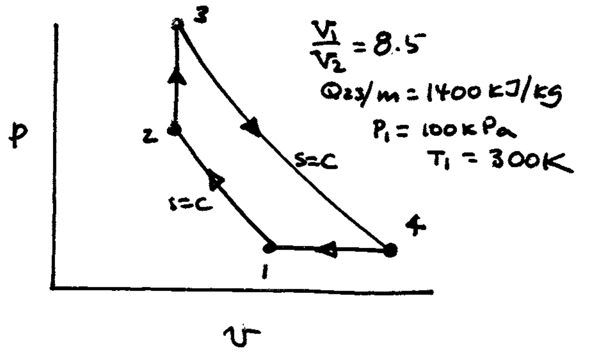
We see from the plot that the net work increases with T_3 at fixed compression ratio. Also, the plots cross each other, indicating the relation of W_{cycle} to r differs depending on T_3.

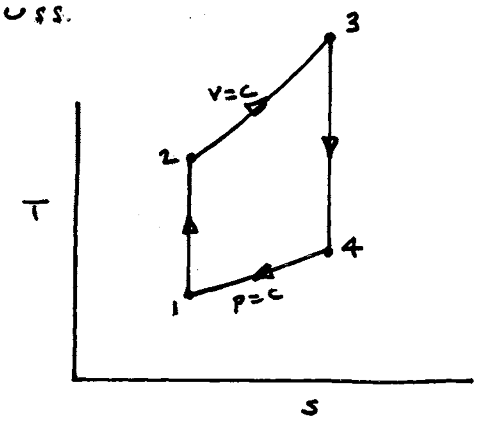
PROBLEM 9.17

KNOWN: An air-standard Atkinson cycle has a known compression ratio for isentropic compression and a specified state at the beginning of compression. The heat addition per unit of mass is also known.

FIND: (a) Sketch the cycle on T-s coordinates. Determine (b) the net work, (c) thermal efficiency, and (d) mean effective pressure. Compare with the Otto cycle of Example 9.1 and discuss.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.1

ANALYSIS: For part (a), see above. For parts (b)-(d), begin by fixing each of the four principle states using data from Table A-22.

State 1: $p_1 = 100$kPa, $T_1 = 300$K $\Rightarrow$ $u_1 = 214.07$ kJ/kg, $v_{r_1} = 621.2$, $p_{r_1} = 1.3860$

State 2: For isentropic compression

$$v_{r_2} = v_{r_1}\frac{V_2}{V_1} = 621.2\left(\frac{1}{8.5}\right) = 73.082$$

Thus, $T_2 = 688.2$K, $u_2 = 503.06\ \frac{kJ}{kg}$, $p_{r2} = 27.034$

and $$p_2 = p_1\frac{p_{r2}}{p_{r1}} = (100\ kPa)\left(\frac{27.034}{1.3860}\right) = 1950.5\ kPa$$

State 3 The specific internal energy u_3 is found using the energy balance for process 2-3

$$m(u_3 - u_2) = Q_{23} - \cancel{W_{23}}^{\,0}$$

$$u_3 = \frac{Q_{23}}{m} + u_2 = 1400 + 503.06 = 1903.06\ kJ/kg$$

Thus, $T_3 = 2231.3$ K, $p_{r3} = 3342.4$

and $$p_3 = p_2\left(\frac{T_3}{T_2}\right) = 1950.5\left(\frac{2231.3}{688.2}\right) = 6324\ kPa$$

State 4 For the isentropic expansion

$$p_{r4} = p_{r3}\left(\frac{p_4}{p_3}\right) = p_{r3}\left(\frac{p_1}{p_3}\right) = 52.853$$

Thus, $T_4 = 821$ K, $h_4 = 845.14$ kJ/kg

(a) To find the net work per unit mass, note that $W_{cycle} = Q_{cycle}$, so

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = \frac{Q_{23}}{m} - (h_4 - h_1)$$

$$= 1400 - (845.14 - 300.19)$$

$$= 855.1\ kJ/kg \longleftarrow \frac{W_{cycle}}{m}$$

PROBLEM 9.17 (Cont'd)

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{855.1}{1400} = 0.611\ (61.1\%) \longleftarrow \eta$$

(c) The overall displacement volume is $V_4 - V_2 = m(v_4 - v_2)$, so the mean effective pressure is given by

$$\text{mep} = \frac{W_{cycle}}{V_4 - V_2} = \frac{W_{cycle}/m}{v_4 - v_2} = \frac{W_{cycle}/m}{v_4(1 - v_2/v_4)}$$

Evaluating v_4

$$v_4 = \frac{RT_4}{p_4} = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(821\ \text{K})}{(100\ \text{kPa})}\left|\frac{1\ \text{kPa}}{10^3\ \text{N/m}^2}\right|\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|$$

$$= 2.356\ \text{m}^3/\text{kg}$$

Now, the specific volume ratio v_2/v_4 is

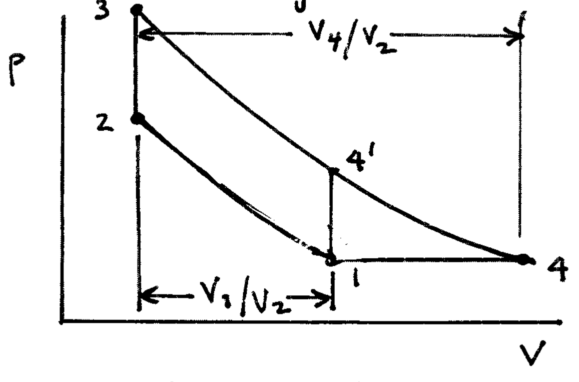
$$\frac{v_2}{v_4} = \frac{v_2}{v_1}\cdot\frac{v_1}{v_4} = \underbrace{\frac{v_2}{v_1}\cdot\frac{T_1}{T_4}}_{p_1 = p_4} = \left(\frac{1}{8.5}\right)\left(\frac{300}{821}\right) = 0.04299$$

Thus

$$\text{mep} = \frac{(855.1\ \text{kJ/kg})}{(2.356\ \text{m}^3/\text{kg})(1 - .04299)}\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\left|\frac{1\ \text{kPa}}{10^3\ \text{N/m}^2}\right|$$

$$= 379.2\ \text{kPa} \longleftarrow \text{mep}$$

DISCUSSION: The particular Atkinson cycle in this problem has the compression ratio for process 1-2 as for the Otto cycle in Problem 9.1. However, the isentropic expansion (process 3-4) has a much higher volume ratio than for the Otto cycle. This is illustrated on the following pressure-volume diagram:



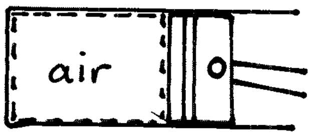
The results indicate that the work and thermal efficiency increase for the Atkinson cycle, as evidenced by the larger enclosed area. Further, the mean effective pressure decreases.

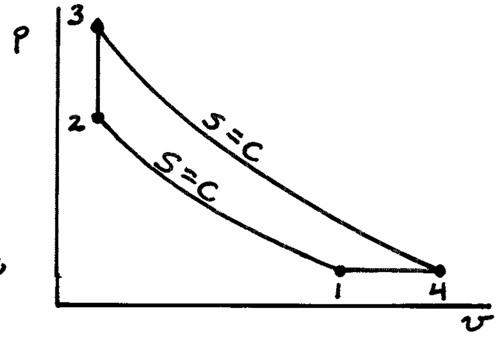
PROBLEM 9.18

KNOWN: Air undergoes a cold air-standard Atkinson cycle.

FIND: Derive an expression for thermal efficiency and compare with the cold air-standard Otto cycle.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.1. Also, assume constant specific heats.

ANALYSIS: To find the net work, note that $W_{cycle} = Q_{cycle}$. Thus

$$\frac{W_{cycle}}{m} = \frac{Q_{in}}{m} - \frac{Q_{out}}{m} = c_v(T_3 - T_2) - c_p(T_4 - T_1)$$

Therefore, the thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = 1 - \frac{c_p(T_4 - T_1)}{c_v(T_3 - T_2)} = 1 - \frac{kT_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)} \quad (1)$$

Process 1-2 is isentropic. Thus $T_2/T_1 = (v_1/v_2)^{k-1} \equiv r^{k-1}$ (2)

Further, for process 2-3: $\frac{T_3}{T_2} = \frac{P_3}{P_2} \equiv r_p$ (3)

And, for process 4-1: $\frac{T_4}{T_1} = \frac{v_4}{v_1} = \frac{v_4}{v_3} \cdot \frac{v_3}{v_1} = \frac{v_4}{v_3} \cdot \frac{1}{r}$ (4)

Now, for the isentropic compression: $\frac{1}{r} = \left(\frac{P_4}{P_2}\right)^{1/k}$ (5)

and, for the isentropic expansion: $\frac{v_4}{v_3} = \left(\frac{P_3}{P_4}\right)^{1/k}$ (6)

Now, combining (4), (5), and (6)

$$\frac{T_4}{T_1} = \left(\frac{P_3}{P_4}\right)^{\frac{1}{k}} \left(\frac{P_4}{P_2}\right)^{\frac{1}{k}} = \left(\frac{P_3}{P_2}\right)^{\frac{1}{k}} = r_p^{\frac{1}{k}} \quad (7)$$

Finally, inserting the results of (2), (3), and (7) into (1)

$$\eta = 1 - \frac{k\,(r_p^{1/k} - 1)}{r^{k-1}\,(r_p - 1)} \longleftarrow$$

n

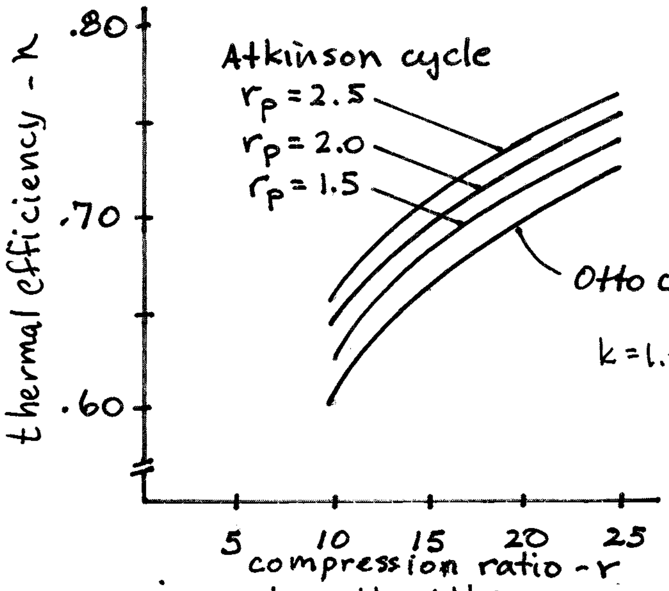
To compare the thermal efficiency of the Atkinson and Otto cycles, recall that

$$\eta_{Otto} = 1 - \frac{1}{r^{k-1}}$$

Note that the thermal efficiency of the Otto cycle depends on r and k, but not on the maximum cycle temperature (nor the

PROBLEM 9.18 (Cont'd)

pressure ratio, r_p.) The following graph can be used to compare the cycles further:



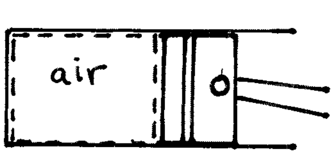
Thus, at any compression ratio, the Atkinson cycle has a higher thermal efficiency than the corresponding Otto cycle. Also, as the pressure ratio r_p increases at fixed compression ratio (r) the thermal efficiency increases as well.

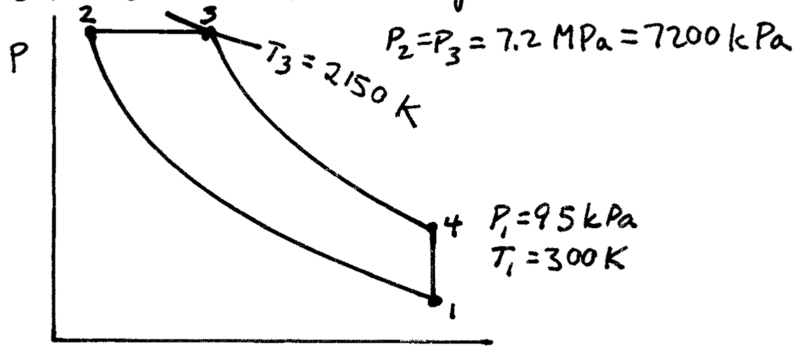
PROBLEM 9.19*

KNOWN: An air-standard Diesel cycle has a specified state at the beginning of compression and a known pressure and temperature at the end of heat addition.

FIND: Determine (a) the compression ratio, (b) the cutoff ratio, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.2.

ANALYSIS: Begin by fixing each principal state in the cycle (Table A-22).

State 1: $T_1 = 300$ K, $p_1 = 95$ kPa ⇒ $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$, $p_{r1} = 1.3860$

State 2: For the isentropic compression

$$p_{r2} = p_{r1}\left(\frac{p_2}{p_1}\right) = (1.3860)\left(\frac{7200}{95}\right) = 105.04$$

Thus, $T_2 = 979.6$K, $v_{r2} = 26.793$, $h_2 = 1022.82$ kJ/kg

State 3: $T_3 = 2150$ K, $p_3 = 7200$ kPa ⇒ $h_3 = 2440.3$ kJ/kg, $v_{r3} = 2.175$

State 4: For the isentropic expansion

$$\frac{v_4}{v_3} = \frac{v_1}{v_2} \cdot \frac{v_2}{v_3} = \frac{v_1}{v_2} \cdot \frac{T_2}{T_3} = \frac{v_{r1}}{v_{r2}} \cdot \frac{T_2}{T_3} = \frac{621.2}{26.793} \cdot \frac{979.6}{2150} = 10.56$$

$$v_{r4} = \frac{v_4}{v_3} \cdot v_{r3} = 22.98 \quad \Rightarrow T_4 = 1031 \text{ K}, \; u_4 = 785.75 \text{ kJ/kg}$$

(a) The compression ratio is

$$r = \frac{v_1}{v_2} = \frac{v_{r1}}{v_{r2}} = \frac{621.2}{26.793} = 23.19 \longleftarrow r$$

(b) The cutoff ratio is

$$r_c = \frac{v_3}{v_2} = \frac{T_3}{T_2} = \frac{2150}{979.6} = 2.19 \longleftarrow r_c$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{(h_3 - h_2) - (u_4 - u_1)}{h_3 - h_2}$$

$$= \frac{(2440.3 - 1022.82) - (785.75 - 214.07)}{(2440.3 - 1022.82)}$$

$$= \frac{845.80}{1417.48} = 0.597 \; (59.7\%) \longleftarrow \eta$$

PROBLEM 9.19 (Cont'd)

(d) The mean effective pressure is given as

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\,\frac{kJ}{kg\cdot K}\right)(300\ K)}{(95\ kPa)}\left|\frac{1\ kPa}{10^3\ N/m^2}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|$$

$$= 0.9063\ m^3/kg$$

Thus

$$mep = \frac{(845.80\ kJ/kg)}{\left(0.9063\ \frac{m^3}{kg}\right)\left(1 - 1/23.19\right)}\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ kPa}{10^3\ N/m^2}\right|$$

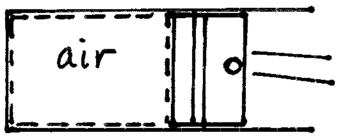
$$= 975\ kPa \xleftarrow{\hspace{8em}} mep$$

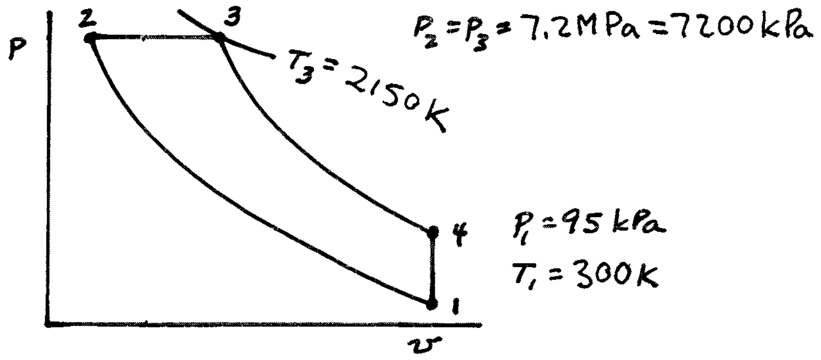
PROBLEM 9.20*

KNOWN: An air-standard Diesel cycle has a specified state at the beginning of compression and a known pressure and temperature at the end of heat addition.

FIND: Determine (a) the compression ratio, (b) the cutoff ratio, (c) the thermal efficiency, and (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.2. Also, assume constant specific heats evaluated at 300 K.

ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 300$ K, $T_3 = 2150$ K are given.

State 2: For the isentropic compression, with $k = 1.4$ from Table A-20

$$T_2 = \left(\frac{P_2}{P_1}\right)^{k-1/k} T_1 = 1033.1 \text{ K}$$

State 4: Similarly, for the expansion

$$\frac{v_4}{v_3} = \frac{v_1}{v_2} \cdot \frac{v_2}{v_3} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{k-1}} \cdot \frac{T_2}{T_3} = 10.57$$

and
$$T_4 = \left(\frac{v_3}{v_4}\right)^{k-1} T_3 = 837.0 \text{ K}$$

(a) The compression ratio is: $r = \dfrac{v_1}{v_2} = \left(\dfrac{T_2}{T_1}\right)^{1/k-1} = 22.01 \longleftarrow$ r

(b) The cutoff ratio is: $r_c = \dfrac{v_3}{v_2} = \dfrac{T_3}{T_2} = 2.081 \longleftarrow$ r

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

With $c_v = .718$ kJ/kg·K and $c_p = 1.005$ kJ/kg·K from Table A-20

$$\eta = \frac{(1.005)(2150 - 1033.1) - (.718)(837.0 - 300)}{(1.005)(2150 - 1033.1)} = \frac{736.9}{1122.5}$$

$= 0.657$ (65.7%) $\longleftarrow$ η

(d) The mean effective pressure is given as

$$\text{mep} = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{v_1(1 - v_2/v_1)}$$

Evaluating v_1

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300 \text{ K})}{(95 \text{ kPa})}\left|\frac{1 \text{ kPa}}{10^3 \text{ N/m}^2}\right|\left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right|$$

$= 0.9063$ m³/kg

PROBLEM 9.20 (Cont'd)

Thus

$$\text{mep} = \frac{(763.9\ \text{kJ/kg})}{(0.9063\ \text{m}^3/\text{kg})\left(1 - \frac{1}{22.01}\right)}\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\left|\frac{1\ \text{kPa}}{10^3\ \text{N/m}^2}\right|$$

$$= 883.0\ \text{kPa} \longleftarrow \underline{\text{mep}}$$

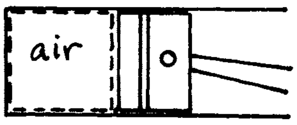
These results can be compared to those of Problem 9.19 to see some effects of the assumption of constant specific heats.

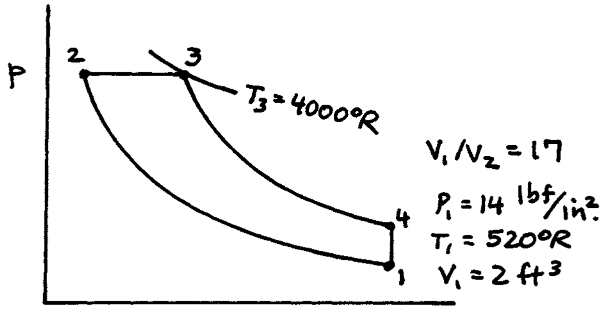
PROBLEM 9.21

KNOWN: An air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression. The maximum cycle temperature is given.

FIND: Determine (a) the net work, (b) the thermal efficiency, (c) the mean effective pressure, and (d) the cutoff ratio.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.2

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22E).

State 1: $T_1 = 520°R \Rightarrow u_1 = 88.62$ Btu/lb , $v_{r_1} = 158.58$

State 2: For the isentropic compression

$$v_{r_2} = \left(\frac{V_2}{V_1}\right) v_{r_1} = \left(\frac{1}{17}\right) 158.58 = 9.3282$$

Thus, $T_2 = 1534.5$ °R , $h_2 = 378.32$ Btu/lb

State 3: $T_3 = 4000°R \Rightarrow h_3 = 1088.3$ Btu/lb , $v_{r_3} = .4518$

State 4: For the isentropic expansion

$$\frac{V_4}{V_3} = \frac{V_1}{V_2} \cdot \frac{V_2}{V_3} = \frac{V_1}{V_2} \cdot \frac{T_2}{T_3} = (17)\,\frac{1534.5}{4000} = 6.522$$

and

$$v_{r_4} = \frac{V_4}{V_3} v_{r_3} = 2.9466$$

Thus, $T_4 = 2253.7$ °R , $u_4 = 421.25$ Btu/lb

(a) For the cycle, $W_{cycle} = Q_{cycle}$. Thus

$$W_{cycle} = Q_{23} - Q_{41} = m\left[(h_3 - h_2) - (u_4 - u_1)\right]$$

Evaluating m

$$m = \frac{P_1 V_1}{R T_1} = \frac{(14.0\ \text{lbf/in.}^2)(2\ \text{ft}^3)}{\left(\frac{1545}{28.97} \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{lb} \cdot °\text{R}}\right)(520°\text{R})} \left| \frac{144\ \text{in.}^2}{1\ \text{ft}^2} \right| = 0.1454\ \text{lb}$$

Thus

$$W_{cycle} = (.1454\ \text{lb})\left[(1088.3 - 378.32) - (421.25 - 88.62)\right] \frac{\text{Btu}}{\text{lb}}$$

$$= 54.87\ \text{Btu} \longleftarrow W_{cycle}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{W_{cycle}}{m(h_3 - h_2)}$$

$$= \frac{(54.87\ \text{Btu})}{(.1454\ \text{lb})(1088.3 - 378.32)\text{Btu/lb}} = 0.531\ (53.1\%) \longleftarrow \eta$$

PROBLEM 9.21 (Contd.)

(c) The mean effective pressure is

$$\text{mep} = \frac{W_{\text{cycle}}}{(V_1 - V_2)} = \frac{W_{\text{cycle}}}{V_1(1 - V_2/V_1)}$$

$$= \frac{(54.87 \text{ Btu})}{(2 \text{ ft}^3)(1 - 1/17)} \left| \frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}} \right| \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right|$$

$$= 157.5 \text{ lbf/in}^2 \longleftarrow \text{mep}$$

(d) The cutoff ratio is determined as follows:

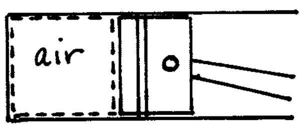
$$r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{4000}{1534.5} = 2.61 \longleftarrow r_c$$

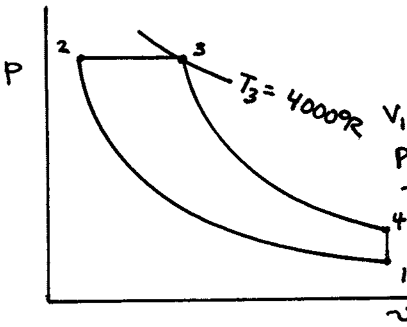
PROBLEM 9.22

KNOWN: A cold air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression. The maximum cycle temperature is given.

FIND: Determine (a) the net work, (b) the thermal efficiency, (c) the mean effective pressure, and (d) the cut off ratio.

SCHEMATIC & GIVEN DATA:





$V_1/V_2 = 17$
$p_1 = 14.0$ lbf/in.²
$T_1 = 520°R$
$V_1 = 2$ ft³

ASSUMPTIONS: See Example 9.2.
Also, assume constant specific heats evaluated at 520°R

ANALYSIS: First, determine the temperature at each principal state of the cycle. $T_1 = 520°R$ and $T_3 = 4000°R$ are given.

State 2: For the isentropic compression, with $k = 1.401$ from Table A-20E

$$T_2 = (V_1/V_2)^{k-1} T_1 = 1619.6\ °R$$

State 3: Similarly, for the expansion

$$\frac{V_4}{V_3} = \frac{V_1}{V_2}\cdot\frac{V_2}{V_3} = \frac{V_1}{V_2}\cdot\frac{T_2}{T_3} = (17)\frac{1619.6}{4000} = 6.8833$$

and

$$T_4 = (V_3/V_4)^{k-1} T_3 = 1845.5\ °R$$

(a) For the cycle, $W_{cycle} = Q_{cycle}$. Thus

$$W_{cycle} = Q_{23} - Q_{41} = m[c_p(T_3 - T_2) - c_v(T_4 - T_1)]$$

Evaluating m

$$m = \frac{p_1 V_1}{R T_1} = \frac{(14.0\ \text{lbf/in.}^2)(2\ \text{ft}^3)}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(520°R)}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 0.1454\ \text{lb}$$

Thus, with $c_p = 0.240$ Btu/lb·°R and $c_v = 0.171$ Btu/lb·°R

$$W_{cycle} = (.1454)[(.240)(4000 - 1619.6) - (.171)(1845.5 - 520)]$$
$$= 50.11\ \text{Btu} \longleftarrow W_{cycle}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{W_{cycle}}{m c_p (T_3 - T_2)} = \frac{(50.11)}{(.1454)(.240)(4000 - 1619.6)} = .603\ (60.3\%) \longleftarrow \eta$$

(c) The mean effective pressure is

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)}$$
$$= \frac{(50.11\ \text{Btu})}{(2\ \text{ft}^3)(1 - 1/17)}\left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right|\left|\frac{1\ \text{ft}^2}{144\ \text{in.}^2}\right| = 143.8\ \text{lbf/in.}^2 \longleftarrow mep$$

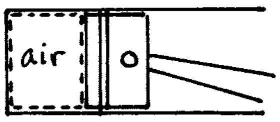
(d) $r_c = V_3/V_2 = T_3/T_2 = 4000/1619.6 = 2.47 \longleftarrow r_c$

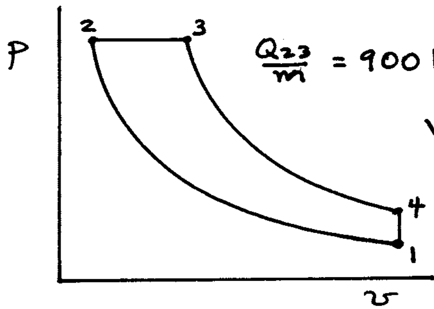
PROBLEM 9.23

KNOWN: An air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression. The heat addition per unit mass is given.

FIND: Determine (a) the maximum temperature, (b) the cutoff ratio, (c) the net work per unit of mass, (d) the thermal efficiency, (e) the mean effective pressure. (f) Plot those quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:





$\frac{Q_{23}}{m} = 900$ kJ/kg

$V_1/V_2 = 20$

$p_1 = 200$ kPa
$T_1 = 380$ K

ASSUMPTIONS: See Example 9.2

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22)

State 1: $T_1 = 380\text{K} \Rightarrow u_1 = 271.69$ kJ/kg, $v_{r1} = 343.4$

State 2: For the isentropic compression

$$v_{r2} = \left(\frac{V_2}{V_1}\right) v_{r1} = \left(\frac{1}{20}\right) 343.4 = 17.17$$

Thus, $T_2 = 1135.2\ °R$, $h_2 = 1202.0$ kJ/kg

State 3: For the heat addition process, $Q_{23}/m = h_3 - h_2$. Thus

$$h_3 = Q_{23}/m + h_2 = 2102.0 \text{ kJ/kg}$$

Thus, $T_3 = 1879.5$ K and $v_{r3} = 3.4202$

State 4: For the isentropic expansion

$$\frac{V_4}{V_3} = \frac{V_1}{V_2} \cdot \frac{V_2}{V_3} = \frac{V_1}{V_2} \cdot \frac{T_2}{T_3} = 12.08$$

and $$v_{r4} = \left(\frac{V_4}{V_3}\right) v_{r3} = 41.316$$

Thus, $T_4 = 843.9$ K, $u_4 = 628.17$ kJ/kg

(a) The maximum temperature in the cycle is $T_3 = 1879.5$ K ← T_{max}

(b) For the constant pressure process

$$r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1879.5}{1135.2} = 1.66 \leftarrow r_c$$

(c) For the cycle, $W_{cycle} = Q_{cycle}$. Thus

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = (h_3 - h_2) - (u_4 - u_1)$$

Inserting values

$$\frac{W_{cycle}}{m} = 900 \text{ kJ/kg} - (628.17 - 271.69) \text{ kJ/kg} = 543.5 \text{ kJ/kg} \leftarrow W_{cycle}/m$$

PROBLEM 9.23 (Cont'd - Page 2)

(d) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = \frac{543.5}{900} = 0.604\ (60.4\%)$$

(e) The mean effective pressure is

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}/m}{v_1\,(1 - V_2/V_1)}$$

Using the ideal gas equation of state

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\,\frac{kJ}{kg\cdot K}\right)(380\,K)}{(200\,kPa)}\left|\frac{1\,kPa}{10^3\,N/m^2}\right|\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|$$

$$= 0.5453\ m^3/kg$$

Thus, the mep becomes

$$mep = \frac{(543.5\ kJ/kg)}{(0.5453\ m^3/kg)\left(1 - \frac{1}{20}\right)}\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|\left|\frac{1\,kPa}{10^3\,N/m^2}\right|$$

$$= 1049\ kPa \longleftarrow \text{mep}$$

(f) The data for the required plots are obtained using IT, as follows:

IT Code

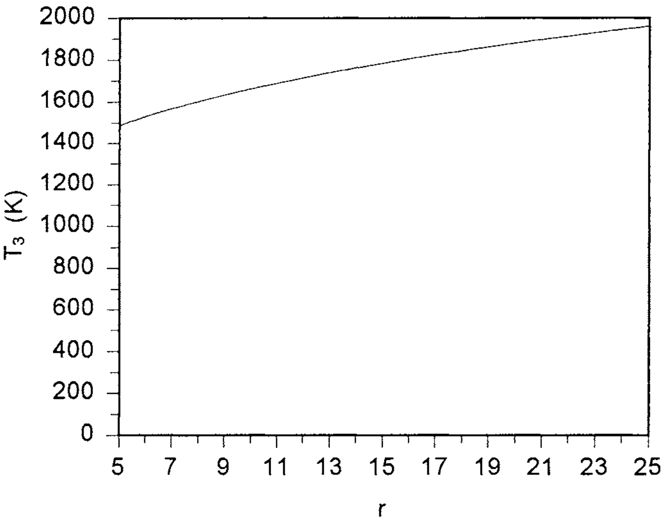
```
p1 = 200  // kPa
T1 = 380  // K
Q23 / m = 900  // kJ/kg
m = 1  // Assume a unit mass of 1 kg.
p3 = p2
v4 = v1
r = 20

v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
r = v1 / v2
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
Q23 = m * (h3 - h2)
h2 = h_T("Air", T2)
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
v3 = v_TP("Air",T3, p3)
u3 = u_T("Air", T3)
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
u4 = u_T("Air", T4)

rc = v3 / v2
Wcycle / m = Q23 / m - (u4 - u1)
eta = Wcycle / Q23
mep = Wcycle / (m * v1 * (1 - 1 / r))
```

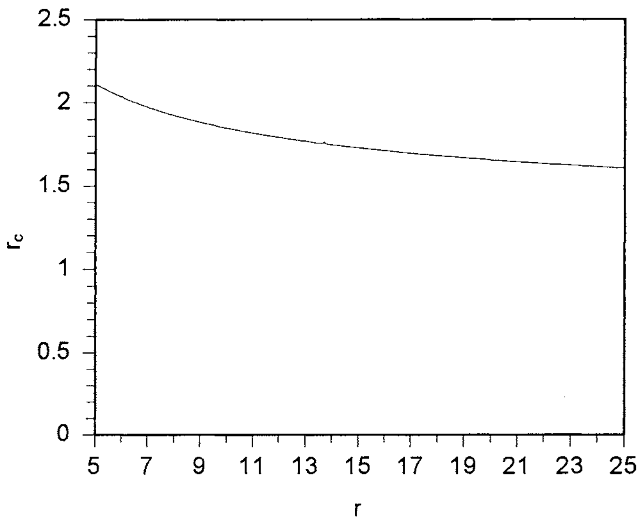
IT Results for r = 20

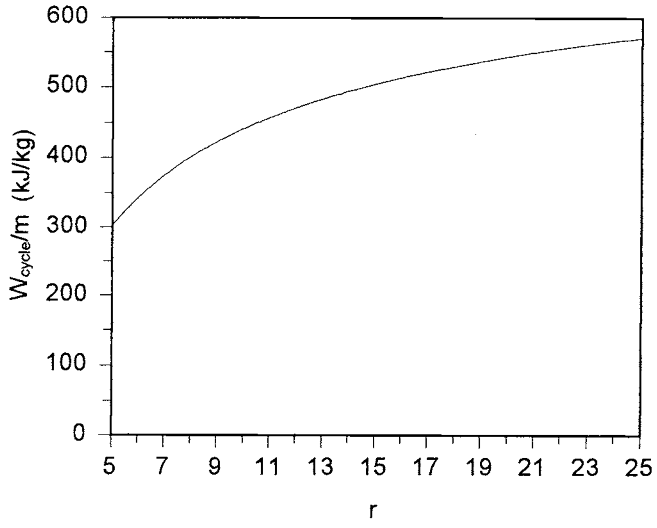
u_1 = 271.5 kJ/kg
T_2 = 1135 K
p_2 = 11950 kPa
h_2 = 1201 kJ/kg
h_3 = 2101 kJ/kg
T_3 = 1881 K
T_4 = 844.1 K
p_4 = 444.3 kPa
u_4 = 628 kJ/kg
r_c = 1.657
W_{cycle}/m = 543.5 kJ/kg
η = 0.6039
mep = 1049 kPa

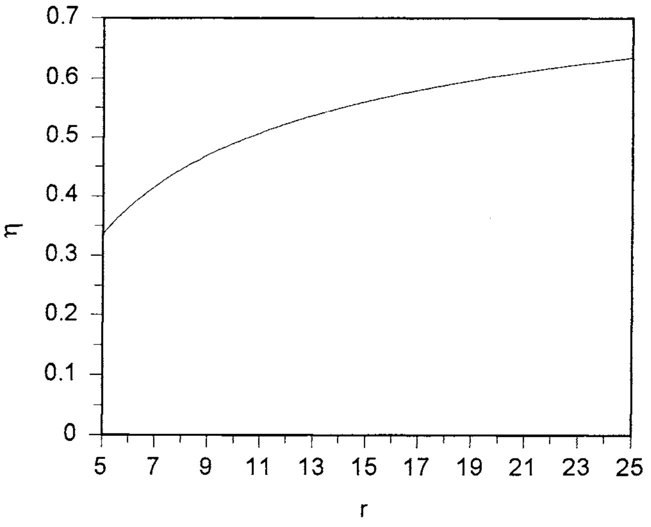
PROBLEM 9.23 (Cont'd. - Page 3)

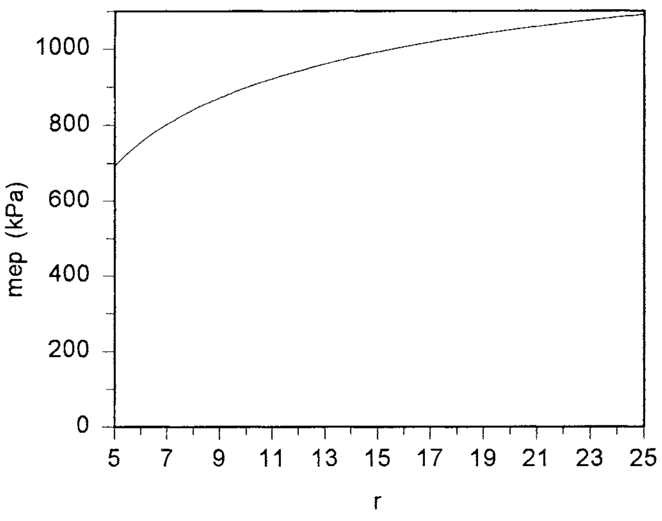
PLOTS:











Discussion:

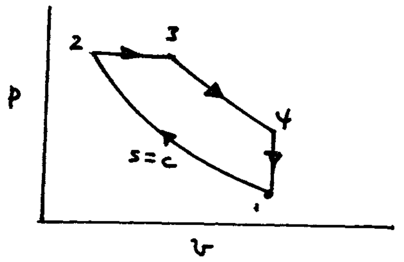
- As r increases, T_2 increases. Thus, for fixed heat input, T_3 increases as does mep.
- The average temperature during the heat addition increases, and thus η increases with r.
- For fixed heat input, the work of the cycle increases with r.

PROBLEM 9.24

KNOWN: Reconsider the air-standard Diesel cycle of Problem 9.23 from an exergy accounting viewpoint.

FIND: (a) Evaluate the exergy transfers accompanying heat and work for each process and (b) devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA: Data given in the table below are from Prob. 9.23 and/or further calculation or interpolation in Table A-22.



State	p (kPa)	T (K)	v (m³/kg)	u (kJ/kg)	s^o (kJ/kg·K)
1	200	380	0.5453	271.69	1.94001
2	11951.1	1135.2	0.02726	876.14	3.11389
3	11951.1	1879.5	0.045133	1563.1	3.7219
4	444.14	843.9	0.5453	628.17	2.77680

ASSUMPTIONS: See example 9.2. Also, $T_0 = 300\,K$, $p_0 = 100\,kPa$.

ANALYSIS: (a) Process 1-2: $Q_{12} = 0$, so there is no accompanying exergy transfer. ←
The exergy transfer accompanying work is, with Eq. 7.13

$$E_W = W - p_0\Delta V \quad \text{or} \quad E_W/m = \underbrace{(W_{12}/m)}_{-[u_2-u_1]\text{ from an energy balance}} - p_0(v_2 - v_1) = u_1 - u_2 - p_0(v_2 - v_1)$$

$$\Rightarrow \quad E_W/m = (271.69 - 876.14)\,\frac{kJ}{kg} - \left(10^5\,\frac{N}{m^2}\right)(0.02726 - 0.5453)\,\frac{m^3}{kg}\left|\frac{1\,kJ}{10^3\,N\cdot m}\right| = -552.65\,\frac{kJ}{kg}$$

Process 2-3: As above, using $W_{23}/m = \int_2^3 p\,dv = p_2(v_3 - v_2)$

$$E_W/m = (W_{23}/m) - p_0(v_3 - v_2) = (p_2 - p_0)[v_3 - v_2] = \left[(119.51 - 1)\times 10^5\,\frac{N}{m^2}\right]\left[0.045133 - 0.02726\right]\frac{m^3}{kg}\left|\frac{kJ}{10^3\,N\cdot m}\right|$$
$$= 211.81\ kJ/kg$$

With Eq. 7.12 and noting that process 2-3 is internally reversible

$$E_Q = \int_2^3\left(1 - \frac{T_0}{T_b}\right)\delta Q = Q - T_0\int_2^3\left(\frac{\delta Q}{T}\right)_{\text{int rev}} = Q - T_0(S_3 - S_2)$$

or

$$(E_Q/m) = Q/m - T_0\left(s_3^o - s_2^o - R\,\overset{0}{\cancel{\ln p_3/p_2}}\right) = 900\,\frac{kJ}{kg} - 300\,K\,(3.7219 - 3.1139) = 717.6\,\frac{kJ}{kg}$$

Process 3-4: $Q_{34} = 0$, so there is no accompanying exergy transfer. ←

$$(E_W/m) = \underbrace{W_{34}/m}_{= -(u_4 - u_3)\text{ from an energy balance}} - p_0(v_4 - v_3) = u_3 - u_4 - p_0(v_4 - v_3)$$

$$\Rightarrow (E_W/m) = (1563.1 - 628.17) - 10^5(0.5453 - 0.0451)\left|\frac{1}{10^3}\right| = 884.91\ kJ/kg \quad ←$$

Process 4-1: $W_{41} = 0$, so there is no accompanying exergy transfer. ←
Reducing an exergy balance, $E_1 - E_4 = E_Q - \overset{0}{\cancel{E_W}} - \overset{0}{\cancel{E_d}} \Rightarrow$

① $$E_Q/m = e_1 - e_4 = (u_1 - u_4) + p_0\overset{0}{\cancel{(v_1 - v_4)}} - T_0(s_1 - s_4)$$

$$E_Q/m = (u_1 - u_4) - T_0\Big(s_1^o - s_4^o - R\underbrace{\ln p_1/p_4}_{= T_1/T_4 \text{ since } v_1 = v_2}\Big) = (u_1 - u_4) - T_0\left(s_1^o - s_4^o - R\ln\frac{T_1}{T_4}\right)$$

$$= (271.69 - 628.17) - 300\left(1.94001 - 2.7768 - \frac{8.314}{28.97}\ln\frac{380}{843.9}\right) = -174.1\,\frac{kJ}{kg} \quad ←$$

Problem 9.24 (Contd.)

(b) An exergetic efficiency in this case can be written as the ratio of the desired exergy output (exergy transfer accompanying net work developed) to the exergy added.

Exergy is added by heat transfer in process 2-3. The exergy transfer accompanying the net work developed is

② $$\frac{(E_W)_{cycle}}{m} = \left(\frac{E_W}{m}\right)_{12} + \left(\frac{E_W}{m}\right)_{23} + \left(\frac{E_W}{m}\right)_{34} + \left(\frac{E_W}{m}\right)_{41}$$

$$= -552.65 + 211.81 + 884.91 + 0 = 544.1 \text{ kJ/kg}$$

③ Thus, $$\epsilon = \frac{544.1}{717.6} = 0.758 \quad (75.8\%)$$ ←

---

1. Alternatively, the approach employed for process 2-3 can be used:

$$E_Q = \int_4^1 \left[1 - \frac{T_0}{T_b}\right]\delta Q = Q_{41} - T_0 \int_4^1 \left(\frac{\delta Q}{T}\right)_{\substack{int\\rev}} = \underbrace{Q_{41}}_{\substack{\text{Energy Balance:}\\ = U_1 - U_4}} - T_0(S_1 - S_4)$$

2. Summarizing

	Ew/m	EQ/m
1-2:	-552.65	0
2-3:	211.81	717.6
3-4:	884.91	0
4-1:	0	-174.1
cycle	544.1	543.5

Accordingly, to within roundoff we get $(E_W/m)_{cycle} = (E_Q/m)_{cycle}$, as expected in this case where no exergy is destroyed.

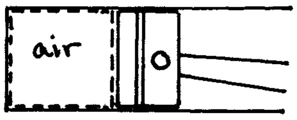
3. Significant internal irreversibilities are present in actual internal combustion engines. Accordingly, significant exergy destruction occurs within such engines and the exergetic efficiency would be much less than determined here.

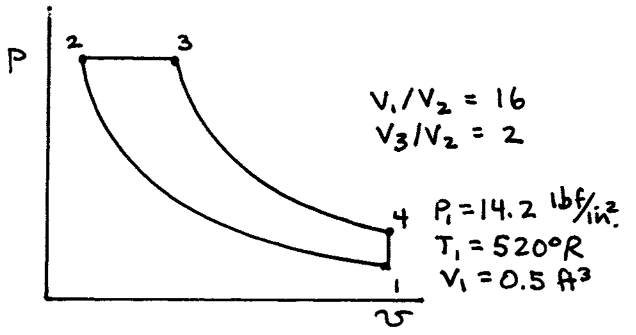
PROBLEM 9.25

KNOWN: An air-standard Diesel cycle has known compression and cutoff ratios and a specified state at the beginning of compression.

FIND: Determine (a) the heat added, (b) the maximum temperature, (c) the thermal efficiency, (d) the mean effective pressure, and (e) plot these quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.2.

ANALYSIS: Begin by fixing each principal state of the cycle (Table A-22E).

State 1: $T_1 = 520°R \Rightarrow u_1 = 88.62$ Btu/lb, $v_{r1} = 158.58$

State 2: For the isentropic compression

$$v_{r2} = \left(\frac{V_2}{V_1}\right) v_{r1} = \left(\frac{1}{16}\right) 158.58 = 9.911$$

Thus, $T_2 = 1502.5$ °R and $h_2 = 369.85$ Btu/lb

State 3: For the constant pressure process, $V_3/V_2 = T_3/T_2$. Thus $T_3 = 2\,(1502.5) = 3005$ °R, and $h_3 = 792.03$ Btu/lb, $v_{r3} = 1.1743$ ← T_{max}

State 4: For the isentropic expansion

$$\frac{V_4}{V_3} = \frac{V_1}{V_2} \cdot \frac{V_2}{V_3} = (16)\left(\frac{1}{2}\right) = 8$$

and

$$v_{r4} = \left(\frac{V_4}{V_3}\right) v_{r3} = 9.3944$$

Thus, $T_4 = 1530.7$ °R and $u_4 = 272.36$ Btu/lb

(a) The heat added is

$$Q_{23} = m(h_3 - h_2)$$

Evaluating m

$$m = \frac{P_1 V_1}{R T_1} = \frac{(14.2 \text{ lbf/in.}^2)(0.5 \text{ ft}^3)}{\left(\frac{1545}{28.97}\frac{\text{ft·lbf}}{\text{lb·°R}}\right)(520°R)} \left| \frac{144 \text{ in.}^2}{1 \text{ ft}^2} \right| = 0.0369 \text{ lb}$$

Thus

$$Q_{23} = (.0369)(792.03 - 369.85) = 15.58 \text{ Btu} \leftarrow Q_{23}$$

(b) $T_{max} = T_3 = 3004.6$ °R (see above)

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = 1 - \frac{Q_{41}}{Q_{23}} = 1 - \frac{6.78}{15.58} = 0.565\ (56.5\%) \leftarrow \eta$$

PROBLEM 9.25 (Cont'd - Page 2)

(d) The mean effective pressure is given by

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{Q_{23} - Q_{41}}{V_1(1 - V_2/V_1)}$$

$$= \frac{(15.58 - 6.78)\ \text{Btu}}{(0.5\ \text{ft}^3)(1 - 1/16)} \left| \frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}} \right| \left| \frac{1\ \text{ft}^2}{144\ \text{in}^2} \right|$$

$$= 101.4\ \text{lbf/in}^2 \longleftarrow \underline{mep}$$

(e) The data for the required plots are obtained using IT, as follows

IT Code

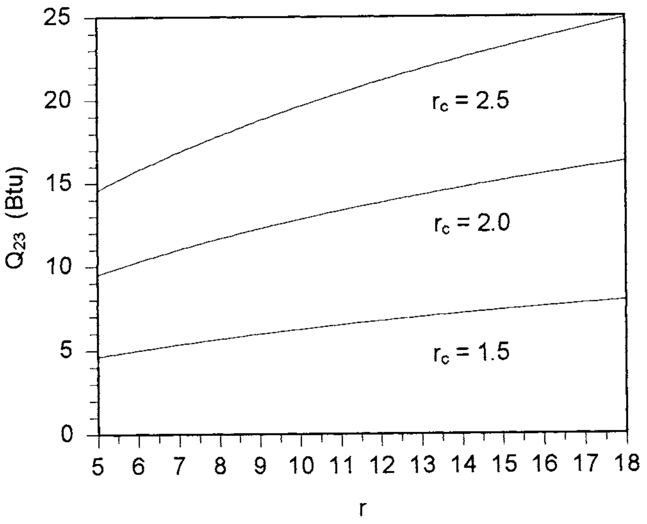
```
p1 = 14.2  // lb/in.²
T1 = 520  // °R
V1 = 0.5  // ft³
p3 = p2
v4 = v1
r = 16
rc = 2.5

v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
r = v1 / v2
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
rc = T3 / T2
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
rc = v3 / v2
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
u4 = u_T("Air", T4)

v1 = V1 / m
Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / Q23
mep = ((Wcycle / m) / (v1 * (1 - 1/ r))) * (778 / 144)
```

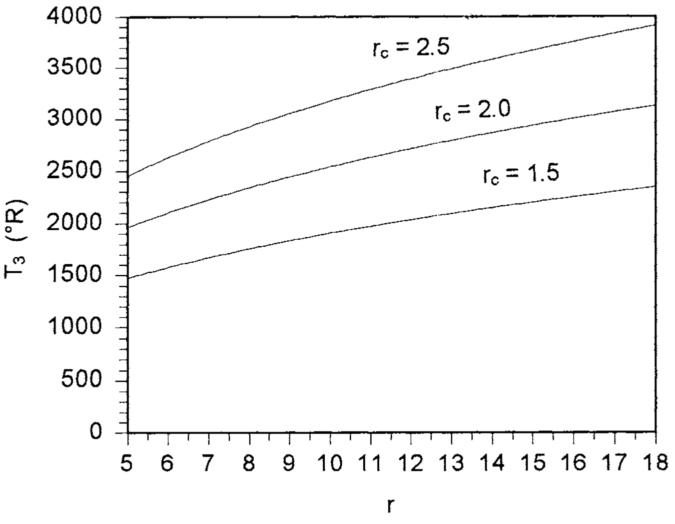
IT Results for r = 16 and rc = 2

T_2 = 1502 °R
T_4 = 1528 °R
h_2 = 369.6 Btu/lb
h_3 = 790.8 Btu/lb
p_2 = 656.3 lbf/in.²
p_4 = 41.73 lbf/in.²
u_1 = 88.73 Btu/lb
u_4 = 271.7 Btu/lb
m = 0.03687 lb
Q_{41} = -6.747 Btu
W_{cycle} = 8.783 Btu

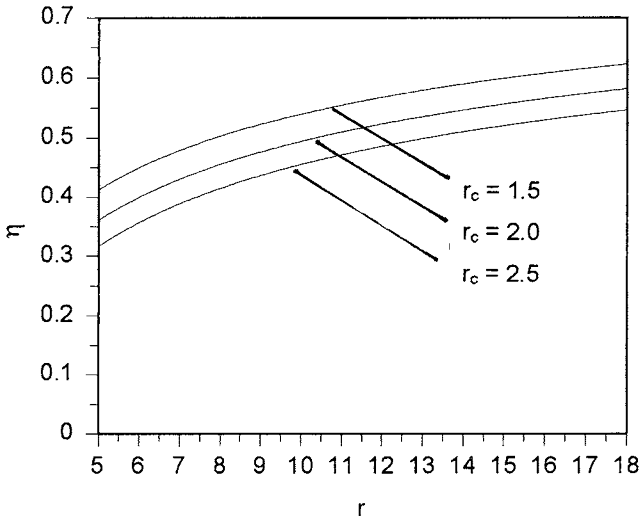
Q_{23} = 15.53 Btu
T_3 = 3004 °R
η = 0.5655
mep = 101.2 lbf/in.²

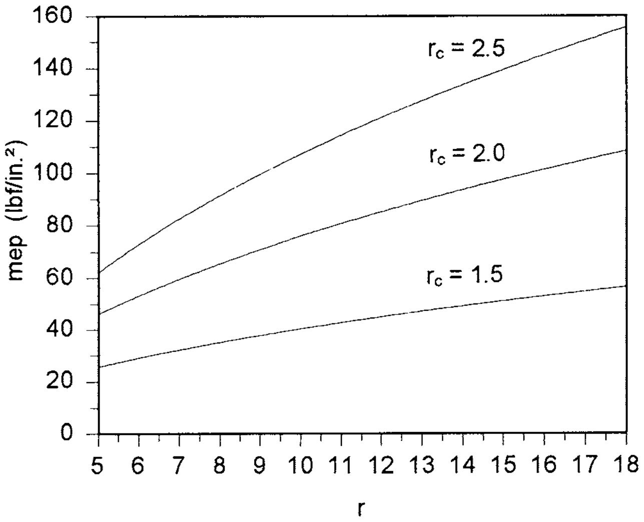
PLOTS:





PROBLEM 9.25 (Cont'd. - page 3)





Discussion

- The thermal efficiency graph exhibit the same trends as in Fig 9.6 for fixed k.
- At constant compression ratio, the values of Q_{23}, T_3, and mep all increase with increasing cut-off ratio r_c, as expected.

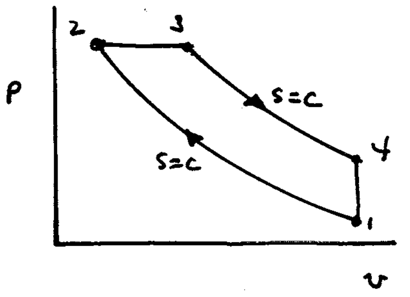
PROBLEM 9.26

KNOWN: Reconsider the air-standard Diesel cycle of Problem 9.25 from an exergy accounting viewpoint.

FIND: (a) Evaluate the exergy transfers accompanying heat and work for each process and (b) devise and evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:

Data from Problem 9.25, further calculation and/or further interpolation in Table A-22E.



State	p(lbf/in²)	T(°R)	v(ft³/lb)	u(Btu/lb)	$s°$(Btu/lb·°R)
1	14.2	520	13.562	88.62	0.59172
2	656.44	1502.5	0.84763	266.84	0.85459
3	656.44	3005	1.6953	586.16	1.05312
4	41.8	1530.7	13.562	272.36	0.85951

ASSUMPTIONS: See Example 9.2. Also, $T_0 = 520°R$, $p_0 = 14.2\,\text{lbf/in}^2$.

ANALYSIS: (a) Process 1-2: $Q_{12} = 0$, so there is no accompanying exergy transfer. With Eq. 7.13 the exergy transfer accompanying work is

$$E_W = \underbrace{W}_{=\, m(u_1-u_2) \text{ from an energy balance}} - p_0\Delta V = m[(u_1-u_2) - p_0(v_2-v_1)] = (0.0369\,\text{lb})\left[(88.62-266.84)\,\text{Btu/lb} - (14.2\times144\,\tfrac{\text{lbf}}{\text{ft}^2})(-12.7143\,\tfrac{\text{ft}^3}{\text{lb}})\left|\tfrac{1\,\text{Btu}}{778\,\text{ft·lbf}}\right|\right]$$

$$= -5.343\,\text{Btu}$$

Process 2-3: As above, using $W_{23} = \int_2^3 p\,dV = p_2(V_3 - V_2)$

$$E_W = p_2[V_3-V_2] - p_0[V_3-V_2] = m(p_2-p_0)(v_3-v_2) = (0.0369)(642.24\times144)(0.84767)\left|\tfrac{1}{778}\right|$$

$$= 3.718\,\text{Btu}$$

With Eq. 7.12 and noting that process 2-3 is internally reversible

$$E_Q = \int_2^3\left[1-\frac{T_0}{T_b}\right]\delta Q = Q_{23} - T_0\int_2^3\left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} = Q_{23} - T_0(S_3-S_2)$$

$$= Q_{23} - T_0 m[s_3^\circ - s_2^\circ - \overset{0}{\cancel{R\ln p_3/p_2}}] = \underset{\text{(from Prob. 9.25)}}{15.58\,\text{Btu}} - (520°R)(0.0369\,\text{lb})(1.05312-0.85459)\,\frac{\text{Btu}}{\text{lb}\cdot°R}$$

$$= 11.771\,\text{Btu}$$

Process 3-4: $Q_{34} = 0$, so there is no accompanying exergy transfer.

$$E_W = W - p_0\Delta V = m[(u_3-u_4) - p_0(v_4-v_3)] = 0.0369\left[313.8 - (14.2\times144)(11.8667)\left|\tfrac{1}{778}\right|\right]$$

$$= 10.428\,\text{Btu}$$

Process 4-1: $W_{41} = 0$, so there is no accompanying exergy transfer.

$$E_Q = \int_4^1\left[1-\frac{T_0}{T_b}\right]\delta Q = Q_{41} - T_0\int_4^1\left(\frac{\delta Q}{T}\right)_{\substack{\text{int}\\\text{rev}}} = \underbrace{Q_{41}}_{=(U_1-U_4) \text{ from an energy balance.}} - T_0(S_1 - S_4)$$

$$= m[(u_1-u_4) - T_0(s_1^\circ - s_4^\circ - R\ln p_1/p_4)] = 0.0369\left[88.62 - 272.36 - 520\left[0.59172 - 0.8591 - \frac{1.986}{28.97}\ln\frac{14.2}{41.8}\right]\right]$$

$$= -3.062\,\text{Btu}$$

Summary:

① Exergy Input accompanying heat transfer in process 2-3 : 11.77 Btu

Disposition of the exergy input :

Transfer out - net work = -5.343 + 3.718 + 10.428	8.80 Btu (74%)
Transfer out - heat transfer in process 4-1	3.06 " (26%)
Exergy destroyed	0
	11.86

---

1. As expected, the exergy input and the disposition of the exergy agree to within roundoff. No exergy is destroyed because each process is internally reversible.

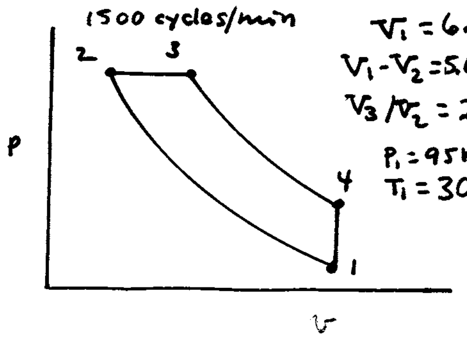
PROBLEM 9.27*

KNOWN: Operating data are provided for an internal combustion engine modeled as an air-standard Diesel cycle.

FIND: Determine the net work per cycle, the power developed, and the thermal efficiency.

SCHEMATIC & GIVEN DATA:

1500 cycles/min



$V_1 = 6.0$ liter

$V_1 - V_2 = 5.6$ liter

$V_3/V_2 = 2.4$

$p_1 = 95$ kPa

$T_1 = 300$ K

ASSUMPTIONS:

See Example 9.2

ANALYSIS: Using given data, the compression ratio is obtained as follows:

$V_2 = V_1 - 5.6 = 6.0 - 5.6 = 0.4$ liter

$$\Rightarrow \frac{V_1}{V_2} = \frac{6.0}{0.4} = 15$$

Next, data are obtained at each principal state:

State 1: From Table A-22 at $T_1 = 300$ K, $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$.

State 2: For the isentropic compression

$$v_{r2} = \frac{V_2}{V_1} v_{r1} = \frac{621.2}{15} = 41.41 \Rightarrow h_2 = 869.63 \text{ kJ/kg}, \; T_2 = 843 \text{ K}$$

State 3: Since pressure is constant for process 2-3,

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} \Rightarrow T_3 = (2.4)(843) = 2023 \text{ K} \Rightarrow h_3 = 2280.85 \frac{\text{kJ}}{\text{kg}}, \; v_{r3} = 2.6743$$

State 4: For the isentropic expansion

$$\frac{v_{r4}}{v_{r3}} = \frac{V_4}{V_3} = \frac{\overset{=V_1}{V_4}}{V_2} \cdot \frac{V_2}{V_3} = (15)\left(\frac{1}{2.4}\right) = 6.25, \Rightarrow v_{r4} = (6.25)(2.6743) = 16.714$$

$$\Rightarrow u_4 = 884.96 \text{ kJ/kg}$$

The mass is obtained using the ideal gas model equation of state

$$m = \frac{p_1 V_1}{R T_1} = \frac{(95 \times 10^3 \text{ N/m}^2)(6.0 \text{ liters})}{\left(\frac{8314}{28.97} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300 \text{ K})} \left| \frac{10^{-3} \text{ m}^3}{1 \text{ liter}} \right| = 6.62 \times 10^{-3} \text{ kg}$$

For any cycle, $W_{cycle} = Q_{cycle}$. Processes 1-2 and 3-4 involve no heat transfer; so, $Q_{cycle} = Q_{23} - Q_{41}$, where $Q_{23} = m[h_3 - h_2]$, $Q_{41} = m(u_4 - u_1)$

Thus, $W_{cycle} = (6.62 \times 10^{-3} \text{ kg})[(2280.85 - 869.63) - (884.96 - 214.07)] \text{ kJ/kg}$

$= 4.901$ kJ per cycle ← W_{cycle}

The power developed is

$$\dot{W}_{cycle} = \left(1500 \frac{\text{cycles}}{\text{min}}\right)\left(4.901 \frac{\text{kJ}}{\text{cycle}}\right) \left| \frac{1 \text{ min}}{60 \text{ s}} \right| \left| \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right| = 122.53 \text{ kW}$$ ← power

The thermal efficiency is

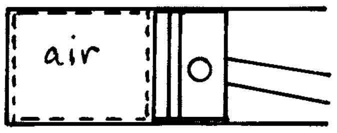
$$\eta = \frac{W_{cycle}}{\underset{\searrow Q_{23}}{Q_{in}}} = \frac{4.901 \text{ kJ/cycle}}{6.62 \times 10^{-3}(2280.85 - 869.63) \text{ kJ/cycle}} = \frac{4.901}{9.3423} = 0.525 \; (52.5\%)$$ ← η

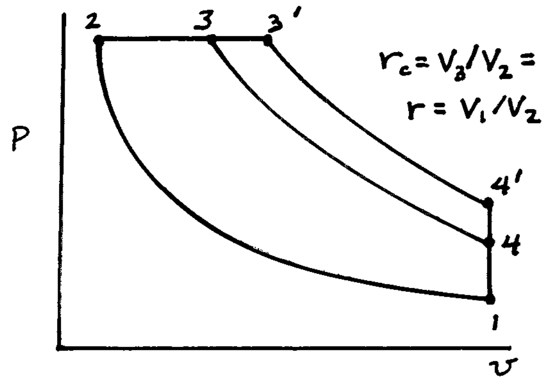
PROBLEM 9.28

KNOWN: An air-standard Diesel cycle has a known compression ratio and a specified state at the beginning of compression.

FIND: Plot (a) the maximum temperature, (b) the pressure at the end of expansion, (c) the net work per unit mass of air, and (d) the thermal efficiency versus cutoff ratio ranging from 1.5 to 2.5

SCHEMATIC & GIVEN DATA:





$r_c = V_3/V_2 = 1.5 - 2.5$

$r = V_1/V_2 = 15$

$P_1 = 100$ kPa

$T_1 = 310$ K

ASSUMPTIONS: See Example 9.2.

ANALYSIS: A sample calculation will be done for $r_c = 1.5$. First, fix each of the principal states of the cycle:

(a) State 1: From Table A-22 at $T_1 = 310$ K ; $u_1 = 221.25$ kJ/kg and $v_{r1} = 572.3$.

State 2: For the isentropic compression

$$v_{r2} = \left(\frac{V_2}{V_1}\right) v_{r1} = \left(\frac{1}{15}\right)(572.3) = 38.153$$

Interpolating in Table A-22; $h_2 = 896.86$ kJ/kg, $T_2 = 867.7$ K

State 3: For each value of cutoff ratio, $r_c = V_3/V_2$, the value of T_3 can be determined from $T_3 = (V_3/V_2)T_2$. In this case

$$T_3 = 1301.5 \text{ K} \; ; \; h_3 = 1397.7 \text{ kJ/kg}, \; v_{r3} = 11.235 \longleftarrow T_{max}$$

State 4: Note that for the isentropic expansion

$$v_{r4} = \left(\frac{V_4}{V_3}\right) v_{r3} = \left(\frac{V_1}{V_2}\cdot\frac{V_2}{V_3}\right) v_{r3} = \left(r\cdot\frac{1}{r_c}\right) v_{r3}$$

Thus $v_{r4} = 112.35$; $T_4 = 586.6$ K, $u_4 = 424.54$ kJ/kg

(b) The pressure at the end of the expansion is

$$P_4 = \left(\frac{T_4}{T_1}\right) P_1 = 189.2 \text{ kPa} \longleftarrow P_4$$

(c) The net work per unit mass of air is

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} - \frac{Q_{41}}{m} = (h_3 - h_2) - (u_4 - u_1) = 297.5 \text{ kJ/kg} \longleftarrow W_{cycle}/m$$

(d) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} = 0.594 \; (59.4\%)$$

The data for the required plots are obtained using IT, as follows:

PROBLEM 9.28 (Cont'd.)

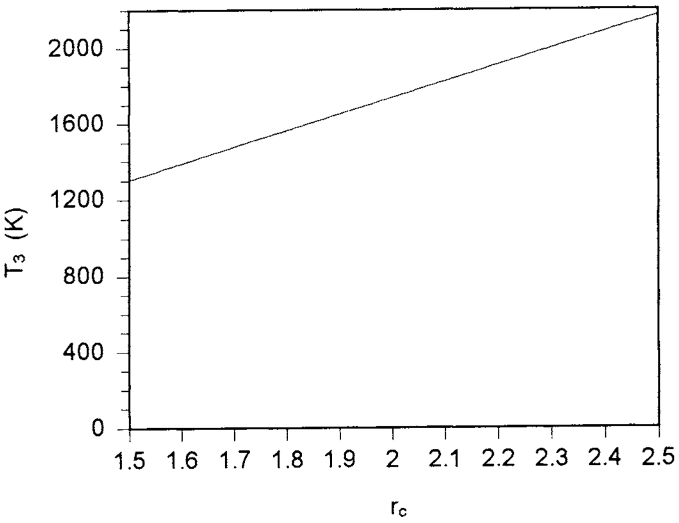
IT Code

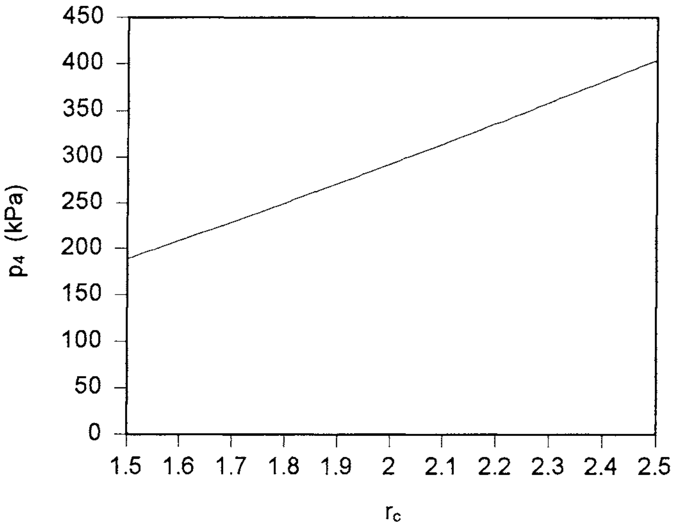
```
p1 = 100  // kPa
T1 = 310  // K
r = v1 / v2
r = 15
rc = v3 / v2
rc = 1.5
m = 1  // Assume a unit mass of 1 kg.

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
v3 = v_TP("Air",T3, p3)
p3 = p2
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
v4 = v1
u4 = u_T("Air", T4)

Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / Q23
```

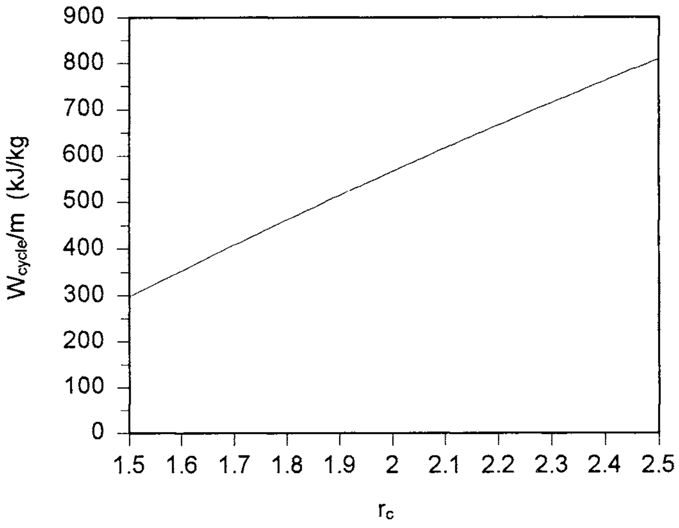
IT Results for r_c = 1.5

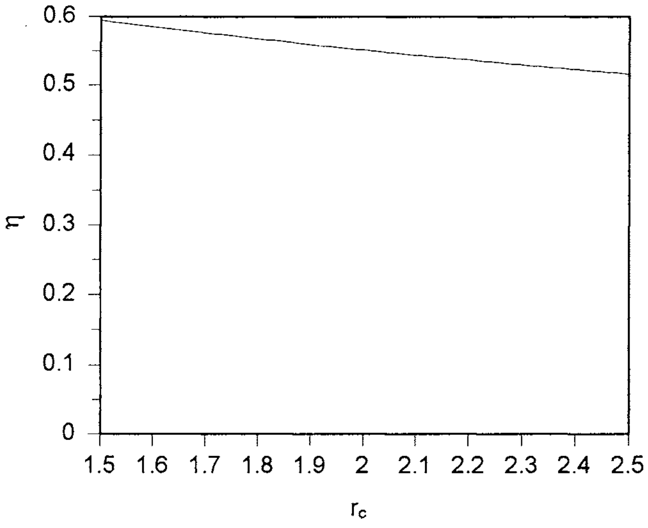
h_2 = 896.6 kJ/kg
p_2 = 4199 kPa
h_3 = 1397 kJ/kg
T_4 = 586.3 K
Q_{23}/m = 500.4 kJ/kg
Q_{41}/m = -203.1 kJ/kg
T_3 = 1302 K
p_4 = 189.1 kPa
W_{cycle}/m = 297.3 kJ/kg
η = 0.5941

PLOTS:







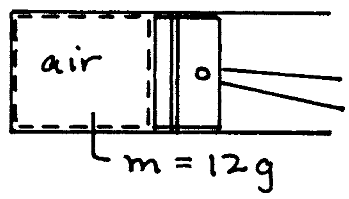


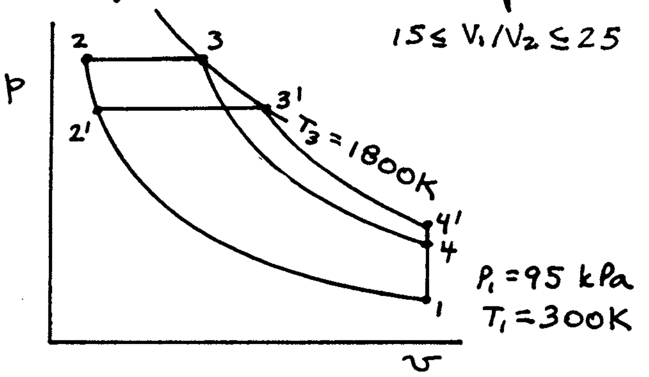
PROBLEM 9.29

KNOWN: An air-standard Diesel cycle has a known maximum temperature and a specified state at the beginning of compression. The mass of air is given.

FIND: Determine for various compression ratios (a) the net work, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.2

ANALYSIS: Consider first a sample calculation using data from Table A-22. for the case of $r = V_1/V_2 = 15$. From Table A-22 at $T_1 = 300K$; $u_1 = 214.07$ kJ/kg, $v_{r_1} = 621.2$. Also, at $T_3 = 1800K$; $h_3 = 2003.3$ kJ/kg, $v_{r_3} = 3.944$. For $r = 15$

$$v_{r_2} = (V_2/V_1)v_{r_1} = \left(\frac{1}{15}\right)621.2 = 41.413$$

Interpolating in Table A-22; $h_2 = 869.63$ kJ/kg, $T_2 = 843.2$ K. Now, for process 3-4

$$v_{r4} = \left(\frac{V_4}{V_3}\right)v_{r_3} \overset{V_4 = V_1}{=} \left(\frac{V_1}{V_2}\right)\left(\frac{V_2}{V_3}\right)v_{r_3} \overset{\frac{V_2}{V_3} = \frac{T_2}{T_3}}{=} \left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_3}\right)v_{r_3} = (15)\left(\frac{843.2}{1800}\right)(3.944) = 27.713$$

Interpolating in Table A-22; $u_4 = 731.99$ kJ/kg.

(a) The net work is calculated from $W_{cycle} = Q_{cycle}$, where $Q_{cycle} = Q_{23} + Q_{34}$. Thus

$$W_{cycle} = m\left[(h_3 - h_2) + (u_1 - u_4)\right]$$

$$= (12\,g)\left|\frac{1\,kg}{10^3\,g}\right|\left[(2003.3 - 869.63) + (214.07 - 731.99)\right]\frac{kJ}{kg} = 7.389\,kJ \leftarrow \frac{W_{cycle}}{(r=15)}$$

(b) The thermal efficiency is

$$\eta = \frac{W_{cycle}}{Q_{23}} = \frac{W_{cycle}}{m(h_3 - h_2)} = \frac{7.389}{(12\times10^{-3})(2003.3 - 869.63)} = 0.543\ (54.3\%) \leftarrow \frac{\eta}{(r=15)}$$

(c) To get the mean effective pressure, first determine V_1.

$$V_1 = \frac{mRT_1}{P_1} = \frac{(12g)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(300K)}{(95\,kPa)}\left|\frac{1\,kg}{10^3 g}\right|\left|\frac{1\,kPa}{10^3\,N/m^2}\right|\left|\frac{10^3\,N\cdot m}{1\,kJ}\right| = 1.088\times10^{-2}\,m^3$$

Thus

$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - 1/r)}$$

$$= \frac{7.389\,kJ}{(1.088\times10^{-2}\,m^3)(1 - 1/15)}\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|\left|\frac{1\,kPa}{10^3\,N/m^2}\right| = 727.6\,kPa \leftarrow \frac{mep}{(r=15)}$$

PROBLEM 9.29 (Cont'd.)

The data for the required plots are obtained using IT, as follows:

IT Code

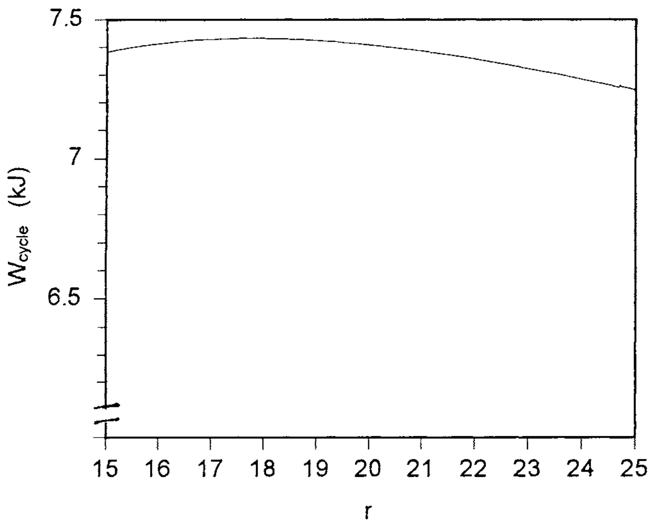
```
p1 = 95  // kPa
T1 = 300  // K
T3 = 1800  // K
r = 15
m  = 12E-03  // kg

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
r = v1 / v2
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
p3 = p2
h3 = h_T("Air", T3)
v3 = v_TP("Air",T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
s4 = s3
v4 = v1
u4 = u_T("Air", T4)
```

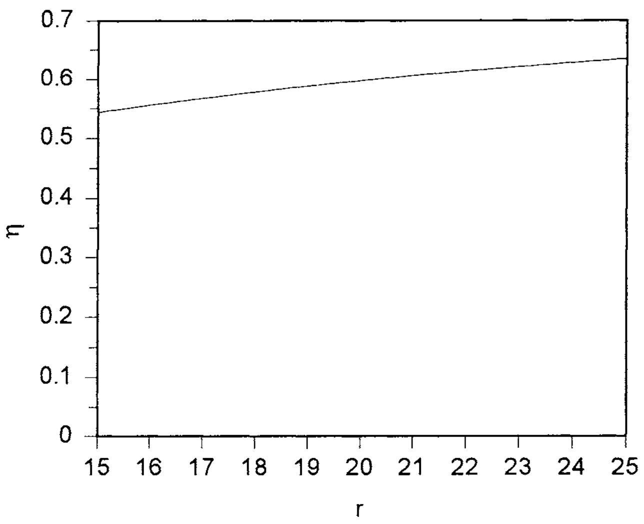
```
Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
eta = Wcycle / (m * (h3 - h2))
V1 = v1 * m
mep = Wcycle / (V1 * (1 - 1 / r))  // kPa
```

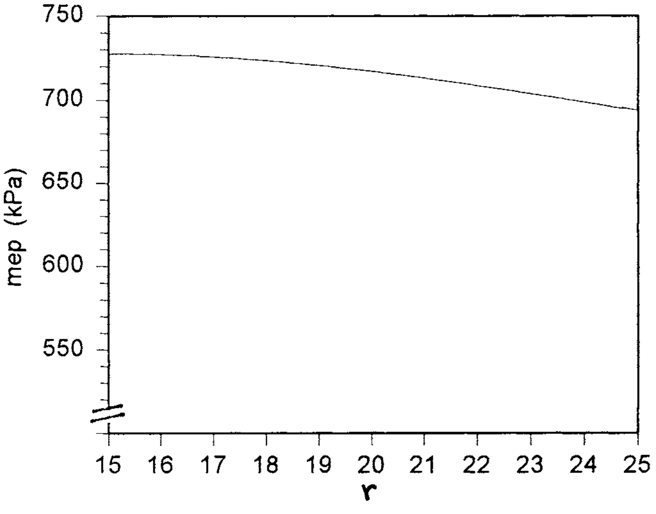
IT Results for r = 15

T_2 = 843.4 K
p_2 = 4006 kPa
h_2 = 869.4 kJ/kg
T_3 = 1800 K
h_3 = 2002 kJ/kg
T_4 = 967.2 K
p_4 = 306.3 kPa
Q_{23} = 13.59 kJ
Q_{41} = -6.202 kJ
W_{cycle} = 7.384 kJ
η = 0.5435
mep = 727.4 kPa

PLOTS:







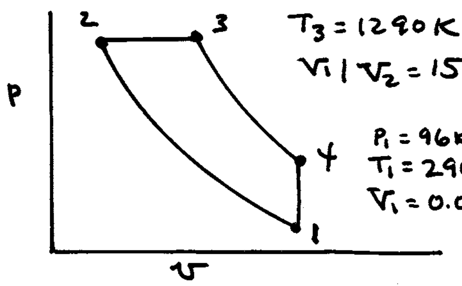
Note that the work doesn't vary greatly with r, but it does exhibit a maximum. Further, thermal efficiency increases with r because the average temperature of heat addition increases and the average temperature of heat rejection decreases, as can be verified by reference to the T-s diagram of the cycle. Also note that the mep decreases with increasing r.

PROBLEM 9.30

KNOWN: Operating data are provided for an air-standard Diesel cycle.

FIND: Determine (a) the mass of air, (b) the heat addition and heat rejection per cycle, (c) the net work and the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$T_3 = 1290\text{K}$

$V_1 / V_2 = 15$

$p_1 = 96\text{kPa}$

$T_1 = 290\text{K}$

$V_1 = 0.016\text{m}^3$

ASSUMPTIONS:

See Example 9.2

ANALYSIS: The mass m is

$$m = \frac{p_1 V_1}{R T_1} = \frac{(96 \times 10^3 \text{N/m}^2)(0.016\text{m}^3)}{\left(\frac{8314}{28.97} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(290\text{K})}$$

$$= 1.846 \times 10^{-2}\,\text{kg} \quad \longleftarrow m$$

Next, fix each of the principal states: state 1: at $T_1 = 290\text{K}$, Table A-22 gives $u_1 = 206.91\ \text{kJ/kg}$, $v_{r1} = 676.1$. State 2: for the isentropic compression $v_{r2} = \frac{V_2}{V_1} v_{r1} = \left(\frac{676.1}{15}\right) = 45.073 \Rightarrow T_2 = 818.6\text{K}$, $h_2 = 842.4\ \frac{\text{kJ}}{\text{kg}}$. State 3: with $T_3 = 1290\text{K}$, $h_3 = 1384.11\ \text{kJ/kg}$, $v_{r3} = 11.555$. State 4: For the isentropic expansion

$$v_{r4} = \frac{V_4}{V_3} v_{r3} = \left(\frac{\overset{= V_1}{V_4}}{V_2}\right)\underbrace{\left(\frac{V_2}{V_3}\right)}_{= \frac{T_2}{T_3} \text{ (since pressure is constant)}} v_{r3} = \left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_3}\right) v_{r3} = (15)\left(\frac{818.6}{1290}\right)(11.555) = 109.99$$

Thus, $u_4 = 428.12\ \text{kJ/kg}$.

The heat addition is $Q_{23} = m(h_3 - h_2) = (1.846 \times 10^{-2})(1384.11 - 842.4) = 10\text{kJ} \longleftarrow Q_{23}$

The heat rejection is $Q_{41} = m(u_4 - u_1) = (1.846 \times 10^{-2})(428.12 - 206.91) = 4.084\ \text{kJ} \longleftarrow Q_{41}$

For any cycle, $W_{cycle} = Q_{cycle}$. Thus

$$W_{cycle} = Q_{23} - Q_{41} = 10\text{kJ} - 4.084\text{kJ} = 5.916\ \text{kJ} \longleftarrow W_{cycle}$$

The thermal efficiency is

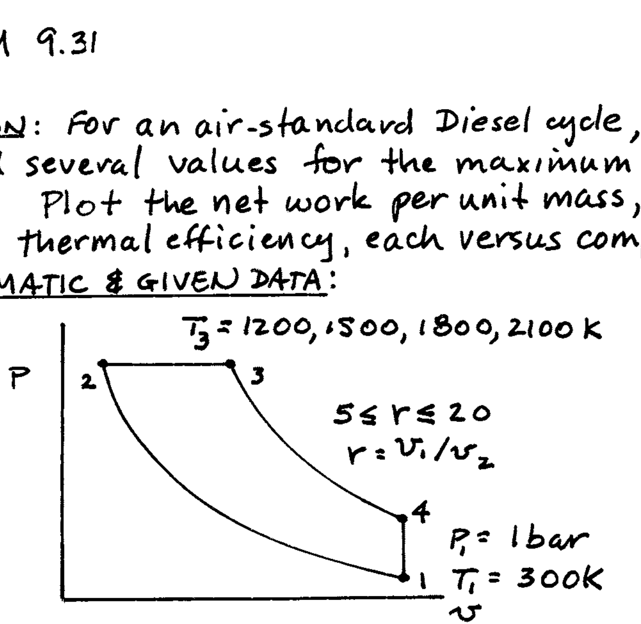
$$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{5.916}{10} = .592 \quad (59.2\%) \longleftarrow \eta$$

PROBLEM 9.31

KNOWN: For an air-standard Diesel cycle, the state at the beginning of compression and several values for the maximum cycle temperature are known.

FIND: Plot the net work per unit mass, the mean effective pressure, and the thermal efficiency, each versus compression ratio ranging from 5 to 20.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 9.2.

ANALYSIS: The following reasoning is used to solve the problem:

1. Given values: P_1, T_1, r, and T_3.
2. Fix state 2 using ideal gas relations and $s(T_1, P_1) = s(T_2, P_2)$. When using Table A-22, this expressed in terms of $v_{r1}/v_{r2} = v_1/v_2$.
3. Using $P_3 = P_2$ and the ideal gas equation: $v_3 = (T_3/T_2)\,v_2$
4. Fix the state at 4 as in step 2: $s(T_3, P_3) = s(T_4, P_4)$; or $v_{r4}/v_{r3} = v_4/v_3$. Also, note that $v_4 = v_1$.

Now, using energy balances and $W_{41} = 0$; $Q_{23} = m(h_3 - h_2)$, $Q_{41} = m(u_1 - u_4)$. (1)

Thus
$$W_{cycle} = Q_{23} + Q_{41} = m[(h_3 - h_2) + (u_1 - u_4)] \quad (2)$$

Further
$$v_1 = \frac{RT_1}{P_1} \; ; \; mep = \frac{W_{cycle}/m}{v_1(1 - 1/r)} \quad (3)$$

and
$$\eta = \frac{W_{cycle}/m}{Q_{23}/m} \quad (4)$$

Sample calculation. $\boxed{r = 15,\ T_3 = 1200K}$ From Table A-22 at $T_1 = 300K$ $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$. Thus $v_{r2} = (v_2/v_1)\,v_{r1} = (1/r)\,v_{r1} = 41.413$

Interpolating in Table A-22; $T_2 = 843.2$ K, $h_2 = 869.63$ kJ/kg. Now, at $T_3 = 1200K$, $h_3 = 1277.79$ kJ/kg, $v_{r3} = 14.470$. Further

$$v_{r4} = (v_4/v_3)\,v_{r3} = (v_1/v_2)(v_2/v_3)\,v_{r3} = (v_1/v_2)(T_2/T_3)\,v_{r3} = 152.514$$

Thus, $T_4 = 522.1$ K, $u_4 = 375.96$ kJ/kg.

Now, from (1) - (4)

$$Q_{23}/m = (1277.79 - 869.63) = 408.2 \text{ kJ/kg}$$

$$W_{cycle} = (408.2) + (214.07 - 375.96) = 246.3 \text{ kJ/kg}$$

$$\eta = 246.3/408.2 = 0.6034 \; (60.34\%)$$

Finally
$$v_1 = \frac{\left(\frac{8.314}{28.97}\frac{kJ}{kg \cdot K}\right)(300K)}{(1 \text{ bar})} \left|\frac{1 \text{ bar}}{10^5 N/m^2}\right| \left|\frac{10^3 N \cdot m}{1 kJ}\right| = 0.861 \text{ m}^3/\text{kg}$$

$$mep = \frac{(246.3 \text{ kJ/kg})}{(0.861 \text{ m}^3/\text{kg})(1 - 1/15)} \left|\frac{10^3 N \cdot m}{1 kJ}\right| \left|\frac{1 \text{ bar}}{10^5 N/m^2}\right| = 3.065 \text{ bar}$$

PROBLEM 9.31 (Cont'd.)

The data for the required plots are obtained using IT, as follows:

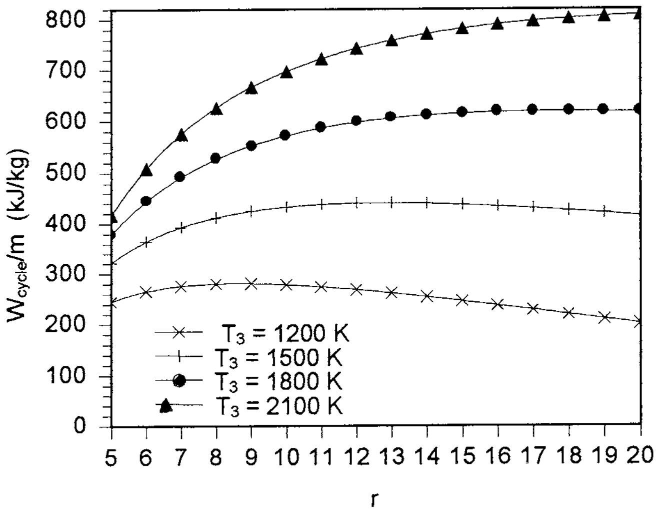
IT Code

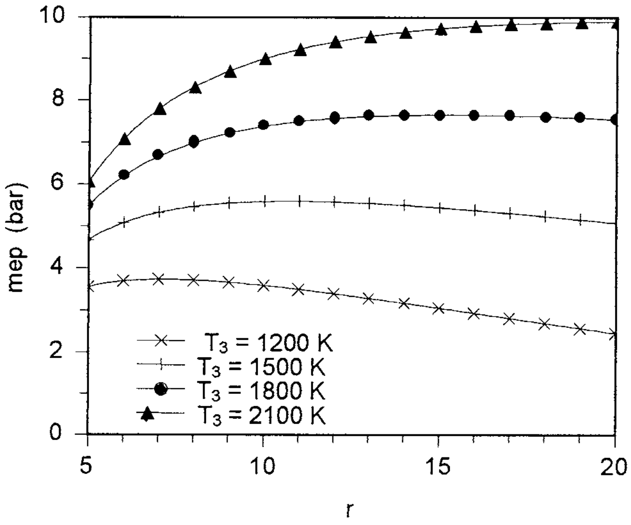
```
p1 = 1  // bar
T1 = 300  // K
T3 = 1200  // K
r = 15
m = 1  // Asume a unit mass of 1 kg.

v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
r = v1 / v2
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
v3 /v2 = T3 / T2
v3 = v_TP("Air",T3, p3)
s3 = s_TP("Air", T3, p3)
v4 = v_TP("Air",T4, p4)
v4 = v1
s4 = s_TP("Air", T4, p4)
s4 = s3
u1 = u_T("Air", T1)
h2 = h_T("Air", T2)
h3 = h_T("Air", T3)
u4 = u_T("Air", T4)
Q23 = m * (h3 - h2)
Q41 = m * (u1 - u4)
Wcycle = Q23 + Q41
mep = (Wcycle / m) / (v1 * (1 - 1 / r)) / 100
eta = Wcycle / Q23
```

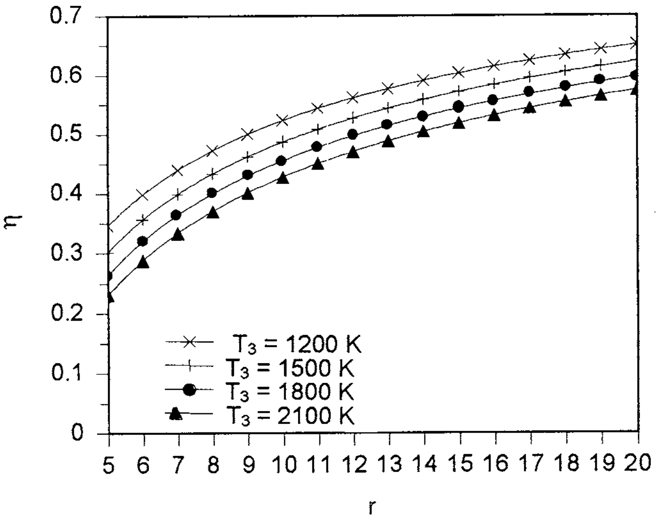
IT Results for r = 15 and T3 = 1200 K

T_2 = 843.4 K
p_2 = 42.17 bar
h_2 = 869.4 kJ/kg
h_3 = 1277 kJ/kg
T_4 = 521.9 K
p4 = 1.74 bar
Q_{23}/m = 407.7 kJ/kg
Q_{41}/m = -161.7 kJ/kg
W_{cycle}/m = 245.9 kJ/kg
mep = 3.06 bar
η = 0.6033

PLOTS:





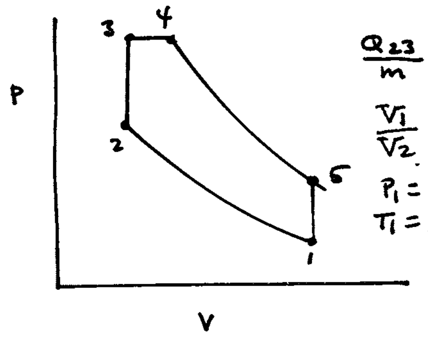


PROBLEM 9.32

KNOWN: Operating data are provided for an air-standard dual cycle.

FIND: Determine (a) the temperatures at the end of each heat addition process, (b) the net work per unit mass, (c) the thermal efficiency, (d) the mean effective pressure.

SCHEMATIC & GIVEN DATA:



$\frac{Q_{23}}{m} = \frac{Q_{34}}{m} = 700 \frac{kJ}{kg}$

$\frac{V_1}{V_2} = 9$

$P_1 = 100 kPa$

$T_1 = 300K$

ASSUMPTIONS:

See Example 9.3

ANALYSIS: Using Table A-22, obtain data at the principal states:

State 1: $T_1 = 300K$, $u_1 = 214.07$, $v_{r1} = 621.2$

State 2: For the isentropic compression

$v_{r2} = \frac{V_2}{V_1} v_{r1} = \frac{621.2}{9} = 69.022$

$\Rightarrow T_2 = 702.7 K$, $u_2 = 514.49 kJ/kg$

State 3: For process 2-3, $Q_{23} = m(u_3 - u_2) \Rightarrow u_3 = \frac{Q_{23}}{m} + u_2 = 700 + 514.49 = 1214.49 kJ/kg. \Rightarrow T_3 = 1509.8K$, $h_3 = 1647.91 kJ/kg$. $\longleftarrow T_3$

State 4: For process 3-4, $Q_{34} = m(h_4 - h_3) \Rightarrow h_4 = \frac{Q_{34}}{m} + h_3 = 700 + 1647.91 = 2347.91 kJ/kg$

$\Rightarrow T_4 = 2076.5K$, $v_{r4} = 2.4495$ $\longleftarrow T_4$

State 5. For the isentropic expansion,

$$v_{r5} = v_{r4}\left(\frac{V_5}{V_4}\right) = v_{r4}\left(\frac{V_1}{V_4}\right) = v_{r4}\left(\frac{V_1}{V_2}\underbrace{\frac{V_3}{V_4}}_{=\frac{T_3}{T_4} \text{ (since } P_4 = P_3)}\right) = 2.4495(9)\left(\frac{1509.8}{2076.5}\right) = 16.029$$

$\Rightarrow u_5 = 898.7 kJ/kg$.

(b) For any cycle, $W_{cycle} = Q_{cycle}$. Thus

$$\frac{W_{cycle}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} - \underbrace{\frac{Q_{51}}{m}}_{=u_5 - u_1} = 700\frac{kJ}{kg} + 700\frac{kJ}{kg} - (898.7 - 214.07)\frac{kJ}{kg} = 715.37\frac{kJ}{kg} \longleftarrow W_{cycle}/m$$

(c) The thermal efficiency is

$$\eta = \frac{W_{cycle}/m}{Q_{in}/m} = \frac{715.37}{1400} = 0.511 \ (51.1\%) \longleftarrow \eta$$

(d) The mean effective pressure is

$$mep = \frac{(W_{cycle}/m)}{v_1 - v_2} = \frac{W_{cycle}/m}{v_1(1 - v_2/v_1)}$$

where

$$v_1 = \frac{RT}{P_1} = \frac{\left(\frac{8314}{28.97}\frac{N \cdot m}{kg \cdot K}\right)(300K)}{(10^5 N/m^2)} = 0.86096 m^3/kg$$

$$\Rightarrow mep = \frac{715.37 kJ/kg}{0.86096 \frac{m^3}{kg}\left(1 - \frac{1}{9}\right)}\left|\frac{10^3 N \cdot m}{1 kJ}\right|\left|\frac{1 kPa}{10^3 N/m^2}\right| = 934.8 kPa \longleftarrow mep$$

PROBLEM 9.33

The data for the required plots are obtained using IT, as follows. See Prob. 9.32 for sample calculations.

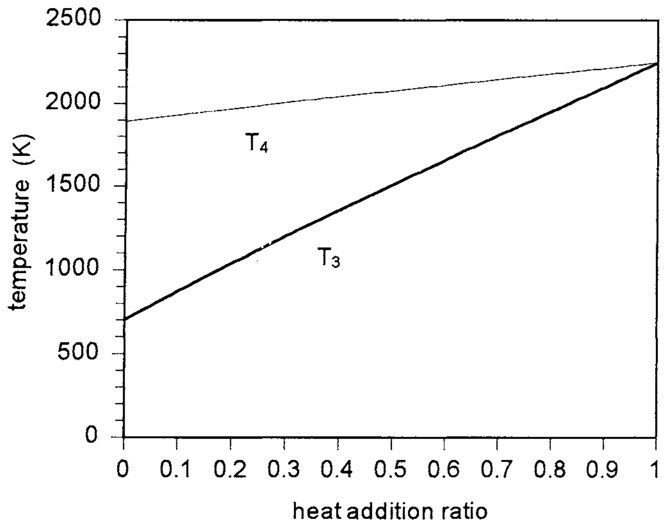
IT Code

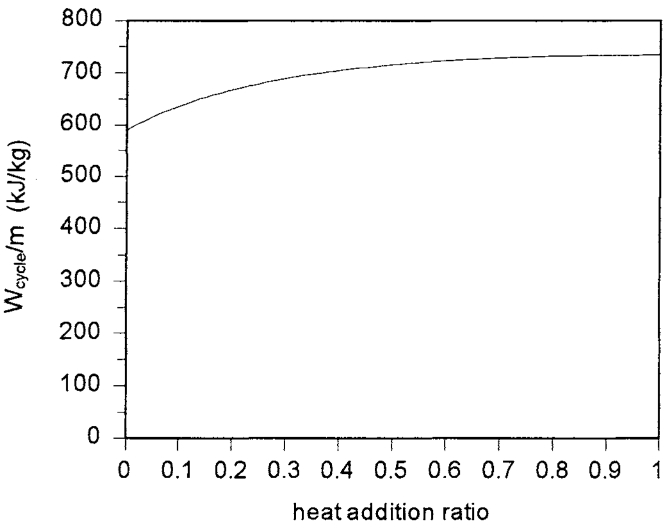
```
p1 = 100  // kPa
T1 = 300  // K
v1 / v2 = 9
Q23 + Q34 = 1400  // kJ/kg
Q23 / (Q23 + Q34) = ratio
ratio = .5
m = 1  // Assume a unit mass of
1 kg.

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
Q23 = m * (u3 - u2)
u2 = u_T("Air", T2)
u3 = u_T("Air", T3)
Q34 = m * (h4 - h3)
h3 = h_T("Air", T3)
h4 = h_T("Air", T4)
p3 / p2 = T3 / T2
p4 = p3
v4 = v_TP("Air",T4, p4)
s4 = s_TP("Air", T4, p4)
v5 = v1
v5 = v_TP("Air",T5, p5)
s5 = s_TP("Air", T5, p5)
s5 = s4
u5 = u_T("Air", T5)

Wcycle / m = Q23 + Q34 - (u5 - u1)
eta = Wcycle / (Q23 + Q34)
mep = (Wcycle / m) / (v1 - v2)
```

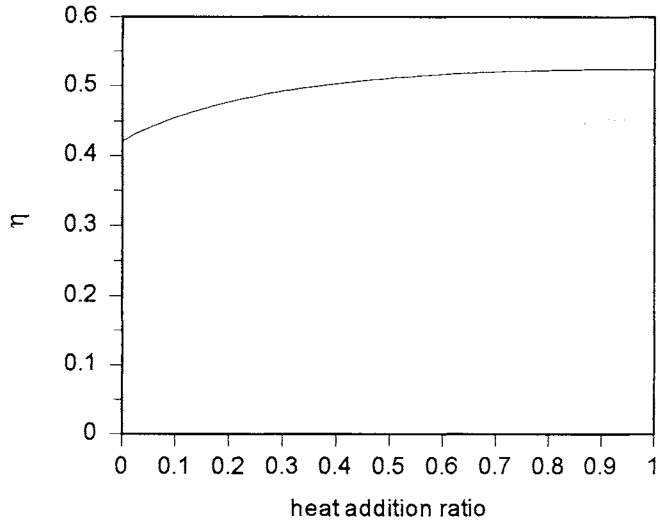
IT Results for $Q_{23}/m = Q_{34}/m = 700$ kJ/kg

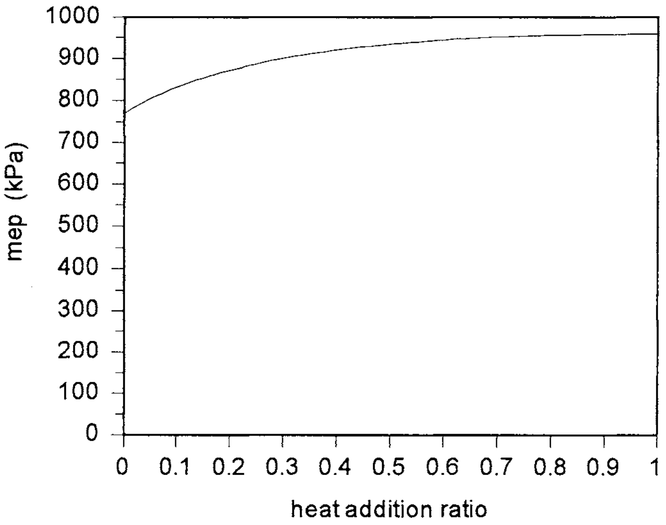
$T_2 = 702.8$ K
$p_2 = 2108$ kPa
$p_3 = 4534$ kPa
$h_3 = 1648$ kJ/kg
$h_4 = 2348$ kJ/kg
$T_5 = 1161$ K
$p_5 = 387$ kPa
$T_3 = 1511$ K
$T_4 = 2078$ K
$W_{cycle}/m = 715.6$ kJ/kg
$\eta = 0.5112$
mep = 935.1 kPa

PLOTS:









PROBLEM 9.34

The IT solution presented below uses the following constant specific heat relations:

$T_2/T_1 = (v_1/v_2)^{k-1}$

$T_5/T_4 = (v_4/v_5)^{k-1}$

$$\left.\begin{array}{l} Q_{23}/m = c_v\,(T_3 - T_2) \\ Q_{34}/m = c_p\,(T_4 - T_3) \\ Q_{51}/m = c_v\,(T_1 - T_5) \end{array}\right\} W_{cycle} = Q_{23} + Q_{34} + Q_{51}$$

IT Code

```
p1 = 100  // kPa
T1 = 300  // K
v1 / v2 = 9
Q23 + Q34 = 1400  // kJ/kg
Q23 / (Q23 + Q34) = ratio
ratio = .5
m = 1  // Assume a unit mass of 1 kg.
cp = cp_T("Air", T1)
cv = cv_T("Air", T1)
k = cp / cv

v1 = v_TP("Air",T1, p1)
T2 / T1 = (v1 / v2)^(k - 1)
v2 = v_TP("Air",T2, p2)
v3 = v2
Q23 = m * cv * (T3 - T2)
v3 = v_TP("Air",T3, p3)
p4 = p3
Q34 = m * cp * (T4 - T3)
v4 = v_TP("Air",T4, p4)
v5 = v1
T5 / T4 = (v4 / v5)^(k - 1)

Wcycle / m = Q23 + Q34 - m *cv * (T5 - T1)
eta = Wcycle / (Q23 + Q34)
mep = (Wcycle / m) / (v1 - v2)
```

IT Results for $Q_{23}/m = Q_{34}/m = 700$ kJ/kg

$c_p = 1.004$ kJ/kg·K

$c_v = 0.7172$ kJ/kg·K

$k = 1.4$

$T_3 = 1699$ K

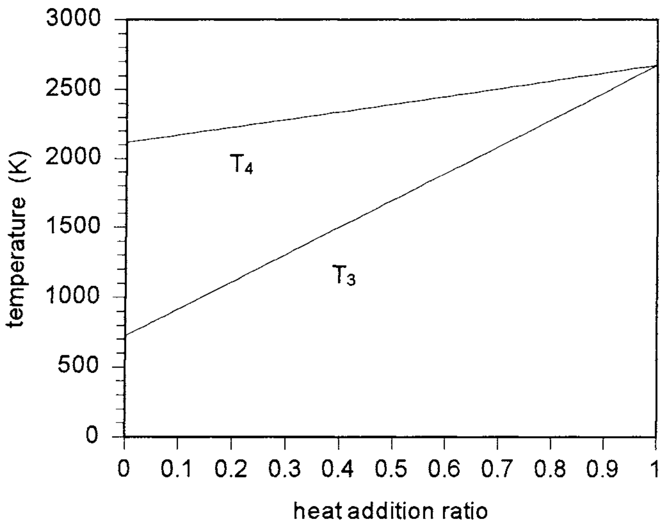
$T_4 = 2396$ K

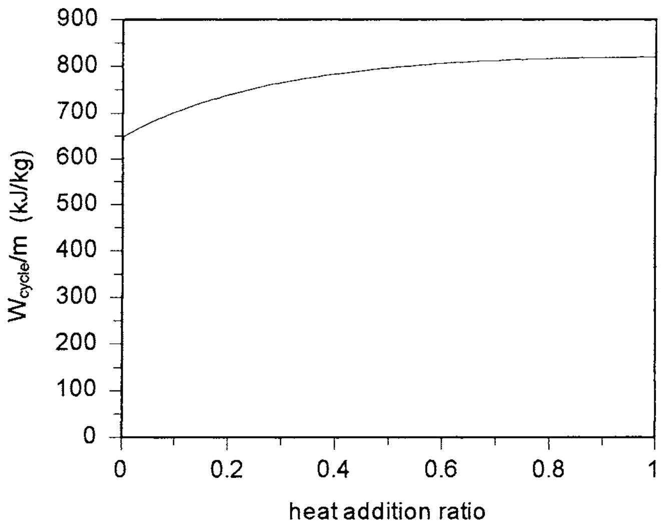
$W_{cycle}/m = 796.7$ kJ/kg

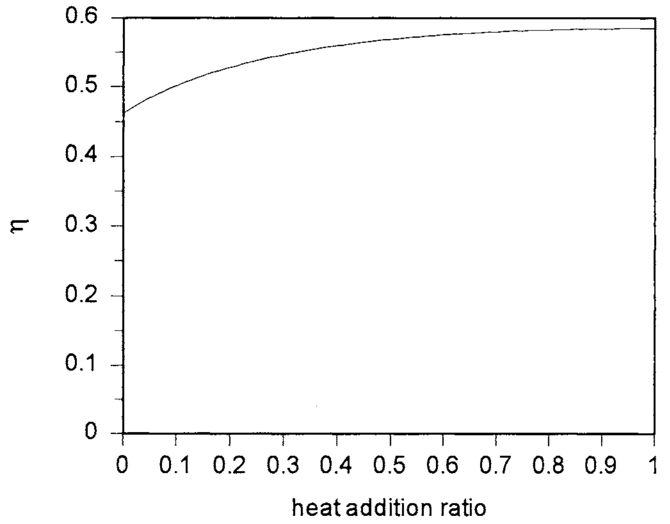
$\eta = 0.5691$

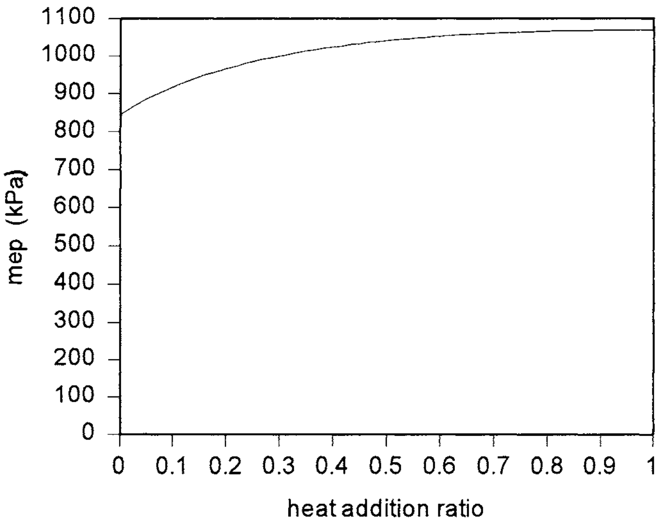
mep = 1041 kPa

PLOTS:









PROBLEM 9.35

KNOWN: An expression is given for the thermal efficiency of a cold air-standard dual cycle in terms of compression ratio, r, cutoff ratio, r_c, and pressure ratio for the constant volume heat addition, r_p.

FIND: Derive the expression.

ASSUMPTIONS: See Example 9.3. Also, the specific heats are constant.

ANALYSIS: Begin with Eq. 9.14

$$\eta = 1 - \frac{(u_5 - u_1)}{(u_3 - u_2) + (h_4 - h_3)}$$

$$= 1 - \frac{c_v\,(T_5 - T_1)}{c_v\,(T_3 - T_2) + c_p\,(T_4 - T_3)} = 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + k\,(T_4 - T_3)} \qquad (*)$$

Note that $r = v_1/v_2$, $r_c = v_4/v_3$, and $r_p = p_3/p_2$. Therefore

$$T_1 = \left(\frac{v_2}{v_1}\right)^{k-1} T_2 = \frac{T_2}{r^{k-1}}$$

$$T_3 = \left(\frac{p_3}{p_2}\right) T_2 = r_p T_2$$

$$T_4 = \left(\frac{v_4}{v_3}\right) T_3 = r_c T_3 = r_c r_p T_2$$

$$T_5 = \left(\frac{v_4}{v_5}\right)^{k-1} T_4 = \left(\frac{v_4}{v_3}\cdot\frac{v_2}{v_1}\right)^{k-1} T_4 = \left(r_c \cdot \frac{1}{r}\right)^{k-1} T_4 = r_p\left(\frac{r_c^{k}}{r^{k-1}}\right) T_2$$

Substituting these relations into (*) gives

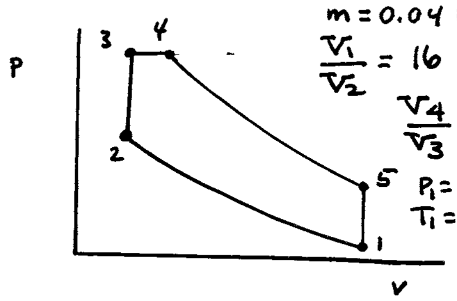
$$\eta = 1 - \frac{1}{r^{k-1}}\left[\frac{r_p r_c^{k} - 1}{(r_p - 1) + k\, r_p (r_c - 1)}\right] \longleftarrow \text{result}$$

PROBLEM 9.36*

KNOWN: Operating data are provided for an air-standard dual cycle.

FIND: Determine (a) the heat addition at constant volume and at constant pressure, (b) the net work, (c) the heat rejected, (d) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



$m = 0.04$ kg

$\frac{V_1}{V_2} = 16$

$\frac{V_4}{V_3} = 1.15$

$P_1 = 95$ kPa

$T_1 = 300$ K

ASSUMPTIONS:

See Example 9.3

ANALYSIS: Using data from Table A-22, fix each of the principal states.

State 1: At $T_1 = 300$ K, $u_1 = 214.07$ kJ/kg, $v_{r1} = 621.2$

State 2: For the isentropic compression

$v_{r2} = \frac{V_2}{V_1} v_{r1} = \frac{621.2}{16} = 38.825 \Rightarrow u_2 = 643.35$ kJ/kg, $T_2 = 862.4$ K

State 3: For the constant volume heat addition, $T_3 = (P_3/P_2)T_2 = 1897.3$ K.

$\Rightarrow u_3 = 1580.0$ kJ/kg, $h_3 = 2124.0$ kJ/kg.

State 4: For the constant pressure heat addition, $T_4 = \frac{V_4}{V_3} T_3 = (1.15)(1897.3) = 2182$ K.

$\Rightarrow h_4 = 2480.6$ kJ/kg, $v_{r4} = 2.0707$

State 5: For the isentropic expansion $\frac{V_5}{V_4} = \frac{V_1}{V_2}\frac{V_3}{V_4} = (16)\left(\frac{1}{1.15}\right) = 13.913$

$\Rightarrow v_{r5} = \frac{V_5}{V_4} v_{r4} = (13.913)(2.0707) = 28.810 \Rightarrow u_5 = 721.2$ kJ/kg

(a) Applying energy balances

$$Q_{23} = m(u_3 - u_2) = 0.04\,\text{kg}\,(1580.0 - 643.35)\,\tfrac{\text{kJ}}{\text{kg}} = 37.47\ \text{kJ/kg}$$ ←

$$Q_{34} = m(h_4 - h_3) = 0.04\,\text{kg}\,(2480.6 - 2124.0)\,\tfrac{\text{kJ}}{\text{kg}} = 14.26\ \text{kJ/kg}$$ ←

(b) For any cycle, $W_{cycle} = Q_{cycle}$. Thus, since processes 1-2, 4-5 are adiabatic,

$$W_{cycle} = (Q_{23} + Q_{34}) - \underbrace{Q_{51}}_{m(u_5-u_1)} = (37.47 + 14.26) - \underbrace{(0.04)(721.2 - 214.07)}_{20.29\,\text{kJ}}$$

$$= 31.44\ \text{kJ}$$ ←

(c) The heat rejection is $Q_{51} = m(u_5 - u_1) = 20.29$ kJ ←

(d) The thermal efficiency is

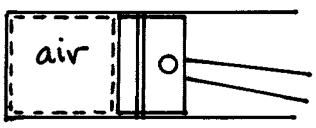
$$\eta = \frac{W_{cycle}}{\underbrace{Q_{in}}_{Q_{23}+Q_{34}}} = \frac{31.44}{37.47 + 14.26} = 0.608\ (60.8\%)$$ ←

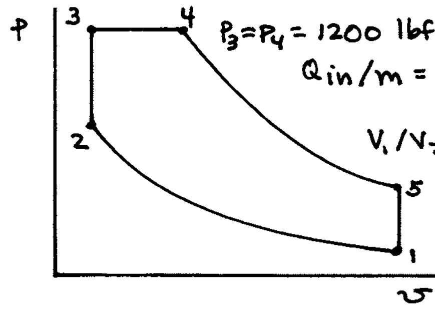
PROBLEM 9.37

KNOWN: An air-standard dual cycle has a known compression ratio and a specified state at the beginning of compression. The total heat addition per unit mass and the pressure at the end of the constant volume heat addition are also known.

FIND: Determine the net work and the heat rejection, each per unit mass of air. Also, the thermal efficiency and cutoff ratio. Plot those quantities versus compression ratio.

SCHEMATIC & GIVEN DATA:





$p_3 = p_4 = 1200$ lbf/in.²

$Q_{in}/m = 800$ Btu/lb

$V_1/V_2 = 15$

$p_1 = 14.0 \frac{lbf}{in^2}$

$T_1 = 520°R$

ASSUMPTIONS: See Example 9.3

ANALYSIS: Begin by fixing each of the principal states of the cycle (Table A-16E).

State 1: $T_1 = 520°R \Rightarrow u_1 = 88.62$ Btu/lb, $v_{r1} = 158.58$

State 2: For the isentropic compression, $v_{r2} = (V_2/V_1)\, v_{r1} = 10.572$.
Thus, $T_2 = 1468.8°R$, $u_2 = 260.26$ Btu/lb, and $p_{r2} = 51.561$

State 3: Since $V_3 = V_2$, $T_3 = (p_3/p_2)T_2$. To get p_2, use

$$p_2 = \left(\frac{p_{r2}}{p_{r1}}\right)p_1 = 594.26 \text{ lbf/in}^2 \Rightarrow T_3 = 2965.9\ °R$$

Thus, $u_3 = 577.4$ Btu/lb and $h_3 = 780.7$ Btu/lb

State 4: The total heat addition is known. Thus

$$\frac{Q_{in}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} = (u_3 - u_2) + (h_4 - h_3)$$

or

$$h_4 = \frac{Q_{in}}{m} - (u_3 - u_2) + h_3 = 1263.56 \text{ Btu/lb}$$

Thus, $T_4 = 4577.6\ °R$ and $v_{r4} = 0.2848$

State 5: For the isentropic expansion

$$\frac{V_5}{V_4} = \frac{V_1}{V_2}\cdot\frac{V_3}{V_4} = \frac{V_1}{V_2}\cdot\frac{T_3}{T_4} = 9.7187$$

and

$$v_{r5} = \left(\frac{V_5}{V_4}\right)v_{r4} = 2.768 \Rightarrow T_5 = 2299.2\ °R,\ u_5 = 431.0 \text{ Btu/lb}$$

Now, the heat rejection per unit mass is

(b) $$\frac{Q_{51}}{m} = u_5 - u_1 = 431.0 - 88.62 = 342.4 \text{ Btu/lb} \longleftarrow \underline{Q_{51}/m}$$

and for the cycle, $W_{cycle} = Q_{cycle}$. Thus

(a) $$\frac{W_{cycle}}{m} = \frac{Q_{in}}{m} - \frac{Q_{51}}{m} = 457.6 \text{ Btu/lb} \longleftarrow \underline{W_{cycle}/m}$$

(c) The thermal efficiency is

$$\eta = (W_{cycle}/m)/(Q_{in}/m) = (457.6)/(800) = 0.572\ (57.2\%) \longleftarrow \underline{\eta}$$

(d) The cutoff ratio is

$$r_c = v_4/v_3 = T_4/T_3 = 4577.6/2965.9 = 1.543 \longleftarrow \underline{r_c}$$

PROBLEM 9.37

The data for the required plots are obtained using IT, as follows:

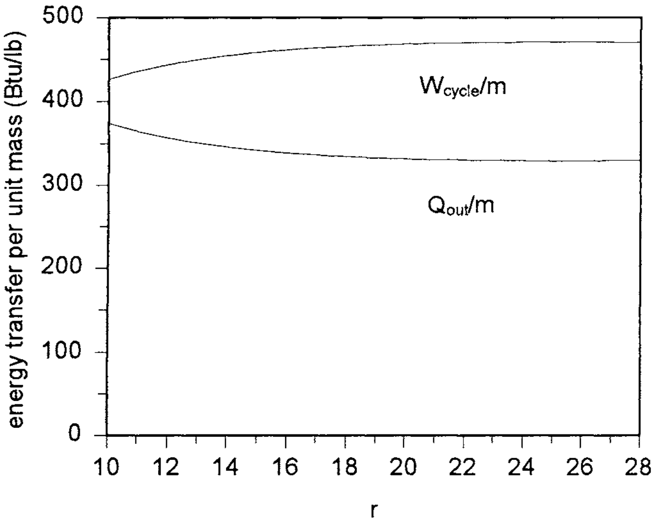
IT Code

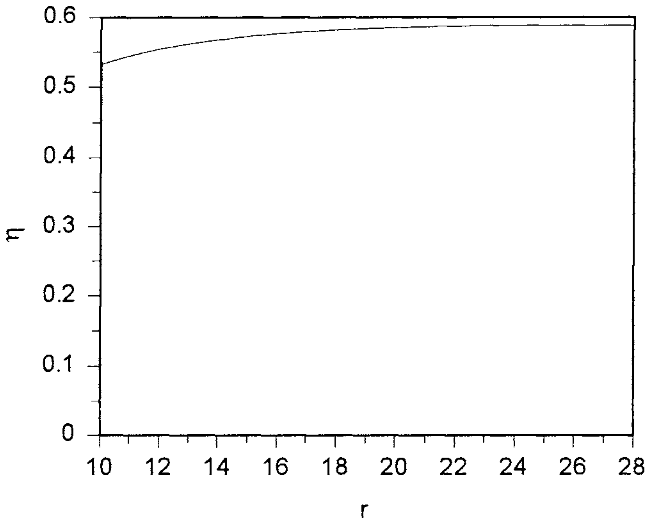
```
p1 = 14  // lbf/in.²
T1 = 520  // °R
r = v1 / v2
r = 15
p3 = 1200  // lbf/in.²
Qin / m = 800  // Btu/lb
m = 1  // Assume a unit mass of 1 lb.

u1 = u_T("Air", T1)
v1 = v_TP("Air",T1, p1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air",T2, p2)
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
T3 / T2 = p3 / p2
u3 = u_T("Air", T3)
h3 = h_T("Air", T3)
v3 = v2
Qin / m = (u3 - u2) + (h4 - h3)
h4 = h_T("Air", T4)
v4 = v_TP("Air",T4, p4)
p4 = p3
s4 = s_TP("Air", T4, p4)
v5 = v_TP("Air",T5, p5)
s5 = s_TP("Air", T5, p5)
s5 = s4
v5 = v1
u5 = u_T("Air", T5)

Qout / m = u5 - u1
Wcycle / m = Qin / m - Qout / m
eta = Wcycle / Qin
rc = v4 / v3
```

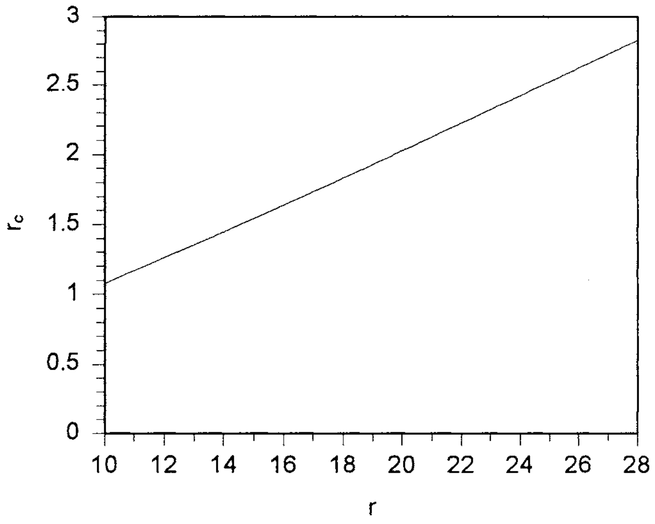
IT Results for r = 15

T_2 = 1469 °R
T_3 = 2971 °R
T_4 = 4582 °R
T_5 = 2301 °R
W_{cycle}/m = 457.9 Btu/lb
Q_{out}/m = 342.1 Btu/lb
η = 0.5724
r_c = 1.542

PLOTS:







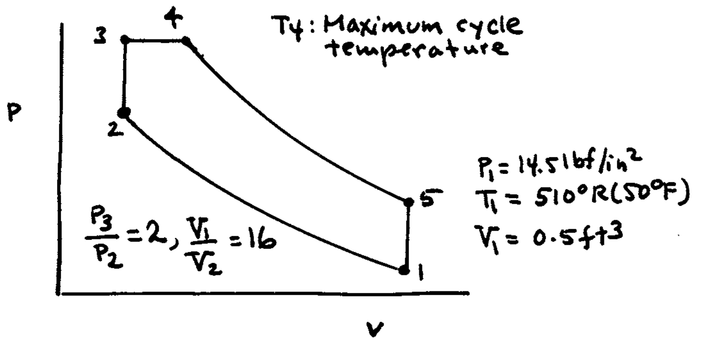
We see from the graphs that the thermal efficiency increases slightly with increasing r. Since Qin is constant, it follows that Wcycle would increase and Qout would decrease. Also, as r increases, the cutoff ratio increases, indication that of the heat input occurs at constant pressure.

PROBLEM 9.38

KNOWN: Operating data are provided for an air-standard dual cycle.

FIND: For a maximum cycle temperature of 3000°R determine (a) the heat added, (b) the net work, (c) the thermal efficiency, (d) the mean effective pressure. (e) Plot the quantities of parts (a)-(d) versus maximum cycle temperature varying from 3000 to 4000°R.

SCHEMATIC & GIVEN DATA:



$P_1 = 14.5\ \text{lbf/in}^2$
$T_1 = 510°R\,(50°F)$
$V_1 = 0.5\ \text{ft}^3$

$\frac{P_3}{P_2} = 2,\ \frac{V_1}{V_2} = 16$

ASSUMPTIONS:

See Example 9.3

ANALYSIS:

$T_4 = 3000°R$ ... begin by fixing the state of each principal state using data from Table A-22E.

State 1: $T_1 = 510°R$, $u_1 = 86.91$ Btu/lb, $v_{r1} = 166.74$.

State 2: For the isentropic compression, $v_{r2} = (V_2/V_1)\,v_{r1} = (166.74/16) = 10.421$

$\Rightarrow T_2 = 1476.1\ °R$, $u_2 = 261.68$ Btu/lb

State 3: For the constant volume heat addition, $T_3/T_2 = P_3/P_2$. Thus,

$T_3 = 2T_2 = 2952.2°R \Rightarrow u_3 = 574.33$ Btu/lb, $h_3 = 776.69$ Btu/lb.

State 4: With $T_4 = 3000°R$, $h_4 = 790.68$ Btu/lb, $v_{r4} = 1.18$

State 5: For the isentropic expansion $v_{r5} = (V_5/V_4)\,v_{r4}$, where

$$\frac{V_5}{V_4} = \frac{V_1}{V_2}\underbrace{\frac{V_3}{V_4}}_{\text{since } P_3 = P_4,\ \frac{V_3}{V_4} = \frac{T_3}{T_4}} = \frac{V_1}{V_2}\cdot\frac{T_3}{T_4} = 16\left(\frac{2952.2}{3000}\right) = 15.745 \Rightarrow v_{r5} = 18.579$$

$\Rightarrow u_5 = 208.76$ Btu/lb

The mass of the system is obtained from

$$m = \frac{P_1 V_1}{R T_1} = \frac{(14.5 \times 144\ \text{lbf/ft}^2)(0.5\ \text{ft}^3)}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(510°\text{R})} = 0.03838\ \text{lb}$$

(a) The heat added is the sum $Q_{23} + Q_{34}$

$$\left.\begin{aligned} Q_{23} &= m(u_3 - u_2) = 0.03838(574.33 - 261.68) = 12\ \text{Btu} \\ Q_{34} &= m(h_4 - h_3) = 0.03838(790.68 - 776.69) = 0.54\ \text{Btu} \end{aligned}\right\} Q_{in} = 12.54\ \text{Btu} \leftarrow Q_{in}$$

(b) Since $W_{cycle} = Q_{cycle}$, and processes 1-2, 4-5 are adiabatic

$W_{cycle} = Q_{in} - Q_{51} = Q_{in} - m(u_5 - u_1) = 12.54 - 0.03838(208.76 - 86.91) = 7.86$ Btu $\leftarrow W_{cycle}$

(d) $$\text{mep} = \frac{W_{cycle}}{(V_1 - V_2)} = \frac{W_{cycle}}{V_1[1 - V_2/V_1]} = \frac{7.86 \times 778\ \text{ft}\cdot\text{lbf}}{0.5\ \text{ft}^3\left[1 - \frac{1}{16}\right]}\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| = 90.59\ \frac{\text{lbf}}{\text{in}^2} \leftarrow \text{mep}$$

(c) $$\eta = \frac{W_{cycle}}{Q_{in}} = \frac{7.86}{12.54} = 0.627\ (62.7\%) \leftarrow \eta$$

PROBLEM 9.38 (cont'd.)

The data for the required plots are obtained using IT, as follows:

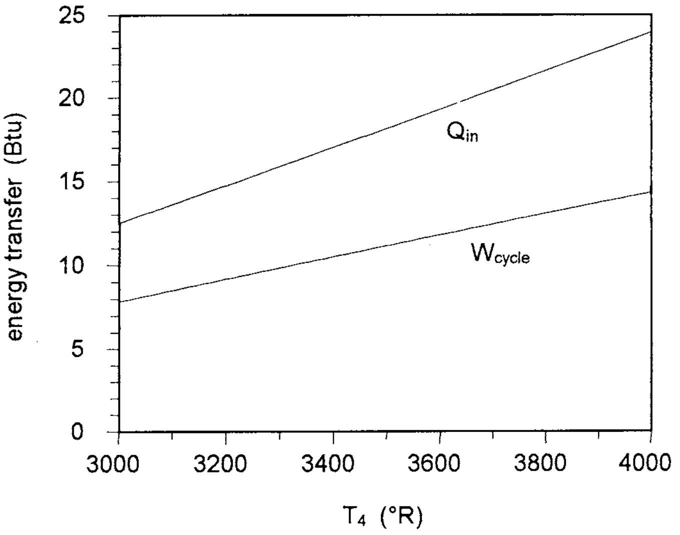
IT Code

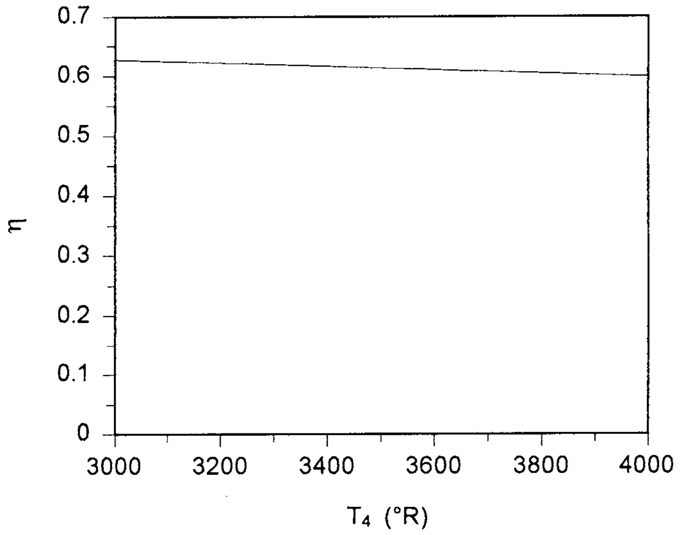
```
p1 = 14.5  // lbf/in.²
T1 = 510  // °R
V1 = 0.5  // ft³
V1 / V2 = 16
p3 / p2 = 2
T4 = 3000  // °R

v1 = v_TP("Air",T1, p1)
u1 = u_T("Air", T1)
s1 = s_TP("Air", T1, p1)
v2 = v_TP("Air",T2, p2)
v1 / v2 = 16
s2 = s_TP("Air", T2, p2)
s2 = s1
u2 = u_T("Air", T2)
T3 / T2 = p3 / p2
u3 = u_T("Air", T3)
h3 = h_T("Air", T3)
v3 = v2
h4 = h_T("Air", T4)
v4 = v_TP("Air",T4, p4)
p4 = p3
s4 = s_TP("Air", T4, p4)
v5 = v_TP("Air",T5, p5)
s5 = s_TP("Air", T5, p5)
s5 = s4
v5 = v1
u5 = u_T("Air", T5)

v1 = V1 / m
Qin = m * (u3 - u2) + m * (h4 - h3)
Q51 = m * (u1 - u5)
Wcycle = Qin + Q51
eta = Wcycle / Qin
mep = Wcycle / (V1 * (1 - V2/ V1)) * (778 / 144)
```

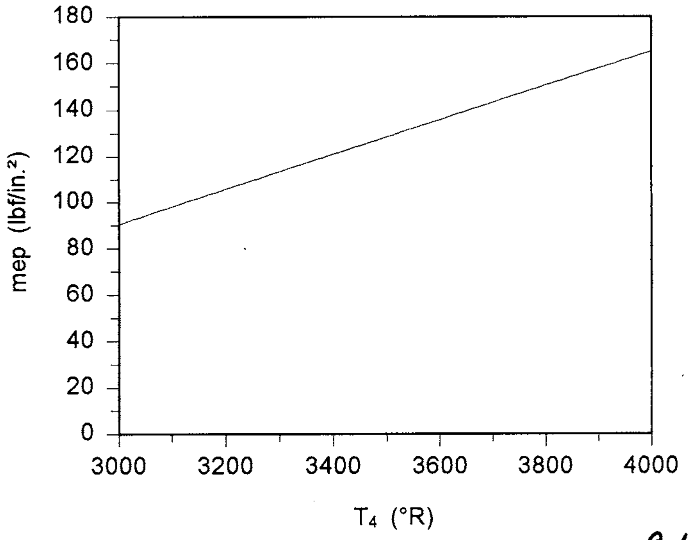
IT Results for T_4 = 3000°R

T_2 = 1477°R
T_3 = 2954°R
T_5 = 1196°R
m = 0.03838 lb
Q_{in} = 12.49 Btu
W_{cycle} = 7.837 Btu
η= 0.6273
mep = 90.33 lbf/in.²

PLOTS:







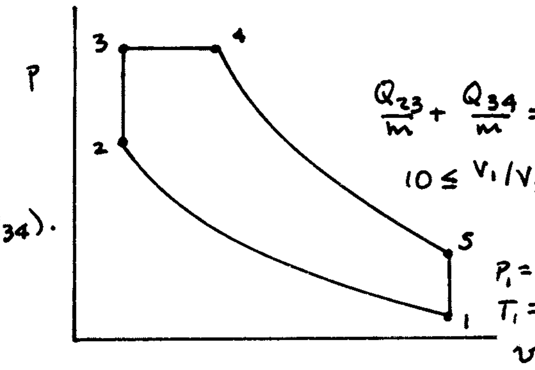
As the maximum cycle temperature increases, we see that the net work and mep both increase as well. However, the heat addition also goes up, and as a result the thermal efficiency decreases slightly.

PROBLEM 9.39

KNOWN: An air-standard dual cycle has a known compression ratio and a specified state at the beginning of compression. The total heat addition is given.

FIND: Plot the net work per unit mass, the mean effective pressure, and the thermal efficiency versus compression ratio ranging from 10 to 20 for various values of the ratio of constant volume to total heat input.

SCHEMATIC & GIVEN DATA:



$\frac{Q_{23}}{m} + \frac{Q_{34}}{m} = 1000$ kJ/kg

$10 \leq V_1/V_2 \leq 20$

$p_1 = 1$ bar

$T_1 = 300$K

① ASSUMPTIONS: See Example 9.3.

ANALYSIS: The following reasoning is used to solve the problem:

1. Given values: p_1, T_1, r, and $Q_{23}/(Q_{23}+Q_{34})$.
2. Fix state 2 using ideal gas relations and $s(T_1, p_1) = s(T_2, p_2)$. When using Table A-22, this expressed in terms of $v_{r_1}/v_{r_2} = v_1/v_2$.
3. Use $Q_{23} = m(u_3 - u_2)$ to get T_3. Then $v_3 = v_2$ gives $p_3 = (T_3/T_2)\,p_2$.
4. For state 4, use $p_4 = p_3$ and $Q_{34} = m(h_4 - h_3)$ to get T_4, v_4.
5. Fix state 5 as in step 2 using $s(T_4, p_4) = s(T_5, p_5)$ or $v_{r4}/v_{r5} = v_4/v_5$. Also, note that $v_5 = v_1$.

Now, using energy balances: $W_{cycle} = Q_{cycle} \Rightarrow \frac{W_{cycle}}{m} = \frac{Q_{23}}{m} + \frac{Q_{34}}{m} - (u_5 - u_1)$ where $Q_{out}/m = u_5 - u_1$. Also

$$\eta = \frac{W_{cycle}/m}{(Q_{23}+Q_{34})/m} \quad \text{and} \quad mep = \frac{W_{cycle}/m}{v_1(1 - 1/r)}$$

Sample calculation. $\boxed{r = 10,\ Q = Q_{23}/(Q_{23}+Q_{34}) = 0.5}$ From Table A-22 at $T_1 = 300$K ; $u_1 = 214.07$ kJ/kg, $v_{r_1} = 621.2$. Thus $v_{r_2} = (v_2/v_1)\,v_{r_1} = 62.12$. Interpolating in Table A-22 ; $T_2 = 730$ K, $u_2 = 536.1$ kJ/kg. Now

$$u_3 = Q_{23}/m + u_2 = 1036.1 \text{ kJ/kg} \Rightarrow T_3 = 1314.7\text{K},\ h_3 = 1413.46 \text{ kJ/kg}$$

Further, $h_4 = Q_{34}/m + h_3 = 1913.46$ kJ/kg $\Rightarrow T_4 = 1727.1$K, $v_{r4} = 4.526$

$$v_{r5} = (v_5/v_4)\,v_{r4} = (v_1/v_2)(v_3/v_4)\,v_{r4} = (v_1/v_2)(T_3/T_4)\,v_{r4} = 34.453$$

Thus, $T_5 = 898.8$ K, $u_5 = 673.55$ kJ/kg.

Now

$$W_{cycle}/m = 1000 - (673.55 - 214.07) = 540.5 \text{ kJ/kg}$$

$$\eta = \frac{W_{cycle}/m}{(Q_{23}+Q_{34})/m} = 0.5405 \ (54.05\%)$$

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300\text{K})}{(1\text{ bar})}\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{kJ}}\right|\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right| = 0.861 \text{ m}^3/\text{kg}$$

$$mep = \frac{(540.5 \text{ kJ/kg})}{(0.861\text{ m}^3/\text{kg})(1 - 1/10)}\left|\frac{10^3\text{N}\cdot\text{m}}{1\text{kJ}}\right|\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right| = 6.975 \text{ bar}$$

The data for the required plots are obtained using IT, as follows:

PROBLEM 9.39 (Cont'd.)

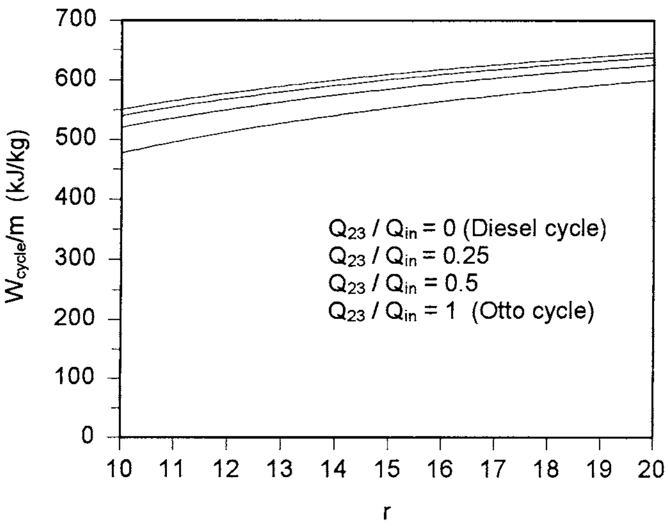
IT Code

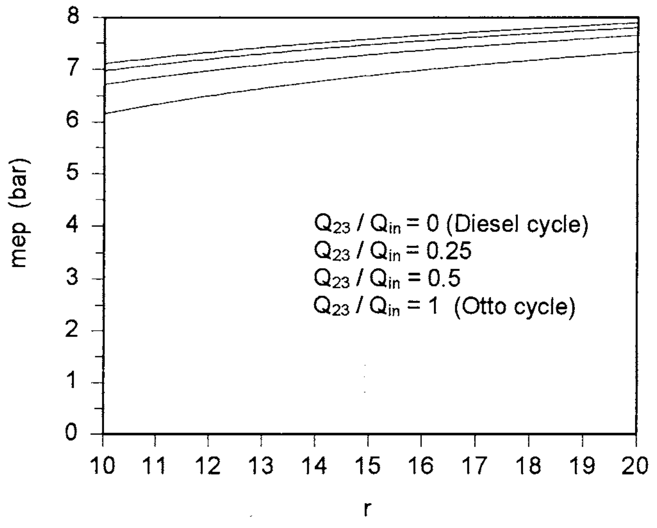
```
p1 = 1   // bar
T1 = 300   // K
(Q23 + Q34) / m = 1000   // kJ/kg
Q = Q23 / (Q23 + Q34)
Q = .5
r = 10
m = 1   // Assume a unit mass of 1
kg.

v1 = v_TP("Air",T1,p1)
s1 = s_TP("Air", T1, p1)
u1 = u_T("Air", T1)
s2 = s1
v2 = v1 / r
v2 = v_TP("Air",T2,p2)
s2 = s_TP("Air", T2, p2)
u2 = u_T("Air", T2)
m * (u3 - u2) = Q23
u3 = u_T("Air", T3)
v3 = v2
v3 = v_TP("Air",T3,p3)
s3 = s_TP("Air", T3, p3)
h3 = h_T("Air", T3)
Q34 / m = h4 - h3
h4 = h_T("Air", T4)
p4 = p3
v4 = v_TP("Air",T4,p4)
s4 = s_TP("Air", T4, p4)
u4 = u_T("Air", T4)
v5 = v1
s5 = s4
v5 = v_TP("Air",T5,p5)
s5 = s_TP("Air", T5, p5)
u5 = u_T("Air", T5)

Wcycle = Q23 / m + Q34 / m + Q51 / m
Q51 = m * (u1 - u5)
eta = Wcycle / (Q23 + Q34)
mep = (Wcycle / m) / (v1 * (1 - 1 / r)) / 100
```

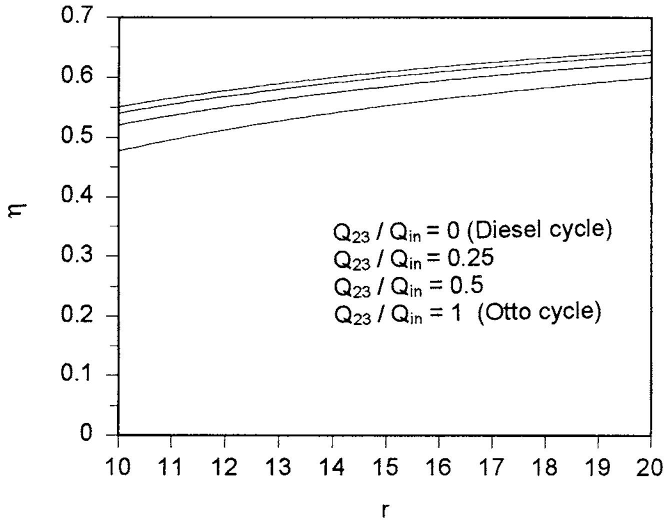
IT Results for Q23 / Q1n = 0.5 and r = 10

T_2 = 730.2 K
p_2 = 24.34 bar
T_3 = 1316 K
p_3 = 43.86 bar
T_4 = 1729 K
T_5 = 899.3 K
W_{cycle}/m = 540.2 kJ/kg
η = 0.5402
mep = 6.972 bar

PLOTS:





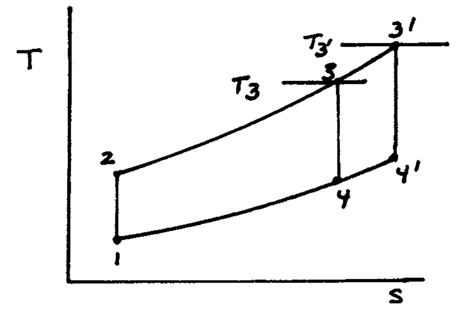


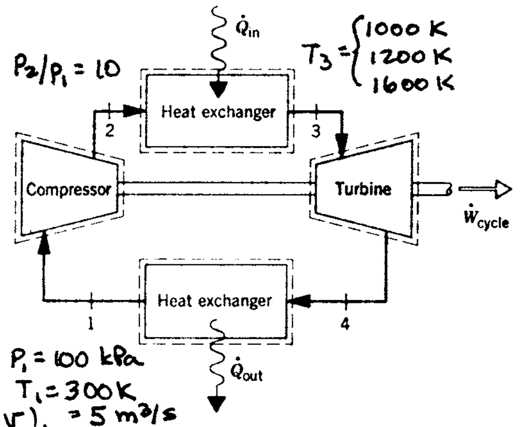
PROBLEM 9.40

KNOWN: Air enters the compressor of an ideal Brayton cycle with known conditions. The compressor pressure ratio is also known.

FIND: Plot for various turbine inlet temperatures (a) the thermal efficiency, (b) the back work ratio, and (c) the net power developed.

SCHEMATIC & GIVEN DATA:





$P_2/P_1 = 10$

$T_3 = \begin{cases} 1000\text{ K} \\ 1200\text{ K} \\ 1600\text{ K} \end{cases}$

$P_1 = 100$ kPa

$T_1 = 300$K

$(AV)_1 = 5\ m^3/s$

ASSUMPTIONS: See Example 9.4

ANALYSIS: Sample calculations are given below for several turbine inlet temperatures. First, fix each of the principal states (Table A-22).

State 1 $T_1 = 300K \Rightarrow h_1 = 300.19$ kJ/kg , $P_{r1} = 1.3860$

State 2 For the isentropic compression, $P_{r2} = (P_2/P_1)\,P_{r1} = 13.860$.
Thus, $T_2 = 574.1$ K and $h_2 = 579.86$ kJ/kg

State 3

$$T_3 = \begin{cases} 1000\text{ K} \Rightarrow h_3 = 1046.04\text{ kJ/kg}, & P_{r3} = 114.0 \\ 1200\text{ K} \Rightarrow h_3 = 1277.79\text{ kJ/kg}, & P_{r3} = 238.0 \\ 1600\text{ K} \Rightarrow h_3 = 1757.57\text{ kJ/kg}, & P_{r3} = 791.2 \end{cases}$$

State 4 For the isentropic expansion, $P_{r4} = (P_4/P_3)\,P_{r3} \Rightarrow T_4, h_4$.
Thus,

$$P_{r4} = \begin{cases} 11.4, & T_4 = 543.9\text{ K}, & h_4 = 548.45\text{ kJ/kg} & (1000\text{ K}) \\ 23.8, & T_4 = 665\text{ K}, & h_4 = 675.84\text{ kJ/kg} & (1200\text{ K}) \\ 79.12, & T_4 = 911.3\text{ K}, & h_4 = 945.65\text{ kJ/kg} & (1600\text{ K}) \end{cases}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

(b) The back work ratio is

$$bwr = \frac{h_2 - h_1}{h_3 - h_4} = \frac{279.67}{h_3 - h_4}$$

(c) The power is

$$\dot{W}_{net} = \dot{m}\,[(h_3 - h_4) - (h_2 - h_1)] = \frac{(AV)_1 P_1}{RT_1}\,[(h_3 - h_4) - 279.67]$$

The results are summarized in the plots on the next page, which give each of these quantities versus turbine inlet temperatures ranging from 1000 to 1600K.

PROBLEM 9.40 (Cont'd)

The data for the required plots are generated by the following IT program:

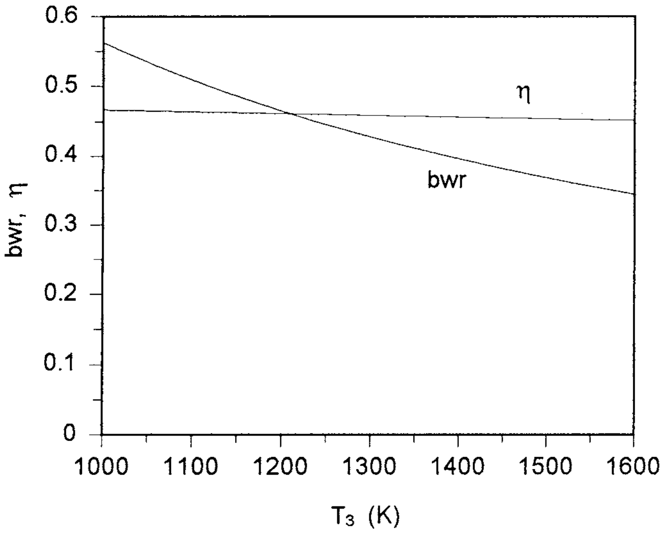
IT Code

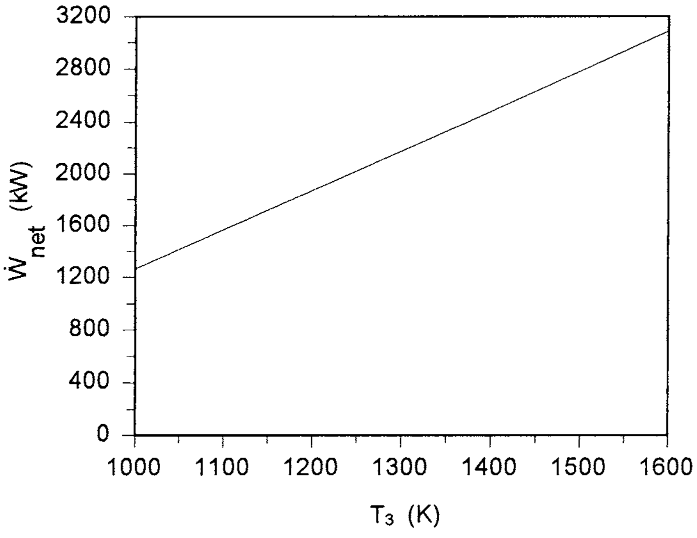
```
p1 = 100  // kPa
T1 = 300  // K
p2 / p1 = 10
p3 = p2
T3 = 1000  // K
p4 = p1
AV1 = 5  // m³/s

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4 = s_TP("Air", T4, p4)
s4 = s3
h4 = h_T("Air", T4)

mdot = AV1 / v1
v1 = v_TP("Air",T1,p1)
Wdotnet = mdot * ((h3 - h4) - (h2 - h1))
eta = Wdotnet /  (mdot * (h3 - h2))
bwr = (h2 - h1) / (h3 - h4)
```

IT Results for T_3 = 1000K

h_1 = 300 kJ/kg
h_2 = 579.7 kJ/kg
h_3 = 1046 kJ/kg
h_4 = 548.3 kJ/kg
$\dot{m}$ = 5.807 kg/s
$\dot{W}_{net}$ = 1264 kW
bwr = 0.5623
η = 0.4672

PLOTS:





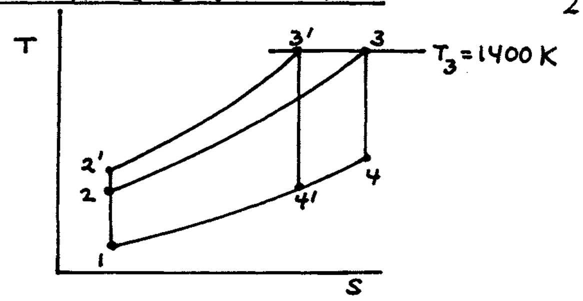
From the plots we see that the net work of the cycle increases significantly and the back-work ratio decreases as turbine inlet temperature increases, for fixed compressor pressure ratio. However, the heat input increases significantly, and the result is a very slight decrease in thermal efficiency.

PROBLEM 9.41

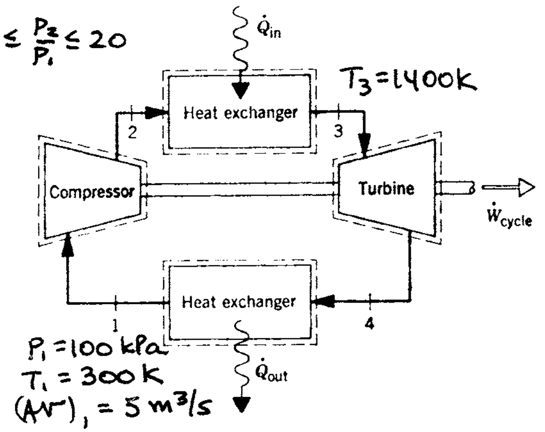
KNOWN: Air enters the compressor of an ideal Brayton cycle with known conditions. The turbine inlet temperature is specified.

FIND: Plot for a given range of compressor pressure ratios (a) the thermal efficiency, (b) the back work ratio, and (c) the net power developed.

SCHEMATIC & GIVEN DATA:



$2 \le \frac{P_2}{P_1} \le 20$



$P_1 = 100$ kPa
$T_1 = 300$ K
$(AV)_1 = 5$ m³/s

ASSUMPTIONS: See Example 9.4

ANALYSIS: Sample calculations are given here. First, fix each of the principal states (Table A-22)

State 1 $T_1 = 300$ K $\Rightarrow h_1 = 300.19$ kJ/kg, $Pr_1 = 1.3860$

State 2 For isentropic compression, $Pr_2 = (P_2/P_1)\,Pr_1 \Rightarrow T_2, h_2$

Thus,

$$P_2/P_1 = \begin{cases} 6 \Rightarrow Pr_2 = 8.316, & T_2 = 498.4 \text{ K}, h_2 = 501.36 \text{ kJ/kg} \\ 8 \Rightarrow Pr_2 = 11.088, & T_2 = 539.8 \text{ K}, h_2 = 544.18 \text{ kJ/kg} \\ 12 \Rightarrow Pr_3 = 16.632, & T_2 = 603.4 \text{K}, h_2 = 610.65 \text{ kJ/kg} \end{cases}$$

State 3 $T_3 = 1400$K $\Rightarrow h_3 = 1515.42$ kJ/kg, $Pr_3 = 450.5$

State 4 For the isentropic expansion, $Pr_4 = (P_4/P_3)\,Pr_3 \Rightarrow T_4, h_4$

Thus,

$$Pr_4 = \begin{cases} 75.083, & T_4 = 899.3 \text{ K}, & h_4 = 932.20 \text{ kJ/kg} \\ 56.312, & T_4 = 834.9 \text{K}, & h_4 = 860.40 \text{ kJ/kg} \\ 37.542, & T_4 = 751 \text{ K}, & h_4 = 768.38 \text{ kJ/kg} \end{cases}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2}$$

(b) The back work ratio is

$$bwr = \frac{h_2 - h_1}{h_3 - h_4}$$

(c) the net power developed is

$$\dot{W}_{cycle} = \dot{m}[(h_3 - h_4) - (h_2 - h_1)] = \frac{(AV)_1 P_1}{RT_1}[(h_3 - h_4) - (h_2 - h_1)]$$

The results are summarized in the following table:

P_2/P_1	η(%)	bwr	$\dot{W}_{net}$ (kW)
6	37.67	.345	2218.6
8	42.32	.372	2386.8
12	48.25	.416	2535.2

$\dot{m} = 5.807$ kg/s

The data for the required plots are obtained using IT, as follows:

PROBLEM 9.41

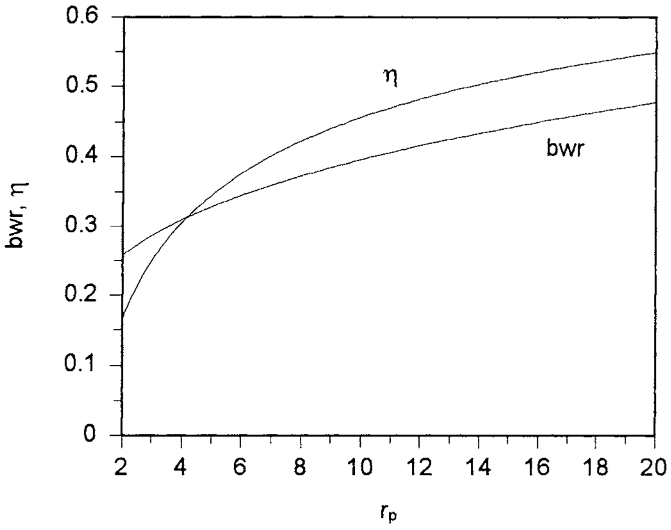
IT Code

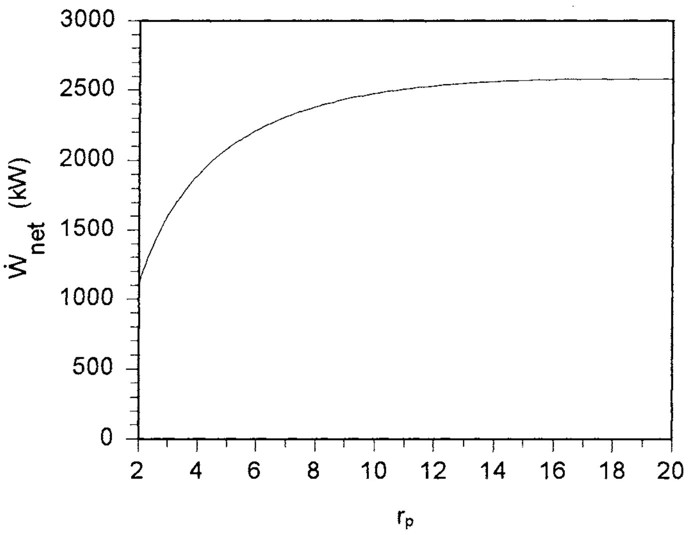
```
p1 = 100  // kPa
T1 = 300  // K
p2 / p1 = rp
rp = 8
p3 = p2
T3 = 1400  // K
p4 = p1
AV1 = 5  // m³/s

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2 = s_TP("Air", T2, p2)
s2 = s1
h2 = h_T("Air", T2)
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4 = s_TP("Air", T4, p4)
s4 = s3
h4 = h_T("Air", T4)

mdot = AV1 / v1
v1 = v_TP("Air",T1,p1)
Wdotnet = mdot * ((h3 - h4) - (h2 - h1))
eta = Wdotnet /  (mdot * (h3 - h2))
bwr = (h2 - h1) / (h3 - h4)
```

IT Results for $r_p = 8$

$h_1 = 300$ kJ/kg
$h_2 = 544.1$ kJ/kg
$h_3 = 1514$ kJ/kg
$h_4 = 859.9$ kJ/kg
$\dot{m} = 5.807$ kg/s
$\dot{W}_{net} = 2382$ kW
bwr = 0.373
$\eta = 0.4228$

PLOTS:





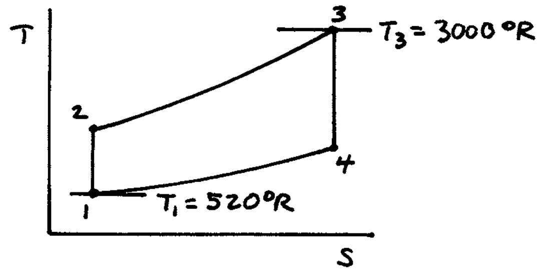
In the given range of compressor pressure ratios, the net power increases with r_p, as does the thermal efficiency. Yet, the back work ratio also increases with r_p.

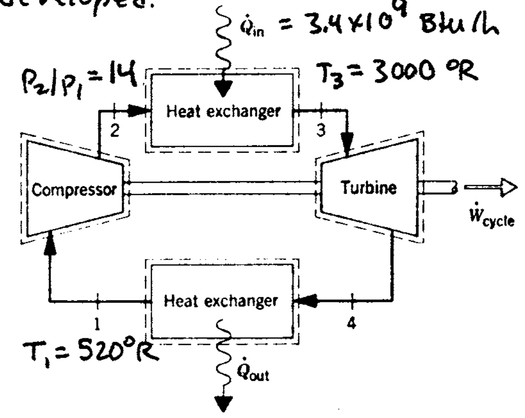
PROBLEM 9.42*

KNOWN: The net power developed and the pressure ratio are known for an ideal air-standard Brayton cycle. The minimum and maximum temperatures are also known.

FIND: Determine (a) the thermal efficiency, (b) the mass flow rate of air, (c) the net power developed.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.4

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 520°R \Rightarrow h_1 = 124.27$ Btu/lb, $p_{r1} = 1.2147$

State 2 For the isentropic compression, $p_{r2} = (P_2/P_1)\,p_{r1} = 17.0058$
Thus, $T_2 = 1092.5\ °R$ and $h_2 = 264.12$ Btu/lb

State 3 $T_3 = 3000\ °R \Rightarrow h_3 = 790.68$ Btu/lb, $p_{r3} = 941.4$

State 4 For the isentropic expansion, $p_{r4} = (P_4/P_3)\,p_{r3} = 67.24$
Thus, $T_4 = 1573\ °R$ and $h_4 = 388.63$ Btu/lb

(a) The thermal efficiency is

$$\eta = 1 - \frac{\dot{Q}_{out}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 1 - \frac{(h_4 - h_1)}{(h_3 - h_2)}$$

$$= 1 - \frac{(388.63 - 124.27)}{(790.68 - 264.12)} = 0.498\ (49.8\,\%) \longleftarrow \underline{\qquad\qquad \eta}$$

(b) To determine the mass flow rate of air, start with the rate of heat addition

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_3 - h_2)} = \frac{(3.4 \times 10^9\ \text{Btu/h})}{(790.68 - 264.12)\ \text{Btu/lb}}$$

$$= 6.457 \times 10^6\ \text{lb/h} \longleftarrow \underline{\qquad\qquad \dot{m}}$$

(c) The net power is

$$\dot{W}_{cycle} = \dot{m}\,[(h_3 - h_4) - (h_2 - h_1)]$$

$$= (6.457 \times 10^6)[(790.68 - 388.63) - (264.12 - 124.27)]$$

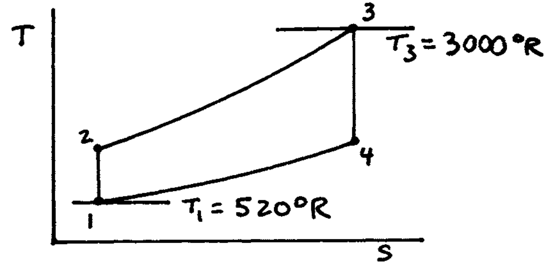
$$= 1.693 \times 10^9\ \text{Btu/h} \longleftarrow \underline{\qquad\qquad \dot{W}_{cycle}}$$

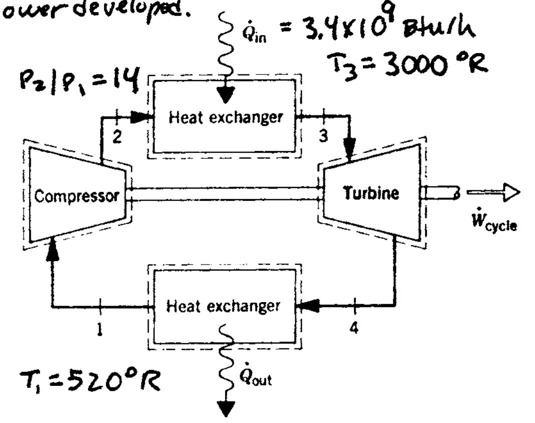
PROBLEM 9.43*

KNOWN: The net power developed and the pressure ratio are known for a cold air-standard ideal Brayton cycle. The minimum and maximum temperatures are also known.

FIND: Determine (a) the thermal efficiency, (b) the mass flow rate of air, (c) the net power developed.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.4
Also, assume constant specific heats evaluated at 520°R.

ANALYSIS: From Table A-20E, $k = 1.401$ and $c_p = 0.240$ Btu/lb·°R. $T_1 = 520°R$ and $T_3 = 2800°R$ are given. For the isentropic compression process

$$T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{(k-1)}{k}} T_1 = 1106.8 \text{ °R}$$

Also, for the isentropic expansion

$$T_4 = \left(\frac{P_4}{P_3}\right)^{\frac{(k-1)}{k}} T_3 = 1409.5 \text{ °R}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{1}{(P_2/P_1)^{(k-1)/k}} = 0.530 \ (53.0\ \%) \longleftarrow \eta$$

(b) To determine the mass flow rate of air, start with the rate of heat addition

$$\dot{m} = \frac{\dot{Q}_{in}}{c_p(T_3 - T_2)} = \frac{(3.4 \times 10^9 \text{ Btu/h})}{(0.240 \text{ Btu/lb·°R})(3000 - 1106.8)\text{°R}}$$

$$= 7.483 \times 10^6 \text{ lb/h} \longleftarrow \dot{m}$$

(c) The net power is

$$\dot{W}_{cycle} = \dot{m}[(h_3 - h_4) - (h_2 - h_1)] = \dot{m} c_p [(T_3 - T_4) - (T_2 - T_1)]$$

$$= (7.483 \times 10^6)(0.240)[(3000 - 1409.5) - (1106.8 - 520)]$$

$$= 1.803 \times 10^9 \text{ Btu/h} \longleftarrow \dot{W}_{cycle}$$

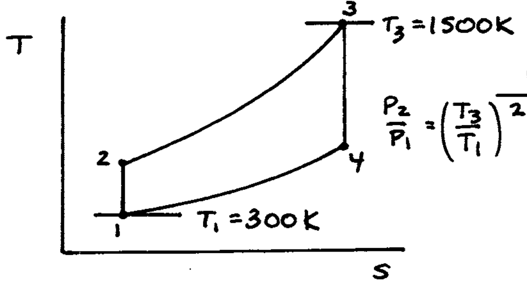
Discussion: The results of this problem can be compared with those of Problem 9.42 to see some effects of the assumption of constant specific heats.

PROBLEM 9.44

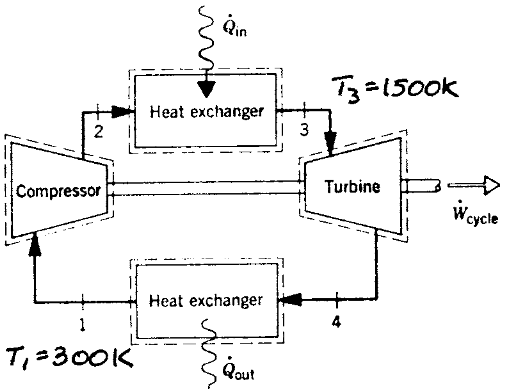
KNOWN: An ideal cold air-standard Brayton cycle has known minimum and maximum temperatures a pressure that corresponds to the maximum net work per unit mass of air flow.

FIND: Determine (a) the compressor and turbine work per unit of mass flow, (b) thermal efficiency. (c) Plot the thermal efficiency versus the maximum cycle temperature ranging from 1200 to 1800K.

SCHEMATIC & GIVEN DATA:



$$\frac{P_2}{P_1} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$



ASSUMPTIONS: See Example 9.5.

ANALYSIS: First, determine the unknown temperatures at principal states 2 and 4. From the solution to Example 9.5

$$\frac{P_2}{P_1} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} = \left(\frac{1500}{300}\right)^{\frac{1.4}{2(1.4-1)}} = 16.719$$

where $k = 1.4$ from Table A-20. For the isentropic compression

$$T_2 = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} T_1 = 670.8\ ^\circ R$$

and for the isentropic expansion

$$T_4 = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} T_3 = 670.8\ ^\circ R$$

(a) The compressor work per unit mass of air flow is

$$\frac{\dot{W}_{comp}}{\dot{m}} = h_2 - h_1 = c_p(T_2 - T_1)$$

$$= (1.005\ kJ/kg\cdot K)(670.8 - 300)K = 372.6\ \frac{kJ}{kg} \longleftarrow \dot{W}_{comp}/\dot{m}$$

Similarly, for the turbine

$$\frac{\dot{W}_{turb}}{\dot{m}} = c_p(T_3 - T_4)$$

$$= (1.005)(1500 - 670.8) = 833.3\ kJ/kg \longleftarrow \dot{W}_{turb}/\dot{m}$$

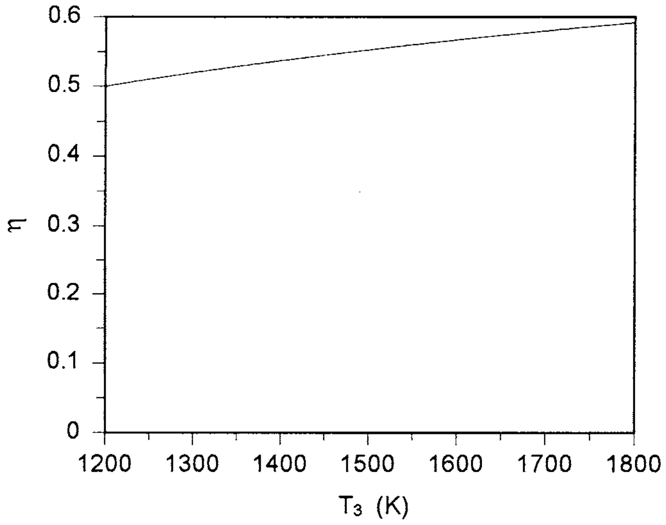
(b) Using Eq. 9.25 together with $P_2/P_1 = (T_3/T_1)^{\frac{k}{2(k-1)}}$, we get

$$\eta = 1 - \frac{1}{(P_2/P_1)^{(k-1)/k}} = 1 - \frac{1}{\left[\left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}\right]^{\frac{k-1}{k}}} = 1 - \left(\frac{T_1}{T_3}\right)^{1/2}$$

$$= 1 - \left(\frac{300}{1500}\right)^{1/2} = 0.553 \quad (55.3\%)$$

PROBLEM 9.44 (Cont'd.)

(c) The expression for η in part (b) is plotted using IT or any plotting package.



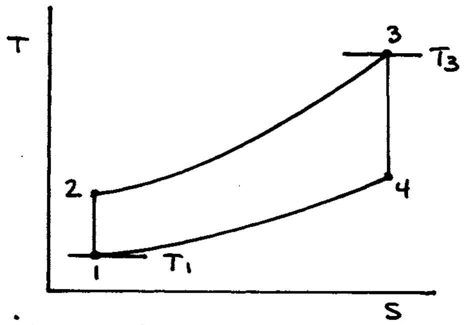
We see that thermal efficiency increases with increasing maximum cycle temperature. Also, note that for this idealized case η is independant of the value of k.

PROBLEM 9.45

KNOWN: An ideal cold air-standard Brayton cycle is analyzed.

FIND: Show that the back work ratio equals the ratio of absolute temperatures at the compressor inlet and turbine outlet.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 9.5

ANALYSIS: The back work ratio is given as

$$bwr = \frac{h_2 - h_1}{h_3 - h_4} = \frac{\cancel{c_p}(T_2 - T_1)}{\cancel{c_p}(T_3 - T_4)}$$

or

$$bwr = \frac{T_1\,(T_2/T_1 - 1)}{T_4\,(T_3/T_4 - 1)}$$

For the isentropic processes

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \frac{T_3}{T_4}$$

thus

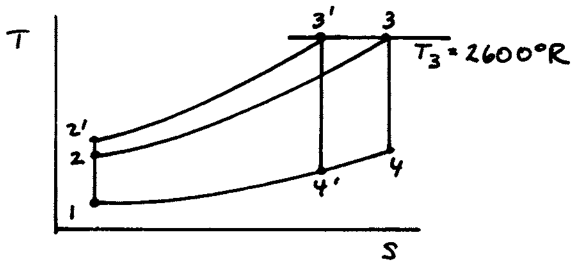
$$bwr = \frac{T_1}{T_4} \longleftarrow \text{bwr}$$

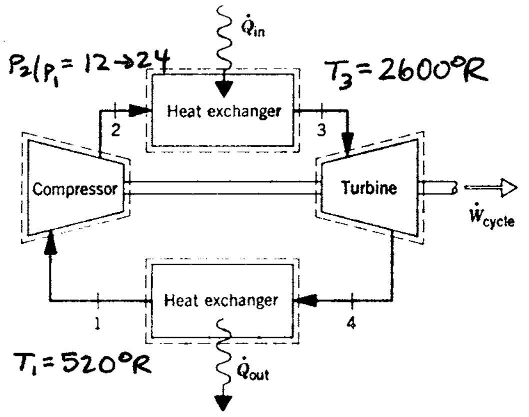
PROBLEM 9.46

KNOWN: An ideal air-standard Brayton cycle has a known compressor inlet temperature and a known maximum turbine inlet temperature.

FIND: Plot the net work per unit mass of air flow and the thermal efficiency versus compressor pressure ratio. Estimate the pressure ratio for maximum net work and the corresponding thermal efficiency and compare with a cold air-standard analysis.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.4

ANALYSIS: Sample calculation. $P_2/P_1 = 12$ Using data from Table A-22E to fix each of the principal states:

State 1 $T_1 = 520°R \Rightarrow h_1 = 124.27$ Btu/lb, $p_{r1} = 1.2147$

State 2 For the isentropic compression, $p_{r2} = (P_2/P_1)\,p_{r1} = 14.5764$
Thus, $T_2 = 1047.5°R$ and $h_2 = 252.84$ Btu/lb

State 3 $T_3 = 2600°R \Rightarrow h_3 = 674.49$ Btu/lb, $p_{r3} = 513.5$

State 4 For the isentropic expansion, $p_{r4} = (p_4/p_3)\,p_{r3} = 42.792$
$T_4 = 1399.2°R$ and $h_4 = 342.69$ Btu/lb

The net work per unit mass of air flow is

$$\dot{W}_{cycle}/\dot{m} = (h_3 - h_4) - (h_2 - h_1) = 203.23 \text{ Btu/lb}$$

and the thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{\dot{W}_{cycle}/\dot{m}}{(h_3 - h_2)} = 0.4820 \; (48.20\%)$$

The data for the required plots are obtained using *IT*, as follows:

IT Code

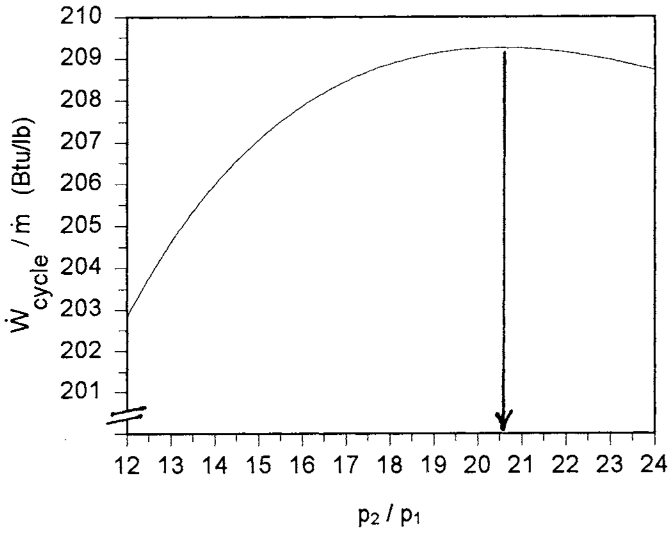
```
T1 = 520   // °R
T3 = 2600  // °R
rp = 12
mdot = 1  // Assume a mass flow rate of unity.
p1 = 1  // Assume a value for pressure
//          (any value would work).

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2 = s1
p2 = p1 * rp
s2 = s_hP("Air", h2, p2)
p3 = p2
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4 = s3
p4 = p1
s4 = s_hP("Air", h4, p4)
Wdotnet = mdot * ((h3 - h4) - (h2 - h1))
eta = Wdotnet / Qdotin
Qdotin = mdot * (h3 - h2)
```

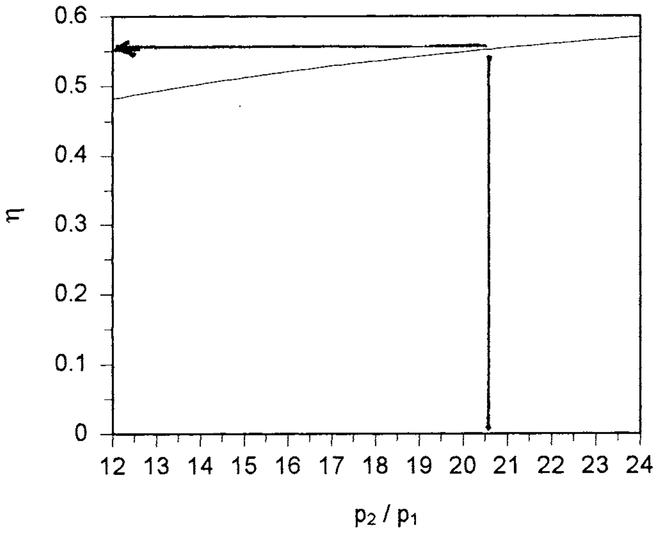
IT Results for $r_p = 12$

$h_1 = 124.3$ Btu/lb
$h_2 = 252.9$ Btu/lb
$h_3 = 673.6$ Btu/lb
$h_4 = 342.2$ Btu/lb
$\dot{W}_{cycle}/\dot{m} = 202.8$ Btu/lb
$\eta = 0.4821$

PROBLEM 9.46 (Cont'd.)

PLOTS:





From the plots: $(P_2/P_1)_{\substack{max\\work}} \approx 20.6$ ←

$(\eta)_{\substack{max\\work}} \approx 55.2\%$ ←

Using the result of Example 9.5, on a cold air-standard basis, the pressure ratio is

$$\left(\frac{P_2}{P_1}\right)_{\substack{max\\work}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} = 16.72 \leftarrow$$

and the corresponding thermal efficiency is

$$\eta = 1 - \frac{1}{(P_2/P_1)^{(k-1)/k}} = 0.553 \leftarrow$$

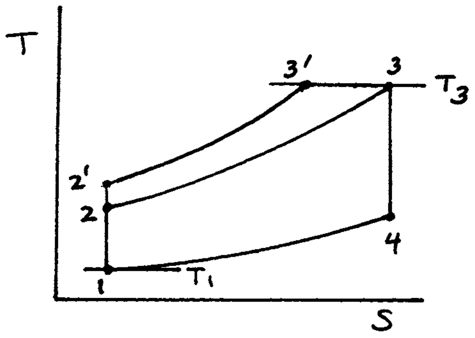
Discussion: The cold air-standard analysis predicts a pressure ratio for maximum net work that is quite different than the value obtained graphically. Interestingly, the corresponding thermal efficiencies are nearly equal.

PROBLEM 9.47

KNOWN: An ideal Brayton cycle is analyzed on a cold air-standard basis. The compressor inlet temperature is T_1 and the turbine inlet temperature is T_3.

FIND: Show that the compressor exit temperature that maximizes net work per unit mass of air flow is given by $T_2 = (T_1 \cdot T_3)^{1/2}$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 9.5

ANALYSIS: From the solution to Example 9.5, the compressor pressure ratio corresponding to maximum net work per unit mass of air flow is

$$\left(\frac{P_2}{P_1}\right)_{\substack{\text{max}\\ \text{work}}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$

Further, for the isentropic compression

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$$

Equating these expressions yields the desired result

$$\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}} \Rightarrow \frac{T_2}{T_1} = \left(\frac{T_3}{T_1}\right)^{\frac{1}{2}}$$

or $$T_2 = (T_1 \cdot T_3)^{1/2} \longleftarrow T_2$$

PROBLEM 9.48

KNOWN: The ideal air-standard Brayton cycle of Problem 9.41 is modified to include that the turbine and compressor each have isentropic efficiencies less than 100%.

FIND: For each of three efficiency values, plot (a) the thermal efficiency, (b) the back work ratio, (c) the net power developed, and (d) the compressor and turbine rates of exergy destruction versus compressor pressure ratios ranging from 2 to 20.

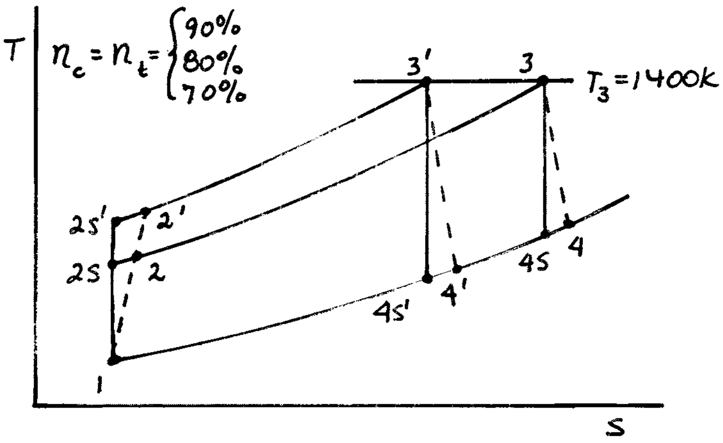
SCHEMATIC & GIVEN DATA:

$P_1 = 100\,kPa$

$T_1 = 300\,K$

$(AV)_1 = 5\,m^3/s$

$2 \le P_2/P_1 \le 20$


T
$\eta_c = \eta_t = \begin{cases} 90\% \\ 80\% \\ 70\% \end{cases}$
3'
3
$T_3 = 1400K$
2s'
2'
2s
2
4s'
4'
4s
4
1
s

ASSUMPTIONS: Same as in Example 9.6. Also, let $T_0 = 300K$.

ANALYSIS: Consider the sample calculations for $P_2/P_1 = 8$, $\eta_t = \eta_c = 80\%$. First, fix each of the principal states (Table A-22).

State 1: $P_1 = 100\,kPa$, $T_1 = 300K \Rightarrow h_1 = 300.19\,kJ/kg$

State 2: From Prob. 9.41; $h_{2s} = 544.18\,kJ/kg$. Using the compressor efficiency: $h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 605.18\,kJ/kg$

State 3: $T_3 = 1400K \Rightarrow h_3 = 1515.42\,kJ/kg$

State 4: Prob. 9.41: $h_{4s} = 860.40\,kJ/kg$. Using the turbine efficiency

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 991.4\,kJ/kg$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 0.241\ (24.1\%)$$

(b) The back work ratio is

$$bwr = \frac{h_2 - h_1}{h_3 - h_4} = 0.582$$

(c) The net power developed is

$$\dot{W}_{cycle} = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$

$$\dot{m} = \frac{(AV)_1 P_1}{RT_1} = 5.807\ kg/s$$

$$\dot{W}_{cycle} = 1271.9\ kW$$

(d) The rate of exergy destruction for the compressor is

PROBLEM 9.48 (Cont'd - Page 2)

$$\dot{E}_{d,comp} = T_o \dot{\sigma}_{comp} = T_o \dot{m} (s_2 - s_1)$$

$$= T_o \dot{m} \left[(s_2^\circ - s_1^\circ) - R \ln p_2/p_1 \right]$$

From Table A-22; $s_1^\circ = 1.70203$ kJ/kg·K and $s_2^\circ = 2.40593$ kJ/kg·K. Thus

$$\dot{E}_{d,comp} = 186.6 \text{ kW}$$

Similarly, for the turbine

$$\dot{E}_{d,turb} = \dot{m} T_o \left[(s_4^\circ - s_3^\circ) - R \ln \frac{p_4}{p_3} \right]$$

From Table A-22; $s_3^\circ = 3.36200$ and $s_4^\circ = 2.91166$. Thus

$$\dot{E}_{d,turb} = 255.1 \text{ kW}$$

The data for the required plots are obtained using IT, as follows:

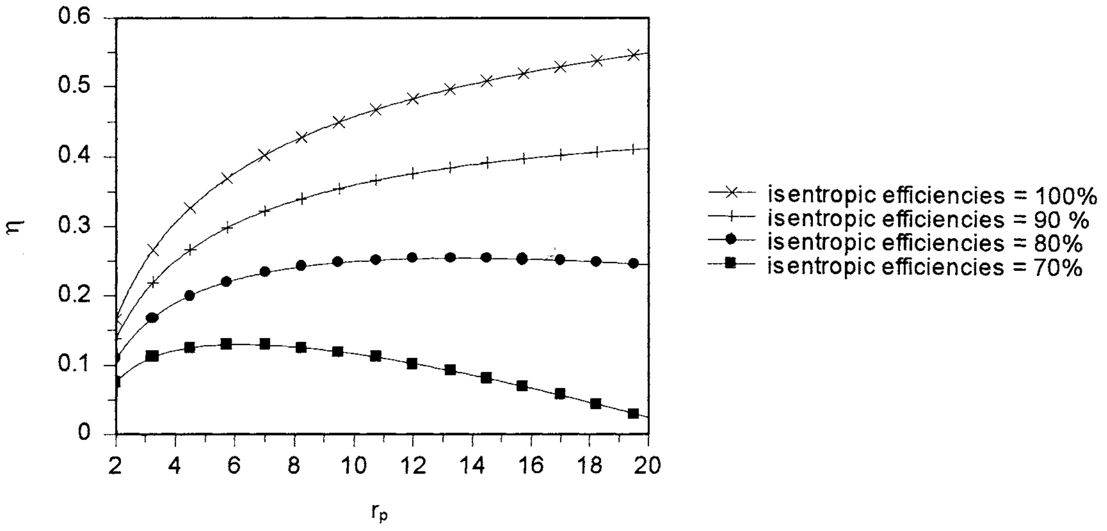
IT Code

```
p1 = 100  // kPa
T1 = 300  // K
p2 / p1 = rp
rp = 8
p3 = p2
T3 = 1400  // K
p4 = p1
AV1 = 5  // m³/s
eff_t = 0.7
eff_c = 0.7
To = 300 // K

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / eff_c
s2 = s_hP("Air", h2, p2)
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eff_t
s4 = s_hP("Air", h4, p4)

mdot = AV1 / v1
v1 = v_TP("Air",T1,p1)
Wdotnet = mdot * ((h3 - h4) - (h2 - h1))
eta = Wdotnet /  (mdot * (h3 - h2))
bwr = (h2 - h1) / (h3 - h4)
Edot_c = To * mdot * (s2 - s1)
Edot_t = To * mdot * (s4 - s3)
```

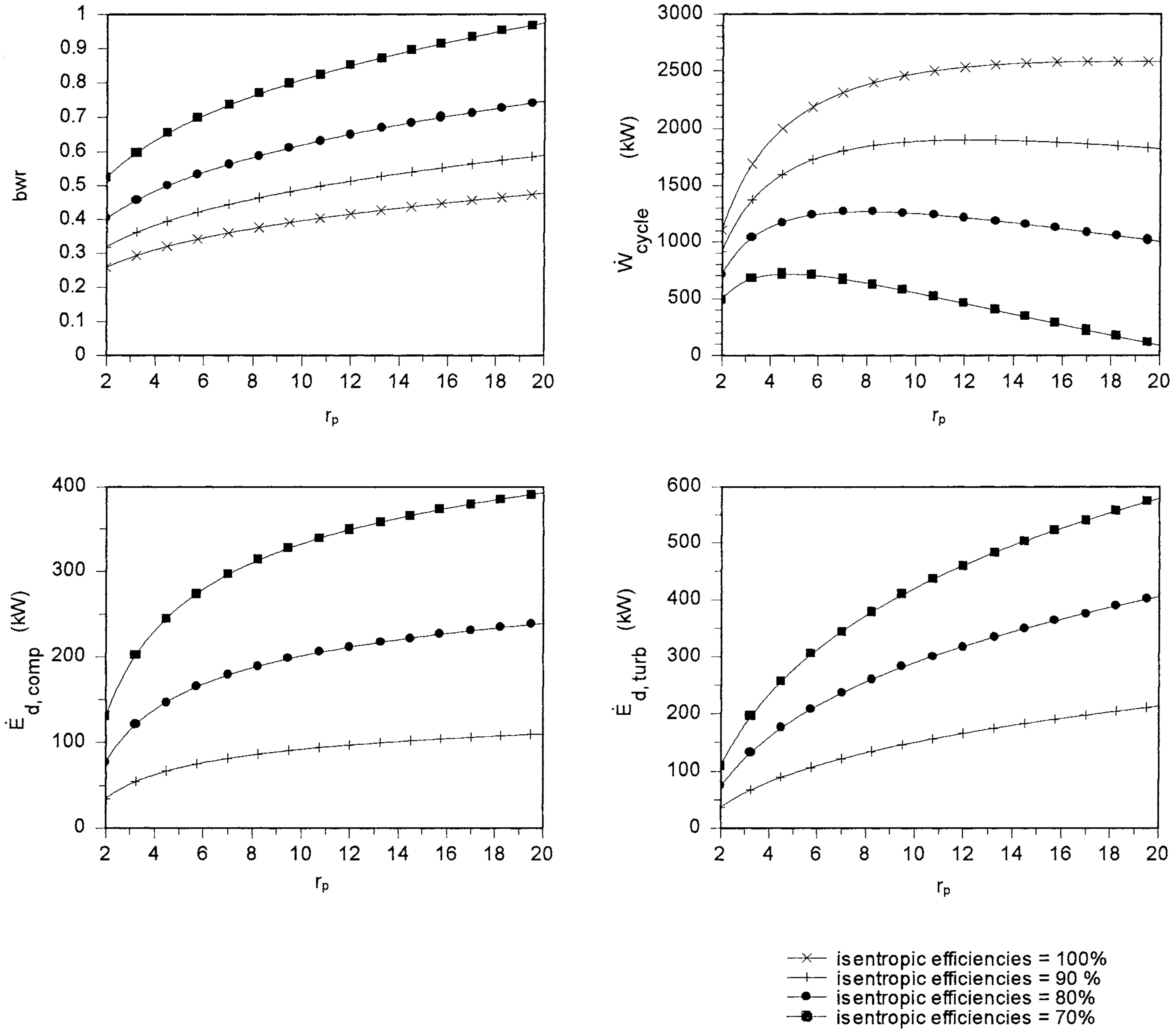
IT Results for r_p= 8, $\eta_c = \eta_t$ = 80%

h_1 = 300 kJ/kg
h_2 = 605.1 kJ/kg
h_{2s} = 544.1 kJ/kg
h_3 = 1514 kJ/kg
h_4 = 990.8 kJ/kg
h_{4s} = 859.9 kJ/kg
$\dot{m}$ = 5.807 kg/s
η = 0.2402
bwr = 0.5828
$\dot{W}_{cycle}$ = 1268 kW
$\dot{E}_{d,comp}$ = 186.9 kW
$\dot{E}_{d,turb}$ = 255.5 kW

PLOTS:



PROBLEM 9.48 (Cont'd. - page 3)



Observations:

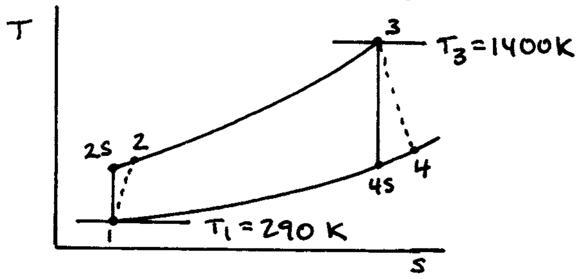
- For $\eta_c = \eta_t < 100\%$, thermal efficiency and $\dot{W}_{cycle}/\dot{m}$ exhibit maximum values as compressor pressure ratio varies.
- For $\eta_t = \eta_c = 70\%$ the work goes to zero as $r_p \rightarrow 20$. Note that the back work ratio approaches unity. That is, all of the turbine work is consumed to run the compressor.
- The exergy destruction rates in the compressor and turbine are highest for $\eta_c = \eta_t = 70\%$, as expected.

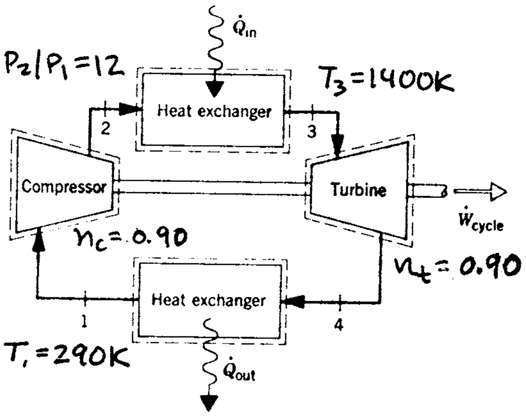
PROBLEM 9.49

KNOWN: An air-standard gas turbine cycle has a known compressor pressure ratio and specified minimum and maximum temperatures. The compressor and turbine each have isentropic efficiencies of 82%.

FIND: Determine (a) the net work per unit mass of air flow, (b) the heat rejected per unit mass of air flow, and (c) the thermal efficiency and compare them to the same quantities evaluated for an ideal cycle.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.6

ANALYSIS: First, fix each of the principal states for each cycle using data from Table A-22.

State 1 $T_1 = 290K \Rightarrow h_1 = 290.16$ kJ/kg, $p_{r1} = 1.2311$

State 2 First, determine h_{2s}. For isentropic compression

$$p_{r2s} = (p_2/p_1)\,p_{r1} = 14.7732 \Rightarrow h_{2s} = 590.47 \text{ kJ/kg}$$

Using the compressor efficiency; $\eta_c = (h_{2s} - h_1)/(h_2 - h_1)$;

$$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 623.84 \text{ kJ/kg}$$

State 3 $T_3 = 1400K \Rightarrow h_3 = 1515.42$ kJ/kg, $p_{r3} = 450.5$

State 4 First, determine h_{4s}. For isentropic expansion

$$p_{r4s} = (p_4/p_3)\,p_{r3} = 37.542 \Rightarrow h_{4s} = 768.38 \text{ kJ/kg}$$

Using the turbine efficiency; $\eta_t = (h_3 - h_4)/(h_3 - h_{4s})$;

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 843.08 \text{ kJ/kg}$$

(a) The net work per unit mass of air flowing is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_3 - h_4) - (h_2 - h_1)$$

$$= (1515.4 - 843.08) - (623.84 - 290.16)$$

$$= 338.64 \text{ kJ/kg} \longleftarrow \dot{W}_{cycle}/\dot{m}$$

For the ideal cycle

$$\left(\frac{\dot{W}_{cycle}}{\dot{m}}\right)_{ideal} = (h_3 - h_{4s}) - (h_{2s} - h_1)$$

$$= 446.7 \text{ kJ/kg} \longleftarrow \left(\begin{matrix}\text{ideal}\\ \text{cycle}\end{matrix}\right)$$

Thus, irreversibilities reduce the net work significantly.

PROBLEM 9.49 (Contd.)

(b) The heat rejected per unit mass of air flowing is

$$\frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1 = 843.08 - 290.16 = 552.92 \text{ kJ/kg} \longleftarrow \dot{Q}_{out}/\dot{m}$$

For the ideal cycle

$$\left(\frac{\dot{Q}_{out}}{\dot{m}}\right)_{ideal} = h_{4s} - h_1 = 478.2 \text{ kJ/kg} \longleftarrow \text{(ideal cycle)}$$

Thus, less heat is rejected for the ideal cycle.

(c) The thermal efficiency is

$$\eta = 1 - \frac{h_4 - h_1}{h_3 - h_2} = 1 - \frac{552.92}{(1515.4 - 623.84)}$$

$$= 0.38 \; (38\%) \longleftarrow \eta$$

For the ideal cycle

$$\eta = 1 - \frac{h_{4s} - h_1}{h_3 - h_{2s}} = 0.483 \; (48.3\%) \longleftarrow \text{(ideal cycle)}$$

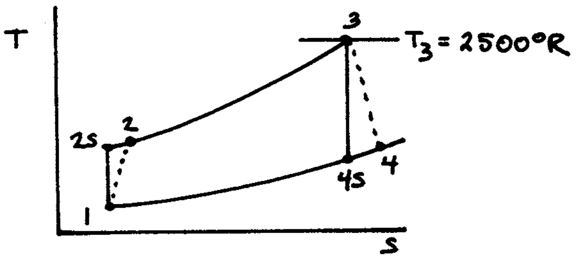
Thus, irreversibilities cause a substantial decrease in thermal efficiency.

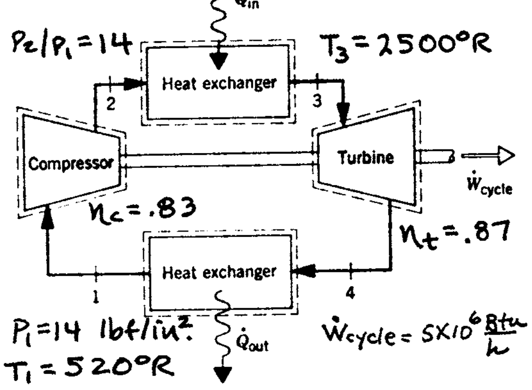
PROBLEM 9.50

KNOWN: A simple gas turbine is analyzed on an air-standard basis. The conditions entering the compressor and the compressor and turbine efficiencies are known. The compressor pressure ratio and the temperature at the turbine inlet are specified as well.

FIND: Determine (a) the volumetric flow rate entering the compressor, (b) the temperatures at the compressor and turbine exits, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.6

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 520\,°R \Rightarrow h_1 = 124.27$ Btu/lb, $p_{r1} = 1.2147$

State 2 For an isentropic compression: $p_{r2s} = (p_2/p_1)\,p_{r1} = 17.006$
Thus, $h_{2s} = 264.12$ Btu/lb. Using the compressor efficiency,

$$\eta_t = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s}-h_1}{\eta_c} = 292.76 \text{ Btu/lb},$$

$$T_2 = 1206\,°R \longleftarrow T_2$$

State 3 $T_3 = 2500\,°R \Rightarrow h_3 = 645.78$ Btu/lb, $p_{r3} = 435.7$

State 4 For an isentropic expansion, $p_{r4s} = (p_4/p_3)\,p_{r3} = 31.121$
Thus, $h_{4s} = 313.36$ Btu/lb. Using the turbine efficiency,

$$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3-h_{4s}) = 356.57 \text{ Btu/lb},$$

$$T4 = 1452\,°R \longleftarrow T_4$$

The thermal efficiency is

$$\eta = 1 - \frac{h_4-h_1}{h_3-h_2} = 1 - \frac{356.57-124.27}{645.78-292.76} = 0.342\ (34.2\%) \longleftarrow \eta$$

To find the volumetric flow rate at the inlet to the compressor, use

$$\dot{W}_{cycle} = \dot{m}[(h_3-h_4) - (h_2-h_1)] = \overset{\text{lb/h}}{\dot{m}}\left[(645.78-356.57)-(292.76-124.27)\right]\frac{\text{Btu}}{\text{lb}}$$

$$= \dot{m}\,[120.72 \text{ Btu/lb}]\ \frac{\text{Btu}}{\text{h}}$$

where

$$\dot{m} = \frac{(AV)_1 p_1}{RT_1} = \frac{\left[(AV)_1 \frac{\text{ft}^3}{\text{min}}\right]\left(14 \times 144 \frac{\text{lbf}}{\text{ft}^2}\right)}{\left(\frac{1545}{28.97}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(520\,°R)}\left|\frac{60 \text{ min}}{1 \text{ h}}\right| = 4.362\left[(AV)_1 \frac{\text{ft}^3}{\text{min}}\right]\frac{\text{lb}}{\text{h}}$$

Thus

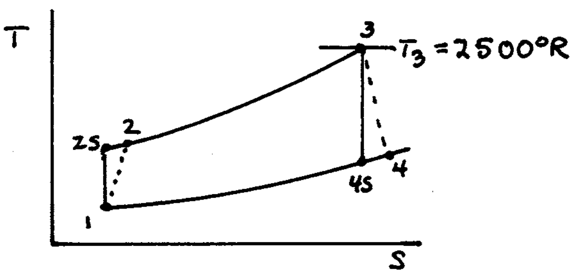
$$(AV)_1 = \frac{5\times10^6 \text{ Btu/h}}{\left(4.362 \frac{\text{lb/h}}{\text{ft}^3/\text{min}}\right)\left[120.72 \frac{\text{Btu}}{\text{lb}}\right]} = 9495 \frac{\text{ft}^3}{\text{min}} \longleftarrow (AV)_1$$

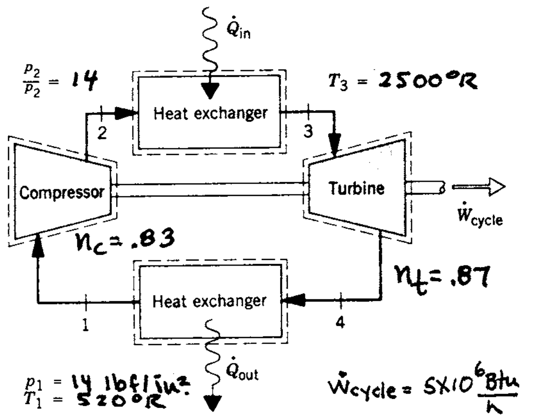
PROBLEM 9.51

KNOWN: A simple gas turbine is analyzed on a cold air-standard basis. The conditions entering the turbine and the turbine and compressor efficiencies are known. The compressor pressure ratio and the temperature at the turbine inlet are specified as well.

FIND: Determine (a) the volumetric flow rate entering the compressor, (b) the temperatures at the compressor and turbine exits, and (c) the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 9.6. Also, assume constant specific heats evaluated at 520°R.

ANALYSIS: From Table A-20E, $k = 1.401$, $c_p = 0.240\ Btu/lb\cdot°R$. $T_1 = 520°R$ is given and $T_3 = 2500°R$ is given.

State 2 For an isentropic compression, $T_{2s} = (P_2/P_1)^{\frac{k-1}{k}} T_1 = 1106.8\,°R$

Using the compressor efficiency,

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \Rightarrow T_2 = T_1 + \frac{T_{2s}-T_1}{\eta_c} = 1227.0\,°R \longleftarrow T_2$$

State 4 For an isentropic expansion, $T_{4s} = (P_4/P_3)^{\frac{k-1}{k}} T_3 = 1174.6\,°R$

Using the turbine efficiency,

$$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} = \frac{c_p(T_3-T_4)}{c_p(T_3-T_{4s})} \Rightarrow T_4 = T_3 - \eta_t(T_3-T_{4s}) = 1346.9\,°R \longleftarrow T_4$$

The thermal efficiency is

$$\eta = 1 - \frac{h_4-h_1}{h_3-h_2} = 1 - \frac{T_4-T_1}{T_3-T_2} = 0.3504\ (35.04\%) \longleftarrow \eta$$

The net power developed is

$$\dot{W}_{cycle} = \dot{m}\left[(h_3-h_4)-(h_2-h_1)\right] = \dot{m}\,c_p\left[(T_3-T_4)-(T_2-T_1)\right]$$

$$= \dot{m}\left[\left(0.24\ \frac{Btu}{lb\cdot°R}\right)(446.1\ °R)\right]$$

where

$$\dot{m} = \frac{(AV)_1 P_1}{R\,T_1} = \frac{\left[(AV)_1\ \frac{ft^3}{min}\right]\left[14\times144\ \frac{lbf}{ft^2}\right]}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot°R}\right)(520°R)}\left|\frac{60\ min}{1\ h}\right| = 4.362\left[(AV)_1\ \frac{ft^3}{min}\right]\frac{lb}{h}$$

So,

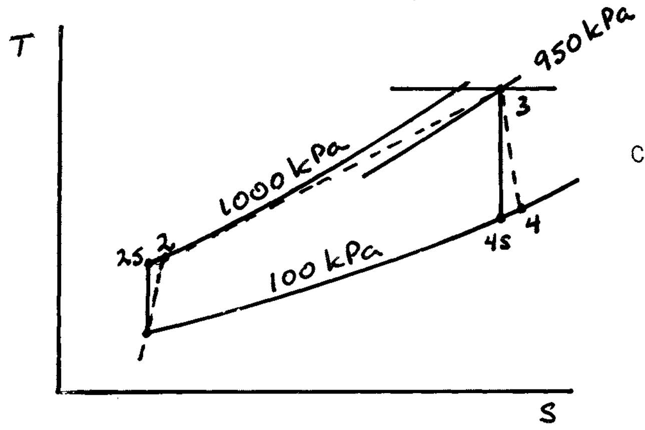
$$(AV)_1 = \frac{5\times10^6\ Btu/h}{\left[4.362\ \frac{lb/h}{ft^3/min}\right]\left[(0.24)(446.1)\ \frac{Btu}{lb}\right]} = 10{,}706\ ft^3/min \longleftarrow (AV)_1$$

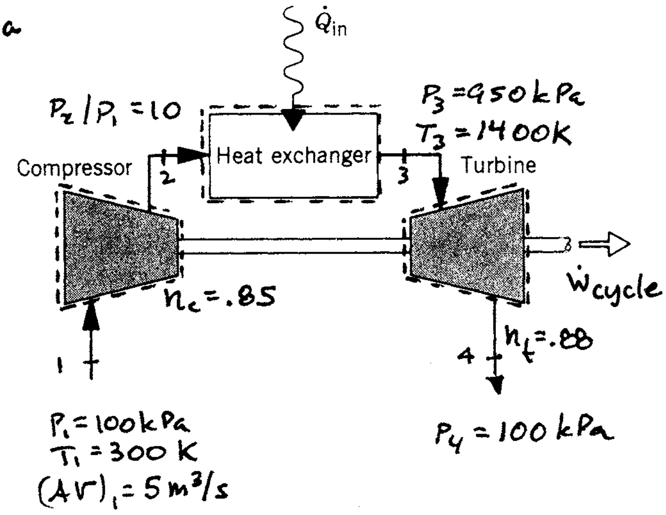
PROBLEM 9.52

KNOWN: A simple gas turbine is analyzed on an air-standard basis from an exergy viewpoint. Data are known at various locations.

FIND: (a) Develop a full exergy accounting of the net exergy increase of the air passing through the gas turbine combustor, (b) devise and evaluate an exergetic efficiency for the gas turbine cycle.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The compressor and turbine are adiabatic. (3) Kinetic and potential energy effects are negligible. (4) The working fluid is air modeled as an ideal gas. (5) Let $T_0 = 300$ K and $p_0 = 100$ kPa.

ANALYSIS: Data are obtained for each principal state from Table A-22:

State	T (K)	p(kPa)	h(kJ/kg)	s°(kJ/kg·K)
1	300	100	300.19	1.70203
2	621.1	1000	629.21	2.44539
3	1400	950	1515.42	3.36200
4	873.9	100	903.72	2.81558

The increase in flow exergy of the air passing through the heat exchanger is taken as the net input of exergy to the gas turbine.

Input: $\dot m(e_{f3}-e_{f2}) = \dot m\left[(h_3-h_2) - T_0\left(s_3^\circ - s_2^\circ - R\ln p_3/p_2\right)\right]$

Evaluating $\dot m$:

$$\dot m = \frac{(AV)_1 P_1}{RT_1} = \frac{(5\ m^3/s)(100\,kPa)}{\left(\frac{8.314\ kJ}{28.97\ kg\cdot K}\right)(300\,k)}\left|\frac{10^3\ N/m^2}{1\ kPa}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 5.807\ kg/s$$

Thus

$$\dot m(e_{f3}-e_{f2}) = \left(5.807\ \frac{kg}{s}\right)\Big[(1515.42-629.21)\ kJ/kg$$

$$-(300\,k)(3.36200-2.44539 - \frac{8.314}{28.97}\ln\frac{950}{1000})\frac{kJ}{kg\cdot K}\Big]\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 3524\ kW\ (\text{Input})$$

PROBLEM 9.52 (Cont'd)

Exergy is destroyed due to irreversibilities in the compressor and turbine. Thus, using $\dot{E}_d = T_0 \dot{\sigma}$, we get

Destructions:

$$\dot{E}_{d,comp} = T_0 \dot{m}(s_2 - s_1) = T_0 \dot{m}\left[(s_2^\circ - s_1^\circ) - R \ln p_2/p_1\right] = 143.8 \text{ kW}$$

$$\dot{E}_{d,turb} = T_0 \dot{m}\left[(s_4^\circ - s_3^\circ) - R \ln p_4/p_3\right] = 173.6 \text{ kW}$$

The net power developed by the cycle represents the output of exergy from the cycle, or

$$\dot{W}_{cycle} = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$
$$= 1641.5 \text{ kW (Output)}$$

Finally, the air enters the gas turbine at p_0 and T_0. Thus, the change in flow exergy from inlet to exit reprepresents the loss due to the hot air being discharged. Thus

Loss: $\dot{m}(e_{f4} - e_{f1}) = \dot{m}\left[(h_4 - h_1) - T_0(s_4^\circ - s_1^\circ - R \ln p_4/p_1)\right]$

$$= 1564.8 \text{ kW}$$

Summarizing

- Input: 3524 kW
- Disposition:
  - Net Power: 1641.5 kW (46.6%)
  - Destroyed: 317.4 (9 %)
  - ① Loss: 1564.8 (44.4%)

3523.7 kW

Since the objective of the gas turbine is to produce power, an exergetic efficiency expressed as the ratio of the desired output (net power) to the exergy input is, by inspection of the summary,

$$\epsilon = 46.6\%$$

---

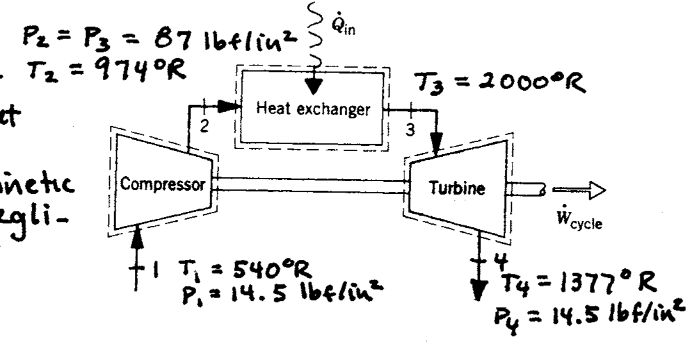
1. Considerable exergy is carried out by the air discharged at 4. This might be exploited by the regenerative approach discussed in Sec. 9.7 or by means of a combined cycle as discussed in Sec. 9.10.

PROBLEM 9.53

KNOWN: A simple gas turbine is analyzed on an air-standard basis from an exergy viewpoint. Data are known at various locations.

FIND: (a) Develop a full exergy accounting of the net exergy increase of the air passing through the gas turbine combustor, (b) devise and evaluate an exergetic efficiency for the gas turbine cycle.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The compressor and turbine are adiabatic. (3) Kinetic and potential energy effect are negligible. (4) The working fluid is air modeled as an ideal gas. (5) Let $T_0 = 540$°R, $p_0 = 14.5$ lbf/in².

ANALYSIS: Data are obtained at each principal state from Table A-22E:

State	T (°R)	p (lbf/in.²)	h (Btu/lb)	s° (Btu/lb·°R)
1	540	14.5	129.06	0.60078
2	974	87	234.53	0.74387
3	2000	87	504.71	0.93205
4	1377	14.5	336.91	0.83169

The input of exergy to the gas turbine is equal to the increase in flow exergy of the working fluid passing through the heat exchanger

Input: $(e_{f3} - e_{f2}) = (h_3 - h_2) - T_0[(s_3^\circ - s_2^\circ) - R\ln \overset{0}{\cancel{p_3/p_2}}]$

$= 168.6$ Btu/lb ←

Exergy is destroyed due to irreversibilities in the compressor and turbine. Thus, with $\dot{E}_d = T_0\dot{\sigma}$

Destructions: $(\dot{E}_d/\dot{m})_{comp} = T_0[(s_2^\circ - s_1^\circ) - R\ln p_2/p_1] = 10.94$ Btu/lb ←

$(\dot{E}_d/\dot{m})_{turb} = T_0[(s_4^\circ - s_3^\circ) - R\ln p_4/p_3] = 12.13$ Btu/lb ←

Exergy is carried out via the net work:

$\dot{W}_{cycle}/\dot{m} = (h_3 - h_4) - (h_2 - h_1) = 62.33$ Btu/lb ←

Exergy also is carried out with the stream exiting at 4. The net exergy loss is then

$e_{f4} - e_{f3} = (h_4 - h_1) - T_0((s_4^\circ - s_1^\circ) - R\,\overset{0}{\cancel{\ln p_4/p_1}})$

$= 83.16$ Btu/lb ←

Summary:

- Input : 168.6 Btu/lb
- Disposition:

– net work	62.33 Btu/lb	(37%)
– destroyed	23.07 Btu/lb	(13.7%)
① – loss	83.16 Btu/lb	(49.3%)
	168.6 Btu/lb	

$\varepsilon = \left(\frac{\text{net power output}}{\text{input}}\right) = 37\%$ ←

---

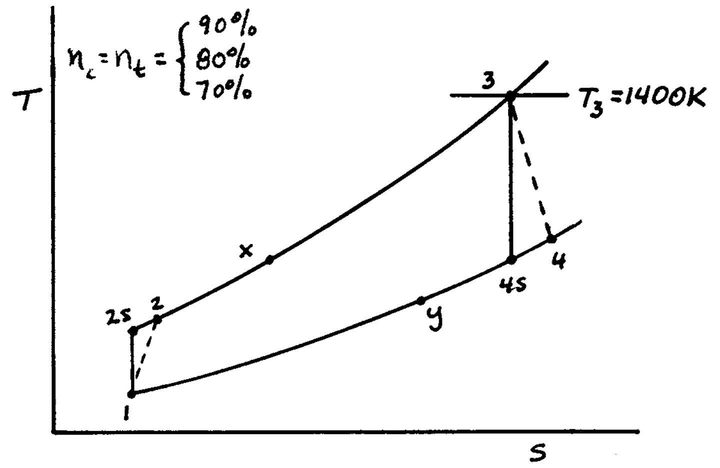
1. Considerable exergy is carried out by the air discharged at 4. This might be exploited by the regenerative approach discussed in Sec. 9.7 or by means of a combined cycle as discussed in Sec. 9.10.

PROBLEM 9.54

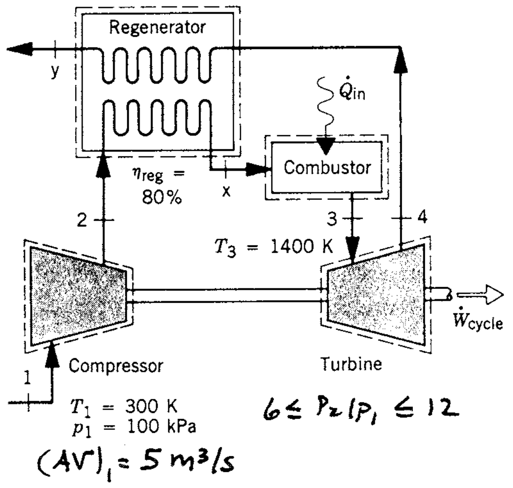
KNOWN: A regenerator of known effectiveness is inorporated into the cycle of Problem 9.48.

FIND: For each of three compressor and turbine efficiency values, plot (a) the thermal efficiency, (b) the back work ratio, (c) the net power developed, and (d) the rate of exergy destruction in the regenerator.

SCHEMATIC & GIVEN DATA:



$\eta_c = \eta_t = \begin{cases} 90\% \\ 80\% \\ 70\% \end{cases}$
T
3
$T_3 = 1400K$
x
4
4s
2s
2
y
1
s



ASSUMPTIONS: Same as in Example 9.7, except $\eta_c = \eta_t < 100\%$. Also, let $T_0 = 300k$.

ANALYSIS: Consider the same sample calculation case as in Problem 9.48. That is, $\eta_c = \eta_t = 80\%$ and $p_2/p_1 = 8$. The numbered states are unchanged:

$$h_1 = 300.19 \text{ kJ/kg}$$
$$h_2 = 605.18 \text{ kJ/kg}$$
$$h_3 = 1515.42 \text{ kJ/kg}$$
$$h_4 = 991.4 \text{ kJ/kg}$$

The specific enthalpy h_x is found using the regenerator effectiveness

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{reg}(h_4 - h_2) + h_2 = 914.16 \text{ kJ/kg}$$

For the regenerator: $0 = (h_x - h_2) + (h_y - h_4)$. Thus

$$h_y = h_4 - (h_x - h_2) = 682.42 \text{ kJ/kg}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_y - h_1}{h_3 - h_x} = 0.364 \ (36.4\%)$$

(b) (c) As the numbered states are unchanged, the back work ratio and the power developed are the same as in Problem 9.48. Thus, refer to the plots in Problem 9.48 for bwr and $\dot{W}_{cycle}$.

(d) For the regenerator, the rate of exergy destruction is

$$\dot{E}_d = T_0 \dot{\sigma}_{reg} = T_0 \dot{m} \left[(s_x - s_2) + (s_y - s_4) \right]$$

$$= T_0 \dot{m} \left[(s_x^\circ - s_2^\circ) - R \cancelto{0}{\ln p_x/p_2} + (s_y^\circ - s_4^\circ) - R \cancelto{0}{\ln p_y/p_4} \right]$$

PROBLEM 9.54 (Cont'd.)

Interpolating in Table A-22; $s_x^\circ = 2.82748$ kJ/kg·K and $s_y^\circ = 2.52779$. From Problem 9.48; $s_2^\circ = 2.40593$ and $s_4^\circ = 2.91166$. Thus, with $\dot{m} = 5.807$ kg/s

$$\dot{E}_{d,reg} = 65.64 \text{ kW}$$

The data for the required plots are obtained using IT, as follows:

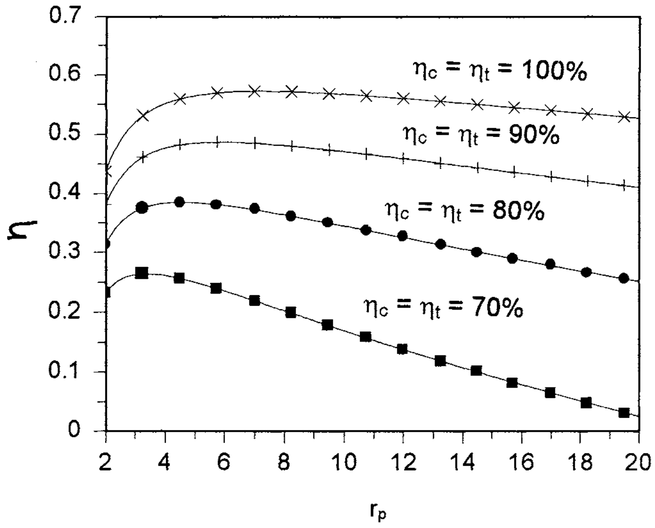
IT Code

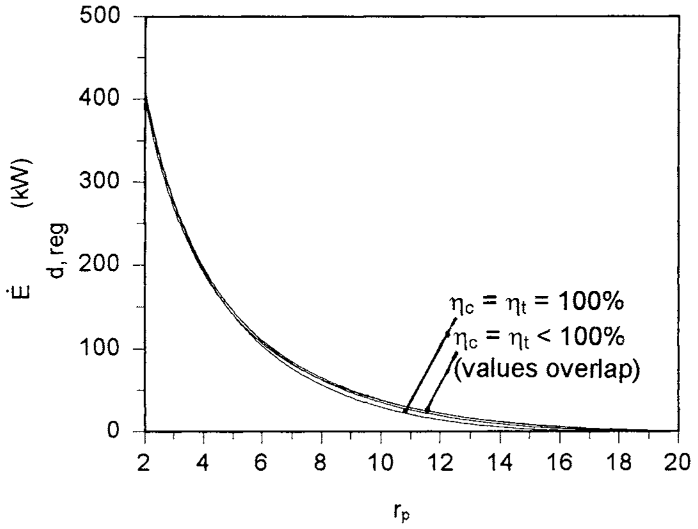
```
p1 = 100  // kPa
T1 = 300  // K
p2 / p1 = rp
rp = 8
p3 = p2
T3 = 1400  // K
p4 = p1
AV1 = 5  // m³/s
eff_t = 0.8
eff_c = 0.8
eff_reg = 0.8
To = 300 // K

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / eff_c
s2 = s_hP("Air", h2, p2)

h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eff_t
s4 = s_hP("Air", h4, p4)
hx = eff_reg * (h4 - h2) + h2
sx = s_hP("Air", hx,p2)
hy = h4 - (hx - h2)
sy = s_hP("Air", hy, p1)

mdot = AV1 / v1
v1 = v_TP("Air",T1,p1)
Wdotnet = mdot * ((h3 - h4) - (h2 - h1))
eta = Wdotnet /  (mdot * (h3 - hx))
bwr = (h2 - h1) / (h3 - h4)
Edot_reg = To * mdot * ((sx - s2) + (sy - s4))
```

IT Results for $r_p = 8$, $\eta_c = \eta_t = 0.8$

$h_1 = 300$ kJ/kg
$h_2 = 605.1$ kJ/kg
$h_{2s} = 544.1$ kJ/kg
$h_3 = 514$ kJ/kg
$h_4 = 990.8$ kJ/kg
$h_{4s} = 859.9$ kJ/kg
$h_x = 913.6$ kJ/kg
$h_y = 682.2$ kJ/kg
$\eta = 0.3636$
bwr = 0.5828
$\dot{W}_{cycle} = 1268$ kW
$\dot{E}_{d,reg} = 65.37$ kW

PLOTS:





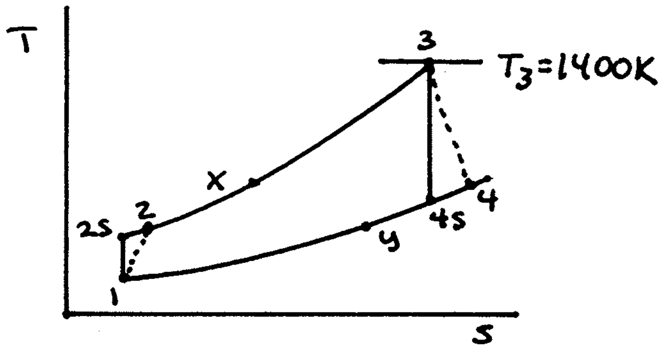
Note that as regenerator effectiveness increases, there is less heat input to the cycle, and hence greater thermal efficiency (power is unaffected by regenerator effectiveness).

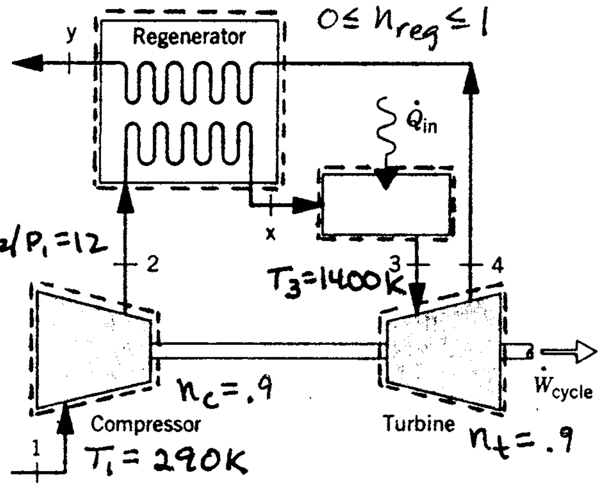
PROBLEM 9.55

KNOWN: A regenerator is incorporated into the cycle of Problem 9.49.

FIND: Plot (a) the heat addition per unit mass of air flowing and (b) the thermal efficiency, each versus regenerator effectiveness varying from 0 to 1.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.7.

ANALYSIS: The numbered states are the same as in Problem 9.49. Thus
$h_1 = 290.16$ kJ/kg, $h_2 = 623.84$, $h_3 = 1515.42$, $h_4 = 843.08$

Consider the sample case of $\eta_{reg} = 0.8$. The specific enthalpy h_x is found using the regenerator effectiveness;

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{reg}(h_4 - h_2) + h_2 = 799.23 \text{ kJ/kg}$$

From an energy balance on the regenerator

$$h_y = (h_2 - h_x) + h_4 = 667.69 \text{ kJ/kg}$$

(a) The heat addition per unit mass of air flowing is

$$\dot{Q}_{in}/\dot{m} = h_3 - h_x = 716.2 \text{ kJ/kg}$$

(b) The thermal efficiency is

$$\eta = 1 - \frac{h_y - h_1}{h_3 - h_x} = 0.473 \ (47.3\%)$$

The data for the required plots are obtained using IT, as follows:

IT Code

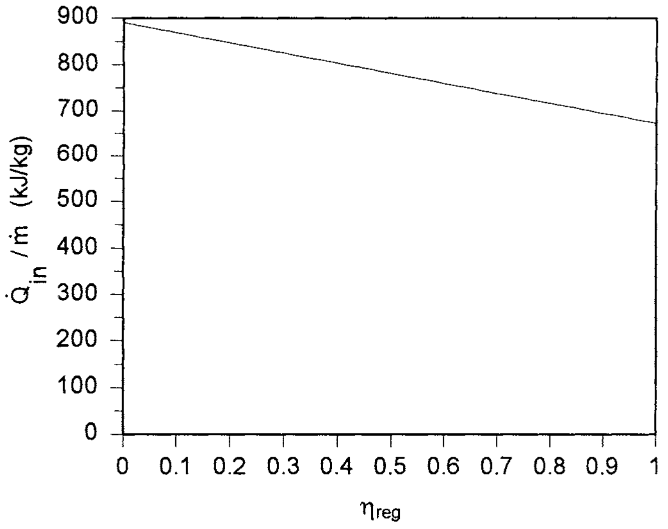
```
T1 = 290  // K
p2 / p1 = 12
p1 = 100  // Assume p1 = 100 kPa
//        (actual value doesn't matter)
p3 = p2
T3 = 1400  // K
p4 = p1
eta_c = 0.9
eta_t = 0.9
eta_reg = 0.8

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air",h2s,p2)
s2s = s1
h2 = h1 + (h2s - h1) / eta_c
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air",h4s,p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eta_t
hx = eta_reg * (h4 - h2) + h2
hy = (h2 - hx) + h4

Qdot / mdot = (h3 - hx)
mdot = 1  // Assume a unit mass flow
rate.
eta = 1 - (hy - h1) / Qdot
```

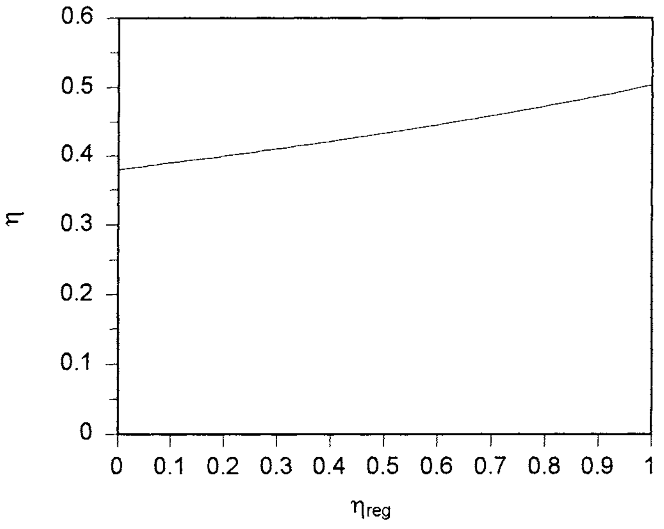
PROBLEM 9.55 (Cont'd.)

IT Results for $\eta_{reg} = 0.8$

$h_1 = 290$ kJ/kg
$h_2 = 623.7$ kJ/kg
$h_{2s} = 590.3$ kJ/kg
$h_3 = 1514$ kJ/kg
$h_4 = 842.4$ kJ/kg
$h_{4s} = 767.8$ kJ/kg
$h_x = 798.7$ kJ/kg
$h_y = 667.4$ kJ/kg
$\eta = 0.4725$
$\dot{Q}_{in}/\dot{m} = 715.5$ kJ/kg

PLOTS:





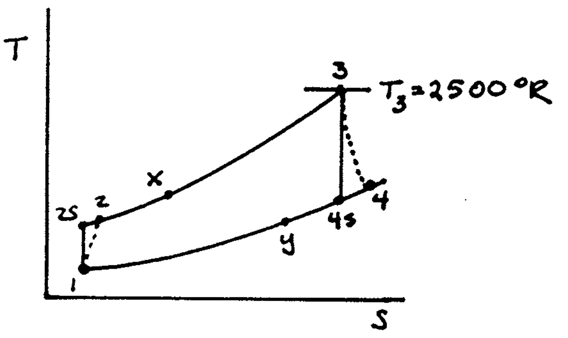
The needed heat input decreases as $\eta_{reg} \to 1$. Accordingly, the thermal efficiency increases as $\eta_{reg} \to 1$, as expected.

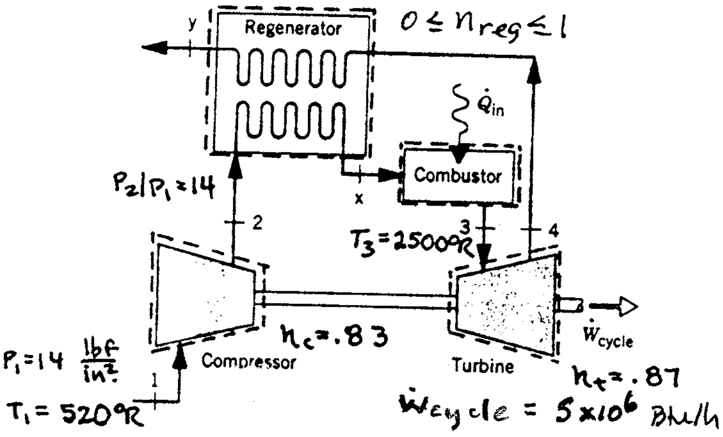
PROBLEM 9.56

KNOWN: A regenerator is incorporated into the cycle of Problem 9.50.

FIND: Plot (a) the thermal efficiency and (b) the percent decrease in heat addition, each versus regenerator effectiveness ranging from 0 to 1.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.7

ANALYSIS: Consider the sample case of $\eta_{reg} = 0.78$. The numbered states are the same as in Problem 9.50. Thus

$$h_1 = 124.27 \text{ Btu/lb},\ h_2 = 292.76,\ h_3 = 645.78,\ h_4 = 356.57$$

The specific enthalpy h_x is found using the regenerator effectiveness;

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{reg}(h_4 - h_2) + h_2 = 342.53 \text{ Btu/lb}$$

From an energy balance on the regenerator

$$h_y = (h_2 - h_x) + h_4 = 306.8 \text{ Btu/lb}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{h_y - h_1}{h_3 - h_x} = 0.398\ (39.8\%)$$

(b) The percent decrease in heat addition is

$$\%\,\text{decrease} = \frac{(h_3 - h_2) - (h_3 - h_x)}{(h_3 - h_2)} \times 100 = 14.1\%$$

The data for the required plots are obtained using IT, as follows:

IT Code

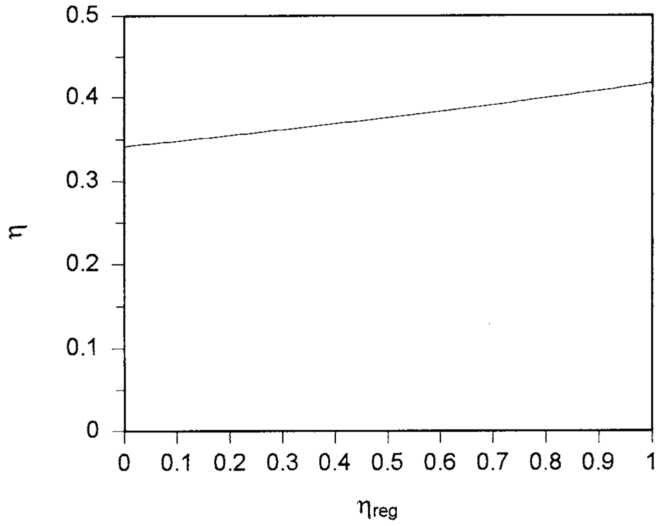
```
p1 = 14 // lbf/in.²
T1 = 520 // °R
p2 / p1 = 14
p3 = p2
T3 = 2500 // °R
p4 = p1
eta_c = 0.83
eta_t = 0.87
eta_reg = 0.78

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / eta_c
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eta_t
hx = eta_reg * (h4 - h2) + h2
hy = h2 - hx + h4
eta = 1 - (hy - h1) / (h3 - hx)
pct = (((h3 - h2) - (h3 - hx)) / (h3 - h2)) * 100
```

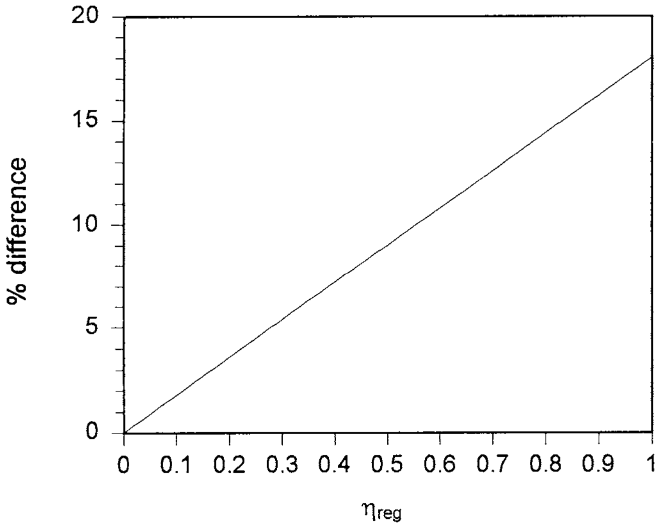
PROBLEM 9.56 (Cont'd.)

IT Results for η_{reg} = 0.78

$h_{1=}$ = 124.3 Btu/lb
h_2 = 292.8 Btu/lb
h_{2s} = 264.2 Btu/lb
h_3 = 644.9 Btu/lb
h_4 = 356.2 Btu/lb
h_{4s} = 313 Btu/lb
h_x = 342.2 Btu/lb
h_y = 306.7 Btu/lb
% difference = 14.04
η = 0.3975

PLOTS:





The heat input to the cycle decreases by about 18% with an ideal regenerator. Since the power is constant (see Problem 9.50), the thermal efficiency increases by a comparable amount.

PROBLEM 9.57

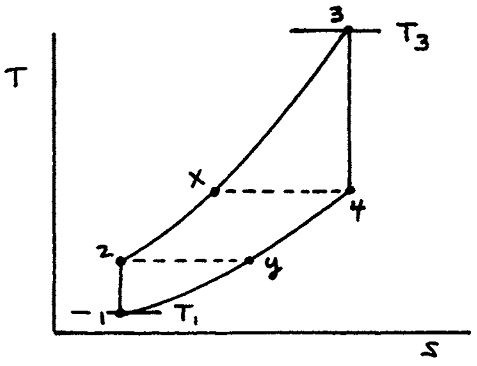
KNOWN: An ideal regenerative gas turbine is analyzed on a cold air-standard basis.

FIND: Show that the thermal efficiency can be expressed as

$$\eta = 1 - \left(\frac{T_1}{T_3}\right)(r)^{(k-1)/k}$$

where r is pressure ratio, and T_1 and T_3 denote the compressor and turbine inlet temperatures, respectively.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as in Example 9.7 except $\eta_{reg} = 100\%$. Also, assume constant specific heats.

ANALYSIS: The thermal efficiency of the ideal cold air-standard cycle can be expressed as

$$\eta = 1 - \frac{\cancel{c_p}(T_y - T_1)}{\cancel{c_p}(T_3 - T_x)}$$

With $\eta_{reg} = 100\%$, $T_x = T_4$ and $T_y = T_2$. Thus

$$\eta = 1 - \frac{T_2 - T_1}{T_3 - T_4} = 1 - \frac{T_1\,(T_2/T_1 - 1)}{T_3\,(1 - T_4/T_3)}$$

If $r = P_2/P_1 = P_3/P_4$, the isentropic relations apply;

$$\frac{T_2}{T_1} = r^{\left(\frac{k-1}{k}\right)} \quad \text{and} \quad \frac{T_4}{T_3} = r^{-\left(\frac{k-1}{k}\right)}$$

Thus

$$\eta = 1 - \frac{T_1}{T_3}\left[\frac{(r^{k-1/k} - 1)}{(1 - r^{-(k-1/k)})}\right]$$

The bracketed term is of the form

$$\left[\frac{a-1}{1-\frac{1}{a}}\right] = \left[\frac{a-1}{\left(\frac{a-1}{a}\right)}\right] = a = r^{\frac{k-1}{k}}$$

Finally

$$\eta = 1 - \left(\frac{T_1}{T_3}\right)(r)^{\left(\frac{k-1}{k}\right)} \longleftarrow \eta$$

PROBLEM 9.58

KNOWN: Operating data are provided for an air-standard Brayton cycle.

FIND: Determine the thermal efficiency and net power developed, if (a) $\eta_t = \eta_c = 100\%$, (b) $\eta_t = 88\%$, $\eta_c = 84\%$, (c) $\eta_t = 88\%$, $\eta_c = 84\%$, and a regenerator with $\eta_{regen} = 80\%$ is incorporated.

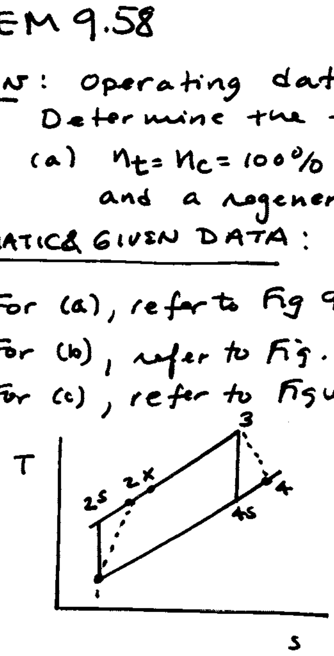
SCHEMATIC & GIVEN DATA:

For (a), refer to Fig 9.10

For (b), refer to Fig. 9.13b

For (c), refer to Figure below

$p_2/p_1 = 10$, $p_1 = 14.7\ \text{lbf/in}^2$, $T_1 = 70°F$, $\dot{m} = 9\times10^4\ \text{lb/h}$, $T_3 = 2200°R$



ASSUMPTIONS:
See Examples 9.4, 9.6, 9.7.

ANALYSIS: Using data from Table A-22E

State 1: $T_1 = 530°R$
$\Rightarrow h_1 = 126.66\ \text{Btu/lb}$, $p_{r1} = 1.2998$

State 2s: $p_{r,2s} = (p_2/p_1)p_{r1} = 12.998$
$\Rightarrow h_{2s} = 244.68\ \text{Btu/lb}$

State 3: $T_3 = 2200°R$, $h_3 = 560.59\ \text{Btu/lb}$, $p_{r3} = 256.6$

State 4s: $p_{r,4s} = (p_4/p_3)p_{r,3} = 25.66 \Rightarrow h_{4s} = 296.7\ \text{Btu/lb}$

(a) **Ideal cycle.** Since $\dot{W}_{cycle} = \dot{Q}_{in} - \dot{Q}_{out}$ and $\eta = \dot{W}_{cycle}/\dot{Q}_{in}$, we get

$$\eta = 1 - \frac{\dot{Q}_{out}}{\dot{Q}_{in}} = 1 - \frac{(h_{4s}-h_1)}{(h_3-h_{2s})} = 1 - \left(\frac{296.7-126.66}{560.59-244.68}\right) = 0.462\ (46.2\%) \longleftarrow \text{(a)}$$

The net power developed is

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = \dot{m}\left[(h_3-h_{4s})-(h_{2s}-h_1)\right]$$
$$= \left(9\times10^4\ \frac{\text{lb}}{\text{h}}\right)\left[(560.59-296.7)-(244.68-126.66)\right]\frac{\text{Btu}}{\text{lb}}\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right|$$
$$= 5158\ \text{hp} \longleftarrow \text{(a)}$$

(b) $\eta_t = 88\%$, $\eta_c = 84\%$. Using data given above

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s}-h_1)}{\eta_c} = 267.16\ \frac{\text{Btu}}{\text{lb}}$$

$$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3-h_{4s}) = 328.4\ \text{Btu/lb}$$

$$\eta = 1 - \left(\frac{h_4-h_1}{h_3-h_2}\right) = 1 - \left(\frac{328.4-126.66}{560.59-267.16}\right) = 0.312\ (31.2\%) \longleftarrow \text{(b)}$$

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = \dot{m}\left[(h_3-h_4)-(h_2-h_1)\right]$$
$$= (9\times10^4)\left[(560.59-328.4)-(267.16-126.66)\right]\left|\frac{1}{2545}\right|$$
$$= 3242\ \text{hp} \longleftarrow \text{(b)}$$

PROBLEM 9.58 (Contd.)

(c) $\eta_t = 88\%$, $\eta_c = 84\%$, $\eta_{regen} = 80\%$

The specific enthalpy h_x is found using the regenerator effectiveness:

$$\eta_{regen} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{regen}(h_4 - h_2) + h_2 = 316.5 \text{ Btu/lb}$$

The addition of a regenerator does not affect the net power out, which remains the same as in (b):

$\dot{W}_{cycle} = 3242$ hp ← (c)

The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_3 - h_2) - (h_2 - h_1)}{(h_3 - h_x)}$$

$$= \frac{(560.59 - 328.4) - (267.16 - 126.66)}{(560.59 - 316.5)}$$

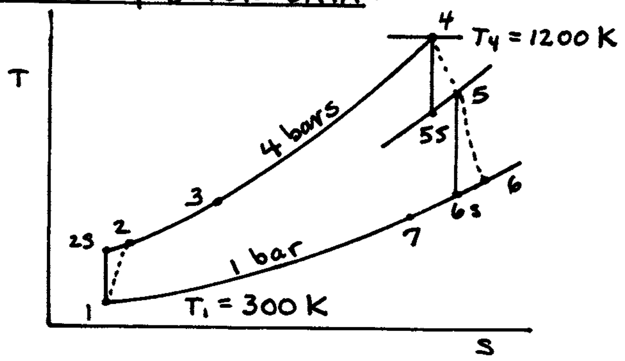
$= 0.376$ (37.6%) ← (c)

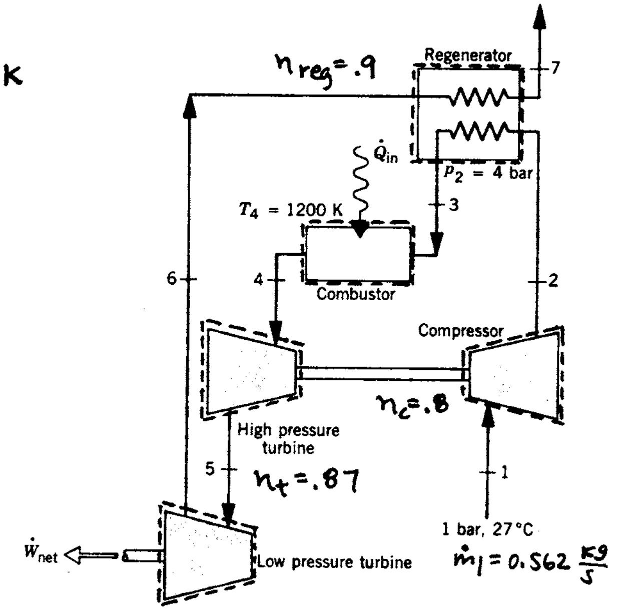
PROBLEM 9.59

KNOWN: In a regenerative gas turbine power plant, a high pressure turbine runs the compressor and the net power output is provided by a low pressure turbine. Data are known at various locations.

FIND: Determine (a) the net power output, (b) the thermal efficiency, and (c) the temperature at states 2, 3, 5, 6, 7.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The compressor, turbines, and regenerator are adiabatic. (3) There are no pressure drops for flow through the heat exchangers. (4) Kinetic and potential energy effects are negligible. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1: $T_1 = 300K \Rightarrow h_1 = 300.19$ kJ/kg, $Pr_1 = 1.3860$

State 2: For an iseutropic compression, $Pr_{2s} = (P_2/P_1)\,Pr_1 = 5.544 \Rightarrow h_{2s} = 446.49 \frac{kJ}{kg}$
Using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 483.06 \text{ kJ/kg},\ T_2 = 481\text{ K} \longleftarrow$$

State 4: $T_4 = 1200K \Rightarrow h_4 = 1277.79$ kJ/kg

State 5: The work of the compressor and the high pressure turbine are equal. Thus; $h_2 - h_1 = h_4 - h_5 \Rightarrow h_5 = 1094.92$ kJ/kg
From the turbine efficiency; $h_4 - h_5 = \eta_t\,(h_4 - h_{5s}) \Rightarrow h_{5s} = 1067.59$
Thus, $Pr_{5s} = 122.86$ and $p_5 = (Pr_{5s}/Pr_4)\,P_4 = 2.065$ bars
From $h_5 = 1094.92 \Rightarrow Pr_5 = 134.714$, $T_5 = 1043$ K $\longleftarrow$

State 6: For iseutropic expansion through the low pressure turbine
$Pr_{6s} = (P_6/P_5)\,Pr_5 = (1/2.065)(134.714) = 65.237 \Rightarrow h_{6s} = 896.40$
Using the turbine efficiency, $h_6 = h_5 - \eta_t(h_5 - h_{6s}) = 922.21$ kJ/kg, $T_6 = 890K \longleftarrow$

State 3: Now, using the regenerator effectiveness

$$\eta_{reg} = \frac{h_3 - h_2}{h_6 - h_2} \Rightarrow h_3 = \eta_{reg}(h_6 - h_2) + h_2 = 878.3 \text{ kJ/kg},$$

$T_3 = 851$ K

State 7: Applying mass and energy rate balances to a control volume enclosing the regenerator, $0 = (h_2 - h_3) + (h_6 - h_7)$ or
$h_7 = (h_2 - h_3) + h_6 = 483.06 - 878.3 + 922.21 = 526.97$ kJ/kg,
$T_7 = 523$ K $\longleftarrow$

PROBLEM 9.59 (Contd.)

The net power output is the power developed by the low pressure turbine:

$$\dot{W}_{net} = \dot{m}\,(h_5 - h_6)$$

$$= \left(0.562\ \frac{kg}{s}\right)\left(1094.92 - 922.21\right)\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 97.1\ kW \longleftarrow$$

The thermal efficiency is

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{m}\,(h_4 - h_3)}$$

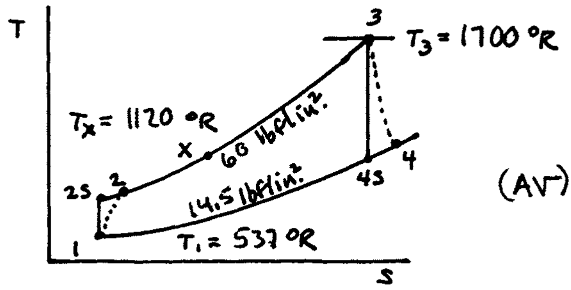
$$= \frac{(97.1\ kJ/s)}{\left(0.562\ \frac{kg}{s}\right)\left[1277.79 - 878.3\right]\frac{kJ}{kg}} = 0.432\ (43.2\%) \longleftarrow$$

PROBLEM 9.60*

KNOWN: A regenerative gas turbine is analyzed on an air-standard basis. Data are known at various locations, and the net power output is given. The isentropic efficiencies of the compressor and turbine are also specified.

FIND: Determine (a) the thermal efficiency, (b) the regenerator effectiveness, and (c) the net power output.

SCHEMATIC & GIVEN DATA:



$(AV)_1 = 3.2 \times 10^5\ ft^3/min$

ASSUMPTIONS: See Example 9.7, except $\eta_t = \eta_c = 0.84$.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 537\,°R \Rightarrow h_1 = 128.34$ Btu/lb, $p_{r1} = 1.3593$

State 2 For an isentropic compression, $p_{r2s} = (p_2/p_1)\,p_{r1} = 5.625$ Thus, $h_{2s} = 192.76$ Btu/lb. Using the compressor efficiency

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s}-h_1}{\eta_c}\right) = 205.03 \text{ Btu/lb}$$

State 3 $T_3 = 1700\,°R \Rightarrow h_3 = 422.59$ Btu/lb, $p_{r3} = 90.95$

State 4 For an isentropic expansion, $p_{r4} = (p_4/p_3)\,p_{r3} = 21.980$. Thus, $h_{4s} = 284.01$ Btu/lb. Using the turbine efficiency

$$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 306.18 \text{ Btu/lb}$$

State x $T_x = 1120\,°R \Rightarrow h_x = 271.03$ Btu/lb

(a) The thermal efficiency is

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_3-h_4)-(h_2-h_1)}{h_3-h_x} = 0.262\ (26.2\%) \longleftarrow \eta$$

(b) The regenerator effectiveness is

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} = 0.652\ (65.2\%) \longleftarrow \eta_{reg}$$

(c) The net power output is

$$\dot{W}_{cycle} = \dot{m}\left[(h_3-h_4)-(h_2-h_1)\right]$$

where

$$\dot{m} = \frac{(AV)_1 p_1}{(RT_1)} = \frac{(3.2\times10^5\ ft^3/min)(14.5\times144\ lbf/ft^2)}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(537°R)}\left|\frac{60\ min}{1h}\right| = 1.400\times10^6\ \frac{lb}{h}$$

Thus

$$\dot{W}_{cycle} = \left(1.400\times10^6\ \frac{lb}{h}\right)\left[(422.59-306.18)-(205.03-128.34)\right]\frac{Btu}{lb}$$

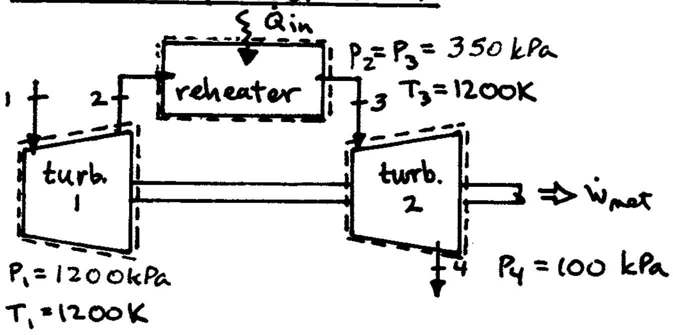
$$= 5.56\times10^7\ Btu/h \longleftarrow \dot{W}_{cycle}$$

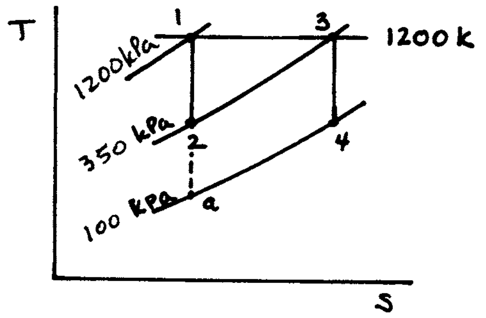
PROBLEM 9.61

KNOWN: Air expands in two stages through a turbine with reheat between the stages. The states are specified at the inlet and exit of each component.

FIND: Determine, per unit mass of air flowing, (a) the work developed by each stage, (b) the heat transfer for reheat, and (c) the increase in net work compared to a single stage of expansion with no reheat.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each control volume is at steady state. (2) The turbines operate isentropically. (3) Kinetic and potential energy effects are negligible. (4) The working fluid is air modeled as an ideal gas.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1 $T_1 = 1200\text{K} \Rightarrow h_1 = 1277.79$ kJ/kg, $p_{r1} = 238.0$

State 2 $p_{r2} = (p_2/p_1)\, p_{r1} = 69.417 \Rightarrow h_2 = 912.11$ kJ/kg

State 3 $T_3 = 1200\text{K} \Rightarrow h_3 = h_1 = 1277.79$, $p_{r3} = p_{r1} = 238.0$

State 4 $p_{r4} = (p_4/p_3)\, p_{r3} = 68 \Rightarrow h_4 = 906.85$ kJ/kg

(a) The work developed by each stage is

$\dot{W}_{t1}/\dot{m} = h_1 - h_2 = 365.68$ kJ/kg ← $\dot{W}_{t1}/\dot{m}$

$\dot{W}_{t2}/\dot{m} = h_3 - h_4 = 370.94$ kJ/kg ← $\dot{W}_{t2}/\dot{m}$

(b) For the reheater

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = (1277.79 - 912.11) = 365.7 \text{ kJ/kg}$$ ← $\dot{Q}_{in}/\dot{m}$

(c) To determine the work for a single stage of expansion, determine h_a, as follows.

$p_{ra} = (p_a/p_1)\, p_{r1} = 19.833 \Rightarrow h_a = 638.58$ kJ/kg

Thus $\dot{W}/\dot{m} = (h_1 - h_a) = 639.21$ kJ/kg

and

$$\%\text{ increase} = \frac{(365.68 + 370.94) - 639.21}{639.21} \times 100 = 15.2\%$$ ← % inc.

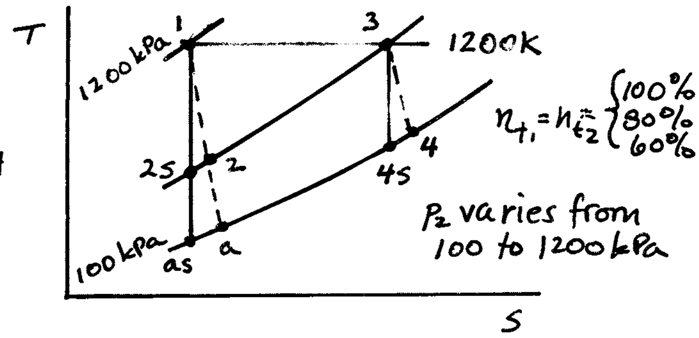
PROBLEM 9.62

KNOWN: The two-stage turbine with reheat in Problem 9.61 is reconsidered with turbine stage efficiencies less than 100%.

FIND: Plot (a) the work developed by each stage, (2) the heat transfer for the reheat process, and (c) the percent increase in work for the two-stage turbine with reheat compared to a single expansion, for each efficiency, and for interstage pressures ranging from 100 to 1200 kPa.

SCHEMATIC & GIVEN DATA:

See solution to Problem 9.61 for schematic.


T
1
3
1200 kPa
1200 K
$\eta_{t_1} = \eta_{t_2}$ 100% 80% 60%
2s
2
4
4s
p_2 varies from 100 to 1200 kPa
100 kPa
as
a
s

ASSUMPTIONS: (1) Each control volume is at steady state. (2) The turbines operate adiabatically. (3) Kinetic and potential energy effects can be neglected. (4) The working fluid is air as an ideal gas.

ANALYSIS: Consider the sample case of $p_2 = 350$ kPa and $\eta_{t_1} = \eta_{t_2} = 0.8$. Fixing each of the principal states:

State 1: Problem 9.61: $T_1 = 1200$ K, $h_1 = 1277.79$ kJ/kg

State 2: With $h_{2s} = 912.11$ from Problem 9.61, and using the turbine efficiency

$$h_2 = h_1 - \eta_{t_1}(h_1 - h_{2s}) = 985.25 \text{ kJ/kg}$$

State 3: $h_3 = 1277.79$ kJ/kg

State 4: With $h_{4s} = 906.85$ kJ/kg ; $h_4 = h_3 - \eta_{t_2}(h_3 - h_{4s}) = 981.04$ kJ/kg

State a: $h_{as} = 638.58$ kJ/kg ; $h_a = h_1 - \eta_{t_1}(h_1 - h_{as}) = 766.42$ kJ/kg

(a) The work developed by each stage is, respectively

$$\dot{W}_{t_1}/\dot{m} = h_1 - h_2 = 292.54 \text{ kJ/kg}$$

$$\dot{W}_{t_2}/\dot{m} = h_3 - h_4 = 296.75 \text{ kJ/kg}$$

(b) For the reheater

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2 = 292.54 \text{ kJ/kg}$$

(c) For comparison to a single stage, the percent increase in work is

$$\dot{W}/\dot{m} = h_1 - h_a = 511.37 \text{ kJ/kg}$$

Thus $$\% \text{ increase} = \frac{(292.54 + 296.75) - 511.37}{511.37} \times 100 = 15.2\ \%$$

The plots on the next page show how each of these quantities vary with interstage pressure for each of three efficiency values. Note that only one curve is needed for the % increase in work, since the efficiency value cancels out if the same value is used for each stage.

The data for the required plots were obtained using IT, as follows:

PROBLEM 9.62 (Cont'd.)

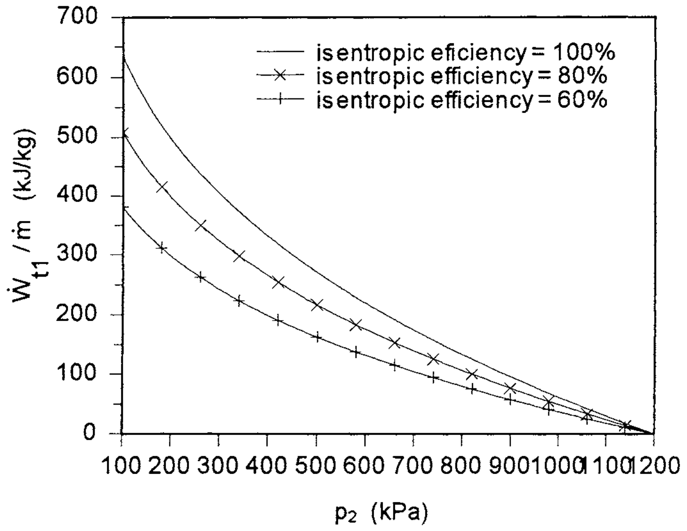
IT Code

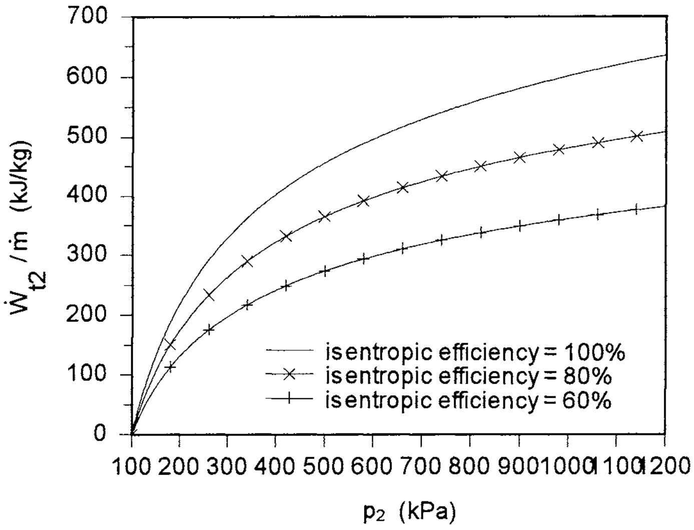
```
p1 = 1200  // kPa
T1 = 1200  // K
p2 = 350  // kPa
T3 = 1200  // K
p3 = p2
p4 = 100  // kPa
eta_t = 0.6

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 - (h1 - h2s) * eta_t
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eta_t

Wdot1 / mdot = (h1 - h2)
Wdot2 / mdot = (h3 - h4)
mdot = 1  // Assume a unit mass flow rate.
Qdot_in / mdot = h3 - h2

Wdots / mdot = h1 - ha
sas = s_hP("Air", has, p4)
sas = s1
ha = h1 - (h1 - has) * eta_t
pct = (((Wdot1 + Wdot2) - Wdots) / Wdots) * 100
```

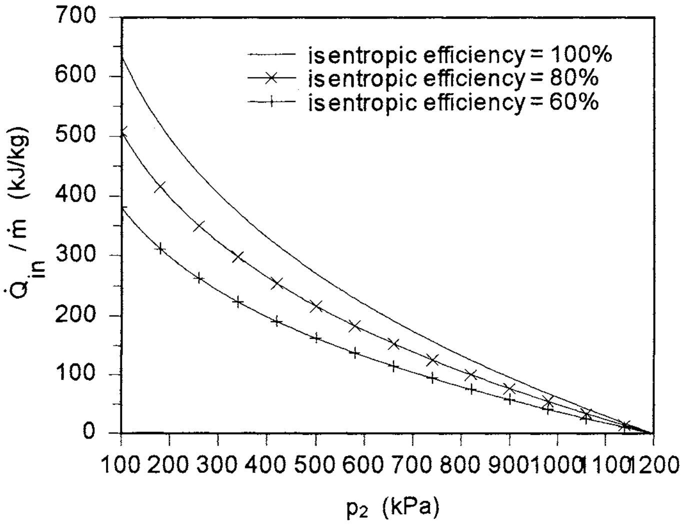
IT Results for $\eta_t = 0.8$

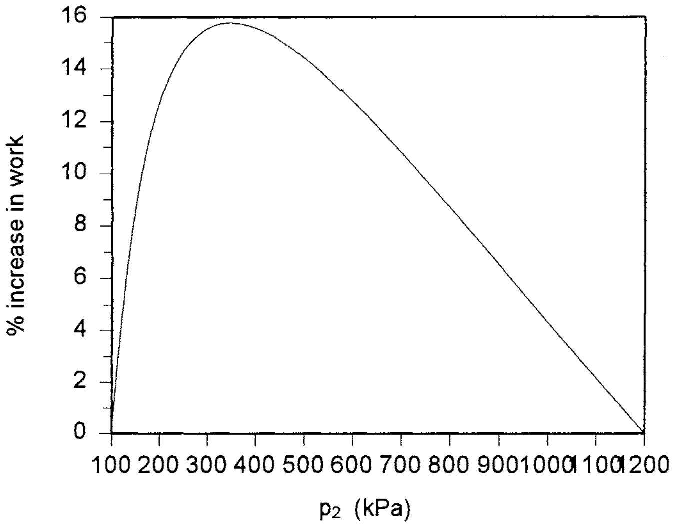
h_1 = 1277 kJ/kg
h_2 = 984.9 kJ/kg
h_{2s} = 911.9 kJ/kg
h_3 = 1277 kJ/kg
h_4 = 980.8 kJ/kg
h_{4s} = 906.7 kJ/kg
h_a = 768.7 kJ/kg
h_{as} = 641.7 kJ/kg
$\dot{W}_{t1}/\dot{m}$ = 292.2 kJ/kg
$\dot{W}_{t2}/\dot{m}$ = 296.3 kJ/kg
$\dot{Q}_{in}/\dot{m}$ = 292.2 kJ/kg
% increase = 15.77

PLOTS:







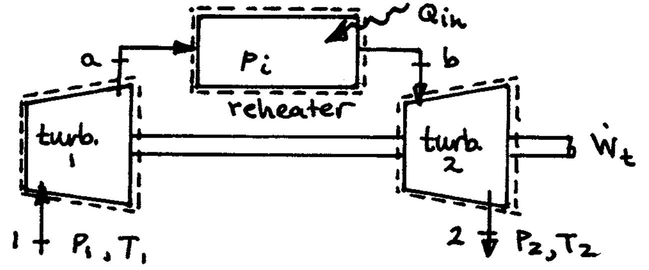


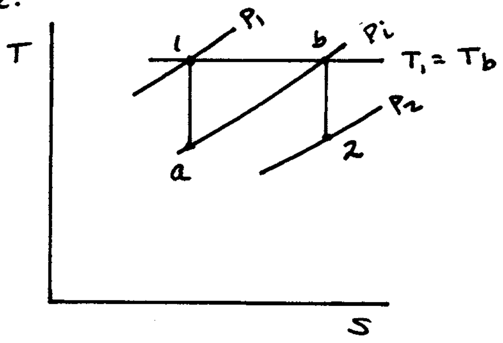
PROBLEM 9.63

KNOWN: A two-stage turbine with reheating operates at steady state under specified conditions.

FIND: Show that the maximum work is developed when the pressure ratio is the same across each stage.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The turbine expansions are isentropic. (3) There is no pressure drop through the reheater and the temperatures at the turbine inlets are equal. (4) Kinetic and potential energy effects are negligible. (5) The working fluid is air modeled as an ideal gas with constant c_p and k.

ANALYSIS: The turbine work developed is

$$\frac{\dot{W}_t}{\dot{m}} = (h_1 - h_a) + (h_b - h_2) = c_p\left[(T_1 - T_a) + (T_b - T_2)\right]$$

With $T_1 = T_b$

$$\frac{\dot{W}_t}{\dot{m}} = c_p\left[2T_1 - T_a - T_2\right] = c_p T_1\left[2 - \frac{T_a}{T_1} - \frac{T_2}{T_1}\right]$$

For the isentropic expansions

$$\frac{T_a}{T_1} = \left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} \quad \text{and} \quad \frac{T_2}{T_b} = \frac{T_2}{T_1} = \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}$$

Thus

$$\dot{W}_t/\dot{m} = c_p T_1\left[2 - \left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}\right]$$

To determine the pressure p_i that maximizes the total, form the derivative

$$\frac{\partial(\dot{W}_t/\dot{m})}{\partial p_i} = -c_p T_1\left(\frac{k-1}{k}\right)\left[\left(\frac{p_i}{p_1}\right)^{-\frac{1}{k}}\left(\frac{1}{p_1}\right) + \left(\frac{p_2}{p_i}\right)^{-\frac{1}{k}}\left(-\frac{p_2}{p_i^2}\right)\right]$$

$$= -c_p T_1\left(\frac{k-1}{k}\right)\left(\frac{1}{p_i}\right)\left[\left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}\right]$$

For $\frac{\partial(\dot{W}_t/\dot{m})}{\partial p_i} = 0 \Rightarrow \frac{p_i}{p_1} = \frac{p_2}{p_i}$ ←

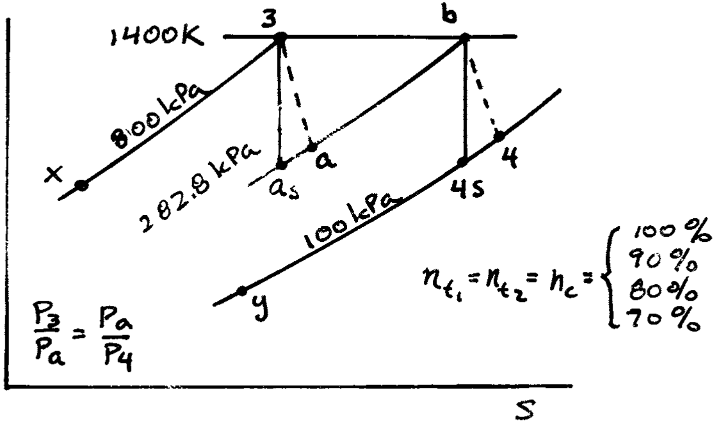
By checking the second derivative it can be verified that the total turbine work is a maximum.

PROBLEM 9.64

KNOWN: The regenerative cycle of Problem 9.54 is modified to include a two-stage turbine with reheat.

FIND: Construct plots of (a) the thermal efficiency, (b) the back work ratio, and (c) the net power developed for each of the given compressor and turbine efficiencies versus a range of compressor pressure ratios.

SCHEMATIC & GIVEN DATA:


T
1400K
3
b
800 kPa
282.8 kPa
100 kPa
x
a
a_s
4
4s
y
$\eta_{t_1} = \eta_{t_2} = \eta_c = \{100\%, 90\%, 80\%, 70\%$
$\frac{P_3}{P_a} = \frac{P_a}{P_4}$
s

ASSUMPTIONS: Same as in Example 9.7, except $\eta_c = \eta_{t_1} = \eta_{t_2} \le 100\%$.

ANALYSIS: Consider the same sample calculation case as in Problem 9.54, with $\eta_c = \eta_{t_1} = \eta_{t_2} = 0.8$ and $p_2/p_1 = 8$.

The following states are unchanged:

$h_1 = 300.19$ kJ/kg

$h_2 = 605.18$

$h_3 = 1515.42$

State a: $P_3/P_a = P_a/P_4 \Rightarrow P_a = 282.8$. Also $P_{ra} = P_{r3}(P_a/P_3) = 159.28$. Thus $h_{as} = 1145.8$. With the turbine efficiency; $h_a = h_3 - (h_3 - h_{as})\eta_{t_1} = 1219.7$ kJ/kg

State b: $T_b = T_3 = 1400$ K. $h_b = h_3 = 1515.42$ kJ/kg

State 4s: $P_{r4s} = P_{rb}(P_4/P_b) = 159.28 \Rightarrow h_{4s} = h_{as} = 1145.8$ kJ/kg. And, with the turbine efficiency; $h_4 = h_a = 1219.7$ kJ/kg.

The specific enthalpy h_x is

$$h_x = \eta_{reg}(h_4 - h_2) + h_2 = 1096.8 \text{ kJ/kg}$$

And, evaluating h_y

$$h_y = h_4 - (h_x - h_2) = 728.08 \text{ kJ/kg}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{(h_y - h_1)}{(h_3 - h_x) + (h_b - h_a)} = 0.401\ (40.1\%)$$

(b) The back work ratio is

$$bwr = \frac{(h_2 - h_1)}{2(h_3 - h_a)} = 0.5157$$

(c) The net power developed by the cycle is found from

$$\dot{W}_{cycle} = \dot{W}_{t_1} + \dot{W}_{t_2} - \dot{W}_c = \dot{m}\left[(h_3 - h_a) + (h_b - h_4) - (h_2 - h_1)\right]$$

With $\dot{m} = 5.807$ kg/s from Problem 9.54,

$$\dot{W}_{cycle} = 1663.4 \text{ kW}$$

The data for the required plots are obtained using IT, as follows:

PROBLEM 9.64 (Cont'd.)

IT Code

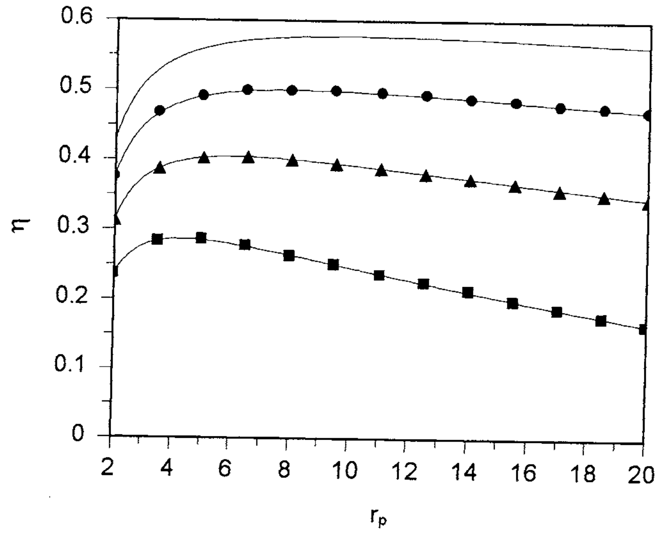
```
p1 = 100  // kPa
T1 = 300  // K
p2 / p1 = rp
rp = 8
p3 = p2
T3 = 1400  // K
p4 = p1
AV1 = 5  // m³/s
eff_t = 0.8
eff_c = 0.8
eff_reg = 0.8
To = 300 // K

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / eff_c
s2 = s_hP("Air", h2, p2)
```

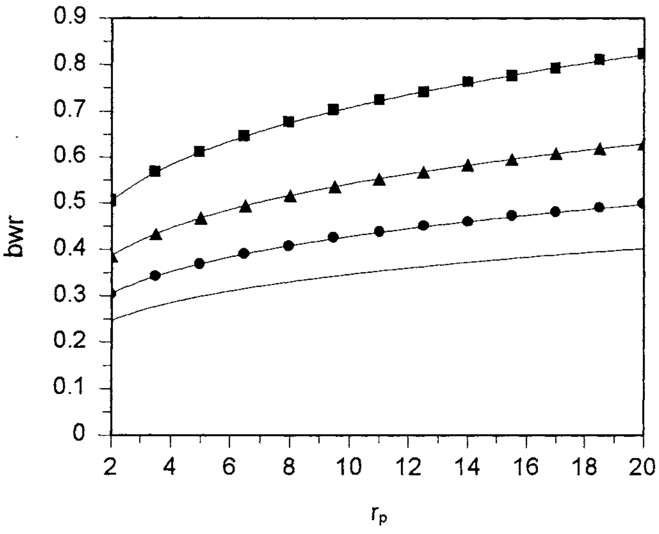
```
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
p3 / pa = pa / p4
sas = s_hP("Air", has, pa)
sas = s3
ha = h3 - (h3 - has) * eff_t
Tb = T3
hb = h_T("Air", Tb)
sb = s_TP("Air", Tb, pa)
s4s = s_hP("Air", h4s, p4)
s4s = sb
h4 = hb - (hb - h4s) * eff_t
hx = eff_reg * (h4 - h2) + h2
hy = h4 - (hx - h2)

mdot = AV1 / v1
v1 = v_TP("Air",T1,p1)
Wdotnet = mdot * ((h3 - ha) + (hb - h4) - (h2 - h1))
eta = Wdotnet /  (mdot * ((h3 - hx) + (hb - ha)))
bwr = (h2 - h1) / ((h3 - ha) + (hb - h4))
```

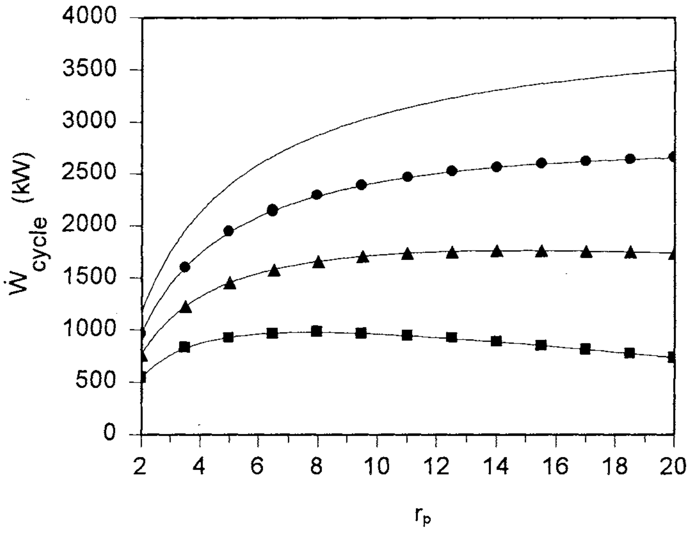
IT Results for $r_p = 8$, $\eta_c = \eta_t = 0.8$

h_1 = 300 kJ/kg
h_2 = 605.1 kJ/kg
h_{2s} = 544.1 kJ/kg
h_3 = 1514 kJ/kg
h_4 = 1219 kJ/kg
h_{4s} = 1145 kJ/kg
h_a = 1219 kJ/kg
h_{as} = 1145 kJ/kg
h_b = 1514 kJ/kg
h_x = 1096 kJ/kg
h_y = 727.8 kJ/kg
η = 0.4005
bwr = 0.5163
$\dot{W}_{cycle}$ = 1660 kW

PLOTS:





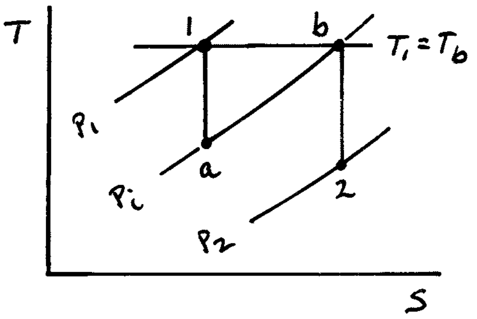


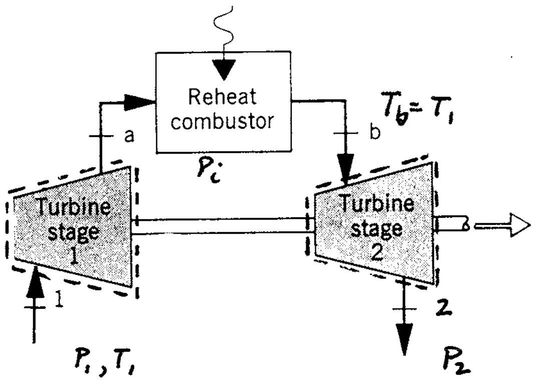
PROBLEM 9.65

KNOWN: A two-stage turbine with reheat operates at steady state under specified conditions.

FIND: Show that the maximum total work output is obtained when the pressure ratio is the same across each stage.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The turbine stages are analyzed as control volumes at steady state. (2) The expansion processes are isentropic. (3) $p_a = p_b \equiv p_i$. (4) $T_1 = T_b$, (5) Kinetic and potential energy effects are negligible. (6) The working fluid is air modeled as an ideal gas with constant specific heats.

ANALYSIS: The total turbine work output per unit mass of air flow is

$$\frac{\dot{W}_t}{\dot{m}} = \frac{\dot{W}_{t1}}{\dot{m}} + \frac{\dot{W}_{t2}}{\dot{m}} = (h_1 - h_a) + (h_b - h_2)$$

or

$$= c_p\left[(T_1 - T_a) + (T_b - T_2)\right]$$

$$= c_p T_1 \left[\left(1 - \frac{T_a}{T_1}\right) + \left(\frac{T_b}{T_1} - \frac{T_2}{T_1}\right)\right]$$

With $T_1 = T_b$,

$$\frac{\dot{W}_t}{\dot{m}} = c_p T_1 \left[2 - \frac{T_a}{T_1} - \frac{T_2}{T_1}\right]$$

For the isentropic processes: $T_a/T_1 = (p_a/p_1)^{\frac{k-1}{k}}$ and $T_2/T_b = (p_2/p_b)^{\frac{k-1}{k}}$. Since $p_a = p_b = p_i$

$$\frac{\dot{W}_t}{\dot{m}} = c_p T_1 \left[2 - \left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}\right]$$

Differentiating with respect to p_i and after re-arrangement

$$\frac{\partial(\dot{W}_t/\dot{m})}{\partial p_i} = c_p T_1 \left(\frac{k-1}{k}\right) \frac{1}{p_i} \left[\left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}\right]$$

When the partial derivative is set to zero, the desired relationship is obtained

$$\frac{p_i}{p_1} = \frac{p_2}{p_i}$$

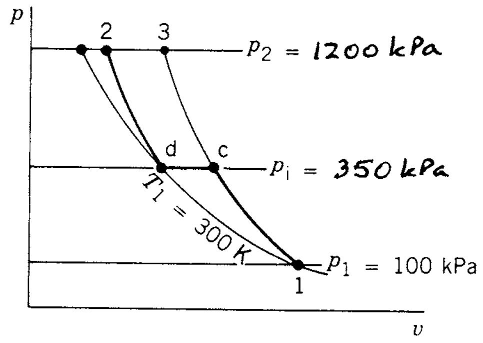
By checking the sign of the second derivative, it can be verified the the total turbine work output is indeed maximized at this pressure ratio.

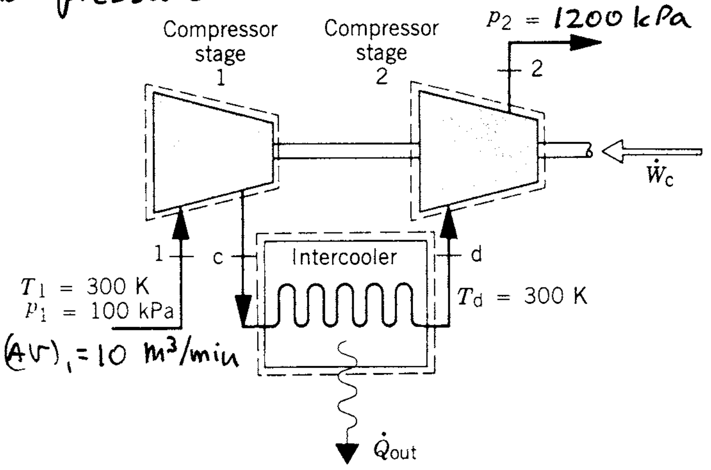
PROBLEM 9.66

KNOWN: Air is compressed in a two-stage compressor with intercooling between the stages. Operating pressures and temperatures are given.

FIND: Determine the power to run the compressor and compare this to the power required for isentropic compression from the same inlet state to the same final pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 9.9.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1: $T_1 = 300\text{K} \Rightarrow h_1 = 300.19\ \text{kJ/kg},\ p_{r1} = 1.3860$

State c: $p_{rc} = (p_c/p_1)\,p_{r1} = 4.851 \Rightarrow h_c = 429.77\ \text{kJ/kg}$

State d: $T_d = T_1 = 300\text{K} \Rightarrow h_d = 300.19\ \text{kJ/kg},\ p_{rd} = 1.3860$

State 2: $p_{r2} = (p_2/p_d)\,p_{rd} = 4.752 \Rightarrow h_2 = 427.21\ \text{kJ/kg}$

The mass flow rate is

$$\dot m = \frac{(AV)_1 p_1}{RT_1} = \frac{(10\ \text{m}^3/\text{min})(100\ \text{kPa})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300\text{K})}\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{10^3\ \text{N/m}^2}{1\ \text{kPa}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = 0.1936\ \text{kg/s}$$

The compressor power is

$$\dot W_c = \dot W_{c1} + \dot W_{c2} = \dot m(h_c - h_1) + \dot m(h_2 - h_d)$$

$$= \left(0.1936\ \frac{\text{kg}}{\text{s}}\right)(429.77 - 300.19)\ \frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| + (0.1936)(427.21 - 300.19)$$

$$= 25.087\ \text{kW} + 24.591\ \text{kW} = 49.68\ \text{kW} \longleftarrow \dot W_c$$

To find the power a single stage of compression, determine h_3 as follows:

$$p_{r3} = (p_2/p_1)\,p_{r1} = 16.632 \Rightarrow h_3 = 610.65\ \text{kJ/kg}$$

Thus $\dot W_c = \dot m(h_3 - h_1) = 60.11\ \text{kW}$

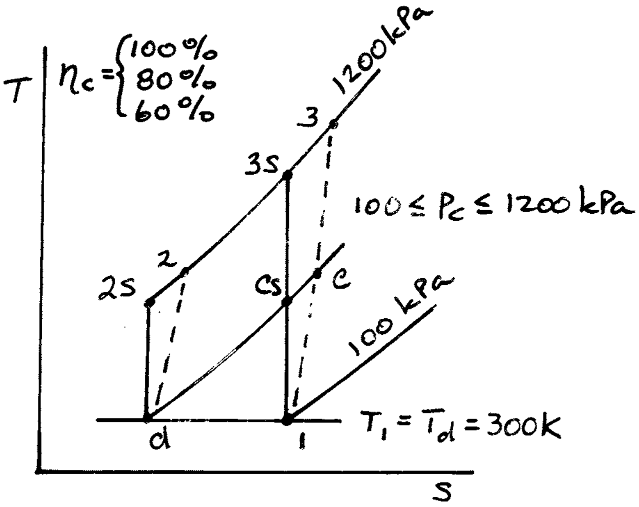
The decrease in power with two-stage, intercooled compression is

$$\%\ \text{decrease} = \frac{60.11 - 49.68}{60.11} \times 100 = 17.35\% \longleftarrow \text{\% reduction in power}$$

PROBLEM 9.67

KNOWN: The two-stage compressor with intercooling in Problem 9.66 is reconsidered with compressor stage efficiencies less than 100%.

FIND: Plot the (a) power input to each stage, (b) heat transfer rate for the intercooler, and (c) percent decrease in power for two-stage compression compared a single compression stage, for each efficiency value given, and for interstage pressures ranging from 100 to 1200 kPa.


T
$\eta_c = \begin{cases} 100\% \\ 80\% \\ 60\% \end{cases}$
1200 kPa
3
3s
$100 \le P_c \le 1200$ kPa
2
2s
cs
c
100 kPa
d
1
$T_1 = T_d = 300$K
s

SCHEMATIC & GIVEN DATA:

See solution to Problem 9.66 for schematic.

ASSUMPTIONS: Same as in Example 9.9, except $\eta_{c1} = \eta_{c2} \le 100\%$.

ANALYSIS: Consider the sample case of $P_c = P_d = 350$ kPa and $\eta_{c1} = \eta_{c2} = 0.8$. Fixing each of the principal states:

State 1: $h_1 = 300.19$ kJ/kg (Problem 9.66)

State c: With $h_{cs} = 429.77$ kJ/kg from Problem 9.66, and using the compressor efficiency $h_c = h_1 + (h_{cs} - h_1)/\eta_{c1} = 462.16$ kJ/kg

State d: $h_d = h_1 = 300.19$ kJ/kg

State 2: $h_{2s} = 427.21$ kJ/kg ; $h_2 = h_d + (h_{2s} - h_d)/\eta_{c2} = 459.0$ kJ/kg

(a) The power input for each stage is, for $\dot{m} = 0.1936$ kg/s

$$\dot{W}_{c1} = \dot{m}(h_c - h_1) = 31.36 \text{ kW}$$

$$\dot{W}_{c2} = \dot{m}(h_2 - h_d) = 30.75 \text{ kW}$$

also $\dot{W}_c = \dot{W}_{c1} + \dot{W}_{c2} = 62.11$ kW

(b) For the intercooler

$$\dot{Q}_{out} = \dot{m}(h_c - h_d) = 31.36 \text{ kW}$$

(c) Determine h_3 as follows: $h_{3s} = 610.65$ kJ/kg (Problem 9.66). Thus

$$h_3 = h_1 + (h_{3s} - h_1)/\eta_c = 688.3 \text{ kJ/kg}$$

Thus $\dot{W} = \dot{m}(h_3 - h_1) = 75.14$ kW

Finally

$$\% \text{ decrease} = \frac{75.14 - 62.11}{75.14} \times 100 = 17.34\%$$

The plots on the next page show how each of these quantities varies with interstage pressure for each of the efficiency values. Note that only one curve is needed for the % decrease in power input, since the efficiency value cancels out if the same value is used for each stage.

PROBLEM 9.67 (Cont'd.)

The data for the required plots are obtained using IT, as follows:

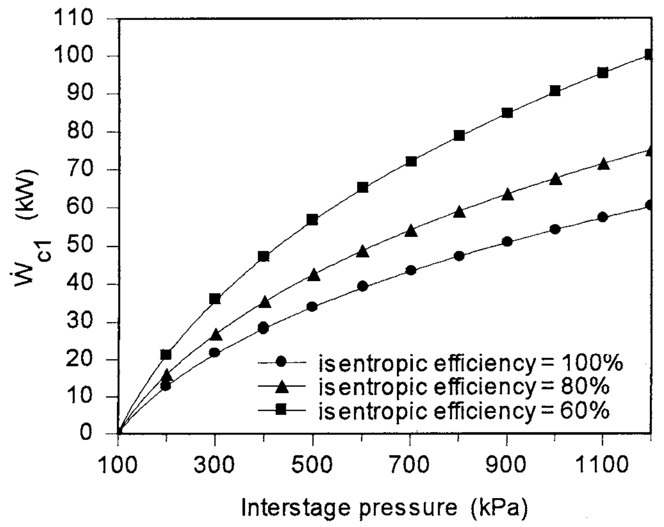
IT Code

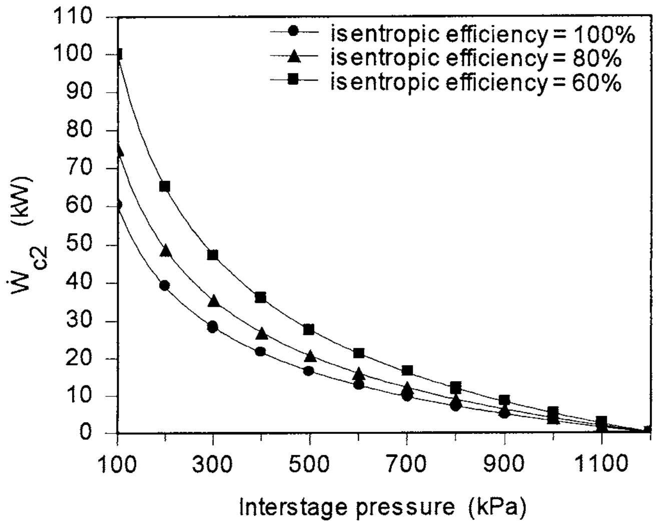
```
p1 = 100  // kPa
T1 = 300  // K
pc = 350  // kPa
pd = pc
Td = T1
p2 = 1200  // kPa
p3 = p2
eta_c = 0.8
AV1 = 10  // m³/min

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
scs = s_hP("Air", hcs, pc)
scs = s1
hc = h1 + (hcs - h1) / eta_c
hd = h_T("Air", Td)
sd = s_TP("Air", Td, pd)
s2s = s_hP("Air", h2s, p2)

s2s = sd
h2 = hd + (h2s - hd) / eta_c
s3s = s_hP("Air", h3s, p3)
s3s = s1
h3 = h1 + (h3s - h1) / eta_c

mdot = (AV1 / v1) * (1 / 60)  // kg/s
v1 = v_TP("Air",T1, p1)
Wdotc1 = mdot * (hc - h1)
Wdotc2 = mdot * (h2 - hd)
Wdotc = Wdotc1 + Wdotc2
Qdotout = mdot * (hc - hd)
Wdot = mdot * (h3 - h1)
pct = ((Wdot - Wdotc) / Wdot) * 100
```

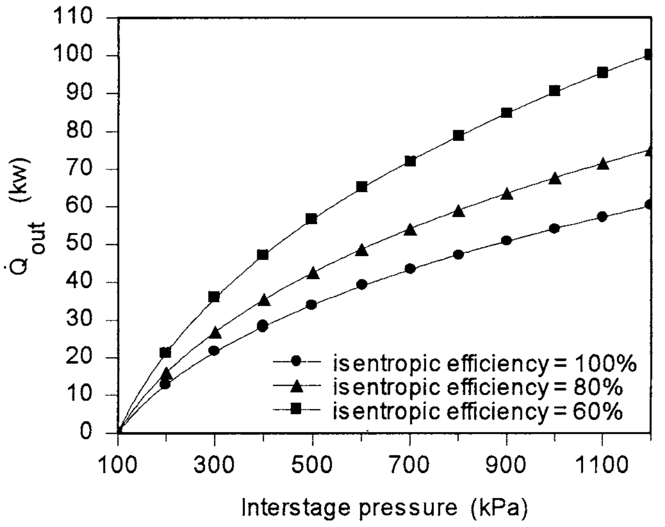
IT Results for $p_c = p_d = 350$ kPa, $\eta_{c1} = \eta_{c2} = 80\%$

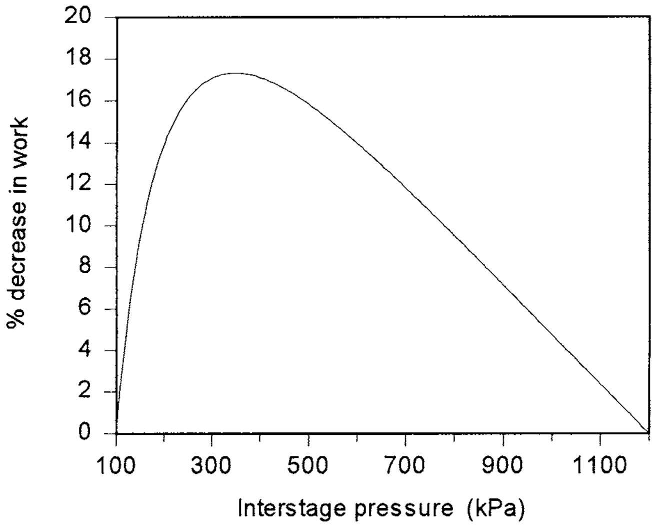
$h_1 = 300$ kJ/kg
$h_2 = 458.9$ kJ/kg
$h_{2s} = 427.1$ kJ/kg
$h_3 = 688.2$ kJ/kg
$h_{3s} = 610.5$ kJ/kg
$h_c = 462.1$ kJ/kg
$h_{cs} = 429.7$ kJ/kg
$h_d = 300$ kJ/kg
$\dot{m} = 0.1936$ kg/s
$\dot{W}_{c1} = 31.37$ kW
$\dot{W}_{c2} = 30.76$ kW
$\dot{Q}_{out} = 31.37$ kW
% decrease = 17.32

PLOTS:







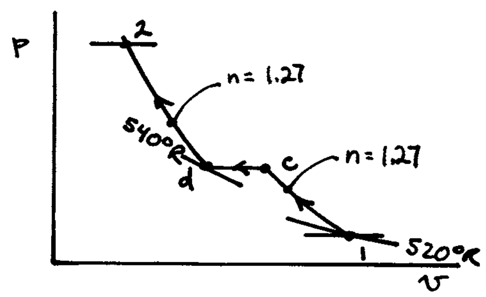


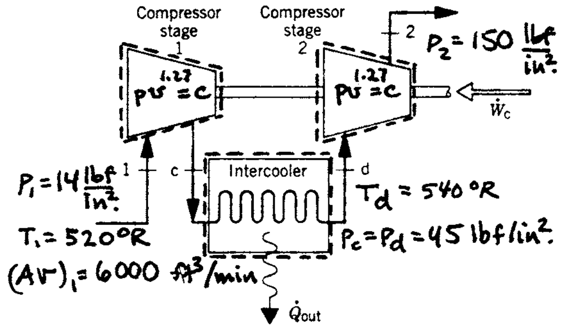
PROBLEM 9.68*

KNOWN: Air is compressed at steady state in a two-stage compressor with intercooling between the stages. Operating pressures and temperatures are known.

FIND: Determine (a) the power and heat transfer rate for each compressor stage and (b) the intercooler heat transfer rate.

SCHEMATIC & GIVEN DATA:





Compressor stage 1
Compressor stage 2
$pv^{1.27} = c$
$pv^{1.27} = c$
2
$p_2 = 150\ \frac{\text{lbf}}{\text{in}^2}$
$\dot{W}_c$
$p_1 = 14\ \frac{\text{lbf}}{\text{in}^2}$
1
c
Intercooler
d
$T_d = 540\ °R$
$T_1 = 520\ °R$
$p_c = p_d = 45\ \text{lbf/in}^2$
$(AV)_1 = 6000\ \text{ft}^3/\text{min}$
$\dot{Q}_{out}$

ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The compression processes are polytropic with $n = 1.27$. (3) There is no pressure drop through the intercooler. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) For compressor stage 1, $T_c = (p_c/p_1)^{\frac{n-1}{n}} T_1 = 666.51\ °R$. From Table A-22E, $h_1 = 124.27$ Btu/lb and $h_c = 159.49$ Btu/lb.

Equation 6.66a, for a polytropic process of an ideal gas, applies

$$\frac{\dot{W}_{c1}}{\dot{m}} = \frac{nR}{n-1}(T_c - T_1) = \frac{(1.27)\left(\frac{1.986}{28.97}\ \frac{\text{Btu}}{\text{lb}\cdot°R}\right)(666.51 - 520)\,°R}{(.27)}$$

$$= 47.24\ \text{Btu/lb}$$

Evaluating $\dot{m}$

$$\dot{m} = \frac{(AV)_1\, p_1}{RT_1} = \frac{(6000\ \text{ft}^3/\text{min})(14\ \text{lbf/in}^2)}{\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(520\,°R)}\left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| = 436.2\ \text{lb/min}$$

The power for stage 1 is

$$\dot{W}_{c1} = (436.2\ \text{lb/min})(47.24\ \text{Btu/lb}) = 20{,}606\ \text{Btu/min} \longleftarrow \dot{W}_{c1}$$

Now, applying an energy balance to stage 1, the heat transfer rate is

$$\dot{Q}_{c1} = \dot{m}(h_c - h_1) - \dot{W}_{c1}$$

$$= (436.2)(159.49 - 124.27) - 20{,}606 = -5243\ \text{Btu/min} \longleftarrow \dot{Q}_{c1}$$

For stage 2; $T_2 = (p_2/p_d)^{n-1/n}\, T_d = (150/45)^{.27/1.27}(540) = 697.5\ °R$. From Table A-22E, $h_d = 129.06$ Btu/lb, $h_2 = 166.96$ Btu/lb. Then

$$\dot{W}_{c2} = \dot{m}\left[\frac{nR}{n-1}\right][T_2 - T_d] = (436.2)\left[\frac{(1.27)(1.986/28.97)}{.27}\right](697.5 - 540) = 22{,}153\ \text{Btu/min} \swarrow \dot{W}_{c2}$$

$$\dot{Q}_{c2} = \dot{m}(h_2 - h_d) - \dot{W}_{c2} = (436.2)(166.96 - 129.06) - 22{,}153 = -5621\ \text{Btu/min} \longleftarrow \dot{Q}_{c2}$$

(b) $$\dot{Q}_{out} = \dot{m}(h_c - h_d)$$

$$= 436.2(159.49 - 129.06) = 13{,}274\ \text{Btu/min} \qquad \longleftarrow \dot{Q}_{out}$$

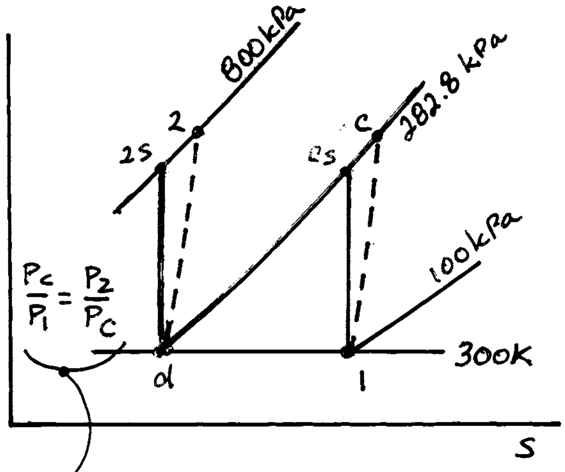
PROBLEM 9.69

KNOWN: The regenerative cycle of Problem 9.54 is modified to include a two-stage compressor with intercooling.

FIND: Construct plots of the (a) thermal efficiency, (b) back work ratio, and (c) net power developed for the given compressor stage and turbine efficiencies versus a range of compressor pressure ratios.

SCHEMATIC & GIVEN DATA:

$$\eta_{c_1} = \eta_{c_2} = \eta_t = \begin{cases} 100\% \\ 90\% \\ 80\% \\ 70\% \end{cases}$$



ASSUMPTIONS: Same as in Example 9.7, except $\eta_{c_1} = \eta_{c_2} = \eta_t \leq 100\%$.

ANALYSIS: Consider the same sample calculation case as in Problem 9.54; with $\eta_{c_1} = \eta_{c_2} = \eta_t = 0.8$ and $P_2/P_1 = 8$.

The following states are unchanged:

$h_1 = 300.19$ kJ/kg, $P_{r_1} = 1.3860$

$h_3 = 1515.42$

$h_4 = 991.4$

State c: $P_{rcs} = (P_c/P_1)\, P_{r_1}$. With $P_c = \sqrt{P_1 P_2} = 282.8$ kPa; $P_{rcs} = 3.920$ and $h_{cs} = 404.32$ kJ/kg. Using the compressor stage efficiency,

$$h_c = h_1 + (h_{cs} - h_1)/\eta_{c_1} = 430.35 \text{ kJ/kg}$$

State d: $T_d = T_1 = 300K \Rightarrow h_d = h_1 = 300.19$ kJ/kg, $P_{rd} = 1.3860$

State 2: $P_{r2s} = (P_2/P_d)\, P_{rd} = 3.920 \Rightarrow h_{2s} = 404.32$, $h_2 = 430.35$ kJ/kg

The specific enthalpy h_x is

$$h_x = \eta_{reg}(h_4 - h_2) + h_2 = 879.19 \text{ kJ/kg}$$

And, evaluating h_y

$$h_y = h_4 - (h_x - h_2) = 542.56 \text{ kJ/kg}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{(h_y - h_1) + (h_c - h_d)}{(h_3 - h_x)} = 0.414 \ (41.4\%)$$

(b) The back work ratio is

$$bwr = \frac{2(h_c - h_1)}{h_3 - h_4} = 0.497$$

(c) The net power developed by the cycle is found from

$$\dot{W}_{cycle} = \dot{W}_t - (\dot{W}_{c1} + \dot{W}_{c2}) = \dot{m}\,[(h_3 - h_4) - 2(h_c - h_1)]$$

With $\dot{m} = 5.807$ kg/s from Problem 9.54

$$\dot{W}_{cycle} = 1531.3 \text{ kW}$$

PROBLEM 9.69 (Cont'd.)

The data for the required plots are obtained using IT, as follows:

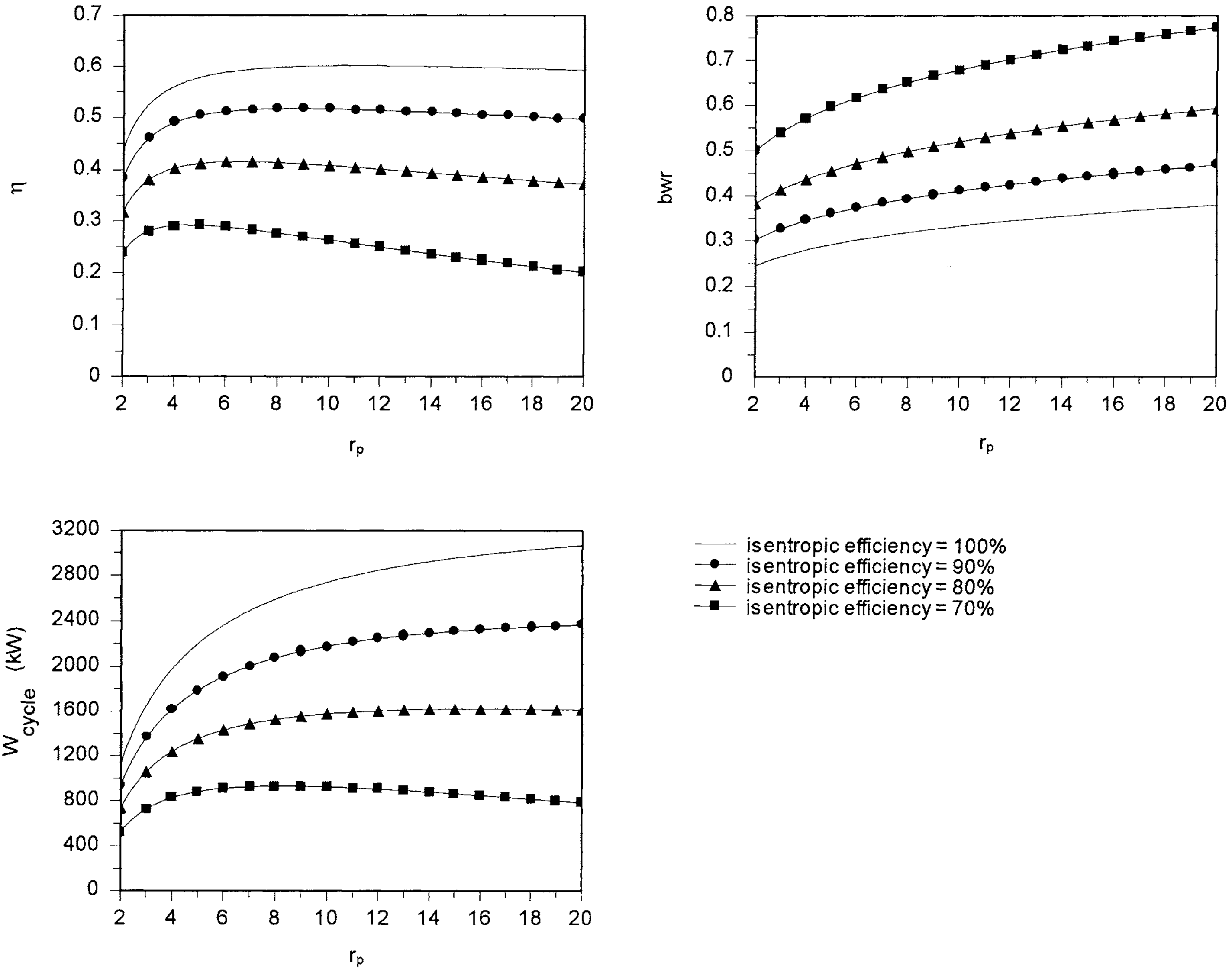
IT Code

```
p1 = 100  // kPa
T1 = 300  // K
p2 / p1 = rp
rp = 8
pc / p1 = p2 / pc
pd = pc
Td = T1
p3 = p2
T3 = 1400  // K
p4 = p1
AV1 = 5  // m³/s
eff_t = .7
eff_c = .7
eff_reg = 0.8

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
scs = s_hP("Air", hcs, pc)
scs = s1
hc = h1 + (hcs - h1) / eff_c
hd = h_T("Air", Td)
sd = s_TP("Air", Td, pd)
s2s = s_hP("Air", h2s, p2)
s2s = sd
h2 = hd + (h2s - hd) / eff_c
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eff_t
hx = eff_reg * (h4 - h2) + h2
hy = h4 - (hx - h2)

mdot = AV1 / v1
v1 = v_TP("Air", T1, p1)
eta = 1 - ((hy - h1) + (hc - hd)) / (h3 - hx)
bwr = ((hc - h1) + (h2 - hd)) / (h3 - h4)
Wdotnet = mdot * ((h3 - h4) - 2 * (hc - h1))
```

IT Results for $r_p = 8$, $\eta_{c1} = \eta_{c2} = \eta_t = 80\%$

$h_1 = 300$ kJ/kg
$h_2 = 430.3$ kJ/kg
$h_{2s} = 404.2$ kJ/kg
$h_3 = 1514$ kJ/kg
$h_4 = 990.8$ kJ/kg
$h_{4s} = 859.9$ kJ/kg
$h_c = 430.3$ kJ/kg
$h_{cs} = 404.2$ kJ/kg
$h_d = 300$ kJ/kg
$h_x = 878.7$ kJ/kg
$h_y = 542.4$ kJ/kg
$\eta = 0.4136$
bwr = 0.4978
$\dot{W}_{cycle} = 1526$ kW

PLOTS:



PROBLEM 9.70

KNOWN: A two-stage compressor with intercooling operates at steady state with known inlet conditions and a given exit pressure.

FIND: Referring to Example 9.10, derive an expression for the relation between pressure ratios across the two stages for the case where $T_d > T_i$.

SCHEMATIC & GIVEN DATA: See Example 9.10.

ASSUMPTIONS: See Example 9.10, except $T_d > T_i$.

ANALYSIS: The total compressor work input per unit mass flow is

$$\frac{\dot{W}_c}{\dot{m}} = (h_c - h_1) + (h_2 - h_d) = c_p\,(T_c - T_1) + c_p\,(T_2 - T_d)$$

$$= c_p T_1 \left(\frac{T_c}{T_1} - 1\right) + c_p T_d \left(\frac{T_2}{T_d} - 1\right)$$

Introducing the isentropic relations

$$\frac{T_c}{T_1} = \left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}}, \quad \frac{T_2}{T_d} = \left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}$$

we get

$$\frac{\dot{W}_c}{\dot{m}} = c_p T_1 \left[\left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - 1\right] + c_p T_d \left[\left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}} - 1\right]$$

To determine the pressure p_i that minimizes the total work, form the derivative

$$\frac{\partial(\dot{W}_c/\dot{m})}{\partial p_i} = c_p T_1 \left(\frac{k-1}{k}\right)\left(\frac{p_i}{p_1}\right)^{-\frac{1}{k}}\left(\frac{1}{p_1}\right) - c_p T_d \left(\frac{k-1}{k}\right)\left(\frac{p_2}{p_i}\right)^{-\frac{1}{k}}\left(\frac{p_2}{p_i}\right)$$

$$= c_p T_1 \left(\frac{k-1}{k}\right)\frac{1}{p_i}\left\{\left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - \left(\frac{T_d}{T_1}\right)\left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}\right\}$$

For the minimum, $\partial(\dot{W}_c/\dot{m})/\partial p_i = 0$. Thus

$$0 = \left(\frac{p_i}{p_1}\right)^{\frac{k-1}{k}} - \left(\frac{T_d}{T_1}\right)\left(\frac{p_2}{p_i}\right)^{\frac{k-1}{k}}$$

and finally

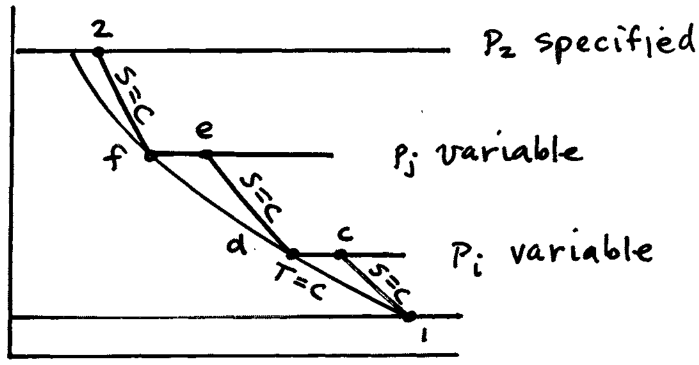
$$\frac{p_i}{p_1} = \left(\frac{T_d}{T_1}\right)^{\frac{k}{k-1}}\left(\frac{p_2}{p_i}\right)$$

PROBLEM 9.71

KNOWN: A multi-stage compressor with intercooling between the stages has known inlet conditions and a given exit pressure.

FIND: Show that the minimum total work input is required when the pressure ratio is the same across each stage.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 9.10.

ANALYSIS: From the result of Example 9.10;

$$\frac{p_i}{p_1} = \frac{p_j}{p_i}$$

Similarly

$$\frac{p_j}{p_i} = \frac{p_2}{p_j}$$

Thus

$$\frac{p_i}{p_1} = \frac{p_j}{p_i} = \frac{p_2}{p_j}$$

Finally, solving for p_i and p_j

$$p_i = \sqrt[3]{p_1^2 p_2}$$

$$p_j = \sqrt[3]{p_2^2 p_1}$$

← optimum pressures

PROBLEM 9.72

KNOWN: The regenerative cycle of Problem 9.54 is modified to include two-stage compression and expansion, with intercooling and reheat, respectively, between the stages.

FIND: Construct plots of the (a) thermal efficiency, (b) back work ratio, and (c) net power developed for given compressor and turbine stage efficiencies versus a range of compressor pressure ratios.

SCHEMATIC & GIVEN DATA: See Probems 9.64 and 9.69.

ASSUMPTIONS: See above-referenced problems.

ANALYSIS: Consider the same sample case as in Problems 9.54, 9.64, and 9.69; with $\eta_{c1} = \eta_{c2} = \eta_{t1} = \eta_{t2} = 0.8$ and $p_2/p_1 = 8$. The following states are unchanged:

$h_1 = 300.19$ kJ/kg $\quad h_3 = 1515.42$

$h_c = 430.35 \quad h_a = 1219.7$

$h_d = 300.19 \quad h_b = 1515.42$

$h_2 = 430.35 \quad h_4 = 1219.7$

Now, the specific enthalpy h_x is

$$h_x = \eta_{reg}(h_4 - h_2) + h_2 = 1061.8 \text{ kJ/kg}$$

And, evaluating h_y

$$h_y = h_4 - (h_x - h_2) = 588.25 \text{ kJ/kg}$$

(a) The thermal efficiency is

$$\eta = 1 - \frac{(h_y - h_1) + (h_c - h_d)}{(h_3 - h_x) + (h_b - h_a)} = 0.442 \; (44.2\%)$$

(b) The back work ratio is

$$bwr = \frac{\cancel{2}(h_c - h_1)}{\cancel{2}(h_3 - h_a)} = 0.440$$

(c) The net power developed by the cycle is

$$\dot{W}_{cycle} = \dot{m}\left[\dot{W}_{t1} + \dot{W}_{t2} - \dot{W}_{c1} - \dot{W}_{c2}\right] = \dot{m}\left[2(h_3 - h_a) - 2(h_c - h_1)\right]$$

with $\dot{m} = 5.807$ kg/s,

$$\dot{W}_{cycle} = 1922.8 \text{ kW}$$

The data for the required plots are obtained using IT, as follows:

PROBLEM 9.72 (Cont'd.)

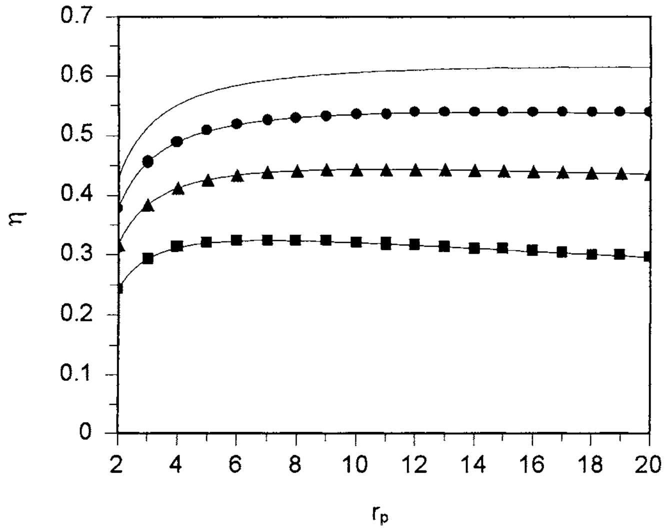
IT Code

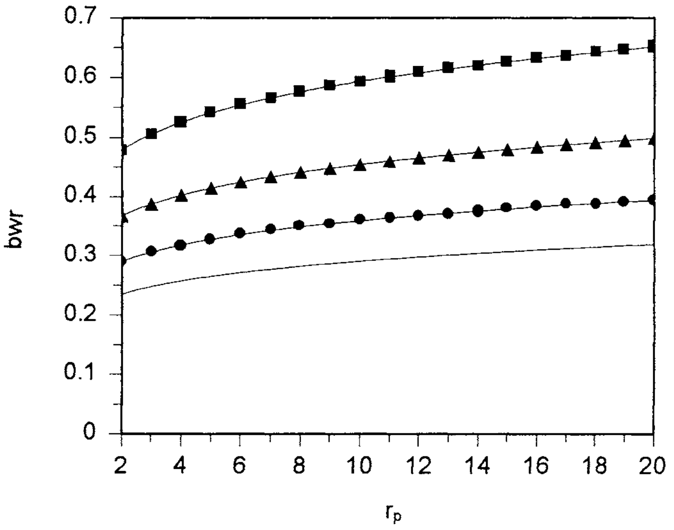
```
p1 = 100  // kPa
T1 = 300  // K
p2 / p1 = rp
rp = 8
pc / p1 = p2 / pc
pd = pc
Td = T1
p3 = p2
T3 = 1400  // K
p4 = p1
AV1 = 5  // m³/s
eff_t = .8
eff_c = .8
eff_reg = 0.8

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
scs = s_hP("Air", hcs, pc)
scs = s1
hc = h1 + (hcs - h1) / eff_c
hd = h_T("Air", Td)
sd = s_TP("Air", Td, pd)
s2s = s_hP("Air", h2s, p2)
s2s = sd
h2 = hd + (h2s - hd) / eff_c
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
p3 / pa = pa / p4
sas = s_hP("Air", has, pa)
sas = s3

ha = h3 - (h3 - has) * eff_t
Tb = T3
hb = h_T("Air", Tb)
sb = s_TP("Air", Tb, pa)
s4s = s_hP("Air", h4s, p4)
s4s = sb
h4 = hb - (hb - h4s) * eff_t
hx = eff_reg * (h4 - h2) + h2
hy = h4 - (hx - h2)

mdot = AV1 / v1
v1 = v_TP("Air",T1,p1)
Wdotnet = mdot * (2 * (h3 - ha) - 2 * (hc - h1))
eta = Wdotnet /  (mdot * ((h3 - hx) + (hb - ha))
bwr = (hc - h1) / (h3 - ha)
```

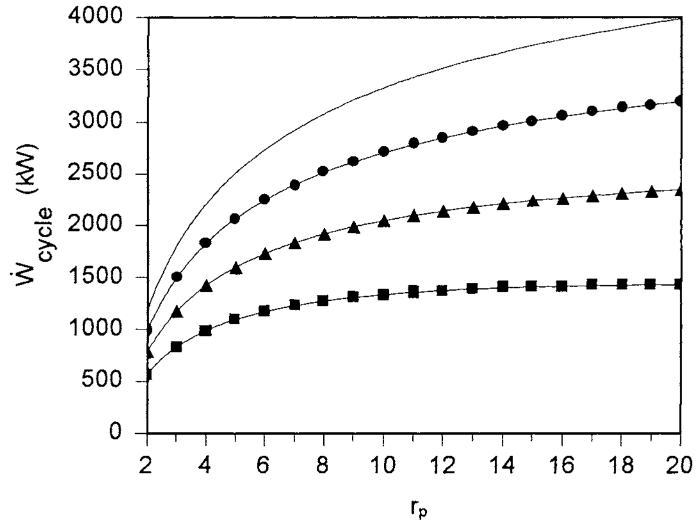
IT Results for $r_p = 8$,
$\eta_{c1} = \eta_{c2} = \eta_{t1} = \eta_{t2} = 80\%$

h_1 = 300 kJ/kg
h_2 = 430.3 kJ/kg
h_{2s} = 404.2 kJ/kg
h_3 = 1514 kJ/kg
h_4 = 1219 kJ/kg
h_{4s} = 1145 kJ/kg
h_a = 1219 kJ/kg
h_{as} = 1145 kJ/kg
h_b = 1514 kJ/kg
h_c = 430.3 kJ/kg
h_{cs} = 404.2 kJ/kg
h_d = 300 kJ/kg
h_x = 1061 kJ/kg
h_y = 588 kJ/kg
η = 0.4413
bwr = 0.4409
$\dot{W}_{cycle}$ =1918 kW

PLOTS:





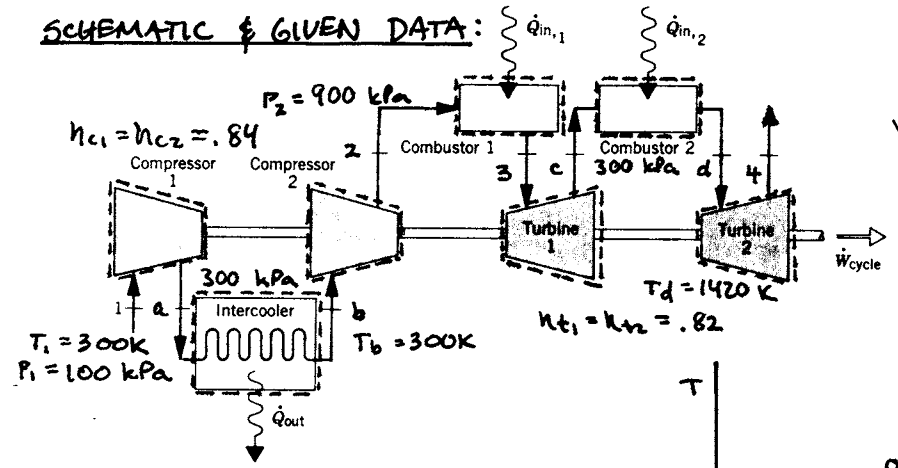


PROBLEM 9.73*

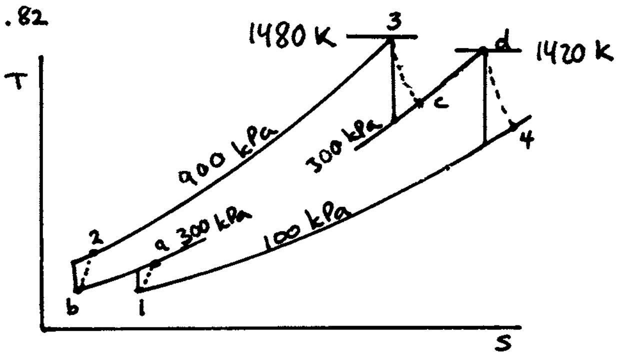
KNOWN: A gas turbine has two stages of compression with intercooling and a two-stage turbine with reheat. Data are known at various locations. Turbine and compressor stage efficiencies are known.

FIND: Determine (a) the volumetric flow rates at 1 and b, (2) the thermal efficiency, (c) the back work ratio.

SCHEMATIC & GIVEN DATA:



$\dot{W}_{cycle} = 1800$ kW



ASSUMPTIONS: Same as in Example 9.11

ANALYSIS: First, fix each of the principal states. (Table A-22)

State 1 $T_1 = 300$K $\Rightarrow h_1 = 300.19$ kJ/kg, $p_{r1} = 1.3860$

State a For isentropic compression, $p_{ras} = (300/100)\,1.3860 = 4.158$. Thus $h_{as} = 411.26$ kJ/kg. Using the compressor efficiency

$$\eta_{c1} = \frac{h_{as} - h_1}{h_a - h_1} \Rightarrow h_a = h_1 + \left(\frac{h_{as} - h_1}{\eta_c}\right) = 432.42 \text{ kJ/kg}$$

State b $h_b = 300.19$ kJ/kg, $p_{rb} = 1.3860$

State 2 $p_{r2s} = (900/300)(1.3860) = 4.158 \Rightarrow h_{2s} = 411.26$ kJ/kg

Thus, with $\eta_{c2} = .84$, $h_2 = h_b + \left(\frac{h_{2s} - h_b}{\eta_c}\right) = 432.42$ kJ/kg

State 3 $T_3 = 1480$ K $\Rightarrow h_3 = 1611.79$ kJ/kg, $p_{r3} = 568.8$

State c For isentropic expansion, $p_{rcs} = (p_c/p_3)p_{r3} = 189.60 \Rightarrow h_{cs} = 1201.5\ \frac{\text{kJ}}{\text{kg}}$

Using the turbine stage efficiency,

$$\eta_{t1} = \frac{h_3 - h_c}{h_3 - h_{cs}} \Rightarrow h_c = h_3 - \eta_{t1}(h_3 - h_{cs}) = 1275.4 \text{ kJ/kg}$$

State d $T_d = 1420$ K $\Rightarrow h_d = 1539.44$, $p_{rd} = 478.0$

State 4 $p_{r4s} = (100/300)(478.0) = 159.33 \Rightarrow h_{4s} = 1145.94$ kJ/kg

Thus, $h_4 = h_d - \eta_{t2}(h_d - h_{4s}) = 1216.77$ kJ/kg

(a) To determine the volumetric flow rate, first find $\dot{m}$ using

$$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_3 - h_c) + (h_d - h_4) - (h_a - h_1) - (h_2 - h_b) = 394.6 \text{ kJ/kg}$$

$$\Rightarrow \quad \dot{m} = \frac{(1800 \text{ kW})}{(394.6 \text{ kJ/kg})}\left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| = 4.562\ \frac{\text{kg}}{\text{s}}$$

Then, $$(AV)_1 = \dot{m}\left(\frac{RT_1}{P_1}\right) = \left(4.562\frac{\text{kg}}{\text{s}}\right)\left(\frac{8314}{28.97}\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)\left(\frac{300 \text{ K}}{10^5 \text{ N/m}^2}\right) = 3.93 \text{ m}^3\text{/s} \longleftarrow (AV)_1$$

PROBLEM 9.73(Contd.)

and

$$(AV)_b = \dot{m}\frac{RT_b}{p_b} = (4.562)\left(\frac{8314}{28.97}\right)\left(\frac{300}{3\times10^5}\right) = 1.31\ \frac{m^3}{s} \longleftarrow \underline{\qquad\qquad (AV)_b}$$

(b) The thermal efficiency is obtained as follows

The total heat added is

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_2) + (h_d - h_c) = 1443.41\ kJ/kg$$

Thus

$$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{394.6}{1443.41} = 0.2734\ (27.34\,\%) \longleftarrow \underline{\qquad\qquad \eta}$$

(c) The back work ratio is

$$bwr = \frac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{(h_a - h_1) + (h_2 - h_b)}{(h_3 - h_c) + (h_d - h_4)}$$

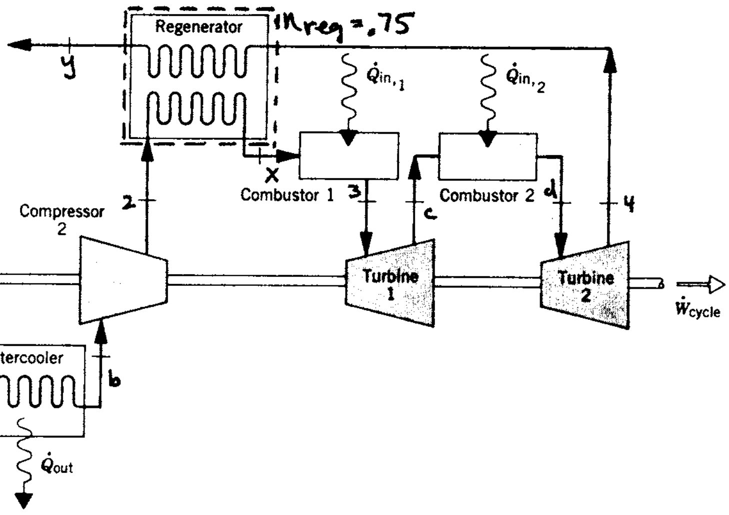
$$= \frac{264.46}{659.06} = 0.401 \longleftarrow \underline{\qquad\qquad bwr}$$

PROBLEM 9.74*

KNOWN: The gas turbine of Problem 9.73 is modified to include a regenerator with effectiveness 75%.

FIND: Answer the same questions for the modified gas turbine as in Problem 9.73.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as in Problem 9.73. Also, there is no pressure drop across the regenerator.

ANALYSIS: From the solution to Problem 9.73.

$h_1 = h_b = 300.19$ kJ/kg $\qquad h_c = 1275.4$ kJ/kg

$h_a = 432.42$ kJ/kg $\qquad h_d = 1539.44$ kJ/kg

$h_2 = 432.42$ kJ/kg $\qquad h_4 = 1216.77$ kJ/kg

$h_3 = 1611.79$ kJ/kg

State x Using the regenerator effectiveness

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{reg}(h_4 - h_2) + h_2 = 1020.68 \text{ kJ/kg}$$

(a) The net work of the gas turbine is unchanged. Thus

$\dot{W}_{cycle}/\dot{m} = 394.6$ kJ/kg

Since the states at 1 and b remain the same, the volumetric flow rates also remain unchanged: $(AV)_1 = 3.93 \text{ m}^3/\text{s}$, $(AV)_b = 1.31 \text{ m}^3/\text{s}$. ← (AV)

(b) However, the heat addition becomes

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_x) + (h_d - h_c) = 855.15 \text{ kJ/kg}$$

① Thus

$$\eta = \frac{394.6}{855.15} = 0.461 \; (46.1\%) \longleftarrow \eta$$

(c) The back work ratio also is unchanged.

$$\text{bwr} = 0.411 \longleftarrow \text{bwr}$$

---

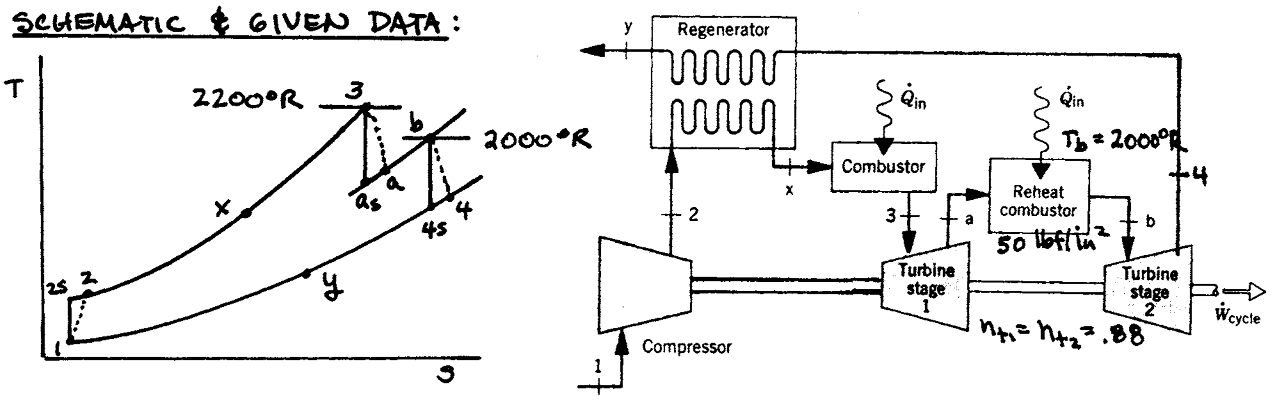
1. As a result of the introduction of the regenerator, there is a significant increase in thermal efficiency.

PROBLEM 9.75

KNOWN: Steady-state operating data are provided in Problem 9.58 (c) for a regenerative gas turbine cycle.

FIND: For each of three modifications of the cycle, determine the thermal efficiency and net power developed.

(a) SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as Example 9.7, except $\eta_{t_1} = \eta_{t_2} = .88$ and $\eta_c = .84$. Also, reheat occurs with no pressure drop.

ANALYSIS: From the solution of Problem 9.58, $h_1 = 126.66$ Btu/lb, $h_2 = 267.16$ Btu/lb, $h_3 = 560.59$ Btu/lb, $p_{r3} = 256.6$. For turbine stage 1,

$$p_{r a_s} = (p_b/p_3)\,p_{r_3} = (50/147)(256.6) = 87.279 \Rightarrow h_{as} = 417.68 \text{ Btu/lb}$$

Thus $$\eta_{t_1} = \frac{h_3 - h_a}{h_3 - h_{as}} \Rightarrow h_a = h_3 - \eta_{t_1}(h_3 - h_{as}) = 434.83 \text{ Btu/lb}$$

Similarly, for turbine state 2; $p_{rb} = 174.0$ and $p_{r4s} = 51.156 \Rightarrow h_{4s} = 360.14 \frac{\text{Btu}}{\text{lb}}$
With $h_b = 504.71$ Btu/lb, $h_4 = h_b - \eta_{t_2}(h_b - h_{4s}) = 377.49$ Btu/lb

The specific enthalpy h_x is calculated from

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{reg}(h_4 - h_2) + h_2 = 355.42 \text{ Btu/lb}$$

The thermal efficiency is found from

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_3 - h_x) + (h_b - h_a) = 275.05 \text{ Btu/lb}$$

and $$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_3 - h_a) + (h_b - h_4) - (h_2 - h_1) = 112.48 \text{ Btu/lb}$$

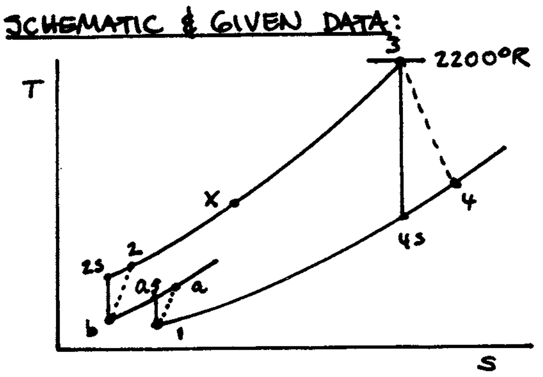
Thus $$\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = 0.409 \;(40.9\%) \leftarrow \qquad \eta$$

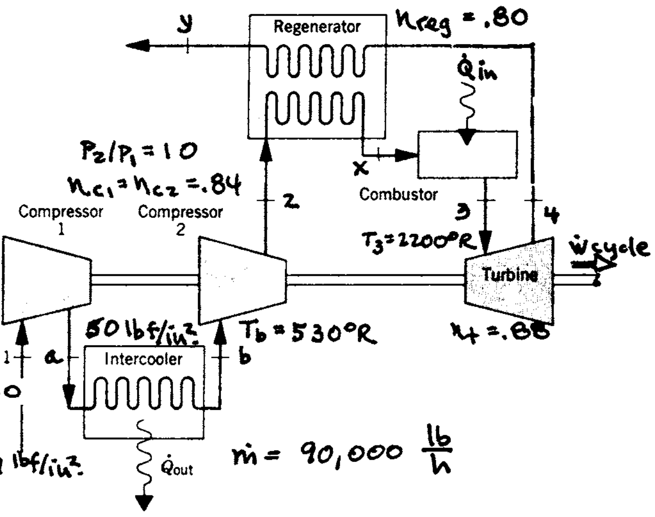
From Problem 9.58, $\dot{m} = 90{,}000$ lb/h. Thus, the net power developed is

$$\dot{W}_{cycle} = \left(90{,}000 \frac{\text{lb}}{\text{h}}\right)(112.48)\frac{\text{Btu}}{\text{lb}}\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$
$$= 3978 \text{ hp} \leftarrow \qquad \dot{W}_{cycle}$$

PROBLEM 9.75 (Contd)

(b) SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Examples 9.7 and 9.9, except for the turbine and compressor efficiencies. $T_1 = 530$ $P_1 = 14.7$ lbf/in².

ANALYSIS: States 1, 3, and 4 are the same as in Problem 9.58. Thus $h_1 = h_b = 126.66$, $h_3 = 560.59$, and $h_4 = 328.4$ Btu/lb.

For the first compressor stage, $p_{ras} = (p_a/p_1)\,p_{r1} = (50/14.7)(1.2997) = 4.4207$ and $h_{as} = 179.90$ Btu/lb. Thus, with the compressor stage efficiency

$$\eta_{c1} = \frac{h_{as} - h_1}{h_a - h_1} \Rightarrow h_a = h_1 + \left(\frac{h_{as} - h_1}{\eta_{c1}}\right) = 190.04 \text{ Btu/lb}$$

Similarly, for stage 2; $p_{r2s} = (p_2/p_b)\,p_{rb} = 3.8211 \Rightarrow h_{2s} = 172.58$ Btu/lb ✓ and

$$h_2 = h_b + \left(\frac{h_{2s} - h_b}{\eta_{c2}}\right) = 181.33 \text{ Btu/lb}$$

For state x use the regenerator effectiveness

$$\eta_{reg} = \frac{h_x - h_2}{h_4 - h_2} \Rightarrow h_x = \eta_{reg}(h_4 - h_2) + h_2 = 298.99 \text{ Btu/lb}$$

To find the thermal efficiency

$$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_3 - h_4) - (h_a - h_1) - (h_2 - h_b) = 114.14 \text{ Btu/lb}$$

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_x = 261.6 \text{ Btu/lb}$$

Thus, $\eta = \dfrac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \dfrac{114.14}{261.6} \doteq 0.436$ (43.6%) ⟵ η

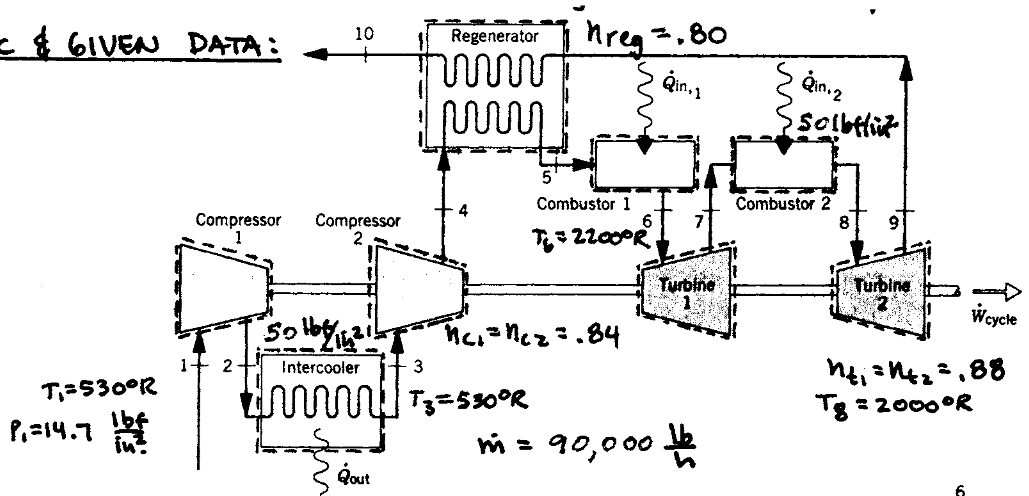
(b) The net power developed is

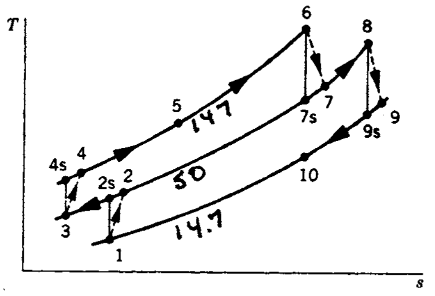
$$\dot{W}_{cycle} = \dot{m}\left(\frac{\dot{W}_{cycle}}{\dot{m}}\right) = \left(90{,}000 \frac{lb}{h}\right)\left(114.14 \frac{Btu}{lb}\right)\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 4036 \text{ hp}$$ ⟵ $\dot{W}_{cycle}$

PROBLEM 9.75 (Cont'd - page 3)

(c) SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 9.11

ANALYSIS: Fix each of the principal states.

State 1 $T_1 = 530°R \Rightarrow h_1 = 126.66$ Btu/lb, $p_{r1} = 1.2997$

State 2 For isentropic compression, $p_{r2} = 4.4207$ and $h_{2s} = 179.90$ Btu/lb. With the compressor efficiency

$$\eta_{c1} = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_{c1}}\right) = 190.04 \text{ Btu/lb}$$

State 3 $T_3 = T_1 \Rightarrow h_3 = 126.66$ Btu/lb, $p_{r3} = 1.2997$

State 4 $p_{r4s} = (p_4/p_3)\, p_{r3} = 3.8211 \Rightarrow h_{4s} = 172.58$ Btu/lb. With the compressor efficiency, $h_4 = h_3 + \left(\frac{h_{4s} - h_3}{\eta_{c2}}\right) = 181.33$ Btu/lb

State 6 $T_6 = 2200°R \Rightarrow h_6 = 560.59$ Btu/lb, $p_{r6} = 256.6$. For turbine stage 1 $p_{r7s} = (p_7/p_6)\, p_{r6} = 87.279 \Rightarrow h_{7s} = 417.68$ Btu/lb. With the turbine efficiency

State 7 $$\eta_{t1} = \frac{h_6 - h_7}{h_6 - h_{7s}} \Rightarrow h_7 = h_6 - \eta_{t1}(h_6 - h_{7s}) = 434.83 \text{ Btu/lb}$$

State 8 $T_8 = 2000°R \Rightarrow h_8 = 504.71$ Btu/lb, $p_{r8} = 174.0$

State 9 $p_{r9s} = 51.156 \Rightarrow h_{9s} = 360.14$ Btu/lb

$$h_9 = h_8 - \eta_{t2}(h_8 - h_{9s}) = 377.49 \text{ Btu/lb}$$

State 5 For the regenerator

$$\eta_{reg} = \frac{h_5 - h_4}{h_9 - h_4} \Rightarrow h_5 = \eta_{reg}(h_9 - h_4) + h_4 = 338.26 \text{ Btu/lb}$$

(a) The net work per unit mass of air flowing is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = (h_6 - h_7) + (h_8 - h_9) - (h_2 - h_1) - (h_4 - h_3) = 134.93 \text{ Btu/lb}$$

and $$\frac{\dot{Q}_{in}}{\dot{m}} = (h_6 - h_5) + (h_8 - h_7) = 292.27 \text{ Btu/lb}$$

$$\Rightarrow \eta = \frac{134.93}{292.27} = 0.462 \quad (46.2\%) \longleftarrow \eta$$

(b) The net power developed is

$$\dot{W}_{cycle} = \left(90{,}000 \frac{\text{lb}}{\text{h}}\right)\left(134.93 \frac{\text{Btu}}{\text{lb}}\right)\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

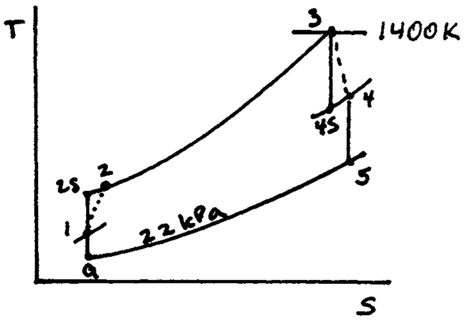
$$= 4772 \text{ hp} \longleftarrow \dot{W}_{cycle}$$

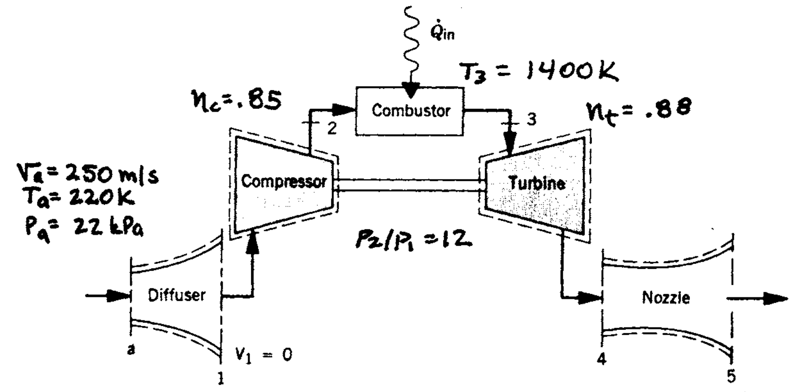
PROBLEM 9.76

KNOWN: A turbojet engine is analyzed on an air-standard basis. Data are known at various locations.

FIND: Determine (a) the pressures and temperatures at each principal state and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 9.12, except $\eta_c = 85\%$ and $\eta_t = 88\%$.

ANALYSIS: (a) First, fix each of the principal states (Table A-22).

State a $T_a = 220K$, $p_a = 22$ kPa $\Rightarrow h_a = 219.97$ kJ/kg, $p_{ra} = 0.4690$

State 1 An energy balance for the diffuser gives; $0 = h_a + \frac{V_a^2}{2} - h_1$. Thus

$$h_1 = h_a + \frac{V_a^2}{2} = 219.97\ \frac{\text{kJ}}{\text{kg}} + \frac{250^2\ \text{m}^2/\text{s}^2}{2}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$
$$= 251.22\ \text{kJ/kg}$$

Since the process is isentropic, $p_1 = (p_{r1}/p_{ra})\,p_a = 34.97$ kPa

State 2 For isentropic compression, $p_{r2s} = p_{r1}\,(p_2/p_1) = 8.9453$ and $h_{2s} = 511.9$ kJ/kg. With the compressor efficiency

$$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 557.9\ \text{kJ/kg}\ ,\quad p_2 = 12 \cdot p_1 = 419.64\ \text{kPa}$$

State 3 $T_3 = 1400K$, $p_3 = p_2 \Rightarrow h_3 = 1515.42$ kJ/kg, $p_{r3} = 450.5$

State 4 For a turbojet, $\dot{W}_c/\dot{m} = \dot{W}_t/\dot{m}$. Thus

$$(h_2 - h_1) = \eta_t\,(h_3 - h_{4s}) \Rightarrow h_{4s} = 1166.92\ \text{kJ/kg}$$

Thus, $p_{r4s} = 170.28$ and $p_4 = p_3\,(p_{r4s}/p_{r3}) = 158.62$ kPa

and $h_4 = h_3 - \eta_t\,(h_3 - h_{4s}) = 1208.74$ kJ/kg $\Rightarrow p_{r4} = 193.81$

State 5 $p_{r5} = p_{r4}\,(p_5/p_4) = 26.881$; $h_5 = 699.48$ kJ/kg

(b) An energy balance on the nozzle gives

$$0 = h_4 - \left(h_5 + \frac{V_5^2}{2}\right)$$

or

$$V_5 = \sqrt{2(h_4 - h_5)}$$
$$= \sqrt{2(1208.74 - 699.48)\frac{\text{kJ}}{\text{kg}}\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|}$$
$$= 1009\ \text{m/s} \longleftarrow V_5$$

PROBLEM 9.77

See Problem 9.76 for sample calculations. The data for the required plots are obtained using IT, as follows:

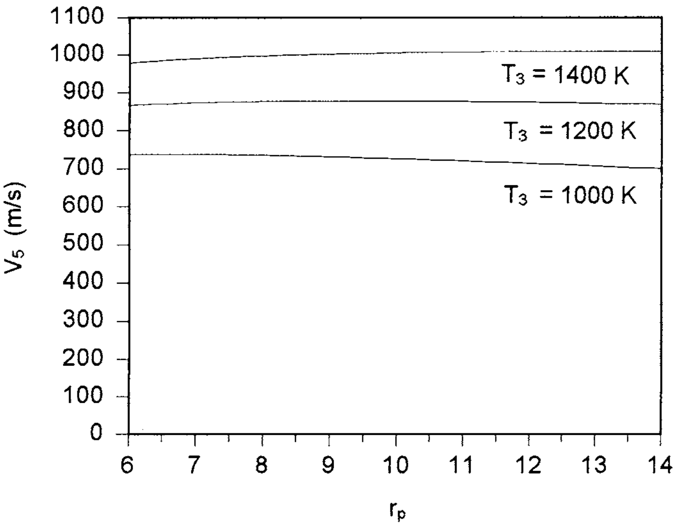
IT code

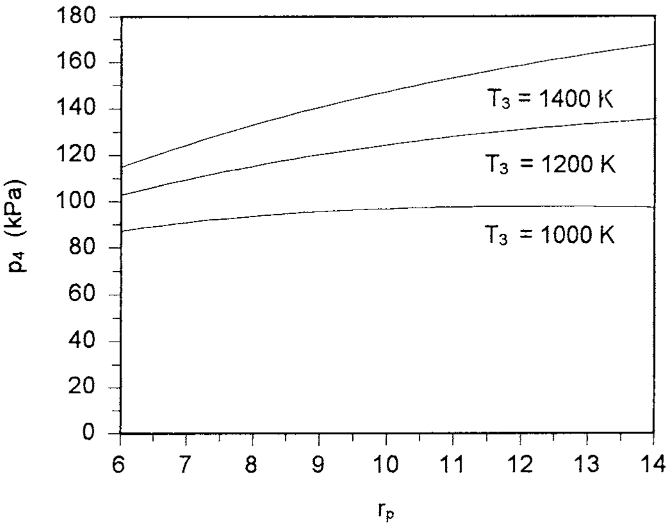
```
pa = 22   // kPa
Ta = 220   // K
Va = 250   // m/s
rp = 12
T3 = 1400  // K
p5 = 22  // kPa
eta_t = 0.88
eta_c = 0.85
mdot = 1   // assume a mass
// flow rate of 1 kg/s

ha = h_T("Air", Ta)
sa = s_TP("Air", Ta, pa)
h1 = ha + (Va^2 / 2) * (1 / 1000)
s1 = s_hP("Air", h1, p1)
s1 = sa
h1 = h_T("Air", T1)
p2 = p1 * rp
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / eta_c
h2 = h_T("Air", T2)
p3 = p2
h3 = h_T("Air", T3)
s3 = s_hP("Air",h3,p3)
s4s = s3
h2 - h1 = eta_t * (h3 - h4s)
s4s = s_hP("Air", h4s, p4)
h4 = h3 - (h3 - h4s) * eta_t
h4 = h_T("Air", T4)
s4 = s_hP("Air", h4, p4):
s5 = s4
s5 = s_hP("Air", h5, p5)
h5 = h_T("Air", T5)

h4 = h5 + (V5^2 / 2) * (1 / 1000)
Qdotin / mdot =  (h3 - h2)
```

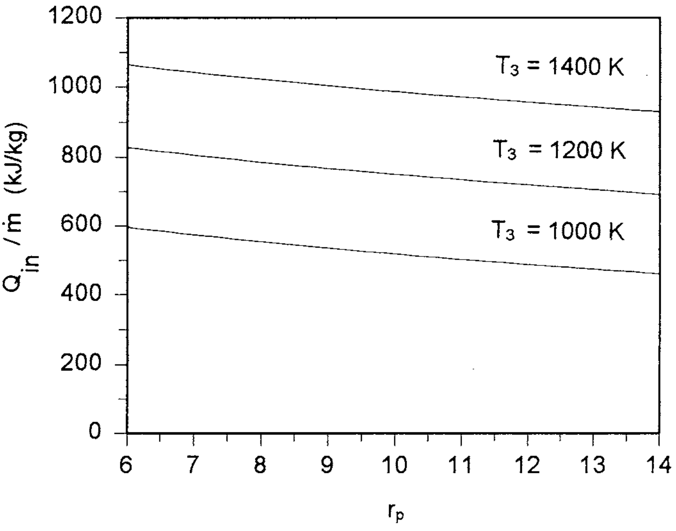
IT Results for $r_p = 12$, $T_3 = 1400$ K

$T_1 = 251.1$ K
$h_1 = 251$ kJ/kg
$h_{2s} = 511.6$ kJ/kg
$T_2 = 552.8$ K
$p_2 = 419.4$ kPa
$h_2 = 557.5$ kJ/kg
$h_3 = 1514$ kJ/kg
$T_4 = 1140$ K
$h_{4s} = 1166$ kJ/kg
$h_4 = 1208$ kJ/kg
$T_5 = 687$ K
$h_5 = 699.1$ kJ/kg
$p_4 = 158.5$ kPa
$V_5 = 1008$ m/s
$\dot{Q}_{in} / \dot{m} = 956.6$ kJ/kg

PLOTS:







Discussion.

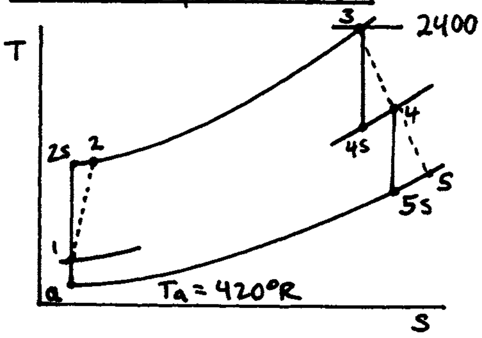
- Generally, as T_3 is increased there is greater heat input, resulting in increased V_5 and p_4.
- As compressor pressure ratio is increased at fixed T_3, less heat input is required, but V_5 changes very little.

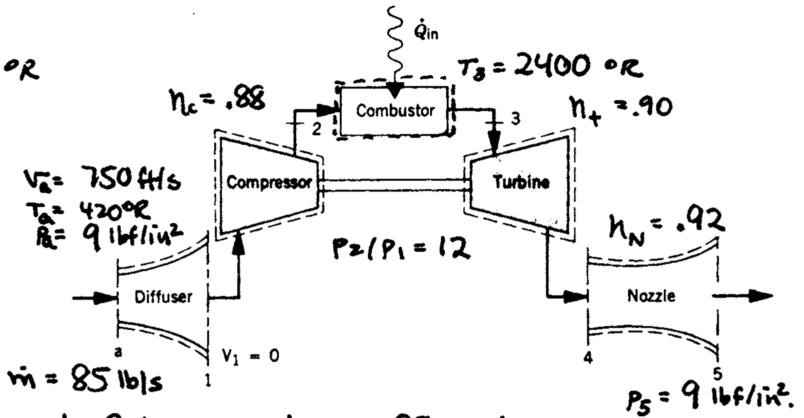
PROBLEM 9.78*

KNOWN: A turbojet engine is analyzed on an air-standard basis. Data are know at various locations.

FIND: Determine (a) the rate of heat addition, (b) the pressure at the turbine exit, (c) the compressor power input, (d) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 9.12, except $\eta_c = .88$ and $\eta_t = .90$.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State a $T_a = 420°R \Rightarrow h_a = 100.32$ Btu/lb, $p_{ra} = 0.5760$

State 1 For the diffuser, $0 = (h_a + V_a^2/2) - h_1$. Thus

$$h_1 = 100.32 \frac{\text{Btu}}{\text{lb}} + \frac{750^2\ \text{ft}^2/\text{s}^2}{2} \left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 111.55 \frac{\text{Btu}}{\text{lb}}$$

$$p_1 = \left(\frac{p_{r1}}{p_{ra}}\right) p_a = \left(\frac{.83492}{.5760}\right) 9 = 13.05\ \text{lbf/in}^2.$$

State 2 For isentropic compression, $p_{r2s} = p_{r1}(p_2/p_1) = 10.019$ and $h_{2s} = 227.29$ Btu/lb. With the compressor efficiency

$$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 243.07\ \text{Btu/lb}, \quad p_2 = 12 p_1 = 156.6\ \text{lbf/in}^2.$$

State 3 $T_3 = 2400\ °R \Rightarrow h_3 = 617.22$ Btu/lb, $p_{r3} = 367.6$

State 4 For a turbojet, $\dot{W}_c/\dot{m} = \dot{W}_t/\dot{m}$. Thus $h_2 - h_1 = \eta_t (h_3 - h_{4s})$.

$$h_{4s} = h_3 - (h_2 - h_1)/\eta_t = 471.09\ \text{Btu/lb} \Rightarrow p_{r4s} = 135.24$$

Thus, $p_4 = (p_{r4s}/p_{r3}) p_3 = 57.61\ \text{lbf/in}^2.$ ← (b)

Using the turbine efficiency, $h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 485.70$ Btu/lb

State 5s $p_{r5} = p_{r4}(p_5/p_4) = (151.24)(9/57.61) = 23.627 \Rightarrow h_{5s} = 289.93 \frac{\text{Btu}}{\text{lb}}$

(a) The rate of heat addition is

$$\dot{Q}_{in} = \dot{m}(h_3 - h_2) = (85 \tfrac{\text{lb}}{\text{s}})(617.22 - 243.07)\ \text{Btu/lb} \left|\frac{3600\ \text{s}}{1\ \text{h}}\right| = 1.145 \times 10^8 \frac{\text{Btu}}{\text{h}} \leftarrow \text{(a)}$$

(c) The compressor power is $\dot{W}_c = \dot{m}(h_2 - h_1) = 85(243.07 - 111.55)|3600| = 4.025 \times 10^7 \frac{\text{Btu}}{\text{h}}$ ← (c)

(d) An energy rate balance for an isentropic expansion gives

$$(V_5^2)_{s} = 2(h_4 - h_{5s}) = 2(485.70 - 289.93) \frac{\text{Btu}}{\text{lb}} \left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right|\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right| = 9.809 \times 10^6 \frac{\text{ft}^2}{\text{s}^2}$$

Then, with the nozzle efficiency, Eq. 6.49

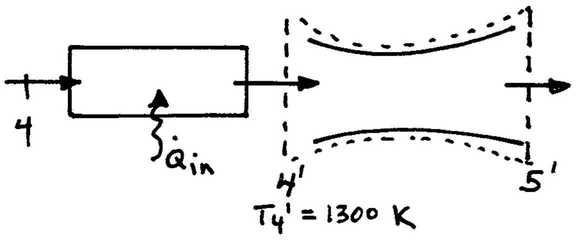
$$V_5 = (V_5)_s \sqrt{\eta_N} = 3004.0\ \text{ft/s}$$ ← (d)

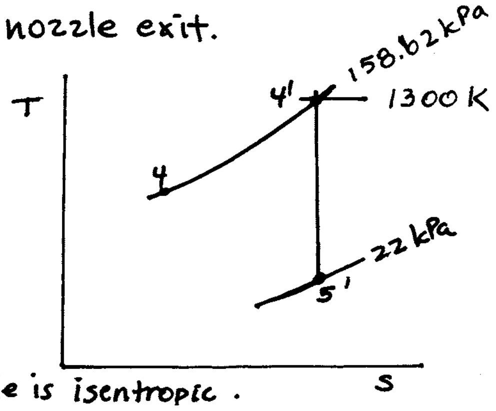
PROBLEM 9.79

KNOWN: An afterburner is added to the turbojet in Problem 9.76

FIND: Determine the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Expansion through the nozzle is isentropic.

ANALYSIS: For the isentropic process

$$p_{r5'} = (p_5/p_4)\, p_{r4'} = (22/158.62)(330.9) = 45.895$$

$$h_{5'} = 812.71 \text{ kJ/kg}$$

Thus, the nozzle exit velocity is

$$V_5 = \sqrt{2(h_{4'} - h_{5'})}$$

① $$= \sqrt{2(1395.97 - 812.71)\,\frac{\text{kJ}}{\text{kg}} \left|\frac{10^3\,\text{N}\cdot\text{m}}{1\,\text{kJ}}\right| \left|\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right|} = 1080\,\text{m/s} \longleftarrow$$

---

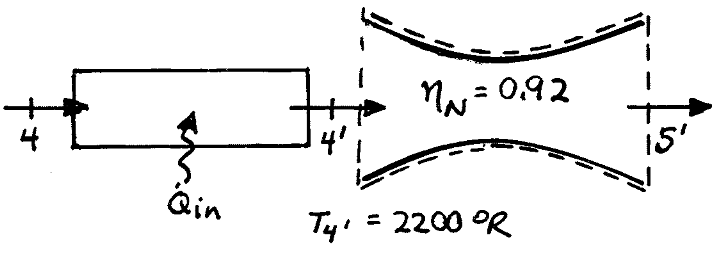
1. Comparing this result with that of Problem 9.76, we see that the use of an afterburner increases the exit velocity, and thus the engine thrust.

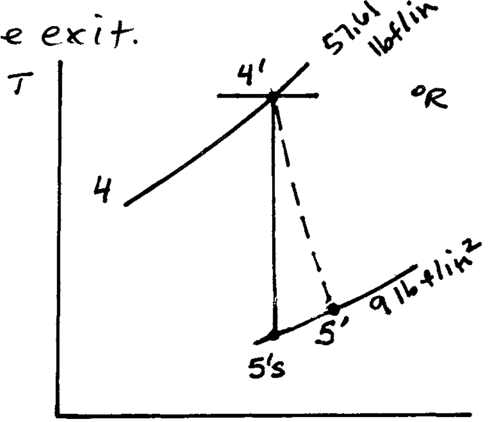
PROBLEM 9.80*

KNOWN: An afterburner is added to the turbojet in Problem 9.78

FIND: Determine the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:





ASSUMPTION: Nozzle operation is adiabatic with $\eta_N = 0.92$.

ANALYSIS: For ideal expansion through the nozzle

$$p_{r5s'} = (p_5/p_4)\, p_{r4'}$$

From Table A-22E; $p_{r4'} = 256.6$. Thus

$$p_{r5s'} = 40.087 \Rightarrow h_{5s'} = 336.39 \text{ Btu/lb}$$

In the ideal case

$$0 = h_{4'} - \left(h_{5s'} + \left(\frac{V_5^2}{2}\right)_s\right)$$

$$\left(\frac{V_5^2}{2}\right)_s = h_{4'} - h_{5s'}$$

or

$$(V_5)_s = \sqrt{2(h_{4'} - h_{5s'})}$$

$$= \sqrt{2(560.59 - 336.39)\,\frac{\text{Btu}}{\text{lb}}\left|\frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}}\right|\left|\frac{32.2 \text{ lb}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right|}$$

$$= 3352 \text{ ft/s}$$

Using the nozzle efficiency

$$\frac{V_5^2}{2} = \eta_N\left(\frac{V_5^2}{2}\right)_s$$

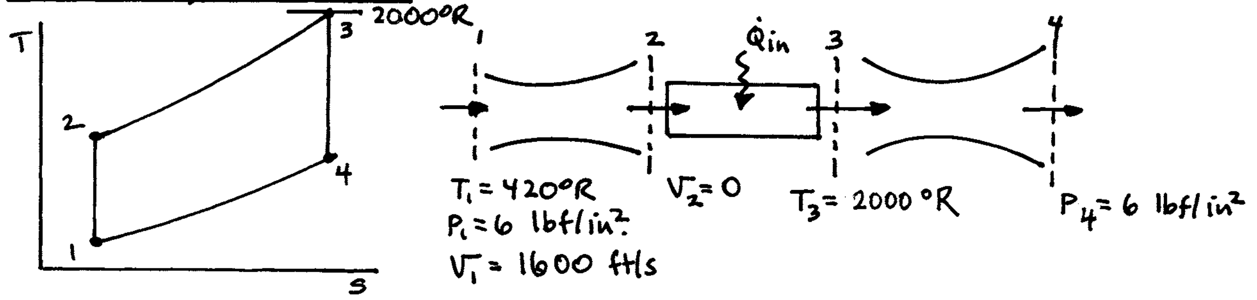
$$V_5 = \sqrt{\eta_N}\,(V_5)_s = 3215 \text{ ft/s} \longleftarrow \underline{\hspace{6cm}} \; V_5$$

PROBLEM 9.81

KNOWN: Air enters the diffuser of a ramjet with known conditions. The temperature after combustion is specified.

FIND: Using an air-standard analysis, determine (a) the pressure at the diffuser exit and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The combustion process is modeled as a constant pressure heat addition. (3) The diffuser and nozzle operate isentropically. (4) Neglect kinetic energy at locations 2 and 3. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: (a) For the diffuser; $0 = \left(h_1 + \frac{V_1^2}{2}\right) - h_2$. Thus,

$$h_2 = h_1 + \frac{V_1^2}{2} = 100.32 + \frac{1600^2 \text{ ft}^2/\text{s}^2}{2}\left|\frac{1 \text{ lbf}}{32.2 \text{ lb}\cdot\text{ft/s}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$

$$= 151.41 \text{ Btu/lb}$$

Thus, $p_{r2} = 2.4214$ and $P_2 = \left(p_{r2}/p_{r1}\right) P_1 = 25.22$ lbf/in.² ⟵ P_2

(b) At $T_3 = 2000°R$, Table A-22E gives $h_3 = 504.71$ Btu/lb, $p_{r3} = 174$. For an isentropic expansion through the nozzle

$$p_{r4} = p_{r3}(P_4/P_3) = (174)(6/25.22) = 41.4$$

$$h_4 = 339.45 \text{ Btu/lb}$$

The velocity is found from

$$0 = h_3 - \left(h_4 + \frac{V_4^2}{2}\right)$$

or

$$V_4 = \sqrt{2(h_3 - h_4)}$$

$$= \sqrt{2\,(504.71 - 339.45)\,\frac{\text{Btu}}{\text{lb}}\left|\frac{32.2 \text{ lb}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right|\left|\frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}}\right|}$$

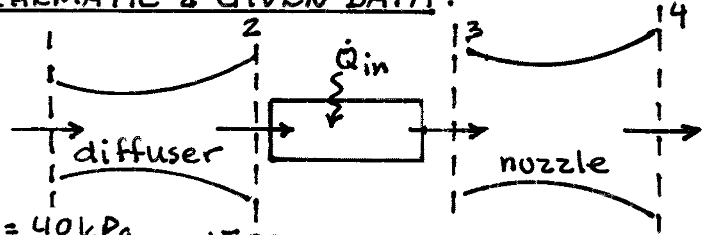
$$= 2878 \text{ ft/s}$$ ⟵ V_4

PROBLEM 9.82*

KNOWN: Air enters the diffuser of a ramjet with known conditions. The heat addition per unit mass of air flowing is specified.

FIND: Using air-standard analysis, determine (a) the pressure at the diffuser exit and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:

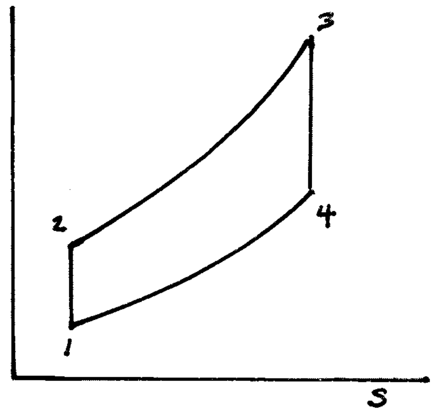


$p_1 = 40\ \text{kPa}$
$T_1 = 240\ \text{K}$
$V_1 = 2500\ \text{km/h}$
$= 694.4\ \text{m/s}$

$V_2 = 0$

$\frac{\dot{Q}_{in}}{\dot{m}} = 1080\ \text{kJ/kg}$

$p_4 = 40\ \text{kPa}$



ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The combustion process is modeled as a constant pressure heat addition. (3) The diffuser and nozzle operate isentropically. (4) Neglect kinetic energy at locations 2 & 3 and neglect potential energy throughout. (5) The working fluid is air modeled as an ideal gas.

ANALYSIS: (a) For the diffuser; $0 = (h_1 + \frac{V_1^2}{2}) - h_2$

Thus $$h_2 = h_1 + \frac{V_1^2}{2} = 240.02\ \frac{\text{kJ}}{\text{kg}} + \frac{(694.4)^2}{2}\ \frac{\text{m}^2}{\text{s}^2}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 481.1\ \text{kJ/kg}$$

Interpolating in Table A-22; $T_2 \approx 479\ \text{K}$ and $p_{r2} = 7.197$

Therefore $$p_2 = \left(\frac{p_{r2}}{p_{r1}}\right) p_1 = \left(\frac{7.197}{0.6355}\right)(40\ \text{kPa}) = 453.0\ \text{kPa} \longleftarrow p_2$$

(b) State 3 is fixed by considering the heat addition process

$$0 = \dot{Q}_{in} + \dot{m}(h_2 - h_3)$$

or $$h_3 = \frac{\dot{Q}_{in}}{\dot{m}} + h_2 = 1080 + 481.1 = 1561.1\ \text{kJ/kg}$$

Again, from Table A-22; $T_3 \approx 1438\ \text{K}$, $p_{r3} = 504.0$

For isentropic expansion through the nozzle

$$p_{r4} = (p_4/p_3)\, p_{r3} = (40/453.0)(504.0) = 44.50 \Rightarrow h_4 = 805.78\ \frac{\text{kJ}}{\text{kg}}$$

The velocity is obtained from; $0 = h_3 - (h_4 + \frac{V_4^2}{2})$

or $$V_4 = \sqrt{2(h_3 - h_4)}$$

$$= \sqrt{2(1561.1 - 805.78)\ \frac{\text{kJ}}{\text{kg}}\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|}$$

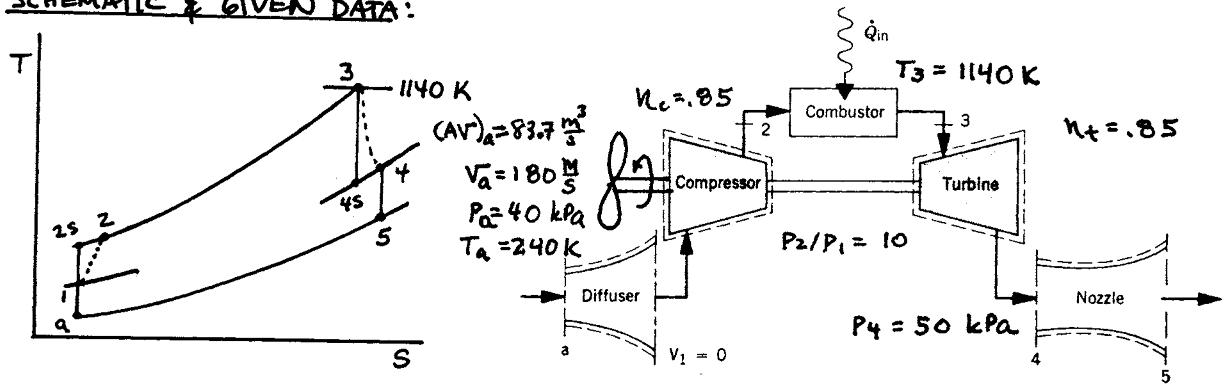
$$= 1229\ \text{m/s} \longleftarrow V_4$$

PROBLEM 9.83

KNOWN: A turboprop engine is analyzed on an air-standard basis. Data are known at various locations.

FIND: Determine (a) the power delivered to the propeller, and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as in Example 9.12, except $\eta_c = \eta_t = 0.85$.

ANALYSIS: First, fix each of the principal states (Table A-22).

State a $T_a = 240\text{ K} \Rightarrow h_a = 240.02\text{ kJ/kg}$, $p_{ra} = 0.6355$

State 1 An energy balance for the diffuser gives; $0 = (h_a + \frac{V_a^2}{2}) - h_1$. Thus

$$h_1 = h_a + V_a^2/2 = 240.02\text{ kJ/kg} + \frac{180^2\text{ m}^2/\text{s}^2}{2}\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= 256.22\text{ kJ/kg} \Rightarrow p_{r1} = 0.79902$$

$$p_1 = (p_{r1}/p_{ra})\,p_a = 50.29\text{ kPa}$$

State 2 For an isentropic compression, $p_{r2s} = (p_2/p_1)\,p_{r1} = 7.9902 \Rightarrow h_{2s} = 495.65\ \frac{\text{kJ}}{\text{kg}}$
Using the compressor efficiency, $h_2 = h_1 + \left(\frac{h_{2s}-h_1}{\eta_c}\right) = 537.9\text{ kJ/kg}$

State 3 $T_3 = 1140 \Rightarrow h_3 = 1207.57\text{ kJ/kg}$, $p_{r3} = 193.1$

State 4 For isentropic expansion, $p_{r4s} = (p_4/p_3)\,p_{r3} = \left(\frac{50}{502.9}\right)193.1 = 19.199$.
Thus, $h_{4s} = 634.06\text{ kJ/kg}$. Using the turbine efficiency,

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 720.09\text{ kJ/kg},\quad p_{r4} = 29.80$$

State 5 $p_{r5} = (p_5/p_4)\,p_{r4} = 23.84 \Rightarrow h_5 = 676.17\text{ kJ/kg}$

(a) The power delivered to the propeller is

$$\dot W_p = \dot W_t - \dot W_c = \dot m\left[(h_3 - h_4) - (h_2 - h_1)\right]$$

where

$$\dot m = \frac{(AV)_a\,p_a}{RT_a} = \frac{(83.7\text{ m}^3/\text{s})(4\times10^4\text{ N/m}^2)}{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(240\text{ K})} = 48.6\ \frac{\text{kg}}{\text{s}}$$

$$\Rightarrow \dot W_p = \left(48.6\ \frac{\text{kg}}{\text{s}}\right)\left[(1207.57 - 720.09) - (537.9 - 256.22)\right]\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ MW}}{10^3\text{ kJ/s}}\right| = 10\text{ MW} \longleftarrow \dot W_p$$

(b) An energy balance for the nozzle gives

$$V_5 = \sqrt{2(h_4 - h_5)}$$

$$= \sqrt{2(720.09 - 676.17)\ \frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right|\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right|}$$

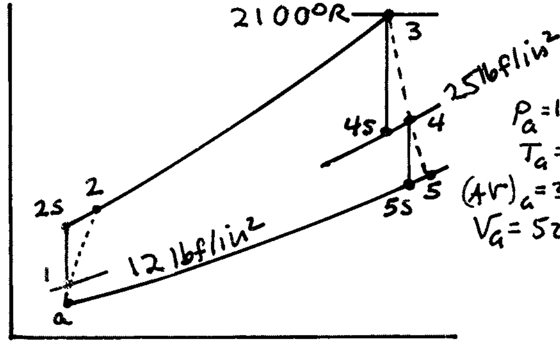
$$= 296.4\text{ m/s} \longleftarrow V_5$$

PROBLEM 9.84*

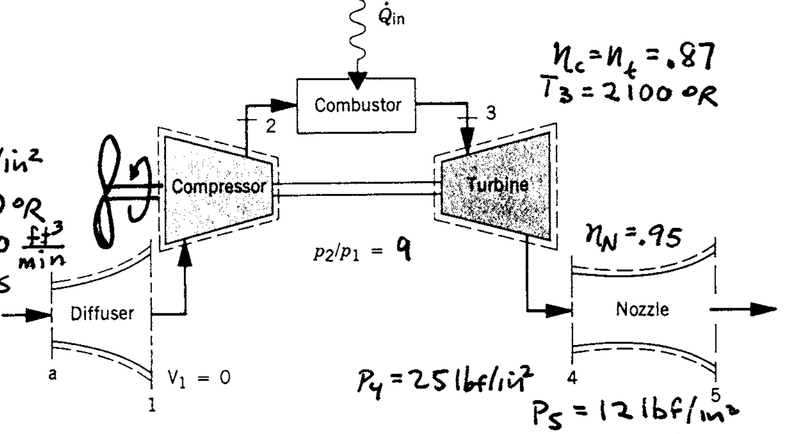
KNOWN: A turboprop engine is analyzed on an air standard basis. Data are known at various locations.

FIND: Determine (a) the power delivered to the propeller, and (b) the velocity at the nozzle exit.

SCHEMATIC & GIVEN DATA:



$P_a = 12\ \text{lbf/in}^2$
$T_a = 460\ °R$
$(AV)_a = 30{,}000\ \frac{ft^3}{min}$
$V_a = 520\ ft/s$



ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The diffuser process is isentropic and the turbine, compressor, and nozzle operate adiabatically. (3) Except at the inlet and exit of the engine, kinetic energy effects can be neglected. Potential energy effects are negligible throughout. (4) The working fluid is air modeled as an ideal gas.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State a: $T_a = 460°R \Rightarrow h_a = 109.90\ \text{Btu/lb},\ p_{ra} = 0.7913$

State 1: An energy balance for the diffuser gives

$$h_1 = h_a + \frac{V_a^2}{2} = 109.90\ \frac{\text{Btu}}{\text{lb}} + \left(\frac{520^2\ ft^2/s^2}{2}\right)\left|\frac{1\ lbf}{32.2\ lb\cdot ft/s^2}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|$$

$$= 115.30\ \text{Btu/lb} \Rightarrow p_{r1} = 0.93604$$

$$p_1 = (p_{r1}/p_{ra})\,p_a = 14.195\ \text{lbf/in}^2$$

State 2: For ideal compression; $p_{r2s} = (p_2/p_1)\,p_{r1} = 8.42436 \Rightarrow h_{2s} = 216.36\ \frac{\text{Btu}}{\text{lb}}$

Using the compressor efficiency

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c = 231.46\ \text{Btu/lb}$$

State 3: $T_3 = 2100\ °R \Rightarrow h_3 = 532.55\ \text{Btu/lb},\ p_{r3} = 212.1$

State 4: For ideal expansion; $p_{r4s} = (p_4/p_3)\,p_{r3} = 41.505 \Rightarrow h_{4s} = 339.7\ \text{Btu/lb}$

Using the turbine efficiency

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 364.77\ \text{Btu/lb} \Rightarrow p_{r4} = 53.515$$

State 5s: $p_{r5s} = (p_5/p_4)\,p_{r4} = 25.687 \Rightarrow h_{5s} = 296.79\ \text{Btu/lb}$

(a) The power delivered to the propeller is

$$\dot{W}_p = \dot{W}_t - \dot{W}_c = \dot{m}\left[(h_3 - h_4) - (h_2 - h_1)\right]$$

Evaluating the mass flow rate

$$\dot{m} = \frac{(AV)_a\,p_a}{RT_a} = \frac{(30{,}000\ ft^3/min)(12\ lbf/in^2)}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(460\ °R)}\left|\frac{144\ in^2}{1\ ft^2}\right| = 2113.1\ \text{lb/min}$$

PROBLEM 9.84 (Cont'd)

Thus, the power is

$$\dot{W}_p = \left(2113.1 \frac{\text{lb}}{\text{min}}\right)\left(\frac{60 \text{ min}}{\text{h}}\right)\left[(532.55-364.77)-(231.46-115.30)\right]\frac{\text{Btu}}{\text{lb}}\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 2571.6 \text{ hp} \longleftarrow\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!- \underline{\dot{W}_p}$$

(b) For an ideal expansion through the nozzle

$$0 = h_4 - \left(h_{5s} + \frac{V_{5s}^2}{2}\right)$$

or

$$V_{5s} = \sqrt{2\,(h_4 - h_{5s})}$$

$$= \sqrt{2\,(364.77-296.79)\,|32.2|\,|778|} = 1845.5 \text{ ft/s}$$

From the definition of nozzle efficiency

$$V_5 = \sqrt{\eta_N}\, V_{5s} = \sqrt{(0.95)}\,(1845.5)$$

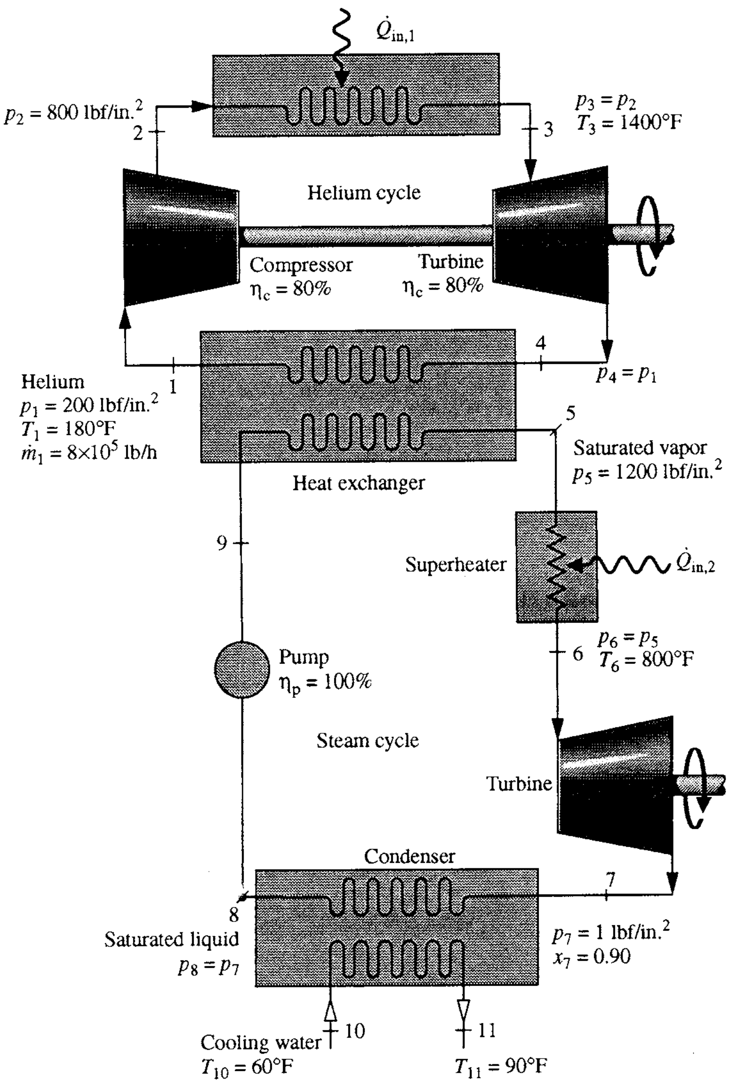
$$= 1798.8 \text{ ft/s} \longleftarrow\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!- \underline{V_5}$$

PROBLEM 9.85

KNOWN: Steady-state operating data are provided for a combined cycle power plant using helium and water as the working fluids.

FIND: Determine (a) the mass flow rates of the steam and cooling water, (b) the net power developed by the gas turbine and vapor cycles, (c) the thermal efficiency of the combined cycle.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. Control volumes enclosing the principal components are at steady state.
2. There are no stray heat transfers, and kinetic and potential energy effects can be ignored.
3. Helium is modeled as an ideal gas with constant $c_p = 5/2 R$, $k = 1.67$ (Table A-2E E)

ANALYSIS: Begin by evaluating data at principal states.

Helium: Using Eq. 6.45

$$T_{2s} = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} = 640K \left(\frac{800}{200}\right)^{\frac{1.67-1}{1.67}}$$

$$= 1116.16°R$$

$$T_{4s} = T_3 \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}} = 1860 \left(\frac{200}{800}\right)^{\frac{1.67-1}{1.67}}$$

$$= 1066.51°R$$

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{\cancel{c_p}(T_3 - T_4)}{\cancel{c_p}(T_3 - T_{4s})}$$

$$\Rightarrow T_4 = T_3 - \eta_t (T_3 - T_{4s})$$

$$= 1860 - 0.8(1860 - 1066.51)$$

$$= 1225.2°R$$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{\cancel{c_p}(T_{2s} - T_1)}{\cancel{c_p}(T_2 - T_1)}$$

$$\Rightarrow T_2 = T_1 + \frac{(T_{2s} - T_1)}{\eta_c}$$

$$= 640 + \frac{(1116.16 - 640)}{0.8}$$

$$= 1235.2°R$$

Steam: Table A-3E, $h_5 = 1183.9$ Btu/lb. Table A-4E, $h_6 = 1378.4$ Btu/lb. Table A-3E, $h_8 = 69.74$ Btu/lb

$h_7 = h_f + x_7 h_{fg} = 69.74 + 0.9(1036) = 1002.14$ Btu/lb.

Also

$$h_9 \approx h_8 + v_8 \Delta p = 69.74 \frac{\text{Btu}}{\text{lb}} + \left(.01614 \frac{\text{ft}^3}{\text{lb}}\right)\left(1199 \times 144 \frac{\text{lbf}}{\text{ft}^2}\right)\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 73.32\ \text{Btu/lb}$$

Cooling Water: $h_{10} \sim h_f(T_{10}) = 28.08$ Btu/lb, $h_{11} \approx h_f(T_{11}) = 58.07$ Btu/lb

(a) Evaluate the mass flow rate of the steam by study of a control volume enclosing the heat exchanger. Mass and energy rate balances reduce to give

$$\dot{m}_s = \frac{\dot{m}_g [h_4 - h_1]}{[h_5 - h_9]} = \frac{\dot{m}_g c_p (T_4 - T_1)}{[h_5 - h_9]}$$

PROBLEM 9.85 (Contd.)

Inserting values, $c_p = 5/2\,R = \frac{5}{2}\left(\frac{1.986}{4.003}\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right) = 1.24\ \text{Btu/lb}\cdot{}^\circ\text{R}$

$$\dot{m}_s = \frac{(8\times10^5\ \text{lb/h})(1.24\ \text{Btu/lb}\cdot{}^\circ\text{R})[1225.2 - 640]}{[1183.9 - 73.32]} = 5.23\times10^5\ \frac{\text{lb}}{\text{h}} \quad \longleftarrow (a)$$

For a control volume enclosing the condenser, mass and energy rate balances reduce to give

$$\dot{m}_{cw} = \frac{\dot{m}_s(h_7 - h_8)}{h_{11} - h_{10}} = \frac{(5.23\times10^5)[1002.14 - 69.74]}{(58.07 - 28.08)} = 1.63\times10^7\ \frac{\text{lb}}{\text{h}} \quad \longleftarrow (a)$$

(b) The net power developed by the gas turbine is

$$\dot{W}_g = \dot{W}_t - \dot{W}_c = \dot{m}_g[(h_3 - h_4) - (h_2 - h_1)] = \dot{m}_g c_p[(T_3 - T_4) - (T_2 - T_1)]$$

$$= (8\times10^5\ \text{lb/h})\left(1.24\ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)[(1860 - 1225.2) - (1235.2 - 640)]\ {}^\circ\text{R} = 3.93\times10^7\ \frac{\text{Btu}}{\text{h}} \quad \longleftarrow (b)$$

The net power developed by the vapor cycle is

$$\dot{W}_s = \dot{W}_t - \dot{W}_p = \dot{m}_s[(h_6 - h_7) - (h_9 - h_8)] = (5.23\times10^5)[(1378.4 - 1002.14) - (73.32 - 69.74)]$$
$$= 19.49\times10^7\ \text{Btu/h} \quad \longleftarrow (b)$$

(c) The thermal efficiency for the combined cycle is

① $$\eta = \frac{\dot{W}_g + \dot{W}_s}{\dot{Q}_{in}} = \frac{\dot{W}_g + \dot{W}_s}{\dot{Q}_{in,1} + \dot{Q}_{in,2}}$$

where

$$\dot{Q}_{in,1} = \dot{m}_g(h_3 - h_2) = \dot{m}_g c_p(T_3 - T_2) = \left(8\times10^5\ \frac{\text{lb}}{\text{h}}\right)\left(1.24\ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)(1860 - 1235.2)\ {}^\circ\text{R}$$
$$= 61.98\times10^7\ \text{Btu/h}$$

$$\dot{Q}_{in,2} = \dot{m}_s(h_6 - h_5) = (5.23\times10^5)(1378.4 - 1183.9) = 10.17\times10^7\ \text{Btu/h}$$

Thus

$$\eta = \frac{(3.93 + 19.49)\times10^7}{(61.98 + 10.17)\times10^7} = 0.324 \quad (32.4\%) \quad \longleftarrow (c)$$

---

1. An energy balance for the overall combined cycle gives

$$\dot{W}_g + \dot{W}_s = \dot{Q}_{in} - \dot{m}_{cw}[h_{11} - h_{10}]$$

Thus.

$$\eta = 1 - \frac{\overbrace{\dot{m}_{cw}[h_{11} - h_{10}]}^{=\dot{m}_s(h_7 - h_8)}}{\dot{Q}_{in}} = 1 - \frac{(48.76\times10^7)}{(72.15\times10^7)} = 0.324$$

PROBLEM 9.86

KNOWN: operating data are provided for a combined gas turbine–vapor power plant.

FIND: Determine (a) the mass flow rates of the air, steam, and cooling water, (b) the net power developed by the gas turbine and the vapor cycle, respectively, (c) the thermal efficiency of the combined cycle, (d) the net rate exergy is carried out with the exhaust air, and (e) the net rate exergy is carried out with the cooling water.

Schematic & Given Data:

See Figure 9.23

Gas Turbine

State	T(°C)	p(bar)
1	25	1
2	–	14
3	1250	14
4	–	1
5	200	1

$\eta_c = 85\%$, $\eta_t = 87\%$

$\dot{Q}_{in} = 50\,MW$

Vapor cycle

State	T(°C)	p(bar)
6	–	125
7	500	125
8	–	0.1
9	sat. liq.	0.1

$\eta_t = 90\%$, $\eta_p = 100\%$

Cooling water

In @ state 10, $T_{10} = 20°C$
Out @ state 11, $T_{11} = 35°C$

ASSUMPTIONS:

1. See assumptions 1–5 of Example 9.13
2. $T_0 = 20°C$, $p_0 = 1\,bar$

ANALYSIS: Begin by evaluating essential property data.

Gas Turbine: At 298K, Table A-22 gives $h_1 = 298.2\ kJ/kg$, $p_{r,1} = 1.3543$, $s_1^o = 1.6953\ \frac{kJ}{kg{\cdot}K}$

$p_{r,2s} = p_{r,1}\left(\frac{p_2}{p_1}\right) = 1.3543(14) = 18.96 \Rightarrow h_{2s} = 632.4\ \frac{kJ}{kg}$

Using the isentropic compressor efficiency,

$$h_2 = h_1 + \frac{(h_{2s}-h_1)}{\eta_c} = 298.2 + \frac{(632.4-298.2)}{.85} = 691.4\ \frac{kJ}{kg}$$

At $T_3 = 1523K$, $h_3 = 1663.9\ \frac{kJ}{kg}$, $p_{r,3} = 641.95$.

$p_{r,4s} = p_{r,3}\left(\frac{p_4}{p_3}\right) = 641.95\left(\frac{1}{14}\right) = 45.85 \Rightarrow h_{4s} = 812.48\ kJ/kg$

Using the isentropic turbine efficiency,

$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 1663.9 - 0.87(1663.9 - 812.48) = 923.2\ kJ/kg$

At $T_5 = 473K$, $h_5 = 475.3\ kJ/kg$, $s_5^o = 2.1625\ kJ/kg{\cdot}K$

① Vapor Cycle: Steam Tables give the following data:

$p_7 = 125\,bar$, $T_7 = 500°C$; $h_7 = 3341.8\ kJ/kg$, $s_7 = 6.4618\ kJ/kg{\cdot}K$

$s_{8s} = s_7 \Rightarrow x_{8s} = \frac{6.4618 - 0.6493}{7.5009} = 0.7749$

$h_{8s} = h_f + x_{8s}(h_{fg}) = 191.83 + (0.7749)(2392.8) = 2046.01\ kJ/kg$

$h_8 = h_7 - \eta_t(h_7 - h_{8s}) = 3341.8 - 0.9(3341.8 - 2046.01) = 2175.6\ kJ/kg$

$h_9 = 191.83\ kJ/kg$

$$h_6 \approx h_9 + v_9 \Delta p = 191.83\ \frac{kJ}{kg} + \left(\frac{1.0102}{10^3}\frac{m^3}{kg}\right)\left(124.9\times10^5\frac{N}{m^2}\right)\left|\frac{1\,kJ}{10^3\,N{\cdot}m}\right| = 204.5\ kJ/kg$$

Cooling water: $h_{10} \approx h_f(20°C) = 83.96\ kJ/kg$, $h_{11} \approx h_f(35°C) = 146.68\ kJ/kg$
$s_{10} \approx s_f(20°C) = 0.2966\ kJ/kg{\cdot}K$, $s_{11} \approx s_f(35°C) = 0.5053\ kJ/kg{\cdot}K$

PROBLEM 9.86 (Contd.)

(a) For a control volume enclosing the combustor, mass and energy rate balances reduce to give

$$\dot{Q}_{in} = \dot{m}_g (h_3 - h_2) \Rightarrow \dot{m}_g = \frac{\dot{Q}_{in}}{(h_3 - h_2)} = \frac{50 \times 10^3 \text{ kJ/s}}{(1663.9 - 691.4)\text{ kJ/kg}} = 51.41 \frac{\text{kg}}{\text{s}}$$

For a control volume enclosing the heat exchanger, we get

$$\dot{m}_s = \dot{m}_g \frac{(h_4 - h_5)}{h_7 - h_6} = \frac{(51.41 \text{ kg/s})(923.2 - 475.3)}{(3341.8 - 204.5)} = 7.34 \frac{\text{kg}}{\text{s}}$$

For a control volume enclosing the condenser

$$\dot{m}_{cw} = \frac{\dot{m}_s (h_8 - h_9)}{h_{11} - h_{10}} = \frac{7.34 (2175.6 - 191.83)}{(146.68 - 83.96)} = 232.16 \text{ kg/s}$$

(b) The net power developed by the gas turbine is

$$\dot{W}_g = \dot{W}_t - \dot{W}_c = \dot{m}_g [(h_3 - h_4) - (h_2 - h_1)] = \left(51.41 \frac{\text{kg}}{\text{s}}\right)\left[(1663.9 - 923.2) - (691.4 - 298.2)\right] \frac{\text{kJ}}{\text{kg}}$$

$$= 17{,}865 \frac{\text{kJ}}{\text{s}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| = 17.865 \text{ MW}$$

For the vapor cycle, the net power developed is

$$\dot{W}_s = \dot{W}_t - \dot{W}_p = \dot{m}_s [(h_7 - h_8) - (h_6 - h_9)]$$

$$= (7.34)\left[(3341.8 - 2175.6) - (204.5 - 191.83)\right] \left| \frac{1}{10^3} \right| = 8.467 \text{ MW}$$

(c) The thermal efficiency is

$$\eta = \frac{\dot{W}_g + \dot{W}_s}{\dot{Q}_{in}} = \frac{17.865 + 8.467}{50} = 0.527 \quad (52.7\%)$$

(d) The net rate exergy is carried out by the exhaust air is

$$\dot{E}_{f5} - \dot{E}_{f1} = \dot{m}_g (e_{f5} - e_{f1}) = \dot{m}_g \left[(h_5 - h_1) - T_0\left(s_5^\circ - s_1^\circ - \overset{0}{\cancel{R \ln p_5/p_1}}\right)\right]$$

$$= \left(51.41 \frac{\text{kg}}{\text{s}}\right)\left[475.3 - 298.2 - 293(2.1625 - 1.6953)\right] \frac{\text{kJ}}{\text{kg}} \left| \frac{1 \text{ MW}}{10^3 \text{ kJ/s}} \right| = 2.07 \text{ MW}$$

(e) The net rate exergy is carried out by the cooling water is

② $$\dot{E}_{f11} - \dot{E}_{f10} = \dot{m}_{cw} (e_{f11} - e_{f10}) = \dot{m}_{cw} [h_{11} - h_{10} - T_0 (s_{11} - s_{10})]$$

$$= (232.16)\left[(146.68 - 83.96) - 293(0.5053 - 0.2966)\right] \left| \frac{1}{10^3} \right| = 0.36 \text{ MW}$$

---

1. Since state 7 requires a double interpolation in Table A-4, steam table data are obtained from the source listed on p. 805. Alternatively, IT can be invoked to retrieve needed data.
2. Considerable energy is carried out with the cooling water: $\dot{m}_{cw}(h_{11} - h_{10}) = 14.56$ MW. But since the cooling water is at a relatively low temperature its utility, as gauged by exergy, is low. Also, note that considerable exergy is carried out at 5. This might be exploited by the regenerative approach discussed in Sec. 9.7 or by other means.

PROBLEM 9.87

KNOWN: Operating data are provided for a combined gas turbine - vapor power plant in which air and water vapor are the working fluids.

FIND: Determine (a) the mass flow rates of air and steam, (b) the thermal efficiency of the combined cycle, and (c) a full exergy accounting of the net exergy increase of the air passing through the combustor of the gas turbine.

SCHEMATIC & GIVEN DATA:

See Figure 9.23. $\dot{W}_{cycle} = 100\,\text{MW}$

Gas Cycle $\eta_t = 88\%$, $\eta_c = 84\%$

State	h (kJ/kg)	s° (kJ/kg·K)
1	300.19	1.70203
2	669.78	2.5088
3	1515.42	3.3620
4	858	2.76195
5	482.49	2.17760

Vapor Cycle $\eta_t = 90\%$, $\eta_p = 80\%$

State	h	s
6	183.96	0.5975
7	3138.3	6.3634
8	2104.74	6.7282
9	173.88	0.5926

ASSUMPTIONS: See Example 9.13 assumptions.

ANALYSIS: Begin by obtaining needed data at each of the principal states: These values are summarized in the tables above.

Gas Cycle

State 1: $T_1 = 300\,\text{K}$. Table A-22 gives $h_1 = 300.19$ kJ/kg, $p_{r1} = 1.3860$, $s_1^\circ = 1.70203$ kJ/kg·K.

State 2: $p_{r,2s} = p_{r,1}(p_2/p_1) = 1.3860\left(\frac{1200\,\text{kPa}}{100\,\text{kPa}}\right) = 16.632 \Rightarrow h_{2s} = 610.65\,\text{kJ/kg}$

and $h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 669.78\,\frac{\text{kJ}}{\text{kg}}$, $s_2^\circ = 2.5088$ kJ/kg·K

State 3: $T_3 = 1400\,\text{K}$. $h_3 = 1515.42$ kJ/kg, $p_{r,3} = 450.5$, $s_3^\circ = 3.3620$ kJ/kg·K

State 4: $p_{r,4s} = (p_4/p_3)\,p_{r,3} = \left(\frac{100}{1200}\right)(450.5) = 37.542 \Rightarrow h_{4s} = 768.38$ kJ/kg

and $h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 1515.42 - 0.88(1515.42 - 768.38) = 858$ kJ/kg,

$s_4^\circ = 2.76195$ kJ/kg·K.

State 5: $T_5 = 480\,\text{K}$, $h_5 = 482.49$ kJ/kg, $s_5^\circ = 2.17760$ kJ/kg·K.

Vapor Cycle

State 7: $p_7 = 80\,\text{bar}$, $T_7 = 400°\text{C}$. Table A-4 gives $h_7 = 3138.3$ kJ/kg,

$s_7 = 6.3634$ kJ/kg·K

State 8: $s_{8s} = s_7 \Rightarrow x_{8s} = \frac{6.3634 - 0.5926}{8.2287 - 0.5926} = 0.7557$, $h_{8s} = 173.88 + .7557(2403.1) = 1989.9$ kJ/kg

and $h_8 = h_7 - \eta_t(h_7 - h_{8s}) = 3138.3 - 0.9(3138.3 - 1989.9) = 2104.74$ kJ/kg

$x_8 = \frac{2104.74 - 173.88}{2403.1} = 0.8035$, $s_8 = 0.5926 + 0.8035(8.2287 - 0.5926) = 6.7282\,\frac{\text{kJ}}{\text{kg·K}}$

State 9: Saturated liquid at 0.08 bar. $h_9 = 173.88\,\frac{\text{kJ}}{\text{kg}}$, $s_9 = 0.5926\,\frac{\text{kJ}}{\text{kg·K}}$

State 6: $h_{6s} \approx h_9 + v_9\Delta p = 173.88 + \left(\frac{1.0084}{10^3}\right)(79.92 \times 10^5)\left|\frac{1}{10^3}\right| = 181.94$ kJ/kg

$h_6 = h_9 + \frac{(h_{6s} - h_9)}{\eta_p} = 173.88 + \left(\frac{8.06}{0.8}\right) = 183.96\,\frac{\text{kJ}}{\text{kg}}$. Then, interpolating in Table A-5 gives $s_6 = 0.5975$ kJ/kg·K.

PROBLEM 9.87(Contd.)

(a) To find the mass flow rates of the air and steam, begin with mass and energy rate balances on the heat exchanger, which reduce to give

$$\frac{\dot{m}_s}{\dot{m}_g} = \frac{h_4 - h_5}{h_7 - h_6} = \frac{858 - 482.49}{3138.3 - 183.96} = 0.1271$$

For the gas turbine, the net power developed is

$$\dot{W}_g = \dot{m}_g \left[(h_3 - h_4) - (h_2 - h_1) \right] = \dot{m}_g \left(287.83 \ \frac{kJ}{kg} \right) \qquad (1)$$

and for the vapor cycle

$$\dot{W}_s = \dot{m}_s \left[(h_7 - h_8) - (h_6 - h_9) \right] = \dot{m}_s \left(1023.5 \ \frac{kJ}{kg} \right) \qquad (2)$$

The net power developed, $\dot{W}_{net} = \dot{W}_g + \dot{W}_s$, is given as 100 MW. Thus, collecting results we get

$$\dot{W}_{net} = \dot{m}_g \left[(287.83) + \overset{0.1271}{\frac{\dot{m}_s}{\dot{m}_g}} (1023.5) \right] \Rightarrow$$

$$\dot{m}_g = \frac{100 \times 10^3 \ kJ/s}{(287.83) + (0.1271)(1023.5)} = 239.3 \ kg/s$$

$$\Rightarrow \quad \dot{m}_s = 30.4 \ kg/s$$

(b) The thermal efficiency is $\eta = (\dot{W}_g + \dot{W}_s)/\dot{Q}_{in}$, where $\dot{Q}_{in} = \dot{m}_g (h_3 - h_2)$. Thus

$$\eta = \frac{287.83 + 0.1271(1023.5)}{(1515.42 - 669.78)} = 0.494 \quad (49.4\%)$$

(c) The net exergy increase of the air passing through the combustor of the gas turbine is

$$\dot{E}_{f3} - \dot{E}_{f2} = \dot{m}_g \left[(h_3 - h_2) - T_0 (s_3^\circ - s_2^\circ - \cancel{R \ln} \, P_3/P_2) \right] = \left(239.3 \ \frac{kg}{s} \right) \left[(1515.42 - 669.78) - 300(3.3620 - 2.5088) \right] \frac{kJ}{kg} \left| \frac{1 \ MW}{10^3 \ kJ/s} \right|$$

$$= 141.1 \ MW$$

Losses:

- **air out**: $(\dot{E}_{f5} - \dot{E}_{f1}) = \dot{m}_g \left[(h_5 - h_1) - T_0 (s_5^\circ - s_1^\circ) \right] = (239.3) \left[482.49 - 300.19 - 300(2.1776 - 1.70203) \right] \left| \frac{1}{10^3} \right|$
  $= 9.48 \ MW$
- **condenser**: $(\dot{E}_{f8} - \dot{E}_{f9}) = \dot{m}_s \left[(h_8 - h_9) - T_0 (s_8 - s_9) \right] = 30.4 \left[(2104.74 - 173.88) - 300(6.7282 - 0.5926) \right] \left| \frac{1}{10^3} \right|$
  $= 2.74 MW$

Destructions: Using $\dot{E}_d = T_0 \dot{\sigma} = T_0 \dot{m} \Delta s$

- **Gas turbine**: $\dot{E}_d = \dot{m}_g T_0 (s_4 - s_3) = (239.3)(300) \left[2.76195 - 3.3620 - \frac{8.314}{28.97} \ln \frac{100}{1200} \right] \left| \frac{1}{10^3} \right| = 8.12 MW$
- **compressor**: $\dot{E}_d = \dot{m}_g T_0 (s_2 - s_1) = (239.3)(300) \left[2.5088 - 1.70203 - \frac{8.314}{28.97} \ln \frac{1200}{100} \right] \left| \frac{1}{10^3} \right| = 6.72 MW$
- **Steam turbine**: $\dot{E}_d = \dot{m}_s T_0 (s_8 - s_7) = (30.4)(800) \left[6.7282 - 6.3634 \right] \left| \frac{1}{10^3} \right| = 3.33 MW$
- **pump**: $\dot{E}_d = \dot{m}_s T_0 (s_6 - s_9) = (30.4)(300) \left[0.5975 - 0.5926 \right] \left| \frac{1}{10^3} \right| = 0.04 MW$
- **heat exchanger**: $\dot{E}_d = T_0 \left[\dot{m}_g (s_5^\circ - s_4^\circ) + \dot{m}_s (s_7 - s_6) \right] = 300 \left[239.3(-0.58435) + 30.4(5.7659) \right] \left| \frac{1}{10^3} \right|$
  $= 10.63 MW$

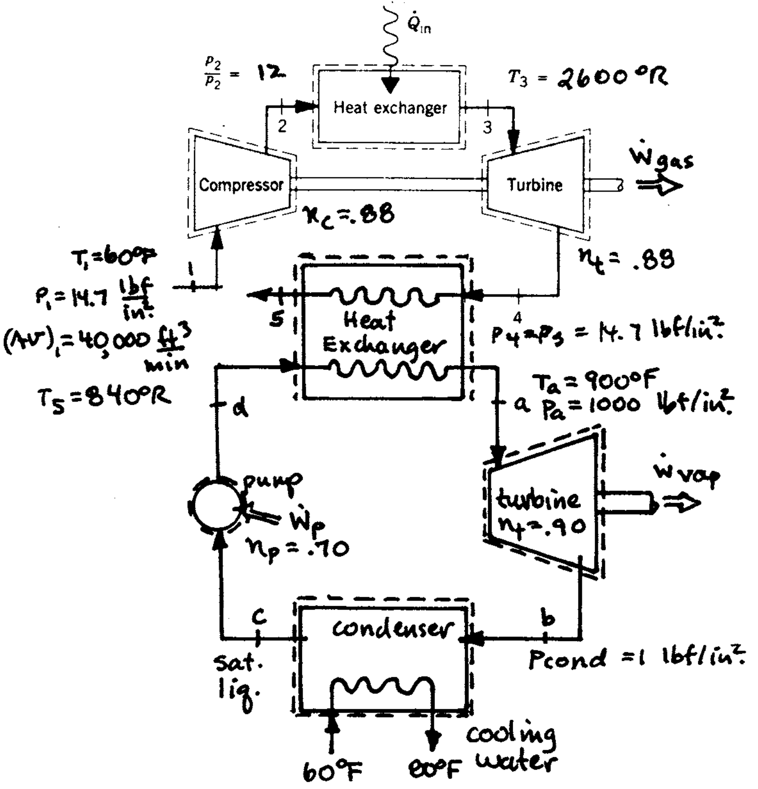
Summary: Input: 141.4 MW
Disposition: $\dot{W}_{net}$: (100 MW) + Losses: (12.22) + Destructions: (28.84) = 141.1 MW, which agrees to within roundoff

PROBLEM 9.88

KNOWN: Operating data are provided for a combined gas turbine – vapor power plant in which air and water vapor are the working fluids.

FIND: Determine (a) the mass flow rates of the air, steam, and cooling water, (b) the net power developed by the gas turbine and vapor cycle, respectively, (c) the thermal efficiency of the combined cycle, and (d) a full accounting of the net exergy increase of the air passing through the combustor of the gas turbine.

SCHEMATIC & GIVEN DATA:



Gas Cycle

State	h(Btu/lb)	s°(Btu/lb·°R)
1	124.27	0.59172
2	270.37	0.77820
3	674.49	1.00623
4	382.51	0.86289
5	201.56	0.70747

Vapor Cycle

State	h(Btu/lb)	s(Btu/lb·°R)
a	1448.1	1.6120
b	955.0	1.7095
c	69.74	.1327
d	74.16	.13497

ASSUMPTIONS:

1. See Example 9.13, assumptions 1-5.
2. $T_0 = 520°R$, $P_0 = 14.7\ lbf/in^2$

ANALYSIS: First, fix each of the principal states. For the gas turbine cycle:

State 1 $T_1 = 520°R \Rightarrow h_1 = 124.27$ Btu/lb

$p_{r1} = 1.2147$, $s^\circ_1 = 0.59172$ Btu/lb·°R

State 2 For isentropic compression,

$p_{r2s} = (p_2/p_1)\,p_{r1} = 14.5764 \Rightarrow h_{2s} = 252.84\ \frac{Btu}{lb}$

Using the compressor efficiency,

$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 270.37$ Btu/lb , $s^\circ_2 = 0.77820$ Btu/lb·°R

State 3 $T_3 = 2600°R \Rightarrow h_3 = 674.49$ Btu/lb , $p_{r3} = 513.5$; $s^\circ_3 = 1.06623$ Btu/lb·°R

State 4 $p_{r4} = (p_4/p_3)\,p_{r3} = 42.792 \Rightarrow h_{4s} = 342.7$ Btu/lb; with $\eta_t = .88$,

$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 382.51$ Btu/lb , $s^\circ_4 = 0.86289$ Btu/lb·°R

State 5 $T_5 = 840°R \Rightarrow h_5 = 201.56$ Btu/lb , $s^\circ_4 = 0.70747$ Btu/lb·°R

For the vapor cycle:

State a $p_a = 1000\ lbf/in^2$, $T_a = 900°F \Rightarrow h_a = 1448.1\ \frac{Btu}{lb}$, $s_a = 1.6120\ \frac{Btu}{lb \cdot °R}$

State b $p_b = 1\ lbf/in^2$, $s_{bs} = s_a \Rightarrow h_{bs} = 900.26$ Btu/lb. With $\eta_t = .90$,

$h_b = h_a - \eta_t(h_a - h_{bs}) = 955.0$ Btu/lb $\Rightarrow x_b = 0.8545$, $s_b = 1.7095$ Btu/lb·°R

State c $p_c = 1\ lbf/in^2$, sat. liquid $\Rightarrow h_c = 69.74$ Btu/lb , $s_c = .1327$ Btu/lb·°R

State d $p_d = 1000\ lbf/in^2$, $s_{ds} = s_c \Rightarrow$ Table A-5E ; $h_{ds} = 72.84$ Btu/lb

with $\eta_p = .70$, $h_d = h_c + \left(\frac{h_{ds} - h_c}{\eta_p}\right) = 74.16$ Btu/lb , $s_d = 0.13497$ Btu/lb·°R (Table A-5E)

PROBLEM 9.88 (Contd.) - page 2

(a) The mass flow rate of the air is

$$\dot{m}_{air} = \frac{(AV)_1\, p_1}{RT_1} = \frac{(40{,}000\ \text{ft}^3/\text{min})\,(14.7\ \frac{\text{lbf}}{\text{in}^2})}{\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(520^\circ\text{R})} \left|\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right| \left|\frac{60\ \text{min}}{1\ \text{h}}\right|$$

$$= 1.832\times10^{5}\ \text{lb/h}$$

Reducing mass and energy rate balances for a control volume enclosing the heat exchanger

$$0 = \dot{m}_{air}(h_4-h_5) + \dot{m}_{st}(h_d-h_a)$$

or

$$\dot{m}_{st} = \dot{m}_{air}\left(\frac{h_4-h_5}{h_a-h_d}\right) = 2.413\times10^{4}\ \text{lb/h}$$

Combining energy and mass balances for the condenser

$$0 = \dot{m}_{st}(h_b-h_c) + \dot{m}_{cw}(h_{cw,in} - h_{cw,out})$$

Solving for $\dot{m}_{cw}$ and with $h_{cw} \approx h_f(T)$

$$\dot{m}_{cw} = \dot{m}_{st}\left(\frac{h_b-h_c}{h_f(80)-h_f(60)}\right) = 1.07\times10^{6}\ \text{lb/h} \longleftarrow \underline{\dot{m}_{cw}}$$

(b) the net power of the gas turbine cycle is

$$\dot{W}_{gas} = \dot{m}_{air}\left[(h_3-h_4)-(h_2-h_1)\right] = 2.67\times10^{7}\ \text{Btu/h}$$

For the vapor power cycle

$$\dot{W}_{vap} = \dot{m}_{st}\left[(h_a-h_b)-(h_d-h_c)\right] = 1.179\times10^{7}\ \text{Btu/h}$$

(c) The rate of heat transfer into the combined cycle is

$$\dot{Q}_{in} = \dot{m}_{air}(h_3-h_2) = 7.403\times10^{7}\ \text{Btu/h}$$

thus

$$\eta = \frac{\dot{W}_{gas}+\dot{W}_{vap}}{\dot{Q}_{in}} = .5199\ (51.99\,\%) \longleftarrow$$

(d) The net increase in exergy of the air passing through the combustor of the gas turbine is

$$\dot{E}_{f3}-\dot{E}_{f2} = \dot{m}_{air}\left[e_{f3}-e_{f2}\right] = \dot{m}_{air}\left[(h_3-h_2) - T_0\left(s_3^\circ - s_2^\circ - \overset{0}{\cancel{R\ln P_3/P_2}}\right)\right] = 5.231\times10^{7}\ \frac{\text{Btu}}{\text{h}}$$

Losses:

- Air out $\dot{E}_{f5}-\dot{E}_{f1} = \dot{m}_{air}\left[(h_5-h_1) - T_0\left(s_5^\circ - s_1^\circ - \overset{0}{\cancel{R\ln P_5/P_1}}\right)\right] = 3.133\times10^{6}\ \text{Btu/h}$

- Condenser:

$$\dot{E}_{f,cw_{out}} - \dot{E}_{f,cw_{in}} = \dot{m}_{cw}\left[h_f(80^\circ\text{F}) - h_f(60^\circ\text{F}) - T_0\left(s_f(80^\circ) - s_f(60^\circ\text{F})\right)\right]$$

$$= 4.103\times10^{5}\ \text{Btu/h}$$

PROBLEM 9.88 (Contd) - page 3

Destructions: Using $\dot{E}_d = T_0 \dot{\sigma} = T_0 \dot{m}(\Delta s)$

Gas turbine: $\dot{E}_d = 2.573 \times 10^6$ Btu/h
Compressor: $\dot{E}_d = 1.537 \times 10^6$ "
Steam turbine: $\dot{E}_d = 1.223 \times 10^6$ "
Pump: $\dot{E}_d = 2.848 \times 10^4$ "
subtotal: $\dot{E}_d = 5.361 \times 10^6$ Btu/h

Condenser: $\dot{E}_d = T_0 [\dot{m}_{st}(s_c - s_6) + \dot{m}_{cw}(s_{cw,out} - s_{cw,in})] = 1.23 \times 10^6$ Btu/h

Heat exchanger: $\dot{E}_d = T_0 [\dot{m}_{st}(s_a - s_d) + \dot{m}_{air}(s^\circ_5 - s^\circ_4)] = 3.727 \times 10^6$ Btu/h

Output:
$\dot{W}_{net} = \dot{W}_{gas} + \dot{W}_{vap} = 3.849 \times 10^7$ Btu/h

Summary

- Exergy Input 5.231×10^7 Btu/h

- Disposition of the exergy input ...

① – Net power developed	3.849×10^7 Btu/h	(73.5%)
– Exergy Destruction	10.318×10^6 Btu/h	(19.7%)
– Exergy Losses	3.543×10^6 Btu/h	(6.8%)
	5.235×10^7 Btu/h	

---

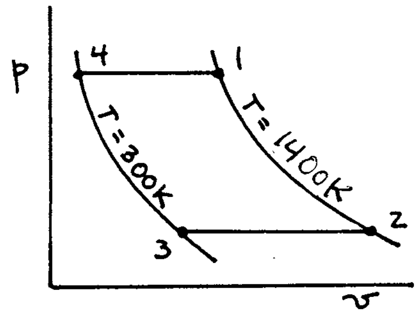
1. The exergy destruction associated with combustion is not considered in the present analysis, which is based on the air-standard model. See the discussion in note 2 of the solution to Example 9.13.

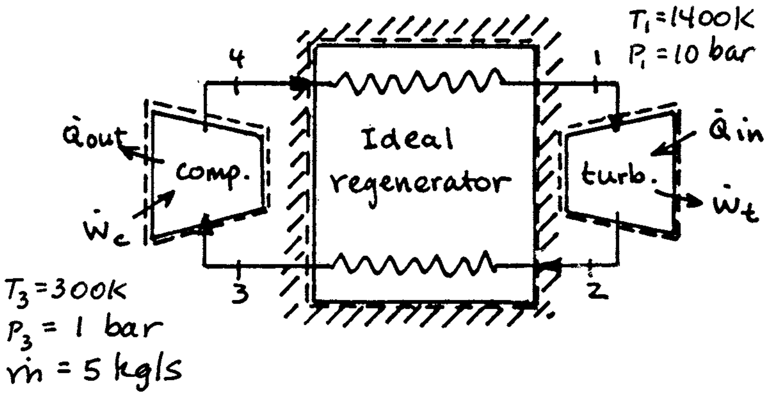
PROBLEM 9.89

KNOWN: Air is the working fluid in an Ericsson cycle with data known at various locations.

FIND: Determine (a) the net power developed, (b) the thermal efficiency, and (c) the back-work ratio.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion processes are isothermal. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) The turbine power is evaluated using

$$\dot{W}_t = -\dot{m}\int_1^2 v\,dp = -\dot{m}RT_1 \ln(P_2/P_1)$$

$$= -(5\ \text{kg/s})\left(\frac{8.314}{28.97}\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(1400\text{K})\ \ln\left(\frac{1}{10}\right)\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 4626\ \text{kW}$$

and for the compressor

$$\dot{W}_c = \dot{m}RT_3 \ln P_4/P_3 = 991.2\ \text{kW}$$

Thus $\dot{W}_{cycle} = \dot{W}_t - \dot{W}_c = 3635$ kW ← $\dot{W}_{cycle}$

(b) The thermal efficiency is

$$\eta = 1 - \frac{T_3}{T_1} = 1 - \frac{300}{1400} = 0.786\ (78.6\%)$$ ← η

Alternatively, from an energy balance on the turbine; $\dot{Q}_{in} = \dot{W}_t$. Thus

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} = \frac{3635}{4626} = 0.786$$

(c) The back work ratio is

$$bwr = \frac{\dot{W}_c}{\dot{W}_t} = \frac{991.2}{4626} = 0.214$$ ← bwr

PROBLEM 9.90

See Problem 9.89 for sample calculations. The data for the required plot are obtained using IT, as follows:

IT Code

```
T1 = 1400  // K
T3 = 300  // K
p3 = 1  // bar
rp = 10
p4 = rp * p3
p1 = p4
p2 = p3
mdot = 5  // kg/s
R = 8.314 / 28.97  // kJ/kg·K

Wdot_t = - mdot * R * T1 *  ln (p2 / p1)
Wdot_c = mdot * R * T3 * ln (p4 / p3)
Wdot_cycle = Wdot_t - Wdot_c
```

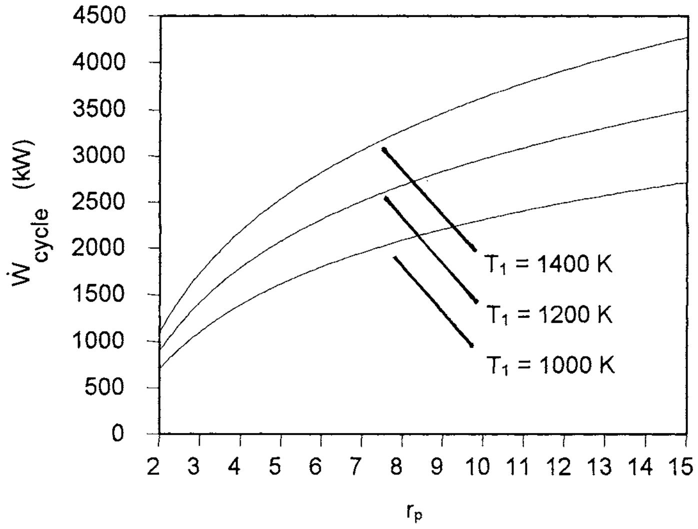
IT Results for r_p = 10, T_1 1400 K

$\dot{W}_t$ = 4626 kW

$\dot{W}_c$ = 991.2 kW

$\dot{W}_{cycle}$ = 3634 kW

PLOT:



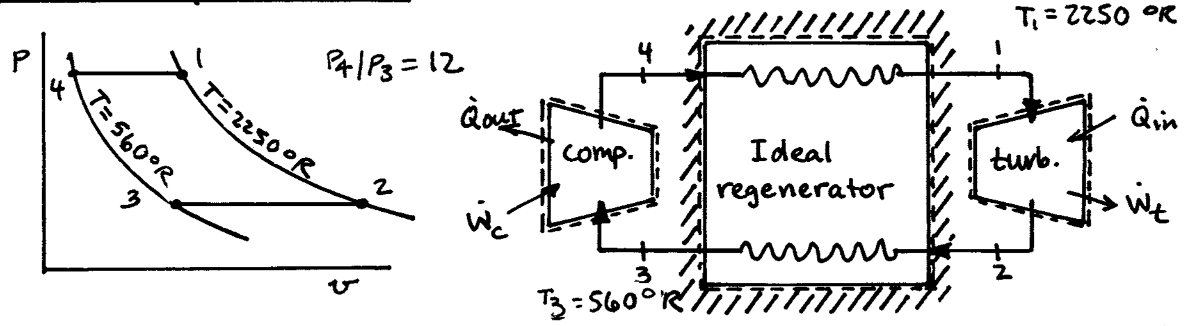
We see that the power increases significantly with compressor pressure ratio and with maximum cycle temperature for this idealized cycle.

PROBLEM 9.91*

KNOWN: Air is the working fluid in an Ericsson cycle with data known at various locations.

FIND: Determine (a) the net work per unit mass of air flowing and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion processes are isothermal. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) The turbine work is evaluated using Eq. 6.67

$$\frac{\dot{W}_t}{\dot{m}} = -\int_1^2 v\,dp = -RT_1 \ln\frac{p_2}{p_1}$$

$$= -\left(\frac{1.986}{28.97}\cdot\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)(2250\ °\text{R})\ln\left(\frac{1}{12}\right) = 383.3\ \text{Btu/lb}$$

For the compressor

$$\frac{\dot{W}_c}{\dot{m}} = RT_3 \ln\left(\frac{P_4}{P_3}\right) = \left(\frac{1.986}{28.97}\right)(560)\ln 12 = 95.4\ \text{Btu/lb}$$

Thus, the net work is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}} - \frac{\dot{W}_c}{\dot{m}} = 383.3 - 95.4 = 287.9\,\frac{\text{Btu}}{\text{lb}} \longleftarrow \dot{W}_{cycle}/\dot{m}$$

(b) The thermal efficiency is

$$\eta = 1 - \frac{T_3}{T_1} = 0.751 \longleftarrow \eta$$

Alternatively, $\dot{Q}_{in}/\dot{m} = \frac{\dot{W}_t}{\dot{m}} + \cancel{(h_2 - h_1)}^{0} = 383.3$ Btu/lb.

thus

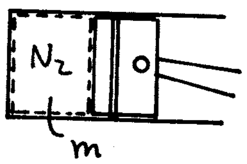
$$\eta = \frac{287.9}{383.3} = 0.751$$

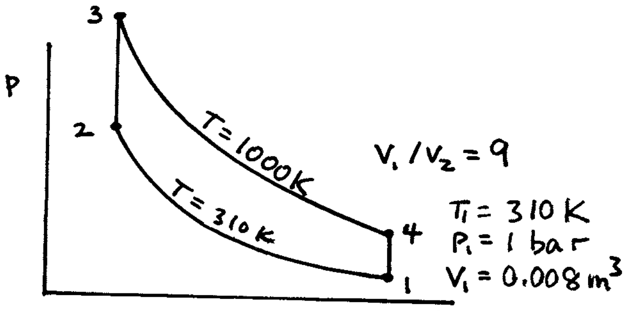
PROBLEM 9.92*

KNOWN: N_2 undergoes a Stirling cycle with a known compression ratio. Other data are also given for the cycle.

FIND: Determine (a) the net work, (b) the thermal efficiency, and (c) the mean effective pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The air is a closed system. (2) The compression and expansion are isothermal. (3) All processes are internally reversible. (4) An ideal regenerator is available to store the energy rejected at constant volume to be used as heat input during the constant volume heat transfer process. (5) The air behaves as an ideal gas.

ANALYSIS: The mass m is found as follows:

$$m = \frac{P_1 V_1}{RT_1} = \frac{(10^5\ N/m^2)(0.008\ m^3)}{\left(\frac{8314}{28.01}\ \frac{N\cdot m}{kg\cdot K}\right)(290\ K)} = 0.0093\ kg$$

(a) For the isothermal compression

$$W_{12} = \int_1^2 p\,dV = mRT_1 \ln \frac{V_2}{V_1}$$
$$= (0.0093\ kg)\left(\frac{8.314}{28.01}\ \frac{kJ}{kg\cdot K}\right)(310\ K) \ln\left(\frac{1}{9}\right) = -1.88\ kJ$$

Similarly, for the expansion

$$W_{34} = mRT_3 \ln \frac{V_4}{V_3} = (0.0093)\left(\frac{8.314}{28.01}\right)(1000) \ln(9) = 6.07\ kJ$$

Thus, $W_{cycle} = W_{12} + W_{34} = 4.19\ kJ \longleftarrow W_{cycle}$

(b) The thermal efficiency is

$$\eta = 1 - \frac{T_1}{T_3} = 1 - \frac{310}{1000} = 0.690 \longleftarrow \eta$$

or

$$\eta = \frac{W_{cycle}}{\underbrace{Q_{34}}_{= W_{34}\ \text{(energy balance)}}} = \frac{4.19\ kJ}{6.07\ kJ} = 0.690$$

(c) The mean effective pressure is

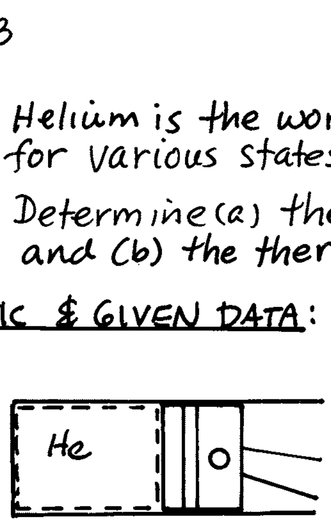
$$mep = \frac{W_{cycle}}{V_1 - V_2} = \frac{W_{cycle}}{V_1(1 - V_2/V_1)}$$
$$= \frac{(4.19\ kJ)}{(0.008\ m^3)(1 - \frac{1}{9})}\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ bar}{10^5\ N/m^2}\right| = 5.89\ bar \longleftarrow mep$$

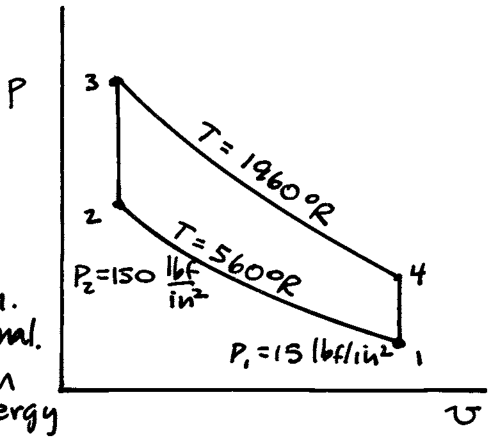
PROBLEM 9.93

KNOWN: Helium is the working fluid in a Stirling cycle. Data are given for various states in the cycle.

FIND: Determine (a) the work and heat transfer for each process and (b) the thermal efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The helium is a closed system. (2) The compression and expansion are isothermal. (3) All processes are internally reversible. (4) An ideal regenerator is available to store the energy transferred by heat during constant volume processes. (5) The helium behaves as an ideal gas.

ANALYSIS: (a) Considering each process

Process 1-2: The work is calculated using

$$\frac{W_{12}}{m} = \int_1^2 p\,dv = RT_1 \ln\frac{v_2}{v_1} = RT_1 \ln\left(\frac{p_1}{p_2}\right)$$

$$= \left(\frac{1545}{4.003}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)\left|\frac{1\text{ Btu}}{778\text{ ft}\cdot\text{lbf}}\right|(560^\circ\text{R})\ln\left(\frac{15}{150}\right) = -639.7\ \frac{\text{Btu}}{\text{lb}} \longleftarrow \frac{W_{12}}{m}$$

From an energy balance

$$\frac{Q_{12}}{m} = \cancelto{0}{(u_2 - u_1)} + \frac{W_{12}}{m} = -639.7\ \frac{\text{Btu}}{\text{lb}} \longleftarrow \frac{Q_{12}}{m}$$

Process 3-4:

$$\frac{W_{34}}{m} = RT_3 \ln\frac{v_4}{v_3} = RT_3 \ln\frac{v_1}{v_2} = RT_3 \ln\frac{p_2}{p_1}$$

$$= \left(\frac{1545}{4.003}\right)\left|\frac{1}{778}\right|(1960)\ln\left(\frac{150}{15}\right) = 2238.9\ \text{Btu/lb} \longleftarrow \frac{W_{34}}{m}$$

and

$$\frac{Q_{34}}{m} = \cancelto{0}{(u_4 - u_3)} + \frac{W_{34}}{m} = 2238.9\ \text{Btu/lb} \longleftarrow \frac{Q_{34}}{m}$$

Processes 2-3 and 4-1:

$$\left.\begin{aligned}\frac{Q_{23}}{m} &= u_3 - u_2\\ \frac{Q_{41}}{m} &= u_1 - u_4\end{aligned}\right\}\ \frac{Q_{23}}{m} = -\frac{Q_{41}}{m}$$

Thus, if the regenerator is ideal, there is no net exchange of energy with the surroundings.

(b) The thermal efficiency is

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{in}}} = \frac{W_{12}/m + W_{34}/m}{Q_{34}/m} = 0.714\ (71.4\%)$$

Aternatively

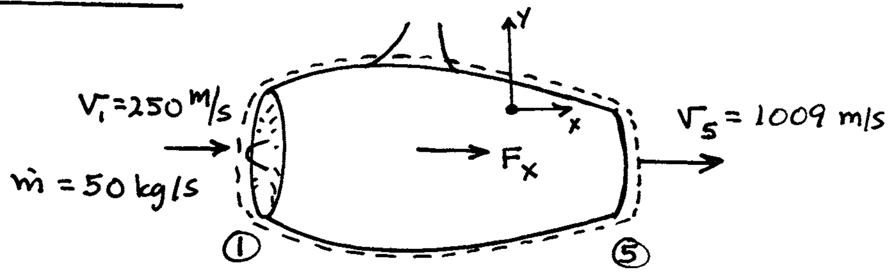
$$\eta = 1 - \frac{T_1}{T_3} = 1 - \frac{560}{1960} = 0.714$$

PROBLEM 9.94

KNOWN: The mass flow rate is known for the cycle in Problem 9.76

FIND: Calculated the thrust developed by the engine.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) The force
① F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m}\,(V_5 - V_1)$$

$$= \left(50\ \frac{kg}{s}\right)(1009 - 250)\frac{m}{s}\left|\frac{1\ kN}{10^3\ kg\cdot m/s^2}\right|$$

$$= 37.95\ kN$$

Thus, the thrust is the force developed by the control volume

$$F_{thrust} = -F_x = \underset{\uparrow\ \text{to the left}}{-37.95\ kN} \longleftarrow \text{thrust}$$

---

1. The effect of the surrounding pressure on the control volume boundary cancels.

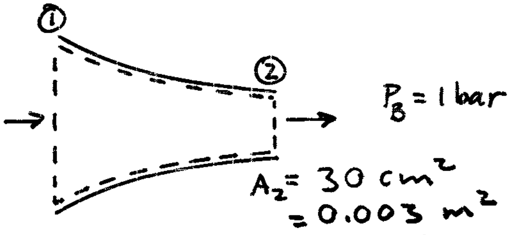
PROBLEM 9.115*

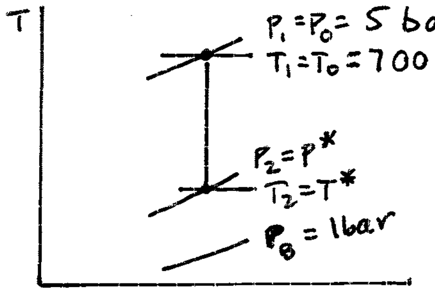
KNOWN: An ideal gas mixture flows isentropically through a converging nozzle. The inlet conditions, back pressure, and exit area are specified.

FIND: Determine (a) the exit temperature, (b) the exit velocity, and (c) the mass flow rate.

SCHEMATIC & GIVEN DATA:

$p_1 = p_0 = 5$ bar
$T_1 = T_0 = 700$ K
$k = 1.31$
Mol. wt. = 23





ASSUMPTIONS: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The gas expands isentropically within the nozzle. (3) The velocity at the inlet is negligible, and potential energy effects can be ignored. (4) The mixture behaves as an ideal gas with constant specific heats.

ANALYSIS: First, it is necessary to determine if the nozzle is choked. Using Eq. 9.51, with M = 1

$$\frac{p_0}{p^*} = \left[1 + \left(\frac{k-1}{2}\right)(1)^2\right]^{k/(k-1)} = \left(\frac{k+1}{2}\right)^{k/(k-1)} = 1.838$$

Thus, $p^* = p_0/1.838 = 2.720$ bar. $p_B < p^* \Rightarrow$ choked conditions $\Rightarrow p_2 = p^*$.

(a) $T_2 = T^*$. Using Eq. 9.50, with $M_2 = 1$

$$\frac{T^0}{T^*} = 1 + \left(\frac{k-1}{2}\right)(1)^2 = \frac{k+1}{2} = 1.155 \Rightarrow T^* = 606.1\ K \longleftarrow T_2$$

(b) Since $M_2 = 1$, $V_2 = c_2 = \sqrt{kRT_2}$

$$V_2 = \sqrt{(1.31)\left(\frac{8.314}{23}\frac{kJ}{kg\cdot K}\right)(606.1\ K)\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ kg\cdot m/s^2}{1\ N}\right|}$$

$$= 535.7\ m/s \longleftarrow V_2$$

(c) The mass flow rate is

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{p_2 A_2 V_2}{RT_2}$$

$$= \frac{(2.720\ bar)(0.003\ m^2)(535.7\ m/s)}{\left(\frac{8.314}{23}\frac{kJ}{kg\cdot K}\right)(606.1\ K)}\left|\frac{10^5\ N/m^2}{1\ bar}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right|$$

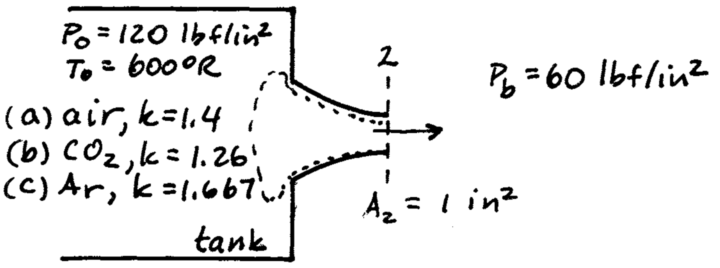
$$= 1.995\ kg/s \longleftarrow \dot{m}$$

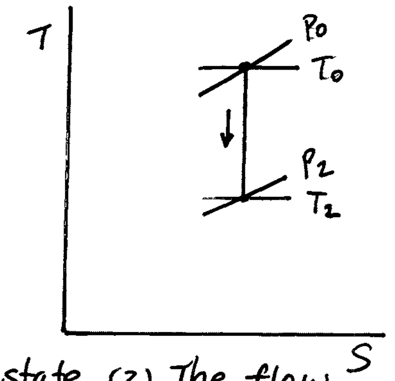
PROBLEM 9.116

KNOWN: An ideal gas expands isentropically through a converging nozzle from a large tank. The conditions in the tank, the pressure in the back, and the nozzle exit area are known.

FIND: Determine the mass flow rate if the gas is (a) air, (b) carbon dioxide, and (c) argon.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state. (2) The flow is isentropic. (3) The gas behaves as an ideal gas with constant specific heats.

ANALYSIS: First, it is necessary to determine if the nozzle is choked. Using Eq. 9.51 with $M_2 = 1$

$$\frac{p_0}{p^*} = \left[1 + \left(\frac{k-1}{2}\right)(1)^2\right]^{k/k-1} = \left(\frac{k+1}{2}\right)^{k/k-1} \qquad (*)$$

(a) For air, with $k = 1.4$

$$\frac{p_0}{p^*} = 1.89293 \Rightarrow p^* = 63.39 \text{ lbf/in}^2$$

Since $p_b < p^*$, the nozzle is choked. Thus, $p_2 = p^* = 63.39$ lbf/in².

Now, for the isentropic expansion

$$\frac{T_2}{T_0} = \left(\frac{p_2}{p_0}\right)^{\frac{k-1}{k}} \Rightarrow T_2 = 500°R$$

Using the energy balance: $0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{m}\left[(h_0 - h_2) + \left(\frac{\cancel{V_0^2}^{0} - V_2^2}{2}\right) + g(\cancel{z_0 - z_2}^{0})\right]$

$$V_2 = \sqrt{2(h_0 - h_2)} = \sqrt{2c_p(T_0 - T_2)}$$

$$= \sqrt{(2)\left(0.24 \frac{\text{Btu}}{\text{lb}\cdot°R}\right)(600-500)°R \left|\frac{32.2 \text{ lb}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right| \left|\frac{778 \text{ ft}\cdot\text{lbf}}{1 \text{ Btu}}\right|}$$

$$= 1096.6 \text{ ft/s}$$

All of the conditions are known at the exit. Thus

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{A_2 V_2 p_2}{R T_2}$$

$$= \frac{(1 \text{ in}^2)(1096.6 \text{ ft/s})(63.39 \text{ lbf/in}^2)}{\left(\frac{1545}{28.97} \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°R}\right)(500°R)} = 2.61 \text{ lb/s} \longleftarrow \dot{m}$$

PROBLEM 9.116 (Cont'd)

(b) For CO_2, with $k = 1.26$ ($c_p = 0.2187$ Btu/lb·°R)

$$\frac{p_0}{p^*} = 1.8081 \Rightarrow p^* = 66.37 \text{ lbf/in}^2$$

Again, $p_b < p^* \Rightarrow$ choked conditions, and $p_2 = p^* = 66.37$ lbf/in².

Thus

$$\frac{T_2}{T_0} = \left(\frac{p_2}{p_0}\right)^{\frac{k-1}{k}} \Rightarrow T_2 = 531.0\,°R$$

and

$$V_2 = \sqrt{(2)(0.2187)(600-531.0)\left|32.2\right|\left|778\right|} = 869.5 \text{ ft/s}$$

Finally

$$\dot{m} = \frac{(1)(869.5)(66.37)}{\left(\frac{1545}{44.01}\right)(531.0)} = 3.096 \text{ lb/s} \longleftarrow \dot{m}$$

(c) For Ar, $k = 1.667$ ($c_p = 0.1243$ Btu/lb·°R)

$$\frac{p_0}{p^*} = 2.053 \Rightarrow p^* = 58.45 \text{ lbf/in}^2$$

In this case, $p_b > p^*$, which means the nozzle is not choked. Thus, the exit plane pressure is equal to the back region pressure. That is

$$p_2 = p_b = 60 \text{ lbf/in}^2$$

and

$$\frac{T_2}{T_0} = \left(\frac{p_2}{p_0}\right)^{\frac{k-1}{k}} \Rightarrow T_2 = 454.7\,°R$$

The exit velocity is

$$V_2 = \sqrt{(2)(0.1243)(600-454.7)\left|32.2\right|\left|778\right|} = 951.3 \text{ ft/s}$$

Finally, the mass flow rate is

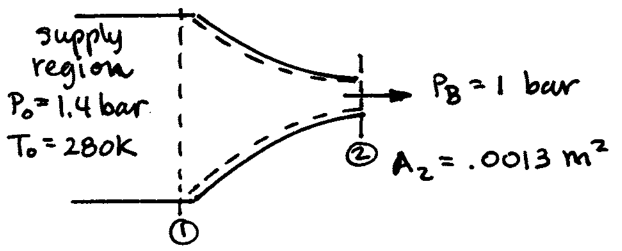
$$\dot{m} = \frac{(1)(951.3)(60)}{\left(\frac{1545}{39.94}\right)(454.7)} = 3.245 \text{ lb/s} \longleftarrow \dot{m}$$

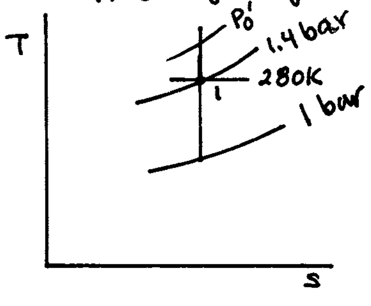
PROBLEM 9.117

KNOWN: Air expands isentropically through a converging nozzle and discharges to the atmosphere. The inlet conditions are specified and the exit plane area is given.

FIND: Determine the mass flow rate. Is it possible to increase the mass flow rate by increasing the supply region pressure?

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air expands isentropically. (3) The air behaves as an ideal gas. (4) Because of the small pressure difference, the temperature range will be limited. Therefore, assume $k = 1.4$ is constant.

ANALYSIS: (a) First, determine if the nozzle is choked. Using Eq. 9.51 with $M = 1$ gives

$$\frac{p_0}{p^*} = \left(\frac{k+1}{2}\right)^{k/k-1} = 1.8929 \Rightarrow p^* = 0.7396 \text{ bar}$$

NOT CHOKED

$\therefore P_2 = 1$ bar

For the isentropic expansion

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 254.3 \text{ K}$$

The exit velocity is, with $c_p = 1.004$ kJ/kg·K,

$$V_2 = \sqrt{2 c_p (T_1 - T_2)}$$
$$= \sqrt{2 \left(1.004 \frac{kJ}{kg \cdot K}\right)(280 - 254.3)K \left|\frac{10^3 N\cdot m}{1 kJ}\right| \left|\frac{1 kg\cdot m/s^2}{1N}\right|}$$
$$= 227.2 \text{ m/s}$$

Thus, the mass flow rate is

$$\dot{m} = \frac{A_2 V_2 p_2}{R T_2} = \frac{(.0013 m^2)(227.2 m/s)(1 bar)}{\left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K}\right)(254.3 K)} \left|\frac{10^5 N/m^2}{1 bar}\right| \left|\frac{1 kJ}{10^3 N \cdot m}\right|$$

$= 0.4047$ kg/s ← $\dot{m}$ (a)

(b) Next, consider the case for which $p_0 = 2$ bar, $T_0 = 280$K and $p_B = 1$bar. With Eq. 9.51

$$\frac{p_0}{p^*} = \left(\frac{k+1}{2}\right)^{k/k-1} = 1.8929 \Rightarrow p^* = 1.0566 \text{ bar}$$

$P_B < p^* \Rightarrow$ choked conditions $\Rightarrow P_2 = p^* = 1.0566$ bar

PROBLEM 9.117 (Cont'd.)

Thus, $T_2 = T^*$. From Eq. 9.50 with $M=1$

$$\frac{T_0}{T^*} = \frac{k+1}{2} = 1.2 \Rightarrow T^* = 233.33\,K$$

The exit velocity is

$$V_2 = \sqrt{2(1.004)(280-233.33)\left|10^3\right|} = 306.1\ m/s$$

Thus, the mass flow rate is

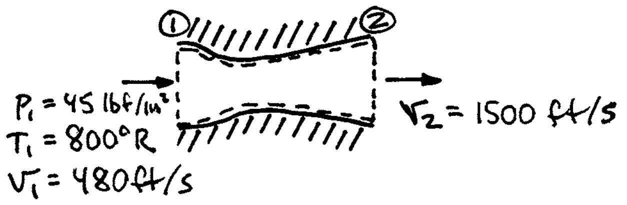
$$\dot{m} = \frac{(0.0013)(306.1)(1)}{\left(\frac{8.314}{28.97}\right)(233.33)}\left|\frac{10^5}{10^3}\right| = 0.5943\ kg/s \longleftarrow \underset{(b)}{\dot{m}}$$

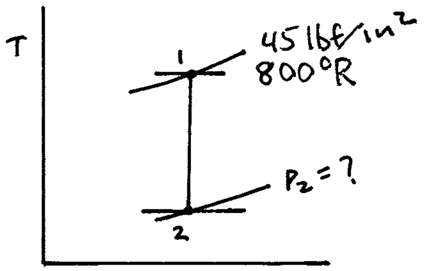
PROBLEM 9.118*

KNOWN: Air expands isentropically through a nozzle from known inlet conditions to a known exit velocity.

FIND: Determine (a) the exit pressure, (b) the ratio of exit area to inlet area, (c) whether the nozzle is converging-diverging or converging only.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air undergoes an isentropic process. (3) The air behaves as an ideal gas.

ANALYSIS: To fix the exit state, determine h_2 using an energy balance. Thus

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\overset{0}{\cancel{(z_1 - z_2)}}\right]$$

or

$$h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

$$= 191.81\,\frac{\text{Btu}}{\text{lb}} + \left(\frac{480^2 - 1500^2}{2}\right)\frac{\text{ft}^2}{\text{s}^2}\left|\frac{1\text{ lbf s}^2}{32.2\text{ lb ft}}\right|\left|\frac{1\text{ Btu}}{778\text{ ft lbf}}\right|$$

$$= 151.50\,\frac{\text{Btu}}{\text{lb}} \Rightarrow p_{r2} = 2.4265,\quad T_2 = 633.4\,°\text{R}$$

Finally

$$p_2 = p_1\left(\frac{p_{r2}}{p_{r1}}\right) = \left(45\,\frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{2.4265}{5.526}\right) = 19.760\,\frac{\text{lbf}}{\text{in}^2} \longleftarrow$$ p_2

(b) From the mass balance; $\frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$. With $v = RT/p$,

$$\frac{A_1 V_1 p_1}{RT_1} = \frac{A_2 V_2 p_2}{RT_2} \Rightarrow \frac{A_2}{A_1} = \frac{V_1}{V_2}\cdot\frac{p_1}{p_2}\cdot\frac{T_2}{T_1} = 0.577 \longleftarrow$$ A_2/A_1

(c) To determine the nozzle type, calculate the inlet and exit Mach numbers.

$$M_1 = \frac{V_1}{\sqrt{kRT_1}} = \frac{(480\text{ ft/s})}{\sqrt{(1.39)\left(\frac{1545}{28.97}\right)\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}(800°\text{R})\left|\frac{32.2\text{ lb}\cdot\text{ft}}{1\text{ lbf s}^2}\right|}}$$

$$= 0.347 \text{ (subsonic)}$$

$$M_2 = \frac{(1500)}{\sqrt{(1.4)\left(\frac{1545}{28.97}\right)(633.4)|32.2|}} = 1.216 \text{ (supersonic)}$$

Thus, a converging-diverging nozzle is needed.

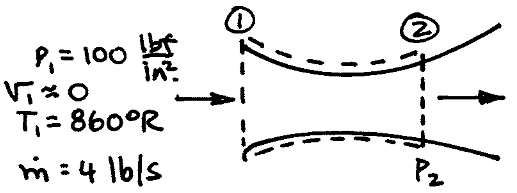
Nozzle type

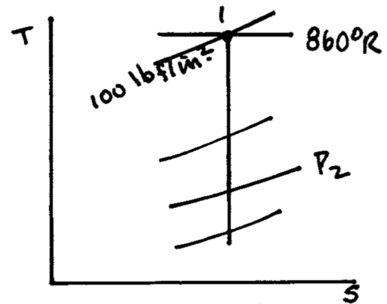
PROBLEM 9.119

KNOWN: Air enters a nozzle with specified conditions and expands isentropically. The mass flow rate is known.

FIND: Determine the velocity, Mach number, and area at locations where the pressure is (a) 80, (b) 60, (c) 50, and (d) 40 lbf/in². Also, plot the results and determine the pressure at which the flow beomes sonic.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The air expands isentropically. (3) The air behaves as an ideal gas.

ANALYSIS: For each case to be considered, $p_1 = 100$ lbf/in², $T_1 = 860°R$. Thus, $h_1 = 206.46$ Btu/lb, $p_{r1} = 7.149$. With p_2 specified

$$p_{r2} = (p_2 / p_1)\, p_{r1} \Rightarrow h_2,\ T_2$$

Then, the velocity is found using an energy balance

$$V_2 = \sqrt{2(h_1 - h_2)} \qquad (1)$$

and the Mach number is

$$M_2 = V_2 / \sqrt{kRT_2} \qquad (2)$$

Finally, area A_2 is

$$A_2 = \frac{\dot{m} v_2}{V_2} = \frac{\dot{m} R T_2}{V_2\, p_2} \qquad (3)$$

(a) $p_{r2} = (80/100)(7.149) = 5.7192 \Rightarrow h_2 = 193.67\ \frac{\text{Btu}}{\text{lb}},\ T_2 = 807.6°R$

$$V_2 = \sqrt{2(206.46 - 193.67)\frac{\text{Btu}}{\text{lb}}\left|\frac{778\ \text{ft·lbf}}{1\ \text{Btu}}\right|\left|\frac{32.2\ \text{lb·ft/s}^2}{1\ \text{lbf}}\right|}$$

$= 800.5$ ft/s $\longleftarrow V_2$

From Table A-20, $k \approx 1.386$. Thus

$$M_2 = \frac{(800.5\ \text{ft/s})}{\sqrt{(1.386)\left(\frac{1545}{28.97}\,\frac{\text{ft·lbf}}{\text{lb·°R}}\right)(807.6\ °R)\left|\frac{32.2\ \text{lb·ft/s}^2}{1\ \text{lbf}}\right|}} = 0.577 \longleftarrow M_2$$

$$A_2 = \frac{(4\ \text{lb/s})\left(\frac{1545}{28.97}\,\frac{\text{ft·lbf}}{\text{lb·°R}}\right)(807.6\ °R)}{(800.5\ \text{ft/s})(80\ \text{lbf/in}^2)}\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| = 1.868 \times 10^{-2}\ \text{ft}^2 \longleftarrow A_2$$

PROBLEM 9.119 (Cont'd.)

The remaining cases are handled in a similar manner. The results are summarized in the following table

	p_2 ($\frac{lbf}{in^2}$)	h_2 ($\frac{Btu}{lb}$)	T_2 (°R)	V_2 ($\frac{ft}{s}$)	M_2	A_2 (ft^2)
(a)	80	193.67	807.6	800.5	.577	1.868×10^{-2}
(b)	60	178.36	744.7	1186.5	.889	1.55×10^{-2}
(c)	50	169.28	707.14	1364.9	1.048	1.535×10^{-2}
(d)	40	158.83	663.78	1544.8	1.224	1.591×10^{-2}

(e) The data for the required plots are obtained using IT, as follows:

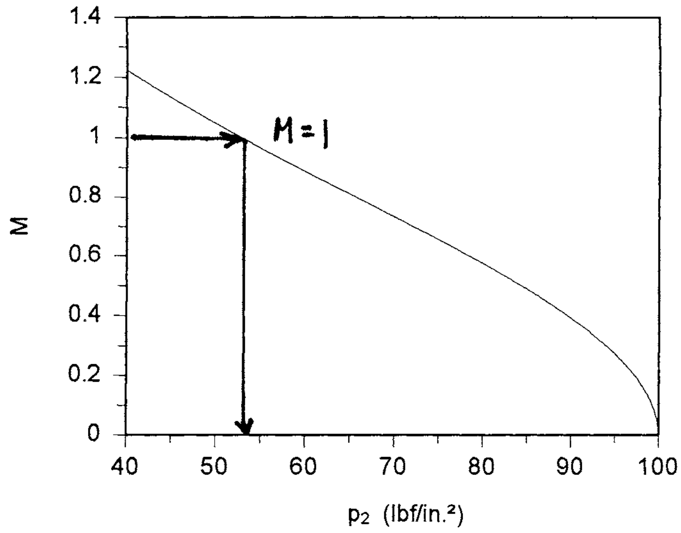
IT Code

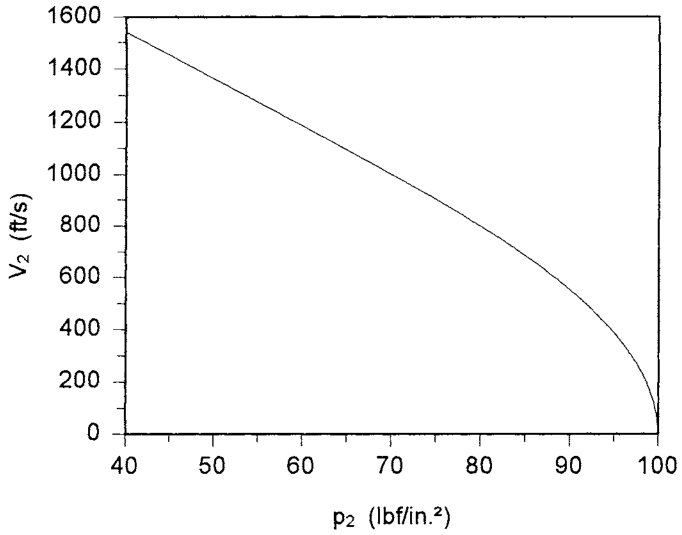
```
p1 = 100   // lbf/in²
T1 = 860   // °R
Vel1 = 0
mdot = 4   // lb/s
p2 = 40   // lbf/in²
Rbar = 1545
MW = 28.97
R = Rbar / MW

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2 = s1
s2 = s_hP("Air",h2,p2)
T2 = T_h("Air", h2)
Vel2 = sqrt (2 * (h1 - h2) * 778 * 32.17)
M2 = Vel2 / c
c = sqrt (k * R * T2 * 32.17)
cp = cp_T("Air", T2)
cv = cv_T("Air", T2)
k = cp / cv
v2 = v_TP("Air",T2,p2)
A2 = (mdot * v2) / Vel2
```

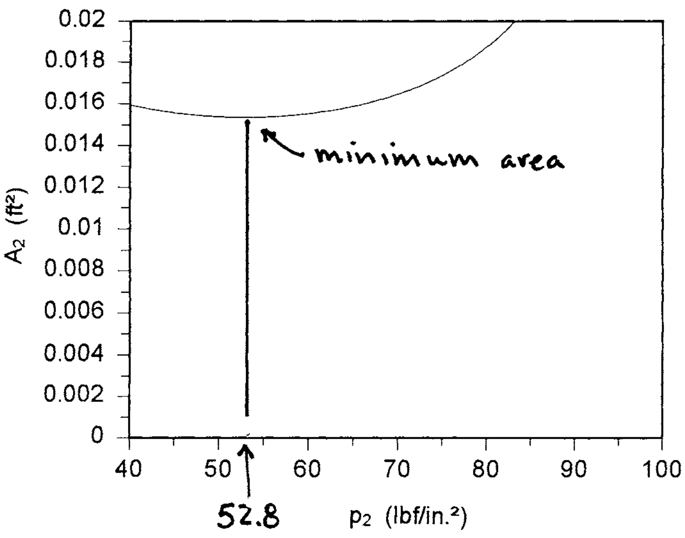
IT Results for p_2 = 40 lbf/in.²

h_1 = 206.5 Btu/lb
h_2 = 158.9 Btu/lb
T_2 = 663.9°R
c = 1262 ft/s
k = 1.397
V_2 = 1543 ft/s
M_2 = 1.223
A_2 = 0.01593 ft²

PLOTS:





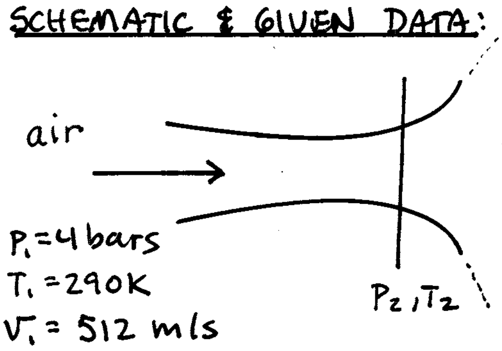


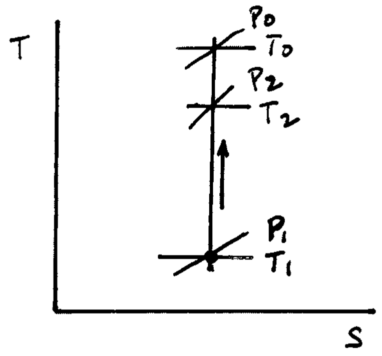
PROBLEM 9.120

KNOWN: Air with $k=1.4$ flows isentropically through a diffuser.

FIND: Plot velocity, Mach number, and area ratio A/A^* for locations in the flow corresponding to a range of pressures.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Isentropic flow. (2) Ideal gas model with $k=1.4$.

ANALYSIS: The following expressions are used to calculate the required quantities:

1. $c = \sqrt{kRT}$
2. $M = V/c$
3. $p_0/p = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1}$ (Eq. 9.51)
4. $T_0/T = \left[p_0/p\right]^{k-1/k}$ (Eq. 9.50)
5. $A/A^* = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}\right)M^2\right]^{(k+1)/2(k-1)}$ (Eq. 9.52)

First, use given values of T_1, p_1, V_1 to get c_1, M_1, p_0, T_0. Then, vary p to get $M, T, A/A^*$, and V.

The data for the required plots are obtained using IT, as follows:

IT Code

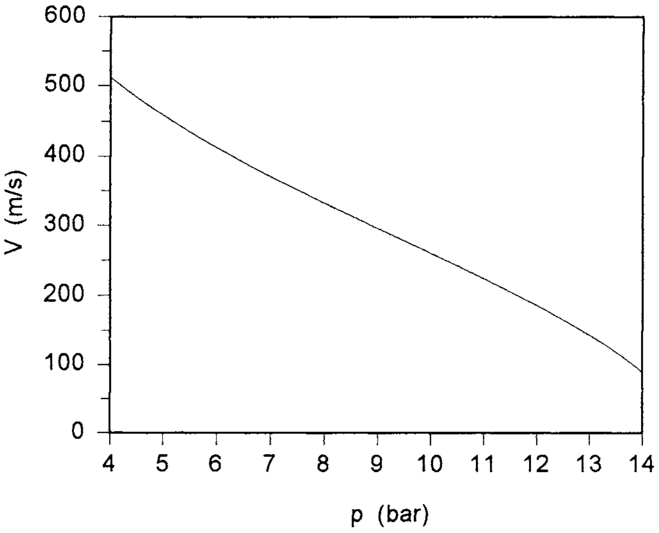
```
p1 = 4  // bar
T1 = 290  // K
V1 = 512  // m/s
k = 1.4
Rbar = 8.314  // kJ/kmol·K
MW = 28.97  // kg/kmol

c1 = sqrt(k * (Rbar / MW) * T1 * 1000)
M1 = V1 / c1
po / p1 = (1 + ((k - 1) / 2) * M1^2)^(k / (k - 1))
To / T1 = (po / p1)^((k - 1) / k)

p = 10
c = sqrt(k * (Rbar / MW) * T * 1000)
M = V / c
Aratio = (1 / M) * ((2 / (k+1)) * ((1 + ((k - 1) / 2) * M^2) ))^((k + 1) / (2 * (k - 1)))
po / p = (1 + ((k - 1) / 2) * M^2)^(k / (k - 1))
To / T = (po / p1)^((k - 1) / k)
```

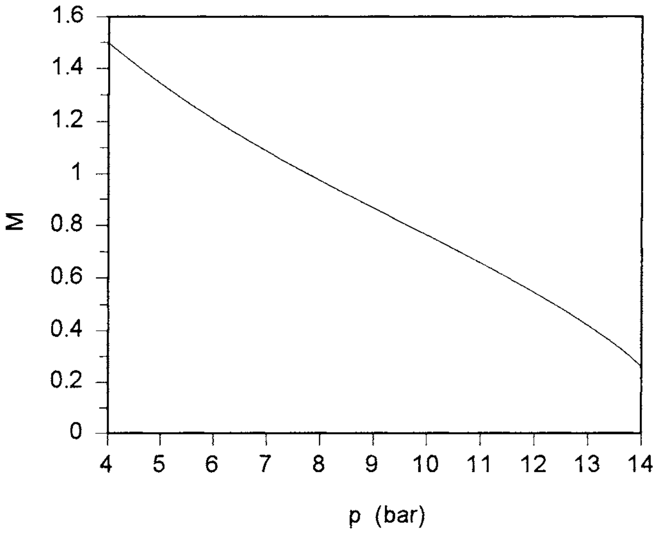
IT Results for p = 10 bar

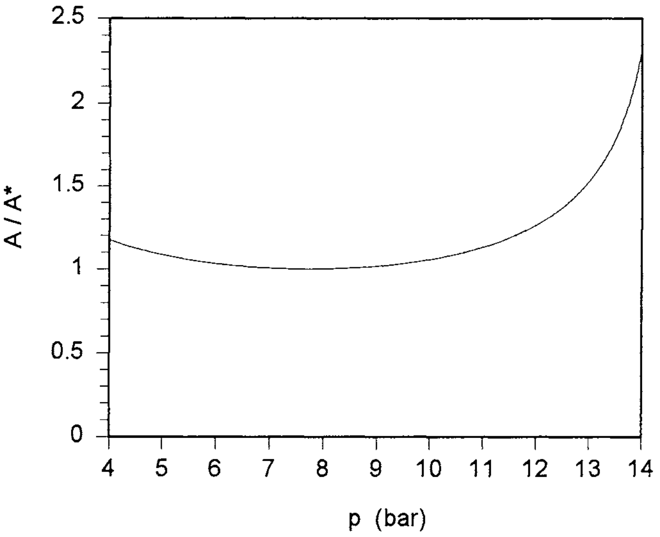
T_o = 420.5 K
p_o = 14.6 bar
T = 290 K
c = 341.3 m/s
V = 260 m/s
M = 0.7616
A/A* = 1.056

PROBLEM 9.120 (Cont'd.)

PLOTS:







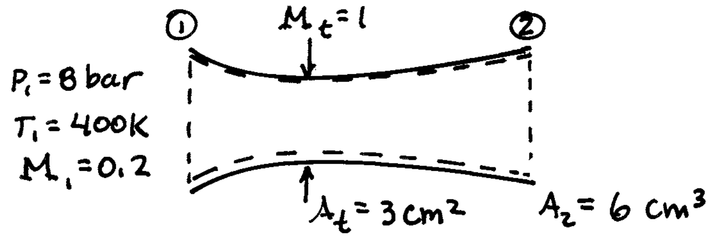
Note that the minimum value of A/A* occurs at a Mach number of unity, as expected.

PROBLEM 9.121

KNOWN: A converging-diverging nozzle operates at steady state with isentropic flow of air.

FIND: Determine specified conditions at the nozzle exit and throat.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The air behaves as an ideal gas, with $k = 1.4$.

ANALYSIS: Use Table 9.1.

First, find the stagnation state.

$$M_1 = 0.2 \Rightarrow T_1/T_0 = 0.99206 \Rightarrow T_0 = 403.2\text{K}$$

$$P_1/P_0 = 0.97250 \Rightarrow P_0 = 8.226 \text{ bar}$$

If the nozzle flow is choked, $M_t = 1$, and

$$T_t = T_0(.83333) = 336.0\text{K}$$

$$P_t = P_0(.52828) = 4.3456 \text{ bar}$$

Further, $v_{th} = \dfrac{RT_t}{P_t} = 0.2219 \text{ m}^3/\text{kg}$

$$V_{th} = (1)\sqrt{kRT_{th}} = 367.4 \text{ m/s}$$

Thus, the mass flow rate is

$$\dot{m} = \frac{A_t V_t}{v_t} = 0.497 \text{ kg/s} \longleftarrow \dot{m}$$

Now, with $A_2/A^* = 6\text{ cm}^2/3\text{ cm}^2 = 2$, there are two cases in Table 9.1.

Supersonic

$$M_2 \approx 2.20 \Rightarrow P_2 = (.09352)P_0 = 0.769 \text{ bar} \longleftarrow P_2$$

$$T_2 = (.50813)T_0 = 204.9\text{ K} \longleftarrow T_2$$

Subsonic

$$M_1 \approx 0.3079 \Rightarrow P_2 = (.93601)P_0 = 7.70 \text{ bar} \longleftarrow P_2$$

$$T_2 = (.98127)T_0 = 395.6\text{ K} \longleftarrow T_2$$

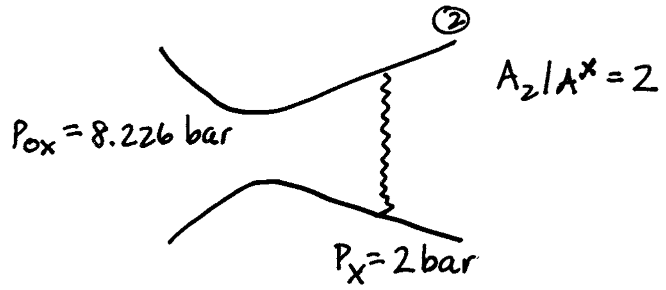
PROBLEM 9.122

$M_x = 2.2 \Rightarrow M_y = 0.54706$ and $P_y/P_x = 5.48$

with $P_x = 0.769$; $P_y = P_b = 4.214$ bar ⟵ back pressure

PROBLEM 9. 123

(See Problem 9.121 for details.)



$$\frac{P_x}{P_{ox}} = \frac{2}{8.226} = 0.24313 \Rightarrow M_x = 1.579$$

Using this value in Table 9.2 gives $P_{oy}/P_{ox} = 0.90246$. Thus, with $P_{ox} = 8.226$; $P_{oy} = 7.424$ bar.

Now, for $A_2/A^* = 2$, and subsonic flow after the shock, from Table 9.1

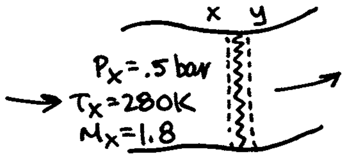
$$P_2/P_o = 0.93601 \Rightarrow P_2 = 6.949 \text{ bar} \longleftarrow \underline{\text{back}} \text{ pressure}$$

PROBLEM 9.124

KNOWN: Air undergoes a normal shock. Conditions upstream are known.

FIND: Determine (a) p_y, (b) p_{ox}, (c) T_{ox}, and (d) the change in specific entropy across the shock, $s_y - s_x$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: The air is an ideal gas with constant specific heat ratio, k = 1.4.

ANALYSIS: (a) Using Table 9.2, at $M_x = 1.8$;

$$p_y/p_x = 3.6133 \Rightarrow p_y = (3.6133)(.5\text{ bar}) = 1.8066\text{ bar} \leftarrow p_y$$

(b) Using Table 9.1, at $M_x = 1.8$;

$$p_x/p_{ox} = 0.17404 \Rightarrow p_{ox} = \frac{(.5\text{ bars})}{.17404} = 2.873\text{ bar} \leftarrow p_{ox}$$

(c) Also from Table 9.1; $T_x/T_{ox} = .60680 \Rightarrow T_{ox} = 461.4\text{ K} \leftarrow T_{ox}$

(d) To calculate $s_y - s_x$,

$$s_y - s_x = c_p \ln(T_y/T_x) - R \ln(p_y/p_x)$$

These ratios can be read directly from Table 9.2;

$$s_y - s_x = 1.004 \ln(1.5316) - \frac{8.314}{28.97} \ln(3.6133)$$
$$= 0.05935\text{ kJ/kg}\cdot\text{K} \leftarrow \Delta s$$

The data for the required plots are obtained using IT, as follows:

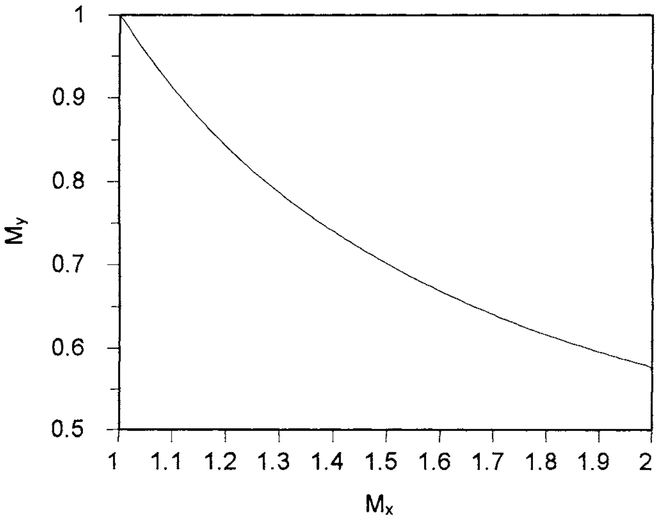
IT Code

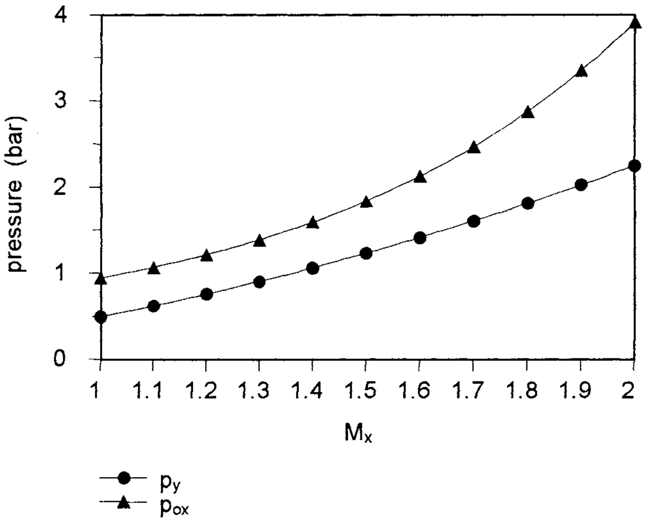
```
px = 0.5  // bar
Tx = 280  // K
Mx = 1.8
k = 1.4

My^2 = (Mx^2+(2/(k-1)))/(((2*k)/(k-1))*Mx^2-1)
py / px = (1+k*Mx^2)/(1+k*My^2)
Ty / Tx = (1+((k-1)/2)*Mx^2)/(1+((k-1)/2)*My^2)
pox / px = (1+((k-1)/2)*Mx^2)^(k/(k-1))
Tox / Tx = (pox / px)^((k-1)/k)
dels = 1.004 * ln(Ty/Tx) - (8.314 / 28.97) * ln(py/px)
```

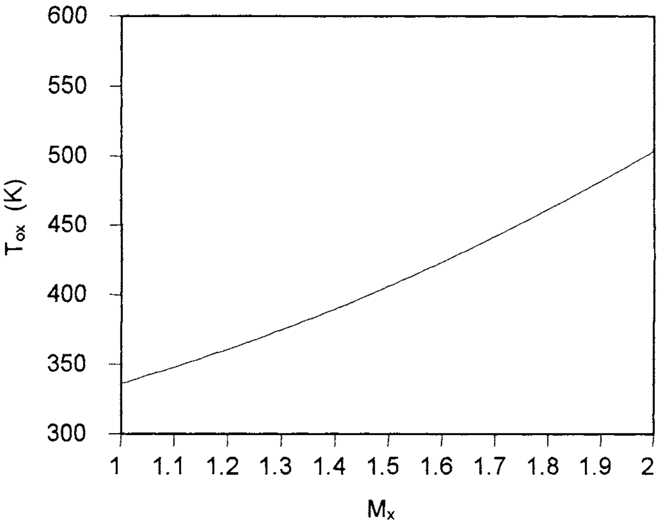
IT Results for Mx = 1.8

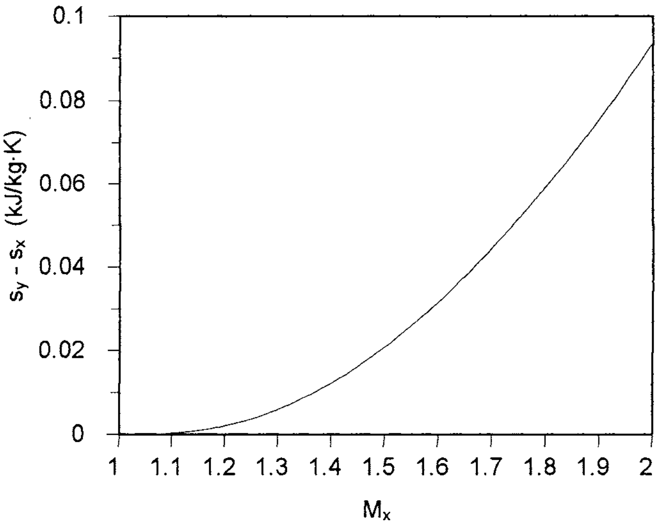
```
My = 0.6165
Tox = 461.4 K
Ty = 428.8 K
pox = 2.873 bar
py = 1.807 bar
sy - sx = 0.05933 kJ/kg·K
```

PROBLEM 9.124 (Cont'd.)







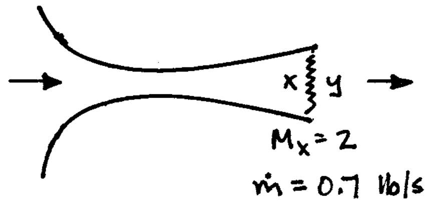


PROBLEM 9.125

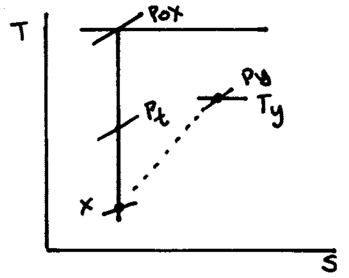
KNOWN: Air flows through a converging-diverging nozzle and discharges to the atmosphere. The flow is isentropic up to the exit plane where a normal shock stands.

FIND: Determine (a) the stagnation pressure, p_{ox}, (b) the stagnation temperature, T_{ox}, and (c) the nozzle exit area.

SCHEMATIC & GIVEN DATA:



$p_y = 14.7$ lbf/in²
$T_y = 540°R$



ASSUMPTIONS: (1) The air behaves as an ideal gas, with constant $k = 1.4$. (2) The flow up to the shock is isentropic.

ANALYSIS: (a) Using Table 9.2, at $M_x = 2.0$;

$$p_y/p_x = 4.5000 \Rightarrow p_x = 3.267 \text{ lbf/in}^2.$$

and, from Table 9.1; $p_x/p_{ox} = 0.12780 \Rightarrow p_{ox} = 25.56\ \frac{\text{lbf}}{\text{in}^2} \longleftarrow$ p_{ox}

(b) Similarly, $T_y/T_x = 1.6875 \Rightarrow T_x = 320°R$

and $T_x/T_{ox} = .55556 \Rightarrow T_{ox} = 576°R \longleftarrow$ T_{ox}

(c) The velocity at x is

$$V_x = M_x\sqrt{kRT_x} = 2\sqrt{(1.4)\left(\frac{1545}{28.97}\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(320\,°\text{R})\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|}$$
$$= 1754 \text{ ft/s}$$

Thus

$$A_x = \frac{\dot{m}RT_x}{V_x p_x} = \frac{(.7\ \text{lb/s})\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(320°\text{R})}{(1754\ \text{ft/s})(3.267\ \text{lbf/in}^2)} = 2.08\ \text{in}^2 \longleftarrow$$ A_x

---

PROBLEM 9.126

KNOWN: See Problem 9.125

FIND: Determine the throat area and the entropy produced per unit mass of air flowing.

ANALYSIS: From Table 9.1 ; $A_x/A^* = 1.6875 \Rightarrow A^* = 1.185$ in.² $\longleftarrow$ throat area

the entropy produced is

$$\dot{\sigma}/\dot{m} = s_y - s_x = c_p \ln(T_y/T_x) - R\ln(p_y/p_x)$$
$$= \left(0.240\ \frac{\text{Btu}}{\text{lb}°\text{R}}\right)\ln(1.6875) - \left(\frac{1.986}{28.97}\cdot\frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln(4.5000)$$
$$= 0.0225\ \text{Btu/lb}\cdot°\text{R} \longleftarrow$$ $\dot{\sigma}/\dot{m}$

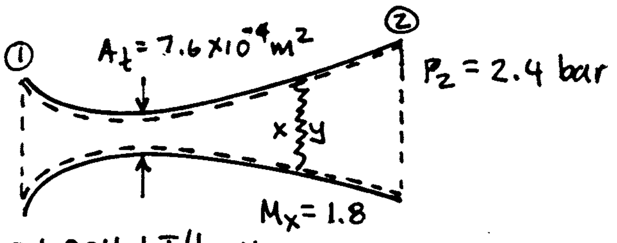
PROBLEM 9.127

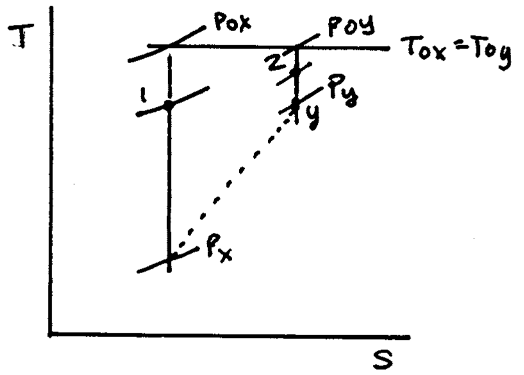
KNOWN: Air flows through a converging diverging nozzle. A normal shock stands in the diverging section, and the flow is isentropic elsewhere.

FIND: Determine (a) T_{ox}, (b) p_{ox}, (c) p_x, (d) p_y, (e) p_{oy}, and (f) T_{oy}. Also, determine the mass flow rate and the exit plane area.

SCHEMATIC & GIVEN DATA:

$p_1 = 3.4$ bar
$T_1 = 530$ K
$M_1 = 0.4$
$k = 1.4$, $c_p = 1.004$ kJ/kg·K





ASSUMPTIONS: The air behaves as an ideal gas with constant specific heats ($k = 1.4$).

ANALYSIS: Use Tables 9.1 and 9.2.

(a) $M_1 = 0.4 \Rightarrow T_1/T_{ox} = 0.96899 \Rightarrow T_{ox} = 547.0 \longleftarrow T_{ox}$

(b) $p_1/p_{ox} = 0.89562 \Rightarrow p_{ox} = 3.796$ bar $\longleftarrow p_{ox}$

(c) at $M_x = 1.8$; $p_x/p_{ox} = 0.17404 \Rightarrow p_x = 0.6607$ bar $\longleftarrow p_x$

(d) at $M_x = 1.8$; $p_y/p_x = 3.6133 \Rightarrow p_y = 2.387$ bar $\longleftarrow p_y$

(e) $M_y = 0.61650 \Rightarrow p_y/p_{oy} = 0.77359 \Rightarrow p_{oy} = 3.086$ bar $\longleftarrow p_{oy}$

(f) $T_{oy} = T_{ox} = 547.0$ K $\longleftarrow T_{oy}$

To determine the mass flow rate, we first determine the conditions at the throat, where the area is known. That is

$$T^* = (0.83333)(547.0\,\text{K}) = 455.8\,\text{K}$$

$$V_{th} = c^* = \sqrt{kRT^*} = \sqrt{(1.4)\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(455.8\,\text{K})\left|\frac{10^3\,\text{N}\cdot\text{m}}{1\,\text{kJ}}\right|\left|\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right|} = 427.9\,\text{m/s}$$

$$p^* = (0.52828)(3.796\,\text{bar}) = 2.005\,\text{bar}$$

Thus

$$\dot{m} = \frac{p^* A_{th} V_{th}}{RT^*} = \frac{(2.005\,\text{bar})(7.6\times10^{-4}\,\text{m}^2)(427.9\,\text{m/s})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(455.8\,\text{K})}\left|\frac{10^5\,\text{N/m}^2}{1\,\text{bar}}\right|\left|\frac{1\,\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right| = 0.498\,\frac{\text{kg}}{\text{s}} \longleftarrow \dot{m}$$

Now, downstream of the shock, $p_{oy} = 3.086$ bar. Thus $p_2/p_{oy} = 0.7777 \Rightarrow M_2 = 0.61$

Also, $T_2/T_{oy} = 0.93064 \Rightarrow T_2 = 509.1$ K. Now

$$V_2 = M_2\sqrt{kRT_2} = (0.61)\sqrt{(1.4)\left(\frac{8.314}{28.97}\right)(509.1)\left|10^3\right|} = 275.9\,\text{m/s}$$

Finally

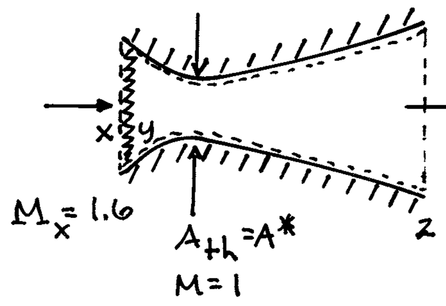
$$A_2 = \frac{\dot{m}RT_2}{p_2V_2} = \frac{(0.498)\left(\frac{8.314}{28.97}\right)(455.8)}{(2.4)(275.9)}\left|\frac{10^3}{10^5}\right| = 9.84\times10^{-4}\,\text{m}^2 \longleftarrow A_2$$

PROBLEM 9.128*

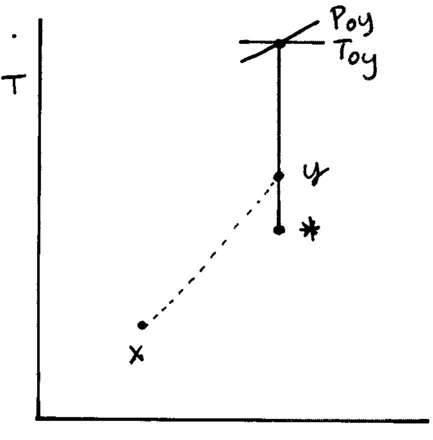
KNOWN: Air enters a converging-diverging channel at a supersonic Mach number. A normal shock stands at the inlet, and downstream of the shock, the flow is isentropic. The mass flow rate, and conditions at the exit are known.

FIND: Determine the inlet and throat areas.

SCHEMATIC & GIVEN DATA:



$p_2 = p_{oy} = 20$ lbf/in²
$T_2 = T_{oy} = 700$ °R
$\dot{m} = 45$ lb/s
$V_2 \approx 0$



ASSUMPTIONS: (1) The control volume is at steady state. (2) Downstream of the normal shock, the flow is isentropic. (3) Potential energy effects and the exit kinetic energy can be neglected. (4) The air behaves as an ideal gas with k=1.4.

ANALYSIS: Using data from Table 9.2; $M_y = 0.66844$. Thus, interpolating in Table 9.1 at $M = 0.66844$

$$T_y/T_{oy} = 0.91772$$
$$p_y/p_{oy} = 0.74083$$
$$A_y/A^* = 1.12398$$

With $T_{oy} = 700$ °R ; $T_y = 642.4$ °R and with $p_{oy} = 20$ lbf/in²; $p_y = 14.817$ lbf/in²

To find V_y

$$V_y = M_y\sqrt{kRT_y} = (0.66844)\sqrt{(1.4)\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(642.4\,{}^\circ\text{R})\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|}$$
$$= 830.7\ \text{ft/s}$$

Also,
$$v_y = \frac{RT_y}{p_y} = \frac{\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(642.4\,{}^\circ\text{R})}{(14.817\ \text{lbf/in}^2)}\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| = 16.057\ \text{ft}^3/\text{lb}$$

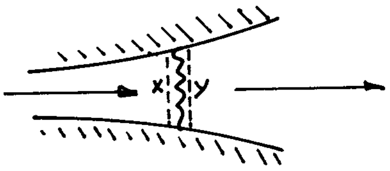
Thus
$$A_y = \frac{\dot{m}v_y}{V_y} = \frac{(45\ \text{lb/s})(16.057\ \text{ft}^3/\text{lb})}{(830.7\ \text{ft/s})} = 0.8698\ \text{ft}^2 \longleftarrow A_y$$

And, finally
$$A^* = A_y/1.12398 = 0.7739\ \text{ft}^2 \longleftarrow A^*$$

PROBLEM 9.129 (Derivations)

FIND: Derive (a) Eq. 9.55, (b) Eq. 9.56, (c) Eq. 9.57

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state. (2) The gas flowing is modeled as an ideal gas with constant specific heat ratio k. (3) $A_x = A_y$.

ANALYSIS: (a) Conservation of mass is expressed by Eq. 9.46. Conservation of energy is expressed by Eq. 9.47 which can be rewriten as Eq. 9.53. Newton's Second Law of motion is expressed as Eq. 9.48 which can be rewritten as Eq. 9.48. Using ideal gas relations these can be combined as follows:

$$\rho_x V_x = \rho_y V_y$$

$$\frac{p_x}{RT_x} M_x \sqrt{kRT_x} = \frac{p_y}{RT_y} M_y \sqrt{kRT_y}$$

$$\frac{p_x M_x}{\sqrt{T_x}} = \frac{p_y M_y}{\sqrt{T_y}} \qquad (1)$$

$$\frac{M_x \left(1 + \frac{k-1}{2} M_x^2\right)^{1/2}}{1 + kM_x^2} = \frac{M_y \left(1 + \frac{k-1}{2} M_y^2\right)^{1/2}}{1 + kM_y^2}$$

One solution to this equation is $M_y = M_x$, corresponding to no change in properties in a flow through a control volume with $A_x = A_y$. This trivial solution is of no interest here. When the above equation is squared it becomes

$$\underbrace{\frac{M_x^2 \left(1 + \frac{k-1}{2} M_x^2\right)}{(1 + kM_x^2)^2}}_{L} = \frac{M_y^2 \left(1 + \frac{k-1}{2} M_y^2\right)}{(1 + kM_y^2)^2}$$

or, when expressed as a quadratic equation in M_y^2

$$M_y^4 \left[\left(\frac{k-1}{2}\right) - k^2 L\right] + M_y^2 \left[1 - 2kL\right] - L = 0$$

Solving the quadratic equation for M_y^2 gives Eq. 9.55.

(b). With Eq. 9.51

$$p_{oy} = p_y \left[1 + \left(\frac{k-1}{2}\right) M_y^2\right]^{k/(k-1)}$$

$$p_{ox} = p_x \left[1 + \left(\frac{k-1}{2}\right) M_x^2\right]^{k/(k-1)}$$

Forming a ratio

$$\frac{p_{oy}}{p_x} = \frac{p_y}{p_x} \left[\frac{1 + \left(\frac{k-1}{2}\right) M_y^2}{1 + \left(\frac{k-1}{2}\right) M_x^2}\right]^{k/(k-1)}$$

Introducing Eq. (1) above together with Eq. 9.53

PROBLEM 9.129 (Cont'd)

$$\frac{p_{0y}}{p_{0x}} = \frac{M_x}{M_y}\left[\frac{1+\left(\frac{k-1}{2}\right)M_x^2}{1+\left(\frac{k-1}{2}\right)M_y^2}\right]^{1/2}\left[\frac{1+\left(\frac{k-1}{2}\right)M_y^2}{1+\left(\frac{k-1}{2}\right)M_x^2}\right]^{k/(k-1)}$$

which simplifies to yield Eq. 9.56.

(c) With Eq. 9.52

$$A_x^* = \frac{M_x A_x}{\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}M_x^2\right)\right]^{(k+1)/2(k-1)}}$$

$$A_y^* = \frac{M_y A_y}{\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}M_y^2\right)\right]^{(k+1)/2(k-1)}}$$

Then, since $A_y = A_x$

$$\frac{A_x^*}{A_y^*} = \frac{M_x}{M_y}\left[\frac{1+\left(\frac{k-1}{2}\right)M_y^2}{1+\left(\frac{k-1}{2}\right)M_x^2}\right]^{(k+1)/2(k-1)}$$

Finally, introducing Eq. 9.56 gives Eq. 9.57.

PROBLEMS 9.130 and 9.131

The computer programs to be written in these problems involve straightforward evaluations of equations in the text.

9.130 Eqs. 9.50, 9.51, and 9.52

9.131 Eqs. 9.53, 9.54, 9.55, and 9.56

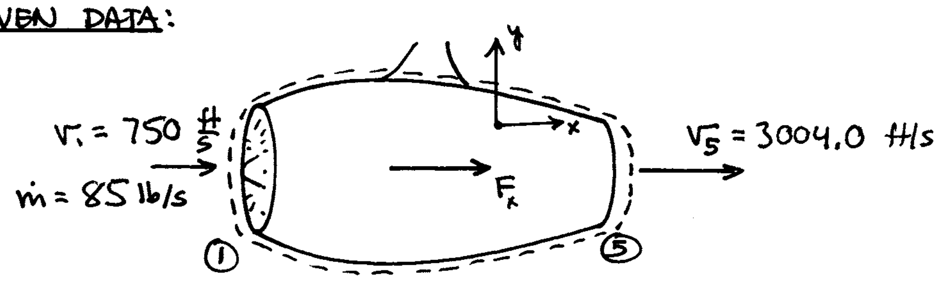
As computers and software vary, the details of the programs are left to the reader.

PROBLEM 9.95*

KNOWN: Conditions are known for the turbojet engine analyzed in Problem 9.78.

FIND: Calculate the thrust developed by the engine.

SCHEMATIC & GIVEN DATA:



① ASSUMPTIONS: (1) The control volume is at steady state. (2) The force F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m}\,(V_5 - V_1)$$

$$= \left(85\ \frac{\text{lb}}{\text{s}}\right)(3004.0 - 750)\ \frac{\text{ft}}{\text{s}}\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|$$

$$= 5950\ \text{lbf}$$

The thrust is the force developed by the engine. Thus

$$F_{\text{thrust}} = -F_x = \underset{\uparrow\ \text{to the}\ \underline{\text{left}}}{-}\ 5950\ \text{lbf} \longleftarrow \underline{\text{thrust}}$$

---

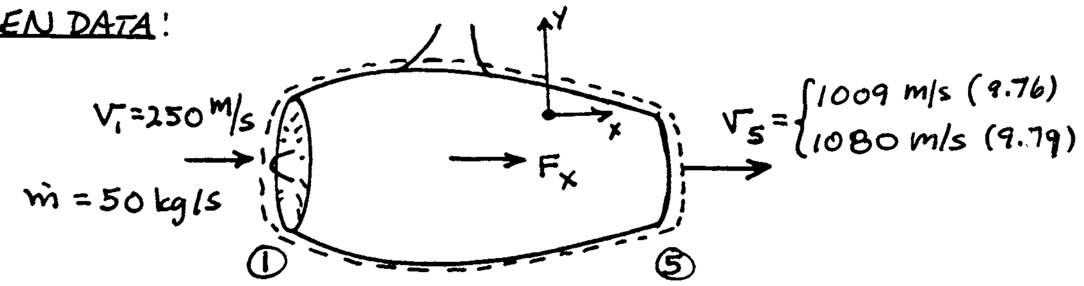
1. The effect of the surrounding pressure on the control volume cancels.

PROBLEM 9.96

KNOWN: The mass flow rate is given for the cycle in Problem 9.76 and for the modified cycle in Problem 9.79.

FIND: Calculate the thrust developed by the engine in each case. Discuss.

SCHEMATIC & GIVEN DATA:



$V_1 = 250$ m/s
$\dot{m} = 50$ kg/s
F_x
y
x
①
⑤
$V_5 = \begin{cases} 1009 \text{ m/s } (9.76) \\ 1080 \text{ m/s } (9.79) \end{cases}$

ASSUMPTIONS: (1) The control volume is at steady state. (2) The force ① F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m}(V_5 - V_1)$$

The thrust is the force developed by the control volume. Thus

$$F_{thrust} = -\dot{m}(V_5 - V_1)$$

Inserting values, first for Problem 9.81 (turbojet)

$$F_{thrust} = -\left(50 \tfrac{kg}{s}\right)(1009 - 250)\tfrac{m}{s} \left| \frac{1 \text{ kN}}{10^3 \text{ kg} \cdot \text{m/s}^2} \right|$$

$$= -37.95 \text{ kN (to the left)} \longleftarrow \underset{(\text{Prob. } 9.76)}{\text{thrust}}$$

Similarly, for Problem 9.84 (turbojet + afterburner)

$$F_{thrust} = -(50)(1080 - 250)\left|\frac{1}{10^3}\right|$$

$$= -41.5 \text{ kN (to the left)} \longleftarrow \underset{(\text{Prob. } 9.79)}{\text{thrust}}$$

The addition of the afterburner increases the velocity of gases leaving the engine. In this case, the result is a 9.3% increase in thrust developed by the engine.

---

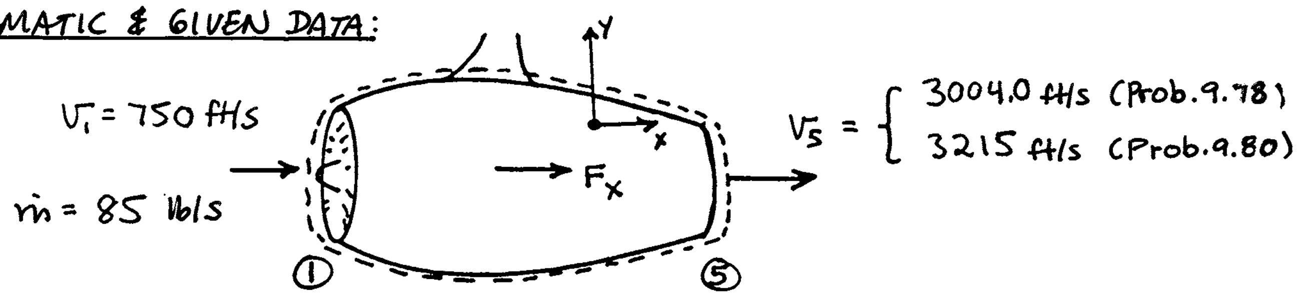
1. The effect of the surrounding pressure on the control volume boundary cancels.

PROBLEM 9.97*

KNOWN: The turbojet in Problem 9.78 and the modified turbojet in Problem 9.80 are reconsidered.

FIND: Calculate the thrust developed by each engine. Discuss.

SCHEMATIC & GIVEN DATA:



$V_1 = 750$ ft/s
$\dot m = 85$ lb/s
F_x
y
x
①
⑤
$V_5 = \begin{cases} 3004.0 \text{ ft/s (Prob. 9.78)} \\ 3215 \text{ ft/s (Prob. 9.80)} \end{cases}$

ASSUMPTIONS: (1) The control volume is at steady state. (2) The force ① F_x is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot m (V_5 - V_1)$$

The thrust is the force developed by the control volume. Thus

$$F_{thrust} = -F_x = -\dot m (V_5 - V_1)$$

Inserting values, first for Problem 9.78 (turbojet)

$$F_{thrust} = -\left(85 \frac{lb}{s}\right)(3004.0 - 750)\frac{ft}{s}\left|\frac{1 \text{ lbf}}{32.2 \text{ lb}\cdot\text{ft/s}^2}\right|$$

$$= -5950 \text{ lbf (to the left)} \longleftarrow \underline{\text{thrust}} \text{ (Prob. 9.78)}$$

Similarly, for Problem 9.80 (turbojet + afterburner)

$$F_{thrust} = -(85)(3215 - 750)\left|\frac{1}{32.2}\right|$$

$$= -6507 \text{ lbf (to the left)} \longleftarrow \underline{\text{thrust}} \text{ (Prob. 9.80)}$$

The addition of an afterburner increases the velocity of gases leaving the engine. In this case, the result is a 9.4% increase in thrust developed by the engine.

---

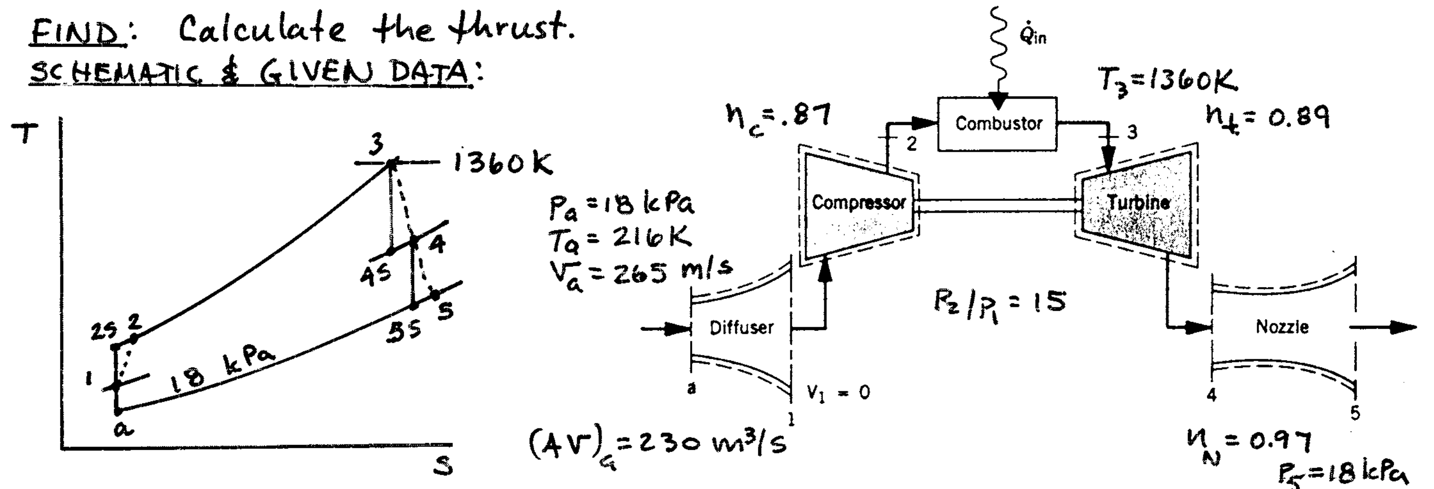
1. The effect of the surrounding pressure on the control volume boundary cancels.

PROBLEM 9.98

KNOWN: A turbojet engine is analyzed on an air-standard basis. Data are known at various locations.

FIND: Calculate the thrust.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: Same as in Example 9.12, except $\eta_c = 0.87$, $\eta_t = 0.89$, $\eta_N = 0.97$.

ANALYSIS: To determine the thrust, we begin by finding the velocity at the nozzle exit, V_5. First, fix each of the principal states (Table A-22).

State a $T_a = 216$ K $\Rightarrow$ $h_a = 215.97$ kJ/kg, $p_{ra} = 0.44088$

State 1 Applying the energy balance to the diffuser; $0 = h_a + \frac{V_a^2}{2} - h_1$. Thus

$$h_1 = h_a + \frac{V_a^2}{2} = 215.97\ \frac{kJ}{kg} + \frac{265^2\ m^2/s^2}{2}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kJ}{10^3\ N\cdot m}\right| = 251.08\ kJ/kg$$

Since the diffuser is assumed to operate isentropically

$$p_1 = (p_{r1}/p_{ra})\,p_a = (0.74394/0.44088)(18\ kPa) = 30.373\ kPa$$

State 2 For isentropic compression, $p_{r2s} = p_{r1}(p_2/p_1) = 11.159$ and $h_{2s} = 545.16\ \frac{kJ}{kg}$ with the compressor efficiency

$$h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 589.10\ kJ/kg\ ,\quad p_2 = 15 \cdot p_1 = 455.6\ kPa$$

State 3 $T_3 = 1360$ K $\Rightarrow$ $h_3 = 1467.49$ kJ/kg, $p_{r3} = 399.1$

State 4 For a turbojet, $\dot{W}_c/\dot{m} = \dot{W}_t/\dot{m} \Rightarrow (h_2 - h_1) = \eta_t(h_3 - h_{4s})$. Thus $h_{4s} = 1087.69$ kJ/kg $\Rightarrow$ $p_{r4s} = 131.51$ and $p_4 = (p_{r4s}/p_{r3})\,p_3 = 150.13$ kPa

Now, with the turbine efficiency

$$h_4 = h_3 - (h_3 - h_{4s})\eta_t = 1129.47\ kJ/kg \Rightarrow p_{r4} = 151.07$$

State 5 For isentropic expansion through the nozzle

$$p_{r5s} = (p_5/p_4)\,p_{r4} = 18.113 \Rightarrow h_{5s} = 625.61\ kJ/kg$$

Now, with the isentropic nozzle efficiency

$$V_5 = \sqrt{\eta_N}\,V_{5s} = \sqrt{\eta_N\, 2(h_4 - h_{5s})}$$

$$= \sqrt{(0.97)\,2\,(1129.47 - 625.61)\frac{kJ}{kg}\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ kg\cdot m/s^2}{1\ N}\right|} = 988.7\ m/s$$

Now, applying Eq. 9.31 to a control volume enclosing the entire engine

$$F_x = \dot{m}\,(V_5 - V_a)$$

Where F_x is the net force acting on the control volume.

PROBLEM 9.98 (Cont'd.)

The mass flow rate is found from $\dot{m} = (AV)_a/v_a$, where

$$v_a = \frac{RT_a}{p_a} = \frac{\left(\frac{8.314}{28.97}\ \frac{kJ}{kg\cdot K}\right)(216\ K)}{(18\ kPa)}\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|\left|\frac{1\ kPa}{10^3\ N/m^2}\right| = 3.444\ m^3/kg$$

and $$\dot{m} = \frac{230\ m^3/s}{3.444\ m^3/kg} = 66.78\ kg/s$$

Thus $$F_x = \left(66.78\ \frac{kg}{s}\right)(988.7\ m/s - 265\ m/s)\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{1\ kN}{10^3\ N}\right| = 48.33\ kN$$

Finally, the thrust is the force developed by the engine, or

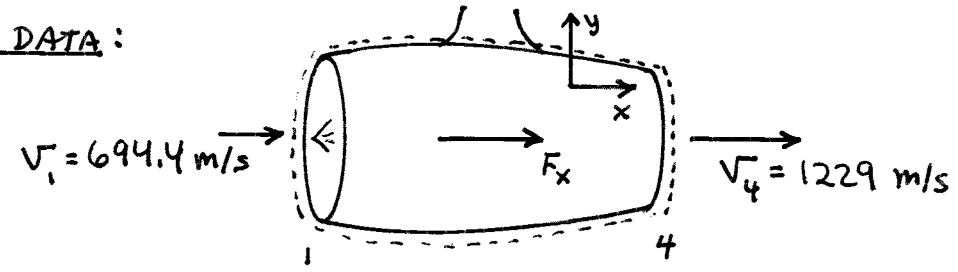
$$F_{thrust} = -F_x = \underset{\uparrow\ \text{to the }\underline{\text{left}}}{-}48.33\ kN \xleftarrow{\hspace{6cm}} F_{thrust}$$

PROBLEM 9.99*

KNOWN: Conditions are known for the ramjet analyzed in Problem 9.82.

FIND: Determine the ratio of the thrust developed to the mass flow rate of air.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) The force F_x ① is the net force acting on the control volume.

ANALYSIS: Applying Eq. 9.31

$$F_x = \dot{m}\,(V_4 - V_1)$$

or

$$\frac{F_x}{\dot{m}} = V_4 - V_1$$

$$= (1229 - 694.4)\,\frac{\text{m}}{\text{s}}\left|\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right|$$

$$= 534.6\ \frac{\text{N}}{\text{kg/s}}$$

The thrust is the force developed by the engine. Thus

$$\frac{F_{\text{thrust}}}{\dot{m}} = -\frac{F_x}{\dot{m}} = \underset{\text{to the left}}{-}\,534.6\ \frac{\text{N}}{\text{kg/s}} \longleftarrow \frac{F_{\text{thrust}}}{\dot{m}}$$

---

1. The effect of the surrounding pressure on the control volume boundary cancels.

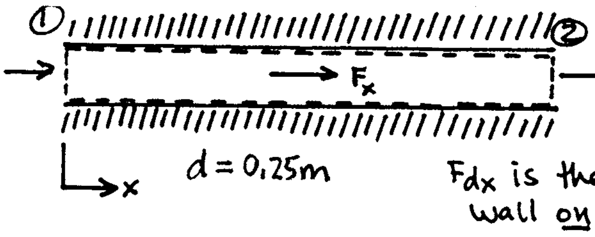
PROBLEM 9.100*

KNOWN: Air flows through a horizontal, well-insulated duct of constant cross-sectional area. Data are known at the inlet and exit.

FIND: Determine the magnitude and direction of the net horizontal force exerted by the duct wall on the air.

SCHEMATIC & GIVEN DATA:

$p_1 = 2.4$ bar
$T_1 = 430$K



$T_2 = 370$K
$\dot{m} = 600$ kg/min
$= 10$ kg/s

F_{dx} is the force of the duct wall on the air.

ASSUMPTIONS: (1) The control volume is at steady state, and $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) Potential energy effects are negligible. (3) The air behaves as an ideal gas.

ANALYSIS: To find the force F_x, we need to evaluate the velocities at ① and ② and p_2. First

$$v_1 = \frac{RT_1}{p_1} = \frac{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(430)}{(2.4\text{ bars})}\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right|\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right| = 0.5142\text{ m}^3/\text{kg}$$

$$A = \pi d^2/4 = 0.0491\text{ m}^2$$

and
$$V_1 = \frac{\dot{m}v_1}{A} = \frac{(10\text{ kg/s})(0.5142\text{ m}^3/\text{kg})}{(0.0491\text{ m}^2)} = 104.73\text{ m/s}$$

To find V_2, begin with the energy balance at steady state

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1-h_2) + \left(\frac{V_1^2-V_2^2}{2}\right) + g\cancelto{0}{(z_1-z_2)}\right]$$

or
$$V_2 = \sqrt{2(h_1-h_2) + V_1^2}$$

From Table A-22; $h_1 = 431.43$ kJ/kg and $h_2 = 370.67$ kJ/kg. Thus

$$V_2 = \sqrt{2(431.43-370.67)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right|\left|\frac{10^3\text{ N}\cdot\text{m}}{1\text{ kJ}}\right| + 104.73^2\text{ m}^2/\text{s}^2}$$
$$= 364.0\text{ m/s}$$

Now, with $AV_1/v_1 = AV_2/v_2 = \dot{m}$

$$v_2 = \frac{AV_2}{\dot{m}} = \frac{(0.0491\text{ m}^2)(364.0\text{ m/s})}{(10\text{ kg/s})} = 1.7872\text{ m}^3/\text{kg}$$

And
$$p_2 = \frac{RT_2}{v_2} = \frac{\left(\frac{8.314}{28.97}\right)(370)}{(1.7872)}\left|\frac{10^3}{10^5}\right| = 0.594\text{ bar} = 59.4\text{ kPa}$$

Finally, using Eq. 9.31; $p_1A - p_2A + F_x = \dot{m}(V_2 - V_1)$. Thus

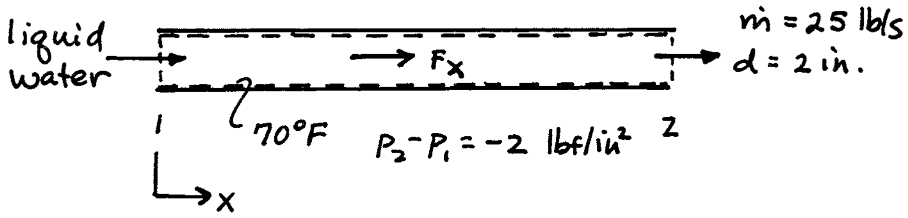
$$F_x = \dot{m}(V_2-V_1) + (p_2-p_1)A$$
$$= \left(10\frac{\text{kg}}{\text{s}}\right)(364.0-104.73)\frac{\text{m}}{\text{s}}\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right| + (0.594-2.4)\text{ bar}\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|(0.0491\text{ m}^2)$$
$$= -6275\text{ N (to the left)} \xleftarrow{\hspace{6em}} F_x$$

PROBLEM 9.101

KNOWN: Water flows at steady state through a constant-diameter horizontal pipe. Data are known for flow through the pipe.

FIND: Determine the magnitude and direction of the horizontal force needed to hold the pipe in place.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state. (2) The water is modeled as incompressible, with $v = v_{f@70°F}$. (3) The area of the pipe is constant.

ANALYSIS: Begin with the mass balance

$$\frac{\cancel{A_1}V_1}{\cancel{v_1}} = \frac{\cancel{A_2}V_2}{\cancel{v_2}} \Rightarrow V_1 = V_2$$

Now, applying Eq. 9.31

$$F_x + P_1A - P_2A = \dot{m}(\cancelto{0}{V_2 - V_1})$$

or

$$F_x = (P_2 - P_1)A = (P_2 - P_1)\left(\frac{\pi d^2}{4}\right)$$

$$= (-2 \text{ lbf/in}^2)\left(\frac{\pi\, 2^2 \text{ in}^2}{4}\right)$$

$$= -6.283 \text{ lbf (left)} \xleftarrow{\qquad\qquad\qquad F_x}$$

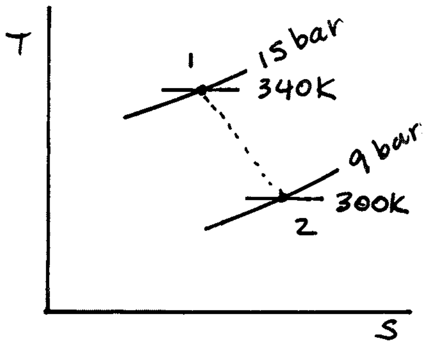
This is the force of the pipe on the fluid, which acts to the left. Since the fluid force on the pipe is to the right, this is also the direction of the force required to hold the pipe in place.

PROBLEM 9.102

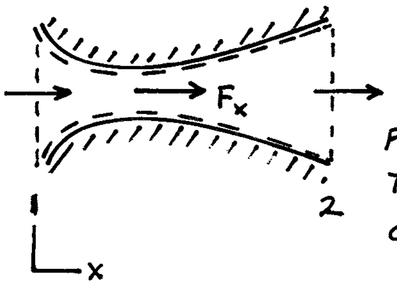
KNOWN: Air flows through a horizontal, well-insulated duct of varying cross-sectional area. Data are known at the inlet and exit.

FIND: Determine (a) the net force exerted by the duct on the air, and (b) the exergy destruction rate.

SCHEMATIC & GIVEN DATA:



$p_1 = 15$ bar
$T_1 = 340$K
$V_1 = 20$ m/s



$p_2 = 9$ bar
$T_2 = 300$K
$d_2 = 1$ cm $= 0.01$ m

ASSUMPTIONS: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) Potential energy effects are negligible. (3) The air is modeled as an ideal gas, which can readily be verified. (4) For part (b), let $T_0 = 300$K and $p_0 = 1$ bar.

ANALYSIS: (a) To find the force F_x, use Eq. 9.31

$$F_x + p_1 A_1 - p_2 A_2 = \dot{m}\,(V_2 - V_1) \qquad (*)$$

Now, find V_2 using the energy balance for the steady-state control volume

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g\,\cancelto{0}{(z_1 - z_2)}\right]$$

where $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$. Thus, with data from Table A-22

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

$$= \sqrt{2(340.42 - 300.19)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right| + 20^2\ \text{m}^2/\text{s}^2}$$

$$= 284.4\ \text{m/s}$$

Next, calculate A_2

$$A_2 = \frac{\pi d_2^2}{4} = 7.854\times10^{-5}\ \text{m}^2$$

and

$$\dot{m} = \frac{A_2 V_2}{v_2} = \frac{A_2 V_2 p_2}{R T_2}$$

$$= \frac{(7.854\times10^{-5}\ \text{m}^2)(284.4\ \text{m/s})(9\ \text{bar})}{\left(\frac{8.314}{28.97}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(300\text{K})}\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 0.2335\ \text{kg/s}$$

Further

$$A_1 = \frac{\dot{m} v_1}{V_1} = \frac{\dot{m} R T_1}{V_1 p_1} = \frac{(0.2335)\left(\frac{8.314}{28.97}\right)(340)}{(20)(15)}\left|\frac{10^3}{10^5}\right| = 7.595\times10^{-4}\ \text{m}^2$$

PROBLEM 9.102

Solving (*) for F_x

$$F_x = \dot{m}(V_2 - V_1) + p_2 A_2 - p_1 A_1$$

Inserting values

$$F_x = \left(0.2335\ \frac{kg}{s}\right)(284.4 - 20)\frac{m}{s}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|$$

$$+ (9\ bar)(7.854\times10^{-5}\ m^2)\left|\frac{10^5\ N/m^2}{1\ bar}\right| - (15)(7.595\times10^{-4})\left|10^5\right|$$

$$= -1007\ N \longleftarrow F_x$$

↑ to the left

(b) Writing the entropy balance at steady state

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv}$$

With $\dot{E}_d = T_0\dot{\sigma}_{cv}$, and solving the entropy balance for $\dot{\sigma}_{cv}$

$$\dot{E}_d = T_0\,\dot{m}(s_2 - s_1)$$

$$= T_0\,\dot{m}\left[(s_2^\circ - s_1^\circ) - R \ln p_2/p_1\right]$$

From Table A-22 ; $s_1^\circ = 1.82790$ kJ/kg·K and $s_2^\circ = 1.70203$. Thus

$$\dot{E}_d = (300\ K)\left(0.2335\ \frac{kg}{s}\right)\left[(1.70203 - 1.82790) - \left(\frac{8.314}{28.97}\right)\ln\left(\frac{9}{15}\right)\right]\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 1.452\ kW \longleftarrow \dot{E}_d$$

PROBLEM 9.103

KNOWN: Ideal gas at a known temperature.

FIND: Determine the sonic velocity.

ANALYSIS: Eq. 9.37 gives the sonic velocity.

(a) Air at 60°F = 520°R ⇒ $k = 1.4$ (Table A-20E)

$$c = \sqrt{kRT} = \sqrt{(1.4)\left(\frac{1545}{28.97}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(520^\circ\text{R})\left|\frac{32.2\ \text{ft}\cdot\text{lb/s}^2}{1\ \text{lbf}}\right|}$$

$$= 1118\ \text{ft/s} \longleftarrow c$$

(b) O_2 at 900°R ⇒ $k = 1.365$

$$c = \sqrt{(1.365)\left(\frac{1545}{32.00}\right)(900)\left|32.2\right|} = 1382\ \text{ft/s} \longleftarrow c$$

(c) Ar at 540°R ⇒ For a monotomic gas, $k = 5/3$. Thus

$$c = \sqrt{(5/3)\left(\frac{1545}{39.94}\right)(540)\left|32.2\right|} = 1059\ \text{ft/s} \longleftarrow c$$

PROBLEM 9.104

KNOWN: A flash of lightning is sighted, and 3 seconds later thunder is heard.

FIND: Estimate how far away the strike was.

ASSUMPTIONS: (1) The air is at 20°C = 293 K. (2) The ideal gas model is applicable.

ANALYSIS: From Table A-20, $k = 1.4$. With Eq. 9.37

$$c = \sqrt{(1.4)\left(\frac{8.314}{28.97}\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(293\ \text{K})\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|}$$

$$= 343\ \text{m/s}$$

Estimating the distance d

$$d = c\cdot t = (343\ \text{m/s})(3\ \text{s}) = 1029\ \text{m} = 0.64\ \text{mi} \longleftarrow d$$

PROBLEM 9.105

KNOWN: Steam at a given state.

FIND: Estimate the sonic velocity using data from Table A-4 and using the ideal gas model. Compare.

ANALYSIS: Beginning with Eq. 9.36b

$$c = \sqrt{(-v^2)\left.\frac{\partial p}{\partial v}\right)_s} \approx \sqrt{(-v^2)\left.\frac{\Delta p}{\Delta v}\right)_s}$$

where the partial derivative is estimated by using differences in tabular data. At $T = 360°C$, $p = 60$ bar : $v = 0.04331$ m³/kg
$s = 6.3782$ kJ/kg·K

Interpolating in adjacent table columns at the same entropy

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	p = 40 bar = 4.0 MPa (T_{sat} = 250.4°C)			
Sat.	0.04978	2602.3	2801.4	6.0701
280	$\boxed{0.05546}$	2680.0	2901.8	6.2568
320	$\boxed{0.06199}$	2767.4	3015.4	6.4553
360	0.06788	2845.7	3117.2	6.6215
	p = 80 bar = 8.0 MPa (T_{sat} = 295.06°C)			
Sat.	0.02352	2569.8	2758.0	5.7432
320	0.02682	2662.7	2877.2	5.9489
360	0.03089	2772.7	3019.8	6.1819
400	$\boxed{0.03432}$	2863.8	3138.3	6.3634
440	$\boxed{0.03742}$	2946.7	3246.1	6.5190

(40 bar) → $s = 6.3782$; $v = 0.05945$ m³/kg

(80 bar) → $s = 6.3782$; $v = 0.03461$ m³/kg

Using these values

$$c = \sqrt{-(0.04331^2)\frac{m^3}{kg}\frac{(80-40)\text{ bar}}{(.03461-.05945)}\left|\frac{10^5 N/m^2}{1 \text{ bar}}\right|\left|\frac{1 \text{ kg}\cdot m/s^2}{1 N}\right|}$$

$$= 549.6 \text{ m/s} \longleftarrow \text{c}$$

Using the function for H_2O in Table A-21 : $c_p = 2.035$ kJ/kg·K and with $c_p = kR/(k-1)$, $k = 1.293$. Thus, assuming the ideal gas model

$$c = kRT = \sqrt{(1.293)\left(\frac{8.314}{18.02}\right)(633)\left|10^3\right|} = 614.5 \text{ m/s}$$

The ideal gas model appears to significantly over-estimate the sonic velocity at the given state.

PROBLEM 9.106

KNOWN: Carbon dioxide at 1 bar and a given velocity.

FIND: Plot the Mach number versus temperature ranging from 250 to 1000K.

ASSUMPTIONS: The carbon dioxide is modeled as an ideal gas, as can be verified readily.

ANALYSIS: Sample calculation for T = 250 K. From Eq. 9.37

$$c = \sqrt{kRT}$$

From Table A-20, at T = 250K; k = 1.314. Thus

$$c = \sqrt{(1.314)\left(\frac{8.314}{44.01}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(250\text{K})\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right|\left|\frac{10^3\ \text{N}\cdot\text{m}}{1\ \text{kJ}}\right|} = 249.1\ \text{m/s}$$

Now, with $V = 460$ m/s

$$M = V/c = 1.847$$

The data for the required plot are obtained using IT, as follows:

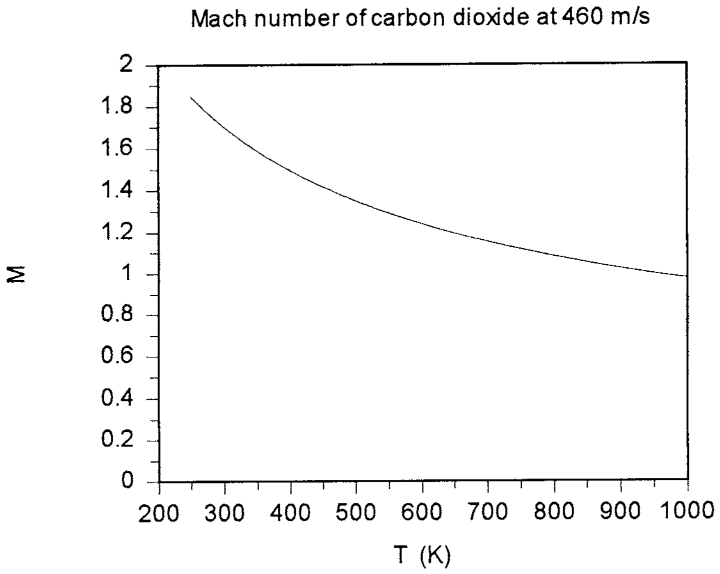
IT Code

```
T = 250  // K
V = 460  // m/s

cp = cp_T("CO2", T)
cv = cv_T("CO2", T)
k = cp / cv
R = 8.314 / 44.01
c = sqrt (k * R * T * 1000)
M = V / c
```

IT Results for T = 250 K

```
cp = 0.7944 kJ/kg·K
cv = 0.6054 kJ/kg·K
k = 1.312
c = 248.9 m/s
M = 1.848
```

PLOT:

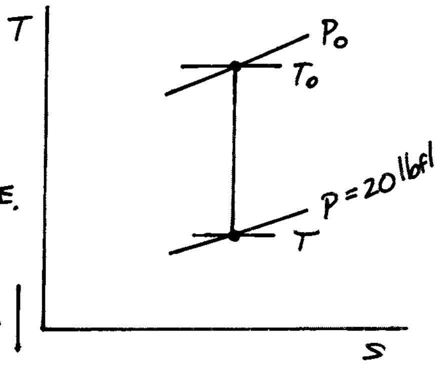


Note that the sonic velocity increases with temperature. Thus, the Mach number decreases, as expected.

PROBLEM 9.107

KNOWN: Ideal gas data are known, including T, p, V, and c.

FIND: Determine the Mach number, stagnation temperature, and stagnation pressure.



ANALYSIS: (a) $T = 520°R$, $p = 20\ \text{lbf/in}^2$

$$M = \frac{V}{c} = \frac{900}{1118} = 0.805$$

To find the stagnation state, use data from Table A-22E.

$$h_o = h + \frac{V^2}{2}$$

$$= 124.27\ \frac{\text{Btu}}{\text{lb}} + \left(\frac{900^2\ \text{ft}^2/\text{s}^2}{2}\right)\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 140.44\ \text{Btu/lb}$$

Interpolating in Table A-22E; $T_o = 587.4\ °R$ ← T_o

Also, $p_r(T) = 1.2147$ and $p_r(T_o) = 1.8633$. Thus

$$p_o = \frac{p_r(T_o)}{p_r(T)}\, p = 30.68\ \text{lbf/in}^2$$ ← p_o

(b) O_2 at $900°R$, $20\ \text{lbf/in}^2$

$$M = \frac{V}{c} = \frac{900}{1382} = 0.651$$ ← M

With data from Table A-23E

$$\bar{h}_o = \bar{h} + \frac{V^2}{2}(\text{mol. wt.})$$

$$= 6337.9\ \frac{\text{Btu}}{\text{lbmol}} + \left(\frac{900^2\ \text{ft}^2/\text{s}^2}{2}\right)\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|\left|\frac{32.00\ \text{lb}}{1\ \text{lbmol}}\right|$$

$$= 6855.2\ \text{Btu/lbmol} \Rightarrow T_o = 969.2\ °R$$ ← T_o

To find p_o

$$0 = \bar{s}°(T_o) - \bar{s}°(T) - \bar{R}\ln(p_o/p)$$

$$\ln\frac{p_o}{p} = \frac{\bar{s}°(T_o) - \bar{s}°(T)}{\bar{R}} = \frac{53.242 - 52.688}{1.986} = 0.278953$$

$$\Rightarrow p_o = 26.43\ \text{lbf/in}^2$$ ← p_o

(c) Ar at $540°R$, $20\ \text{lbf/in}^2$

$$M = V/c = 900/1059 = 0.85$$ ← M

For the monatomic gas, $c_p = \frac{5}{2}R = \frac{5}{2}\frac{(1.986)}{(39.94)} = 0.1243\ \text{Btu/lb}\cdot°R$

Thus
$$T_o = T + \frac{V^2}{2c_p} = 900°R + \frac{900^2\ \text{ft}^2/\text{s}^2}{2(.1243\ \text{Btu/lb}\cdot°R)}\left|\frac{1\ \text{lbf}}{32.2\ \text{lb}\cdot\text{ft/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 1030°R$$ ← T_o

Also
$$p_o = \left(\frac{T_o}{T}\right)^{k/k-1} p = 28.02\ \text{lbf/in}^2$$ ← p_o

-47

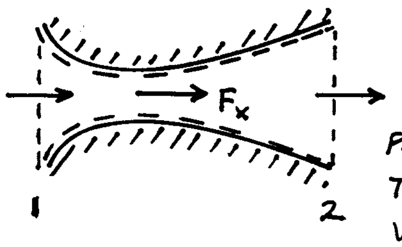
PROBLEM 9.108

KNOWN: Air flows through a duct as in Problem 9.102.

FIND: Determine the values of Mach number, stagnation temperature, and stagnation pressure at the inlet and exit of the duct.

SCHEMATIC & GIVEN DATA:

$p_1 = 15$ bar
$T_1 = 340$K
$V_1 = 20$ m/s



$p_2 = 9$ bar
$T_2 = 300$K
$V_2 = 284.4$ m/s

ASSUMPTIONS: See solution to Problem 9.102.

ANALYSIS: From Table A-20; $k = 1.399$ in the temperature range of this problem. Thus

$$c_1 = \sqrt{kRT_1} = \sqrt{(1.399)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(340\,K)\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|}$$
$$= 369.5 \text{ m/s}$$
$$M_1 = V_1/c_1 = 0.054 \longleftarrow M_1$$

and

$$c_2 = \sqrt{(1.399)\left(\frac{8.314}{28.97}\right)(300)\left|10^3\right|} = 347.1 \text{ m/s}$$
$$M_2 = V_2/c_2 = 0.819 \longleftarrow M_2$$

From the energy balance, the stagnation enthalpy is the same at the inlet and exit:

$$h_0 = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$
$$= 300.19 + \frac{284.4^2}{2}\left|\frac{1}{10^3}\right| = 340.63 \text{ kJ/kg}$$

From Table A-22; $T_{o_1} = T_{o_2} = 340.2$ K $\longleftarrow$ T_{o_1}, T_{o_2}

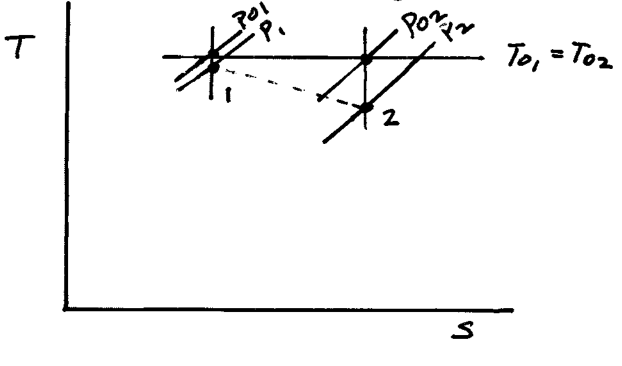
At the inlet, $p_r(T_1) = 1.82790$. Also, $p_r(T_o) = 1.82851$. Thus

$$p_{o_1} = [p_r(T_o)/p_r(T_1)]\,p_1 = 15.005 \text{ bar} \longleftarrow p_{o_1}$$

Further, $p_r(T_2) = 1.70203$. Thus

$$p_{o_2} = [p_r(T_o)/p_r(T_2)]\,p_2 = 9.669 \text{ bar} \longleftarrow p_{o_2}$$

Visualizing these states on a T-s diagram:



PROBLEM 9.109

KNOWN: Water vapor flows with known pressure, temperature, and velocity.

FIND: Using Fig. A-8E, determine (a) the stagnation enthalpy, (b) the stagnation temperature, and (c) the stagnation pressure.

SCHEMATIC & GIVEN DATA: $T = 600°F$, $p = 500\ \text{lbf/in}^2$, $V = 1000\ \text{ft/s}$

ANALYSIS:

(a) $h \approx 1298\ \text{Btu/lb}$

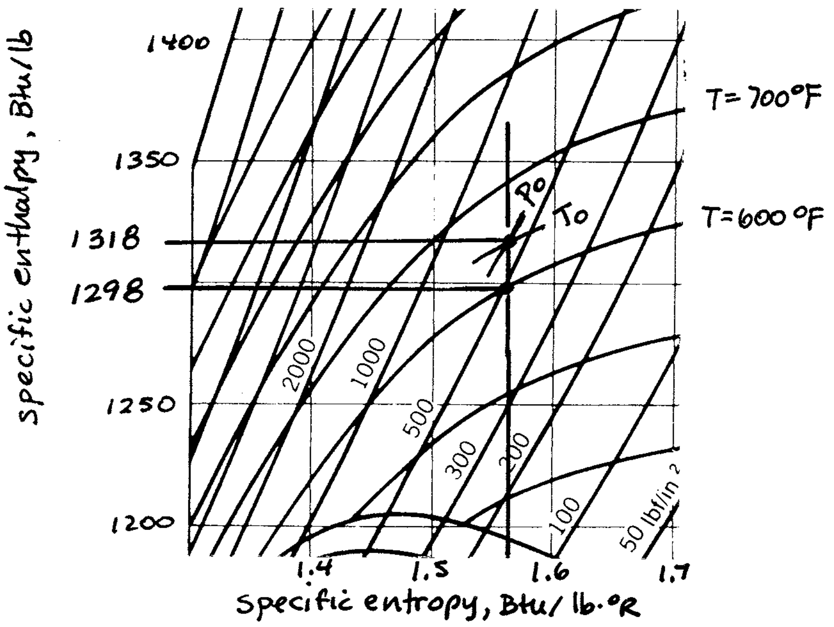
$$h_o = h + \frac{V^2}{2}$$

$$= 1298\ \frac{\text{Btu}}{\text{lb}} + \left(\frac{1000^2\ \text{ft}^2/\text{s}^2}{2}\right)\left|\frac{1\ \text{lbf}}{32.2\ \text{ft}\cdot\text{lb/s}^2}\right|\left|\frac{1\ \text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right|$$

$$= 1318\ \text{Btu/lb}$$

(b) $T_o \approx 645\ °F$

(c) $p_o \approx 600\ \text{lbf/in}^2$

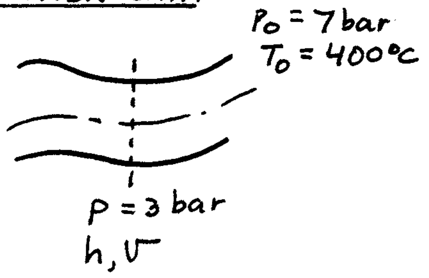


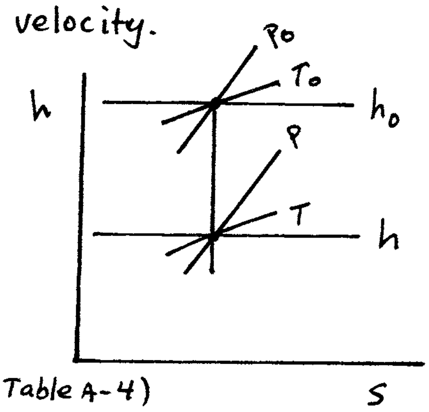
PROBLEM 9.110

KNOWN: Conditions of steam flowing at a particular location in a passageway are known.

FIND: Determine the specific enthalpy and velocity.

SCHEMATIC & GIVEN DATA:





ANALYSIS: $h_0 = 3268.7$, $s_0 = 7.6350$ kJ/kg·K (Table A-4)

At $p = 3$ bar, $s = s_0 = 7.6350 \Rightarrow h = 3031.5$ kJ/kg $\longleftarrow$ h

Thus, from $h_0 = h + V^2/2$

$$V = \sqrt{2(h_0 - h)}$$

$$= \sqrt{2(3268.7 - 3031.5)\frac{kJ}{kg}\left|\frac{1\ kg \cdot m/s^2}{1\ N}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right|}$$

$$= 688.8\ m/s \longleftarrow V$$

PROBLEM 9.111

KNOWN: Isentropic flow of an ideal gas with constant specific heat ratio.

FIND: Develop the relation: $T^*/T_0 = 2/(k+1)$.

ANALYSIS: Begin with Eq. 9.50

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right) M^2$$

For $M = 1$, $T = T^*$. Thus

$$\frac{T_0}{T^*} = 1 + \left(\frac{k-1}{2}\right) = \frac{k+1}{2}$$

① and

$$\frac{T^*}{T_0} = \frac{2}{k+1} \longleftarrow T^*/T_0$$

---

1. It follows directly that

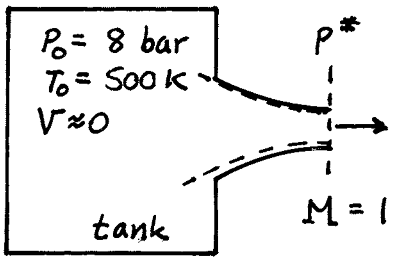
$$\frac{p^*}{p_0} = \left(\frac{T^*}{T_0}\right)^{\frac{k}{k-1}} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

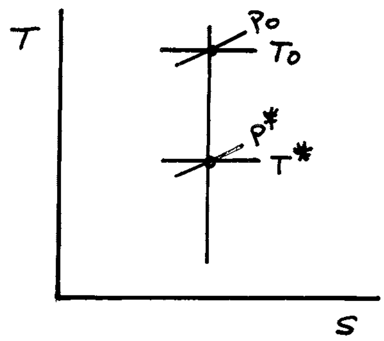
PROBLEM 9.112*

KNOWN: A gas expands isentropically through a converging nozzle from a large tank at known pressure and temperature.

FIND: Determine the critical pressure p^* and the corresponding temperature T^* if the gas is (a) air, (b) carbon dioxide, and (c) water vapor.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The gas expands isentropically. (3) The velocity in the tank is negligible. (4) The gas follows the ideal gas model. To avoid iteration, the specific heats are taken as constant, evaluated at 450 K.

ANALYSIS: With assumption (4), Eq's 9.50 and 9.51 apply. Evaluating each one for M = 1, respectively, gives

$$\frac{T_o}{T^*} = \frac{k+1}{2}$$

$$\frac{p_o}{p^*} = \left(\frac{k+1}{2}\right)^{k/k-1}$$

(a) From Table A-20; at T = 450 K; k = 1.391 (air).

$T^* = 418.2$ K

$p^* = 4.238$ bar ← T^*, p^*

(b) From Table A-20; at T = 450 K; k = 1.239 (CO_2)

$T^* = 446.6$ K

$p^* = 4.456$ bar ← T^*, p^*

(c) Evaluating c_p using the relation in Table A-21 for H_2O; $c_p = 1.926 \frac{kJ}{kg \cdot K}$

Also

$$k = \frac{c_p}{c_p - R} = 1.315$$

Thus $T^* = 432.0$ K

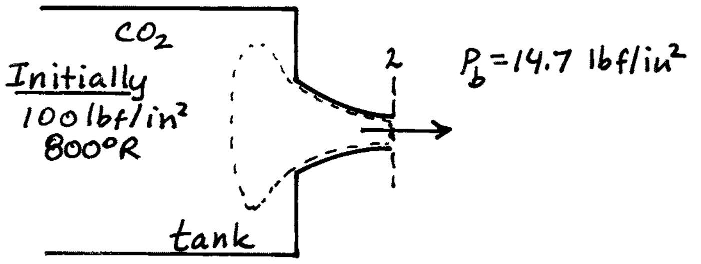
$p^* = 4.344$ bar ← T^*, p^*

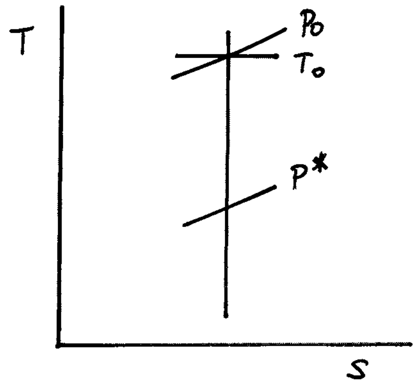
PROBLEM 9.113

KNOWN: Carbon dioxide gas discharges from a large tank through a converging nozzle. The initial conditions in the tank and the pressure of the surroundings are known.

FIND: Estimate the pressure in the tank when the flow first ceases to be choked.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The tank pressure changes slowly, and the control volume can be considered to be at steady state for short periods of time. (2) The expansion can be considered isentropic at any time. (3) The carbon dioxide is modeled as an ideal gas with constant specific heats.

ANALYSIS: Under these assumptions, Eq. 9.51 can be applied, with $M=1$, to see if the flow is choked.

$$\frac{p_0}{p^*} = \left[1 + \frac{k-1}{2}(1)\right]^{k/k-1} = \left(\frac{k+1}{2}\right)^{k/k-1} = 1.80$$

where $k = 1.246$ from Table A-20E at $T = 760$ °R. For the initial conditions in the tank, $p^* = 55.56$ lbf/in². This is substantially greater than the pressure in the surroundings, $p_b = 14.7$ lbf/in².

As the pressure in the tank drops, the pressure at the nozzle exit also drops. Therefore, a plausible estimate of the tank pressure when the nozzle ceases to be choked is obtained when

$$P_b = p^* = 14.7 \text{ lbf/in}^2$$

Then

$$(P_0)_{\substack{\text{choking}\\ \text{ceases}}} = (14.7)(1.8) = 26.46 \text{ lbf/in}^2 \longleftarrow$$

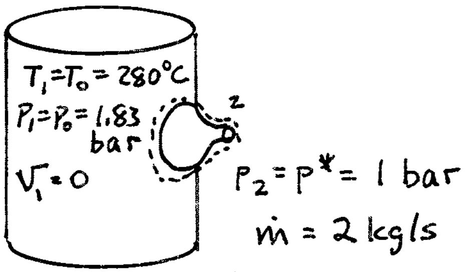
For tank pressures lower than this value, the exit plane pressure will be 14.7 lbf/in², and the nozzle flow will not be choked.

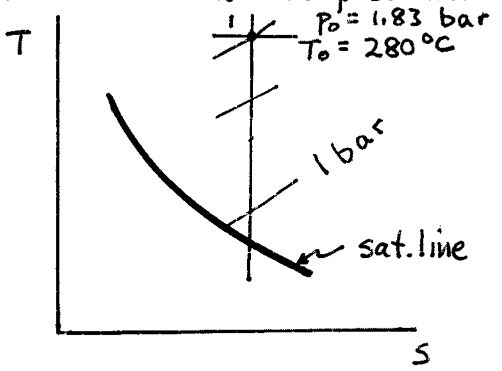
PROBLEM 9.114*

KNOWN: Steam expands isentropically through a converging nozzle from a large tank. The flow is choked, and the mass flow rate and exit plane pressure are known.

FIND: Determine the nozzle diameter at locations where various pressures are specified.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown is at steady state, with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The steam expands isentropically. (3) The nozzle is choked. (4) The velocity in the tank is negligible, and potential energy effects can be ignored.

ANALYSIS: For each case to be considered, $P_1 = P_0 = 1.83$ bar and $T_1 = T_0 = 280$ °C. Thus, from Table A-4, $h_1 = h_0 = 3031.9$ kJ/kg and $s_1 = s_0 = 7.8839$ kJ/kg·K. Further, the exit velocity is found from

$$V_2 = \sqrt{2(h_0 - h_2)} \tag{1}$$

and

$$A_2 = \frac{\dot{m}\, v_2}{V_2} \Rightarrow d_2 = \sqrt{\frac{4A_2}{\pi}} \tag{2}$$

$P_2 = 1.5$ bar $h_2 = 2994.6$ kJ/kg, $v_2 = 1.6355$ m³/kg

$$V_2 = \sqrt{2(3031.9 - 2994.6)\,\frac{\text{kJ}}{\text{kg}} \left|\frac{10^3\,\text{N}\cdot\text{m}}{1\,\text{kJ}}\right| \left|\frac{1\,\text{kg}\cdot\text{m/s}^2}{1\,\text{N}}\right|} = 273.13 \text{ m/s}$$

$$A_2 = \frac{(2\,\text{kg/s})(1.6355\ \text{m}^3/\text{kg})}{(273.13\ \text{m/s})} = 0.01198\ \text{m}^2$$

$$d_2 = \sqrt{\frac{(4)(0.01198)\ \text{m}^2}{\pi} \left|\frac{10^4\,\text{cm}^2}{1\,\text{m}^2}\right|} = 12.348 \text{ cm} \longleftarrow \frac{d_2}{(1.5\text{ bar})}$$

$P_2 = 1$ bar $h_2 = 2899.8$ kJ/kg, $v_2 = 2.2298$ m³/kg

$V_2 = 514.0$ m/s, $A_2 = 0.008676$ m², $d_2 = 10.510$ cm $\longleftarrow \frac{d_2}{(1\text{ bar})}$

CHAPTER TEN

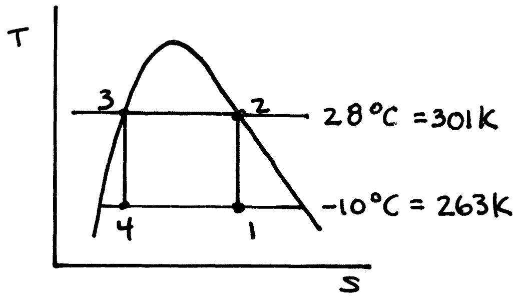
REFRIGERATION AND HEAT PUMP SYSTEMS

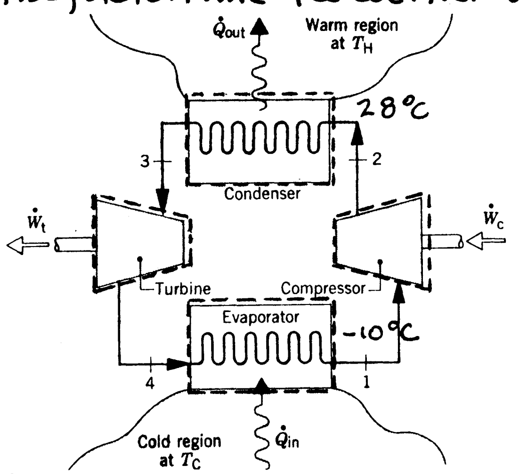
PROBLEM 10.1

KNOWN: Refrigerant 134a is the working fluid in a Carnot vapor refrigeration cycle. The states of the refrigerant entering and leaving the condenser, and the evaporator temperature are specified.

FIND: Determine (a) the work input to the compressor, (b) the work developed by the turbine, and (c) the heat transfer to the refrigerant passing through the evaporator, each per unit mass of refrigerant flow. Also, determine the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible. (3) The compression and expansion are adiabatic. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 2 $T_2 = 28°C$, sat. vapor $\Rightarrow h_2 = 262.50$ kJ/kg, $s_2 = .9076$ kJ/kg·K

State 1 $T_1 = -10°C$, $s_1 = s_2 \Rightarrow x_1 = 0.9773$, $h_1 = 236.7$ kJ/kg

State 3 $T_3 = 28.°C$, sat. liquid $\Rightarrow h_3 = 88.61$ kJ/kg, $s_3 = .3302$ kJ/kg·K

State 4 $T_4 = -10°C$, $s_4 = s_3 \Rightarrow x_4 = 0.2339$, $h_4 = 84.76$ kJ/kg

(a) For the compressor

$$\frac{\dot{W}_c}{\dot{m}} = (h_2 - h_1) = 25.8 \text{ kJ/kg} \longleftarrow \dot{W}_c/\dot{m}$$

(b) For the turbine

$$\frac{\dot{W}_t}{\dot{m}} = (h_3 - h_4) = 3.85 \text{ kJ/kg} \longleftarrow \dot{W}_t/\dot{m}$$

(c) For the evaporator

$$\frac{\dot{Q}_{in}}{\dot{m}} = (h_1 - h_4) = 151.94 \text{ kJ/kg} \longleftarrow \dot{Q}_{in}/\dot{m}$$

The coefficient of performance is: $\beta = \dfrac{\dot{Q}_{in}/\dot{m}}{\dot{W}_c/\dot{m} - \dot{W}_t/\dot{m}} = 6.922 \longleftarrow \beta$

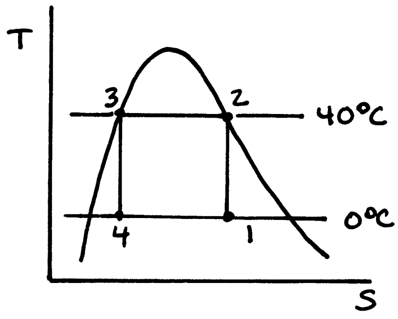
Alternatively; $\beta = \dfrac{T_c}{T_H - T_c} = 6.92$

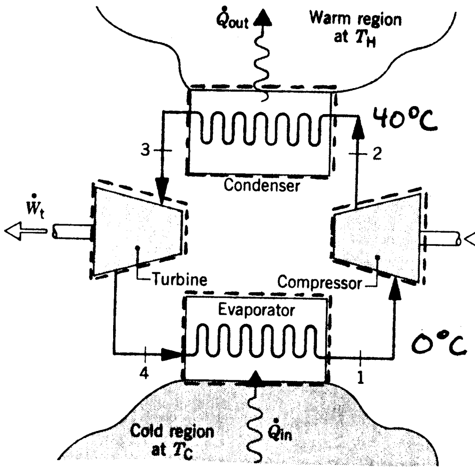
PROBLEM 10.2

KNOWN: Refrigerant 22 is the working fluid in a Carnot vapor refrigration cycle. Data are known at various locations and the mass flow rate is specified.

FIND: Determine (a) the rate of heat transfer to the R-22 passing through the evaporator, (b) the net power input, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible. (3) The compression and expansion are adiabatic. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states (Table A-7).

State 2 $T_2 = 40°C$, sat. vapor $\Rightarrow h_2 = 260.79$ kJ/kg, $s_2 = 0.8738$ kJ/kg·K

State 1 $T_1 = 0°C$, $s_1 = s_2 \Rightarrow x_1 = .9289$, $h_1 = 235.37$ kJ/kg

State 3 $T_3 = 40°C$, sat. liquid $\Rightarrow h_3 = 94.53$, $s_3 = 0.3429$ kJ/kg·K

State 4 $T_4 = 0°C$, $s_4 = s_3 \Rightarrow x_4 = 0.22086$, $h_4 = 90.35$ kJ/kg

(a) For the evaporator

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4)$$

$$= \left(3\ \frac{kg}{min}\right)\left|\frac{1\ min}{60\ s}\right|(235.37 - 90.35)\ \frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right| = 7.251\ kW \longleftarrow \dot{Q}_{in}$$

(b) The net power input is

$$\dot{W}_{cycle} = \dot{W}_c - \dot{W}_t$$

$$= \dot{m}\left[(h_2 - h_1) - (h_3 - h_4)\right]$$

$$= (3)\left|\frac{1}{60}\right|\left[(260.79 - 235.37) - (94.53 - 90.35)\right]$$

$$= 1.062\ kW \longleftarrow \dot{W}_{cycle}$$

(c) Thus, the coefficient of performance is

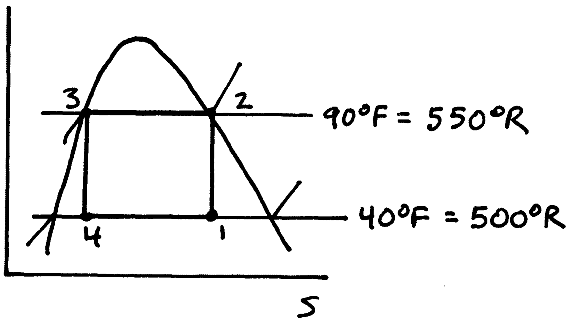
$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{7.251}{1.062} = 6.83 \longleftarrow \beta$$

PROBLEM 10.3

KNOWN: A carnot vapor refrigeration cycle operates between known reservoir temperatures.

FIND: Determine operating pressures in the condenser and evaporator for (a) Refrigerant 134a, (b) propane, (c) water, (d) refrigerant 22, and (e) ammonia as the working fluid. Calculate the coefficient of performance.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each component operates at steady state. (2) All processes are internally reversible. (3) The condenser and evaporator operate at the respective reservoir temperatures. (4) The compression and expansion are adiabatic.

ANALYSIS: The operating pressures are the saturation pressures at the respective temperatures.

(a) Refrigerant 134a

① $p_{cond} = p_{sat}$ @ 90°F = 118.99 lbf/in.² ← p_{cond}

$p_{evap} = p_{sat}$ @ 40°F = 49.738 lbf/in.² ← p_{evap}

(b) Propane

$p_{cond} = p_{sat}$ @ 90°F = 165.2 lbf/in.² ← p_{cond}

$p_{evap} = p_{sat}$ @ 40°F = 78.6 lbf/in.² ← p_{evap}

(c) Water

② $p_{cond} = p_{sat}$ @ 90°F = 0.6988 lbf/in.² ← p_{cond}

$p_{evap} = p_{sat}$ @ 40°F = 0.1217 lbf/in.² ← p_{evap}

(d) Refrigerant 22

$p_{cond} = p_{sat}$ @ 90°F = 183.16 lbf/in.² ← p_{cond}

$p_{evap} = p_{sat}$ @ 40°F = 83.278 lbf/in.² ← p_{evap}

(e) Ammonia

$p_{cond} = p_{sat}$ @ 90°F = 180.73 lbf/in.² ← p_{cond}

$p_{evap} = p_{sat}$ @ 40°F = 73.359 lbf/in.² ← p_{evap}

The coefficient of performance is

$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{500}{550-500} = 10 \leftarrow \beta_{max}$$

---

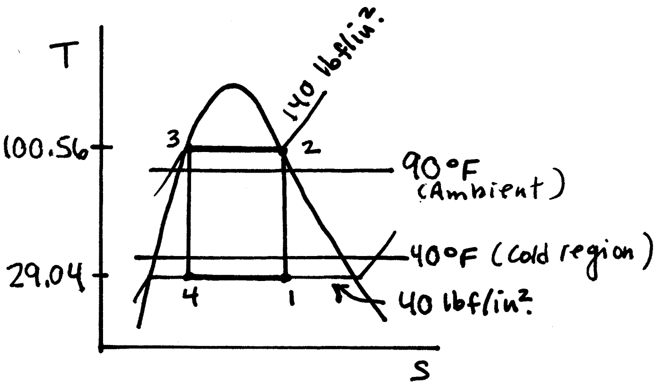
1. Refrigerant 134a has lower operating pressures than propane, Refrigerant 22, and ammonia in this cycle.
2. Water operates at very low pressures and cannot be used to achieve temperatures lower than 32°F.

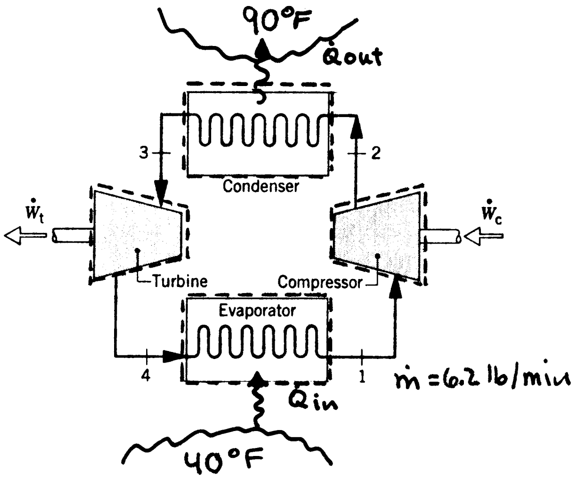
PROBLEM 10.4*

KNOWN: Refrigerant 134a is the working fluid in a Carnot vapor refrigeration cycle. Operating data are specified.

FIND: Determine (a) the compressor and turbine power and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion are adiabatic. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states (*Table A-11E*).

State 2 $\quad p_2 = 140\ \text{lbf/in}^2$, sat. vapor $\Rightarrow h_2 = 114.95\ \frac{\text{Btu}}{\text{lb}}$, $s_2 = 0.2161\ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$

State 1 $\quad p_1 = 40\ \text{lbf/in}^2$, $s_1 = s_2 \Rightarrow x_1 = 0.9794$, $h_1 = 104.12$ Btu/lb

State 3 $\quad p_3 = 140\ \text{lbf/in}^2$, sat. liquid $\Rightarrow h_3 = 44.43\ \frac{\text{Btu}}{\text{lb}}$, $s_3 = 0.0902\ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$

State 4 $\quad p_4 = 40\ \text{lbf/in}^2$, $s_4 = s_3 \Rightarrow x_4 = 0.2579$, $h_4 = 42.57$ Btu/lb

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = 6.2\ \frac{\text{lb}}{\text{min}}(114.95 - 104.12)\ \frac{\text{Btu}}{\text{lb}}$$

$$= 67.1\ \text{Btu/min} \longleftarrow \dot{W}_c$$

The turbine power is

$$\dot{W}_t = \dot{m}(h_3 - h_4) = (6.2)(44.43 - 42.57) = 11.53\ \text{Btu/min} \longleftarrow \dot{W}_t$$

(b) The evaporator heat transfer rate is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = 381.6\ \text{Btu/min}$$

thus

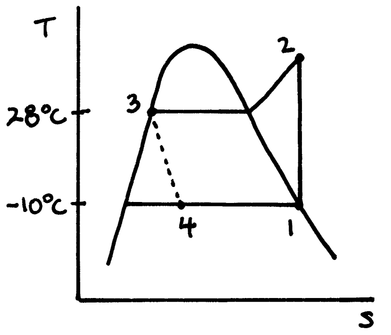
$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c - \dot{W}_t} = 6.87 \longleftarrow \beta$$

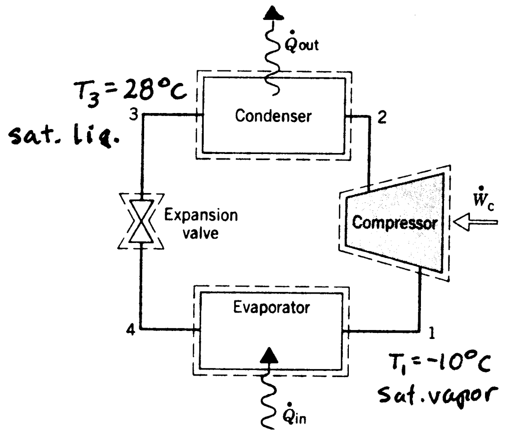
PROBLEM 10.6

KNOWN: Refrigerant 134a is the working fluid in an ideal vapor-compression refrigeration cycle. Operating data are known.

FIND: Determine (a) the compressor power, (b) the refrigerating capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The expansion through the valve is a throttling process. All other processes are internally reversible. (3) The compressor and valve operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -10°C$, sat. vapor ⇒ $h_1 = 241.35$ kJ/kg, $s_1 = 0.9253$ kJ/kg·K

State 2 $p_2 = p_{sat} @ 28°C = 7.2675$ bar, $s_2 = s_1$ ⇒ Double interpolation in Table A-12 gives $h_2 \approx 267.9$ kJ/kg

State 3 $T_3 = 28°C$, sat. liquid ⇒ $h_3 = 88.61$ kJ/kg

State 4 Throttling process ⇒ $h_4 = h_3 = 88.61$ kJ/kg

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(5\,\frac{kg}{min}\right)\left|\frac{1\,min}{60\,s}\right|(267.9 - 241.35)\frac{kJ}{kg}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 2.212\text{ kW} \longleftarrow \dot{W}_c$$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(5\,\frac{kg}{min}\right)(241.35 - 88.61)\frac{kJ}{kg}\left|\frac{1\text{ ton}}{211\text{ kJ/min}}\right|$$

$$= 3.62\text{ tons} \longleftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

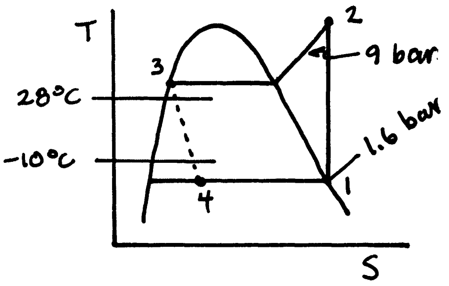
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 5.75 \longleftarrow \beta$$

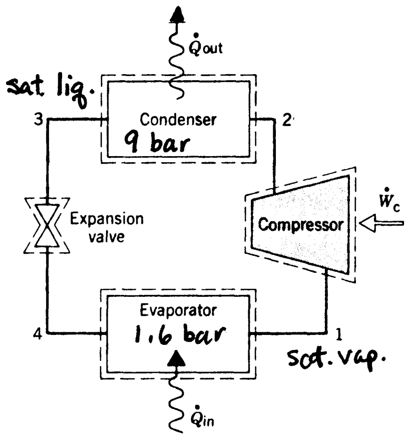
PROBLEM 10.7

KNOWN: The cycle in Problem 10.6 is modified by changing the evaporator and condenser pressure.

FIND: Determine for the modified cycle (a) the compressor power, (b) the refrigerating capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.2.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 1.6$ bar, sat. vapor $\Rightarrow h_1 = 237.97$ kJ/kg, $s_1 = .9295$ kJ/kg·K

State 2 $p_2 = 9$ bar, $s_2 = s_1$ $\Rightarrow h_2 = 273.73$ kJ/kg

State 3 $p_3 = 9$ bar, sat. liquid $\Rightarrow h_3 = 99.56$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 99.56$ kJ/kg

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(5\,\frac{kg}{min}\right)\left|\frac{1\ min}{60\ s}\right|(273.73 - 237.97)\,\frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 2.98\ kW \longleftarrow \dot{W}_c$$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(5\,\frac{kg}{min}\right)(237.97 - 99.56)\,\frac{kJ}{kg}\left|\frac{1\ ton}{211\ kJ/min}\right|$$

$$= 3.28\ tons \longleftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 3.87 \longleftarrow \beta$$

PROBLEM 10.8

The data for the required plots are obtained using IT, as follows: (See the solution to Problem 10.7 for sample calculations.)

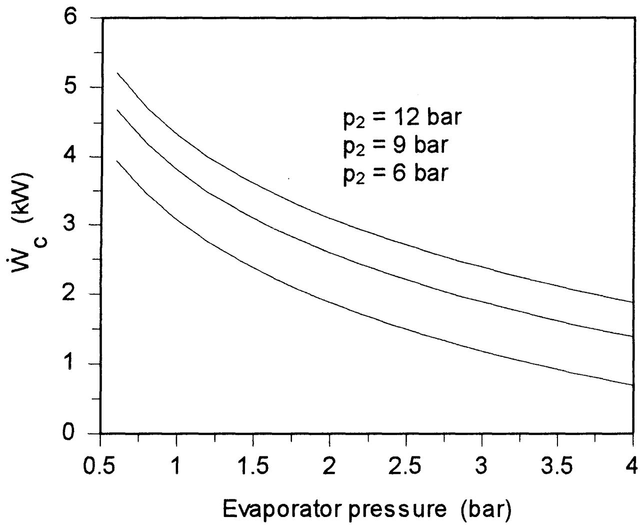
IT Code

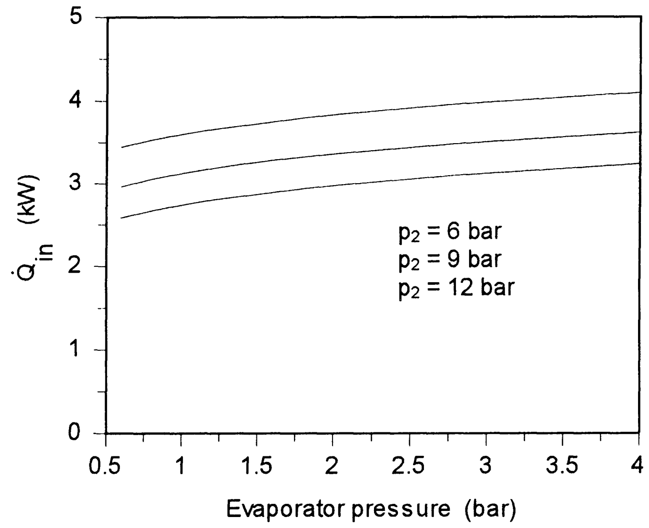
```
p1 = 1.6    // bar
p3 = 9   // bar
mdot = 5    // kg/min

x1 = 1
h1 = hsat_Px("R134A", p1, x1)
s1 = ssat_Px("R134A", p1, x1)
s2 = s1
p2 = p3
h2 = h_Ps("R134A", p2, s2)
x3 = 0
h3 = hsat_Px("R134A", p3, x3)
h4 = h3

Wdot = (mdot / 60) * (h2 - h1)
Qdotin = mdot * (h1 - h4) * (1 / 211)  // ton
beta = (h1 - h4) / (h2 - h1)
```

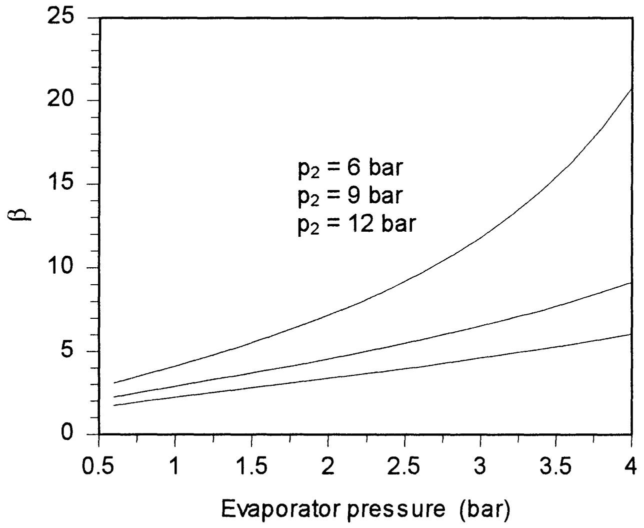
IT Results for p_{evap} = 1.6, p_{cond} = 9 bar
(See Problem 10.7 for sample calculation.)

h_1 = 238 kJ/kg
h_2 = 273.7 kJ/kg
h_3 = 99.56 kJ/kg
h_4 = 99.56 kJ/kg
$\dot{W}_c$ = 2.977 kW
$\dot{Q}_{in}$ = 3.28 ton
β = 3.874

PLOTS:







For fixed condenser pressure –
The power requirement goes down and the capacity increases as evaporator pressure increases. As a result, the coefficient of performance increases.

For fixed evaportator pressure –
The power requirement increases and the capacity decreases. As a result, the coefficient of performance goes down.

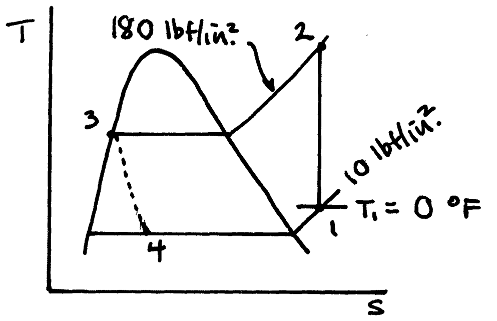
These conclusions are confirmed with reference to the T-s diagram.

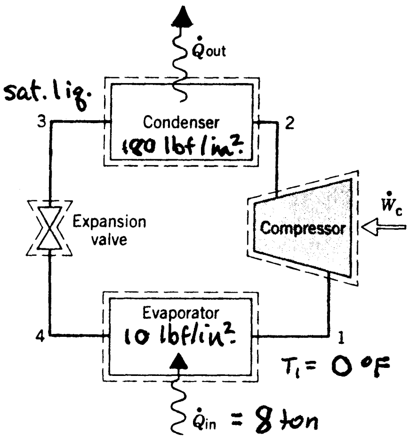
PROBLEM 10.9*

KNOWN: Refrigerant 134a is the working fluid in an ideal vapor-compression refrigeration cycle. Operating data are known, and the refrigerating capacity is given.

FIND: Determine (a) the compressor power, (b) the rate of heat transfer from the refrigerant passing through the condenser, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.1, assumptions 1-4.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 10$ lbf/in.², $T_1 = 0$ °F $\Rightarrow h_1 = 102.94$ Btu/lb, $s_1 = 0.2391$ Btu/lb·°R

State 2 $p_2 = 180$ lbf/in.², $s_2 = s_1 \Rightarrow h_2 = 131.04$ Btu/lb

State 3 $p_3 = 180$ lbf/in.², sat. liquid $\Rightarrow h_3 = 50.64$ Btu/lb

State 4 Throttling process $\Rightarrow h_4 = h_3 = 50.64$ Btu/lb

(a) The refrigerating capacity is used to find the mass flow rate.

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) \Rightarrow \dot{m} = \frac{\dot{Q}_{in}}{(h_1 - h_4)} = \frac{8 \text{ ton}}{(102.94 - 50.64)\frac{\text{Btu}}{\text{lb}}} \left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right|$$
$$= 30.59 \text{ lb/min}$$

The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(30.59 \frac{\text{lb}}{\text{min}}\right) \left|\frac{60 \text{ min}}{1 \text{ h}}\right| (131.04 - 102.94) \frac{\text{Btu}}{\text{lb}} \left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$
$$= 20.27 \text{ hp} \longleftarrow \dot{W}_c$$

(b) For the condenser

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) = (30.59)(131.04 - 50.64) = 2459.4 \text{ Btu/min} \longleftarrow \dot{Q}_{out}$$

(c) The coefficient of performance is

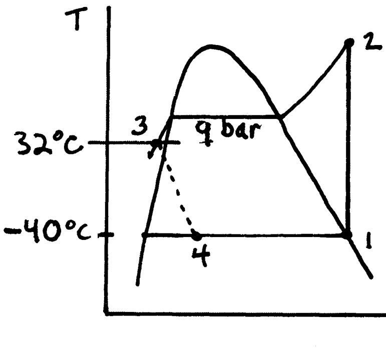
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 1.86 \longleftarrow \beta$$

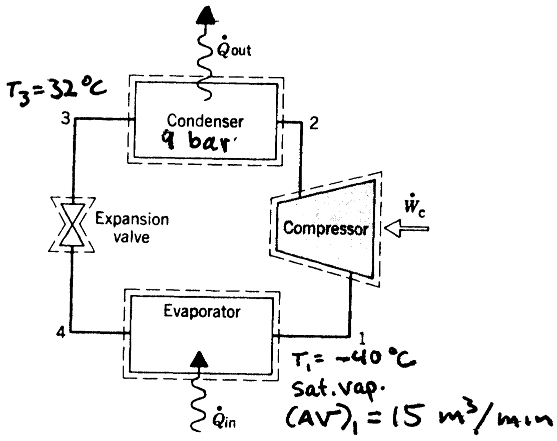
PROBLEM 10.10*

KNOWN: Refrigerant 22 is the working fluid in an ideal vapor-compression refrigeration cycle. Operating data are known.

FIND: Determine (a) the compressor power, (b) the refrigerating capacity, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.1, assumptions 1-4.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -40\,°C$, sat. vapor $\Rightarrow h_1 = 233.27$ kJ/kg, $s_1 = 1.0005$ kJ/kg·K

State 2 $p_2 = 9$ bar, $s_2 = s_1 \Rightarrow h_2 = 287.54$ kJ/kg

State 3 $h_3 \approx h_f(32\,°C) = 84.14$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 84.14$ kJ/kg

(a) From Table A-7, $v_1 = 0.2052$ m³/kg. The mass flow rate is

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{15\ \text{m}^3/\text{min}}{0.2052\ \text{m}^3/\text{kg}} = 73.10\ \text{kg/min}$$

The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(73.10\,\frac{\text{kg}}{\text{min}}\right)\left|\frac{1\ \text{min}}{60\ \text{s}}\right|(287.54 - 233.27)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 66.12\ \text{kW} \longleftarrow \dot{W}_c$$

(b) the refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(73.10\,\frac{\text{kg}}{\text{min}}\right)(233.27 - 84.14)\,\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{ton}}{211\ \text{kJ/min}}\right|$$

$$= 51.67\ \text{tons} \longleftarrow \dot{Q}_{in}$$

(c) the coefficient of performance is

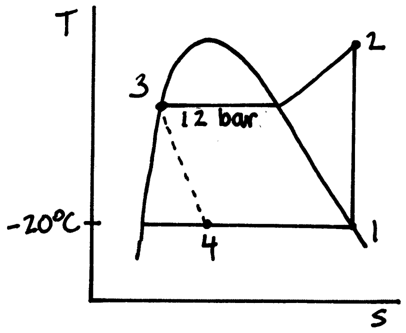
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 2.75 \longleftarrow \beta$$

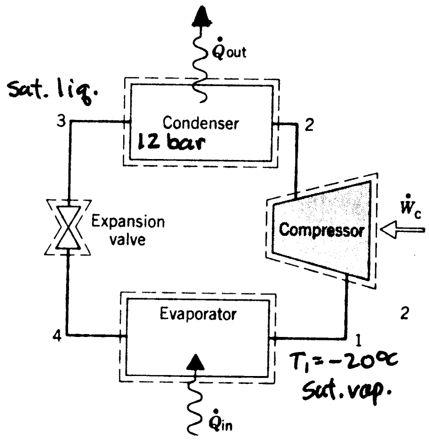
PROBLEM 10.11

KNOWN: An ideal vapor-compression refrigeration cycle uses ammonia as the working fluid. Operating data are known.

FIND: Determine (a) the coefficient of performance and (b) the refrigerating capacity.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.1.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -20°C$, sat. vapor $\Rightarrow h_1 = 1417.79$ kJ/kg, $s_1 = 5.6144$ kJ/kg·K

State 2 $p_2 = 12$ bar, $s_2 = s_1$ $\Rightarrow h_2 = 1689.5$ kJ/kg

State 3 $p_2 = 12$ bar, sat. liquid $\Rightarrow h_3 = 327.01$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 327.01$ kJ/kg

(a) The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = \frac{1417.79 - 327.01}{1689.5 - 1417.79} = 4.015 \longleftarrow \beta$$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4)$$

$$= \left(3\ \frac{\text{kg}}{\text{min}}\right)(1417.79 - 327.01)\ \frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{ton}}{211\ \text{kJ/min}}\right|$$

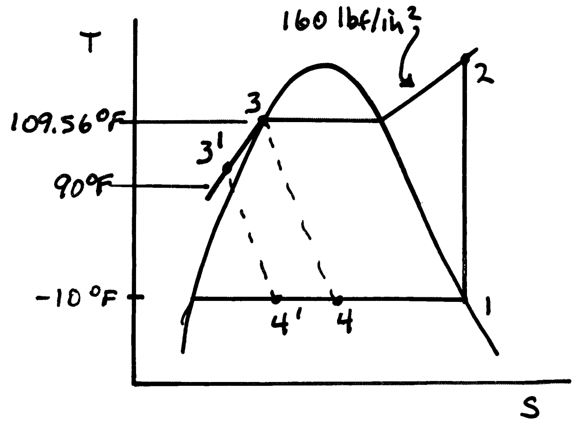
$$= 15.51\ \text{tons} \longleftarrow \dot{Q}_{in}$$

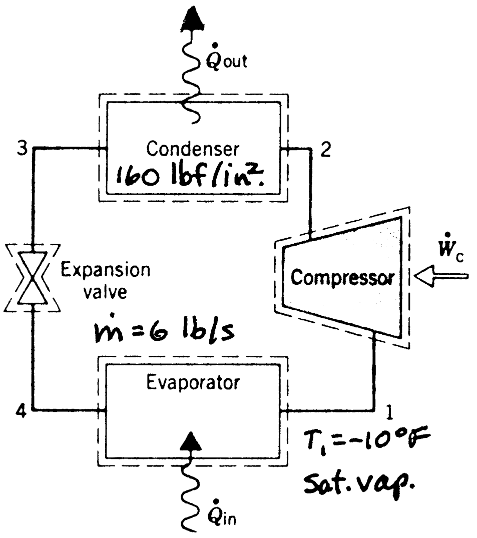
PROBLEM 10.12

KNOWN: An ideal vapor-compression refrigeration cycle uses refrigerant 134a as the working fluid. Operating data are known.

FIND: Plot the coefficient of performance and the refrigerating capacity for a range of condenser exit temperatures.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.1, assumptions 1-4.

ANALYSIS: For a sample calculation, consider state 3 to be saturated liquid. Fixing each principal state:

State 1: $T_1 = -10°F$, sat. vapor ⇒ $h_1 = 100.29$ Btu/lb, $s_1 = 0.2236$ Btu/lb·°R

State 2: $p_2 = 160$ lbf/in², $s_2 = s_1$ ⇒ $h_2 = 120.48$ Btu/lb

State 3: $p_2 = 160$ lbf/in², sat. liquid ⇒ $h_3 = 47.65$ Btu/lb

State 4: Throttling process ⇒ $h_4 = h_3 = 47.65$ Btu/lb

The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 2.61$$

The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(6\,\frac{lb}{min}\right)(100.29 - 47.65)\,\frac{Btu}{lb}\left|\frac{1\ ton}{200\ Btu/min}\right| = 1.58\ tons$$

The data for the required plots are obtained using *IT* as follows:

IT Code

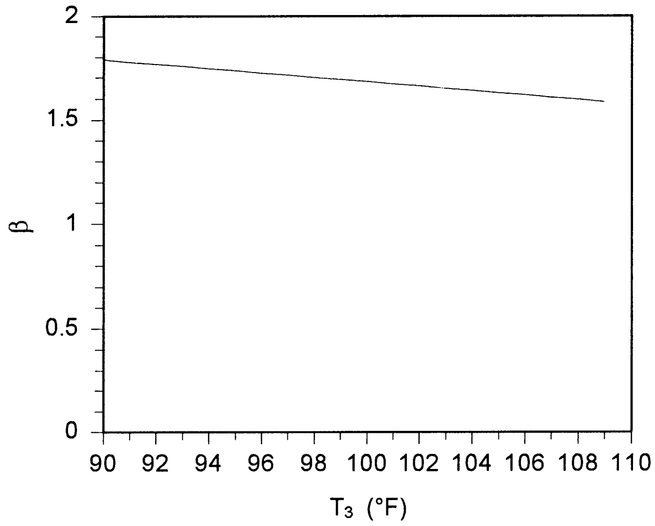
```
T1 = -10  // °F
p3 = 160  // lbf/in²
mdot = 6  // lb/min
T3 = 109.5  // °F

x1 = 1
p1 = Psat_T("R134A", T1)
h1 = hsat_Px("R134A", p1, x1)
s1 = ssat_Px("R134A", p1, x1)
s2 = s1
p2 = p3
h2 = h_Ps("R134A", p2, s2)
h3 = h_PT("R134A", p3, T3)
h4 = h3

beta = (h1 - h4) / (h2 - h1)
Qdotin = mdot * (h1 - h4) * (1 / 200)
```

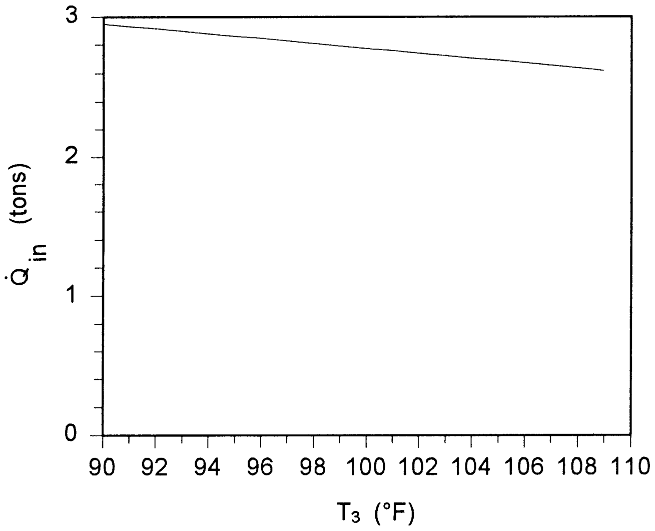
IT Results for T_3 = 109.5 °F

h_1 = 100.3 Btu/lb
h_2 = 120.5 Btu/lb
h_3 = 47.63 Btu/lb
h_4 = 47.63 Btu/lb
β = 2.608
$\dot{Q}_{in}$ = 1.58 tons

PROBLEM 10.12 (Cont'd.)

PLOTS:





As T_3 decreases (more subcooling) point 4 on the T-s diagram moves to the left, increasing the change in specific enthalpy across the evaporator. For constant mass flow rate, the refrigeration capacity increases accordingly. Also, since the compressor power is constant, the coefficient of performance is greater as subcooling increases.

PROBLEM 10.13

The data for the required plots are obtained using IT, as follows: (See the solution to Problem 10.11 for a sample calculation.)

IT Code

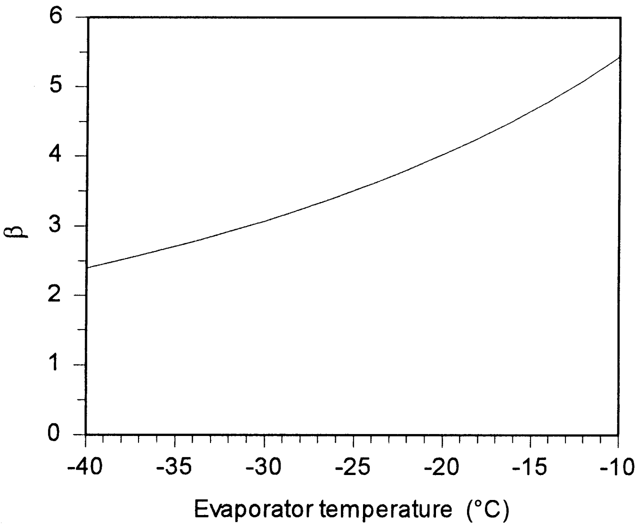
```
T1 = -20   // °C
p3 = 12   // bar
mdot = 3   // kg/min

x1 = 1
p1 = Psat_T("Ammonia", T1)
h1 = hsat_Px("Ammonia", p1, x1)
s1 = ssat_Px("Ammonia", p1, x1)
s2 = s1
p2 = p3
s2 = s_PT("Ammonia", p2, T2)
h2 = h_PT("Ammonia", p2, T2)
x3 = 0
h3 = hsat_Px("Ammonia", p3, x3)
 h4 = h3
```

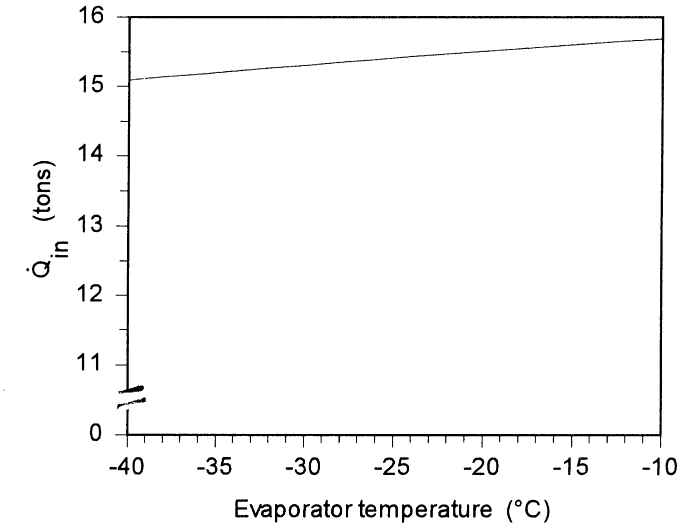
```
beta = (h1 - h4) / (h2 -h1)
Qdotin = mdot * (h1 - h4) * ( 1 / 211)  // tons
```

IT Results for T_{evap} = -20 °C

h_1 = 1418 kJ/kg
h_2 = 1689 kJ/kg
h_3 = 327 kJ/kg
h_4 = 327 kJ/kg
β = 4.019
$\dot{Q}_{in}$ = 15.51 tons

PLOTS:



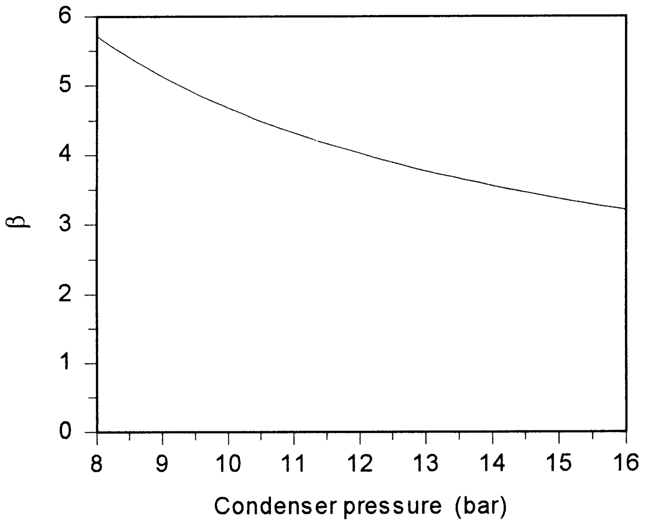


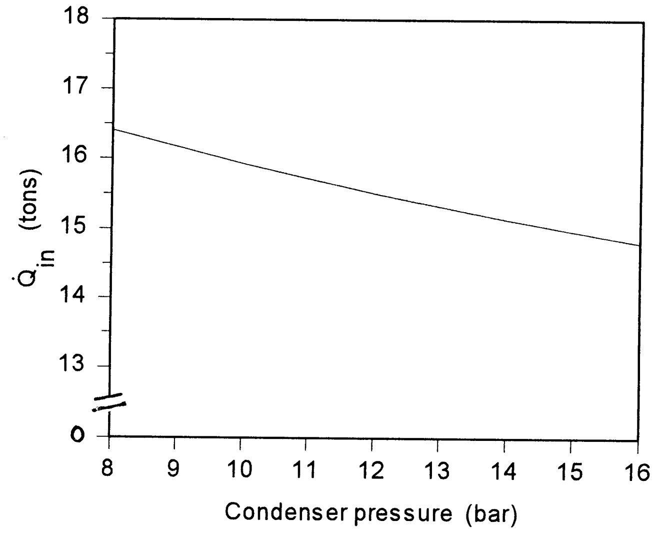
In this case, the condenser pressure is constant. Thus, $h_3=h_4$ remains unchanged as evaporator temperature changes. Since h_1 decreases as evaporator temperature decreases, the change in specifically across the evaporator decreases as T_1 goes down. Accordingly, the capacity decreases. Furthermore, the compressor power is greater as the evaporator temperature goes down, which contributes to the dramatic decrease in coefficient of performance shown on the graph.

PROBLEM 10.14

The data for the required plots are obtained using the IT code given in the solution of Problem 10.13. In this case, the variable to sweep is p_3, the condenser pressure.

PLOTS:





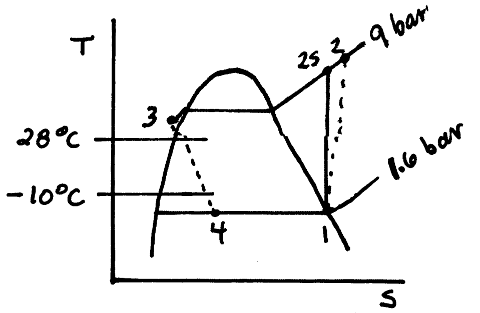
In this case, with the evaporator pressure constant and the compressor inlet state fixed, the specific enthalpy h_1 is constant. As condenser pressure is increased, state 4 on the T-s diagram moves to the right, thereby decreasing Δh across the evaporator. With mass flow rate fixed, the capacity decreases accordingly. Also, the compressor power increases, resulting in the dramatic decrease in coefficient of performance.

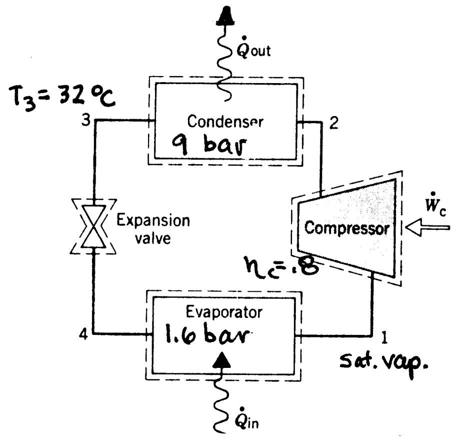
PROBLEM 10.15

KNOWN: The ideal vapor-compression refrigeration cycle of Problem 10.7 is modified to have a compressor efficiency of 80% and subcooled liquid exiting the condenser.

FIND: Answer the same questions as in Problem 10.7 and determine (d) the rates of exergy destruction in the compressor and valve.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.3, assumptions 1-4. Also, let $T_0 = 301$ K.

ANALYSIS: From the Solution of Problem 10.7, $h_1 = 237.97$ kJ/kg, $s_1 = 0.9295$ kJ/kg·K and $h_{2s} = 273.73$ kJ/kg. Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 282.67 \text{ kJ/kg}, \; s_2 = 0.9576 \text{ kJ/kg·K}$$

State 3 $p_3 = 9$ bar, $T_3 = 32°C \Rightarrow$ subcooled liquid; $h_3 \approx h_f(32°C) = 94.39$ kJ/kg

$$s_3 \approx s_f(32°C) = .3490 \text{ kJ/kg·K}$$

State 4 Throttling process $\Rightarrow h_4 = h_3 = 94.39$ kJ/kg

and; $x_4 = .31034$; $s_4 = 0.37198$ kJ/kg·K

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(5 \frac{\text{kg}}{\text{min}}\right)\left|\frac{1 \text{ min}}{60 \text{ s}}\right|(282.67 - 237.97)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 3.725 \text{ kW} \longleftarrow \dot{W}_c$$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = \left(5 \frac{\text{kg}}{\text{min}}\right)(237.97 - 94.39)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ ton}}{211 \text{ kJ/min}}\right| = 3.402 \text{ tons} \longleftarrow \dot{Q}_{in}$$

(c) The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 3.212 \longleftarrow \beta$$

(d) For the compressor: $0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{comp}$. Thus

$$(\dot{E}_d)_{comp} = T_0 \dot{m}(s_2 - s_1)$$

$$= (301 \text{ K})\left(5 \frac{\text{kg}}{\text{min}}\right)(.9576 - .9295)\frac{\text{kJ}}{\text{kg·K}}\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 0.705 \text{ kW} \longleftarrow (\dot{E}_d)_{comp}$$

For the valve, similarly

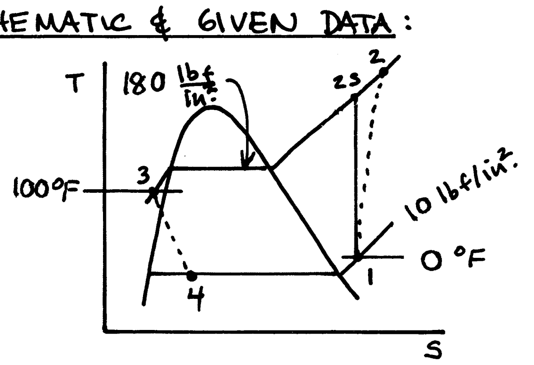
$$(\dot{E}_d)_{valve} = T_0 \dot{\sigma}_{valve} = T_0 \dot{m}(s_4 - s_3) = 0.576 \text{ kW} \longleftarrow (\dot{E}_d)_{valve}$$

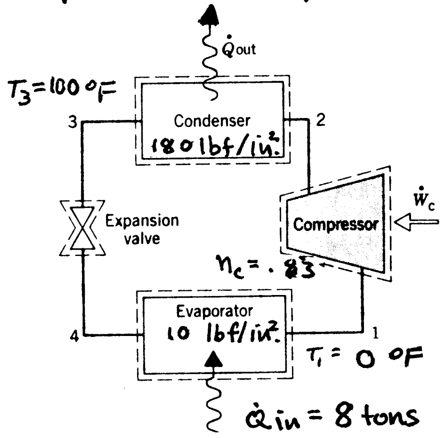
PROBLEM 10.16 *

KNOWN: The ideal vapor-compression refrigeration cycle of Problem 10.9 is modified to have a compressor efficiency of 83 % and subcooled liquid exiting the condenser.

FIND: Answer the same questions as in Prob. 10.9 and determine (d) the exergy destruction rates in the compressor and expansion valve.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.3, assumptions 1-4, except $\eta_c = .83$. Also, Let $T_0 = 550°R$.

ANALYSIS: First, fix each of the principal states.

From the solution of Prob. 10.9; $h_1 = 102.94 \frac{Btu}{lb}$, $s_1 = 0.2391 \frac{Btu}{lb \cdot °R}$, and $h_{2s} = 131.04$ Btu/lb. Using the compressor efficiency

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s}-h_1}{\eta_c}\right) = 136.80 \text{ Btu/lb}, \quad s_2 = 0.2481 \text{ Btu/lb}\cdot°R$$

State 3 $p_3 = 180 \frac{lbf}{in^2}$, $T_3 = 100$ °F $\Rightarrow$ subcooled liq.; $h_3 \approx h_f(100°F) = 44.23$ Btu/lb

$s_3 \approx s_f(100°F) = 0.0898$ Btu/lb·°R

State 4 $h_4 = h_3 = 44.23 \frac{Btu}{lb}$ and $s_4 = 0.1029$ Btu/lb·°R

(a) The mass flow rate is

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{(8 \text{ tons})}{(102.94 - 44.23)\frac{Btu}{lb}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 27.30 \text{ lb/min}$$

and the compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(27.30 \frac{lb}{min}\right)\left|\frac{60 \text{ min}}{1 h}\right| (136.80 - 102.94)\frac{Btu}{lb}\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 21.79 \text{ hp} \longleftarrow \dot{W}_c$$

(b) $\dot{Q}_{out} = \dot{m}(h_2 - h_3) = 2527.2$ Btu/min $\longleftarrow \dot{Q}_{out}$

(c) $\beta = \frac{h_1 - h_4}{h_2 - h_1} = 1.73 \longleftarrow \beta$

(d) For the compressor: $0 = \cancelto{0}{\sum_j \left(\frac{\dot{Q}}{T}\right)_j} + \dot{m}(s_2 - s_1) + \dot{\sigma}_{comp}$. Thus

$$(\dot{E}_d)_{comp} = T_0 \dot{\sigma}_{comp} = T_0 \dot{m}(s_2 - s_1)$$

$$= (550°R)\left(27.30 \frac{lb}{min}\right)(.2481 - .2391)\frac{Btu}{lb \cdot °R} = 135.14 \text{ Btu/min} \longleftarrow (\dot{E}_d)_{comp}$$

Similarly, for the valve

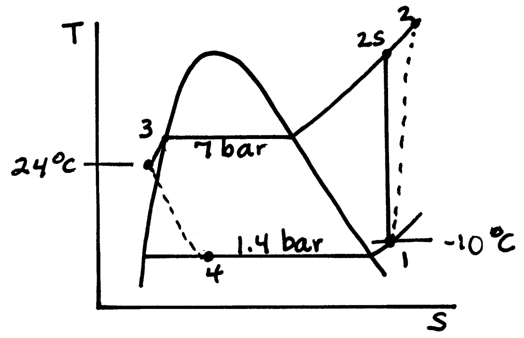
$$(\dot{E}_d)_{valve} = T_0 \dot{\sigma}_{valve} = T_0 \dot{m}(s_4 - s_3) = 196.70 \text{ Btu/min} \longleftarrow (\dot{E}_d)_{valve}$$

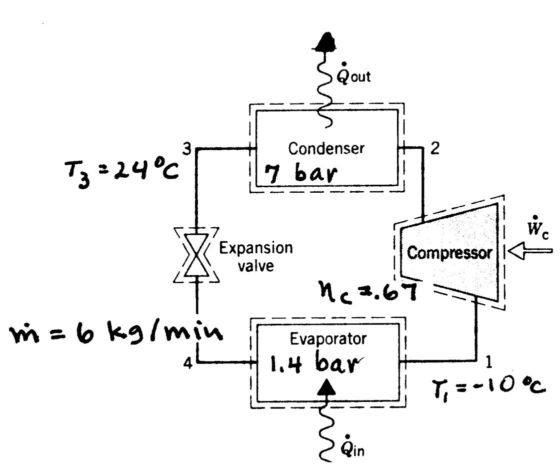
PROBLEM 10.17

KNOWN: A vapor-compression refrigeration system circulates R134 with a known mass flow rate. Data are given at various locations and the compressor efficiency is specified.

FIND: Determine (a) the coefficient of performance, (b) the refrigerating capacity, (c) the rates of exergy destruction in the compressor and valve, (d) the changes in specific flow exergy of the refrigerant passing through the evaporator and condenser.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 10.3, assumptions 1-4, except $\eta_c = .67$. Also, let $T_0 = 294$ K.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -10°C$, $p_1 = 1.4$ bar $\Rightarrow h_1 = 243.40$ kJ/kg, $s_1 = .9606$ kJ/kg·K

State 2 For isentropic compression, $p_2 = 7$ bar, $s_{2s} = s_1 \Rightarrow h_{2s} = 278.06 \frac{kJ}{kg}$

Using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{(h_{2s} - h_1)}{\eta_c} = 295.13 \text{ kJ/kg}$$

$$s_2 = 1.0135 \text{ kJ/kg·K}$$

State 3 $p_3 = 7$ bar, $T = 24°C \Rightarrow h_3 \approx h_{f@24} = 82.90$ kJ/kg, $s_3 \approx .3113$ kJ/kg·K

State 4 Throttling procecc $\Rightarrow h_4 = h_3 = 82.90$ kJ/kg, $s_4 = .33011$ kJ/kg·K

(a) The coefficient of performance is

$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 3.10 \longleftarrow \beta$$

(b) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4)$$

$$= \left(6 \frac{kg}{min}\right)(243.40 - 82.90) \frac{kJ}{kg} \left|\frac{1 \text{ ton}}{211 \text{ kJ/min}}\right| = 4.564 \text{ tons} \longleftarrow \dot{Q}_{in}$$

(c) For the compressor : $0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{comp}$. Thus, the rate of exergy destruction is

$$(\dot{E}_d)_{comp} = T_0 \dot{\sigma}_{comp} = T_0 \dot{m}(s_2 - s_1)$$

$$= (294 \text{ K})\left(6 \frac{kg}{min}\right)\left|\frac{1 \text{ min}}{60 \text{ s}}\right|(1.0135 - .9606) \frac{kJ}{kg}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 1.55 \text{ kW} \longleftarrow (\dot{E}_d)_{comp}$$

PROBLEM 10.17 (Cont'd)

Similarly, for the valve

$$(\dot{E}_d)_{valve} = T_o \dot{m} (s_4 - s_3)$$

$$= (294\ K)(6) \left|\frac{1}{60}\right| (.33011 - .3113\ ) = .5530\ kW \longleftarrow (\dot{E}_d)_{valve}$$

(d) The change in specific flow exergy for refrigerant passing through the evaporator is

$$(e_{f1} - e_{f4}) = (h_1 - h_4) - T_o(s_1 - s_4)$$

$$= (243.40 - 82.90) - (294)(.9606 - .33011\ )$$

① $$= -\ 24.86\ kJ/kg \longleftarrow (e_{f1} - e_{f4})$$

For the condenser

$$(e_{f3} - e_{f2}) = (h_3 - h_2) - T_o(s_3 - s_2) = -5.78\ kJ/kg \longleftarrow (e_{f3} - e_{f2})$$

---

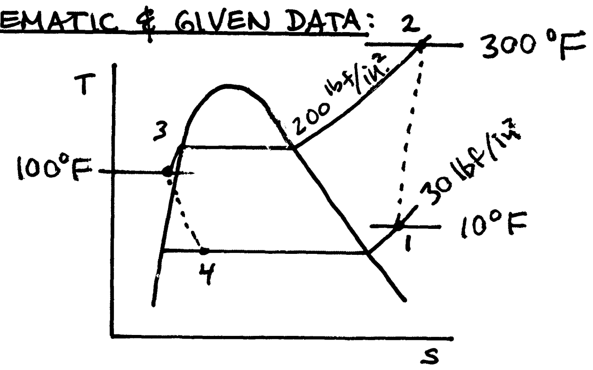
1. Although there is heat transfer to the refrigerant passing through the evaporator, the specific flow exergy decreases. This can be explained by noting that the state of the working fluid moves closer to the dead state as it is heated at a temperature below T_o.

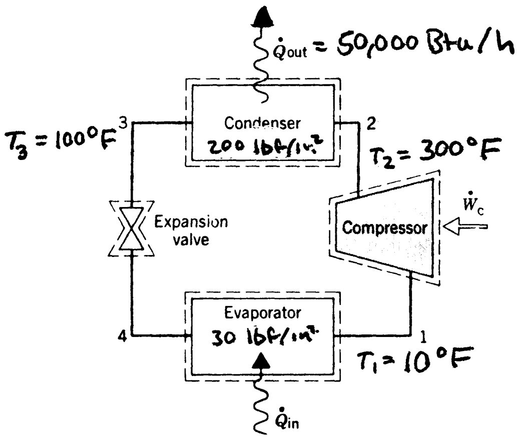
PROBLEM 10.18*

KNOWN: A vapor-compression refrigeration system uses ammonia as the working fluid. Data are known at various locations, and the heat transfer rate from the working fluid passing through the condenser is specified.

FIND: Determine (a) the compressor power and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 10.3, assumptions 1-4, except the compressor efficiency is not specified.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 30$ lbf/in², $T_1 = 10°F \Rightarrow h_1 = 617.07$ Btu/lb, $s_1 = 1.3479$ Btu/lb°R

State 2 $p_2 = 200$ lbf/in², $T_2 = 300°F \Rightarrow h_2 = 763.74$ Btu/lb, $s_2 = 1.3774$ Btu/lb°R

State 3 $p_3 = 200$ lbf/in², $T_3 = 100°F \Rightarrow h_3 \approx h_f$ @100°F $= 155.05$ Btu/lb

State 4 $p_4 = 30$ lbf/in²; Throttling process $\Rightarrow h_4 = h_3 = 155.05$ Btu/lb

(a) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1)$$

Evaluating $\dot{m}$,

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) \Rightarrow \dot{m} = \frac{\dot{Q}_{out}}{(h_2 - h_3)} = \frac{(50{,}000 \text{ Btu/h})}{(763.74 - 155.05) \text{ Btu/lb}}$$

$$= 82.14 \text{ lb/h}$$

thus

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(82.14 \frac{\text{lb}}{\text{h}}\right)(763.74 - 617.07)\frac{\text{Btu}}{\text{lb}}\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 4.73 \text{ hp} \longleftarrow \dot{W}_c$$

(b) The coefficient of performance is

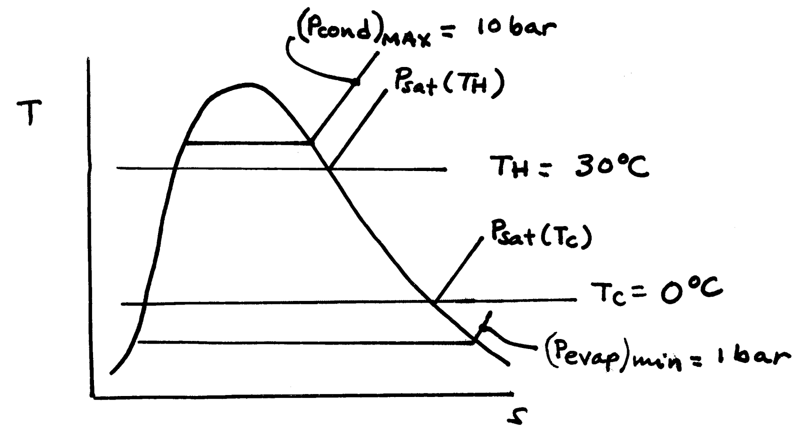
$$\beta = \frac{h_1 - h_4}{h_2 - h_1} = 3.15 \longleftarrow \beta$$

PROBLEM 10.19

KNOWN: For a vapor-compression refrigeration cycle, the minimum and maximum allowed refrigeration pressures are 1 and 10 bar, respectively. The cold and warm region temperatures are also specified.

FIND: Among several candidate working fluids, determine which (if any) can be used for this duty.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The refrigeration cycle adheres to the model presented for the vapor-compression cycle in Sec. 10.2

ANALYSIS: For heat transfer to the working fluid passing through the evaporator from the cold region, we must have

$$1 \text{ bar} \leq P_{evap} \leq P_{sat}(T_C)$$

For heat transfer from the working fluid passing through the condenser to the warm region, we must have

$$P_{sat}(T_H) \leq P_{cond} \leq 10 \text{ bar}$$

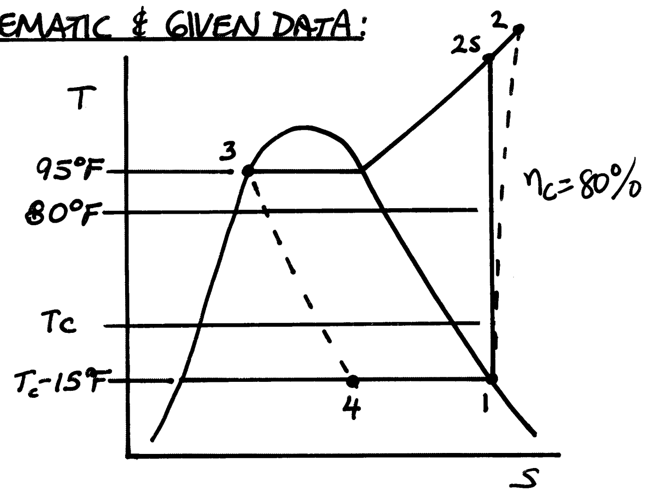
working fluid	table	$P_{sat}(0°C)$	$P_{sat}(30°C)$	feasible
R22	A-7	4.98 bar	11.93 bar	No - $P_{sat}(30°C)$ too high
R134a	A-10	2.93	7.7	yes
Ammonia	A-13	4.3	11.69	No - $P_{sat}(30°C)$ too high
Propane	A-16	4.74	10.8	No - $P_{sat}(30°C)$ too high

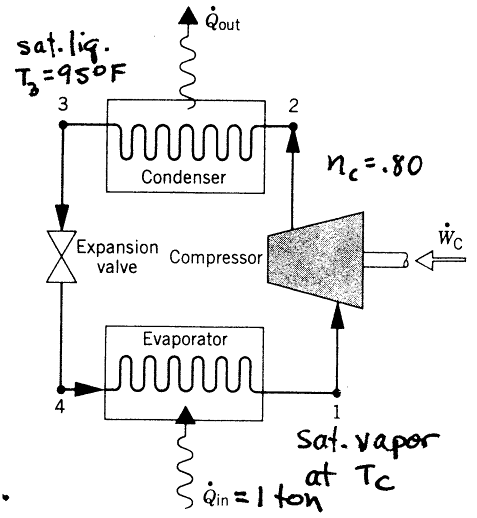
PROBLEM 10.20

KNOWN: A vapor-compression refrigeration cycle is used to maintain a cold region at temperature T_c. Refrigerant conditions are given at various points. The refrigerating capacity is specified.

FIND: Plot refrigerant mass flow rate, coefficient of performance, and refigerating efficiency versus a range of T_c values and for the refrigerants (a) R134a, (b) propane, (c) R22, (d) ammonia.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.2, except $\eta_c = 80\%$.

ANALYSIS: The procedure is the same for each refrigerant. The principal states are fixed as follows:

State 1: $T_1 = T_c - 15$, sat. vapor; h_g and s_g at T_1

State 2: Using T_3, determine $p_3 = p_2 = p_{sat@T_3}$. Then, with p_2 and $s_1 = s_{2s}$ determine h_{2s}. When using the superheated vapor table, double interpolation is likely. Next, use η_c to get h_2;

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c$$

State 3: $T_3 = 95\,^\circ F$. $h_3 = h_{f@95°F}$.

State 4: Throttling process: $h_4 = h_3$

To get the refrigerant mass flow rate

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4}$$

The coefficient of performance is

$$\beta = \frac{h_2 - h_1}{h_1 - h_4}$$

Finally; $\beta_{carnot} = \dfrac{T_c\,(°R)}{540 - T_c(°R)}$

and $\eta_{ref} = \beta / \beta_{carnot}$

PROBLEM 10.20 (Cont'd.)

① The data for the required plots are obtained using IT, as follows. Shown here is the code for ammonia. The code for other refrigerants is similar.

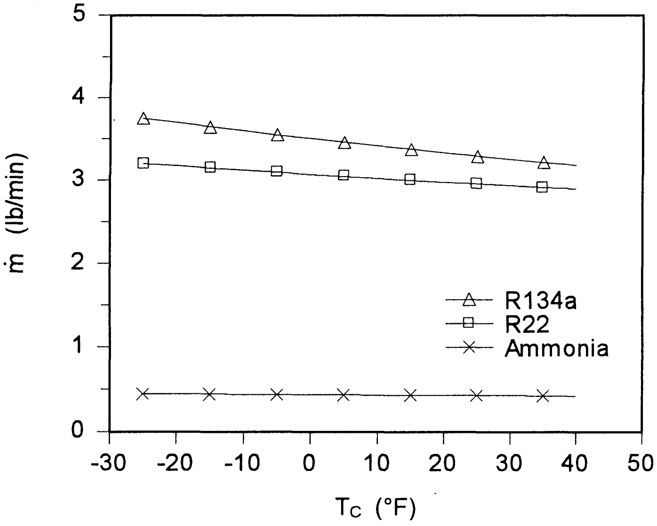
IT Code

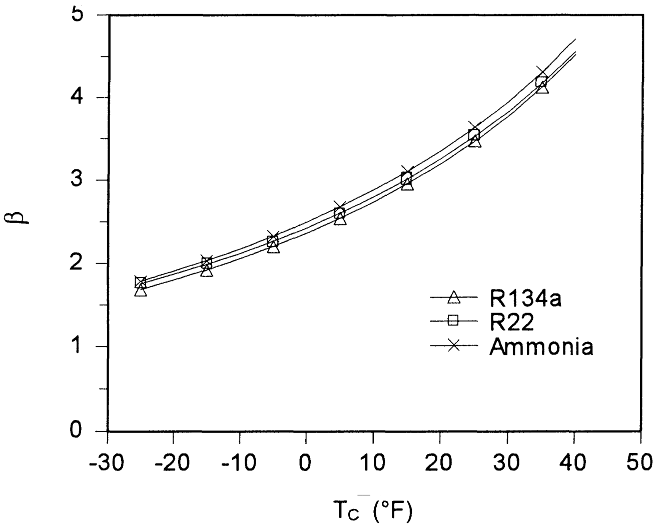
```
T1 = TC - 15   // °F
eta_c = 0.8
T3 = 95   // °F
Qdotin = 1    // ton
TC = -25    // °F
TH = 80    // °F

x1 = 1
p1 = Psat_T("Ammonia", T1)
h1 = hsat_Px("Ammonia", p1, x1)
s1 = ssat_Px("Ammonia", p1, x1)
s2s = s1
p2 = p3
s2s = s_PT("Ammonia", p2, T2s)
h2s = h_PT("Ammonia", p2, T2s)
h2 = h1 + (h2s - h1) / eta_c
p3 = Psat_T("Ammonia", T3)
x3 = 0
h3 = hsat_Px("Ammonia", p3, x3)
h4 = h3

mdot = (Qdotin * 200) / (h1 - h4)
beta = (h1 - h4) / (h2 - h1)
betacarnot = (TC + 460)  / ((TH + 460) - (TC + 460))
eff = beta / betacarnot
```

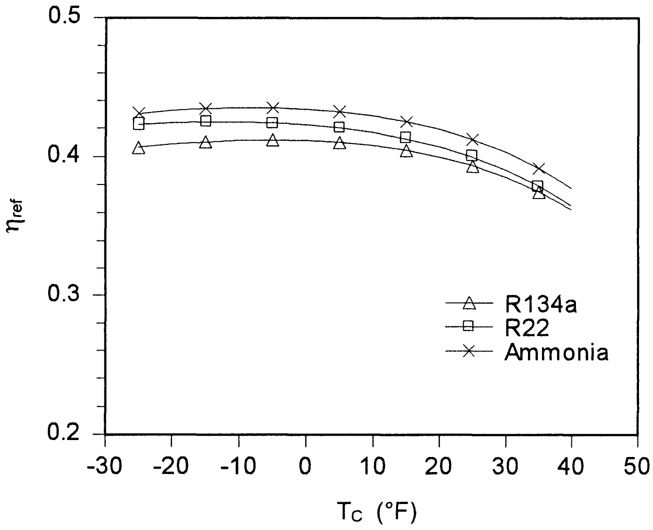
IT Results for Ammonia, T_C = -25 °F

h_1 = 597 Btu/lb
h_2 = 847.8 Btu/lb
h_{2s} = 797.7 Btu/lb
h_3 = 149.1 Btu/lb
h_4 = 149.1 Btu/lb
β_{carnot} = 4.143
$\dot{m}$ = 0.4465 lb/min
β = 1.785
η_{ref} = 0.431

PLOTS:







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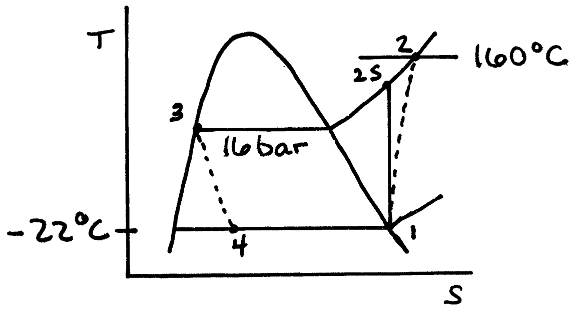
1. The version of IT available with early printings of the 4th edition does not include property routines for the propane tables. Thus, propane is not shown on these plots.

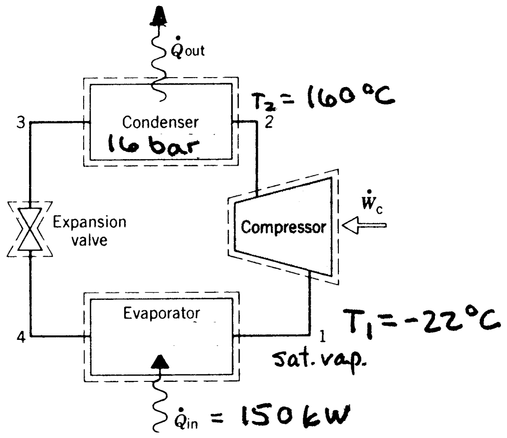
PROBLEM 10.21*

KNOWN: Ammonia is the working fluid in a vapor-compression refrigeration system. Data are known at various locations and the refrigerating capacity is given.

FIND: Determine (a) the mass flow rate of refrigerant, (b) the compressor power, (c) the coefficient of performance, and (d) the isentropic compressor efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.3, assumptions 1-4, except the compressor efficiency is unknown.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -22°C$, sat. vapor $\Rightarrow h_1 = 1415.08$ kJ/kg, $s_1 = 5.6457$ kJ/kg K

State 2 $P_2 = 16$ bar, $T_2 = 160°C \Rightarrow h_2 = 1798.45$ kJ/kg

State 3 $P_3 = 16$ bar, sat. liquid $\Rightarrow h_3 = 376.46$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 376.46$ kJ/kg

(a) The mass flow rate is determined using the refrigerating capacity

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4)$$

Thus

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{(150\ \text{kW})}{(1415.08 - 376.46)\frac{\text{kJ}}{\text{kg}}}\left|\frac{1\ \text{kJ/s}}{1\ \text{kW}}\right| = 0.1444\ \text{kg/s} \quad \longleftarrow \dot{m}$$

(b) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(0.1444\frac{\text{kg}}{\text{s}}\right)(1798.45 - 1415.08)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 55.36\ \text{kW} \quad \longleftarrow \dot{W}_c$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{150\ \text{kW}}{55.36\ \text{kW}} = 2.71 \quad \longleftarrow \beta$$

(d) For isentropic compression, $P_{2s} = 16$ bar, $s_{2s} = s_1 \Rightarrow h_{2s} = 1755.38$ kJ/kg

Thus, the isentropic efficiency is

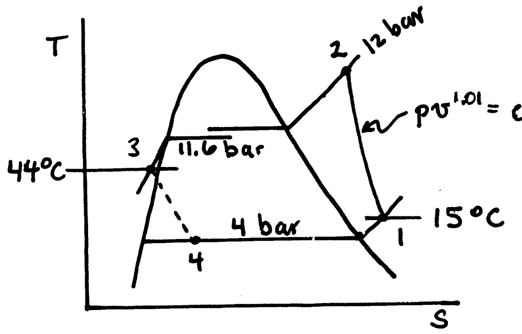
$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{1755.38 - 1415.08}{1798.45 - 1415.08} = 0.888\ (88.8\%) \quad \longleftarrow \eta$$

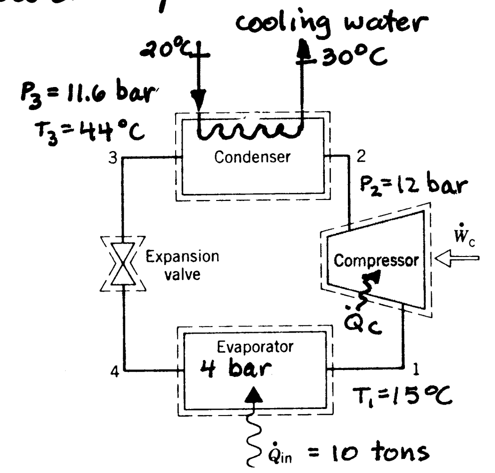
PROBLEM 10.22

KNOWN: R134a is the working fluid in a vapor-compression refrigeration system having a water cooled condenser. The capacity is known and the compression process is described by $pv^{1.01}$ = constant.

FIND: Determine (a) the mass flow rate of R134a, (b) power input and heat transfer rate for the compressor, (c) coefficient of performance, (d) cooling water mass flow rate, (e) rates of exergy destruction in the condenser and expansion valve, each expressed as a percentage of the power input.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The compression is polytropic with n = 1.01. (3) The expansion through the valve is a throttling process. (4) Heat transfer from the outside of the condenser can be neglected. (5) Kinetic and potential energy effects are negligible. (6) Let T_0 = 293K.

ANALYSIS: First, fix each of the principal states.

State 1 p_1 = 4 bar, T_1 = 15°C ⇒ $h_1 = 258.15\ \frac{kJ}{kg}$, $s_1 = .9348$ kJ/kg·K

$v_1 = 0.05258$ m³/kg

State 2 For the polytropic process, P_2 = 12 bar and

$$v_2 = \left(\frac{P_1}{P_2}\right)^{1/1.01} v_1 = 0.01772\ m^3/kg$$

From Table A-12 ; $h_2 = 281.33$ kJ/kg , $s_2 = .9341$ kJ/kg·K

State 3 T_3 = 44°C, compressed liquid ⇒ $h_3 \approx h_f(44°C) = 112.22$ kJ/kg

$s_3 \approx s_f(44°C) = 0.4054$ kJ/kg·K

State 4 Throttling process ⇒ $h_4 = h_3 = 112.22$ kJ/kg

Equation 6.55 is used to determine the compressor work input

$$\frac{\dot{W}_c}{\dot{m}} = \int_1^2 v\,dp = \left(\frac{n}{n-1}\right)(p_2v_2 - p_1v_1)$$

$$= \left(\frac{1.01}{.01}\right)\left[(12\text{ bar})(.01772)\frac{m^3}{kg} - (4)(.05258)\right]\left|\frac{10^5\ N/m^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\ N\cdot m}\right|$$

$$= 23.432\text{ kJ/kg}$$

The mass flow rate is found using the refrigerating capacity

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{(10\text{ tons})}{(258.15 - 112.22)\text{ kJ/kg}}\left|\frac{211\text{ kJ/min}}{1\text{ ton}}\right|\left|\frac{1\text{ min}}{60\text{ s}}\right| = 0.241\text{ kg/s} \leftarrow \dot{m}$$

Thus, the compressor power input is

$$\dot W_c = \dot m\left(\frac{\dot W_c}{\dot m}\right) = \left(0.241\ \frac{kg}{s}\right)\left(23.432\ \frac{kJ}{kg}\right)\left|\frac{1\ kW}{1\ kJ/s}\right| = 5.647\ kW \longleftarrow \dot W_c$$

The heat transfer rate is

$$\dot Q_{cv} = -\dot W_{cv} + \dot m\,(h_2 - h_1)$$

$$= -5.647 + (.241)(281.33 - 258.15) = -0.0606\ kW \longleftarrow \dot Q_{cv}$$

The coefficient of performance is

$$\beta = \frac{\dot Q_{in}}{\dot W_c} = \frac{(10\ tons)}{(5.647\ kW)}\left|\frac{211\ kJ/min}{1\ ton}\right|\left|\frac{1\ min}{60\ s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 6.227 \longleftarrow \beta$$

Evaluating $\dot m_{cw}$ from an energy balance

$$0 = \dot m\,(h_2 - h_3) + \dot m_{cw}\,(h_{cw,in} - h_{cw,out})$$

$$\dot m_{cw} = \dot m\left(\frac{h_2 - h_3}{h_{cw,out} - h_{cw,in}}\right) = \left(0.241\ \frac{kg}{s}\right)\left(\frac{281.33 - 112.22}{125.79 - 83.96}\right)$$

$$= 0.9743\ kg/s \longleftarrow \dot m_{cw}$$

where $h_{cw} = h_f(T)$.

The rates of exergy destruction can be found using $\dot E_d = T_0\,\dot\sigma_{cv}$.

- For the condenser.

$$0 = \cancel{\sum\left(\frac{\dot Q}{T}\right)_j}^{\,0} + \dot m\,(s_2 - s_3) + \dot m_{cw}\,(s_{cw,in} - s_{cw,out}) + \dot\sigma_{cond}$$

$$\rightarrow (\dot E_d)_{cond} = T_0\,\dot\sigma_{cond} = T_0\left[\dot m\,(s_3 - s_2) + \dot m_{cw}\,(s_{cw,out} - s_{cw,in})\right]$$

$$= (293\ K)\Big[(0.241)\ \frac{kg}{s}\ (0.4054 - 0.9341)\ \frac{kJ}{kg}$$

$$+ (.9743)(.4369 - .2966)\Big]\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 2.718\ kW$$

Expressed as a percentage of the compressor power, $\% = \left(\frac{2.718}{5.647}\right)(100) = 48 \longleftarrow (e)$

- For the valve.

$(\dot E_d)_{valve} = T_0\,\dot m\,(s_4 - s_3)$. To find s_4, use $h_4 = h_3$ with data from Table A-11 at 4 bar: $x_4 = \frac{112.22 - 62}{190.32} = 0.2639 \Rightarrow s_4 = 0.2399 + 0.2639(0.9145 - 0.2399) = 0.4179\ kJ/kg\cdot K$

Thus

$$(\dot E_d)_{valve} = (293)(0.241)(0.4179 - 0.4054) = 0.88\ kW$$

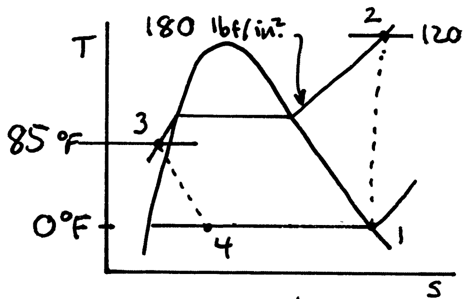
$$\% = \left(\frac{0.88}{5.647}\right)(100) = 16 \longleftarrow (e)$$

PROBLEM 10.23*

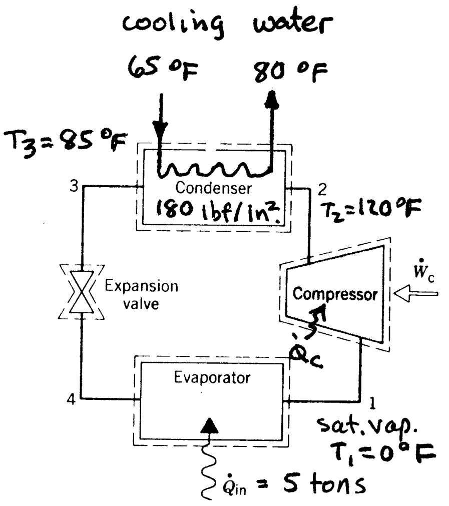
KNOWN: A vapor-compression refrigeration system uses propane as the working fluid. Data are known at various locations and the refrigerating capacity is given. The condenser is water-cooled.

FIND: Determine (a) the compressor power, (b) the mass flow rate of condenser cooling water, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:



$$\frac{\dot{Q}_c}{\dot{m}} = -3.5\ \frac{\text{Btu}}{\text{lb}}$$



ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) There are no pressure drops through the heat exchangers. (3) Expansion through the valve is a throttling process. (4) Kinetic and potential energy effects are negligible. (5) Heat transfer from the outside of the condenser can be neglected.

ANALYSIS: First, fix each of the principal states (Tables A-16E to A-18E).

State 1 $T_1 = 0°F$, sat. vapor $\Rightarrow h_1 = 193.2$ Btu/lb, $s_1 = 0.422$ Btu/lb·°R

State 2 $p_2 = 180$ lbf/in.², $T_2 = 120°F \Rightarrow h_2 = 229.8$ Btu/lb, $s_2 = 0.431$ Btu/lb·°R

State 3 $p_3 = 180$ lbf/in.², $T_3 = 85°F \Rightarrow$ compressed liquid ; $h_3 \approx h_f(85°F) = 74.41$ Btu/lb

State 4 Throttling process $\Rightarrow h_4 = h_3 = 74.41$ Btu/lb

(a) Evaluating the refrigerant mass flow rate

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_1 - h_4)} = \frac{5\ \text{tons}}{(193.2 - 74.41)\frac{\text{Btu}}{\text{lb}}} \left|\frac{200\ \text{Btu/min}}{1\ \text{ton}}\right| = 8.42\ \frac{\text{lb}}{\text{min}}$$

Thus $\dot{W}_c = \dot{m}\,[-(\dot{Q}_c/\dot{m}) + (h_2 - h_1)] = (8.42\frac{\text{lb}}{\text{min}})[-(-3.5) + (229.8 - 193.2)]\frac{\text{Btu}}{\text{lb}}$

$$= 337.6\ \text{Btu/min} \longleftarrow \dot{W}_c$$

(b) An energy balance on the condenser gives, with $h_{cw} \approx h_f(T)$

$$\dot{m}_{cw} = \dot{m}\left(\frac{h_2 - h_3}{h_f(80) - h_f(65)}\right) = (8.42\frac{\text{lb}}{\text{min}})\left(\frac{229.8 - 74.41}{48.09 - 33.09}\right) = 87.23\ \frac{\text{lb}}{\text{min}} \longleftarrow \dot{m}_{cw}$$

(c) The coefficient of performance is

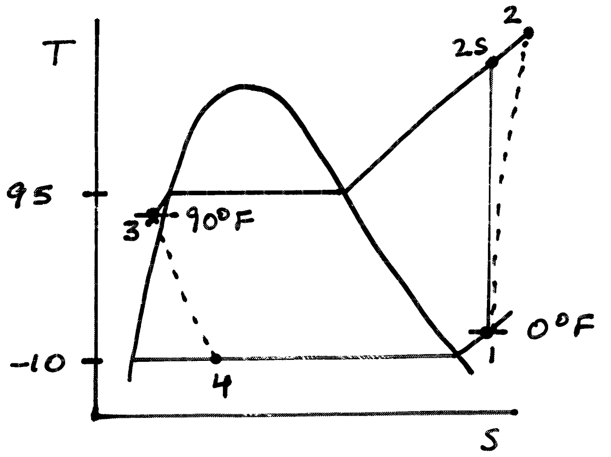
$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{5\ \text{tons}}{337.6\ \text{Btu/min}} \left|\frac{200\ \text{Btu/min}}{1\ \text{ton}}\right| = 2.96 \longleftarrow \beta$$

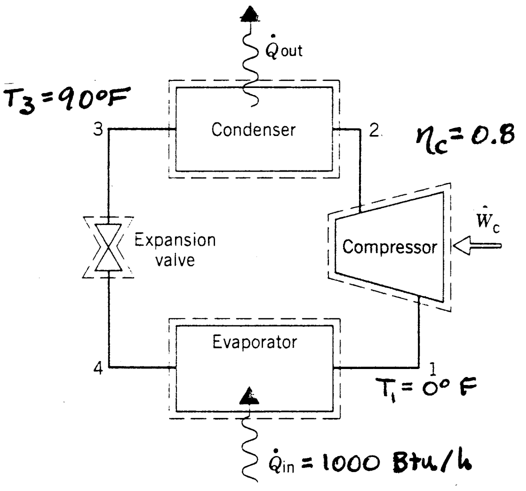
PROBLEM 10.24

KNOWN: Data are known at various locations in a vapor-compression refrigeration cycle. The refrigerating capacity is 1000 Btu/h.

FIND: Determine the evaporator and condenser pressures, mass flow rate of refrigerant, compressor power input, and coefficient of performance for (a) Refrigerant 134a and (b) propane as the working fluid.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.3, assumptions 1-4.

ANALYSIS: First, fix each principal state.

(a) Refrigerant 134a

State 1: $P_1 = P_{sat} @ -10°F = 16.674\ lbf/in^2$ ← $P_{evap}(a)$

Interpolating in Table A-12E at $T_1 = 0°F \Rightarrow h_1 = 102.2\ \frac{Btu}{lb}$, $s_1 = 0.2281$ Btu/lb°R

State 2: $P_2 = P_{sat} @ 95°F = 128.62\ lbf/in^2$ ← $P_{cond}(a)$

$$h_{2s} = 121.01\ Btu/lb$$

$$h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 125.7\ Btu/lb$$

State 3: $h_3 \approx h_f(T_3) = 40.72$ Btu/lb

State 4: throttling process $\Rightarrow h_3 = h_4 = 40.72$ Btu/lb

(b) Propane

State 1 $P_1 = P_{sat} @ -10°F = 31.9\ lbf/in.^2$ ← $P_{evap}(b)$

① Double interpolation in Table A-18E gives; $h_1 = 194.2$ Btu/lb, $s_1 = 0.4331$ Btu/lb·°R

State 2 $P_2 = P_{sat} @ 95°F = 176.9\ lbf/in.^2$ ← $P_{cond}(b)$

For isentropic compression, $s_{2s} = s_1 = 0.4331 \Rightarrow h_{2s} = 230.61$ Btu/lb.

Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_c} = 239.7$$

State 3 $h_3 \approx h_f(90°F) = 77.74$ Btu/lb

State 4 throttling process $\Rightarrow h_4 = h_3 = 77.74$ Btu/lb

Mass flow rate

The refrigerating capacity is used to find the mass flow rate.

$$\dot{m}_{R\text{-}134a} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{(1000\ \text{Btu/h})}{(102.2 - 40.72)\ \text{Btu/lb}} \left| \frac{1\ \text{h}}{60\ \text{min}} \right|$$

$$= 0.271\ \text{lb/min} \longleftarrow \dot{m}\ (a)$$

Similarly, for propane; $\dot{m}_{propane} = 0.143\ \text{lb/min} \longleftarrow \dot{m}\ (b)$

Compressor power input

For Refrigerant 134a

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(0.271\ \frac{\text{lb}}{\text{min}}\right)(125.7 - 102.2)\frac{\text{Btu}}{\text{lb}} \left| \frac{60\ \text{min}}{1\ \text{h}} \right| \left| \frac{1\ \text{hp}}{2545\ \text{Btu/h}} \right|$$

② $$= 0.150\ \text{hp} \longleftarrow \dot{W}_c\ (a)$$

Similarly, for propane; $\dot{W}_c = 0.153 \longleftarrow \dot{W}_c\ (b)$

Coefficient of performance

$$\beta_{R\text{-}134a} = \frac{h_1 - h_4}{h_2 - h_1} = \frac{102.2 - 40.72}{125.7 - 102.2} = 2.62 \longleftarrow \beta\ (a)$$

Similarly, for propane; $\beta_{propane} = 2.56 \longleftarrow \beta\ (b)$

---

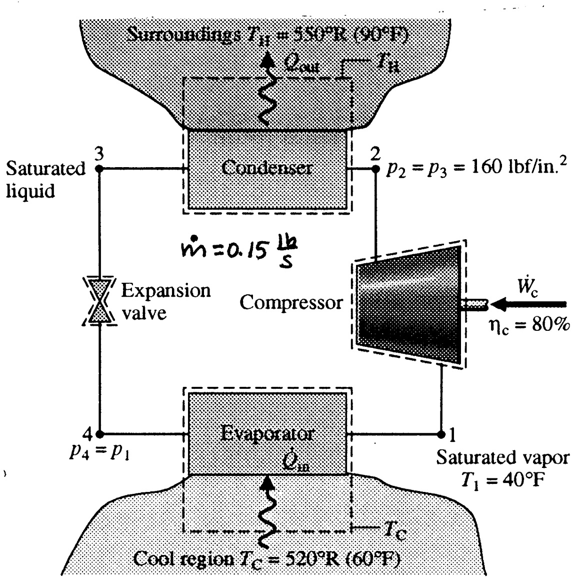
1. Note that the propane cycle operates at higher pressures than the R-134a cycle.
2. The propane cycle has a much lower mass flow rate than the R-134a cycle. However, the power and coefficients of performance are nearly the same for the two working fluids in this case.

PROBLEM 10.25

KNOWN: Steady-state operating data are provided for a vapor-compression air conditioning system.

FIND: Determine (a) the power required by the compressor, (b) the coefficient of performance, (c) the rates of exergy destruction in the compressor and valve, and the rates of exergy destruction and exergy transfer accompanying heat transfer for control volumes enclosing (d) the evaporator and a portion of the cool region, (e) the condenser and a portion of the surroundings.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volumes shown in the schematic are at steady state.
2. Kinetic and potential energy changes are negligible as are pressure changes for flow through the evaporator and condenser.
3. The expansion across the valve is a throttling process.
4. There is no stray heat transfer for any control volume.
5. T_0 = 550°R.

ANALYSIS: For the principal states, essential property data can be obtained from the tables for R134a

State	T(°F)	p(lbf/in²)	h (Btu/lb)	s (Btu/lb·°R)	
1	40	49.7	107.39	0.2189	(Table A-10E)
2	–	160	120.33	0.2234	
3	–	160	47.65	0.0958	(Table A-10E)
4	–	49.7	47.65	0.0994	

Using $s_{2s} = s_1$ and interpolating in Table A-12E gives h_{2s} = 117.74 Btu/lb. Then, using the isentropic efficiency $h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 107.39 + \left(\frac{117.74 - 107.39}{0.8}\right) = 120.33$ Btu/lb

Interpolation in Table A-12E then yields s_2 = 0.2234 Btu/lb·°R.

Since $h_4 = h_3$, $x_4 = \frac{47.65 - 24.05}{83.34} = 0.2832 \Rightarrow s_4 = 0.0522 + 0.2832(0.2189 - 0.0522) = 0.0994 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

(a) An energy rate balance for the compressor gives the compressor power input:

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(0.15 \frac{\text{lb}}{\text{s}}\right)(120.33 - 107.39) \frac{\text{Btu}}{\text{lb}} = 1.94 \frac{\text{Btu}}{\text{s}} \longleftarrow \text{(a)}$$

(b) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_c} = \frac{\dot{m}(h_1 - h_4)}{\dot{W}_c} = \frac{(0.15)(107.39 - 47.65)\ \text{Btu/s}}{1.94\ \text{Btu/s}} = \frac{8.96\ \text{Btu/s}}{1.94\ \text{Btu/s}} = 4.62 \longleftarrow \text{(b)}$$

PROBLEM 10.25 (Contd.)

To evaluate exergy destruction rates, we use $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production obtained from an entropy rate balance. Thus

(c) For the compressor

$$(\dot{E}_d)_{comp} = T_0 \dot{m}(s_2 - s_1) = (550°R)(0.15 \tfrac{lb}{s})(0.2234 - 0.2189)\frac{Btu}{lb \cdot °R} = 0.371 \frac{Btu}{s}$$

For the valve

$$(\dot{E}_d)_{valve} = T_0 \dot{m}(s_4 - s_3) = (550)(0.15)(0.0994 - 0.0958) = 0.297 \frac{Btu}{s}$$

(d) For a control volume enclosing the evaporator, as shown in the schematic, an entropy rate balance reads

$$0 = \frac{\dot{Q}_{in}}{T_C} + \dot{m}(s_4 - s_1) + \dot{\sigma}_{cv}$$

$$\Rightarrow \quad (\dot{E}_d)_{evap} = T_0\left[-\frac{\dot{Q}_{in}}{T_C} + \dot{m}(s_1 - s_4)\right] = 550°R\left[-\frac{8.96\ Btu/s}{520°R} + (0.15 \tfrac{lb}{s})(0.2189 - 0.0994)\frac{Btu}{lb \cdot °R}\right]$$

$$= 0.382 \frac{Btu}{s}$$

The transfer of exergy accompanying heat transfer is

① $\dot{E}_q = \left[1 - \frac{T_0}{T_C}\right]\dot{Q}_{in} = \left[1 - \frac{550}{520}\right](8.96) = -0.517 \frac{Btu}{s}$. The exergy transfer is opposite to the direction of the heat transfer because heat transfer occurs at $T_C < T_0$.

(e) For a control volume enclosing the condenser, as shown in the schematic, an entropy rate balance reads

$$0 = -\frac{\dot{Q}_{out}}{T_H} + \dot{m}(s_2 - s_3) + \dot{\sigma}_{cv}$$

$$\boxed{= \dot{m}(h_2 - h_3) = 0.15(120.33 - 47.65) = 10.9\ Btu/s}$$

$$\Rightarrow \quad (\dot{E}_d)_{COND} = T_0\left[\frac{\dot{Q}_{out}}{T_H} + \dot{m}(s_3 - s_2)\right] = 550\left[\frac{10.9}{550} + 0.15(0.0958 - 0.2234)\right]$$

$$= 0.373\ Btu/s$$

The transfer of exergy accompanying heat transfer is

$$\dot{E}_q = \left[1 - \frac{T_0}{T_H}\right]\dot{Q}_{out} = 0, \text{ since } T_0 = T_H.$$

---

1. The exergy evaluations can be summarized as follows:

- Rate of exergy in:
  - compressor power input 1.94 $\frac{Btu}{s}$
- Disposition of the exergy:

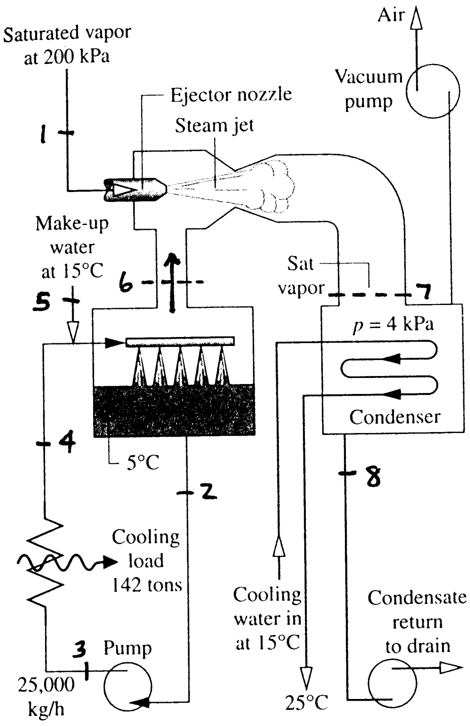
– Transfer to cool region	0.517 Btu/s	(26.6%)
– Transfer to warm region	0	
– Destructions compressor	0.371 "	(19.1%)
valve	0.297 "	(15.3%)
evaporator	0.382 "	(19.7%)
condenser	0.373 "	(19.2%)
	1.94 Btu/s	

PROBLEM 10. 26

KNOWN: A refrigeration system produces chilled water in a flash chamber maintained at a vacuum by a steam ejector. Data are known at various locations.

FIND: Determine the make-up water and cooling water flow rates.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The pump work is negligible. (3) Kinetic and potential energy effects are negligible at the numbered locations. (4) There are no stray heat transfers with the surroundings.

ANALYSIS: First, fix each of the principal states.

State 1 Sat. vapor, 200 kPa $\Rightarrow h_1 = 2706.7$ kJ/kg

State 2 sat. liquid, 5°C $\Rightarrow h_2 = 20.98$ kJ/kg

State 3 $h_3 = h_2 + \overset{0}{\cancel{\dot{W}_p}}/\dot{m}_2 \approx h_2$

State 4 $\dot{Q}_{load} = 142 \text{ tons} = 1.798 \times 10^6$ kJ/h

$h_4 = h_3 + \dot{Q}_{load}/\dot{m} = 20.98 + (1.798 \times 10^6/25{,}000) = 92.9$ kJ/kg

State 5 $T_5 = 15°C$; $h_5 \approx h_f(T_5) = 62.99$ kJ/kg

State 6 sat. vapor, 5°C $\Rightarrow h_6 = 2510.6$ kJ/kg

State 7 sat. vapor, 4 kPa $\Rightarrow h_7 = 2554.4$ kJ/kg

State 8 sat. liquid, 4 kPa $\Rightarrow h_8 = 121.46$ kJ/kg

Now, consider a control volume enclosing the flash chamber:

$$\dot{m}_4 h_4 + \dot{m}_5 h_5 = \dot{m}_2 h_2 + \dot{m}_6 h_6$$

and

$$\dot{m}_2 = \dot{m}_4$$

$$\dot{m}_5 = \dot{m}_6$$

Thus

$$\dot{m}_5 (h_5 - h_6) = \dot{m}_2 (h_2 - h_4)$$

PROBLEM 10.26 (Cont'd)

or
$$\dot{m}_5 = \dot{m}_2\left(\frac{h_2-h_4}{h_5-h_6}\right) = \left(25{,}000\ \frac{\text{kg}}{\text{h}}\right)\left(\frac{20.98-92.9}{62.99-2510.6}\right)$$

$$= 734.6\ \text{kg/h} \longleftarrow \underline{\dot{m}_5}$$

Next, consider a control volume enclosing the steam jet ejector.

$$\dot{m}_1 h_1 + \dot{m}_6 h_6 = \dot{m}_7 h_7$$

or
$$\left(1-\frac{\dot{m}_6}{\dot{m}_7}\right)h_1 + \frac{\dot{m}_6}{\dot{m}_7}h_6 = h_7$$

$$\frac{\dot{m}_6}{\dot{m}_7} = \left(\frac{h_7-h_1}{h_6-h_1}\right) = \left(\frac{2554.4-2706.7}{2510.6-2706.7}\right) = 0.7766$$

$$\dot{m}_7 = 734.6/.7766 = 945.9\ \text{kg/h}$$

Finally, for the condenser

$$\dot{m}_7(h_7-h_8) = \dot{m}_{cw}(h_{cw,out}-h_{cw,in})$$

From Table A-3; $h_{cw,in} \approx h_f(15\,°\text{C}) = 62.99$ kJ/kg and $h_{cw,out} = 104.89$ kJ/kg.

Thus
$$\dot{m}_{cw} = \dot{m}_7\left(\frac{h_7-h_8}{h_{cw,out}-h_{cw,in}}\right) = \left(945.9\ \frac{\text{kg}}{\text{h}}\right)\left(\frac{2554.4-121.46}{104.89-62.99}\right)$$

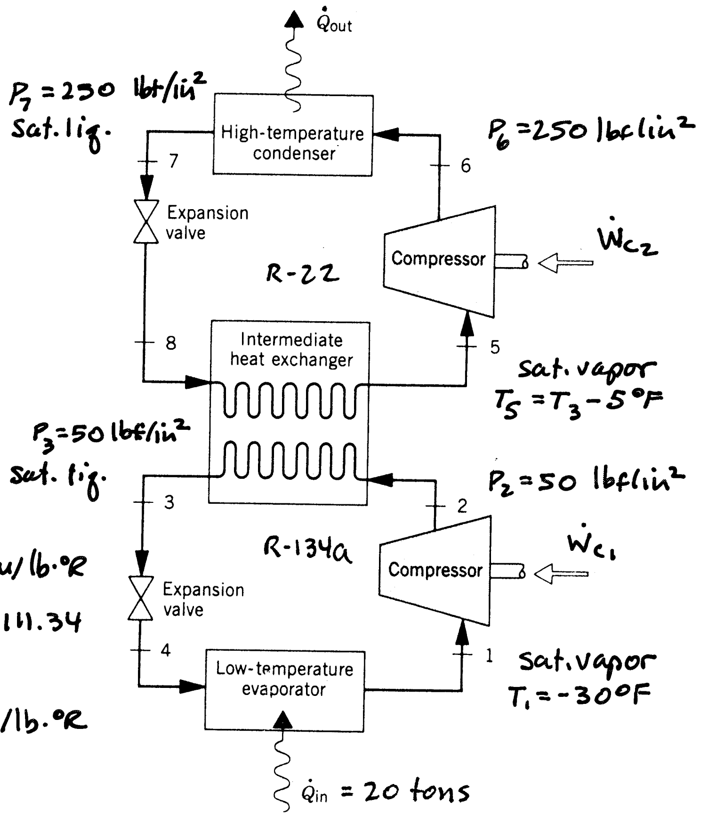
$$= 54{,}920\ \text{kg/h} \longleftarrow \underline{\dot{m}_{cw}}$$

PROBLEM 10.27

KNOWN: A cascade refrigeration system is composed of two ideal vapor-compression cycles with an inter-connecting heat exchanger. Data are known at various locations in each cycle, and the refrigerating capacity is given.

FIND: Determine (a) the power input to each compressor, (b) the coefficient of performance, and (c) the rate of exergy destruction in the intermediate heat exchanger.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The assumptions of Example 10.1 apply for each cycle. (2) There is no heat transfer from the outside of the intermediate heat exchanger. (3) Let $T_0 = 80°F = 540°R$ and $p_0 = 14.7$ lbf/in².

ANALYSIS: First, fix each of the principal states.

For the R-134a cycle:

State 1: $T_1 = -30°F$, sat. vapor
$\Rightarrow h_1 = 97.32$ Btu/lb, $s_1 = 0.2266$ Btu/lb·°R

State 2: $p_2 = 50$ lbf/in², $s_2 = s_1 \Rightarrow h_2 = 111.34$

State 3: $p_3 = 50$ lbf/in², sat. liquid
$\Rightarrow h_3 = 24.14$ Btu/lb, $s_3 = 0.0523$ Btu/lb·°R
$T_3 = 40.27$ °F

State 4: $p_4 = p_1$, throttling process; $h_4 = h_3 = 24.14 \Rightarrow s_4 = 0.0563$ Btu/lb·°R

For the R-22 cycle:

State 5: $T_5 = T_3 - 5 = 35.27°F$, sat. vapor $\Rightarrow h_5 = 107.8$ Btu/lb, $s_5 = 0.22095 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

State 6: $p_6 = 250$ lbf/in², $s_6 = s_5 \Rightarrow h_6 = 120.51$ Btu/lb

State 7: $p_7 = 250$ lbf/in², sat. liquid $\Rightarrow h_7 = 43.46$ Btu/lb

State 8: $T_8 = 35.27$ °F; throttling process $\Rightarrow h_8 = h_7 = 43.46$ Btu/lb
and $s_8 = 0.09088$ Btu/lb·°R

(a) The mass flow rate of R-134a is determined using the given refrigerating capacity

$$\dot{Q}_{in} = \dot{m}_{R\text{-}134a}(h_1 - h_4) \Rightarrow \dot{m}_{R\text{-}134a} = \frac{\dot{Q}_{in}}{(h_1 - h_4)} = \frac{(20 \text{ tons})}{(97.32 - 24.14)\frac{\text{Btu}}{\text{lb}}}\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right|$$

$$= 54.66 \text{ lb/min}$$

Thus, for the R-134a compressor

$$\dot{W}_{c1} = \dot{m}_{R\text{-}134a}(h_2 - h_1) = (54.66 \tfrac{\text{lb}}{\text{min}})(111.34 - 97.32)\tfrac{\text{Btu}}{\text{lb}}$$

$$= 766.3 \text{ Btu/min} \longleftarrow \dot{W}_{c1}$$

To find the refrigerant 22 mass flow rate, analyze the intermediate heat exchanger.

$$0 = \dot{m}_{R\text{-}134a}(h_2 - h_3) + \dot{m}_{R\text{-}22}(h_8 - h_5)$$

or

$$\dot{m}_{R\text{-}22} = \dot{m}_{R\text{-}134a}\left(\frac{h_2 - h_3}{h_5 - h_8}\right) = \left(54.66\ \frac{\text{lb}}{\text{min}}\right)\left(\frac{111.34 - 24.14}{107.8 - 43.46}\right)$$

$$= 74.08\ \text{lb/min}$$

Thus, for the R-22 compressor

$$\dot{W}_{c2} = \dot{m}_{R\text{-}22}(h_6 - h_5) = \left(74.08\ \frac{\text{lb}}{\text{min}}\right)(120.51 - 107.8)\,\frac{\text{Btu}}{\text{lb}} = 941.6\ \text{Btu/min} \longleftarrow \dot{W}_{c2}$$

(b) The overall coefficient of performance is

$$\beta = \frac{Q_{in}}{\dot{W}_{c1} + \dot{W}_{c2}} = \frac{(20\ \text{tons})}{(766.3 + 941.6)\ \text{Btu/min}}\left|\frac{200\ \text{Btu/min}}{1\ \text{ton}}\right|$$

$$= 2.34 \longleftarrow \beta$$

(c) For the heat exchanger

$$0 = \cancel{\sum_j \left(\frac{\dot{Q}}{T}\right)_j}^{\,0} + \dot{m}_{R\text{-}134a}(s_2 - s_3) + \dot{m}_{R\text{-}22}(s_8 - s_5) + \dot{\sigma}_{HX}$$

Thus

$$\dot{E}_d = T_0\dot{\sigma}_{HX} = T_0\left[\dot{m}_{R\text{-}134a}(s_3 - s_2) + \dot{m}_{R\text{-}22}(s_5 - s_8)\right]$$

$$= (540^\circ\text{R})\left[(54.66)(.0523 - .2266)\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} + (74.08)(.22095 - .09088)\right]$$

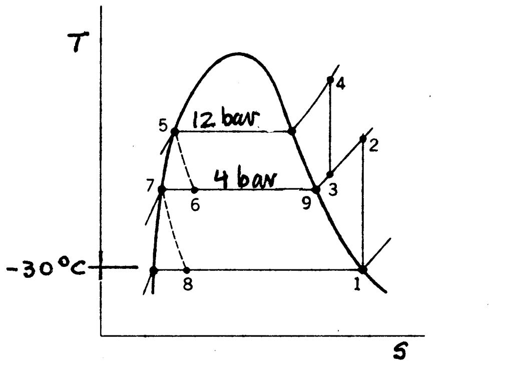
$$= 58.51\ \text{Btu/min}$$

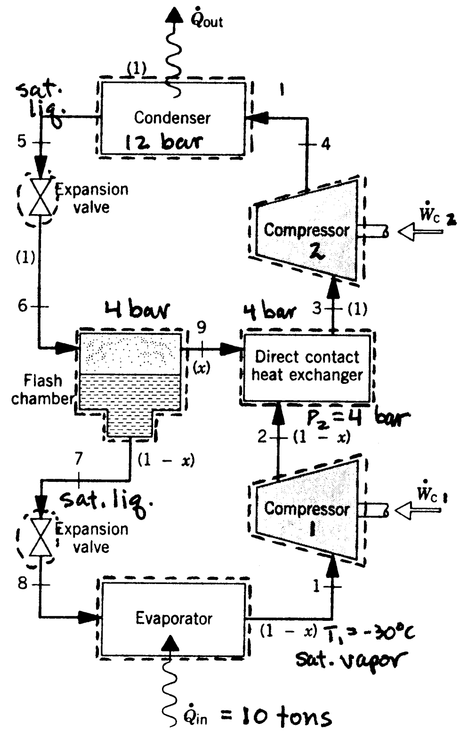
PROBLEM 10.28

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression refrigeration system using two-stage compression with intercooling between the stages. The refrigerating capacity is given, and data at various locations are known.

FIND: Determine (a) the power input to each compressor, and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The compressors operate isentropically. (3) There are no pressure drops for flow through any of the heat exchangers or the flash chamber. (4) The flash chamber and direct contact heat exchanger operate adiabatically. (5) The expansion in the valve is a throttling process. (6) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -30\ °C$, sat. vapor $\Rightarrow h_1 = 229.14$ kJ/kg, $s_1 = 0.9434$ kJ/kg·K.

State 2 $P_2 = 4$ bar, $s_2 = s_1 \Rightarrow h_2 = 260.62$ kJ/kg

State 5 $P_5 = 12$ bar, sat. liquid $\Rightarrow h_5 = 115.76$ kJ/kg

State 6 Throttling process $\Rightarrow h_6 = h_5 = 115.76$ kJ/kg, $x_6 = 0.2825$

State 7 $P_7 = 4$ bar, sat. liquid $\Rightarrow h_7 = 62.00$ kJ/kg

State 8 Throttling process $\Rightarrow h_8 = h_7 = 62.00$ kJ/kg

State 9 $P_9 = 4$ bar, sat. vapor $\Rightarrow h_9 = 252.32$ kJ/kg

State 3 The fraction of the flow into the flash chamber at 6 that exits as saturated vapor at 9 is equal to the quality at 6. The liquid leaving the flash chamber at 7 is the fraction $1-x_6$. With these flow rate ratios

$$0 = (1-x_6)h_2 + x_6 h_9 - 1\, h_3$$

or

$$h_3 = (1-x_6)h_2 + x_6 h_9$$

$$= (1-0.2825)(260.62) + 0.2825(252.32) = 258.27 \text{ kJ/kg}$$

and by interpolation in Table A-12 with h_3 we get $s_3 = 0.9352$ kJ/kg·K.

State 4. $P_4 = 12$ bar, $s_4 = s_3 \Rightarrow h_4 = 281.69$ kJ/kg.

PROBLEM 10.28 (Contd.)

(a) To determine the compressor power, first determine the mass flow rates. For the evaporator

$$\dot{Q}_{in} = \dot{m}_1(h_1 - h_8)$$

or

$$\dot{m}_1 = \frac{\dot{Q}_{in}}{(h_1 - h_8)} = \frac{(10 \text{ tons})}{(229.14 - 62.00)\frac{\text{kJ}}{\text{kg}}}\left|\frac{211 \text{ kJ/min}}{1 \text{ ton}}\right|\left|\frac{1 \text{ min}}{60 \text{ s}}\right|$$

$$= 0.21 \text{ kg/s}$$

Also, since $1 - x_6$ is the fraction of the total flow passing through the evaporator

$$\frac{\dot{m}_1}{\dot{m}_3} = 1 - x_6 \Rightarrow \dot{m}_3 = \frac{\dot{m}_1}{1 - x_6} = 0.293 \text{ kg/s}$$

Thus,

$$\dot{W}_{c1} = \dot{m}_1(h_2 - h_1) = \left(0.21 \frac{\text{kg}}{\text{s}}\right)(260.62 - 229.14)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 6.61 \text{ kW} \longleftarrow \dot{W}_{c1}$$

and

$$\dot{W}_{c2} = \dot{m}_3(h_4 - h_3) = (0.293)(281.69 - 258.27) = 6.86 \text{ kW} \longleftarrow \dot{W}_{c2}$$

(b) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{c1} + \dot{W}_{c2}} = \frac{(10 \text{ tons})}{(6.61 + 6.86)\text{ kW}}\left|\frac{211 \text{ kJ/min}}{1 \text{ ton}}\right|\left|\frac{1 \text{ min}}{60 \text{ s}}\right|\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

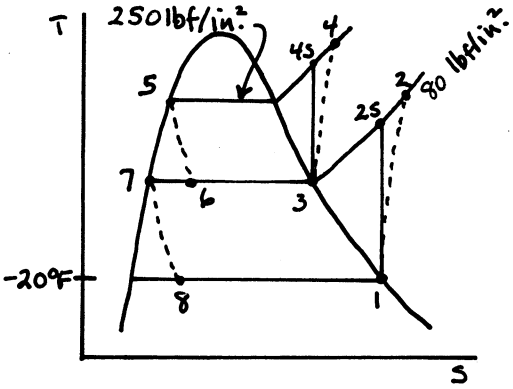
$$= 2.61 \longleftarrow \beta$$

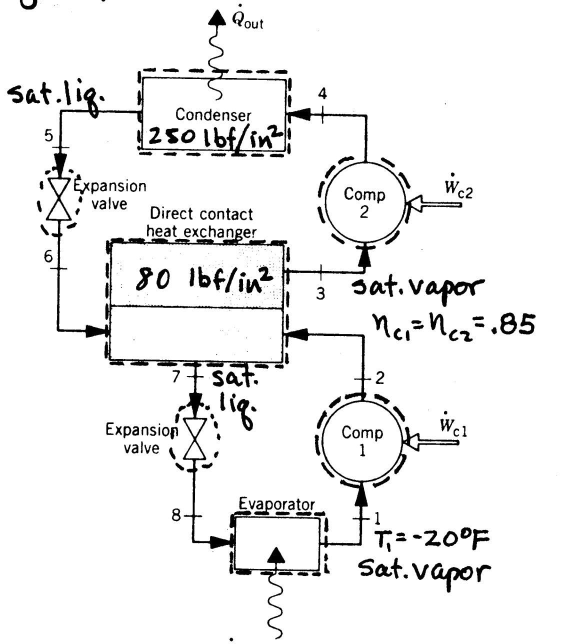
PROBLEM 10.29

KNOWN: Ammonia is the working fluid in a two-stage vapor-compression refrigeration system using a direct contact heat exchanger to achieve intercooling. Data are known at various locations, and the refrigerating capacity is given.

FIND: Determine (a) the ratio of the mass flow rates $\dot{m}_3/\dot{m}_1$ (see figure), (b) the power input to each compressor stage, and (c) the coefficient of performance. (d) Plot each of the quantities in parts (a)-(c) versus the direct-contact heat exchanger pressure. Discuss.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The compressors, valves, and direct contact heat exchanger each operate adiabatically. (3) The expansions through the valves are throttling processes. (4) There are no significant pressure drops for flow through the heat exchangers. (5) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1 $T_1 = -20°F$, sat. vapor $\Rightarrow h_1 = 604.61$ Btu/lb, $s_1 = 1.3762$ Btu/lb·°R

State 2 For isentropic compression, $p_2 = 80$ lbf/in.², $s_{2s} = s_1 \Rightarrow h_{2s} = 692.14\ \frac{\text{Btu}}{\text{lb}}$
Using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 707.59 \text{ Btu/lb}$$

State 3 $p_3 = 80$ lbf/in.², sat. vapor $\Rightarrow h_3 = 623.32$ Btu/lb, $s_3 = 1.2529$ Btu/lb·°R

State 4 $p_4 = 250$ lbf/in.², $s_{4s} = s_3 \Rightarrow h_{4s} = 693.53$ Btu/lb. thus

$$h_4 = h_3 + \left(\frac{h_{4s} - h_3}{\eta_c}\right) = 705.92 \text{ Btu/lb}$$

State 5 $p_5 = 250$ lbf/in.², sat. liquid $\Rightarrow h_5 = 167.77$ Btu/lb

State 6 Throttling process $\Rightarrow h_6 = h_5 = 167.77$ Btu/lb

State 7 $p_7 = 80$ lbf/in.², sat. liquid $\Rightarrow h_7 = 91.22$ Btu/lb

State 8 Throttling process $\Rightarrow h_8 = h_7 = 91.22$ Btu/lb

(a) The mass flow rate through the evaporator is

$$\dot{m}_1 = \frac{\dot{Q}_{in}}{h_1 - h_8} = \frac{(30 \text{ tons})}{(604.61 - 91.22)\frac{\text{Btu}}{\text{lb}}} \left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right| = 11.69 \frac{\text{lb}}{\text{min}}$$

PROBLEM 10.29 (Contd.) - Page 2

The mass flow rate into the second turbine stage is determined by using an energy balance on the direct contact heat exchanger

$$0 = \dot m_6 h_6 + \dot m_2 h_2 - \dot m_3 h_3 - \dot m_7 h_7$$

But $\quad \dot m_6 = \dot m_3$ and $\dot m_7 = \dot m_2 = \dot m_1$

Thus $\quad 0 = \dot m_3 (h_6 - h_3) + \dot m_1 (h_2 - h_7)$

or $\quad \dot m_3 = \dot m_1 \left(\frac{h_7 - h_2}{h_6 - h_3}\right) = (11.69)\left(\frac{91.22 - 707.59}{167.77 - 623.32}\right) = 15.82 \text{ lb/min}$

The mass flow rate ratio is

$$\dot m_3/\dot m_1 = 1.353 \longleftarrow \dot m_3/\dot m_1$$

(b) The power input to the first compressor stage is

$$\dot W_{c1} = \dot m_1 (h_2 - h_1)$$

$$= \left(11.69 \frac{\text{lb}}{\text{min}}\right)\left|\frac{60 \text{ min}}{1 \text{ h}}\right|(707.59 - 604.61)\frac{\text{Btu}}{\text{lb}}\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|$$

$$= 28.38 \text{ hp} \longleftarrow \dot W_{c1}$$

And, the power input to the second compressor stage is

$$\dot W_{c2} = \dot m_3 (h_4 - h_3)$$

$$= (15.82)(705.92 - 623.32)\left|\frac{60}{2545}\right| = 30.81 \text{ hp} \longleftarrow \dot W_{c2}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot Q_{in}}{\dot W_{c1} + \dot W_{c2}} = \frac{(30 \text{ tons})}{(28.38 + 30.81)\text{hp}}\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right|\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right|\left|\frac{60 \text{ min}}{1 \text{ h}}\right|$$

$$= 2.390 \longleftarrow \beta$$

(d) The data for the required plots are obtained using IT, as follows:

IT Code

```
Qdotin = 30    // tons
T1 = -20    // °F
p2 = 80    // lbf/in²
p3 = p2
p4 = 250    // lbf/in²
eta_c1 = 0.85
eta_c2 = 0.85

x1 = 1
p1 = Psat_T("Ammonia", T1)
h1 = hsat_Px("Ammonia", p1, x1)
```

```
s1 = ssat_Px("Ammonia", p1, x1)
s2s = s1
h2s = h_Ps("Ammonia", p2, s2s)
h2 = h1 + (h2s - h1) / eta_c1
x3 = 1
h3 = hsat_Px("Ammonia", p3, x3)
s3 = ssat_Px("Ammonia", p3, x3)
s4s = s3
h4s = h_Ps("Ammonia", p4, s4s)
h4 = h3 + (h4s - h3) / eta_c2
p5 = p4
h5 = hsat_Px("Ammonia", p5, 0)
h6 = h5
```

```
p7 = p3
h7 = hsat_Px("Ammonia", p7, 0)
h8 = h7


mdot1 = (Qdotin * 200) / (h1 - h8)  // Btu/min
0 = mdot3 * (h6 - h3) + mdot1 * (h2 - h7)
mratio = mdot3 / mdot1
Wdotc1 = mdot1 * (h2 - h1) * (60 / 2545)
Wdotc2 = mdot3 * (h4 - h3) * (60 / 2545)
Wdotc = Wdotc1 + Wdotc2
beta = ((Qdotin*200)/(Wdotc1+Wdotc2))* (60/2545)
```

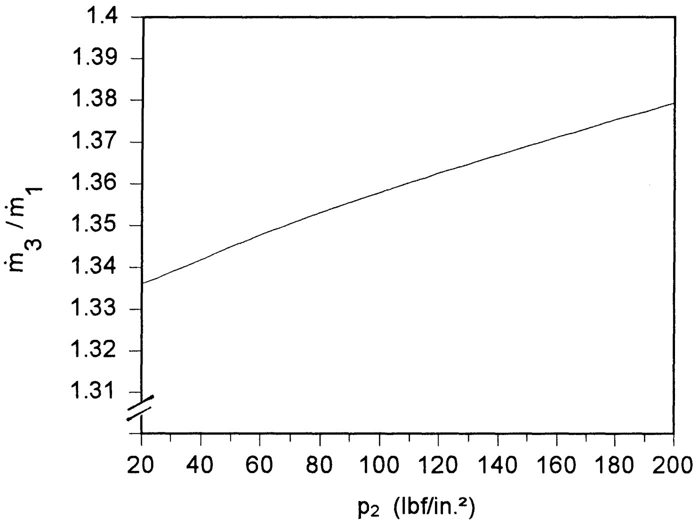
IT Results for p_2 = 80 lbf/in.²

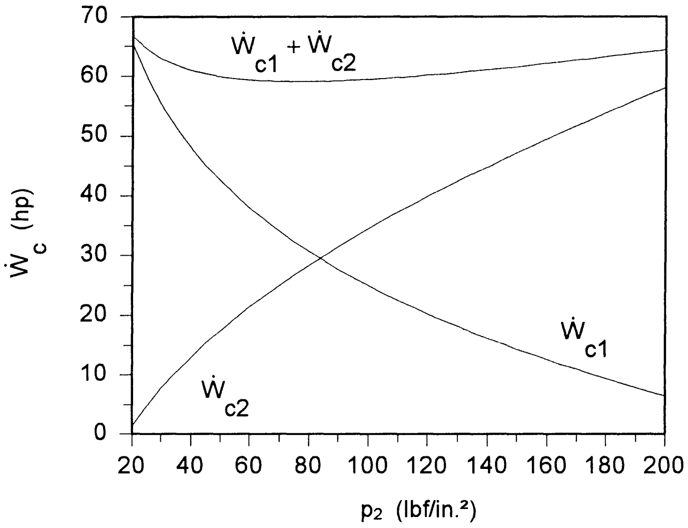
h_1 = 604.2 Btu/lb
h_2 = 707.1 Btu/lb
h_{2s} = 691.7 Btu/lb
h_3 = 622.9 Btu/lb
h_4 = 705.2 Btu/lb
h_{4s} = 692.9 Btu/lb
h_5 = 167.7 Btu/lb
h_6 = 167.7 Btu/lb

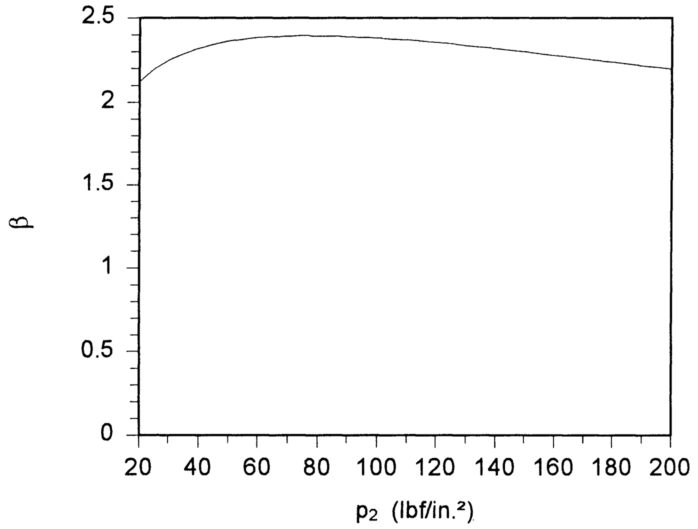
h_7 = 91.16 Btu/lb
h_8 = 91.16 Btu/lb
$\dot m_3/\dot m_1$ = 1.353
$\dot W_{c1}$ = 28.37 hp
$\dot W_{c2}$ = 30.71 hp
$\dot W_c$ = 59.08 hp
β = 2.394

PROBLEM 10.29 (Cont'd.) - page 3

PLOTS:







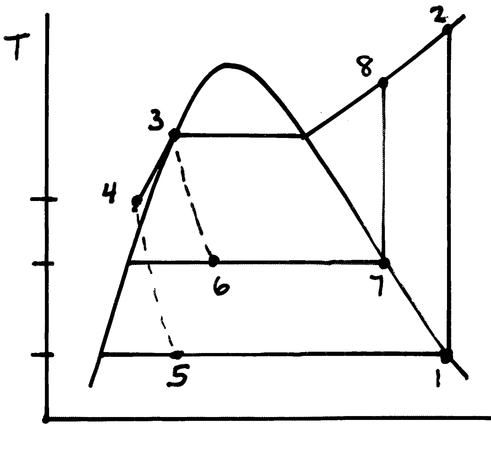
From the plots we see that as the interstage pressure increases, the fraction of the flow entering the first compressor stage that enters the second stage increases. Also, the power required by the first stage decreases and that required by the second stage increases. The curve that shows the total compressor power exhibits a minimum value. Accordingly, the coefficient of performance exhibits a maximum value.

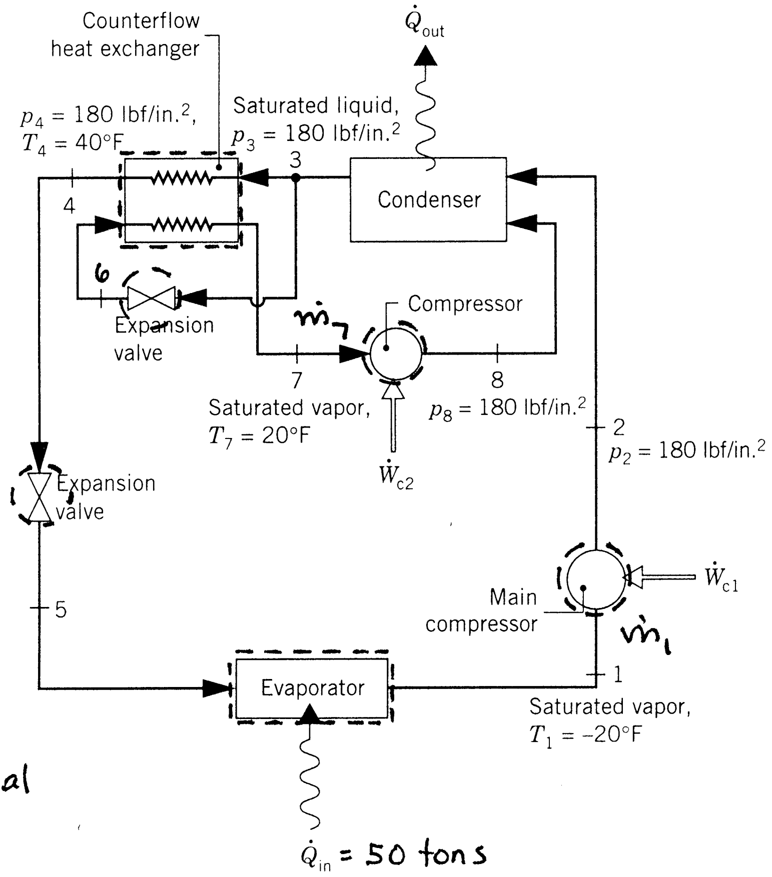
PROBLEM 10.30

KNOWN: A vapor-compression refrigeration system with mechanical subcooling uses refrigerant 22. Data are known at various locations, and the refrigerating capacity is given.

FIND: Determine (a) the mass flow rate into each compressor, (b) the power input into each compressor, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is modeled as a control volume at steady state. (2) The compressors operate isentropically, (3) There are no pressure drops through the heat exchangers. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 1: $T_1 = -20°F$, sat. vapor $\Rightarrow h_1 = 102.50$ Btu/lb, $s_1 = 0.2334$ Btu/lb·°R

State 2: $p_2 = 180$ lbf/in^2, $s_2 = s_1$ $\Rightarrow h_2 = 124.23$ Btu/lb

State 3: $p_3 = 180$ lbf/in^2, sat. liquid $\Rightarrow h_3 = 35.95$ Btu/lb

State 4: $p_4 = 180$ lbf/in^2, $T_4 = 40°F$ $\Rightarrow$ subcooled $\Rightarrow h_4 \approx h_f(40°F) = 21.66$ Btu/lb

State 5: Throttling process $\Rightarrow h_5 = h_4 = 21.66$ Btu/lb

State 6: Throttling process $\Rightarrow h_6 = h_3 = 35.95$ Btu/lb

State 7: $T_7 = 20°C$, sat. vapor $\Rightarrow h_7 = 106.45$ Btu/lb, $s_7 = 0.2240$ Btu/lb·°R

State 8: $p_8 = 180$ lbf/in^2, $s_8 = s_7$ $\Rightarrow h_8 = 118.64$ Btu/lb

(a) To find the mass flow rate into the first compressor, $\dot{m}_1$, use the refrigerating capacity

$$\dot{Q}_{in} = \dot{m}_1 (h_1 - h_5)$$

PROBLEM 10.30 (Cont'd)

Solving

$$\dot{m}_1 = \frac{Q_{in}}{(h_1 - h_5)} = \frac{(50 \text{ tons})}{(102.50 - 21.66)\ \frac{\text{Btu}}{\text{lb}}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right|$$

$$= 123.7 \text{ lb/min} \longleftarrow \dot{m}_1$$

To get $\dot{m}_7$, analyze the counterflow heat exchanger

$$0 = \dot{m}_1 (h_3 - h_4) + \dot{m}_7 (h_6 - h_7)$$

$$\dot{m}_7 = m_1 \left(\frac{h_4 - h_3}{h_6 - h_7} \right) = \left(\frac{21.66 - 35.95}{35.95 - 106.45} \right) (123.7 \tfrac{\text{lb}}{\text{min}})$$

$$= 25.073 \text{ lb/min} \longleftarrow \dot{m}_7$$

(b) The power of each compressor is, respectively

$$\dot{W}_{c1} = \dot{m}_1 (h_2 - h_1) = (123.7)(124.23 - 102.50)$$

$$= 2688 \text{ Btu/min} \longleftarrow \dot{W}_{c1}$$

$$\dot{W}_{c2} = \dot{m}_7 (h_8 - h_7) = (25.073)(118.64 - 106.45)$$

$$= 305.6 \text{ Btu/min} \longleftarrow \dot{W}_{c2}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{c1} + \dot{W}_{c2}} = \frac{(50 \text{ tons})}{(2688 + 305.6) \text{ Btu/min}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right|$$

$$= 3.34 \longleftarrow \beta$$

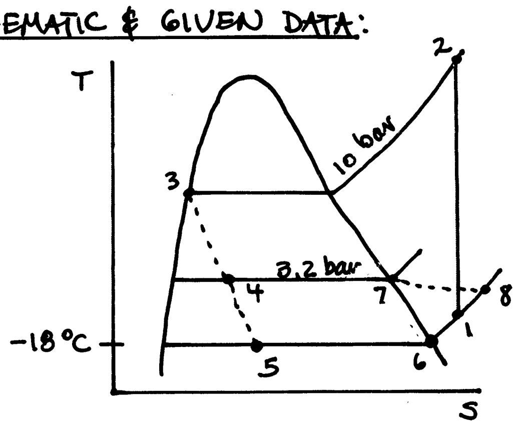
1146

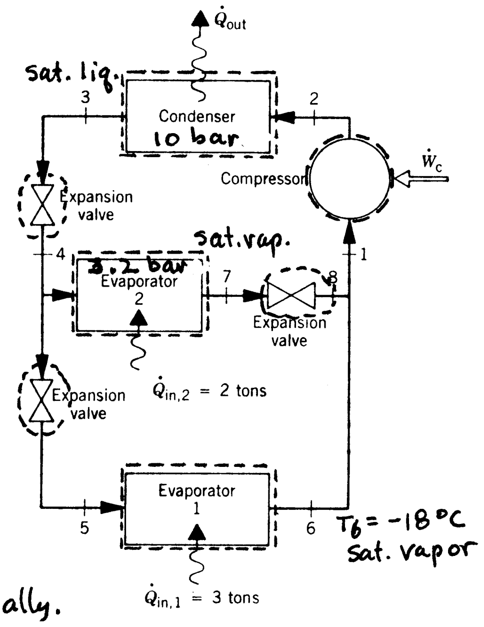
PROBLEM 10.31

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression refrigeration system with two evaporators. Data are known at various locations, and the refrigerating capacity of each evaporator is specified. The system uses only one compressor.

FIND: Determine (a) the mass flow rates through each evaporator, (b) the compressor power input, and (c) the heat transfer from the refrigerant passing through the condenser.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes of the working fluid are internally reversible, except for the expansion through each valve, which is a throttling process. (3) The compressor and valves operate adiabatically. (4) Kinetic and potential energy effects are negligible.

ANALYSIS: First, fix each of the principal states.

State 3 $p_3 = 10$ bar, sat. liquid $\Rightarrow h_3 = 105.29$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 105.29$ kJ/kg

State 5 Throttling process $\Rightarrow h_5 = h_4 = 105.29$ kJ/kg

State 6 $T_6 = -18\,°C$, sat. vapor $\Rightarrow h_6 = 236.53$ kJ/kg

State 7 $p_7 = 3.2$ bar, sat. vapor $\Rightarrow h_7 = 248.66$ kJ/kg

State 8 Throttling process $\Rightarrow h_8 = h_7 = 248.66$ kJ/kg

To fix states 1 and 2 requires the mass flow rates through the evaporators. Thus

(a) $$\dot{Q}_{in,1} = \dot{m}_6 (h_6 - h_5)$$

or $$\dot{m}_6 = \frac{\dot{Q}_{in,1}}{(h_6 - h_5)} = \frac{(3 \text{ tons})}{(236.53 - 105.29)\,\frac{kJ}{kg}} \left| \frac{211 \text{ kJ/min}}{1 \text{ ton}} \right|$$

$$= 4.823 \text{ kg/min} \longleftarrow \dot{m}_6$$

PROBLEM 10.31 (Cont'd.)

and

$$\dot{m}_8 = \frac{\dot{Q}_{in,2}}{(h_7 - h_4)} = \frac{(2)(211)}{(248.46 - 105.29)} = 2.943\ \frac{kg}{min} \longleftarrow \dot{m}_8$$

State 1 Now, for the adiabatic mixing streams 6 and 8 to form Stream 1 : $0 = \dot{m}_6 h_6 + \dot{m}_8 h_8 - (\dot{m}_6 + \dot{m}_8)\, h_1$. Thus

$$h_1 = \frac{\dot{m}_6 h_6 + \dot{m}_8 h_8}{\dot{m}_6 + \dot{m}_8} = 241.13 \quad kJ/kg$$

The specific entropy s_1 is found using double interpolation in Table A-12 to be ; $s_1 \approx 0.9493$ kJ/kg·K

State 2 $p_2 = 10$ bar , $s_2 = s_1 \Rightarrow h_2 = 282.3$ kJ/kg

(b) The compressor power is

$$\dot{W}_c = (\dot{m}_6 + \dot{m}_8)(h_2 - h_1)$$

$$= \left(7.766\ \frac{kg}{min}\right)\left|\frac{1\ min}{60\ s}\right|(282.3 - 241.13)\ \frac{kJ}{kg}\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 5.329\ kW \longleftarrow \dot{W}_c$$

For the condenser

$$\dot{Q}_{out} = (\dot{m}_6 + \dot{m}_8)(h_2 - h_3)$$

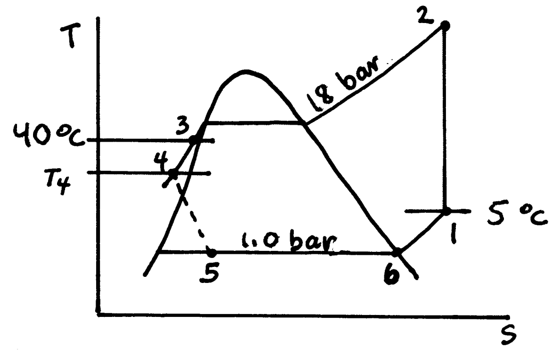
$$= (7.766)\left|\frac{1}{60}\right|(282.3 - 105.29) = 22.91\ kW \longleftarrow \dot{Q}_{out}$$

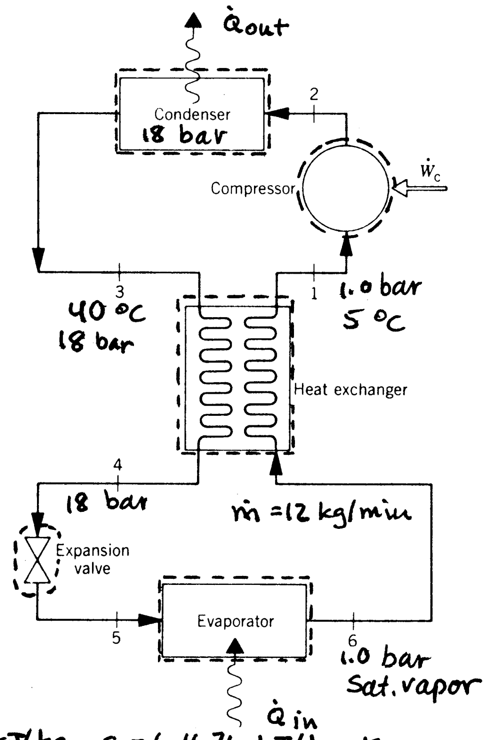
PROBLEM 10.32*

KNOWN: An ideal vapor-compression refrigeration cycle is modified to include a counterflow heat exchanger between the streams exiting the condenser and evaporator. Ammonia is the working fluid. Data are known at various locations, and the mass flow rate is given.

FIND: Determine (a) the refrigerating capacity, (b) the compressor power input, (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: Same as in Example 10.1, assumptions 1-4. Also, assume no heat loss from the outside of the heat exchanger.

ANALYSIS: First, fix each of the principal states.

State 6 $p_6 = 1.0$ bar, sat. vapor
$\Rightarrow h_6 = 1398.41$ kJ/kg

State 1 $p_1 = 1.0$ bar, $T_1 = 5$ °C $\Rightarrow h_1 = 1483.25$ kJ/kg, $s_1 = 6.1676$ kJ/kg·K

State 2 $p_2 = 18$ bar, $s_2 = s_1$ $\Rightarrow$ $h_2 = 2025.54$ kJ/kg

State 3 $p_3 = 18$ bar, $T_3 = 40$ °C $\Rightarrow$ comp. liquid; $h_3 \approx h_f(40°C) = 371.35$ kJ/kg

State 4 Using an energy balance on the heat exchanger control volume

$$0 = \dot{m}(h_6 - h_1) + \dot{m}(h_3 - h_4)$$

or $\quad h_4 = h_6 - h_1 + h_3 = 286.51$ kJ/kg

State 5 Throttling process $\Rightarrow h_5 = h_4 = 286.51$ kJ/kg

(a) The refrigerating capacity is

$$\dot{Q}_{in} = \dot{m}(h_6 - h_5) = \left(12\frac{kg}{min}\right)(1398.41 - 286.5)\frac{kJ}{kg}\left|\frac{1 \text{ ton}}{211 \text{ kJ/min}}\right| = 63.24 \text{ tons} \longleftarrow \dot{Q}_{in}$$

(b) The compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(12\frac{kg}{min}\right)\left|\frac{1 \text{ min}}{60 \text{ s}}\right|(2025.54 - 1483.25)\frac{kJ}{kg}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 108.5 \text{ kW} \longleftarrow \dot{W}_c$$

(c) The coefficient of performance is $\beta = (h_6 - h_5)/(h_2 - h_1) = 2.05 \longleftarrow \beta$

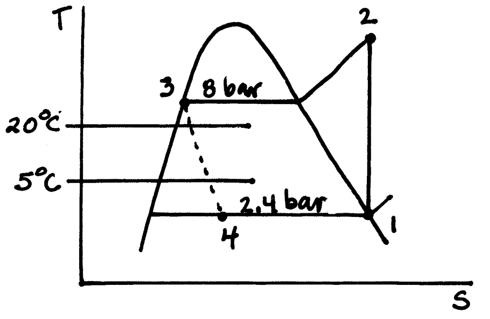
Discussion: The heat exchanger (1) tends to increase the capacity, (2) helps ensure superheated vapor entering the compressor, and (3) increases the average specific volume in the compressor, thereby increasing the power required.

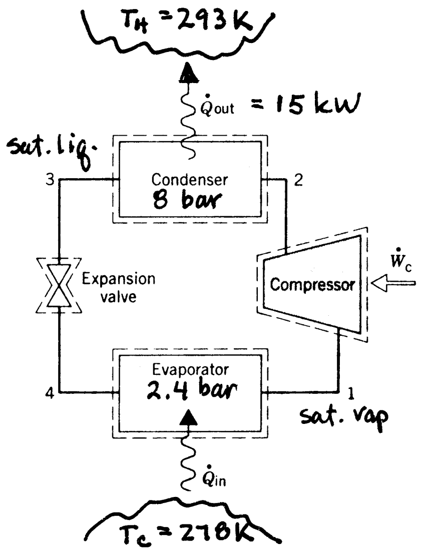
PROBLEM 10.33

KNOWN: An ideal vapor-compression heat pump cycle uses Refrigerant 134a as the working fluid and provides a known energy output to heat a building. Data are known at various locations.

FIND: Determine (a) the compressor power, (b) the coefficient of performance, and (c) the maximum theoretical coefficient of performance for a heat pump operating between reservoirs at 20°C and 5°C.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.1.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 2.4$ bar, sat. vapor $\Rightarrow h_1 = 244.09$ kJ/kg, $s_1 = 0.9222$ kJ/kg·K

State 2 $p_2 = 8$ bar, $s_2 = s_1 \Rightarrow h_2 = 268.97$ kJ/kg

State 3 $p_3 = 8$ bar, sat. liquid $\Rightarrow h_3 = 93.42$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 93.42$ kJ/kg

(a) To determine the compressor, first find the mass flow rate from

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3)$$

or
$$\dot{m} = \frac{\dot{Q}_{out}}{h_2 - h_3} = \frac{(15\text{ kW})}{(268.97 - 93.42)\frac{\text{kJ}}{\text{kg}}}\left|\frac{1\text{ kJ/s}}{1\text{ kW}}\right| = 0.08544\text{ kg/s}$$

thus,
$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(.08544\frac{\text{kg}}{\text{s}}\right)(268.97 - 244.09)\frac{\text{kJ}}{\text{kg}}\left|\frac{1\text{ kW}}{1\text{ kJ/s}}\right|$$

$$= 2.126\text{ kW} \longleftarrow \dot{W}_c$$

(b) The coefficient of performance is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{15}{2.126} = 7.055 \longleftarrow \gamma$$

(c) For a reversible heat pump operating between reservoirs at $T_H = 293$ K and $T_C = 278$ K

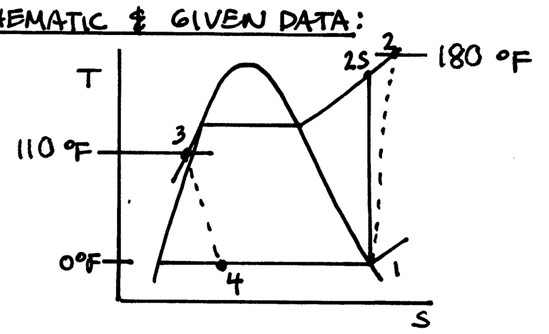
$$\gamma_{max} = \frac{T_H}{T_H - T_C} = 19.53 \longleftarrow \gamma_{max}$$

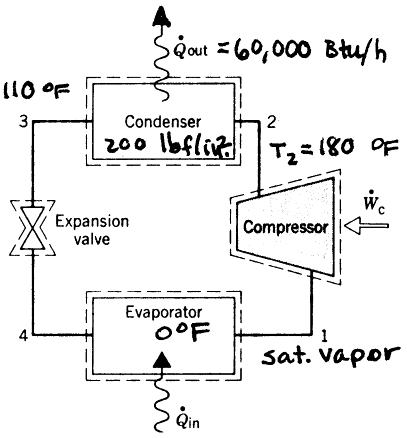
PROBLEM 10.34*

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression heat pump system with know heating capacity. Data are known at various locations.

FIND: Determine (a) the mass flow rate, (b) the compressor power, (c) the isentropic compressor efficiency, and (d) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.3, assumptions 1-4, except the compressor efficiency is unknown.

ANALYSIS: First, fix each of the principal states.

State 1 T_1 = 0°F, sat. vapor ⇒ $h_1 = 101.75$ Btu/lb, $s_1 = 0.2224$ Btu/lb·°R

State 2 $p_2 = 200$ lbf/in², $T_2 = 180$ °F ⇒ $h_2 = 133.28$ Btu/lb

State 3 $p_3 = 200$ lbf/in², $T_3 = 110$ °F ⇒ comp. liq.; $h_3 \approx h_f(110) = 47.81$ Btu/lb

State 4 Throttling process ⇒ $h_4 = h_3 = 47.81$ Btu/lb

(a) The mass flow rate is

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3)$$

or

$$\dot{m} = \frac{\dot{Q}_{out}}{h_2 - h_3} = \frac{60{,}000 \text{ Btu/h}}{(133.28 - 47.81)\ \frac{\text{Btu}}{\text{lb}}}\left|\frac{1\text{ h}}{60\text{ min}}\right| = 11.70 \text{ lb/min} \longleftarrow \dot{m}$$

(b) Thus, the compressor power is

$$\dot{W}_c = \dot{m}(h_2 - h_1)$$

$$= \left(11.70\ \frac{\text{lb}}{\text{min}}\right)\left|\frac{60\text{ min}}{1\text{ h}}\right|(133.28 - 101.75)\ \frac{\text{Btu}}{\text{lb}}\left|\frac{1\text{ hp}}{2554\text{ Btu/h}}\right| = 8.70 \text{ hp} \longleftarrow \dot{W}_c$$

(c) The compressor efficiency is found as follows. First, for isentropic compression, $p_2 = 200$ lbf/in², $s_2 = s_1$ ⇒ $h_{2s} = 121.80$ Btu/lb. Then

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{133.28 - 101.75}{138.75 - 101.75} = 0.852\ (85.2\%) \longleftarrow \eta_c$$

(d) The coefficient of performance is

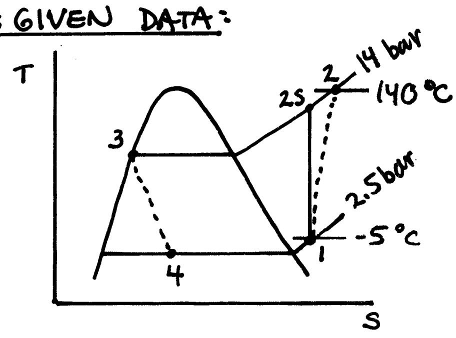
$$\gamma = \frac{h_2 - h_3}{h_2 - h_1} = 2.711 \longleftarrow \gamma$$

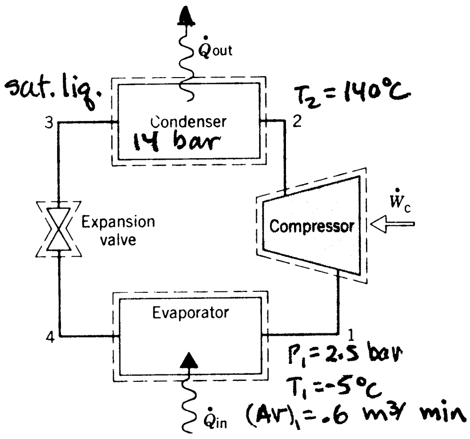
PROBLEM 10.35*

KNOWN: Ammonia is the working fluid in a vapor-compression heat pump system. The volumetric flow rate entering the compressor and data at various locations are known.

FIND: Determine (a) the compressor power, (b) the heating capacity, (c) the coefficient of performance, and (d) the isentropic compressor efficiency.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.3, assumptions 1-4, except the compressor efficiency is unknown.

ANALYSIS: First, fix each of the principal states.

State 1 $p_1 = 2.5$ bar, $T_1 = -5°C \Rightarrow h_1 = 1447.10$ kJ/kg, $s_1 = 5.5989$ kJ/kg·K

State 2 $p_2 = 14$ bar, $T_2 = 140°C \Rightarrow h_2 = 1752.52$ kJ/kg

State 3 $p_3 = 14$ bar, sat. liquid $\Rightarrow h_3 = 352.97$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 352.97$ kJ/kg

(a) First, find the mass flow rate. The specific volume at the compressor inlet is $v_1 = 0.50180$ m³/kg. Thus

$$\dot{m} = \frac{(AV)_1}{v_1} = \frac{(0.6 \text{ m}^3/\text{min})}{(0.50180 \text{ m}^3/\text{kg})}\left|\frac{1 \text{ min}}{60 \text{ s}}\right| = 0.0199 \text{ kg/s}$$

The power is

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(0.0199\frac{\text{kg}}{\text{s}}\right)(1752.52 - 1447.10)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 6.09 \text{ kW} \longleftarrow \dot{W}_c$$

(b) The heating capacity is

$$\dot{Q}_{out} = \dot{m}(h_2 - h_3) = \left(0.0199\frac{\text{kg}}{\text{s}}\right)(1752.52 - 352.97)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right| = 27.85 \text{ kW}$$

$$(7.92 \text{ tons}) \longleftarrow \dot{Q}_{out}$$

(c) The coefficient of performance is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{27.85}{6.09} = 4.57 \longleftarrow \gamma$$

(d) For isentropic compression, $p_2 = 14$ bar, $s_{2s} = s_1 \Rightarrow h_{2s} = 1711.24$ kJ/kg. Thus, the compressor efficiency is

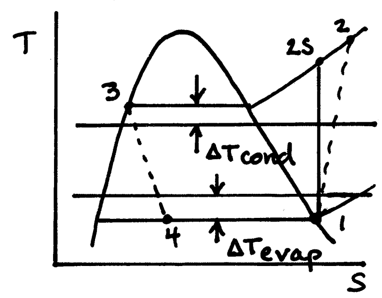
$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{1711.24 - 1447.10}{1752.52 - 1447.10} = 0.865 \ (86.5\%) \longleftarrow \eta_c$$

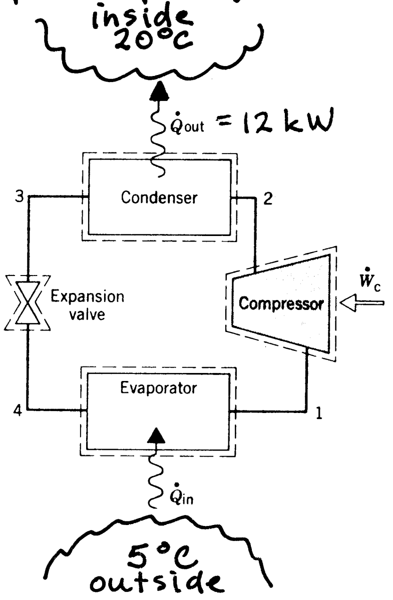
PROBLEM 10.36

KNOWN: A vapor-compression heat pump with Refrigerant 22 as the working fluid is to be used to provide 12 kW to heat a house to an inside temperature of 20°C when the outside temperature is 5°C.

FIND: Specify evaporator and condenser pressures and calculate (a) the mass flow rate, (b) the compressor power, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.3, assumptions 1-4.

ANALYSIS: The evaporator and condenser pressures must be chosen to allow for sufficient ΔT's to avoid excessive heat exchanger sizes (surface area). For a preliminary design, assume

$$\Delta T_{evap} = 10°\text{C} \Rightarrow T_1 = -5\,°\text{C}$$

$$p_{cond} = 12\text{ bar} \quad (T_{sat} = 30.25\,°\text{C})$$

Also, let $\eta_c = 80\%$ for the purpose of this analysis. With these specifications

State 1 $h_1 = 248.82$, $s_1 = 0.9315$ kJ/kg·K

State 2 $h_{2s} = 273.01$ kJ/kg $\Rightarrow h_2 = 279.06$ kJ/kg

State 3 $h_3 = 81.90$ kJ/kg (sat. liq. at 12 bar)

State 4 $h_4 = h_3 = 81.90$ kJ/kg

(a)

$$\dot{m} = \frac{\dot{Q}_{out}}{h_2 - h_3} = \frac{(12\text{ kW})}{(279.06 - 81.90)\text{ kJ/kg}}\left|\frac{1\text{ kJ/s}}{1\text{ kW}}\right|\left|\frac{60\text{ s}}{1\text{ min}}\right| = 3.652\,\frac{\text{kg}}{\text{min}}$$

(b)

$$\dot{W}_c = \dot{m}(h_2 - h_1) = (3.652)\left|\frac{1}{60}\right|(279.06 - 248.82) = 1.84\text{ kW}$$

(c)

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{12}{1.84} = 6.52$$

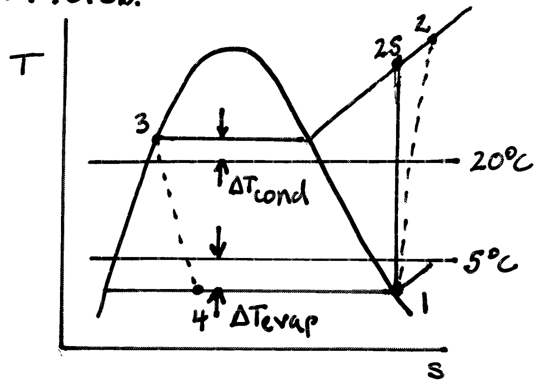
Discussion: These values are preliminary estimates. In practice, other factors, such as economics, would play an important role in the design process.

PROBLEM 10.37

KNOWN: Reconsider the cycle of Problem 10.36, but use Refrigerant 134a as the working fluid.

FIND: Answer the same questions as in Problem 10.36, and compare the results with those of Problem 10.36.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 10.3, assumptions 1-4.

ANALYSIS: For a preliminary design, we use similar consideration to those in Problem 10.36. Thus, let

$$\Delta T_{evap} = 10°C \Rightarrow T_1 = -5°C$$

$$P_{cond} = 7 \text{ bar} \quad (T_{sat} = 26.72°C)$$

Also, let $\eta_c = 80\%$. With these specifications

State 1 $\quad h_1 = 244.31, \; s_1 = .92195$ kJ/kg·K

State 2 $\quad h_{2s} = 266.06$ kJ/kg $\Rightarrow h_2 = 271.50$ kJ/kg

State 3 $\quad h_3 = 86.78$ kJ/kg

State 4 $\quad h_4 = h_3 = 86.78$ kJ/kg

(a) $$\dot{m} = \frac{\dot{Q}_{out}}{h_2 - h_3} = \frac{(12 \text{ kW})}{(271.50 - 86.78) \text{ kJ/kg}} \left|\frac{1 \text{ kJ/s}}{1 \text{ kW}}\right| \left|\frac{60 \text{ s}}{1 \text{ min}}\right| = 3.90 \text{ kg/min}$$

(b) $$\dot{W}_c = \dot{m}(h_2 - h_1) = (3.90)\left|\frac{1}{60}\right|(271.50 - 244.31) = 1.77 \text{ kW}$$

(c) $$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{12}{1.77} = 6.78$$

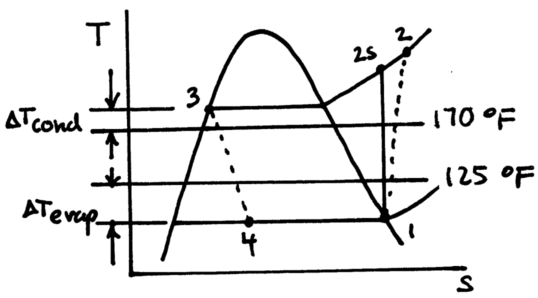
The performance of the two cycles is quite comparable in terms of $\dot{m}$, $\dot{W}_c$, and γ. The only significant difference is the somewhat higher operating pressures of the R-22 cycle.

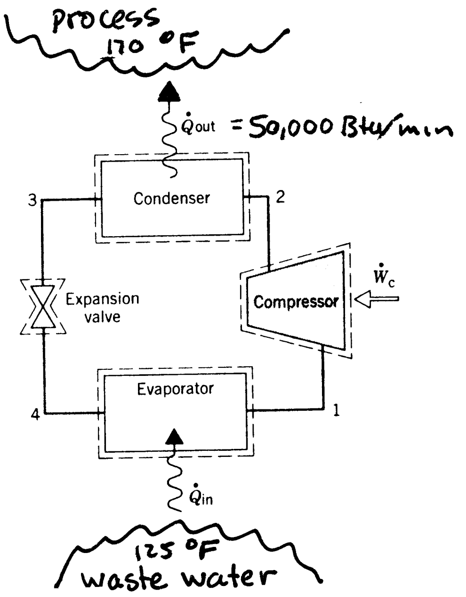
PROBLEM 10.38*

KNOWN: A Refrigerant 134a vapor-compression heat pump is proposed to develop process heat at a specified rate and temperature from a waste water stream at a known temperature.

FIND: Specify evaporator and condenser pressures and calculate (a) the mass flow rate, (b) the compressor power, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.3, assumptions 1-4.

ANALYSIS: The evaporator and condenser pressures must be chosen to allow for sufficient ΔT's to avoid excessive heat exchanger sizes (surface area). For a preliminary design, assume

$$P_{evap} = 180 \text{ lbf/in}^2 \rightarrow T_1 = 117.74\ °F$$

$$P_{cond} = 400 \text{ lbf/in}^2 \rightarrow T_{sat} = 179.95\ °F$$

Also, let $\eta_c = 80\%$ for the purpose of this analysis. With these specifications

State 1 $h_1 = 116.74$ Btu/lb, $s_1 = 0.2154$ Btu/lb·°R

State 2 $s_{2s} = s_1 \Rightarrow h_{2s} = 123.32 \text{ Btu/lb} \Rightarrow h_2 = h_1 + \frac{(h_{2s}-h_1)}{\eta_c} = 124.97$ Btu/lb

State 3 $h_3 = 50.64$ Btu/lb

State 4 $h_4 = h_3 = 50.64$ Btu/lb

(a) $$\dot m = \frac{\dot Q_{out}}{h_2-h_3} = \frac{50{,}000 \text{ Btu/min}}{(124.97-50.64)\text{ Btu/lb}}\left|\frac{60 \text{ min}}{1 \text{ h}}\right| = 4.036\times10^4 \frac{\text{lb}}{\text{h}} \quad \longleftarrow \dot m$$

(b) $$\dot W_c = \dot m\,(h_2-h_1) = \left(4.036\times10^4 \frac{\text{lb}}{\text{h}}\right)(\underbrace{124.97-116.74}_{8.23})\frac{\text{Btu}}{\text{lb}}\left|\frac{1 \text{ hp}}{2545 \text{ Btu/h}}\right| = 130.5 \text{ hp} \longleftarrow \dot W_c$$

(c) $$\gamma = \frac{\dot Q_{out}}{\dot W_c} = \frac{\left(50{,}000 \frac{\text{Btu}}{\text{min}}\right)\left|\frac{60 \text{ min}}{1 \text{ h}}\right|}{\left(4.036\times10^4 \frac{\text{lb}}{\text{h}}\right)(8.23)\frac{\text{Btu}}{\text{lb}}} = 9.03 \quad \longleftarrow \gamma$$

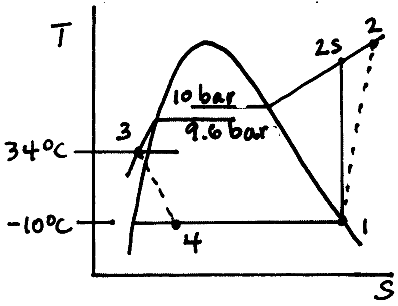
Discussion: The use of heat pumps to produce process heat from waste streams can enhance overall energy resource use in a plant. However, other factors such as economics weigh heavily in decisions to utilize such techniques.

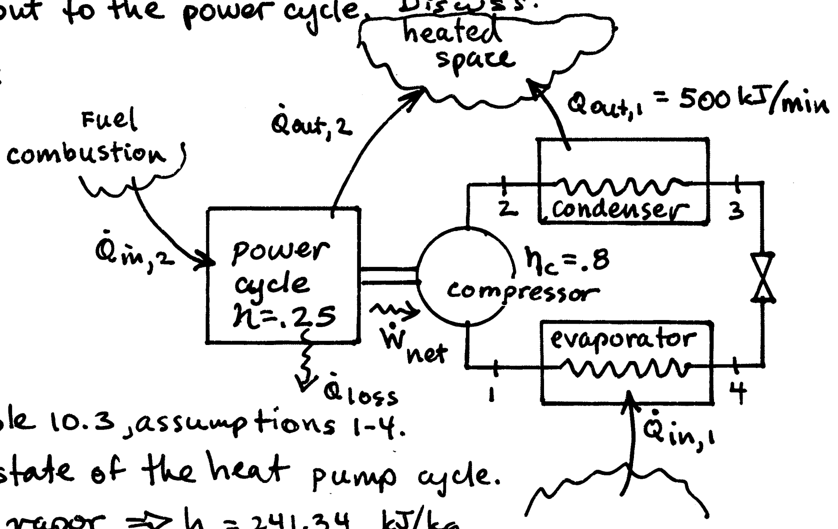
PROBLEM 10.39

KNOWN: Refrigerant 134a is the working fluid in a vapor-compression heat pump driven by a power cycle. Operating data are specified for the heat pump and the power cycle.

FIND: Determine (a) the heat pump compressor power and (b) the ratio of the total rate heat is delivered to the heated space to the rate of heat input to the power cycle. Discuss.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.3, assumptions 1-4.

ANALYSIS: First, fix each state of the heat pump cycle.

State 1 $T_1 = -10°C$, sat. vapor $\Rightarrow h_1 = 241.34$ kJ/kg
$s_1 = .9253$ kJ/kg·K

State 2 For isentropic compression, $p_2 = 10$ bar, $s_{2s} = s_1 \Rightarrow h_{2s} = 274.63$ kJ/kg
Using the compressor efficiency

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s}-h_1}{\eta_c}\right) = 282.95 \text{ kJ/kg}$$

State 3 $p_3 = 9.6$ bar, $T_3 = 34°C \Rightarrow$ comp. liq.; $h_3 \approx h_f(34°C) = 97.31$ kJ/kg

State 4 Throttling process $\Rightarrow h_4 = h_3 = 97.31$ kJ/kg

(a) The mass flow rate of refrigerant is

$$\dot{m} = \frac{\dot{Q}_{out,1}}{h_2-h_3} = \frac{(500 \text{ kJ/min})}{(282.95 - 97.31)\text{ kJ/kg}}\left|\frac{1 \text{ min}}{60 \text{ s}}\right| = 0.04489 \text{ kg/s}$$

The compressor power becomes

$$\dot{W}_c = \dot{m}(h_2 - h_1) = \left(.04489\frac{\text{kg}}{\text{s}}\right)(282.95 - 241.34)\frac{\text{kJ}}{\text{kg}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$
$$= 1.868 \text{ kW} \longleftarrow \dot{W}_c$$

(b) For the power cycle, $\eta = 0.25$. With $\dot{W}_{\substack{power\\cycle}} = \dot{W}_{\substack{heat\\pump}} = 1.868$ kW

$$\dot{Q}_{in,2} = \frac{\dot{W}_{cycle}}{\eta} = 7.472 \text{ kW}$$

The total heat rejected is, $\dot{Q}_{rej} = 7.472\text{ kW} - 1.868 \text{ kW} = 5.604$ kW

thus, $\dot{Q}_{out,2} = (.8)\dot{Q}_{rej} = 4.483$ kW

Finally, $$\frac{\dot{Q}_{out,1} + \dot{Q}_{out,2}}{\dot{Q}_{in,2}} = \frac{(500/60) + 4.483}{7.472} = 1.687 \longleftarrow (b)$$

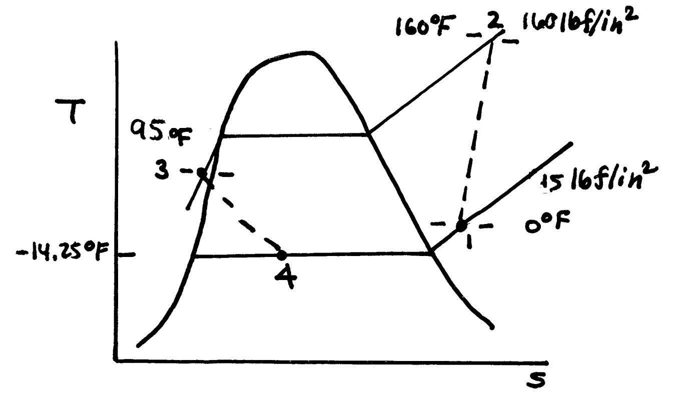
Discussion: The engin-driven heat pump delivers more energy to the heated space than could be obtained by burning the fuel directly.

PROBLEM 10.40*

KNOWN: Operating data are provided for a vapor compression heat pump using R134a as the working fluid.

FIND: Determine (a) the isentropic compressor efficiency, and (b) the coefficient of performance. (c) Perform an exergy accounting of the compressor power input.

SCHEMATIC & GIVEN DATA:



State	h (Btu/lb)	s (Btu/lb·°R)
1	102.42	0.2303
2	129.78	0.2391
3	42.47	0.0867
4	42.47	0.0958

ASSUMPTIONS: 1. Control volumes enclosing each of the principal components are at steady state. 2. Kinetic and potential energy effects can be ignored as can stray heat transfers and pressure drops for flow through the evaporator and condenser. 3. The expansion across the valve is a throttling process. 4. $T_0 = 480°R$

ANALYSIS: First, fix each of the principal states and obtain the data shown in the table above.

State 1: $P_1 = 15\ \text{lbf/in}^2$, $T_1 = 0\ °F$. Table A-12E gives $h_1 = 102.42\ \text{Btu/lb}$, $s_1 = 0.2303\ \frac{\text{Btu}}{\text{lb}\cdot °R}$

State 2: $P_2 = 160\ \text{lbf/in}^2$, $T_2 = 160°F$, Table A-12E gives $h_2 = 129.78\ \text{Btu/lb}$, $s_2 = 0.2391\ \frac{\text{Btu}}{\text{lb}\cdot °R}$

Also, $s_{2s} = s_1 \Rightarrow h_{2s} = 124.41\ \text{Btu/lb}$

State 3: $h_3 = h_f(95°F) = 42.47\ \text{Btu/lb}$, $s_3 = s_f(95°F) = 0.0867\ \text{Btu/lb}\cdot°R$ from Table A-10E.

State 4: $h_4 = h_3 = 42.47\ \text{Btu/lb}$

$$\Rightarrow x_4 = \frac{42.47 - 7.40}{92.27} = 0.380 \Rightarrow s_4 = 0.0171 + 0.380[0.2242 - 0.0171] = 0.0958\ \frac{\text{Btu}}{\text{lb}\,°R}$$

(a) The isentropic compressor efficiency is

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{124.41 - 102.42}{129.78 - 102.42} = 0.804\ (80.4\%) \longleftarrow$$

(b) The coefficient of performance is

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{\cancel{\dot{m}}(h_2 - h_3)}{\cancel{\dot{m}}(h_2 - h_1)} = \frac{129.78 - 42.47}{129.78 - 102.42} = 3.19 \longleftarrow$$

(c) The compressor work input per unit of mass flowing is

$$\dot{W}_c/\dot{m} = h_2 - h_1 = 27.36\ \text{Btu/lb}$$

PROBLEM 10.40 (Contd.)

The rates of exergy destruction for the compressor and valve can be found using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$. That is,

- Compressor

$$(\dot{E}_d/\dot{m})_{comp} = T_0(s_2 - s_1) = 480°R(0.2391 - 0.2303)\frac{Btu}{lb \cdot °R} = 4.22 \, Btu/lb$$

- Valve

$$(\dot{E}_d/\dot{m})_{valve} = T_0(s_4 - s_3) = (480°R)(0.0958 - 0.0867)\frac{Btu}{lb \cdot °R} = 4.37 \, Btu/lb$$

① The change in flow exergy as the refrigerant passes through the evaporator is

$$e_{f1} - e_{f4} = h_1 - h_4 - T_0(s_1 - s_4)$$

$$= (102.42 - 42.47) - 480(0.2303 - 0.0958) = -4.61 \, Btu/lb$$

The change in flow exergy as the refrigerant passes through the condenser is

$$e_{f3} - e_{f2} = h_3 - h_2 - T_0(s_3 - s_2)$$

$$= (42.47 - 129.78) - 480(0.0867 - 0.2391) = -14.16 \, Btu/lb$$

Summary:

- Exergy input– compressor work 27.36 $\frac{Btu}{lb}$
- Disposition of the exergy input:

-Exergy destruction		
compressor	4.22 Btu/lb	(15.4 %)
valve	4.37 Btu/lb	(16.0 %)
- Exergy transfer out		
evaporator	4.61 Btu/lb	(16.8 %)
condenser	14.16 Btu/lb	(51.8 %)
	27.36 Btu/lb	

The summary indicates that there are two significant sources of irreversibility: the compression process and the expansion process. The compressor irreversibility can be reduced by means of a compressor having a higher isentropic efficiency than found in part (a). Economic consequences should be carefully considered, however.

---

1. Since there is no work (except flow work) and no internal irreversibilities (pressure is constant), an exergy rate balance for the refrigerant side of the evaporator, and of the condenser, reads

$$0 = \dot{E}_q - \cancel{\dot{W}_{cv}} + \dot{m}(e_{f,in} - e_{f,out}) - \cancel{\dot{E}_d} \Rightarrow \dot{E}_q = \dot{m}(e_{f,out} - e_{f,in}).$$

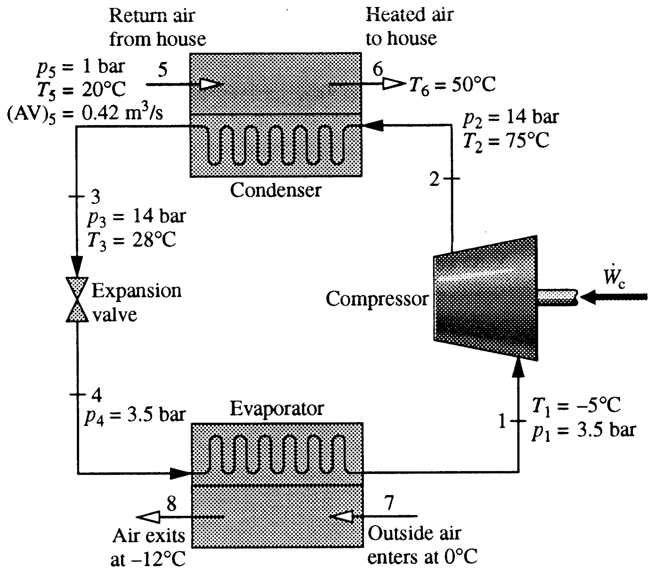
Thus, the change in flow exergy equals the exergy transfer accompanying heat transfer. Since heat transfer occurs at $T < T_0$ in the evaporator case, the exergy transfer accompanying heat is opposite to the direction of the heat transfer: from the refrigerant to the cold space. For the condenser case, heat transfer and exergy transfer are in the same direction: from the refrigerant to the heated space.

PROBLEM 10.41

KNOWN: Steady-state operating data are provided for a residential heat pump system using R22 as the working fluid.

FIND: Determine (a) the power required by the compressor and the isentropic compressor efficiency, and (b) the coefficient of performance. (c) Perform an exergy accounting of the compressor power input. Discuss. (d) Devise and evaluate an exergetic efficiency for the heat pump system.

SCHEMATIC & GIVEN DATA



NOTE: This system is recognized from Examples 6.8, 6.14 and 7.9, except here an evaporator is considered explicitly.

ASSUMPTIONS: See assumptions listed for Examples 6.8, 6.14 and 7.9

ANALYSIS: (a) The power required by the compressor is evaluated in Ex. 6.14 as $\dot{W}_c = 3.11$ kW. In Ex. 6.14, we get $\eta_c = 81\%$.

(b) Using data from Ex. 6.8 and Ex. 6.14

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_c} = \frac{\dot{m}(h_2-h_3)}{\dot{W}_c} = \frac{h_2-h_3}{h_2-h_1} = \frac{294.17-79.05}{294.17-249.75} = 4.84$$

(c) From Ex. 7.9 we get the following exergy destruction values:

compressor: $\dot{E}_d = 0.478$ kW
valve: $\dot{E}_d = 0.271$ kW
condensor: $\dot{E}_d = 0.217$ kW

To find the exergy destruction in the evaporator, first determine the outside air mass flow rate, $\dot{m}_7$, using mass and energy rate balances applied to a control volume enclosing the evaporator to get

$$\dot{m}_7 = \frac{\dot{m}(h_1-h_4)}{h_7-h_8} = \frac{\dot{m}(h_1-h_4)}{c_{pa}(T_7-T_8)} = \frac{(0.07\ \text{kg/s})[249.75-79.05]\ \text{kJ/kg}}{(1.005\ \text{kJ/kg·K})[12\ \text{K}]} = 0.99\ \text{kg/s}$$

Then, with $\dot{E}_d = T_0\dot{\sigma}_{cv}$ and $\dot{\sigma}_{cv}$ from an entropy rate balance

$$\dot{E}_d = T_0\left[\dot{m}_7(s_8-s_7)+\dot{m}(s_1-s_4)\right] = T_0\left[\dot{m}_7\left(c_p\ln\frac{T_8}{T_7} - R\cancelto{0}{\ln\frac{p_8}{p_7}}\right)+\dot{m}(s_1-s_4)\right]$$

$$= 273\text{K}\left[\left(0.99\frac{\text{kg}}{\text{s}}\right)\left(1.005\frac{\text{kJ}}{\text{kg·K}}\right)\ln\frac{261}{273} + \left(0.07\frac{\text{kg}}{\text{s}}\right)(0.9572-0.3078)\frac{\text{kJ}}{\text{kg·K}}\right]\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 0.202\ \text{kW}$$

PROBLEM 10.41 (Contd.)

The net rate exergy is carried out via heated air to the house is

$$\dot{E}_{f6} - \dot{E}_{f5} = \dot{m}_{air}\left[(h_6 - h_5) - T_0(s_6 - s_5)\right] = \dot{m}_{air} c_{pa}\left[(T_6 - T_5) - T_0 \ln \frac{T_6}{T_5}\right]$$

$$= (0.5)(1.005)\left[30 - 273 \ln \frac{323}{293}\right] = 1.698 \text{ kW}$$

The net rate exergy is carried out via the outside air is

$$\dot{E}_{f8} - \dot{E}_{f7} = \dot{m}_7\left[(h_8 - h_7) - T_0(s_8 - s_7)\right] = \dot{m}_7 c_p\left[(T_8 - T_7) - T_0 \ln \frac{T_8}{T_7}\right]$$

$$= (0.99)(1.005)\left[(-12) - 273 \ln \frac{261}{273}\right] = 0.27 \text{ kW}$$

Exergy Summary:

	• Rate of exergy input — compressor work:	3.11 kW		
	• Disposition of exergy input:			
	– Net rate exergy is transferred to the house	1.698 kW	(54.6%)	←
	– net rate exergy is carried out to the surroundings	0.27 kW	(8.7%)	
①	– exergy destruction			
	compressor	0.478 kW	(15.4%)	~38%
	valve	0.271 kW	(8.7%)	
	condenser	0.217 kW	(7%)	
	evaporator	0.202 kW	(6.5%)	

(d) In this case an exergetic efficiency in the form (output/input) is given by the ratio of the exergy delivered to the house to the exergy input. This can be obtained by inspection of the summary:

$$\epsilon = \frac{(\dot{E}_{f6} - \dot{E}_{f5})}{\dot{W}_c} = 54.6\%$$

---

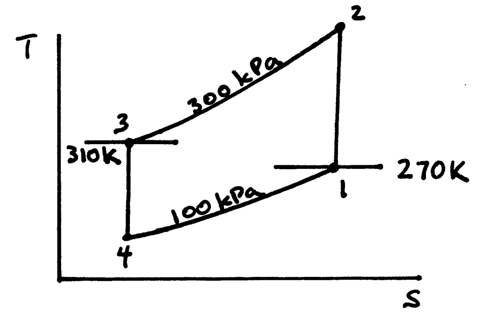
1. Better thermodynamic performance could be achieved by reducing the exergy destruction in the compressor with a compressor having a higher isentropic efficiency than determined in the solution. However, the effect on the overall system of a change in one component must be carefully considered as should be the economic consequences of such a step.

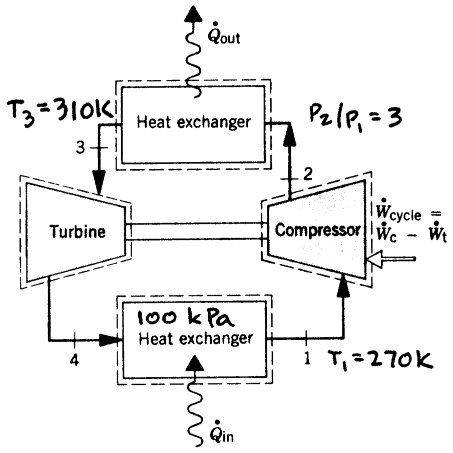
PROBLEM 10.42

KNOWN: Air is the working fluid in an ideal Brayton refrigeration cycle. Operating data are specified.

FIND: Determine (a) the net work input, per unit mass of air flow, (b) the refrigerating capacity, per unit mass of air flow, (c) the coefficient of performance, and (d) the coefficient of performance of a reversible cycle operating between reservoirs at $T_C = 270\text{ K}$ and $T_H = 310\text{ K}$.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.4.

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1 $T_1 = 270\text{K} \Rightarrow h_1 = 270.11\text{ kJ/kg}$ $p_{r1} = 0.9590$

State 2 $p_{r2} = p_{r1}(P_2/P_1) = 2.877 \Rightarrow h_2 = 370.10\text{ kJ/kg}$

State 3 $T_3 = 310\text{K} \Rightarrow h_3 = 310.24\text{ kJ/kg}$, $p_{r3} = 1.5546$

State 4 $p_{r4} = p_{r3}(p_4/p_3) = 0.5182 \Rightarrow h_4 = 226.25\text{ kJ/kg}$

(a) The net work is

$$\frac{\dot W_{cycle}}{\dot m} = \frac{\dot W_c}{\dot m} - \frac{\dot W_t}{\dot m} = (h_2 - h_1) - (h_3 - h_4)$$

$$= (370.10 - 270.11) - (310.24 - 226.25)\ \frac{\text{kJ}}{\text{kg}}$$

$$= 16.0\text{ kJ/kg} \longleftarrow \dot W_{cycle}/\dot m$$

(b) The refrigerating capacity is

$$\frac{\dot Q_{in}}{\dot m} = h_1 - h_4 = 270.11 - 226.25 = 43.86\text{ kJ/kg}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot Q_{in}/\dot m}{\dot W_{cycle}/\dot m} = 2.74 \longleftarrow \beta$$

(d) For a reversible cycle with $T_C = 270\text{K}$, $T_H = 310\text{K}$

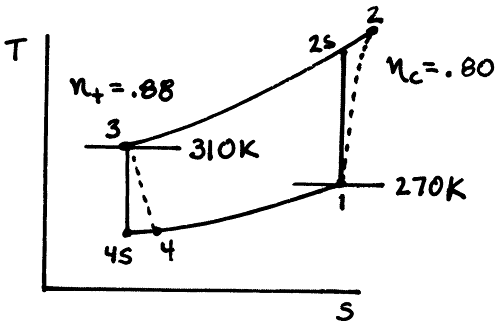
$$\beta_{max} = \frac{T_C}{T_H - T_C} = \frac{270}{310 - 270} = 6.75 \longleftarrow \beta_{max}$$

PROBLEM 10.43

KNOWN: The ideal Brayton refrigeration cycle of Problem 10.42 is modified to include compressor and turbine efficiencies of 80 and 88%, respectively.

FIND: Determine (a) the coefficient of performance and (b) develop an exergy accounting of the net work input. Discuss.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 10.5. Also, let $T_0 = 310$ K.

ANALYSIS: First, fix each of the principal states (Table A-22). From the solution to Problem 10.42; $h_1 = 270.11$ kJ/kg, $h_{2s} = 370.10$ kJ/kg, $h_3 = 310.24$ kJ/kg, $h_{4s} = 226.25$ kJ/kg.

State 2 Using the compressor efficiency,

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 395.1 \text{ kJ/kg}$$

State 4 Using the turbine efficiency,

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 236.3 \text{ kJ/kg}$$

(a) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_{cycle}/\dot{m}} = \frac{(h_1 - h_4)}{(h_2 - h_1) - (h_3 - h_4)} = 0.662 \longleftarrow \beta$$

(b) The exergy destruction rate for the compressor is

$$(\dot{E}_d/\dot{m})_c = T_0 \dot{\sigma}_c/\dot{m} = T_0 \left[(s_2^\circ - s_1^\circ) - R \ln p_2/p_1\right]$$

From Table A-22; $s_1^\circ = 1.59634$ and $s_2^\circ = 1.95805$. Thus

$$(\dot{E}_d/\dot{m})_c = (310)\left[(1.97703 - 1.59634) - \left(\frac{8.314}{28.97}\right) \ln (3)\right]$$

$$= 20.27 \text{ kJ/kg} \longleftarrow (\dot{E}_d/\dot{m})_c$$

Similarly, for the turbine, with $s_3^\circ = 1.73498$ and $s_4^\circ = 1.46237$,

$$(\dot{E}_d/\dot{m})_t = T_0 \left[(s_4^\circ - s_3^\circ) - R \ln p_4/p_3\right]$$

$$= (310)\left[(1.46237 - 1.73498) - \frac{8.314}{28.97} \ln \left(\frac{1}{3}\right)\right]$$

$$= 13.23 \text{ kJ/kg} \longleftarrow (\dot{E}_d/\dot{m})_t$$

PROBLEM 10.43 (Cont'd.)

The net change in exergy as air passes through the high-temperature heat exchanger is

① $$e_{f3} - e_{f2} = (h_3 - h_2) - T_0(s_3 - s_2) = (h_3 - h_2) - T_0\left(s_3^\circ - s_2^\circ - \cancel{R \ln p_3/p_2}^{\,0}\right)$$

$$= (310.24 - 395.1) - 310(1.73498 - 1.97703) = -9.83 \text{ kJ/kg} \quad \leftarrow$$

The net change in exergy as air passes through the low-temperature heat exchanger is

$$e_{f1} - e_{f4} = (h_1 - h_4) - T_0(s_1 - s_4) = (h_1 - h_4) - T_0\left(s_1^\circ - s_4^\circ - \cancel{R \ln p_1/p_4}^{\,0}\right)$$

$$= (270.11 - 236.3) - 310(1.59634 - 1.46237) = -7.72 \text{ kJ/kg} \quad \leftarrow$$

The net work input is

$$\dot W_{net}/\dot m = (h_2 - h_1) - (h_3 - h_4)$$

$$= (395.1 - 270.11) - (310.24 - 236.3) = 51.05 \text{ kJ/kg}$$

Summary:

• Net exergy input by work	51.05 kJ/kg	
• Disposition of the exergy input		
– Exergy transfers out		
✓ high-temp. HX	9.83 kJ/kg	(19.3 %)
✓ low-temp. HX	7.72 kJ/kg	(15.1 %)
– Exergy destruction		
✓ compressor	20.27 kJ/kg	(39.7 %)
✓ turbine	13.23 kJ/kg	(25.9 %)
	51.05 kJ/kg	

In this case, nearly two-thirds of the exergy input is destroyed in achieving a fairly modest refrigerating effect, as gauged by the coefficient of performance.

---

1. For the heat exchangers there is no work (except for flow work). Also, there are no internal irreversibilities (pressure is constant). Thus, an exergy rate balance for the air side of each heat exchanger takes the form

$$0 = \dot E_q - \cancel{\dot W_{cv}}^{\,0} + \dot m(e_{f,in} - e_{f,out}) - \cancel{\dot E_d}^{\,0}$$

$\Rightarrow \dot m(e_{f,out} - e_{f,in}) = \dot E_q$. That is, the change in flow exergy is equal to the exergy transfer accompanying heat transfer. For the low-temperature heat exchanger, heat transfer occurs at $T < T_0$, and thus the exergy transfer is opposite to the heat transfer: from the air to the cold space. For the high-temperature heat exchanger, heat transfer and exergy transfer are in the same direction: from the air to the warm space.

PROBLEM 10.44

The data for the required plots are obtained using IT, as follows:

IT Code

```
T1 = 270   // K
p1 = 100    // kPa
rp = 3
T3 = 310   // K
eta_t = 1
eta_c = 1
mdot = 1   // Assume a unit mass
//              flow rate of 1 kg/s.

h1 = h_T("Air", T1)
s1 = s_hP("Air",h1,p1)
s2s = s1
p2 = rp * p1
s2s = s_hP("Air", h2s, p2)
h2 = h1 + (h2s - h1) / eta_c
h3 = h_T("Air", T3)
p3 = p2
s3 = s_hP("Air",h3,p3)
s4s = s3
p4 = p1
s4s = s_hP("Air",h4s,p4)
h4 = h3 - (h3 - h4s) * eta_t

Wdotcycle / mdot = Wdotc / mdot - Wdott / mdot
Wdott / mdot =  (h3 - h4)
Wdotc / mdot  = (h2 - h1)
Qdotin / mdot = (h1 - h4)
beta = (h1 - h4) / ((h2 - h1) - (h3 - h4))
```

IT Results for $p_2/p_1 = 3$ and $\eta_c = \eta_t = 100\%$

h1 269.9 kJ/kg

h2 369.9 kJ/kg

h3 310.1 kJ/kg

h4 226.2 kJ/kg

$\dot{W}_c/\dot{m}$ = 100 kJ/kg

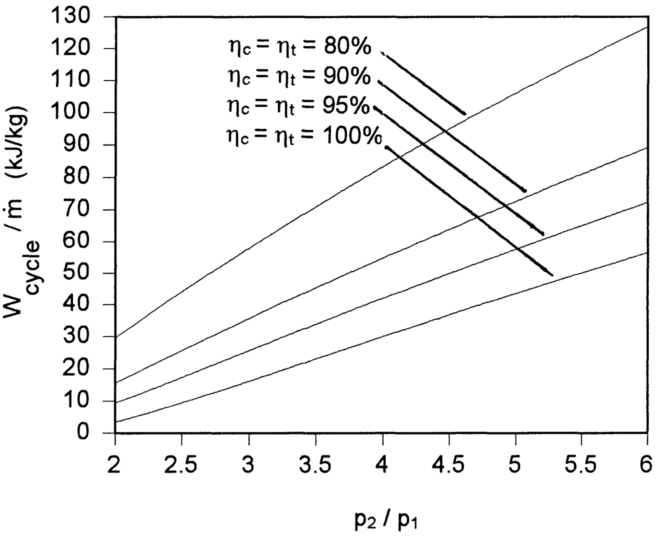
$\dot{W}_t/\dot{m}$ = 83.88 kJ/kg

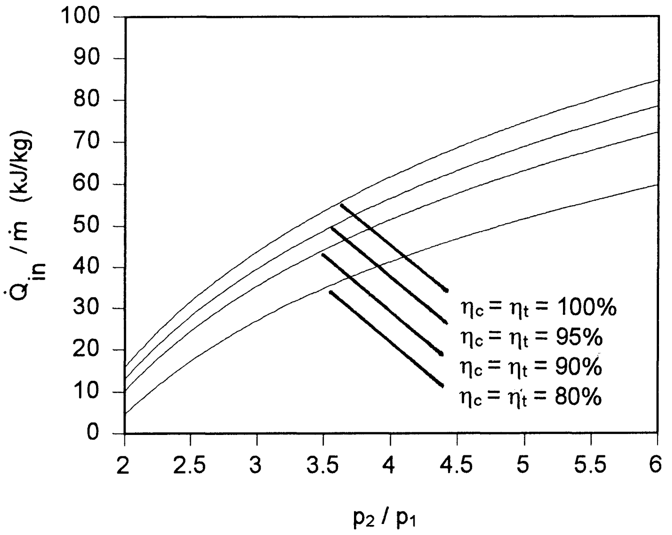
$\dot{W}_{cycle}/\dot{m}$ = 16.13 kJ/kg

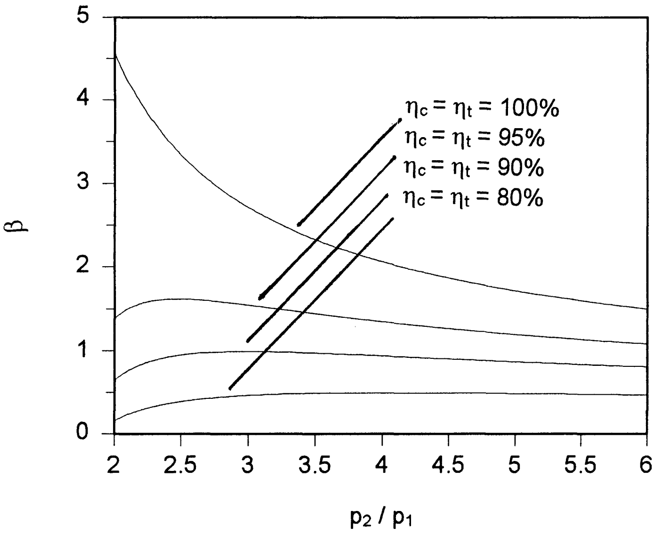
$\dot{Q}_{in}/\dot{m}$ = 43.73 kJ/kg

β = 2.711

PLOTS:







Discussion.

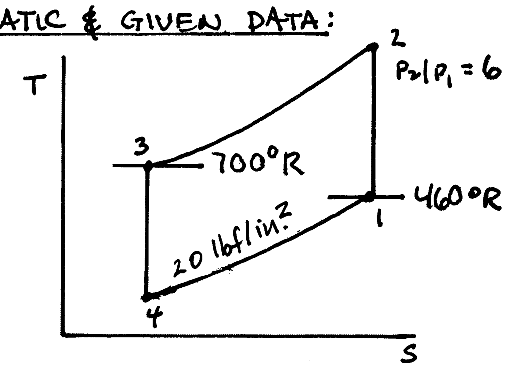
- Compressor and turbine irreversibility decrease the capacity and increase the work input to the cycle. As a result, the coefficient of performance decreases significantly for $\eta_c = \eta_t \leq 100\%$.
- Increasing compressor pressure ratio increases capacity but also the net work input. The coefficient of performance exhibits a maximum and then decreases with increasing pressure ratio.

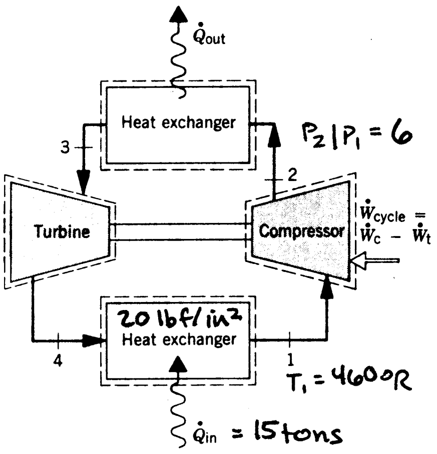
PROBLEM 10.45*

KNOWN: Air is the working fluid in an ideal Brayton refrigeration cycle. Operating data and the refrigerating capacity are specified.

FIND: Determine (a) the air mass flow rate, (b) the net power input, (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.4.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 460\ °R \Rightarrow h_1 = 109.90$ Btu/lb, $p_{r1} = 0.7913$

State 2 $p_{r2} = p_{r1}(P_2/P_1) = 4.7478 \Rightarrow h_2 = 183.62$ Btu/lb

State 3 $T_3 = 700\ °R \Rightarrow h_3 = 167.56$ Btu/lb, $p_{r3} = 3.446$

State 4 $p_{r4} = p_{r3}(P_4/P_3) = (3.446)(\frac{1}{6}) = 0.5743 \Rightarrow h_4 = 100.23$ Btu/lb

(a) The mass flow rate of air is

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1 - h_4} = \frac{15 \text{ tons}}{(109.90 - 100.23)\frac{\text{Btu}}{\text{lb}}}\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right| = 310.24 \frac{\text{lb}}{\text{min}} \longleftarrow \dot{m}$$

(b) Thus, the net power input is

$$\dot{W}_{cycle} = \dot{m}\left[(h_2 - h_1) - (h_3 - h_4)\right]$$
$$= (310.24)\left[(183.62 - 109.90) - (167.69 - 100.23)\right]$$
$$= 1{,}942.1 \text{ Btu/min} \longleftarrow \dot{W}_{cycle}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{15 \text{ tons}}{1{,}942.1 \text{ Btu/min}}\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right| = 1.54 \longleftarrow \beta$$

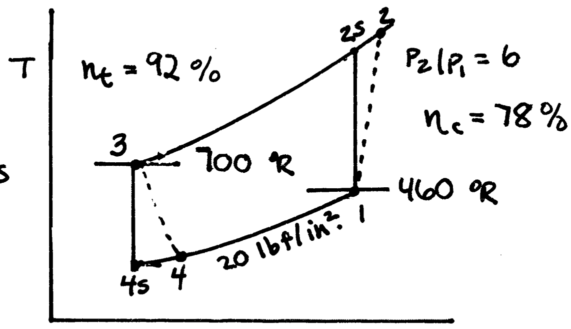
PROBLEM 10.46*

KNOWN: The Brayton refrigeration cycle of Problem 10.45 is modified to include that the compressor and turbine have isentropic efficiencies of 78% and 92%, respectively.

FIND: Determine (a) the air mass flow rate, (b) the net power input, (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:

$\dot{Q}_{in} = 15$ tons



ASSUMPTIONS: See Example 10.5.

ANALYSIS: First, fix each of the principal states (Table A-22E).

From the solution of Problem 10.45

$h_1 = 109.80$ Btu/lb
$h_{2s} = 183.62$ Btu/lb
$h_3 = 167.56$ Btu/lb
$h_{4s} = 100.23$ Btu/lb

State 2 Using the compressor efficiency,

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s}-h_1}{\eta_c}\right) = 204.44 \text{ Btu/lb}$$

State 4 Using the turbine efficiency,

$$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3-h_{4s}) = 105.62 \text{ Btu/lb}$$

(a) Using the refrigerating capacity

$$\dot{m} = \frac{\dot{Q}_{in}}{h_1-h_4} = \frac{(15 \text{ tons})}{(109.80-105.62)\ \frac{\text{Btu}}{\text{lb}}}\left|\frac{200 \text{ Btu/min}}{1 \text{ ton}}\right|$$

$$= 717.7 \ \frac{\text{lb}}{\text{min}} \longleftarrow \dot{m}$$

(b) The net power input is

$$\dot{W}_{net} = \dot{m}\left[(h_2-h_1)-(h_3-h_4)\right] = (717.7)[(204.44-109.80)-(167.56-105.62)]$$

$$= 23{,}469 \text{ Btu/min} \longleftarrow \dot{W}_{net}$$

(c) The coefficient of performance is

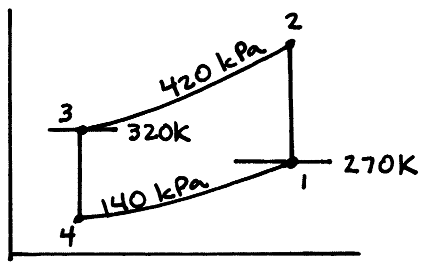
$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{net}} = \frac{(15 \times 200) \text{ Btu/min}}{(23{,}469) \text{ Btu/min}} = 0.128 \longleftarrow \beta$$

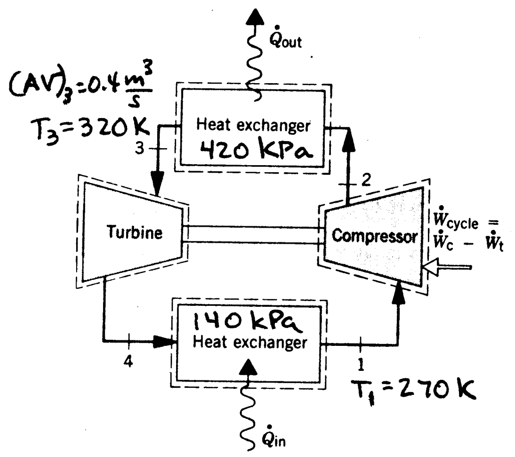
PROBLEM 10.47

KNOWN: Operating data are provided for a Brayton refrigeration cycle using air as the working fluid.

FIND: Determine (a) the air mass flow rate, (b) the net power input, (c) the refrigerating capacity, and (d) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.4

ANALYSIS: First, fix each of the principal states (Table A-22).

State 1 $T_1 = 270K \Rightarrow h_1 = 270.11$ kJ/kg, $p_{r1} = 0.9590$

State 2 $p_{r2} = p_{r1}(p_2/p_1) = 2.877 \Rightarrow h_2 = 370.10$ kJ/kg

State 3 $T_3 = 320K \Rightarrow h_3 = 320.29$ kJ/kg, $p_{r3} = 1.7375$

State 4 $p_{r4} = p_{r3}(p_4/p_3) = 0.5792 \Rightarrow h_4 = 233.61$ kJ/kg

(a) The mass flow rate is

$$\dot{m} = \frac{(AV)_3}{v_3} = \frac{(AV)_3 \, p_3}{R T_3} = \frac{(0.4 \, m^3/s)(420 \, kPa)}{\left(\frac{8.314}{28.97} \frac{kJ}{kg \cdot K}\right)(320K)} \left| \frac{10^3 N/m^2}{1 \, kPa} \right| \left| \frac{1 \, kJ}{10^3 \, N \cdot m} \right| = 1.829 \, \frac{kg}{s}$$

Thus, the net power is

(b) $\dot{W}_{cycle} = \dot{m}[(h_2 - h_1) - (h_3 - h_4)]$

$$= \left(1.829 \frac{kg}{s}\right)[(370.1 - 270.11) - (320.29 - 233.61)] \frac{kJ}{kg} \left| \frac{1 \, kW}{1 \, kJ/s} \right| = 24.34 \, kW \longleftarrow \dot{W}_{cycle}$$

(c) The refrigerating capacity is

$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = 66.76$ kW $\longleftarrow \dot{Q}_{in}$

(d) The coefficient of performance is

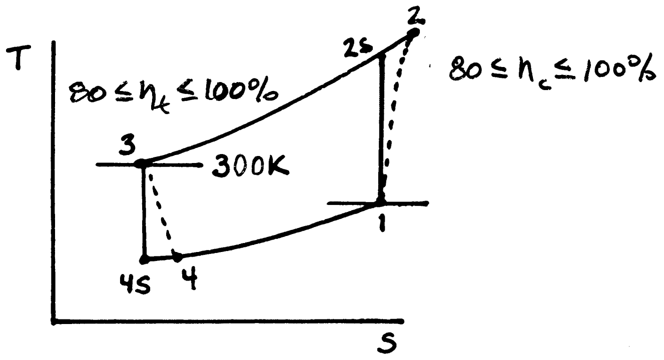
$$\beta = \frac{\dot{Q}_{in}}{\dot{W}_{cycle}} = \frac{66.76 \, kW}{24.34 \, kW} = 2.743 \longleftarrow \beta$$

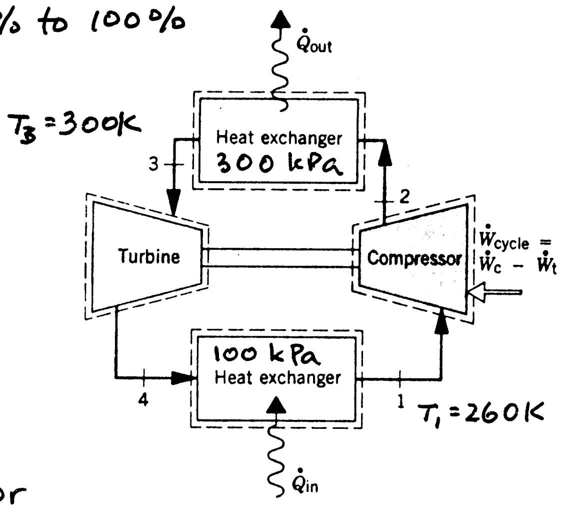
PROBLEM 10.48

KNOWN: Air is the working fluid in a Brayton refrigeration cycle. Operating data are specified.

FIND: (a) Determine the net work per unit mass of air flow and the coefficient of performance for $\eta_c = \eta_t = 100\%$. (b) Plot the same quantities for $\eta_c = \eta_t$ ranging from 80% to 100%

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: For part (a), See Example 10.4. For part (b), see Example 10.5.

ANALYSIS: Sample calculations using data from Table A-22:

State 1: $T_1 = 260K \Rightarrow h_1 = 260.09$ kJ/kg, $p_{r1} = 0.8405$

State 2: $p_{r2s} = p_{r1}(p_2/p_1) = 2.5215 \Rightarrow h_{2s} = 356.31$ kJ/kg

Using the isentropic compressor efficiency

$$h_2 = h_1 + (h_{2s} - h_1)/\eta_c \; ; \; \eta_c = 100\% \Rightarrow h_2 = 356.31 \text{ kJ/kg} \quad (a)$$

$$\eta_c = 80\% \Rightarrow h_2 = 380.37 \text{ kJ/kg} \quad (b)$$

State 3: $T_3 = 300K \Rightarrow h_3 = 300.19$ kJ/kg, $p_{r3} = 1.3860$

State 4: $p_{r4s} = p_{r3}(p_4/p_3) = 0.462 \Rightarrow h_{4s} = 218.97$ kJ/kg

Using the isentropic turbine efficiency

$$h_4 = h_3 - (h_3 - h_{4s})\eta_t \; ; \; \eta_t = 100\% \Rightarrow h_4 = 218.97 \text{ kJ/kg} \quad (a)$$

$$\eta_t = 80\% \Rightarrow h_4 = 235.21 \text{ kJ/kg} \quad (b)$$

The net work is

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4)$$

$$\eta_c = \eta_t = 100\% \Rightarrow \dot{W}_{cycle}/\dot{m} = 15 \text{ kJ/kg} \longleftarrow \dot{W}_{cycle}/\dot{m} \text{ (part a)}$$

$$\eta_c = \eta_t = 80\% \Rightarrow \dot{W}_{cycle}/\dot{m} = 55.3 \text{ kJ/kg}$$

Now, the coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_{cycle}/\dot{m}} = \frac{(h_1 - h_4)}{\dot{W}_{cycle}/\dot{m}}$$

$$\eta_c = \eta_t = 100\% \Rightarrow \beta = 2.74 \longleftarrow \beta \text{ (part a)}$$

$$\eta_c = \eta_t = 80\% \Rightarrow \beta = 0.45$$

PROBLEM 10.48 (Cont'd.)

(b) The data for the required plots are obtained using IT, as follows:

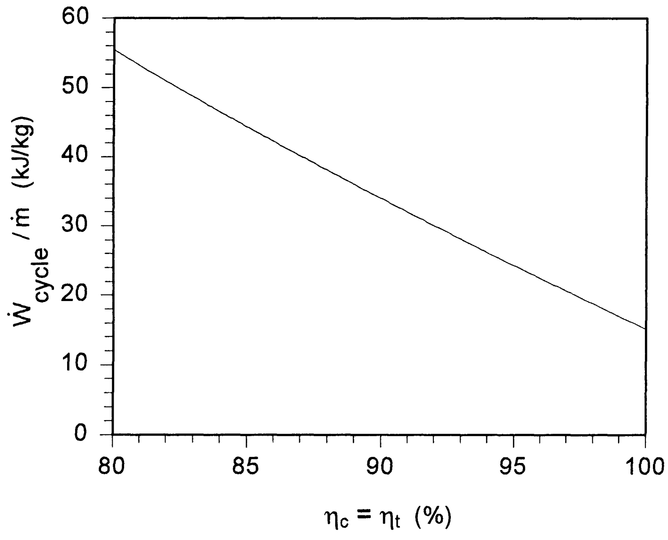
IT Code

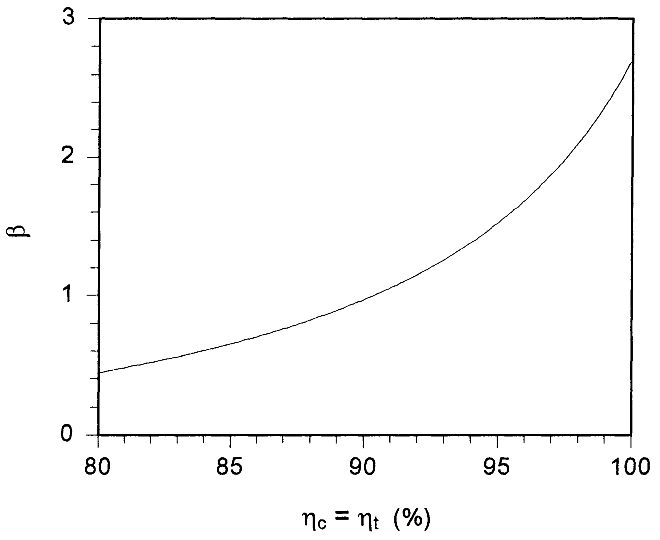
```
T1 = 260  // K
p1 = 100  // kPa
p2 = 300  // kPa
T3 = 300  // K
p3 = p2
p4 = p1
eta_c = 80  // %
eta_t = eta_c
mdot = 1  // Assume a unit mass
//              flow rate of 1 kg/s.

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / (eta_c / 100)
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * (eta_t / 100)

Wdot_c / mdot = (h2 - h1)
Wdot_t / mdot = (h3 - h4)
Wdotcycle / mdot = Wdot_c / mdot - Wdot_t / mdot
beta = (h1 - h4) / (Wdotcycle / mdot)
```

IT Results for $\eta_c = \eta_t = 80\%$

$h_1 = 259.9$ kJ/kg
$h_2 = 380.3$ kJ/kg
$h_{2s} = 356.2$ kJ/kg
$h_3 = 300$ kJ/kg
$h_4 = 235.1$ kJ/kg
$h_{4s} = 218.9$ kJ/kg
$\dot{W}_c/\dot{m} = 120.4$ kJ/kg
$\dot{W}_t/\dot{m} = 64.94$ kJ/kg
$\dot{W}_{cycle}/\dot{m} = 55.45$ kJ/kg
$\beta = 0.4471$

PLOTS:





Discussion:

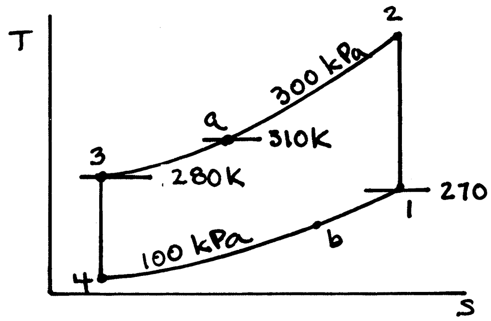
- As the isentropic compressor and turbine efficiencies decrease, the net work input to the cycle increases.
- As turbine efficiency decreases, point 4 on the accompanying T-s diagram moves to the right, thereby decreasing the capacity. The decrease in capacity, coupled with the increase in net work, result in a dramatic drop in coefficient of performance compared to the ideal case (isentropic compression and expansion.)

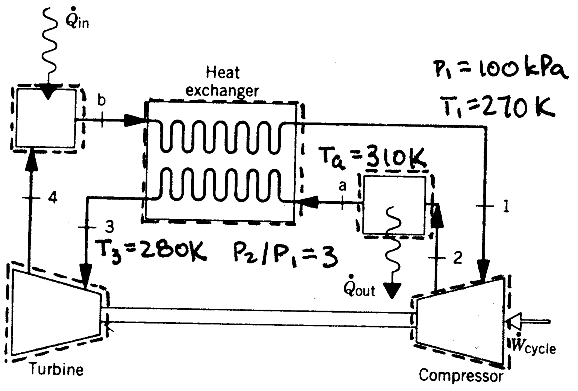
PROBLEM 10.49

KNOWN: The ideal Brayton refrigeration cycle of Problem 10.42 is modified to include a regenerative heat exchanger.

FIND: Determine (a) the lowest temperature, (b) the net work input per unit mass of air flow, (c) the refrigerating capacity, per unit mass of air flow, (d) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.4. Also, there is no heat transfer from the heat exchanger to its surroundings.

ANALYSIS: First, fix each of the principal states (Table A-22). From the solution to Problem 10.42, $h_1 = 270.11$ kJ/kg, $h_2 = 370.10$ kJ/kg, and $h_a = 310.24$ kJ/kg.

State 3 $T_3 = 280\text{K} \Rightarrow h_3 = 280.13$ kJ/kg , $p_{r3} = 1.0889$

State 4 $p_{r4} = p_{r3}(p_4/p_3) = 0.36297 \Rightarrow h_4 = 204.24 \frac{\text{kJ}}{\text{kg}}$, $T_4 = 204.3\text{K}$ ← T_{min} (a)

State b For the heat exchanger

$$0 = (h_b - h_1) + (h_a - h_3) \Rightarrow h_b = 240 \text{ kJ/kg}$$

(b) $\frac{\dot{W}_{cycle}}{\dot{m}} = (h_2 - h_1) - (h_3 - h_4) = 24.1$ kJ/kg ← $\dot{W}_{cycle}/\dot{m}$

(c) The refrigerating capacity becomes

$\frac{\dot{Q}_{in}}{\dot{m}} = h_b - h_4 = 35.76$ kJ/kg ← $\dot{Q}_{in}/\dot{m}$

(d) $\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_{cycle}/\dot{m}} = 1.484$ ← β

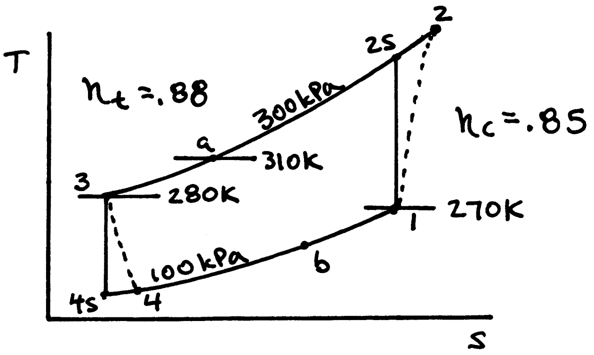
Discussion: Comparing these results with those of Problem 10.42, we see that introducing a regenerator has reduced T_4 significantly. However, comparing the other quantities indicates that the net work has increased, the capacity has decreased, and the coefficient of performance has gone down.

PROBLEM 10.50

KNOWN: Reconsider the cycle of Problem 10.49 and include that the compressor and turbine have efficiencies of 85 and 88%, respectively.

FIND: Answer the same questions as in Problem 10.49.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 10.5. Also, there is no heat transfer from the heat exchanger to its surroundings.

ANALYSIS: First, fix each of the principal states (Table A-22). From the solution to Problem 10.49

$h_1 = 270.11$ kJ/kg $\qquad h_3 = 280.13$ kJ/kg

$h_{2s} = 370.10$ kJ/kg $\qquad h_{4s} = 204.24$ kJ/kg

$h_a = 310.24$ kJ/kg $\qquad h_b = 240$ kJ/kg

State 2 Using the compressor efficiency,

$$\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s}-h_1}{\eta_c}\right) = 387.7 \text{ kJ/kg}$$

State 4 Using the turbine efficiency

$$\eta_t = \frac{h_3-h_4}{h_3-h_{4s}} \Rightarrow h_4 = h_3 - \eta_t(h_3-h_{4s}) = 213.35 \text{ kJ/kg}$$

$\therefore T_4 = 213.4$ K ◄———— T_{min} (a)

(b) $\frac{\dot{W}_{cycle}}{\dot{m}} = (h_2-h_1)-(h_3-h_4) = 50.81$ kJ/kg ◄———— $\dot{W}_{cycle}/\dot{m}$

(c) $\frac{\dot{Q}_{in}}{\dot{m}} = h_b - h_4 = 26.65$ kJ/kg ◄———— $\dot{Q}_{in}/\dot{m}$

(d) $\beta = \frac{26.65}{50.81} = 0.524$ ◄———— β

Discussion: Comparing these results with those of Problem 10.49, we see the penalties associated with irreversibilities in the compressor and turbine: increased T_{min}, increased net work, decreased capacity, and decreased coefficient of performance.

PROBLEM 10.51

The data for the required plots are obtained using IT, as follows. (See Problem 10.49 for sample calculations.)

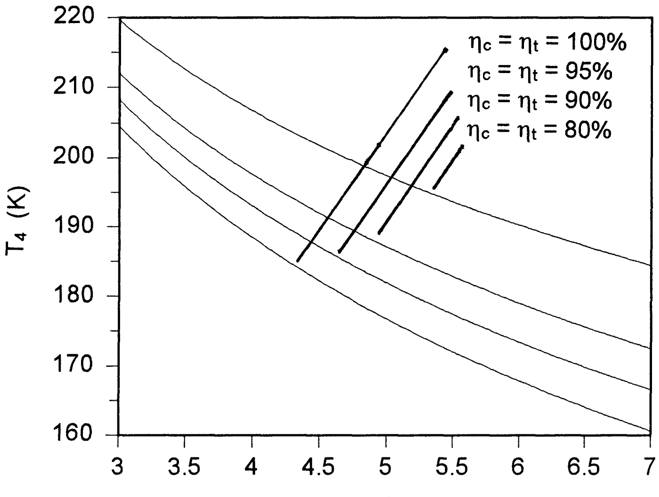
IT Code

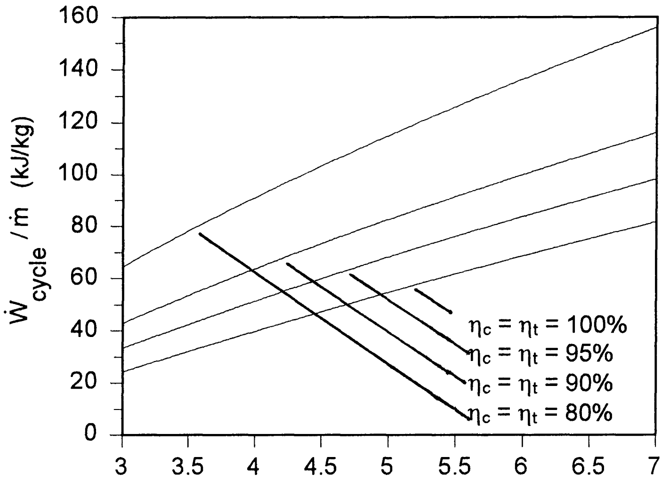
```
p1 = 100  // kPa
T1 = 270  // K
Ta = 310  // K
rp = 3
rp = p2 / p1
p3 = p2
T3 = 280  // K
p4 = p1
eta_c = .8
eta_t = eta_c
mdot = 1  // Assume a unit mass flow rate.

h1 = h_T("Air", T1)
s1 = s_TP("Air", T1, p1)
s2s = s_hP("Air", h2s, p2)
s2s = s1
h2 = h1 + (h2s - h1) / eta_c
ha = h_T("Air", Ta)
h3 = h_T("Air", T3)
s3 = s_TP("Air", T3, p3)
s4s = s_hP("Air", h4s, p4)
s4s = s3
h4 = h3 - (h3 - h4s) * eta_t
T4 = T_h("Air", h4)
0 = (hb - h1) + (ha - h3)

Wdotcycle / mdot = (h2 - h1) - (h3 - h4)
Qdotin / mdot = hb - h4
beta = Qdotin / Wdotcycle
```

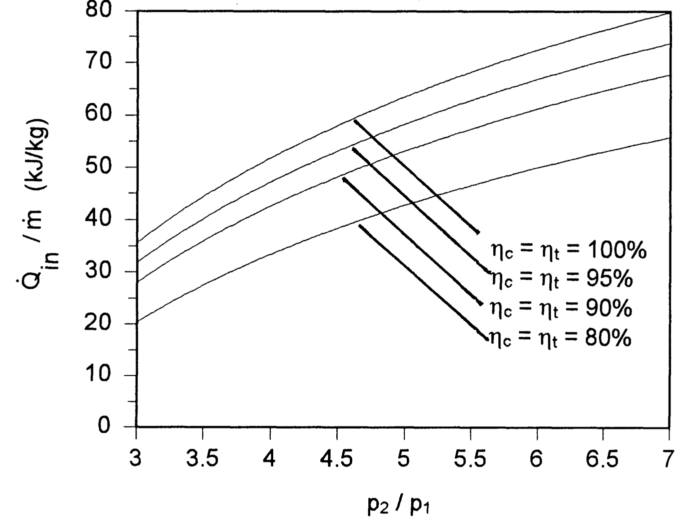
IT Results for $p_2/p_1 = 3$ and $\eta_c = \eta_t = 100\%$

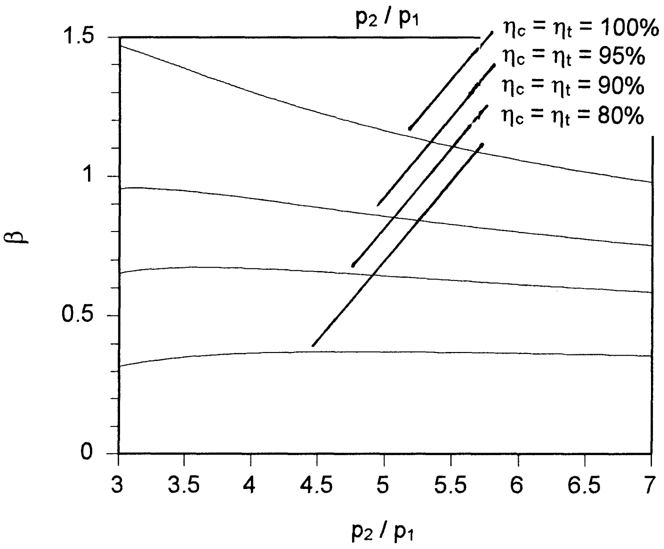
$h_1 = 269.9$ kJ/kg
$h_2 = 369.9$ kJ/kg
$h_{2s} = 369.9$ kJ/kg
$h_3 = 280$ kJ/kg
$h_4 = 204.2$ kJ/kg
$h_{4s} = 204.2$ kJ/kg
$h_a = 310.1$ kJ/kg
$h_b = 239.8$ kJ/kg
$T_4 = 204.5$ K
$\dot{W}_{cycle}/\dot{m} = 24.25$ kJ/kg
$\dot{Q}_{in}/\dot{m} = 35.6$ kJ/kg
$\beta = 1.468$

PLOTS:







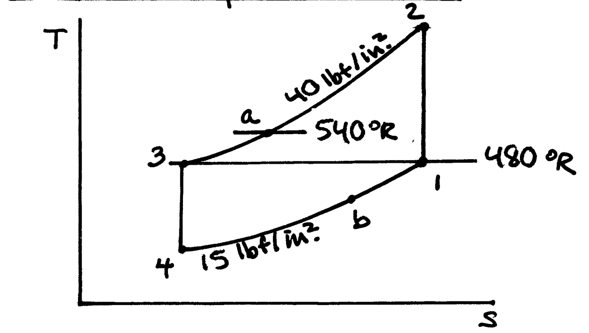


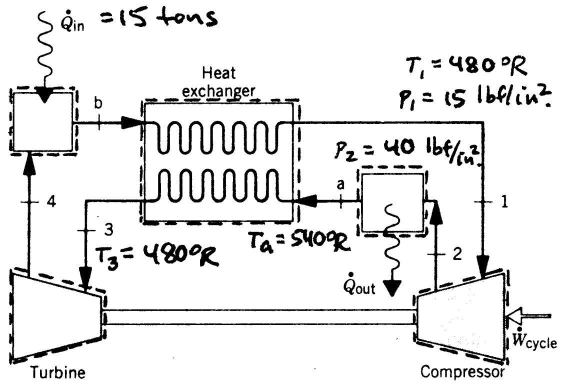
PROBLEM 10.52*

KNOWN: Air is the working fluid in a Brayton refrigeration cycle with a regenerative heat exchanger. Data are known at various locations and the refrigeration capacity is given.

FIND: Determine (a) the volumetric flow rate at the turbine inlet, and (b) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: See Example 10.4. Also, there is no heat transfer from the heat exchanger to its surroundings.

ANALYSIS: First, fix each of the principal states (Table A-22E).

State 1 $T_1 = 480\ °R \Rightarrow h_1 = 114.69$ Btu/lb, $p_{r1} = 0.9182$

State 2 $p_{r2} = p_{r1}(p_2/p_1) = 2.449 \Rightarrow h_2 = 151.90$ Btu/lb

State a $T_a = 540\ °R \Rightarrow h_a = 129.06$ Btu/lb

State 3 $T_3 = 480\ °R \Rightarrow h_3 = 114.69$ Btu/lb, $p_{r3} = 0.9182$

State 4 $p_{r4} = p_{r3}(p_4/p_3) = 0.3443 \Rightarrow h_4 = 86.52$ Btu/lb

State b For the heat exchanger,

$$0 = (h_b - h_1) + (h_a - h_3) \Rightarrow h_b = 100.32 \text{ Btu/lb}$$

(a) With the given refrigeration capacity, the air mass flow rate is

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_b - h_4)} = \frac{(15 \text{ tons})}{(100.32 - 86.52)\text{Btu/lb}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 217.4 \frac{\text{lb}}{\text{min}}$$

and the volumetric flow rate at the compressor inlet is

$$(AV)_1 = \frac{\dot{m} R T_1}{p_1} = \frac{(217.4)\frac{\text{lb}}{\text{min}}\left(\frac{1545}{28.97}\right)\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}(480)°\text{R}}{(20)\ \text{lbf/in}^2} \left| \frac{1\ \text{ft}^2}{144\ \text{in}^2} \right| = 1932 \frac{\text{ft}^3}{\text{min}} \longleftarrow (AV)_1$$

(b) The coefficient of performance is

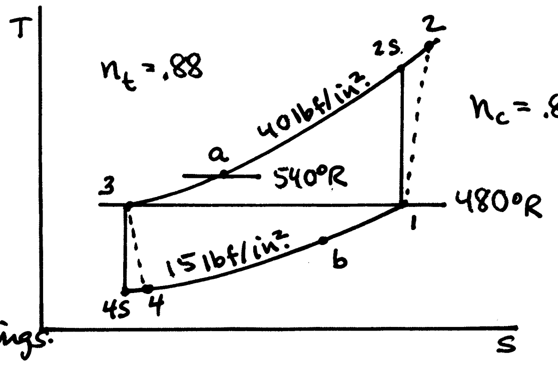
$$\beta = \frac{(h_b - h_4)}{(h_2 - h_1) - (h_3 - h_4)} = \frac{13.80}{37.21 - 28.17} = 1.527 \longleftarrow \beta$$

PROBLEM 10.53*

KNOWN: Reconsider the cycle of Problem 10.52, but include compressor and turbine efficiencies of 88 % each in the analysis.

FIND: Answer the same questions as in Problem 10.52

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: See Example 10.5. Also, there is no heat transfer between the heat exchanger and its surroundings.

ANALYSIS: First, fix each of the principal states (Table A-22E). From the solution to Problem 10.52

$h_1 = 114.69$ Btu/lb $\qquad h_3 = 114.69$ Btu/lb

$h_{2s} = 151.90$ Btu/lb $\qquad h_{4s} = 86.52$ Btu/lb

$h_a = 129.06$ Btu/lb $\qquad h_b = 100.32$ Btu/lb

State 2 Using the compressor efficiency

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \Rightarrow h_2 = h_1 + \left(\frac{h_{2s} - h_1}{\eta_c}\right) = 156.97 \text{ Btu/lb}$$

State 4 Using the turbine efficiency

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 89.90 \text{ Btu/lb}$$

(a) With the given refrigeration capacity, the air mass flow rate is

$$\dot{m} = \frac{\dot{Q}_{in}}{(h_b - h_4)} = \frac{(15 \text{ tons})}{(100.32 - 89.90) \text{Btu/lb}} \left| \frac{200 \text{ Btu/min}}{1 \text{ ton}} \right| = 287.9 \frac{\text{lb}}{\text{min}}$$

and the volumetric flow rate at the compressor inlet is

$$(AV)_1 = \frac{\dot{m} RT_1}{P_1} = \frac{(287.9)\frac{\text{lb}}{\text{min}} \left(\frac{1545}{28.97}\right)\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}} (480)^\circ\text{R}}{(20) \text{ lbf/in}^2} \left| \frac{1 \text{ ft}^2}{144 \text{ in}^2} \right| = 2559 \frac{\text{ft}^3}{\text{min}} \longleftarrow (AV)_1$$

(b)

$$\beta = \frac{(h_b - h_4)}{(h_2 - h_1) - (h_3 - h_4)} = \frac{10.42}{(42.28 - 24.79)} = 0.596 \longleftarrow \beta$$

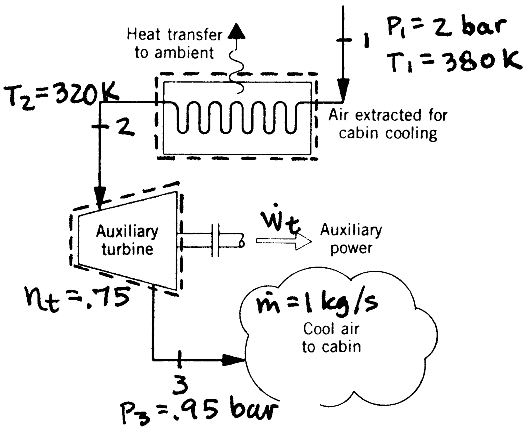
Note, the flow rate is higher and the coefficient of performance is lower than in Problem 10.52 due to irreversibilities in the compressor and turbine.

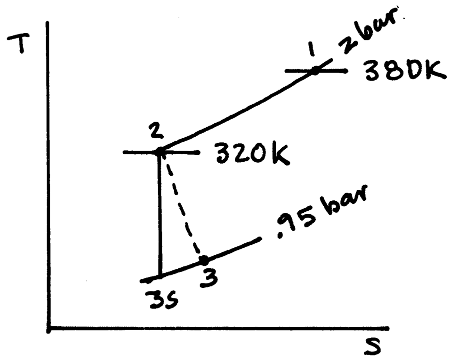
PROBLEM 10.54

KNOWN: Air is extracted from a main jet engine for cabin cooling. The air passes through a heat exchanger and turbine before being discharged into the cabin.

FIND: Determine (a) the power developed by the turbine and (b) the heat transfer rate from the air passing through the heat exchanger.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is at steady state. (2) The air experiences no pressure drop in passing through the heat exchanger. (3) Kinetic and potential energy effects can be neglected. (4) The turbine operates adiabatically. (5) The air is modeled as an ideal gas.

ANALYSIS: (a) From Table A-22; $h_2 = 320.29$ kJ/kg and $p_{r2} = 1.7375$. For an isentropic expansion, $p_{r3s} = p_{r2}(P_3/P_2) = 0.82531$. Thus, $h_{3s} = 258.67$ kJ/kg. Using the turbine efficiency

$$\dot{W}_t = \dot{m}\,\eta_t\,(h_2 - h_{3s})$$

$$= \left(1\ \frac{\text{kg}}{\text{s}}\right)(.75)(320.29 - 258.67)\ \frac{\text{kJ}}{\text{kg}}\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 46.2\ \text{kW} \longleftarrow \dot{W}_t$$

(b) From Table A-22; $h_1 = 380.77$ kJ/kg. Thus

$$0 = \dot{Q}_{cv} + \dot{m}(h_1 - h_2)$$

or

$$\dot{Q}_{cv} = \dot{m}(h_2 - h_1) = (1)(320.29 - 380.77)(1)$$

$$= -60.48\ \text{kW}$$

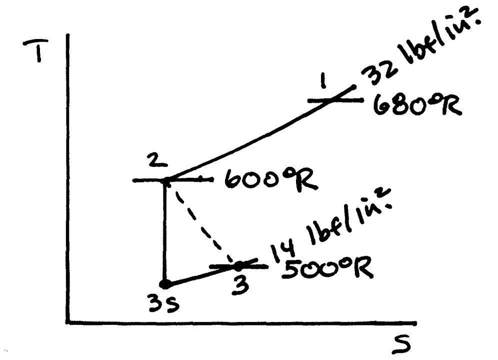
$$\dot{Q}_{out} = 60.48\ \text{kW} \longleftarrow \dot{Q}_{out}$$

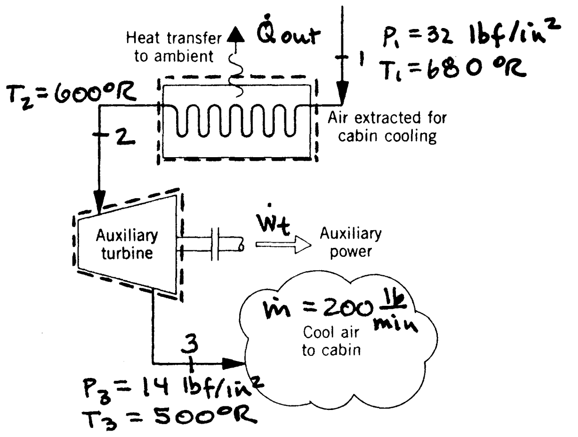
PROBLEM 10.55

KNOWN: Air is extracted from a main jet engine for cabin cooling. The air passes through a heat exchanger and turbine before being discharged into the cabin.

FIND: Determine (a) the power developed by the turbine, (b) the isentropic turbine efficiency, and (c) the heat transfer rate from the air passing through the heat exchanger.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is at steady state. (2) The air passes through the heat exchanger with no pressure drop. (3) Kinetic and potential energy effects are negligible. (4) The turbine operates adiabatically. (5) The air is modeled as an ideal gas.

ANALYSIS: (a) From Table A-22E, $h_2 = 143.47$ Btu/lb and $h_3 = 119.48$ Btu/lb. Thus, the power developed by the turbine is

$$\dot{W}_t = \dot{m}(h_2 - h_3) = \left(200 \frac{lb}{min}\right)\left|\frac{60\ min}{1\ h}\right|(143.47 - 119.48)\frac{Btu}{lb}\left|\frac{1\ hp}{2545\ Btu/h}\right|$$

$$= 113.1\ hp \longleftarrow \dot{W}_t$$

(b) For an isentropic expansion, $p_{r2} = 2.005$ and $p_{r3s} = p_{r2}(P_3/P_2) = 0.87719$. Thus, $h_{3s} = 113.14$ Btu/lb, and

$$\eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} = 0.791\ (79.1\ \%) \longleftarrow \eta_t$$

(c) From Table A-22E, $h_1 = 162.73$ Btu/lb. Thus

$$\dot{Q}_{out} = \dot{m}(h_1 - h_2)$$

$$= \left(200 \frac{lb}{min}\right)(162.73 - 143.47)\frac{Btu}{lb}$$

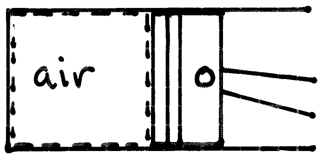
$$= 3852\ Btu/min \longleftarrow \dot{Q}_{out}$$

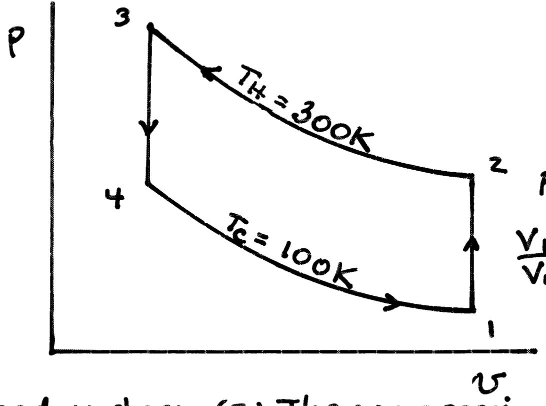
PROBLEM 10.56

KNOWN: Air undergoes a Stirling refrigeration cycle with a known compression ratio. Other data are also given for the cycle.

FIND: Determine (a) the heat transfer during the isothermal expansion, per unit mass of air, (b) the net work for the cycle, per unit mass of air, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





$P_2 = 100$ kPa

$\frac{V_1}{V_4} = \frac{V_2}{V_3} = 6$

ASSUMPTIONS: (1) The air is a closed system. (2) The compression and expansion are isothermal. (3) All processes are internally reversible. (4) An ideal regenerator is available to store the energy rejected during process 3-4 to be used as the energy input during process 1-2. (5) The air behaves as an ideal gas. (6) Kinetic and potential energy effects are negligible.

ANALYSIS: (a) The energy balance for process 4-1 reduces to

$$\frac{Q_{41}}{m} = \cancelto{0}{(u_1 - u_4)} + \frac{W_{41}}{m}$$

The work is calculated using Eq. 3.58

$$\frac{W_{41}}{m} = \int_4^1 p\,dv = RT_c \ln\frac{V_1}{V_4}$$
$$= \left(\frac{8.314}{28.97} \cdot \frac{kJ}{kg \cdot K}\right)(100K) \ln(6) = 51.42 \text{ kJ/kg}$$

Thus $\frac{Q_{41}}{m} = \frac{W_{41}}{m} = 51.42$ kJ/kg ◄ Q_{41}/m

(b) The work for process 2-3 is

$$\frac{W_{23}}{m} = \int_2^3 p\,dv = RT_H \ln\frac{V_3}{V_2} = \left(\frac{8.314}{28.97}\right)(300)\ln\left(\frac{1}{6}\right) = -154.26 \frac{kJ}{kg}$$

Thus $\frac{W_{cycle}}{m} = \left|\frac{W_{23}}{m}\right| - \frac{W_{41}}{m} = 154.26 - 51.42 = 102.84$ kJ/kg ◄ W_{cycle}/m

(c) The coefficient of performance is

① $$\beta = \frac{Q_{41}/m}{W_{cycle}/m} = \frac{51.42}{102.84} = 0.5$$ ◄ β

---

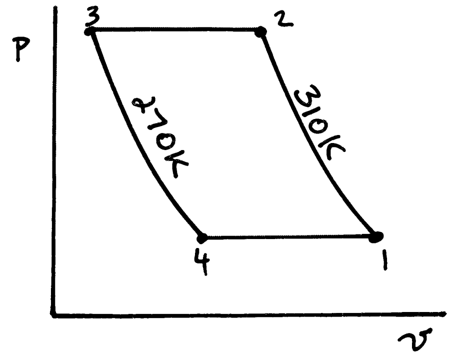
1. Alternatively, using Eq. 5.9, $\beta = \beta_{max} = \frac{T_c}{T_H - T_c} = 0.5$.

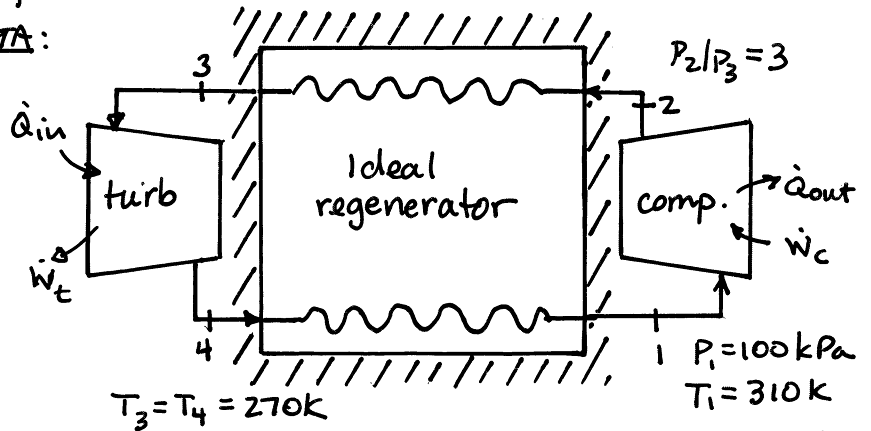
PROBLEM 10.57

KNOWN: Air is the working fluid in an Ericsson refrigeration cycle. Data are known at various locations.

FIND: Determine (a) the heat transfer for the isothermal expansion, (b) the net work per unit mass of air flow, and (c) the coefficient of performance.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) All processes are internally reversible. (3) The compression and expansion processes are isothermal. (4) Kinetic and potential energy effects are negligible. (5) The air behaves as an ideal gas.

ANALYSIS: (a) Determine $\dot{Q}_{in}/\dot{m}$ from an energy balance on the turbine

$$0 = \dot{Q}_{in} - \dot{W}_t + \dot{m}\,\overset{0}{\cancel{(h_3 - h_4)}}$$

or

$$\frac{\dot{Q}_{in}}{\dot{m}} = \frac{\dot{W}_t}{\dot{m}}$$

The turbine work is evaluated using Eq. 6.67

$$\frac{\dot{W}_t}{\dot{m}} = -\int_3^4 v\,dp = -RT_3 \ln P_4/P_3 = RT_3 \ln \frac{P_2}{P_1}$$

$$= \left(\frac{8.314}{28.97}\,\frac{kJ}{kg\cdot K}\right)(270\,K)\ln(3) = 85.13\,kJ/kg$$

$$\frac{\dot{Q}_{in}}{\dot{m}} = 85.13\,kJ \quad \xleftarrow{\hspace{6em}} \quad \dot{Q}_{in}/\dot{m}$$

(b) The compressor work is evaluated similarly

$$\frac{\dot{W}_c}{\dot{m}} = RT_1 \ln \frac{P_2}{P_1} = \left(\frac{8.314}{28.97}\right)(310)\ln(3) = 97.69\,kJ/kg$$

$$\frac{\dot{W}_{cycle}}{\dot{m}} = \frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_t}{\dot{m}} = 12.56\,kJ/kg \quad \dot{W}_{cycle}/\dot{m}$$

(c) The coefficient of performance is

$$\beta = \frac{\dot{Q}_{in}/\dot{m}}{\dot{W}_{cycle}/\dot{m}} = 6.78$$

CHAPTER ELEVEN

THERMODYNAMIC RELATIONS

PROBLEM 11.1

KNOWN: 100 lb of CO_2 is at 212°F in a 19.3-ft^3 cylinder.

FIND: Determine the pressure using (a) the van der Waals equation, (b) the compressibility chart, (c) the ideal gas model.

ANALYSIS: Using the given data $v = 0.193\ ft^3/lb$.

(a) The van der Waals equation is given by Eq. 11.2

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2}$$

From Table A-24E

$$a = 926\ atm\left(\frac{ft^3}{lbmol}\right)^2, \quad b = 0.686\ \frac{ft^3}{lbmol}$$

$$p = \frac{\left(1545\ \frac{ft\cdot lbf}{lbmol\cdot °R}\right)(672°R)\left|\frac{ft^2}{144 in^2}\right|\left|\frac{atm}{14.696\ lbf/in^2}\right|}{\left(0.193\ \frac{ft^3}{lb}\right)\left(44.01\ \frac{lb}{lbmol}\right) - 0.686\ \frac{ft^3}{lbmol}} - \frac{926\ atm\ (ft^3/lbmol)^2}{\left[(0.193)(44.01)\ \frac{ft^3}{lbmol}\right]^2} = 50\ atm \longleftarrow \underline{vdW}$$

(b) Compressibility Chart

From Table A-1E, $T_c = 548\ °R$, $P_c = 72.9\ atm$. Thus, $T_R = 672/548 = 1.226$ and

$$v_R' = \frac{\bar{v}P_c}{\bar{R}T_c} = \frac{\left[(0.193)(44.01)\ ft^3/lbmol\right]\left[72.9\ atm\right]\left|14.696\ \frac{lbf/in^2}{atm}\right|\left|\frac{144 in^2}{ft^2}\right|}{\left(1545\ \frac{ft\ lbf}{lbmol\cdot °R}\right)(548°R)} = 1.548$$

Then Fig. A-1 gives $p_R \approx 0.685 \Rightarrow p = 49.94\ atm.$ $\longleftarrow$ z chart

(c) With the ideal gas equation of state

$$p = \frac{\bar{R}T}{\bar{v}} = \frac{\left(1545/(144)(14.696)\right)672}{(0.193)(44.01)} = 57.76\ atm \longleftarrow \text{ideal gas}$$

Discussion: Methods (a), (b) suggest that the pressure level would be safe, but at the high end of the allowed range. The ideal gas model suggests that the pressure would not be satisfactory.

PROBLEM 11.2

① KNOWN: Ten lb of C_3H_8 has a volume of 2 ft^3 and a pressure of 600 lbf/in.^{2}

FIND: Determine the temperature using (a) the Van der Waals equation, (b) the compressibility chart, (c) the ideal gas equation of state, and (d) the propane tables.

ANALYSIS: $p = \left(600 \frac{\text{lbf}}{\text{in}^2}\right)\left|\frac{1 \text{ atm}}{14.696 \text{ lbf/in.}^2}\right| = 40.83 \text{ atm}$, $\bar{v} = \left(\frac{2 \text{ ft}^3}{10 \text{ lb}}\right)\left|\frac{44.09 \text{ lb}}{1 \text{ lb·mol}}\right| = 8.818 \frac{\text{ft}^3}{\text{lb·mol}}$

(a) With a and b from Table A-24E, the Vander Waals Equation gives

$$T = \frac{(p + a/\bar{v}^2)(\bar{v} - b)}{\bar{R}}$$

$$= \frac{\left[40.83 \text{ atm} + \frac{2369 (\text{ft}^3/\text{lbmol})^2}{(8.818 \text{ ft}^3/\text{lbmol})^2}\right](8.818 - 1.444)\frac{\text{ft}^3}{\text{lbmol}}}{\left(1543 \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}\right)}\left|\frac{14.696 \text{ lbf/in}^2}{1 \text{ atm}}\right|\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right|$$

$$= 720.1 \text{ °R} \longleftarrow \text{(a)}$$

(b) From Table A-1E; $T_c = 66$ °R, $p_c = 42.1$ atm. Thus $P_R = 40.83/42.1 = 0.97$, and

$$v_R' = \frac{\bar{v} p_c}{\bar{R} T_c} = \frac{(8.818 \text{ ft}^3/\text{lbmol})(42.1 \text{ atm})}{\left(1545 \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot{}^\circ\text{R}}\right)(666 \text{ °R})}\left|\frac{14.696 \text{ lbf/in.}^2}{1 \text{ atm}}\right|\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right| = 0.764$$

Then, Fig A-1 gives $T_R \approx 1.08 \Rightarrow T = 719.3$ °R $\longleftarrow$ (b)

(c) The ideal gas equation gives

$$T = \frac{p\bar{v}}{\bar{R}} = \frac{(600 \text{ lbf/in.}^2)(8.818 \text{ ft}^3/\text{lbmol})}{\left(1545 \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol °R}}\right)}\left|\frac{144 \text{ in.}^2}{1 \text{ ft}^2}\right| = 493 \text{ °R} \longleftarrow \text{(c)}$$

(d) $p = 600$ lbf/in.^{2}, $v = 0.2$ ft^3/lb. Interpolating in Table A-18E, we get

$$T = 261.5 \text{ °F} = 721.5 \text{ °R} \longleftarrow \text{(d)}$$

Discussion. The answers for parts (a), (b) and (d) agree well. The ideal gas value deviates significantly from the others.

PROBLEM 11.3

KNOWN: 1000 kg of water vapor fills a 23.3-m^3 tank at 360°F.

FIND: Estimate the pressure using (a) the ideal gas equation, (b) the van der Waals equation, (c) the Redlich-Kwong equation, (d) the compressibility chart, (e) the steam tables.

ANALYSIS:

(a) ideal gas equation.

$$p = \frac{RT}{v} = \frac{(8314/18.02)\left(\frac{N\cdot m}{kg\cdot K}\right)(633\ K)}{(23.31/10^3)\ (m^3/kg)}\left|\frac{1\ bar}{10^5 N/m^2}\right| = 125.29\ bar \longleftarrow (a)$$

(25% high)

(b) van der Waals equation. With a and b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} = \left[\frac{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(633\ K)}{(0.42-0.0305)\ m^3/kmol}\right]\left|\frac{1\ bar}{10^5 N/m^2}\right| - \frac{5.531\ bar\,(m^3/kmol)^2}{(0.42\ m^3/kmol)^2} = 103.76\ bar \longleftarrow (b)$$

(~4% high)

where $\bar{v} = vM = \left(\frac{23.31}{10^3}\right)\left(\frac{m^3}{kg}\right)\left(18.02\frac{kg}{kmol}\right) = 0.42\frac{m^3}{kmol}$.

(c) Redlich-Kwong equation. with a and b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)T^{1/2}} = \frac{(8314)(633)}{(0.42-0.02111)}\left|\frac{1}{10^5}\right| - \frac{142.59}{(0.42)(0.42+0.02111)(633)^{1/2}} = 101.34\ bar \longleftarrow (c)$$

(~ 1% high)

(d) Compressibility chart. From Table A-1, $T_c = 647.3\ K$, $p_c = 220.9$ bar. Thus, $T_R = 633/647.3 = 0.978$. And

$$v_R' = \frac{\bar{v}\,p_c}{\bar{R}T_c} = \frac{(0.42\ m^3/kmol)(220.9\times10^5\ N/m^2)}{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(647.3\ K)} = 1.724$$

Then, Fig A-1 gives $Z \approx 0.81$, so

$$p = Z\frac{RT}{v} = 0.81(125.29) = 101.48\ bar \longleftarrow (d)$$

(~ 1.5% high)

(e) Steam tables. Table A-4 gives 100 bar.

Discussion: All but the ideal gas model suggest that the pressure would be in the safe range. Using the steam table value, 100 bar, as the standard, the various methods depart by the following percentages:

ideal gas:	25% high
van der Waals:	4% high
Redlich-Kwong:	10% high
Compressibility chart:	1.5% high

PROBLEM 11.4

KNOWN: Water vapor is at $T = 500°C$, $\rho = 24\ kg/m^3$.

FIND: Determine the pressure using (a) the steam tables, (b) the compressibility chart, (c) the Redlich–Kwong equation, (d) the van der Waals equation, (e) the ideal gas model.

ANALYSIS: (a) At 500°C and $v = 1/\rho = 0.0417\ m^3/kg$, Steam Table data gives $p = 80.1$ bar. ← (a)

(b) Compressibility chart. $T_R = \frac{773}{647} = 1.19$, $\bar{v}_R' = \frac{\bar{v}\,p_c}{\bar{R}T_c} = \frac{[(0.0417)(18.02)\frac{m^3}{kmol}][220.9 \times 10^5 \frac{N}{m^2}]}{(8314 \frac{N\cdot m}{kmol\cdot K})(647K)}$

or $\bar{v}_R' = 3.08$. Then, Figure A-1 gives $Z \sim 0.93$. So

$$p = \frac{ZRT}{v} = \frac{0.93\left[\frac{8314}{18.02}\frac{N\cdot m}{kg\cdot K}\right][773K]}{(0.0417 \frac{m^3}{kg})\left|10^5 N/m^2/bar\right|} = 79.5\ \text{bar}$$

(1% low) ← (b)

(c) the Redlich Kwong Equation. With data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)\sqrt{T}} = \frac{(8314 \frac{N\cdot m}{kmol\cdot K})(773K)\left|1bar/10^5 N/m^2\right|}{[\underbrace{(0.0417)(18.02)}_{0.7514\ m^3/kmol} - 0.02111]\frac{m^3}{kmol}} - \frac{142.59\ bar\left(\frac{m^3}{kmol}\right)^2(K)^{1/2}}{(0.7514\frac{m^3}{kmol})(0.77251\frac{m^3}{kmol})(773K)^{1/2}}$$

$$= 88.0 - 8.84 = 79.2\ bar$$

(~1% low) ← (c)

(d) the van der Waals Equation. With the data from Table **A-24**

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} = \frac{(8314 \frac{N\cdot m}{kmol\cdot K})(773K)}{(0.7514\frac{m^3}{kmol} - 0.0305\frac{m^3}{kmol})}\left|\frac{1bar}{10^5 N/m^2}\right| - \frac{5.531\ bar\ (m^3/kmol)^2}{(0.7514\frac{m^3}{kmol})^2}$$

$$= 89.15 - 9.8 = 79.35\ bar$$

(~1% low) ← (d)

(e) ideal gas model

$$p = \frac{RT}{v} = \frac{(8314/18.02\ \frac{N\cdot m}{kg\cdot K})(773K)}{(0.0417\ m^3/kg)}\left|\frac{1bar}{10^5 N/m^2}\right| = 85.53\ bar$$

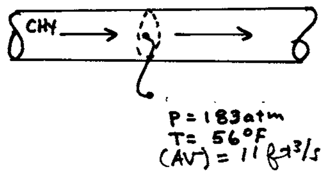
(~7% high) ← (e)

PROBLEM 11.5

KNOWN: CH_4 flows through a pipeline at a pressure of 183 atm, a temperature of 56°F, and a volumetric flow rate of 11 ft³/s.

FIND: Determine the mass flow rate using (a) the ideal gas equation, (b) the van der Waals equation, (c) the compressibility chart.

SCHEMATIC & GIVEN DATA:



ANALYSIS: The mass flow rate and volumetric flow rate are related by

$$\dot{m} = \frac{(AV)}{v} \qquad (1)$$

(a) Ideal Gas Equation:

$$v = \frac{RT}{p} = \frac{(1545/16.04)\left(\frac{ft\cdot lbf}{lb\cdot °R}\right)(516°R)}{(183\ atm)\left|14.696 \times 144 \frac{lbf/ft^2}{atm}\right|} = 0.128\ \frac{ft^3}{lb}$$

Thus, Eq. (1) gives

$$\dot{m} = \frac{11\ ft^3/s}{0.128\ ft^3/lb} = 85.9\ lb/s \longleftarrow (a)$$

(b) van der Waals Equation: With a and b from Table A-24E

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} \Rightarrow 183\ atm = \left[\frac{\left(1545 \frac{ft\ lbf}{lbmol\cdot °R}\right)(516°R)}{(\bar{v} - 0.685)\ ft^3/lbmol}\right]\left|\frac{atm}{(14.696)(144)\frac{lbf}{ft^2}}\right| - \frac{581}{\bar{v}^2}$$

or

$$183 = \frac{376.718}{\bar{v} - 0.685} - \frac{581}{\bar{v}^2}$$

Using an equation solver, $\bar{v} = 1.611$ ft³/lbmol. Thus, $v = \bar{v}/M = (1.611/16.04) =$ 0.1004 ft³/lb, and Eq. (1) gives

$$\dot{m} = \frac{11\ ft^3/s}{0.1004\ ft^3/lb} = 109.6\ lb/s \longleftarrow (b)$$

(c) Generalized Chart. From Table A-1E $T_c = 344°R$, $P_c = 45.8$ atm. Thus

$$T_R = \frac{516}{344} = 1.5,\quad P_R = 4.0$$

Then, from Fig. A-2, Z = 0.81. Accordingly

$$v = Z\frac{RT}{p} = (0.81)\underbrace{(0.128)}_{\text{part(a)}} = 0.1037\ \frac{ft^3}{lb}$$

Finally, with Eq. (1)

$$\dot{m} = \frac{11\ ft^3/s}{0.1037\ ft^3/lb} = 106.1\ lb/s \longleftarrow (c)$$

PROBLEM 11.6

KNOWN: Water vapor is at 20 MPa, 400°C.

FIND: v in m^3/kg using (a) steam tables, (b) compressibility chart, (c) Redlich-Kwong equation, (d) van der Waal equation, (e) ideal gas model.

ANALYSIS: (a) Steam Tables. Table A-4 gives, $v = 0.00994\ m^3/kg$. $\longleftarrow$ (a)

(b) Compressibility Chart. With p_c, T_c from Table A-1

$$T_R = \frac{673}{647} = 1.04,\quad P_R = \frac{20}{22.09} = 0.91.$$

Figure A-1 gives $Z \sim 0.65$. Then

$$v = \frac{ZRT}{p} = 0.65\underbrace{\left[\frac{(8314/18.02)\frac{N\cdot m}{kg\cdot K}\cdot 673K}{20\times10^6\ N/m^2}\right]}_{0.0155\ m^3/kg\ \longleftarrow\ (e)} = 0.0101\ m^3/kg \quad\longleftarrow\quad (b)$$

(c) Redlich-Kwong Equation. With a, b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)\sqrt{T}} \Rightarrow 200\,bar = \frac{(8314\frac{N\cdot m}{kmol\cdot K})(673K)}{(\bar{v}-0.02111)\frac{m^3}{kmol}}\left|\frac{1\,bar}{10^5 N/m^2}\right| - \frac{142.59\ bar}{\bar{v}(\bar{v}+0.02111)(673)^{1/2}}$$

or

$$200 = \frac{55.953}{(\bar{v}-0.02111)} - \frac{5.496}{\bar{v}(\bar{v}+0.0211)}.$$

Using an equation solver, $\bar{v} = 0.18\ m^3/kmol$.

$$\Rightarrow \bar{v} = \frac{0.18\ m^3/kmol}{18.02\ kg/kmol} = 0.00999\ m^3/kg \quad\longleftarrow\quad (c)$$

(d) van der Waals Equation. With a, b from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} \Rightarrow 200\,bar = \frac{(8314)(673)}{(\bar{v}-0.0305)}\left|\frac{1}{10^5}\right| - \frac{5.531}{\bar{v}^2}$$

or

$$200 = \frac{55.953}{\bar{v}-0.0305} - \frac{5.531}{\bar{v}^2}.$$

Using an equation solver, $\bar{v} = 0.186\ m^3/kmol$.

$$\Rightarrow \bar{v} = \frac{0.186}{18.02} = 0.01032\ \frac{m^3}{kg} \quad\longleftarrow$$

Discussion: Comparing the calculated values with the Steam Table result,

compressibility chart ~ 2% high
Redlich-Kwong equation ~ ½% high
van der Waals equation ~ 4% high
ideal gas model ~ 6% high

PROBLEM 11.7

KNOWN: A vessel whose volume is 1 m^3 contains 4 kmol of CH_4 at 100°C.

FIND: Estimate the pressure using (a) the ideal gas model, (b) the Redlich – Kwong equation, (c) the Benedict - Webb - Rubin equation.

ANALYSIS: (a) Ideal Gas Model

$$p = \frac{\bar{R}T}{\bar{v}} = \frac{\left(8314 \frac{N \cdot m}{kmol \cdot K}\right)(373K)}{(1 m^3/4 kmol)} \left| \frac{1 bar}{10^5 \frac{N}{m^2}} \right| = 124.04 \text{bar} \longleftarrow \quad (a)$$

(b) Redlich-Kwong. With data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)(T)^{1/2}} = \frac{\left(8314 \frac{N \cdot m}{kmol \cdot K}\right)(373K)}{(0.25 - 0.02965)\frac{m^3}{kmol}} \left| \frac{1 bar}{10^5 N/m^2} \right| - \frac{32.11 \text{ bar}}{0.25(0.25+0.02965)(373)^{1/2}}$$

$$= 140.74 - 23.78 = 116.96 \text{ bar} \longleftarrow \quad (b)$$

(c) Benedict-Webb-Rubin. With data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}} + \left[B\bar{R}T - A - \frac{C}{T^2}\right]\frac{1}{\bar{v}^2} + \left[\frac{b\bar{R}T - a}{\bar{v}^3}\right] + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3T^2}\left(1+\frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$p = \frac{\left(0.08314 \frac{bar \cdot m^3}{kmol \cdot K}\right)(373K)}{0.25 \; m^3/kmol} + \left[(0.04260)(0.08314)(373) - 1.8796 - \frac{2.287 \times 10^4}{(373)^2}\right]\frac{1}{(0.25)^2}$$

$$+ \left[\frac{(0.00338)(0.08314)(373) - 0.0501}{(0.25)^3}\right] + \frac{(0.0501)(1.244 \times 10^{-4})}{(0.25)^6}$$

$$+ \left[\frac{(2.579 \times 10^3)}{(0.25)^3(373)^2}\left(1 + \frac{0.0060}{(0.25)^2}\right)\exp\left(-\frac{0.0060}{(0.25)^2}\right)\right]$$

$$= 124.04 - 11.57 + 3.50 + 0.03 + 1.18 = 117.18 \text{ bar} \longleftarrow \quad (c)$$

Discussion: The Redlich-Kwong and Benedict-Webb-Rubin equations suggest that the pressure is in the safe range. Using P_c and T_c from Table A-1,

$$P_R = \frac{120 \text{ bar}}{46.4 \text{bar}} = 2.59 \;, \; T_R = \frac{373K}{191K} = 1.95$$

Figure A-2 gives $Z \approx 0.96$, and so $p = Z\bar{R}T/\bar{v} = (0.96)\overbrace{(124.04)}^{\text{part(a)}}\text{bar} =$ 119.1 bar, which is also in the safe range

PROBLEM 11.8

KNOWN: A 10-m³ tank contains CH_4 at 100 atm and −18°C.

FIND: Determine the mass contained in the tank using (a) the ideal gas model, (b) the van der Waals equation, the Benedict-Webb-Rubin equation.

ANALYSIS: (a) Ideal Gas Model

$$\bar{v} = \frac{\bar{R}T}{p} = \frac{\left(8314 \frac{N \cdot m}{kmol \cdot K}\right)(255 K)}{(100\,atm)\left|1.01325 \times 10^5\,N/m^2 / 1\,atm\right|} = 0.209 \frac{m^3}{kmol}$$

Thus

$$m = \frac{V}{v} = \frac{V}{\bar{v}/M} = \left(\frac{10\,m^3}{0.209 \frac{m^3}{kmol}}\right)\left(\frac{16.04\,kg}{kmol}\right) = 767.5\,kg \longleftarrow \text{(a)}$$

(b) van der Waals equation. With data from Table A-24

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2} \Rightarrow 1.01325 \times 10^7 \frac{N}{m^2} = \frac{\left(8314 \frac{N \cdot m}{kmol \cdot K}\right)(255 K)}{(\bar{v}-0.0428)\left(\frac{m^3}{kmol}\right)} - \frac{2.293 \times 10^5\,N/m^2}{\bar{v}^2}$$

Using an equation solver, $\bar{v} = 0.14\,m^3/kmol$.

$$\Rightarrow m = \frac{VM}{\bar{v}} = \frac{(10)(16.04)}{0.14} = 1145.7\,kg \longleftarrow \text{(b)}$$

(c) Benedict-Webb-Rubin equation. With data from Table A-24

$$101.325\,bar = \frac{(0.08314)(255)}{\bar{v}} + \left[\frac{(0.0426)(0.08314)(255) - 1.8796 - \frac{(2.287)(10^4)}{(255)^2}}{\bar{v}^2}\right] +$$

$$\frac{[(0.00338)(0.08314)(255) - 0.0501]}{\bar{v}^3} + \frac{(0.0501)(1.244 \times 10^{-4})}{\bar{v}^6} +$$

$$\left[\frac{2.579 \times 10^3}{\bar{v}^3\,(255)^2}\left(1 + \frac{0.0060}{\bar{v}^2}\right)\exp\left(-\frac{0.0060}{\bar{v}^2}\right)\right]$$

Using an equation solver, $\bar{v} = 0.149\,m^3/kmol$.

① $$\Rightarrow m = \frac{VM}{\bar{v}} = \frac{(10)(16.04)}{0.149} = 1076.5\,kg \longleftarrow \text{(c)}$$

---

1. With p_c, T_c from Table A-1

$$p_R = \frac{100\,atm}{45.8\,atm} = 2.18, \quad T_R = \frac{255\,K}{191\,K} = 1.34$$

Figure A-2 gives $Z \sim 0.72$. Thus,

$$\bar{v} = Z\frac{\bar{R}T}{p} = 0.72\underbrace{(0.209)}_{\text{part (a)}} = 0.15\,m^3/kmol$$

$$\Rightarrow m = \frac{VM}{\bar{v}} = \frac{(10)(16.04)}{0.15} = 1069.3\,kg$$

This agrees closely with the result of part (c).

PROBLEM 11.9

KNOWN: 165 kg of CH_4 is at 200 atm, 400K.

FIND: Using the Benedict-Webb-Rubin equation, determine the volume in m^3. Compare with the results obtained from the ideal gas model and the compressibility chart.

ANALYSIS: With the ideal gas equation of state

$$\bar{v} = \frac{\bar{R}T}{p} = \frac{\left(8314 \frac{N\cdot m}{kmol\cdot K}\right)(400\,K)}{(200\ atm)\left|1.01325\times10^5\ N/m^2/atm\right|} = 0.164\ \frac{m^3}{kmol}$$

Thus,

$$V = m\frac{\bar{v}}{M} = \frac{(165\,kg)(0.164\ m^3/kmol)}{(16.04\ kg/kmol)} = 1.69\ m^3$$ ← ideal gas.

With T_c, p_c from Table A-1, $T_R = 400/191 = 2.09$, $p_R = 200/45.8 = 4.37$. Then, Fig A-2 gives $Z \approx 0.98$. Thus

$$v = Z\frac{\bar{R}T}{p} = 0.98(0.164) = 0.161\ \frac{m^3}{kmol}$$

So

$$V = m\frac{\bar{v}}{M} = \frac{(165)(0.161)}{(16.04)} = 1.66\ m^3$$ ← compressibility chart

With constants from Table A-24 and $\bar{R} = 0.08314\ bar\cdot m^3/kmol\cdot K$, Eq. 11.12 gives with $p = (200)(1.01325) = 202.65$ bar

$$p = \frac{\bar{R}T}{\bar{v}} + \left[B\bar{R}T - A - \frac{C}{T^2}\right]\frac{1}{\bar{v}^2} + \frac{(b\bar{R}T - a)}{\bar{v}^3} + \frac{a\alpha}{\bar{v}^6} + \frac{c}{\bar{v}^3T^2}\left(1 + \frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$202.65 = \frac{(0.08314)(400)}{\bar{v}} + \left[(0.04260)(0.08314)(400) - 1.8796 - \frac{2.287\times10^4}{(400)^2}\right]\frac{1}{\bar{v}^2} +$$
$$\left[\frac{(0.00338)(0.08314)(400) - 0.0501}{\bar{v}^3}\right] + \frac{(0.0501)(1.244\times10^{-4})}{\bar{v}^6} +$$
$$\frac{2.579\times10^3}{\bar{v}^3(400)^2}\left(1 + \frac{0.0060}{\bar{v}^2}\right)\exp\left(-\frac{0.0060}{\bar{v}^2}\right)$$

$$202.65 = \frac{33.256}{\bar{v}} - \frac{0.6058}{\bar{v}^2} + \frac{0.0623}{\bar{v}^3} + \frac{(0.0623\times10^{-4})}{\bar{v}^6} + \frac{0.0161\left(1 + \frac{0.0060}{\bar{v}^2}\right)\exp\left(-\frac{0.0060}{\bar{v}^2}\right)}{\bar{v}^3}$$

Using an equation solver, $\bar{v} = 0.161\ m^3/kmol$, giving $V = 1.66\,m^3$. ← B-W-R equation

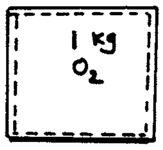
Discussion: The B-W-R equation and compressibility chart values agree. Since $Z \sim 0.98$, the ideal gas value gives a reasonable approximation as well.

PROBLEM 11.10

KNOWN: A rigid tank contains 1 kg of O_2 at $p_1 = 40$ bar, $T_1 = 180$K. The gas is cooled to a final state where $T_2 = 150$ K.

FIND: Determine the tank volume and p_2 using (a) the ideal gas equation, (b) the Redlich-Kwong equation, (c) the compressibility chart.

SCHEMATIC & GIVEN DATA:



$p_1 = 40$ bar, $T_1 = 180$K
$T_2 = 150$ K

ASSUMPTION: As shown in the accompanying figure, the O_2 is taken as the system.

ANALYSIS: (a) Ideal Gas. The ideal gas equation of state gives

$$V = \frac{m R T}{P} = \frac{(1\,kg)(8314/32)\left(\frac{N\cdot m}{kg\cdot K}\right)(180K)}{(40\times10^5\,N/m^2)} = 5.846\times10^{-3}\,m^3 \longleftarrow (a)$$

Since V is constant

$$\left.\begin{array}{l} p_1 V = m R T_1 \\ p_2 V = m R T_2 \end{array}\right\rangle \quad \frac{T_2}{T_1} = \frac{p_2}{p_1} \Rightarrow p_2 = \frac{T_2}{T_1} p_1 = \frac{150}{180}(40\,bar) = 33.33\,bar$$

(b) Redlich-Kwong. With values for a and b from Table A-24 and $\bar{R} = 0.08314 \frac{bar\cdot m^3}{kmol\cdot K}$

$$p_1 = \frac{\bar{R}\,T_1}{\bar{v}_1 - b} - \frac{a}{\bar{v}_1(\bar{v}_1+b)(T_1)^{1/2}} \Rightarrow 40\,bar = \frac{\left(0.08314\,\frac{bar\cdot m^3}{kmol\cdot K}\right)(180K)}{(\bar{v}_1 - 0.02197)\,m^3/kmol} - \frac{17.22}{\bar{v}_1(\bar{v}_1+0.02197)(180)^{1/2}}$$

Simplifying

$$40 = \frac{14.9652}{\bar{v}_1 - 0.02197} - \frac{1.2835}{\bar{v}_1(\bar{v}_1+0.02197)} \qquad (1)$$

Since Eq(1) is implicit in $\bar{v}_1$, an iterative solution is required if the result is pursued using hand calcutations. Alternatively, an equation-solver such as IT can be used. The result is $\bar{v}_1 = 0.3015\,m^3/kmol$. Thus

$$V = m v_1 = \frac{m\bar{v}_1}{M} = \frac{(1)(0.3015)}{32} = 9.422\times10^{-3}\,m^3 \longleftarrow (b)$$

To find p_2 write, using $\bar{v}_2 = \bar{v}_1$

$$p_2 = \frac{\bar{R}T_2}{\bar{v}_2 - b} - \frac{a}{\bar{v}_2(\bar{v}_2+b)(T_2)^{1/2}} \Rightarrow p_2 = \frac{(0.08314)(150)}{(0.3015-0.02197)} - \frac{17.22}{(0.3015)(.3015+.02197)(150)^{1/2}}$$

$$= 30.2\,bar$$

(c) Compressibility Chart. With T_c and p_c from Table A-1

$$(T_R)_1 = 180/154 = 1.169, \quad (P_R)_1 = 40/50.5 = 0.792$$

Then Fig. A-1 gives $(v_R')_1 \approx 1.22$. Thus

$$v_1 = (v_R')_1\left(\frac{RT_c}{p_c}\right) = 1.22\left[\frac{\left(\frac{8314}{32}\,\frac{N\cdot m}{kg\cdot K}\right)(154K)}{50.5\times10^5\,N/m^2}\right] = 9.666\times10^{-3}\,\frac{m^3}{kg}$$

Accordingly, $V = m v_1 = 9.666\times10^{-3}\,m^3 \longleftarrow (c)$

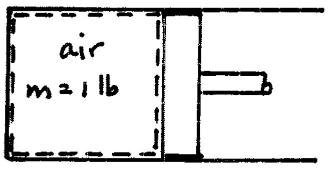
To find p_2, use $v_{R2}' = v_{R1}'$ and $T_{R2} = 150/154 = 0.97$ to locate state 2 on Fig. A-1, giving $P_{R2} \approx 0.6$. Accordingly $p_2 = p_c P_{R2} = (50.5)(0.6) = 30.3$ bar.

PROBLEM 11.11

KNOWN: One lb of air initially at $V_1 = 0.4\ ft^3$ and $1000\ \frac{lbf}{in^2}$ expands isothermally until $V_2 = 2\ ft^3$.

FIND: Using the Van der Waals equation of state, determine (a) T, (b) p_2, and (c) the work.

SCHEMATIC & GIVEN DATA:



$V_1 = 0.4\ ft^3$
$P_1 = 1000\ lbf/in^2$
$V_2 = 2\ ft^3$

ASSUMPTIONS: (1) The closed system consists of 1 lb of air. (2) The expansion takes place without irreversibilities.
(3) The air is modeled by the Van der Waals equation of state.

ANALYSIS: $\bar{v}_1 = \frac{(0.4\ ft^3)}{(1\ lb)}\left(\frac{28.97\ ft\cdot lbf}{lb\cdot mol\cdot °R}\right) = 11.588\ \frac{ft^3}{lbmol}$, $\bar{v}_2 = \frac{(2)(28.97)}{(1)} = 57.94\ \frac{ft^3}{lbmol}$

$$P_1 = (1000\ lbf/in^2)\left|\frac{1\ atm}{14.696\ lbf/in^2}\right| = 68.05\ atm$$

(a) With constants from Table A-24E and $\bar{R} = 0.73\ \frac{atm\cdot ft^3}{lbmol\cdot °R}$

$$T_1 = \left(\frac{\bar{v}_1 - b}{\bar{R}}\right)\left(P_1 + \frac{a}{\bar{v}_1^2}\right) = \frac{(11.588 - 0.586)\ ft^3/lbmol}{\left(0.73\ \frac{atm\cdot ft^3}{lbmol\cdot °R}\right)}\left[68.05\ atm + \frac{345\ \frac{atm\cdot lbmol^2}{ft^6}}{(11.588\ ft^3/lbmol)^2}\right]$$

$$= 1064.3\ °R \longleftarrow T_1$$

(b) Since $T_2 = T_1$

$$P_2 = \frac{\bar{R}T_2}{\bar{v}_2 - b} - \frac{a}{\bar{v}_2^2} = \frac{(0.73)(1064.3)}{(11.588 - 0.586)} - \frac{345}{11.588^2} = 68.05\ atm$$

$$P_2 = (68.05\ atm)\left|\frac{14.696\ lbf/in^2}{1\ atm}\right| = 1000\ lbf/in^2 \longleftarrow P_2$$

(c) The work can be evaluated from

$$W = \int_1^2 p\,dV = n\int_1^2\left[\frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}^2}\right]d\bar{v} = n\left[\bar{R}T\ln\left(\frac{\bar{v}_2 - b}{\bar{v}_1 - b}\right) + a\left(\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1}\right)\right]$$

Thus

$$\frac{W}{n} = \left(1.986\ \frac{Btu}{lbmol\cdot °R}\right)(1064.3\ °R)\left[\ln\left(\frac{57.94 - 0.586}{11.588 - 0.586}\right)\right]$$
$$+ \left(345\ \frac{atm\cdot ft^3}{lbmol}\right)\left[\frac{1}{57.94} - \frac{1}{11.588}\right]\left|\frac{14.696\ lbf/in^2}{1\ atm}\right|\left|\frac{1\ Btu}{778\ ft\cdot lbf}\right|\left|\frac{144\ in^2}{1\ ft^2}\right|$$

$= 3425.2\ Btu/lbmol$

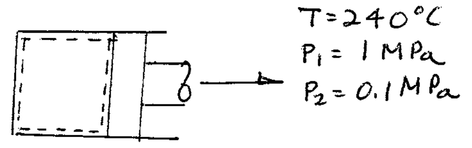
or $W = \frac{(1\ lb)}{\left(28.97\ \frac{lb}{lbmol}\right)}\left(3425.2\ \frac{Btu}{lbmol}\right) = 118.2\ Btu \longleftarrow W$

PROBLEM 11.12

KNOWN: Water vapor expands isothermally and without irreversibilities from 240°C, 1 MPa to 0.1 MPa. A truncated virial equation of state is specified.

FIND: Determine the work.

SCHEMATIC & GIVEN DATA:



$$Z = 1 + \frac{B}{v} + \frac{C}{v^2}$$

ASSUMPTIONS: (1) The water vapor is the closed system. (2) The process is isothermal with no irreversibilities. (3) Volume change is the only work mode. (4) The p-v-T relation is given by the truncated virial series above.

ANALYSIS: The work is given by

$$\frac{W}{m} = \int_{v_1}^{v_2} p\,dv$$

Then, with

$$\frac{pv}{RT} = 1 + \frac{B}{v} + \frac{C}{v^2} \Rightarrow p = RT\left(\frac{1}{v} + \frac{B}{v^2} + \frac{C}{v^3}\right) \qquad (1)$$

$$\frac{W}{m} = RT\left[\ln\frac{v_2}{v_1} - B\left(\frac{1}{v_2} - \frac{1}{v_1}\right) - \frac{C}{2}\left[\frac{1}{v_2^2} - \frac{1}{v_1^2}\right]\right] \qquad (2)$$

where from Table A-4, $v_1 = 0.2275\ \text{m}^3/\text{kg}$, $v_2 = 2.359\ \text{m}^3/\text{kg}$.

The coefficients B, C can be evaluated using steam table data by noting that

$$Z = 1 + \frac{B}{v} + \frac{C}{v^2} \Rightarrow (Z-1)v = B + C(1/v)$$

① Thus, B and C are the intercept and slope, respectively, of a plot of $(Z-1)v$ vs $(1/v)$. The outcome of this procedure using steam table data is

$$B = -84.0 \times 10^{-4}\ \text{m}^3/\text{kg}, \quad C = -1.25 \times 10^{-4}\ \text{m}^6/\text{kg}^2.$$

as can be verified by using Eq. (1) to check steam Table pressure values at 240°C.

Calculating

$$\frac{W}{m} = \left(\frac{8.314}{18.02}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(513.15\,\text{K})\left[\ln\left(\frac{2.359}{0.2275}\right) + \frac{84}{10^4}\left[\frac{1}{2.359} - \frac{1}{0.2275}\right] + \frac{1.25}{(2)(10^4)}\left[\frac{1}{(2.359)^2} - \frac{1}{(0.2275)^2}\right]\right]$$

$$= 545.6\ \frac{\text{kJ}}{\text{kg}} \longleftarrow$$

---

1. The use of appropriate computer software facilitates this step. Alternatively, table data can be plotted by hand, and the values of B and C determined graphically.

PROBLEM 11.13

KNOWN: The compressibility factor Z can be expressed in the forms

$$Z = 1 + \hat{B}(T)\,p + \hat{C}(T)\,p^2 + \hat{D}(T)\,p^3 + \cdots \qquad (3.29)$$

$$Z = 1 + \frac{B(T)}{\bar{v}} + \frac{C(T)}{\bar{v}^2} + \frac{D(T)}{\bar{v}^3} + \cdots \qquad (3.30)$$

FIND: Show that

$$\hat{B} = B/\bar{R}T\,, \quad \hat{C} = (C - B^2)/\bar{R}^2T^2$$

ANALYSIS: With $Z = p\bar{v}/\bar{R}T$, Eq. 3.30 gives

$$p = \frac{\bar{R}T}{\bar{v}} + \frac{\bar{R}TB}{\bar{v}^2} + \frac{\bar{R}TC}{\bar{v}^3} + \cdots$$

Inserting this into Eq. 3.29

$$Z = 1 + \hat{B}\left[\frac{\bar{R}T}{\bar{v}} + \frac{\bar{R}TB}{\bar{v}^2} + \frac{\bar{R}TC}{\bar{v}^3} + \cdots\right] + \hat{C}\left[\frac{\bar{R}T}{\bar{v}} + \frac{\bar{R}TB}{\bar{v}^2} + \cdots\right]^2 + \cdots$$

$$= 1 + \frac{\hat{B}\bar{R}T}{\bar{v}} + \frac{\hat{B}\bar{R}TB + \hat{C}(\bar{R}T)^2}{\bar{v}^2} + \underbrace{\cdots}_{\text{terms in } \bar{v}^3 \text{ or higher}}$$

Comparing this with Eq. 3.30

$$B = \hat{B}\bar{R}T \Rightarrow \hat{B} = B/\bar{R}T \longleftarrow$$

and

$$\hat{B}\bar{R}TB + \hat{C}(\bar{R}T)^2 = C(T)$$

or

$$\left(\frac{B}{\bar{R}T}\right)\bar{R}TB + \hat{C}(\bar{R}T)^2 = C(T)$$

① $$B^2 + \hat{C}(\bar{R}T)^2 = C(T) \Rightarrow \hat{C} = \frac{C(T) - (B(T))^2}{\bar{R}^2T^2} \longleftarrow$$

1. This procedure can be contined to obtain expressions for $\hat{D}$ and higher terms.

PROBLEM 11.14

KNOWN: The van der Waals equation expressed in terms of Z is

$$Z = \frac{v_R'}{v_R' - 1/8} - \frac{27/64}{T_R v_R'}$$

FIND: Express this (a) as a virial series in v_R', (b) as a virial series in p_R. (c) Dropping higher terms in the expression of part (b), obtain an approximate form, and (d) compare with data from the literature.

ANALYSIS: (a) The term $1/(v_R' - 1/8)$ can be written as

$$\frac{1}{(v_R' - 1/8)} = \frac{1}{v_R'} + \frac{1}{8(v_R')^2} + \frac{1}{(8)^2(v_R')^3} + \cdots$$

Accordingly

$$Z = v_R'\left[\frac{1}{v_R'} + \frac{1}{8(v_R')^2} + \frac{1}{(8)^2(v_R')^3} + \cdots\right] - \frac{27/64}{T_R v_R'}$$

$$= 1 + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{1}{v_R'} + \frac{1}{(8)^2(v_R')^2} + \underbrace{\cdots}_{\text{terms in } (v_R')^3 \text{ and higher}}$$

This is the desired form:

$$Z = 1 + \frac{B}{v_R'} + \frac{C}{(v_R')^2} + \cdots \longleftarrow (a)$$

(b) A virial series in p_R has the form

$$Z = 1 + \hat{B}(T)\,p_R + \hat{C}(T)\,(p_R)^2 + \cdots$$

With $Z = p_R v_R'/T_R$, the result of part (a) becomes

PROBLEM 11.14 (Continued)

$$\frac{P_R V_R'}{T_R} = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{1}{V_R'} + \frac{1}{(8)^2(V_R')^2} + \cdots\cdots$$

or

$$P_R = \frac{T_R}{V_R'} + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{T_R}{(V_R')^2} + \frac{T_R}{(8)^2(V_R')^3} + \cdots$$

Inserting this into the series in P_R

$$Z = 1 + \hat{B}\left[\frac{T_R}{V_R'} + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{T_R}{(V_R')^2} + \frac{T_R}{(8)^2(V_R')^3} + \cdots\right] + \hat{C}\left[\frac{T_R}{V_R'} + \cdots\right]^2$$

$$= 1 + \frac{\hat{B}\,T_R}{V_R'} + \left[\frac{\hat{B}\left[\frac{1}{8} - \frac{27/64}{T_R}\right]T_R + \hat{C}\,T_R^2}{(V_R')^2}\right] + \underbrace{\cdots\cdots}_{\text{terms in } (V_R')^3 \text{ or higher}}$$

Comparing this with the result of part (a)

$$\hat{B}\,T_R = \left[\frac{1}{8} - \frac{27/64}{T_R}\right] \Rightarrow \hat{B} = \frac{1}{T_R}\left[\frac{1}{8} - \frac{27/64}{T_R}\right]$$

$$\hat{B}\left[\frac{1}{8} - \frac{27/64}{T_R}\right]T_R + \hat{C}\,T_R^2 = \frac{1}{(8)^2} \Rightarrow \left[\frac{1}{8} - \frac{27/64}{T_R}\right]^2 + \hat{C}\,T_R^2 = \frac{1}{(8)^2}$$

$$\Rightarrow \hat{C} = \left[1/(8)^2 - \left[\frac{1}{8} - \frac{27/64}{T_R}\right]^2\right]\frac{1}{(T_R)^2}$$

(c) The series obtained in part (b) has the form

$$Z = 1 + \hat{B}\,P_R + \underbrace{\hat{C}\,(P_R)^2 + \cdots}_{\text{Terms in } (P_R)^2,\ (P_R)^3,\ \cdots}$$

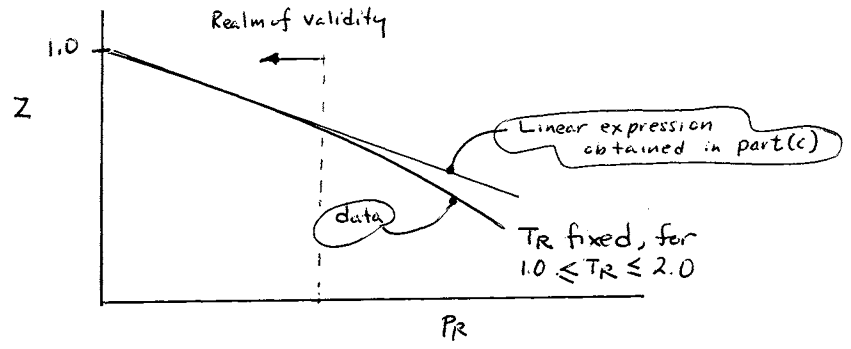
Dropping terms involving $(P_R)^2$, and higher

$$Z = 1 + \hat{B}\,P_R$$

or

$$Z = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{P_R}{T_R} \qquad \longleftarrow$$

(d) Calculating compressibility factors from the result of part (c) and comparing them with values obtained from the literature ①, the realm of validity can be determined. This procedure is indicated by the following sketch:



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1. See, for example, R.C. Reid and T.K. Sherwood, The Properties of Gases and Liquids, 2nd ed., McGraw-Hill, New York, 1966, pp. 587-595.

PROBLEM 11.15

KNOWN: The Berthelot equation of state has the form

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{T\bar{v}^2}$$

FIND: (a) Using Eqs. 11.3, evaluate a and b in terms of T_c and p_c, (b) Express the equation in terms of Z, T_R, and v_R'.

ANALYSIS: (a) Differentiating

$$\left(\frac{\partial p}{\partial \bar{v}}\right)_T = -\frac{\bar{R}T}{(\bar{v}-b)^2} + \frac{2a}{T\bar{v}^3}$$

At the critical point: $(\partial p/\partial \bar{v})_T = 0$. Thus

$$-\frac{\bar{R}T_c}{(\bar{v}_c-b)^2} + \frac{2a}{T_c\bar{v}_c^3} = 0 \Rightarrow -\bar{R}T_c^2\bar{v}_c^3 + 2a(\bar{v}_c-b)^2 = 0 \qquad (1)$$

Differentiating

$$\left(\frac{\partial^2 p}{\partial \bar{v}^2}\right)_T = +\frac{2\bar{R}T}{(\bar{v}-b)^3} - \frac{6a}{T\bar{v}^4}$$

At the critical point: $(\partial^2 p/\partial \bar{v}^2)_T = 0$. Thus

$$\frac{2\bar{R}T_c}{(\bar{v}_c-b)^3} - \frac{6a}{T_c\bar{v}_c^4} = 0 \Rightarrow 2\bar{R}T_c^2\bar{v}_c^4 - 6a(\bar{v}_c-b)^3 = 0 \qquad (2)$$

At the critical point:

$$p_c = \frac{\bar{R}T_c}{\bar{v}_c-b} - \frac{a}{T_c\bar{v}_c^2} \qquad (3)$$

Combining (1) and (2), $b = \bar{v}_c/3$. Inserting this into Eq. (1)

$$-\bar{R}T_c^2\bar{v}_c^3 + 2a\left(\bar{v}_c - \frac{\bar{v}_c}{3}\right)^2 = 0 \Rightarrow a = 9\bar{R}T_c^2\bar{v}_c/8$$

Inserting the expressions for a and b into Eq. (3), $(p_c v_c/\bar{R}T_c) = 3/8$, or $v_c = \frac{3}{8}\frac{\bar{R}T_c}{p_c}$. Finally

$$a = \frac{9\bar{R}T_c^2}{8}\left[\frac{3}{8}\frac{\bar{R}T_c}{p_c}\right] = \frac{27}{64}\frac{\bar{R}^2T_c^3}{p_c}, \quad b = \frac{1}{8}\frac{\bar{R}T_c}{p_c} \longleftarrow \text{(a)}$$

(b) With $Z = p\bar{v}/\bar{R}T$ and the above expressions for a and b

$$\frac{p\bar{v}}{\bar{R}T} = \frac{\bar{v}}{\bar{v}-b} - \frac{a}{\bar{R}T^2\bar{v}} = \frac{\bar{v}}{\bar{v}-\left[\frac{\bar{R}T_c}{8p_c}\right]} - \frac{(27/64)(\bar{R}^2T_c^3/p_c)}{\bar{R}T^2\bar{v}}$$

or

$$Z = \frac{1}{1-\frac{1}{8}\left[\frac{\bar{R}T_c}{\bar{v}p_c}\right]} - \frac{(27/64)(\bar{R}T_c/p_c\bar{v})}{T_R^2}$$

Thus

$$Z = \frac{v_R'}{v_R' - \frac{1}{8}} - \frac{27/64}{v_R' T_R^2} \longleftarrow \text{(b)}$$

PROBLEM 11.16

KNOWN: The Beattie – Bridgeman equation of state can be expressed as

$$p = \frac{RT(1-\epsilon)(v+B)}{v^2} - \frac{A}{v^2}$$

where

$$A = A_0\left(1-\frac{a}{v}\right), \quad B = B_0\left(1-\frac{b}{v}\right), \quad \epsilon = \frac{c}{vT^3}$$

FIND: Express the equation in terms of P_R, T_R, v_R', and dimensionless constants.

ANALYSIS: With $T = T_R T_c$, $p = P_R P_c$, and $v = v_R'(RT_c/P_c)$, the equation becomes

$$P_R P_c = \frac{RT_R T_c(1-\epsilon)(v_R' RT_c/P_c + B)}{(v_R')^2 (RT_c/P_c)^2} - \frac{A}{(v_R')^2(RT_c/P_c)^2}$$

where

$$A = A_0\left(1 - \frac{a}{v_R'(RT_c/P_c)}\right), \quad B = B_0\left(1 - \frac{b}{v_R'(RT_c/P_c)}\right), \quad \epsilon = \frac{c}{v_R' T_R^3 (RT_c/P_c)(T_c)^3}$$

Define the following dimensionless quantities

$$a' \equiv \frac{a}{RT_c/P_c}, \quad b' \equiv \frac{b}{RT_c/P_c}, \quad c' \equiv \frac{c}{RT_c^4/P_c}$$

$$A_0' \equiv \frac{A_0}{R^2T_c^2/P_c}, \quad B_0' \equiv \frac{B_0}{RT_c/P_c}, \quad A' \equiv \frac{A}{R^2T_c^2/P_c}, \quad B' \equiv \frac{B}{RT_c/P_c}$$

Then

$$A' = A_0'\left(1 - \frac{a'}{v_R'}\right), \quad B' = B_0'\left(1 - \frac{b'}{v_R'}\right), \quad \epsilon = \frac{c'}{v_R' T_R^3}$$

Upon simplification, the equation of state becomes

$$P_R = \frac{RT_R T_c\left(\frac{RT_c}{P_c}\right)(1-\epsilon)\left(v_R' + \frac{BP_c}{RT_c}\right)}{P_c(RT_c/P_c)^2 (v_R')^2} - \frac{A}{P_c(v_R')^2(RT_c/P_c)^2}$$

or

$$P_R = \frac{T_R(1-\epsilon)(v_R' + B')}{(v_R')^2} - \frac{A'}{(v_R')^2}$$

which is the required result.

PROBLEM 11.17

KNOWN: The Dieterici equation of state is

$$p = \left(\frac{RT}{v-b}\right)\exp\left(\frac{-a}{RTv}\right)$$

FIND: (a) Evaluate a and b in terms of T_c and p_c using Eqs. 11.3. (b) Express the equation in terms of Z, T_R, and v_R'. (c) Convert the result of part (b) into a virial series in v_R'.

ANALYSIS: (a) Differentiating twice with respect to v, we get

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2}\exp\left(-\frac{a}{RTv}\right) + \left(\frac{RT}{v-b}\right)\exp\left(-\frac{a}{RTv}\right)\cdot\left(+\frac{a}{RTv^2}\right)$$

$$= -\frac{RT}{(v-b)^2}\exp\left(-\frac{a}{RTv}\right) + \frac{a}{v^2(v-b)}\exp\left(-\frac{a}{RTv}\right) \qquad (1)$$

$$\left(\frac{\partial^2 p}{\partial v^2}\right)_T = \frac{2RT}{(v-b)^3}\exp\left(-\frac{a}{RTv}\right) - \frac{a}{v^2(v-b)^2}\exp\left(-\frac{a}{RTv}\right) -$$

$$\frac{a(3v-2b)}{v^3(v-b)^2}\exp\left(-\frac{a}{RTv}\right) + \frac{a^2}{RTv^4(v-b)}\exp\left(-\frac{a}{RTv}\right) \qquad (2)$$

PROBLEM 11.17 (Contd.)

With $(\partial p/\partial v)_T = 0$ at the critical point, Eq. (1) gives

$$0 = -\frac{RT_c}{(v_c - b)} + \frac{a}{v_c^2} \Rightarrow a = \frac{RT_c (v_c)^2}{(v_c - b)} \qquad (3)$$

with $(\partial^2 p/\partial v^2)_T = 0$ at the critical point, Eq. (2) gives

$$0 = \frac{2RT_c}{(v_c - b)^2} - \frac{a}{v_c^2 (v_c - b)} - \frac{a(3v_c - 2b)}{v_c^3 (v_c - b)} + \frac{a^2}{RT_c v_c^4} \qquad (4)$$

Inserting Eq. (3) into Eq. (4) and simplifying, $b = v_c/2$. Thus, $a = 2RT_c v_c$.
At the critical point, the equation of state reads

$$p_c = \frac{RT_c}{v_c - b} \exp\left(-\frac{a}{RT_c v_c}\right) = \frac{RT_c}{(v_c - \frac{v_c}{2})} \exp\left(-\frac{2RT_c v_c}{RT_c v_c}\right)$$

Upon reduction, $v_c = 2RT_c/p_c e^2$, giving

$$a = \frac{4R^2 T_c^2}{p_c e^2}, \quad b = \frac{RT_c}{p_c e^2} \qquad \longleftarrow$$

(b) With the given equation of state, the results of part (a), and $T = T_R T_c$, $v = v_R' RT_c/p_c$, we get

$$\frac{pv}{RT} = \frac{v}{v-b} \exp\left(-\frac{a}{RTv}\right)$$

Thus

$$Z = \frac{v_R' RT_c/p_c}{\left[v_R' RT_c/p_c - RT_c/p_c e^2\right]} \exp\left(-\frac{\frac{4R^2 T_c^2}{p_c e^2}}{R(T_R T_c)(v_R' RT_c/p_c)}\right)$$

$$Z = \left[\frac{v_R'}{v_R' - 1/e^2}\right] \exp\left(-\frac{4}{T_R v_R' e^2}\right) \qquad \longleftarrow \qquad (b)$$

(c) The term $1/(v_R' - 1/e^2)$ can be written as

$$\frac{1}{v_R' - 1/e^2} = \frac{1}{v_R'} + \frac{1}{e^2 (v_R')^2} + \frac{1}{e^4 (v_R')^3} + \cdots$$

The exponential term can also be written as a series. Collecting results and reduced to read

$$Z = 1 + \frac{1}{e^2}\left[1 - \frac{4}{T_R}\right]\frac{1}{v_R'} + \frac{1}{e^4}\left[1 - \frac{4}{T_R} - \frac{8}{T_R^2}\right]\frac{1}{(v_R')^2} + \cdots \qquad \longleftarrow \quad (c)$$

PROBLEM 11.18

KNOWN: The Peng-Robinson equation of state is specified.

FIND : Evaluate the constants a, b, c in terms of P_c, T_c, Z_c, using Eqs. 11.3

ANALYSIS: Evaluating the Peng-Robinson equation of state at the critical point and using Eqs. 11.3 gives the following three expressions

$$P_c = \frac{RT_c}{V_c - b} - \frac{a}{V_c^2 - c^2} \qquad (1)$$

$$\frac{-RT_c}{(V_c - b)^2} + \frac{2aV_c}{(V_c^2 - c^2)^2} = 0 \qquad (2)$$

$$\frac{2RT_c}{(V_c - b)^3} + \frac{2a(V_c^2 - c^2) - 8aV_c^2}{(V_c^2 - c^2)^3} = 0 \qquad (3)$$

Solving these expressions simultaneously

① $$a = \frac{8V_c^3 P_c^2}{RT_c} = 8V_c^2 P_c Z_c \qquad (4)$$

$$b = 3V_c - \frac{RT_c}{P_c} = V_c\left[3 - \frac{1}{Z_c}\right] \qquad (5)$$

$$c^2 = V_c^2\left[8\frac{P_cV_c}{RT_c} - 3\right] = V_c^2\left[8Z_c - 3\right] \qquad (6)$$

as can be verified by substitution into Eqs. (1)-(3) and reduction of the resulting expressions.

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1. Note that when $Z_c = 3/8$, Eqs. (4, 5) reduce to Eqs. (11.4a, 4b), respectively. The equation of state reduces to the vander Waals equation.

PROBLEM 11.19

KNOWN: The Carnhan-Starling-De Santis equation of state is provided, together with values of the constants in the equation for the cases of Refrigerants 12 and 13.

FIND: Specify which of the two refrigerants would allow the smaller amount of mass to be stored in a 10-m^3 vessel at 0.2 MPa, 80°C. Determine the mass.

ANALYSIS:

For the case of R12, using the values of the constants provided,

$$b = (0.15376) + (-1.84195 \times 10^{-4})(353) + (-5.03644 \times 10^{-8})(353)^2$$

$$= 8.2463 \times 10^{-2} \frac{L}{mol} = 8.2463 \times 10^{-2} \frac{m^3}{kmol}$$

$$a = (3.52412 \times 10^3) \exp\left((-2.7723 \times 10^{-3})(353) + (-0.67318 \times 10^{-6})(353)^2\right)$$

$$= 1217.896 \frac{J \cdot L}{(mol)^2} = 1.2179 \times 10^6 \frac{J \cdot m^3}{(kmol)^2}$$

Inserting values into the equation of state

$$\frac{(0.2 \times 10^6\ N/m^2)\,\bar{v}}{\left(8314 \frac{J}{kmol \cdot K}\right)(353K)} = \frac{1 + \beta + \beta^2 - \beta^3}{(1+\beta)^3} - \frac{1.2179 \times 10^6 \frac{J \cdot m^3}{(kmol)^2}}{\left(8314 \frac{J}{kmol \cdot K}\right)(353K)\left(\bar{v} + 8.2463 \times 10^{-2}\right)\frac{m^3}{kmol}}$$

where $\beta = 8.2463 \times 10^{-2} / 4\bar{v}$.

Using an equation solver, we get $\bar{v} = 14.21\ m^3/kmol$. Then, with M = 120.92 for R12,

$$v = \frac{14.21}{120.92} = 0.1175\ m^3/kg$$

① This value is within 1% of the value obtained from the literature: $v = 0.1187\ m^3/kg$.

Similarly, for R13 at the same state, $v = 0.1391\ m^3/kg$. Thus, a smaller mass of R13 can be stored:

$$m = \frac{V}{v} = \frac{10\ m^3}{0.1391\ m^3/kg} = 71.89\ kg \qquad \longleftarrow$$

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1. For example, R12 and R13 data can be obtained from C. Borgnakke and R. E. Sonntag, Thermodynamic and Transport Properties, Wiley, New York, 1997, Tables B.12, B.13.

PROBLEM 11.20

KNOWN: Two expressions for dp in terms of v and T are provided.
FIND: Determine which of the two expressions is the differential of an equation of state: $p = p(v,T)$, and obtain the equation.
ANALYSIS: For the first of the two equations, let

$$M = \frac{2(v-b)}{RT}, \quad N = \frac{(v-b)^2}{RT^2}$$

If this is the differential of an equation of state, the test for exactness must be satisfied: $\partial M/\partial T)_v = \partial N/\partial v)_T$. Checking

$$\frac{\partial M}{\partial T}\Big)_v = \frac{-2(v-b)}{RT^2}, \quad \frac{\partial N}{\partial v}\Big)_T = \frac{2(v-b)}{RT^2}$$

As these are unequal, the first expression given cannot be the differential of an equation of state.

For the second of the two equations, let

$$M = -\frac{RT}{(v-b)^2}, \quad N = \frac{R}{v-b}$$

Applying the test for exactness: $(\partial M/\partial T)_v = -R/(v-b)^2$, $(\partial N/\partial v)_T = -R/(v-b)^2$. As these are equal, a function $p = p(T,v)$ exists for which the second differential form is the exact differential. To obtain the function, write

$$dp = \left(\frac{\partial p}{\partial v}\right)_T dv + \left(\frac{\partial p}{\partial T}\right)_v dT$$

Comparing with the given differential form

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2}, \quad \left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

Integrating the second of these

$$p = \frac{RT}{v-b} + f(v)$$

Inserting this trial form into the first gives

$$-\frac{RT}{(v-b)^2} + \frac{df}{dv} = -\frac{RT}{(v-b)^2} \Rightarrow \frac{df}{dv} = 0 \Rightarrow f = \text{constant}.$$

Accordingly, the equation of state has the form: $p = \frac{RT}{(v-b)} + \text{constant}.$ ←

PROBLEM 11.21

KNOWN: $\delta Q_{\substack{int\\rev}} = dU + p\,dV$

FIND: Use the given expression together with the test for exactness to demonstrate that $Q_{\substack{int\\rev}}$ is not a property.

ASSUMPTION: A simple compressible system of fixed mass is under consideration.

ANALYSIS: For a simple compressible system of fixed mass, $U = U(V,T)$. The differential of this is

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV = M\,dT + N\,dV \qquad (1)$$

As U, V, and T are properties, this is an exact differential. Applying the test for exactness:

$$\frac{\partial}{\partial V}\Big[\underbrace{\left(\frac{\partial U}{\partial T}\right)_V}_{M}\Big]_T = \frac{\partial}{\partial T}\Big[\underbrace{\left(\frac{\partial U}{\partial V}\right)_T}_{N}\Big]_V \qquad (2)$$

Inserting Eq.(1) into the given expression for $\delta Q_{\substack{int\\rev}}$

$$\delta Q_{\substack{int\\rev}} = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV + p\,dV$$

$$= \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right] dV \qquad (3)$$

If $Q_{\substack{int\\rev}}$ is a property, the last expression would be an exact differential. Applying the test for exactness to Eq.(3)

$$M = \left(\frac{\partial U}{\partial T}\right)_V \Rightarrow \left(\frac{\partial M}{\partial V}\right)_T = \frac{\partial}{\partial V}\left[\left(\frac{\partial U}{\partial T}\right)_V\right]_T$$

$$N = \left(\frac{\partial U}{\partial V}\right)_T + p \Rightarrow \left(\frac{\partial N}{\partial T}\right)_V = \frac{\partial}{\partial T}\left[\left(\frac{\partial U}{\partial V}\right)_T\right]_V + \left(\frac{\partial p}{\partial T}\right)_V$$

Accordingly

$$\left(\frac{\partial M}{\partial V}\right)_T = \left(\frac{\partial N}{\partial T}\right)_V \Rightarrow \underbrace{\frac{\partial}{\partial V}\left[\left(\frac{\partial U}{\partial T}\right)_V\right]_T = \frac{\partial}{\partial T}\left[\left(\frac{\partial U}{\partial V}\right)_T\right]_V}_{\text{equal by Eq.(2)}} + \left(\frac{\partial p}{\partial T}\right)_V$$

Using Eq.(2) in this, as indicated, the test for exactness is satisfied only when

$$\left(\frac{\partial p}{\partial T}\right)_V \equiv 0$$

However, reference to Fig. 11.1 shows that this expression is not generally satisfied. It can be concluded therefore that the test for exactness is not satisfied and $Q_{\substack{int\\rev}}$ cannot be a property. The use of the symbol δ with $Q_{\substack{int\\rev}}$ is to signal this fact.

PROBLEM 11.22

KNOWN: An equation of state has the form $p = [RT/(v-b)] + a$

FIND: Show that Eq. 11.16 is satisfied.

ANALYSIS: Eq. 11.16 has the form

$$\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -1$$

Identifying $p : x,\ v : y,\ z : T$, then

$$\left(\frac{\partial y}{\partial z}\right)_x = \left(\frac{\partial v}{\partial T}\right)_p$$
$$\left(\frac{\partial z}{\partial x}\right)_y = \left(\frac{\partial T}{\partial p}\right)_v$$
$$\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial p}{\partial v}\right)_T$$

Using the equation of state, $(\partial p/\partial v)_T = -RT(v-b)^{2}$ (1)

Solving,

$$v = b + \frac{RT}{p-a} \Rightarrow \left.\frac{\partial v}{\partial T}\right)_p = \frac{R}{p-a} \Rightarrow \left.\frac{\partial v}{\partial T}\right)_p = \frac{v-b}{T} \qquad (2)$$

Solving

$$T = \frac{(p-a)(v-b)}{R} \Rightarrow \left.\frac{\partial T}{\partial p}\right)_v = \frac{v-b}{R} \qquad (3)$$

With (1)-(3), Eq. 11.16 takes the form

$$\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v \left(\frac{\partial p}{\partial v}\right)_T = \left(\frac{v-b}{T}\right)\left(\frac{v-b}{R}\right)\left(-\frac{RT}{(v-b)^2}\right) = -1$$

Thus, Eq. 11.16 is satisfied.

PROBLEM 11.23

KNOWN: $x = x(y,w)$, $y = y(z,w)$, $z = z(x,w)$.

FIND: Show that

$$\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w = 1$$

ANALYSIS: Forming differentials

$$dx = \left(\frac{\partial x}{\partial y}\right)_w dy + \left(\frac{\partial x}{\partial w}\right)_y dw \qquad (1)$$
$$dy = \left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw \qquad (2)$$
$$dz = \left(\frac{\partial z}{\partial x}\right)_w dx + \left(\frac{\partial z}{\partial w}\right)_x dw \qquad (3)$$

Replacing dy in Eq. (1) with Eq. (2), and then replacing dz in the resulting expression by Eq. (3):

$$dx = \left(\frac{\partial x}{\partial y}\right)_w \left[\left(\frac{\partial y}{\partial z}\right)_w dz + \left(\frac{\partial y}{\partial w}\right)_z dw\right] + \left(\frac{\partial x}{\partial w}\right)_y dw = \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w dz + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

$$= \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left[\left(\frac{\partial z}{\partial x}\right)_w dx + \left(\frac{\partial z}{\partial w}\right)_x dw\right] + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

$$= \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w dx + \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial w}\right)_x + \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

On rearrangement

$$\left[1 - \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w\right] dx = \left[\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial w}\right)_x + \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial w}\right)_z + \left(\frac{\partial x}{\partial w}\right)_y\right] dw$$

Since x and w are independent, hold w constant and vary x. That is, $dw = 0$ and $dx \neq 0$. Then

$$\left[1 - \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w\right] dx = 0 \Rightarrow \left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w \left(\frac{\partial z}{\partial x}\right)_w = 1. \qquad \longleftarrow$$

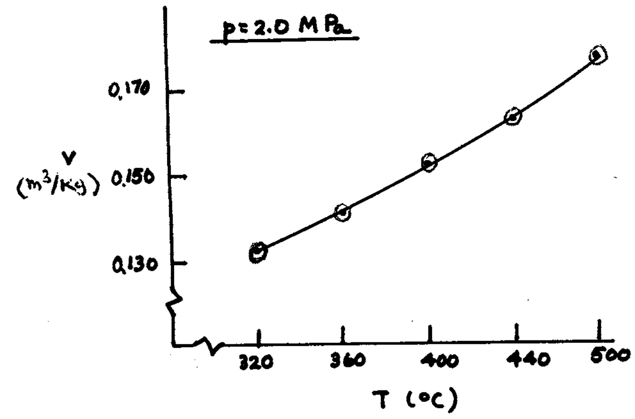
PROBLEM 11.24

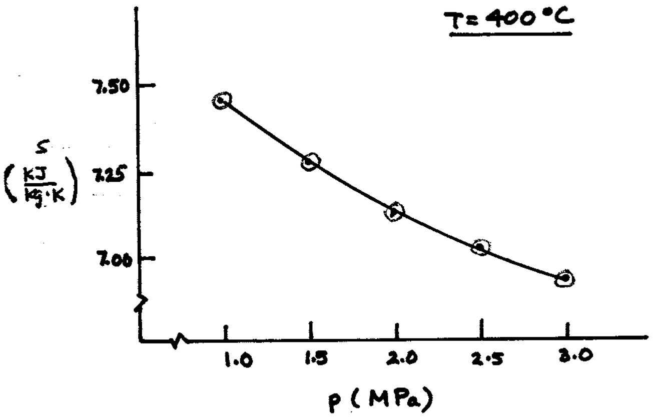
KNOWN:

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T \qquad (11.35)$$

FIND: Using Eq. (11.35), check the consistency of (a) the steam tables at 2 MPa, 400°C, (b) the R134a tables at 2 bar, 50°C.

① ANALYSIS: We select a graphical approach here. (a) Using data from Table A-4





From the v vs. T plot, at 400°C

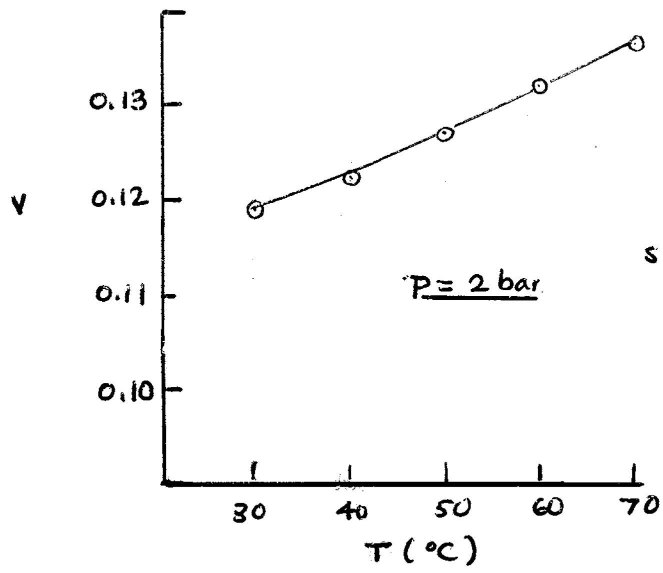
$$\left(\frac{\partial v}{\partial T}\right)_p \approx 0.25 \times 10^{-3}\ \frac{m^3}{kg \cdot K}$$

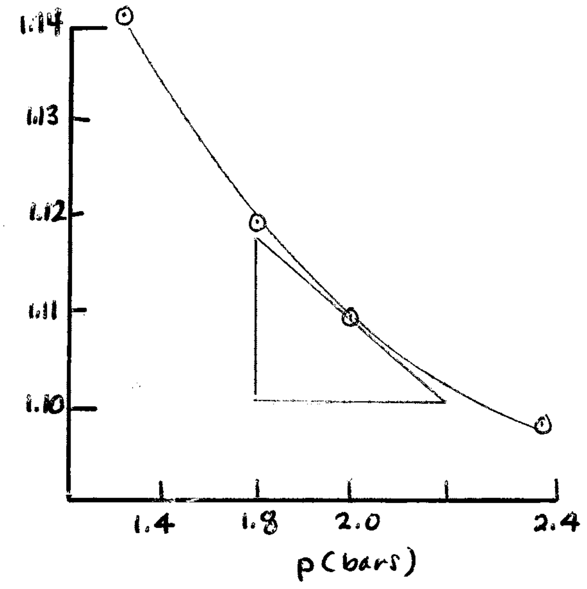
From the s vs p plot, at 2MPa

$$\left(\frac{\partial s}{\partial p}\right)_T \approx -0.249\ \frac{kJ/kg \cdot K}{MPa} = -0.249\ \frac{(10^3\ N \cdot m)}{kg \cdot K}\left(\frac{1}{10^6\ N/m^2}\right) = -0.249 \times 10^{-3}\ \frac{m^3}{kg \cdot K}$$

These values are in good agreement, as required by 11.35.

(b) Using data from Table A-12





From the v vs. T plot, at 50°C

$$\left(\frac{\partial v}{\partial T}\right)_p \approx 4.45 \times 10^{-4}\ m^3/kg \cdot K$$

from the s vs p plot, at 2 bar

$$\left(\frac{\partial s}{\partial p}\right)_T \approx 4.45 \times 10^{-4}\ m^3/kg \cdot K$$

In accordance with Eq. 11.35, the data are in agreement.

PROBLEM 11.24 (Cont'd.)

An alternative solution using IT follows:

IT Code - part (a)

```
// (a)  steam at 2 MPa, 400°C
p = 2000  // kPa
T = 400  // °C
dT = 0.001
dp = 0.001
T1 = T - dT
T2 = T + dT
p1 = p - dp
p2 = p + dp
v1 = v_PT("Water/Steam", p, T1)
v2 = v_PT("Water/Steam", p, T2)
dvdT_a = (v2 - v1) / (T2 - T1)
s1 = s_PT("Water/Steam", p1, T)
s2 = s_PT("Water/Steam", p2, T)
dsdp = (s2 - s1) / (p2 - p1)
```

IT Results - part (a)

$(\partial v/\partial T)_p = 0.2491 \times 10^{-3}\ \mathrm{m^3/kg\cdot K}$

$(\partial s/\partial p)_T = -\ 0.2492 \times 10^{-3}\ \mathrm{m^3/kg\cdot K}$

IT Code - part (b)

```
(b) R-134a at 2 bar, 50°C
p_b = 200  // kPa
T_b = 50  // °C
v = v_PT("R134A", p_b, T_b)
T1_b = T_b - dT
T2_b = T_b + dT
p1_b = p_b - dp
p2_b = p_b + dp
v1_b = v_PT("R134A", p_b, T1_b)
v2_b = v_PT("R134A", p_b, T2_b)
dvdT_b = (v2_b - v1_b) / (T2_b - T1_b)
s2_b = s_PT("R134A", p2_b, T_b)
s1_b = s_PT("R134A", p1_b, T_b)
dsdp_b = (s2_b - s1_b) / (p2_b - p1_b)
```

IT Results - part (b)

$(\partial v/\partial T)_p = 4.449 \times 10^{-4}\ \mathrm{m^3/kg\cdot K}$

$(\partial s/\partial p)_T = -\ 4.449 \times 10^{-4}\ \mathrm{m^3/kg\cdot K}$

The results of the IT solution compare favorably with the graphical results.

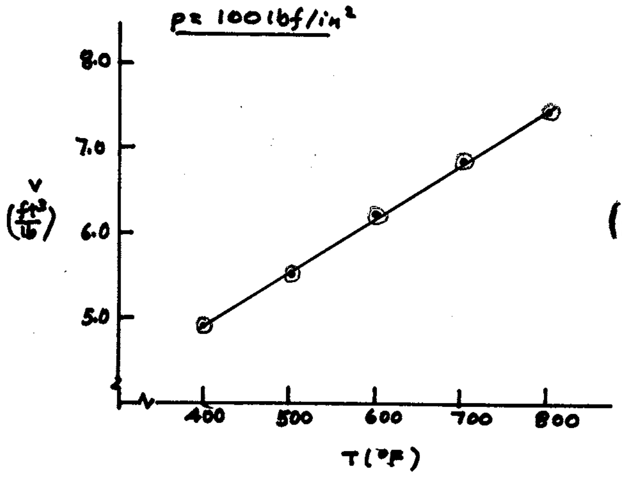
PROBLEM 11.25

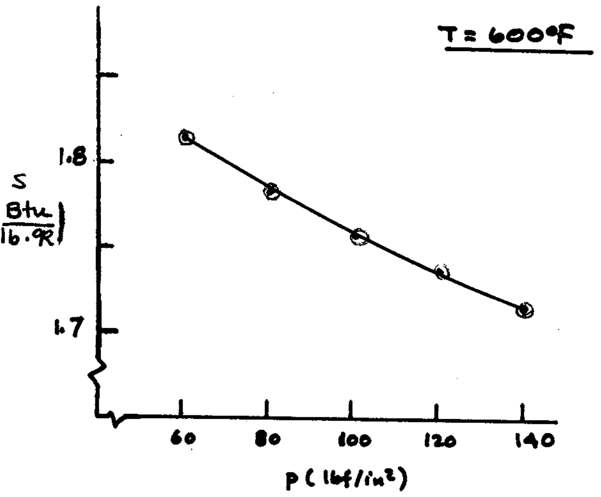
KNOWN:

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T \qquad (11.35)$$

FIND: Using Eq. 11.35, check the consistency of (a) the steam tables at 100 lbf/in², 600°F, (b) the R134a tables at 40 lbf/in², 100°F

① **ANALYSIS:** We select a graphical approach here. (a) Using data from Table A-4E,





From the v vs T plot at 600°F

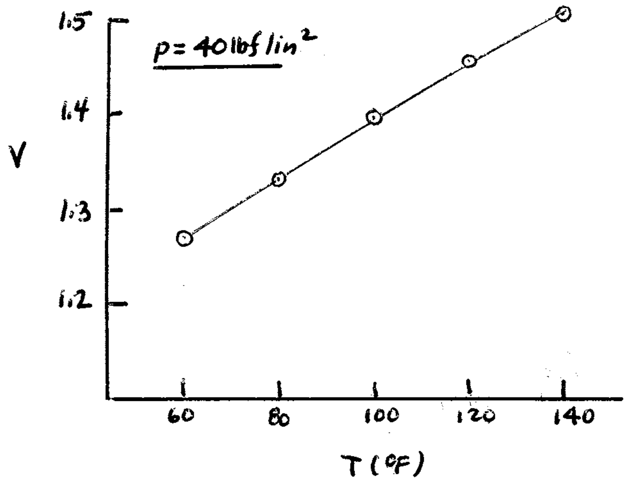
$$\left(\frac{\partial v}{\partial T}\right)_p = 6.24\times10^{-3}\ \frac{ft^3}{lb\cdot°R}$$

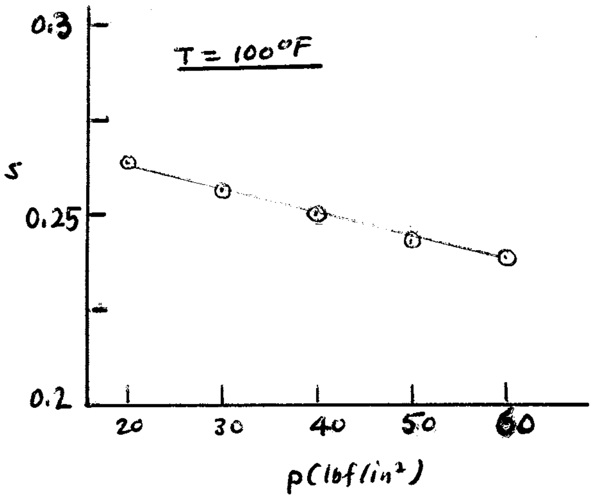
From the s vs p plot at 100 lbf/in²

$$\left(\frac{\partial s}{\partial T}\right)_p = -1.168\times10^{-3}\ \frac{Btu/lb\cdot°R}{lbf/in^2}\left(\frac{778\ ft\cdot lbf}{Btu}\right)\left(\frac{ft^2}{144\ in^2}\right) = -6.31\times10^{-3}\ \frac{ft^3}{lb\cdot°R}$$

These values are in good agreement, as required by Eq. 11.35.

(b) With data from Table A-12E





From the v vs. T plot, at 100°F:

$$\left(\frac{\partial v}{\partial T}\right)_p \approx 3.05\times10^{-3}\ \frac{ft^3}{lb\cdot°R}$$

From the s vs. p plot, at 40 lbf/in²

$$\left(\frac{\partial s}{\partial p}\right)_T \approx -3.11\times10^{-3}\ \frac{ft^3}{lb\cdot°R}$$

} again, we have good agreement, in accordance with Eq. 11.35.

PROBLEM 11.25 (Cont'd.)

An alternative solution using IT follows:

IT Code - part (a)

```
// (a)  steam at 100 lbf/in², 600°F
pa = 100  //  lbf/in²
Ta = 600  //  °F
dTa = 0.1
T1a = Ta - dTa
T2a = Ta + dTa
dpa = 0.1
p1a = pa - dpa
p2a = pa + dpa
v1a = v_PT("Water/Steam", pa, T1a)
v2a = v_PT("Water/Steam", pa, T2a)
dvdTa = (v2a - v1a) / (T2a - T1a)
s1a = s_PT("Water/Steam", p1a, Ta)
s2a = s_PT("Water/Steam", p2a, Ta)
dsdpa = ((s2a - s1a) / (p2a - p1a)) * (778.17 / 144)
```

IT Results - part (a)

$(\partial v/\partial t)_p = 6.225 \times 10^{-3}$ ft³/lb·°R

$(\partial s/\partial p)_T = -6.226 \times 10^{-3}$ ft³/lb·°R

IT Code - part (b)

```
// (b)  R134a at 40 lbf/in², 100°F
pb = 40  //  lbf/in²
Tb = 100  //  °F
dTb = 0.1
T1b = Tb - dTb
T2b = Tb + dTb
dpb = 0.1
p1b = pb - dpb
p2b = pb + dpb
v1b = v_PT("R134A", pb, T1b)
v2b = v_PT("R134A", pb, T2b)
dvdTb = (v2b - v1b) / (T2b - T1b)
s1b = s_PT("R134A", p1b, Tb)
s2b = s_PT("R134A", p2b, Tb)
dsdpb = ((s2b - s1b) / (p2b - p1b)) * (778.17 / 144)
```

IT Results - part (b)

$(\partial v/\partial t)_p = 3.042 \times 10^{-3}$ ft³/lb·°R

$(\partial s/\partial p)_T = -3.041 \times 10^{-3}$ ft³/lb·°R

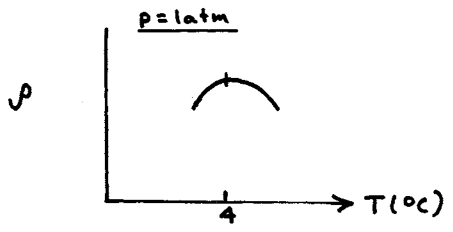
The results of the IT solution compare favorably with the graphical results.

PROBLEM 11.26

KNOWN: At 1atm, liquid water has a state of maximum density at about 4°C.

FIND: Determine what can be concluded about $(\partial s/\partial p)_T$ at (a) 3°C, (b) 4°C, (c) 5°C?

SCHEMATIC & GIVEN DATA:



ANALYSIS: From Eq. 11.35

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Since $\left(\frac{\partial \rho}{\partial T}\right)_p = \left(\frac{\partial v}{\partial T}\right)_p \frac{d\rho}{dv} = -\frac{1}{v^2}\left(\frac{\partial v}{\partial T}\right)_p \Rightarrow -\left(\frac{\partial v}{\partial T}\right)_p = \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial T}\right)_p$

Thus $\left(\frac{\partial s}{\partial p}\right)_T = \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial T}\right)_p \Rightarrow$

$(\partial s/\partial p)_T > 0$ for $T < 4°C$
$(\partial s/\partial p)_T = 0$ for $T = 4°C$
$(\partial s/\partial p)_T < 0$ for $T > 4°C$

PROBLEM 11.27

KNOWN: A gas enters a compressor and is compressed isentropically.

FIND: Does the specific enthalpy increase or decrease as the gas passes from inlet to exit?

ASSUMPTION: The compression occurs isentropically.

ANALYSIS: From Eq. 11.27

$$\left(\frac{\partial h}{\partial p}\right)_s = v$$

As v is positive and pressure p increases, the specific enthalpy must also increase.

PROBLEM 11.28

KNOWN: A fundamental thermodynamic function $u = u(s,v)$ is provided.

FIND: Show that T, p, h, ψ, and g can each be determined using $u(s,v)$.

ANALYSIS: From Eqs. 11.24 and 11.25 T and p can be found by differentiation

$$T = \left(\frac{\partial u}{\partial s}\right)_v$$
← T

$$p = -\left(\frac{\partial u}{\partial v}\right)_s$$
← p

Then, by definition $h = u + pv$, giving

$$h = u(s,v) + \left(-\left(\frac{\partial u}{\partial v}\right)_s\right)v$$
← h

Also, by definition $\psi = u - Ts$, giving

$$\psi = u(s,v) - \left(\frac{\partial u}{\partial s}\right)_v s$$
← ψ

Too, $g = h - Ts$, giving $g = u(s,v) - v(\partial u/\partial v)_s - s(\partial u/\partial s)_v$
← g

PROBLEM 11.29

KNOWN: The Helmholtz function is known for a certain substance.

FIND : Evaluate p, s, u, h, c_v, and c_p.

ANALYSIS: The Helmholtz function has the form

$$\psi = -RT\ln\frac{v}{v'} - cT'\left[1-\frac{T}{T'}+\frac{T}{T'}\ln\frac{T}{T'}\right]$$

where v', T' define the reference state and c is a constant. Note that at the reference state: when $v = v'$ and $T = T'$, $\psi = 0$.

With Eq. 11.28

$$p = -\left(\frac{\partial\psi}{\partial v}\right)_T = -\left(-\frac{RT}{v}\right) \Rightarrow \quad p = RT/v \qquad \longleftarrow p$$

With Eq. 11.29

$$s = -\left(\frac{\partial\psi}{\partial T}\right)_v = -\left(-R\ln\frac{v}{v'} - cT'\left[-\frac{1}{T'}+\frac{1}{T'}\ln\frac{T}{T'}+\frac{T}{T'}\left(\frac{1}{T}\right)\right]\right)$$

$$= R\ln\frac{v}{v'} + cT'\left[-\frac{1}{T'}+\frac{1}{T'}\ln\frac{T}{T'}+\frac{1}{T'}\right]$$

$$= R\ln\frac{v}{v'} + c\ln\frac{T}{T'} \qquad \longleftarrow s$$

Note that $s = 0$ when $v = v'$, $T = T'$.

Then, since $\psi = u - Ts$, $u = \psi + Ts$

$$u = \left[-RT\ln\frac{v}{v'} - cT'\left[1-\frac{T}{T'}+\frac{T}{T'}\ln\frac{T}{T'}\right]\right] + T\left[R\ln\frac{v}{v'} + c\ln\frac{T}{T'}\right]$$

$$= \cancel{-RT\ln\frac{v}{v'}} - cT' + cT - \cancel{cT\ln\frac{T}{T'}} + \cancel{RT\ln\frac{v}{v'}} + \cancel{cT\ln\frac{T}{T'}}$$

$$u = c\left[T - T'\right] \qquad \longleftarrow u$$

So, $u = 0$ when $T = T'$.

With $h = u + pv$,

$$h = c\left[T - T'\right] + RT \qquad \longleftarrow h$$

Further, $c_v = \partial u/\partial T)_v$, so

$$c_v = c \qquad \longleftarrow c_v$$

and $c_p = \partial h/\partial T)_p$, so

① $$c_p = c + R \qquad \longleftarrow c_p$$

---

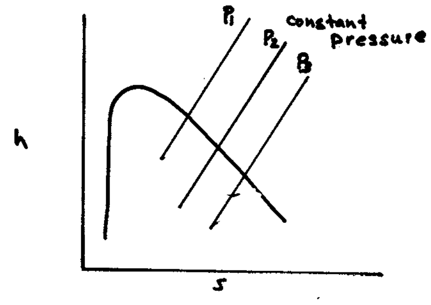
1. This substance adheres to the ideal gas model with constant c_v.

PROBLEM 11.30

KNOWN: The Mollier diagram gives a fundamental thermodynamic function $h = h(s, p)$.

FIND: Show that at any state fixed by s and p, the properties T, v, u, ψ, and g can be evaluated from the diagram.

SCHEMATIC & GIVEN DATA



ANALYSIS: From Eqs. 11.26 and 11.27 T and v can be found by differentiation (which can be performed graphically)

$$T = \left(\frac{\partial h}{\partial s}\right)_p \quad , \quad v = \left(\frac{\partial h}{\partial p}\right)_s$$

Then, since $h = u + pv$

$$u = h - pv = h(s,p) - p\left(\frac{\partial h}{\partial p}\right)_s$$

And with $\psi = u - Ts$

$$\psi = h(s,p) - p\left(\frac{\partial h}{\partial s}\right)_p - s\left(\frac{\partial h}{\partial s}\right)_p$$

Finally, using $g = h - Ts$

$$g = h(s,p) - s\left(\frac{\partial h}{\partial s}\right)_p$$

PROBLEM 11.31

KNOWN:

$$c_p = -T\left(\frac{\partial^2 g}{\partial T^2}\right)_p$$

FIND: Derive the above expression.

ANALYSIS: Since T and p are regarded as independent, the following functions can be considered: $u(T,p)$, $h(T,p)$, $s(T,p)$, etc. As $c_p = \partial h/\partial T)_p$, form the differential of $h(T,p)$

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

$$= c_p\, dT + \left(\frac{\partial h}{\partial p}\right)_T dp \qquad (1)$$

Also, forming the differential of $s(T,p)$

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \qquad (2)$$

Substituting Eqs. (1) and (2) into Eq 11.19 : $dh = T\,ds + v\,dp$

$$\left[c_p\, dT + \left(\frac{\partial h}{\partial p}\right)_T dp\right] = T\left[\left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp\right] + v\,dp$$

Collecting terms

$$\left[c_p - T\left(\frac{\partial s}{\partial T}\right)_p\right] dT = \left[T\left(\frac{\partial s}{\partial p}\right)_T + v - \left(\frac{\partial h}{\partial p}\right)_T\right] dp$$

As T and p are independent, fix p while varying T: $dp = 0$ and $dT \neq 0$; then

$$\left[c_p - T\left(\frac{\partial s}{\partial T}\right)_p\right] dT = 0 \quad \Rightarrow \quad \left[c_p - T\left(\frac{\partial s}{\partial T}\right)_p\right] \equiv 0$$

or

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p \qquad (3)$$

Next, introduce Eq. 11.31 : $-s = (\partial g/\partial T)_p$ to obtain on differentiation

$$-\left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial^2 g}{\partial T^2}\right)_p \qquad (4)$$

Finally, combining Eqs. (3) and (4)

$$c_p = -T\left(\frac{\partial^2 g}{\partial T^2}\right)_p \qquad \longleftarrow$$

which is the result to be obtained.

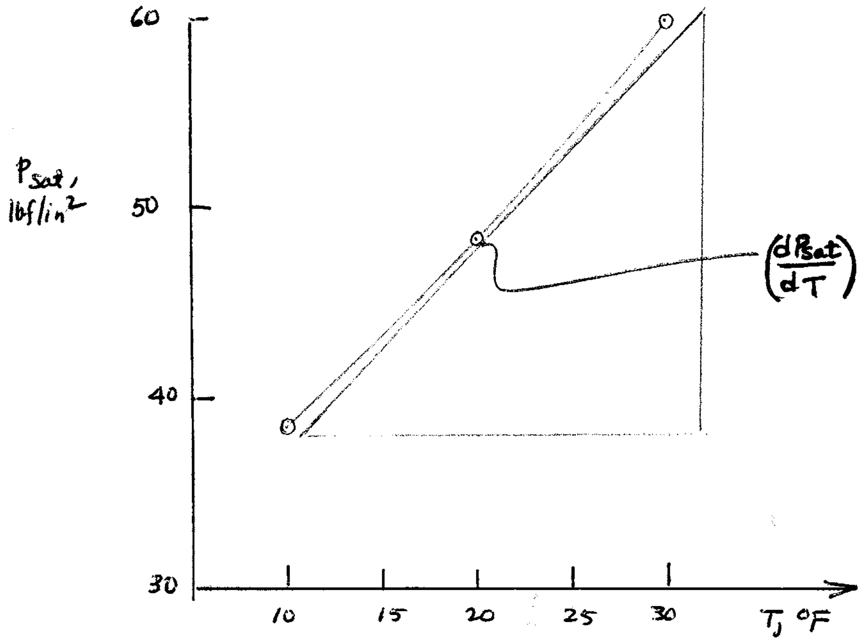
PROBLEM 11.32

KNOWN: p-v-T data for saturated ammonia are available from Table A-13E.

FIND: Determine at 20°F h_{fg}, u_{fg}, s_{fg}, and compare with table values.

ANALYSIS: From Table A-13, h_{fg} = 552.95 Btu/lb, u_{fg} = 500.46 Btu/lb, s_{fg} = 1.1528 Btu/lb·°R.

The value of h_{fg} can be determined from saturated p-v-T data using the Clapeyron equation, Eq. 11.40: $h_{fg} = T\, v_{fg} \left(\frac{dp}{dT}\right)_{sat}$. (1)
A graphical method can be used to obtain $(dp/dT)_{sat}$, as follows:



$$\left(\frac{dP_{sat}}{dT}\right) \approx \frac{(60-38)|144|}{(32-11)} \left(\frac{lbf/ft^2}{°R}\right)$$
$$= 150.9 \left(\frac{lbf/ft^2}{°R}\right)$$

Inserting values into Eq. (1)

$$h_{fg} = \frac{(479.67°R)(5.878\ \frac{ft^3}{lb})(150.9\ \frac{lbf}{°R\,ft^2})}{|778.17\ ft \cdot lbf/Btu|} = 546.7 \quad Btu/lb \longleftarrow h_{fg}$$

Then, with Eq. 11.38 $\longleftarrow s_{fg}$

$$s_{fg} = \frac{h_{fg}}{T} = \frac{546.7\ Btu/lb}{479.67\ °R} = 1.1397\ \frac{Btu}{lb \cdot °R}$$

Finally, with $h = u + pv$

$$u_{fg} = h_{fg} - p_{sat}\, v_{fg} = 546.7 - \frac{(48.224)|144|(5.878)}{|778.17\ ft \cdot lbf/Btu|} = 494.2\ \frac{Btu}{lb} \longleftarrow u_{fg}$$

Each of these is about 1% less than the corresponding table value. The values obtained using the graphical approach are sensitive to the accuracy of the slope $(dP/dT)_{sat}$ determined graphically, as above.

Alternatively, an IT solution like that of Example 11.4 can be employed:

PROBLEM 11.32 (Cont'd.)

<u>*IT* Code</u>

```
T = 20  // °F
dT = 0.01
T1 = T - dT
T2 = T + dT

p2 = Psat_T("Ammonia", T2)
p1 = Psat_T("Ammonia", T1)
dpdT_sat = ((p2 - p1) / (T2 - T1)) * 144
p = Psat_T("Ammonia", T)
vg = vsat_Px("Ammonia", p, 1)
vf = vsat_Px("Ammonia", p, 0)

hfg = ((T + 459.67) * (vg - vf) * dpdT_sat) / 778.17
sg = ssat_Px("Ammonia", p, 1)
sf = ssat_Px("Ammonia", p, 0)
sfg = hfg / (T + 459.67)
ufg = hfg - p * (vg - vf) * 144 / 778.67
```

<u>*IT* Results</u>

$(\partial p/\partial T)_{sat}$ = 152.5 lbf/ft^2·°R
h_{fg} = 552.6 Btu/lb
s_{fg} = 1.152 Btu/lb·°R
u_{fg} = 500.2 Btu/lb

PROBLEM 11.33

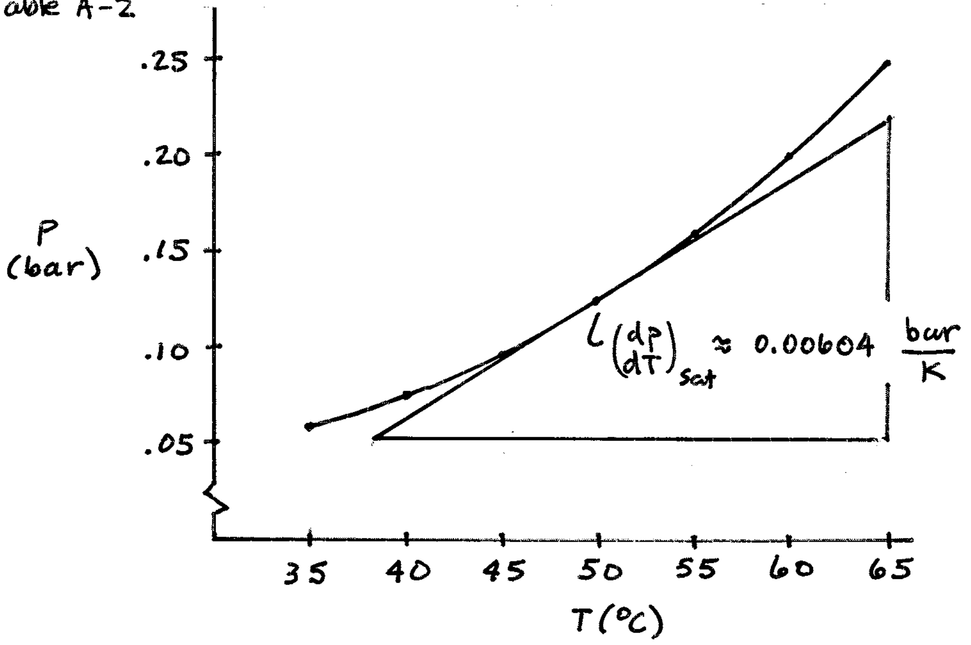
KNOWN: p-v-T data for water are available from the steam tables.

FIND: Determine at 50°C, (a) $h_g - h_f$, (b) $u_g - u_f$, and (c) $s_g - s_f$, and compare with table values.

ANALYSIS: The value of $(h_g - h_f)$ can be calculated using saturation data and the Clapeyron equation, Eq 11.40

$$h_g - h_f = T(v_g - v_f)\left(\frac{dp}{dT}\right)_{sat} \qquad (1)$$

To obtain the value for $(dp/dT)_{sat}$, prepare the plot shown below using data from Table A-2



Inserting values into Eq.(1)

$$h_g - h_f = (50 + 273.15)K(12.032 - 1.0121\times10^{-3})\frac{m^3}{kg}(0.00604\frac{bar}{K})\left|\frac{10^5 N/m^2}{1 bar}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right|$$

$$= 2348 \text{ kJ/kg}$$

With Eq. 11.38

$$s_g - s_f = \frac{h_g - h_f}{T} = \frac{2348}{323.15} = 7.266 \text{ kJ/kg}\cdot K$$

Further, using $u = h - pv$

$$u_g - u_f = (h_g - h_f) - p(v_g - v_f)$$

$$= (2348 \frac{kJ}{kg}) - (0.1235 \text{ bar})(12.032 - 1.021\times10^{-3})\frac{m^3}{kg}\left|\frac{10^5 N/m^2}{1 bar}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right|$$

$$= 2199.4 \text{ kJ/kg}$$

The respective table values are $h_g - h_f = 2382.7$ kJ/kg, $s_g - s_f = 7.3725$ kJ/kg·K, and $u_g - u_f = 2234.2$ kJ/kg. The percent difference between calculated and table values is about 1.5% for all quantities.

An altenative solution using IT follows:

PROBLEM 11.33 (Cont'd.)

IT Code

```
T = 50  // °C
dT = 0.001
T1 = T - dT
T2 = T + dT

p1 = Psat_T("Water/Steam", T1)
p2 = Psat_T("Water/Steam", T2)
dpdT_sat = ((p2 - p1) / (T2 - T1))  //  bar/K
p = Psat_T("Water/Steam", T)
vg = vsat_Px("Water/Steam", p, 1)
vf = vsat_Px("Water/Steam", p, 0)

hfg = (T + 273.15) * (vg - vf) * dpdT_sat * 100
sg = ssat_Px("Water/Steam", p, 1)
sf = ssat_Px("Water/Steam", p, 0)
sfg = sg - sf
ufg = hfg - p * (vg - vf) * 100
```

IT Results

$(\partial p/\partial T)_{sat}$ = 0.006127 bar
h_{fg} = 2382 kJ/kg
s_{fg} = 7.372 kJ/kg·K
u_{fg} = 2234 kJ/kg

The IT results compare very favorably with the table data.

PROBLEM 11.34

<u>KNOWN</u>: h_{fg}, v_{fg}, and p_{sat} at 10°F are known from the R134a tables.

<u>FIND</u>: Estimate p_{sat} at 20°F. Comment on the accuracy of this estimate.

<u>ASSUMPTIONS</u>: (1) The ratio h_{fg}/v_{fg} does not change significantly with temperature over the interval from 10° to 20°F. (2) v_f can be ignored relative to v_g, and v_g can be evaluated using the ideal gas model.

<u>ANALYSIS</u>: Table A-10E at 20°F gives $p_{sat} = 33.137\ \text{lbf/in}^2$.

<u>Method 1</u>: Assuming h_{fg}/v_{fg} is constant, Eq. 11.40 gives upon integration

$$(\Delta p)_{sat} = \frac{h_{fg}}{v_{fg}} \ln \frac{T_2}{T_1}$$

With data from Table A-10E at 10°F

$$(\Delta p)_{sat} = \left[\frac{88.53\ \text{Btu/lb}}{(1.7251 - 0.012)\ \text{ft}^3/\text{lb}}\right] \left|\frac{778\ \text{ft}\cdot\text{lbf}}{1\ \text{Btu}}\right| \left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right| \ln\left(\frac{480}{470}\right)$$

$$= 5.878\ \text{lbf/in}^2$$

At 10°F, $p_{sat} = 26.651\ \text{lbf/in}^2$. So,

$$p_{sat}(20°\text{F}) \approx 26.651 + 5.878 = 32.529\ \text{lbf/in}^2 \quad \longleftarrow$$

This value is about 1.8% less than the table value.

<u>Method 2</u>: With assumption 2, Eq. 11.40 can be converted to the Clausius-Clapeyron equation: Eq. 11.42, which upon integration gives

$$\ln \frac{p_2}{p_1} = -\frac{h_{fg}}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

Thus

$$\ln \frac{p_2}{p_1} = -\frac{(88.53\ \text{Btu/lb})}{\left(\frac{1.986}{120.92}\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)}\left[\frac{1}{480} - \frac{1}{470}\right]\left(\frac{1}{°\text{R}}\right)$$

$$\Rightarrow \quad \frac{p_2}{p_1} = 1.27 \Rightarrow p_2 = 33.85\ \text{lbf/in}^2 \quad \longleftarrow$$

This is about 2% above the table value.

<u>Discussion</u>. Even though the assumptions are only satisfied approximately, satisfactory results are obtained with both methods. Excellent agreement with the table value is achieved with the following IT approach based on integrating Eq. 11.40 without making the above assumptions:

<u>*IT* Code</u>

```
hg = hsat_Px("R134A", p, 1)
hf = hsat_Px("R134A", p, 0)
vf = vsat_Px("R134A", p, 0)
vg = vsat_Px("R134A", p, 1)
p = Psat_T("R134A", T)

// Clapeyron Eq. 11.40
dpdT_sat = ((hg - hf) / ((vg - vf) * (T + 459.67))) * (778.17 / 144)
psat = Integral(dpdT_sat,T) + 26.651
```

<u>*IT* Result (Sweep T from 10 to 20°F in steps of 0.1.)</u>

$p_{sat}(20°\text{F}) = 33.14\ \text{lbf/in.}^2$

PROBLEM 11.35

KNOWN: h_{fg}, v_{fg}, and p_{sat} at 26°C are known from the ammonia tables.

FIND: Estimate p_{sat} at 30°C. Comment on the accuracy of this estimate.

ASSUMPTIONS: (1) The ratio h_{fg}/v_{fg} does not change significantly with temperature over the interval from 26°C to 30°C. (2) v_f can be ignored relative to v_g, and v_g can be evaluated using the ideal gas model.

ANALYSIS: Table A-13 gives at 26°C,

$$p_{sat} = 10.3602 \text{ bar}, \quad v_{fg} = 0.1229 \frac{m^3}{kg}, \quad h_{fg} = 1160.7 \text{ kJ/kg}$$

Method #1. With assumption (1), Eq. 11.40 gives on integration

$$(\Delta p)_{sat} = \frac{h_{fg}}{v_{fg}} \ln \frac{T_2}{T_1} = \left(\frac{1160.7 \text{ kJ/kg}}{0.1229 \text{ m}^3/\text{kg}}\right) \left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right| \left|\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right| \ln\left(\frac{303}{299}\right)$$

$$= 1.255 \text{ bar}$$

$$\Rightarrow \quad p_{sat}(30°C) \approx 10.3602 + 1.255 = 11.615 \text{ bar}.$$

Comparing with the table value: 11.6865 bar, this is about 0.6% low.

Method #2. With assumption (2), Eq. 11.40 can be converted to the Clausius - Clapeyron equation: Eq. 11.42, which on integration gives

$$\ln \frac{p_2}{p_1} = -\frac{h_{fg}}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right] = -\left(\frac{1160.7 \text{ kJ/kg}}{\frac{8314}{17.04} \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}}\right)\left(\frac{1}{303\text{K}} - \frac{1}{299\text{K}}\right)\left|\frac{10^3 \text{ N}\cdot\text{m}}{1 \text{ kJ}}\right|$$

$$\Rightarrow \quad \frac{p_2}{p_1} = 1.11 \Rightarrow p_2 = 11.5 \text{ bar}$$

This is about 1.6% below the table value.

Discussion. Even though the assumptions are only satisfied approximately, satisfactory results are obtained with both methods. Excellent agreement with the table value is achieved with the following IT approach based on integrating Eq. 11.40 without making the above assumptions.

IT Code

```
hg = hsat_Px("Ammonia", p, 1)
hf = hsat_Px("Ammonia", p, 0)
vg = vsat_Px("Ammonia", p, 1)
vf = vsat_Px("Ammonia", p, 0)
p = Psat_T("Ammonia", T)
```

```
// Clapeyron Eq. 11.40
dpdT_sat = ((hg - hf) / ((vg - vf) * (T + 273.15))) / 100
psat = Integral(dpdT_sat,T) + 10.3602
```

IT Result (Sweep T from 26 to 30 in steps of 0.1.)

$p_{sat}(30°C) = 11.69$ bar

PROBLEM 11.36

KNOWN: Triple point data for water are available from Table A-6E.

FIND: Estimate P_{sat} at $-40°F$ and compare with the table value, which is 0.0019 lbf/in^2 from Table A-6E.

ASSUMPTIONS: (1) The value of h_{ig} is constant independent of temperature over the interval from the triple point temperature to $-40°F$, (2) v_i can be ignored relative to v_g and v_g can be evaluated using the ideal gas model.

ANALYSIS: With the assumptions listed, Eq. 11.41 can be converted to give

$$\left(\frac{d \ln p}{dT}\right)_{sat} = \frac{h_{ig}}{RT^2}$$

which gives upon integration

$$\ln \frac{P_2}{P_1} = \frac{h_{ig}}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

With data from Table A-6E: $h_{ig} = 1218.7$ Btu/lb, $P_1 = 0.0887$ lbf/in^2

$$\ln \frac{P_2}{P_1} = \frac{(1218.7 \text{ Btu/lb})}{\left(\frac{1.986}{18.02}\frac{\text{Btu}}{\text{lb}\cdot°R}\right)}\left[\frac{1}{492°R} - \frac{1}{420°R}\right]$$

$$\Rightarrow \frac{P_2}{P_1} = 0.02122$$

$$\Rightarrow P_2 = 0.00188 \text{ lbf/in}^2$$

Discussion: The calculated value agrees closely with the table value. This is an expected outcome, for by reference to Table A-6E data the two assumptions are closely adhered to in this case.

PROBLEM 11.37

KNOWN: For water at 0°C

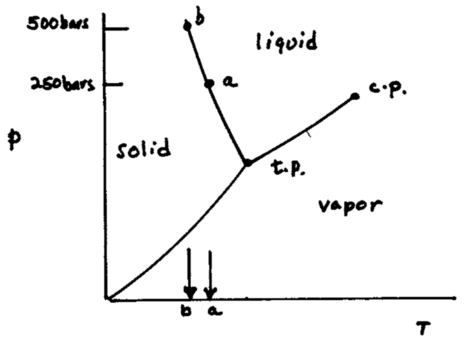
$v_i = 1.0911 \times 10^{-3}\ m^3/kg$

$v_f = 1.0002 \times 10^{-3}\ m^3/kg$

$h_{if} = 333.4\ kJ/kg$

FIND: Estimate the melting temperature of ice at (a) 250 bar, (b) 500 bar.

SCHEMATIC & GIVEN DATA:



Not to scale

ASSUMPTIONS: h_{if} and v_{if} do not vary significantly with temperature.

ANALYSIS: With the assumptions listed, Eq 11.41 gives upon integration

$$\Delta p = \left(\frac{h_f - h_i}{v_f - v_i}\right) \ln \frac{T_2}{T_1}$$

$$= \left[\frac{333.4\ kJ/kg}{\left(\frac{1.0002 - 1.0911}{10^3}\right)\frac{m^3}{kg}}\right]\left|\frac{10^3\ N \cdot m}{kJ}\right|\left|\frac{bar}{10^5\ N/m^2}\right| \ln\frac{T_2}{T_1}$$

or

$$\Delta p = -(36,677.7\ bars) \ln \frac{T_2}{T_1}$$

At 0°C, the pressure is negligible relative to P_2: $\Delta p \approx P_2$.

(a) $P_2 = 250$ bar

$$\ln\frac{T_2}{273.15} = -\frac{250}{36,677.7} \Rightarrow T_2 = 271.29\ K\ (-1.86\ °C) \longleftarrow \text{(a)}$$

(b) $P_2 = 500$ bar.

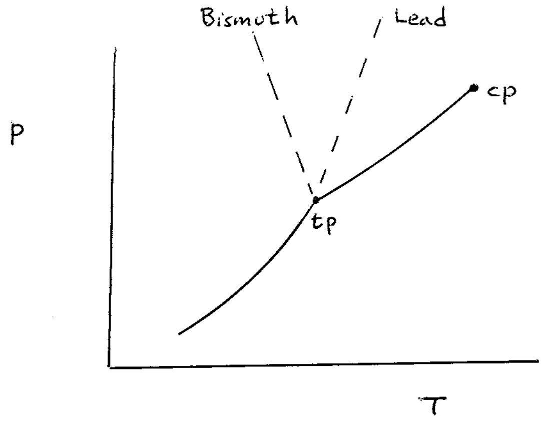
$$\ln\frac{T_2}{273.15} = \frac{-500}{36,677.7} \Rightarrow T_2 = 269.45\ K\ (-3.7\ °C) \longleftarrow \text{(b)}$$

PROBLEM 11.38

KNOWN: Bismuth and lead are the substances under consideration.

FIND: Confirm that the line representing the two-phase solid-liquid region on the phase diagram slopes to the left for bismuth and to the right for lead.

SCHEMATIC & GIVEN DATA:



ANALYSIS: The key expression is Eq 11.41

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{h'' - h'}{T(v'' - v')}$$

where ′ denotes solid and ″ denotes liquid. As $(h'' - h')$ would be positive, the slope is determined by the difference $(v'' - v')$.

The CRC Handbook of Chemistry and Physics, 37th Ed, 1955-1956, gives a table of the change in volume due to fusion for selected substances, p. 2138. For the substances under consideration

Bismuth: change in volume = $-0.0034\ cm^3/g$
Lead: " " " = $+0.0034\ cm^3/g$

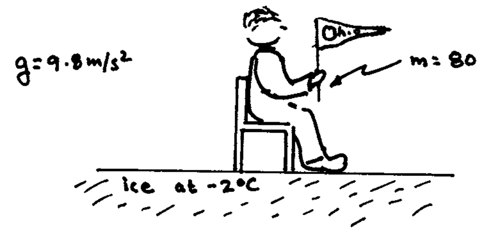
Accordingly, $(dP/dT)_{sat} < 0$ for bismuth and $(dP/dT)_{sat} > 0$ for lead.

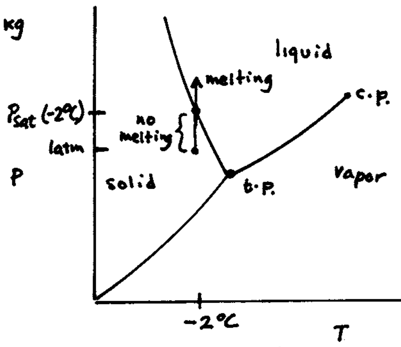
PROBLEM 11.39

KNOWN: An occupied chair of total mass equal to 80 kg is at rest on an ice rink. The ice temperature is −2°C.

FIND: Determine the minimum total area the tips of the chair legs can have before the ice in contact with the legs would melt.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The ice temperature remains at −2°C. (2) The local acceleration of gravity is 9.8 m/s². (3) At 0°C, $h_{if} = 333.4\ kJ/kg$, $v_i = 1.0911 \times 10^{-3}\ m^3/kg$, $v_f = 1.0002 \times 10^{-3}\ m^3/kg$, h_{if} and v_{if} remain constant with temperature.

ANALYSIS: As shown by the phase diagram, melting would take place if the pressure exerted on the ice (at −2°C) would exceed $p_{sat}(-2°C)$.

The pressure $p_{sat}(-2°C)$ can be estimated using Eq. 11.41, together with the assumption that h_{if} and v_{if} remain constant with temperature, for then the equation becomes on integration

$$\Delta p = \left(\frac{h_f - h_i}{v_f - v_i}\right) \ln \frac{T_2}{T_1}$$

Inserting values for h_{if}, v_i, v_f and taking $T_1 = 273.15\ K\ (0°C)$, $T_2 = 271.15\ K\ (-2°C)$

$$\Delta p = \left[\frac{\left(333.4\ \frac{kJ}{kg}\right)\left|\frac{1000\ N\cdot m}{kJ}\right|}{\left(\frac{1.0002 - 1.0911}{1000}\right)\left(\frac{m^3}{kg}\right)}\right]\left|\frac{bar}{10^5\ N/m^2}\right| \ln \frac{271.15}{273.15} = 269.54\ bar$$

Since $p_{sat}(0°C) = 0.61\ kPa$, $p_{sat}(-2°C) = 269.55$ bar.

The pressure exerted on the ice by the chair/occupant and the atmosphere is

$$p = p_{atm} + \frac{mg}{A}$$

where A is the total area of the tips of the 4 chair legs. The pressure p can be no greater than 269.54 bar. Thus the minimum total area A is

$$269.55\ bar = 1.01325\ bar + \frac{(80\ kg)(9.8\ m/s^2)}{A\ (m^2)}\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|\left|\frac{bar}{10^5\ N/m^2}\right|$$

or

$$A_{min} = (2.92 \times 10^{-5}\ m^2)\left|\frac{100\ cm}{m}\right|^2$$

$$= 0.292\ cm^2 \xleftarrow{\qquad A_{min} \qquad}$$

PROBLEM 11.40

KNOWN: Over a certain temperature interval the saturation pressure-temperature curve of a substance is represented by

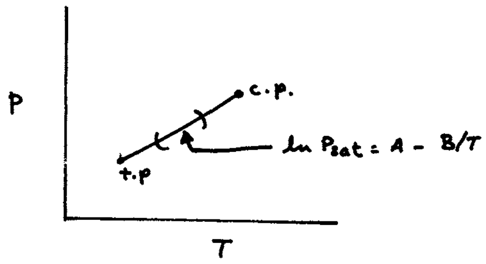
$$\ln p_{sat} = A - B/T$$

where A and B are constants.

FIND: (a) Obtain expressions for h_{fg} and s_{fg} in terms of p-v-T data and B.

(b) Using the result of (a), determine h_{fg} and s_{fg} for water vapor at 25°C and compare with steam table data.

SCHEMATIC & GIVEN DATA:



ANALYSIS: (a) The Clapeyron equation takes the form

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{h_{fg}}{T\,v_{fg}} \qquad (1)$$

To obtain $(dp/dT)_{sat}$, differentiate the given expression for $\ln p_{sat}$

$$\frac{d}{dT}(\ln p_{sat}) = +\frac{B}{T^2}$$

$$\frac{1}{p_{sat}}\frac{dp_{sat}}{dT} = \frac{B}{T^2}$$

or

$$\left(\frac{dp}{dT}\right)_{sat} = \frac{p_{sat}(T)\,B}{T^2} \qquad (2)$$

Combining Eqs. (1) and (2)

$$h_{fg} = \frac{p_{sat}(T)\,v_{fg}(T)\,B}{T} \qquad (3) \longleftarrow h_{fg}$$

Then, with Eq. 11.38: $s_{fg} = h_{fg}/T$

$$s_{fg} = \frac{p_{sat}(T)\,v_{fg}(T)\,B}{T^2} \qquad (4) \longleftarrow s_{fg}$$

(b) Using standard computer routines, the values of A and B are found such that

$$\ln p_{sat} = \overbrace{28.83593}^{Ⓐ} - \overbrace{\frac{5299.62}{T}}^{Ⓑ} \qquad (p_{sat}: \text{Pa},\ T: \text{K}) \qquad (5)$$

Then, with Eq. (3), B from Eq. (5), and p_{sat}, v_{fg} from Table A-2 at 25°C

① $$h_{fg} = \frac{(0.03169\times10^5\ \frac{N}{m^2})(43.359\ \frac{m^3}{kg})(5299.62\ K)}{298.15\ K}\left|\frac{kJ}{10^3\ N\cdot m}\right| = 2442.4\ \frac{kJ}{kg} \quad \text{(table value: 2442.3 kJ/kg)}$$

Similarly, Eq. (4) gives

$$s_{fg} = \frac{(0.03169\times10^5)(43.359)(5299.62)}{(298.15)^2}\left|\frac{1}{10^3}\right| = 8.192\ \frac{kJ}{kg\cdot K} \quad \text{(table value: 8.1906 kJ/kg·K)}$$

---

1. In this case, Excellent agreement is realized with steam table data.

PROBLEM 11.41

KNOWN: Data for water from Table A-2 is available for use.

FIND: Determine the constants A and B to give the best fit in a least-squares sense in the interval from 20° to 30°C by the equation $\ln p_{sat} = A - B/T$. Using this equation determine dp_{sat}/dT at 25°C. Calculate h_{fg} at 25°C and compare with the steam-table value.

ANALYSIS: Using standard computer routines, the values of A and B are found so that

$$\ln p_{sat} = -1.999728 - \frac{35.767963}{T} \qquad (p_{sat}: \text{bar}, \ T: {}^\circ\text{C}) \qquad (1)$$

or

$$\ln p_{sat} = 25.83593 - \frac{5299.62}{T} \qquad (p_{sat}: \text{Pa}, \ T: \text{K}) \qquad (2)$$

Using Eq. (2),

$$p_{sat} = \exp\left(25.83593 - \frac{5299.62}{T}\right)$$

Thus

$$\frac{dp_{sat}}{dT} = \frac{5299.62}{T^2} \exp\left(25.83593 - \frac{5299.62}{T}\right)$$

At 298.15K, $dp_{sat}/dT = 188.88$ Pa/K.

Using Eq. 11.40 and v_{fg} from Table A-2 at 25°C

$$h_{fg} = T v_{fg} \left(\frac{dp}{dT}\right)_{sat}$$

$$= (298.15\,\text{K})\left(\frac{43360 - 1.0029}{10^3}\right)\left(\frac{\text{m}^3}{\text{kg}}\right)\left(188.88\,\frac{\text{N/m}^2}{\text{K}}\right)\left|\frac{\text{kJ}}{10^3\,\text{N}\cdot\text{m}}\right|$$

$$= 2441.7 \text{ kJ/kg}$$

Table A-2 gives at 25°C, 2442.3 kJ/kg. Accordingly the calculated and table values of h_{fg} agree extremely well.

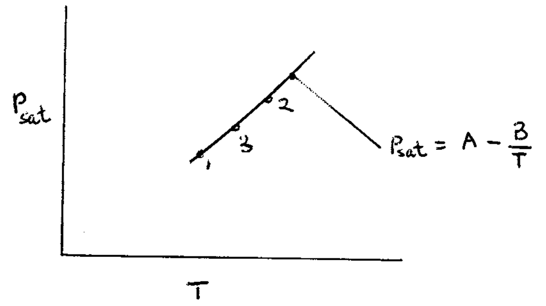
PROBLEM 11.42

KNOWN: The saturation pressure-temperature curve for two-phase liquid-vapor states is represented by $\ln p_{sat} = A - B/T$ where A, B are constants.

FIND: Show that any three points on the cure are related by

$$\frac{p_{sat,3}}{p_{sat,1}} = \left[\frac{p_{sat,2}}{p_{sat,1}}\right]^{\tau} \quad \text{where} \quad \tau = \frac{T_2\,[T_3 - T_1]}{T_3\,[T_2 - T_1]}$$

SCHEMATIC & GIVEN DATA:



ANALYSIS: Using the given expression

$$\ln(p_{sat,1}) = A - \frac{B}{T_1} \Rightarrow A = \ln(p_{sat,1}) + \frac{B}{T_1}$$

$$\ln(p_{sat,2}) = A - \frac{B}{T_2} \Rightarrow \ln(p_{sat,2}) = \ln(p_{sat,1}) + \frac{B}{T_1} - \frac{B}{T_2}$$

$$\Rightarrow \ln\left(\frac{p_{sat,2}}{p_{sat,1}}\right) = B\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\ln(p_{sat,3}) = A - \frac{B}{T_3} \Rightarrow \ln\left(\frac{p_{sat,3}}{p_{sat,1}}\right) = B\left[\frac{1}{T_1} - \frac{1}{T_3}\right]$$

Combining,

$$\frac{\ln\left(\frac{p_{sat,2}}{p_{sat,1}}\right)}{\ln\left(\frac{p_{sat,3}}{p_{sat,1}}\right)} = \frac{\frac{1}{T_1} - \frac{1}{T_2}}{\frac{1}{T_1} - \frac{1}{T_3}} = \frac{\frac{T_2 - T_1}{T_1 T_2}}{\frac{T_3 - T_1}{T_1 T_3}} = \frac{T_3}{T_2} \cdot \frac{T_2 - T_1}{T_3 - T_1} = \frac{1}{\tau}$$

where $\tau \equiv \frac{T_2}{T_3} \cdot \frac{T_2 - T_1}{T_3 - T_1}$. Then

$$\ln\left(\frac{p_{sat,3}}{p_{sat,1}}\right) = \tau \ln\left(\frac{p_{sat,2}}{p_{sat,1}}\right)$$

$$\Rightarrow \quad \frac{p_{sat,3}}{p_{sat,1}} = \left[\frac{p_{sat,2}}{p_{sat,1}}\right]^{\tau} \quad \longleftarrow$$

PROBLEM 11.43

KNOWN: The result of Problem 11.42 is applicable.

FIND: Using saturation pressure temperature data from Table A-2
(a) evaluate p_{sat} at 30°C and compare with the tabulated value.
(b) evaluate T_{sat} at 0.006 MPa and compare with the tabulated value.

ANALYSIS: The result of Problem 11.42 is

$$\frac{p_{sat,3}}{p_{sat,1}} = \left[\frac{p_{sat,2}}{p_{sat,1}}\right]^{\tau} \quad \text{where} \quad \tau = \frac{T_2}{T_3}\left[\frac{T_3 - T_1}{T_2 - T_1}\right]$$

(a) $T_1 = 20°C$, $T_2 = 30°C$, $T_3 = 40°C$. From Table A-2,

$$p_{sat,1} = 0.02339 \text{ bar}$$
$$p_{sat,3} = 0.07384 \text{ bar}$$

$$\tau = \frac{T_2}{T_3}\left(\frac{T_3 - T_1}{T_2 - T_1}\right) = \frac{303.15}{313.15}\left(\frac{20}{10}\right) = 1.9361$$

Accordingly

$$\ln\frac{p_{sat,2}}{p_{sat,1}} = \frac{1}{\tau}\ln\frac{p_{sat,3}}{p_{sat,1}} = \frac{1}{1.9361}\ln\left(\frac{0.07384}{0.02339}\right)$$

$$\Rightarrow \quad \frac{p_{sat,2}}{p_{sat,1}} = 1.8108 \quad \Rightarrow \quad p_{sat,2} = 0.04235 \text{ bar} \quad \longleftarrow$$

Table A-2 gives $p_{sat,2} = 0.04246$ bar. Accordingly the percent difference is

$$\% = \left(\frac{0.04235 - 0.04246}{0.04246}\right)(100) = -0.26$$

(b) $T_1 = 20°C$, $p_2 = 0.006$ MPa, $T_3 = 40°C$. $p_{sat,1}$ and $p_{sat,3}$ are given in part (a).
Accordingly

$$\tau = \frac{\ln\left(\frac{p_{sat,3}}{p_{sat,1}}\right)}{\ln\left(\frac{p_{sat,2}}{p_{sat,1}}\right)} = \frac{\ln\left(\frac{0.07384}{0.02339}\right)}{\ln\left(\frac{0.06}{0.02339}\right)} = 1.22033$$

And

$$\tau = \frac{T_2\,[20]}{313.15\,[T_2 - 293.15]}$$

So

$$20\,T_2 = 1.22033\left[313.15\,[T_2 - 293.15]\right]$$

$$\Rightarrow \quad T_2 = 309.34\text{ K} \quad (36.19°C) \quad \longleftarrow$$

Table A-3 gives $T = 36.16°C$. The percent difference is

$$\% = \left(\frac{36.19 - 36.16}{36.16}\right)(100) = 0.08$$

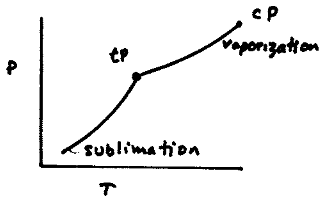
PROBLEM 11.44

KNOWN: Four exercises dealing with slopes are described.

FIND: (a) Using steam table data, evaluate the ratio of the slope of the vaporization line to the slope of the sublimation line. (b) On a T-s diagram, show that the slope of a constant specific volume line is greater than the slope of a constant pressure line through the same superheated vapor state. (c) For an h-s diagram, obtain an expression for the slope of a constant pressure line in terms of p-v-T data only. (d) For a p-h diagram, obtain an expression for the slope of an isentropic line in terms of p-v-T data only.

ANALYSIS: (a) Using the Clapeyron equation, Eq. 11.40

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{1}{T}\frac{\Delta h}{\Delta v}$$



Accordingly at the triple point

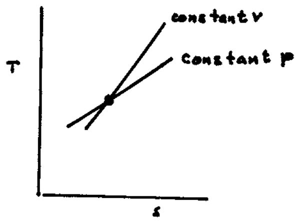
$$\frac{(dP/dT)_{sat}]_{subl.}}{(dP/dT)_{sat}]_{vap}} = \frac{[h_g - h_i]/[v_g - v_i]}{[h_g - h_f]/[v_g - v_f]}$$

With data from Tables A-2 and A-6 at 0.01 °C

$$\frac{(dp/dT)_{sat}]_{subl}}{(dp/dT)_{sat}]_{vap}} = \frac{[2834.8]/(206100 - 1.0908)}{[2501.3]/(206136 - 1.0002)} = 1.134 \longleftarrow \text{(a)}$$

(b) With Eqs. 11.46 and 11.55

$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{c_v}{T}, \quad \left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p}{T}$$



Or, using Eq. 11.15

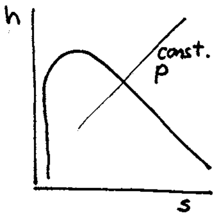
$$\left(\frac{\partial T}{\partial s}\right)_v = \frac{T}{c_v}, \quad \left(\frac{\partial T}{\partial s}\right)_p = \frac{T}{c_p}$$

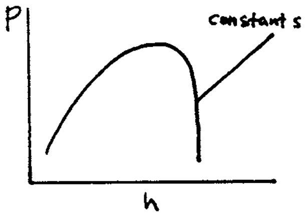
Forming a ratio

$$\frac{(\partial T/\partial s)_v}{(\partial T/\partial s)_p} = \frac{T/c_v}{T/c_p} = \frac{c_p}{c_v} = k > 1 \longleftarrow \text{(b)}$$

(c) and (d) Eq. 11.19 reads

$$dh = T\,ds + v\,dp$$





The function h(s,p) gives

$$dh = \left(\frac{\partial h}{\partial s}\right)_p ds + \left(\frac{\partial h}{\partial p}\right)_s dp$$

By comparison

(11.26): $$\left(\frac{\partial h}{\partial s}\right)_p = T \longleftarrow \text{(c)}$$

(11.27): $$\left(\frac{\partial h}{\partial p}\right)_s = v$$

With Eq. 11.15, the second of these gives

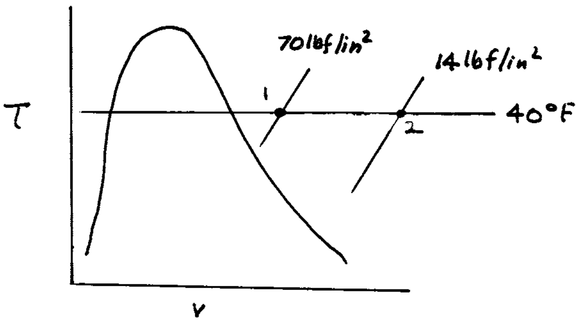
$$\left(\frac{\partial p}{\partial h}\right)_s = \frac{1}{v} \longleftarrow \text{(d)}$$

PROBLEM 11.45

KNOWN: NH_3 undergoes a process from 70 lbf/in², 40°F to 14 lbf/in², 40°F.

FIND: Using p-v-T data, evaluate (h_2-h_1) and (s_2-s_1), and compare with table values.

SCHEMATIC & GIVEN DATA:



ANALYSIS: Since the states under consideration are in the superheated vapor region, temperature and pressure can be selected as the independent properties that fix the state. Then, Eqs. 11.59 and 11.60 give, respectively

① $$s(T,p_2)-s(T,p_1)=\int_{p_1}^{p_2}\left(\frac{\partial v}{\partial T}\right)_p(T,p)\,dp \tag{1}$$

$$h(T,p_2)-h(T,p_1)=\int_{p_1}^{p_2}\left[v(T,p)-T\left(\frac{\partial v}{\partial T}\right)_p(T,p)\right]dp \tag{2}$$

Consequently, the analysis reduces to the evaluation of $(\partial v/\partial T)_p$ as a function of pressure at 40°F, and the subsequent use of these data to obtain the required integrals. A numerical procedure using data obtained from the source listed for Tables A-13 is suggested. The values of Δh and Δs obtained directly from table data are, respectively

$$h_2-h_1 = 14.46 \text{ Btu/lb}$$
$$s_2-s_1 = 0.2103 \text{ Btu/lb}\cdot°R$$

---

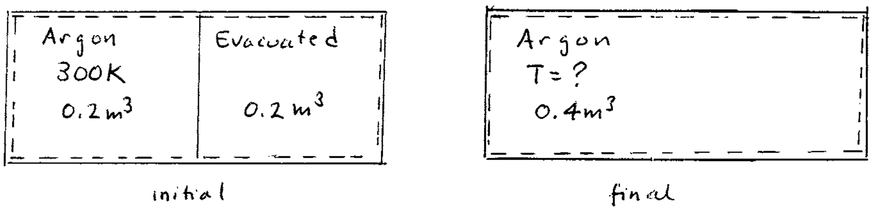
1. Since the integrations are at fixed temperature, the terms of Eqs. 11.59 and 11.60 involving the specific heat c_p drop out of consideration, thereby allowing the required property changes to be found using only p-v-T data. These integrations require appropriate software.

PROBLEM 11.46

KNOWN: One kmol of Ar initially occupies one side of a container divided by a partition. The other side is evacuated. The partition is removed and the Ar expands to fill the entire volume.

FIND: Find the final temperature of the argon using (1) the van der Waals equation of state, (2) the ideal gas model.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is shown in the figure. (2) For the system, $Q = W = 0$, kinetic and potential energy effects are absent, and the partition can be ignored. (3) The argon is modeled by the van der Waals equation of state and then by the ideal gas model.

ANALYSIS: Using indicated assumptions, the energy balance reduces to give $\Delta U = 0$, or since mass is fixed $\Delta u = 0$. Invoking Eq. 11.51

$$\Delta u = \int_{①}^{②} c_v\,dT + \int_{①}^{②} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv = 0 \qquad (1)$$

- Van der Waals Equation. Since $p = \dfrac{RT}{v-b} - \dfrac{a}{v^2}$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}, \qquad \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] = \frac{RT}{v-b} - \left[\frac{RT}{v-b} - \frac{a}{v^2}\right] = \frac{a}{v^2}$$

To investigate c_v, the test for exactness applied to Eq 11.48 gives

$$\left.\frac{\partial(c_v/T)}{\partial v}\right)_T = \frac{\partial}{\partial T}\left(\left(\frac{\partial p}{\partial T}\right)_v\right)_v \Rightarrow \left.\frac{\partial(c_v/T)}{\partial v}\right)_T = \frac{\partial}{\partial T}\left(\frac{R}{v-b}\right) = 0 \Rightarrow \left.\frac{\partial c_v}{\partial v}\right)_T \equiv 0.$$ This implies

c_v is independent of v and can depend only on T. Accordingly, it can be concluded that $c_v = 3/2\,R$, which is the limiting value for a monatomic gas such as Ar. Inserting results into Eq. (1), and integrating

$$\frac{3}{2}R\,[T_2 - T_1] - a\left[\frac{1}{v_2} - \frac{1}{v_1}\right] = 0$$

$$\Rightarrow \quad T_2 - T_1 = \frac{2a}{3R}\left[\frac{1}{v_2} - \frac{1}{v_1}\right]$$

$$= \frac{2\left(\frac{27}{64}\frac{\bar{R}^2 T_c^2}{p_c}\right)}{3\bar{R}}\left[\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1}\right] = \frac{9}{32}\left[\frac{\bar{R}T_c^2}{p_c}\right]\left[\frac{1}{\bar{v}_2} - \frac{1}{\bar{v}_1}\right]$$

With $\bar{v}_1 = 0.2\ \text{m}^3/\text{kmol}$, $\bar{v}_2 = 0.4\ \text{m}^3/\text{kmol}$, and data from Table A-1

$$T_2 - T_1 = \frac{9}{32}\left[8314\frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}\right]\left[151\,\text{K}\right]^2\left[\frac{1}{48.6\times10^5\ \text{N/m}^2}\right]\left[\frac{1}{0.4} - \frac{1}{0.2}\right]\frac{\text{kmol}}{\text{m}^3}$$

$$= -27.43\ \text{K} \Rightarrow T_2 = 272.6\ \text{K} \quad \longleftarrow$$

- ideal gas model. Since $p = RT/v$, $(\partial p/\partial T)_v = R/v$ and $[T(\partial p/\partial T)_v - p] \equiv 0$. Thus, with $c_v = 5/2\,R$, Eq. (1) becomes

$$\frac{3}{2}R\,[T_2 - T_1] = 0 \Rightarrow T_2 = T_1 = 300\,\text{K} \quad \longleftarrow$$

PROBLEM 11.47

KNOWN: A gas obeys the equation of state $p(v-b) = RT$

FIND: Obtain the relationship between c_p and c_v for this gas.

ANALYSIS:

Method 1. Using Eq. 11.47

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p \qquad (1)$$

The given equation of state can be rearranged to read

$$p = \frac{RT}{v-b}$$

Then

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b} \qquad (2)$$

Combining Eqs. (1) and (2)

$$\left(\frac{\partial u}{\partial v}\right)_T = \frac{RT}{v-b} - p \equiv 0 \Rightarrow u = u(T) \quad (u \text{ depends on } T \text{ only})$$

Since $h = u + pv$, the above equation indicates that

$$h(T,p) = u(T) + pv$$

Then, since $c_p = (\partial h/\partial T)_p$ and $c_v = (\partial u/\partial T)_v = du/dT$

$$c_p = \frac{du}{dT} + \left.\frac{\partial(pv)}{\partial T}\right)_p = c_v + p\left(\frac{\partial v}{\partial T}\right)_p \qquad (3)$$

The equation of state can be expressed as

$$v = \frac{RT}{p} + b \Rightarrow \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p}$$

Inserting this into Eq. (3)

① $$c_p = c_v + p\left(\frac{R}{p}\right) = c_v + R \qquad \longleftarrow$$

Method 2. Using Eq. 11.68

$$c_p - c_v = -T\left(\frac{\partial v}{\partial T}\right)_p^2 \left(\frac{\partial p}{\partial v}\right)_T$$

From above, $(\partial v/\partial T)_p = R/p$, and using $p = RT/(v-b)$

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2}$$

Collecting results

$$c_p - c_v = -T\left(\frac{R}{p}\right)^2\left(-\frac{RT}{(v-b)^2}\right)$$

$$= -T\left(\frac{R}{RT/(v-b)}\right)^2\left(\frac{-RT}{(v-b)^2}\right) = R$$

or

$$c_p = c_v + R \qquad \longleftarrow$$

---

1. Method 1 also brings out the conclusion that the specific internal energy, u, can only vary with temperature. Thus, c_v (and c_p) can only vary with temperature as well.

PROBLEM 11.48

KNOWN: The p-v-T relation for a certain gas is

$$v = \frac{RT}{p} + B - \frac{A}{RT}$$

where A and B are constants.

FIND: Obtain expressions for $[h(p_2,T) - h(p_1,T)]$, $[u(p_2,T) - u(p_1,T)]$, and $[s(p_2,T) - s(p_1,T)]$.

ANALYSIS: As T and p are the independent properties that fix the state and an equation of state explicit in v is known, Eqs. 11.59 and 11.60 are convenient to use here. Each of these requires $(\partial v/\partial T)_p$. Thus, with the given equation of state

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{A}{RT^2}$$

Then, Eq. 11.59 reads

① $$s(p_2,T) - s(p_1,T) = -\int_{p_1}^{p_2} \left(\frac{\partial v}{\partial T}\right)_p dp = -\int_{p_1}^{p_2} \left[\frac{R}{p} + \frac{A}{RT^2}\right] dp$$

$$= -R \ln\frac{p_2}{p_1} - \frac{A}{RT^2}[p_2 - p_1] \longleftarrow \Delta s$$

Similarly, Eq. 11.60 gives

$$h(p_2,T) - h(p_1,T) = \int_{p_1}^{p_2} \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$= \int_{p_1}^{p_2} \left[\left(\frac{RT}{p} + B - \frac{A}{RT}\right) - T\left(\frac{R}{p} + \frac{A}{RT^2}\right)\right] dp$$

$$= \int_{p_1}^{p_2} \left(B - \frac{2A}{RT}\right) dp = \left[B - \frac{2A}{RT}\right](p_2 - p_1) \longleftarrow \Delta h$$

Then, with $h = u + pv$

$$h(p_2,T) - h(p_1,T) = [u(p_2,T) - u(p_1,T)] + [p_2 v(p_2,T) - p_1 v(p_1,T)]$$

$$\left[B - \frac{2A}{RT}\right](p_2 - p_1) = [u(p_2,T) - u(p_1,T)] + p_2\left[\frac{RT}{p_2} + B - \frac{A}{RT}\right] - p_1\left[\frac{RT}{p_1} + B - \frac{A}{RT}\right]$$

$$\left(B - \frac{2A}{RT}\right)(p_2 - p_1) = [u(p_2,T) - u(p_1,T)] + (p_2 - p_1)\left[B - \frac{A}{RT}\right]$$

$$\therefore\ u(p_2,T) - u(p_1,T) = (p_2 - p_1)\left[\left(B - \frac{2A}{RT}\right) - \left(B - \frac{A}{RT}\right)\right]$$

$$= -\frac{A}{RT}(p_2 - p_1) \longleftarrow \Delta u$$

---

1. Since the integrations are at fixed temperature, the terms of Eqs. 11.59 and 11.60 involving c_p drop out of consideration, thereby allowing the required property changes to be evaluated in terms of T_1, T_2, p_1, and p_2.

PROBLEM 11.49

KNOWN: The equations of state are the van der Waals equation and the Redlich-Kwong equation.

FIND: For each equation of state, determine $[h(v_2,T)-h(v_1,T)]$, $[u(v_2,T)-u(v_1,T)]$, $[s(v_2,T)-s(v_1,T)]$.

ANALYSIS: Since the equations of state are explicit in pressure, and temperature is fixed, it is convenient to use Eqs. 11.50 and 11.51, which reduce to read

$$u(v_2,T)-u(v_1,T)=\int_{v_1}^{v_2}\left[T\left(\frac{\partial p}{\partial T}\right)_v-p\right]dv \tag{1}$$

$$s(v_2,T)-s(v_1,T)=\int_{v_1}^{v_2}\left(\frac{\partial p}{\partial T}\right)_v dv \tag{2}$$

Also, Δh can be found using Eq. 11.61:

$$h_2-h_1=(u_2-u_1)+(p_2v_2-p_1v_1) \tag{3}$$

(a) van der Waals. This equation is explicit in pressure. So $(\partial p/\partial T)_v = R/(v-b)$. Eq. (1) then becomes

$$u(v_2,T)-u(v_1,T)=\int_{v_1}^{v_2}\left(\frac{RT}{v-b}-p\right)dv=\int_{v_1}^{v_2}\left(\frac{RT}{v-b}-\left(\frac{RT}{v-b}-\frac{a}{v^2}\right)\right)dv=\int_{v_1}^{v_2}\frac{a}{v^2}dv$$

$$=-a\left(\frac{1}{v_2}-\frac{1}{v_1}\right) \longleftarrow \Delta u$$

Thus

$$h(v_2,T)-h(v_1,T)=p(v_2,T)v_2-p(v_1,T)v_1-a\left(\frac{1}{v_2}-\frac{1}{v_1}\right) \longleftarrow \Delta h$$

Also, Eq. (2) becomes

$$s(v_2,T)-s(v_1,T)=\int_{v_1}^{v_2}\left(\frac{R}{v-b}\right)dv=R\ln\left(\frac{v_2-b}{v_1-b}\right) \longleftarrow \Delta s$$

(b) Redlich-Kwong. This equation is explicit in pressure. So, $(\partial p/\partial T)_v = \frac{R}{v-b}+\frac{a}{2v(v+b)T^{3/2}}$. Eq. (1) then becomes

$$u(v_2,T)-u(v_1,T)=\int_{v_1}^{v_2}\left(\left(\frac{RT}{v-b}+\frac{a}{2v(v+b)T^{1/2}}\right)-\left(\frac{RT}{v-b}-\frac{a}{v(v+b)T^{1/2}}\right)\right)dv$$

$$=\frac{3}{2}\int_{v_1}^{v_2}\frac{a}{v(v+b)T^{1/2}}dv=\frac{-3a}{2bT^{1/2}}\ln\left[\frac{v_2+b}{v_1+b}\cdot\frac{v_1}{v_2}\right] \longleftarrow \Delta u$$

Thus

$$h(v_2,T)-h(v_1,T)=p(v_2,T)v_2-p(v_1,T)v_1-\frac{3a}{2bT^{1/2}}\ln\left[\frac{v_2+b}{v_1+b}\cdot\frac{v_1}{v_2}\right] \longleftarrow \Delta h$$

Also, Eq. (2) becomes

$$s(v_2,T)-s(v_1,T)=\int_{v_1}^{v_2}\left(\frac{R}{v-b}+\frac{a}{2v(v+b)T^{3/2}}\right)dv$$

$$=R\ln\left(\frac{v_2-b}{v_1-b}\right)-\frac{a}{2bT^{3/2}}\ln\left[\frac{v_2+b}{v_1+b}\cdot\frac{v_1}{v_2}\right] \longleftarrow \Delta s$$

PROBLEM 11.50

KNOWN: At certain states the p-v-T data of a gas can be described as

$$Z = 1 - \frac{Ap}{T^4}$$

where A is a constant.

FIND: Obtain an expression for (a) $(\partial p/\partial T)_v$ in terms of p, T, A and R, (b) the quantity $[s(p_2,T) - s(p_1,T)]$, and (c) $[h(p_2,T) - h(p_1,T)]$.

ANALYSIS: Since $Z = pv/RT$, the equation can be expressed as

$$\frac{pv}{RT} = 1 - \frac{Ap}{T^4} \Rightarrow v = \frac{RT}{p} - \frac{AR}{T^3} \qquad (1)$$

(a) To obtain $(\partial p/\partial T)_v$, Eq. (1) can be solved for p and then differentiated:

$$p = \frac{RT}{v + AR/T^3}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R\left[v + \frac{AR}{T^3}\right] - RT\left[-\frac{3AR}{T^4}\right]}{\left(v + \frac{AR}{T^3}\right)^2} = \frac{Rv + \frac{4AR^2}{T^3}}{\left(v + \frac{AR}{T^3}\right)^2} = \frac{R\left[\frac{RT}{p} - \frac{AR}{T^3}\right] + \frac{4AR^2}{T^3}}{(RT/p)^2}$$

$$= \frac{p^2}{R^2T^2}\left[\frac{R^2T}{p} + \frac{3AR^2}{T^3}\right] = \frac{p^2}{RT}\left[\frac{R}{p} + \frac{3AR}{T^4}\right] \longleftarrow \text{(a)}$$

Alternatively, Eq. 11.16 can be used:

$$\left(\frac{\partial p}{\partial v}\right)_T\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial T}{\partial p}\right)_v = -1$$

$$\left(\frac{\partial p}{\partial v}\right)_T = \frac{-RT}{\left[v + \frac{AR}{T^3}\right]^2} = -\frac{p^2}{RT}$$

Then using Eq. 11.15

$$\left(\frac{\partial p}{\partial T}\right)_v = -\left(\frac{\partial p}{\partial v}\right)_T\left(\frac{\partial v}{\partial T}\right)_p$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3AR}{T^4}$$

$$= -\left[-\frac{p^2}{RT}\right]\left[\frac{R}{p} + \frac{3AR}{T^4}\right] = \frac{p^2}{RT}\left[\frac{R}{p} + \frac{3AR}{T^4}\right]$$

(b) Using Eq. 11.59

$$s(p_2,T) - s(p_1,T) = \int_{p_1}^{p_2} -\left(\frac{\partial v}{\partial T}\right)_p dp = -\int_{p_1}^{p_2}\left[\frac{R}{p} + \frac{3AR}{T^4}\right]dp$$

$$= -R\ln\frac{p_2}{p_1} + \frac{3AR}{T^4}[p_2 - p_1] \longleftarrow \text{(b)}$$

(c) Using Eq. 11.60

$$h(p_2,T) - h(p_1,T) = \int_{p_1}^{p_2}\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

From (a)

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3AR}{T^4} \Rightarrow T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p} + \frac{3AR}{T^3} \Rightarrow \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] = \left[\frac{RT}{p} - \frac{AR}{T^3}\right] - \left[\frac{RT}{p} + \frac{3AR}{T^3}\right] = -\frac{4AR}{T^3}$$

So

$$h(p_2,T) - h(p_1,T) = \int_{p_1}^{p_2}\left(-\frac{4AR}{T^3}\right)dp = -\frac{4AR}{T^3}(p_2 - p_1) \longleftarrow \text{(c)}$$

PROBLEM 11.51

KNOWN: The p-v-T behavior of a gas is described by

$$Z = 1 + \frac{B(T)p}{RT}$$

FIND: Obtain expressions for (a) $[h(p_2,T)-h(p_1,T)]$, (b) $[u(p_2,T)-u(p_1,T)]$ (c) $[s(p_2,T)-s(p_1,T)]$.

ANALYSIS: Since $Z = pv/RT$, the equation becomes

$$\frac{pv}{RT} = 1 + \frac{B(T)p}{RT}$$

$$v = \frac{RT}{p} + B(T) \qquad (1)$$

As T and p are the independent properties that fix the state and an equation of state explicit in v is known, Eqs. 11.59 and 11.60 are convenient to use here. Each of these requires $(\partial v/\partial T)_p$. Thus, from Eq.(1)

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{dB}{dT}$$

Eq. 11.59 then gives

$$s(p_2,T) - s(p_1,T) = -\int_{p_1}^{p_2}\left(\frac{\partial v}{\partial T}\right)_p dp = -\int_{p_1}^{p_2}\left[\frac{R}{p} + \frac{dB}{dT}\right]dp$$

$$= -R\ln\frac{p_2}{p_1} - \frac{dB}{dT}[p_2-p_1] \longleftarrow \Delta s$$

Similarly, using Eq. 11.60

$$h(p_2,T) - h(p_1,T) = \int_{p_1}^{p_2}\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

$$= \int_{p_1}^{p_2}\left[\left(\frac{RT}{p} + B(T)\right) - T\left(\frac{R}{p} + \frac{dB}{dT}\right)\right]dp$$

$$= \int_{p_1}^{p_2}\left[B - T\frac{dB}{dT}\right]dp$$

$$= \left[B - T\frac{dB}{dT}\right][p_2 - p_1] \longleftarrow \Delta h$$

Then

$$\Delta h = \Delta u + \Delta(pv) = \Delta u + [p_2 v(p_2,T) - p_1 v(p_1,T)]$$

$$= \Delta u + p_2\left[\frac{RT}{p_2} + B\right] - p_1\left[\frac{RT}{p_1} + B\right]$$

$$= \Delta u + B(p_2 - p_1)$$

$$\Rightarrow \Delta u = \underbrace{\left[B - T\frac{dB}{dT}\right](p_2-p_1)}_{\Delta h} - B(p_2-p_1) \Rightarrow \Delta u = -T\frac{dB}{dT}(p_2-p_1) \longleftarrow \Delta u$$

PROBLEM 11.52

KNOWN: The p-v-T behavior of a gas is described by

$$Z = 1 + \frac{B(T)}{v} + \frac{C(T)}{v^2}$$

FIND: Obtain an expression for $[s(v_2,T) - s(v_1,T)]$.

ANALYSIS: Since $Z = pv/RT$, the equation becomes

$$p = \frac{RT}{v} + \frac{RT\,B(T)}{v^2} + \frac{RT\,C(T)}{v^3} \qquad (1)$$

As T and v are the independent properties that fix the state and an equation of state explicit in p is known, Eq. 11.50 is convenient to use here. This expression requires $(\partial p/\partial T)_v$. Thus, from Eq. (1)

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v} + \frac{R\,B(T) + RT\,B'(T)}{v^2} + \frac{R\,C(T) + RT\,C'(T)}{v^3}$$

Then, with Eq. 11.50

$$s(v_2,T) - s(v_1,T) = \int_{v_1}^{v_2} \left(\frac{\partial p}{\partial T}\right)_v dv$$

$$= \int_{v_1}^{v_2} \left[\frac{R}{v} + \frac{[RB + RTB']}{v^2} + \frac{[RC + RTC']}{v^3} \right] dv$$

$$= R \ln \frac{v_2}{v_1} - [RB + RTB']\left[\frac{1}{v_2} - \frac{1}{v_1}\right] - \frac{[RC + RTC']}{2}\left[\frac{1}{v_2^2} - \frac{1}{v_1^2}\right]$$

$$= R \ln \frac{v_2}{v_1} - R[B + TB']\left[\frac{1}{v_2} - \frac{1}{v_1}\right] - \frac{R}{2}[C + TC']\left[\frac{1}{v_2^2} - \frac{1}{v_1^2}\right] \longleftarrow \Delta s$$

or

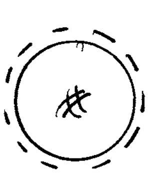
$$s(v_2,T) - s(v_1,T) = R \ln \frac{v_2}{v_1} + R\left[\frac{1}{v_1} - \frac{1}{v_2}\right]\left\{ \frac{d(BT)}{dT} + \frac{1}{2}\frac{d(CT)}{dT}\left[\frac{1}{v_1} + \frac{1}{v_2}\right] \right\}$$

PROBLEM 11.53

KNOWN: A 1-kg copper sphere is at 300K and an initial pressure of 10bar.

FIND: If the volume of the sphere is not allowed to vary by more than 0.1 %, determine the maximum allowed pressure the sphere can attain, in bar.

SCHEMATIC & GIVEN DATA:



$m = 1\,kg$
$T = 300K$
$p_1 = 10\ bar$
$\rho = 8888\ kg/m^3$
$\beta = 49.2\times10^{-6}(K)^{-1}$
$\kappa = 0.776\times10^{-11} m^2/N$

ASSUMPTION: (1) The copper sphere is the system. (2) Temperature remains constant at 300K. (3) ρ, β, κ do not vary as pressure changes.

ANALYSIS: As the volume would decrease as pressure increases, the restriction on volume change reads

$$\frac{(\Delta V)_{MAX}}{V} = -0.001 \qquad (1)$$

With Eq. 11.63

① $$\left(\frac{\partial v}{\partial p}\right)_T = -\kappa v = -\kappa/\rho$$

and at fixed T

$$\Delta v = -\int_{p_1}^{p_2} \frac{\kappa}{\rho} dp = -\frac{\kappa}{\rho}(p_2 - p_1)$$

and

$$\Delta V = \left[\frac{-\kappa}{\rho}(p_2 - p_1)\right] m = -\kappa \Delta p\, V$$

$$\Rightarrow \frac{\Delta V}{V} = -\kappa \Delta p \qquad (2)$$

Combining (1), (2)

$$-\kappa(\Delta p)_{MAX} = -0.001$$

or

$$(\Delta p)_{MAX} = \left(\frac{0.001}{0.776\times10^{-11}\ m^2/N}\right)\left|\frac{1\ bar}{10^5\ N/m^2}\right| = 1289\ bar$$

② $$\Rightarrow (p_2)_{MAX} = 1299\ bar \longleftarrow$$

---

1. Alternatively, this equation can be expressed as

$$\left(\frac{\partial \ln V}{\partial p}\right)_T = -\kappa$$

Since κ is constant with p (assumption 3), integration gives

$$\frac{V_2}{V_1} = exp\left(-\kappa(p_2 - p_1)\right)$$

So

$$\Delta V = V_2 - V_1 = V_1\left[exp(-\kappa(p_2 - p_1)) - 1\right]$$

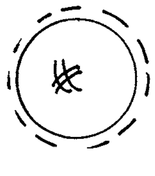
2. β is not required for the solution.

PROBLEM 11.54

KNOWN: A 1-lb copper sphere is at 80°F and an initial pressure of 1atm.

FIND: If the volume of the sphere is not allowed to vary by more than 0.1%, determine the maximum allowed pressure the sphere can attain, in atm.

SCHEMATIC & GIVEN DATA:



$m = 1 \text{ lb}$
$T = 80°F$
$p_1 = 1 \text{ atm}$
$\rho = 555 \text{ lb/ft}^3$
$\beta = 2.75\times10^{-5} (°R)^{-1}$
$\kappa = 3.72\times10^{-10} \text{ ft}^2/\text{lbf}$

ASSUMPTIONS: (1) The copper sphere is the system. (2) Temperature remains constant at 80°F. (3) ρ, β, κ do not vary as pressure changes.

ANALYSIS: As the volume would decrease as pressure increases, the restriction on volume change reads

$$\frac{(\Delta V)_{MAX}}{V} = -0.001 \qquad (1)$$

With Eq. 11.63

① $$\left(\frac{\partial v}{\partial p}\right)_T = -\kappa v = -\kappa/\rho$$

and at fixed T

$$\Delta v = -\int_{p_1}^{p_2} \frac{\kappa}{\rho}\, dp = -\frac{\kappa}{\rho}\Delta p$$

and

$$\Delta V = \left[-\frac{\kappa}{\rho}\Delta p\right] m = -\kappa\, \Delta p\, V$$

$$\Rightarrow \frac{\Delta V}{V} = -\kappa\, \Delta p \qquad (2)$$

Combining (1), (2)

$$-\kappa(\Delta p)_{MAX} = -0.001$$

$$(\Delta p)_{MAX} = \left[\frac{0.001}{3.72\times10^{-10} \text{ ft}^2/\text{lbf}}\right]\left|\frac{1 \text{ atm}}{2116 \text{ lbf/ft}^2}\right|$$

$$= 1270 \text{ atm}$$

② ➡ $(p_2)_{MAX} = 1271$ atm. ◀———

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1. Alternatively, this equation can be expressed as

$$\left(\frac{\partial \ln V}{\partial p}\right)_T = -\kappa$$

Since κ is constant with pressure (assumption 3), integration gives

$$\frac{V_2}{V_1} = \exp(-\kappa(p_2-p_1)) \Rightarrow \Delta V = V_2 - V_1 = V_1\left[\exp(-\kappa(p_2-p_1)) - 1\right]$$

2. β is not required for the solution

PROBLEM 11.55

KNOWN: Three cases are under consideration: (a) an ideal gas, (b) a gas whose equation of state is $p(v-b) = RT$, (c) a gas obeying the van der Waals equation.

FIND: Derive expressions for β and κ for each case.

ANALYSIS: From Eqs. 11.62 and 11.63

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \quad \text{and} \quad \kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

(a) Ideal Gas: $v = RT/p$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} \quad \text{and} \quad \left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2}$$

Thus

$$\beta = \frac{1}{v}\left[\frac{R}{p}\right] = \frac{1}{T} \qquad \longleftarrow \text{(a)}$$

$$\kappa = -\frac{1}{v}\left[-\frac{RT}{p^2}\right] = \frac{1}{p}\left[\frac{RT}{pv}\right] = \frac{1}{p}$$

(b) $v = (RT/p) + b$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} \quad , \quad \left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2}$$

Thus

$$\beta = \frac{1}{v}\left[\frac{R}{p}\right] = \frac{R}{v}\left[\frac{v-b}{RT}\right] = \frac{1}{T}\left[\frac{v-b}{v}\right] \qquad \longleftarrow \text{(b)}$$

$$\kappa = -\frac{1}{v}\left[-\frac{RT}{p^2}\right] = \frac{1}{p}\left[\frac{RT}{pv}\right] = \frac{1}{p}\left[\frac{RT/v}{RT/(v-b)}\right] = \frac{1}{p}\left[\frac{v-b}{v}\right]$$

Note: when $b=0$ these expressions reduce to those of part (a).

(c) van der Waals: $p = \frac{RT}{(v-b)} - \frac{a}{v^2}$.

As the van der Waals equation is not explicit in v, the required partial derivatives are not so easily found as in parts (a) and (b). Thus, following the procedure explained in Example 11.2, an expression for $(\partial v/\partial T)_p$ is obtained as

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{-R/(v-b)}{\left[2a/v^3 - RT/(v-b)^2\right]}$$

So

$$\beta = \frac{-R(v-b)v^2}{2a(v-b)^2 - RTv^3}$$

Using Eqs. 11.15 and 11.16

$$\left(\frac{\partial v}{\partial p}\right)_T = \frac{-(\partial v/\partial T)_p}{(\partial p/\partial T)_v} = \frac{\dfrac{R/(v-b)}{\left[2a/v^3 - RT/(v-b)^2\right]}}{R/(v-b)} = \frac{1}{\left[2a/v^3 - RT/(v-b)^2\right]} \qquad \longleftarrow \text{(c)}$$

Thus

$$\kappa = -\frac{1}{v\left[2a/v^3 - RT(v-b)^2\right]} = \frac{-v^2(v-b)^2}{\left[2a(v-b)^2 - RTv^3\right]}$$

PROBLEM 11.56

FIND: Derive expressions for β and κ in terms of $T, p, Z, (\partial Z/\partial T)_p$, $(\partial Z/\partial p)_T$. For gas states with $p_R < 3$, $T_R < 2$, determine the sign of κ.

ANALYSIS: From Eqs. 11.62 and 11.63

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p \quad , \quad \kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

The compressibility factor is

$$Z(T,p) = \frac{pv}{RT}$$

$$\Rightarrow \qquad v = \frac{RT}{p} \cdot Z(T,p)$$

Then

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} Z + \frac{RT}{p}\left(\frac{\partial Z}{\partial T}\right)_p$$

and

$$\left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2} Z + \frac{RT}{p}\left(\frac{\partial Z}{\partial p}\right)_T$$

Accordingly

$$\beta = \frac{1}{v}\left[\frac{R}{p} Z + \frac{RT}{p}\left(\frac{\partial Z}{\partial T}\right)_p\right]$$

$$= \frac{R}{pv}\left(\frac{pv}{RT}\right) + \frac{1}{Z}\left(\frac{\partial Z}{\partial T}\right)_p$$

$$= \frac{1}{T} + \frac{1}{Z}\left(\frac{\partial Z}{\partial T}\right)_p \qquad \longleftarrow \beta$$

And

$$\kappa = -\frac{1}{v}\left[-\frac{RT}{p^2} Z + \frac{RT}{p}\left(\frac{\partial Z}{\partial p}\right)_T\right]$$

$$= \left[\frac{1}{p}\cdot\frac{RT}{pv}\cdot Z - \frac{RT}{pv}\left(\frac{\partial Z}{\partial p}\right)_T\right]$$

$$= \frac{1}{p} - \frac{1}{Z}\left(\frac{\partial Z}{\partial p}\right)_T \qquad \longleftarrow \kappa$$

Referring to Figure A-2, for $p_R < 3$, $T_R < 0$ all isotherms have a negative slope: $(\partial Z/\partial p)_T < 0$. Thus, it is evident that κ must be positive at all such states.

PROBLEM 11.57

FIND: Show that $\kappa > \alpha$.

ANALYSIS: From the development of Sec. 11.5.2, the following expressions are obtained

$$c_p - c_v = v\,\frac{T\beta^2}{\kappa} \qquad (11.69)$$

$$k = \frac{\kappa}{\alpha} \qquad (11.73)$$

where $k = c_p/c_v$. Following the discussion of Eq. 11.69 $c_p > c_v$ except when $\beta = 0$ and as $T \to 0$. Accordingly, except for such special circumstances, $k > 1$ and so $\kappa/\alpha > 1$, as was to be established.

PROBLEM 11.58

FIND: Prove that $(\partial\beta/\partial p)_T = -(\partial\kappa/\partial T)_p$.

ANALYSIS: Consider $v = v(T, p)$ and form the differential

$$dv = \left(\frac{\partial v}{\partial T}\right)_p dT + \left(\frac{\partial v}{\partial p}\right)_T dp$$

With Eqs. 11.62 and 11.63 this becomes

$$dv = \underbrace{(v\beta)}_{M}\,dT + \underbrace{(-v\kappa)}_{N}\,dp \qquad (1)$$

Since this is an exact differential, the test for exactness: Eq. 11.14 must be satisfied. That is

$$\left(\frac{\partial M}{\partial p}\right)_T = \left(\frac{\partial N}{\partial T}\right)_p$$

Thus

$$\left.\frac{\partial(v\beta)}{\partial p}\right]_T = \left.\frac{\partial(-v\kappa)}{\partial T}\right)_p$$

$$\beta\left(\frac{\partial v}{\partial p}\right)_T + v\left(\frac{\partial\beta}{\partial p}\right)_T = -\kappa\left(\frac{\partial v}{\partial T}\right)_p - v\left(\frac{\partial\kappa}{\partial T}\right)_p$$

$$\underline{\frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial v}{\partial p}\right)_T} + v\left(\frac{\partial\beta}{\partial p}\right)_T = +\underline{\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T\left(\frac{\partial v}{\partial T}\right)_p} - v\left(\frac{\partial\kappa}{\partial T}\right)_p$$

The underlined terms cancel, leaving

① $$\left(\frac{\partial\beta}{\partial p}\right)_T = -\left(\frac{\partial\kappa}{\partial T}\right)_p$$

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1. The desired result is obtained more directly by writing Eq. (1) as

$$d\ln v = \underbrace{\beta}_{M}\,dT + \underbrace{(-\kappa)}_{N}\,dp$$

and then applying the test for exactness.

PROBLEM 11.59

KNOWN: Data are provided for aluminum at 0°C.

FIND: Determine the percent error in c_v that would result if it were assumed that $c_p = c_v$.

SCHEMATIC & GIVEN DATA:

$T = 0°C$

$\rho = 2700\ \text{kg/m}^3$

$c_p = 0.9211\ \text{kJ/kg}\cdot\text{K}$

$\beta = 7.14\times10^{-8}\ \text{K}^{-1}$

$\kappa = 1.34\times10^{-13}\ \text{m}^2/\text{N}$

ANALYSIS: Eq 11.69 can be used to find $c_p - c_v$:

$$c_p - c_v = v\,\frac{T\beta^2}{\kappa} = \frac{T\beta^2}{\rho\kappa}$$

$$= \frac{(273\ \text{K})(7.14\times10^{-8}\ \text{K}^{-1})^2}{(2700\ \text{kg/m}^3)(1.34\times10^{-13}\ \text{m}^2/\text{N})}\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right| = 3.846\times10^{-6}\ \text{kJ/kg}\cdot\text{K}$$

Thus, $c_p \approx c_v$. That is, the difference is negligible. Specifically

$$\%\ \text{error} = \left(\frac{3.846\times10^{-6}}{0.9211}\right)(100) = 4.2\times10^{-4}\,\% \longleftarrow \text{error}$$

Note that specific heat data reported for solids and liquids are normally c_p values, since these are more easily determined for solids and liquids than c_v.

PROBLEM 11.60

KNOWN: Data are provided for mercury at 0°c, 1 bar.
FIND: Estimate the temperature rise for an isentropic process to 1000 bar.
SCHEMATIC & GIVEN DATA:

$T_1 = 0°C$ $\quad \bar c_p = 28.0$ kJ/kmol·K
$p_1 = 1$ bar $\quad \bar v = 0.0147$ m^3/kmol
$p_2 = 1000$ bar $\quad \beta = 17.8 \times 10^{-5}$ K^{-1}
$s_1 = s_2$

ASSUMPTION: the temperature change is small, and $\bar c_p$, $\bar v$, and β can be taken as constant.

ANALYSIS: Begin with Eq. 11.33: $\partial T/\partial p)_s = \frac{\partial \bar v}{\partial \bar s}\Big)_p$. The derivative on the right hand side of this expression can be expanded as follows:

$$\left.\frac{\partial T}{\partial p}\right|_s = \left.\frac{\partial \bar v}{\partial T}\right)_p \left.\frac{\partial T}{\partial \bar s}\right)_p$$

$$\left.\frac{\partial \bar v}{\partial T}\right)_p = \bar v \beta \qquad \left.\frac{\partial T}{\partial \bar s}\right)_p = \frac{T}{\bar c_p}$$

(from Eq. 11.62) (from Eq. 11.55)

Thus
$$\left.\frac{\partial T}{\partial p}\right)_s = \frac{\bar v \beta T}{\bar c_p}$$

Using the assumption, we can integrate

$$\int_{T_1}^{T_2} dT = \int_{p_1}^{p_2} \frac{\bar v \beta T}{\bar c_p} dp \Rightarrow \Delta T = \left(\frac{\bar v \beta T}{\bar c_p}\right) \Delta p$$

Inserting values

$$\Delta T = \frac{(0.0147\ \text{m}^3/\text{kmol})(17.8\times 10^{-5}\ \text{K}^{-1})(273.15\ \text{K})(999\ \text{bar})}{(28.0\ \text{kJ/kmol}\cdot\text{K})}\left|\frac{10^5\ \text{N/m}^2}{1\ \text{bar}}\right|\left|\frac{1\ \text{kJ}}{10^3\ \text{N}\cdot\text{m}}\right|$$

$$= 2.55\ \text{K} \longleftarrow \Delta T$$

PROBLEM 11.61

KNOWN: The p-v-T data for a certain gas can be represented as

$$Z = 1 - \frac{Ap}{T^4}$$

where A is a constant.

FIND: obtain an expression for c_p and verify that the expression reduces to Eq. 3.47a when $Z = 1$.

ANALYSIS: Using Eq. 11.68

$$c_p - c_v = -T\left(\frac{\partial v}{\partial T}\right)_p^2\left(\frac{\partial p}{\partial v}\right)_T$$

With Eq. 11.15

$$\left(\frac{\partial p}{\partial v}\right)_T = \frac{1}{(\partial v/\partial p)_T}$$

Accordingly

$$c_p - c_v = -T\,\frac{(\partial v/\partial T)_p^2}{(\partial v/\partial p)_T} \qquad (1)$$

This requires an equation of state explicit in v. With $Z = pv/RT$, the given equation becomes

$$v = \frac{RT}{p} - \frac{AR}{T^3}$$

Thus

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3AR}{T^4} \qquad (2)$$

$$\left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2} \qquad (3)$$

Combining Eqs. (1)-(3)

$$c_p - c_v = \frac{T\left[\frac{R}{p} + \frac{3AR}{T^4}\right]^2}{RT/p^2}$$

$$= R\left[1 + \frac{3Ap}{T^4}\right]^2$$

With $(Ap/T^4) = 1 - Z$

$$c_p - c_v = R\left[1 + 3(1-Z)\right]^2 = R(4-3Z)^2$$

with $k = c_p/c_v$, $c_v = c_p/k$ and this expression can be written as

$$c_p = \frac{kR}{k-1}\left[4-3Z\right]^2 \qquad \longleftarrow$$

if $Z = 1$, this becomes Eq. 3.47a:

$$c_p = \frac{kR}{k-1} \qquad \longleftarrow$$

PROBLEM 11.62

KNOWN: A gas obeys the van der Waals equation of state.

FIND: (a) Show that $(\partial c_v/\partial v)_T = 0$. (b) Develop an expression for $(c_p - c_v)$.

(c) Develop expressions for $[u(T_2, v_2) - u(T_1, v_1)]$, $[s(T_2, v_2) - s(T_1, v_1)]$.

(d) Complete the results of (c) if $c_v = a + bT$. (Note: a, b are constants, but do not correspond to the constants of the Equation of State.)

ANALYSIS: (a) $c_v = (\partial u/\partial T)_v$. Then

$$\left(\frac{\partial c_v}{\partial v}\right)_T = \frac{\partial}{\partial v}\left[\frac{\partial u}{\partial T}\right)_v\Big]_T = \frac{\partial}{\partial T}\left[\frac{\partial u}{\partial v}\right)_T\Big]_v$$

Inserting Eq. 11.47

$$\left(\frac{\partial c_v}{\partial v}\right)_T = \frac{\partial}{\partial T}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]_v = \left(\frac{\partial p}{\partial T}\right)_v + T\left(\frac{\partial^2 p}{\partial T^2}\right)_v - \frac{\partial p}{\partial T}\Big)_v = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

Using the van der Waals equation

$$\frac{\partial p}{\partial T} = \frac{R}{v-b}, \quad \frac{\partial^2 p}{\partial T^2} \equiv 0 \Rightarrow \left(\frac{\partial c_v}{\partial v}\right)_T \equiv 0. \quad \longleftarrow \text{(a)}$$

That is, c_v is independent of v and depends only on T: $c_v = c_v(T)$.

(b) Begin with Eq. 11.68, and note that $(\partial v/\partial T)_p$ is evaluated for the van der Waals equation in Ex 11.2. Further,

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

Thus

$$c_p - c_v = -T\left[\frac{R/(v-b)}{2a/v^3 - RT/(v-b)^2}\right]^2\left[\frac{2a}{v^3} - \frac{RT}{(v-b)^2}\right]$$

$$= \frac{-R^2T}{(v-b)^2\left[\frac{2a}{v^3} - \frac{RT}{(v-b)^2}\right]}$$

$$= \frac{R}{1 - 2a(v-b)^2/RTv^3} \quad \longleftarrow (c_p - c_v)$$

(c) Using the result of part (a), Eqs. 11.50 and 11.51 give

$$s(T_2,v_2) - s(T_1,v_1) = \int_{T_1}^{T_2} \frac{c_v(T)}{T}\,dT + \int_{v_1}^{v_2}\left(\frac{R}{v-b}\right)dv = \int_{T_1}^{T_2}\frac{c_v(T)}{T}\,dT + R\ln\left(\frac{v_2-b}{v_1-b}\right) \quad \longleftarrow \Delta s$$

$$u(T_2,v_2) - u(T_1,v_1) = \int_{T_1}^{T_2} c_v(T)\,dT + \int_{v_1}^{v_2}\left[T\left[\frac{R}{v-b}\right] - \left(\frac{RT}{v-b} - \frac{a}{v^2}\right)\right]dv$$

$$= \int_{T_1}^{T_2} c_v(T)\,dT - a\left[\frac{1}{v_2} - \frac{1}{v_1}\right] \quad \longleftarrow \Delta u$$

To evaluate these expressions requires only the function $c_v(T)$.

(d) If $c_v = a + bT$, then

$$\Delta s = \int_{T_1}^{T_2}\left(\frac{a+bT}{T}\right)dT + R\ln\left(\frac{v_2-b}{v_1-b}\right) = a\ln\frac{T_2}{T_1} + b(T_2 - T_1) + R\ln\left(\frac{v_2-b}{v_1-b}\right)$$

$$\Delta u = \int_{T_1}^{T_2}(a+bT)\,dT - a\left[\frac{1}{v_2} - \frac{1}{v_1}\right] = a(T_2 - T_1) + \frac{b^2}{2}(T_2^2 - T_1^2) - a\left[\frac{1}{v_2} - \frac{1}{v_1}\right]$$

PROBLEM 11.63

KNOWN: For air $c_v = 0.1965$ Btu/lb·°R at $T_1 = 1000°F$, $v_1 = 36.8\ ft^3/lb$.

FIND: Using the Berthelot equation of state, determine c_v at $T_2 = 1000°F$, $v_2 = 0.0555\ ft^3/lb$.

ASSUMPTION: The Bethelot equation describes the behavior of air at the specified states:

$$p = \frac{RT}{v-b} - \frac{a}{Tv^2} \quad \text{where} \quad a = \frac{27}{64}\frac{R^2T_c^3}{p_c}, \quad b = \frac{1}{8}\frac{RT_c}{p_c}$$

ANALYSIS: $c_v = \partial u/\partial T)_v$. Also, Eq. 11.47 gives $(\partial u/\partial v)_T = T(\partial p/\partial T)_v - p$. Thus

$$\frac{\partial}{\partial T}\left[\left(\frac{\partial u}{\partial v}\right)_T\right]_v = \frac{\partial}{\partial T}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]$$

$$\frac{\partial}{\partial v}\left[\left.\frac{\partial u}{\partial T}\right)_v\right]_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v + \left(\frac{\partial p}{\partial T}\right)_v - \left(\frac{\partial p}{\partial T}\right)_v$$

or

$$\left(\frac{\partial c_v}{\partial v}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v \tag{1}$$

Differentiation of the equation of state gives

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b} + \frac{a}{T^2v^2}, \quad \left(\frac{\partial^2 p}{\partial T^2}\right)_v = -\frac{2a}{T^3v^2}$$

Inserting this into Eq. (1) and integrating

$$c_v(T, v_2) - c_v(T, v_1) = \left.\frac{2a}{T^2 v}\right]_{v_1}^{v_2}$$

$$= \frac{2a}{T^2}\left[\frac{1}{v_2} - \frac{1}{v_1}\right]$$

The value of a is determined using T_c and p_c from Table A-1E

$$a = \frac{27}{64}\frac{R^2T_c^3}{p_c} = \frac{27}{64}\frac{\left(1545/28.97\ \frac{ft\cdot lbf}{lb\cdot °R}\right)^2 (239°R)^3}{(37.2)\left|14.696 \times 144\right|\ lbf/ft^2}$$

$$= 208{,}081\ \frac{lbf\cdot ft^4\cdot(°R)^2}{(lb)^2}\left|\frac{Btu}{778\ ft\cdot lbf}\right|$$

$$= 267.46\ \frac{ft^3\cdot(°R)^2\cdot Btu}{(lb)^2}$$

Accordingly, for $T = 1460°R$

$$c_v(T, 0.0555\ ft^3/lb) = 0.1965 + \frac{2(267.46)}{(1460)^2}\left[\frac{1}{0.0555} - \frac{1}{36.8}\right]$$

$$= 0.1965 + 0.0045$$

$$= 0.2010\ \text{Btu/lb·°R} \quad \longleftarrow$$

PROBLEM 11.64

KNOWN: The specific heat ratio can be expressed as

$$k = \frac{c_p \kappa}{c_p \kappa - T v \beta^2}$$

FIND: Derive this expression and use it together with steam table data to evaluate k at 200 lbf/in², 500°F.

ANALYSIS: Begin with Eq. 11.69, divide each side by c_p, giving

$$1 - \frac{c_v}{c_p} = \frac{vT\beta^2}{c_p \kappa} \Rightarrow 1 - \frac{1}{k} = \frac{vT\beta^2}{c_p \kappa}$$

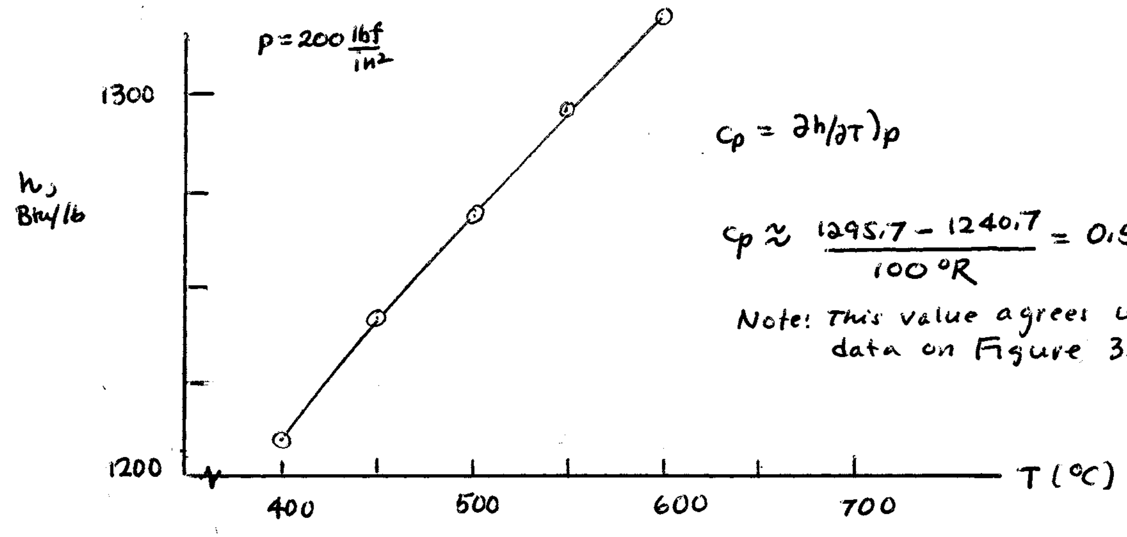
Then

$$1 - \frac{vT\beta^2}{c_p \kappa} = \frac{1}{k} \Rightarrow k = \frac{c_p \kappa}{c_p \kappa - vT\beta^2} = \frac{1}{1 - (vT\beta^2/c_p \kappa)} \qquad (1)$$

To evaluate k requires c_p, κ, β, or alternatively c_p, $\left.\frac{\partial v}{\partial T}\right)_p$, $\left.\frac{\partial v}{\partial p}\right)_T$

GRAPHICAL SOLUTION:

To find c_p

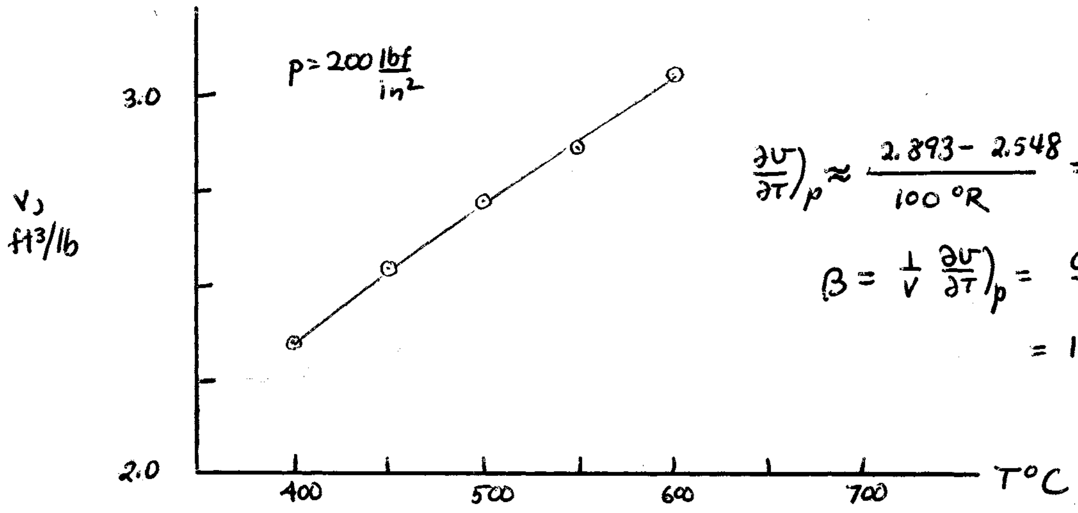


$$c_p = \partial h/\partial T)_p$$

$$c_p \approx \frac{1295.7 - 1240.7}{100\ °R} = 0.55\ \frac{\text{Btu}}{\text{lb}\cdot °R}$$

Note: This value agrees with data on Figure 3.9

To find $\partial v/\partial T)_p$

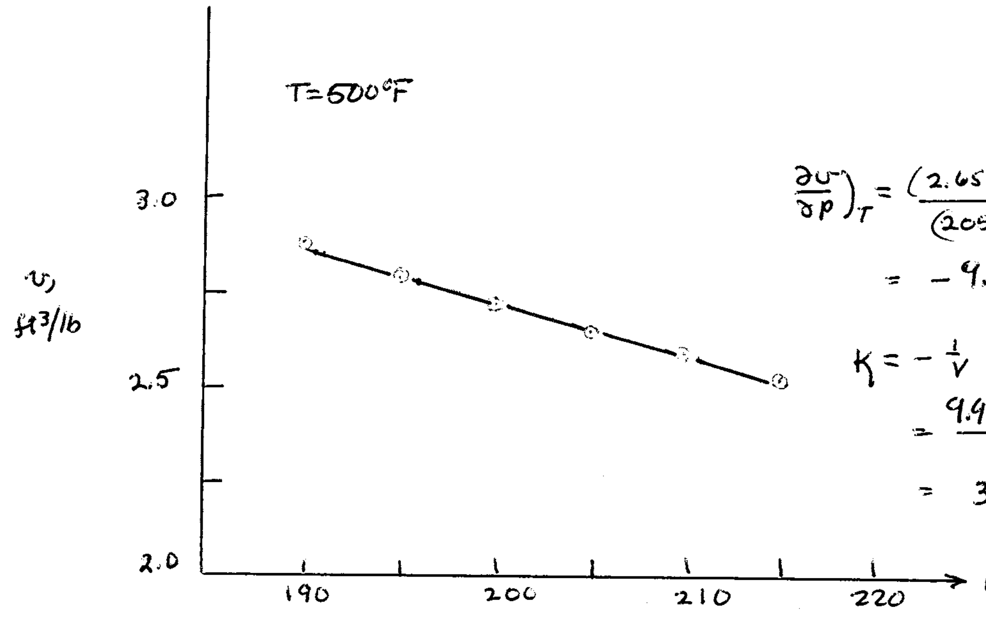


$$\left.\frac{\partial v}{\partial T}\right)_p \approx \frac{2.893 - 2.548}{100\ °R} = 0.00345\ \frac{ft^3}{lb \cdot °R}$$

$$\beta = \frac{1}{v}\left.\frac{\partial v}{\partial T}\right)_p = \frac{0.00345}{2.724} = 1.267 \times 10^{-3} (°R)^{-1}$$

PROBLEM 11.64 (Cont'd)

To find $(\partial v/\partial p)_T$



$$\left.\frac{\partial v}{\partial p}\right)_T = \frac{(2.654 - 2.797)\ \text{ft}^3/\text{lb}}{(205 - 195)(144)\ \text{lbf/ft}^2}$$

$$= -9.93 \times 10^{-5} \frac{\text{ft}^5}{\text{lb} \cdot \text{lbf}}$$

$$\kappa = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$$

$$= \frac{9.93 \times 10^{-5}\ \text{ft}^5/\text{lb} \cdot \text{lbf}}{2.724\ \text{ft}^3/\text{lb}}$$

$$= 3.645 \times 10^{-5}\ \text{ft}^2/\text{lbf}$$

With these values

$$\frac{v\,T\beta^2}{c_p\,\kappa} = \frac{(2.724\ \text{ft}^3/\text{lb})(959.67\,°\text{R})(1.267 \times 10^{-3}\ (°\text{R})^{-1})^2}{\left(0.55\ \frac{\text{Btu}}{\text{lb} \cdot °\text{R}}\right)\left(778.17\ \frac{\text{ft} \cdot \text{lbf}}{\text{Btu}}\right)\left(3.645 \times 10^{-5}\ \frac{\text{ft}^2}{\text{lbf}}\right)} = 0.269$$

and Eq. (1) gives $k = 1.37$, which agrees with the value determined from Fig. 6 of <u>Steam Tables</u> (English Units) by Keenan et.al., Wiley, New York, 1969.

IT Code

```
p = 200  // lbf/in²
T = 500  // °F

dT = 0.001
dp = 0.001
T1 = T - dT
T2 = T + dT
p1 = p - dp
p2 = p + dp
h1 = h_PT("Water/Steam", p, T1)
h2 = h_PT("Water/Steam", p, T2)
c_p = (h2 - h1) / (T2 - T1)
v2 = v_PT("Water/Steam", p, T2)
v1 = v_PT("Water/Steam", p, T1)
v = v_PT("Water/Steam", p, T)
beta = (1 / v) * ((v2 - v1) / (T2 - T1))
v22 = v_PT("Water/Steam", p2, T)
v11 = v_PT("Water/Steam", p1, T)
kappa = - (1 / v) * ((v22 - v11) / (p2 - p1)) / 144
k = 1 / (1 - (v * (T + 459.67) * beta^2) / (c_p * kappa * 778.17))
```

IT Results

β = 0.001265 (1/°R)
c_p = 0.548 Btu / lb · °R
κ = 3.656 x 10^{-5} ft² / lbf
k = 1.366

PROBLEM 11.65

KNOWN: Liquid water at 40 °C, 1atm is under consideration. Data are available from Table 11.2.

FIND: Determine (a) c_v, (b) velocity of sound.

ANALYSIS: (a) Using Eq. 11.69

$$c_v = c_p - v\frac{T\beta^2}{\kappa} = c_p - \frac{T\beta^2}{\rho\kappa} \qquad (1)$$

From Table 11.2 at 40°C,

$$\rho = 992.22\ \frac{kg}{m^3}\ ,\quad \beta = \frac{385.4}{10^6}(K)^{-1}\ ,\quad \kappa = \frac{44.24}{10^6}\ (bar)^{-1}$$

① From Table A.19, $c_p = 4.18\ kJ/kg\cdot K$.

Inserting values in Eq. (1)

$$c_v = \left(4.18\ \frac{kJ}{kg\cdot K}\right) - \frac{(313.15K)(385.4\times10^{-6})^2\,(bar)}{\left(992.22\ \frac{kg}{m^3}\right)(K)^2\,(44.24\times10^{-6})}\left|\frac{10^5 N/m^2}{1 bar}\right|\left|\frac{1 kJ}{10^3 N\cdot m}\right|$$

② $$= (4.18 - 0.11) = 4.07\ kJ/kg\cdot K \qquad \longleftarrow$$

(b) With Eqs. 11.65, 11.73

$$c = \sqrt{\frac{v}{\alpha}} = \sqrt{\frac{k}{\rho\kappa}}$$

with the result of (a), $k = \frac{c_p}{c_v} = \frac{4.18}{4.07} = 1.027$

$$c = \sqrt{\frac{(1.027)\ (bar)}{\left(992.22\ \frac{kg}{m^3}\right)(44.24\times10^{-6})}\left|\frac{10^5 N/m^2}{1 bar}\right|\left|\frac{1 kg\cdot m/s^2}{1 N}\right|}$$

$$= 1530\ m/s \qquad \longleftarrow$$

---

1. Following the discussion of Sec. 3.3.6, we take c_p at 1atm, 40°C as the saturated liquid value at 40°C.

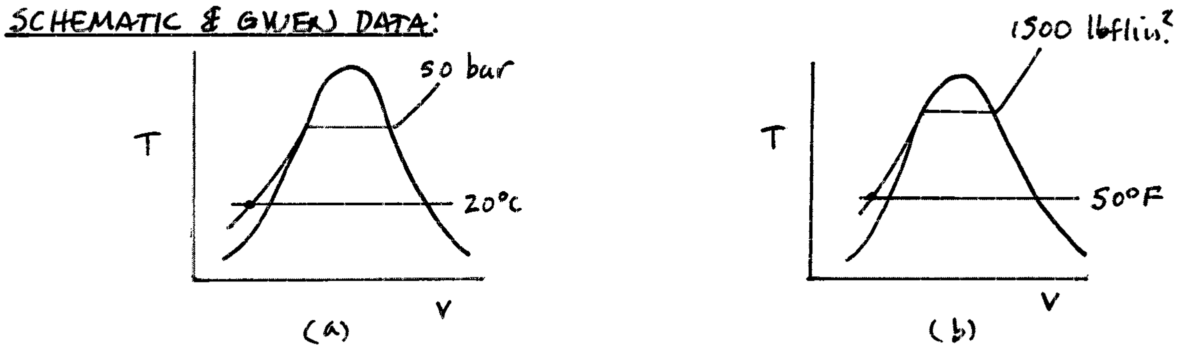
2. The result of part (a) shows that for liquid water at the given state, c_p and c_v are closely equal.

PROBLEM 11.66

KNOWN: Liquid water is at specified states.

FIND: Using steam table data, estimate the velocity of sound at (a) 20°C, 50 bar, (b) 50°F, 1500 lbf/in.^{2}

SCHEMATIC & GIVEN DATA:



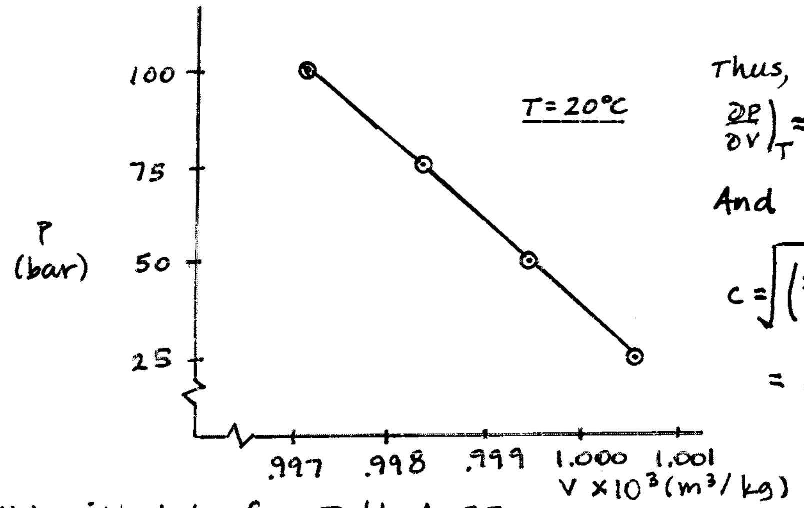
(a) (b)

ASSUMPTION: For liquid water in the range of temperatures under consideration, the ratio of specific heats is $k \approx 1.0$. (See discussion in Sec. 11.5.2.)

ANALYSIS: With Eq. 11.74 and the above assumption

$$c = \sqrt{-k v^2 \left.\frac{\partial p}{\partial v}\right)_T} \approx \sqrt{-v^2 \left.\frac{\partial p}{\partial v}\right)_T}$$

(a) With data from Table A-5



Thus,

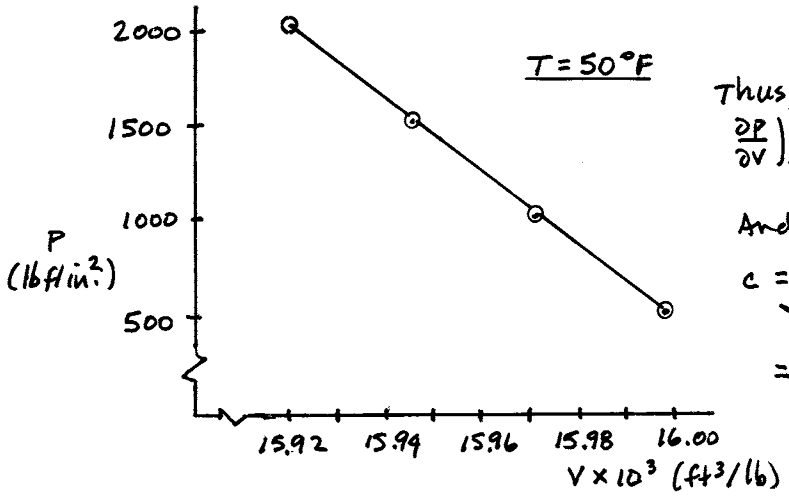
$$\left.\frac{\partial p}{\partial v}\right)_T \approx \frac{(75-25)\ 10^5\ \text{N/m}^2}{\dfrac{(.9984-1.0006)}{10^3}\dfrac{\text{m}^3}{\text{kg}}} = -22727 \times 10^8\ \frac{\text{N}\cdot\text{kg}}{\text{m}^5}$$

And

$$c = \sqrt{\left(\frac{.9995}{10^3}\frac{\text{m}^3}{\text{kg}}\right)^2 (22727\times 10^8)\ \frac{\text{N}\cdot\text{kg}}{\text{m}^5}\left|\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\text{N}}\right|}$$

$= 1507$ m/s ←——— (a)

(b) With data from Table A-5E



Thus,

$$\left.\frac{\partial p}{\partial v}\right)_T \approx \frac{(2000-1000)\ 144\ \text{lbf/ft}^2}{(0.015920-0.015972)\ \text{ft}^3/\text{lb}} = -2.7692\times 10^9\ \frac{\text{lbf}\cdot\text{lb}}{\text{ft}^5}$$

And

$$c = \sqrt{\left(\frac{15.946}{10^3}\frac{\text{ft}^3}{\text{lb}}\right)^2 (2.7692\times 10^9)\ \frac{\text{lbf}\cdot\text{lb}}{\text{ft}^5}\left|\frac{32.2\ \text{lb}\cdot\text{ft/s}^2}{1\ \text{lbf}}\right|}$$

$= 4762$ ft/s ←——— (b)

PROBLEM 11.67

KNOWN: At a point within a stream of air $T = 500°F$, $p = 1\,\text{atm}$, $V = 2115\,\text{ft/s}$.

FIND: Determine the Mach number.

ASSUMPTION: Air is modeled as an ideal gas.

ANALYSIS: The Mach number is $M = V/c$, where c is the velocity of sound. For an ideal gas, c is given by Eq. 9.37, obtained by reduction of Eq. 11.74. Then

$$M = \frac{V}{c} = \frac{V}{\sqrt{kRT}} = \frac{2115\,\text{ft/s}}{\sqrt{(1.383)\left(\frac{1545}{28.97}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(960°\text{R})\left|32.2\,\frac{\text{lb}\cdot\text{ft}}{\text{lbf}\cdot\text{s}^2}\right|}}$$

$$= \frac{2115}{1510} = 1.4 \longleftarrow$$

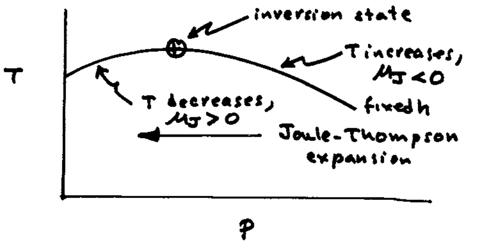
where k is obtained from Table A-20E.

PROBLEM 11.68

KNOWN: A gas obeys the equation of state, $p(v-b) = RT$, where b is a positive constant.

FIND: Determine if the temperature can be reduced in a Joule-Thompson expansion.

SCHEMATIC & GIVEN DATA



$$\mu_J = \left(\frac{\partial T}{\partial p}\right)_h$$

ANALYSIS: Using Eq. 11.77

$$\mu_J = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right]$$

With the given equation of state $(\partial v/\partial T)_p = R/p$. Thus

$$\mu_J = \frac{1}{c_p}\left[\frac{RT}{p} - v\right]$$

$$= \frac{1}{c_p}\left[\frac{RT}{p} - \left(\frac{RT}{p} + b\right)\right] = -\frac{b}{c_p} < 0$$

Accordingly, the Joule-Thompson coefficient is negative, and so the temperature can only increase in a Joule-Thompson expansion. ⟵

PROBLEM 11.69

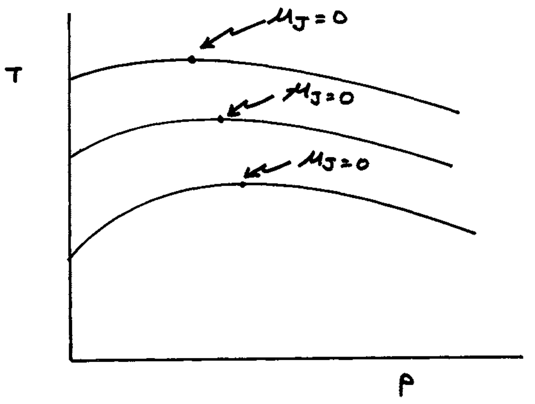
KNOWN: A gas is described by the equation of state

$$v = \frac{RT}{p} - \frac{A}{T} + B$$

where A and B are constants.

FIND: (a) Obtain an expression for the temperatures at the Joule-Thompson inversion states. (b) obtain an expression for $(c_p - c_v)$.

SCHEMATIC & GIVEN DATA:



At Joule-Thompson inversion states, $\mu_J = 0$

ANALYSIS: (a) At the Joule-Thompson inversion states, the Joule-Thompson coefficient vanishes. Thus, Eq. 11.77 becomes

$$0 = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] \Rightarrow T\left(\frac{\partial v}{\partial T}\right)_p - v \equiv 0$$

With the given equation of state

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{A}{T^2} \qquad (1)$$

$$T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p} + \frac{A}{T}$$

Accordingly

$$\left(\frac{RT}{p} + \frac{A}{T}\right) - \left(\frac{RT}{p} - \frac{A}{T} + B\right) = 0 \Rightarrow \frac{2A}{T} - B \Rightarrow T = \frac{2A}{B} \longleftarrow$$

① (b) To find $c_p - c_v$, employ Eq. 11.66. This requires $(\partial v/\partial T)_p$ and $(\partial p/\partial T)_v$. $(\partial v/\partial T)_p$ is given by Eq. (1). Rewriting the equation of state

$$p = RT\left[v + \frac{A}{T} - B\right]^{-1}, \quad \left(\frac{\partial p}{\partial T}\right)_v = R\left[v + \frac{A}{T} - B\right]^{-1} - RT\left(v + \frac{A}{T} - B\right]^{-2}\left[-\frac{A}{T^2}\right]$$

$$= \frac{R(v-B) + 2RA/T}{\left[v + \frac{A}{T} - B\right]^2}$$

② $$\Rightarrow \quad c_p - c_v = T\left[\frac{R}{p} + \frac{A}{T^2}\right]\left[\frac{R(v-B) + 2RA/T}{\left(v + \frac{A}{T} - B\right)^2}\right] \longleftarrow$$

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ernatively, $(c_p - c_v)$ can be evaluated from Eq. 11.68.
ir an ideal gas $A = B = 0$. Then, this expression for $(c_p - c_v)$ reduces to $(c_p - c_v) = R$, which is Eq. 3.44.

PROBLEM 11.70

KNOWN: Three equations of state are under consideration: (a) van der Waals equation, (b) Redlich-Kwong equation, (c) Dieterici equation.

FIND: Determine the maximum Joule-Thompson inversion temperature in terms of the critical temperature, T_c, predicted by each of these equations.

ANALYSIS: Eq. 11.77 can be rearranged to give

$$\mu_J = \frac{[T(\partial v/\partial T)_p - v]}{c_p}$$

The maximum inversion temperature is determined in accordance with $\lim_{p\to 0} \mu_J = 0$. Since $\lim_{p\to 0} c_p(T,p) \longrightarrow c_p(T)$ (ideal gas), this reduces to a consideration of

$$\lim_{p\to 0}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = 0$$

Also, note that the limit as pressure tends to zero corresponds to the limit as $v \to \infty$. Accordingly, the condition to be considered is

$$\lim_{v\to\infty}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = 0$$

(a) van der Waals. The partial derivative $(\partial v/\partial T)_p$ is evaluated in Example 11.2 as

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{-R/(v-b)}{\left[\frac{2a}{v^3} - \frac{RT}{(v-b)^2}\right]} = \frac{Rv^3(v-b)}{RTv^3 - 2a(v-b)^2}$$

Thus

$$\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{RTv^3(v-b)}{RTv^3 - 2a(v-b)^2} - v$$

$$= \frac{+RTv^4 - bRTv^3 - vRTv^3 + 2av(v-b)^2}{RTv^3 - 2a(v-b)^2}$$

$$= \frac{2av(v-b)^2 - bRTv^3}{RTv^3 - 2a(v-b)^2} = \frac{2a - \frac{bRT}{(1-(b/v))^2}}{\frac{RT}{(1-(b/v))^2} - \frac{2a}{v}}$$

Considering the limit as $v \to \infty$

$$\lim_{v\to\infty}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{2a - bRT}{RT} = 0 \quad \Rightarrow \quad T = \frac{2a}{Rb}$$

Introducing Eqs. 11.4a and 11.4b

$$T = \frac{2\left(\frac{27}{64}\frac{R^2T_c^2}{p_c}\right)}{\left(\frac{1}{8}\frac{RT_c}{p_c}\right)} = \frac{27}{4}T_c = 6.75\,T_c \longleftarrow \text{(a)}$$

(b) Redlich-Kwong. To evaluate $(\partial v/\partial T)_p$, use the relation

$$\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial v}\right)_T\left(\frac{\partial T}{\partial p}\right)_v = -1 \quad \Rightarrow \quad \left(\frac{\partial v}{\partial T}\right)_p = -\frac{(\partial p/\partial T)_v}{(\partial p/\partial v)_T}$$

For the Redlich-Kwong equation, the partial derivatives are

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{(v-b)} + \frac{a}{2v(v+b)T^{3/2}}, \quad \left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{a(2v+b)}{v^2(v+b)^2T^{1/2}}$$

So

$$\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = -T\left[\frac{\frac{R}{(v-b)} + \frac{a}{2v(v+b)T^{3/2}}}{\frac{a(2v+b)}{v^2(v+b)^2T^{1/2}} - \frac{RT}{(v-b)^2}}\right] - v$$

PROBLEM 11.70 (Contd.)

Upon rearrangement

$$\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{-\frac{RT}{(v-b)} - \frac{a}{2v(v+b)T^{1/2}} - \frac{a(2v+b)}{v(v+b)^2T^{1/2}} + \frac{vRT}{(v-b)^2}}{\left(\frac{a(2v+b)}{v^2(v+b)^2T^{1/2}} - \frac{RT}{(v-b)^2}\right)}$$

$$= \frac{\frac{bRT}{(v-b)^2} - \frac{a}{2(v+b)^2T^{1/2}}\left[5+3\frac{b}{v}\right]}{\left(\frac{a(2v+b)}{v^2(v+b)^2T^{1/2}} - \frac{RT}{(v-b)^2}\right)} = \left[\frac{\frac{bRT}{(1-\frac{b}{v})^2} - \frac{a\left[5+3\frac{b}{v}\right]}{2(1+\frac{b}{v})^2T^{1/2}}}{\frac{a(2+\frac{b}{v})}{v(v+b)^2T^{1/2}} - \frac{RT}{(1-\frac{b}{v})^2}}\right]$$

Considering the limit as $v \to \infty$

$$\lim_{v\to\infty}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{bRT - \frac{5a}{2T^{1/2}}}{-RT} = 0 \Rightarrow T = \left(\frac{5a}{2bR}\right)^{2/3}$$

With Eqs. 11.8

① $$T = \left(\frac{5(0.42748)R^2T_c^{5/2}/p_c}{(2)(0.08664)\left(\frac{RT_c}{p_c}\right)R}\right)^{2/3} = 5.34\,T_c \longleftarrow \text{(b)}$$

(c) Dieterici. As above

$$\left(\frac{\partial v}{\partial T}\right)_p = -\frac{(\partial p/\partial T)_v}{(\partial p/\partial v)_T}$$

For the Dieteric equation, the partial derivatives are

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{(v-b)}\exp\left(-\frac{a}{RTv}\right)\left[1+\frac{a}{RTv}\right], \quad \left(\frac{\partial p}{\partial v}\right)_T = \frac{RT}{(v-b)}\exp\left(-\frac{a}{RTv}\right)\left[\frac{a}{RTv^2} - \frac{1}{(v-b)}\right]$$

Thus

$$\left[T\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{\left[1+\frac{a}{RTv}\right]}{\left[\frac{a}{RTv^2} - \frac{1}{(v-b)}\right]} - v = \frac{\frac{v}{v-b} - \frac{2a}{RTv} - 1}{\frac{a}{RTv^2} - \frac{1}{(v-b)}}$$

As $\lim_{v\to\infty}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{0}{0}$, apply L'Hospital's rule to obtain

$$\lim_{v\to\infty}\left[\frac{\frac{-b}{(v-b)^2} + \frac{2a}{RTv^2}}{-\frac{2a}{RTv^3} + \frac{1}{(v-b)^2}}\right] = \lim_{v\to\infty}\left[\frac{-\frac{b}{(1-\frac{b}{v})^2} + \frac{2a}{RT}}{\frac{-2a}{RTv} + \frac{1}{(1-\frac{b}{v})^2}}\right] = -b + \frac{2a}{RT}$$

$$\Rightarrow \quad T = \frac{2a}{bR}$$

Introducing the expressions for a and b given in Problem 11.17

$$T = \frac{2\left(\frac{4R^2T_c^2}{p_ce^2}\right)}{R\left(\frac{RT_c}{p_ce^2}\right)} = 8\,T_c \longleftarrow \text{(c)}$$

---

1. The maximum inversion temperature for a wide range of gases is about $5T_c$. Of the three equations of state considered here, only the Redlich-Kwong equation predicts a maximum inversion temperature in the neighborhood of $5T_c$.

PROBLEM 11.71

KNOWN: A certain gas obeys the van der Waals equation of state and has a specific heat c_v

①
$$c_v = A + BT + CT^2$$

where A, B, and C are constants.

FIND: Derive an equation for μ_J as a function of v and T. Evaluate the Joule-Thompson inversion state temperatures in terms of v, R, and the van der Waals constants a and b.

ANALTSIS: Using Eq. 11.77: $\mu_J = \left[T(\partial v/\partial T)_p - v\right]/c_p$. This requires $(\partial v/\partial T)_p$ which is evaluated for the van der Waals equation in Example 11.2 as

$$\left(\frac{\partial v}{\partial T}\right)_p = -\frac{R/(v-b)}{\left[2a/v^3 - RT/(v-b)^2\right]} = \frac{-Rv^3(v-b)}{2a(v-b)^2 - RTv^3}$$

Thus

$$\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{-RTv^3(v-b)}{(2a(v-b)^2 - RTv^3)} - v = \frac{2av(v-b)^2 - bRTv^3}{RTv^3 - 2a(v-b)^2}$$

An expression for c_p is also required. This can be found using Eq. 11.66

$$c_p - c_v = T\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial p}{\partial T}\right)_v$$

From the van der Waals equation, $(\partial p/\partial T)_v = R/(v-b)$. Thus

$$c_p = c_v + T\left[\frac{-Rv^3(v-b)}{2a(v-b)^2 - RTv^3}\right]\frac{R}{(v-b)}$$

$$= (A + BT + c^2) - \frac{R^2Tv^3}{2a(v-b)^2 - RTv^3}$$

$$= \frac{(A + BT + c^2)\left[RTv^3 - 2a(v-b)^2\right] + R^2Tv^3}{RTv^3 - 2a(v-b)^2}$$

Collecting results

$$\mu_J = \left\{\frac{\left(\dfrac{2av(v-b)^2 - bRTv^3}{RTv^3 - 2a(v-b)^2}\right)}{\dfrac{(A+BT+c^2)\left[RTv^3 - 2a(v-b)^2\right] + R^2Tv^3}{RTv^3 - 2a(v-b)^2}}\right\}$$

$$= \frac{2av(v-b)^2 - bRTv^3}{(A+BT+c^2)(RTv^3 - 2a(v-b)^2) + R^2Tv^3} \longleftarrow \mu_J$$

At Joule-Thompson inversion states, $\mu_J = 0$. Accordingly, the inversion temperatures T_i are determined by

② $0 = 2av(v-b)^2 - bRT_iv^3 \Rightarrow T_i = \dfrac{2a(v-b)^2}{bRv^2} = \dfrac{2a}{bR}\left[1 - \left(\dfrac{b}{v}\right)\right]^2 \longleftarrow T_i$

---

1. For a van der Waals gas, $c_v = f(T)$. See Problem 11.62a.
2. In the limit as $p \rightarrow 0$, $v \rightarrow \infty$. Thus, the maximum inversion temperature is $(T_i)_{MAX} = \frac{2a}{bR}$, which is in agreement with the result of Problem 11.70(a).

PROBLEM 11.72

KNOWN: Equation 11.77 is written in an alternative form.

FIND: Verify the alternative form. (a) Using this result, obtain an expression for μ_J for a gas obeying

$$v = \frac{RT}{p} - \frac{Ap}{T^2} \quad \text{where } A \text{ is a constant.}$$

(b) Using the expression obtained in (b), determine c_p for CO_2 at a state where $T = 400\text{ K}$, $p = 1\text{ atm}$, $\mu_J = 0.57\text{ K/atm}$.

ANALYSIS: The alternative expression for μ_J is

$$\mu_J = \frac{T^2}{c_p}\left(\frac{\partial(v/T)}{\partial T}\right)_p \Rightarrow \mu_J = \frac{T^2}{c_p}\left[\frac{1}{T}\left(\frac{\partial v}{\partial T}\right)_p - \frac{v}{T^2}\right] = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right],$$

which agrees with Eq. 11.77.

(b) Using the given relationship, we get $(v/T) = (R/p) - (Ap/T^3)$. Thus

$$\left(\frac{\partial(v/T)}{\partial T}\right)_p = \frac{3Ap}{T4} \Rightarrow \mu_J = \frac{T^2}{c_p}\left[\frac{3Ap}{T4}\right] = \frac{3Ap}{c_p T^2} \longleftarrow$$

(c) Solving for c_p, and using $A = 2.78 \times 10^{-3}\ m^5 \cdot K^2/kg \cdot N$ and other known data

$$c_p = \frac{3AP}{\mu_J T^2} = \frac{3(2.78 \times 10^{-3}\ m^5 \cdot K^2/kg \cdot N)(1.01325 \times 10^5\ N/m^2)}{\left(0.57\ \frac{K}{atm}\right)\left|\frac{1\ atm}{1.01325 \times 10^5\ N/m^2}\right|(400K)^2}\left|\frac{1\ kJ}{10^3\ N \cdot m}\right|$$

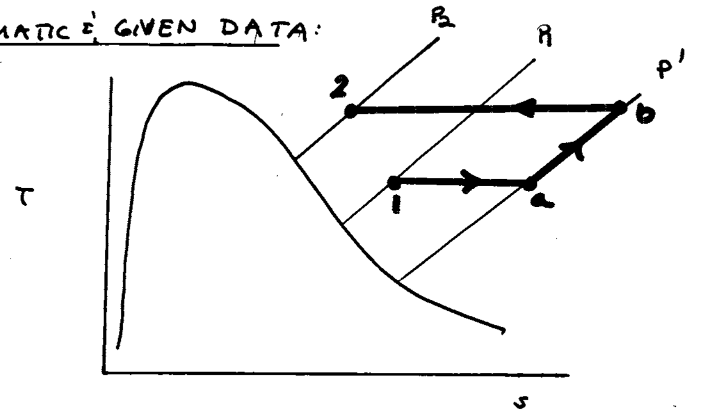
$$= 0.939\ \frac{kJ}{kg \cdot K} \longleftarrow$$

PROBLEM 11.73

KNOWN: For a gas obeying the van der Waals equation, at pressure p' c_v is given by $c_v = A + BT$, where A and B are constants.

FIND: Develop an expression for $[s(T_2,p_2) - s(T_1,p_1)]$

SCHEMATIC & GIVEN DATA:



At 1, $v_1 = v(T_1, p_1)$

At a, $v_a = v(T_1, p')$

At b, $v_b = v(T_2, p')$

At 2, $v_2 = v(T_2, p_2)$

$c_v = A + BT$ when $p = p'$

ASSUMPTION: The gas is described by the van der Waals equation and the given c_v function.

ANALYSIS: For any choice of states 1, 2 for which the assumption is satisfied.

$$s(T_2,p_2) - s(T_1,p_1) = \underbrace{[s(T_2,p_2) - s(T_2,p')]}_{(b\rightarrow 2)} + \underbrace{[s(T_2,p') - s(T_1,p')]}_{(a\rightarrow b)} + \underbrace{[s(T_1,p') - s(T_1,p_1)]}_{(1\rightarrow a)}$$

The specific volumes v_1, v_a, v_b, v_2 can be found by solving the van der Waals equation using the respective temperature and pressure.

Using the van der Waals equation, $(\partial p/\partial T)_v = R/(v-b)$. Thus, with Eq. 11.50

$$s_a - s_1 = \int_1^a \left(\frac{\partial p}{\partial T}\right)_v dv = \int_1^a \frac{R}{v-b}\, dv = R \ln\left(\frac{v_a - b}{v_1 - b}\right) \qquad (1)$$

Similarly

$$s_2 - s_b = R \ln\left(\frac{v_2 - b}{v_b - b}\right) \qquad (2)$$

Then for process a → b, Eq. 11.50 gives

$$s_b - s_a = \int_{T_1}^{T_2} \frac{c_v}{T}\, dT = \int_{T_1}^{T_2} \frac{A + BT}{T} = A \ln\frac{T_2}{T_1} + B(T_2 - T_1) \qquad (3)$$

Then, summing Eqs. (1) - (3)

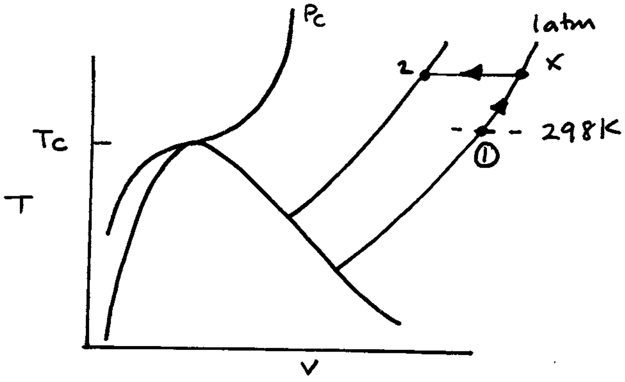
$$s_2 - s_1 = \underbrace{R \ln\left(\frac{v_a - b}{v_1 - b}\right)}_{(1\rightarrow a)} + \underbrace{A \ln\frac{T_2}{T_1} + B(T_2 - T_1)}_{(a\rightarrow b)} + \underbrace{R \ln\left(\frac{v_2 - b}{v_b - b}\right)}_{(b\rightarrow 2)} \longleftarrow$$

PROBLEM 11.74

KNOWN: Air is modeled by the van der Waals equation and $c_p(T)$ from Table A-21.

FIND: Write a computer program evaluating Δh between a state where ($T_1 = 298\,K$, $P_1 = 1\,atm$) and (T_2, P_2).

SCHEMATIC & GIVEN DATA:



From Table A-1, $P_c = 37.2\,atm$, $T_c = 133\,K$

ASSUMPTION: The ideal gas model is valid for the path 1–X (Check Fig. A-1) and the specific ① heat function of Table A-21 is valid for $298 \le T \le 1000\,K$

ANALYSIS: The evaluation of Δh is conveniently accomplished by considering two paths: 1–X and X–2. As noted, the ideal gas model applies for path 1–X, and so

$$h_x - h_1 = \int_{T_1}^{T_2 = T_x} c_p(T)\,dT = R\left[\alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) + \frac{\gamma}{3}(T_2^3 - T_1^3) + \frac{\delta}{4}(T_2^4 - T_1^4) + \frac{\epsilon}{5}(T_2^5 - T_1^5)\right] \tag{1}$$

where $\alpha, \beta, \gamma, \delta,$ and ϵ are obtained from Table A-21

Since the van der Waals equation is explicit in pressure, it is convenient to evaluate $h_2 - h_x$ using

$$h_2 - h_x = (u_2 - u_x) + P_2 v_2 - P_x v_x \tag{2}$$

where $(u_2 - u_x)$ is evaluated from Eq. 11.51, which reduces to give

$$u_2 - u_x = \int_{v_x}^{v_2}\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv = \int_{v_x}^{v_2}\left(T\left(\frac{R}{v-b}\right) - p\right)dv = \int_{v_x}^{v_2}\frac{a}{v^2} = -a\left[\frac{1}{v_2} - \frac{1}{v_x}\right] \tag{3}$$

With Eqs. (2) and (3)

$$h_2 - h_x = P_2 v_2 - P_x v_x - a\left[\frac{1}{v_2} - \frac{1}{v_x}\right] \tag{4}$$

The value of v_x can be obtained using the ideal gas equation of state: $v_x = RT_x/P_x = RT_2/P_1$. The value of v_2 can be obtained iteratively from the van der Waals equation using the specified values for P_2, T_2.

The overall change $(h_2 - h_1)$ is then obtained from Eqs. (1) and (4):

$$(h_2 - h_1) = \underbrace{(h_2 - h_x)}_{Eq.(4)} + \underbrace{(h_x - h_1)}_{Eq.(1)}$$

This provides the required relations for finding Δh. Computer program details are left to the reader.

---

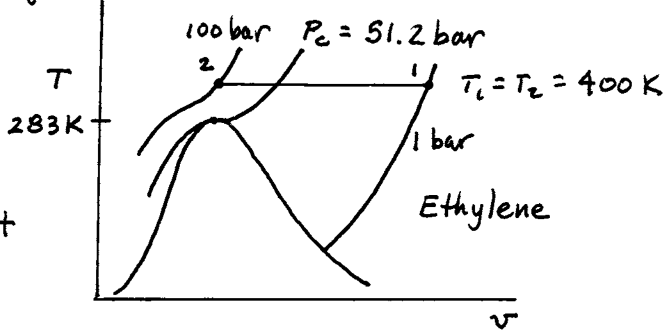
1. For temperatures out of this range, other representations of $c_p(T)$ may be required.

PROBLEM 11.75

KNOWN: Two states of ethylene (C_2H_4) are under consideration: $T_1 = T_2 = 400\,K$, $p_1 = 1$ bar, $p_2 = 100$ bar.

FIND: Using the Redlich-Kwong equation of state, evaluate $\Delta\bar{s}$ and $\Delta\bar{h}$.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The Redlich-Kwong equation of state is applicable at these states.

ANALYSIS: The results of Problem 11.49(b) are quoted for present use:

$$\bar{h}_2 - \bar{h}_1 = p(\bar{v}_2, T)\,\bar{v}_2 - p(\bar{v}_1, T)\,\bar{v}_1 - \frac{3a}{2bT^{1/2}} \ln\left[\frac{\bar{v}_2 + b}{\bar{v}_1 + b} \cdot \frac{\bar{v}_1}{\bar{v}_2}\right] \quad (1)$$

$$\bar{s}_2 - \bar{s}_1 = \bar{R} \ln\left(\frac{\bar{v}_2 - b}{\bar{v}_1 - b}\right) - \frac{a}{2bT^{3/2}} \ln\left[\frac{\bar{v}_2 + b}{\bar{v}_1 + b} \cdot \frac{\bar{v}_1}{\bar{v}_2}\right] \quad (2)$$

where the values of a and b are obtained using Eq. 11.8 as

$$a = 77.76 \text{ bar} \left(\frac{m^3}{kmol}\right)^2 K^{1/2}, \quad b = 0.03981 \frac{m^3}{kmol}$$

The values of $\bar{v}_1$ and $\bar{v}_2$ can be found using IT to solve the Redlich-Kwong equation knowing the pressure and temperature at each state. The results are

$$\bar{v}_1 = 33.18 \text{ m}^3/\text{kmol}, \quad \bar{v}_2 = 0.2636 \text{ m}^3/\text{kmol}$$

Inserting numerical values into Eq. (2)

$$\bar{s}_2 - \bar{s}_1 = \left(8.314 \frac{kJ}{kmol \cdot K}\right) \ln\left(\frac{.22379}{32.9164}\right) - \frac{77.76 \text{ bar} \left(\frac{m^3}{kmol}\right)^2 K^{1/2}}{2\left(.03981 \frac{m3}{kmol}\right)(400\,K)^{3/2}} \left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right| \ln\left[\frac{0.30341}{33.21981} \cdot \frac{33.18}{0.2636}\right]$$

$$= -43.20 \text{ kJ/kmol}\cdot\text{K} \longleftarrow \Delta\bar{s}$$

$$\bar{h}_2 - \bar{h}_1 = \left[(100 \text{ bar})(0.2636 \text{ m}^3/\text{kmol}) - (1)(33.18)\right] \left|\frac{10^5 \text{ N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{ N}\cdot\text{m}}\right|$$

$$- \frac{(3)\left(77.76 \text{ bar} \left(\frac{m^3}{kmol}\right)^2 K^{1/2}\right)}{(2)(0.03981 \text{ m}^3/\text{kmol})(400\,K)^{1/2}} \ln\left[\frac{0.30341}{33.21981} \cdot \frac{33.18}{0.2636}\right]$$

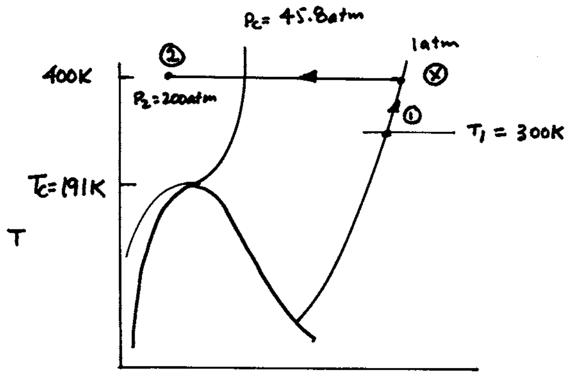
$$= -2270.6 \text{ kJ/kmol} \longleftarrow \Delta\bar{h}$$

PROBLEM 11.76

KNOWN: Two states of CH_4 are under consideration: $T_1 = 300K$, $P_1 = 1atm$, $T_2 = 400K$, $P_2 = 200$ atm.

FIND: Using the Benedict-Webb-Rubin equation of state together with an appropriate specific heat relation, determine $\Delta\bar{h}$.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The ideal gas model applies along path 1-x.

ANALYSIS: The evaluation of $\Delta\bar{h}$ can be accomplished conveniently by considering two paths: 1-x followed by x-2. For $p = 1atm$, $P_R = 1/45.8 = 0.022$. Reference to Fig A-1 suggests that at this pressure the ideal gas model is applicable. Thus, since T_1 and $T_x = T_2$ are in the range of Table A-21

$$\bar{h}_x - \bar{h}_1 = \int_{T_1}^{T_x=T_2} \bar{c}_p(T)\,dT = \bar{R}\left[\alpha(T_2-T_1) + \frac{\beta}{2}(T_2^2-T_1^2) + \frac{\gamma}{3}(T_2^3-T_1^3) + \frac{\delta}{4}(T_2^4-T_1^4) + \frac{\epsilon}{5}(T_2^5-T_1^5)\right] \quad (1)$$

where $\alpha, \beta, \gamma, \delta$, and ϵ are given in the table.

Since the Benedict-Webb-Rubin equation is explicit in pressure, it is convenient to evaluate $h_2 - h_x$ using

$$h_2 - h_x = (u_2 - u_x) + P_2 v_2 - P_x v_x \quad (2)$$

where $(u_2 - u_x)$ is evaluated from Eq. 11.51, which reduces to give

$$\bar{u}_2 - \bar{u}_x = \int_{\bar{v}_x}^{\bar{v}_2} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] d\bar{v}$$

Differentiating Eq 11.12

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{\bar{R}}{\bar{v}} + \left[B\bar{R} + \frac{2C}{T^3}\right]\frac{1}{\bar{v}^2} + \frac{b\bar{R}}{\bar{v}^3} - \frac{2c}{\bar{v}^3 T^3}\left(1 + \frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$T\left(\frac{\partial p}{\partial T}\right)_v = \frac{\bar{R}T}{\bar{v}} + \left[B\bar{R}T + \frac{2C}{T^2}\right]\frac{1}{\bar{v}^2} + \frac{b\bar{R}T}{\bar{v}^3} - \frac{2c}{\bar{v}^3 T^2}\left(1 + \frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

$$\left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] = \left[A + \frac{3C}{T^2}\right]\frac{1}{\bar{v}^2} + \frac{a}{\bar{v}^3} - \frac{a\alpha}{\bar{v}^6} - \frac{3c}{\bar{v}^3 T^2}\left(1 + \frac{\gamma}{\bar{v}^2}\right)\exp\left(-\frac{\gamma}{\bar{v}^2}\right)$$

Thus

$$u_2 - u_x = \left[A + \frac{3C}{T_2^2}\right]\left[\frac{1}{\bar{v}_x} - \frac{1}{\bar{v}_2}\right] + \frac{a}{2}\left[\frac{1}{\bar{v}_x^2} - \frac{1}{\bar{v}_2^2}\right] + \frac{a\alpha}{5}\left[\frac{1}{\bar{v}_2^5} - \frac{1}{\bar{v}_x^5}\right]$$
$$- \frac{3c}{T_2^2}\int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^3}\exp\left(-\frac{\gamma}{\bar{v}^2}\right)d\bar{v} - \frac{3c\gamma}{T_2^2}\int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^5}\exp\left(-\frac{\gamma}{\bar{v}^2}\right)d\bar{v} \quad (3)$$

PROBLEM 11.76 (Contd.)

In Eq. (3)

$$\int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^3} \exp\left(-\frac{\gamma}{\bar{v}^2}\right) d\bar{v} = \frac{1}{2\gamma}\left[\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \exp\left(-\frac{\gamma}{\bar{v}_x^2}\right)\right]$$

$$\int_{\bar{v}_x}^{\bar{v}_2} \frac{1}{\bar{v}^5} \exp\left(-\frac{\gamma}{\bar{v}^2}\right) d\bar{v} = \frac{1}{2\gamma^2}\left(\frac{\gamma}{\bar{v}_2^2}+1\right)\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \frac{1}{2\gamma^2}\left(\frac{\gamma}{\bar{v}_x^2}+1\right)\exp\left(-\frac{\gamma}{\bar{v}_x^2}\right)$$

Collecting results

$$u_2 - u_x = \left[A + \frac{3C}{T_2^2}\right]\left[\frac{1}{\bar{v}_x} - \frac{1}{\bar{v}_2}\right] + \frac{a}{2}\left[\frac{1}{\bar{v}_x^2} - \frac{1}{\bar{v}_2^2}\right] + \frac{a\alpha}{5}\left[\frac{1}{\bar{v}_2^5} - \frac{1}{\bar{v}_x^5}\right] -$$

$$\frac{3c}{2\gamma T_2^2}\left[\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \exp\left(-\frac{\gamma}{\bar{v}_x^2}\right)\right] - \frac{3c}{2\gamma T_2^2}\left[\left(\frac{\gamma}{\bar{v}_2^2}+1\right)\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right) - \left(\frac{\gamma}{\bar{v}_x^2}+1\right)\exp\left(-\frac{\gamma}{\bar{v}_x^2}\right)\right]$$

Finally

$$u_2 - u_x = \left[A + \frac{3C}{T_2^2}\right]\left[\frac{1}{\bar{v}_x} - \frac{1}{\bar{v}_2}\right] + \frac{a}{2}\left[\frac{1}{\bar{v}_x^2} - \frac{1}{\bar{v}_2^2}\right] + \frac{a\alpha}{5}\left[\frac{1}{\bar{v}_2^5} - \frac{1}{\bar{v}_x^5}\right] - \frac{3c}{2\gamma T_2^2}\left(\frac{\gamma}{\bar{v}_2^2}+2\right)\exp\left(-\frac{\gamma}{\bar{v}_2^2}\right)$$

$$+\frac{3c}{2\gamma T_2^2}\left(\frac{\gamma}{\bar{v}_x^2}+2\right)\exp\left(-\frac{\gamma}{\bar{v}_x^2}\right) \quad (4)$$

The value of v_x can be found using the ideal gas equation of state: $v_x = RT_x/p_x = RT_2/p_1$. The value of v_2 can be obtained by solving the Benedict-Webb-Rubin equation. The results are

$$v_x = 32.807\ \frac{m^3}{kmol}, \quad v_2 = 0.1607\ \frac{m^3}{kmol}$$

The values of the Benedict-Webb-Rubin constants are obtained from Table A-24. Accordingly, all values that are required to evaluate Eqs. (1), (2), and (4) are known. The final result is

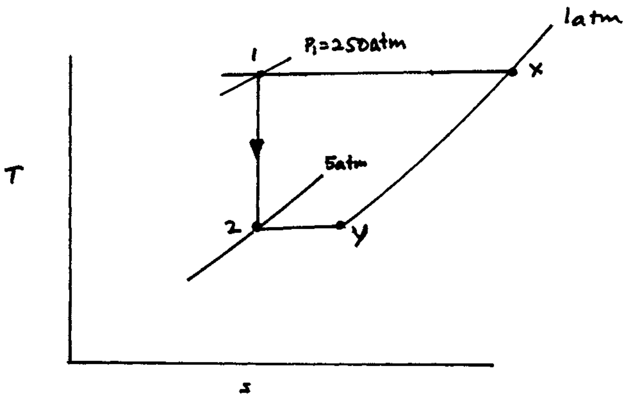
$$h_2 - h_1 \approx 2305\ \frac{kJ}{kmol} \quad \left(\approx 144\ \frac{kJ}{kg}\right) \longleftarrow$$

PROBLEM 11.77

KNOWN: Nitrogen expands isentropically from $T_1 = 400K$, $P_1 = 250$ atm to $P_2 = 5$atm.

FIND: Using the Redlich-Kwong equation of state together with an appropriate specific heat relation, determine T_2.

SCHEMATIC & GIVEN DATA:



ANALYSIS: For an isentropic expansion, $\bar{s}_2 - \bar{s}_1 = 0$. In the present case it is convenient to consider three paths in series: constant temperature to 1atm (1-x), constant pressure to T_2 (x-y), constant temperature to state 2: y-2. Then

$$0 = \bar{s}_2 - \bar{s}_1 = (\bar{s}_2 - \bar{s}_y) + (\bar{s}_y - \bar{s}_x) + (\bar{s}_x - \bar{s}_1) \tag{1}$$

Since $P_c = 33.5$ atm, $P_R = 1/33.5 = 0.03$, indicating that the ideal gas model is appropriate for path x-y. As 1-x and y-2 are at fixed temperature, and the Redlich-Kwong equation is explicit in pressure, it is convenient to select Eq. 11.50 which reduces to give

$$\bar{s}_x - \bar{s}_1 = \int_1^x \left(\frac{\partial p}{\partial T}\right)_v d\bar{v} \quad , \quad \bar{s}_2 - \bar{s}_y = \int_y^2 \left(\frac{\partial p}{\partial T}\right)_v d\bar{v}$$

In Problem 11.49b, an expression for these integrals is obtained. Thus

$$\bar{s}_x - \bar{s}_1 = \bar{R} \ln\left[\frac{\bar{v}_x - b}{\bar{v}_1 - b}\right] - \frac{a}{2b T_1^{3/2}} \ln\left[\frac{\bar{v}_x + b}{\bar{v}_1 + b} \cdot \frac{\bar{v}_1}{\bar{v}_x}\right] \tag{2}$$

$$\bar{s}_2 - \bar{s}_y = \bar{R} \ln\left[\frac{\bar{v}_2 - b}{\bar{v}_y - b}\right] - \frac{a}{2b T_2^{3/2}} \ln\left[\frac{\bar{v}_2 + b}{\bar{v}_y + b} \cdot \frac{\bar{v}_y}{\bar{v}_2}\right] \tag{3}$$

The values of the constants a and b can be obtained from Table A-24. The value of $\bar{v}_x$ can be found using the ideal gas equation of state: $\bar{v}_x = \bar{R}T_1/P_x$, where $P_x = 1$atm. Similarly, $v_y = \bar{R}T_2/P_y$, where $P_y = 1$atm. The value of $\bar{v}_1$ can be obtained by solving the Redlich-Kwong equation of state, knowing $P_1 = 250$ atm, $T_1 = 400K$. Similarly, $\bar{v}_2$ can be found by solving the Redlich-Kwong equation of state, knowing T_2 and $P_2 = 5$atm.

To obtain $(\bar{s}_y - \bar{s}_x)$, Eq. 6.19 can be used:

$$\bar{s}_y - \bar{s}_x = \int_{T_x = T_1}^{T_y = T_2} \frac{\bar{c}_p(T)}{T} dT \tag{4}$$

where $\bar{c}_p(T)$ is the ideal gas specific heat c_p. The $c_p(T)$ relation given in Table A-21 is valid for $300 < T < 1000K$, so a relation that is more suited for the temperatures under present consideration is needed. For example, Table 1-23 of the Handbook of Tables for Applied Engineering Science, 2nd Ed., CRC Press, Cleveland, OH, 1977 (R.E. Bolz and G.L. Tuve, Eds.) gives c_p at 1atm for $250 < T < 1500$ K. Another source of data is p. 2-91 of the Handbook of Heat Transfer, McGraw-Hill, New York, 1973 (W.M. Rohsenow and J.P. Harnett, Eds.). Also, note that the data of Table A-23 suggests that c_p of N_2 varies only slightly for temperatures below about 300K. Using tabular data from the literature, and appropriate computer software, a polynomial expression for c_p can be developed. To illustrate, the equation form of Table A-21 might be employed. Then, Eq. (4) would give

$$\bar{s}_y - \bar{s}_x = R\left[\alpha \ln\frac{T_2}{T_1} + \beta(T_2-T_1) + \frac{\gamma}{2}(T_2^2-T_1^2) + \frac{\delta}{3}(T_2^3-T_1^3) + \frac{\epsilon}{4}(T_2^4-T_1^4)\right] \quad (5)$$

where α, β, γ, δ, and ϵ are determined as indicated above.

Collecting Eqs. (1), (2), (3), and (5)

$$0 = \left\{\bar{R}\ln\left[\frac{\bar{v}_x - b}{\bar{v}_1 - b}\right] - \frac{a}{2bT_1^{3/2}}\left[\frac{\bar{v}_x+b}{\bar{v}_1+b}\cdot\frac{\bar{v}_1}{\bar{v}_x}\right]\right\} + \bar{R}\left[\alpha\ln\frac{T_2}{T_1} + \beta(T_2-T_1) + \frac{\gamma}{2}(T_2^2-T_1^2) + \right.$$

$$\left.\frac{\delta}{3}\left[T_2^3-T_1^3\right] + \frac{\epsilon}{4}(T_2^4-T_1^4)\right] + \left\{\bar{R}\ln\left[\frac{\bar{v}_2-b}{\bar{v}_y-b}\right] - \frac{a}{2bT_2^{3/2}}\ln\left[\frac{\bar{v}_2+b}{\bar{v}_y+b}\cdot\frac{\bar{v}_y}{\bar{v}_2}\right]\right\} \quad (6)$$

The following values are known: T_1, $\bar{v}_1$, $\bar{v}_x$, a, b, α, β, γ, δ, ϵ, $\bar{R}$. As noted above, $\bar{v}_y$ and $\bar{v}_2$ can be determined from the appropriate equation of state using T_2 and P_y = 1atm and P_2 = 5atm, respectively. Accordingly, the computational procedure is to specify T_2, find $\bar{v}_y$ and $\bar{v}_2$, and check for closure using Eq. (6). The use of a computer is clearly indicated. The final result is $T_2 \approx$ 125K. ◄

PROBLEM 11.78

KNOWN: A substance has the property relations

(vapor phase) $v = \dfrac{RT}{p} - \dfrac{Bp}{T^2}$

(sat. p vs. T) $\ln P_{sat} = 12 - \dfrac{2400}{T}$

(ideal gas) $c_{po} =$ constant $0 < T < 300^\circ F$

(Joule-Thomson) $\mu_J(10\ \text{lbf/in}^2, 200^\circ F) = 0.004\ {}^\circ R \cdot \text{ft}^2/\text{lbf}$

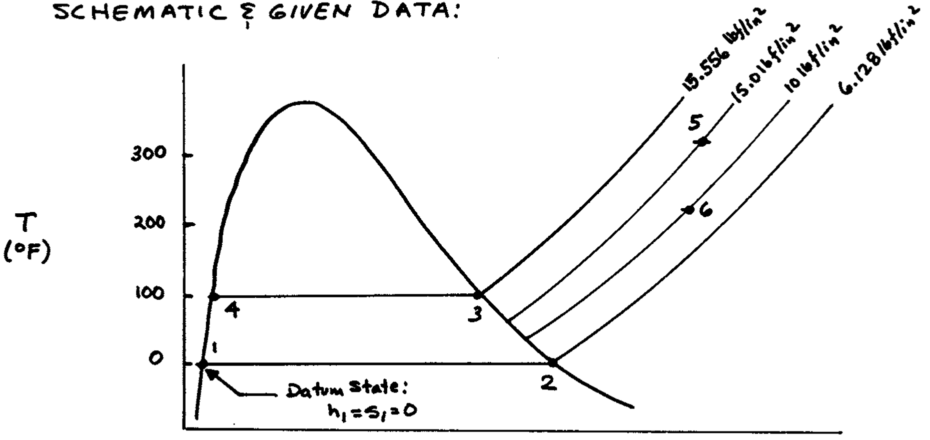
where $v: \text{ft}^3/\text{lb}$, $T: {}^\circ R$, $p: \text{lbf/ft}^2$, $R = 50\ \text{ft}\cdot\text{lbf/lb}\cdot{}^\circ R$, $B = 100\ \text{ft}^5({}^\circ R)^2/\text{lb}\cdot\text{lbf}$

FIND: (a) Complete the table

T(°F)	p (lbf/in²)	v (ft³/lb)		h (Btu/lb)		s (Btu/lb·°R)	
–	–	v_f	v_g	h_f	h_g	s_f	s_g
0		0.03		0		0	
100		0.03					

(b) Evaluate v, h, s at 300°F, 15 lbf/in²

SCHEMATIC & GIVEN DATA:



State 5: 300°F, 15 lbf/in²

State 6: 200°F, 10 lbf/in²

ANALYSIS: (a) Using the given expression for ln P_{sat}

at 0°F: $\ln P_{sat} = 12 - \dfrac{2400}{460} \Rightarrow P_2 = P_1 = 882.37\ \text{lbf/ft}^2\ (6.128\ \text{lbf/in}^2)$

at 100°F: $\ln P_{sat} = 12 - \dfrac{2400}{560} \Rightarrow P_3 = P_4 = 2240.12\ \text{lbf/ft}^2\ (15.556\ \text{lbf/in}^2)$

State 2: With the above pressure value, and the vapor phase equation of state,

$$v_2 = \frac{RT_2}{P_2} - \frac{BP_2}{T_2^2} = \frac{(50\ \text{ft}\cdot\text{lbf/lb}\cdot{}^\circ R)(460^\circ R)}{882.37\ \text{lbf/ft}^2} - \frac{(100\ \text{ft}^5({}^\circ R)^2/\text{lb}\cdot\text{lbf})(882.37\ \text{lbf/ft}^2)}{(460^\circ R)^2}$$

$$= 25.65\ \text{ft}^3/\text{lb}$$

Similarly

$$v_3 = \frac{RT_3}{P_3} - \frac{BP_3}{T_3^2} = \frac{(50)(560)}{2240.12} - \frac{(100)(2240.12)}{(560)^2} = 11.79\ \text{ft}^3/\text{lb}$$

$$v_5 = \frac{RT_5}{P_5} - \frac{BP_5}{T_5^2} = \frac{(50)(760)}{2160} - \frac{(100)(2240.12)}{(760)^2} = 17.20\ \text{ft}^3/\text{lb}$$

PROBLEM 11.78 (Contd. – 2/4)

Using the Clapeyron equation, Eq. 11.40,

$$h_2 = \cancelto{0}{h_1} + T_2(v_g - v_f)\left(\frac{dp}{dT}\right)_{sat}$$

Since

$$p_{sat} = \exp\left(12 - \frac{2400}{T}\right)$$

$$\frac{dp_{sat}}{dT} = \frac{2400}{T^2}\exp\left(12 - \frac{2400}{T}\right) = \frac{2400\, p_{sat}}{T^2}$$

So

$$h_2 = T_2(v_g - v_f)\left[\frac{2400\, p_2}{T_2^2}\right] = \frac{(2400)(v_g - v_f)\, p_2}{T_2}$$

$$= \frac{(2400\,°R)(25.65 - 0.03)\left(\frac{ft^3}{lb}\right)(882.37\ lbf/ft^2)}{460\,°R}\left|\frac{Btu}{778\,ft\,lbf}\right| = 151.60\ \frac{Btu}{lb}$$

Then, with Eq. 11.38

$$s_2 - \cancelto{0}{s_1} = \frac{h_2 - \cancelto{0}{h_1}}{T} \Rightarrow s_2 = \frac{151.60}{460} = 0.330\ \frac{Btu}{lb \cdot °R}$$

State 3. With Eq. 11.82

$$h(T_3, p_3) = \bar{h}^*(T_3) + \int_0^{p_3}\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp \qquad (1)$$

$$h(T_2, p_2) = \bar{h}^*(T_2) + \int_0^{p_2}\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]dp \qquad (2)$$

where with the given equation of state

$$\left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] = \left(\frac{RT}{p} - \frac{Bp}{T^2}\right) - T\left(\frac{R}{p} + \frac{2Bp}{T^3}\right) = -\frac{3Bp}{T^2} \qquad (3)$$

Accordingly, with Eqs. (1)-(3)

$$h_3 - h_2 = \bar{h}^*(T_3) - \bar{h}^*(T_2) + \int_0^{p_3}\left(-\frac{3Bp}{T_3^2}\right)dp - \int_0^{p_2}\left(-\frac{3Bp}{T_2^2}\right)dp$$

$$= \int_{T_2}^{T_3} c_{po}\,dT - \frac{3Bp_3^2}{2T_3^2} + \frac{3Bp_2^2}{2T_2^2} = c_{po}[T_3 - T_2] - \frac{3B}{2}\left[\left(\frac{p_3}{T_3}\right)^2 - \left(\frac{p_2}{T_2}\right)^2\right] \qquad (4)$$

where c_{po} is given in the problem statement as constant.

To calculate $h_3 - h_2$ requires the value of c_{po}. This can be determined as follows: Applying the test for exactness to Eq. 11.57 gives

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p$$

Then, paralleling the development of Eq 11.82

$$c_p(T,p) - c_{po}(T) = -T\int_0^p\left(\frac{\partial^2 v}{\partial T^2}\right)_p dp$$

From the given equation of state $(\partial^2 v/\partial T^2)_p = -6Bp/T^4$. Thus,

$$c_p(T,p) - c_{po}(T) = -T\int_0^p\left(-\frac{6Bp}{T^4}\right)dp = \frac{3Bp^2}{T^3}$$

Rearranging this

$$c_{po} = c_p(T,p) - \frac{3Bp^2}{T^3} \qquad (5)$$

Thus c_{po} can be found from the value of c_p at any state. The value of c_p at State 5 can be found using the Joule-Thomson coefficient. Thus, with Eq. 11.77

$$c_p = \frac{1}{\mu_J}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{3Bp}{\mu_J T^2}$$

At T = 660°R, p = 1440 lbf/ft² this gives

$$c_p = \frac{3\,(100\ \text{ft}^5\cdot(°\text{R})^2/\text{lb}\cdot\text{lbf})(1440\ \text{lbf/ft}^2)}{(0.004\ °\text{R}\cdot\text{ft}^2/\text{lbf})(660°\text{R})^2}\left|\frac{\text{Btu}}{778\ \text{ft}\cdot\text{lbf}}\right| = 0.3187\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

Then, with Eq.(5)

$$c_{po} = 0.3187 - \frac{(3)(100)(1440)^2}{(660)^3}\left|\frac{1}{778}\right| = 0.3159\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

Returning to Eq.(4)

$$h_3 - h_2 = 0.3159\,(100) - \frac{3\,(100\ \text{ft}^3(°\text{R})^2/\text{lb}\cdot\text{lbf})}{2}\left[\left(\frac{2240.12}{560}\right)^2 - \left(\frac{882.37}{460}\right)^2\right]\left(\frac{\text{lbf/ft}^2}{°\text{R}}\right)^2\left|\frac{\text{Btu}}{778\,\text{ft}\,\text{lbf}}\right|$$

$$= 31.59 - 2.38 = 29.21\ \text{Btu/lb}$$

Then, with the previously determined value for h_2, $h_3 = 180.81$ Btu/lb.

With Eq. 11.90

$$s(T,p) - s^*(T,p) = \int_0^p\left[\frac{R}{p} - \left(\frac{\partial v}{\partial T}\right)_p\right]dp$$

Using the given equation of state

$$s(T,p) - s^*(T,p) = \int_0^p\left[\frac{R}{p} - \left(\frac{R}{p} + \frac{2Bp}{T^3}\right)\right]dp = -\frac{Bp^2}{T^3} \qquad (6)$$

Using Eq.(6),

$$s_3 - s_3^* = -\frac{Bp_3^2}{T_3^2}\ , \quad s_2 - s_2^* = -\frac{Bp_2^2}{T_2^3}$$

Accordingly

$$s_3 - s_2 = s_3^* - s_2^* - B\left[\frac{p_3^2}{T_3^2} - \frac{p_2^2}{T_2^3}\right]$$

$$= c_{po}\ln\frac{T_3}{T_2} - R\ln\frac{p_3}{p_2} - B\left[\frac{p_3^2}{T_3^2} - \frac{p_2^2}{T_2^3}\right]$$

Substituting values

$$s_3 - s_2 = 0.3159\ln\left[\frac{560}{460}\right] - \left(\frac{50}{778}\right)\ln\frac{2240.12}{882.37} - \left(\frac{100}{778}\right)\left[\frac{(2240.12)^2}{(560)^3} - \frac{(882.37)^2}{(460)^3}\right]$$

$$= -0.00038\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$$

Thus, with the previously determined value for s_2, $s_3 = 0.3296$ Btu/lb·°R.

PROBLEM 11.78 (Contd. - 4/4)

State 4. Using the Clapeyron equation

$$h_3 - h_4 = T_3 (v_3 - v_4) \left(\frac{dP}{dT}\right)_{sat}$$

Thus

$$h_4 = h_3 - \frac{2400(v_3 - v_4) P_3}{T_3} = 180.81 - \frac{2400(11.79 - 0.03)(2240.12)}{(560)|778|}$$

$$= 35.69 \text{ Btu/lb}$$

And with Eq. 11.38

$$s_3 - s_4 = \frac{h_3 - h_4}{T_3} = 0.259 \Rightarrow s_4 = 0.0705 \ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}$$

SUMMARY:

T(°F)	P (lbf/in²)	v_f	v_g	h_f	h_g	s_f	s_g
0	6.128	0.03	25.65	0	151.6	0	0.330 ←
100	15.556	0.03	11.79	35.69	180.81	0.0705	0.3296

(b) To evaluate v_5, h_5, s_5.

With the equation of state, $v_5 = 17.20$ ft³/lb (see page 1).
Following the same procedures as used to evaluate h_3 and s_3

$$h_5 = h_2 + c_{p0}(T_5 - T_2) - \frac{3B}{2}\left[\left(\frac{P_5}{T_5}\right)^2 - \left(\frac{P_2}{T_2}\right)^2\right]$$

$$= 151.6 + 0.3159(300) - \frac{(3)(100)}{(2)(778)}\left[\left(\frac{2160}{760}\right)^2 - \left(\frac{882.37}{460}\right)^2\right]$$

$$= 245.52 \ \frac{\text{Btu}}{\text{lb}} \qquad \xleftarrow{\quad h_5 \quad}$$

$$s_5 = s_2 + c_{p0} \ln\frac{T_5}{T_2} - R \ln\frac{P_5}{P_2} - B\left[\frac{P_5^2}{T_5^3} - \frac{P_2^2}{T_2^3}\right]$$

$$= 0.3296 + 0.3159 \ln\left(\frac{760}{460}\right) - \left(\frac{50}{778}\right) \ln\frac{2160}{882.37} - \left(\frac{100}{778}\right)\left[\frac{(2160)^2}{(760)^3} - \frac{(882.37)^2}{(460)^3}\right]$$

$$= 0.4303 \ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} \qquad \xleftarrow{\quad s_5 \quad}$$

PROBLEM 11.79

FIND: Beginning with Eq. 11.90 derive Eq. 11.91.

ANALYSIS:

$$s(T,p) - s^*(T,p) = \int_0^p \left[\frac{R}{p} - \left(\frac{\partial v}{\partial T}\right)_p \right] dp$$

With $v = ZRT/p$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{RZ}{p} + \frac{RT}{p}\left(\frac{\partial Z}{\partial T}\right)_p$$

Thus

$$\left[\frac{R}{p} - \left(\frac{\partial v}{\partial T}\right)_p\right] = \frac{R}{p}(1-Z) - \frac{RT}{p}\left(\frac{\partial Z}{\partial T}\right)_p$$

With $T = T_c T_R$, $p = p_c p_R$ $(\partial Z/\partial T)_p = 1/T_c\,(\partial Z/\partial T_R)_{p_R}$

$$\left[\frac{R}{p} - \left(\frac{\partial v}{\partial T}\right)_p\right] = \frac{R}{p_R p_c}(1-Z) - \frac{R T_R T_c}{p_R p_c} \cdot \frac{1}{T_c}\left(\frac{\partial Z}{\partial T_R}\right)_{p_R}$$

Collecting results

$$s(T,p) - s^*(T,p) = \int_0^p \frac{R}{p_R p_c}(1-Z)\,dp - \int_0^p \frac{R T_R}{p_R p_c}\left(\frac{\partial Z}{\partial T_R}\right)_{p_R} dp$$

$$= R\left[\int_0^{p_R}(1-Z)\frac{dp_R}{p_R} - \underbrace{\int_0^{p_R} T_R\left(\frac{\partial Z}{\partial T_R}\right)_{p_R}\frac{dp_R}{p_R}}_{\dfrac{h^*(T)-h(T,p)}{R T_c T_R}}\right] \quad (\text{Eq. } 11.84)$$

So

$$\frac{s^*(T,p) - s(T,p)}{R} = \frac{h^*(T) - h(T,p)}{R T_R T_c} + \int_0^{p_R}(Z-1)\frac{dp_R}{p_R}$$

Or on a per mole basis

$$\frac{\bar{s}^*(T,p) - \bar{s}(T,p)}{\bar{R}} = \frac{\bar{h}^*(T) - \bar{h}(T,p)}{\bar{R} T_c T_c} + \int_0^{p_R}(Z-1)\frac{dp_R}{p_R} \longleftarrow$$

PROBLEM 11.80

FIND: (a) Derive an expression giving $[u(T,v) - u^*(T)]$. (b) Derive an expression giving $[s(T,v) - s^*(T,v)]$.

ANALYSIS: (a) Eq. 11.47 gives

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p$$

Integrating from specific volume v′ to v at fixed T

$$u(T,v) - u(T,v') = \int_{v'}^{v} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv$$

Adding and subtracting $u^*(T)$ on the left side

$$[u(T,v) - u^*(T)] - [u(T,v') - u^*(T)] = \int_{v'}^{v} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv \qquad (1)$$

As pressure tends to zero at fixed temperature, the internal energy of a substance approaches that of its ideal gas model. However, $\lim_{p\to 0}$ (at fixed T) is equivalent to $\lim_{v\to\infty}$ (fixed T). Accordingly, Eq. (1) becomes in the limit as $v' \to \infty$ (at fixed T)

$$\lim_{v'\to\infty}\left\{[u(T,v) - u^*(T)] - [u(T,v') - u^*(T)]\right\} = \int_{\infty}^{v} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv$$

or

$$u(T,v) - u^*(T) = \int_{\infty}^{v} \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right] dv \longleftarrow \qquad (a)$$

(b) Eq. 11.34 gives

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

Integrating from specific volume v′ to v at fixed T

$$s(T,v) - s(T,v') = \int_{v'}^{v} \left(\frac{\partial p}{\partial T}\right)_v dv \qquad (2)$$

For an ideal gas $p = RT/v$, so $(\partial p/\partial T)_v = R/v$. Using this, Eq. (2) gives for an ideal gas

$$s^*(T,v) - s^*(T,v') = \int_{v'}^{v} \frac{R}{v} dv \qquad (3)$$

Subtracting Eq. (3) from Eq. (2)

$$[s(T,v) - s^*(T,v)] - [s(T,v') - s^*(T,v')] = \int_{v'}^{v} \left[\left(\frac{\partial p}{\partial T}\right)_v - \frac{R}{v}\right] dv \qquad (4)$$

As noted in part (a), as specific volume tends to infinity at fixed temperature, the specific entropy of a substance approaches that of its ideal gas model. Thus, in the limit as $v' \to \infty$ (at fixed T), Eq. (4) gives

$$s(T,v) - s^*(T,v) = \int_{\infty}^{v} \left[\left(\frac{\partial p}{\partial T}\right)_v - \frac{R}{v}\right] dv \longleftarrow \qquad (b)$$

PROBLEM 11.81

KNOWN: The equation of state is

$$Z = 1 + B(T_R)\,p_R$$

FIND: Derive expressions for the enthalpy and entropy departures.

ANALYSIS: Beginning with Eq. 11.84

$$\frac{\bar{h}^*(T) - \bar{h}(T,p)}{\bar{R}T_c} = T_R^2 \int_0^{p_R} \left(\frac{\partial Z}{\partial T_R}\right)_{p_R} \frac{dp_R}{p_R}$$

where

$$\left(\frac{\partial Z}{\partial T_R}\right)_{p_R} = \frac{dB}{dT_R}\,p_R$$

Thus

$$\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} = T_R^2 \int_0^{p_R} \left(\frac{dB}{dT_R}\,p_R\right) \frac{dp_R}{p_R}$$

$$= T_R^2 \left(\frac{dB}{dT_R}\right) p_R \qquad \longleftarrow$$

Beginning with Eq. 11.91

$$\frac{\bar{s}^*(T,p) - \bar{s}(T,p)}{\bar{R}} = \frac{\bar{h}^* - \bar{h}}{\bar{R}T_c\,T_R} + \underbrace{\int_0^{p_R} (Z-1)\,\frac{dp_R}{p_R}}_{B\,p_R}$$

$$= \frac{T_R^2\left(\frac{dB}{dT_R}\right)p_R}{T_R} + B\,p_R$$

$$= p_R T_R \frac{dB}{dT_R} + B\,p_R$$

$$= p_R \frac{d\,[T_R B]}{d\,T_R} \qquad \longleftarrow$$

PROBLEM 11.82

KNOWN: An expression is provided for the enthalpy departure for use when an equation of state explicit in pressure is in hand.

FIND: (a) Derive the expression. (b) Using (a), obtain an expression for the enthalpy departure for a gas obeying the Redlich-Kwong equation. (c) Using (b), evaluate $\Delta\bar{h}$ for CO_2 in a process at 300K from 50 to 20 bar.

ANALYSIS: (a) Beginning with $h = h(T, v)$, write $dh = \left.\frac{\partial h}{\partial T}\right)_v dT + \left.\frac{\partial h}{\partial v}\right)_T dv$. Then, at fixed T

$$dh\Big]_T = \left.\frac{\partial h}{\partial v}\right)_T dv \Rightarrow h(T, v) - h(T, v') = \int_{v'}^{v} \left(\frac{\partial h}{\partial v}\right)_T dv \qquad (1)$$

With $h = u + pv$, $(\partial h/\partial v)_T = (\partial u/\partial v)_T + (\partial(pv)/\partial v)_T$. Introducing Eq. 11.47, this becomes

$$\left.\frac{\partial h}{\partial v}\right)_T = \left[T \left.\frac{\partial p}{\partial T}\right)_v - p\right] + \left.\frac{\partial(pv)}{\partial v}\right)_T \qquad (2)$$

Inserting Eq. (2) into Eq. (1)

$$h(T, v) - h(T, v') = \int_{v'}^{v} \left[T \left.\frac{\partial p}{\partial T}\right)_v - p\right] dv + [pv](T, v) - [pv](T, v') \qquad (3)$$

In the limit as $p \to 0$, $v' \to \infty$ and $[pv](T, v') \to RT$, $h(T, v') \to \bar{h}^*(T)$; so

$$h(T, v) - h^*(T) = \int_{\infty}^{v} \left[T \left.\frac{\partial p}{\partial T}\right)_v - p\right] dv + pv - RT$$

or

$$h^*(T) - h(T, v) = RT\Big[1 - \underbrace{\frac{pv}{RT}}_{Z} - \frac{1}{RT}\int_{\infty}^{v} \left[T \left.\frac{\partial p}{\partial T}\right)_v - p\right] dv\Big]$$

With $T = T_R T_c$, and expressing this on a molar basis

$$\frac{\bar{h}^*(T) - \bar{h}(T, \bar{v})}{\bar{R}\, T_c} = T_R\left[1 - Z - \frac{1}{\bar{R}T}\int_{\infty}^{\bar{v}} \left[T \left.\frac{\partial p}{\partial T}\right)_v - p\right] d\bar{v}\right] \longleftarrow \quad (a)$$

(b) Using the Redlich-Kwong equation of state,

$$T\left(\frac{\partial p}{\partial T}\right)_v = \frac{\bar{R}T}{\bar{v} - b} + \frac{1}{2}\frac{a}{\bar{v}(\bar{v} + b)\sqrt{T}} = p + \frac{3}{2}\frac{a}{\bar{v}(\bar{v} + b)\sqrt{T}} \Rightarrow T\left.\frac{\partial p}{\partial T}\right)_v - p = \frac{3}{2}\frac{a}{\bar{v}(\bar{v} + b)\sqrt{T}}$$

$$\therefore \quad \frac{\bar{h}^* - \bar{h}(T, \bar{v})}{\bar{R}T_c} = T_R\Big[1 - Z - \frac{3/2\, a}{\bar{R}\, T^{3/2}} \underbrace{\int_{\infty}^{\bar{v}} \frac{d\bar{v}}{\bar{v}(\bar{v} + b)}}_{-\frac{1}{b}\ln\left(1 + \frac{b}{\bar{v}}\right)\Big]_{\infty}^{\bar{v}} = -\frac{1}{b}\ln\left(1 + \frac{b}{\bar{v}}\right)}\Big]$$

So

$$\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c} = T_R\left[1 - Z + \frac{3a}{2b\bar{R}\,T^{3/2}}\ln\left(1 + \frac{b}{\bar{v}}\right)\right] \qquad (4) \longleftarrow \quad (b)$$

(c) At 300 K, $p_1 = 50$ bar, $p_2 = 20$ bar. Obtaining a, b from Table A-24, the Redlich-Kwong equation can be solved to give $\bar{v}_1 = 0.3475\ m^3/kmol$, $\bar{v}_2 = 1.119\ m^3/kmol$. The corresponding values of Z are $Z_1 = 0.697$, $Z_2 = 0.897$. With Eq (4),

$$\bar{h}_2 - \bar{h}_1 = \bar{R}T\left[Z_2 - Z_1 + \frac{3a}{2b\bar{R}T^{3/2}}\ln\left[\frac{1 + b/\bar{v}_1}{1 + b/\bar{v}_2}\right]\right] \qquad (5)$$

Inserting values

$$\bar{h}_2 - \bar{h}_1 = \left(8.314\frac{kJ}{kmol \cdot K}\right)(300K)\left[0.897 - 0.697 + \frac{3\left(64.43\ bar\left(\frac{m^3}{kmol}\right)^2(K)^{1/2}\right)\left|\frac{10^5 N/m^2}{bar}\right|}{2\left(0.02963\frac{m^3}{kmol}\right)\left(8314\frac{N \cdot m}{kmol \cdot K}\right)(300)^{3/2}}\ln\left[\frac{1 + \frac{0.02963}{0.3475}}{1 + \frac{0.02963}{1.119}}\right]\right]$$

$$= 1548\ \frac{kJ}{kmol} \longleftarrow \quad (c)$$

PROBLEM 11.83

KNOWN: The equation of state has the form

$$Z = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{P_R}{T_R}$$

FIND: For water vapor at 550°C, 20 MPa, evaluate v and c_p. Compare with data from Table A-4 and Fig. **3.9**, respectively.

ANALYSIS: At 550°C, 20 MPa, Table A-4 gives v = 0.01655 m³/kg. Fig. **3.9** gives $c_p \approx 2.95$ kJ/kg·K.

The equation of state can be expressed alternatively as

$$\frac{pv}{RT} = 1 + \left[\frac{1}{8} - \frac{27/64}{T/T_c}\right]\left(\frac{P/P_c}{T/T_c}\right) \Rightarrow v = \frac{RT}{p} + \left[\frac{1}{8} - \frac{(27/64)T_c}{T}\right]\left[\frac{RT_c}{P_c}\right]$$

Substituting values from Table A-1

$$v = \left(\frac{8314}{18.02}\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)\left(\frac{823.15\text{ K}}{200\times10^5\text{ N/m}^2}\right) + \left(\frac{1}{8} - \frac{(27/64)(647.3)}{823.15}\right)\left(\frac{8314}{18.02}\,\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)\left(\frac{647.3\text{ K}}{220.9\times10^5\text{ N/m}^2}\right)$$

$$= 0.0162\text{ m}^3/\text{kg} \qquad \longleftarrow$$

Comparing with the steam table value

$$\% = \left(\frac{0.0162 - 0.01655}{0.01655}\right)(100) = -2.1 \qquad \longleftarrow$$

Differentiating the given equation of state

$$\left(\frac{\partial Z}{\partial T_R}\right)_{P_R} = -\frac{1}{8}P_R T_R^{-2} + \frac{27}{32}P_R T_R^{-3} \Rightarrow \frac{1}{P_R}\left(\frac{\partial Z}{\partial T_R}\right)_{P_R} = -\frac{1}{8}T_R^{-2} + \frac{27}{32}T_R^{-3}$$

Substituting this into Eq. 11.84

$$\frac{h^*(T) - h(T,p)}{RT_c} = T_R^2\int_0^{P_R}\left[-\frac{1}{8}T_R^{-2} + \frac{27}{32}T_R^{-3}\right]dP_R = \left[-\frac{1}{8} + \frac{27}{32}T_R^{-1}\right]P_R$$

or

$$h(T,p) - h^*(T) = RT_c\left[\frac{1}{8} - \frac{27}{32}\frac{1}{T_R}\right]P_R$$

But with $c_p = \partial h/\partial T)_p$, $c_{p0} = dh^*/dT$, this yields upon differentiation

$$c_p - c_{p0} = \frac{\partial}{\partial T_R}\left[RT_c\left[\frac{1}{8} - \frac{27}{32}\frac{1}{T_R}\right]P_R\right]_{P_R}\frac{dT_R}{dT}$$

$$= \left[\frac{27}{32}R\right]\left[\frac{P_R}{T_R^2}\right]$$

Using P_c, T_c from Table A-1, $P_R = (200/220.9) = 0.905$, $T_R = (823.15/647.3) = 1.272$, So

$$c_p - c_{p0} = \left(\frac{27}{32}\right)\left(\frac{8.314}{18.02}\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left(\frac{.905}{(1.272)^2}\right) = 0.218\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$$

Evaluating c_{p0} from the appropriate expression given in Table A-**21**, $c_{p0} = 2.163$ kJ/kg·K. Finally

$$c_p = 2.163 + 0.218 = 2.381\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \qquad \longleftarrow$$

Comparing with the **value** from **Fig. 3.9**

① $$\% = \left(\frac{2.381 - 2.95}{2.95}\right)(100) = -19.3 \qquad \longleftarrow$$

---

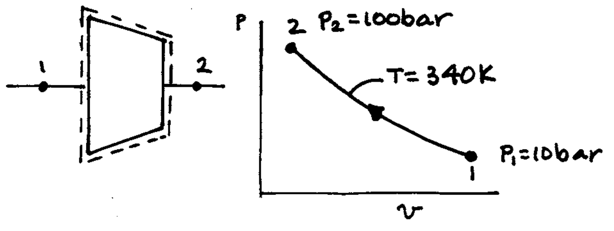
1. The simple equation of state used does a fair job in representing the p-v-T variation but does not suffice to yield other properties, such as c_p, accurately.

PROBLEM 11.84

KNOWN: Steady-state operating data are provided for a compressor for which ethylene is the working fluid.

FIND: Evaluate per kg of ethylene flowing the (a) work required, (b) heat transfer.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:
1. The control volume is at steady state.
2. Compression occurs without internal irreversibilities at fixed temperature.
3. Kinetic and potential energy effects can be ignored.

ANALYSIS: For this case, Eq. 6.51 is applicable. Since temperature is constant, $(\dot{Q}_{cv}/\dot{m})_{\substack{int\\rev}} = T(s_2-s_1)$, where (s_2-s_1) is obtained from Eq. 11.92.

From Table A-1, $p_c = 51.2$ bar, $T_c = 283$ K. Thus, $T_R = (340/283) = 1.2$, $P_{R1} = (10/51.2) = 0.2$, $P_{R2} = (100/51.2) = 1.95$. Then, from Fig. A-5, we get $\left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_1 = 0.08$, $\left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_2 = 1.25$.

Thus

$$\bar{s}_2-\bar{s}_1 = \underbrace{\bar{s}^\circ(T_2)-\bar{s}^\circ(T_1)}_{=0 \;\; T_1=T_2} - \bar{R}\ln\frac{P_2}{P_1} - \bar{R}\left[\left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_1\right] = -\bar{R}\left[\ln\frac{P_2}{P_1} + \left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_1\right]$$

$$= -\left(8.314\frac{kJ}{kg\cdot K}\right)\left[\ln 10 + 1.25 - 0.08\right] = -28.87\frac{kJ}{kmol\cdot K}$$

$$\Rightarrow \left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = 340K\left[-28.87\frac{kJ}{kmol\cdot K}\right]\left[\frac{1}{28.05\,kg/kmol}\right] = -350 \text{ kJ/kg} \qquad \text{(b)}$$

Reduction of mass and energy rate balances gives $(\dot{W}_{cv}/\dot{m}) = (\dot{Q}_{cv}/\dot{m}) + (h_1-h_2)$. From Fig. A-4, we get $\left(\frac{\bar{h}^*-\bar{h}}{\bar{R}T_c}\right)_1 = 0.15$, $\left(\frac{\bar{h}^*-\bar{h}}{\bar{R}T_c}\right)_2 = 2$. Then, with Eq. 11.85

$$\bar{h}_2-\bar{h}_1 = \underbrace{\bar{h}_2^*-\bar{h}_1^*}_{=0 \;\; T_1=T_2} - \bar{R}T_c\left[\left(\frac{\bar{h}^*-\bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^*-\bar{h}}{\bar{R}T_c}\right)_1\right] = -(8.314)(283)\left[2-0.15\right] = -4352.8\frac{kJ}{kmol}$$

$$\Rightarrow h_1-h_2 = \frac{4352.8}{28.05} = 155 \frac{kJ}{kg}$$

Finally, (a)

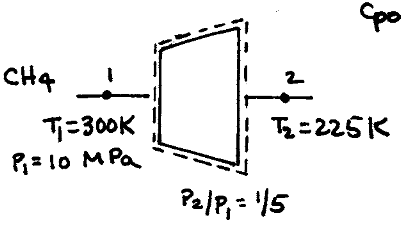
$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -350 + 155 = -195 \text{ kJ/kg}$$

PROBLEM 11.85

KNOWN: Steady-state operating data are provided for the adiabatic expansion of CH_4 through a turbine.

FIND: Determine the work developed per unit of mass flowing and compare with the value obtained using the ideal gas model only.

SCHEMATIC & GIVEN DATA:



$\bar{c}_{po} = 35 \frac{kJ}{kmole \cdot K}$

ASSUMPTIONS:
1. The control volume is at steady state.
2. $\dot{Q}_{cv}=0$, kinetic & potential energy effects can be ignored.
3. The ideal gas specific heat $\bar{c}_{po}$ is constant.

ANALYSIS: Reducing mass and energy rate balances, we get

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = \cancel{\left(\frac{\dot{Q}_{cv}}{\dot{m}}\right)}^{0} + (h_1 - h_2) \Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = h_1 - h_2$$

, where $(h_1 - h_2)$ can be found using Eq. 11.85:

$$h_1 - h_2 = \frac{1}{M}\left[\underbrace{(\bar{h}_1^* - \bar{h}_2^*)}_{\bar{c}_{po}(T_1 - T_2)} - \bar{R}T_c\left(\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2\right)\right]$$

With data from Table A-1, M = 16.04 kg/kmole, $T_c = 191$ K, $P_c = 46.4$ bar. Thus

$$T_{R1} = \frac{300}{191} = 1.57,\quad T_{R2} = \frac{225}{191} = 1.18,\quad P_{R1} = \frac{100}{46.4} = 2.16,\quad P_{R2} = \frac{20}{46.4} = 0.43.$$

Then, from Fig. A-4

$$\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 = 1,\quad \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 = 0.34$$

So,

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = \left[\frac{1}{16.04\ kg/kmol}\right]\left[\left(35\frac{kJ}{kmol \cdot K}\right)(300-225)K - \left(8.314\frac{kJ}{kmol \cdot K}\right)(191\ K)\left[1 - 0.34\right]\right]$$

$$= \left(\frac{1}{16.04}\right)\left[2625 - 1048\right] = 98.3\ kJ/kg \quad \longleftarrow$$

The ideal gas value is

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = \frac{1}{M}\left[\bar{h}_1^* - \bar{h}_2^*\right] = \frac{1}{M}\left[\underbrace{\bar{c}_{po}(T_1 - T_2)}_{2625}\right] = 163.7\ \frac{kJ}{kg} \quad \longleftarrow$$

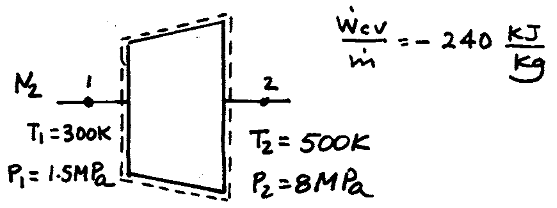
which is 67% greater than the value found using Eq. 11.85.

PROBLEM 11.86

KNOWN: Steady-state operating data is provided for a compressor for which N_2 is the working fluid.

FIND: Determine the heat transfer per unit mass of N_2 flowing.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume is at steady state.
2. Kinetic & potential energy changes can be ignored.

ANALYSIS: Reducing mass and energy rate balances, we get

$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1),$$ where $(h_2 - h_1)$ can be found using Eq. 11.85.

With data from Table A-1, $M = 28.01$, $T_c = 126K$, $P_c = 33.9$ bar,

$$T_{R1} = \frac{300}{126} = 2.38, \quad T_{R2} = \frac{500}{126} = 3.97, \quad P_{R1} = \frac{15}{33.9} = 0.44, \quad P_{R2} = \frac{80}{33.9} = 2.36.$$

From Figure A-4

$$\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 = 0.12, \quad \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 = 0.09$$

From Table A-23, $\bar{h}_1^* = 8723$ kJ/kmol, $\bar{h}_2^* = 14{,}581$ kJ/kmol.

Thus

$$h_2 - h_1 = \frac{1}{M}\left[\bar{h}_2^* - \bar{h}_1^* - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right]\right]$$

① $$= \frac{1}{28.01}\left[\underbrace{(14{,}581 - 8723)}_{5858} - \underbrace{(8.314)(126)\left[0.09 - 0.12\right]}_{-31.4}\right]$$

$$= 210.3 \text{ kJ/kg}$$

Finally,

$$\frac{\dot{Q}_{cv}}{\dot{m}} = -240 \frac{\text{kJ}}{\text{kg}} + 210.3 \frac{\text{kJ}}{\text{kg}} = -29.7 \frac{\text{kJ}}{\text{kg}}$$ ←

---

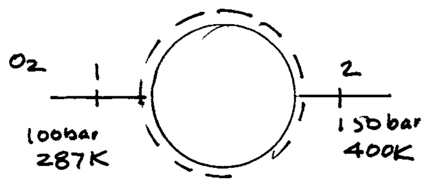
1. In this case, the ideal gas model contribution dominates. As can be seen from Fig A-2, the large T_R values give $Z \sim 1$ without regard for P_R.

PROBLEM 11.87

KNOWN: O_2 enters a control volume at steady state with a mass flow rate of 9 kg/min at 100 bar, 287K and is compressed adiabatically to 150 bar, 400K

FIND: Determine the power required and the rate of entropy production.

SCHEMATIC & GIVEN DATA:



$\dot{m} = 9$ kg/min

ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible.

ANALYSIS: Reducing mass, energy, and entropy balances

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad , \quad \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_1 - s_2$$

Using Eq. 11.85 and Eq. 11.92 to evaluate these differences via the generalized plots,

$$T_{R1} = \frac{287K}{154K} = 1.86 \;,\; P_{R1} = \frac{100\text{ bar}}{50.5\text{ bar}} = 1.98 \Rightarrow \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_1 \approx 0.63 \;,\; \left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_1 \approx 0.27$$

$$T_{R2} = \frac{400}{154} = 2.60 \;,\; P_{R2} = \frac{150}{50.5} = 2.97 \Rightarrow \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 \approx 0.43 \;,\; \left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_2 \approx 0.17$$

For O_2, Table A-23 gives $\bar{h}_1^* - \bar{h}_2^* = 8355 - 11{,}711 = -3356$ kJ/kmol. Then

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\bar{h}_1^* - \bar{h}_2^* - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2\right]}{M} = -\frac{3356}{32} - \left(\frac{8.314}{32}\right)(154)\left[0.63 - 0.43\right]$$

$$= -104.9 - 8 = -112.9 \text{ kJ/kg}$$

Thus

$$\dot{W}_{cv} = \left(9\,\frac{kg}{min}\right)\left(-112.9\,\frac{kJ}{kg}\right)\left|\frac{min}{60s}\right|\left|\frac{1kW}{1kJ/s}\right| = -16.94\,kW \longleftarrow \dot{W}_{cv}$$

With $\Delta\bar{s}^* = \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R}\ln P_2/P_1$ and data from Table A-23

$$\Delta\bar{s}^* = 213.765 - 203.91 - 8.314 \ln\frac{150}{100} = 6.484 \frac{kJ}{kmol \cdot K}$$

Accordingly

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \frac{\Delta\bar{s}^* - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1\right]}{M} = \frac{6.484 - 8.314(0.17 - 0.27)}{32} = 0.2286 \frac{kJ}{kg \cdot K}$$

Finally

① $$\dot{\sigma}_{cv} = \left(9\,\frac{kg}{min}\right)\left(0.2286\,\frac{kJ}{kg \cdot K}\right)\left|\frac{min}{60s}\right|\left|\frac{1kW}{1kJ/s}\right| = 0.034\,\frac{kW}{K} \longleftarrow \dot{\sigma}_{cv}$$

---

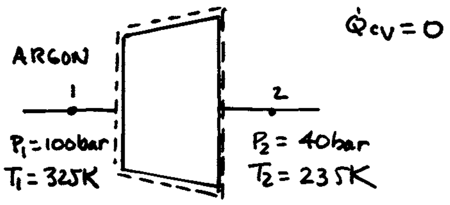
1. Owing to the approximate nature of retrieving data from the generalized charts, extreme accuracy in the calculated results should not be expected. This applies with greater effect for entropy evaluations, which are generally sensitive to roundoff.

PROBLEM 11.88

KNOWN: Steady-state operating data are provided for a turbine in which argon is the working fluid.

FIND: Determine per unit mass of argon flowing (a) the work, (b) the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume is at steady state.
2. $\dot{Q}_{cv}=0$, and kinetic & potential energy changes can be ignored.

ANALYSIS: Reducing mass, energy and entropy balances, we get

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \quad (1) \quad , \quad \frac{\dot{\sigma}_{cv}}{\dot{m}} = s_2 - s_1 \quad (2)$$

where Eqs. 11.85, 11.92 are used to find (h_1-h_2), (s_2-s_1). With data from Table A-1, $M = 39.94$ kg/kmol, $T_c = 151$ K, $P_c = 48.6$ bar. Then,

$$T_{R1} = \frac{325}{151} = 2.15, \quad T_{R2} = \frac{235}{151} = 1.56, \quad P_{R1} = \frac{100}{48.6} = 2.06, \quad P_{R2} = \frac{40}{48.6} = 0.82$$

Also, from Table A-21, $\bar{c}_p^* = \frac{5}{2}\bar{R}$, and from Figs. A-4, A-5

$$\left(\frac{\bar{h}^*-\bar{h}}{\bar{R}T_c}\right)_1 = 0.4, \quad \left(\frac{\bar{h}^*-\bar{h}}{\bar{R}T_c}\right)_2 = 0.38, \quad \left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_1 = 0.2, \quad \left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_2 = 0.18$$

Then,

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{1}{M}\left[\underbrace{(\bar{h}_1^* - \bar{h}_2^*)}_{2.5\bar{R}(T_1-T_2)} - \bar{R}T_c\left[\left(\frac{\bar{h}^*-\bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^*-\bar{h}}{\bar{R}T_c}\right)_1\right]\right] = \frac{8.314}{39.94}\left[2.5(90) - 151\left[0.4-0.38\right]\right]$$

$$= 46.2 \text{ kJ/kg} \longleftarrow \text{(a)}$$

And with Eq. 6.23

$$\frac{\dot{\sigma}_{cv}}{\dot{m}} = \frac{1}{M}\left[\left(\overset{2.5\bar{R}}{\bar{c}_p^*} \ln\frac{T_2}{T_1} - \bar{R}\ln\frac{P_2}{P_1}\right) - \bar{R}\left[\left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^*-\bar{s}}{\bar{R}}\right)_1\right]\right]$$

① $$= \frac{8.314}{39.94}\left[\left(2.5\ln\frac{235}{325} - \ln\frac{40}{100}\right) + \left[0.2 - 0.18\right]\right] = 0.026 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} \longleftarrow \text{(b)}$$

---

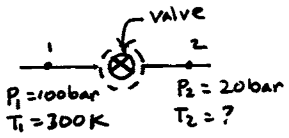
1. Owing to the approximate nature of retrieving data from the generalized charts, extreme accuracy in the calculated results should not be expected. This applies with greater effect for entropy evaluations, which are generally sensitive to roundoff.

PROBLEM 11.89

KNOWN: O_2 undergoes a throttling process from 100 bar, 300K to 20 bar.

FIND: Determine the temperature after throttling, and compare with the value obtained using the ideal gas model.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:
1. The expansion across the valve adheres to the throttling process model of Sec. 4.3, for which $h_2 = h_1$.

ANALYSIS: Using Eq. 11.85 with $h_2 = h_1$, we get

$$\bar{h}^*(T_2) - \bar{R}T_c\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 = \bar{h}^*(T_1) - \bar{R}T_c\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_1 \qquad (1)$$

with data from Table A-1, $T_c = 154$ K, $P_c = 50.5$ bar,

$$P_{R1} = \frac{100}{50.5} = 1.98 \;, \; P_{R2} = \frac{20}{50.5} = 0.4 \;, \; T_{R1} = \frac{300}{154} = 1.95$$

Fig. A-4 gives $\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_1 = 0.5$. Table A-23 gives $\bar{h}_1^* = 8736$ kJ/kmol.

Thus, Eq. (1) becomes

$$\bar{h}^*(T_2) - (8.314)(154)\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 = 8736\frac{\text{kJ}}{\text{kmol}} - \underbrace{\left(8.314\frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(154\,\text{K})(0.5)}_{640.2}$$

$$= 8096\ \frac{\text{kJ}}{\text{kmol}} \qquad (2)$$

Equation (2) can then be used in a trial procedure with data from
① Table A-23: $\bar{h}_2^*$ and from Fig. A-4: $\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2$. Iterating, $T_2 = 280$K. ⟵

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1. At $T_2 = 280$K, Table A-23 gives $\bar{h}_2^* = 8150$ kJ/kmol. Using $T_{R2} = (280/154) = 1.82$ and $P_{R2} = 0.4$, Fig. A-4 gives $\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 = 0.1$.

With these values the left side of Eq. 2 is

$$\bar{h}^*(T_2) - (8.314)(154)\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 = 8150 - (8.314)(154)(0.1) = 8022\ \frac{\text{kJ}}{\text{kmol}}$$

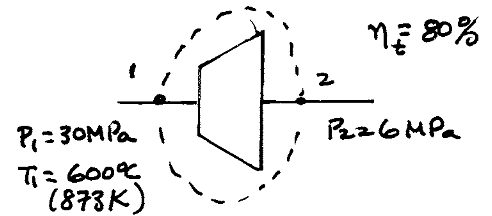
which is within 1% of the value 8096 kJ/kmol on the right side of Eq. (2). The approximate nature of obtaining data from the generalized charts does not justify additional iteration.

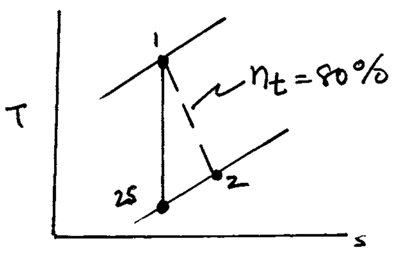
PROBLEM 11.90

KNOWN: Steady state operating data are provided for a steam turbine.

FIND: Using the generalized charts, determine the work developed per kg of steam flowing, and compare with the steam table result.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible.

ANALYSIS: Reducing mass and energy rate balances, $\dot{W}_{cv}/\dot{m} = h_1 - h_2$. Then, with the isentropic turbine efficiency

$$\frac{\dot{W}_{cv}}{\dot{m}} = \eta_t (h_1 - h_{2s})$$

STEAM TABLE SOLUTION. With steam table data, $h_1 = 3443.9$ kJ/kg, $s_1 = 6.2331$ kJ/kg·K. Then, with $s_{2s} = s_1$, $h_{2s} = 2981.6$ kJ/kg, $T_{2s} = 329$ °C. Finally

$$\frac{\dot{W}_{cv}}{\dot{m}} = 0.8(3443.9 - 2981.6) = 369.8 \text{ kJ/kg} \longleftarrow \frac{\text{steam}}{\text{tables}}$$

GENERALIZED CHART SOLUTION.

$$T_{R1} = \frac{873\text{K}}{647.3\text{K}} = 1.35 \;,\; P_{R1} = \frac{300\text{bar}}{220.9\text{bar}} = 1.36 \Rightarrow \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 \approx 0.91 \;,\; \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1 \approx 0.5$$

(Fig A-4) (Fig. A-5)

$$P_{R2} = \frac{60}{220.9} = 0.27$$

To fix state 2s, use $\bar{s}_2 - \bar{s}_{2s} = 0$. That is, with Eq. 11.92

$$0 = \left[\bar{s}°(T_{2s}) - \bar{s}°(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right] - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_{2s} - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1\right]$$

with $\bar{s}°(T_1)$ from Table A-23, this takes the form

$$\bar{s}°(T_{2s}) - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_{2s}\right] = 227.11 + 8.314\ln\frac{6}{30} - 8.314(0.5) = 209.57 \frac{\text{kJ}}{\text{kg·K}} \quad (1)$$

① Eq. (1) can be used iteratively to determine state 2s. We get $T_{2s} = 600$K. Thus, $(T_r)_{2s} = 0.93$. Table A-23 gives $\bar{h}_1^* = 30754$ kJ/kmol, $\bar{h}_{2s}^* = 20402$, and Fig. A-4 gives $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_{2s} = 0.45$. Thus, Eq. 11.85 yields

$$\bar{h}_1 - \bar{h}_{2s} = \bar{h}_1^* - \bar{h}_{2s}^* - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_{2s}\right] = 30754 - 20402 - \underbrace{8.314(647.3)[0.91 - 0.45]}_{2476}$$

$$= 7876 \text{ kJ/kmol}$$

And $$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{0.8}{18.02}(7876) = 349.7 \frac{\text{kJ}}{\text{kg}} \longleftarrow \frac{\text{generalized}}{\text{charts}}$$

which is about 5% less than the steam table result.

---

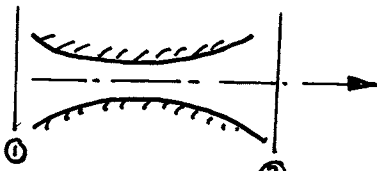
1. Using $T_{2s} = 600$K, Table A-23 gives $\bar{s}^{\circ}_{2s} = 212.92$. Figure A-5 gives $\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_{2s} = 0.3$. Thus, the left side of Eq. (1) is $[(212.92) - (8.314)(0.3)] = 210.4 \frac{\text{kJ}}{\text{kg·K}}$, which is about 0.4% greater than the value on the right side of Eq. (1).

PROBLEM 11.91

KNOWN: O_2 expands isentropically through a nozzle. Steady-state operating data are provided.

FIND: Determine the velocity at the exit.

SCHEMATIC & GIVEN DATA:



$p_1 = 60\,\text{bar}$
$T_1 = 300\,K$
$V_1 = 1\,m/s$

$p_2 = 30\,\text{bar}$
$s_2 = s_1$

ASSUMPTION:

1. A control volume enclosing the nozzle is at steady state.
2. The expansion is isentropic
3. Potential energy change can be ignored. $\dot{W}_{cv} = \dot{Q}_{cv} = 0$.

ANALYSIS: Reducing mass and energy rate balances $V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$, (1) where $(h_1 - h_2)$ can be evaluated using Eq. 11.85. First, state 2 is fixed using $s_2 = s_1$ and Eq. 11.92.

From Table A-1, $M = 32$, $T_c = 154\,K$, $p_c = 50.5$ bar. Thus,

$$p_{R1} = \frac{60}{50.5} = 1.19, \quad p_{R2} = \frac{30}{50.5} = 0.59, \quad T_{R1} = \frac{300}{154} = 1.95$$

When $s_2 = s_1$, Eq. 11.92 reduces to read

$$\bar{s}^\circ(T_2) - \bar{R}\left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_2 = \underbrace{\bar{s}^\circ(T_1)}_{\substack{\text{Table A-23} \\ 205.213}} + \bar{R}\ln\frac{p_2}{p_1} - \bar{R}\underbrace{\left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_1}_{\substack{\text{Fig A-5} \\ = 0.14}} = 205.213 + 8.314(\ln 0.5 - 0.14) = 198.29\,\frac{kJ}{kmol\cdot K} \quad (2)$$

① This result can be used to determine T_2 using trial with data from Table A-23, Fig A-5. We get $T_2 = 245\,K$.

Then, to find $(h_1 - h_2)$ use the following expression with data from Table A-23, Fig A-4,

$$\bar{h}_1 - \bar{h}_2 = \bar{h}_1^* - \bar{h}_2^* - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2\right] = 8736 - 7130 - (8.314)(154)\left[0.35 - 0.26\right]$$

$$= 1491\,\frac{kJ}{kmol}$$

Then, returning to Eq. (1)

$$V_2 = \sqrt{\left(1\,\frac{m}{s}\right)^2 + 2\left[\frac{1491\,kJ/kmol}{32\,kg/kmol}\right]\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|} = 305.3\,m/s$$

---

1. At 245 K, $T_{R2} = 1.59$. Then, with $p_{R2} = 0.59$, Fig. A-5 gives $\left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_2 = 0.12$. Table A-23 gives $\bar{s}^\circ(T_2) = 199.29$. Thus, the left-side of Eq (2) reads

$$\bar{s}^\circ(T_2) - \bar{R}\left[\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right]_2 = 199.29 - 8.314(0.12) = 198.29\,kJ/kmol\cdot K,$$

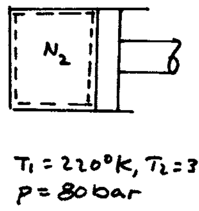
which agrees with the term on the right side of Eq. (2).

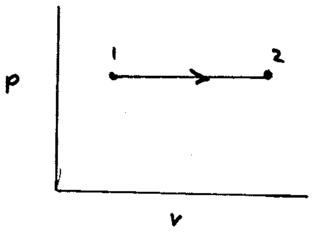
PROBLEM 11.92

KNOWN: A quantity of N_2 expands at a constant pressure of 80 bar from $T_1 = 220K$ to $T_2 = 300K$.

FIND: Determine W/n and Q/n.

SCHEMATIC & GIVEN DATA:





ASSUMPTION: The system shown in the accompanying figure experiences no change in kinetic or potential energy.

ANALYSIS: As pressure is fixed the work is given by

$$\frac{W}{n} = \int_1^2 p\,d\bar{v} = p[\bar{v}_2 - \bar{v}_1]$$

Then, since $Z = p\bar{v}/\bar{R}T$, this equation becomes

$$\frac{W}{n} = \bar{R}\,[T_2 Z_2 - T_1 Z_1] \qquad (1)$$

where Z is the compressibility factor. From Table A-1, $T_c = 126K$, $p_c = 33.9 bar$; so $p_R = \frac{80}{33.9} = 2.36$, $T_{R1} = \frac{220}{126} = 1.75$, $T_{R2} = 2.38$. From Fig. A-2, $Z_1 = 0.91$ and $Z_2 = 0.99$. Then, Eq. (1) gives

$$\frac{W}{n} = \left(8.314\,\frac{kJ}{kg\cdot K}\right)\Big[(300)(.99) - (220)(0.91)\Big]K = 805\ \frac{kJ}{kmol} \qquad \longleftarrow W/n$$

An energy balance reduces to $\Delta U = Q - W$ or $Q = \Delta U + W$. Thus

$$\frac{Q}{n} = (\bar{u}_2 - \bar{u}_1) + \frac{W}{n} = (\bar{u}_2 - \bar{u}_1) + (p\bar{v}_2 - p\bar{v}_1) = \bar{h}_2 - \bar{h}_1$$

Then, with Eq. 11.85

$$\frac{Q}{n} = \bar{h}^*(T_2) - \bar{h}^*(T_1) - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right]$$

with data from Table A-23 and Fig. A-4

$$\frac{Q}{n} = 8723 - 6391 - (8.314)(126)\big[0.43 - 0.86\big] = 2782\,\frac{kJ}{kmol} \qquad \longleftarrow Q/n$$

PROBLEM 11.93

KNOWN: O_2 contained in a closed, rigid, insulated vessel is stirred by paddle wheel. State data are provided.

FIND: Determine (a) the final temperature, (b) the work, and (c) the amount of exergy destroyed.

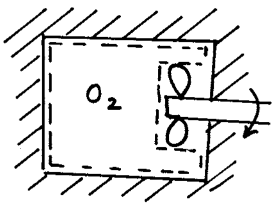
SCHEMATIC & GIVEN DATA:

$V = 0.142\,m^3$

$P_1 = 100\,bar$

$T_1 = 280\,K$

$P_2 = 150\,bar$



ASSUMPTIONS: (1) The system shown in the accompanying figure experiences no changes in kinetic and potential energy between the initial and final states. (2) The generalized property charts provide data with sufficient accuracy. (3) $T_0 = 7°C\ (280K)$

ANALYSIS: From Table A-1, $T_c = 154K$, $p_c = 50.5\,bar$. Thus, $P_{R1} = \frac{100}{50.5} = 1.98$, $P_{R2} = \frac{150}{50.5} = 2.97$, $T_{R1} = 1.82$. The gen. compressibility chart gives $Z_1 = 0.932$, $Z_2 = 1.01$

(a) Since volume is constant, $V_2 = V_1 \Rightarrow \left(\frac{ZRT}{P}\right)_2 = \left(\frac{ZRT}{P}\right)_1 \Rightarrow$

$$T_2 = \left(\frac{Z_1}{Z_2}\right)\left(\frac{P_2}{P_1}\right)T_1 = \left(\frac{0.932}{1.01}\right)\left(\frac{150}{100}\right)(280) = 387.6K \longleftarrow (a)$$

(b) An energy balance reduces to give $\Delta U = \cancel{Q} - W \Rightarrow W = -\Delta U$, or $W = -m[u_2 - u_1] = -m[(h - pv)_2 - (h - pv)_1] = -m[(h_2 - h_1) - v(P_2 - P_1)]$. Or since $pv = ZRT$, $W = -m[(h_2 - h_1) - R(Z_2T_2 - Z_1T_1)]$. To find m, use

$$Z_1 = \frac{P_1V}{mRT_1} \Rightarrow m = \frac{P_1V}{Z_1RT_1} = \frac{(100 \times 10^5 N/m^2)(0.142m^3)}{(0.932)\left(\frac{8314}{32}\frac{N\cdot m}{kg\,K}\right)(280K)} = 20.94kg$$

$(h_2 - h_1)$ is evaluated using Eq. 11.85 with data from Table A-23, Fig. A-4:

$$h_2 - h_1 = \frac{1}{M}\left[h_2^* - h_1^* - \bar{R}T_c\left[\left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_2 - \left[\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right]_1\right]\right] = \frac{1}{32}\left[11{,}337 - 8150 - (8.314)(154)[.45 - .66]\right]$$
$$= 108\ kJ/kg$$

Then

$$W = -20.94\left[108 - \frac{8.314}{32}\left((1.01)(387.6) - (0.932)(280)\right)\right] = -1551\ kJ \longleftarrow (b)$$

(c) Using $E_d = T_0\sigma$, where from an entropy balance $\sigma = m(s_2 - s_1)$, together with Eq. 11.92 and data from Table A.23, Fig. A-5, we get

$$E_d = mT_0(s_2 - s_1) = \frac{mT_0}{M}\left[\left(\bar{s}^\circ(T_2) - s^\circ(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right) - \bar{R}\left[\left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_2 - \left(\frac{\bar{s}^* - \bar{s}}{\bar{R}}\right)_1\right]\right]$$

$$= \frac{(20.94)(280)}{32}\left[212.82 - 203.19 - 8.314\ln 1.5 - 8.314[0.18 - 0.29]\right]$$

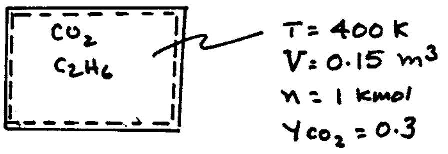
$$= 1314.4\ kJ \longleftarrow (c)$$

PROBLEM 11.94

KNOWN: A 1 kmol mixture of CO_2 and C_2H_6 occupies a volume of $0.15 m^3$ at $T = 400 K$. $y_{CO_2} = 0.3$. Pressure should not exceed 180 bar.

FIND: Determine p using (a) the ideal gas equation of state, (b) Kay's rule and the compressibility chart, (c) the additive pressure rule and the compressibility chart.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system is the mixture.

ANALYSIS: (a) Ideal Gas.

$$p = \frac{n\bar{R}T}{V} = \frac{(1\,kmol)(8314\,\frac{N\cdot m}{kmol\cdot K})(400K)}{0.15\,m^3}\left|\frac{bar}{10^5 N/m^2}\right| = 221.7\,bar \longleftarrow (a)$$

(b) Kay's Rule plus Z chart. From Table A-1

		T_c (K)	P_c (bar)
1.	CO_2	304	73.9
2.	C_2H_6	305	48.8

Then, with Eqs. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.3)(304) + (0.7)(305) = 304.7\,K$$

$$P_c = y_1 P_{c1} + y_2 P_{c2} = (0.3)(73.9) + (0.7)(48.8) = 56.33\,bar$$

Accordingly, $T_R = 400/304.7 = 1.313$ and

$$v_R' = \frac{\bar{v} P_c}{\bar{R} T_c} = \frac{(0.15\,m^3/kmol)(56.33\times10^5\,N/m^2)}{(8314\,\frac{N\cdot m}{kmol\cdot K})(304.7\,K)} = 0.334$$

With these values, Fig. A-2 gives $Z \approx 0.67$, $P_R \approx 2.65$. With the Z value

$$p = Z\frac{n\bar{R}T}{V} = Z\,P_{ideal} = 0.67(221.7) = 148.5\,bar \longleftarrow (b)$$

With the P_R value

$$p = P_c P_R = 56.33(2.65) = 149.3\,bar$$

(c) Additive Pressure Rule plus Z chart. Selecting Eq. 11.99b

$$Z = y_1 Z_1(T,V) + y_2 Z_2(T,V)$$

$$T_{R1} = 400/304 = 1.32,\quad v_{R1}' = \frac{(0.15 m^3/0.3 kmol)(73.9\times10^5 N/m^2)}{(8314\,\frac{N\cdot m}{kmol\cdot K})(304 K)} = 1.46 \Rightarrow Z_1 \approx 0.9.$$

$$T_{R2} = 400/305 = 1.31,\quad v_{R2}' = \frac{(0.15 m^3/0.7 kmol)(48.8\times10^5 N/m^2)}{(8314\,\frac{N\cdot m}{kmol\cdot K})(305 K)} = 0.412 \Rightarrow Z_2 \approx 0.69$$

Then, $Z = (0.3)(0.9) + (0.7)(0.69) = 0.753$

and

$$p = Z\frac{n\bar{R}T}{V} = Z\,P_{ideal} = 0.753(221.7\,bar) = 166.9\,bar \longleftarrow$$

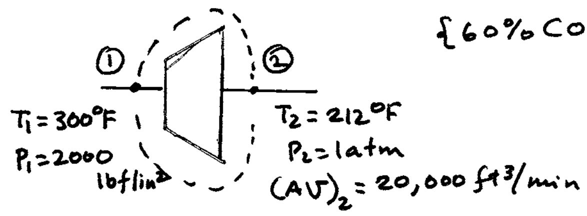
Discussion: At this state, where Z = 0.67, the ideal gas model is not applicable. Although methods (b), (c) yield different predictions, the methods do suggest that operation in the safe range might be realized.

PROBLEM 11.95

KNOWN: A gaseous mixture with the molar composition {60% CO, 40% H_2} enters a turbine operating at steady state at 300°F, 2000 lbf/in^2 and exits at 212°F, 1atm with a volumetric flow rate of 20,000 ft^3/min.

FIND: Using Kay's rule estimate the volumetric flow rate at the turbine inlet, and compare with the value yielded by the ideal gas model.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The control volume shown in the figure is at steady state.

ANALYSIS: At steady state a mass rate balance reduces to give

$$\frac{(AV)_1}{v_1} = \frac{(AV)_2}{v_2} \Rightarrow (AV)_1 = \frac{v_1}{v_2}(AV)_2 \qquad (1)$$

Ideal Gas Model. With $v = RT/p$, Eq(1) gives

$$(AV)_1 = \left(\frac{T_1}{T_2}\right)\left(\frac{p_2}{p_1}\right)(AV)_2 = \left(\frac{760°R}{672°R}\right)\left(\frac{14.7\ lbf/in^2}{2000\ lbf/in^2}\right)\left(20,000\ \frac{ft^3}{min}\right) = 166.3\ \frac{ft^3}{min}$$

Kay's Rule. With $Z = pv/RT$, $v = ZRT/p$, and Eq. (1) becomes

$$(AV)_1 = \frac{Z_1}{Z_2}\underbrace{\left(\frac{T_1}{T_2}\right)\left(\frac{p_2}{p_1}\right)(AV)_2}_{\text{ideal gas value}} = \frac{Z_1}{Z_2}\underbrace{\left(166.3\ \frac{ft^3}{min}\right)}_{\text{ideal gas value}} \qquad (2)$$

With T_c, p_c data from Tables A-1 using the corrections for H_2 footnoted in Sec. 3.4

CO: $T_c = 133K$ $\quad$ H_2: $T_c' = 33.2 + 8 = 41.2K$
$p_c = 34.5atm$ $\quad$ $p_c' = 12.8 + 8 = 20.8atm$

Then with Kay's rule

$T_c = (0.6)(133) + (0.4)(41.2) = 96.3K$
$p_c = (0.6)(34.5) + (0.4)(20.8) = 29atm$

Thus,

$$T_{R1} = \frac{760/1.8}{96.3} = 4.38, \quad P_{R1} = \frac{2000/14.7}{29} = 4.69 \Rightarrow Z_1 \approx 1.08 \qquad \text{(Fig A-2)}$$

$$T_{R2} = \frac{672/1.8}{96.3} = 3.88, \quad P_{R2} = 1/29 = 0.03 \Rightarrow Z_2 \approx 1.0 \qquad \text{(Fig. A-1)}$$

Finally, using Eq. (2)

$$(AV)_1 = \left(\frac{1.08}{1.0}\right)\left(166.3\ \frac{ft^3}{min}\right) = 179.6\ \frac{ft^3}{min} \longleftarrow$$

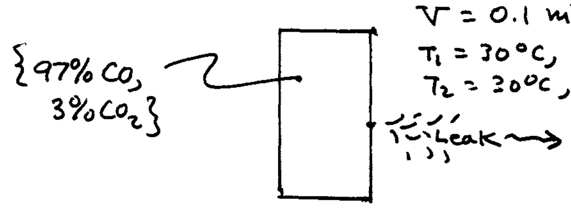
The ideal gas value is about 8% less than the volumetric flow rate predicted by Kay's rule.

PROBLEM 11.96

KNOWN: A 0.1-m^3 cylinder contains a gaseous mixture with the molar composition {97% CO, 3 % CO_2} initially at 30°C and 138 bar. Mixture leaks from the cylinder, eventually the cylinder contents are at 129 bar, 30°C.

FIND: Using Kay's rule, estimate the amount of mixture that leaks from the cylinder.

SCHEMATIC & GIVEN DATA:



$V = 0.1\ m^3$

$T_1 = 30°C$, $p_1 = 138$ bar

$T_2 = 30°C$, $p_2 = 129$ bar

ASSUMPTION: The system is the mixture present within the tank initially and finally.

ANALYSIS: With $Z = pV/n\bar{R}T$, $n = pV/Z\bar{R}T$. Thus, the amount that leaks is

$$n_1 - n_2 = \frac{p_1 V}{Z_1 \bar{R} T} - \frac{p_2 V}{Z_2 \bar{R} T} = \frac{V}{\bar{R} T}\left[\frac{p_1}{Z_1} - \frac{p_2}{Z_2}\right] \qquad (1)$$

With data from Table A-1

CO: $T_c = 133$ K, $p_c = 35$ bar

CO_2: $T_c = 304$ K, $p_c = 73.9$ bar

Kay's Rule

$$T_c = 0.97(133) + 0.03(304) = 138.1\ K$$

$$p_c = 0.97(35) + 0.03(73.9) = 36.2\ bar.$$

Thus

$$p_{R1} = \frac{138}{36.2} = 3.81,\quad T_{R1} = \frac{303}{138.1} = 2.19,\quad Z_1 \approx 0.98$$

$$p_{R2} = \frac{129}{36.2} = 3.56,\quad T_{R2} = T_{R1} = 2.19,\quad Z_2 \approx 0.97$$

Accordingly, Eq. (1) gives

$$n_1 - n_2 = \frac{0.1\ m^3}{\left(8314 \frac{N\cdot m}{kmol\cdot K}\right)(303 K)}\left[\frac{138}{0.98} - \frac{129}{0.97}\right] bar \left|\frac{10^5\ N/m^2}{1\ bar}\right|$$

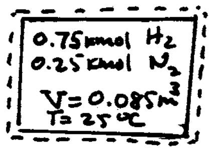
$$= 0.031\ kmol \longleftarrow \text{leak}$$

PROBLEM 11.97

KNOWN: A gaseous mixture consisting of 0.75 kmol H_2 and 0.25 kmol N_2 occupies 0.085 m^3 at 25°C.

FIND: Estimate the pressure using (a) the ideal gas equation of state, (b) Kay's rule with the gen. compressibility chart, (c) van der Waals equation, (d) rule of additive pressure using the gen. compressibility chart.

SCHEMATIC & GIVEN DATA:



ANALYSIS:

(a) Ideal gas equation of state

$$p = \frac{n\bar{R}T}{V} = \frac{(1\,kmol)(8314\,\frac{N\cdot m}{kmol\cdot K})(298K)}{0.085\,m^3}\left|\frac{1\,bar}{10^5\,\frac{N}{m^2}}\right| = 291.5\ bar \quad \xleftarrow{} (a)$$

(b) Kay's rule plus gen. compressibility chart. With data from Table A-1 and using the corrections for H_2 indicated in the footnote of Sec. 3.4.

H_2: $T'_{c1} = 41.2K$ $\quad N_2$: $T_{c2} = 126K$

$P'_{c1} = 21.08\,bar$ $\quad$: $P_{c2} = 33.9\,bar$

$$T_c = y_1 T'_{c1} + y_2 T_{c2} = (0.75)(41.2) + (0.25)(126) = 62.4\,K$$

$$P_c = y_1 P'_{c1} + y_2 P_{c2} = (0.75)(21.08) + (0.25)(33.9) = 24.3\ bar$$

$$\therefore T_R = \frac{298}{62.4} = 4.78, \quad v'_R = \frac{(V/n)P_c}{\bar{R}T_c} = \frac{(0.085\,m^3/kmol)(24.3\times10^5\,N/m^2)}{(8314\,\frac{N\cdot m}{kmol\cdot K})(62.4\,K)} = 0.398$$

Figure A-3 gives $Z \cong 1.3$. Then, $p = Z\left[\frac{n\bar{R}T}{V}\right] = Z\,P_{id} = 1.3\,(291.5) = 379\,bar \longleftarrow$ (b)

(c) van der Waals equation. From Table A-24, for N_2

$a_2 = 1.366\,bar\left(\frac{m^3}{kmol}\right)^2$, $b_2 = 0.0386\,m^3/kmol$. Using Eqs. 11.4 for H_2,

$a_1 = 0.247$, $b_1 = 0.0265$. Then, with Eqs. 11.96

$$a = [y_1 a_1^{1/2} + y_2 a_2^{1/2}]^2 = [\,0.75(0.247)^{1/2} + 0.25\,(1.366)^{1/2}\,] = 0.442$$

$$b = [y_1 b_1 + y_2 b_2] = [\,0.25(0.0265) + 0.75\,(0.0386)\,] = 0.0296$$

Then, Eq. 11.2 gives

$$p = \frac{(8314\,\frac{N\cdot m}{kmol\cdot K})(298K)}{(0.085-0.0296)\,\frac{m^3}{kmol}}\left|\frac{1\,bar}{10^5\,N/m^2}\right| - \frac{0.442\ bar\,(m^3/kmol)^2}{(0.085\ m^3/kmol)^2} = 386\,bar \quad \longleftarrow (c)$$

(d) Additive pressure rule plus gen. compressibility chart. Using T'_{c1}, P'_{c1} for H_2 from part (b),

$$\left.\begin{aligned} T_{R1} &= 298/41.2 = 7.23, \\ v'_{R1} &= \frac{\bar{v}\,P'_{c1}}{\bar{R}\,T'_{c1}} = \left(\frac{0.085}{0.75}\,\frac{m^3}{kmol}\right)\frac{(21.08\times10^5\,N/m^2)}{(8314\,\frac{N\cdot m}{kmol\cdot K})(41.2K)} = 0.697 \end{aligned}\right\} \text{Fig. A-2, } Z_1 \sim 1.05$$

For N_2,

$$\left.\begin{aligned} T_{R2} &= 298/126 = 2.37 \\ v'_{R2} &= \left(\frac{0.085}{0.25}\right)\frac{(33.9\times10^5)}{(8314)(126)} = 1.1 \end{aligned}\right\} \text{Fig A-2, } Z_2 \sim 0.99$$

Then, with Eq. 11.99b,

$$Z = 0.75(1.05) + (0.25)(0.99) = 1.04$$

① $\Rightarrow \quad p = Z\,\frac{\bar{n}RT}{V} = Z\,P_{id} = 1.04\,(291.5) = 303\ bar \quad \longleftarrow (d)$

---

1. Parts (a) and (d) are roughly in agreement. Also, parts (c) and (d) agree fairly well. A conservative choice dictated by the application at hand would govern the value that would be used for the estimated pressure.

PROBLEM 11.98

KNOWN: A gaseous mixture of 0.5 lbmol CH_4 and 0.5 lbmol of C_3H_8 occupies a volume of 7.65 ft³ at 194 °F. The measured pressure is 50 atm.

FIND: Estimate the pressure using each of six specified procedures and compare with the measured pressure.

SCHEMATIC & GIVEN DATA

$V = 7.65\,ft^3$
$T = 654\,°R$
$p = 50\,atm$

		Y
1:	CH_4	0.5
2.	C_3H_8	0.5

ASSUMPTION: The mixture is the system.

ANALYSIS: (a) Ideal Gas Equation of State.

$$p = \frac{n\bar{R}T}{V} = \frac{(1.0\,lbmol)(1545\,ft{\cdot}lbf/lbmol{\cdot}°R)(654°R)}{7.65\,ft^3}\left|\frac{atm}{14.696\times 144\,lbf/ft^2}\right| = 62.4\,atm \longleftarrow (a)$$

(b) Kay's Rule plus Z chart. With data from Table A-1E and Eqs. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = 0.5(344 + 666) = 505°R$$

$$p_c = y_1 p_{c1} + y_2 p_{c2} = 0.5(45.8 + 42.1) = 43.95\,atm$$

Then

$$T_R = 654/505 = 1.3$$

$$v_R' = \frac{\bar{v}\,p_c}{\bar{R}T_c} = \frac{(7.65/1\,\frac{ft^3}{lbmol})(43.95\times 14.696\times 144)\frac{lbf}{ft^2}}{\left(1545\frac{ft\,lbf}{lbmol{\cdot}°R}\right)(505°R)} = 0.911$$

Fig. A-2, $Z \simeq .825$. Thus

$$p = Z\frac{n\bar{R}T}{V} = Z\,p_{ideal} = (.825)(62.4) = 51.48\,atm \longleftarrow (b)$$

(c) Van der Waals Equation of State. With data from Table A-24E and Eqs. 11.96

$$a = \left[y_1 (a_1)^{1/2} + y_2 (a_2)^{1/2}\right]^2 = 0.25\left[(581)^{1/2} + (2369)^{1/2}\right]^2 = 1323.9\,atm\left(\frac{ft^3}{lbmol}\right)^2$$

$$b = y_1 b_1 + y_2 b_2 = 0.5[0.685 + 1.444] = 1.065\,\frac{ft^3}{lbmol}$$

Then, Eq. 11.2 gives

$$p = \frac{\left(1545\frac{ft\,lbf}{lbmol{\cdot}°R}\right)(654°R)}{(7.65 - 1.065)\frac{ft^3}{lbmol}}\left|\frac{atm}{14.696\times 144\,\frac{lbf}{ft^2}}\right| - \frac{1323.9\,atm}{(7.65)^2} = 49.9\,atm \longleftarrow (c)$$

(d) Additive Pressure Rule plus van der Waals Equation. Selecting Eq. 11.99a

$$p = p_1\left(T, \frac{V}{n_1}\right) + p_2\left(T, \frac{V}{n_2}\right)$$

with data from Table A-24E and Eq. 11.2

$$p_1 = \frac{(1545)(654)}{\left[\left(\frac{7.65}{0.5}\right) - 0.685\right]|14.696\times 144|} - \frac{581}{(7.65/0.5)^2} = 30.19\,atm$$

$$p_2 = \frac{(1545)(654)}{\left[\left(\frac{7.65}{0.5}\right) - 1.444\right]|14.696\times 144|} - \frac{2369}{(7.65/0.5)^2} = 24.34\,atm$$

Then

$$p = 30.19 + 24.34 = 54.5\,atm \longleftarrow (d)$$

PROBLEM 11.98 (Contd.)

(e) Additive Pressure Rule plus Z chart. Selecting Eq. 11.99a

$$Z = y_1 Z_1(T,V) + y_2 Z_2(T,V) = 0.5\left[Z_1 + Z_2\right]$$

$$T_{R1} = \frac{654}{344} = 1.90,\quad v'_{R1} = \frac{\left(\frac{7.65}{0.5}\right)(45.8\times 14.696\times 144)}{(1545)(344)} = 2.79 \Rightarrow Z_1 \approx 0.98$$

$$T_{R2} = \frac{654}{666} = 0.982,\quad v'_{R2} = \frac{\left(\frac{7.65}{0.5}\right)(42.1)(14.696)(144)}{(1545)(666)} = 1.325 \Rightarrow Z_2 \approx 0.77$$

Then

$$Z = 0.5(0.98 + 0.77) = 0.875 \Rightarrow p = Z\frac{n\bar{R}T}{V} = Z\,p_{ideal} = (0.875)(62.4)$$

$$= 54.6 \text{ atm} \longleftarrow (e)$$

(f) Additive Volume Rule plus Vander Waals Equation. Selecting Eq. 11.100a

$$V = V_1(p,T) + V_2(p,T) \qquad (1)$$

With data from Table A-24E Eq 11.2 gives

$$p = \frac{(1545)(654)}{\left(\frac{V_1}{0.5} - 0.685\right)(14.696)(144)} - \frac{581}{(V_1/0.5)^2} \Rightarrow p = \frac{238.73}{V_1 - 0.3425} - \frac{145.25}{V_1^2} \qquad (2)$$

$$p = \frac{(1545)(654)}{\left(\frac{V_2}{0.5} - 1.444\right)(14.696)(144)} - \frac{2369}{(V_2/0.5)^2} \Rightarrow p = \frac{238.73}{V_2 - 0.722} - \frac{592.25}{V_2^2} \qquad (3)$$

Setting Eqs. (2) and (3) equal

$$\frac{238.73}{V_1 - 0.3425} - \frac{145.25}{V_1^2} = \frac{238.73}{V_2 - 0.722} - \frac{592.25}{V_2^2}$$

Introducing Eq. (1): $V_1 = V - V_2 = 7.65 - V_2$

$$\frac{238.73}{7.3075 - V_2} - \frac{145.25}{(7.65 - V_2)^2} = \frac{238.73}{V_2 - 0.722} - \frac{592.25}{V_2^2}$$

Using an equation solver, $V_2 = 1.76$ ft³. Eq. (3) then gives, $p = 38.8$ atm ⟵ (f)

Discussion: Comparing the calculated values with the measured value, 50atm, the result is summarized as follows.

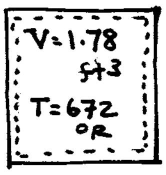
(a)	Ideal Gas Equation	62.4 atm	+24.8 %
(b)	Kay's Rule / Z chart	51.48 atm	+ 3.0 %
(c)	van der Waals Equation	49.9 atm	—
(d)	Additive pressure / van der Waals	54.5 atm	+ 9.0 %
(e)	Additive pressure / Z chart	54.6 atm	+ 9.0 %
(f)	Additive volume / van der Waals	38.8 atm	− 22.4 %

PROBLEM 11.99

KNOWN: One lbmol of {69.5% CO_2, 30.5 C_2H_4} occupies 1.78 ft^3 at 212°F.

FIND: Estimate the pressure using (a) the ideal gas equation of state, (b) Kay's rule with gen. compressibility chart, (c) additive pressure rule with gen. compressibility chart, (d) van der Waals equation.

SCHEMATIC & GIVEN DATA:



CO_2: $y_1 = 0.695$

C_2H_4: $y_2 = 0.305$

ASSUMPTION: The mixture is the system

ANALYSIS: (a) Ideal Gas Model

$$p = \frac{n\bar{R}T}{V} = \frac{(1\,lbmol)(1545\,\frac{ft\cdot lbf}{lbmol\cdot °R})(672°R)}{(1.78\,ft^3)\left|(144)\times(14.7)\frac{lbf}{ft^2}/atm\right|}$$

$= 275.5$ atm ⟵ (a)

(b) Kay's rule with gen. compressibility chart. With data from Table A-1E, Eqs. 11.97 give

$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.695)(548°R) + (0.305)(510°R) = 536.4°R$

$P_c = y_1 P_{c1} + y_2 P_{c2} = (0.695)(72.9\,atm) + (0.305)(50.5\,atm) = 66.07\,atm$

Then, $T_R = (672/536.4) = 1.25$

$$v_R' = \frac{\bar{v}\,P_c}{\bar{R}\,T_c} = \frac{(1.78\,ft^3/lbmol)(66.07\times14.7\times144\,lbf/ft^2)}{(1545\,\frac{ft\,lbf}{lbmol\,°R})(536.4°R)} = 0.3$$

} Fig A-2 $Z \sim 0.61$

$\Rightarrow \quad p = Z\frac{n\bar{R}T}{V} = Z\,p_{id} = 0.61\,(275.5\,atm) = 168.1\,atm.$ ⟵ (b)

(c) Additive pressure rule with gen. compressibility chart.

$T_{R1} = 672/548 = 1.23$

$$v_{R1}' = \left(\frac{1.78}{0.695}\right)\frac{(72.9\times14.7\times144)}{(1545)(548)} = 0.47$$

}: Fig. A-2 $Z_1 \approx 0.68$

$T_{R2} = 672/510 = 1.32$

$$v_{R2}' = \left(\frac{1.78}{0.305}\right)\frac{(50.5\times14.7\times144)}{(1545)(510)} = 0.79$$

}: Fig. A-2 $Z_2 \approx 0.81$

With Eq. 11.99b,

$Z = (0.695)(0.68) + (0.305)(0.81) = 0.72$

$\therefore\ p = Z\,p_{id} = (0.72)(275.5\,atm)$

$= 198.4$ atm ⟵ (c)

(d) van der Waals equation. From Table A-24E, for CO_2

$a_1 = 926\,atm(ft^3/lbmol)^2$, $b_1 = 0.686\,ft^3/lbmol$. Then, using Eqs. 11.4 for C_2H_4,

$a_2 = 1158$, $b_2 = 0.921$. With these values, Eqs. 11.96 give

$a = \left[(0.695)(926)^{1/2} + (0.305)(1158)^{1/2}\right] = 994\,atm(ft^3/lbmol)^2$

$b = \left[(0.695)(0.686) + (0.305)(0.921)\right] = 0.758\,ft^3/lbmol$

Eq. 11.2 then gives

① $$p = \frac{(1545\,ft\cdot lbf/lbmol\cdot °R)(672°R)}{(1.78 - 0.758)(ft^3/lbmol)}\left|\frac{1\,atm}{(144\times14.7)\,lbf/ft^2}\right| - \frac{994\,atm(ft^3/lbmol)^2}{(1.78\,ft^3/lbmol)^2}$$

$= 166.2$ atm ⟵ (d)

---

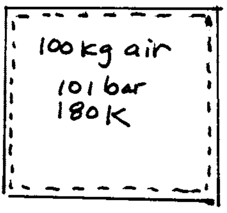
1. Although the results of (b) and (d) are in fair agreement, the results obtained with the four methods are scattered. A conservative choice dictated by the application at hand would govern the value that would be used for the estimated pressure.

PROBLEM 11.100

KNOWN: 100 kg of air with the molar composition {79% N_2, 21% O_2} fills a 0.36-m^3 vessel at a measured pressure and temperature of 101 bar, 180 K, respectively.

FIND: Compare the measured pressure with the pressure predicted by (a) the ideal gas model, (b) Kay's rule, (c) additive pressure rule with the Redlich-Kwong equation, (d) additive volume rule with the Redlich-Kwong equation.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The system is the 100 kg of air.

① ANALYSIS: (a) Ideal gas model, $p = m\bar{R}T/MV$, where $M = y_1M_1 + y_2M_2$ or

$$M = (0.79)(28.01) + (0.21)(32) = 28.85.$$ Thus

$$p = \frac{(100\text{ kg})\left(\frac{8314}{28.85}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(180\text{K})}{0.36\text{ m}^3}\left|\frac{1\text{ bar}}{10^5\text{ N/m}^2}\right| = 144.1\text{ bar} \quad (\sim 43\%\text{ high})$$

(b) With data from Table A-1

N_2: $T_c = 126$ K, $p_c = 33.9$ bar O_2: $T_c = 154$ K, $p_c = 50.5$ bar $\Rightarrow$ $T_c = (0.79)(126) + (0.21)(154) = 131.9$ K
$p_c = (0.79)(33.9) + (0.21)(50.5) = 37.39$ bar

Then, $T_R = 180/131.9 = 1.36$

$$\left.\begin{aligned} &T_R = 180/131.9 = 1.36 \\ &\bar{v}_R' = \frac{\bar{v}\,p_c}{\bar{R}T_c} = \left[\frac{0.36\text{ m}^3}{(100/28.85)\text{ kmol}}\right]\left[\frac{37.39\times10^5\text{ N/m}^2}{\left(8314\frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}\right)(131.9\text{K})}\right] = 0.354\end{aligned}\right\} \begin{aligned}&\text{Figure A-2}\\ &Z \sim 0.7\end{aligned}$$

With $Z = pV/mRT \Rightarrow$

$$p = Z\underbrace{\left(\frac{mRT}{V}\right)}_{\text{ideal gas value}} = 0.7(144.1\text{ bar}) = 100.9\text{ bar}$$ (within 0.1% of the measured value)

(c) Additive pressure model: $p = p_1 + p_2$, where p_1 denotes the pressure of N_2 if it occupied the full volume at 180 K and p_2 denotes the pressure of O_2 if it occupied the full volume at 180 K.

With $n = \frac{m}{M} = \frac{100\text{ kg}}{28.85\text{ kg/kmol}} = 3.466$ kmol. Then

$n_{N_2} = 0.79(3.466) = 2.738$ kmol, $n_{O_2} = 0.728$ kmol.

Then, with data from Table A-24 for the Redlich-Kwong equation...

Nitrogen: $\bar{v} = 0.36\text{m}^3/2.738\text{kmol} = 0.131\text{ m}^3/\text{kmol}$

$$p_1 = \frac{\left(8314\frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}\right)(180\text{ K})}{(0.131 - 0.02677)\text{ m}^3/\text{kmol}}\left|\frac{\text{bar}}{10^5\text{ N/m}^2}\right| - \frac{(15.53)\text{ bar}}{(0.131)(0.131 + 0.02677)(180)^{1/2}}$$

$$= 143.58\text{ bar} - 56.01\text{ bar} = 87.57\text{ bar}$$

PROBLEM 11.100 (Continued)

Oxygen: $\bar{v} = 0.36/0.728 = 0.495\ \text{m}^3/\text{kmol}$

$$p_2 = \frac{(8314)(180)}{(0.495-0.02197)}\left|\frac{1}{10^5}\right| - \frac{17.22}{(0.495)(0.495+0.02197)(180)^{1/2}}$$

$$= 31.64 - 5.02 = 26.62\ \text{bar}$$

Then, $p = p_1 + p_2 = 87.57 + 26.62 = 114.2\ \text{bar}$ (13% higher than the measured value)

(d) Additive volume model: $V = V_1 + V_2$, where V_1 is the volume occupied by N_2 if it were at the mixture temperature and pressure and V_2 is the volume occupied by O_2 if it were at the mixture temperature and pressure. Thus, $V = n_1\bar{v}_1(T,p) + n_2\bar{v}_2(T,p)$, or

$$\frac{V}{n} = y_1\bar{v}_1(T,p) + y_2\bar{v}_2(T,p) \qquad (1)$$

Also, with the Redlich-Kwong equation, when each gas is at the same T, p

$$p = \frac{\bar{R}T}{\bar{v}_1 - b_1} - \frac{a_1}{\bar{v}_1(\bar{v}_1+b_1)\sqrt{T}} = \frac{\bar{R}T}{\bar{v}_2 - b_2} - \frac{a_2}{\bar{v}_2(\bar{v}_2+b)\sqrt{T}} \qquad (2)$$

Eqs. (1), (2) are simultaneous equations for $\bar{v}_1, \bar{v}_2$. Inserting values

$$\left\{\begin{array}{l} 0.1039 = 0.79\,\bar{v}_1 + 0.21\,\bar{v}_2 \qquad (1)' \\[2ex] \dfrac{(8314)(180)}{(\bar{v}_1 - 0.02677)(10^5)} - \dfrac{15.53}{\bar{v}_1(\bar{v}_1+0.02677)(180)^{1/2}} = \dfrac{(8314)(180)}{(\bar{v}_2-0.02197)(10^5)} - \dfrac{17.22}{\bar{v}_2(\bar{v}_2+0.02197)(180)^{1/2}} \qquad (2)' \end{array}\right.$$

With an equation solver, $\bar{v}_1 = 0.11$, $\bar{v}_2 = 0.078$. Using $\bar{v}_1$, Eq (2) gives $p = 102.86$ bars. Using $\bar{v}_2$, Eq. (2) gives $p = 102.49$ bar. Thus, $p \approx 102.7$ bar, which is about 1.7% higher than the measured value.

---

1. $M_{mix} = \dfrac{m_{mix}}{n_{mix}} = \dfrac{m_1 + m_2}{n_{mix}} = \dfrac{M_1 n_1 + M_2 n_2}{n_{mix}} = y_1 M_1 + y_2 M_2$

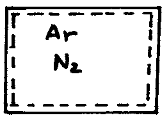
PROBLEM 11.101

KNOWN: A gaseous mixture with the molar analysis below is contained in a tank at 20 atm, 320°R.

1. Argon : $y_1 = .50$
2. N_2 : $y_2 = .50$

FIND: Estimate the specific volume using each of four specified procedures.

SCHEMATIC & GIVEN DATA:



$T = 320°R$
$p = 20$ atm

$y_1 = y_2 = 0.5$

ASSUMPTION: The mixture is the system.

ANALYSIS: (a) Ideal Gas Model

$$v = \frac{(\bar{R}/M)T}{p} = \frac{\left(\frac{1545}{33.975}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(320°R)}{(20)(14.696)(144)\ lbf/ft^2} = 0.344\ ft^3/lb$$

where the mixture molecular weight is found using

$$M_{mix} = \frac{m_{mix}}{n_{mix}} = \frac{m_1 + m_2}{n_{mix}} = \frac{M_1 n_1 + M_2 n_2}{n_{mix}} = y_1 M_1 + y_2 M_2$$
$$= 0.5(39.94 + 28.01) = 33.975$$

(b) Kay's Rule plus Z chart. With data from Table A-1E and Eqs. 11.97

$$T_c = y_1 T_{c1} + y_2 T_{c2} = 0.5[272 + 227] = 249.5°R$$

$$p_c = y_1 p_{c1} + y_2 p_{c2} = 0.5[47.97 + 33.5] = 40.74\ atm$$

Then

$$\left.\begin{aligned} T_R &= \frac{320}{249.5} = 1.283 \\ p_R &= \frac{20}{40.74} = 0.49 \end{aligned}\right\} : \ Z \approx 0.92,\ v_R' \approx 2.40$$

With $Z = 0.92$

$$\bar{v} = Z \frac{\bar{R}T}{p} = Z\, v_{ideal} = 0.92(1.03) = 0.95\ ft^3$$

⟵ (b)

With $v_R' = 2.4$

$$v = v_R' \frac{RT_c}{p_c} = \frac{2.4\,(1545/33.975)(249.5)}{(40.74)(14.696)(144)} = 0.316\ \frac{ft^3}{lb}$$

(c). Redlich - Kwong Equation. Using data from Table A-24E for N_2

$a_2 = 5280\ atm\left(\frac{ft^3}{lbmol}\right)^2(°R)^{1/2}$, $b_2 = 0.4286\ ft^3/lbmol$

For argon use Eqs. 11.8 together with critical data from Table A-1E

$$a_1 = \frac{0.42748\left[0.73\ \frac{atm\cdot ft^3}{lbmol\cdot °R}\right]^2[272°R]^{5/2}}{47.97\ atm} = 5794.5\ atm\left(\frac{ft^3}{lbmol}\right)^2(°R)^{1/2}$$

$$b_1 = 0.08664\,\frac{(0.73)(272)}{47.97} = 0.3586\ ft^3/lbmol$$

Then, using Eqs. 11.96

$$a = \left[y_1 a_1^{1/2} + y_2 a_2^{1/2}\right]^2 = 0.25\left[(5280)^{1/2} + (5794.5)^{1/2}\right]^2 = 5534.3\ atm\left(\frac{ft^3}{lbmol}\right)^2(°R)^{1/2}$$

$$b = y_1 b_1 + y_2 b_2 = 0.5[0.4286 + 0.3586] = 0.3936\ ft^3/lbmol$$

PROBLEM 11.101 (Contd.)

Then, the Redlich - Kwong equation of state becomes

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)T^{1/2}}$$

$$20\,\text{atm} = \frac{\left(0.73\ \frac{\text{atm}\cdot\text{ft}^3}{\text{lbmol}\cdot{}^\circ\text{R}}\right)(320^\circ\text{R})}{(\bar{v}-0.3936)\ \text{ft}^3/\text{lbmol}} - \frac{5534.3\ \text{atm}\left(\frac{\text{ft}^3}{\text{lbmol}}\right)({}^\circ\text{R})^{1/2}}{\bar{v}(\bar{v}+0.3936)\left(\frac{\text{ft}^3}{\text{lbmol}}\right)^2(320^\circ\text{R})^{1/2}}$$

or

$$20 = \frac{233.6}{\bar{v}-0.3936} - \frac{309.37}{\bar{v}(\bar{v}+0.3936)}$$

With an equation solver, $\bar{v} = 10.73\ \text{ft}^3/\text{lbmol}$. Finally,

$$v = \frac{10.73}{33.975} = 0.316\ \text{ft}^3/\text{lb} \quad \longleftarrow \text{(c)}$$

(d) Additive volume rule with gen. compressibility chart. With data from Table A-1E

ARGON: $T_{R1} = \frac{320}{272} = 1.18$, $p_{R1} = \frac{20}{47.97} = 0.42$ $\Big\}\ Z_1 = 0.92$

N_2: $T_{R2} = \frac{320}{227} = 1.41$, $p_{R2} = \frac{20}{33.5} = 0.6$ $\Big\}\ Z_2 = 0.94$

Then, with Eq. 11.100b, $Z = (0.5)(.92) + (0.5)(0.94) = 0.93$. Since $Z = p\bar{V}/mRT$

① $$v = \frac{Z\,RT}{p} = Z\,v_{id} = 0.93\left(0.344\ \frac{\text{ft}^3}{\text{lb}}\right) = 0.32\ \text{ft}^3/\text{lb} \quad \longleftarrow \text{(d)}$$

---

1. Methods (b)-(d) give estimates for specific volume of about 0.32 ft3/lb.

PROBLEM 11.102

KNOWN: A mixture rule is provided for the Carnahan-Starling-DeSantis equation of state.

FIND: Evaluate the pressure, in kPa, for a mixture of R12 and R13, in which R12 is 40% by mass, at $v = 0.005\ \text{m}^3/\text{kg}$, $T = 180°\text{C}$.

ANALYSIS: Rearranging the equation of state

$$p = \frac{\bar{R}T}{\bar{v}}\left\{\frac{1+\beta+\beta^2-\beta^3}{(1+\beta)^3} - \frac{a}{\bar{R}T(\bar{v}+b)}\right\}$$

The term in brackets {...} is seen to "correct" the pressure that an ideal gas would exhibit at the same state: $P_{id} = \bar{R}T/\bar{v}$. Begin by calculating this pressure.

The molar masses are $M_{12} = 120.92$, $M_{13} = 104.5$. Considering a typical 1 kg of mixture, then, $m_{12} = 0.4\ \text{kg}$, $m_{13} = 0.6\ \text{kg}$, and so

$$n_{12} = \frac{0.4}{120.92},\quad n_{13} = \frac{0.6}{104.5} \Rightarrow y_{12} = \frac{\left(\frac{0.4}{120.92}\right)}{\left(\frac{0.4}{120.92}\right)+\left(\frac{0.6}{104.5}\right)} = 0.3655$$

$$y_{13} = 1 - y_{12} = 0.6345$$

Accordingly, for the mixture

① $$M = y_{12}M_{12} + y_{13}M_{13} = 110.5\ \frac{\text{kg}}{\text{kmol}}$$

Then, $\bar{v} = Mv = \left(110.5\ \frac{\text{kg}}{\text{kmol}}\right)\left(0.005\ \frac{\text{m}^3}{\text{kg}}\right) = 0.5525\ \text{m}^3/\text{kmol}$. Using this,

$$P_{id} = \frac{\bar{R}T}{\bar{v}} = \left(\frac{8314\ \frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}}{0.5525\ \text{m}^3/\text{kmol}}\right)(453\ \text{K})\left|\frac{1\ \text{kPa}}{10^3\ \text{N/m}^2}\right| = \mathbf{6817\ kPa}$$

Turning next to the term in brackets {...}, the individual values of a, b must be determined for each component. That is, for R12

$$a_{12} = a_0 \exp(a_1T + a_2T^2) = 3524.12 \exp\left(-2.77230\left(\frac{453}{10^3}\right) - 0.67318\left(\frac{453}{10^3}\right)^2\right)$$

$$= 3524.12 \exp(-1.25585 - 0.13814) = 874.276\ \frac{\text{J}\cdot\text{L}}{(\text{mol})^2}$$

$$= 874{,}276\ \frac{\text{J}\cdot\text{m}^3}{(\text{kmol})^2}$$

$$b_{12} = b_0 + b_1T + b_2T^2 = 0.15376 - \frac{1.84195}{10}\left(\frac{453}{10^3}\right) - \frac{5.03644}{10^2}\left(\frac{453}{10^3}\right)^2$$

$$= 0.15376 - 0.08344 - 0.01034 = 0.05998\ \frac{\text{m}^3}{\text{kmol}}$$

And for R13

$$a_{13} = 2298.13 \exp\left(-3.41828(.453) - 1.52430(.453)^2\right)$$

$$= 2298.13 \exp(-1.54848 - 0.3128) = 357.298\ \frac{\text{J}\cdot\text{L}}{(\text{mol})^2} = 357{,}298\ \frac{\text{J m}^3}{(\text{kmol})^2}$$

$$b_{13} = 0.12814 - \frac{1.84474}{10}(.453) - \frac{10.7951}{10^2}(.453)^2$$

$$= 0.12814 - 0.08357 - 0.02215 = 0.0224\ \frac{\text{m}^3}{\text{kmol}}$$

PROBLEM 11.102 (Contd.)

Next, using the given mixture rule, mixture values for a & b are calculated as follows:

$$b = y_{12}\, b_{12} + y_{13}\, b_{13} = (0.3655)(0.05998) + (0.6345)(0.0224)$$
$$= 0.03614 \ \frac{m^3}{kmol}$$

$$a = y_{12}^2 a_{12} + 2 y_{12} y_{13} (1-f)(a_{12} a_{13})^{1/2} + y_{13}^2\, a_{13}$$
$$= \left[(.3655)^2(.874276) + 2(.3655)(.6345)(1-.035)\left[(.874276)(.357298)\right]^{1/2} + (0.6345)^2(.357298)\right] \times 10^6$$
$$= \left[0.116795 + 0.250159 + 0.143845\right] \times 10^6 = 510{,}799 \ \frac{J \cdot m^3}{(kmol)^2}$$

Also,

$$\beta = \frac{b}{4\bar{v}} = \frac{0.03614}{4(0.5525)} = 0.01635$$

Accordingly

$$\left\{ \frac{1+\beta+\beta^2-\beta^3}{(1+\beta)^3} - \frac{a}{\bar{R}T(\bar{v}+b)} \right\} = 0.96831 - \frac{510{,}799 \ \frac{J \cdot m^3}{(kmol)^2}}{\left(8314 \ \frac{J}{kmol \cdot K}\right)(453K)\left(0.58864 \ \frac{m^3}{kmol}\right)}$$
$$= 0.7379.$$

Collecting results

$$p = (6817\ kPa)(.7379)$$

② $$= 5030\ kPa \qquad \longleftarrow$$

---

1. See Sec. 12.1, Eq. 12.9.

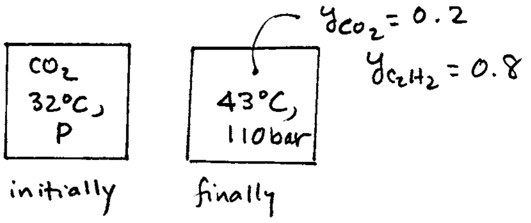
2. The calculated value is within 9% of the value read from a thermodynamic diagram for the mixture: ≈5500 kPa. See "Thermodynamic Diagrams for Refrigerant Mixtures" by J.S. Gallagher et. al. ASHRAE TRANS., 1988 (Part 2), 2119-2135.

PROBLEM 11.103

KNOWN: A rigid vessel initially contains CO_2 at 32°C and pressure p. C_2H_4 enters the tank until a mixture of 20% CO_2, 80% C_2H_4 (molar basis) is contained at 43°C, 110 bar.

FIND: Determine the pressure p.

SCHEMATIC & GIVEN DATA:



ASSUMPTION:
1. Kay's rule together with the gen. compressibility chart suffices.

ANALYSIS: The pressure p can be found, in principle, from the gen. compressibility chart using $p = p_R\, p_c$, where $p_c = 73.9$ bar from Table A-1. p_R is fixed by $T_R = T/T_c = 305/304 = 1.0$ and

$$v_R' = \frac{\bar{v}\, p_c}{\bar{R}\, T_c} = \frac{\bar{v}\,(73.9 \times 10^5\ \text{N/m}^2)}{(8314\ \text{N·m/kmol·K})(304\ \text{K})} = 2.924\, \bar{v} \qquad (1)$$

where $\bar{v}$ is the molar specific volume of CO_2 initially. The specific volume $\bar{v}$ is determined next.

For the final mixture, $$\frac{V}{n_{mix}} = Z_{mix} \frac{\bar{R}\, T_{mix}}{p_{mix}} = Z_{mix}\left(\frac{(8314)(316)}{110 \times 10^5}\right) = 0.239\, Z_{mix}$$

Since the amount of CO_2 present initially and finally is the same, we can write $n_{CO_2} = 0.2\, n_{mix}$. Thus

$$\bar{v} = \frac{V}{n_{CO_2}} = \frac{V}{0.2\, n_{mix}} = 5\left(\frac{V}{n_{mix}}\right).$$

Combining the last two expressions, $\bar{v} = 5(0.239)\, Z_{mix}$. (2)

To find Z_{mix}, use Kay's rule with Table A-1 data:

$$T_c = (yT_c)_{CO_2} + (yT_c)_{C_2H_4} = (0.2)(304) + (0.8)(283) = 287.2\,\text{K}$$

$$p_c = (y p_c)_{CO_2} + (y p_c)_{C_2H_4} = (0.2)(73.9) + (0.8)(51.2) = 55.74\ \text{bar}$$

Then, for the final mixture

$$(T_R)_{mix} = \frac{316}{287.2} = 1.1, \quad (p_R)_{mix} = \frac{110}{55.74} = 1.97 \Longrightarrow Z_{mix} = 0.4$$
(Fig A-2)

Accordingly, Eq. (2) gives $\bar{v} = 5(0.239)(0.4) = 0.478\ \text{m}^3/\text{kmol}\,(CO_2)$. And Eq. (1) gives

$$v_R' = (2.924)(0.478) = 1.4$$

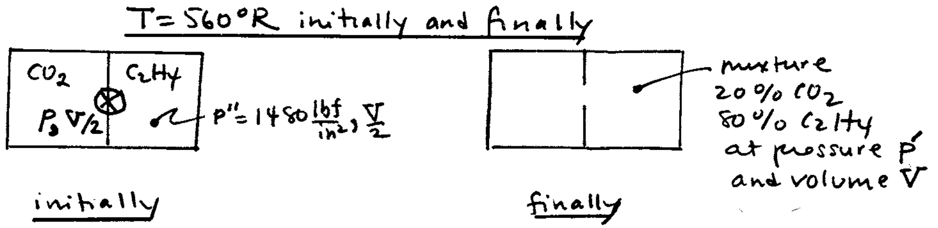
With $T_R = 1$, Fig. A-1 gives $p_R = 0.57$. Then $p = (0.57)(73.9\ \text{bar}) = 42.1\ \text{bar}$

PROBLEM 11.104

KNOWN: Two equal-volume tanks are connected by a valve. Initially, one tank contains CO_2 at 560°R, pressure p. The other tank contains C_2H_4 at 560°R, 1480 lbf/in². The gases are allowed to mix until a final mixture of 20% CO_2, 80% C_2H_4 is achieved at 560°R and pressure p′.

FIND: Determine the pressures p and p′.

SCHEMATIC & GIVEN DATA:



ANALYSIS: Let's begin by evaluating the specific volume of the ethylene in the initial condition. With data from Table A-1E for C_2H_2, $T_R = (560°R/510°R) = 1.1$, $P_R = \frac{(1480/14.7)}{50.5} = 1.99$. Then, Fig A-2 gives $Z = 0.4$. Using this

$$\bar{v}_{C_2H_4} = \frac{Z\bar{R}T}{p''} = \frac{(0.4)(1545\ ft\cdot lbf/lbmol\cdot°R)(560°R)}{(1480\times144\ lbf/ft^2)} = 1.62\ \frac{ft^3}{lbmol}$$

Since the amount of C_2H_4 present initially and finally is the same, we can write $n_{C_2H_4} = 0.8\,n$, where n is the total amount of mixture at the final condition. Accordingly, the specific volume $\bar{v}_{C_2H_4}$ is

$$\bar{v}_{C_2H_4} = \frac{V/2}{n_{C_2H_4}} = \frac{V/2}{0.8n} = \frac{1}{1.6}\left(\frac{V}{n}\right) \Rightarrow \frac{V}{n} = 1.6\,\bar{v}_{C_2H_4} = 1.6\left(1.62\ \frac{ft^3}{lbmol}\right) = 2.59\ \frac{ft^3}{lbmol}$$

where V/n is the specific volume of the mixture at the final condition. Next, using Kay's rule

$$T_c = (yT_c)_{CO_2} + (yT_c)_{C_2H_4} = (0.2)(548) + (0.8)(510) = 518°R$$
$$P_c = (yP_c)_{CO_2} + (yP_c)_{C_2H_4} = (0.2)(72.9) + (0.8)(50.5) = 55\ atm$$

Thus, for the final condition, $T_R = (560/518) = 1.08$ and

$$v_R' = \frac{\bar{v}P_c}{\bar{R}T_c} = \frac{(2.59\ ft^3/lbmol)(55\times14.7\times144\ lbf/ft^2)}{(1545\ ft\,lbf/lbmol\cdot°R)(518°R)} = 0.377$$

Referring again to Fig. A-2, we get $P_R = 1.4$, and thus $p' = P_R P_c = 1.4\ (55\,atm) = 77\,atm$ ←
or $p' = (77\times14.7) = 1132\ lbf/in^2$.

Considering next CO_2 in the initial condition, its specific volume is

$$\bar{v}_{CO_2} = \frac{V/2}{n_{CO_2}} = \frac{V/2}{0.2n} = \left(\frac{1}{0.4}\right)\left(\frac{V}{n}\right) = \left(\frac{1}{0.4}\right)(2.59) = 6.48\ ft^3/lbmol.$$

Accordingly, for the CO_2, $T_R = (560/548) = 1.02$ and

$$v_R' = \frac{(6.48)(72.9\times14.7\times144)}{(1545)(560)} = 1.16$$

Reffering to Fig. A-1, $P_R = 0.67$. Then, $p = P_R P_c = 0.67(72.9)(14.7) = 718\ lbf/in^2$ ←

PROBLEM 11.105

KNOWN: A binary solution at 25°C consists of 59 kg of C_2H_5OH and 41 kg of water. The partial molal volumes are 0.0573 and 0.0172 m³/kmol, respectively.

FIND: Determine the total volume using (a) the partial molal volumes and (b) the molar specific volumes of the pure components, each a liquid at 25°C.

ASSUMPTION: The solution is the system.

ANALYSIS: (a) With Eq. 11.104

$$V = n_1 \overline{V}_1 + n_2 \overline{V}_2$$

$$= \left(\frac{59\,\text{kg}}{46.07\,\text{kg/kmol}}\right)\left(0.0573\,\frac{m^3}{\text{kmol}}\right) + \left(\frac{41\,\text{kg}}{18.02\,\text{kg/kmol}}\right)\left(0.0172\,\frac{m^3}{\text{kmol}}\right)$$

$$= 0.0734 + 0.0391 = 0.1125\ m^3 \longleftarrow$$

(b) Table A-2 gives for liquid water at 25°C, $v = \frac{1.0029}{10^3}\,\frac{m^3}{kg}$. A handbook value for the specific volume of liquid C_2H_5OH at 25°C is $\frac{1.2738}{10^3}\,\frac{m^3}{kg}$. Thus

$$V = m_1 v_1 + m_2 v_2$$

$$= (59\,\text{kg})\left(\frac{1.2738}{10^3}\,\frac{m^3}{kg}\right) + (41\,\text{kg})\left(\frac{1.0029}{10^3}\,\frac{m^3}{kg}\right)$$

$$= 0.0752 + 0.0411 = 0.1163\ m^3$$ (about 3% above the value obtained previously)

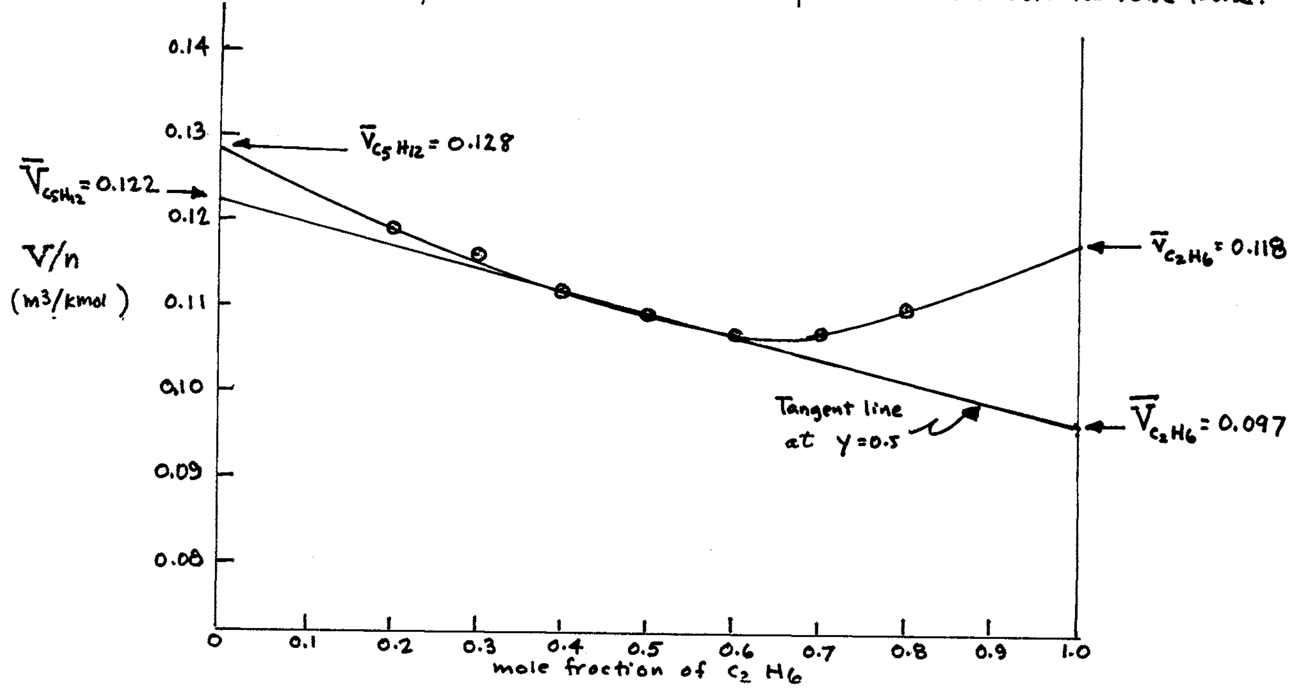
PROBLEM 11.106

KNOWN: Data are provided for a solution of C_2H_6 and C_5H_{12}.

FIND: Estimate (a) the specific volumes of C_2H_6 and C_5H_{12}, (b) the partial molal volumes of C_2H_6 and C_5H_{12} for an equimolar solution.

ASSUMPTION: The solution is the system.

ANALYSIS: Using the data provided, the following plot can be developed. The method of intercepts (Sec. 11.9.1) allows the partial molal volumes to be found.



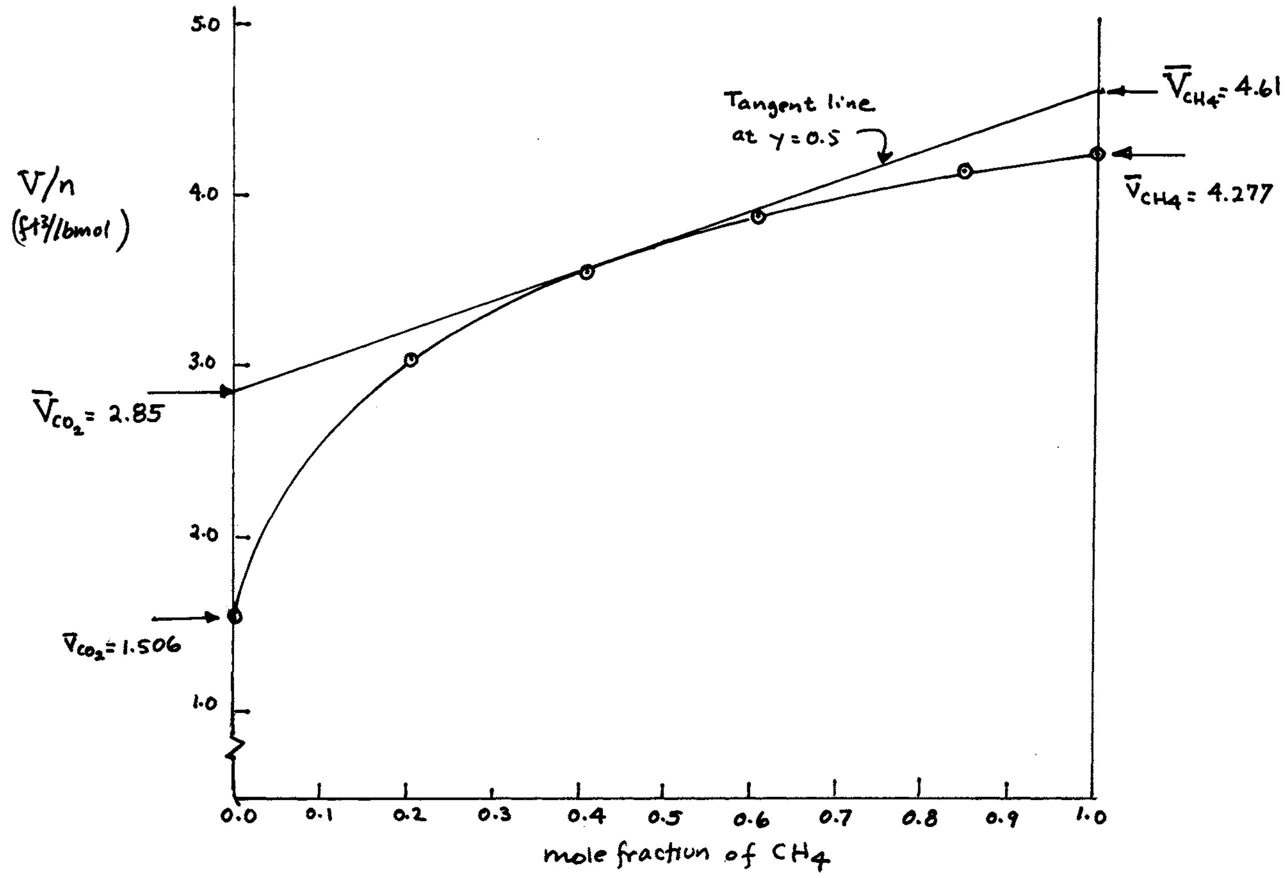
PROBLEM 11.107

KNOWN: Data are provided for a mixture of CO_2 and CH_4.

FIND: Estimate (a) the specific volumes of CO_2 and CH_4, (b) the partial molal volumes of CO_2 and CH_4 for an equimolar mixture.

ASSUMPTION: The mixture is the system.

ANALYSIS: Using the data provided, the following plot can be developed. The method of intercepts (Sec. 11.9.1) allows the partial molal volumes to be found.



PROBLEM 11.108

KNOWN: Water as a saturated vapor at (a) 280°C, (b) 500°F is under consideration.

FIND: Using p-v-T data from the steam tables determine the fugacity at each state and compare with the value yielded by the generalized fugacity chart.

ANALYSIS: (a) Saturated vapor at 280°C.

From Table A-1 $T_c = 647.3$ K, $P_c = 220.9$ bar. Then $T_R = 553/647.3 = 0.854$ ($P_R = 64.12/220.9 = 0.29$) Then, Fig. A-6 gives $f/p \approx 0.825$. Thus

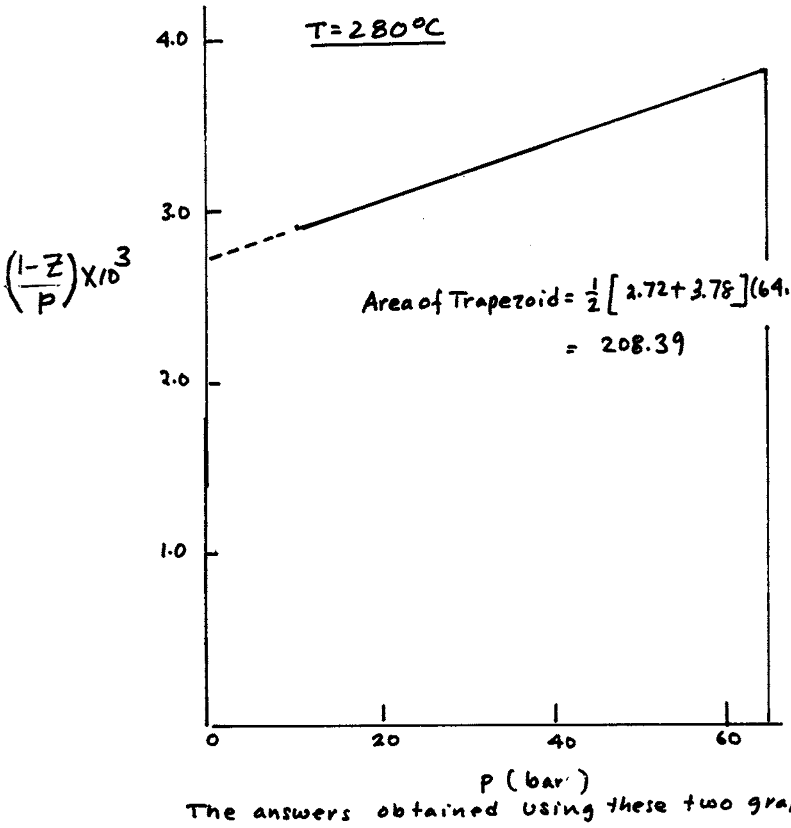
$f = 0.825(64.12) = 52.9$ bar ← chart

From the discussion of Sec. 11.9.4

$$\ln\frac{f}{p} = \int_0^p \left(\frac{Z-1}{p}\right)dp$$

Graphical Solution:

Using p-v-T data from the steam tables the following plot can be drawn at T = 280°C



Area of Trapezoid $= \frac{1}{2}\left[2.72 + 3.78\right](64.12)$ $= 208.39$

$\Rightarrow \ln\frac{f}{p} = -208.39\times 10^{-3}$

$\Rightarrow \frac{f}{p} = 0.812$

$\Rightarrow f = 52.1$ bar ← table data

The answers obtained using these two graphical methods agree to within 2%.

(b) Saturated vapor at 500°F

From Table A-1E $T_c = 1165$°R, $P_c = 218$ atm. Then, $T_R = 960/1165 = 0.82$ ($P_R = \frac{680/14.696}{218} = 0.212$). Fig A-6 gives $f/p \approx 0.87$ Thus

$f = 0.87\ (680) = 592$ lbf/in² ← chart

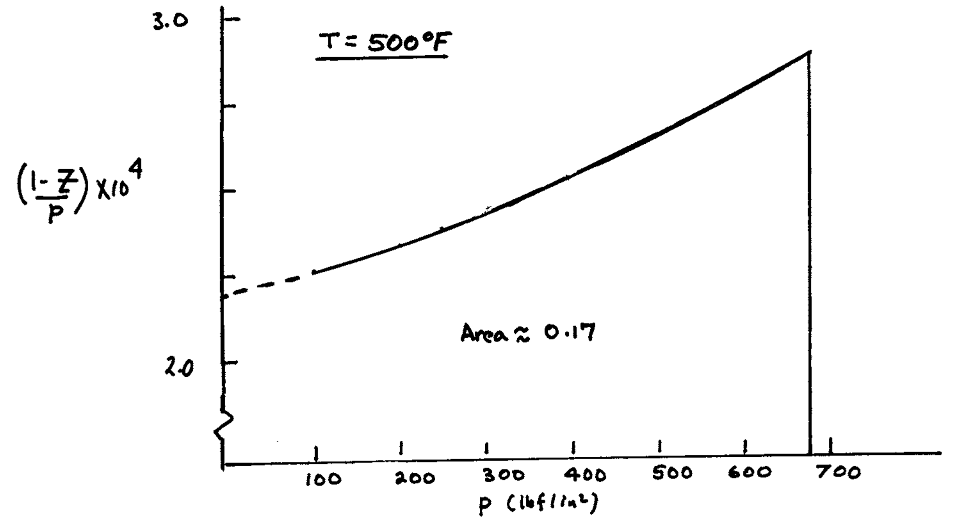
From the discussion of Sec. 11.9.4

$$\ln\frac{f}{p} = \int_0^p \left(\frac{Z-1}{p}\right)dp$$

PROBLEM 11.108 Continued)

Graphical Solution:

Using p-v-T data from the steam tables the following plot can be drawn at T = 500°F



Thus $\ln\frac{f}{p} = -0.17 \Rightarrow \frac{f}{p} = 0.844 \Rightarrow f = 574\ \frac{\text{lbf}}{\text{in}^2}$

The answers obtained using these two graphical methods agree to within 3%.

The following IT code provides an alternative to using Steam Table data for part(a). A similar approach can be used for part(b).

IT Code

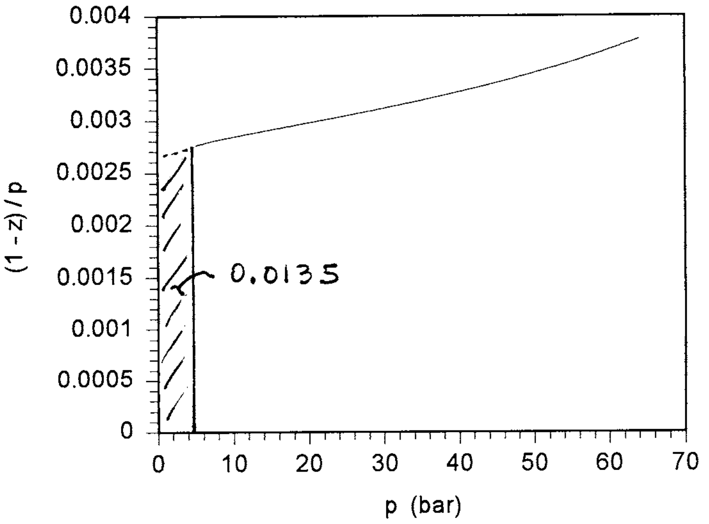
```
T = 280  // °C
psat = 64.12  // bar

R = 8.314/18.02  // kJ/kg·K
Z = (p * v * 100) / (R * (T + 273.15))
v = v_PT("Water/Steam",p,T)
Int  = - (Z - 1) / p
Inf_p = Integral(Int,p)
f_p = exp(Inf_p + 0.0135)
f = f_p * p
```

```
/* The constant 0.0135 is added to account for the
area from 0 to 5 bar.  The computer cannot evaluate
Int for that range because the pressure is too low.  The
value was determined by extrapolating the data from the
BROWSE table to low pressure.  */
```

IT Results

```
ln f/p = - 0.1898
f/p = 0.816
f = 52.31 bar
```



PROBLEM 11.109

KNOWN: Systems at specified states are given.

FIND: For each case determine the fugacity, in atm.

ANALYSIS:

(a) Butane 555 K, 150 bar

For Butane $T_c = 425\,K$, $p_c = 38\,bar$. Then

$$T_R = \frac{555}{425} = 1.31, \quad P_R = \frac{150}{38} = 3.95$$

Figure A-6, $\frac{f}{p} = 0.62 \Rightarrow f = 0.62(150\,bar)\left|\frac{1\,atm}{1.01325\,bar}\right| = 91.8\,atm$ $\longleftarrow$ (a)

(b) Methane 120°F, 800 lbf/in²

For Methane $T_c = 344°R$, $P_c = 45.8\,atm$. Then

$$T_R = \frac{580}{344} = 1.69, \quad P_R = \frac{\left(800\,lbf/in^2 / 14.7\,\frac{lbf/in^2}{atm}\right)}{45.8\,atm} = 1.19$$

Figure A-6, $\frac{f}{p} = 0.95 \Rightarrow f = 51.7\,atm$ $\longleftarrow$ (b)

(c) Benzene 890°R, 135 atm

For Benzene $T_c = 1013°R$, $P_c = 48.7\,atm$. Then

$$T_R = \frac{890}{1013} = 0.88, \quad P_R = \frac{135\,atm}{48.7\,atm} = 2.77$$

Figure A-6, $\frac{f}{p} = 0.18 \Rightarrow f = 24.3\,atm$ $\longleftarrow$ (c)

PROBLEM 11.110

KNOWN: The equation of state has the form

$$Z = 1 + \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{p_R}{T_R}$$

FIND: Using the equation of state evaluate the fugacity of ammonia at 750K, 100 atm am compare with the value obtained from Fig. A-6.

ANALYSIS: Eq. 11.124 is the applicable expression:

$$\ln\frac{f}{p} = \int_0^{p_R}(Z-1)\,d\ln p_R = \int_0^{p_R}\left(\frac{Z-1}{p_R}\right)dp_R$$

The equation of state gives

$$\frac{Z-1}{p_R} = \left[\frac{1}{8} - \frac{27/64}{T_R}\right]\frac{1}{T_R}$$

Combining these two results

$$\ln\frac{f}{p} = \int_0^{p_R}\frac{1}{T_R}\left[\frac{1}{8} - \frac{27/64}{T_R}\right]dp_R = \frac{p_R}{T_R}\left[\frac{1}{8} - \frac{27/64}{T_R}\right] \qquad (1)$$

For ammonia, Table A-1 gives $T_c = 406\text{ K}$, $p_c = 112.8\text{ bar}$. Thus,

$$p_R = \frac{(100\text{ atm})(1.01325\text{ bar/atm})}{112.8\text{ bar}} = 0.898$$

$$T_R = \frac{750\text{K}}{406\text{ K}} = 1.847$$

Inserting values into Eq. (1)

$$\ln\frac{f}{p} = \frac{0.898}{1.847}\left[\frac{1}{8} - \frac{27/64}{1.847}\right] \Rightarrow \frac{f}{p} = 0.951$$

Thus, $f = 95.1$ atm. ←

By inspection of Fig. A-6, $f/p = 0.97$, giving $f = 97$ atm.

$$\%\text{ difference} = \left(\frac{95.1 - 97}{97}\right)(100) = -1.96$$ ←

PROBLEM 11.111

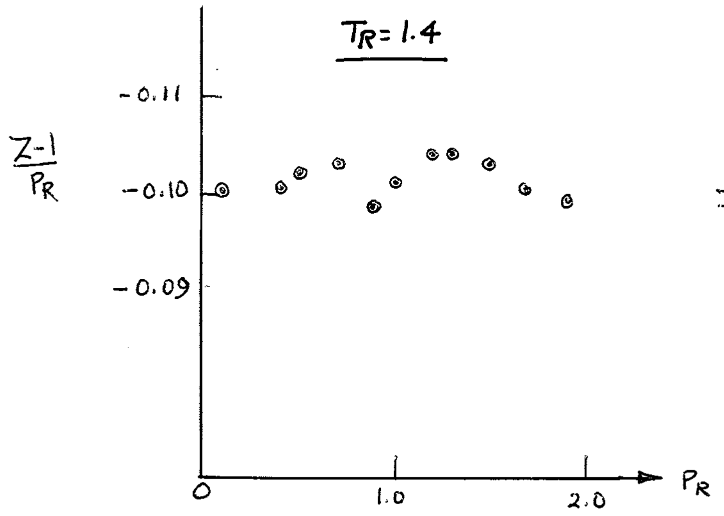
KNOWN: Tabulated compressibility data from the literature is to be used.

FIND: Evaluate f/p at $T_R = 1.4$, $P_R = 2.0$, and compare with the value from Fig. A-6.

ANALYSIS: From Fig. A-6, $f/p = 0.824$. For calculation, Eq. 11.124 is the applicable expression:

$$\ln \frac{f}{p} = \int_0^{P_R} (Z-1)\, d \ln P_R = \int_0^{P_R} \left(\frac{Z-1}{P_R}\right) dP_R$$

The plan is to evaluate $(Z-1)/P_R$ at $T_R = 1.4$ in the range $0 < P_R \leq 2.0$, and then plot vs P_R. (Data from "The Properties of Gases and Liquids" 2nd ed., by R.C. Reid and T.K. Sherwood, McGraw-Hill, 1966, pp. 587–595.)



$$\Rightarrow \quad \frac{Z-1}{P_R} \approx -0.102$$

Accordingly

$$\ln \frac{f}{p} \approx (-0.102)(2.0) = -0.204$$

$$\Rightarrow \frac{f}{p} \approx 0.815$$

Comparison

$$\% = \left(\frac{0.815 - 0.824}{0.824}\right)(100) = -1.1$$

PROBLEM 11.112

KNOWN: A truncated virial expansion is specified:

$$Z = 1 + \hat{B}(T_R)P_R + \hat{C}(T_R)P_R^2 + \hat{D}(T_R)P_R^3$$

FIND: (a) Using tabulated compressibility data, evaluate $\hat{B}, \hat{C}, \hat{D}$ for $0 < P_R < 1.0$ and $T_R = 1.0, 1.2, 1.4, 1.6, 1.8, 2.0$. (b) Obtain an expression for $\ln f/p$ in terms of T_R, P_R. Then, using the coefficients of part (a), evaluate f/p at selected states and compare with tabulated values.

ANALYSIS: (a) Eq. 11.124 is the applicable expression:

$$\ln\frac{f}{p} = \int_0^{P_R}(Z-1)\,d\ln P_R = \int_0^{P_R}\left(\frac{Z-1}{P_R}\right)dP_R$$

Using the given equation of state

$$\ln\frac{f}{p} = \int_0^{P_R}\left(\hat{B}(T_R) + \hat{C}(T_R)P_R + \hat{D}(T_R)P_R^2\right)dP_R$$

$$= \hat{B}(T_R)P_R + \hat{C}(T_R)\frac{P_R^2}{2} + \hat{D}(T_R)\frac{P_R^3}{3} \qquad (1)$$

where the coefficients $\hat{B}, \hat{C}, \hat{D}$ are obtained for selected T_R values and P_R ranging from 0 to 1.0 by linear regression using tabulated compressibility data. The coefficients evaluated this way are

T_R	$\hat{B}$	$\hat{C}$	$\hat{D}$
1	-0.66404	1.274314	-1.30854
1.0	-0.32199	0.081732	-0.14687
1.1	-0.24781	-0.02425	-0.02729
1.2	-0.17771	0.012029	-0.04160
1.3	-0.12188	-0.05410	0.034815
1.4	-0.09219	-0.02849	0.021567
1.5	-0.07307	-0.00706	0.010698
1.6	-0.06488	0.028939	-0.01516
1.8	-0.05442	0.068857	-0.04020
2	-0.03951	0.070563	-0.04350

← (a)

(b) As a sample calculation, at $P_R = 0.5$, $T_R = 1.2$, $f/p = 0.92$. Inserting values into Eq. (1)

$$\ln\frac{f}{p} = -0.17771(0.5) + 0.012029\frac{(.5)^2}{2} - 0.04160\frac{(.5)^3}{3}$$

$$\Rightarrow \frac{f}{p} = 0.915 \qquad \longleftarrow (b)$$

The % difference is

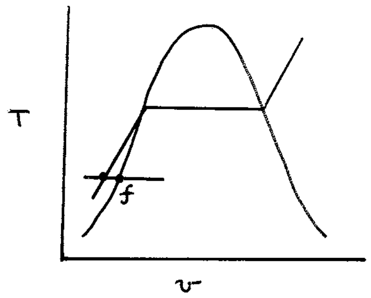
$$\% = \left(\frac{0.915 - 0.92}{0.92}\right)(100) = -0.54$$

PROBLEM 11.113

KNOWN: Approximate expressions for the fugacity of a liquid are specified.

FIND: Derive the expressions and comment.

SCHEMATIC & GIVEN DATA:



ANALYSIS: Beginning with Eq. 11.122

$$RT\left(\frac{\partial \ln f}{\partial p}\right)_T = v$$

Then, approximating v as $v \approx v_f(T)$

$$\left(\frac{\partial \ln f}{\partial p}\right)_T \approx \frac{v_f(T)}{RT} \Rightarrow \ln\frac{f}{f^L_{sat}} \approx \frac{v_f(T)}{RT}\left[p - p_{sat}\right]$$

or

$$f(T,p) \approx f^L_{sat}(T)\exp\left\{\frac{v_f(T)}{RT}(p - p_{sat})\right\} \longleftarrow$$

Letting $x = \frac{v_f(T)}{RT}[p - p_{sat}]$ and expanding $\exp(x)$

$$\frac{f(T,p)}{f^L_{sat}(T)} \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Accordingly, $f(T,p) \approx f^L_{sat}$ when the terms in powers of x are small enough to ignore. For example, considering the case of water at 27°C, Table A-2 gives $p_{sat} = 0.03567$ bar, $v_f = 1.0035 \times 10^{-3}$ m³/kg

$$x = \frac{v_f(T)}{RT}[p - p_{sat}] = \frac{1.0035\times10^{-3}\,\text{m}^3/\text{kg}\,[p - 0.03567]\,\text{bar}}{\left(\frac{8314}{18.02}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300.15\,\text{K})}\left|\frac{1\,\text{bar}}{10^5\,\text{N/m}^2}\right| \longleftarrow$$

$$= (7.25\times10^{-4})[p - 0.03567]$$

The pressure range for which $x < 10^{-n}$, where n is an integer, is

$$7.25\times10^{-4}[p - 0.03567] < 10^{-n}$$

$$\Rightarrow [p - 0.03567] < 0.13793\;10^{4-n}$$

n	p (bar)
2	< 13.83
3	< 1.41

PROBLEM 11.114*

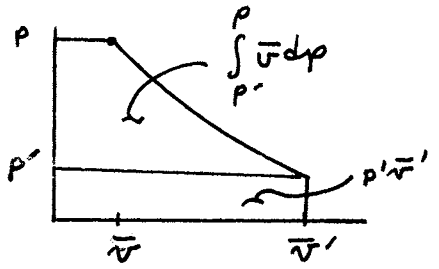
KNOWN: The Redlich-Kwong equation of state describes the $p-\bar{v}-T$ behavior of a gas.

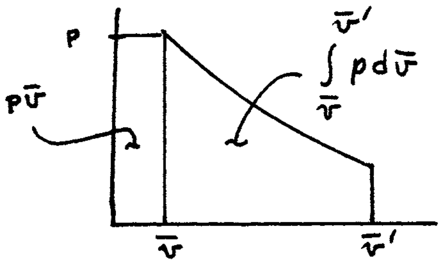
FIND: (a) Evaluate $\ln f$ for the gas. (b) Using the result of part (a), evaluate f, in bar, for Refrigerant 134a at 90°C, 10 bar, and compare with the value obtained from Fig. A-6, the generalized fugacity chart.

ANALYSIS: (a) Beginning with Eq. 11.122, integrate at fixed T from $(p', \bar{v}')$ to $(p, \bar{v})$

$$\bar{R}T \ln f \Big]_{p'}^{p} = \int_{p'}^{p} \bar{v}\,dp \qquad (1)$$

Considering areas,





$$\int_{p'}^{p} \bar{v}dp + p'\bar{v}' = \int_{\bar{v}}^{\bar{v}'} pdv + p\bar{v}$$

$$\Rightarrow \int_{p'}^{p} \bar{v}dp = p\bar{v} - p'\bar{v}' + \int_{\bar{v}}^{\bar{v}'} pd\bar{v}$$

$$= p\bar{v} - p'\bar{v}' - \int_{\bar{v}'}^{\bar{v}} pd\bar{v} \qquad (2)$$

Combining Eqs. (1) and (2)

$$\bar{R}T \ln f \Big]_{p'}^{p} = p\bar{v} - p'\bar{v}' - \int_{\bar{v}'}^{\bar{v}} pd\bar{v} \qquad (3)$$

Introducing the Redlich-Kwong equation in the integral

$$\bar{R}T \ln f \Big]_{p'}^{p} = p\bar{v} - p'\bar{v}' - \int_{\bar{v}'}^{\bar{v}} \left[\frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\sqrt{T}\,\bar{v}(\bar{v}+b)} \right] d\bar{v}$$

$$\Rightarrow \ln\frac{f}{f'} = \frac{p\bar{v}}{\bar{R}T} - \frac{p'\bar{v}'}{\bar{R}T} - \ln\left[\frac{\bar{v}-b}{\bar{v}'-b}\right] - \frac{a}{b\bar{R}T^{3/2}} \ln\left[\frac{\bar{v}+b}{\bar{v}}\left(\frac{1}{1+b/\bar{v}'}\right)\right]$$

or

$$\ln f = \frac{p\bar{v}}{\bar{R}T} - \frac{p'\bar{v}'}{\bar{R}T} + \ln\left[\frac{f'(\bar{v}'-b)}{\bar{v}-b}\right] - \frac{a}{b\bar{R}T^{3/2}} \ln\left[\frac{\bar{v}+b}{\bar{v}}\left(\frac{1}{1+b/\bar{v}'}\right)\right]$$

PROBLEM 11.114 (continued)

Next, consider the limit as $p' \to 0$ $(\bar{v}' \to \infty)$ at fixed T:

$$f' \to p' \to 0, \quad p'\bar{v}' \to \bar{R}T, \quad p'(\bar{v}'-b) \to \bar{R}T, \quad 1+\frac{b}{\bar{v}'} \to 1.$$

Thus

$$\ln f = Z - 1 + \ln\left[\frac{\bar{R}T}{\bar{v}-b}\right] - \frac{a}{b\bar{R}T^{3/2}}\ln\left(\frac{\bar{v}+b}{\bar{v}}\right) \qquad (4)$$

Eq (4) can be rewritten using the Redlich-Kwong equation:

$$p = \frac{\bar{R}T}{\bar{v}-b} - \frac{a}{\bar{v}(\bar{v}+b)T^{1/2}} \Rightarrow \underbrace{\frac{p\bar{v}}{\bar{R}T}}_{Z} = \frac{\bar{v}}{\bar{v}-b} - \frac{a}{(\bar{v}+b)\bar{R}T^{3/2}}$$

$$\Rightarrow Z = \frac{\bar{v}}{\bar{v}-b} - \frac{a}{(\bar{v}+b)\bar{R}T^{3/2}} \Rightarrow Z-1 = \frac{b}{\bar{v}-b} - \frac{a}{(\bar{v}+b)\bar{R}T^{3/2}}$$

Thus

$$\ln f = \frac{b}{\bar{v}-b} + \ln\left[\frac{\bar{R}T}{\bar{v}-b}\right] - \frac{a}{\bar{R}T^{3/2}}\left[\frac{1}{(\bar{v}+b)} + \frac{1}{b}\ln\left(\frac{\bar{v}+b}{\bar{v}}\right)\right] \quad (5) \leftarrow$$

(b) From Table A-1, for R134a, $T_c = 374\,K$, $p_c = 40.7\,bar$. Thus

$$T_R = \frac{363\,K}{374\,K} = 0.97, \quad p_R = \frac{10\,bar}{40.7\,bar} = 0.25$$

Figure A-6 gives, $\frac{f}{p} \sim 0.92 \Rightarrow f = 9.2\,bar$ ←

Evaluating a and b for R134a via Eqs. 11.8

$$a = 196.39\,bar\left(\frac{m^3}{kmol}\right)^2(K)^{1/2}, \quad b = 0.0662\,m^3/kmol$$

Solving the Redlich-Kwong equation for $\bar{v}$ when $T = 363\,K$ and $p = 10\,bar$ gives $\bar{v} = 2.724\,m^3/kmol$.

Finally, inserting values into Eq. (5), using $\bar{R} = 0.08314\,\frac{bar \cdot m^3}{kmol \cdot K}$

$$\ln f = \frac{0.0662}{2.6578} + \ln\left[\frac{(0.08314)(363)}{2.6578}\right] - \frac{196.39}{(0.08314)(363)^{3/2}}\left[\frac{1}{2.7902} + \frac{1}{0.0662}\ln\left(\frac{2.7902}{2.724}\right)\right]$$

$$= 2.208 \Rightarrow f = 9.1\,bar \qquad \leftarrow$$

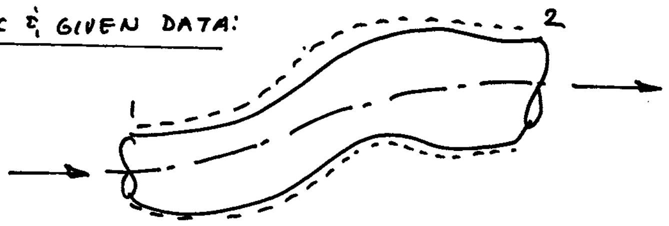
The calculated and chart values for f agree to within $\approx 1\%$.

PROBLEM 11.115

KNOWN: Under consideration is a one-inlet one-exit control volume at steady state through which the flow is internally reversible and isothermal.

FIND: Derive an expression for the work per unit of mass flowing in terms of the fugacity and other relevant quantities.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying sketch is at steady state. (2) The flow is internally reversible and isothermal. (3) A single component (or a mixture with known properties such as air) is flowing.

ANALYSIS:

mass rate balance: $\dot{m}_1 = \dot{m}_2 \equiv \dot{m}$

energy rate balance: $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left[h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)\right]$

entropy rate balance (with assumption 2):

$$0 = \frac{\dot{Q}_{cv}}{T} + \dot{m}(s_1 - s_2) + \cancel{\dot{\sigma}_{cv}}^{\,0}$$

Eliminating $\dot{Q}_{cv}$ between the energy and entropy rate balances

$$\frac{\dot{W}_{cv}}{\dot{m}} = T(s_2 - s_1) + (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)$$

The Gibbs function is $g = h - Ts$, thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = (g_1 - g_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

For a single component, the chemical potential equals the Gibbs function per mole. Then, with Eq. 11.121

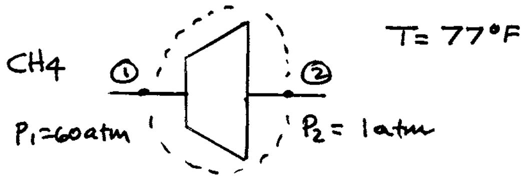
$$\frac{\dot{W}_{cv}}{\dot{m}} = -\bar{R}T \ln \frac{f_2}{f_1} + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2) \longleftarrow$$

PROBLEM 11.116

KNOWN: Methane expands isothermally and without irreversibilities through a turbine operating at steady state.

FIND: Using the generalized fugacity chart, determine the work developed per lb of CH4 flowing.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) The expansion is isothermal and internally reversible. (3) Kinetic and potential energy effects can be ignored

ANALYSIS: Using the result of Problem 11.115 and data from Table A-1E and Figure A-6,

$$p_{R1} = \frac{60\text{ atm}}{45.8\text{ atm}} = 1.31,\quad T_{R1} = T_{R2} = \frac{537°R}{344°R} = 1.56$$

$$p_{R2} = \frac{1}{45.8} = 0.02$$

$$\left(\frac{f}{p}\right)_1 \approx 0.92,\ \left(\frac{f}{p}\right)_2 \approx 1.0 \Rightarrow \left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{\substack{int\\rev}} = -RT\ln\frac{f_2}{f_1} = -\left(\frac{1.986}{16.04}\frac{\text{Btu}}{\text{lb}\cdot°R}\right)(537°R)\ln\left(\frac{1.0\times 1\text{atm}}{0.92\times 60\text{atm}}\right)$$

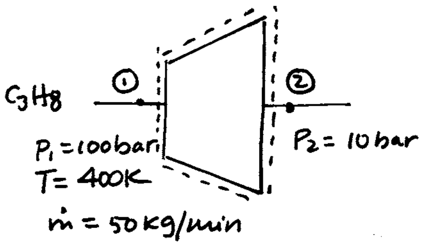
$$= 266.7\text{ Btu/lb} \longleftarrow$$

PROBLEM 11.117

KNOWN: Propane expands isothermally and without irreversibilities through a turbine operating at steady state.

FIND: Using the generalized fugacity chart, determine the power developed.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume shown operates at steady state.
2. The expansion is isothermal and internally reversible
3. Kinetic & Potential energy effects can be ignored.

ANALYSIS: Using the result of Problem 11.115 and data from Table A-1 and Fig A-6,

$$p_{R1} = \frac{100\text{ bar}}{42.7\text{ bar}} = 2.34,\quad p_{R2} = \frac{10}{42.7} = 0.23,\quad T_R = \frac{400}{370} = 1.08$$

$\left(\frac{f}{p}\right)_1 = 0.48$, $\left(\frac{f}{p}\right)_2 = 0.96$. Then

$$\dot{W}_{cv} = -\dot{m}RT\ln\left(\frac{f_2}{f_1}\right) = -\dot{m}\left(\frac{\bar{R}}{M}\right)T\ln\left(\left(\frac{f_2}{p_2}\right)\left(\frac{p_2}{p_1}\right)\left(\frac{p_1}{f_1}\right)\right)$$

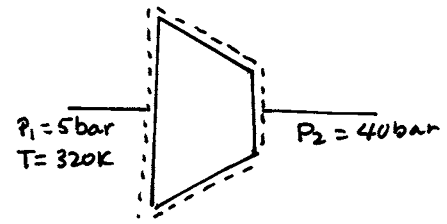
$$= -\left(\frac{50}{60}\frac{\text{kg}}{\text{s}}\right)\left(\frac{8.314}{44.09}\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(400\text{K})\ln\left((0.96)(0.1)\left(\frac{1}{0.48}\right)\right)\left|\frac{\text{kW}}{\text{kJ/s}}\right| = 101.1\text{ kW} \longleftarrow$$

PROBLEM 11.118

KNOWN: C_2H_6 is compressed isothermally without internal irreversibilities.

FIND: Determine the work and heat transfer, each per unit mass of C_2H_6 flowing.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume is at steady state.
2. Kinetic & potential energy effects can be ignored.
3. The process occurs isothermally and without internal irreversibilities.

ANALYSIS: Using the result of Prob. 11.115

$$\frac{\dot{W}_{cv}}{\dot{m}} = -RT \ln\left(\left(\frac{f}{p}\right)_2 \left(\frac{P_2}{P_1}\right)\left(\frac{P}{f}\right)_1\right)$$

With data from Table A-1 and Fig. A-6,

$$T_R = \frac{320\,K}{305\,K} = 1.05 \quad P_{R1} = \frac{5\text{ bar}}{48.8\text{ bar}} = 0.1,\ P_{R2} = \frac{40}{48.8} = 0.82,\ \left(\frac{f}{p}\right)_1 = 0.98,\ \left(\frac{f}{p}\right)_2 = 0.79$$

Thus

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\left(\frac{8.314}{30.07}\frac{kJ}{kg\cdot K}\right)(320K) \ln\left((0.79)\left(\frac{40}{5}\right)\left(\frac{1}{0.98}\right)\right) = -164.9\frac{kJ}{kg} \quad \longleftarrow$$

Reducing mass and energy rate balances gives $\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$.
Invoking Eq. 11.85 and Fig A-4 data

$$\bar{h}_2 - \bar{h}_1 = \underbrace{\bar{h}_2^* - \bar{h}_1^*}_{\substack{=0 \\ \text{since } T \\ \text{is const}}} - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right]$$

$$\Rightarrow h_2 - h_1 = -\left(\frac{8.314}{30.07}\right)(305)\left[(1.0) - (0.09)\right] = -76.7\frac{kJ}{kg}$$

Then

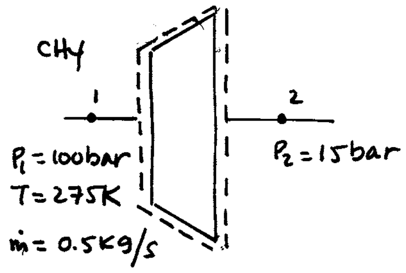
$$\frac{\dot{Q}_{cv}}{\dot{m}} = \frac{\dot{W}_{cv}}{\dot{m}} + (h_2 - h_1) = -164.9\frac{kJ}{kg} + (-76.7)\frac{kJ}{kg} = -241.6\frac{kJ}{kg} \quad \longleftarrow$$

PROBLEM 11.119

KNOWN: Steady-state operating data are provided for methane expanding isothermally and without internal irreversibilities through a turbine.

FIND: Determine the power developed and the rate of heat transfer.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. The control volume is at steady state.
2. Kinetic & potential energy effects can be ignored.

ANALYSIS: Using the result of Prob. 11.115 together with data from Table A-1 and Fig A-6: $M = 16.04$, $T_R = \frac{275K}{191K} = 1.44$, $P_{R1} = \frac{100bar}{46.4bar} = 2.16$, $P_{R2} = \frac{15}{46.4} = 0.32$, $\left(\frac{f}{p}\right)_1 = 0.82$, $\left(\frac{f}{p}\right)_2 = 0.97$.

Then

$$\dot{W}_{cv} = -\dot{m}\, RT \ln\left(\frac{f_2}{f_1}\right) = -\dot{m}\left(\frac{\bar{R}}{M}\right) T \ln\left(\left(\frac{f}{p}\right)_2\left(\frac{P_2}{P_1}\right)\left(\frac{p}{f}\right)_1\right)$$

$$= -\left(0.5\,\frac{kg}{s}\right)\left(\frac{8.314}{16.04}\right)(275) \ln\left((0.97)\left(\frac{15}{100}\right)\left(\frac{1}{0.82}\right)\right) = 123.3\,kW \longleftarrow$$

Mass and energy rate balances reduce to give $\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}(h_2 - h_1)$. $(h_2 - h_1)$ can be found using Eq. 11.85 and data from Fig A-4

$$\bar{h}_2 - \bar{h}_1 = \underbrace{\bar{h}_2^* - \bar{h}_1^*}_{=0 \text{ since } T \text{ is const}} - \bar{R}\,T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right]$$

$$\Rightarrow \quad h_2 - h_1 = -\left(\frac{8.314}{16.04}\right)(191)\left[(0.17) - (1.18)\right] = 100\,\frac{kJ}{kg}$$

Thus

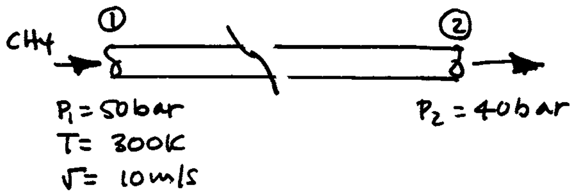
$$\dot{Q}_{cv} = (123.3\,kW) + \left(0.5\,\frac{kg}{s}\right)\left(100\,\frac{kJ}{kg}\right)\left|\frac{kW}{kJ/s}\right| = 173.3\,kW \longleftarrow$$

PROBLEM 11.120

KNOWN: CH_4 flows isothermally and without irreversibilities through a horizontal pipe. Steady-state data are provided.

FIND: Determine the exit velocity.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: 1. A control volume enclosing the pipe is at steady state.
2. For the control volume, potential energy effects can be ignored and $\dot{W}_{cv} = 0$.

ANALYSIS: Using the result of Problem 11.115

$$\cancel{\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)}^{0}_{\substack{int\\rev}} = -RT\ln\left(\frac{f_2}{f_1}\right) + \frac{V_1^2 - V_2^2}{2} + g\cancel{(z_1 - z_2)}^{0}$$

$$\Rightarrow \quad V_2^2 = V_1^2 - 2RT\ln\left(\frac{f_2}{f_1}\right) \qquad (1)$$

With data from Table A-1 and Fig A-6

$M = 16.04$, $T_R = \frac{300K}{191K} = 1.57$, $P_{R1} = \frac{50}{46.4} = 1.08$, $P_{R2} = \frac{40}{46.4} = 0.86$

$\left(\frac{f}{p}\right)_1 = 0.93$, $\left(\frac{f}{p}\right)_2 = 0.95$

Thus, Eq. (1) gives

$$V_2^2 = (10\,m/s)^2 - 2\left(\frac{8314}{16.04}\,\frac{N\cdot m}{kg\cdot K}\right)(300K)\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|\ln\left((0.95)\left(\frac{40}{50}\right)\left(\frac{1}{0.93}\right)\right)$$

$$= (10\,m/s)^2 - (-62820\,m^2/s^2) = 251\,m/s \quad \longleftarrow$$

PROBLEM 11.121

KNOWN: Ethane (C_2H_6) is (a) pure at 310K, 20.4 atm., (b) a component of an ideal solution at 310K, 20.4 atm with a mole fraction of 0.35.

FIND: In each case, evaluate the fugacity.

ANALYSIS: (a) From Table A-1, $T_c = 305\text{ K}$, $p_c = 48.8\text{ bar}$. Then, $T_R = 310/305 = 1.02$,

$$p_R = \frac{20.4\text{ atm}}{48.8\text{ bar}}\left|\frac{1.01325\text{ bar}}{1\text{ atm}}\right| = 0.42. \quad \text{Fig. A-6 gives } \frac{f}{p} = 0.89 \Rightarrow f = 18.2\text{ atm} \longleftarrow \text{(a)}$$

(b) For ethane as a member of an ideal solution, Eq. 11.134 is applicable:

$\bar{f}_i = y_i f_i$, where f_i is the fugacity of pure i at the same T, p. Thus,

$$\bar{f}_i = 0.35(18.2\text{ atm}) = 6.37\text{ atm.} \qquad \longleftarrow \text{(b)}$$

PROBLEM 11.122

KNOWN: The fugacity of the solute (denoted by 2) in a dilute binary liquid solution at T, p is proportional to its mole fraction in the solution: $\bar{f}_2 = K y_2$, where K is a constant.

FIND: Show that the fugacity of the solvent (denoted by 1) is $\bar{f}_1 = y_1 f_1$, where y_1 is the mole fraction of the solvent and f_1 is the fugacity of pure 1 at T, p.

ANALYSIS: The intensive state of the solution is fixed by T, P, y_1 and y_2. But with T, p assumed known, either y_1 or y_2 can be viewed as the independent variable since $y_1 + y_2 = 1$. Accordingly, the Gibbs-Duhem equation (Eq. 11.113) reduces at fixed T, p and y_2 as the independent variable to read

$$n_1 \frac{d\mu_1}{dy_2} + n_2 \frac{d\mu_2}{dy_2} = 0 \Longrightarrow y_1 \frac{d\mu_1}{dy_2} + y_2 \frac{d\mu_2}{dy_2} = 0$$

Introducing $\mu_i = \bar{R}T \ln \bar{f}_i + C_i(T)$ (Eq 11.125),

$$y_1 \frac{d\ln \bar{f}_1}{dy_2} + y_2 \frac{d\ln \bar{f}_2}{dy_2} = 0 \qquad (1)$$

Since $\bar{f}_2 = K y_2 \Rightarrow \ln \bar{f}_2 = \ln y_2 + \ln K$, and so $\frac{d\ln \bar{f}_2}{dy_2} = \frac{1}{y_2}$ (2)

Combining Eqs. (1), (2)

$$y_1 \frac{d\ln \bar{f}_1}{dy_2} + 1 = 0 \Rightarrow \frac{d\ln \bar{f}_1}{dy_2} = -\frac{1}{y_1} \longleftarrow (1 = y_1 + y_2)$$

$$= -\frac{1}{(1-y_2)}$$

Accordingly,

$$\ln \bar{f}_1 = \ln(1-y_2) + \ln K' \Longrightarrow \bar{f}_1 = K'(1-y_2)$$

$$\bar{f}_1 = K' y_1 \qquad (3)$$

Since $\bar{f}_1 \to f_1$ as $y_1 \to 1$, $K' = f_1$, and thus Eq. (3) becomes

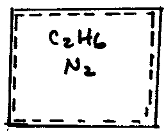
$$\bar{f}_1 = y_1 f_1 \qquad \longleftarrow$$

PROBLEM 11.123

KNOWN: A tank contains 310 kg of a gaseous mixture with the molar analysis {70% C_2H_6, 30% N_2} at 311 K, 170 atm.

FIND: Determine the tank volume using the compressibility chart and (a) Kay's rule, (b) the ideal solution model. Compare with the measured value: 1 m³.

SCHEMATIC & GIVEN DATA:



$T = 311$ K
$p = 170$ atm
$m = 310$ kg

1. C_2H_6 $y_1 = 0.7$
2. N_2 $y_2 = 0.3$

ASSUMPTION: The system is the mixture.

ANALYSIS: (a) Kay's Rule plus Z chart. With data from Table A-1, Eqs. 11.97 give

$$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.7)(305) + (0.3)(126) = 251.3\,K$$

$$p_c = y_1 p_{c1} + y_2 p_{c2} = (0.7)(48.2) + (0.3)(33.5) = 43.79\,atm$$

Thus, $T_R = 311/251.3 = 1.24$, $P_R = 170/43.79 = 3.88$. Then, Fig. A-2 gives, $Z \approx 0.65$, so

$$V = Z\left[\frac{mRT}{p}\right] = 0.65\left[\frac{(310\,kg)(8314/29.45)(N\cdot m/kg\cdot K)(311\,K)}{170 \times 1.01325 \times 10^5\ N/m^2}\right] = 1.03\,m^3 \quad \longleftarrow (a)$$

① where $M = y_1 M_1 + y_2 M_2 = (0.7)(30.07) + (0.3)(28.01) = 29.45$ kg/kmol of mixture.

(b) Ideal Solution Model plus Z chart. Using Eqs. 11.136 for an ideal solution

$$V = n_1 \bar{V}_1 + n_2 \bar{V}_2 = n_1 \bar{v}_1 + n_2 \bar{v}_2$$

where $\bar{v}_i$ is the molar specific volume of pure i at the temperature and pressure of the solution. Dividing by n, the number of moles of solution gives

$$\frac{V}{n} = y_1 \bar{v}_1 + y_2 \bar{v}_2 \qquad (1)$$

To find $\bar{v}_1$, calculate $T_{R1} = 311/305 = 1.02$, $P_{R1} = 170/48.2 = 3.53$. Then, from Fig. A-2 $Z_1 \approx 0.51$. Accordingly

$$\bar{v}_1 = \frac{Z_1 \bar{R} T}{p} = \frac{(0.51)(8314\ N\cdot m/kmol\cdot K)(311\,K)}{(170 \times 1.01325 \times 10^5\ N/m^2)} = 0.0766\,\frac{m^3}{kmol}$$

To find $\bar{v}_2$, calculate $T_{R2} = 311/126 = 2.47$, $P_{R2} = 170/33.5 = 5.07$. From Fig. A-2, $Z_2 \approx 1.05$. Accordingly

$$\bar{v}_2 = \frac{Z_2 \bar{R} T}{p} = \frac{(1.05)(8314)(311)}{(170 \times 1.01325 \times 10^5)} = 0.1576\,\frac{m^3}{kmol}$$

Then, with Eq. (1), $V = n[y_1 \bar{v}_1 + y_2 \bar{v}_2]$, or

$$V = \left(\frac{310\ kg}{29.45\,kg/kmol}\right)\left[(0.7)(0.0766) + (0.3)(0.1576)\right]\frac{m^3}{kmol} = 1.06\,m^3 \quad \longleftarrow (b)$$

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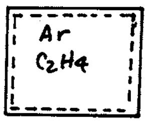
1. Using Eq. 12.9 of Sec. 12.1

PROBLEM 11.124

KNOWN: A tank contains 157 lb of a mixture of 75% Ar and 25% C_2H_2 (molar basis) at 77°F, 81.42 atm.

FIND: Estimate the tank volume using (a) the ideal gas equation of state, (b) Kay's rule with the gen. compressibility chart, (c) ideal solution model with the compressibility chart.

SCHEMATIC & GIVEN DATA:



T = 537°R
p = 81.42 atm
m = 157 lb

1. Argon $y_1 = 0.75$
2. C_2H_4 $y_2 = 0.25$

ASSUMPTION: The mixture is the system.

(a) Ideal gas model.

$$V_{id} = \frac{mRT}{p} = \frac{(157\ \text{lb})(1545/36.97)(\text{ft·lbf/lb·°R})(537°\text{R})}{(81.42)(14.7)(144)\ \text{lbf/ft}^2} = 20.44\ \text{ft}^3 \longleftarrow (a)$$

① where $M = y_1M_1 + y_2M_2 = (0.75)(39.94) + (0.25)(28.05) = 36.97$ lb/lbmol.

(b) Kay's rule with gen. compressibility chart. With data from Table A-1E, Eqs. 11.97 give

$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.75)(272) + (0.25)(510) = 331.5°R$

$p_c = y_1 p_{c1} + y_2 p_{c2} = (0.75)(47.97) + (0.25)(50.5) = 48.61$ atm

Thus, $T_R = 537/331.5 = 1.62$, $p_R = 1.67$. Fig. A-2 gives $Z \approx 0.91$, so

$$V = Z\frac{mRT}{p} = Z\,V_{id} = 0.91(20.44\ \text{ft}^3) = 18.6\ \text{ft}^3 \longleftarrow (b)$$

(c) Ideal solution with gen. compressibility chart. Using Eq. 11.136

$$V = n_1\bar{V}_1 + n_2\bar{V}_2 = n_1\bar{v}_1 + n_2\bar{v}_2$$

where $\bar{v}_i$ is the molar specific volume of pure i at the temperature and pressure of the solution. Dividing by n, the number of moles of solution gives

$$\frac{V}{n} = y_1\bar{v}_1 + y_2\bar{v}_2 \qquad (1)$$

To find $\bar{v}_1$, calculate $T_{R1} = 537/272 = 1.97$, $p_{R1} = 81.42/47.97 = 1.7$. From Fig. A-2 $Z_1 \approx 0.97$. Accordingly

$$\bar{v}_1 = \frac{Z_1\bar{R}T}{p} = \frac{(0.97)(1545)(537)}{[(81.42)(14.696)(144)]} = 4.671\ \frac{\text{ft}^3}{\text{lbmol}}$$

To find $\bar{v}_2$, calculate $T_{R2} = 537/510 = 1.053$, $p_{R2} = 81.42/50.5 = 1.612$. From Fig. A-2, $Z_2 \approx 0.32$. Accordingly

$$\bar{v}_2 = \frac{Z_2\bar{R}T}{p} = \frac{(0.32)(1545)(537)}{[(81.42)(14.696)(144)]} = 1.541\ \frac{\text{ft}^3}{\text{lbmol}}$$

With Eq. (1)

$$V = n[y_1\bar{v}_1 + y_2\bar{v}_2] = \left(\frac{157}{36.97}\right)\left[0.75(4.671) + 0.25(1.541)\right] = 16.51\ \text{ft}^3 \longleftarrow (c)$$

---

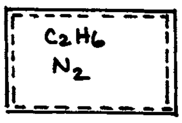
1. Using Eq. 12.9 of Sec. 12.1

PROBLEM 11.125

KNOWN: A tank contains 2130 kg of a mixture of 70% C_2H_6, 30% N_2 at 400K, 200 atm.

FIND: Estimate the tank volume using (a) the ideal gas equation of state, (b) Kay's rule with the gen. compressibility chart, (c) the ideal solution model with the gen. compressibility chart.

SCHEMATIC & GIVEN DATA:



T = 400K
p = 200 atm
m = 2130 kg

1. C_2H_6, $y_1 = 0.7$
2. N_2, $y_2 = 0.3$

ASSUMPTION: The mixture is the system.

ANALYSIS: (a) Ideal Gas Model.

$$V_{id} = \frac{mRT}{p} = \frac{(2130\,kg)(8314/29.45)(N\cdot m/kg\cdot K)(400K)}{(200\times 1.01325\times 10^5\ N/m^2)} = 11.87\,m^3 \quad \longleftarrow (a)$$

① where $M = y_1M_1 + y_2M_2 = (0.7)(30.07) + (0.3)(28.01) = 29.45$ kg/kmol.

(b) Kay's rule with gen. compressibility chart. With Eqs. 11.97

$T_c = y_1 T_{c1} + y_2 T_{c2} = (0.7)(305) + (0.3)(126) = 251.3\,K$

$P_c = y_1 p_{c1} + y_2 p_{c2} = (0.7)(48.2) + (0.3)(33.5) = 43.79$ atm

Then, $T_R = 400/251.3 = 1.59$, $P_R = 200/43.79 = 4.57$. Fig. A-2 gives $Z \approx 0.87$. So

$$V = Z\left[\frac{mRT}{p}\right] = Z\,V_{id} = (0.87)(11.87) = 10.33\,m^3 \quad \longleftarrow (b)$$

(c) Ideal Solution plus Z chart. Using Eqs. 11.136 for an ideal solution

$$V = n_1 \bar{V}_1 + n_2 \bar{V}_2 = n_1\bar{v}_1 + n_2\bar{v}_2$$

where $\bar{v}_i$ is the molar specific volume of pure i at the temperature and pressure of the solution. Dividing by n, the number of moles of solution gives

$$\frac{V}{n} = y_1\bar{v}_1 + y_2\bar{v}_2 \qquad (1)$$

To find $\bar{v}_1$, calculate $T_{R1} = 400/305 = 1.31$, $P_{R1} = 200/48.2 = 4.15$. Then, Fig. A-2 gives $Z_1 \approx 0.7$. Accordingly

$$\bar{v}_1 = \frac{Z_1\bar{R}T}{p} = \frac{(0.7)(8314\ \frac{N\cdot m}{kmol\cdot K})(400K)}{[(200)(1.01325\times 10^5\ N/m^2)]} = 0.1149\ \frac{m^3}{kmol}$$

To find $\bar{v}_2$, calculate $T_{R2} = 400/126 = 3.175$, $P_{R2} = 200/33.5 = 5.97$. Fig. A-2 gives $Z_2 \approx 1.08$. Accordingly

$$\bar{v}_2 = \frac{Z_2\bar{R}T}{p} = \frac{(1.08)(8314)(400)}{[(200)(1.01325\times 10^5)]} = 0.1772\ \frac{m^3}{kmol}$$

With Eq. (1)

$$V = n\left[y_1\bar{v}_1 + y_2\bar{v}_2\right] = \left(\frac{2130}{29.45}\right)\left[(0.7)(0.1149) + (0.3)(0.1772)\right] = 9.66\,m^3 \quad \longleftarrow (c)$$

---

1. Using 12.9 of Sec. 12.1

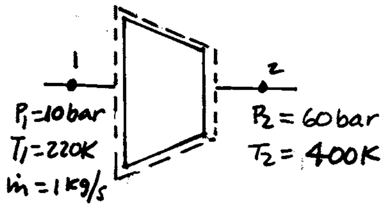
PROBLEM 11.126

KNOWN: Steady-state operating data are provided for a compressor handling an equimolar mixture of O_2 and N_2.

FIND: Determine (a) the power required, (b) the rate of entropy production.

SCHEMATIC & GIVEN DATA:

Mixture: $y_{O_2} = y_{N_2} = 0.5$



For the pure components

	10 bar, 220K		60 bar, 400K	
	h (kJ/kg)	s (kJ/kg·K)	h (kJ/kg)	s (kJ/kg·K)
O_2	195.6	5.521	358.2	5.601
N_2	224.1	5.826	409.8	5.911

ASSUMPTIONS: 1. The control volume shown is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$, kinetic & potential energy effects can be ignored. (3) The mixture adheres to the ideal solution model.

ANALYSIS: Reducing mass, energy, and entropy balances, we get

$$\dot{W}_{cv} = \dot{m}(h_1 - h_2) = \frac{\dot{m}}{M_{mix}}\left[\bar{h}_1 - \bar{h}_2\right], \quad \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1) = \frac{\dot{m}}{M_{mix}}(\bar{s}_2 - \bar{s}_1)$$

① where $M_{mix} = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 30.01$.

(a) With Eq. 11.138,

$$\dot{W}_{cv} = \frac{\dot{m}}{M_{mix}}\left[y_{O_2}(h_1 - h_2)_{O_2} M_{O_2} + y_{N_2}(h_1 - h_2)_{N_2} M_{N_2}\right]$$

$$= \left(\frac{1 \text{ kg/s}}{30.01 \text{ kg/kmol}}\right)\left[(0.5)\left[\underbrace{(195.6 - 358.2)}_{-162.6}(32) + \underbrace{(224.1 - 409.8)}_{-185.7}(28.01)\right]\right]\frac{\text{kJ}}{\text{kmol}}\left|\frac{1 \text{ kW}}{1 \text{ kJ/s}}\right|$$

$$= 173.4 \text{ kW} \qquad \longleftarrow \dot{W}_{cv}$$

(b) Similarly

$$② \quad \dot{\sigma}_{cv} = \left(\frac{1}{30.01}\right)\left[(0.5)\left\{\underbrace{(5.601 - 5.521)}_{0.08}(32) + \underbrace{(5.911 - 5.826)}_{0.085}(28.01)\right]\right]$$

$$= 0.082 \frac{\text{kW}}{\text{K}} \qquad \longleftarrow \dot{\sigma}_{cv}$$

---

1. Using Eq. 12.9 of Sec. 12.1

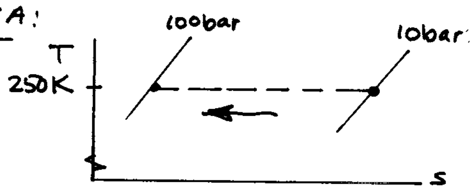
2. Since composition remains constant, this approach suffices. For the special case of ideal gas mixture processes at constant composition, see the discussion of Sec. 12.4.1 leading to Eq. 12.36.

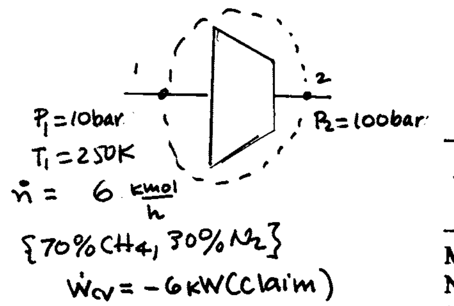
PROBLEM 11.127

KNOWN: A gaseous mixture with the molar analysis {70% CH_4, 30% N_2} enters a compressor operating at steady state and exits at a higher pressure. During compression the temperature of the mixture departs by no more than 0.1 K from a temperature of 250 K. The power required is claimed to be 6 kW.

FIND: Determine if the claimed power requirement can be correct.

SCHEMATIC & GIVEN DATA:





At 250 K	h (kJ/kg) 10 bar	h (kJ/kg) 100 bar	s (kJ/kg·K) 10 bar	s (kJ/kg·K) 100 bar
Methane	506.0	358.6	10.003	8.3716
Nitrogen	256.18	229.68	5.962	5.188

ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) Kinetic and potential energy changes can be ignored. (3) The mixture is modeled as an ideal solution. (4) The temperature at which heat transfer takes place is 250K.

ANALYSIS: Reducing mass, energy, and entropy balances

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{n}(\bar{h}_1 - \bar{h}_2), \quad 0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{n}(\bar{s}_1 - \bar{s}_2) + \dot{\sigma}_{cv}$$

Combining these expressions, we get

$$(-\dot{W}_{cv}) = \dot{n}\left[(\bar{h}_2 - \bar{h}_1) - T_b(\bar{s}_2 - \bar{s}_1)\right] + T_b\dot{\sigma}_{cv}$$

Since states 1 and 2 are fixed, the minimum theoretical power input corresponds to $\dot{\sigma}_{cv} = 0$: $(-\dot{W}_{cv})_{min} = \dot{n}[(\bar{h}_2 - \bar{h}_1) - T_b(\bar{s}_2 - \bar{s}_1)]$, where T_b is the temperature at which heat transfer occurs. Since the mixture is modeled as an ideal solution, use of Eq. 11.138 gives

$$(\bar{h}_2 - \bar{h}_1) = y_{CH_4}[\bar{h}_2 - \bar{h}_1]_{CH_4} + y_{N_2}[\bar{h}_2 - \bar{h}_1]_{N_2}$$
$$= (0.7)[358.6 - 506.0](16.04) + (0.3)[229.68 - 256.18](28.01) = -1877.7 \frac{kJ}{kmol}$$

Similarly,

① $$(\bar{s}_2 - \bar{s}_1) = y_{CH_4}[\bar{s}_2 - \bar{s}_1]_{CH_4} + y_{N_2}[\bar{s}_2 - \bar{s}_1]_{N_2}$$
$$= (0.7)[8.3716 - 10.003](16.04) + (0.3)[5.188 - 5.962](28.01) = -24.82 \frac{kJ}{kmol \cdot K}$$

Collecting results

$$(-\dot{W}_{cv})_{min} = \left(6 \frac{kmol}{h}\right)\left|\frac{1h}{3600s}\right|\left[-1877.7 - 250(-24.82)\right]\left(\frac{kJ}{kmol \cdot K}\right)\left|\frac{kW}{kJ/s}\right|$$
$$= 7.2 kW$$

Since the claimed value for $(-\dot{W}_{cv})$ is less than the minimum theoretical value, the claim cannot be valid. ◄

---

1. Since composition remains constant, this approach suffices. For the special case of ideal gas mixture processes at constant composition, see the discussion of Sec. 12.4.1 leading to Eq. 12.36

PROBLEM 11.128

Known: The departure of a solution from ideal behavior is represented by the activity coefficient $\gamma_i = a_i/y_i$, or with Eq. 11.140

$$\gamma_i = \frac{\bar{f}_i}{y_i f_i^{\circ}}$$

Find: Using the Gibbs - Duhem equation, obtain the following expression for a binary solution, $i = 1, 2$, relating the activity coefficients and the composition at temperature T and pressure p

$$\left[y_1 \frac{d \ln \gamma_1}{d y_1} \right]_{p,T} = \left[y_2 \frac{d \ln \gamma_2}{d y_2} \right]_{p,T}$$

How might this expression be useful?

Analysis:

The Gibbs - Duhem equation is given by

$$\sum_{i=1}^{j} n_i \, d\mu_i = V \, dp - S \, dT$$

where $j = 2$ for a binary mixture.

Considering T and p to be the independent thermodynamic properties,

$$d\mu_i = \left. \frac{d\mu_i}{d y_i} \right|_{T,p,y_\ell} dy_i + \left. \frac{d\mu_i}{dT} \right|_{p,y} dT + \left. \frac{d\mu_i}{dp} \right|_{T,y} dp$$

For p = constant and T = constant, $dT = dp = 0$

$$d\mu_i = \left. \frac{d\mu_i}{d y_i} \right|_{T,p,y_\ell} dy_i \qquad (1)$$

Also, for dp = dT = 0, the Gibbs-Duhem equation reduces to

$$\sum_{i=1}^{2} n_i \, d\mu_i = 0 \qquad (2)$$

for a binary mixture.

Also for a binary mixture,

$$y_1 + y_2 = 1$$

$$y_1 = 1 - y_2$$

$$\Rightarrow dy_1 = -dy_2 \qquad (3)$$

The difference in the chemical potential of i between a specified state of the mixture and the reference state is given by Eq. 11.139,

$$\mu_i - \mu_i^\circ = RT \ln\left(\frac{\bar{f}_i}{f_i^\circ}\right)$$

Inserting the activity coefficient and rearranging

$$\mu_i = \mu_i^\circ + RT \ln\left(\gamma_i y_i\right)$$

Performing the differentiation indicated in (1),

$$\frac{d\mu_i}{dy_i} = RT \left.\frac{d \ln \gamma_i}{dy_i}\right|_{T,p} + \frac{RT}{y_i}$$

$$\Rightarrow d\mu_i = RT\left[dy_i \left.\frac{d \ln \gamma_i}{dy_i}\right|_{T,p} + \frac{dy_i}{y_i}\right]$$

Substituting this result into (2)

$$n_1 RT\left[dy_1 \left.\frac{d \ln \gamma_1}{dy_1}\right|_{T,p} + \frac{dy_1}{y_1}\right] + n_2 RT\left[dy_2 \left.\frac{d \ln \gamma_2}{dy_2}\right|_{T,p} + \frac{dy_2}{y_2}\right] = 0$$

Recalling that $dy_2 = -dy_1$,

$$n_1 RT\left[dy_1 \left.\frac{\partial \ln \gamma_1}{\partial y_1}\right|_{T,p} + \frac{dy_1}{y_1}\right] + n_2 RT\left[-dy_1 \left.\frac{\partial \ln \gamma_2}{\partial y_2}\right|_{T,p} - \frac{dy_1}{y_2}\right] = 0$$

$$n_1\left[dy_1 \left.\frac{\partial \ln \gamma_1}{\partial y_1}\right|_{T,p} + \frac{dy_1}{y_1}\right] = n_2\left[dy_1 \left.\frac{\partial \ln \gamma_2}{\partial y_2}\right|_{T,p} + \frac{dy_1}{y_2}\right]$$

$$n_1\left[\left.\frac{\partial \ln \gamma_1}{\partial y_1}\right|_{T,p} + \frac{1}{y_1}\right] = n_2\left[\left.\frac{\partial \ln \gamma_2}{\partial y_2}\right|_{T,p} + \frac{1}{y_2}\right]$$

$$y_i = \frac{n_i}{n} \Rightarrow \frac{n_i}{y_i} = n$$

$$\Rightarrow n_1\left[\frac{\partial \ln \gamma_1}{\partial y_1}\right]_{T,p} + n = n_2\left[\frac{\partial \ln \gamma_2}{\partial y_2}\right]_{T,p} + n$$

$$n_1\left[\frac{\partial \ln \gamma_1}{\partial y_1}\right]_{T,p} = n_2\left[\frac{\partial \ln \gamma_2}{\partial y_2}\right]_{T,p}$$

Dividing through by n gives the desired relation,

$$y_1\left[\frac{\partial \ln \gamma_1}{\partial y_1}\right]_{T,p} = y_2\left[\frac{\partial \ln \gamma_2}{\partial y_2}\right]_{T,p}$$

This expression is of particular value in minimizing the number of experimental data necessary to evaluate the properties of a system and for detecting inconsistent or erroneous measurements.

CHAPTER TWELVE

IDEAL GAS MIXTURE AND PSYCHROMETRIC APPLICATIONS

PROBLEM 12.1

KNOWN: The problem concerns a mixture of two gases.

FIND: Determine (a) when the analysis in terms of mass fractions would be identical to the analysis in terms of mole fractions, (b) when the apparent molecular weight of the mixture would equal the average of the molecular weights of the gases.

ANALYSIS: (a) Using Eq. 12.3

$$mf1 = \frac{m_1}{m} \quad , \quad mf2 = \frac{m_2}{m}$$

and Eqs. 12.1, 6

$$y_1 = \frac{n_1}{n} = \frac{m_1/M_1}{\left[\frac{m_1}{M_1} + \frac{m_2}{M_2}\right]} = \frac{m_1}{m}\left[\frac{m/M_1}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}\right]$$

$$\Rightarrow \quad y_1 = mf_1 \left[\frac{\frac{m_1+m_2}{M_1}}{\frac{m_1}{M_1} + \frac{m_2}{M_2}}\right]$$

For $y_1 = mf_1$, the term in brackets must equal unity. That is,

$$\frac{m_1+m_2}{M_1} = \frac{m_1}{M_1} + \frac{m_2}{M_2} \Rightarrow \frac{m_2}{M_1} = \frac{m_2}{M_2} \Rightarrow M_1 = M_2 \longleftarrow$$

Accordingly, for $y_1 = mf1$, $y_2 = mf2$, it is necessary for $M_1 = M_2$.

(b) Using Eq. 12.9, $M = y_1 M_1 + y_2 M_2$. Then, since $1 = y_1 + y_2$

$$M = y_1 M_1 + (1-y_1) M_2 \quad \Rightarrow \quad M = y_1(M_1 - M_2) + M_2$$

If $M = (M_1+M_2)/2$, the last expression becomes

$$\frac{M_1+M_2}{2} = y_1(M_1 - M_2) + M_2$$

or

$$y_1(M_1-M_2) = \frac{M_1+M_2}{2} - M_2$$

$$= \frac{M_1 - M_2}{2}$$

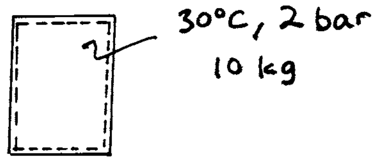
Accordingly, $y_1 = 1/2$. In sum, for $M = (M_1+M_2)/2$, it is necessary for the mole fractions to be equal : $y_1 = y_2 = 1/2$. $\longleftarrow$

PROBLEM 12.2*

KNOWN: The molar analysis of a gas mixture is specified.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, and the volume occupied by 10 kg of mixture.

SCHEMATIC & GIVEN DATA:



i	y_i
N_2	0.4
CO_2	0.5
CH_4	0.1

ASSUMPTIONS: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1 kmol of mixture in part (a).

ANALYSIS (a) Considering a typical 1 kmol of mixture

①

i	n_i	M_i	$m_i = n_i \cdot M_i$	mf_i
N_2	0.4	28.01	11.204	0.3218
CO_2	0.5	44.01	22.005	0.6321
CH_4	0.1	16.04	1.604	0.0461
			34.813 $\frac{kg}{kmol}$	1.0000

← (a)

(b) With Eq. 12.12

$$p_{N_2} = y_{N_2}\, p = (0.4)(2\,\text{bar}) = 0.8\ \text{bar}$$
$$p_{CO_2} = y_{CO_2}\, p = (0.5)(2\,\text{bar}) = 1.0\ \text{bar}$$
$$p_{CH_4} = y_{CH_4}\, p = (0.1)(2\,\text{bar}) = 0.2\ \text{bar}$$

← (b)

(c) With the ideal gas equation of state applied to the overall mixture.

$$V = \frac{m\,(\bar{R}/M)\,T}{p}$$
$$= \frac{(10\ \text{kg})\left(\frac{8314}{34.813}\ \frac{N\cdot m}{kg\cdot K}\right)(303\,K)}{2\times 10^5\ N/m^2} = 3.618\ m^3$$

← (c)

---

1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used: $M = \sum y_i M_i$

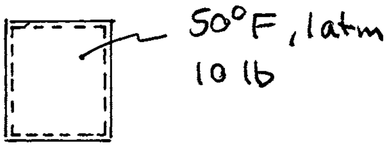
$$M = y_{N_2} M_{N_2} + y_{CO_2} M_{CO_2} + y_{CH_4} M_{CH_4}$$
$$= (0.4)(28.01) + (0.5)(44.01) + (0.1)(16.04) = 34.813\ \frac{kg}{kmol}$$

PROBLEM 12.3*

KNOWN: The molar analysis of a gas mixture is specified.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, the volume occupied by 10 lb of mixture.

SCHEMATIC & GIVEN DATA:



i	y_i
Ar	0.2
CO_2	0.35
O_2	0.45

ASSUMPTIONS: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1 lbmol of mixture in part(a).

ANALYSIS: (a) Considering a typical 1 lbmol of mixture

①

i	n_i	M_i	$m_i = n_i M_i$	mf_i
Ar	0.2	39.94	7.988	0.2114
CO_2	0.35	44.01	15.404	0.4076
O_2	0.45	32.00	14.400	0.3810
			37.792 $\frac{lb}{lbmol}$	

← (a)

(b)

$$P_{Ar} = y_{Ar} P = (0.2)(14.7) = 2.940 \text{ lbf/in}^2$$

$$P_{CO_2} = y_{CO_2} P = (0.35)(14.7) = 5.145 \text{ lbf/in}^2$$

$$P_{O_2} = y_{O_2} P = (0.45)(14.7) = 6.615 \text{ lbf/in}^2$$

← (b)

(c) Using the ideal gas equation of state

$$V = \frac{m\,\bar{R}/M\,T}{p} = \frac{(10\text{ lb})\left(\frac{1545}{37.792}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(510^\circ\text{R})}{(14.7 \times 144\ \text{lbf/ft}^2)}$$

$$= 98.50 \text{ ft}^3$$

← (c)

---

1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used: $M = \sum y_i M_i$

$$M = y_{Ar} M_{Ar} + y_{CO_2} M_{CO_2} + y_{O_2} M_{O_2}$$

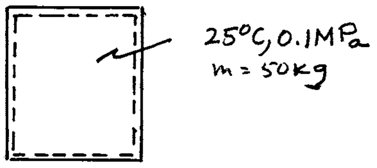
$$= (0.2)(39.94) + (0.35)(44.01) + (0.45)(32.00) = 37.792 \frac{\text{lb}}{\text{lbmol}}$$

PROBLEM 12.4

KNOWN: The molar analysis of a gas mixture is specified.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, the volume occupied by 50kg of mixture.

SCHEMATIC & GIVEN DATA:



i	y_i
N_2	0.6
CO_2	0.3
O_2	0.1

ASSUMPTIONS: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1kmol of mixture in part (a).

ANALYSIS: (a) Considering a typical 1 kmol of mixture

①

i	n_i	M_i	$m_i = n_i M_i$	mf_i
N_2	0.6	28.01	16.806	0.5061
CO_2	0.3	44.01	13.203	0.3976
O_2	0.1	32.00	3.200	0.0964
			33.209 $\frac{kg}{kmol}$	1.000

(b)

$$p_{N_2} = y_{N_2}\, p = 0.6\,(0.1\,MPa) = 0.06\ MPa$$

$$p_{CO_2} = y_{CO_2}\, p = 0.3\,(0.1\,MPa) = 0.03\ MPa$$

$$p_{O_2} = y_{O_2}\, p = 0.1\,(0.1\,MPa) = 0.01\ MPa$$

(c) Using the ideal gas equation of state

$$V = \frac{mRT}{p} = \frac{(50\,kg)\left(\frac{8314}{33.209}\ \frac{N\cdot m}{kg\cdot K}\right)(298K)}{(0.1\times 10^{6}\ N/m^2)}$$

$$= 37.3\ m^3$$

---

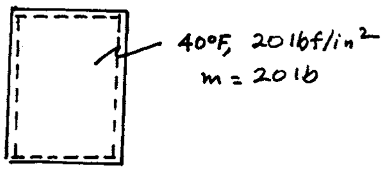
1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used: $M = \sum y_i M_i$.

PROBLEM 12.5

KNOWN: An analysis on a mass basis is specified for a gas mixture.

FIND: Determine the analysis in terms of mole fractions, the partial pressure of each component, the volume occupied by 20 lb of mixture.

SCHEMATIC & GIVEN DATA:



i	mf_i
CO_2	0.6
CO	0.25
O_2	0.15

ASSUMPTIONS: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) Calculations are based on 1 lb of mixture in part (a).

ANALYSIS: (a) Considering a typical 1 lb of mixture.

①

i	m_i	M_i	$n_i = m_i/M_i$	y_i
CO_2	0.6	44.01	0.01363	0.5002
CO	0.25	28.01	0.00893	0.3277
O_2	0.15	32.00	0.00469	0.1721
			0.02725	

← (a)

$$M = \frac{m}{n} = \frac{1}{0.02725} = 36.697$$

(b)

$$p_{CO_2} = y_{CO_2}\, p = (.5002)(20) = 10.004 \text{ lbf/in}^2$$

$$p_{CO} = y_{CO}\, p = (.3277)(20) = 6.554 \text{ lbf/in}^2$$

$$p_{O_2} = y_{O_2}\, p = (.1721)(20) = 3.442 \text{ lbf/in}^2$$

← (b)

(c) Using the ideal gas equation of state

$$V = \frac{m\,R\,T}{p} = \frac{(20\text{ lb})\left(\frac{1545}{36.697}\ \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(500°\text{R})}{(20 \times 144 \text{ lbf/ft}^2)} = 146.2 \text{ ft}^3 \quad \leftarrow \text{(c)}$$

---

1. The apparent molecular weight of the mixture is obtained in the calculations of part (a). Equivalently, Eq. 12.9 can be used:

$$M = y_{CO_2} M_{CO_2} + y_{CO} M_{CO} + y_{O_2} M_{O_2}$$

$$= (.5002)(44.01) + (.3277)(28.01) + (.1721)(32)$$

$$= 36.7$$

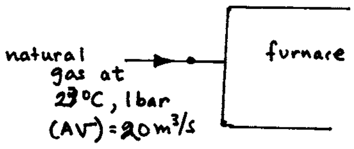
The difference between this value and the one obtained above is due to rounoff.

PROBLEM 12.6

KNOWN: An analysis on a molar basis is specified for a natural gas.

FIND: Determine the analysis in terms of mass fractions, the partial pressure of each component, the mass flow rate for a volumetric flow rate of 20 m³/s.

SCHEMATIC & GIVEN DATA:



i	y_i
C_3H_8	0.4
C_2H_6	0.4
CH_4	0.2

ASSUMPTIONS: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the full volume of the mixture at the mixture temperature. (3) In part (a), calculations are based on 1 kmol of mixture.

ANALYSIS: (a) Considering a typical 1 kmol of mixture

i	n_i	M_i	$m_i = n_i M_i$	mf_i
C_3H_8	0.4	44.09	17.636	0.5365
C_2H_6	0.4	30.07	12.028	0.3659
CH_4	0.2	16.04	3.208	0.0976
			32.872	1.0000

← (a)

(b)

$p_{C_3H_8} = y_{C_3H_8}\, p = 0.4\,(1\,\text{bar}) = 0.4\ \text{bar}$

$p_{C_2H_6} = y_{C_2H_6}\, p = 0.4\,(1\,\text{bar}) = 0.4\ \text{bar}$ ← (b)

$p_{CH_4} = y_{CH_4}\, p = 0.2\,(1\,\text{bar}) = 0.2\ \text{bar}$

(c) Using the ideal gas equation of state, $v = RT/p$

$$\dot{m} = \frac{(AV)}{v}$$

$$= \frac{(AV)\,p}{(\bar{R}/M)\,T} = \frac{(20\,m^3/s)(10^5\,N/m^2)}{\left(\frac{8314}{32.872}\,\frac{N\cdot m}{kg\cdot K}\right)(296\,K)}$$

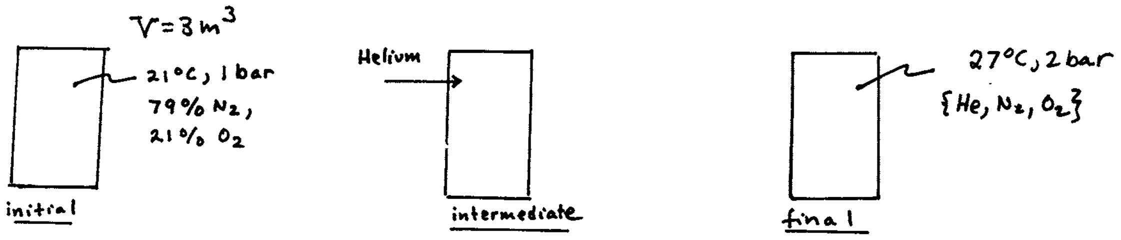
$$= 26.71\ \frac{kg}{s}$$ ← $\dot{m}$

PROBLEM 12.7

KNOWN: A rigid vessel whose volume is 3 m^3, initially contains a mixture of N_2 and O_2 at a specified pressure, temperature, and molar analysis. Helium is allowed to flow in until the pressure attains a specified value.

FIND: If the final temperature is 21°C, determine the mass of each component present.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS: Using the ideal gas equation of state, $V = n\bar{R}T/p$. Since the total volume is constant: $V_1 = V_2$

$$\frac{n_1 \bar{R} T_1}{p_1} = \frac{n_2 \bar{R} T_2}{p_2} \qquad (1)$$

where 1 denotes the initial total number of moles present, temperature, and pressure, and 2 denotes the final total number of moles present, temperature, and pressure. From Eq. 1

$$n_2 = \frac{T_1}{T_2}\frac{p_2}{p_1} n_1 \qquad (2)$$

Initially

$$n_1 = \frac{p_1 V}{\bar{R} T_1} = \frac{(10^5 N/m^2)(3 m^3)}{\left(8314 \frac{N\cdot m}{kmol\cdot K}\right)(294 K)} = 0.1227 \text{ kmol (mix)}$$

The amounts of N_2 and O_2 present initially and finally are

$$n_{O_2} = y_{O_2} n_1 = (0.21)(0.1227) = 0.0258 \text{ kmol} (O_2)$$
$$n_{N_2} = y_{N_2} n_1 = (0.79)(0.1227) = 0.0969 \text{ kmol} (N_2)$$

And with Eq. (2) the total amount of mixture finally is

$$n_2 = \left(\frac{294 K}{300 K}\right)\left(\frac{2 bar}{1 bar}\right)(0.1227 \text{ kmol}) = 0.2405 \text{ kmol (mix)}$$

Then, since $n_2 = \underbrace{n_{O_2} + n_{N_2}}_{n_1} + n_{He}$,

$$n_{He} = 0.2405 - \underbrace{0.0258 - 0.0969}_{n_1} = 0.1178 \text{ kmo (He)}$$

Finally, with molecular weights from Table A-1 we get

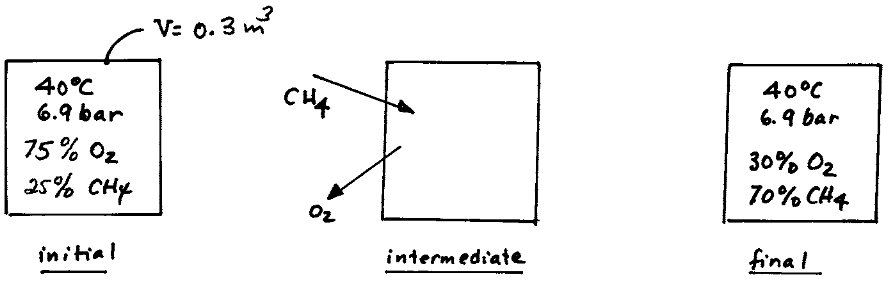
$$m_{O_2} = (32)(0.0258) = 0.8256 \text{ kg} (O_2)$$
$$m_{N_2} = (28.01)(0.0969) = 2.7142 \text{ kg} (N_2)$$
$$m_{He} = (4.003)(0.1178) = 0.4716 \text{ kg (He)}$$

PROBLEM 12.8

KNOWN: A vessel of known volume contains a mixture of O_2, CH_4 at a specified temperature, pressure, and molar analysis.

FIND: Determine the mass of CH_4 that would have to be added, and the mass of O_2 that would have to be removed, to produce a mixture at the same temperature and pressure, but with a different specified molar analysis.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS: As the initial and final mixtures are at the same temperature and pressure, and occupy the same total volume, the total number of moles present initially and finally is the same. Using the ideal gas equation of state

$$n = \frac{pV}{\bar{R}T} = \frac{(6.9\times10^5\ N/m^2)(0.3\ m^3)}{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(313\ K)} = 0.0795\ kmol(mixture)$$

Thus, the changes in the number of moles of O_2 and CH_4 present are

$$\Delta n\big]_{O_2} = [(y_{O_2})_{final} - (y_{O_2})_{initial}]\, n = [0.3 - 0.75](0.0795) = -0.0358\ kmol$$

$$\Delta n\big]_{CH_4} = [(y_{CH_4})_{final} - (y_{CH_4})_{initial}]\, n = [0.70 - 0.25](0.0795) = +0.0358$$

Then, with molecular weights from Table A-1, $M_{O_2} = 32$, $M_{CH_4} = 16.04$

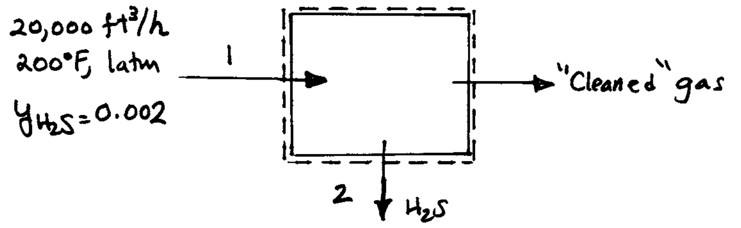
$$\Delta m\big]_{O_2} = -1.1456\ kg \quad , \quad \Delta m\big]_{CH_4} = +0.5742\ kg \quad \longleftarrow$$

PROBLEM 12.9

KNOWN: A flue gas containing H_2S enters a scrubber at a specified state and volumetric flow rate. The scrubber removes 92% (molar basis) of the H_2S.

FIND: Determine the rate H_2S is removed.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The control volume shown in the figure is at steady state.

ANALYSIS: The molar flow rate of the entering mixture, $\dot{n}$, can be found as follows:

$$\dot{n}_1 = \frac{(AV)_1}{\bar{v}_1} = \frac{(AV)_1 p_1}{\bar{R}T_1} = \frac{(20{,}000\ \text{ft}^3/\text{h})(14.7\times 144\ \text{lbf/ft}^2)}{(1545\ \text{ft}\cdot\text{lbf/lbmol}\cdot{}^\circ\text{R})(660^\circ\text{R})}$$

$$= 41.518\ \frac{\text{lbmol (mixture)}}{\text{h}}$$

The rate H_2S enters is then

$$\dot{n}_{H_2S,1} = (0.002)(41.518) = 0.083\ \frac{\text{lbmol} (H_2S)}{\text{h}}$$

If 92% is removed by the scrubber

$$\dot{n}_{H_2S,2} = (0.92)(0.083) = 0.0764\ \frac{\text{lbmol} (H_2S)}{\text{h}}$$

Then with M = 34.08 for H_2S

$$\dot{m}_{H_2S,2} = (0.0764)(34.08) = 2.6\ \frac{\text{lb} (H_2S)}{\text{h}} \longleftarrow$$

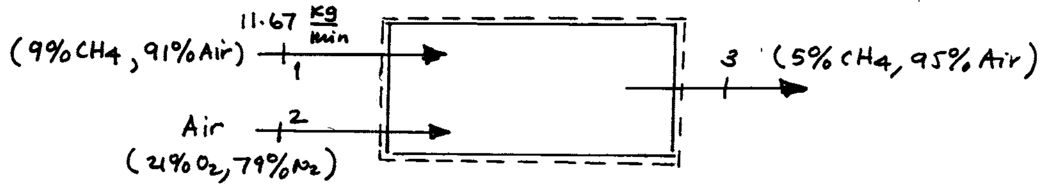
H_2S should be removed because it can combine with water to form a corrosive product. Acid rain can be traced to the presence of sulfur in combustion gases.

PROBLEM 12.10

KNOWN: A control volume at steady state has two entering streams and a single exiting stream. The molar analysis of each entering stream is provided. The mole fraction of CH_4 in the exiting stream is also provided.

FIND: Determine (a) the molar flow rate of the entering air stream, (b) the mass flow rate of O_2 in the exiting stream.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) Air has the molar analysis: 21% O_2, 79% N_2.

ANALYSIS: (a) For the control volume, a mass rate balance reads $\dot m_1 + \dot m_2 = \dot m_3$, or $\dot m_2 = \dot m_3 - \dot m_1$. Also, $\dot m = M\dot n$. So, $\dot n_2 = \frac{\dot m_3 - \dot m_1}{M_2}$. (1)

where $M_2 = (0.21)(32) + (0.79)(28.01) = 28.848$.

Since the rate that CH_4 enters at 1 equals the rate CH_4 exits at 3, the relationship between the molar flow rates at 1 and 3 is, $0.09\dot n_1 = 0.05\dot n_3$. Or, with $\dot n = \dot m/M$

$$0.09\frac{\dot m_1}{M_1} = 0.05\frac{\dot m_3}{M_3} \Rightarrow \dot m_3 = \frac{0.09}{0.05}\frac{M_3}{M_1}\dot m_1 \qquad (2)$$

where

$$M_1 = 0.09(16.04) + 0.91[28.848] = 27.696$$
$$M_3 = 0.05(16.04) + 0.95[28.848] = 28.208$$

Then, Eq. (2) gives

$$\dot m_3 = \left(\frac{0.09}{0.05}\right)\left(\frac{28.208}{27.696}\right)(11.67) = 21.39\ \frac{kg}{min}$$

Then, Eq. (1) gives

$$\dot n_2 = \frac{21.39 - 11.67}{28.848} = 0.337\ \frac{kmol(air)}{min} \blacktriangleleft$$

(b) The molar flow rate of O_2 in the exiting stream is

$$\dot n_{O_2} = y_{O_2}\dot n_3$$

① where y_{O_2} is the mole fraction of O_2 in the exiting mixture and $\dot n_3$ is the molar flow rate of the mixture. With $\dot m_{O_2} = \dot n_{O_2} M_{O_2}$, this becomes

$$\dot m_{O_2} = M_{O_2}(y_{O_2}\dot n_3) = M_{O_2}\, y_{O_2}\left(\frac{\dot m_3}{M_3}\right)$$
$$= (32)(.1995)\left(\frac{21.39}{28.208}\right) = 4.84\ \frac{kg(O_2)}{min} \blacktriangleleft$$

---

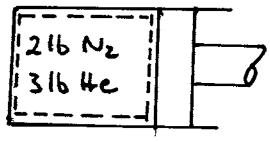
1. Note that the exiting mixture is $(5\%\ CH_4, 95\%\ Air) = (5\%\ CH_4, \underbrace{(0.95)(21\%\ O_2)}_{19.95\%}, (.95)(79\%\ N_2))$

PROBLEM 12.11*

KNOWN: A gas mixture of 2 lb of N_2 and 3 lb of He is maintained at a constant pressure.

FIND: Determine the mixture analysis in terms of mass fractions and mole fractions, the heat transfer required to increase the mixture temperature from 70 to 150 °F, and the change in entropy of the mixture for this process.

SCHEMATIC & GIVEN DATA:



$T_1 = 70$ °F
$T_2 = 150$ °F
$p =$ constant

ASSUMPTIONS: (1) For the system shown in the accompanying figure, changes in kinetic and potential energy are negligible. (2) The overall mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. (3) The specific heats of the gases are constants.

ANALTSIS: (a) The analysis of the mixture in terms of mass fractions is determined with $(mf)_i = m_i/m$

$$(mf)_{N_2} = \frac{2}{2+3} = 0.40 \quad , \quad (mf)_{He} = \frac{3}{2+3} = 0.6 \qquad \longleftarrow mf$$

(b) The molar analysis of the mixture is determined with $n_i = m_i/M_i$

$$\left.\begin{aligned} n_{N_2} &= \frac{2}{28.01} = 0.0714 \text{ lbmol} \\ n_{He} &= \frac{3}{4.003} = 0.7494 \text{ lbmol} \end{aligned}\right\} \Rightarrow \begin{aligned} y_{N_2} &= \frac{0.0714}{0.0714+0.7494} = 0.0870 \\ y_{He} &= \frac{0.7494}{0.0714+0.7494} = 0.9130 \end{aligned} \qquad \longleftarrow y$$

(c) With assumption 1 an energy balance reduces to $\Delta U = Q - W$ where $W = \int p\,dV = p\Delta V$. Thus,

$$Q = \Delta U + p\Delta V = (U_2 - U_1) + p(V_2 - V_1) = (U_2 + pV_2) - (U_1 + pV_1) = \Delta H$$

The change in enthalpy of the mixture is the sum of the enthalpy changes of the components:

$$\Delta H = m_{N_2}\,\Delta h_{N_2} + m_{He}\,\Delta h_{He}$$

with assumption 3, $\Delta h_{N_2} = c_{p,N_2}(T_2 - T_1)$ and $\Delta h_{He} = c_{p,He}(T_2 - T_1)$. Thus, with c_p data for N_2 from Table A-20E and for He, according to Table A-21E, $c_p = (2.5)\left(\frac{1.986}{4.003}\right) = 1.240$ Btu/lb·°R

$$Q = [m_{N_2}c_{p,N_2} + m_{He}c_{p,He}](T_2 - T_1) = \left[(2\text{ lb})\left(0.248\ \tfrac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right) + (3\text{ lb})\left(1.240\ \tfrac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\right](80\ °\text{R})$$

$$= 337.3 \text{ Btu} \qquad \longleftarrow Q$$

(d) Since pressure remains constant, the entropy change for the mixture is

$$\Delta S = m\left[c_p \ln\frac{T_2}{T_1} - R\ln\overset{0}{\cancel{\frac{p_2}{p_1}}}\right] = m c_p \ln T_2/T_1$$

where $c_p = (m_{N_2}c_{p,N_2} + m_{He}c_{p,He})/m = [(2)(.248)+(3)(1.240)]/5 = 0.8432$ Btu/lb°R

Thus

$$\Delta S = (5\text{ lb})\left(0.8432\ \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}\right)\ln\frac{610}{530} = 0.593\ \frac{\text{Btu}}{°\text{R}} \qquad \longleftarrow \Delta S$$

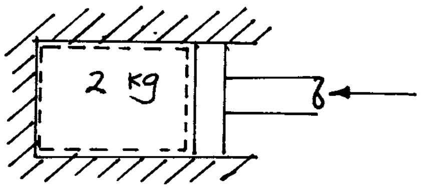
PROBLEM 12.12*

KNOWN: A mixture having a specified analysis on a mass basis is compressed adiabatically between specified states.

FIND: Determine the work and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:

$mf_{N_2} = 0.3$
$mf_{CO_2} = 0.4$
$mf_{O_2} = 0.3$



$P_1 = 1$ bar, $T_1 = 300$ K
$P_2 = 4$ bar, $T_2 = 500$ K

ASSUMPTIONS: (1) For the system shown in the accompanying figure, $Q = 0$ and changes in kinetic and potential energy are negligible. (2) The overall mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS: An energy balance reduces with assumption 1 to give $\Delta U = \cancel{Q}^{0} - W$. The internal energy change of the mixture, ΔU, equals the sum of the internal energy changes of the components. Thus, on a unit mass of mixture basis

$$-\frac{W}{m} = (mf)_{N_2}\,\Delta u_{N_2} + (mf)_{CO_2}\,\Delta u_{CO_2} + (mf)_{O_2}\,\Delta u_{O_2}$$

With data from the ideal gas tables and molecular weights from Table A-1

$$-\frac{W}{m} = 0.3\left[\frac{10423-6229}{28.01}\right] + 0.4\left[\frac{13521-6939}{44.01}\right] + 0.3\left[\frac{10614-6242}{32}\right]$$

Thus

$$\frac{W}{m} = -(44.92 + 59.82 + 40.99) = -145.73\ \text{kJ/kg}$$

Then, with m = 2 kg

$$W = -291.46\ \text{kJ} \longleftarrow W$$

In this calculation, the relationship $\Delta u = \Delta\bar{u}/M$ is used for each of the three components.

An entropy rate balance reduces as follows

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma \quad \Rightarrow \quad \sigma = \Delta S$$

The entropy change of the mixture, ΔS, equals the sum of the entropy changes of the components:

$$\Delta S = (\Delta S)_{N_2} + (\Delta S)_{CO_2} + (\Delta S)_{O_2} \Rightarrow \frac{\Delta S}{m} = (mf)_{N_2}\left(\frac{\Delta\bar{s}}{M}\right)_{N_2} + (mf)_{CO_2}\left(\frac{\Delta\bar{s}}{M}\right)_{CO_2} + (mf)_{O_2}\left(\frac{\Delta\bar{s}}{M}\right)_{O_2}$$

where each of the $\Delta\bar{s}$ terms is determined using Eq. 12.36. Thus, with $\bar{s}^\circ$ data

$$\frac{\sigma}{m} = 0.3\left[\frac{206.630 - 191.682 - 8.314\ln 4/1}{28.01}\right] + 0.4\left[\frac{234.814 - 213.915 - 8.314\ln 4/1}{44.01}\right] + 0.3\left[\frac{220.589 - 205.213 - 8.314\ln 4/1}{32}\right]$$

$$= (0.03665 + 0.08519 + 0.03610)\ \frac{\text{kJ/K}}{\text{kg}}$$

$$= 0.15794\ \frac{\text{kJ/K}}{\text{kg}}$$

or

$$\sigma = 0.3159\ \frac{\text{kJ}}{\text{K}} \longleftarrow \sigma$$

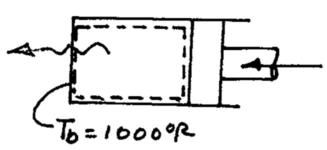
PROBLEM 12.13

KNOWN: A specified mixture is compressed isothermally.

FIND: Determine (a) the work, (b) the heat transfer, and (c) the amount of entropy produced, for each of two choices for the boundary.

SCHEMATIC & GIVEN DATA:

System #1



$n_{N_2} = 0.6$ lbmol

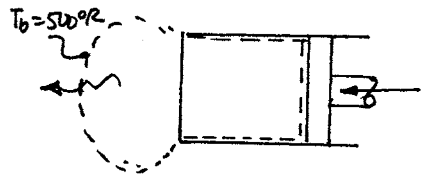
$n_{O_2} = 0.4$ lbmol

$T = 1000°R$

$p_1 = 1 atm$

$p_2 = 3 atm$

System #2



ASSUMPTIONS: (1) For the system shown in the accompanying figure, changes in kinetic and potential energy are negligible. (2) The overall mixture acts as an ideal gas. Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.
(3) The immediate surroundings experience no net change in state.
(4) In system #1, heat transfer occurs at $T_b = 1000°R$. In system #2, heat transfer occurs at $T_b = 500°R$.

ANALYSIS: (a), (b). With assumption (1), an energy balance reduces to give $\Delta U = Q - W$, where ΔU is the change in internal energy of the mixture. Since the overall mixture acts as an ideal gas $\Delta U = 0$ because the mixture temperature does not change. Thus, $Q = W$. The work is ← Q

$$W = \int_1^2 p\,dV = \int_1^2 \frac{n\bar{R}T}{V}\,dV = n\bar{R}T \ln\frac{V_2}{V_1}$$

With $V = nRT/p$ and $T_1 = T_2$

$$W = n\bar{R}T \ln\frac{p_1}{p_2} = \left[(0.6+0.4)\,\text{lbmol}\right]\left[1545\,\frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°R}\right](1000°R)\left(\ln\frac{1}{3}\right)\left(\frac{\text{Btu}}{778\,\text{ft}\cdot\text{lbf}}\right)$$

$$= -2181.7\ \text{Btu}$$ ← W

(c) An entropy balance for the system #1 takes the form

$$\Delta S = \frac{Q}{\underset{\nwarrow\boxed{=T}}{T_b}} + \sigma \Rightarrow \sigma = \Delta S - \frac{Q}{T} = \Delta S + n\bar{R}\ln\frac{p_2}{p_1} \qquad (1)$$

where ΔS is the sum of the entropy changes of the mixture components:

$$\Delta S = n_{N_2}\Big[\underbrace{\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1)}_{=0 \text{ since } T_2 = T_1} - \bar{R}\ln\frac{p_2}{p_1}\Big] + n_{O_2}\Big[\underbrace{\bar{s}^\circ(T_2) - \bar{s}^\circ(T_1)}_{=0 \text{ since } T_2 = T_1} - \bar{R}\ln\frac{p_2}{p_1}\Big] = -n\bar{R}\ln\frac{p_2}{p_1} \qquad (2)$$

Inserting this into Eq. (1) gives $\sigma = 0$. That is the process is internally reversible. ← σ

Taking an enlarged system that includes the mixture and enough of its immediate surroundings so that heat transfer occurs at $T_b = 500°R$, an entropy balance takes the form

$$\Delta S + \cancelto{0}{\Delta S\big]_{\substack{\text{immed.}\\\text{surr.}}}} = \frac{Q}{T_b} + \sigma$$

where σ now accounts for the entropy produced in the enlarged system and $\Delta S\big]_{\text{surr}}$ vanishes by assumption 3. Thus, with Eq. (2) and $Q = W = n\bar{R}T\ln p_1/p_2$

①
$$\sigma = \Delta S - \frac{Q}{T_b} = -n\bar{R}\ln\frac{p_2}{p_1} - \frac{Q}{T_b}$$
$$= Q\left[\frac{1}{T} - \frac{1}{T_b}\right] = -2181.7\ \text{Btu}\left[\frac{1}{1000°R} - \frac{1}{500°R}\right] = 2.182\ \frac{\text{Btu}}{°R}$$ ←

---

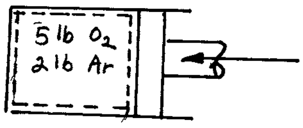
1. In this case entropy is produced in the immediate surroundings owing to heat transfer from the mixture at 1000°R to the surroundings at 500°R. This is an external irreversibility for system #1.

PROBLEM 12.14

KNOWN: A specified mixture of O_2 and Ar is compressed between two specified states.

FIND: Determine if the process can be adiabatic.

SCHEMATIC & GIVEN DATA:



$P_1 = 20$ lbf/in² $\quad$ $P_2 = 80$ lbf/in²
$T_1 = 80°F = 540°R$ $\quad$ $T_2 = 240°F = 700°R$

ASSUMPTIONS: (1) The overall mixture acts as an ideal gas. (2) Each mixture component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature.

ANALYSIS: Since W is not specified and cannot be determined from the given data, an energy balance cannot be invoked to determine if heat transfer occurs. However, if the process were adiabatic an entropy balance would give

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b} + \sigma \Rightarrow \Delta S \geq 0 \Rightarrow \left(\text{the entropy change cannot be negative}\right)$$

where ΔS is the entropy change of the mixture: the sum of the entropy changes of the components. That is

$$\Delta S = n_{O_2}(\Delta \bar{s})_{O_2} + n_{Ar}(\Delta \bar{s})_{Ar}$$

Since argon is a monatomic gas, Table A-21E indicates $\bar{c}_{p,Ar} = 5/2\,\bar{R}$, so

$$\Delta S = \underbrace{n_{O_2}}_{= \frac{m_{O_2}}{M_{O_2}}}\left[\bar{s}^\circ_{O_2}(T_2) - \bar{s}^\circ_{O_2}(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right] + \underbrace{n_{AR}}_{= \frac{m_{AR}}{M_{AR}}}\left[\bar{c}_{p,AR}\ln\frac{T_2}{T_1} - \bar{R}\ln\frac{P_2}{P_1}\right]$$

where $\bar{s}^\circ_{O_2}$ is obtained from Table A-24E. In writing this, note that the mole fractions cancel in each of the log terms: the ratio of partial pressures equals the ratio of mixture pressures.

Inserting values

$$\Delta S = \left(\frac{5}{32}\ \text{lbmol}\right)\left(50.858 - 49.021 - 1.986\ln\frac{80}{20}\right)\frac{\text{Btu}}{\text{lbmol}\cdot°R} +$$

$$\left(\frac{2}{39.94}\right)(1.986)\left[\frac{5}{2}\ln\frac{700}{540} - \ln\frac{80}{20}\right]$$

$$= (-0.1432 - 0.0733)\ \frac{\text{Btu}}{°R}$$

$$= -0.2165\ \text{Btu}/°R$$

As ΔS is negative, the process cannot be adiabatic. There must be a heat transfer from the mixture.

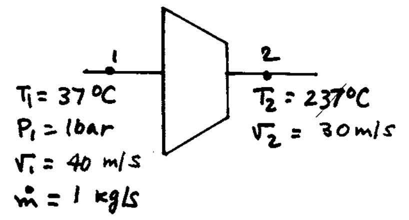
PROBLEM 12.15

KNOWN: A mixture with a specified molar analysis enters a compressor operating at steady state at a specified mass flow rate and state and exits at a specified temperature and velocity. The rate of heat transfer from the compressor to its surroundings is also specified.

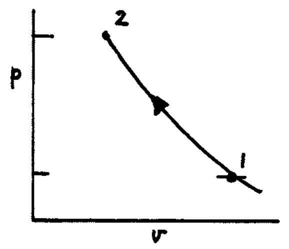
FIND: (a) Determine the power input to the compressor. (b) If the compression were polytropic, evaluate the polytropic exponent n and the exit pressure.

SCHEMATIC & GIVEN DATA:

① $\dot{Q}_{cv} = 0.5\,\dot{W}_{cv}$



i	y_i
CO_2	0.5
CO	0.333
O_2	0.167



ASSUMPTIONS: (1) The compressor operates at steady state. (2) Potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model. (4) The mixture composition remains constant.

ANALYSIS: (a) At steady state, mass and energy rate balances reduce to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = \frac{\dot{Q}_{cv}}{\dot{m}} + h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}$$

Since $\dot{Q}_{cv} = 0.5\,\dot{W}_{cv}$, this ~~becomes~~

$$0.95\,\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} \qquad (1)$$

In this equation $(h_1 - h_2)$ represents a change in the specific enthalpy of the mixture, per unit of mass of mixture. This can be evaluated as follows

$$h_1 - h_2 = \frac{\bar{h}_1 - \bar{h}_2}{M} = \frac{1}{M}\left[y_{CO_2}(\bar{h}_1 - \bar{h}_2)_{CO_2} + y_{CO}(\bar{h}_1 - \bar{h}_2)_{CO} + y_{O_2}(\bar{h}_1 - \bar{h}_2)_{O_2} \right]$$

where M is the mixture molecular weight:

$$M = y_{CO_2} M_{CO_2} + y_{CO} M_{CO} + y_{O_2} M_{O_2} = 0.5(44.01) + 0.333(28.01) + 0.167(32) = 36.68$$

With data from the ideal gas tables

$$h_1 - h_2 = \frac{1}{36.68\,\frac{kg(mix)}{kmol(mix)}}\left[0.5\,\frac{kmol(CO_2)}{kmol(mix)}(9807 - 18126)\frac{kJ}{kmol(CO_2)} + 0.333(9014 - 14898) + 0.167(9030 - 15082) \right]$$

$$= -194.37\ kJ/kg(mix)$$

Substituting values into Eq. (1)

$$0.95\,\frac{\dot{W}_{cv}}{\dot{m}} = (-194.37)\frac{kJ}{kg(mix)} + \underbrace{\frac{[(40\,m/s)^2 - (30\,m/s)^2]}{2}\left|\frac{1\,N}{1\,kg\cdot m/s^2}\right|\left|\frac{1\,kJ}{10^3\,N\cdot m}\right|}_{0.35\ kJ/kg} = -194.02\,\frac{kJ}{kg\cdot mix}$$

or

$$\frac{\dot{W}_{cv}}{\dot{m}} = -204.23\ kJ/kg(mix) \xleftarrow{\hspace{6cm}} \frac{\dot{W}_{cv}}{\dot{m}}$$

(b) If the process adheres to $pv^n = \text{constant}$, the work required would be given by Eq. 6.57a:

$$\frac{\dot{W}_{cv}}{\dot{m}} = -\frac{nR}{n-1}(T_2 - T_1) \Rightarrow \frac{n-1}{n} = \frac{R(T_2 - T_1)}{(-\dot{W}_{cv}/\dot{m})}$$

PROBLEM 12.15 (Contd.)

Inserting known values

$$\frac{n-1}{n} = \frac{(8.314/36.68)(200)}{204.23} \Rightarrow n = 1.285 \longleftarrow n$$

Then, with Eq. 3.56

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{n/(n-1)} \Rightarrow p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{n/(n-1)}$$

or, with n = 1.285

$$p_2 = (1\ \text{bar})\left(\frac{510\text{K}}{310\text{K}}\right)^{n/(n-1)} = 9.4\ \text{bar} \longleftarrow p_2$$

---

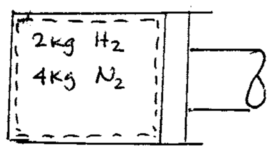
1. Since a power input is required $\dot{W}_{cv}$ takes on a negative value. Thus, in this expression $\dot{Q}_{cv}$ is negative, which is in accord with a heat loss during the compression.

PROBLEM 12.16

KNOWN: A mixture of H_2 and N_2 is compressed in a polytropic process.

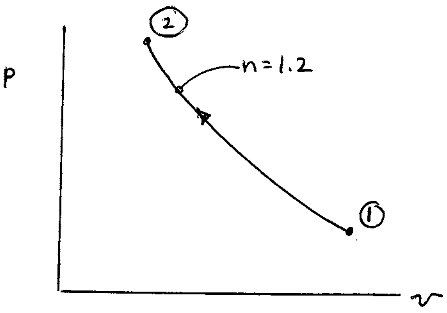
FIND: Determine the heat transfer and entropy change.

SCHEMATIC & GIVEN DATA:



$pv^n = \text{const.}$, $n = 1.2$

$T_1 = 22°C$, $T_2 = 150°C$



ASSUMPTIONS: (1) The system is shown in the figure above. (2) For the system, the composition remains constant and kinetic/potential energy effects are negligible. (2) Each component behaves as if it were an ideal gas occupying the entire volume at the mixture temperature. The overall mixture acts as an ideal gas. (3) The compression obeys $pv^{1.2} = \text{constant}$. (4) Specific heats are constant at the mean temperature.

ANALYSIS: An energy balance reduces to give $Q = \Delta U + W$, where

$$W = \int_1^2 p\,dv = \frac{m(\bar{R}/M)(T_2 - T_1)}{1-n}$$

In this expression, $m = 2 + 4 = 6$ kg and M is given by $y_1 M_1 + y_2 M_2$, where

$$H_2: \quad n_1 = \frac{2}{2.018} = 0.991 \text{ kmol} \quad \Rightarrow \quad y_1 = \frac{0.991}{1.134} = 0.874 \quad \Rightarrow \quad M = (0.874)(2.018) + (0.126)(28.01)$$

$$N_2: \quad n_2 = \frac{4}{28.01} = 0.143 \text{ kmol} \qquad y_2 = 1 - .874 = 0.126 \qquad = 5.29$$

Then,

$$W = (6\,\text{kg})\left(\frac{8.314}{5.29}\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(128\,\text{K})/(1-1.2) = -6035.1\text{ kJ}$$

To find ΔU, write $\Delta U = m_1 c_{v1}[T_2 - T_1] + m_2 c_{v2}[T_2 - T_1] = \overbrace{(m_1 c_{v1} + m_2 c_{v2})}^{m c_v}(T_2 - T_1)$ where the c_v terms are evaluated from Table A-20 at 359 K: $c_{v1} = 0.745$ kJ/kg·K, $c_{v2} = 10.311$ kJ/kg K. Then

$$\Delta U = [(4)(0.745) + (2)(10.311)]\left(\frac{\text{kJ}}{\text{K}}\right)(128\,\text{K}) = \overbrace{(23.602)}^{m\,c_v}(128) = 3021.1\text{ kJ}$$

Collecting results, $Q = \Delta U + W = 3021.1 + (-6035.1) = -3014\text{ kJ} \longleftarrow Q$

To find ΔS combine Eqs. (3.56), (6.22) to obtain

$$\Delta S = m\left[c_v \ln\frac{T_2}{T_1} + \frac{\bar{R}}{M}\ln\frac{V_2}{V_1}\right] = m\left[c_v \ln\frac{T_2}{T_1} + \frac{\bar{R}}{M}\ln\left(\frac{T_1}{T_2}\right)^{\frac{1}{n-1}}\right]$$

$$= m\left[c_v \ln\frac{T_2}{T_1} - \frac{\bar{R}/M}{n-1}\ln\frac{T_2}{T_1}\right]$$

$$= m\left[c_v - \frac{\bar{R}/M}{n-1}\right]\ln\frac{T_2}{T_1}$$

$$= 6\,\text{kg}\left[\left(\frac{23.602}{6}\right)\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right) - \frac{(8.314/5.29)}{(1.2-1)}\left|\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right|\right]\ln\frac{423}{295}$$

$$= 6\,[3.934 - 7.858]\ln\frac{423}{295}$$

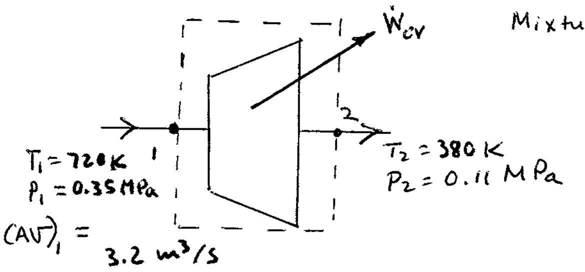
$$= -8.485\text{ kJ/K} \qquad \longleftarrow \Delta S$$

PROBLEM 12.17*

KNOWN: A specified mixture expands through a turbine for which steady state operating data are provided.

FIND: Determine the power developed.

SCHEMATIC & GIVEN DATA:



Mixture: 10% CO_2, 19% H_2O, 71% N_2

ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS: Reducing mass and energy balances at steady state

$$\dot{W}_{cv} = \dot{m}\left[h_1 - h_2\right] = \dot{m}\left[\frac{\bar{h}_1 - \bar{h}_2}{M_{mix}}\right] = \dot{m}\left[\frac{y_{CO_2}(\bar{h}_1-\bar{h}_2)_{CO_2} + y_{H_2O}(\bar{h}_1-\bar{h}_2)_{H_2O} + y_{N_2}(\bar{h}_1-\bar{h}_2)_{N_2}}{M_{mix}}\right]$$

And

$$\dot{m} = \frac{(A\bar{V})_1}{v_1} = \frac{(A\bar{V})_1 P_1}{(\bar{R}/M)_{mix} T_1} \Rightarrow \frac{\dot{m}}{M_{mix}} = \frac{(A\bar{V})_1 P_1}{\bar{R}\ T_1} = \frac{(3.2\,kg/s)(0.35\times10^6\,\frac{N}{m^2})}{(8314\,\frac{N\cdot m}{kmol\cdot K})(720K)} = 0.1871\ \frac{kmol(mix)}{s}$$

Then, with data from the ideal gas tables

$$\dot{W}_{cv} = 0.1871\left[0.10(28{,}121-12{,}552)+0.19(24{,}840-12{,}672)+0.71(21{,}220-11{,}055)\right]\frac{kJ}{s}\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 2074.2\ kW$$

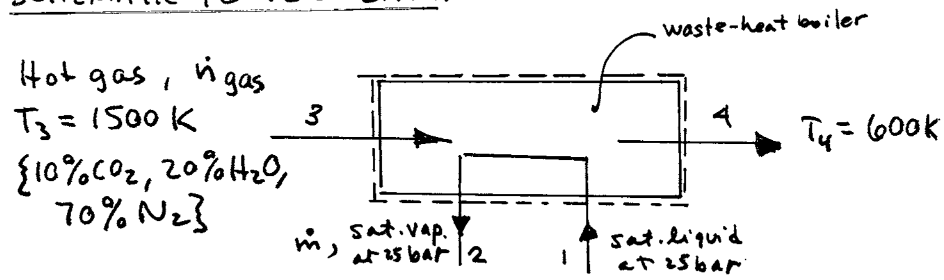
$\longleftarrow \dot{W}_{cv}$

PROBLEM 12.18*

KNOWN: Operating data are provided for a waste-heat boiler operating at steady state.

FIND: Determine the mass flow rate of the steam generated, in kg per kmol of entering hot gas.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and kinetic/potential energy effects are negligible. (3) The entering and exiting gas mixtures are modeled as ideal gas mixtures.

ANALYSIS: With the indicated assumptions, the mass and energy rate balances reduce to read

$$0 = \dot{m}[h_1 - h_2] + \dot{n}_{gas}\Big[y_{CO_2}\big(\bar{h}_{CO_2}(T_3) - \bar{h}_{CO_2}(T_4)\big) + y_{H_2O}\big(\bar{h}_{H_2O}(T_3) - \bar{h}_{H_2O}(T_4)\big) + y_{N_2}\big(\bar{h}_{N_2}(T_3) - \bar{h}_{N_2}(T_4)\big)\Big]$$

or

$$\frac{\dot{m}}{\dot{n}_{gas}} = \frac{y_{CO_2}[\bar{h}_{CO_2}(T_3) - \bar{h}_{CO_2}(T_4)] + y_{H_2O}[\bar{h}_{H_2O}(T_3) - \bar{h}_{H_2O}(T_4)] + y_{N_2}[\bar{h}_{N_2}(T_3) - \bar{h}_{N_2}(T_4)]}{h_2 - h_1}$$

With data from Table A-23 for the numerator and from Table A-3 for the denominator

$$\frac{\dot{m}}{\dot{n}_{gas}} = \frac{0.1[71{,}078 - 22{,}280] + 0.2[57{,}999 - 20{,}402] + 0.7[47{,}073 - 17{,}563]\ \frac{\text{kJ}}{\text{kmol (mix)}}}{1841.0\ \text{kJ/kg (steam)}}$$

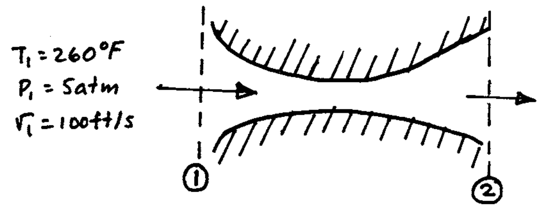
$$= 17.96\ \frac{\text{kg (steam)}}{\text{kmol (mix)}} \quad \triangleleft$$

PROBLEM 12.19

KNOWN: An equimolar mixture of He and CO_2 expands isentropically from a specified inlet state to a specified exit pressure.

FIND: Determine the exit temperature and velocity.

SCHEMATIC & GIVEN DATA:



i	y_i	M_i
He	0.5	4.003
CO_2	0.5	44.01

ASSUMPTIONS: (1) The nozzle operates at steady state and the expansion is isentropic. (2) Potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS: Since He is monatomic, Table A-21 gives $\bar{c}_{p,He} = 5/2\bar{R}$, independent of temperature.

The expansion is isentropic. Thus $\bar{s}_2 - \bar{s}_1 = 0$. This can be expressed as

$$y_{He}(\bar{s}_2-\bar{s}_1)_{He} + y_{CO_2}(\bar{s}_2-\bar{s}_1)_{CO_2} = 0$$

or

$$y_{He}\left[\bar{c}_{p,He}\ln\frac{T_2}{T_1} - \bar{R}\ln\frac{p_2}{p_1}\right] + y_{CO_2}\left[\bar{s}^\circ_{CO_2}(T_2) - \bar{s}^\circ_{CO_2}(T_1) - \bar{R}\ln\frac{p_2}{p_1}\right] = 0$$

Solving

$$\ln\frac{p_2}{p_1} = \frac{y_{He}\,\bar{c}_{p,He}\ln\frac{T_2}{T_1} + y_{CO_2}(\bar{s}^\circ_{CO_2}(T_2) - \bar{s}^\circ_{CO_2}(T_1))}{\bar{R}}$$

With data from Table A-23E at $T_1 = 720°R$

$$\ln\left(\frac{3.24}{5.0}\right) = 0.5\left[\frac{5}{2}\ln\frac{T_2}{720} + \frac{\bar{s}^\circ_{CO_2}(T_2) - 53.78}{1.986}\right]$$

① This is a single equation in one unknown, T_2. Solving iteratively using table data gives $T_2 = 640°R$ ◄———————————————— T_2

Reduction of mass and energy rate balances at steady state result in

$$0 = \cancel{\dot{Q}_{cv}} - \cancel{\dot{W}_{cv}} + \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2} + \cancel{g(z_1 - z_2)}\right)$$

Thus

$$h_2 - h_1 + \frac{(V_2^2 - V_1^2)}{2} = 0 \Rightarrow \frac{\bar{h}_2 - \bar{h}_1}{M} + \frac{(V_2^2 - V_1^2)}{2} = 0$$

where M is the mixture molecular weight: $M = y_{He}M_{He} + y_{CO_2}M_{CO_2} = 24.01$, and

$$\bar{h}_2 - \bar{h}_1 = y_{He}(\bar{h}_2 - \bar{h}_1)_{He} + y_{CO_2}(\bar{h}_2 - \bar{h}_1)_{CO_2} = y_{He}\bar{c}_{p,He}(T_2 - T_1) + y_{CO_2}(\bar{h}_2 - \bar{h}_1)_{CO_2}$$

With $\bar{c}_{p,He} = 5/2\,\bar{R}$ and $\bar{h}$ for CO_2 from Table A-23E

$$0 = \frac{(0.5)\left[(5/2)(1.986)[640-720] + [4974.9 - 5748.4]\right]}{24.01}\frac{Btu}{lb} + \left[\frac{V_2^2 - (100\,ft/s)^2}{2}\right]\left|\frac{1\,lbf}{32.2\,lb\cdot ft/s^2}\right|\left|\frac{Btu}{778\,ft\cdot lbf}\right|$$

Solving, $V_2 = 1110\ ft/s$ ◄————————————————

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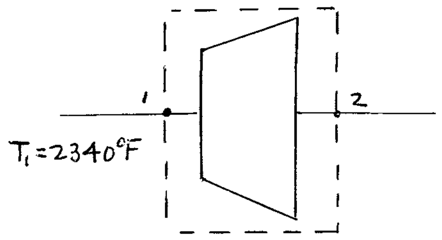
1. An iterative solution using table data can be avoided using IT. The results for T_2 and V_2 obtained using IT agree with the values given here.

PROBLEM 12.20

KNOWN: An equimolar mixture of He and Ar expands adiabatically through a turbine for which steady state operating data are known.

FIND: Determine the power developed, in Btu per lb of mixture flowing.

SCHEMATIC & GIVEN DATA:



Mixture: (He : H, Ar : A) $y_H = y_A = 0.5$

$P_2/P_1 = 1/7.5$

$\eta_t = 80\%$

ASSUMPTIONS: (1) The control volume shown operates at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS: Using the isentropic turbine efficiency η_t, $\dot{W}_{cv}/\dot{m} = \eta_t(\dot{W}_{cv}/\dot{m})_{s=c}$. Accordingly, the value of $(\dot{W}_{cv}/\dot{m})_{s=c}$ is required. Reducing mass and energy balances at steady state

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{s=c} = h_1 - h_{2s} = \frac{1}{M}[\bar{h}_1 - \bar{h}_{2s}] = \frac{1}{M}\left[y_H[\bar{h}_1 - \bar{h}_{2s}]_H + y_A[\bar{h}_1 - h_{2s}]_A\right]$$

By reference to Table A-21E, for each of the monatomic gases He and Ar, $\bar{c}_p = \frac{5}{2}\bar{R}$. Moreover, $y_A = y_H = 0.5$. So, in summary

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right)_{s=c} = \frac{1}{M}\left[\frac{5}{2}\bar{R}\right]\left[T_1 - T_{2s}\right] \qquad (1)$$

where $M = (M_{He} + M_{Ar})/2 = (4.003 + 39.94)/2 = 21.97$ and T_{2s} is the exit temperature for an isentropic expansion.

For an isentropic expansion, $\bar{s}_2 - \bar{s}_1 = 0$. Thus,

$$y_A[\bar{s}_1 - \bar{s}_{2s}]_A + y_H[\bar{s}_1 - \bar{s}_{2s}]_{He} = 0 \Rightarrow y_A\left[\bar{c}_{pA}\ln\frac{T_{2s}}{T_1} - \bar{R}\ln\frac{P_2}{P_1}\right] + y_H\left[\bar{c}_{pH}\ln\frac{T_{2s}}{T_1} - \bar{R}\ln\frac{P_2}{P_1}\right] = 0$$

With $y_A = y_2 = 0.5$, $\bar{c}_{pA} = \bar{c}_{pH} = 5/2\,\bar{R}$,

$$0.5\left[\frac{5}{2}\bar{R}\ln\frac{T_{2s}}{T_1} - \bar{R}\ln\frac{P_2}{P_1}\right] + 0.5\left[\frac{5}{2}\bar{R}\ln\frac{T_{2s}}{T_1} - \bar{R}\ln\frac{P_2}{P_1}\right] = 0$$

Reducing this

$$\frac{5}{2}\ln\frac{T_{2s}}{T_1} = \ln\frac{P_2}{P_1}$$

$$\Rightarrow \ln\frac{T_{2s}}{T_1} = 0.4\ln(1/7.5) \Rightarrow T_{2s} = 1251\,°R$$

Finally, substituting values into Eq. (1)

$$\left(\frac{\dot{W}_{cv}}{\dot{m}}\right) = (0.80)\left[\frac{(5/2)(1.986)}{21.97}\ \frac{\text{Btu}}{\text{lb}\cdot°R}\right]\left[2800 - 1251\right]°R$$

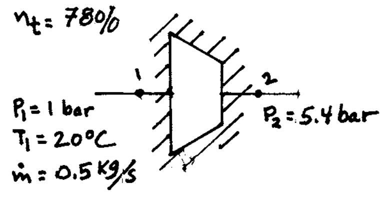
$$= 280.05\ \frac{\text{Btu}}{\text{lb}} \longleftarrow \dot{W}_{cv}/\dot{m}$$

PROBLEM 12.21

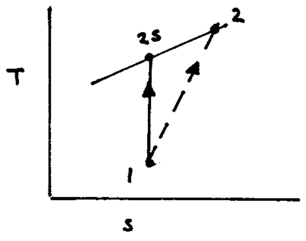
KNOWN: A gas mixture with a specified molar analysis enters a compressor at a specified mass flow rate and state and exits at a specified pressure. The isentropic compressor efficiency is also known.

FIND: Determine the temperature at the exit, the power required, and the rate of entropy production.

SCHEMATIC & GIVEN DATA:



i	y_i	M_i
O_2	0.6	32
N_2	0.4	28.01



ASSUMPTIONS: (1) The compressor is well insulated and operates at steady state. (2) Kinetic and potential energy effects can be ignored. (3) The mixture adheres to the idealizations of the Dalton model and composition remains constant.

ANALYSIS: The temperature T_2 can be determined using the isentropic compressor efficiency, but first the temperature at state 2s, T_{2s}, must be found. Since $\bar{s}_{2s} - \bar{s}_1 = 0$, Eq. 12.36 can be invoked to write

$$y_{O_2}(\bar{s}_{2s}-\bar{s}_1)_{O_2} + y_{N_2}(\bar{s}_{2s}-\bar{s}_1)_{N_2} = y_{O_2}\left(\bar{s}^\circ_{2s} - \bar{s}^\circ_1 - \bar{R}\ln\frac{P_2}{P_1}\right)_{O_2} + y_{N_2}\left(\bar{s}^\circ_{2s} - s^\circ_1 - \bar{R}\ln\frac{P_2}{P_1}\right)_{N_2} = 0$$

Accordingly, with $\bar{s}^\circ$ data from the ideal gas tables

$$y_{O_2}(\bar{s}^\circ_{2s})_{O_2} + y_{N_2}(\bar{s}^\circ_{2s})_{N_2} = y_{O_2}(\bar{s}^\circ_1)_{O_2} + y_{N_2}(\bar{s}^\circ_1)_{N_2} + \bar{R}\ln\frac{P_2}{P_1}$$

$$= (0.6)(204.524) + (0.4)(190.998) + 8.314 \ln 5.4 = 213.13 \ \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$$

① Solving iteratively using table data, T_{2s} = 470 K.

In accord with listed assumptions, the isentropic compressor efficiency is

$$\eta_c = \frac{\bar{h}_{2s} - \bar{h}_1}{\bar{h}_2 - \bar{h}_1}$$

Thus

$$\bar{h}_2 - \bar{h}_1 = \frac{\bar{h}_{2s} - \bar{h}_1}{\eta_c} = \frac{y_{O_2}(\bar{h}_{2s}-\bar{h}_1)_{O_2} + y_{N_2}(\bar{h}_{2s}-\bar{h}_1)_{N_2}}{\eta_c} = \frac{0.6(13842-8533)+0.4(13693-8521)}{0.78}$$

$$= 6736 \text{ kJ/kmol}$$

Accordingly

$$y_{O_2}(\bar{h}_2-\bar{h}_1)_{O_2} + y_{N_2}(\bar{h}_2-\bar{h}_1)_{N_2} = 6736 \text{ kJ/kmol}$$

or

$$y_{O_2}(\bar{h}_2)_{O_2} + y_{N_2}(\bar{h}_2)_{N_2} = y_{O_2}(\bar{h}_1)_{O_2} + y_{N_2}(\bar{h}_1)_{N_2} + 6736$$

$$= (0.6)(8533) + (0.4)(8521) + 6736 = 15{,}264 \text{ kJ/kmol}$$

Solving, T_2 = 520K (247°C) $\longleftarrow$ T_2

The power required is

$$\dot{W}_c = -\dot{m}\,(h_2 - h_1) = -\dot{m}\left(\frac{\bar{h}_2 - \bar{h}_1}{M}\right)$$

where M is the mixture molecular weight: $M = y_{O_2}M_{O_2} + y_{N_2}M_{N_2} = (0.6)(32) + (0.4)(28.01) = 30.4$. Therefore

PROBLEM 12.21 (Cont'd.)

$$\dot{W}_c = -\left(0.5\,\frac{\text{kg}}{\text{s}}\right)\left[\frac{6736\ \text{kJ/kmol}}{30.4\ \ \text{kg/kmol}}\right]\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = -110.8\ \text{kW} \quad \longleftarrow \dot{W}_c$$

Mass and entropy rate balances reduce at steady state to give

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}\,(s_1 - s_2) + \dot{\sigma}_{cv}$$

Thus

$$\dot{\sigma}_{cv} = \dot{m}\,(s_2 - s_1)$$

$$= \dot{m}\,\frac{(\bar{s}_2 - \bar{s}_1)}{M} = \dot{m}\left[\frac{y_{O_2}(\bar{s}_2-\bar{s}_1)_{O_2} + y_{N_2}(\bar{s}_2-\bar{s}_1)_{N_2}}{M}\right]$$

With $\bar{s}^\circ$ data from Table A-23

$$\dot{\sigma}_{cv} = \left(0.5\,\frac{\text{kg}}{\text{s}}\right)\left[\frac{0.6\,(221.812 - 204.524) + 0.4\,(207.792 - 190.998) - 8.314 \ln 5.4}{30.4}\right]\left(\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= 0.0505\ \frac{\text{kW}}{\text{K}} \quad \longleftarrow \dot{\sigma}_{cv}$$

---

1. An iterative solution with table data can be avoided by using IT. The resuts for T_{2s}, T_2, $\dot{W}_{cv}$, and $\dot{\sigma}_{cv}$ obtained using IT agree with the values given here.

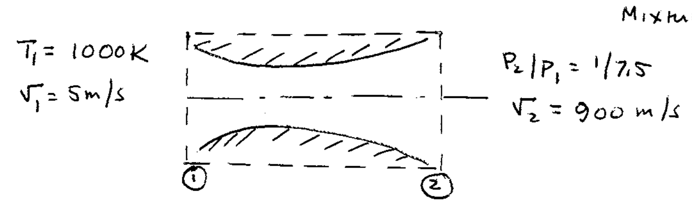
PROBLEM 12.22

KNOWN: A specified mixture expands through a nozzle for which steady state operating data are provided.

FIND: Determine the isentropic nozzle efficiency.

SCHEMATIC & GIVEN DATA:

Mixture: $mf_{N_2} = 0.80 \quad mf_{CO_2} = 0.20$



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and potential energy effects are negligible. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant.

ANALYSIS: By definition of isentropic nozzle efficiency, $\eta_{nozzle} = (V_2^2/2)/(V_{2s}^2/2) = (V_2/V_{2s})^2$, where V_{2s} is the exit velocity for an isentropic expansion. Reducing mass and energy rate balances at steady state

$$\frac{V_{2s}^2}{2} = \frac{V_1^2}{2} + h_1 - h_{2s} = \frac{V_1^2}{2} + \left[mf_{N_2} \frac{[\bar{h}_1 - \bar{h}_{2s}]_{N_2}}{M_{N_2}} + mf_{CO_2} \frac{[\bar{h}_1 - \bar{h}_{2s}]_{CO_2}}{M_{CO_2}} \right] \qquad (1)$$

where h_{2s} denotes the enthalpy at the exit for an isentropic expansion.

Consider next, $s_2 - s_1 = 0 \Rightarrow mf_{N_2}[s_1 - s_{2s}]_{N_2} + mf_{CO_2}[s_1 - s_{2s}]_{CO_2} = 0$

Or

$$0.8\left[\frac{\bar{s}^\circ(T_{2s}) - \bar{s}^\circ(T_1) - \bar{R}\ln P_2/P_1}{M_{N_2}}\right]_{N_2} + 0.2\left[\frac{\bar{s}^\circ(T_{2s}) - \bar{s}^\circ(T_1) - \bar{R}\ln P_2/P_1}{M_{CO_2}}\right]_{CO_2} = 0$$

With data from Table A-23

$$0.8\left[\frac{\bar{s}^\circ_{N_2}(T_{2s}) - 228.057 + 8.314\ln 7.5}{28.01}\right] + 0.2\left[\frac{\bar{s}^\circ_{CO_2}(T_{2s}) - 269.215 + 8.314\ln 7.5}{44.01}\right] = 0$$

$$0.8\left[\frac{\bar{s}^\circ_{N_2}(T_{2s}) - 211.305}{28.01}\right] + 0.2\left[\frac{\bar{s}^\circ_{CO_2}(T_{2s}) - 252.463}{44.01}\right] = 0$$

① Solving iteratively using $\bar{s}^\circ$ data from Table A-23, $T_{2s} = 611.5\,K$.

Returning to Eq. (1) and using $\bar{h}$ data from Table A-23

$$\frac{V_{2s}^2}{2} = \frac{(5\,m/s)^2}{2} + \left[\frac{0.8(30{,}129 - 17{,}909)}{28.01} + \frac{0.2(42{,}769 - 22{,}826)}{44.01}\right]\left(\frac{kJ}{kg}\right)\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|$$

$$= 12.5\left(\frac{m}{s}\right)^2 + 439{,}650\left(\frac{m}{s}\right)^2 \Rightarrow V_{2s} = 937.7\,m/s$$

Finally

$$\eta_{nozzle} = \left(\frac{900}{937.7}\right)^2 = 0.92 \quad (92\%) \qquad \longleftarrow$$

---

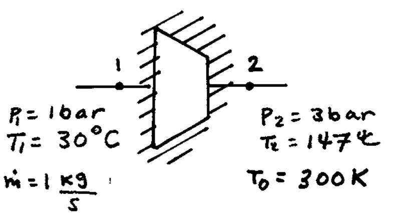
1. An iterative solution with table data can be avoided by using IT. The result for η_{NOZZLE} obtained using IT agrees with the value given here.

PROBLEM 12.23

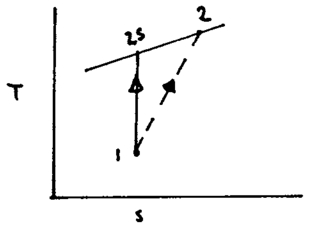
KNOWN: A mixture with a specified molar analysis enters a compressor operating at steady state at a specified mass flow rate and state and exits at a specified temperature and pressure.

FIND: Determine the power required, the isentropic compressor efficiency, and the rate of exergy destruction.

SCHEMATIC & GIVEN DATA



i	y_i	M_i
N_2	0.6	28.01
CO_2	0.4	44.01



ASSUMPTIONS: (1) The compressor is well insulated and operates at steady state. (2) Kinetic and potential energy effects can be ignored. (3) The mixture adheres to the idealizations of the Dalton model and the composition remains constant. (4) For the environment, $T_0 = 300K$.

ANALYSIS: Reduction of the mass and energy rate balances at steady state gives

$$\dot{W}_c = \dot{m}[h_1 - h_2] = \dot{m}\left[\frac{y_{N_2}(\bar{h}_1 - \bar{h}_2)_{N_2} + y_{CO_2}(\bar{h}_1 - \bar{h}_2)_{CO_2}}{M}\right]$$

where M is the mixture molecular weight: $M = y_{N_2} M_{N_2} + y_{CO_2} M_{CO_2} = 0.6(28.01) + 0.4(44.01) = 34.41$. Then, with data from Table A-23

$$\dot{W}_c = (1 \text{ kg/s})\left[\frac{0.6(8810.3 - 12225) + 0.4(9543.8 - 14206)}{34.41}\right]\left|\frac{kJ}{kg}\right|\left|\frac{kW}{kJ/s}\right| = -113.7 \text{ kW} \longleftarrow \dot{W}_c$$

To determine the isentropic compressor efficiency requires the power for an isentropic compression from state 1 to the pressure p_2 — that is, from 1 to state 2s. For this process, $\bar{s}_{2s} - \bar{s}_1 = 0$. Or

$$y_{N_2}[\bar{s}_{2s} - \bar{s}_1]_{N_2} + y_{CO_2}[\bar{s}_{2s} - \bar{s}_1]_{CO_2} = 0 \Rightarrow y_{N_2}\left[\bar{s}^o_{2s} - \bar{s}^o_1 - \bar{R}\ln\frac{p_2}{p_1}\right]_{N_2} + y_{CO_2}\left[\bar{s}^o_{2s} - \bar{s}^o_1 - \bar{R}\ln\frac{p_2}{p_1}\right]_{CO_2} = 0$$

Then, with data from Table: A-23

$$y_{N_2}(\bar{s}^o_{2s})_{N_2} + y_{CO_2}(\bar{s}^o_{2s})_{CO_2} = (0.6)(191.969) + (0.4)(214.288) + 8.314 \ln 3 = 210.029 \frac{kJ}{kmol \cdot K}$$

① Solving this equation by iteration with table data; $T_{2s} = 399$ K. Accordingly

$$(\dot{W}_c)_s = \dot{m}(h_1 - h_{2s}) = \dot{m}\left[\frac{y_{N_2}(\bar{h}_1 - \bar{h}_{2s})_{N_2} + y_{CO_2}(\bar{h}_1 - \bar{h}_{2s})_{CO_2}}{M}\right]$$

$$= (1)\left[\frac{0.6(8810.3 - 11610.7) + 0.4(9543.8 - 13330.8)}{34.41}\right] = -92.85 \text{ kW}$$

The isentropic compressor efficiency is then

$$\eta_c = \frac{(\dot{W}_c)_s}{(\dot{W}_c)} = \frac{(-92.85)}{(-113.7)} = 0.817 \; (81.7\%) \longleftarrow \eta_c$$

The rate of exergy destruction can be found using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production obtained from an entropy rate balance, which reduces at steady state as follows:

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{\,0} + \dot{m}(s_1 - s_2) + \dot{\sigma}_{cv} \Rightarrow \dot{\sigma}_{cv} = \dot{m}(s_2 - s_1)$$

$$\Rightarrow \quad \dot{\sigma}_{cv} = \dot{m}\left[\frac{y_{N_2}(\bar{s}_2 - \bar{s}_1)_{N_2} + y_{CO_2}(\bar{s}_2 - \bar{s}_1)_{CO_2}}{M}\right]$$

PROBLEM 12.23 (Contd.)

The terms $(\bar{s}_2 - \bar{s}_1)_{N_2}$ and $(\bar{s}_2 - \bar{s}_1)_{CO_2}$ are evaluated using Eq. 12.36, so

$$\dot{\sigma}_{cv} = \dot{m}\left[\frac{y_{N_2}\left(\bar{s}^{\circ}_{N_2}(T_2) - \bar{s}^{\circ}_{N_2}(T_1) - \bar{R}\ln\frac{p_2}{p_1}\right) + y_{CO_2}\left(\bar{s}^{\circ}_{CO_2}(T_2) - \bar{s}^{\circ}_{CO_2}(T_1) - \bar{R}\ln p_2/p_1\right)}{M}\right]$$

$$= \dot{m}\left[\frac{y_{N_2}\left(\bar{s}^{\circ}_{N_2}(T_2) - \bar{s}^{\circ}_{N_2}(T_1)\right) + y_{CO_2}\left(\bar{s}^{\circ}_{CO_2}(T_2) - \bar{s}^{\circ}_{CO_2}(T_1)\right) - \bar{R}\ln p_2/p_1}{M}\right]$$

Then, with $\bar{s}^{\circ}$ data from Table A-23

$$\dot{\sigma}_{cv} = \left(1\,\frac{kg}{s}\right)\left[\frac{0.6(201.499 - 191.969) + 0.4(227.258 - 214.284) - 8.314\ln 3}{34.41}\right]\left|\frac{kJ}{kg \cdot K}\right|\left|\frac{1\,kW}{1\,kJ/s}\right|$$

$$= 0.0516\ kW/K$$

Multiplying this by T_0

② $$\dot{E}_d = T_0\,\dot{\sigma}_{cv} = (300\,K)\left(0.0516\,\frac{kW}{K}\right) = 15.48\,kW \longleftarrow \dot{E}_d$$

---

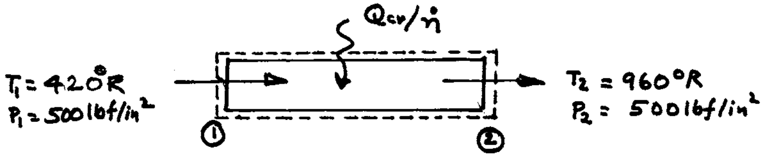
1. An iterative solution using table data can be avoided by using IT. The results for $\dot{W}_c$, T_{2s}, h_c, and $\dot{E}_d$ obtained using IT agree with the values given here.
2. When expressed as a percentage of the power input to the compressor
   $\% = [\dot{E}_d/(-\dot{W}_{cv})](100) = (15.48/113.7)(100) = 13.6$

PROBLEM 12.24

KNOWN: An equimolar mixture of N_2 and CO_2 enters a heat exchanger at a known temperature and pressure and exits with no change in composition at a known temperature and pressure.

FIND: Determine the rate of heat transfer to the mixture per lbmol of mixture flowing using (a) ideal gas mixture principles, (b) the generalized enthalpy chart together with Kay's rule.

SCHEMATIC & GIVEN DATA:



i	y_i
N_2	0.5
CO_2	0.5

ASSUMPTIONS: (1) For the control volume shown in the accompanying figure $\dot{W}_{cv}=0$ and kinetic and potential energy effects are negligible. (2) The control volume is at steady state. (3) The composition of the mixture remains constant. (4) In part (a), the mixture adheres to the Dalton model. (5) In Part (b), Kay's rule applies.

ANALYSIS: At steady state, mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}\left[h_1 - h_2 + \cancelto{0}{\frac{V_1^2 - V_2^2}{2}} + \cancelto{0}{g(z_1 - z_2)}\right] \Rightarrow \dot{Q}_{cv} = \dot{m}(h_2 - h_1)$$

or, when expressed in terms of the molar flow rate of the mixture

$$\dot{Q}_{cv} = \dot{n}(\bar{h}_2 - \bar{h}_1) \Rightarrow \frac{\dot{Q}_{cv}}{\dot{n}} = \bar{h}_2 - \bar{h}_1 \qquad (1)$$

(a) When the ideal gas mixture principles of Chap. 12 are used, the change in specific enthalpy of the mixture can be expressed in terms of the enthapy changes of the components evaluated at the inlet and exit temperatures:

$$\frac{\dot{Q}_{cv}}{\dot{n}} = y_{N_2}\left[\bar{h}_{N_2}(T_2) - \bar{h}_{N_2}(T_1)\right] + y_{CO_2}\left[\bar{h}_{CO_2}(T_2) - \bar{h}_{CO_2}(T_1)\right]$$

With data from Table A-23

$$\frac{\dot{Q}_{cv}}{\dot{n}} = 0.5\left[6693.1 - 2916.1\right] + 0.5\left[8243.8 - 3035.7\right] = 4492.6 \text{ Btu/lbmol} \longleftarrow \text{(a)}$$

(b) Invoking Eq. 11.85 for the mixture:

$$\frac{\dot{Q}_{cv}}{\dot{n}} = \underline{\bar{h}_2^* - \bar{h}_1^*} - \bar{R}T_c\left[\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 - \left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1\right] \qquad (2)$$

The underlined term is the enthalpy change for the mixture evaluated in part (a) using ideal gas mixture principles. The remaining terms can be evaluated from Figure A-4 using T_c and P_c for the mixture calculated via Kay's rule:

$$T_c = y_{N_2}(T_c)_{N_2} + y_{CO_2}(T_c)_{CO_2} = 0.5(227°R + 548°R) = 388°R$$

$$P_c = y_{N_2}(P_c)_{N_2} + y_{CO_2}(P_c)_{CO_2} = 0.5(33.5\,atm + 72.9\,atm) = 53.2\,atm$$

Thus

$$(T_R)_1 = \frac{420}{388} = 1.08, \quad (T_R)_2 = \frac{960}{388} = 2.47, \quad P_{R1} = P_{R2} = \left(\frac{500/14.7}{53.2}\right) = 0.64$$

Fig. A-4 gives $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_1 = 0.7$, $\left(\frac{\bar{h}^* - \bar{h}}{\bar{R}T_c}\right)_2 \simeq 0.1$. And from Eq. (2) we get

① $$\left(\dot{Q}_{cv}/\dot{n}\right) = 4492.6 \text{ Btu/lbmol} - (1.986 \text{ Btu/lbmol}\cdot°R)(388°R)(0.1 - 0.7) = 4954.9 \text{ Btu/lbmol}.$$

---

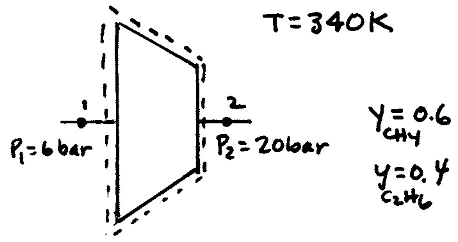
1. The result of part (b) is about 10% greater than that of part (a).

PROBLEM 12.25

KNOWN: Natural gas with a specified composition is compressed isothermally without internal irreversibilities from 6 to 20 bar. The compressor operates at steady state.

FIND: Determine the work and heat transfer per unit mole of mixture flowing (a) using ideal gas mixture principles, (b) using ideal solution principles with given data for the pure components.

SCHEMATIC & GIVEN DATA:



	h (kJ/kg)		s (kJ/kg·K)	
	6bar	20bar	6bar	20bar
CH_4	715.33	704.40	10.9763	10.3275
C_2H_6	469.39	439.13	7.3493	6.9680

ASSUMPTIONS: (1) The control volume operates at steady state with negligible kinetic and potential energy changes. (2) The mixture composition remains constant. (3) In part (a) the mixture adheres to the Dalton model. The ideal solution model applies in part (b).

ANALYSIS: At steady state, mass and energy rate balances reduce to give

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}\left(h_1 - h_2 + \cancel{\frac{V_1^2 - V_2^2}{2}}^{\,0} + \cancel{g(z_1 - z_2)}^{\,0}\right) \Rightarrow \dot{W}_{cv} = \dot{Q}_{cv} + \dot{m}\,(h_1 - h_2)$$

Mass and entropy rate balances reduce to give

$$0 = \frac{\dot{Q}_{cv}}{T} + \dot{m}(s_1 - s_2) + \cancel{\dot{\sigma}_{cv}} \Rightarrow \dot{Q}_{cv} = \dot{m}\,T(s_2 - s_1) = \dot{n}\,T(\bar{s}_2 - \bar{s}_1) \qquad (1)$$

Combining these results

$$\dot{W}_{cv} = \dot{m}\left[h_1 - h_2 - T(s_1 - s_2)\right]$$

or, in terms of the molar flow rate of the mixture $\dot{n}$

$$\dot{W}_{cv} = \dot{n}\left[\bar{h}_1 - \bar{h}_2 - T(\bar{s}_1 - \bar{s}_2)\right] \Longrightarrow \frac{\dot{W}_{cv}}{\dot{n}} = \bar{h}_1 - \bar{h}_2 - T(\bar{s}_1 - \bar{s}_2) \qquad (2)$$

(a) When the ideal gas mixture principles of Chap. 12 are used, the enthalpy change of the mixture vanishes since temperature remains constant. The specific entropy change of the mixture can be expressed in terms of the specific entropy changes of the components using Eq. 12.36:

① $$\bar{s}_1 - \bar{s}_2 = y_{CH_4}\Big[\underbrace{\bar{s}_1^\circ - \bar{s}_2^\circ}_{=0 \text{ since } T_1 = T_2} - \bar{R}\ln\frac{P_1}{P_2}\Big]_{CH_4} + y_{C_2H_6}\Big[\underbrace{\bar{s}_1^\circ - \bar{s}_2^\circ}_{=0 \text{ since } T_1 = T_2} - \bar{R}\ln\frac{P_1}{P_2}\Big]_{C_2H_6} = -\bar{R}\ln\frac{P_1}{P_2}$$

Collecting results, Eq. (1) becomes

$$\frac{\dot{W}_{cv}}{\dot{n}} = \bar{R}\,T\ln\frac{P_1}{P_2} = \left(8.314\,\frac{kJ}{kmol\cdot K}\right)(340K)\ln\frac{6}{20} = -3403\,\frac{kJ}{kmol}$$

Since $(h_1 - h_2) = 0$, $(\dot{Q}_{cv}/\dot{n}) = (\dot{W}_{cv}/\dot{n}) = -3403\,\frac{kJ}{kmol}$

(a)

(b) Using the ideal solution model together with the data above for the pure components, we get

$$\bar{h}_1 - \bar{h}_2 = 0.6\big[(16.04)[715.3 - 704.4]\big] + 0.4\big[(30.07)[462.39 - 439.13]\big] = 385\,\frac{kJ}{kmol}$$

PROBLEM 12.25 (Contd)

① $\bar{s}_1 - \bar{s}_2 = 0.6\left[(16.04)(10.9763 - 10.3275)\right] + 0.4\left[(30.07)\left[7.3493 - 6.9680\right]\right] = 10.83 \frac{kJ}{kmol \cdot K}$

Then, with Eq. (2)

② $$\left(\frac{\dot{W}_{cv}}{\dot{n}}\right) = \bar{h}_1 - \bar{h}_2 - T(\bar{s}_1 - \bar{s}_2) = 385 - 340(10.83) = -3297 \frac{kJ}{kmol}$$

and with Eq. (1)

$$\left(\frac{\dot{Q}_{cv}}{\dot{n}}\right) = T(s_2 - s_1) = 340(-10.83) = -3682 \frac{kJ}{kmol}$$

---

1. Since composition remains constant, this approach suffices. See discussion leading up to Eq. 12.36.

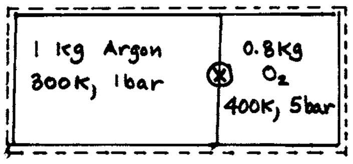
2. Comparing the results of part (a) in terms of magnitudes,
$\left(\frac{-\dot{W}_{cv}}{\dot{n}}\right)$ in part (a) is about 3% greater than the part (b) result.
$\left(-\frac{\dot{Q}_{cv}}{\dot{n}}\right)$ in part (a) is about 8% loss than the part (b) result.

PROBLEM 12.26

KNOWN: Argon at a specified state is allowed to mix with O_2 at a specified state to achieve a mixture at a known temperature.

FIND: Determine (a) the volume of each gas initially, (b) the final pressure, (c) the heat transfer, and (d) the entropy change of each gas.

SCHEMATIC & GIVEN DATA:



$T_f = 360\text{K}$

ASSUMPTIONS: (1) For the system shown in the accompanying figure there is no change in kinetic or potential energy between initial and final states. (2) Each gas and the overall mixture behaves as an ideal gas. The Dalton mixture model applies.

ANALYSIS: (a) The volume of each tank can be determined using the ideal gas equation of state together with known initial data for each gas:

$$V_{AR} = \frac{m_{AR}(\bar{R}/M_{Ar})T_{Ar}}{P_{Ar}} = \frac{(1\,\text{kg})\left(\frac{8314}{39.94}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(300\text{K})}{(10^5\,\text{N/m}^2)} = 0.624\ \text{m}^3 \qquad \longleftarrow V_{Ar},\ V_{O_2}$$

$$V_{O_2} = \frac{m_{O_2}(\bar{R}/M_{O_2})T_{O_2}}{P_{O_2}} = \frac{(0.8)\left(\frac{8314}{32.0}\right)(400)}{(5\times10^5)} = 0.166\ \text{m}^3$$

where P_{Ar} and P_{O_2} represent the initial pressures of the argon and oxygen, respectively, and not partial pressures.

(b) The final pressure, P_f, can also be determined using the ideal gas equation of state:

$$P_f = \frac{n\bar{R}T_f}{V}$$

where $V = 0.624\,\text{m}^3 + 0.166\,\text{m}^3 = 0.79\ \text{m}^3$ and

$$n = n_{Ar} + n_{O_2} = \frac{m_{Ar}}{M_{Ar}} + \frac{m_{O_2}}{M_{O_2}} = \frac{1}{39.94} + \frac{0.8}{32.0} = 0.025 + 0.025 = 0.05\ \text{kmol}$$

Finally

$$P_f = \frac{(0.05\,\text{kmol})\left(8314\,\frac{\text{N}\cdot\text{m}}{\text{kmol}\cdot\text{K}}\right)(360\text{K})}{0.79\ \text{m}^3}\left|\frac{\text{bar}}{10^5\,\text{N/m}^2}\right| = 1.89\,\text{bar} \qquad \longleftarrow P_f$$

(c) An energy balance reduces to give $\Delta U = Q - \overset{0}{\cancel{W}}$, or

$$Q = \Delta U = \Delta U]_{Ar} + \Delta U]_{O_2} = n_{Ar}\bar{c}_{v,Ar}[T_f - T_{Ar}] + n_{O_2}[\bar{u}_{O_2}(T_f) - \bar{u}_{O_2}(T_{O_2})]$$

From Table A-21, for argon $\bar{c}_p = 5/2\,\bar{R}$. Then, with Eq. 3.45, $\bar{c}_v = \bar{c}_p - \bar{R} = 3/2\,\bar{R}$. With $\bar{u}$ data from Table A-23

$$Q = (0.025)\left[\tfrac{3}{2}\times 8.314\right][360 - 300] + (0.025)[7518 - 8384]$$

$$= -2.94\ \text{kJ} \qquad \longleftarrow Q$$

(d) Argon initially is at 300K, 1bar and finally at 360K and the partial pressure, $y_{Ar}P_f = (0.5)(1.89\,\text{bar}) = 0.945\,\text{bar}$. Thus, with $\bar{c}_p = 5/2\,\bar{R}$

$$(\Delta S)_{Ar} = (0.025)\left[(2.5)(8.314)\ln\frac{360}{300} - (8.314)\ln\left(\frac{0.945}{1.00}\right)\right] = 0.083\ \frac{\text{kJ}}{\text{K}} \qquad \longleftarrow (\Delta S)_{Ar}$$

PROBLEM 12.26 (Contd.)

Oxygen: initially is at 400 K, 5 bar and finally at 360 K and the partial pressure 0.945 bar. With $\bar{s}^{o}$ data from Table A-23

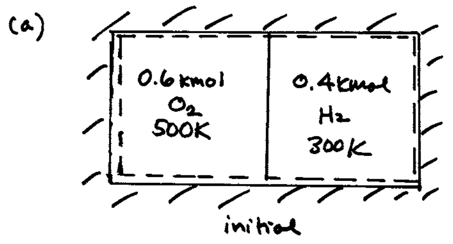
$$(\Delta S)_{O_2} = 0.025\left[210.604 - 213.765 - 8.314 \ln \frac{0.945}{5}\right] = 0.2673 \frac{kJ}{K} \leftarrow (\Delta S)_{O_2}$$

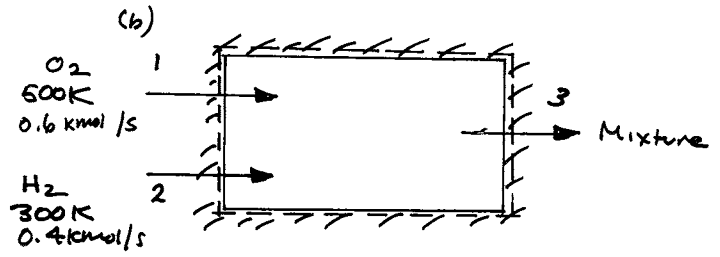
PROBLEM 12.27

KNOWN: Two cases involving the mixing of O_2 and H_2 are specified.

FIND: For each case determine the mixture temperature.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) For case (a), a closed system consisting of the O_2 and N_2 is the system. For case (b), a control volume at steady state is the system. (2) For each system, $\dot{Q}=0$, $\dot{W}=0$ (except for flow work in (b)), and there are no significant kinetic/potential energy effects. (3) Each gas and the overall mixture behaves as an ideal gas. The Dalton mixture model applies. (4) Appropriate constant specific heats are employed.

ANALYSIS: (a) An energy balance reduces to read $\Delta U = \cancel{Q}^{0} - \cancel{W}^{0}$, or $\Delta U = 0$. That is

$$[n\Delta\bar{u}]_{O_2} + [n\Delta\bar{u}]_{H_2} = 0 \qquad (1)$$

Since the initial temperature difference is just 200K, and c_v does not vary significantly over such an interval, an appropriate constant c_v can be used for each gas. Thus, with data from Table A-20 at 400K, Eq. (1) reads

$$[n(c_vM)[T_f - T_i]]_{O_2} + [n(c_vM)(T_f - T_i)]_{H_2} = 0$$

$$\Rightarrow \quad (0.6)(0.681 \times 32)[T_f - 500] + (0.4)(10.352 \times 2.016)[T_f - 300] = 0 \Rightarrow T_f = 422K \leftarrow$$

(b) An energy rate balance reduces to read

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \dot{n}_{O_2}\bar{h}_{O_2}(T_1) + \dot{n}_{H_2}\bar{h}_{H_2}(T_2) - [\dot{n}_{O_2}\bar{h}_{O_2}(T_3) + \dot{n}_{H_2}\bar{h}_{H_2}(T_3)]$$

or

$$0 = \dot{n}_{O_2}[\bar{h}_{O_2}(T_1) - \bar{h}_{O_2}(T_3)] + \dot{n}_{H_2}[\bar{h}_{H_2}(T_2) - \bar{h}_{H_2}(T_3)] \qquad (2)$$

Then, with c_p for each gas from Table A-20 at 400K we get

$$0 = \dot{n}_{O_2}[(c_pM)_{O_2}[T_1 - T_3]] + \dot{n}_{H_2}[(c_pM)_{H_2}[T_2 - T_3]]$$

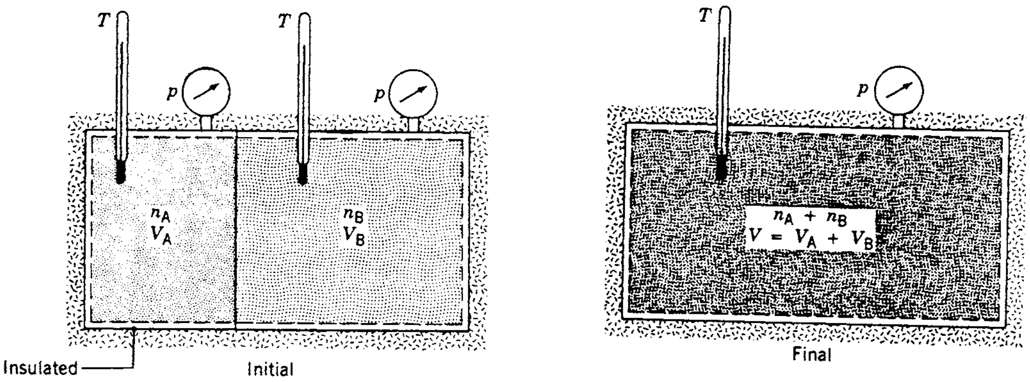
$$0 = (0.6)[(0.941 \times 32)[500 - T_3]] + (0.4)[(14.476)(2.016)[300 - T_3]] \Rightarrow T_f = 422K \leftarrow$$

PROBLEM 12.28

KNOWN: A system consists initially of n_A moles of gas A at pressure p and temperature T and n_B moles of gas B separate from gas A but at the same pressure and temperature. The gases are allowed to mix at fixed total volume. The final temperature is T and the final pressure is p.

FIND: (a) obtain an expression for the entropy produced in terms of $\bar{R}$, n_A and n_B. (b) Using the result of part (a) demonstrate that $\sigma > 0$. (c) Would entropy be produced if the gases were identical?

SCHEMATIC & GIVEN DATA:



Assumptions:

1. The system consists of gases A and B together.
2. Each gas behaves as an ideal gas. The final mixture also acts as an ideal gas with each mixture component occupying the total volume and exhibiting the mixture temperature.
3. No heat or work interactions with the surroundings occur.

Analysis: An entropy balance for the closed system reduces to

$$S_2 - S_1 = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma$$

where the entropy transfer term drops out in the adiabatic process. S_1 and S_2 denote the initial and final entropies of the system, respectively.

The initial entropy of the system is the sum of the entropies of the gases *when separate*. That is

$$S_1 = n_A \bar{s}_A(T, p) + n_B \bar{s}_B(T, p)$$

Since each gas initially exists separately at T and p, each of the specific entropies appearing in this expression is evaluated at T, p.

The entropy of the system at the final state equals the sum of the entropies of the gases A and B evaluated at the conditions at which they exist in the mixture. That is

$$S_2 = n_A \bar{s}_A(T, y_A p) + n_B \bar{s}_B(T, y_B p)$$

In this expression, the specific entropy of each gas is evaluated at its partial pressure in the mixture and at the mixture temperature.

Combining the last three equations and solving for the entropy production

$$\sigma = n_A[\bar{s}_A(T, y_A p) - \bar{s}_A(T, p)] + n_B[\bar{s}_B(T, y_B p) - \bar{s}_B(T, p)]$$

The specific entropy changes can be evaluated using Eq. 6.25b. Since there is no change in temperature

$$\bar{s}_A(T, y_A p) - \bar{s}_A(T, p) = -\bar{R} \ln \frac{y_A p}{p} = -\bar{R} \ln y_A$$

$$\bar{s}_B(T, y_B p) - \bar{s}_B(T, p) = -\bar{R} \ln \frac{y_B p}{p} = -\bar{R} \ln y_B$$

Introducing the foregoing into the entropy production expression gives ← (a)

①② $$\sigma = -\bar{R}(n_A \ln y_A + n_B \ln y_B)$$

Since y_A and y_B are each less than unity, σ would be positive, in accordance with the second law. ← (b)

PROBLEM 12.28 (Contd.)

If the gases were identical, the initial entropy of the system would be

$$S_1 = n_A s(T, p) + n_B s(T, p)$$

where n_A and n_B denote the amounts of the gas present initially in the two tanks. The final entropy of the system would be

$$S_2 = (n_A + n_B)\, s(T, p)$$

Clearly, $\sigma = 0$ for this case. No entropy would be produced. $\longleftarrow$ (c)

---

1. The irreversibility in this case is due to the *free expansion* each individual gas would undergo from its initial volume, V_A or V_B, and initial pressure p to the total volume V and its partial pressure in the mixture, p_A or p_B. The mixture would be formed spontaneously; however, once formed, energy would be required from the surroundings to separate the gases and return them to their respective initial conditions.

2. For the case of mixing several components, each initially at the same temperature and pressure, the expression for the entropy production takes the form

$$\sigma = -\bar{R} \sum_{i=1}^{j} n_i \ln y_i$$

where n_i and y_i are the number of moles and the mole fraction, respectively, of component i. The result obtained in the solution is a special case of this equation.

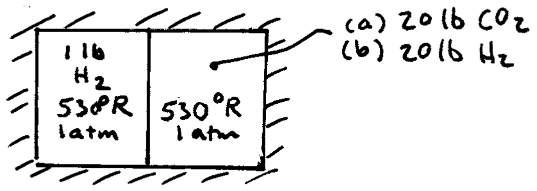
PROBLEM 12.29*

KNOWN: Hydrogen (H_2) mixes adiabatically with (a) CO_2, (b) H_2, each at the same T, p, initially.

FIND: Determine the amount of entropy produced.

SCHEMATIC & GIVEN DATA:

initially:



ASSUMPTIONS:

1. The system is the total quantity of gas.
2. Each gas behaves as an ideal gas and the final mixture is described by the Dalton model.
3. $Q = 0$

ANALYSIS: An entropy balance reduces with assumption 3 to give $\sigma = \Delta S$.

(a) In this case each gas undergoes a process from 530 °R, 1atm to 530°R and its partial pressure in the mixture. To obtain the respective partial pressures ...

$$n_{CO_2} = \frac{20\ \text{lb}}{44.01\ \text{lb/lbmol}} = 0.4544\ \text{lbmol} \qquad y_{CO_2} = \frac{0.4544}{0.9504} = 0.4781,\quad p_{CO_2} = 0.4781\ \text{atm}$$

$$n_{H_2} = \frac{1}{2.016} = 0.4960\ \text{lbmol} \qquad y_{H_2} = \frac{0.4960}{0.9504} = 0.5219,\quad p_{H_2} = 0.5219\ \text{atm}$$

$$n = 0.9504\ \text{lbmol}$$

Thus,

$$\sigma = n_{CO_2}\Big[\underbrace{\Delta\bar{s}^{\circ}_{CO_2}}_{\substack{=0 \\ \text{since } T \\ \text{is constant}}} - \bar{R}\ln\frac{p_{CO_2}}{p}\Big] + n_{H_2}\Big[\underbrace{\Delta\bar{s}^{\circ}_{H_2}}_{\substack{=0 \\ \text{since } T \\ \text{is constant}}} - \bar{R}\ \frac{p_{H_2}}{p}\Big]$$

$$= (0.4544)\Big[-1.986\ln\frac{0.4781}{1}\Big] + (0.4960)\Big[-1.986\ln\frac{0.5219}{1}\Big]$$

$$= 1.307\ \frac{\text{Btu}}{°\text{R}} \qquad \leftarrow \sigma$$

(b) If H_2 is present initially in each subcompartment, the initial entropy is

$$S_1 = n'\,\bar{s}(T,p) + n\,\bar{s}(T,p)$$

where n' and n denote the amounts present. At the final condition

$$S_f = (n'+n)\,\bar{s}(T,p)$$

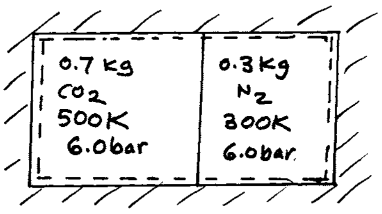
Clearly, then, $\sigma = 0$. That is, no entropy is produced $\leftarrow \sigma$ when samples of the same gas at the same temperature and pressure are allowed to mix adiabatically.

PROBLEM 12.30

KNOWN: CO_2 and N_2, initially separate, are allowed to mix without heat or work interactions with the surroundings until a final equilibrium state is attained.

FIND: Determine the final temperature and pressure, and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is the CO_2 and N_2 together. (2) For the system, $Q=0$, $W=0$, and kinetic/potential energy effects are absent.
(3) Each gas behaves as an ideal gas. The mixture adheres to the Dalton model.
(4) The specific heats are taken as constant at 400K.

ANALYSIS: An energy balance reduces to $\Delta U = \cancel{Q} - \cancel{W}$ or $\Delta U = 0$. That is

$$n_{CO_2} \Delta\bar{u})_{CO_2} + n_{N_2}\Delta\bar{u})_{N_2} = 0 \quad \text{or} \quad m_{CO_2}\Delta u)_{CO_2} + m_{N_2}\Delta u)_{N_2} = 0$$

As the final temperature will fall between 300 and 500K, an interval over which the variation in c_v is nearly linear, c_v for each gas is taken as constant at 400K. This eliminates an iterative solution with the ideal gas tables. From Table A-20, $c_{v,CO_2} = 0.75\,kJ/kg\cdot K$, $c_{v,N_2} = 0.747\,kJ/kg\,K$. The energy balance becomes

$$m_{CO_2} c_{v,CO_2}[T_f - 500] + m_{N_2} c_{v,N_2}[T_f - 300] = 0$$
$$(0.7)(0.75)(T_f - 500) + (0.3)(0.747)(T_f - 300) = 0 \Rightarrow T_f = 440.17\,K \longleftarrow T_f$$

The final pressure is found using the ideal gas equation of state

$$p_f = \frac{n\bar{R}T_f}{V} = \frac{(n_{CO_2}+n_{N_2})\bar{R}T_f}{V}, \quad V = V_{CO_2} + V_{N_2} = \left(\frac{n\bar{R}T}{p}\right)_{CO_2} + \left(\frac{n\bar{R}T}{p}\right)_{N_2}$$

Combining

$$p_f = \overset{6\,bar}{p}\left[\frac{(n_{CO_2}+n_{N_2})\overset{440.17K}{T_f}}{n_{CO_2}\underset{500K}{T_{CO_2}} + n_{N_2}\underset{300K}{T_{N_2}}}\right], \quad n_{CO_2} = \frac{0.7}{44.01} = 0.01591, \; n_{N_2} = \frac{0.3}{28.01} = 0.01071 \Big\} n = 0.02662$$

$$= 6\,bar\left[\frac{(0.02662)(440.17)}{(0.01591)(500) + (0.01071)(300)}\right] = 6.295\,bar \longleftarrow p_f$$

Reducing an entropy balance, $\Delta S = \sigma$, or

$$\sigma = m_{CO_2}\left[c_{p,CO_2}\ln\frac{T_f}{T_{CO_2}} - \frac{\bar{R}}{M_{CO_2}}\ln\frac{y_{CO_2}p_f}{p}\right] +$$
$$m_{N_2}\left[c_{p,N_2}\ln\frac{T_f}{T_{N_2}} - \frac{\bar{R}}{M_{N_2}}\ln\frac{y_{N_2}p_f}{p}\right]$$

PROBLEM 12.30 (Continued)

Substituting values

$$\sigma = (0.7\,\text{kg})\left[\left(0.939\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\ln\frac{440.17}{500} - \frac{8.314}{44.01}\ln\left(\frac{(0.5977)(6.295)}{6}\right)\right]$$

$$(0.3)\left[(1.044)\ln\frac{440.17}{300} - \frac{8.314}{28.01}\ln\left(\frac{(0.4023)(6.295)}{6}\right)\right]$$

where $y_{CO_2} = 0.01591/0.02662 = 0.5977$, $y_{N_2} = 0.01071/0.02662 = 0.4023$, and c_{p,CO_2}, c_{p,N_2} are evaluated at 400K. Calculating

$$\sigma = -0.0221\,\frac{\text{kJ}}{\text{K}} + 0.1969\,\frac{\text{kJ}}{\text{K}}$$

$$= 0.1748\,\frac{\text{kJ}}{\text{K}} \longleftarrow \sigma$$

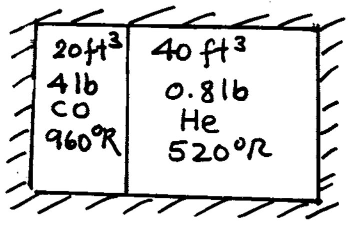
PROBLEM 12.31

KNOWN: Carbon monoxide and helium, initially separate at different states, are allowed to mix adiabatically to equilibrium.

FIND: Determine (a) the final temperature, (b) the final pressure, (c) the exergy destruction.

SCHEMATIC & GIVEN DATA:

Initially,



ASSUMPTIONS:

1. The system is the total quantity of gas present.
2. For the system, $Q = 0$, and kinetic and potential energy effects can be ignored.
3. Each gas behaves as an ideal gas. The mixture adheres to the Dalton model.
4. $T_0 = 520°R$.

ANALYSIS: To begin, the number of moles of each gas and initial pressures are found...

$$\left.\begin{array}{l} n_{CO} = \dfrac{4}{28.01} = 0.1428 \\ n_{He} = \dfrac{0.8}{4.003} = 0.1999 \end{array}\right\} n = 0.3427 \Rightarrow \quad \begin{array}{l} y_{CO} = \dfrac{0.1428}{0.3427} = 0.4167 \\ y_{He} = \dfrac{0.1999}{0.3427} = 0.5833 \end{array}$$

Using the ideal gas model equation of state the initial pressures are

$$p_{CO} = \frac{n_{CO}\bar{R}T_{CO}}{V_{CO}} = \frac{(0.1428)(1545)(960)}{(20)}\left|\frac{1\ ft^2}{144 in^2}\right| = 73.542\ \frac{lbf}{in^2}$$

$$p_{He} = \frac{n_{He}\bar{R}T_{He}}{V_{He}} = \frac{(0.1999)(1545)(520)}{(40)|144|} = 27.882\ \frac{lbf}{in^2}$$

} note: these are the initial pressures, not partial pressures.

(a) An energy balance reduces to $\Delta U = \overset{0}{\cancel{Q}} - \overset{0}{\cancel{W}} = 0$, where ΔU is

$$\Delta U = \Delta U]_{CO} + \Delta U]_{He} = n_{CO}[\bar{u}_{CO}(T_f) - \bar{u}_{CO}(T_{CO})] + n_{He}\bar{c}_{v,He}[T_f - T_{He}] = 0$$

Since helium is monatomic, Table A-21 gives $\bar{c}_p = 5/2\,\bar{R}$. So, with $\bar{c}_p = \bar{c}_v + R$, we get $\bar{c}_{v,He} = 1.5\bar{R}$. For CO, $\bar{u}$ is obtained from Table A-23. Rearranging

$$n_{CO}\bar{u}_{CO}(T_f) + n_{He}\bar{c}_{v,He}T_f = n_{CO}\bar{u}_{CO}(T_{CO}) + n_{He}\bar{c}_{v,He}T_{He}$$

$$= (0.1428)(4798.5) + (0.1999)(1.5 \times 1.986)(520) = 994.89$$

① Solving iteratively using table data, $T_f = 762°R$ (302 °F) $\longleftarrow$ T_f

(b) Using the ideal gas equation of state applied to the final mixture

$$p_f = \frac{n\bar{R}T_f}{V} = \frac{(0.3427)(1545)(762)}{60|144|} = 46.696\ \frac{lbf}{in^2}$$ $\longleftarrow$ p_f

(c) The exergy destruction can be found using $E_d = T_0\sigma$, where σ is the amount of entropy produced, obtained from an entropy balance: $\sigma = \Delta S$. Thus

$$E_d = T_0\Delta S = T_0[\Delta S)_{CO} + \Delta S)_{He}]$$, where

$$\Delta S)_{CO} = n_{CO}\left[\bar{s}^{\circ}_{CO}(T_f) - \bar{s}^{\circ}_{CO}(T_{CO}) - \bar{R}\ln\frac{y_{CO}p_f}{p_{CO}}\right] = 0.1428\left[49.715 - 51.353 - 1.986\ln\frac{19.458}{73.542}\right] = 0.1432\ Btu/°R$$

$$\Delta S)_{He} = n_{He}\left[\bar{c}_{p,He}\ln\frac{T_f}{T_{He}} - \bar{R}\ln\frac{y_{He}p_f}{p_{He}}\right] = (0.1999)[1.986]\left(2.5\ln\frac{762}{520} - \ln\frac{27.238}{27.882}\right) = 0.3885$$

Thus,

$$E_d = (520°R)[0.1432 + 0.3885]\ \frac{Btu}{°R} = 276.5\ Btu$$ $\longleftarrow$ E_d

---

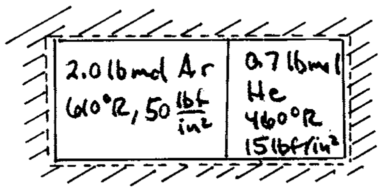
1. An iterative solution using table data can be avoided using IT. The results agree with those given here.

PROBLEM 12.32*

KNOWN: Argon and helium, each initially at different temperatures and pressures are allowed to mix adiabatically to a final equilibrium state.

FIND: Determine the final temperature and pressure, and the amount of entropy produced.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system shown in the accompanying figure experiences no change in kinetic or potential energy, and $Q=0$. (2) Each gas behaves as an ideal gas. The final mixture adheres to the Dalton model.

ANALYSIS: (a) An energy balance reduces as follows, $\Delta U = \overset{0}{\cancel{Q}} - \overset{0}{\cancel{W}}$. Thus

$$\Delta U = \Delta U)_{Ar} + \Delta U)_{He} = n_{Ar}\bar{c}_{v,Ar}[T_f - T_{Ar}] + n_{He}\bar{c}_{v,He}[T_f - T_{He}] = 0 \qquad (1)$$

where T_{Ar}, T_{He} denote the initial temperatures of the argon and helium, respectively. The final temperature is denoted by T_f. From Table A-21E for monotomic gases $\bar{c}_p = 5/2\,\bar{R}$. Then, since $\bar{c}_p = \bar{c}_v + \bar{R}$ (Eq. 3.45), $\bar{c}_v = \frac{3}{2}\bar{R}$, and Eq. (1) reduces to give

$$T_f = \frac{n_{Ar}T_{Ar} + n_{He}T_{He}}{n_{Ar}+n_{He}} = \frac{(2.0)(610)+(0.7)(460)}{2.7} = 571.11\ °R \quad (111\ °F) \longleftarrow T_f$$

(b) The ideal gas equation of state gives $P_f = n\bar{R}T_f/V$, where V is the total volume:

$$V = \frac{n_{Ar}\bar{R}T_{Ar}}{P_{Ar}} + \frac{n_{He}\bar{R}T_{He}}{P_{He}}$$

where P_{Ar}, P_{He} denote the initial pressures of the argon and helium, and not partial pressures. Thus

$$P_f = \frac{(n_{Ar}+n_{He})\bar{R}T_f}{\left[\frac{n_{Ar}\bar{R}T_{Ar}}{P_{Ar}} + \frac{n_{He}\bar{R}T_{He}}{P_{He}}\right]} = \frac{(2.7)(571.11)}{\left[\frac{(2)(610)}{(50)} + \frac{(0.7)(460)}{(15)}\right]} = 33.62\ \frac{lbf}{in^2}$$

$$= 2.29\ atm \longleftarrow P_f$$

(c) An entropy balance reduces as follows

$$\Delta S = \cancel{\int_1^2 \left(\frac{\delta Q}{T}\right)_b}^{\,0} + \sigma \Rightarrow \sigma = \Delta S = \Delta S)_{Ar} + \Delta S)_{He} \qquad (2)$$

The argon undergoes a process from 610 °R, 50 lbf/in² to 571.11 °R and the partial pressure of argon in the final mixture: $y_{Ar}P_f = \left(\frac{2}{2.7}\right)(33.62) = 24.90\ lbf/in^2$. Thus, with $\bar{c}_p = \frac{5}{2}\bar{R}$

$$\Delta S)_{Ar} = (2\ lbmol)\left(1.986\ \frac{Btu}{lbmol\cdot °R}\right)\left[\frac{5}{2}\ln\frac{571.11}{610} - \ln\frac{24.90}{50}\right] = 2.115\ \frac{Btu}{°R}$$

The helium undergoes a process from 460 °R, 15 lbf/in² to 571.11 °R and the partial pressure of helium in the final mixture: $y_{He}P_f = \left(\frac{0.7}{2.7}\right)(33.62) = 8.72\ lbf/in^2$. Thus

$$\Delta S)_{He} = (0.7)(1.986)\left[\frac{5}{2}\ln\frac{571.11}{460} - \ln\frac{8.72}{15}\right] = 1.506\ \frac{Btu}{°R}$$

Inserting values into Eq. (2)

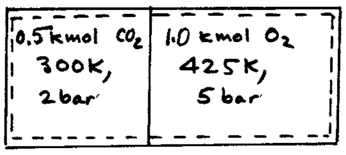
$$\sigma = 2.115 + 1.506 = 3.621\ \frac{Btu}{°R} \longleftarrow \sigma$$

PROBLEM 12.33

KNOWN: CO_2 and O_2, each at initially different temperatures and pressures are allowed to mix adiabatically while a specified amount of energy is added by electrical work.

FIND: Determine the final temperature and pressure, the change in exergy and the exergy destruction.

SCHEMATIC & GIVEN DATA:



$W = -500\ kJ$
$T_0 = 293\ K$

ASSUMPTIONS: (1) For the system shown in the accompanying figure $Q = 0$, and kinetic and potential energy effects are negligible. (2) Each gas behaves as an ideal gas and the final mixture adheres to the Dalton model.

ANALYSIS: (a) An energy balance reduces to the form $\Delta U = \overset{0}{\cancel{Q}} - W$, or

$$-W = \Delta U = \Delta U)_{CO_2} + \Delta U)_{O_2} = n_{CO_2}[\bar{u}_{CO_2}(T_f) - \bar{u}_{CO_2}(T_{CO_2})] + n_{O_2}[\bar{u}_{O_2}(T_f) - \bar{u}_{O_2}(T_{O_2})]$$

where T_f is the final temperature and T_{CO_2}, T_{O_2} denote the initial temperatures of the CO_2 and O_2, respectively. Solving for T_f

$$n_{CO_2}\bar{u}_{CO_2}(T_f) + n_{O_2}\bar{u}_{O_2}(T_f) = n_{CO_2}\bar{u}(T_{CO_2}) + n_{O_2}\bar{u}(T_{O_2}) - W$$

Inserting known values and data from Tables A-24, A-27

$$0.5\bar{u}_{CO_2}(T_f) + 1.0\bar{u}_{O_2}(T_f) = 12902\ kJ$$

① Solving iteratively using table data $T_f = 387\ K$. ← T_f

(b) The ideal gas equation of state gives $p_f = n\bar{R}T_f/V$, where V is the total volume:

$$V = \frac{n_{CO_2}\bar{R}T_{CO_2}}{p_{CO_2}} + \frac{n_{O_2}\bar{R}T_{O_2}}{p_{O_2}}$$

Combining results, the final pressure is

$$p_f = \frac{(n_{CO_2} + n_{O_2})T_f}{\left(\frac{n_{CO_2}T_{CO_2}}{p_{CO_2}} + \frac{n_{O_2}T_{O_2}}{p_{O_2}}\right)} = \frac{(1.5)(387)}{\frac{(0.5)(300)}{2} + \frac{(1)(425)}{5}} = 3.628\ \text{bar}$$ ← p_f

where p_{CO_2}, p_{O_2} denote the initial pressures of the CO_2 and O_2, respectively, and not partial pressures.

(c) The change in exergy is found from Eq. 7.10, which reduces to $\Delta E = \Delta U + \overset{0}{\cancel{p_0\Delta V}} - T_0\Delta S$. Since $W = -\Delta U$, $\Delta E = (-W) - T_0\Delta S$, where

$$\Delta S = \Delta S)_{CO_2} + \Delta S)_{O_2} = n_{CO_2}\left[\bar{s}^\circ_{CO_2}(T_f) - \bar{s}^\circ_{CO_2}(T_{CO_2}) - \bar{R}\ln\frac{y_{CO_2}p_f}{p_{CO_2}}\right] + n_{O_2}\left[\bar{s}^\circ_{O_2}(T_f) - \bar{s}^\circ_{O_2}(T_{O_2}) - \bar{R}\ln\frac{y_{O_2}p_f}{p_{O_2}}\right]$$

with data from Table A-23

$$\Delta S = (0.5)\left[223.864 - 213.915 - 8.314\ln\left(\frac{(1/3)(3.628)}{2}\right)\right] + (1.0)\left[212.768 - 215.598 - 8.314\ln\left(\frac{(2/3)(3.628)}{5}\right)\right] = 10.274\ \frac{kJ}{K}$$

Then

$$\Delta E = 500 - 293(10.274) = -2510.3\ kJ$$ ← ΔE

PROBLEM 12.33 (Contd.)

(d) The exergy destruction can be found using $E_d = T_0 \sigma$, where σ is obtained from an entropy balance. Thus

$$E_d = T_0\, \Delta S = (293\,\text{K})\left(10.274\,\frac{\text{kJ}}{\text{K}}\right) = 3010.3\ \text{kJ} \qquad \xleftarrow{\qquad\qquad\qquad E_d}$$

Alternatively, an exergy balance can be used:

$$\Delta E = \int_1^2 \cancelto{0}{\left[1 - \frac{T_0}{T_b}\right]\delta Q} - [W - p_0 \cancelto{0}{\Delta V}] - E_d \Rightarrow E_d = (-W) - \Delta E$$

or

$E_d = 500\ \text{kJ} - (-2510.3) = 3010.3\ \text{kJ}$. The exergy destruction occurs because different gases at different initial states are allowed to mix.

---

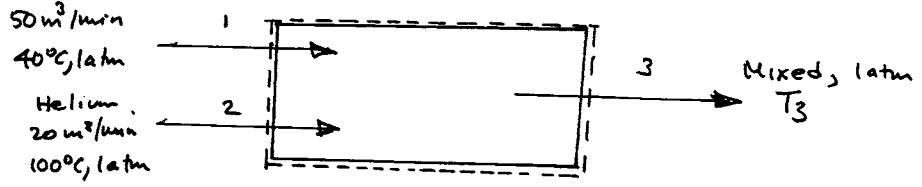
1. An iterative solution using table data can be avoided by using IT. The results agree with those given here.

PROBLEM 12.34

KNOWN: 50 m^3/min of air at 40°C, 1atm is mixed adiabatically and at steady state with 20 m^3/min of helium at 100°C, 1atm to form a mixed stream at 1atm.

FIND: Determine (a) the temperature of the exiting mixture, (b) the rate of entropy production.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and effects of kinetic/potential energy can be ignored. (3) The entering gases each can be modeled as an ideal gas. The exiting mixture adheres to the Dalton model. (4) The specific heat $c_{p,air}$ is taken as a constant at 350K

ANALYSIS: (a) Mass rate balances at steady state indicate that

$$\dot{m}_{a1} = \dot{m}_{a3} \quad (\equiv \dot{m}_a)$$
$$\dot{m}_{h2} = \dot{m}_{h3} \quad (\equiv \dot{m}_h)$$

An energy rate balance reduces at steady state to read

$$0 = [\dot{m}_{a1} h_a(T_1) + \dot{m}_{h2} h_h(T_2)] - [\dot{m}_{a3} h_a(T_3) + \dot{m}_{h3} h_h(T_3)]$$

or

$$0 = \dot{m}_a [h_a(T_3) - h_a(T_1)] + \dot{m}_h [h_h(T_3) - h_h(T_2)] \qquad (1)$$

For Helium Table A-21 indicates that $c_{p,h} = 5/2\,\bar{R}$. Since the final temperature will fall between T_1 and T_2, and the difference is relatively narrow, c_p for air is also taken as constant. The value at 350K is used: $c_{p,air} = 1.008$ kJ/kg·K (Table A-20). Thus, Eq. (1) gives

$$0 = \dot{m}_a\, c_{p,air}(T_3 - T_1) + \dot{m}_h\, c_{p,h}(T_3 - T_2) = 0 \qquad (2)$$

The mass flow rates are found from the volumetric flow rates

$$\dot{m}_a = \frac{(AV)_1 p_1}{RT_1} = \frac{(50\ m^3/min)(1.013\times10^5\ N/m^2)}{\left(\frac{8314}{28.97}\frac{N\cdot m}{kg\cdot K}\right)(313\ K)}\left|\frac{1\ min}{60\ s}\right| = 0.9398\ \frac{kg}{s}$$

$$\dot{m}_h = \frac{(20)(1.013\times10^5)}{\left(\frac{8314}{4}\right)(373)(60)} = 0.0436\ \frac{kg}{s}$$

Solving Eq. (2)

T_3

$$0 = \left(0.9398\ \frac{kg}{s}\right)\left(1.008\ \frac{kJ}{kg\cdot K}\right)(T_3 - 313) + (0.0436)\Big(\overbrace{\tfrac{5}{2}\left(\tfrac{8.314}{4}\right)}^{(c_{p,h} = 5.196\ kJ/kg\cdot K)}\Big)(T_3 - 373) \Rightarrow T_3 = 324.6K$$
(52°C)

(b) Reducing an entropy rate balance

$$0 = \dot{m}_a s_a(T_1, p) + \dot{m}_h s_h(T_2, p) - [\dot{m}_a s_a(T_3, y_a p) + \dot{m}_h s_h(T_3, y_h p)] + \dot{\sigma}$$

Rearranging

$$\dot{\sigma} = \dot{m}_a\left[c_{p,air} \ln\frac{T_3}{T_1} - \frac{\bar{R}}{M_{air}} \ln\frac{y_a \cancel{p}}{\cancel{p}}\right] + \dot{m}_h\left[c_{p,h} \ln\frac{T_3}{T_2} - \frac{\bar{R}}{M_h} \ln\frac{y_h \cancel{p}}{\cancel{p}}\right]$$

PROBLEM 12.34 (Contd)

The mole fractions are evaluated as follows:

$$\dot{n}_a = \frac{\dot{m}_a}{M_{air}} = \frac{0.9398}{28.97} = 0.0324 \frac{\text{kmol}}{s} \quad \Big\rangle \; \dot{n} = 0.0433$$

$$\dot{n}_h = \frac{\dot{m}_h}{M_h} = \frac{0.0436}{4} = 0.0109 \frac{\text{kmol}}{s}$$

So $y_a = \frac{\dot{n}_a}{\dot{n}} = \frac{0.0324}{0.0433} = 0.748$, $y_h = \frac{\dot{n}_h}{\dot{n}} = \frac{.0109}{.0433} = 0.252$

Solving for $\dot{\sigma}$

$$\dot{\sigma} = \left(0.9398 \frac{kg}{s}\right)\left[\left(1.008 \frac{kJ}{kg \cdot K}\right) \ln \frac{324.6}{313} - \frac{8.314}{2897} \ln 0.748\right] \left|\frac{1 kW}{1 kJ/s}\right| +$$

$$(0.0436)\left[(5.196) \ln \frac{324.6}{373} - \frac{8.314}{4} \ln 0.252\right] \frac{kW}{K}$$

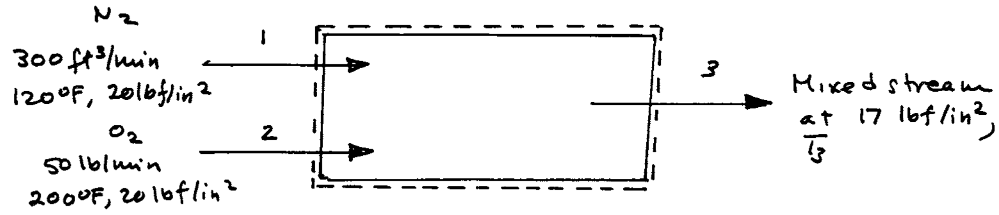
$$= (0.9398)(0.12) + (0.0436)(2.1428) = 0.2062 \frac{kW}{K} \longleftarrow \dot{\sigma}$$

PROBLEM 12.35

KNOWN: 300 ft³/min of N_2 at 120°F, 20 lbf/in² is mixed adiabatically at steady state with 50 lb/min of O_2 at 200°F, 20 lbf/in² to form a mixed stream at 17 lbf/in².

FIND: Determine (a) the temperature of the exiting mixture and (b) the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$ and all kinetic/potential energy effects can be ignored. (3) The entering gases can each be modeled as an ideal gas. The exiting mixture adheres to the Dalton model. (4) The specific heats c_p can be taken as constants. (5) For the exergy reference environment, $T_0 = 500°R$.

ANALYSIS: (a) At steady state $\dot{m}_{N_2,1} = \dot{m}_{N_2,3}$, $\dot{m}_{O_2,2} = \dot{m}_{O_2,3}$. The energy rate balance reduces to read

$$0 = \dot{m}_{N_2} h_{N_2}(T_1) + \dot{m}_{O_2} h_{O_2}(T_2) - [\dot{m}_{N_2} h_{N_2}(T_3) + \dot{m}_{O_2} h_{O_2}(T_3)]$$

or on rearrangement and with assumption (4)

$$0 = \dot{m}_{N_2} c_{p,N_2} [T_3 - T_1] + \dot{m}_{O_2} c_{p,O_2} [T_3 - T_2] \quad (1)$$

PROBLEM 12.35 (Contd.)

As T_3 will fall between T_1 and T_2, and the temperature difference is relatively narrow, c_p for each gas is taken as constant at the average temperature of 160°F. With data from Table A-20E, $c_{p,N_2} = 0.249$ Btu/lb·°R, $c_{p,O_2} = 0.222$ Btu/lb·°R. The mass flow rate of N_2 is found as follows:

$$\dot{m}_{N_2} = \frac{(AV)_1 p_1}{RT_1} = \frac{(300\,\text{ft}^3/\text{min})(20 \times 144\,\text{lbf}/\text{ft}^2)}{\left(\frac{1545}{28.01}\,\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot{}^\circ\text{R}}\right)(580\,{}^\circ\text{R})} = 27\,\text{lb}/\text{min}.$$

Substituting values in Eq.(1) and solving

$$0 = \left(27\,\frac{\text{lb}}{\text{min}}\right)\left(0.249\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}}\right)[T_3 - 580]\,{}^\circ\text{R} + (50)(0.222)(T_3 - 660) \Rightarrow T_3 = 630\,\text{K}$$

$$(170\,{}^\circ\text{F})$$

(b) An entropy rate balance reads

$$0 = \dot{m}_{N_2} s_{N_2}(T_1, p_1) + \dot{m}_{O_2} s_{O_2}(T_2, p_2) - \left[\dot{m}_{N_2} s_{N_2}(T_3, y_{N_2} p_3) + \dot{m}_{O_2} s_{O_2}(T_3, y_{O_2} p_3)\right] + \dot{\sigma}$$

or

$$\dot{\sigma} = \dot{m}_{N_2}\left[s_{N_2}(T_3, y_{N_2} p_3) - s_{N_2}(T_1, p_1)\right] + \dot{m}_{O_2}\left[s_{O_2}(T_3, y_{O_2} p_3) - s_{O_2}(T_2, p_2)\right]$$

$$= \dot{m}_{N_2}\left[c_{p,N_2} \ln \frac{T_3}{T_1} - \frac{\bar{R}}{M_{N_2}} \ln \frac{y_{N_2} p_3}{p_1}\right] + \dot{m}_{O_2}\left[c_{p,O_2} \ln \frac{T_3}{T_2} - \frac{\bar{R}}{M_{O_2}} \ln \frac{y_{O_2} p_3}{p_2}\right]$$

To obtain the mole fractions

$$\left.\begin{aligned} \dot{n}_{N_2} &= \frac{\dot{m}_{N_2}}{M_{N_2}} = \frac{27}{28.01} = 0.9639 \\ \dot{n}_{O_2} &= \frac{\dot{m}_{O_2}}{M_{O_2}} = \frac{50}{32} = 1.5625 \end{aligned}\right\rangle \dot{n} = 2.5264$$

$$y_{N_2} = \frac{0.9639}{2.5264} = 0.3815, \quad y_{O_2} = \frac{1.5625}{2.5264} = 0.6185$$

Thus

$$\dot{\sigma} = \left(27\,\frac{\text{lb}}{\text{min}}\right)\left[0.249\,\frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{R}} \ln \frac{630}{580} - \frac{1.986}{28.01} \ln\left(\frac{(0.3815)(17)}{20}\right)\right] +$$

$$(50)\left[0.222 \ln \frac{630}{660} - \frac{1.986}{32} \ln\left(\frac{(0.6185)(17)}{20}\right)\right]$$

$$= (2.712 + 1.480)\,\frac{\text{Btu}}{\text{min}\cdot{}^\circ\text{R}} = 4.192\,\frac{\text{Btu}}{\text{min}\cdot{}^\circ\text{R}}$$

The rate of exergy destruction can be found as follows:

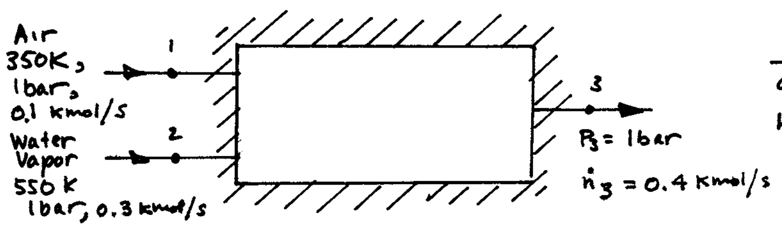
$$\dot{E}_d = T_0 \dot{\sigma} = (500\,{}^\circ\text{R})\left(4.192\,\frac{\text{Btu}}{\text{min}\cdot{}^\circ\text{R}}\right) = 2096\,\frac{\text{Btu}}{\text{min}} \qquad \xleftarrow{\quad \dot{E}_d \quad}$$

PROBLEM 12.36

KNOWN: Air at a specified temperature and pressure enters an insulated chamber at steady state and mixes with water vapor entering at a specified temperature and pressure. The mixture exits at a known pressure and molar analysis.

FIND: Determine the temperature of the exiting mixture and the rate entropy is produced.

SCHEMATIC & GIVEN DATA:



At 3, based on the given molar flow rates,

i	y_i
air	0.25
H_2O	0.75

ASSUMPTIONS: (1) The mixing chamber is well insulated and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) Each gas can be modeled as an ideal gas and the exiting mixture adheres to the Dalton model.

ANALYSIS: (a) An energy rate balance at steady state, expressed on a molar basis, reduces with assumptions 1 and 2 to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{n}_1 \bar{h}_1 + \dot{n}_2 \bar{h}_2 - \dot{n}_3 \bar{h}_3 \Rightarrow \bar{h}_3 = \left(\frac{\dot{n}_1}{\dot{n}_3}\right)\bar{h}_1 + \left(\frac{\dot{n}_2}{\dot{n}_3}\right)\bar{h}_2$$

where $\dot{n}_1$, $\dot{n}_2$, and $\dot{n}_3$ are the molar flow rates of the air, water vapor, and mixture, respectively. Since $\dot{n}_1/\dot{n}_3 = 0.25$ and $\dot{n}_2/\dot{n}_3 = 0.75$, this becomes

$$\bar{h}_3 = 0.25\,\bar{h}_1 + 0.75\,\bar{h}_2 = 0.25\,\bar{h}_{air}(T_1) + 0.75\,\bar{h}_{H_2O}(T_2)$$

The enthalpy of the mixture, per kmol of mixture, is $\bar{h}_3 = 0.25\,\bar{h}_{air}(T_3) + 0.75\,\bar{h}_{H_2O}(T_3)$.
① Accordingly, with data from Table A-23,

$$0.25\,[28.97\,h_{air}(T_3)] + 0.75\,\bar{h}_{H_2O} = (0.25)(28.97)(350.49) + (0.75)(18601) = 16489 \text{ kJ}$$

where $\bar{h}_{air} = M_{air} h_{air}$,
② An iterative solution using data from Table A-23 gives
T_3 = 507 K (234°C). ←———————— T_3

(b) An entropy rate balance at steady state, expressed on a molar basis, reads

$$0 = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{n}_1 \bar{s}_1 + \dot{n}_2 \bar{s}_2 - \dot{n}_3 \bar{s}_3 + \dot{\sigma}_{cv}$$

Thus

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = \bar{s}_3 - \left(\overset{0.25}{\frac{\dot{n}_1}{\dot{n}_3}} \bar{s}_1 + \overset{0.75}{\frac{\dot{n}_2}{\dot{n}_3}} \bar{s}_2 \right)$$

The specific entropy of the mixture is

$$\bar{s}_3 = 0.25\,\bar{s}_{Air}(T_3, y_{air} P_3) + 0.75\,\bar{s}_{H_2O}(T_3, y_{H_2O} P_3)$$

Combining the last two equations and using s° data from Table A-23

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = 0.25\left[\bar{s}_{Air}(T_3, y_{Air} P_3) - \bar{s}_{Air}(T_1, P_1)\right] + 0.75\left[\bar{s}_{H_2O}(T_3, y_{H_2O} P_3) - \bar{s}_{H_2O}(T_2, P_2)\right]$$

$$= (0.25)(28.97)\left[s^\circ_{Air}(T_3) - s^\circ_{Air}(T_1) - \frac{\bar{R}}{M_{air}} \ln \frac{y_{air} P_3}{P_1} \right] +$$

$$0.75\left[\bar{s}^\circ_{H_2O}(T_3) - \bar{s}^\circ_{H_2O}(T_2) - \bar{R} \ln \frac{y_{H_2O} P_3}{P_2} \right]$$

PROBLEM 12.36 (contd.)

Continuing the calculation

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_3} = (0.25)(28.97)\left[2.2338 - 1.85708 - \frac{8.314}{28.97}\ln\frac{(0.25)(1)}{(1)}\right] + 0.75\left[206.902 - 209.795 - 8.314\ln\frac{(0.75)(1)}{(1)}\right]$$

$$= 5.6098 + (-0.3759) = 5.2339\ \frac{kJ}{kmol\cdot K}$$

or

$$\dot{\sigma}_{cv} = \left(0.4\ \frac{kmol}{s}\right)\left(5.2339\ \frac{kJ}{kmol\cdot K}\right)\left|\frac{1\ kW}{1\ kJ/s}\right| = 2.094\ \frac{kW}{K} \qquad \longleftarrow$$

---

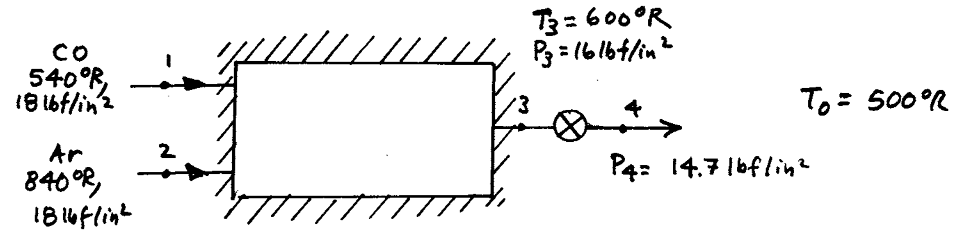
1. Table A-23 can be use here for water vapor because water vapor is modeled as an ideal gas at each of the three states.
2. An iterative solution using table data can be avoided in this case by using constant c_p for dry air and water vapor evaluated at the average of the inlet temperatures: 450 K. Alternatively, IT could be used.

PROBLEM 12.37

KNOWN: Streams of CO and Ar, each at known states, form a mixture that expands across a valve to a specified pressure.

FIND: Determine (a) the mass and molar analyses of the mixture, (b) the temperature at the valve exit, (c) the rates of exergy destruction for the mixing chamber and the valve, each per unit mass of mixture.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The mixing chamber is well insulated and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) The expansion across the valve is a throttling process. (4) Each gas can be modeled as an ideal gas and the mixture adheres to the Dalton model. (5) $T_0 = 500°R$.

ANALYSIS: (a) Reducing mass and energy rate balances for a control volume enclosing the mixing chamber, we get $0 = \dot{m}_1 h_1 + \dot{m}_2 h_2 - \dot{m}_3 h_3$, or expressed alternatively

$$0 = \dot{m}_{CO} h_{CO}(540°R) + \dot{m}_{Ar} h_{Ar}(840°R) - [\dot{m}_{CO} h_{CO}(600°R) + \dot{m}_{Ar} h_{Ar}(600°R)]$$

$$\Rightarrow \dot{m}_{CO}\left[\frac{\bar{h}_{CO}(600°R) - \bar{h}_{CO}(540°R)}{M_{CO}}\right] = \dot{m}_{Ar}\left[\frac{\bar{h}_{Ar}(840) - \bar{h}_{Ar}(600)}{M_{Ar}}\right]$$

For Ar from Table A-21, $\bar{c}_p = 2.5\bar{R}$. Data for CO is obtained from Table A-23. Thus

$$\dot{m}_{CO}\left[\frac{4168 - 3750.3}{28.01}\right] = \dot{m}_{AR}\left[\frac{(2.5)(1.986)[240]}{39.94}\right] \Rightarrow \frac{\dot{m}_{CO}}{\dot{m}_{Ar}} = 2$$

Accordingly, $(mf)_{CO} = 2/3$, $(mf)_{Ar} = 1/3$

Considering a typical unit mass of mixture,

$$n_{CO} = \frac{2/3 \text{ lb}}{28.01 \text{ lb/lbmol}} = 0.0238, \quad n_{Ar} = \frac{1/3}{39.94} = 0.0083 \Rightarrow$$

$$y_{CO} = \frac{0.0238}{0.0321} = 0.741$$

$$y_{Ar} = 0.259$$

(b) Since the expansion across the valve is a throttling process, $h_4 = h_3$. And since the enthalpy of an ideal gas depends only on temperature, $T_4 = T_3 = 600°R$.

(c) The rate of exergy destruction can be found using $\dot{E}_d = T_0 \dot{\sigma}$, where $\dot{\sigma}$ is the rate of entropy production. For a control volume enclosing the mixing chamber

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}_1 s_{CO}(T_1, P_1) + \dot{m}_2 s_{Ar}(T_2, P_2) - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

Thus, with $\bar{s}°$ data from Table A-23

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = s_3 - \left(\frac{\dot{m}_1}{\dot{m}_3}\right) s_{CO}(T_1, P_1) - \left(\frac{\dot{m}_2}{\dot{m}_3}\right) s_{Ar}(T_2, P_2)$$

$$= [(mf)_{CO}\, s_{CO}(T_3, y_{CO}P_3) + (mf)_{Ar}\, s_{Ar}(T_3, y_{Ar}P_3)] - \left(\frac{\dot{m}_1}{\dot{m}_3}\right) s_{CO}(T_1, P_1) - \left(\frac{\dot{m}_2}{\dot{m}_3}\right) s_{Ar}(T_2, P_2)$$

$$= \frac{2}{3}\left[s_{CO}(T_3, y_{CO}P_3) - s_{CO}(T_1, P_1)\right] + \frac{1}{3}\left[s_{Ar}(T_3, y_{Ar}P_3) - s_{Ar}(T_2, P_2)\right]$$

PROBLEM 12.37 (Contd.)

That is

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = \frac{2}{3}\left[\frac{\bar{s}^o_{co}(T_3) - \bar{s}^o_{co}(T_1) - \bar{R}\ln\frac{y_{co}p_3}{p_1}}{M_{co}}\right] + \frac{1}{3}\left[\frac{\bar{c}_{p,Ar}\ln\frac{T_3}{T_2} - \bar{R}\ln\frac{y_{Ar}p_3}{p_2}}{M_{Ar}}\right]$$

$$= \frac{2}{3}\left(\frac{48.044 - 47.310 - 1.986\ln\left(\frac{0.741\times 16}{18}\right)}{28.01}\right] + \frac{1}{3}\left[\frac{(1.986)\left[2.5\ln\frac{600}{840} - \ln\left(\frac{0.259\times 16}{18}\right)\right]}{39.94}\right]$$

$$= 0.0372 + 0.0104 = 0.0476\ \frac{\text{Btu}/^oR}{\text{lb(mix)}}$$

Thus, for the mixing chamber,

$$(\dot{E}_d)_{mix} = T_o\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = (500^oR)\left(0.0476\ \frac{\text{Btu}/^oR}{\text{lb(mix)}}\right) = 23.8\ \frac{\text{Btu}}{\text{lbmix}}$$ ←

For a control volume enclosing the valve, an entropy rate balance at steady state reduces to

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = s_4 - s_3$$

$$= \left[(mf)_{co}\, s_{co}(T_4, y_{co}p_4) + (mf)_{Ar}\, s_{Ar}(T_4, y_{Ar}p_4)\right]$$

$$- \left[(mf)_{co}\, s_{co}(T_3, y_{co}p_3) + (mf)_{Ar}\, s_{Ar}(T_3, y_{Ar}p_3)\right]$$

$$= (mf)_{co}\left[\frac{\bar{s}^o_{co}(T_4) - \bar{s}^o_{co}(T_3) - \bar{R}\ln\frac{y_{co}p_4}{y_{co}p_3}}{M_{co}}\right] + (mf)_{Ar}\left[\frac{\bar{c}_{p,Ar}\ln\frac{T_4}{T_3} - \bar{R}\ln\frac{y_{Ar}p_4}{y_{Ar}p_3}}{M_{Ar}}\right]$$

Since $T_3 = T_4$, the terms involving temperature vanish, leaving

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_3} = -\bar{R}\left[\frac{(mf)_{co}}{M_{co}} + \frac{(mf)_{Ar}}{M_{Ar}}\right]\ln\frac{p_4}{p_3}$$

$$= -1.986\left[0.0238 + 0.0083\right]\ln\frac{14.7}{16} = 0.0054\ \frac{\text{Btu}/^oR}{\text{lb(mix)}}$$

Then

① $$(\dot{E}_d)_{valve} = (500)(0.0054) = 2.7\ \frac{\text{Btu}}{\text{lbmix}}$$ ←

---

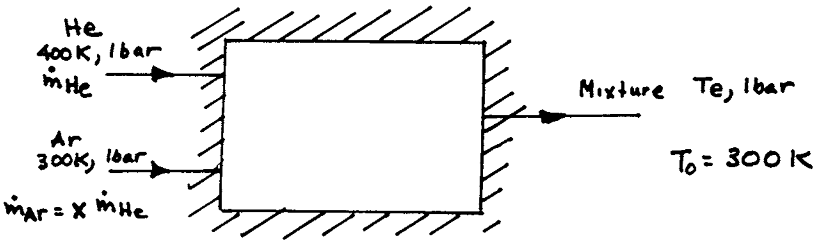
1. Significant exergy destruction accompanies the mixing of different substances initially at different states and the expansion of substances across valves.

PROBLEM 12.38

KNOWN: Helium at 400 K, 1 bar enters an insulated mixing chamber where it mixes with argon entering at 300 K, 1 bar. The mixture exits at a pressure of 1 bar. The argon mass flow rate is x times that of helium.

FIND: Plot versus x the exit temperature and the rate of exergy destruction per unit mass of He entering.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The mixing chamber is well insulated and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) Each gas can be modeled as an ideal gas and the mixture adheres to the Dalton model.

ANALYSIS: At steady state the mass flow rate of the mixture exiting equals the sum of the incoming flow rates:

$$\dot{m}_{mix} = \dot{m}_{He} + \dot{m}_{Ar} = (1+x)\dot{m}_{He} \qquad (1)$$

The energy rate balance reduces at steady state to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_{He}\, h_{He}(400K) + \dot{m}_{Ar}\, h_{Ar}(300K) - \left[\dot{m}_{He}\, h_{He}(T_e) + \dot{m}_{Ar}\, h_{Ar}(T_e)\right]$$

Thus

$$\dot{m}_{He}\left[h_{He}(T_e) - h_{He}(400K)\right] + \dot{m}_{Ar}\left[h_{Ar}(T_e) - h_{Ar}(300K)\right] = 0$$

For the monatomic gases He and Ar, $c_p = 5/2\ \bar{R}/M$ (Table A-21). Accordingly,

$$\dot{m}_{He}\left[\frac{5}{2}\frac{\bar{R}}{M_{He}}(T_e - 400)\right] + x\,\dot{m}_{He}\left[\frac{5}{2}\frac{\bar{R}}{M_{Ar}}(T_e - 300)\right] = 0$$

or

$$\frac{T_e - 400}{M_{He}} + \frac{x}{M_{Ar}}\left[T_e - 300\right] = 0$$

giving

$$T_e = \frac{\dfrac{400}{M_{He}} + \dfrac{300x}{M_{Ar}}}{\dfrac{1}{M_{He}} + \dfrac{x}{M_{Ar}}} = \frac{400\left(\dfrac{M_{Ar}}{M_{He}}\right) + 300x}{\left(\dfrac{M_{Ar}}{M_{He}}\right) + x} \qquad (2)$$

where $M_{Ar} = 39.94$, $M_{He} = 4.003$.

The entropy rate balance reduces at steady state to give

$$0 = \cancelto{0}{\sum \frac{\dot{Q}_j}{T_j}} + \dot{m}_{He}\, s_{He}(400K, 1bar) + \dot{m}_{Ar}\, s_{Ar}(300K, 1bar)$$
$$- \left[\dot{m}_{He}\, s_{He}(T_e, y_{He}\cdot 1bar) + \dot{m}_{Ar}\, s_{Ar}(T_e, y_{Ar}\cdot 1bar)\right] + \dot{\sigma}_{cv}$$

PROBLEM 12.38 (Cont'd.)

Since $y_{Ar} + y_{He} = 1$, $y_{He} = 1 - y_{Ar}$, and $\dot{E}_d/\dot{m}_{He} = T_0 \dot{\sigma}_{cv}/\dot{m}_{He}$. Thus

$$\frac{\dot{E}_d}{\dot{m}_{He}} = T_0 \left\{ \frac{\bar{R}}{M_{He}} \left[\frac{5}{2} \ln \frac{T_e}{400} - \ln(1 - y_{Ar}) \right] + x \frac{\bar{R}}{M_{Ar}} \left[\frac{5}{2} \ln \frac{T_e}{300} - \ln y_{Ar} \right] \right\} \tag{3}$$

To find y_{Ar}, notice that the mass fractions of He and Ar in the exiting mixture are, respectively

$$(mf)_{He} = \frac{\dot{m}_{He}}{\dot{m}_{mix}} = \frac{\dot{m}_{He}}{(1+x)\dot{m}_{He}} = \frac{1}{1+x}$$

$$(mf)_{Ar} = \frac{\dot{m}_{Ar}}{\dot{m}_{mix}} = \frac{x\,\dot{m}_{He}}{(1+x)\dot{m}_{He}} = \frac{x}{1+x}$$

The number of moles of He and Ar in a typical unit mass of mixture is

$$n_{He} = \frac{1/(1+x)}{M_{He}} \quad , \quad n_{Ar} = \frac{x/(1+x)}{M_{Ar}}$$

Thus

$$y_{Ar} = \frac{n_{Ar}}{n_{He} + n_{Ar}} = \frac{\left(\frac{x}{(1+x) M_{Ar}}\right)}{\left[\frac{1}{(1+x) M_{He}} + \frac{x}{(1+x) M_{Ar}}\right]} = \frac{x}{\left[\frac{M_{Ar}}{M_{He}} + x\right]} \tag{4}$$

The data for the required plots are obtained using *IT*, as follows:

IT Code

```
mdotHe = 1
MHe = 4.003  // kg/kmol
MAr = 39.94  // kg/kmol
To = 300  // K
Rbar = 8.314  // kJ/kmol·K
x = 1

mdotHe + mdotAr = (1 +x) * mdotHe
Te = (400 * (MAr / MHe) + 300 * x) / ((MAr / MHe) + x)

Edotd / mdotHe = To * (A + B)
A = (Rbar / MHe) * (2.5 * ln(Te / 400) -  ln(1 - yAr))
B = x * (Rbar / MAr) * (2.5 * ln(Te / 300) - ln(yAr))
yAr = x / ((MAr / MHe) + x)
```

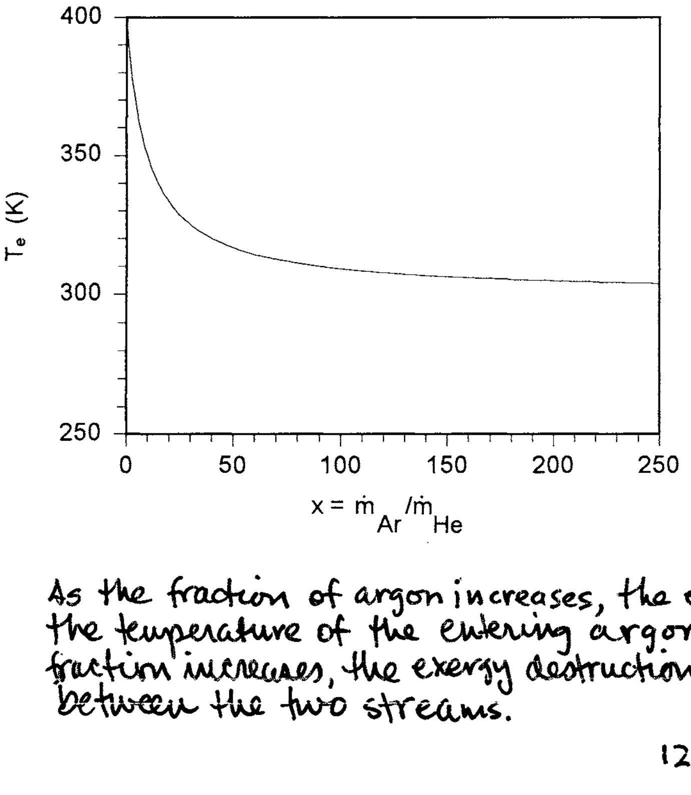
IT Results for x = 1

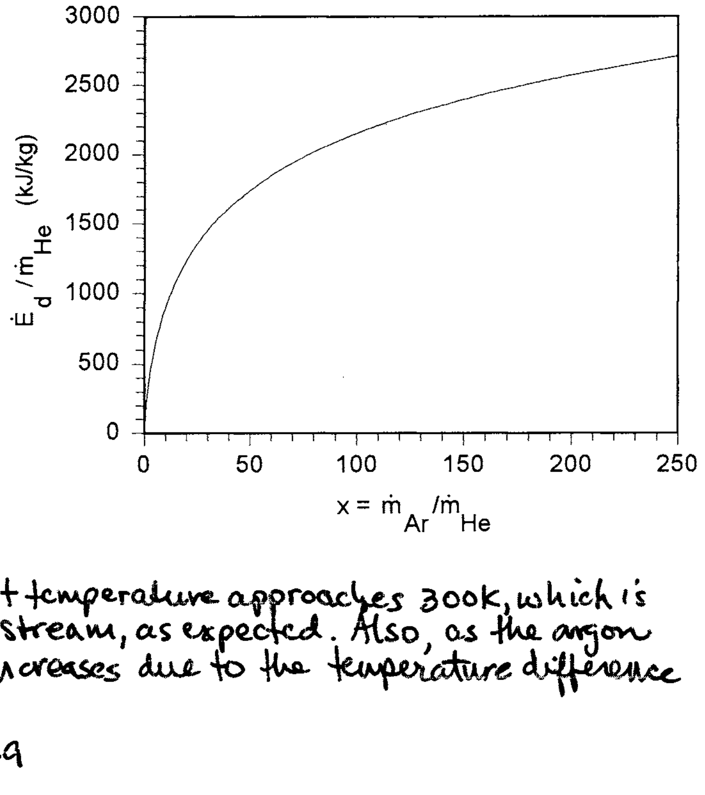
$y_{Ar} = 0.0911$

$T_e = 390.9$ K

$\dot{E}_d/\dot{m}_{He} = 214.6$ kJ/kg

PLOTS:





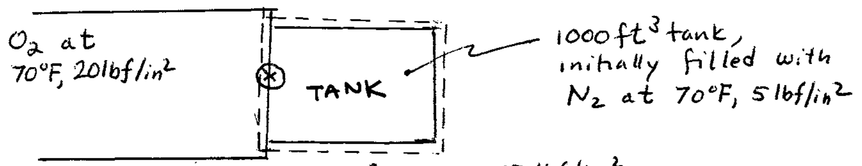
As the fraction of argon increases, the exit temperature approaches 300K, which is the temperature of the entering argon stream, as expected. Also, as the argon fraction increases, the exergy destruction increases due to the temperature difference between the two streams.

PROBLEM 12.39

KNOWN: Oxygen is allowed to flow into a 1000 ft^3 tank, initially filled with N_2, until the pressure in the tank achieves a specified value, **while the temperature in the tank remains constant.**

FIND: Determine (a) the mass of O_2 that enters, (b) the heat transfer

SCHEMATIC & GIVEN DATA:



O_2 at 70°F, 20 lbf/in^2
TANK
1000 ft^3 tank, initially filled with N_2 at 70°F, 5 lbf/in^2

Final pressure in 1000 ft^3 tank = 15 lbf/in^2

ASSUMPTIONS: (1) For the control volume shown in the figure, $\dot{W}_{cv} = 0$. (2) Heat transfer with the surroundings maintains the tank contents at 70°F. (3) Kinetic and potential energy effects are negligible. (4) Each pure component behaves as an ideal gas. The mixture adheres to the ideal gas model.

ANALYSIS: (a) Initially, the tank is filled with N_2 at $T = 530°R$, $p_1 = 5\ \text{lbf/in}^2$: $p_1 V = n_{N_2} \bar{R} T$. Finally, the tank is filled **N_2 and O_2** at $T = 530°R$, $p_2 = 15\ \text{lbf/in}^2$: $p_2 V = (n_{N_2} + n_{O_2}) \bar{R} T$. Combining these expressions

$$\frac{p_2}{p_1} = \frac{n_{N_2} + n_{O_2}}{n_{N_2}} \Rightarrow \frac{15}{5} = 1 + \frac{n_{O_2}}{n_{N_2}} \Rightarrow n_{O_2} = 2 n_{N_2}$$

The amount of N_2 initially present is

$$n_{N_2} = \frac{p_1 V}{\bar{R} T} = \frac{(5 \times 144)\ \text{lbf/ft}^2\ (1000\ \text{ft}^3)}{\left(1545\ \frac{\text{ft}\cdot\text{lbf}}{\text{lbmol}\cdot°R}\right)(530°R)} = 0.879\ \text{lbmol}\ (N_2) \Rightarrow n_{O_2} = 2(0.879) = 1.758\ \text{lbmol}\ (O_2)$$

$$\Rightarrow m_{O_2} = \left(32\ \frac{\text{lb}}{\text{lbmol}}\right)(1.758\ \text{lbmol}) = 56.26\ \text{lb}\ (O_2) \longleftarrow$$

(b) An energy rate balance reduces to

$$\frac{dU_{cv}}{dt} = \dot{Q}_{cv} + \dot{n}_{O_2} h_{O_2}$$

Then, since O_2 enters at a constant temperature (530°R), h_{O_2} is constant. Thus, integration gives

$$\Delta U_{cv} = Q_{cv} + n_{O_2} h_{O_2}(T) \Rightarrow Q_{cv} = \Delta U_{cv} - n_{O_2} h_{O_2}(T)$$

where

$$\Delta U_{cv} = [n_{O_2} \bar{u}_{O_2}(T_2) + n_{N_2} \bar{u}_{N_2}(T_2)] - n_{N_2} \bar{u}_{N_2}(T_1) = n_{O_2} \bar{u}_{O_2}(T)$$

since $T_1 = T_2 = T$. collecting results

① $$Q_{cv} = n_{O_2}\left[\bar{u}_{O_2}(T) - \underbrace{\bar{h}_{O_2}(T)}_{=[\bar{u}_{O_2}(T) + (p\bar{v})_{O_2}]}\right] = n_{O_2}\left[-\overbrace{(p\bar{v})_{O_2}}^{= RT}\right]$$

$$= -n_{O_2} \bar{R} T = -1.758\ \text{lbmol}\ (O_2)\left[1.986\ \frac{\text{Btu}}{\text{lbmol}\cdot°R}\right](530°R)$$

$$= -1850.4\ \text{Btu} \longleftarrow$$

---

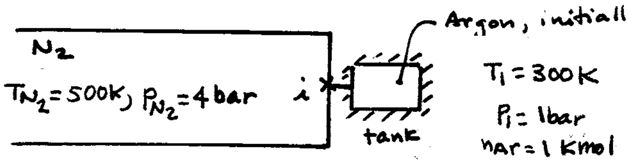
1. To maintain the temperature at 530°R, the energy exiting by heat transfer exactly matches the energy entering by flow work.

PROBLEM 12.40

KNOWN: An insulated, rigid tank initially containing Ar at a known temperature and pressure is connected to a large vessel containing N_2 at a known temperature and pressure. Nitrogen is allowed to flow into the tank, forming an Ar - N_2 mixture at T, p.

FIND: Plot T and p versus the amount of N_2 in the tank.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS:

1. For a control volume enclosing the tank, $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. Kinetic & potential energy can be ignored.
2. At the control volume inlet, N_2 enters at 500K, 4bar.
3. Each pure component behaves as an ideal gas. The mixture adheres to the Dalton model

ANALYSIS: Since the tank volume remains constant

$$V = \frac{n_{Ar}\bar{R}T_1}{p_1} = \frac{[n_{Ar} + \overset{\text{amount of } N_2 \text{ in tank finally}}{n_{N_2}}]\bar{R}T}{p} \Rightarrow p = p_1\left[1 + \frac{n_{N_2}}{n_{Ar}}\right]\left[\frac{T}{T_1}\right] \qquad (1)$$

where $p \leq 4$bar.

Mass and energy rate balances reduce to give (see Sec. 4.4)

$$\Delta U_{cv} = \bar{h}_{N_2}(T_{N_2})\, n_{N_2}$$

or

$$\{[n_{N_2}\bar{u}_{N_2}(T) + n_{Ar}\bar{u}_{Ar}(T)] - [n_{Ar}\bar{u}_{Ar}(T_1)]\} = \bar{h}_{N_2}(T_{N_2})\, n_{N_2}$$

$$\Rightarrow \quad n_{N_2}\bar{u}_{N_2}(T) + n_{Ar}\underbrace{[\bar{u}_{Ar}(T) - \bar{u}_{Ar}(T_1)]}_{\substack{\text{From Table A-21} \\ \bar{c}_v = 1.5\bar{R}}} = \bar{h}_{N_2}(T_{N_2})\, n_{N_2}$$

$$\Rightarrow \quad n_{N_2}\bar{u}_{N_2}(T) + n_{Ar}[1.5\bar{R}(T - T_1)] = \bar{h}_{N_2}(T_{N_2})\, n_{N_2} \qquad (2)$$

Sample Calculation: when $n_{N_2} = 0.2$ kmol, Eq. (2) reads with data from Table A-23

$$(0.2\text{kmol})\,\bar{u}_{N_2}(T) + (1\text{kmol})\left(1.5 \times 8.314 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}\right)(T - 300)\text{K} = (0.2\text{ kmol})\left(14581 \frac{\text{kJ}}{\text{kmol}}\right)$$

$$(0.2)\,\bar{u}_{N_2}(T) + 12.471\,T = 6657.5 \text{ kJ}$$

Solving iteratively with Table A-23 data, T = 400K. Then, Eq. (1) gives

$$p = (1\text{ bar})\left[1 + \frac{0.2}{1.0}\right]\left[\frac{400}{300}\right] = 1.6 \text{ bar}.$$

The data for the required plots are obtained using the following simple IT code:

IT Code

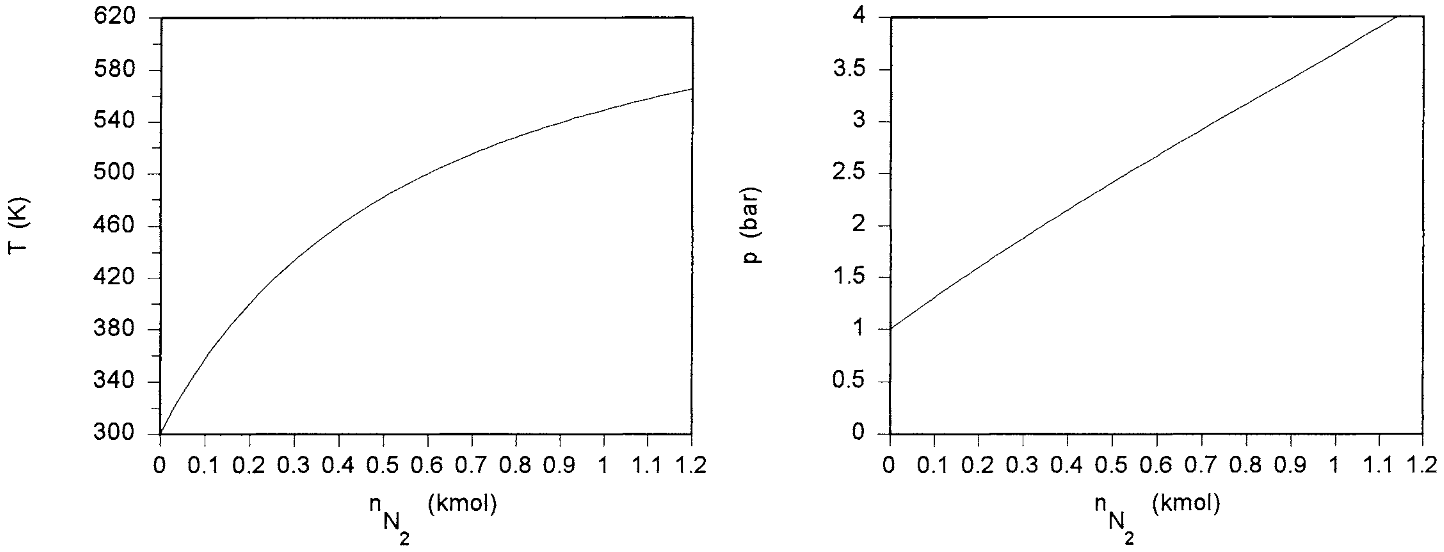
```
p1 = 1  // bar
T1 = 300  // K
TN2 = 500  // K
n_N2 = 0.2  // kmol
 Rbar = 8.314  // kJ/kmol·K

p = p1 * (1 + n_N2) * ( T / T1)
n_Ar = 1  // kmol
n_N2 * u_T("N2",T) + n_Ar * 1.5 * Rbar *(T - T1) = n_N2 * h_T("N2",TN2)
```

IT Results for n_{N2} = 0.2 kmol

```
T = 400.4 K
p = 1.602 bar
```

PROBLEM 12.40 (Cont'd.)

PLOTS:



Note ....

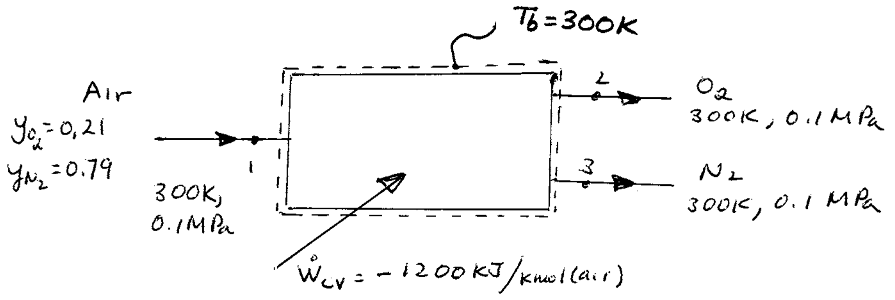
- As more N_2 is introduced, the pressure and temperature both increase.
- The tank temperature increases to values greater than the temperature of the nitrogen entering, due to flow work.
- When n_{N_2} reaches 1.13 kmol, the pressure in the tank reaches 4 bar and the process ceases.

PROBLEM 12.41

KNOWN: A stream of dry air at 300K, 0.1 MPa enters a device operating at steady state and is separated into pure O_2, N_2 streams, each at 300 K, 0.1 MPa. Operating data are provided.

FIND: Determine if the claimed work input value can be correct.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state with negligible kinetic/potential energy effects. (2) Ideal gas principles apply to the air and the pure O_2, N_2 streams exiting. (3) Heat transfer with the surroundings occurs at $T_b = 300K$. (4) $\dot{W}_{cv} = -1200$ kJ/kmol(air)

ANALYSIS: The plan is to evaluate the entropy production rate. This requires the heat transfer rate. Accordingly, reduction of an energy rate balance gives in terms of molar flow rates

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{n}_{Air}\bar{h}_{Air} - \dot{n}_{O_2}\bar{h}_{O_2} - \dot{n}_{N_2}\bar{h}_{N_2}$$

Then, with $\bar{h}_{Air} = y_{O_2}\bar{h}_{O_2} + y_{N_2}\bar{h}_{N_2}$,

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{Air}} - \frac{\dot{W}_{cv}}{\dot{n}_{Air}} + \left[y_{O_2}\bar{h}_{O_2} + y_{N_2}\bar{h}_{N_2}\right] - \overbrace{\left(\frac{\dot{n}_{O_2}}{\dot{n}_{Air}}\right)}^{y_{O_2}}\bar{h}_{O_2} - \overbrace{\left(\frac{\dot{n}_{N_2}}{\dot{n}_{Air}}\right)}^{y_{N_2}}\bar{h}_{N_2}$$

And, since all streams are at 300K, this reduces to give $\dfrac{\dot{Q}_{cv}}{\dot{n}_{Air}} = \dfrac{\dot{W}_{cv}}{\dot{n}_{Air}} = -1200 \dfrac{kJ}{kmol(air)}$

Next, an entropy rate balance at steady state reads

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \dot{n}_{Air}\bar{s}_{Air} - \dot{n}_{O_2}\bar{s}_{O_2} - \dot{n}_{N_2}\bar{s}_{N_2} + \dot{\sigma}_{cv}$$

$\bar{s}_{O_2} \to \bar{s}_{O_2}(T,p)$, $\bar{s}_{N_2} \to \bar{s}_{N_2}(T,p)$

$\bar{s}_{Air} \to \left[y_{O_2}\bar{s}_{O_2}(T, y_{O_2}p) + y_{N_2}\bar{s}_{N_2}(T, y_{N_2}p)\right]$

Thus,

$$0 = \frac{(\dot{Q}_{cv}/\dot{n}_{Air})}{T_b} + \left[y_{O_2}\bar{s}_{O_2}(T, y_{O_2}p) + y_{N_2}\bar{s}_{N_2}(T, y_{N_2}p)\right] - y_{O_2}\bar{s}_{O_2}(T,p) - y_{N_2}\bar{s}_{N_2}(T,p) + \frac{\dot{\sigma}_{cv}}{\dot{n}_{Air}}$$

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{Air}}{T_b} + y_{O_2}\underbrace{\left[\bar{s}_{O_2}(T, y_{O_2}p) - \bar{s}_{O_2}(T,p)\right]}_{-\bar{R}\ln y_{O_2}} + y_{N_2}\underbrace{\left[\bar{s}_{N_2}(T, y_{N_2}p) - \bar{s}_{N_2}(T,p)\right]}_{-\bar{R}\ln y_{N_2}} + \dot{\sigma}_{cv}/\dot{n}_{Air}$$

or

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{Air}} = -\frac{\dot{Q}_{cv}/\dot{n}_{Air}}{T_b} + \bar{R}\left[y_{O_2}\ln y_{O_2} + y_{N_2}\ln y_{N_2}\right]$$

$$= \frac{1200\ kJ/kmol}{300\ K} + 8.314\left[0.21\ln 0.21 + 0.79\ln 0.79\right]$$

$$= (4.00 - 4.273)\ kJ/kmol\cdot K$$

$$= -0.273\ kJ/kmol\cdot K$$

Accordingly, since $\dot{\sigma}_{cv} < 0$, the assumed value for the work input cannot be valid.

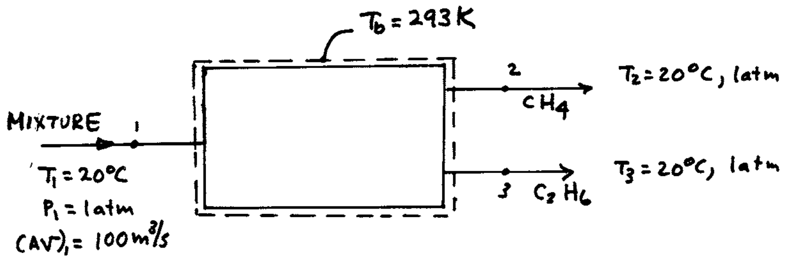
PROBLEM 12.42

KNOWN: Data are provided for a device operating at steady state that separates a natural gas into components.

FIND: Plot the minimum theoretical power input versus y, the mole fraction of C_2H_6

SCHEMATIC & GIVEN DATA:

Mixture	
component	mole fraction
C_2H_6	y
CH_4	$1-y$
$0.05 \le y \le 0.15$	



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates isothermally and at steady state. (2) Kinetic and potential energy effects can be ignored. (3) Ideal gas principles apply for the pure components. The mixture adheres to the Dalton model.

ANALYSIS: The mass flow rate at 1 is obtained using the given volumetric flow rate and the ideal gas equation of state:

$$\dot{m}_1 = \frac{(AV)_1}{v_1} = \frac{P_1 (AV)_1}{(\bar{R}/M) T_1}$$

where $M = y_{CH_4} M_{CH_4} + y_{C_2H_6} M_{C_2H_6} = (0.94)(16.04) + (0.06)(30.07) = 16.88$. Thus

$$\dot{m}_1 = \frac{(1.01325 \times 10^5 \, N/m^2)(100 \, m^3/s)}{\left(\frac{8314}{16.88} \frac{N \cdot m}{kg \cdot K}\right)(293K)} = 70.2 \, kg/s$$

The molar flow rate is

$$\dot{n}_1 = \frac{70.2}{16.88} = 4.1588 \frac{kmol(mix)}{s}$$

Then

$$\begin{cases} \dot{n}_{C_2H_6} = y(4.1588) \\ \dot{n}_{CH_4} = (1-y)(4.1588) \end{cases} \qquad (1)$$

With assumptions 1 and 2, an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \underline{\left[\dot{n}_{CH_4} \bar{h}_{CH_4}(T_1) + \dot{n}_{C_2H_6} \bar{h}_{C_2H_6}(T_1)\right] - \dot{n}_{CH_4} \bar{h}_{CH_4}(T_2) - \dot{n}_{C_2H_6} \bar{h}_{C_2H_6}(T_3)}$$

Since $T_1 = T_2 = T_3$, the underlined term vanishes, leaving $\dot{W}_{cv} = \dot{Q}_{cv}$.

An entropy rate balance reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \left[\dot{n}_{CH_4} \bar{s}_{CH_4}(T_1, y_{CH_4} P_1) + \dot{n}_{C_2H_6} \bar{s}_{C_2H_6}(T_1, y_{C_2H_6} P_1)\right] - \dot{n}_{CH_4} \bar{s}_{CH_4}(T_2, P_2) - \dot{n}_{C_2H_6} \bar{s}_{C_2H_6} + \dot{\sigma}_{cv}$$

Or, upon rearrangement, and inserting $\dot{W}_{cv} = \dot{Q}_{cv}$

$$-\dot{W}_{cv} = T_b \left\{ \dot{n}_{CH_4}\left(\bar{s}_{CH_4}(T_1, y_{CH_4} P_1) - \bar{s}_{CH_4}(T_2, P_2)\right) + \dot{n}_{C_2H_6}\left(\bar{s}_{C_2H_6}(T_1, y_{C_2H_6} P_1) - \bar{s}_{C_2H_6}(T_3, P_3)\right) + \dot{\sigma}_{cv} \right\}$$

In this equation $(-\dot{W}_{cv})$ is the work input. Since $\dot{\sigma}_{cv} \ge 0$, it follows that $(-\dot{W}_{cv})$ is a minimum when $\dot{\sigma}_{cv} = 0$. Accordingly

PROBLEM 12.42 (Continued)

$$(-\dot{W}_{cv})_{MIN} = T_b \left\{ \dot{n}_{CH_4} \left(\underbrace{\bar{s}^\circ_{CH_4}(T_1) - \bar{s}^\circ_{CH_4}(T_2)}_{=0 \text{ since } T_1 = T_2} - \bar{R} \ln \frac{y_{CH_4} \cancel{p_1}}{\cancel{p_2}} \right) + \right.$$

$$\left. \dot{n}_{C_2H_6} \left(\underbrace{\bar{s}^\circ_{C_2H_6}(T_1) - \bar{s}^\circ_{C_2H_6}(T_3)}_{=0 \text{ since } T_1 = T_2} - \bar{R} \ln \frac{y_{C_2H_6} \cancel{p_1}}{\cancel{p_3}} \right) \right\}$$

$$= -\bar{R}\, T_b \left\{ \dot{n}_{CH_4} \ln y_{CH_4} + \dot{n}_{C_2H_6} \ln y_{C_2H_6} \right\} \qquad (2)$$

The governing equations are Equations (1) and (2). That is

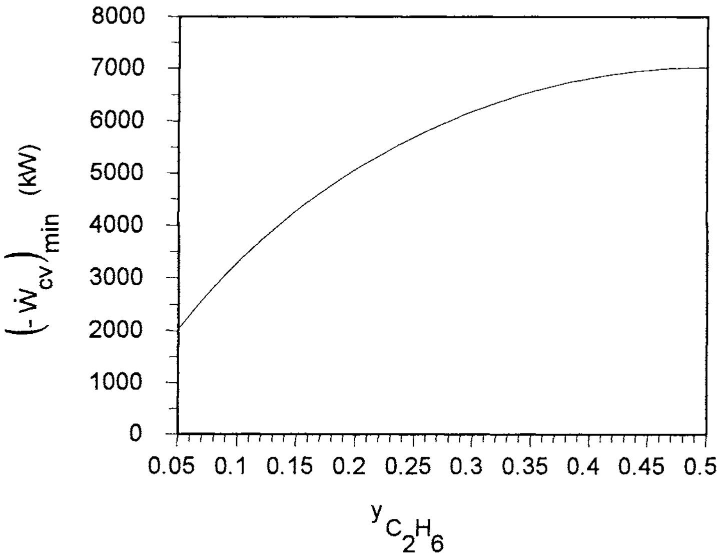
$$(-\dot{W}_{cv})_{min} = \left(-8.314 \frac{kJ}{kmol \cdot K}\right)(293 K)\left(4.1588 \frac{kmol}{s}\right) \left\{ (1-y)\ln(1-y) + y \ln y \right\} \left| \frac{kW}{kJ/s} \right|$$

①

$$= -(10,131 \text{ kW}) \left\{ (1-y)\ln(1-y) + y \ln y \right\} \qquad (3)$$

Eq. (3) can be plotted using available plotting software. The result using IT is presented below.

PLOT:



We see from the plot that more power is required as the mole fraction of C_2H_6 increases.

---

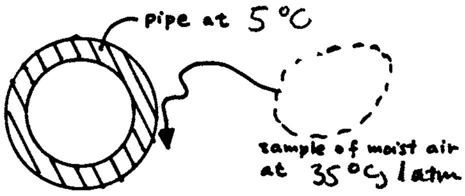
1. Owing to inevitable irreversibilities, the actual power required would exceed the minimum theoretical value obtained from Eq. (3).

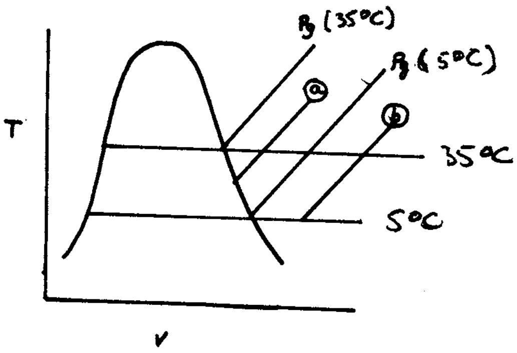
PROBLEM 12.43*

KNOWN: A water pipe at 5°C runs between buildings through air at 35°C

FIND: Determine the maximum relative humidity the air can have before condensation occurs on the wall.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system consists of a sample of moist air initially at 35°C. (2) As the system comes close to the pipe at 5 °C, the system undergoes a cooling process at fixed total pressure from 35°C to 5°C.

ANALYSIS: As the sample of moist air is cooled at fixed total pressure, the partial pressure of the water vapor remains constant as long as no condensation occurs, for $P_v = y_v p$ and y_v remains constant.

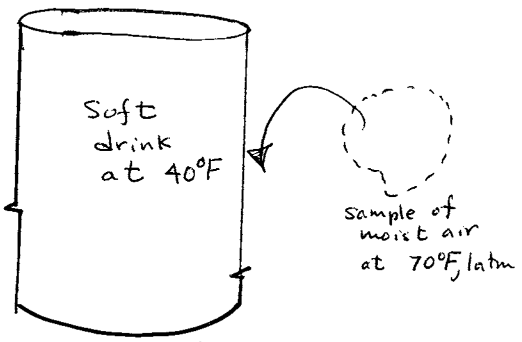
Accordingly, if the initial pressure is less than P_g (5 °C), such as ⓑ shown on the T-v diagram, the sample would be cooled to 5 °C without condensation. However, if the initial partial pressure is greater than P_g (5 °C), such as ⓐ shown on the T-v diagram, the system would be cooled until a saturated mixture is attained. Subsequent cooling to 5 °C would involve condensation. It can be concluded, therefore, that the partial pressure must be less than, or equal to, P_g (5 °C). Thus

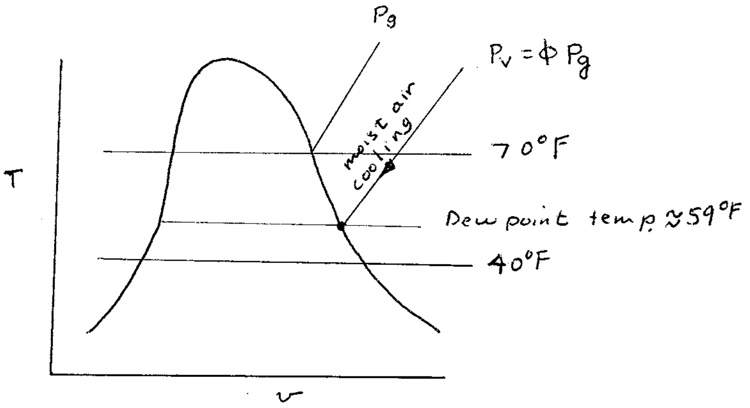
$$\phi = \frac{P_v}{P_g(35°C)} \leq \frac{P_g(5°C)}{P_g(35°C)} = \frac{0.00872 \text{ bar}}{0.05628 \text{ bar}} = 0.155 \ (15.5\%) \text{ ———— } \phi_{max}$$

PROBLEM 12.44

KNOWN: A can of soft drink at 40°F is placed in a room where $T = 70°F$, $\phi = 70\%$.
FIND: Explain why beads of moisture form on the can's surface.

SCHEMATIC & GIVEN DATA:


Soft drink at 40°F
Sample of moist air at 70°F, 1 atm



ASSUMPTIONS: (1) The system consists of a sample of moist air initially at 70°F. (2) As the system comes close to the can at 40°F, the system cools at a fixed total pressure.

ANALYSIS: As the sample of moist air is cooled at fixed total pressure, the partial pressure of the water vapor remains constant as long as no condensation occurs. Once the dew point temperature is attained, however, further cooling results in condensation. Accordingly, in this case beads of moisture form because the dew point temperature is greater than 40°F. That is

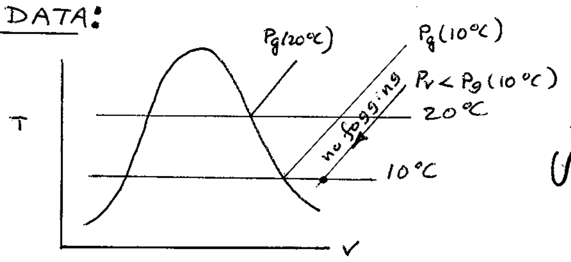
$$p_v = \phi\, p_g(70°F) = (0.7)\left(0.3632\ \frac{lbf}{in^2}\right) = 0.2542\ \frac{lbf}{in^2}$$

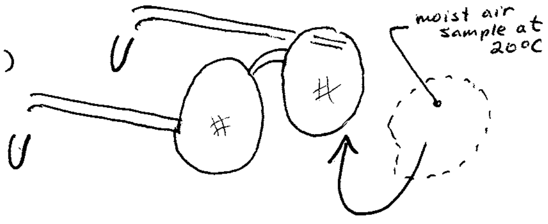
Interpolating in Table A-2E gives $T_{dew\ point} \approx 59°F$.

PROBLEM 12.45

KNOWN: Eyeglasses at 10°C brought into a dwelling at 20°C do not fog up.
FIND: Determine if the relative humidity in the dwelling can be 55%.

SCHEMATIC & GIVEN DATA:




moist air sample at 20°C

ASSUMPTIONS: (1) The system consists of a sample of moist air initially at 20°C. (2) As the system comes close to the eyeglasses at 10°C, the system cools at a fixed mixture pressure.

ANALYSIS: As the sample of moist air is cooled at fixed total pressure, the partial pressure of the water vapor remains constant as long as no condensation occurs. That is, condensation would not occur if the partial pressure p_v satisfies $p_v < p_g(10°C)$. If $\phi = 55\%$,

$$p_v = \phi\, p_g(20°C) = 0.55(0.2339\ bar) = 0.0129\ bar$$

However, at 10°C, $p_g = 0.01228$ bar. Since no fogging occurs, the relative humidity value cannot be correct.

COMMENT: The maximum relative humidity for fogging not to occur is

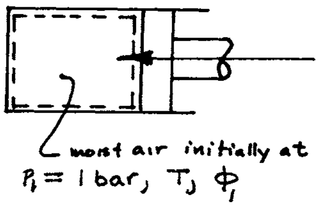
$$\phi_{MAX} = \frac{p_g(10°C)}{p_g(20°C)} = \frac{0.01228}{0.02339} = 0.525\ (52.5\%)$$

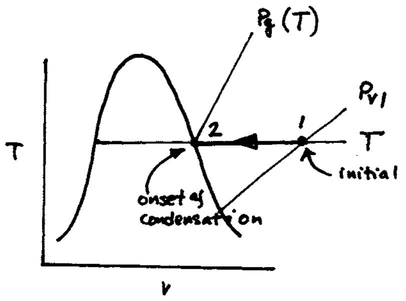
PROBLEM 12.46

KNOWN: A fixed amount of air initially at 1 bar, relative humidity ϕ_1, and temperature T is compressed isothermally until the onset of condensation.

FIND: Determine the mixture pressure when condensation begins, if (a) $\phi_1 = 60\%$, (b) $\phi_1 = 90\%$

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system consists of a fixed amount of moist air, as illustrated in the accompanying figure. (2) The moist air acts as an ideal gas mixture, with each component adhering to the Dalton model.

ANALYSIS: As long as there is no condensation, the mole fraction of the water vapor, y_v, remains constant. Thus, the partial pressure of the water vapor at each state visited during the isothermal compression is $P_v = y_v P$, where P is the corresponding mixture pressure. For example, $P_{v1} = y_v P_1$, where $P_1 = 1$bar. The onset of condensation occurs when the mixture becomes saturated: State 2. At state 2, $P_g = y_v P_2$, where P_2 is the mixture pressure. Forming the ratio of partial pressures

$$\underbrace{\frac{P_{v1}}{P_g}}_{= \phi_1} = \frac{y_v P_1}{y_v P_2} = \frac{P_1}{P_2}$$

When $\phi_1 = 60\%$,

$$P_2 = \frac{P_1}{\phi_1} = \frac{1\text{ bar}}{0.6} = 1.67\text{ bar} \longleftarrow \text{(a)}$$

When $\phi_1 = 90\%$,

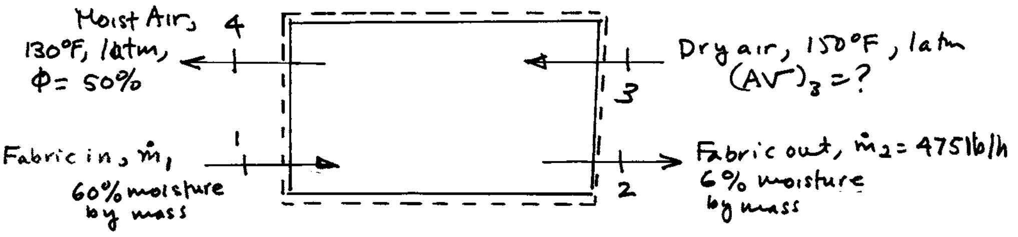
$$P_2 = \frac{P_1}{\phi_2} = \frac{1\text{bar}}{0.9} = 1.11\text{ bar} \longleftarrow \text{(b)}$$

PROBLEM 12.47

KNOWN: Damp fabric containing 60% moisture by mass enters a dryer operating at steady state. Fabric exits the dryer with a moisture content of 6% at the rate of 475 lb/h. Dry air at 150°F, 1atm enters the dryer and moist air at 130°F, 1atm, $\phi = 50\%$ exits.

FIND: Determine the volumetric flow rate of the dry air.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) Ideal gas principles apply to the dry and moist air streams.

ANALYSIS: The volumetric flow rate of the dry air can be found from the molar flow rate of dry air:

$$\dot{n}_{a3} = \frac{(AV)_3}{\bar{v}_3} \Rightarrow (AV)_3 = \dot{n}_{a3}\left[\underbrace{\bar{R}T_3/P_3}_{T_3 = 610\,°R,\ P_3 = 1\,\text{atm}}\right] \qquad (1)$$

The molar flow rate of dry air at 3 and 4 must be equal: $\dot{n}_{a4} = \dot{n}_{a3}$.
At 4, $\dot{n}_{a4} = y_{a4}\dot{n}_4$ and $\dot{n}_{v4} = y_{v4}\dot{n}_4$. Combining these

$$\dot{n}_{a3} = \dot{n}_{a4} = y_{a4}\left[\frac{\dot{n}_{v4}}{y_{v4}}\right] = (1 - y_{v4})\left[\frac{\dot{n}_{v4}}{y_{v4}}\right] \qquad (2)$$

The mole fraction of moisture at 4 can be found using ϕ_4 as follows:

$$\phi_4 = \frac{p_{v4}}{p_{g4}} = \frac{y_{v4}P_4}{p_{g4}} \Rightarrow y_{v4} = \frac{\phi_4 p_{g4}}{P_4} = \frac{(0.5)(\overset{\text{Table A-2E at }130°F}{2.225\ \text{lbf/in}^2})}{14.7\ \text{lbf/in}^2} = 0.0757. \qquad (3)$$

The molar flow rate of vapor at 4 can be found from an overall mass balance for water. If $\dot{m}_1$ is the total mass flow rate of fabric plus moisture at 1,

$$\underbrace{0.60\,\dot{m}_1}_{\text{moisture in at ①}} + \underbrace{0}_{\text{moisture in at ③}} = \underbrace{\dot{m}_{v4}}_{\text{moisture out at ④}} + \underbrace{(0.06)(475)}_{\text{moisture out at ②}} \Rightarrow \dot{m}_{v4} = 0.6\,\dot{m}_1 - 28.5 \qquad (4)$$

The mass flowrate $\dot{m}_1$ can be found from a mass balance on the fabric:

$$\underbrace{0.4\,\dot{m}_1}_{\text{fabric in at ①}} = \underbrace{(0.94)(475)}_{\text{fabric out @ ②}} \Rightarrow \dot{m}_1 = 1116.25\ \text{lb/h}$$

Using this in Eq. (4), gives $\dot{m}_{v4} = 641.25$ lb/h, Thus $\dot{n}_{v4} = 641.25/18.02 = 35.59$ lbmol/h

Then, Eq. (2) gives $\dot{n}_{a3} = (1 - 0.0757)(35.59/0.0757) = 434.56\ \frac{\text{lbmol (a)}}{\text{h}}$

Finally, with Eq. (1)

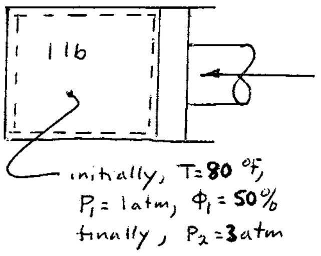
$$(AV_3) = \frac{\left(434.56\ \frac{\text{lbmol (a)}}{\text{h}}\right)\left(1545\ \frac{\text{ft·lbf}}{\text{lbmol·°R}}\right)(610\,°R)}{(14.7 \times 144\ \text{lbf/ft}^2)} = 193{,}477\ \frac{\text{ft}^3}{\text{h}} \longleftarrow$$

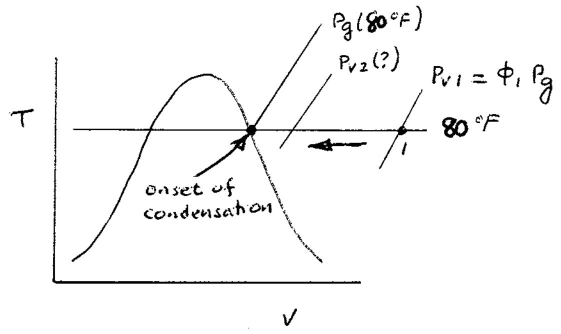
PROBLEM 12.48*

KNOWN: One lb of moist air initially at 80 °F, 1 atm, $\phi = 50\%$ is compressed isothermally to 3 atm.

FIND: Determine if condensation occurs. If so, find the amount condensed. If not, find ϕ_2

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system consists of a 1 lb sample of dry air and water; (2) For the vapor phase, ideal gas mixture principles apply.

ANALYSIS: If no condensation occurs, $P_{v2} \le P_g(80°F)$. To check this, write

$$\omega_1 = \omega_2 \Rightarrow 0.622 \frac{P_{v1}}{P_1 - P_{v1}} = 0.622 \frac{P_{v2}}{P_2 - P_{v2}} \Rightarrow \frac{P_{v1}}{P_1} = \frac{P_{v2}}{P_2} \Rightarrow P_{v2} = \frac{P_2}{P_1} P_{v1}$$

Then, with $P_{v1} = \phi_1 P_g(80°F)$, this expression gives

$$P_{v2} = \left(\frac{3\,\text{atm}}{1\,\text{atm}}\right)\left(0.50\ P_g(80°F)\right) = 1.5\,P_g(80°F)$$

However, for no condensation $P_{v2} \le P_g$, and so condensation occurs in this case. ◀—

At the final state, the system consists of a gas phase (dry air plus saturated water vapor) and a liquid phase (saturated liquid). The amount condensed is

$$\text{Amount Condensed} = m_{v1} - m_{v2} = m_a(\omega_1 - \omega_2)$$

where

$$\omega_1 = 0.622 \frac{P_{v1}}{P_1 - P_{v1}} = \frac{0.622\,(0.50)\,P_g}{P_1 - (0.50)\,P_g} = \frac{(0.622)(0.50)(0.5073\,\text{lbf/in}^2)}{[14.7 - (0.50)(0.5073)]\,\text{lbf/in}^2} = 0.01756\ \frac{\text{lb(v)}}{\text{lb(a)}}$$

$$\omega_2 = 0.622 \frac{P_g}{P_2 - P_g} = \frac{0.622\,(0.5073)}{(3)(14.7) - (0.5073)} = 0.00724\ \frac{\text{lb(v)}}{\text{lb(a)}}$$

The mass of dry air is found from $m_{MIX} = m_a + m_{v1} = m_a(1 + \omega_1)$, giving

$$m_a = \frac{m_{MIX}}{1 + \omega_1} = \frac{1\ \text{lb}}{1.01756\,(\text{lb/lb(a)})} = 0.983\ \text{lb(a)}$$

Collecting result

$$\text{Amount condensed} = 0.983\,[0.01756 - 0.00724]$$

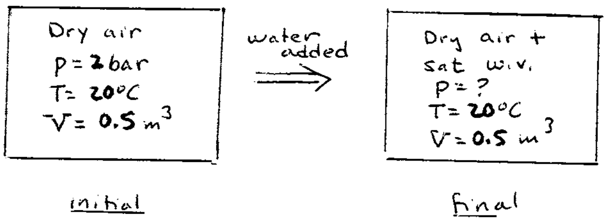
$$= 0.01014\ \text{lb(v)}$$ ◀——

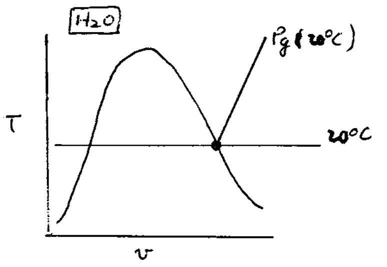
PROBLEM 12.49*

KNOWN: Water is added to a vessel initially containing dry air at 20°C until the air is saturated at 20°C.

FIND: Determine (a) the mass of water added and (b) the final pressure.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system is the final tank contents. (2) Ideal gas principles apply.

ANALYSIS: (a) Since the water vapor is saturated at 20°C at the final state, its partial pressure is p_g (20°C). Thus

$$m_v = \frac{p_g V}{(\bar{R}/M)T} \Rightarrow m_v = \frac{(0.02339 \times 10^5\ N/m^2)(0.5\ m^3)}{\left(\frac{8314}{18.02}\ \frac{N\cdot m}{kg\cdot K}\right)(293\ K)} = 0.00865\ kg \longleftarrow (a)$$

(b) For the overall mixture at the final state

$$p = \frac{(n_a + n_v)\bar{R}T}{V} = \underline{\underline{\frac{n_a \bar{R} T}{V}}} + \underline{\underline{\frac{n_v \bar{R} T}{V}}}$$

Since T & V do not change in the process, this is the initial pressure = 2 bar

This is the pressure of the w.v. at the final state ⇒ $p_g = 0.02339$ bar.

Thus

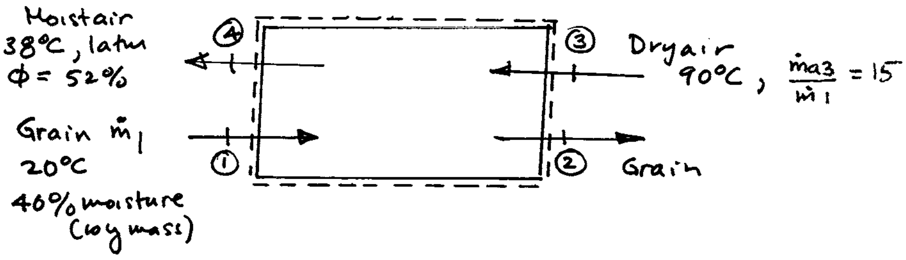
$$p = 2.0234\ bar \longleftarrow (b)$$

PROBLEM 12.50

KNOWN: Grain containing 40% moisture by mass enters a dryer operating at steady state. Dry air also enters and a moist air stream exits.

FIND: For the grain tho exits the dryer, determine the percent moisture by mass.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) Ideal gas principles apply to the dry air and moist air streams.

ANALYSIS: The percent moisture by mass of the exiting grain is evaluated as

$$(1) \qquad \% = (100)\left(\frac{\dot m_{\text{Moisture},2}}{\dot m_2}\right) \begin{array}{l} \leftarrow \text{rate moisture exits at 2} \\ \leftarrow \underline{\text{total}}\text{ rate grain } \underline{\text{plus}} \text{ moisture exits at 2} \end{array}$$

Mass balance on water:

$$\overbrace{\dot m_{\text{moisture},1} + \underset{\text{dry air enters at 3}}{0}}^{\text{moisture in}} = \overbrace{\dot m_{\text{moisture},2} + \dot m_{v4}}^{\text{moisture out}} \Rightarrow \dot m_{\text{moisture},2} = 0.4\dot m_1 - \dot m_{v4} \qquad (2)$$

Mass balance on dry air: $\dot m_{a4} = \dot m_{a3} = 15\,\dot m_1$. Additionally $\omega_4 = \dot m_{v4}/\dot m_{a4}$. So $\dot m_{v4} = \omega_4 \dot m_{a4} = \omega_4 (15\,\dot m_1)$. To find ω_4, calculate $p_{v4} = \phi_4 p_{g4}$ or $p_{v4} = 0.52(0.06632\text{ bar}) = 0.0345\text{ bar}$ and

$$\omega_4 = \frac{.622\ p_{v4}}{p_4 - p_{v4}} = \frac{.622(0.0345)}{1.01325 - 0.0345} = 0.022\ \frac{\text{kg(v)}}{\text{kg(a)}}$$

Thus

$$\dot m_{v4} = (0.022)(15)\,\dot m_1 = 0.33\,\dot m_1 \qquad (3)$$

Also, the total mass flow rates at 1,2 differ by the amount of moisture removed. That is

$$\dot m_2 = \dot m_1 - \dot m_{v4} = \dot m_1 - 0.33\,\dot m_1 = 0.67\,\dot m_1 \qquad (4)$$

Collecting Eqs. (1), (2), (3), (4)

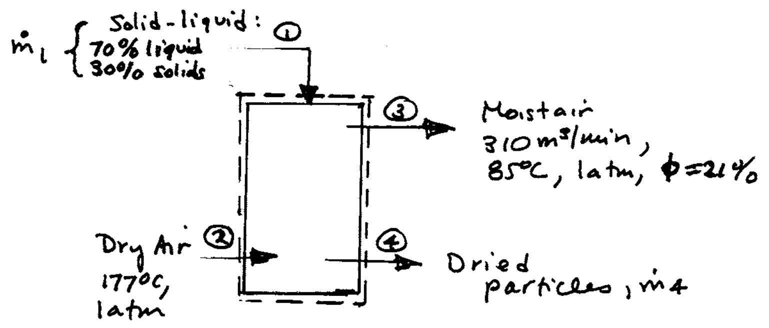
$$\% = \left(\frac{0.4\,\dot m_1 - 0.33\,\dot m_1}{0.67\,\dot m_1}\right)(100) = 10.4 \longleftarrow$$

PROBLEM 12.51

KNOWN: Operating data are provided for a spray dryer at steady state. Dry air enters the dryer at one location. Dried particles exit the dryer.

FIND: Determine (a) the volumetric flow rate of the entering dry air, (b) the rate the dried particles exit.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) Ideal gas principles apply to the dry air and moist air streams.

ANALYSIS: (a) The volumetric flow rate of the dry air can be found from the molar flow rate of dry air:

$$\dot{n}_{a2} = \frac{(A V)_2}{\bar{v}_2} \Rightarrow (AV)_2 = \dot{n}_{a2}\left(\frac{\bar{R}T_2}{P_2}\right) \qquad (1)$$

From a mass balance on dry air: $\dot{n}_{a2} = \dot{n}_{a3}$. To find $\dot{n}_{a3}$,

$$\dot{n}_{a3} = \frac{(AV)_3}{\bar{v}_{a3}} = \frac{(AV)_3\, p_{a3}}{\bar{R}T_3} \qquad (2)$$

where $p_{a3} = p_3 - p_{v3}$. Using ϕ_3, $p_{v3} = \phi_3 p_{g3} = 0.21(0.5783\text{ bar}) = 0.12144$ bar. So $p_{a3} = 1.01325 - 0.12144 = 0.8918$ bar. Combining (1), (2), and p_{a3}

$$(AV)_2 = (AV)_3\left(\frac{p_{a3}}{p_2}\right)\left(\frac{T_2}{T_3}\right) = \left(310\frac{m^3}{min}\right)\left(\frac{0.8918}{1.01325}\right)\left(\frac{450}{358}\right) = 342.96\,\frac{m^3}{min} \longleftarrow \qquad (a)$$

(b) The rate that dried particles exit at 4 must equal the rate at which particles enter at 1. That is,

$$\dot{m}_4 = 0.3\,\dot{m}_1$$

where $\dot{m}_1$ is the total mass flow rate of particles plus moisture entering at 1.
Since $\dot{m}_{moisture,1} = 0.7\,\dot{m}_1$, $\dot{m}_1 = \frac{\dot{m}_{moisture,1}}{0.7}$
Since 1 is the only location that moisture enters, a mass balance on water gives $\dot{m}_{moisture,1} = \dot{m}_{v3}$. Collecting results

$$\dot{m}_4 = 0.3\,\dot{m}_1 = 0.3\left[\frac{\dot{m}_{moisture,1}}{0.7}\right] = \frac{0.3\,\dot{m}_{v3}}{0.7} \qquad (3)$$

To find $\dot{m}_{v3}$, use

$$\dot{m}_{v3} = \frac{(AV)_3}{v_{v3}} = \frac{(AV)_3\, p_{v3}}{R\,T_3} = \frac{(310\,m^3/min)(0.12144\times10^5\,N/m^2)}{\left(\frac{8314}{18.02}\,\frac{N\cdot m}{kg\cdot K}\right)(358\,K)} = 22.79\,\frac{kg}{min}$$

Then Eq. (3) gives

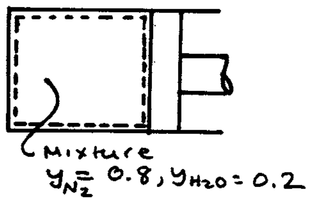
$$\dot{m}_4 = \frac{(0.3)(22.79)}{0.7} = 9.77\,\frac{kg}{min} \longleftarrow \qquad (b)$$

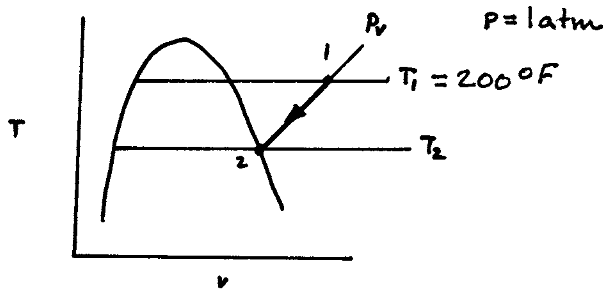
PROBLEM 12.52

KNOWN: A mixture of nitrogen and water vapor having a molar analysis 80% N_2, 20% water vapor is at 200°F, 1atm.

FIND: If the mixture is cooled at constant pressure, determine the temperature at which water vapor begins to condense.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system consists of a fixed amount of a nitrogen/water vapor mixture, as illustrated in the accompanying figure. (2) The mixture acts like an ideal gas, with each component adhering to the Dalton model.

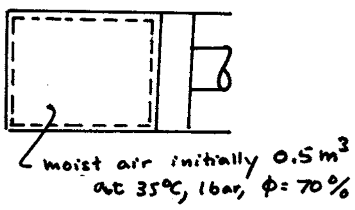
ANALYSIS: As long as there is no condensation, the mole fraction of the water vapor, y_v, remains constant. Moreover, as cooling occurs at fixed mixture pressure, the partial pressure of the water vapor remains constant: $p_v = y_v p$. Thus, cooling takes place at constant p_v, and the onset of condensation occurs at state 2 shown on the accompanying figure. The corresponding temperature is the saturation temperature, T_2. Therefore, $p_v = y_{H_2O} p = (0.2)(14.7\ lbf/in^2) = 2.94\ lbf/in^2$. Interpolating in Table A-3E gives $T_2 = 140.5°F$ ⟵ T_2

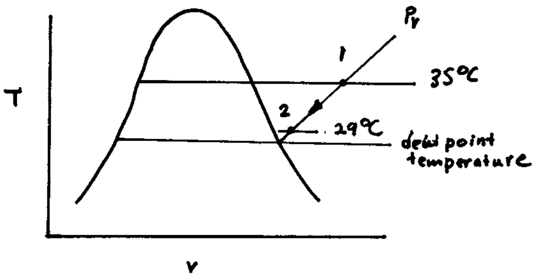
PROBLEM 12.53

KNOWN: A system consisting initially of $0.5\ m^3$ of air at 35°C, 1 bar, $\phi = 70\%$ is cooled at constant pressure to 29°C.

FIND: Determine the work and heat transfer for the process.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) As shown in the accompanying figure, the system consists of the specified quantity of moist air. (2) The mixture acts as an ideal gas, with each component adhering to the Dalton model.

ANALYSIS: The first step is to determine whether condensation occurs. As long as there is no condensation the mole fraction of the water vapor, y_v, remains constant. Thus, as the cooling occurs at fixed mixture pressure, the partial pressure of the water vapor remains constant until a saturated mixture would be attained: $p_v = y_v p$. The onset of condensation in this case corresponds, therefore, to the dew point temperature. Using given data

$$p_v = \phi_1\, p_g(35°C) = (0.7)(0.05628\ bars) = 0.0394\ bar \Rightarrow T_{dp} = 28.7°C$$

Accordingly, condensation does not take place.

PROBLEM 12.53 (Continued)

Since the mixture undergoes a constant pressure process

$$W = \int_1^2 p\,dV = p(V_2 - V_1) = pV_1\left[\frac{V_2}{V_1} - 1\right]$$

The total amount of mixture remains constant, so the ideal gas equation of state gives

$$\left.\begin{array}{l} pV_1 = n\bar{R}T_1 \\ pV_2 = n\bar{R}T_2 \end{array}\right\} \quad \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

Thus

$$W = P_1V_1\left[\frac{T_2}{T_1} - 1\right] = \left(10^5\frac{N}{m^2}\right)(1\,m^3)\left[\frac{302}{308} - 1\right]\left|\frac{kJ}{10^3 N\cdot m}\right| = -1.948\,kJ \longleftarrow \quad W$$

With assumption 3, an energy balance gives $\Delta U = Q - W$, or $Q = \Delta U + W$. The change in internal energy of the system is the sum of the internal energy changes of the dry air and water vapor: $\Delta U = (\Delta U)_a + (\Delta U)_v$. Thus

$$Q = \left\{m_a\left[u_a(T_2) - u_a(T_1)\right] + m_v\left[u_v(T_2) - u_v(T_1)\right]\right\} + W$$

In accordance with the discussion of Sec. 12.5.2, $u_v \approx u_g(T)$ from the steam tables. For air u_a is obtained from Table A-22. The mass amounts m_a and m_v are obtained from the ideal gas equation of state:

$$m_v = \frac{p_{v1}V_1}{\bar{R}/M_v\,T_1} = \frac{(0.0394\times10^5\,N/m^2)(0.5\,m^3)}{(8314/18.02\ N\cdot m/kg\cdot K)(308\,K)} = 0.01387\ kg(v)$$

$$m_a = \frac{p_{a1}V_1}{\bar{R}/M_a\,T_1} = \frac{(P_1 - p_{v1})V_1}{\bar{R}/M_a\,T_1} = \frac{(0.9606\times10^5)(0.5)}{(8314/28.97)(308)} = 0.54338\ kg(a)$$

Substituting values into the expression for Q

$$Q = \left\{0.54338\,kg(a)\left[215.51 - 219.8\right]\frac{kJ}{kg(a)} + (0.01387)\left[2415.2 - 2423.4\right]\frac{kJ}{kg(v)}\right\} + (-1.948\,kJ)$$

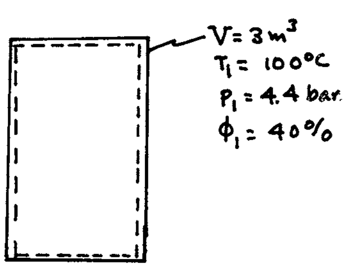
$$= \{-2.331 - 0.114\} + (-1.948) = -4.393\,kJ \longleftarrow \quad Q$$

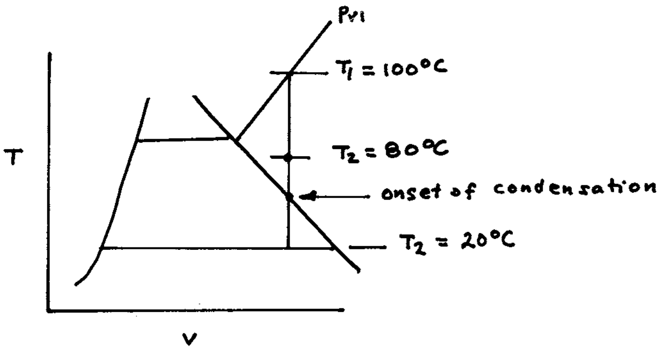
PROBLEM 12.54

KNOWN: A tank with a volume of 3 m^3 initially contains air at 100°C, 4.4 bars, and $\phi_1 = 60\%$. The tank contents are cooled to (a) 80°C, (b) 20°C.

FIND: Determine in each case the heat transfer.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) As shown by the accompanying figure, the system consists of the tank contents. For the system, the changes in kinetic and potential energy are zero in value. (2) The gas mixture acts as an ideal gas, with each component adhering to the Dalton model.

ANALYSIS: Begin by determining if condensation occurs during cooling.

As the total volume of the tank and the total mass of the water present are constant, the water undergoes a constant specific volume process as the tank contents are cooled. This is illustrated by the above T-v diagram. Using the ideal gas equation of state

$$v_{v1} = \frac{(\bar{R}/M_v)\,T_1}{p_{v1}} \qquad \text{where} \quad p_{v1} = \phi_1\, p_g(T_1)$$

Thus, with data from Table A-2, $p_{v1} = (0.40)(1.014 \text{ bars}) = 0.4056$ bars. And

$$v_{v1} = \frac{\left(\frac{8314}{18.02}\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(373 \text{ K})}{(.4056 \times 10^5 \text{ N/m}^2)} = \left(4.2429\ \frac{\text{m}^3}{\text{kg}}\right)\left|\frac{10^6 \text{ cm}^3/\text{m}^3}{10^3 \text{ g/kg}}\right| = 4242.9\ \frac{\text{cm}^3}{\text{g}}$$

Interpolating in Table A-2 with $v_g = v_{v1}$ gives $T = 74.4°C$ as the temperature at which condensation would be begin.

(a) T = 80°C. In this case there is no condensation, an energy balance $\Delta U = Q - \cancel{W}^0$ reduces to $Q = \Delta U$, where $U_1 = m_a u_a(T_1) + m_v u_g(T_1)$, $U_2 = m_a u_a(T_2) + m_v u_g(T_2)$. Then, with data from Tables A-2, A-22

$$Q = m_a\left(u_a(T_2) - u_a(T_1)\right) + m_v\left(u_g(T_2) - u_g(T_1)\right)$$

$$= 11.194(252.19 - 266.63) + 0.7071(2482.2 - 2506.5)$$

$$= (-161.64) + (-17.18) = -178.82 \text{ kJ} \longleftarrow Q$$

(b) T = 20°C In this case condensation does occur. An energy balance reduces to give $\Delta U = Q - \cancel{W}^0$, or $Q = U_2 - U_1$, where

$$U_1 = m_{v1} u_g(T_1) + m_a u_a(T_1)$$

$$U_2 = m_{v2} u_g(T_2) + m_{w2} u_f(T_2) + m_a u_a(T_2)$$

where m_{w2} is the mass of water that condenses. Collecting results

$$Q = m_a\left[u_a(T_2) - u_a(T_1)\right] + \left[m_{v2} u_g(T_2) + m_{w2} u_f(T_2) - m_{v1} u_g(T_1)\right]$$

PROBLEM 12.54 (Continued)

Using the ideal gas equation of state

$$m_a = \frac{p_{a1} V}{\frac{\bar{R}}{M_a} T_1} = \frac{(P_1 - P_{v1}) V}{\frac{\bar{R}}{M_a} T_1} = \frac{[(4.4 - 0.4056) \times 10^5](3)}{\left(\frac{8314}{28.97}\right)(373)} = 11.194 \text{ kg(a)}$$

The mass of water vapor initially is

$$m_{v1} = \frac{V}{v_{v1}} = \left(\frac{3 \text{ m}^3}{4.2429 \text{ m}^3/\text{kg}}\right) = 0.7071 \text{ kg(v)}$$

Since the water undergoes a constant specific volume process, the quality of the two-phase liquid-vapor mixture at T_2 is

$$x_2 = \frac{v_{v2} - v_f}{v_g - v_f} = \frac{4242.9 - 1.0018}{57791 - 1.0018} = 0.0734$$

Thus, the amount of water vapor present in the final mixture is

$$m_{v2} = x_2 m_{v1} = (0.0734)(0.7071) = 0.0519 \text{ kg(v)}$$

The mass of condensate is

$$m_{w2} = m_{v1} - m_{v2} = 0.7071 - 0.0519 = 0.6552 \text{ kg}(\ell)$$

Finally, with data from Tables A-2 and A-22

$$Q = 11.194(209.06 - 266.63) + 0.0519(2402.9) + 0.6552(83.95) - 0.7071(2506.5)$$

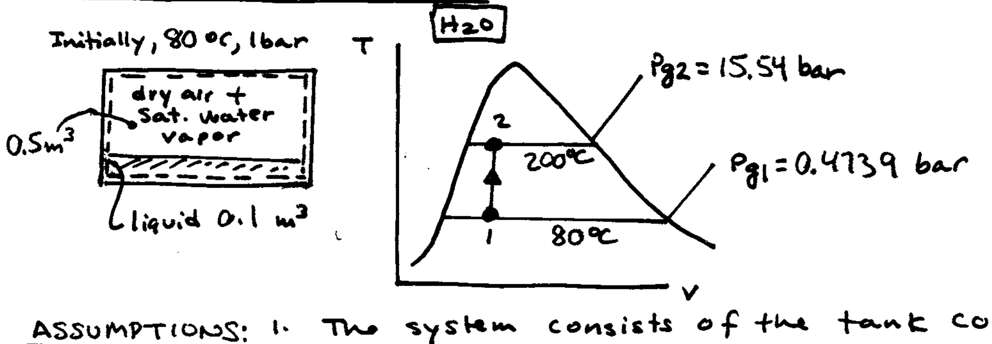
$$= (-644.44 + 124.71 + 55. - 1772.35) = -2237.08 \text{ kJ} \longleftarrow Q$$

PROBLEM 12.55*

KNOWN: A closed, rigid tank, initially containing moist air in equilibrium with liquid water, is heated.

FIND: Determine (a) the final pressure, (b) the heat transfer.

SCHEMATIC & GIVEN DATA



ASSUMPTIONS: 1. The system consists of the tank contents. 2. The gas phase adheres to ideal gas principles. 3. W=0 and kinetic & potential energy effects are absent.

ANALYSIS: (a) Since total volume is constant, the water present as liquid and water vapor, undergoes a constant specific volume process — as shown in the schematic. In stating this, the Dalton model is used: the water vapor occupies the full volume filled with water vapor and dry air. The initial amounts of liquid and vapor are

$$m_{vap} = \frac{V_{vap}}{v_g(80°C)} = \frac{0.5\ m^3}{3.407\ m^3/kg} = 0.1468\ kg, \quad m_{LIQ} = \frac{0.1\ m^3}{(1.0291/10^3)\ m^3/kg} = 97.1723\ kg$$

The total mass of water is then $m_{water} = 97.3191$ kg. The final specific volume is

$$v = \frac{0.60\ m^3}{97.3191\ kg} = 6.165 \times 10^{-3}\ \frac{m^3}{kg} \Rightarrow$$ at state 2 the water is a two-phase liquid-vapor mixture with quality

$$x = \left[\frac{(6.165\times10^{-3}) - (1.1565\times10^{-3})}{0.1274 - (1.1565\times10^{-3})}\right] = 0.03967 \Rightarrow m_{v2} = x \cdot m_{water} = (0.03967)(97.1723) = 3.855\ kg$$

The mass of dry air is

$$m_a = \frac{p_{a1} V_{gas}}{R\, T_1} = \frac{[[1-0.4739] \times 10^5 N/m^2][0.5\ m^3]}{\left(\frac{8314}{28.97}\right)(353\ K)} = 0.2597\ kg \Rightarrow \omega_2 = \frac{m_{v2}}{m_a} = \frac{3.855}{0.259} = 14.846$$

Rearranging Eq. 12.43

$$p_2 = p_{v2}\left[1 + \frac{0.622}{\omega_2}\right] = 15.54\ bar)\left[1 + \frac{0.622}{14.846}\right] = 16.19\ bar = 1.62\ MPa$$ p_2

(b) An energy balance reduces to read $\Delta U = Q - \cancelto{0}{W}$, where

$$U_1 = m_{LIQ}\, u_f(80°C) + m_{VAP}\, u_g(80°C) + m_a u_a(80°C)$$

$$U_2 = m_{water}[u_f(200°C) + x\, u_g(200°C)] + m_a u_a(200°C)$$

or

$$Q = m_{water}[u_f(200°C) + x\, u_g(200°C)] - [m_{LIQ}\, u_f(80°C) + m_{vap}\, u_g(80°C)] + m_a[u_a(200°C) - u_a(80°C)]$$

$$Q = (97.3191)[850.65 + (0.03967)(2595.3)] - [(97.1723)(334.86) + (0.1468)(2482.2)] + (0.2597)[339.5 - 252.2]$$

$$= 92{,}804 - 32{,}904 + 23$$

$$= 59{,}923\ kJ$$ Q

PROBLEM 12.56

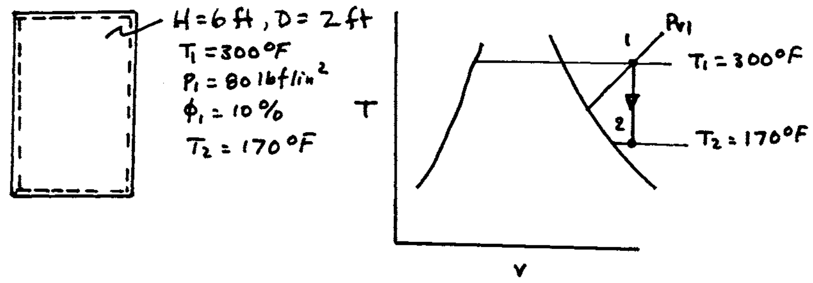
KNOWN: A cylindrical tank having specified dimensions initially contains moist air at known conditions. The tank contents are cooled.

FIND: Determine (a) whether condensation occurs, (b) the final pressure, (c) the heat transfer, and (d) the change in entropy.

KNOWN: A tank with a volume of 100 ft³ initially contains air at 300°F, 80 lbf/in², $\phi_1 = 10\%$. The tank contents are cooled to 170°F.

FIND: Determine the final pressure, the heat transfer, and the entropy change.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) As shown by the accompanying figure, the system consists of the tank contents for which there are no changes in kinetic or potential energy. (2) The mixture acts as an ideal gas and each component adheres to the Dalton model.

ANALYSIS: (a) The first step is to determine if condensation occurs. As the total volume of the tank and the total mass of water present are constant, the water undergoes a constant specific volume process as the tank contents are cooled (see T-v diagram). Using the ideal gas equation of state

$$v_{v1} = \frac{(\bar{R}/M_v)T_1}{p_{v1}} \quad \text{where} \quad p_{v1} = \phi_1 p_g(T_1) = (0.10)\left(66.98\frac{\text{lbf}}{\text{in}^2}\right) = 6.698 \text{ lbf/in}^2 .$$

Thus

$$v_{v1} = \frac{\left(\frac{1545}{18.02}\frac{\text{ft·lbf}}{\text{lb·°R}}\right)(760°\text{R})}{(6.698\times 144)\ \text{lbf/ft}^2} = 67.56 \text{ ft}^3/\text{lb}$$

Interpolating in Table A-2E with $v_{v1} = v_g$ gives $T = 166.3°F$ as the temperature at which condensation would begin. Accordingly, there is no condensation in the present case. $\longleftarrow$ (a)

(b) Treating the overall mixture as an ideal gas at states 1 and 2

$$p_1 = \frac{mRT_1}{V}, \quad p_2 = \frac{mRT_2}{V} \Rightarrow \frac{p_2}{p_1} = \frac{T_2}{T_1} \Rightarrow p_2 = \left(80\frac{\text{lbf}}{\text{in}^2}\right)\left(\frac{630°\text{R}}{760°\text{R}}\right) = 66.32\frac{\text{lbf}}{\text{in}^2} \longleftarrow p_2$$

(c) The volume of moist air is $V = \left(\frac{\pi D^2}{4}\right)(H) = \frac{\pi(2\text{ft})^2(6\text{ft})}{4} = 18.85\text{ft}^3$. The mass of dry air is

$$m_a = \frac{p_{a1}V}{(\bar{R}/M_a)T_1} = \frac{[(80-6.698)\times 144](18.85)}{(1545/28.97)(760)} = 4.91 \text{ lb(a)}$$

An energy balance reduces to $\Delta U = Q - \cancel{W}^{\,0}$, or $Q = \Delta U$. Thus

$$Q = m_a[u_a(T_2) - u_a(T_1)] + \underset{\uparrow\, = V/v_v}{m_v}[u_v(T_2) - u_v(T_1)] \tag{1}$$

① In this case, where there is no condensation, the change in specific internal energy of the water vapor can be evaluated using steam table data and $u_v \approx u_g$ or the ideal gas table, Table A-23. Noting that Table A-23E gives $\bar{u}$, the previous equation would be written

PROBLEM 12.56 (Contd.)

$$Q = m_a [u_a(T_2) - u_a(T_1)] + [V/v_v] \left[\frac{\bar{u}_v(T_2) - \bar{u}_v(T_1)}{M_v} \right]$$

With data from Tables A-22E, 23E

$$Q = (4.91) \cdot [107.5 - 129.49] + (18.85/67.56) \left[\frac{3755.9 - 4557.6}{18.02} \right]$$

$$= -110.43 - 12.41 = -122.84 \text{ Btu} \longleftarrow Q$$

(d) The change in entropy for the process is

$$\Delta S = m_a [s_a(T_2, p_{a2}) - s_a(T_1, p_{a1})] + m_v [s_v(T_2, p_{v2}) - s_v(T_1, p_{v1})]$$

where the specific entropy of each mixture component is evaluated at the mixture temperature and the partial pressure of the component in the mixture. Accordingly

$$\Delta S = m_a \left[s_a^\circ(T_2) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a} \ln \frac{\cancel{y_a} p_2}{\cancel{y_a} p_1} \right] + m_v \left[\frac{\bar{s}_v^\circ(T_2) - \bar{s}_v^\circ(T_1) - \bar{R} \ln \frac{\cancel{y_v} p_2}{\cancel{y_v} p_1}}{M_v} \right]$$

With data from Tables A-22E and A23E

② $$\Delta S = (4.91) \left[.63777 - .68312 - \frac{1.986}{28.97} \ln \frac{66.32}{80} \right] + \left(\frac{18.85}{67.56} \right) \left[\frac{46.3635 - 47.893 - 1.986 \ln \frac{66.32}{80}}{18.02} \right]$$

$$= -0.1595 - 0.0179 = -0.1774 \text{ Btu/}^\circ\text{R} \longleftarrow \Delta S$$

---

1. A solution using steam table data can also be developed. In this case, Eq. (1) would be expressed as

$$Q = m_a [u_a(T_2) - u_a(T_1)] + [V/v_v] [u_g(T_2) - u_g(T_1)]$$

where u_g is obtained from the steam tables.

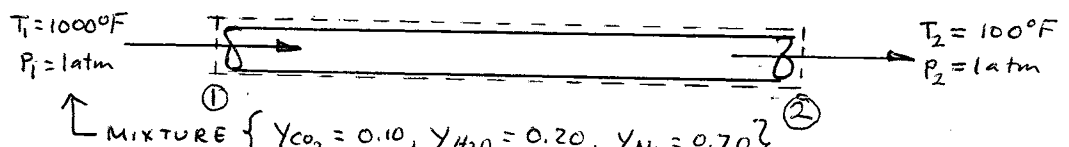
2. Entropy decreases because entropy is carried from the tank accompanying heat transfer.

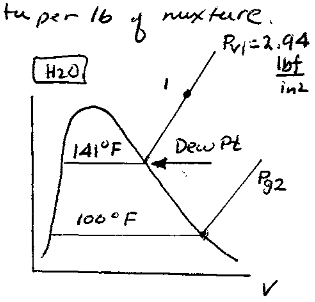
PROBLEM 12.57

KNOWN: A gaseous mixture including water vapor enters an exhaust pipe at 1000°F, 1atm and exits at 100°F, 1atm.

FIND: Determine the heat transfer rate at steady state, in Btu per lb of mixture.

SCHEMATIC & GIVEN DATA:





MIXTURE $\{y_{CO_2} = 0.10,\ y_{H_2O} = 0.20,\ y_{N_2} = 0.70\}$

ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume, $\dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) For the gas phase, ideal gas principles are applicable.

ANALYSIS: To begin, check if condensation would occur by determining the dew point temperature. For the water vapor entering, $p_{v1} = y_v P_1 = 0.2 atm = 2.94\ lbf/in^2$. Thus, the dew point temperature is ≈ 141°F, and so condensation would occur. At the exit, there is a saturated gas phase and saturated liquid.

Next, consider the gas phase exiting. The partial pressure of the water vapor is p_{g2} and the mole fraction of the water vapor is $y_v = n_v/(n_{dry}+n_v)$, where n_v is the amount of water vapor exiting per lbmol of entering mixture and n_{dry} is the amount of CO_2 and N_2 per lbmol of entering mixture. Accordingly

$$p_v = y_v p \Rightarrow p_{g2} = \left(\frac{n_v}{0.8+n_v}\right) p_2 \Rightarrow 0.9503 = \left(\frac{n_v}{0.8+n_v}\right) 14.7 \Rightarrow n_v = 0.0553\ \frac{lbmol(vapor)}{lbmol(mix,1)}$$

Accordingly, at ② there would be $(0.2 - 0.0553) = 0.1447$ lbmol(condensate)/lbmol(mix, 1)

Reducing an energy balance at steady state

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{mix,1}} + \left[0.1\,\bar{h}_{CO_2} + 0.2\,\bar{h}_{H_2O(v)} + 0.7\,\bar{h}_{N_2}\right]_① - \left[0.1\,\bar{h}_{CO_2} + 0.0553\,\bar{h}_{H_2O(v)} + 0.7\,\bar{h}_{N_2}\right]_② - \left[0.1447\,\bar{h}_{H_2O(\ell)}\right]_②$$

Then, with $h_{H_2O(v)} = h_g$ and $h_{H_2O(\ell)} = h_f$ at ② and $\bar{h}_{H_2O(v)}$ at ① from
① the steam tables, and data for CO_2 and N_2 from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{mix,1}} = 0.1\left[\bar{h}_{CO_2}(T_2) - \bar{h}_{CO_2}(T_1)\right] + 0.7\left[\bar{h}_{N_2}(T_2) - \bar{h}_{N_2}(T_1)\right] + M_{H_2O}\left[0.0553\,h_g(T_2) + 0.1447\,h_f(T_2) - 0.2\,h_g(T_1)\right]$$

$$= 0.1\left[4236 - 14081\right] + 0.7\left[3890 - 10347\right] + 18.02\left[0.0553(1105) + 0.1447(68) - 0.2(1535)\right]$$

$$= -984.5 - 4519.9 - 4253.6$$

$$= -9758\ \frac{Btu}{lbmol\,(mix\,in)}$$

For the entering mixture $M_{mix} = (0.1)(44.01) + (0.2)(18.02) + (0.7)(28.01) = 27.6\ \frac{lb(mix)}{lbmol(mix)}$. Accordingly,

$$\frac{\dot{Q}_{cv}}{\dot{m}_{mix,1}} = \frac{-9758}{27.6} = -353.6\ \frac{Btu}{lb(mix\,in)} \longleftarrow$$

---

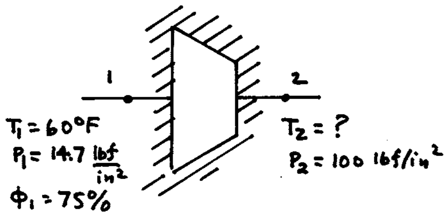
1. For the entering mixture, $P_{v1} = 2.94\ lbf/in^2$. At this pressure, the enthalpy of water vapor varies little with pressure and is determined mainly by temperature.

PROBLEM 12.58

KNOWN: Steady-state operating data are provided for a compressor.

FIND: (a) For $\eta_c = 0.8$, determine the exit temperature, the work and exergy destruction per unit mass of dry air flowing. (b) Plot the quantities of part (a) versus η_c ranging from 0.7 to 1.0.

SCHEMATIC & GIVEN DATA:



Note: Since T_2 would be greater than T_1, as a consequence of compression, there is no possibility of condensation.

ASSUMPTIONS: (1) The compressor operates at steady state and is well insulated. (2) Changes in kinetic and potential energy effects from inlet to exit can be ignored. (3) The moist air acts as an ideal gas and each component adheres to the Dalton model. (4) $T_0 = 520°R$.

ANALYSIS: At steady state mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$ and $\dot{m}_{v1} = \dot{m}_{v2} \equiv \dot{m}_v$. Using assumptions 1-3 an energy rate balance reduces to

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \dot{W}_{cv} + [\dot{m}_a h_a(T_1) + \dot{m}_v h_v(T_1)] - [\dot{m}_a h_a(T_2) + \dot{m}_v h_v(T_2)]$$

or

$$\dot{W}_{cv} = \dot{m}_a [h_a(T_1) - h_a(T_2)] + \dot{m}_v [h_v(T_1) - h_v(T_2)]$$

Noting that $\omega = \dot{m}_v/\dot{m}_a$ this becomes

$$\frac{\dot{W}_{cv}}{\dot{m}_a} = [h_a(T_1) - h_a(T_2)] + \omega [h_v(T_1) - h_v(T_2)] \qquad (1)$$

This applies both for the actual compression and an isentropic compression from the same initial state. For the isentropic compression

$$\left(\frac{\dot{W}_{cv}}{\dot{m}_a}\right)_s = [h_a(T_1) - h_a(T_{2s})] + \omega [h_v(T_1) - h_v(T_{2s})] \qquad (2)$$

where T_{2s} is the temperature at the exit for an isentropic compression.

The actual work requirement can be found using the given isentropic compression efficiency together with Eq. (2):

$$\frac{\dot{W}_{cv}}{\dot{m}_a} = \frac{(\dot{W}_{cv}/\dot{m}_a)_s}{\eta_c} \qquad (3)$$

To determine the numerator of Eq. (3) requires T_{2s} which can be found in an iterative procedure using the requirement that the specific entropy of the mixture does not change from inlet to exit in an isentropic compression. When expressed per unit mass of dry air this condition takes the form

$$[s_a(T_1, p_{a1}) + \omega s_v(T_1, p_{v1})] = [s_a(T_{2s}, p_{a2}) + \omega s_v(T_{2s}, p_{v2})]$$

or

$$[s_a(T_{2s}, p_{a2}) - s_a(T_1, p_{a1})] + \omega [s_v(T_{2s}, p_{v2}) - s_v(T_1, p_{v1})] = 0$$

$$\left[s_a^\circ(T_{2s}) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a} \ln \frac{\cancel{y_a} p_2}{\cancel{y_a} p_1}\right] + \omega \left[\frac{\bar{s}_v^\circ(T_{2s}) - \bar{s}_v^\circ(T_1) - \bar{R} \ln \frac{\cancel{y_v} p_2}{\cancel{y_v} p_1}}{M_v}\right] = 0$$

With values from Tables A-22E, 23E at 520°R

$$0 = \left[s_a^\circ(T_{2s}) - 0.59172 - \frac{1.986}{28.97} \ln \frac{100}{14.7}\right] + \omega \left[\frac{\bar{s}_v^\circ(T_{2s})}{M_v} - \frac{\left(44.821 + 1.986 \ln \frac{100}{14.7}\right)}{18.02}\right] \qquad (4)$$

PROBLEM 12.58 (Contd.) - page 2

ω is found from Eq. 12.43 using $p_v = \phi_1 p_g(T_1) = 0.75(0.2563) = 0.1922$ lbf/in^2

Thus

$$\omega = 0.622\left(\frac{0.1922}{14.7 - 0.1922}\right) = 0.622\left(\frac{0.1922}{14.5078}\right) = 0.0082\ \frac{\text{lb(v)}}{\text{lb(a)}}$$

With this Eq. (4) gives upon rearrangement

$$s_a^\circ(T_{2s}) + \left(\frac{0.0082}{18.02}\right)\bar{s}_v^\circ(T_{2s}) = 0.7453 \qquad (5)$$

Solving Eq (5) iteratively, $T_{2s} \approx 895$ °R.

With T_{2s} known, Eq. (2) yields

$$\left(\frac{\dot{W}_{cv}}{\dot{m}_a}\right)_s = (124.27 - 215.03) + 0.0082\left(\frac{4122 - 7188.9}{18.02}\right)$$
$$= -90.76 - 1.40 = -92.16\ \text{Btu/lb(a)}$$

Equation (3) then gives

$$\left(\frac{\dot{W}_{cv}}{\dot{m}_a}\right) = \frac{-92.16}{0.8} = -115.2\ \frac{\text{Btu}}{\text{lb(a)}} \longleftarrow \frac{\dot{W}_{cv}}{\dot{m}_a}$$

Finally, with $(\dot{W}_{cv}/\dot{m}_a)$ known, Eq. (1) can be used iteratively to find T_2. Thus, upon rearrangement

$$h_a(T_2) + \frac{\omega}{M_v}\bar{h}_v(T_2) = h_a(T_1) + \frac{\omega}{M_v}\bar{h}_v(T_1) - \left(\frac{\dot{W}_{cv}}{\dot{m}_a}\right)$$
$$= 124.27 + \frac{0.0082}{18.02}(4122) + 115.2$$
$$= 241.35\ \text{Btu/lb(a)}$$

Solving this expression iteratively gives $T_2 \approx 987$ °R $\longleftarrow T_2$

The exergy destruction can be obtained using $(\dot{E}_d/\dot{m}_a) = T_0(\dot{\sigma}_{cv}/\dot{m}_a)$, where $\dot{\sigma}_{cv}$ is obtained from an entropy balance:

$$(\dot{\sigma}_{cv}/\dot{m}_a) = \left[s_a^\circ(T_2) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a}\ln\frac{p_2}{p_1}\right] + \omega\left[\frac{\bar{s}_v^\circ(T_2) - \bar{s}_v^\circ(T_1) - \bar{R}\ln\frac{p_2}{p_1}}{M_v}\right]$$

$$= \left[0.7472 - 0.59172 - \frac{1.986}{28.97}\ln\frac{100}{14.7}\right] + (0.0082)\left[\frac{50.079 - 44.821 - 1.986\ln(100/14.7)}{18.02}\right]$$

$$= 0.0240 + 0.0007 = 0.0247\ \text{Btu/lb(a)}\cdot°\text{R}$$

Then, $$(\dot{E}_d/\dot{m}_a) = 520°\text{R}\left(0.0247\ \frac{\text{Btu}}{\text{lb(a)}\cdot°\text{R}}\right) = 12.84\ \frac{\text{Btu}}{\text{lb(a)}} \longleftarrow \frac{\dot{E}_d}{\dot{m}_a}$$

b) The data for the required plots are obtained using IT, as follows:

PROBLEM 12.58 (Cont'd.) - page 3

IT Code

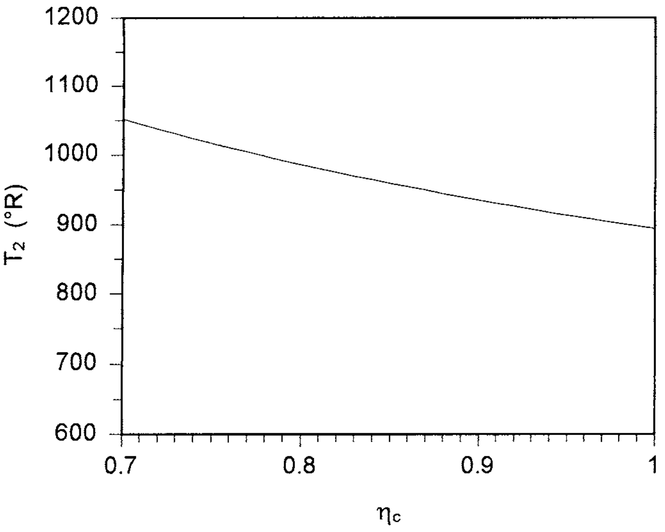
```
T1 = 520
phi1 = 0.75
p1 = 14.7
p2 = 100
w = w_Tphi(T1,phi1,p1)
mdota = 1  // Assume a unit mass flow rate.

Wdot / mdota = (ha1 - ha2) + w * (hv1 - hv2)
Wdots / mdota = (ha1 - ha2s) + w * (hv1 - hv2s)
Wdot / mdota = (Wdots / mdota) / eta_c
eta_c = 0.8

ha1 = h_T("Air", T1)
ha2 = h_T("Air", T2)
hv1 = h_T("H2O", T1)
hv2 = h_T("H2O", T2)
ha2s = h_T("Air", T2s)
hv2s = h_T("H2O", T2s)

pg1 = Psat_T("Water/Steam", T1)
pv1 = phi1 * pg1
yv = pv1 / p1
ya = 1 - yv
pa1 = p1 - pv1
pv2 = yv * p2
pa2 = p2 - pv2

sa1 = s_TP("Air", T1, pa1)
sa2s = s_TP("Air", T2s, pa2)
sv1 = s_TP("H2O", T1, pv1)
sv2s = s_TP("H2O", T2s, pv2)
(sa1 - sa2s) + w * (sv1 - sv2s) = 0
sa2 = s_TP("Air", T2, pa2)
sv2 = s_TP("H2O", T2, pv2)

Edot_d / mdota = To * ((sa2 - sa1) + w * (sv2 - sv1))
To = 520
```

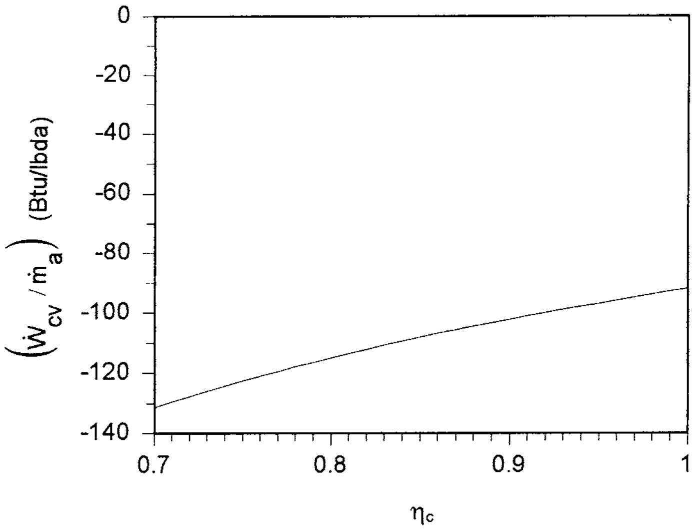
IT Results for $\eta_c = 0.8$

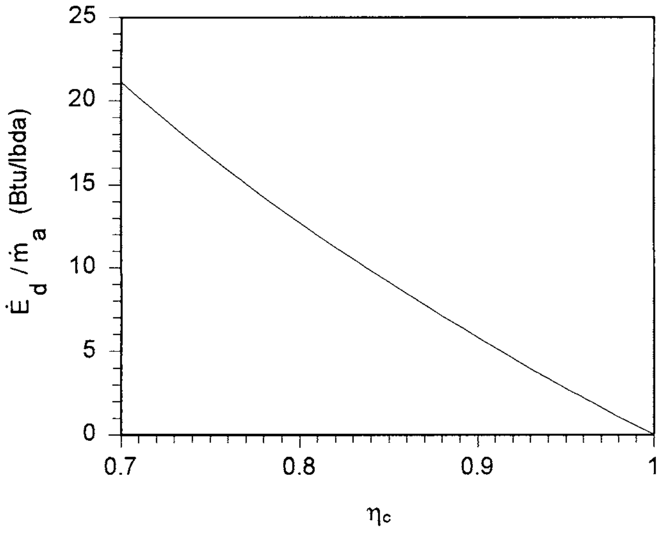
p_{a1} = 14.51 lbf/in.^{2}
p_{a2} = 98.68 lbf/in.^{2}
p_{v1} = 0.1945 lbf/in.^{2}
p_{v2} = 1.323 lbf/in.^{2}
T_{2s} = 894.4 °R
w = 0.008341
$\left(\dot{W}_{cv}/\dot{m}_a\right)_s$ = -91.98 Btu / lb(da)
T_2 = 986.1 °R
$\left(\dot{W}_{cv}/\dot{m}_a\right)$ = -115 Btu / lb(da)

$\dot{E}_d/\dot{m}_a$ = 12.73 Btu / lb(da)

PLOTS:







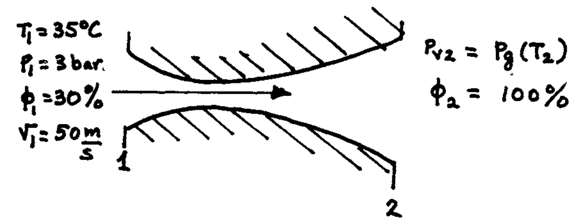
From the plots, we see that lower isentropic compressor efficiency corresponds to increased exit temperature, power input, and exergy destruction, as expected.

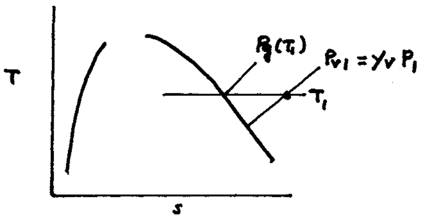
PROBLEM 12.59

KNOWN: Moist air expands isentropically through a nozzle. Conditions at the inlet are specified.

FIND: Determine the lowest exit pressure that can be obtained without condensation and the corresponding exit velocity.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The nozzle is well insulated and at steady state. (2) There are no significant potential energy effects. (3) The moist air acts as an ideal gas, with each component adhering to the Dalton model. (4) At the exit the moist air is saturated.

ANALYSIS: As the expansion is isentropic, $s_2 - s_1 = 0$, where

$$s_1 = mf_a\, s_a(T_1, y_a P_1) + mf_v\, s_v(T_1, p_{v1})$$
$$s_2 = mf_a\, s_a(T_2, y_a P_2) + mf_v\, s_v(T_2, p_{v2})$$

Since $p_{v2} = p_g(T_2)$, it follows that $y_v P_2 = p_g(T_2)$ or $P_2 = p_g(T_2)/y_v$. Also, at 2 the specific entropy of the water vapor is $s_g(T_2)$. Collecting results

$$mf_a\left[s_a(T_2, y_a \underset{\uparrow\, = p_g(T_2)/y_v}{P_2}) - s_a(T_1, y_a P_1)\right] + mf_v\left[s_g(T_2) - \underbrace{s_v(T_1, p_{v1})}_{= s_g(T_1) - \frac{\bar{R}}{M_v} \ln \frac{p_{v1}}{p_g(T_1)} = s_g(T_1) - \frac{\bar{R}}{M_v} \ln \phi_1}\right] = 0$$

or

$$mf_a\left[s_a^\circ(T_2) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a} \ln \frac{p_g(T_2)}{p_{v1}}\right] + mf_v\left[s_g(T_2) - s_g(T_1) + \frac{\bar{R}}{M_v} \ln \phi_1\right] = 0$$

Further, with $mf_a = \frac{m_a}{m_a + m_v} = \frac{1}{1+\omega}$, $mf_v = \frac{m_v}{m_a + m_v} = \frac{\omega}{1+\omega}$, the last equation is

$$\left[s_a^\circ(T_2) - s_a^\circ(T_1) - \frac{\bar{R}}{M_a} \ln \frac{p_g(T_2)}{p_{v1}}\right] + \omega\left[s_g(T_2) - s_g(T_1) + \frac{\bar{R}}{M_v} \ln \phi_1\right] = 0 \qquad (1)$$

With $\phi_1 = 0.30$ and $p_g(T_1)$ from Table A-2, $p_{v1} = 0.3(0.05628 \text{ bars}) = 0.01688$ bar. Too

$$\omega = 0.622 \frac{p_{v1}}{P_1 - p_{v1}} = 0.622 \frac{(0.01688)}{(3 - 0.01688)} = 0.00352 \frac{kg(v)}{kg(a)}$$

Substituting known values into Eq. (1)

$$\left[s_a^\circ(T_2) - 1.7284 - \frac{8.314}{28.97} \ln \frac{p_g(T_2)}{0.01688}\right] + 0.00352\left[s_g(T_2) - 8.3531 + \frac{8.314}{18.02} \ln 0.3\right] = 0$$

$$\left[s^\circ(T_2) - 1.7284 - 0.287 \ln \frac{p_g(T_2)}{0.01688}\right] + 0.00352\left[s_g(T_2) - 8.9086\right] = 0$$

Solving this equation iteratively using data from Tables A-2 and A-22,

① $T_2 \approx 10.4°C$.

PROBLEM 12.59 (Cont'd.)

With $p_{v2} = y_v p_2$ and $p_{v1} = y_v p_1$, $p_{v2}/p_{v1} = p_2/p_1$. Also, $p_{v2} = p_g(T_2)$, so

$$p_2 = \frac{p_g(T_2)}{p_{v1}} p_1 = \left(\frac{0.0126}{0.01688}\right) 3\,\text{bar} = 2.24\,\text{bar} \longleftarrow p_2$$

To evaluate V_2, write an energy rate balance for a control volume enclosing the nozzle. With $\dot{Q}_{cv} = \dot{W}_{cv} = 0$, and no kinetic and potential energy effects, this reduces to

$$0 = \left[\dot{m}_a h_a(T_1) + \dot{m}_v h_v(T_1) + (\dot{m}_a + \dot{m}_v)\frac{V_1^2}{2}\right] - \left[\dot{m}_a h_a(T_2) + \dot{m}_v h_v(T_2) + (\dot{m}_a + \dot{m}_v)\frac{V_2^2}{2}\right]$$

Dividing by $\dot{m}_a$, and using $\omega = \dot{m}_v/\dot{m}_a$, together with $h_v(T_1) \approx h_g(T_1)$ and $h_v(T_2) = h_g(T_2)$,

$$0 = \left[h_a(T_1) + \omega h_g(T_1) + (1+\omega)\frac{V_1^2}{2}\right] - \left[h_a(T_2) + \omega h_g(T_2) + (1+\omega)\frac{V_2^2}{2}\right]$$

Solving for $V_2^2/2$

$$\frac{V_2^2}{2} = \frac{V_1^2}{2} + \frac{h_a(T_1) - h_a(T_2) + \omega(h_g(T_1) - h_g(T_2))}{1+\omega}$$

or

$$V_2 = \sqrt{V_1^2 + \frac{2}{1+\omega}\left[h_a(T_1) - h_a(T_2) + \omega(h_g(T_1) - h_g(T_2))\right]}$$

$$= \sqrt{\left(50\,\frac{m}{s}\right)^2 + \frac{2}{1.00352}\left[308.23 - 283.54 + 0.00352(2565.3 - 2520.5)\right]\frac{kJ}{kg}\left|\frac{10^3\,N\cdot m}{1\,kJ}\right|\left|\frac{1\,kg\cdot m/s^2}{1\,N}\right|}$$

$$= 228.1\,m/s \longleftarrow V_2$$

---

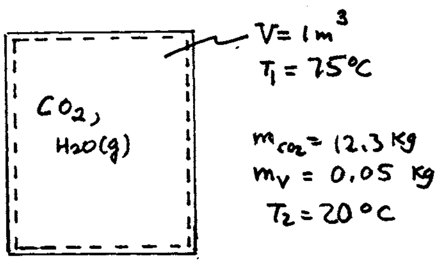
1. An iterative solution using table data can be avoided using IT. The results obtained using LT agree with the values given here.

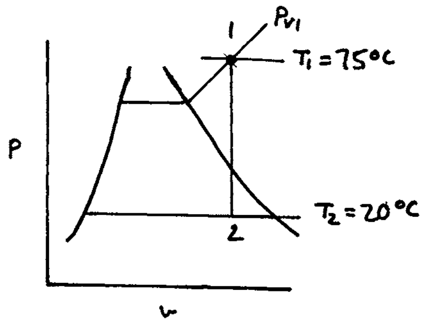
PROBLEM 12.60*

KNOWN: A tank having a known volume initially contains 12.3 kg of CO_2 and 0.05 kg of water vapor at 75°C, 15 bar. The tank contents are cooled to 20°C.

FIND: Determine the heat transfer.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) As shown by the accompanying figure, the system is the tank contents. (2) There are no changes in kinetic or potential energy. (3) The gas mixture behaves as an ideal gas, with each component adhering to the Dalton model.

ANALYSIS: The first step is to determine if condensation occurs. As the total volume of the tank and the total mass of water present are constant, the water undergoes a constant specific volume process as the tank contents are cooled (see T-v diagram). The specific volume of the water is $v = 1 m^3/0.05\ kg = 20.00\ m^3/kg$. By inspection of v_g data in Table A-2, condensation would occur at a temperature between 38 and 40°C. Accordingly, there is condensation in this case.

An energy balance reduces to give $\Delta U = Q - \cancel{W}$, or $Q = \Delta U$:

$$Q = [m_{CO_2} u_{CO_2}(T_2) + m_{v2}\, u_g(T_2) + m_{w2}\, u_f(T_2)] - [m_{CO_2} u_{CO_2}(T_1) + m_{v1}\, u_g(T_1)]$$

$$= \frac{m_{CO_2}[\bar{u}_{CO_2}(T_2) - \bar{u}_{CO_2}(T_1)]}{M_{CO_2}} + [m_{v2}\, u_g(T_2) - m_{v1}\, u_g(T_1) + m_{w2}\, u_f(T_2)] \qquad (1)$$

To find m_{v2} and m_{w2}, note that the quality of the two-phase liquid-vapor mixture of water at the final state is

$$x_2 = \frac{v_2 - v_f}{v_g - v_f} = \frac{20.00 - 0.0010018}{57.791 - 0.0010018} = 0.3461$$

Accordingly, $m_{v2} = x_2 m_{v1} = (0.3461)(0.05) = 0.01731$ kg. So, $m_{w2} = 0.03269$ kg.

Returning to Eq. (1) and using data from Tables A-2 and A-23

$$Q = \left(\frac{12.3}{44.01}\right)(6739 - 8377) + [(0.01731)(2402.9) - (0.05)(2475.9) + (0.03269)(83.95)]$$

$$= -457.79 + [41.59 - 123.80 + 2.74] = -537.3\ kJ \longleftarrow$$

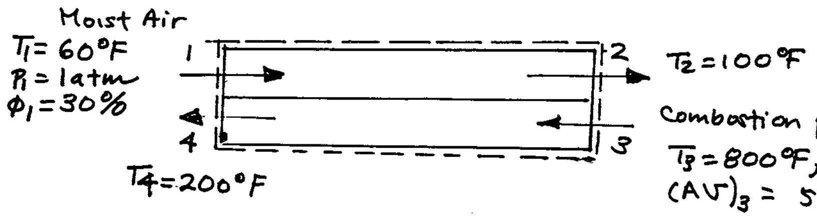
Note: In the first printing of the fifth edition, the mass of water vapor was given incorrectly as 0.5 kg. The correct value of 0.05 kg is used in this solution.

PROBLEM 12.61

KNOWN: Steady state operating data are provided for a counterflow heat exchanger having combustion products flowing on one side and moist air flowing on the other.

FIND: Determine the mass flow rate of the moist air stream.

SCHEMATIC & GIVEN DATA:



Combustion products : { 7.1% CO_2, 4.3% O_2, 14.3% H_2O, 74.3% N_2 }
$T_3 = 800°F$, $p_3 = 1 atm$
$(AV)_3 = 5\ ft^3/s$

ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, kinetic/potential energy effects can be ignored, and there is no significant pressure drop on either side. (3) The combustion products and the moist air adhere to the Dalton model.

ANALYSIS: At steady state mass rate balances read $\dot{m}_{a1} = \dot{m}_{a2}$, $\dot{m}_{v1} = \dot{m}_{v2}$, and $\dot{m}_3 = \dot{m}_4$ (or $\dot{n}_3 = \dot{n}_4$). It follows that $\omega_2 = \omega_1$. The molar flow rate of the combustion products is

$$\dot{n}_3 = \frac{(AV)_3}{\bar{v}_3} = \frac{(AV)_3 p_3}{\bar{R} T_3} = \frac{(5\ ft^3/s)(14.7 \times 144\ lbf/ft^2)}{\left(1545 \frac{ft \cdot lbf}{lbmol \cdot °R}\right)(1260°R)} = 0.00544\ \frac{lbmol}{s}$$

Reducing an energy rate balance

$$0 = \dot{m}_a\left[(h_a + \omega h_g)_1 - (h_a + \omega h_g)_2\right] - \dot{n}_3\left[y_{CO_2}\Delta\bar{h}_{CO_2} + y_{O_2}\Delta\bar{h}_{O_2} + y_{H_2O}\Delta\bar{h}_{H_2O} + y_{N_2}\Delta\bar{h}_{N_2}\right]$$

Or, since $\omega_1 = \omega_2$

$$\dot{m}_a = \frac{-\dot{n}_3\left[y_{CO_2}\Delta\bar{h}_{CO_2} + y_{O_2}\Delta\bar{h}_{O_2} + y_{H_2O}\Delta\bar{h}_{H_2O} + y_{N_2}\Delta\bar{h}_{N_2}\right]}{\left[(h_{a2} - h_{a1}) + \omega(h_{g1} - h_{g2})\right]} \qquad (1)$$

For the combustion products, ideal gas table data gives

	CO_2	O_2	H_2O	N_2
y	0.071	0.043	0.143	0.743
$\bar{h}(1260°R)$	11,661	9,097	10,355	8,859
$\bar{h}(660°R)$	5,165	4,594	5,250	4,586
$y\Delta\bar{h}$	-461.2	-193.6	-730.0	-3174.8

From Table A-22E
$h_a(520°R) = 124.27$ Btu/lb
$h_a(560°R) = 133.86$
From Table A-2E
$h_g(60°F) = 1087.7$ Btu/lb
$h_g(100°F) = 1105.0$ Btu/lb
Also, $p_{v1} = \phi_1 p_{g1} = 0.30(.2563\ lbf/in^2) = 0.07689\ \frac{lbf}{in^2}$
So $\omega = .622 \frac{(0.07689)}{(14.7 - 0.07689)}$
$= 0.0033$

Inserting values into Eq.(1)

$$\dot{m}_a = \frac{0.00544\left[461.2 + 193.6 + 730 + 3174.8\right]}{(133.86 - 124.27) + 0.0033(1105 - 1087.7)} = 2.57\ lb/s$$

The total mass flow rate of the moist air stream is

$$\dot{m} = (1 + \omega)\dot{m}_a = (1.0033)(2.57\ \tfrac{lb}{s}) = 2.58\ \tfrac{lb}{s} \longleftarrow$$

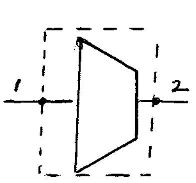
PROBLEM 12.62

KNOWN: Steady state operating data are provided for a compressor handling moist air.

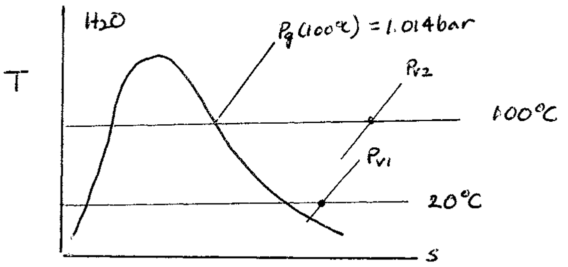
FIND: Determine (a) the relative humidity at the exit, (b) the power input, and (c) the rate of entropy production.

SCHEMATIC & GIVEN DATA:

$T_1 = 20°C$
$p_1 = 1.05$ bar
$\phi_1 = 85\%$
$(AV)_1 = 0.3\ m^3/s$



$T_2 = 100°C$
$p_2 = 2.0$ bar



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The moist air adheres to ideal gas mixture principles.

ANALYSIS: As the partial pressures are only small fractions of a bar in magnitude, the final state in an adiabatic compression would be far enough from the vapor dome to eliminate the chance of condensation.

(a) Mass rate balances reduce to give $\dot{m}_{a1} = \dot{m}_{a2}$, $\dot{m}_{v1} = \dot{m}_{v2}$, or $\omega_2 = \omega_1$. Thus

$$(.622)\frac{p_{v1}}{p_1 - p_{v1}} = (0.622)\frac{p_{v2}}{p_2 - p_{v2}} \Rightarrow \frac{p_{v2}}{p_2} = \frac{p_{v1}}{p_1} \Rightarrow p_{v2} = \phi_1 p_g(T_1)\,\frac{p_2}{p_1} = (0.85)(0.02339)\left(\frac{2}{1.05}\right) = 0.0379\ \text{bar}$$

Accordingly, $\phi_2 = p_{v2}/p_g(T_2) = 0.0379/1.014 = 0.037\ (3.7\%)$ ⟵ (a)

(b) Reducing an energy rate balance gives after rearrangement

① $$\dot{W}_{cv} = \dot{m}_a\left[(h_a + \omega h_v)_1 - (h_a + \omega h_v)_2\right] = \dot{m}_a\left[h_{a1} - h_{a2} + \omega(h_{v1} - h_{v2})\right]$$

where

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{(AV)_1}{\left(\frac{\bar{R}}{M_a}\frac{T}{p_{a1}}\right)} = \frac{(0.3\ m^3/s)(1.05 - 0.0199)\times 10^5\ N/m^2}{\left(\frac{8314}{29}\frac{N\cdot m}{kg\cdot K}\right)(293\ K)} = 0.368\ kg(a)/s$$

$$\omega = .622\frac{p_{v1}}{p_1 - p_{v1}} = \frac{.622(0.0199)}{1.0301} = 0.012\ kg(v)/kg(a)$$

Since there is no condensed phase present, it suffices to use the ideal gas table, Table A-25 for the water and Table A-22 for the air.

$$\dot{W}_{cv} = -0.368\left[373.7 - 293.2 + 0.0120\left[\frac{12433 - 9733}{18}\right]\right] = -30.29\ kW$$

Power input = 30.29 kW ⟵ (b)

(c) An entropy rate balance reduces to give

$$\dot{\sigma}_{cv} = \dot{m}_a\left[(s_{a2} - s_{a1}) + \omega(s_{v2} - s_{v1})\right]$$

$$= \dot{m}_a\left[\left[s_a^\circ(T_2) - s^\circ(T_1) - \frac{\bar{R}}{M_a}\ln\frac{p_{a2}}{p_{a1}}\right] + \omega\left[\bar{s}_v^\circ(T_2) - \bar{s}_v^\circ(T_1) - \frac{\bar{R}}{M_v}\ln\frac{p_{v2}}{p_{v1}}\right]\right]$$

$= p_2/p_1$

$= \frac{p_2}{p_1}$

Or with s° data from Table A-22 and $\bar{s}^\circ$ data from Table A-23

$$\dot{\sigma}_{cv} = 0.368\left[\left[1.921 - 1.678 - \frac{8.314}{29}\ln\left(\frac{2.0}{1.05}\right)\right] + 0.012\left[\frac{196.284 - 188.139 - 8.314\ln(2.0/1.05)}{18}\right]\right]$$

$$= 0.368\left[0.058 + 0.012\left[0.155\right]\right]$$

$$= 0.022\ \frac{kJ/s}{K} = 0.022\ \frac{kW}{K}$$ ⟵ (c)

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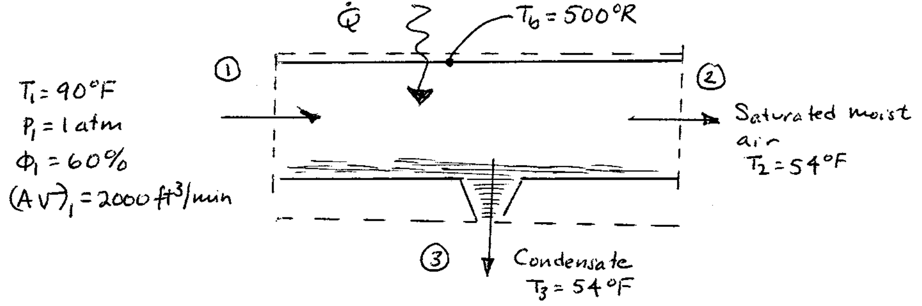
1. Parts (b) and (c) also could be solved on the basis of constant specific heats c_{pa}, c_{pv} evaluated at the mean temperature, for example.

PROBLEM 12.63

KNOWN: Operating data are provided for a control volume at steady state.

FIND: Determine (a) the heat transfer rate, (b) entropy production rate.

SCHEMATIC & GIVEN DATA



ASSUMPTIONS: (1) The control volume is at steady state. (2) For the control volume $\dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The moist air adheres to ideal gas principles. (4) The pressure is uniform at 1atm.

ANALYSIS: Mass rate balances reduce to give $\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$, $\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_3$. Thus, $\dot{m}_3 = \dot{m}_a(\omega_1 - \omega_2)$. Calculating

$$\omega_1 = 0.622\left[\frac{\phi_1 p_{g1}}{p - \phi_1 p_{g1}}\right] = 0.622\left[\frac{(0.6)(0.6988)}{14.7 - 0.4193}\right] = 0.01826 \frac{\text{lb(v)}}{\text{lb(a)}}$$

$$\omega_2 = 0.622\left[\frac{p_{g2}}{p - p_{g2}}\right] = 0.622\left[\frac{0.2064}{14.7 - 0.2064}\right] = 0.00886 \frac{\text{lb(v)}}{\text{lb(a)}}$$

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{(AV)_1}{\frac{\bar{R}}{M_a}\frac{T_1}{p_{a1}}} = \frac{(2000\ \text{ft}^3/\text{min})(14.281)(144\ \text{lbf/ft}^2)}{\left(\frac{1545}{29}\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(550°\text{R})} = 140.36 \frac{\text{lb(a)}}{\text{min}}$$

An energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}_a\left[(h_a + \omega h_v)_1 - (h_a + \omega h_v)_2\right] - \dot{m}_3 h_f(T_3)$$

or

$$\dot{Q}_{cv} = \dot{m}_a\left[(h_{a2} - h_{a1}) + \omega_2 h_{g2} - \omega_1 h_{g1} + (\omega_1 - \omega_2) h_{f3}\right]$$

$$= 140.36\left[0.24(54 - 90) + 0.00886(1085.1) - 0.01826(1100.7) + (0.0094)(22.1)\right] = -2655.5\ \text{Btu/min} \longleftarrow \dot{Q}_{cv}$$

An entropy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \left[\dot{m}_a s_{a1} + \dot{m}_{v1} s_{v1}\right] - \left[\dot{m}_a s_{a2}\ \dot{m}_{v2} s_{v2}\right] - \dot{m}_3 s_3 + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_a\left[s_{a2} - s_{a1}\right] + \underbrace{\dot{m}_{v2}}_{\omega_2 \dot{m}_a} s_{v2} - \underbrace{\dot{m}_{v1}}_{\omega_1 \dot{m}_a} s_{v1} + \underbrace{\dot{m}_3}_{(\omega_1 - \omega_2)\dot{m}_a} s_3$$

At ③ the liquid is saturated, $s_3 = s_f(T_3)$. At ② the vapor is saturated, $s_{v2} = s_{g2}$. At ①, $s_{v1} = s_{g1} - \frac{\bar{R}}{M}\ln\phi_1$ (see Sec. 12.5.2 for discussion). Thus

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_a\left[\left(c_{pa}\ln\frac{T_2}{T_1} - \frac{\bar{R}}{M}\ln\frac{p_{a2}}{p_{a1}}\right) + \omega_2 s_{g2} - \omega_1\left(s_{g1} - \frac{\bar{R}}{M}\ln\phi_1\right) + (\omega_1 - \omega_2)s_{f3}\right]$$

$$= \frac{2655.5}{500} + 140.36\left[\left(0.24\ln\frac{514}{550} - \frac{1.986}{29}\ln\frac{14.444}{14.281}\right) + (0.00886)(2.1131) - 0.01826\left(2.0083 - \frac{1.986}{18}\ln(0.6)\right) + (0.0094)(.04391)\right]$$

$$= 5.311 + 140.36\left[-0.0172 + 0.0187 - 0.0377 + 0.0004\right]$$

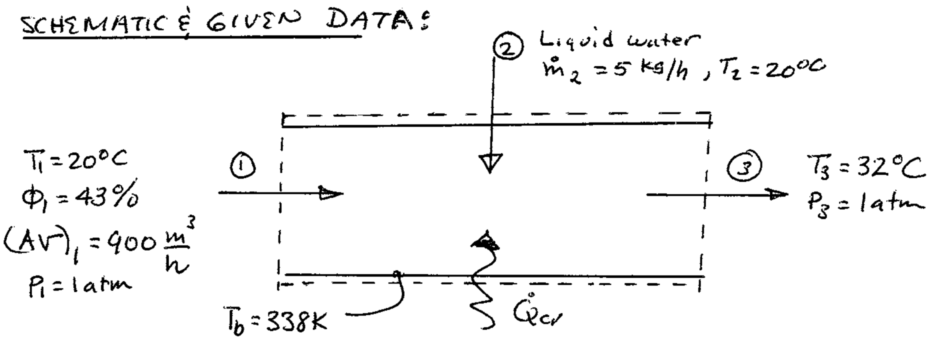
$$= 0.286\ \frac{\text{Btu/min}}{°\text{R}} \longleftarrow \dot{\sigma}_{cv}$$

PROBLEM 12.64

KNOWN: Operating data are provided for a control volume at steady state.

FIND: Determine (a) the heat transfer rate, (b) the entropy production rate.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume $\dot{W}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The moist air adheres to ideal gas mixture principles.

ANALYSIS: Mass rate balances at steady state read $\dot{m}_{a1} = \dot{m}_{a3} \equiv \dot{m}_a$, $\dot{m}_{v1} + \dot{m}_2 = \dot{m}_{v3}$. Calculating, $P_{a1} = P_1 - P_{v1} = P_1 - \phi_1 P_{g1} = 1.01325 - (0.43)(0.02339) = 1.0032\ bar$.

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{(AV)P_{a1}}{\frac{\bar{R}}{M_a}T_1} = \frac{(900\,m^3/h)(1.0032\times10^5\,N/m^2)}{\left(\frac{8314}{29}\frac{N\cdot m}{kg\cdot K}\right)(293K)} = 1074.9\ \frac{kg}{h},\quad \frac{\dot{m}_2}{\dot{m}_a} = \frac{5}{1074.9} = 0.00465\ \frac{kg(\ell)}{kg(a)}$$

$$\omega_1 = 0.622\frac{P_{v1}}{P_1 - P_{v1}} = \frac{(0.622)(0.43)(0.02339)}{1.0032} = 0.0062\ \frac{kg(v)}{kg(a)},\quad \omega_3 = \frac{\dot{m}_2}{\dot{m}_a} + \omega_1 = 0.00465 + 0.0062 = 0.01085$$

$$\Rightarrow P_{v3} = 0.01737\,bar,\ \phi_3 = 0.365.$$

An energy rate balance reduces to read

$$0 = \dot{Q}_{cv} + \dot{m}_a\left[(h_a + \omega h_v)_1 + \frac{\dot{m}_2}{\dot{m}_a}h_2 - (h_a + \omega h_v)_3\right]$$

or

$$\dot{Q}_{cv} = \dot{m}_a\left[(h_{a3} - h_{a1}) + \omega_3 h_{g3} - \omega_1 h_{g1} - \frac{\dot{m}_2}{\dot{m}_a}h_{f2}\right]$$

$$= 1074.9\left[1.005(32-20) + (0.01085)(2559.9) - 0.0062(2538.1) - 0.00465(84)\right]$$

$$= \left(25474.5\ \frac{kJ}{h}\right)\left|\frac{h}{3600s}\right|\left|\frac{1\ kW}{1\ kJ/s}\right| = 7.08\ kW$$

$\longleftarrow \dot{Q}_{cv}$

An entropy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \left[\dot{m}_a s_{a1} + \dot{m}_{v1}s_{v1}\right] + \dot{m}_2 s_2 - \left[\dot{m}_a s_{a3} + \dot{m}_{v3}s_{v3}\right] + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_a\left[s_{a3} - s_{a1}\right] + \underbrace{\dot{m}_{v3}}_{\omega_3\dot{m}_a}\underbrace{s_{v3}}_{= s_{g3} - \frac{\bar{R}}{M_v}\ln\phi_3} - \overbrace{\dot{m}_{v1}}^{\omega_1\dot{m}_a}\overbrace{s_{v1}}^{s_{g1} - \frac{\bar{R}}{M_v}\ln\phi_1} - \dot{m}_2\underbrace{s_2}_{s_{f2}}$$

So

$$\dot{\sigma}_{cv} = -\frac{\dot{Q}_{cv}}{T_b} + \dot{m}_a\left[\left(c_{pa}\ln\frac{T_3}{T_1} - \frac{\bar{R}}{M}\ln\frac{P_{a3}}{P_{a1}}\right) + \omega_3\left(s_{g3} - \frac{\bar{R}}{M_v}\ln\phi_3\right) - \omega_1\left(s_{g1} - \frac{\bar{R}}{M_v}\ln\phi_1\right) - \frac{\dot{m}_2}{\dot{m}_a}s_{f2}\right]$$

$$= \frac{-(7.08kW)}{338K} + \left(\frac{1074.9}{3600}\frac{kg(a)}{s}\right)\left[\left(1.005\ln\frac{305}{293} - \frac{8.314}{29}\ln\frac{0.99588}{1.0032}\right) + 0.01085\left(8.4127 - \frac{8.314}{18}\ln 0.365\right)\right.$$
$$\left. - 0.0062\left(8.6672 - \frac{8.314}{18}\ln 0.43\right) - 0.00465(0.2966)\right]\frac{kJ}{kg(a)}\left|\frac{1\ kW}{kJ/s}\right|$$

$$= -0.0209 + (0.299)\left[0.0424 + 0.0963 - 0.0562 - 0.0014\right]$$

$$= 0.0033\ \frac{kW}{K}$$

$\longleftarrow \dot{\sigma}_{cv}$

PROBLEM 12.65

KNOWN: The dry-bulb and wet-bulb temperatures are provided for each of two cases.

FIND: Using Eq. 12.48, determine ω and ϕ for each case.

ASSUMPTIONS (1) The wet-bulb temperature can be used in place of the adiabatic temperature to evaluate ω. (2) $p = 1\,\text{atm}$.

ANALYSIS: (a) $T_{db} = 24°C$, $T_{wb} = 16°C$, $p = 1\,\text{atm}$

ω can be found from Eq. 12.48. The value of ω' in this expression is found using p_g at the wet bulb temperature from Table A-2

$$\omega' = 0.622\left[\frac{0.01818}{1.01325 - 0.01818}\right] = 0.0114\ \frac{\text{kg(v)}}{\text{kg(a)}}$$

Equation 12.48 then gives with $c_{pa} = 1.005\ \text{kJ/kg·K}$ and data from Table A-2

$$\omega = \frac{c_{pa}(T_{wb} - T) + \omega'[h_g(T_{wb}) - h_f(T_{wb})]}{h_g(T) - h_f(T_{wb})}$$

ω

$$= \frac{(1.005)(16-24) + 0.0114[2463.6]}{(2545.4 - 67.2)} = \frac{-8.04 + 28.09}{2478.2} = 0.0081\ \frac{\text{kg(v)}}{\text{kg(a)}}$$

Solving Eq. 12.43 for p_v

$$p_v = \frac{\omega p}{0.622 + \omega} = \frac{(0.0081)(1.01325\ \text{bar})}{0.622 + 0.0081} = 0.013\ \text{bar}$$

Then

$$\phi = \frac{p_v}{p_g(T)} = \frac{0.013}{0.02985} = 0.436 \quad (43.6\%)$$

(b) $T_{db} = 75°F$, $T_{wb} = 60°F$, $p = 1\,\text{atm}$

ω can be found from Eq. 12.48. The value of ω' in this expression is found using p_g at the wet bulb temperature from Table A-2E

$$\omega' = 0.622\left[\frac{0.2563}{14.7 - 0.2563}\right] = 0.01104\ \frac{\text{lb(v)}}{\text{lb(a)}}$$

Equation 12.48 then gives with $c_{pa} = 0.24\ \text{Btu/lb·°R}$ and data from Table A-2E

$$\omega = \frac{c_{pa}(T_{wb} - T) + \omega'[h_g(T_{wb}) - h_f(T_{wb})]}{h_g(T) - h_f(T_{wb})}$$

$$= \frac{0.24[60-75] + 0.01104[1059.6]}{1094.25 - 28.08} = \frac{-3.6 + 11.698}{1066.17} = 0.0076\ \frac{\text{lb(v)}}{\text{lb(a)}}$$

Solving Eq 12.43 for p_v

$$p_v = \frac{\omega p}{0.622 + \omega} = \frac{(0.0076)(14.696)}{0.622 + 0.0076} \quad 0.1774\ \text{lbf/in}^2$$

Then

$$\phi = \frac{p_v}{p_g(T)} = \frac{0.1774}{0.4302} = 0.412 \quad (41.2\%)$$

PROBLEM 12.65 (Contd.)

(c) Referring to Figure A-9 with the data of part (a), $\phi = 44\%$, $\omega = 0.0081$ kg(v)/kg(a).
Referring to Figure A-9E with the data of part (b), $\phi = 42\%$, $\omega = 0.0078$ lb(v)/lb(a).

(d) <u>*IT* Solution</u>

```
T_a = 24  // °C
Twb_a = 16  // °C
p = 1.01325  // bar
w_a = w_TTwb(T_a,Twb_a, p)
phi_a = phi_Tw(T_a,w_a, p)
```

<u>*IT* Results - Part (a)</u>

```
φ = 0.435
ω = 0.00807
```

```
T_b = 75  // °F
Twb_b = 60  // °F
p = 1  // atm
w_b = w_TTwb(T_b,Twb_b, p)
phi_b = phi_Tw(T_b,w_b, p)
```

<u>*IT* Results - Part (b)</u>

```
φ = 0.4124
ω = 0.00759
```

PROBLEM 12.66

FIND: Using the psychrometric chart, determine

(a) ϕ, ω, $h_a + \omega h_g$ when $T_{db} = 30°C$, $T_{wb} = 25°C$.

(b) ω, $h_a + \omega h_g$, T_{wb} when $T_{db} = 30°C$, $\phi = 60\%$

(c) $T_{\text{dew point}}$ for $T_{db} = 30°C$, $T_{wb} = 20°C$

(d) Repeat (a)-(c) using IT.

ANALYSIS: Using the psychrometric chart ($p = 1\,atm$)

(a) $T_{db} = 30°C$, $T_{wb} = 25°C$

$\phi = 68\%$, $\omega = .0182$, $h_a + \omega h_g = 76.3\ kJ/kg(a)$

(b) $T_{db} = 30°C$, $\phi = 60\%$

$\omega = 0.0163$, $h_a + \omega h_g = 71.5\ kJ/kg(a)$, $T_{wb} \approx 24°C$

(c) $T_{db} = 30°C$, $T_{wb} = 20°C$, $T_{\text{dew pt}} \approx 15°C$

(d) Using IT for parts (a), (b), (c)

```
p = 1.01325  // bar

T_a = 30  // °C
Twb_a = 25  // °C
Twb_a = Twb_Tphi(T_a, phi_a, p)
w_a = w_TTwb(T_a,Twb_a, p)
h_a = ha_Tw(T_a, w_a)
```

IT Results for part (a)

$h_a + \omega h_v = 76.13$ kJ / kg(a)

$\phi = 0.6706$

$\omega = 0.018$

```
T_b = 30  // °C
phi_b = 0.6
w_b = w_Tphi(T_b, phi_b, p)
h_b = ha_Tphi(T_b, phi_b, p)
Twb_b = Twb_Tphi(T_b, phi_b, p)
```

IT Results for part (b)

$T_{wb} = 23.83$ °C

$h_a + \omega h_v = 71.15$ kJ / kg(a)

$\omega = 0.01605$

```
T_c = 30  // °C
Twb_c = 20  // °C
Twb_c = Twb_Tphi(T_c, phi_c, p)
pg = Psat_T("Water/Steam", T_c)
pv = phi_c * pg
T_dewpt = Tsat_P("Water/Steam", pv)
```

IT Results for part (c)

$p_g = 0.04246$ bar

$p_v = 0.0169$ bar

$\phi = 0.398$

$T_{\text{dew point}} = 14.86$ °C

PROBLEM 12.67

FIND: Using the psychrometric chart, determine

(a) $T_{dew\,pt}$ for $T_{db} = 80°F$, $T_{wb} = 70°F$

(b) ω, $h_a + \omega h_g$, T_{wb} for $T_{db} = 80°F$, $\phi = 70\%$

(c) ϕ, ω, $h_a + \omega h_g$ for $T_{db} = 80°F$, $T_{wb} = 65°F$

(d) Repeat (a)-(c) using IT

ANALYSIS: Using the psychrometric chart ($p = 1$ atm)

(a) $T_{db} = 80°F$, $T_{wb} = 70°F$

$T_{dew\,pt} = 65.5°F$

(b) $T_{db} = 80°F$, $\phi = 70\%$

$\omega = 0.0155$

$h_a + \omega h_g = 36.1$ Btu/lb(a)

$T_{wb} = 72.5°F$

(c) $T_{db} = 80°F$, $T_{wb} = 65°F$

$\phi = 45\%$

$\omega = 0.0099$

$h_a + \omega h_g = 30$ Btu/lb(a)

(d) Using IT for parts (a), (b), (c)

```
p = 1  // atm

T_a = 80  // °F
Twb_a = 70  // °F
Twb_a = Twb_Tphi(T_a, phi_a, p)
pg = Psat_T("Water/Steam", T_a)
pv = phi_a * pg
Tdewpt = Tsat_P("Water/Steam", pv)
```

IT Results for part (a)

$p_g = 0.03452$ lbf/in.^{2}

$p_v = 0.02114$ lbf/in.^{2}

$\phi = 0.6124$

$T_{dew\ point} = 65.46°F$

```
T_b = 80  // °F
phi_b = 0.7
w_b = w_Tphi(T_b, phi_b, p)
h_b = ha_Tphi(T_b, phi_b, p)
Twb_b = Twb_Tphi(T_b, phi_b, p)
```

IT Results for part (b)

$\omega = 0.01539$

$h_a + \omega h_v = 36.03$ Btu/lb(a)

$T_{wb} = 72.39°F$

```
T_c = 80  // °F
Twb_c = 65  // °F
Twb_c = Twb_Tphi(T_c, phi_c, p)
w_c = w_TTwb(T_c, Twb_c, p)
h_c = ha_Tphi(T_c, phi_c, p)
```

IT Results for part (c)

$\phi = 0.447$

$\omega = 0.009739$

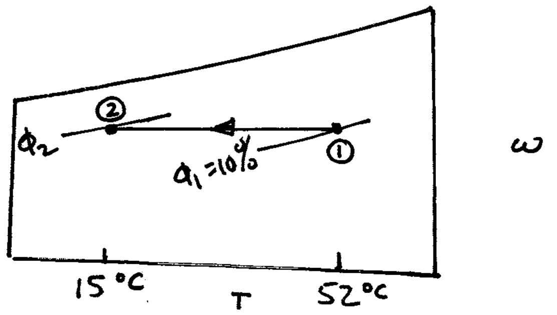
$h_a + \omega h_v = 29.84$ Btu/lb(a)

PROBLEM 12.68*

KNOWN: Moist air at 52°C, 1 atm, $\phi = 10\%$ is cooled at constant pressure to 15 °C.

FIND: Using the psychrometric chart, determine whether condensation occurs. If so, find the amount condensed per kg of dry air. If not, determine ϕ at the final state.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: For moist air, ideal gas principles apply.

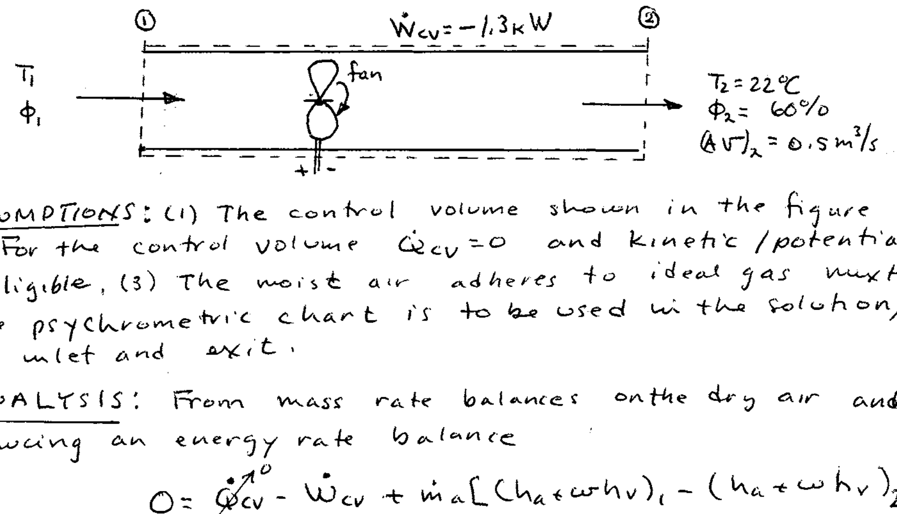
ANALYSIS: As long as there is no condensation the mole fraction of water vapor, and thus p_v remains constant during constant-pressure cooling: $p_v = y_v p$. Since both p_v and p are constant, Eq. 12.43 shows ω remains constant, as shown on the schematic. With the given data, Fig. A-9 indicates that no condensation occurs ← and $\Phi_2 = 80\%$. ←

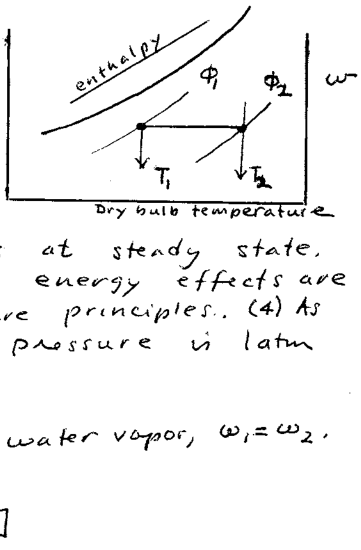
PROBLEM 12.69

KNOWN: Steady state operating data are given for a fan within an insulated duct.

FIND: Determine the temperature and relative humidity at the duct inlet.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The moist air adheres to ideal gas mixture principles. (4) As the psychrometric chart is to be used in the solution, pressure is 1 atm at inlet and exit.

ANALYSIS: From mass rate balances on the dry air and water vapor, $\omega_1 = \omega_2$. Reducing an energy rate balance

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \dot{W}_{cv} + \dot{m}_a[(h_a + \omega h_v)_1 - (h_a + \omega h_v)_2]$$

Thus

$$(h_a + \omega h_v)_1 = (h_a + \omega h_v)_2 + \dot{W}_{cv}/\dot{m}_a \qquad (1)$$

Using Φ_2, T_2, the psychrometric chart gives

$$\omega_2 = 0.0099\ kg(v)/kg(a)$$

$$(h_a + \omega h_v)_2 = 47.3\ kJ/kg(a)$$

$$v_{a2} = 0.849\ m^3/kg(a)$$

PROBLEM 12.69 (Contd.)

Then

$$\dot{m}_a = \frac{(AV)_2}{v_{a2}} = \frac{0.5\ m^3/s}{0.849\ m^3/kg(a)} = \mathbf{0.589\ \frac{kg(a)}{s}}$$

and inserting values in Eq. (1)

$$(h_a + \omega h_v)_1 = 47.3\ \frac{kJ}{kg(a)} + \frac{(-1.3\ kJ/s)}{0.589\ kg(a)/s} = 45.09\ \frac{kJ}{kg(a)}$$

Then, using the known values for $(h_a+\omega h_v)_1$ and $\omega_1 (=\omega_2)$, Fig A-9 gives

$$T_1 \approx 20°C$$
$$\phi_1 \approx 68\% \quad \longleftarrow$$

PROBLEM 12.70

KNOWN: Moist air adheres to ideal gas principles.

FIND: Derive expressions in SI and English units for the mixture enthalpy per unit of mass as used on Figs. A-9, A-9E.

ANALYSIS:

SI CASE.

Turning to the discussion of Fig. A-9 in Sec. 12.9, note that the enthalpy of the dry air is determined relative to a datum of 0°C. That is

$$h_a(T) = \overset{0}{\cancel{h_a(T_{ref})}} + \underset{1.005\ \frac{kJ}{kg(a)\cdot K}}{c_{pa}} \underbrace{[T - T_{ref}]}_{T(°C)}$$

The enthalpy of the water vapor is evaluated as $h_v \approx h_g$ at the dry-bulb temperature. A plot of h_g vs $T(°C)$ using data from Tables A-2, 6 shows a nearly linear variation; with **h_g** being given closely by

$$h_g \approx 2507.1 + \underbrace{1.82}_{\approx c_{pv}} T(°C)$$

ENGLISH CASE.

The enthalpy of the dry air is determined relative to a datum of 0°F. That is

$$h_a(T) = \overset{0}{\cancel{h(T_{ref})}} + \underset{0.24\ \frac{Btu}{lb\cdot °R}}{c_{pa}} \underbrace{[T - T_{ref}]}_{T(°F)}$$

Plotting h_g vs $T(°F)$ using data from Tables A-2E, 6E shows a nearly linear variation, with **h_g** being given closely by

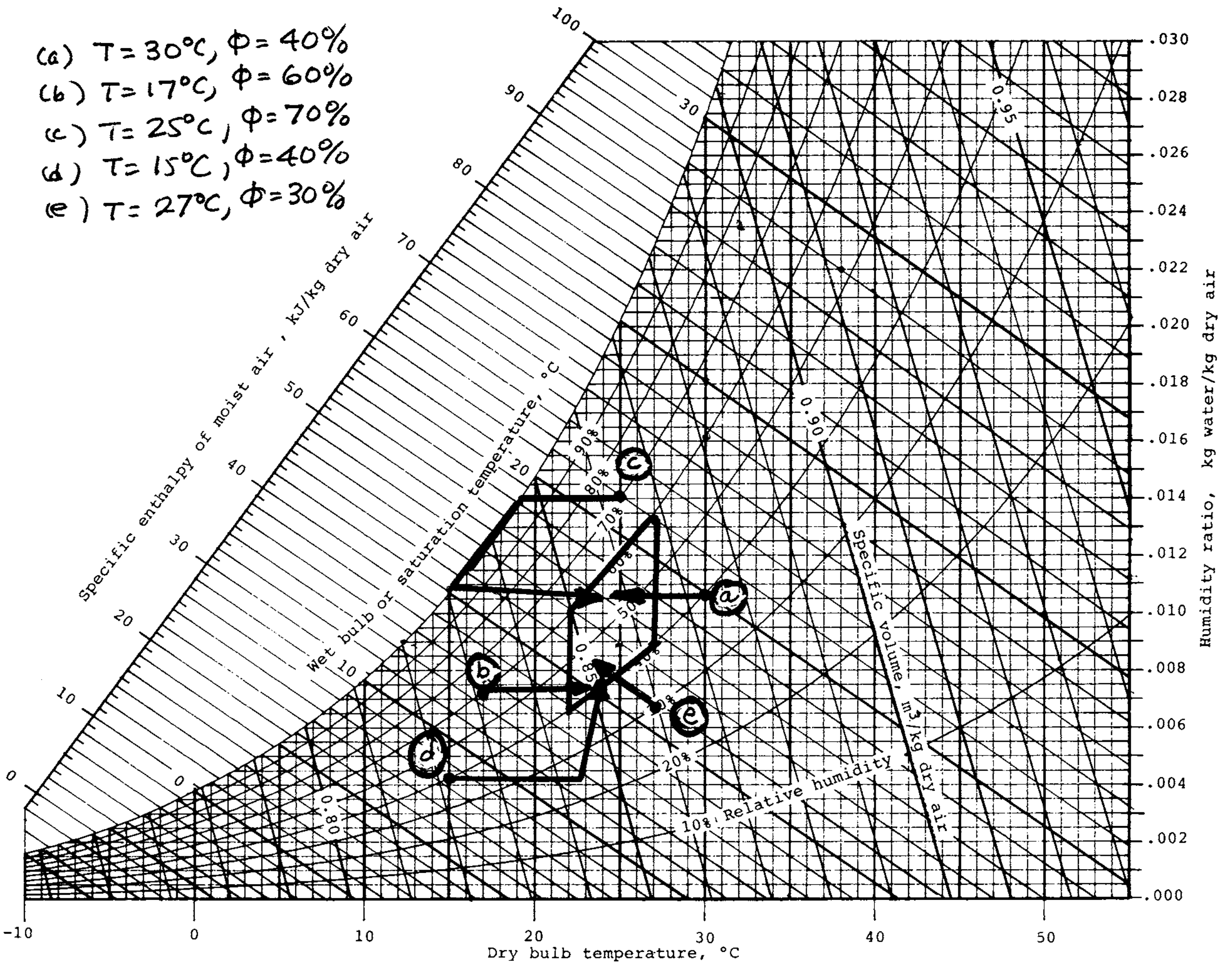
$$h_g \approx 1061 + \underbrace{0.444}_{\approx c_{pv}} T(°F)$$

PROBLEM 12.71

KNOWN: Five cases are provided for the condition of the moist-air stream entering an air-conditioning system, together with the constraints that the exiting moist-air stream must satisfy.

FIND: In each case develop a schematic of equipment and processes from Sec. 12.10 that would achieve the desired result.

SCHEMATIC & GIVEN DATA:



(a) $T = 30°C$, $\phi = 40\%$
(b) $T = 17°C$, $\phi = 60\%$
(c) $T = 25°C$, $\phi = 70\%$
(d) $T = 15°C$, $\phi = 40\%$
(e) $T = 27°C$, $\phi = 30\%$

ASSUMPTION: The principles of Sec. 12.9 are applicable.

ANALYSIS:

(a) Cooling at fixed moisture.

① (b) Heating at fixed moisture.

(c) Dehumidifying followed by reheating. Figure 12.11.

(d) Heating and humidifying. Figure 12.12.

(e) Evaporative cooling. Figure 12.13.

---

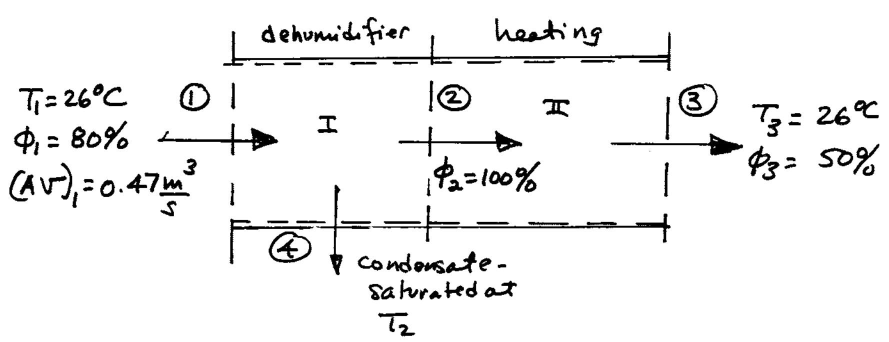
1. The alternatives presented are not the only ones that might be considered. Depending on the condition at the exit, other processes can be proposed.

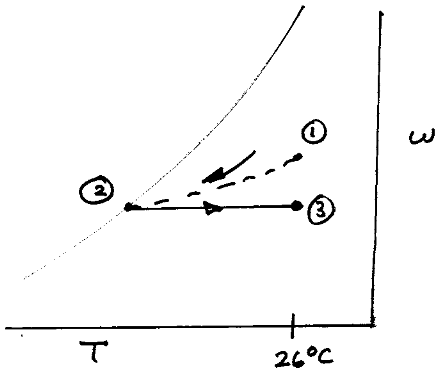
PROBLEM 12.72

KNOWN: Steady state operating data are provided for an air-conditioning system as shown in Fig 12.11.

FIND: Determine (a) the energy removed by heat transfer in the dehumidifier section, in tons, (b) the energy added by heat transfer in the heating section, in kW.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = 0$, $\dot{W}_{cv} = 0$, and kinetic/potential energy effects are negligible. (3) p = 1atm throughout. (4) At ②, $\phi = 100\%$. At ④ the condensate is saturated at T_2. ⑤ The moist air is modeled as an ideal gas mixture.

ANALYSIS: Since no moisture is added or removed in the heating section, $\omega_2 = \omega_3$. And the values of ω_1 and ω_3 can be found from the psychrometric chart, or by calculation:

$$\left.\begin{array}{l} \omega_1 = .622 \frac{p_{v1}}{p - p_{v1}} \\ \phi_1 = p_{v1}/p_{g1} \end{array}\right] \omega_1 = .622 \frac{\phi_1 p_{g1}}{p_1 - \phi_1 p_{g1}} = \frac{(0.622)(0.8)(0.03363)}{1.01325 - (0.8)(0.03363)} = 0.017$$

$$\omega_3 = \frac{(0.622)(0.5)(0.03363)}{1.01325 - (0.5)(0.03363)} = 0.0105$$

Since $\omega_2 = \omega_3$, and the mixture is saturated at ②

$$p_{g2} = \frac{\omega_2 p}{\omega_2 + 0.622} = \frac{(0.0105)(1.01325)}{0.0105 + 0.622} = 0.0168 \text{ bar} \Rightarrow T_2 \cong 15°C$$

Also,

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{(AV)_1 \, p_{a1}}{\left(\frac{\bar{R}}{M_a}\right) T_1} = \frac{(0.47 \, m^3/s)(0.9863 \times 10^5 N/m^2)}{\left(\frac{8314}{28.97} \frac{N \cdot m}{kg \cdot K}\right)(299 \, K)} = 0.54 \, \frac{kg(a)}{s}$$

(a) An energy rate balance for control volume I reads

$$0 = \dot{Q}_{cv} + \dot{m}_a[h_{a1} + \omega_1 h_{g1}] - \dot{m}_a[h_{a2} + \omega_2 h_{g2}] - \dot{m}_4 h_{f4}$$

A mass balance for water gives $\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_4 \Rightarrow \dot{m}_4 = \dot{m}_{v1} - \dot{m}_{v2} = \dot{m}_a(\omega_1 - \omega_2)$

So

$$\dot{Q}_{cv} = \dot{m}_a[(h_{a2} - h_{a1}) + \omega_2 h_{g2} - \omega_1 h_{g1} + (\omega_1 - \omega_2) h_{f4}]$$

With data from Tables A-2, A-22

$$\dot{Q}_{cv} = \left(0.54 \frac{kg(a)}{s}\right)\Big[(288.2 - 299.2) + 0.0105(2528.9) - 0.017(2549) + (0.017 - 0.0105)(62.99)\Big] \frac{kJ}{kg(a)}$$

PROBLEM 12.72 (Continued)

That is

$$\dot{Q}_{cv} = 0.54\left[-11 + 26.55 - 43.33 + 0.41\right] =$$

$$= -14.78 \ \frac{kJ}{s}$$

Then

$$\dot{Q}_{cv} = \left(-14.78 \ \frac{kJ}{s}\right)\left|\frac{1 \text{ ton}}{211 \ kJ/min}\right|\left|\frac{60 s}{1 min}\right| = -4.2 \text{ tons} \longleftarrow$$

(b) An energy rate balance for control volume II reads

$$\dot{Q}_{cv} = \dot{m}_a\left[(h_{a3} - h_{a2}) + \omega(h_{g3} - h_{g2})\right]$$

$$= \left(0.54 \ \frac{kg(a)}{s}\right)\left[(299.2 - 288.2) + 0.0105\left[2549 - 2528.9\right]\right]\frac{kJ}{kg(a)}$$

$$= (0.54)\left[11 + 0.21\right]\left(\frac{kJ}{s}\right)\left|\frac{1 \ kW}{1 \ kJ/s}\right|$$

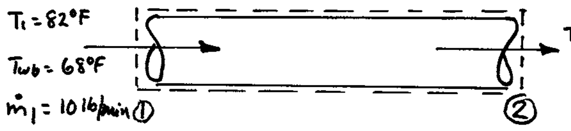
$$= 6.05 \ kW \longleftarrow$$

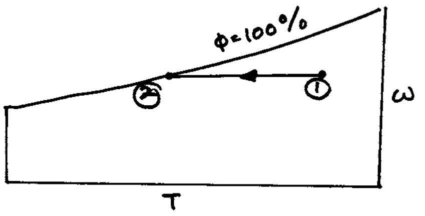
PROBLEM 12.73

KNOWN: Moist air enters a duct with a known mass flow rate and is cooled at constant pressure to a specified temperature.

FIND: Using table data, determine (a) the relative humidity at the duct inlet and (b) the rate of heat transfer. (C) Check the results of parts (a), (b) using the Psychrometric chart. (d) Check the results of parts (a), (b) using IT.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The moist air adheres to ideal gas mixture principles. (3) The wet-bulb temperature can be used in place of T_{as} to evaluate ω.

ANALYSIS: (a) To find ϕ_1, first evaluate ω_1 using Eqs. 12.52 and 12.53. Thus

$$\omega' = 0.622\,\frac{p_g(T_{wb})}{p - p_g(T_{wb})} = 0.622\left(\frac{0.3391}{14.696 - 0.3391}\right) = 0.01469\ \frac{\text{lb(v)}}{\text{lb(a)}}$$

and

$$\omega_1 = \frac{0.24[68 - 82] + 0.01469[1055.1]}{1097.3 - 36.09} = 0.0114\ \frac{\text{lb(v)}}{\text{lb(a)}}$$

Solving Eq. 12.43

$$p_{v1} = \frac{\omega_1\, p}{0.622 + \omega_1} = \frac{(0.0114)(14.696)}{0.622 + 0.0114} = 0.2645\ \frac{\text{lbf}}{\text{in}^2}$$

Then

$$\phi_1 = \frac{p_{v1}}{p_g(82°F)} = \frac{0.2645}{0.5415} = .489 \quad (48.9\%) \qquad \longleftarrow \phi_1$$

(b) Before applying an energy balance it is necessary to determine if condensation occurs upon cooling. Using p_{v1}, the dew point temperature is ≈ 61°F. Accordingly, no condensation takes place. At steady state an energy rate balance reduces to

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + [\dot{m}_a h_{a1} + \dot{m}_v h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}]$$

or

$$\dot{Q}_{cv} = \dot{m}_a\left[(h_{a2} - h_{a1}) + \omega(h_{g2} - h_{g1})\right]$$

The total mass flow rate is $\dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1+\omega)$. Thus, $\dot{m}_a = \dot{m}/(1+\omega)$, so

$$\dot{Q}_{cv} = \frac{\dot{m}}{1+\omega}\left[(h_{a2} - h_{a1}) + \omega(h_{g2} - h_{g1})\right]$$

$$= \left(\frac{10}{1.0114}\ \frac{\text{lb(a)}}{\text{min}}\right)\left[0.24(62 - 82) + 0.0114(1088.6 - 1097.3)\right]\frac{\text{Btu}}{\text{lb(a)}}$$

$$= \left(\frac{10}{1.0114}\right)(-4.9) = -48.4\ \frac{\text{Btu}}{\text{min}} \qquad \longleftarrow \dot{Q}_{cv}$$

PROBLEM 12.73 (Cont'd.)

(c) In the psychrometric chart solution, inspection of Fig. A-9E using T_1 and T_{wb}, $\phi_1 \approx 50\%$. Also, the energy rate balance takes the form ←

$$\dot{Q}_{cv} = \frac{\dot{m}}{1+\omega}\left[(h_a + \omega h_g)_2 - (h_a + \omega h_g)_1\right]$$

Fig. A-9E gives $(h_a + \omega h_g)_2 \approx 27.3$ Btu/lb(a) and $(h_a + \omega h_g)_1 \approx 32.2$ Btu/lb(a). Accordingly

$$\therefore \dot{Q}_{cv} = \left(\frac{10}{1.0114}\right)\left[27.3 - 32.2\right] = -48.4 \ \frac{\text{Btu}}{\text{lb}} \longleftarrow$$

(d) <u>*IT* Code</u>

```
T1 = 82  // °F
Twb1 = 68  // °F
mdot1 = 10  // lb/min
T2 = 62  // °F
p = 1 // atm

// Check for condensation.
Twb1 = Twb_Tphi(T1, phi1, p)
pg1 = Psat_T("Water/Steam", T1)
pv1 = phi1 * pg1
Tdp1 = Tsat_P("Water/Steam", pv1)
// Result:  Tdp = 61.03°F
// No condenstaion.

w = w_TTwb(T1, Twb1, p)
h1 = ha_Tw(T1, w)
h2 = ha_Tw(T2, w)
mdota1 = mdot1 / (1 + w)
Qdot = mdota1 * (h2 - h1)
```

<u>*IT* Results</u>

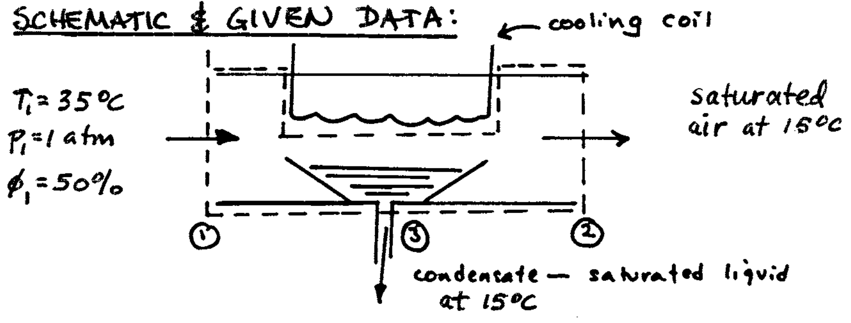
h_1 = 32.24 Btu/lb(a)
h_2 = 27.34 Btu/lb(a)
ω = 0.01148
T_{dp1} = 61.03°F
$\dot{m}_a$ = 9.886 lb/min
ϕ_1= 0.491 ←———— ϕ_1
$\dot{Q}_{cv}$ = -48.39 Btu/min ←———— $\dot{Q}_{cv}$

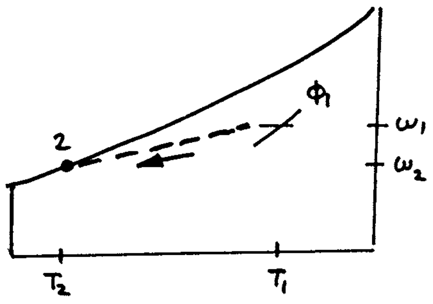
PROBLEM 12.74

KNOWN: Operating data are provided for a dehumidifier at steady state.

FIND: Using table data, determine (a) the heat transfer from the moist air, and (b) the amount of water condensed, each per kg of dry air flowing. Check the results of parts (a) and (b) using (c) the psychrometric chart, and (d) using IT.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot W_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air at 2 is saturated. The condensate at 3 is a saturated liquid. (3) Pressure remains constant throughout at 1 atm.

ANALYSIS: (a) At steady state mass rate balances give $\dot m_{a1} = \dot m_{a2} \equiv \dot m_a$ and $\dot m_{v1} = \dot m_{v2} + \dot m_w$. Thus, the rate water is condensed per unit mass of dry air is

$$\frac{\dot m_w}{\dot m_a} = \omega_1 - \omega_2 . \qquad (1)$$

To find ω_1, use $p_{v1} = \phi_1 p_g(T_1) = (0.5)(0.05628) = 0.02814$ bar. Then

$$\omega_1 = .622\left[\frac{0.02814}{1.01325 - 0.02814}\right] = 0.01777 \ \frac{\text{kg(v)}}{\text{kg(a)}}$$

Since the moist air is saturated at 2, $p_{v2} = p_g(T_2) = 0.01705$ bar, and

$$\omega_2 = .622\left[\frac{0.01705}{1.01325 - 0.01705}\right] = 0.01065 \ \frac{\text{kg(v)}}{\text{kg(a)}}$$

These values are closely the same as obtained from Fig A-9. Substituting into Eq. (1)

$$\frac{\dot m_w}{\dot m_a} = 0.01777 - 0.01065 = 0.00717 \ \frac{\text{kg(v)}}{\text{kg(a)}} \longleftarrow \frac{\dot m_w}{\dot m_a}$$

At steady state an energy rate balance reduces to

$$0 = \dot Q_{cv} - \overset{0}{\cancel{\dot W_{cv}}} + [\dot m_a h_{a1} + \dot m_{v1} h_{v1}] - [\dot m_a h_{a2} + \dot m_{v2} h_{v2}] - \dot m_w h_3$$

Or, with Eq. (1)

$$\frac{\dot Q_{cv}}{\dot m_a} = (h_{a2} - h_{a1}) + \omega_2 h_{g2} - \omega_1 h_{g1} + (\omega_1 - \omega_2) h_{f3} \qquad (2)$$

With $c_{pa} = 1.005$ kJ/kg·K and data from Table A-2 for water vapor

$$\frac{\dot Q_{cv}}{\dot m_a} = 1.005[15 - 35] + 0.01065(2528.9) - 0.01777(2565.3) + 0.00717(62.99)$$

$$= -20.1 + 26.93 - 45.59 + 0.452 = -38.31 \ \frac{\text{kJ}}{\text{kg(a)}} \longleftarrow \frac{\dot Q_{cv}}{\dot m_a}$$

PROBLEM 12.74 (Cont'd.)

(c) In the psychrometric chart solution, the energy rate equation, Eq.(2), takes the form

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_a + \omega h_g)_2 - (h_a + \omega h_g)_1 + (\omega_1 - \omega_2) h_{f3}$$

From Fig A-9; $(h_a + \omega h_g)_1 \approx 80.6$ kJ/kg(a) and $(h_a + \omega h_g)_2 \approx 42$ kJ/kg(a). Thus

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = 42 - 80.6 + 0.452 = -38.15 \longleftarrow \frac{\dot{Q}_{cv}}{\dot{m}_a}$$

(d) *IT* Code

```
T1 = 35  // °C
phi1 = 0.5
T2 = 15 // °C
phi2 = 1
T3 = 15  // °C
p = 1.01325  // bar
mdota = 1  // Assume a unit mass flow rate.

w1 = w_Tphi(T1, phi1, p)
w2 = w_Tphi(T2, phi2, p)
h1 = ha_Tphi(T1, phi1, p)
h2 = ha_Tphi(T2, phi2, p)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 0)

mdotw / mdota = w1 - w2
Qdotcv / mdota = h2 - h1 + (w1 - w2) * h3
```

IT Results

```
w1 = 0.01775 kg(v)/kg(a)
w2 = 0.01066 kg(v)/kg(a)
h1 = 80.66 kJ/kg(a)
h2 = 42.02 kJ/kg(a)
h3 = 61.93 kJ/kg(a)
```

$\dot{m}_w / \dot{m}_a$ = 0.007085 kg/kg(a) ←

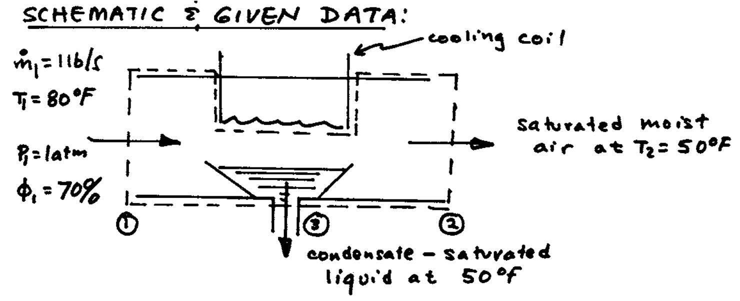
$\dot{Q}_{cv} / \dot{m}_a$ = -38.21 kJ/kg(a) ←

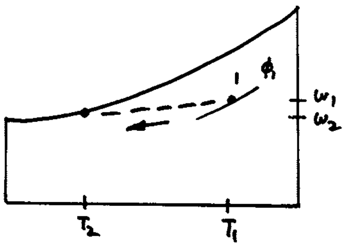
PROBLEM 12.75

KNOWN: Steady-state operating data are provided for a dehumidifier.
FIND: Using table data, determine (a) the rate of heat transfer from the moist air, (b) the rate water is condensed. (c) Check results using the psychrometric chart. (d) Check results using IT.

KNOWN: Operating data are provided for a dehumidifier at steady state.
FIND: Determine the heat transfer from the moist air and the rate water is condensed, in tons and lb/s, respectively.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air at 2 is saturated. The condensate at 3 is saturated liquid. (3) Pressure remains constant throughout at 1 atm.

ANALYSIS: At steady state mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$ and $\dot{m}_{v1} = \dot{m}_{v2} + \dot{m}_w$. Thus, the rate water is condensed per unit mass of dry air is

$$\frac{\dot{m}_w}{\dot{m}_a} = \omega_1 - \omega_2 \qquad (1)$$

To find ω_1, use $p_{v1} = \phi_1 p_g(T_1) = 0.7(0.5073) = 0.3551$ lbf/in². Then

$$\omega_1 = 0.622\left[\frac{0.3551}{14.696 - 0.3551}\right] = 0.0154 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Since the moist air is saturated at 2, $p_{v2} = p_g(T_2) = 0.1780$ lbf/in², and

$$\omega_2 = 0.622\left[\frac{0.1780}{14.696 - 0.1780}\right] = 0.0076 \frac{\text{lb(v)}}{\text{lb(a)}}$$

These values for ω_1 and ω_2 are closely the same as obtained from Fig. A-9E. Substituting into Eq. (1)

$$\frac{\dot{m}_w}{\dot{m}_a} = 0.0154 - 0.0076 = 0.0078 \frac{\text{lb(v)}}{\text{lb(a)}}$$

At the inlet $\dot{m}_1 = \dot{m}_{v1} + \dot{m}_{a1} = \dot{m}_{a1}(1 + \omega_1)$. Thus

$$\dot{m}_{a1} = \frac{\dot{m}_1}{1+\omega} = \frac{1 \text{ lb/s}}{1.0154} = 0.985 \frac{\text{lb(a)}}{\text{s}}$$

Thus

$$\dot{m}_w = \left[0.0078 \frac{\text{lb(v)}}{\text{lb(a)}}\right]\left(0.985 \frac{\text{lb(a)}}{\text{s}}\right] = 0.0077 \frac{\text{lb}}{\text{s}} \longleftarrow (b)$$

At steady state an energy rate balance reduces with Eq. (1) to give

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = (h_{a2} - h_{a1}) + \omega_2 h_{g2} - \omega_1 h_{g1} + (\omega_1 - \omega_2) h_{f3} \qquad (2)$$

with $c_p = 0.24$ Btu/lb·°R

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = 0.24(50-80) + 0.0076(1083.3) - 0.0154(1096.4) + 0.0078(18.06)$$
$$= -7.2 + 8.23 - 16.88 + 0.14 = -15.71 \text{ Btu/lb(a)}$$

Thus

$$\dot{Q}_{cv} = \left[-15.71 \frac{\text{Btu}}{\text{lb(a)}}\right]\left[0.985 \frac{\text{lb(a)}}{\text{s}}\right]\left|\frac{1 \text{ ton}}{200 \text{ Btu/min}}\right|\left|\frac{60 \text{ s}}{\text{min}}\right| = -4.64 \text{ tons} \longleftarrow (a)$$

PROBLEM 12.75 (Contd.)

(c) By inspection of Fig. A-9E, $\omega_1 = 0.0155$, $\omega_2 = 0.0076$, $(h_a + \omega h_g)_1 = 36.1$ Btu/lb(a), $(h_a + \omega h_g)_2 = 20.2$ Btu/lb(a). Thus, Eq. (1) gives

$$\frac{\dot{m}_w}{\dot{m}_a} = \omega_1 - \omega_2 = 0.0078 \, \frac{\text{lb(v)}}{\text{lb(a)}}$$

, which agrees well with the table result. Rearranging Eq. (2) to read

$$(\dot{Q}_{cv}/\dot{m}_a) = (h_a + \omega h_g)_2 - (h_a + \omega h_g)_1 + (\omega_1 - \omega_2) h_{f3}$$

$$= 20.2 - 36.1 + 0.14 = -15.76 \text{ Btu/lb(a)}$$

This also agrees with the table result, as expected.

(d) *IT* Code

```
T1 = 80  // °F
phi1 = 0.7
T2 = 50  // °F
phi2 = 1
T3 = 50  // °F
p = 1  // atm
mdot1 = 1  // lb/s

w1 = w_Tphi(T1, phi1, p)
w2 = w_Tphi(T2, phi2, p)
h1 = ha_Tphi(T1, phi1, p)
h2 = ha_Tphi(T2, phi2, p)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 0)

mdota = mdot1 / ( 1 + w1)
mdotw / mdota = w1 - w2
Qdotcv / mdota = (h2 - h1 + (w1 - w2) * h3) * (60 / 200)  // tons
```

It Results

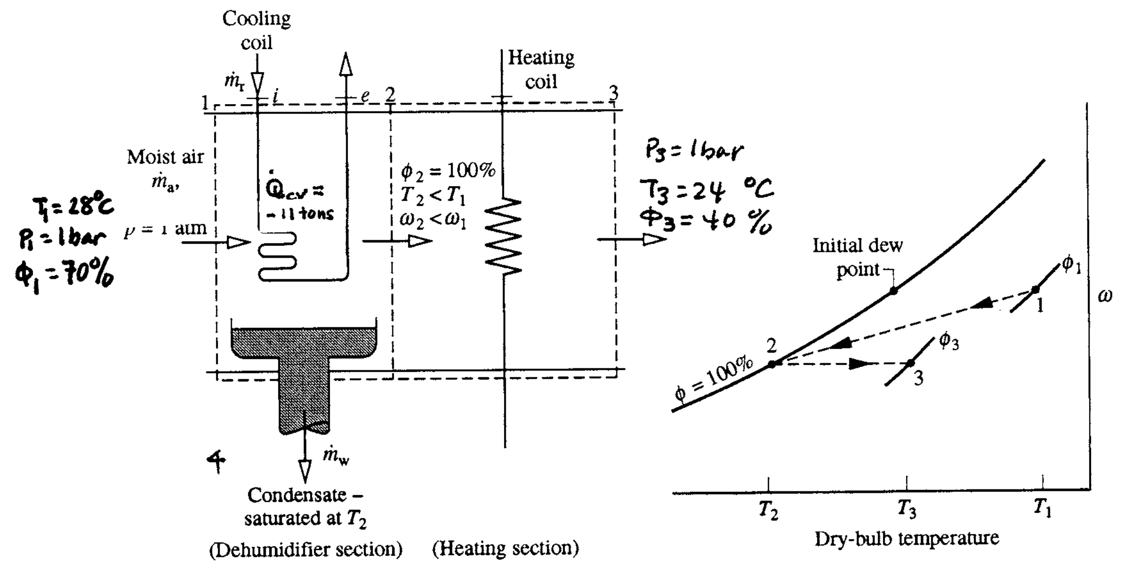
ω_1 = 0.01539 lb(v)/lb(a)
ω_2 = 0.007624 lb(v)/lb(a)
h_1 = 36.03 Btu/lb(a)
h_2 = 20.23 Btu/lb(a)
h_3 = 17.67 Btu/lb(a)
$\dot{m}_a$ = 0.9848 lb/s
$\dot{m}_w$ = 0.007645 lb/s
$\dot{Q}_{cv}$ = -4.628 tons

PROBLEM 12.76

KNOWN: Steady-state operating data are provided for an air conditioner wherein moisture is removed from the moist air, which is then heated.

FIND: Determine (a) the temperature of the air exiting the dehumidifier, (b) the volumetric flow rate at the air conditioner inlet, (c) the rate water is condensed, and (d) the rate of heat transfer to the air passing through the heating unit.

SCHEMATIC & GIVEN DATA



ASSUMPTIONS: 1. The control volumes shown are at steady state. The coils are not included within the control volumes enclosing the two individual sections. 2. For each control volume, $\dot W_{cv} = 0$ and kinetic and potential energy effects can be ignored. 3. At 2, the moist air is saturated. At 4, the liquid is saturated at T_2.

ANALYSIS: At steady state, mass rate balances give $\dot m_{a1} = \dot m_{a2} = \dot m_{a3} \equiv \dot m_a$, $\dot m_{v1} = \dot m_w + \dot m_{v2}$, $\dot m_{v2} = \dot m_{v3}$. Accordingly, $\omega_2 = \omega_3$, and

$$\dot m_w = \dot m_a(\omega_1 - \omega_2) = \dot m_a(\omega_1 - \omega_3) \qquad (1)$$

Using $\omega_2 = \omega_3$,

$$0.622\left[\frac{p_g(T_2)}{p - p_g(T_2)}\right] = 0.622\left[\frac{p_{v3}}{p - p_{v3}}\right] \Rightarrow p_g(T_2) = p_{v3}$$

where $p_{v3} = \phi_3 p_{g3} = 0.4\,(0.02985 \text{ bar}) = 0.01194$ bar. Interpolating in Table A-2 with $p_g(T_2) = 0.01194$ bar gives $T_2 \approx 9.6°C$ ← (a)

Then, with $p_{v1} = \phi_1 p_{g1} = (0.7)(0.03782) = 0.02647$ bar

$$\omega_1 = 0.622\left[\frac{0.02647}{1 - 0.02647}\right] = 0.01691 \frac{\text{kg(v)}}{\text{kg(a)}}, \quad \omega_2 = \omega_3 = 0.622\left[\frac{0.01194}{1 - 0.01194}\right] = 0.007516$$

An energy rate balance on the dehumidifier reads

$$0 = \dot Q_{cv} - \overset{0}{\cancel{\dot W_{cv}}} + [\dot m_a h_{a1} + \dot m_{v1} h_{v1}] - [\dot m_a h_{a2} + \dot m_{v2} h_{v2}] - \dot m_w h_w$$

$$0 = \dot Q_{cv} + \dot m_a[h_{a1} + \omega_1 \underset{h_{g1}}{h_{v1}}] - \dot m_a[h_{a2} + \omega_2 \underset{h_{g2}}{h_{v2}}] - \dot m_a(\omega_1 - \omega_2)\underset{h_{f2}}{h_w}$$

$$0 = \dot Q_{cv} + \dot m_a\left[\underbrace{(h_{a1} - h_{a2})}_{c_{pa}(T_1 - T_2)} + \omega_1 h_{g1} - \omega_2 h_{g2} - (\omega_1 - \omega_2) h_{f2}\right]$$

PROBLEM 12.76 (Contd.)

Solving

$$\dot{m}_a = \frac{\dot{Q}_{cv}}{c_{pa}(T_2-T_1) + \omega_2 h_{g2} - \omega_1 h_{g1} + (\omega_1-\omega_2) h_{f2}}$$

$$= \frac{(-11\ \text{tons})\left|\frac{211\ \text{kJ/min}}{\text{ton}}\right|}{\left[1.005[9.6-28] + 0.007516(2519.06) - 0.01691(2552.6) + (0.0094)(40.33)\right]\frac{\text{kJ}}{\text{kg(a)}}}$$

$$= 54.81\ \text{kg(a)/min}$$

The volumetric flow rate at 1 is then

$$(AV)_1 = \dot{m}_a v_{a1} = \dot{m}_a \frac{RT_1}{(p_1-p_{v1})} = \left(54.81\frac{\text{kg(a)}}{\text{min}}\right)\frac{\left(\frac{8314}{28.97}\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(301\ \text{K})}{(0.97353\times10^5\ \text{N/m}^2)} = 48.63\ \frac{\text{m}^3}{\text{min}} \leftarrow$$

Using Eq. (1), the rate water is condensed is

$$\dot{m}_w = (54.81)(0.01691 - 0.007516) = 0.515\ \frac{\text{kg(l)}}{\text{min}}$$

Finally, an energy rate balance on the heating section reduces to give

$$\dot{Q}_{cv} = \dot{m}_a\left[(h_{a3}-h_{a2}) + \omega_3(h_{g3}-h_{g2})\right]$$

$$= 54.81\left[(1.005)(24-9.6) + 0.007516(2545.4-2519.06)\right]$$

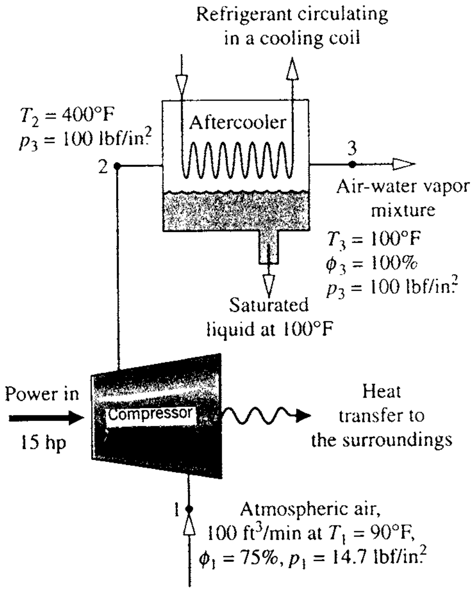
$$= 804.1\ \frac{\text{kJ}}{\text{min}} \leftarrow$$

PROBLEM 12.77

KNOWN: Steady-state operating data are provided for a compressor followed by an aftercooler.

FIND: Determine (a) the rate of heat transfer from the compressor to its surroundings, (b) the mass flow rate of the condensate, (c) the rate of heat transfer from the moist air to the refrigerant.

SCHEMATIC & GIVEN DATA:


Refrigerant circulating in a cooling coil
T_2 = 400°F
p_3 = 100 lbf/in²
2
Aftercooler
3
Air-water vapor mixture
T_3 = 100°F
ϕ_3 = 100%
p_3 = 100 lbf/in²
Saturated liquid at 100°F
Power in
15 hp
Compressor
Heat transfer to the surroundings
1
Atmospheric air, 100 ft³/min at T_1 = 90°F, ϕ_1 = 75%, p_1 = 14.7 lbf/in²

ASSUMPTIONS: (1) The compressor and aftercooler operate at steady state with negligible changes in kinetic and potential energy. (2) There is no stray heat transfer between the aftercooler and its surroundings.

ANALYSIS: At steady state mass rate balances on control volumes enclosing the compressor and aftercooler give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_{a3} = \dot{m}_a$, $\dot{m}_{v1} = \dot{m}_{v2}$, $\dot{m}_{v2} = \dot{m}_{v3} + \dot{m}_w$. Accordingly, $\omega_1 = \omega_2$ and $\dot{m}_w / \dot{m}_a = (\omega_2 - \omega_3) = (\omega_1 - \omega_3)$.

To find ω_1 and ω_3, $p_{v1} = \phi_1 p_{g1} = (0.75)(0.6988) = 0.5241$ lbf/in² and $p_{v3} = \phi_3 p_{g3} = p_{g3} = 0.9503$ lbf/in². Thus

$$\omega_1 = 0.622\left[\frac{0.5241}{14.7 - 0.5241}\right] = 0.023\ \frac{\text{lb(v)}}{\text{lb(a)}}, \quad \omega_3 = 0.622\left[\frac{0.9503}{100 - 0.9503}\right] = 0.00597\ \frac{\text{lb(v)}}{\text{lb(a)}}$$

The mass flow rate of the dry air can be evaluated using the volumetric flow rate at 1 and the ideal gas equation of state:

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{p_{a1}(AV)_1}{RT_1} = \frac{[(14.7 - 0.5241) \times 144\ \text{lbf/ft}^2](100\ \text{ft}^3/\text{min})}{(1545/28.97\ \text{ft}\cdot\text{lbf/lb}\cdot{}^\circ\text{R})(550^\circ\text{R})} = 6.96\ \frac{\text{lb(a)}}{\text{min}}$$

The mass flow rate of condensate is then

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_3) = 6.96(0.023 - 0.00597) = 0.1185\ \frac{\text{lb}}{\text{min}} \longleftarrow \text{(b)}$$

An energy rate balance at steady state for a control volume enclosing the compressor gives

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}]$$

or

$$\dot{Q}_{cv} = \dot{W}_{cv} + \dot{m}_a[(h_{a2} - h_{a1}) + \omega_1(\underset{h_{g2}}{h_{v2}} - \underset{h_{g1}}{h_{v1}})]$$

With data from Tables A-2E, 22E

$$\dot{Q}_{cv} = (-15\ \text{hp})\left|\frac{2545\ \text{Btu/h}}{1\ \text{hp}}\right|\left|\frac{1\ \text{h}}{60\ \text{min}}\right| + \left(6.96\ \frac{\text{lb(a)}}{\text{min}}\right)\left[(206.46 - 131.46) + 0.023(1202 - 1100.7)\right]\frac{\text{Btu}}{\text{lb(a)}}$$

$$= -636.25 + 538.22 = -98.03\ \text{Btu/min} \longleftarrow \text{(a)}$$

PROBLEM 12.77 (Contd.)

An energy rate balance at steady state for a control volume enclosing the aftercooler, but excluding the refrigerant line

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}] - [\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}] - \dot{m}_w h_4$$

Thus, with $h_v \approx h_g$

$$-\dot{Q}_{cv} = \dot{m}_a \{(h_{a2} - h_{a3}) + \omega_2 h_{g2} - \omega_3 h_{g3}\} - \dot{m}_w h_{f4}$$

$$= 6.96\{(206.46 - 133.86) + (0.023)(1202) - (0.00597)(1105)\} - (0.1185)(68.05)$$

$$= 6.96\{\ 72.6 + 27.65 - 6.60\} - 8.06$$

$$= 643.74\ \frac{\text{Btu}}{\text{min}}$$

1 ton of refrigeration = 200 Btu/min (Sec. 10.2.1). Thus

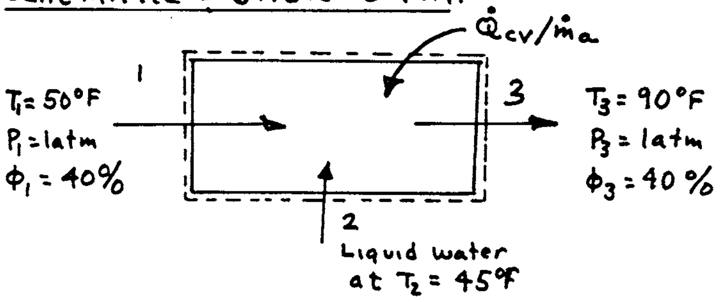
$$-\dot{Q}_{cv} = \left(643.74\ \frac{\text{Btu}}{\text{min}}\right)\left|\frac{1\ \text{ton}}{200\ \text{Btu/min}}\right| = 3.22\ \text{tons} \quad \longleftarrow \quad (c)$$

PROBLEM 12.78

KNOWN: Operating data are provided for an air conditioner at steady state.

FIND: Determine the amount of liquid water injected and the heat transfer rate, each per lb of dry air.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The liquid water enters as saturated liquid and the moist air streams adhere to ideal gas mixture principles.

ANALYSIS: (a) At steady state, mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$, $\dot{m}_{v1} + \dot{m}_w = \dot{m}_{v2}$. Accordingly, the rate liquid is injected is

$$\dot{m}_w = \dot{m}_{v3} - \dot{m}_{v1} = \dot{m}_a(\omega_3 - \omega_1) \qquad (1)$$

To find ω_1 and ω_3, $p_{v1} = \phi_1 p_g(T_1) = 0.4(0.178 \text{ lbf/in}^2) = 0.0712 \text{ lbf/in}^2$ and $p_{v3} = \phi_3 p_g(T_3) = 0.4(0.6988) = 0.2795 \text{ lbf/in.}^2$ Then

$$\omega_1 = 0.622\left[\frac{0.0712}{14.696 - 0.0712}\right] = 0.00303 \ \frac{\text{lb(v)}}{\text{lb (a)}}, \quad \omega_3 = 0.622\left[\frac{0.2795}{14.696 - 0.2795}\right] = 0.01206$$

To get $\dot{m}_a$, use $\dot{m}_a = (AV)_3/v_{a3}$ with the ideal gas equation

$$\dot{m}_a = \frac{(p - p_{v3})(AV)_3}{R_a T_3} = \frac{(14.696 - 0.2795)\frac{\text{lbf}}{\text{in}^2}(1000 \text{ ft}^3/\text{min})}{\left(\frac{1545}{28.97}\cdot\frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(90+460)°\text{R}}\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|$$

$$= 70.77 \text{ lb/min}$$

Inserting values into (1)

$$\dot{m}_w = 70.77(0.01206 - 0.00303) = 0.639 \text{ lb/min} \longleftarrow \dot{m}_w$$

(b) An energy balance at steady state reduces to give

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{\,0} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] + \dot{m}_w h_2 - [\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}]$$

or

$$\dot{Q}_{cv} = \dot{m}_a[(h_{a3} - h_{a1}) + \omega_3 h_{g3} - \omega_1 h_{g1}] - \dot{m}_w h_2$$

With data from Tables A-2E and A-22E

$$\dot{Q}_{cv} = \left(70.77 \frac{\text{lb}}{\text{min}}\right)[(131.46 - 121.88) + (0.01206)(1100.7) - (0.00303)(1083.3)]\frac{\text{Btu}}{\text{lb}} - (.639)(13.04)\frac{\text{Btu}}{\text{min}}$$

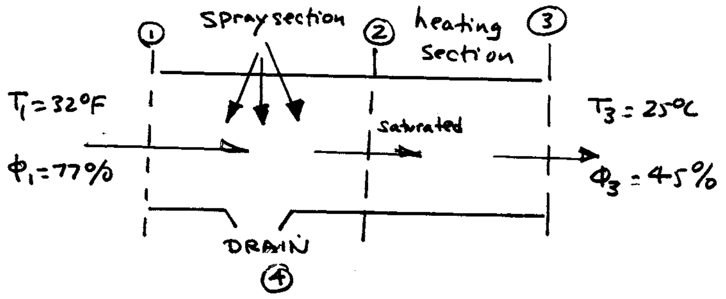
$$= \left(1376.8 \frac{\text{Btu}}{\text{min}}\right)\left|\frac{60 \text{ min}}{1 \text{ h}}\right| = 8.26 \times 10^4 \text{ Btu/h} \longleftarrow \dot{Q}_{cv}$$

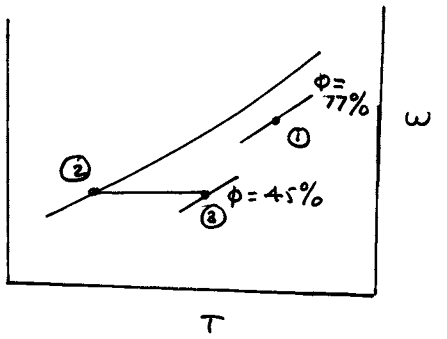
PROBLEM 12.79

KNOWN: An air-conditioning system operating at steady state consists of a spray section followed by a reheater. Moist air at 32°C, $\phi = 77\%$ enters the system and passes through the spray section, exiting saturated. The moist air is then heated at fixed moisture content to 25°C, $\phi = 45\%$

FIND: Determine (a) the temperature of the moist air leaving the spray section, (b) the change in the amount of water vapor contained in the moist air passing through the system. Locate the principal states on a psychrometric chart.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Two control volumes at steady state are under consideration: one enclosing the overall system and one enclosing the heating section. (2) p = 1 atm throughout. (3) At 2 the moist air is saturated. (4) Ideal gas principles apply to the moist air.

ANALYSIS: (a) As no moisture is added or removed from the heating section, $\omega_2 = \omega_3$. To find ω_3

$$\omega_3 = 0.622\,\frac{p_{v3}}{p - p_{v3}} = 0.622\left(\frac{\phi_3\, p_{g3}}{p - \phi_3\, p_{g3}}\right) = 0.622\left(\frac{(0.45)(0.03169)}{1.01325 - (0.45)(0.03169)}\right) = 0.0089$$

Then, since the moist air is saturated at 2

$$\omega_2 = 0.622\,\frac{p_{g2}}{p - p_{g2}} \Rightarrow p_{g2} = \frac{\omega_2\, p}{\omega_2 + 0.622} = \frac{(0.0089)(1.01325)}{0.0089 + 0.622} = 0.01429 \text{ bar}$$

Interpolating in Table A-2, $T_2 \approx 12.3°C$ ◄———— T_2

(b) Considering next the overall system

① $\begin{bmatrix}\text{change in the}\\ \text{amount of water}\\ \text{vapor contained}\\ \text{in the moist air}\end{bmatrix} = \dot{m}_{v3} - \dot{m}_{v1} = \dot{m}_a(\omega_3 - \omega_1)$

Thus

$\begin{bmatrix}\text{change in the}\\ \text{amount of water}\\ \text{vapor contained}\\ \text{in the moist air}\\ \text{per unit mass of}\\ \text{dry air}\end{bmatrix} = \omega_3 - \omega_1$, where $\omega_1 = 0.622\left[\dfrac{\phi_1\, p_{g1}}{p - \phi_1\, p_{g1}}\right]$

$$= 0.622\left[\frac{(.77)(.04759)}{(1.01325) - (.77)(.04759)}\right]$$

$$= 0.0233$$

$= 0.0089 - 0.0233 = -0.0144$ kg/kg(da) ◄———— (Removed)

---

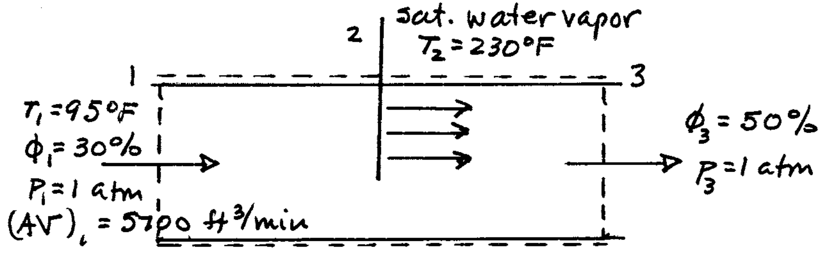
1. Note that neither the temperature nor the flow rate of the spray is provided. The analysis in this case uses the mass balance but not the energy balance.

PROBLEM 12.80

KNOWN: Operating data are provided for a steam-spray humidification device at steady state.

FIND: Determine the temperature of the exiting moist air stream and the rate of steam injection.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air streams adhere to the ideal gas model.

ANALYSIS: Governing Equations. The state at 3 will be determined using mass and energy rate balances. For mass, $\dot{m}_{a_1} = \dot{m}_{a_3} \equiv \dot{m}_a$ and $\dot{m}_{v_1} + \dot{m}_2 = \dot{m}_{v_3}$. Thus

$$\dot{m}_2 = \dot{m}_a(\omega_3 - \omega_1) \tag{1}$$

To get $\dot{m}_a$, use

$$\dot{m}_a = \frac{(AV)_1}{v_{a1}} = \frac{(p - p_{v1})(AV)_1}{R_a T_1} \tag{2}$$

The energy balance at steady state reduces as follows:

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_a\left[(h_{a1} + \omega_1 h_{v1}) + (\omega_3 - \omega_1)h_2 - (h_{a3} + \omega_3 h_{v3})\right]$$

or

$$(h_{a3} + \omega_3 h_{g3}) = (h_{a1} + \omega_1 h_{g1}) - (\omega_3 - \omega_1)h_{g2} \tag{3}$$

Evaluating Properties. To find ω_1, use $p_{v1} = \phi_1 p_g(T_1)$ and $\omega_1 = 0.622\,(p_{v1}/p - p_{v1})$. With T_1 and p_1 known, ω_1, h_{a1}, and h_{g1} can be determined. Also, h_{g2} can be determined since T_2 is known.

The value of ω_3 is related to T_3 using $p_{v3} = \phi_3 p_g(T_3)$ and $\omega_3 = 0.622(p_{v3}/p - p_{v3})$. Also, h_{a3} and h_{g3} are related directly to T_3.

Solution Procedure. We see from (3) and the discussion of property evaluation that T_3 can be found using an iterative procedure with data from Tables A-22E and A-2E. The following IT code provides an alternative to the iterative procedure:

IT Code

```
T1 = 95  // °F
p = 14.696
phi1 = 0.3
AV1 = 5700  // ft³/min
T2 = 230  // °F
x2 = 1
phi3 = 0.5
```

```
w1 = w_Tphi(T1,phi1,p)
h1 = ha_Tw(T1,w1)
va1 = va_Tw(T1,w1,p)
psat = Psat_T("Water/Steam", T2)
h2 = hsat_Px("Water/Steam", psat, x2)
w3 = w_Tphi(T3, phi3, p)
h3 = ha_Tphi(T3, phi3, p)

mdota = AV1 / va1
mdotv = mdota * (w3 - w1)
0 = mdota * (h1 - h3) + mdotv * h2
```

IT Results

ω_1 = 0.01053 lb(v)/lb(a)
ω_3 = 0.01877 lb(v)/lb(a)
h_1 = 34.37 Btu/lb(a)
h_2 = 1157 Btu/lb(a)
h_3 = 43.9 Btu/lb(a)
$\dot{m}_a$ = 400.9 lb/min
$\dot{m}_2$ = 3.304 lb/min ← $\dot{m}_2$
T_3 = 96.79°F ← T_3

To check the validity of the IT results, Fig A-9E gives $h_1 = 34.4$, $\omega_1 = 0.0106$, which agree closely with the respective IT answers. Also, referring to Fig A-9E with ω_3 and h_3 from IT, we get $T_3 \approx 96.5°F$ which also agrees well with the IT value.

PROBLEM 12.81

KNOWN: Consider the steam-spray humidifier of Problem 12.80.

FIND: Determine the exergy destruction rate.

SCHEMATIC & GIVEN DATA: See solution to Problem 12.80 for the schematic.

State 1	State 2	State 3
$T_1 = 95°F$	$T_2 = 230°F$	$T_3 = 96.79°F$
$p_1 = 14.696$ lbf/in.^{2}	Sat. vapor	$p_3 = 14.696$ lbf/in.^{2}
$\omega_1 = 0.01053$ lb(v)/lb(a)	$\dot{m}_2 = 3.304$ lb/min	$\omega_3 = 0.01877$ lb(v)/lb(a)
$\dot{m}_a = 400.9$ lb/min		

ASSUMPTIONS: (1)-(2) See Problem 12.80. (3) Let $T_0 = 95°F = 550°R$.

ANALYSIS: The exergy destruction rate can be evaluated using $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is found from an entropy rate balance

$$0 = \cancel{\sum_j \frac{\dot{Q}_j}{T_j}}^{0} + \dot{m}_a[s_a(T_1, p_{a1}) + \omega_1 s_v(T_1, p_{v1})] + \dot{m}_2 s_g(T_2) - \dot{m}_a[s_a(T_3, p_{a3}) + \omega_3 s_v(T_3, p_{v3})] + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}_a \underbrace{[s_a(T_3, p_{a3}) + \omega_3 s_v(T_3, p_{v3})]}_{s(T_3, \omega_3, p_3)} - \dot{m}_a \underbrace{[s_a(T_1, p_{a1}) + \omega_1 s_v(T_1, p_{v1})]}_{s(T_1, \omega_1, p_3)} - \dot{m}_2 s_g(T_2) \qquad (1)$$

As noted, the terms in brackets are each uniquely determined from the given data. ① These functions are provided by IT. Thus, $\dot{E}_d$ is determined using IT as follows:

IT Code

```
T1 = 95  // °F
p1 = 14.696  // lbf/in.²
w1 = 0.01053  // lb(v)/lb(a)
mdota = 400.9  // lb/min
T2 = 230  // °F
x2 = 1
T3 = 96.79  // °F
p3 = 14.696  // lbf/in.²
w3 = 0.01877  // lb(v)/lb(a)
To = 555 // °R

s1 = sa_Tw(T1,w1,p1)
s3 = sa_Tw(T3,w3,p3)
psat = Psat_T("Water/Steam", T2)
s2 = ssat_Px("Water/Steam", psat, x2)

sigmadot = mdota * (s3 - s1 - (w3 - w1) * s2)
Edotd = To * sigmadot
```

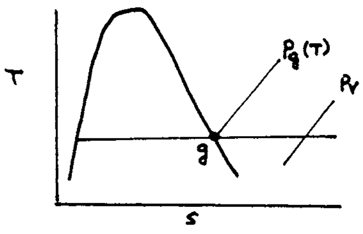
IT Results

$\dot{\sigma}_{cv}$ = 1.526 Btu/min·°R

$\dot{E}_d$ = 846.8 Btu/min ⟵ $\dot{E}_d$

---

1. For states of water vapor where the ideal gas model applies, Eq. 6.19 can be invoked to write



$$s(T, p_v) - s(T, p_g) = \cancel{\int c_p \frac{dT}{T}}^{0} - R \ln \frac{p_v}{p_g}$$

or

$$s(T, p_v) = s_g(T) - \frac{\bar{R}}{M_v} \ln \underbrace{\frac{p_v}{p_g}}_{= \phi}$$

Further, applying Eq. 6.21 to the dry air

$$s(T, p_a) - \cancel{s(T_{ref}, p_{ref})}^{0} = s°(T) - s°(T_{ref}) - R_a \ln \frac{p_a}{p_{ref}}$$

$$s(T, p_a) = s°(T) - s°(T_{ref}) - \frac{\bar{R}}{M_a} \ln \frac{(p - p_v)}{p_{ref}}$$

Thus

$$s(T, \omega, p) = [s°(T) - s°(T_{ref}) - \frac{\bar{R}}{M_a} \ln \frac{(p - p_v)}{p_{ref}}] + \omega [s_g(T) - \frac{\bar{R}}{M_v} \ln \phi]$$

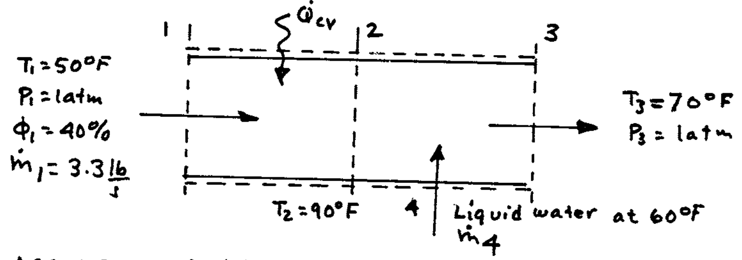
The functions in IT are based on this relationship. The development of a hand calculation to verify the IT result is left to the reader.

PROBLEM 12.82

KNOWN: Operating data are provided for an air conditioner at steady state.

FIND: Determine (a) the rate of heat transfer to the air passing through the heating section, (b) the rate water is injected, (c) the relative humidity at the exit of the humidification section.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volumes shown in the accompanying figure are at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. For the humidification unit, $\dot{Q}_{cv} = 0$. (2) The liquid enters saturated and the moist air streams adhere to the ideal gas model.

ANALYSIS: (a) At steady state, mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_{a3} \equiv \dot{m}_a$, $\dot{m}_{v1} = \dot{m}_{v2}$, $\dot{m}_{v3} = \dot{m}_{v2} + \dot{m}_w$. Thus $\dot{m}_w = \dot{m}_{v3} - \dot{m}_{v2} = \dot{m}_{v3} - \dot{m}_{v1} = \dot{m}_a(\omega_3 - \omega_1)$.

An energy rate balance for the first control volume reduces to give

$$0 = \dot{Q}_{cv} - \overset{0}{\cancel{\dot{W}_{cv}}} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = [h_{a2} - h_{a1}] + \omega[h_{g2} - h_{g1}] \qquad (1)$$

The humidity ratio is found from Eq 12.43: $p_{v1} = \phi_1 p_g(T_1) = 0.40(0.178) = 0.0712 \text{ lbf/in}^2$. So

$$\omega_1 = 0.622 \frac{p_{v1}}{p_1 - p_{v1}} = 0.622\left[\frac{0.0712}{14.696 - 0.0712}\right] = 0.00303 \frac{lb(v)}{lb(a)}$$

Then, Eq. (1) gives with $c_{pa} = 0.24$ Btu/lb·°R

$$\frac{\dot{Q}_{cv}}{\dot{m}_a} = 0.24[90-50] + 0.00303[1100.7 - 1083.3] = 9.65 \text{ Btu/lb(a)}$$

To find $\dot{m}_a$, write $\dot{m}_1 = \dot{m}_{v1} + \dot{m}_{a1} = \dot{m}_{a1}(1+\omega_1) \Rightarrow$

$$\dot{m}_{a1} = \frac{\dot{m}_1}{1+\omega_1} = \frac{3.3 \text{ lb/s}}{1.00303} = 3.29 \frac{lb(a)}{s}$$

So

$$\dot{Q}_{cv} = \left(3.29 \frac{lb(a)}{s}\right)\left(9.65 \frac{Btu}{lb(a)}\right) = 31.75 \text{ Btu/s} \longleftarrow \text{(a)}$$

(c) The relative humidity ϕ_3 can be found from ω_3 which is obtained from an energy rate balance on the the second control volume:

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}] + \dot{m}_4 h_a - [\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}]$$

or with $\dot{m}_4 = \dot{m}_a(\omega_3 - \omega_1)$ and $\omega_2 = \omega_1$

$$0 = (h_{a2} + \omega_1 h_{g2}) + (\omega_3 - \omega_1) h_{f4} - [h_{a3} + \omega_3 h_{g3}]$$

Solving for ω_3

$$\omega_3 = \frac{(h_{a2} - h_{a3}) + \omega_1(h_{g2} - h_{f4})}{h_{g3} - h_{f4}} = \frac{0.24[90-70] + 0.00303[1100.7 - 28.08]}{1092 - 28.08} = 0.00757 \frac{lb(v)}{lb(a)}$$

(c)

Then, solving Eq 12.43

$$p_{v3} = \frac{\omega_3 p_3}{0.622 + \omega_3} = \frac{(0.00757)(14.696)}{0.62957} = 0.1767 \frac{lbf}{in^2} \Rightarrow \phi_3 = \frac{p_{v3}}{p_{g3}} = \frac{0.1767}{0.3632} = 0.487 \; (48.7\%)$$

(b) A mass rate balance on water reads $\dot{m}_4 + \dot{m}_{v1} = \dot{m}_{v3}$, or

$$\dot{m}_4 = \dot{m}_{v3} - \dot{m}_{v1} = \dot{m}_a(\omega_3 - \omega_1) = 3.29 \frac{lb(a)}{s}[0.00757 - 0.00303] \frac{lb(v)}{lb(a)}$$

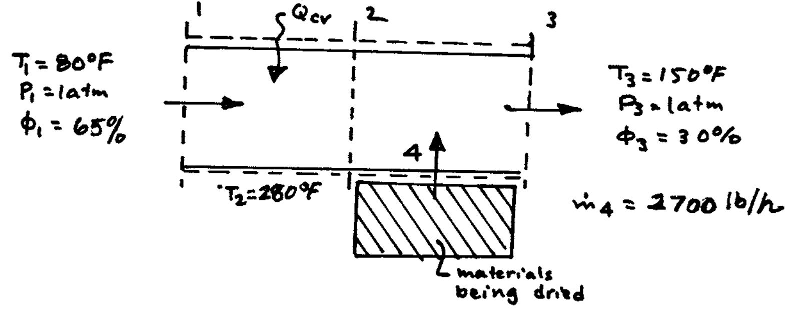
$$= 0.0149 \frac{lb}{s} \longleftarrow \text{(b)}$$

PROBLEM 12.83

KNOWN: Operating data are provided for an industrial dryer at steady state.

FIND: Determine the mass flow rate of dry air required and the rate of heat transfer to the moist air flowing through the drier.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volumes shown on the accompanying figure are at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moist air streams adhere to the ideal gas model.

ANALYSIS: At steady state mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_{a3} \equiv \dot{m}_a$ and $\dot{m}_{v1} = \dot{m}_{v2}$, $\dot{m}_{v3} = \dot{m}_{v2} + \dot{m}_4 = \dot{m}_{v1} + \dot{m}_4$. The last of these can be expressed as

$$\omega_3 \dot{m}_a = \omega_1 \dot{m}_a + \dot{m}_4 \Rightarrow \dot{m}_a = \frac{\dot{m}_4}{\omega_3 - \omega_1}$$

To find ω_1 and ω_3: $p_{v1} = \phi_1 p_g(T_1) = (0.65)(0.5073) = 0.3297$ lbf/in², $p_{v3} = \phi_3 p_g(T_3) = 0.3(3.722) = 1.1166$ lbf/in². Then with Eq. 12.43

$$\omega_1 = 0.622\left[\frac{0.3297}{14.696 - 0.3297}\right] = 0.01427 \frac{lb(v)}{lb(a)}, \quad \omega_3 = 0.622\left[\frac{1.1166}{14.696 - 1.1166}\right] = 0.05115 \frac{lb(v)}{lb(a)}$$

Thus

$$\dot{m}_a = \frac{2700\ lb(v)/h}{(0.05115 - 0.01427)\ \frac{lb(v)}{lb(a)}} = 73,210 \frac{lb(a)}{h} \longleftarrow \dot{m}_a$$

An energy rate balance on the heating unit reduces to give

$$0 = \dot{Q}_{cv} - \cancelto{0}{\dot{W}_{cv}} + [\dot{m}_a h_{a1} + \dot{m}_v h_{v1}] - [\dot{m}_a h_{a2} + \dot{m}_v h_{v2}]$$

or

$$\dot{Q}_{cv} = \dot{m}_a\left[(h_{a2} - h_{a1}) + \omega(h_{g2} - h_{g1})\right]$$

$$= 73,210\left[177.23 - 129.06 + 0.01427(1174.1 - 1096.4)\right]$$

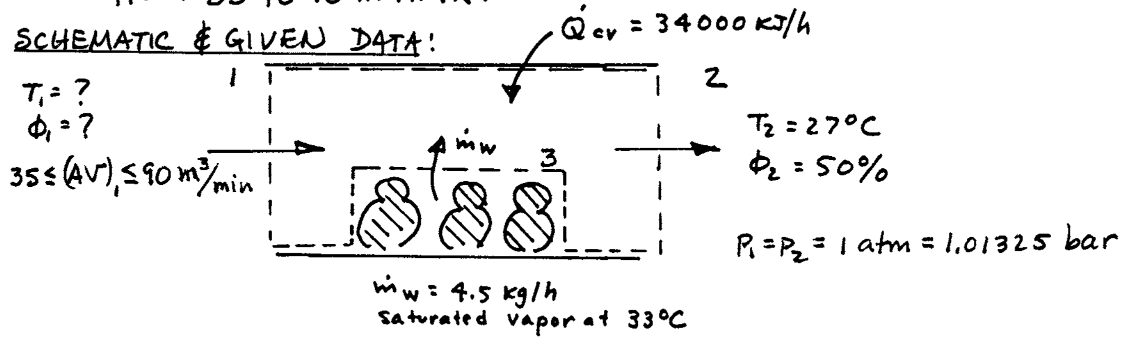
$$= 3.608 \times 10^6\ Btu/h \longleftarrow \dot{Q}_{cv}$$

PROBLEM 12.84

KNOWN: Data are provided for moist air being supplied and removed from an occupied class room.

FIND: (a) For a specified supply air volumetric flow rate, determine the supply air temperature and relative humidity. (b) Plot supply air temperature and relative humidity versus supply air volumetric flow rate ranging from 35 to 90 m³/min.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The moisture entering from the occupants is saturated vapor at 33°C. The moist air streams adhere to the ideal gas model. Pressure is uniform throughout at 1 atm.

ANALYSIS: At steady state, mass rate balances give: $\dot{m}_{a1} = \dot{m}_{a2} \equiv \dot{m}_a$; $\dot{m}_{v1} + \dot{m}_w = \dot{m}_{v2}$. Thus $\dot{m}_w = \dot{m}_a(\omega_2 - \omega_1)$, or

$$\dot{m}_a = \frac{\dot{m}_w}{(\omega_2 - \omega_1)} \qquad (1)$$

The humidity ratio ω_2 is found using $p_{v2} = \phi_2 \, p_g(T_2)$ and $\omega_2 = .622[p_{v2}/(p - p_{v2})]$. To determine ω_1 and T_1 we use an energy rate balance, which reduces at steady state to

$$0 = \dot{Q}_{cv} - \overset{0}{\cancel{\dot{W}_{cv}}} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] + \dot{m}_w h_3 - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}]$$

or

$$0 = \dot{Q}_{cv} + \dot{m}_a[(h_{a1} - h_{a2}) + \omega_1 h_{g1} - \omega_2 h_{g2}] + \dot{m}_w h_g(T_3) \qquad (2)$$

With T_2 and T_3 known, the specific enthalpies h_{g2} and h_{g3} can be determined. Thus Eq. (1) has two unknowns: ω_1 and $\dot{m}_a$. Equation (2) has the same two unknowns and also has T_1 as an unknown.

Another expression relating these quantities is obtained from $(AV)_1 = \dot{m}_a v_{a1}$, as follows:

$$(AV)_1 = \frac{\dot{m}_w}{(\omega_2 - \omega_1)} \left[\frac{(\bar{R}/M_a)T_1}{p_{a1}}\right] = \frac{\dot{m}_w (\bar{R}/M_a) T_1}{(\omega_2 - \omega_1)(p - p_{v1})} \qquad (3)$$

where $\omega_1 = 0.622[p_{v1}/(p - p_{v1})]$. (4)

(a) Sample calculation using table data, $(AV)_1 = 40$ m³/min. To evaluate ω_2

$$p_{v2} = \phi_2 \, p_g(T_2) = 0.5(0.03567) = 0.01784 \text{ bar. Thus}$$

$$\omega_2 = 0.622\left[\frac{(0.01784)}{(1.01325 - 0.01784)}\right] = 0.0111 \frac{\text{kg(v)}}{\text{kg(a)}}$$

Thus

$$\dot{m}_a = \frac{4.5 \text{ kg(v)/h}}{(0.0111 - \omega_1)\frac{\text{kg(v)}}{\text{kg(a)}}} \qquad (a)$$

Solving Eq. (2) for ω_1

$$\omega_1 = \frac{\omega_2[\dot{Q}_{cv}/\dot{m}_w + h_{g3} - h_{g2}] + (h_{a1} - h_{a2})}{(\dot{Q}_{cv}/\dot{m}_w + h_{g3} - h_{g1})}$$

PROBLEM 12.84 (Cont'd.)

From Table A-2E, $h_{g_2} = 2550.8$ and $h_{g_3} = 2561.7$ kJ/kg. With $h_{a_1} - h_{a_2} = c_{pa}(T_1 - T_2)$ and $c_{pa} = 1.005$ kJ/kg·K, we get

$$\omega_1 = \frac{(0.0111)\left[(34000/4.5) + 2561.7 - 2550.8\right] + 1.005(T_1 - 27)}{\left[(34000/4.5) + 2561.7 - h_{g_1}\right]} \qquad (b)$$

From Eq. (3)

$$40\,\frac{m^3}{min} = \frac{(4.5\ kg/h)\left(\frac{8.314}{28.97}\frac{kJ}{kg\cdot K}\right)(T_1 + 273)K}{(0.0111 - \omega_1)(1.01325 - p_{v_1})\ bar}\left|\frac{1\ h}{60\ min}\right|\left|\frac{1\ bar}{10^5\ N/m^2}\right|\left|\frac{10^3\ N\cdot m}{1\ kJ}\right| \qquad (c)$$

From Eq. (4)

$$p_{v_1} = \frac{\omega_1 p}{0.622 + \omega_1} = \frac{\omega_1 (1.01325)}{0.622 + \omega_1} \qquad (d)$$

Solving (a) - (d) simultaneously using an iterative procedure with data for $h_g(T_1)$ from Table A-2: $T_1 = 15.1\,°C \longleftarrow T_1$

$\omega_1 = 0.009546$

$p_{v_1} = 0.01532$ bar

To get ϕ_1,

$$\phi_1 = p_{v_1}/p_{g(T_1)} = 0.01532/0.01762 = 0.869\ (86.9\%) \longleftarrow \phi_1$$

(b) The data for the required plots are obtained using IT, as follows:

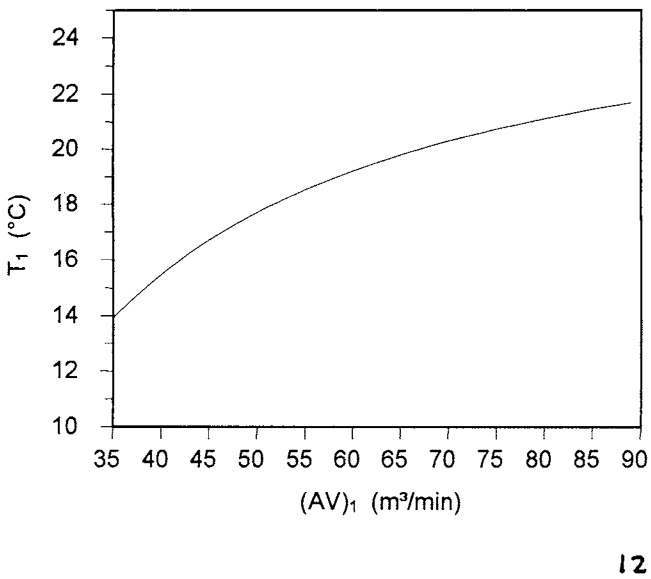
IT Code

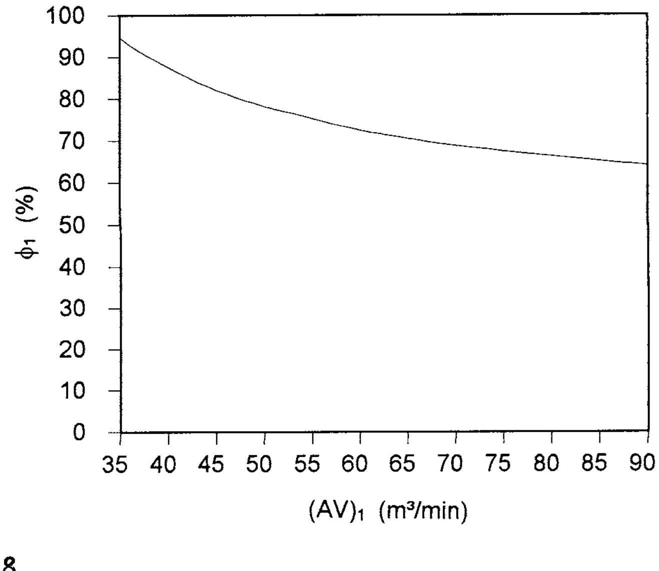
```
p = 1.01325  // bar
T2 = 27
phi2 = 0.5
T3 = 33  // °C
Qdot = 34000  // kJ/h
mdotw = 4.5  // kg/h
AV1 = 40  // m³/min

phi1 = phi_Tw(T1, w1, p)
h1 = ha_Tw(T1, w1)
h2 = ha_Tphi(T2, phi2, p)
w2 = w_Tphi(T2, phi2, p)
psat = Psat_T("Water/Steam", T3)
h3 = hsat_Px("Water/Steam", psat, 1)
va1 = va_Tw(T1,w1,p)
mdota = (AV1 * 60) / va1
mdota * w1 + mdotw = mdota * w2
0 = Qdot + mdota * (h1 - h2) + mdotw * h3
```

IT Results for $(AV)_1 = 40$ m³/min

$\omega_2 = 0.01115$ kg(v)/kg(a)
$h_2 = 55.54$ kJ/kg
$h_3 = 2561$ kJ/kg
$\dot{m}_a = 2891$ kg(a)/min
$h_1 = 39.79$ kJ/kg
$\omega_1 = 0.009592$ kg(v)/kg(a)
$T_1 = 15.47°C$
$\phi_1 = 0.8758$ (87.58%)

PLOTS:



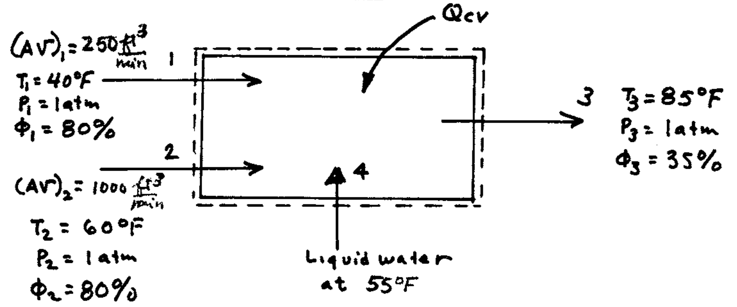


PROBLEM 12.85

KNOWN: Data are provided for a device for heating and humidifying air.

FIND: Determine the rate of heat transfer to the device and the rate liquid water is injected.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady-state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The liquid enters as saturated liquid. The moist air streams adhere to the ideal gas model.

ANALYSIS: At steady state, mass rate balances give

air: $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$

water: $\dot{m}_{v1} + \dot{m}_{v2} + \dot{m}_w = \dot{m}_{v3} \Rightarrow \dot{m}_w = \dot{m}_{v3} - \dot{m}_{v1} - \dot{m}_{v2} = \omega_3 \dot{m}_{a3} - \omega_1 \dot{m}_{a1} - \omega_2 \dot{m}_{a2}$

Combining these results

$$\dot{m}_w = \omega_3(\dot{m}_{a1} + \dot{m}_{a2}) - \omega_1 \dot{m}_{a1} - \omega_2 \dot{m}_{a2} = \dot{m}_{a1}(\omega_3 - \omega_1) + \dot{m}_{a2}(\omega_3 - \omega_2) \qquad (1)$$

To find $\omega_1, \omega_2, \omega_3$: $p_{v1} = \phi_1 p_g(T_1) = 0.8(0.1217) = 0.09736\ lbf/in^2$. $p_{v2} = \phi_2 p_g(T_2) = 0.8(0.2563) = 0.20504\ lbf/in^2$. $p_{v3} = \phi_3 p_g(T_3) = 0.35(0.5967) = 0.20885\ lbf/in^2$. Then

$$\omega_1 = 0.622\left[\frac{0.09736}{14.696 - 0.09736}\right], \quad \omega_2 = 0.622\left[\frac{0.20504}{14.696 - 0.20504}\right], \quad \omega_3 = 0.622\left[\frac{0.20885}{14.696 - 0.20885}\right]$$

$$= 0.004148\ \frac{lb(v)}{lb(a)} \qquad = 0.0088\ \frac{lb(v)}{lb(a)} \qquad = 0.00897\ \frac{lb(v)}{lb(a)}$$

The volumetric flow rates at 1 and 2, together with the ideal gas equation of state give

$$\dot{m}_{a1} = \frac{(AV)_1}{v_{a1}} = \frac{p_{a1}(AV)_1}{\frac{\bar{R}}{M_a}T_1} = \frac{[(14.696 - 0.09736)(144)\frac{lbf}{ft^2}][250\frac{ft^3}{min}]}{\left(\frac{1545}{28.97}\frac{ft \cdot lbf}{lb \cdot °R}\right)(500°R)} = 19.71\ \frac{lb(a)}{min}$$

$$\dot{m}_{a2} = \frac{(AV)_2}{v_{a2}} = \frac{p_{a2}(AV)_2}{\frac{\bar{R}}{M_a}T_2} = \frac{(14.696 - 0.20504)(144)(1000)}{\left(\frac{1545}{28.97}\right)(520)} = 75.24\ \frac{lb(a)}{min}$$

Substituting values into Eq. (1), the rate liquid is injected is

$$\dot{m}_w = (19.71)[0.00897 - 0.004148] + (75.24)[0.00897 - 0.0088] = 0.1078\ \frac{lb}{min} \longleftarrow \dot{m}_w$$

An energy rate equation at steady state gives

$$0 = \dot{Q}_{cv} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1}] + [\dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}] + \dot{m}_w h_4 - [\dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3}]$$

Thus

$$\dot{Q}_{cv} = \dot{m}_{a3}[h_a(T_3) + \omega_3 h_g(T_3)] - \dot{m}_{a1}[h_a(T_1) + \omega_1 h_g(T_1)] - \dot{m}_{a2}[h_a(T_2) + \omega_2 h_g(T_2)] - \dot{m}_w h_f(T_4)$$

$$= 94.95[130.26 + 0.00897(1098.55)] - 19.71[119.48 + 0.004148(1078.9)]$$

$$- 75.24[124.27 + 0.0088(1087.7)] - 0.1078(23.10)$$

$$= 94.95[140.11] - 19.71[123.96] - 75.24[133.84] - 2.5$$

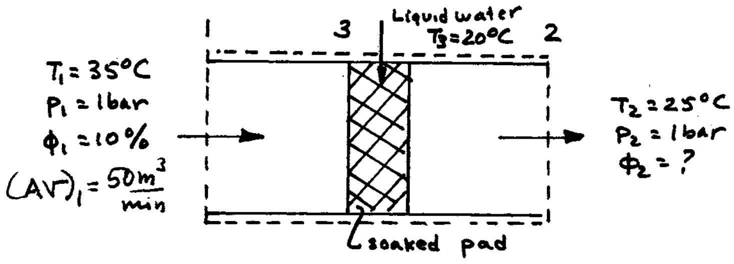
$$= 787.5\ \frac{Btu}{min} \longleftarrow \dot{Q}_{cv}$$

PROBLEM 12.86

KNOWN: Steady-state operating data are provided for an evaporative cooler.

FIND: Determine (a) the rate at which liquid enters, (b) the relative humidity at the exit, (c) the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) All of the entering liquid evaporates into the moist air stream. (3). The liquid enters as saturated liquid. (4) $T_0 = 293$.

ANALYSIS: At steady state mass rate balances give $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$, $\dot{m}_{v1} + \dot{m}_w = \dot{m}_{v2}$. Thus $\dot{m}_w = \dot{m}_a(\omega_2 - \omega_1)$ or $\dot{m}_w/\dot{m}_a = \omega_2 - \omega_1$. To find ω_1, use ϕ_1 to obtain $p_{v1} = \phi_1 p_g(T_1) = (0.10)(0.05628) = 0.005628$ bar. Thus

$$\omega_1 = 0.622\left[\frac{0.005628}{1 - 0.005628}\right] = 0.00352 \frac{kg(v)}{kg(a)}$$

The value of ω_2 can be obtained using an energy rate balance:

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + [\dot{m}_a h_{a1} + \dot{m}_{v1} h_{v1}] + \dot{m}_w h_3 - [\dot{m}_a h_{a2} + \dot{m}_{v2} h_{v2}]$$

or

$$0 = \dot{m}_a\left[(h_{a1} - h_{a2}) + \omega_1 h_{g1} + (\omega_2 - \omega_1) h_{f3} - \omega_2 h_{g2}\right]$$

Solving for ω_2

$$\omega_2 = \frac{(h_{a1} - h_{a2}) + \omega_1(h_{g1} - h_{f3})}{h_{g2} - h_{f3}} = \frac{1.005(35-25) + 0.00352(2565.3 - 83.96)}{2547.2 - 83.96} = 0.00763 \frac{kg(v)}{kg(a)}$$

The mass flow rate of dry air is

$$\dot{m}_a = \frac{(AV)_1 (p_{a1})}{RT_1} = \frac{(50\ m^3/min)[0.994372 \times 10^5\ N/m^2]}{\left(\frac{8314}{28.97} \frac{N \cdot m}{kg \cdot K}\right)(308\ K)} = 56.25 \frac{kg(a)}{min}$$

Thus, $\dot{m}_w = \dot{m}_a(\omega_2 - \omega_1) = (56.25)(0.00763 - 0.00352) = 0.231 \frac{kg(\ell)}{min}$ ⟵ (a)

Solving Eq 12.43

$$p_{v2} = \frac{\omega_2 p}{0.622 + \omega_2} = \frac{(0.00763)(1)}{0.62963} = 0.01212 \text{ bars}, \Rightarrow \phi_2 = \frac{p_v}{p_g(T_2)} = \frac{0.01212}{0.03169} = 0.382 \quad (38.2\%)$$ ⟵ (b)

The rate of exergy destruction is found from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production obtained from an entropy balance:

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_a} = [s_a(T_2, p_{a2}) - s_a(T_1, p_{a1})] + \omega_2 s_v(T_2, p_{v2}) - \omega_1 s_v(T_1, p_{v1}) - (\omega_2 - \omega_1) s_{f3}$$

Using Eq. 6.19, $s_v(T, p_v) = s_g(T) - (\bar{R}/M_v) \ln \phi$ and applying Eq. 6.27 to the dry air

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_a} = \left[c_{pa} \ln \frac{T_2}{T_1} - \frac{\bar{R}}{M_a} \ln \frac{p_{a2}}{p_{a1}}\right] + \omega_2\left[s_g(T_2) - \frac{\bar{R}}{M_v} \ln \phi_2\right] - \omega_1\left[s_g(T_1) - \frac{\bar{R}}{M_v} \ln \phi_1\right] - (\omega_2 - \omega_1) s_{f3}$$

$$= c_{pa} \ln \frac{T_2}{T_1} - \frac{\bar{R}}{M_a} \ln \frac{p_{a2}}{p_{a1}} + \omega_2(s_g(T_2) - s_f(T_3))] - \omega_1(s_g(T_1) - s_f(T_3)] + \frac{\bar{R}}{M_v}[\omega_1 \ln \phi_1 - \omega_2 \ln \phi_2]$$

$$= 1.005 \ln \frac{298}{308} - \frac{8.314}{28.97} \ln \frac{0.98788}{0.99437} + 0.00763[8.558 - 0.2966] - 0.00352[8.3531 - 0.2966] + \frac{8.314}{18.02}[0.00352 \ln 0.1 - 0.00763 \ln .382]$$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_a} = 0.003 \frac{kJ/kg(a)}{°K}$$

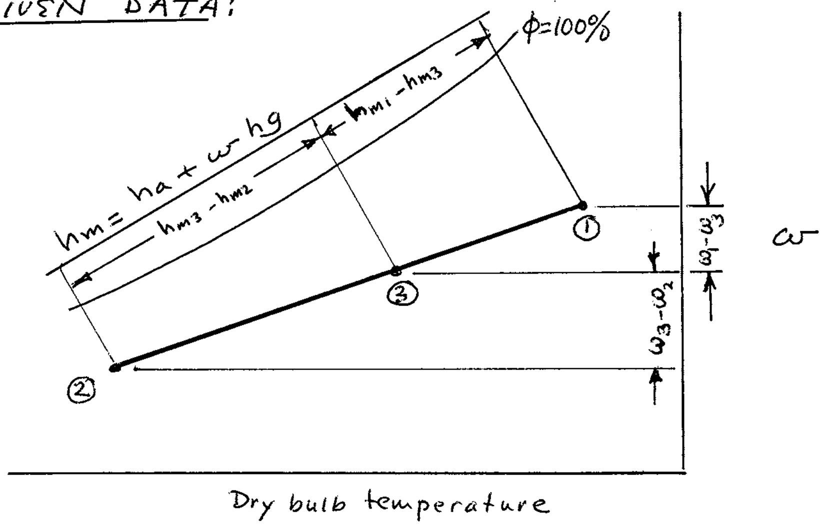
$$\Rightarrow \dot{E}_d = (56.25)(293)(0.003) = 49.44\ kJ/min$$ ⟵ (c)

PROBLEM 12.87

KNOWN: The case of adiabatic mixing of two moist air streams is under consideration.

FIND: Show on a psychrometric ~~chart~~ that exit state 3 is on a straight line connecting inlet states 1 and 2.

SCHEMATIC & GIVEN DATA:



ANALYSIS: From Sec. 12.10.4, and writing for the mixture $h_m = h_a + \omega h_g$,

(12.56b) $\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$

(12.56c) $\dot{m}_{a1} h_{m1} + \dot{m}_{a2} h_{m2} = \dot{m}_{a3} h_{m3}$ **where** $h_m = h_a + \omega h_g$

Since $\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2}$, Eq 12.56b can be rearranged as

$$\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 (\dot{m}_{a1} + \dot{m}_{a2})$$

$$\Rightarrow \quad (\omega_1 - \omega_3)\dot{m}_{a1} + (\omega_2 - \omega_3)\dot{m}_{a2} = 0$$

$$\Rightarrow \quad \frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3} \qquad (1)$$

Similarly, Eq. 12.56c gives

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{h_{m3} - h_{m2}}{h_{m1} - h_{m3}} \qquad (2)$$

Combining Eqs. (1), (2)

$$\frac{\dot{m}_{a1}}{\dot{m}_{a2}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3} = \frac{h_{m3} - h_{m2}}{h_{m1} - h_{m3}} \qquad (3)$$

Since the ω and h_m scales are linear, the following proportionality applies for the sketch above

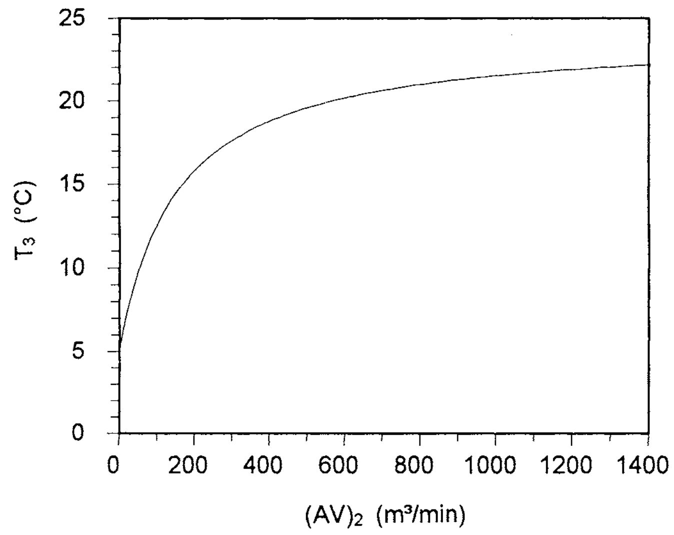
$$\frac{\text{Line 2-3}}{\text{Line 3-1}} = \frac{h_{m3} - h_{m2}}{h_{m1} - h_{m3}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3}$$

which corresponds to Eq. (3).

PROBLEM 12.88

See Example 12.16 for the IT code. To obtain data for the plot, use the **Explore** button to sweep AV2 from 0 to 1400 m³/min in steps of 10.

PLOT:



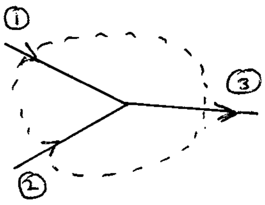
Discussion: When $(AV)_2$ approaches 0, there is no flow in stream 2, and the exit temperature approaches the value of T_1. When $(AV)_2$ becomes large, the effect of stream 1 is reduced and the exit temperature approaches the value of T_2.

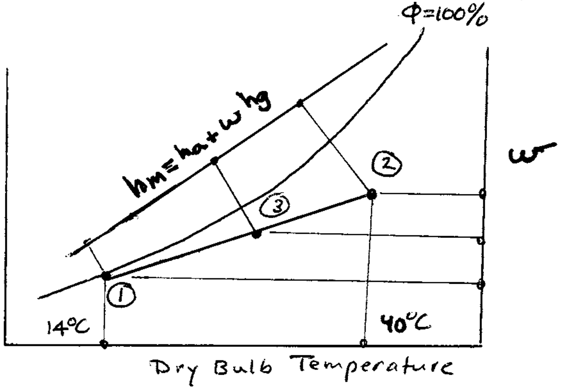
PROBLEM 12.89*

KNOWN: Steady state operating data are provided for the adiabatic mixing of two moist air streams.

FIND: Using the graphical procedure of Problem 12.87, find ϕ_3, T_3.

SCHEMATIC & GIVEN DATA:





$(A V)_1 = 35\ m^3/min$, $T_1 = 14°C$, $\phi_1 = 80\%$
$(A V)_2 = 80\ m^3/min$, $T_2 = 40°C$, $\phi_2 = 40\%$

ASSUMPTIONS: (1) The control volume shown is at steady state. (2) For the control volume $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. Changes in kinetic and potential energy can be neglected. (3) The pressure remains constant at 1 atm.

ANALYSIS: Inspection of Fig A-9 gives

$h_{m1} = 34.0\ kJ/kg(a)$	$h_{m2} = 88.2$
$\omega_1 = 0.008\ kg(v)/kg(a)$	$\omega_2 = 0.0188$
$v_{a1} = 0.824\ m^3/kg(a)$	$v_{a2} = 0.913$

Then

$$\dot{m}_{a1} = \frac{(AV)_1}{v_{a1}} = \frac{35\ m^3/min}{0.824\ m^3/kg(a)} = 42.48\ \frac{m^3}{kg(a)}$$

$$\dot{m}_{a2} = \frac{80}{0.913} = 87.62\ \frac{m^3}{kg(a)}$$

Therefore, the relation of Problem 12.87 becomes

$$\frac{42.48}{87.62} = \frac{\omega_3 - 0.0188}{0.008 - \omega_3} = \frac{h_{m3} - 88.2}{34.0 - h_{m3}}$$

Solving, $\omega_3 = 0.0153\ kg(v)/kg(a)$, $h_{m3} = 70.50\ kJ/kg(a)$. Using these values, Fig. A-9 gives

$$\phi_3 \approx 52\%$$
$$T_3 \approx 32°C$$

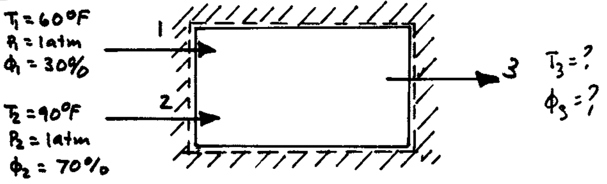
← ϕ_3, T_3

PROBLEM 12.90

KNOWN: A moist air stream at 60°F, 1atm, $\phi = 30\%$ is mixed adiabatically with a stream of moist air at 90°F, 1atm, $\phi = 70\%$. The mass flow rate of the 90°F stream is twice that of the 60°F stream.

FIND: Using the result of Problem 12.70, determine the temperature and relative humidity of the exiting stream.

SCHEMATIC & GIVEN DATA:



$\dot{m}_2 = 2\dot{m}_1$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The pressure at 3 is 1 atm.

ANALYSIS: At steady state, mass rate balances give

mixture : $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \Rightarrow \dot{m}_3 = 3\dot{m}_1$ (1)

air : $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$

water vapor: $\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \Rightarrow \dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2 = \dot{m}_{a3}\omega_3$ (2)

Also, at each location $\dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1+\omega) \Rightarrow \dot{m}_a = \dot{m}/(1+\omega)$ (3)

Combining Eqs. (1), (2), (3) together with $\dot{m}_2 = 2\dot{m}_1$

$$\frac{\dot{m}_1}{(1+\omega_1)}\omega_1 + \frac{\dot{m}_2}{(1+\omega_2)}\omega_2 = \left(\frac{\dot{m}_3}{1+\omega_3}\right)\omega_3 \Rightarrow \left(\frac{\omega_1}{1+\omega_1}\right) + 2\left(\frac{\omega_2}{1+\omega_2}\right) = 3\left(\frac{\omega_3}{1+\omega_3}\right) \qquad (4)$$

To find ω_1 and ω_2, $p_{v1} = \phi_1 p_g(T_1) = (0.3)(0.2563) = 0.07689 \text{ lbf/in}^2$, $p_{v2} = \phi_2 p_g(T_2) = (0.7)(0.6988) = 0.4892 \text{ lbf/in}^2$, so

$$\omega_1 = .622\left[\frac{0.07689}{14.696 - .07689}\right] = 0.0033 \frac{\text{lb(v)}}{\text{lb(a)}}, \quad \omega_2 = 0.622\left[\frac{0.4892}{14.696 - 0.4892}\right] = 0.0214 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Substituting values into Eq.(4)

$$\left(\frac{0.0033}{1.0033}\right) + 2\left(\frac{0.0214}{1.0214}\right) = 3\left(\frac{\omega_3}{1+\omega_3}\right) \Rightarrow \omega_3 = 0.0153 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Solving Eq. 12.43

$$p_{v3} = \frac{\omega_3 p}{0.622 + \omega_3} = \frac{(0.0153)(14.696)}{0.6373} = 0.3528 \frac{\text{lbf}}{\text{in}^2} \Rightarrow \phi_3 = \frac{p_{v3}}{p_g(T_3)} = \frac{0.3528}{p_g(T_3)} \qquad (5)$$

To find T_3, write an energy rate balance for the control volume. With assumption (1) this becomes

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1}] + [\dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}] - [\dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3}]$$

or

$$\dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}] = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}]$$

$$\frac{\dot{m}_3}{1+\omega_3}[h_{a3} + \omega_3 h_{g3}] = \frac{\dot{m}_1}{1+\omega_1}[h_{a1} + \omega_1 h_{g1}] + \frac{\dot{m}_2}{1+\omega_2}[h_{a2} + \omega_2 h_{g2}] \qquad (6)$$

With the result of Problem 12.70

$(h_a + \omega h_g)_1 = 0.24(60) + 0.0033(1061 + 0.444(60)) = 17.99$ Btu/lb(a)

$(h_a + \omega h_g)_2 = 0.24(90) + 0.0214(1061 + 0.444(90)) = 45.16$ "

$(h_a + \omega h_g)_3 = 0.24\underset{\uparrow °F}{T_3} + 0.0153(1061 + 0.444\underset{\uparrow °F}{T_3})$

Thus, Eq.(6) becomes with $\dot{m}_3 = 3\dot{m}_1$, $\dot{m}_2 = 2\dot{m}_1$

$$0.24\,T_3(°F) + 0.0153[1061 + 0.444\,T_3(°F)] = \frac{\left(\frac{1.0153}{1.0033}\right)(17.99) + 2\left(\frac{1.0153}{1.0214}\right)(45.16)}{3} = 36.0$$

or

$$0.2468\,T_3(°F) = 19.77 \Rightarrow T_3(°F) = 80 \text{ °F} \longleftarrow T_3$$

Then, with Eq.(5)

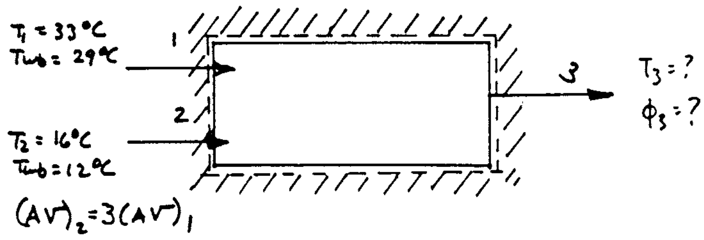
$$\phi_3 = \frac{0.3528}{0.5073} = 0.695 \quad (69.5\%) \longleftarrow \phi_3$$

PROBLEM 12.91

KNOWN: Atmospheric air having dry-bulb and wet-bulb temperatures of 33 and 29°C mixes adiabatically with moist air having dry-bulb and wet-bulb temperatures of 16 and 12°C. The volumetric flow rate of the 16°C stream is twice that of the other.

FIND: For the exiting stream, determine the relative humidity and the temperature.

SCHEMATIC & GIVEN DATA:



$(AV)_2 = 3(AV)_1$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The pressure is uniform throughout at 1 atm. (3) The wet-bulb temperature can be used in place of the adiabatic saturation temperature.

ANALYSIS: At steady state, mass rate balances give

air: $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$

water: $\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \Rightarrow \omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$

Combining these

$$\omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{\omega_1 + \left(\frac{\dot{m}_{a2}}{\dot{m}_{a1}}\right)\omega_2}{1 + \left(\frac{\dot{m}_{a2}}{\dot{m}_{a1}}\right)} \qquad (1)$$

The mass flow rate ratio is obtained using the given volumetric flow rate relation:

$$\dot{m}_{a1} = \frac{(AV)_1}{v_{a1}} = \frac{p_{a1}(AV)_1}{\frac{\bar{R}}{M_a} T_1}, \quad m_{a2} = \frac{p_{a2}(AV)_2}{\frac{\bar{R}}{M_a} T_2} \Rightarrow \frac{\dot{m}_{a2}}{\dot{m}_{a1}} = \frac{p_{a2}}{p_{a1}} \frac{T_1}{T_2} \frac{(AV)_2}{(AV)_1} = 3 \frac{p_{a2}}{p_{a1}} \frac{T_1}{T_2} \qquad (2)$$

The terms ω_1 and ω_2 can be calculated using Eqs. 12.52 and 12.53: First, to find ω_1

$$\omega' = 0.622\left[\frac{0.04008}{0.97817}\right] = 0.0256, \quad \omega_1 = \frac{1.005[29-33] + 0.0256[2432.8]}{2561.7 - 121.61} = 0.0239 \frac{kg(v)}{kg(a)}$$

To find ω_2

$$\omega' = 0.622\left[\frac{0.01402}{0.99923}\right] = 0.0087, \quad \omega_2 = \frac{1.005[12-16] + 0.0087[2523.4]}{2530.8 - 50.41} = 0.0072 \frac{kg(v)}{kg(a)}$$

Solving Eq. 12.43

$$p_{v1} = \frac{\omega_1 p}{0.622 + \omega_1} = \frac{(0.0239)(1.01325)}{0.6459} = 0.0375 \text{ bar} \Rightarrow p_{a1} = 0.97576 \text{ bar}$$

$$p_{v2} = \frac{\omega_2 p}{0.622 + \omega_2} = \frac{(0.0072)(1.01325)}{0.6292} = 0.01159 \text{ bar} \Rightarrow p_{a2} = 1.00166 \text{ bar}$$

Inserting values into Eq. (2) gives

$$\frac{\dot{m}_{a2}}{\dot{m}_{a1}} = 3\left(\frac{1.00166}{0.97576}\right)\left(\frac{306}{289}\right) = 3.261$$

Then, with known values Eq. (1) yields

$$\omega_3 = \frac{0.0239 + (3.261)(0.0072)}{4.261} = 0.0111 \frac{kg(v)}{kg(a)}$$

Accordingly

$$p_{v3} = \frac{\omega_3 p}{0.622 + \omega_3} = \frac{(0.0111)(1.01325)}{0.6331} = 0.0178 \text{ bar.}$$

Then

$$\phi_3 = \frac{p_{v3}}{p_g(T_3)} = \frac{0.0178}{p_g(T_3)} \qquad (3)$$

PROBLEM 12.91 (Cont'd.)

To find T_3 write an energy rate balance for the control volume. With assumption (1)

$$0 = \cancel{\dot{Q}_{cv}}^{\,0} - \cancel{\dot{W}_{cv}}^{\,0} + [\dot{m}_{a1} h_{a1} + \dot{m}_{v1} h_{v1}] + [\dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2}] - [\dot{m}_{a3} h_{a3} + \dot{m}_{v3} h_{v3}]$$

Thus

$$0 = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}] - \dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}]$$

or

$$h_{a3} + \omega_3 h_{g3} = \frac{\dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}]}{\dot{m}_{a1} + \dot{m}_{a2}}$$

$$= \frac{(h_{a1} + \omega_1 h_{g1}) + \left(\frac{\dot{m}_{a2}}{\dot{m}_{a1}}\right)(h_{a2} + \omega_2 h_{g2})}{\left(1 + \frac{\dot{m}_{a2}}{\dot{m}_{a1}}\right)}$$

With data from Tables A-2 and A-22

$$h_{a3} + \omega_3 h_{g3} = \frac{[306.2 + 0.0239(2561.7)] + (3.261)[289.2 + 0.0072(2530.8)]}{4.261}$$

$$= 321.5 \text{ kJ/kg(a)}$$

① This is a single equation in one unknown, T_3. Solving iteratively using table data gives $T_3 \approx 20$ °C. $\longleftarrow$ T_3

Returning to Eq. (3)

$$\phi_3 = \frac{0.0178}{0.02339} = 0.761 \quad (76.1\%) \qquad \longleftarrow \phi_3$$

---

1. An alternative solution that avoids iteration with table data is obtained using IT, as follows:

IT Code

```
T1 = 33  // °C
Twb1 = 29  // °C
T2 = 16  // °C
Twb2 = 12  // °C
AV2 = R * AV1
R = 3
AV1 = 1
p = 1.01325  // bar

w1 = w_TTwb(T1,Twb1,p)
phi1 = phi_Tw(T1,w1,p)
h1 = ha_Tw(T1,w1)
w2 = w_TTwb(T2,Twb2,p)
phi2 = phi_Tw(T2,w2,p)
h2 = ha_Tw(T2,w2)
phi3 = phi_Tw(T3,w3,p)
h3 = ha_Tw(T3,w3)
mdota1 = AV1 / va1
mdota2 = AV2 / va2
va1 = va_Tw(T1,w1,p)
va2 = va_Tw(T2,w2,p)
mdota1 + mdota2 = mdota3
mdota1 * w1 + mdota2 * w2 = mdota3 * w3
0 = mdota1 * h1 + mdota2 * h2 - mdota3 * h3
```

IT Results

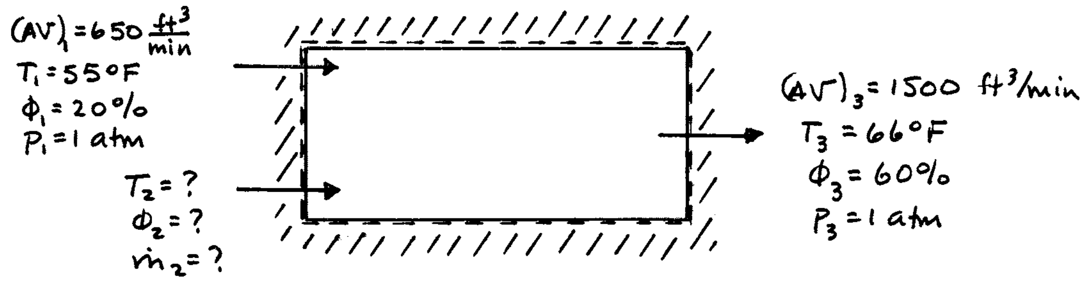
ω_1 = 0.02391 kg(v)/kg(a)
h_1 = 94.37 kJ/kg(a)
ω_2 = 0.007106 kg(v)/kg(a)
h_2 = 34.04 kJ/kg(a)
ω_3 = 0.01105 kg(v)/kg(a)
h_3 = 48.2 kJ/kg(a)
T_3 = 20.08°C
ϕ_3 = 0.7508 (75.08%)

PROBLEM 12.92

KNOWN: A stream consisting of 650 ft³/min of air at 55°F, 1 atm, $\phi = 20\%$ mixes adiabatically with a second stream. A single stream consisting of 1500 ft³/min of air exits at 66°F, 1 atm, $\phi = 60\%$.

FIND: Determine for the second entering stream the relative humidity, temperature, and mass flow rate.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Each stream can be modeled as an ideal gas mixture.

ANALYSIS: At steady state, mass rate balances give

air: $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$

water: $\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \Rightarrow \dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2 = \dot{m}_{a3}\omega_3 \Rightarrow \omega_2 = \dfrac{\dot{m}_{a3}\omega_3 - \dot{m}_{a1}\omega_1}{\dot{m}_{a3} - \dot{m}_{a1}}$ (1)

The mass flow rates are determined using the given volumetric flow rates and $p_{v1} = \phi_1 p_g(T_1) = (0.2)(0.21415) = 0.0428$ lbf/in.², $p_{v3} = \phi_3 p_g(T_3) = (0.6)(0.3165) = 0.1899$ lbf/in.²

$$\dot{m}_{a1} = \frac{(AV)_1}{v_{a1}} = \frac{p_{a1}(AV)_1}{R_a T_1} = \frac{(14.696 - 0.0428)\frac{lbf}{in^2}(650\ \frac{ft^3}{min})}{\left(\frac{1545}{28.97}\ \frac{ft\cdot lbf}{lb\cdot °R}\right)(515°R)}\left|\frac{144\ in^2}{1\ ft^2}\right| = 49.94\ \frac{lb(a)}{min}$$

$$\dot{m}_{a3} = \frac{p_{a3}(AV)_3}{R_a T_3} = \frac{(14.696 - 0.1899)(1500)}{\left(\frac{1545}{28.97}\right)(526)}\left|\frac{144}{1}\right| = 111.7\ \frac{lb(a)}{min}$$

The values of ω_1 and ω_3 are

$$\omega_1 = 0.622\left[\frac{0.0428}{14.696 - 0.0428}\right] = 0.0018\ \frac{lb(v)}{lb(a)},\quad \omega_3 = .622\left[\frac{0.1899}{14.696 - 0.1899}\right] = 0.008143\ \frac{lb(v)}{lb(a)}$$

Accordingly, Eq. (1) gives

$$\omega_2 = \frac{(111.7)(0.008143) - (49.94)(0.0018)}{111.7 - 49.94} = 0.01327\ \frac{lb(v)}{lb(a)}$$

Solving Eq. 12.43

$$p_{v2} = \frac{\omega_2 p_2}{0.622 + \omega_2} = \frac{(0.01327)(14.696)}{0.6353} = 0.307\ \frac{lbf}{in^2} \Rightarrow \phi_2 = \frac{p_{v2}}{p_g(T_2)} = \frac{0.307}{p_g(T_2)} \quad (2)$$

The temperature T_2 can be obtained from an energy rate balance which reduces with assumptions listed above to

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + [\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1}] + [\dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}] - [\dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3}]$$

or

$$0 = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}] - \dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}]$$

Solving for $h_{a2} + \omega_2 h_{g2}$

$$h_{a2} + \omega_2 h_{g2} = \frac{\dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}] - \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}]}{\dot{m}_{a3} - \dot{m}_{a1}}$$

PROBLEM 12.92 (Cont'd.)

With data from Tables A-2E and A-22E

$$h_{a2}+\omega_2 h_{g2} = \frac{(111.7)[125.71+(0.008143)(1090.3)] - (49.94)[123.07+(0.0018)(1085.5)]}{111.7 - 49.94}$$

$$= 142.32 \ \frac{\text{Btu}}{\text{lb(a)}}$$

① This equation has a single unknown, T_2. Solving iteratively using table data gives $T_2 \approx 74.8\ ^\circ F$ $\longleftarrow$ T_2

Returning to Eq. (2) $\phi_2 = \dfrac{0.307}{0.4273} = 0.718\ (71.8\%)$ $\longleftarrow$ ϕ_2

To get the mass flow rate at 2

$$\dot{m}_2 = \dot{m}_{a2} + \dot{m}_{v2} = \dot{m}_{a2}(1+\omega_2) = (\dot{m}_{a3} - \dot{m}_{a1})(1+\omega_2)$$

$$= (111.7 - 49.94)(1 + 0.01327) = 62.58 \ \frac{\text{lb}}{\text{min}}$$ $\longleftarrow$ $\dot{m}_2$

---

1. An alternative solution that avoids iteration with table data is obtained using IT, as follows:

IT Code

```
T1 = 55 // °F
phi1 = 0.2
p = 14.696
AV1 = 650  // ft³/min
T3 = 66  // °F
phi3 = .6
AV3 = 1500  // ft³/min

w1 = w_Tphi(T1,phi1,p)
w2 = w_Tphi(T2,phi2,p)
w3 = w_Tphi(T3,phi3,p)
h1 = ha_Tw(T1,w1)
h2 = ha_Tw(T2,w2)
h3 = ha_Tw(T3,w3)

mdota1 = AV1 / va1
va1 = va_Tw(T1,w1,p)
mdota2 = AV2 / va2
va2 = va_Tw(T2,w2,p)
mdota3 = AV3 / va3
va3 = va_Tw(T3,w3,p)
mdota3 = mdota1 + mdota2
mdota1 * w1 + mdota2 * w2 = mdota3 * w3
mdota1 * h1 + mdota2 * h2 = mdota3 * h3
mdot2 = (mdota3 - mdota1) * (1 + w2)
```

IT Results

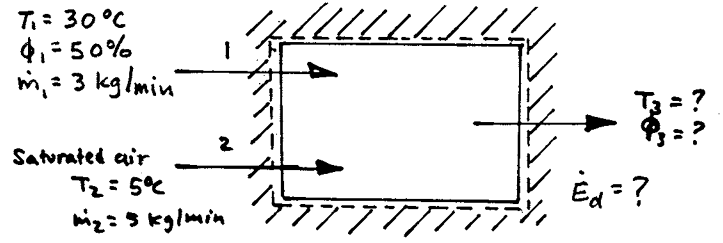
```
ω1 = 0.001816 lb(v)/lb(a)
ω2 = 0.01328 lb(v)/lb(a)
ω3 = 0.008154 lb(v)/lb(a)
ṁa1 = 49.96 lb(a)/min
ṁa2 = 61.79 lb(a)/min
ṁa3 = 111.7 lb(a)/min
(AV)2 = 850 ft³/min
T2 = 74.71°F
φ2 = 0.7215 (72.15%)
ṁ2 = 62.61 lb/min
```

PROBLEM 12.93

KNOWN: Moist air at $T_1 = 30°C$, $p_1 = 1$ bar, $\phi_1 = 50\%$, $\dot{m}_1 = 3$ kg/min mixes adiabatically with saturated air at $T_2 = 5°C$, $p_2 = 1$ bar, $\dot{m}_2 = 5$ kg/min to produce a single mixed stream at $p_3 = 1$ bar.

FIND: Determine ϕ_3, T_3, and the exergy destruction rate.

SCHEMATIC & GIVEN DATA:



$p_1 = p_2 = p_3 = 1$ bar
$T_0 = 293$ K

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Each stream can be modeled as an ideal gas mixture.

ANALYSIS: At steady state, mass rate balances reduce to give $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$ and $\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2}$, $\dot{m}_{v3} = \dot{m}_{v1} + \dot{m}_{v2}$. Also, for each stream $\dot{m} = \dot{m}_a + \dot{m}_v = \dot{m}_a(1+\omega)$. Accordingly, $\dot{m}_{a3}\omega_3 = \dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2$, or

$$\omega_3 = \frac{\dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{\left(\frac{\omega_1}{1+\omega_1}\right)\dot{m}_1 + \left(\frac{\omega_2}{1+\omega_2}\right)\dot{m}_2}{\left(\frac{\dot{m}_1}{1+\omega_1}\right) + \left(\frac{\dot{m}_2}{1+\omega_2}\right)}$$

To find ω_1 and ω_2, $p_{v1} = \phi_1 p_g(T_1) = 0.5(0.04246) = 0.02123$ bar, $p_{v2} = \phi_2 p_g(T_2) = 0.00872$ bar. Then

$$\omega_1 = 0.622\left(\frac{0.02123}{1 - 0.02123}\right) = 0.01349\ \frac{\text{kg(v)}}{\text{kg(a)}}, \quad \omega_2 = 0.622\left(\frac{0.00872}{1 - 0.00872}\right) = 0.00547\ \frac{\text{kg(v)}}{\text{kg(a)}}$$

Accordingly

$$\omega_3 = \frac{\left(\frac{0.01349}{1.01349}\right)(3) + \left(\frac{0.00547}{1.00547}\right)(5)}{\left(\frac{3}{1.01349}\right) + \left(\frac{5}{1.00547}\right)} = 0.008463\ \frac{\text{kg(v)}}{\text{kg(a)}}$$

Solving Eq. 12.43

$$p_{v3} = \frac{p_3\omega_3}{0.622 + \omega_3} = \frac{(1)(0.008463)}{0.63046} = 0.01342 \Rightarrow \phi_3 = \frac{p_{v3}}{p_g(T_3)} = \frac{0.01342}{p_g(T_3)} \qquad (1)$$

To find T_3, an energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1}] + [\dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2}] - [\dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3}]$$

or $$0 = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}] - \dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}]$$

Then

$$h_{a3} + \omega_3 h_{g3} = \frac{\dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}]}{\dot{m}_{a1} + \dot{m}_{a2}}$$

$\dot{m}_{a1} = \dot{m}_1/(1+\omega_1) = 2.96$ kg(a)/min, $\dot{m}_{a2} = \dot{m}_2/(1+\omega_2) = 4.9728$ kg(a)/min. With data from Tables A-2, A-22

$$h_{a3} + \omega_3 h_{g3} = \frac{(2.96)[303.21 + (0.01349)(2556.3)] + 4.9728[278.1 + (0.00547)(2510.6)]}{(2.96 + 4.9728)}$$

$$= 308.95\ \text{kJ/kg(a)}$$

① This equation has a single unknown, T_3. Solving iteratively using table data gives
$T_3 = 14.4\ °C \longleftarrow T_3$

PROBLEM 12.93

Returning to Eq. (1), $\phi_3 = \frac{0.01342}{0.01641} = 0.818\ (81.8\%) \longleftarrow \phi_3$

The exergy destruction rate is obtained from $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance which at steady state reduces to

$$0 = \cancel{\sum \frac{\dot{Q}_j}{T_j}}^{\,0} + [\dot{m}_{a1} s_a(T_1, p_{a1}) + \dot{m}_{v1} s_v(T_1, p_{v1})] + [\dot{m}_{a2} s_a(T_2, p_{a2}) + \dot{m}_{v2} s_g(T_2)] - [\dot{m}_{a3} s_a(T_3, p_{a3}) + \dot{m}_{v3} s_v(T_3, p_{v3})] + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}_{a3}[s_a(T_3, p_{a3}) + \omega_3 s_v(T_3, p_{v3})] - \dot{m}_{a1}[s_a(T_1, p_{a1}) + \omega_1 s_v(T_1, p_{v1})] - \dot{m}_{a2}[s_a(T_2, p_{a2}) + \omega_2 s_g(T_2)]$$

$$= \dot{m}_{a1}[s_a(T_3, p_{a3}) - s_a(T_1, p_{a1})] + \dot{m}_{a2}[s_a(T_3, p_{a3}) - s_a(T_2, p_{a2})] + \dot{m}_{a3}\omega_3 s_v(T_3, p_{v3}) - \dot{m}_{a1}\omega_1 s_v(T_1, p_{v1}) - \dot{m}_{a2}\omega_2 s_g(T_2)$$

From Eq. 6.19, $s_v(T, p_v) = s_g(T) - \bar{R}/M_v \ln\phi$, and using Eq. 6.23 for the dry air

$$\dot{\sigma}_{cv} = \dot{m}_{a1}\left[c_{pa} \ln\frac{T_3}{T_1} - \frac{\bar{R}}{M_a} \ln\frac{p_{a3}}{p_{a1}}\right] + \dot{m}_{a2}\left[c_{pa} \ln\frac{T_3}{T_2} - \frac{\bar{R}}{M_a} \ln\frac{p_{a3}}{p_{a2}}\right] + \dot{m}_{a3}\omega_3\left[s_g(T_3) - \frac{\bar{R}}{M_v}\ln\phi_3\right] - \dot{m}_{a1}\omega_1\left[s_g(T_1) - \frac{\bar{R}}{M_v}\ln\phi_1\right] - \dot{m}_{a2}\omega_2 s_g(T_2)$$

From previous calculations: $p_{a1} = 0.97877$ bar, $p_{a2} = 0.99128$ bar, and $p_{a3} = 1 - 0.01342 = 0.98658$ bar. Evaluating $\dot{\sigma}_{cv}$ with $c_{pa} = 1.005$ kJ/kg·K

$$\dot{\sigma}_{cv} = 2.96\left[1.005 \ln\frac{287.4}{303} - \frac{8.314}{28.97}\ln\frac{0.98658}{0.97877}\right] + 4.9728\left[1.005 \ln\frac{287.4}{278} - \frac{8.314}{28.97}\ln\frac{0.98658}{0.99128}\right] + (7.9323)(0.008463)\left[8.79544 - \frac{8.314}{18.02}\ln 0.818\right] - (2.96)(0.01349)\left[8.4533 - \frac{8.314}{18.02}\ln 0.5\right] - (4.9728)(0.00547)(9.0257)$$

$$= 0.009828\ \frac{\text{kJ/min}}{\text{K}}$$

The exergy destruction rate is then

$$\dot{E}_d = (293\,\text{K})\left(0.009828\ \frac{\text{kJ/min}}{\text{K}}\right)\left|\frac{1\ \text{min}}{60\ \text{s}}\right|\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right| = 0.048\ \text{kW} \longleftarrow \dot{E}_d$$

---

1. An alternative solution that avoids iteration with table data is obtained using IT, as follows:

IT Code

```
T1 = 30  // °C
p = 1  // bar
phi1 = 0.5
mdot1 = 3  // kg/min
T2 = 5  // °C
phi2 = 1
mdot2 = 5  // kg/min

w1 = w_Tphi(T1,phi1,p)
w2 = w_Tphi(T2,phi2,p)
mdota1 = mdot1 / (1 + w1)
mdota2 = mdot2 / (1 + w2)
mdota3 = mdota1 + mdota2
mdota3 * w3 = mdota1* w1 + mdota2 * w2
mdota3 * h3 = mdota1* h1 + mdota2 * h2
h1 = ha_Tw(T1,w1)
h2 = ha_Tw(T2,w2)
h3 = ha_Tw(T3,w3)
w3 = w_Tphi(T3,phi3,p)
0 = mdota1* s1 + mdota2 * s2 - mdota3 * s3 + sigmadot
Edotd = To * sigmadot / 60
To = 293
s1 = sa_Tw(T1,w1,p)
s2 = sa_Tw(T2,w2,p)
s3 = sa_Tw(T3,w3,p)
```

IT Results

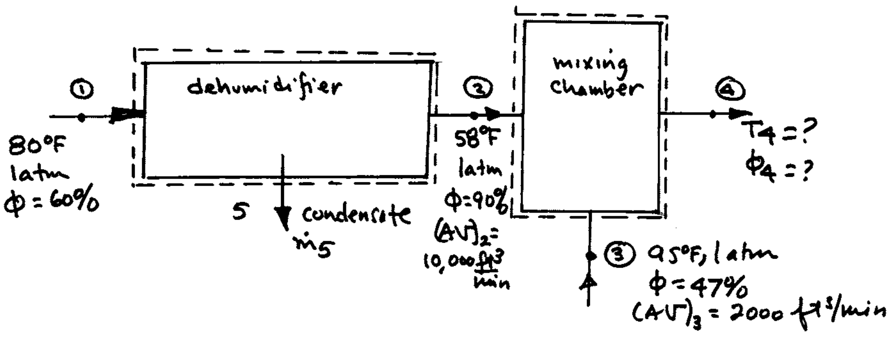
$\omega_1 = 0.0135$ kg(v)/kg(a)
$\omega_2 = 0.005486$ kg(v)/kg(a)
$\omega_3 = 0.008477$ kg(v)/kg(a)
$\dot{m}_{a1} = 2.96$ kg(a)/min
$\dot{m}_{a2} = 4.973$ kg(a)/min
$\dot{m}_{a3} = 7.933$ kg(a)/min
$\dot{\sigma}_{cv} = 0.01009$ kJ/min·K
$T_3 = 14.41$°C
$\phi_3 = 0.8167$ (81.67%)
$\dot{E}_d = 0.04926$ kW

PROBLEM 12.94

KNOWN: Steady state operating data are provided for a system consisting of a dehumidifier followed by adiabatic mixing with another air stream.

FIND: Determine (a) the rate water is removed from the moist air passing through the dehumidifier, (b) the temperature and relative humidity of the stream exiting the mixing chamber.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volumes shown in the figure are at steady state. (2) For the control volumes $\dot{Q}_{cv}=0$, $\dot{W}_{cv}=0$, and all kinetic/potential energy effects can be ignored. (3) The moist air streams are modeled as ideal gas mixtures. (4) p = 1atm throughout

ANALYSIS: (a) Mass rate balances for dry air and water applied to the dehumidifier reduce to give $\dot{m}_5 = \dot{m}_a(\omega_1 - \omega_2)$.

Using ϕ_1, ϕ_2 and p_g at T_1, T_2

$$\omega_1 = \frac{0.622(0.6)(0.5073)}{14.7-(0.6)(0.5073)} = 0.01315, \quad \omega_2 = \frac{0.622(0.9)(0.2386)}{14.7-(0.9)(0.2386)} = 0.00922$$

Also

$$\dot{m}_{a2} = \frac{(AV)_2}{v_{a2}} = \frac{(AV)_2\, \overset{[14.7-(0.9)(0.2386)]\ \text{lbf/in}^2}{p_{a2}}}{\frac{\bar{R}}{M_a}T_2} = \frac{(10{,}000\ \text{ft}^3/\text{min})(14.485\times 144)\ \text{lbf/ft}^2}{\left(\frac{1545}{28.97}\frac{\text{ft·lbf}}{\text{lb·°R}}\right)(518°\text{R})}\left|\frac{60\ \text{min}}{1\ \text{h}}\right| = 45{,}302.5\ \frac{\text{lb}}{\text{h}}$$

Thus, $\dot{m}_5 = 45{,}302.5[0.01315 - 0.00922] = 178$ lb ◄——— (a)

(b) Following the development of Sec. 12.9.4, mass rate and energy rate balances give

$$\omega_4 = \frac{\omega_2\dot{m}_{a2}+\omega_3\dot{m}_{a3}}{\dot{m}_{a2}+\dot{m}_{a3}}, \quad (h_a+\omega h_g)_4 = \frac{\dot{m}_{a2}(h_{a2}+\omega_2 h_{g2}) + \dot{m}_{a3}(h_{a3}+\omega_3 h_{g3})}{\dot{m}_{a2}+\dot{m}_{a3}} \qquad (1),(2)$$

For stream 3

$$\omega_3 = \frac{(0.622)(.47)(0.8165)}{14.7-(0.47)(0.8165)} = 0.0167, \quad \dot{m}_{a3} = \frac{(AV)_3\, \overset{[14.7-0.47(0.8165)]\ \text{lbf/in}^2}{p_{a3}}}{\frac{\bar{R}}{M_a}T_3} = \frac{(2000)(14.3162)(144)(60)}{\left(\frac{1545}{28.97}\right)(555)} = 8358\ \frac{\text{lb}}{\text{h}}$$

Thus, Eq. (1) gives

$$\omega_4 = \frac{(0.00922)(45302.5)+(0.0167)(8358)}{(45302.5+8358)} = 0.01039$$

① Using the result of Problem 12.70, Eq. (2) gives

$$0.24\,T_4(°\text{F}) + 0.01039[1061 + 0.444\,T_4(°\text{F})] = \left(\frac{45{,}302.5}{53{,}660.5}\right)\left[(0.24)(58) + 0.00922(1061+(0.444)(58))\right] + \left(\frac{8358}{53{,}660.5}\right)\left[(0.24)(95) + 0.0167(1061+(0.444)(95))\right]$$

$$= 26.63\ \text{Btu/lb(da)}$$

PROBLEM 12.94 (Continued)

or

$$0.245\ T_4(°F) = 15.606 \Rightarrow T_4 = 63.7°F \longleftarrow \qquad T_4$$

To find Φ_4, solve Eq. 12.43 to give

$$p_{v4} = \frac{\omega_4\ p_4}{\omega_4 + 0.622} = \frac{(0.01039)(14.7)}{(0.01039 + 0.622)} = 0.2415\ \frac{lbf}{in^2}$$

Then

② $$\Phi_4 = \frac{p_{v4}}{p_{g4}} = \frac{0.2415}{0.2922} = 0.826 \quad (82.6\%)$$

---

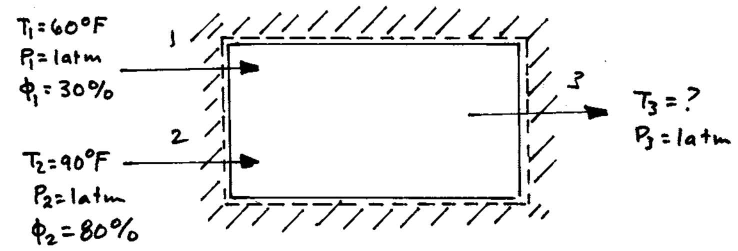
1. Using this expression avoids an iterative solution of Eq.(2) using table data.
2. The problem is readily solved using the psychrometric chart, thereby reducing considerably the computation required throughout.

PROBLEM 12.95

KNOWN: A stream of air at 60°F, 1atm, $\phi = 30\%$ is mixed adiabatically with a stream at 90°F, 1atm, $\phi = 80\%$. A single mixed stream exits.

FIND: Plot the temperature of the exiting stream versus the ratio of the mass flow rates of the dry air present in the two incoming streams.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy.

ANALYSIS: (a) At steady state mass rate balances reduce to give

air: $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$

water: $\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \Rightarrow \omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$

Thus

$$\omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}} = \frac{\left(\frac{\dot{m}_{a1}}{\dot{m}_{a2}}\right)\omega_1 + \omega_2}{\left(\frac{\dot{m}_{a1}}{\dot{m}_{a2}}\right) + 1} = \frac{r\omega_1 + \omega_2}{r+1} \qquad (1)$$

where $r = \dot{m}_{a1}/\dot{m}_{a2}$. To find ω_1 and ω_2, $p_{v1} = \phi_1\, p_g(T_1) = 0.3(0.2563) = 0.07689\ lbf/in^2$, $p_{v2} = \phi_2\, p_g(T_2) = 0.8(0.6988) = 0.55904\ lbf/in^2$. Then

$$\omega_1 = 0.622 \frac{p_{v1}}{p_1 - p_{v1}} = \frac{0.622\,(0.07689)}{14.696 - 0.07689} = 0.00327 \frac{lb(a)}{lb(v)}$$

$$\omega_2 = 0.622 \frac{p_{v2}}{p_2 - p_{v2}} = \frac{(0.622)(0.55904)}{14.13696} = 0.024597 \frac{lb(a)}{lb(v)}$$

At steady state an energy rate balance reduces to

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + [\dot{m}_{a1} h_{a1} + \dot{m}_{v1} h_{g1}] + [\dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2}] - [\dot{m}_{a3} h_{a3} + \dot{m}_{v3} h_{v3}]$$

Thus

$$0 = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}] - \dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}]$$

or

$$h_{a3} + \omega_3 h_{g3} = \frac{\dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}]}{\dot{m}_{a1} + \dot{m}_{a2}}$$

$$= \frac{\left(\frac{\dot{m}_{a1}}{\dot{m}_{a2}}\right)[h_{a1} + \omega_1 h_{g1}] + [h_{a2} + \omega_2 h_{g2}]}{(\dot{m}_{a1}/\dot{m}_{a2}) + 1}$$

$$= \frac{r[h_{a1} + \omega_1 h_{g1}] + [h_{a2} + \omega_2 h_{g2}]}{r+1} \qquad (2)$$

Combining Eqs. (1) and (2)

$$h_{a3} + \left[\frac{r\omega_1 + \omega_2}{r+1}\right] h_{g3} = \frac{r[h_{a1} + \omega_1 h_{g1}] + [h_{a2} + \omega_2 h_{g2}]}{r+1} \qquad (3)$$

Using the result of Problem 12.70, $(h_a + \omega h_g) = 0.24\,T(°F) + \omega(1061 + 0.444\,T(°F))$

PROBLEM 12.95

Accordingly

$h_{a1} = 0.24\,(60) = 14.4$ Btu/lb(a) , $h_{a2} = 0.24(90) = 21.6$ Btu/lb(a)
$h_{g1} = 1061 + 0.444\,(60) = 1087.64$ Btu/lb(v)
$h_{g2} = 1061 + 0.444\,(90) = 1100.96$ Btu/lb(v)

Equation (3) becomes

$$0.24\,T_3 + \left[\frac{r\omega_1 + \omega_2}{r+1}\right]\left[1061 + 0.444\,T_3\right] = \frac{17.957\,r + 48.6803}{r+1}$$

or

$$\left\{0.24 + 0.444\left[\frac{r\omega_1 + \omega_2}{r+1}\right]\right\} T_3 = \frac{14.488\,r + 22.583}{r+1}$$

where $\omega_1 = 0.00327$ and $\omega_2 = 0.024597$. For $r = 2$, $T_3 = 70.26\ °F \longleftarrow T_3$

(b) The data for the required plot are obtained using IT, as follows:

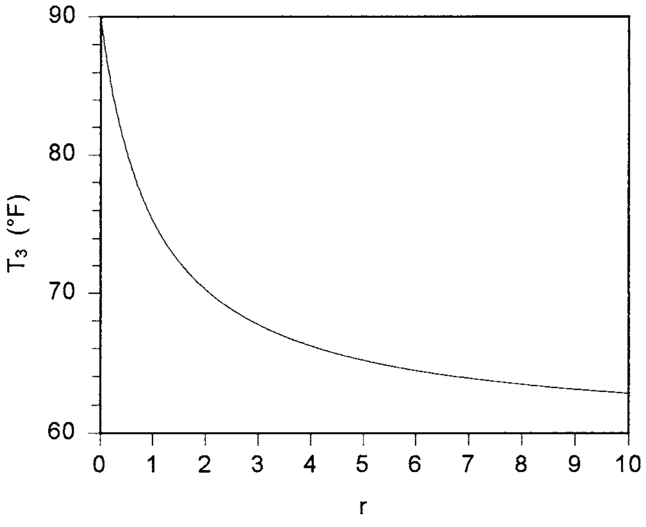
IT Code

```
T1 = 60  // °F
p = 1  // atm
phi1 = 0.3
T2 = 90  // °F
phi2 = 0.8
r = 2

w1 = w_Tphi(T1,phi1,p)
w2 = w_Tphi(T2,phi2,p)
w3 = w_Tphi(T3,phi3,p)
h1 = ha_Tw(T1,w1)
h2 = ha_Tw(T2,w2)
h3 = ha_Tw(T3,w3)

r * w1 +  w2 = (1 + r) * w3
r * h1 + h2 = (1 + r) * h3
```

IT Results

$\omega_1 = 0.003269$ lb(v)/lb(a)
$\omega_2 = 0.02457$ lb(v)/lb(a)
$\omega_3 = 0.01037$ lb(v)/lb(a)
$\phi_3 = 0.6577$ (65.77%)
$T_3 = 70.26°F$

PLOT:



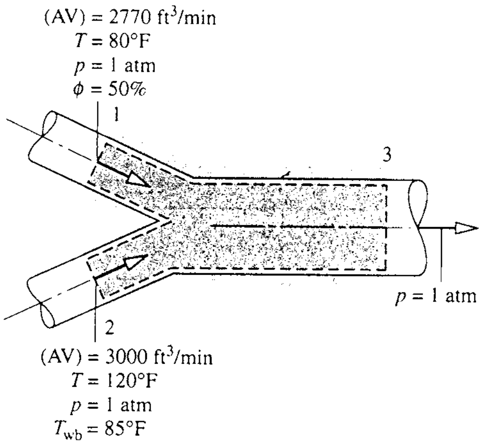
As $r \to \infty$, $T_3 \to T_1 = 60°F$, as expected.

PROBLEM 12.96

KNOWN: Operating data are provided for two moist air streams mixing adiabatically at steady state.

FIND: Determine the exergy destruction rate.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Each stream is modeled as an *ideal gas mixture*. (3) Let $T_0 = 555°R$.

ANALYSIS: To determine the exergy destruction rate, we first fix state 3 using mass and energy balances. At steady state

air: $\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3}$

water: $\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \Rightarrow \omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2} = \omega_3 \dot{m}_{a3}$

Thus

$$\omega_3 = \frac{\omega_1 \dot{m}_{a1} + \omega_2 \dot{m}_{a2}}{\dot{m}_{a1} + \dot{m}_{a2}} \qquad (1)$$

At state 1, $p_{v1} = \phi_1 \, p_g(T_1) = 0.5\,(0.5073) = 0.25365$ lbf/in.²; $\omega_1 = .622\left(\frac{p_{v1}}{p - p_{v1}}\right) = 0.01092 \frac{\text{lb(v)}}{\text{lb(a)}}$

To find ω_2, use Eqs. 12.53 and 12.52, respectively, with $T_{wb2} = T_{as}$.

$$w' = .622\left[\frac{p_g(T_{wb2})}{p - p_g(T_{wb2})}\right] = .622\left[\frac{0.5967}{14.696 - .5967}\right] = 0.02632$$

$$\omega_2 = \frac{c_{pa}(T_{wb2} - T_2) + w' \, h_{fg@T_{wb2}}}{h_g(T_2) - h_f(T_{wb2})} = \frac{0.24(85-120) + (0.02632)(1045.45)}{1113.5 - 53.08}$$

$$= 0.01803 \frac{\text{lb(v)}}{\text{lb(a)}}$$

Also

$$p_{v2} = \frac{\omega_2 p}{.622 + \omega_2} = \frac{(0.01803)(14.696)}{0.64} = 0.414 \text{ lbf/in.}^2$$

The mass flow rates $\dot{m}_{a1}$ and $\dot{m}_{a2}$ are

$$\dot{m}_{a1} = \frac{(p - p_{v1})(AV)_1}{R_a T_1} = \frac{(14.696 - 0.25365)\,\text{lbf/in.}^2\,(2770\ \text{ft}^3/\text{min})}{\left(\frac{1545}{28.97} \cdot \frac{\text{ft}\cdot\text{lbf}}{\text{lb}\cdot°\text{R}}\right)(540°\text{R})}\left|\frac{144\ \text{in.}^2}{1\ \text{ft}^2}\right| = 200 \frac{\text{lb(a)}}{\text{min}}$$

$$\dot{m}_{a2} = \frac{(14.696 - 0.414)(3000)}{\left(\frac{1545}{28.97}\right)(580)}\left|\frac{144}{1}\right| = 199.5 \frac{\text{lb(a)}}{\text{min}}$$

$$\dot{m}_{a3} = \dot{m}_{a1} + \dot{m}_{a2} = 399.5 \frac{\text{lb(a)}}{\text{min}}$$

At steady state, the energy rate balance reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_{a1} h_{a1} + \dot{m}_{v1} h_{v1}] + [\dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2}] - [\dot{m}_{a3} h_{a3} + \dot{m}_{v3} h_{v3}]$$

Thus

$$0 = \dot{m}_{a1}[h_{a1} + \omega_1 h_{g1}] + \dot{m}_{a2}[h_{a2} + \omega_2 h_{g2}] - \dot{m}_{a3}[h_{a3} + \omega_3 h_{g3}]$$

or

$$h_{a3} + \omega_3 h_{g3} = \frac{\dot{m}_{a1}}{\dot{m}_{a3}}[h_{a1} + \omega_1 h_{g1}] + \frac{\dot{m}_{a2}}{\dot{m}_{a3}}[h_{a2} + \omega_2 h_{g2}] \qquad (2)$$

PROBLEM 12.96 (Cont'd.)

Using the result of Problem 12.70, $h_a + \omega h_g = 0.24\,T + \omega(1061 + 0.444\,T)$. Thus

$$h_{a_1} + \omega_1 h_{g_1} = 31.174 \text{ Btu/lb(a)}$$
$$h_{a_2} + \omega_2 h_{g_2} = 48.89 \text{ Btu/lb(a)}$$

From (2), and using Eq.(1) to get $\omega_3 = 0.01447$ lb(v)/lb(a)

$$h_{a_3} + \omega_3 h_{g_3} = \left(\frac{200}{399.5}\right)(31.174) + \left(\frac{199.5}{399.5}\right)(48.89) = 40.021 \Rightarrow T_3 = 100.1\,°F \longleftarrow T_3$$

The exergy destruction rate is $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is the entropy production rate from an entropy balance which at steady state takes the form

$$0 = \cancel{\sum_j \dot{Q}_j / T_j}^{\,0} + [\dot{m}_{a1} s_a(T_1, p_{a1}) + \dot{m}_{v1} s_v(T_1, p_{v1})] + [\dot{m}_{a2} s_a(T_2, p_{a2}) + \dot{m}_{v2} s_v(T_2, p_{v2})] - [\dot{m}_{a3} s_a(T_3, p_{a3}) + \dot{m}_{v3} s_v(T_3, p_{v3})] + \dot{\sigma}_{cv}$$

or

$$\dot{\sigma}_{cv} = \dot{m}_{a3}[s_a(T_3, p_{a3}) + \omega_3 s_v(T_3, p_{v3})] - \dot{m}_{a1}[s_a(T_1, p_{a1}) + \omega_1 s_v(T_3, p_{v1})] - \dot{m}_{a2}[s_a(T_2, p_{v2}) + \omega_2 s_v(T_2, p_{v2})]$$

$$= \dot{m}_{a1}[s_a(T_3, p_{a3}) - s_a(T_1, p_{a1})] + \dot{m}_{a2}[s_a(T_3, p_{a3}) - s_a(T_2, p_{a2})] + \dot{m}_{a3}\omega_3 s_v(T_3 p_{v3}) - \dot{m}_{a1}\omega_1 s_v(T_1, p_{v1}) - \dot{m}_{a2}\omega_2 s_v(T_2, p_{v2})$$

From Eq. 6.19, $s_v(T, p_v) = s_g(T) - \bar{R}/M_v \ln\phi$, and using Eq. 6.23 for the dry air

$$\dot{\sigma}_{cv} = \dot{m}_{a1}\left[c_{pa}\ln\frac{T_3}{T_1} - \frac{\bar{R}}{M_a}\ln\frac{p_{a3}}{p_{a1}}\right] + \dot{m}_{a2}\left[c_{pa}\ln\frac{T_3}{T_2} - \frac{\bar{R}}{M_a}\ln\frac{p_{a3}}{p_{a2}}\right] + \dot{m}_{a3}\omega_3\left[s_g(T_3) - \frac{\bar{R}}{M_v}\ln\phi_3\right] - \dot{m}_{a1}\omega_1\left[s_g(T_1) - \frac{\bar{R}}{M_v}\ln\phi_1\right] - \dot{m}_{a2}\omega_2\left[s_g(T_2) - \frac{\bar{R}}{M_v}\ln\phi_2\right] \quad (3)$$

From above $p_{a1} = p - p_{v1} = 14.696 - 0.25365 = 14.442$ and $p_{a2} = p - p_{v2} = 14.282$. Now

$$p_{v3} = \frac{\omega_3 p}{.622 + \omega_3} = 0.3341 \text{ lbf/in}^2 \Rightarrow p_{a3} = p - p_{v3} = 14.362 \text{ lbf/in}^2$$

Inserting values into Eq.(3)

①
$$\dot{\sigma}_{cv} = 200\left[0.24\ln\frac{560.1}{540} - \frac{1.986}{28.97}\ln\frac{14.362}{14.442}\right] + 199.5\left[0.24\ln\frac{560.1}{580} - \frac{1.986}{28.97}\ln\frac{14.362}{14.282}\right] + 399.5(0.01447)\left[1.98195 - \frac{1.986}{18.02}\ln 0.3504\right] - 200(0.01092)\left[2.0356 - \frac{1.986}{18.02}\ln 0.5\right] - 199.5(0.01803)\left[1.9336 - \frac{1.986}{18.02}\ln 0.244\right] = 0.08079 \ \frac{\text{Btu/min}}{°\text{R}}$$

Finally

$$\dot{E}_d = T_0\dot{\sigma}_{cv} = (555)(0.08079) = 44.84 \text{ Btu/min} \longleftarrow \dot{E}_d$$

---

1. The following is a much simpler solution that uses the psychrometric functions in IT:

IT Code

```
T1 = 80  // °F
p = 1  // atm
phi1 = 0.5
AV1 = 2770  // ft³/min
va1 = va_Tw(T1,w1,p)
mdota1 = AV1 / va1
w1 = w_Tphi(T1,phi1,p)

T2 = 120  // °F
Twb2 = 85  // °F
phi12 = phi_Tw(T2,w2,p)
w2 = w_TTwb(T2,Twb2,p)
AV2 = 3000  // ft³/min
va2 = va_Tw(T2,w2,p)
```

```
mdota2 = AV2 / va2
mdota3 = mdota1 + mdota2
mdota3 * w3 = mdota1 * w1 + mdota2 * w2

mdota3 * h3 = mdota1 * h1 + mdota2 * h2
h1 = ha_Tw(T1,w1)
h2 = ha_Tw(T2,w2)
h3 = ha_Tw(T3,w3)

0 = mdota1 * s1 + mdota2 * s2 - mdota3 * s3 + sigmadot
s1 = sa_Tw(T1,w1,p)
s2 = sa_Tw(T2,w2,p)
s3 = sa_Tw(T3,w3,p)

Edotd = To * sigmadot
To = 80 + 460
```

IT Results

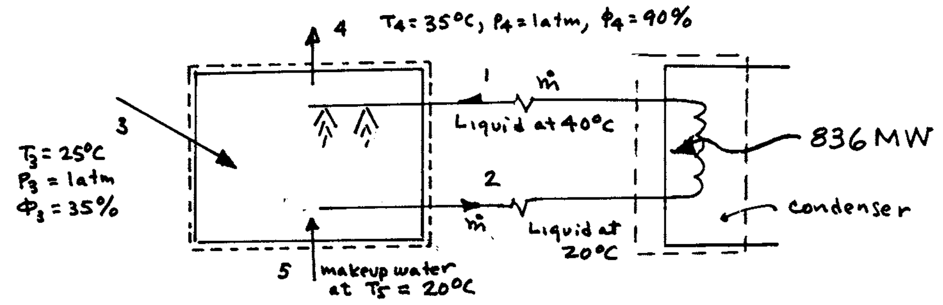
ω_1 = 0.01091 lb(v)/lb(a)
ω_2 = 0.01799 lb(v)/lb(a)
ϕ_2 = 0.2441 (24.41%)
ω_3 = 0.01445 lb(v)/lb(a)
$\dot{m}_{a1}$ = 200.1 lb(a)/min
$\dot{m}_{a2}$ = 199.6 lb(a)/min
$\dot{m}_{a3}$ = 399.7 lb(a)/min
h1 = 31.13 Btu/lb(a)
h2 = 48.8 Btu/lb(a)
h3 = 39.95 Btu/lb(a)
$\dot{\sigma}_{cv}$ = 0.08154 Btu/min·°R
T3 = 100.1°F
$\dot{E}_d$ = 45.26 Btu/min

PROBLEM 12.97

KNOWN: Operating data are provided for a cooling tower that services a power plant.

FIND: Determine the mass flow rates of the entering atmospheric air and the make up water.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volumes shown in the figure are at steady state with negligible $\dot{Q}_{cv}$, $\dot{W}_{cv}$, and kinetic/potential energy effects. (2) For the streams 1, 2, 5, $h \approx h_f(T)$.

ANALYSIS: The mass flow rates at 1, 2 are equal, $\dot{m}$. An energy balance on the cooling water side of the condenser is $\dot{Q}_{cv} = \dot{m}[h_1 - h_2]$, or

$$\dot{m} = \frac{\dot{Q}_{cv}}{h_1 - h_2} = \frac{836 \times 10^3 \text{ kJ/s}}{(167.6 - 84) \text{ kJ/kg}} = 10^4 \text{ kg/s}$$

Next, for a control volume enclosing the cooling tower, mass rate balances give $\dot{m}_{a3} = \dot{m}_{a4} \equiv \dot{m}_a$ and

$$\dot{m}_1 + \dot{m}_{v3} + \dot{m}_5 = \dot{m}_2 + \dot{m}_{v4}$$

Since $\dot{m}_1 = \dot{m}_2$

$$\dot{m}_5 = \dot{m}_{v4} - \dot{m}_{v3} = \dot{m}_a(\omega_4 - \omega_3) \qquad (1)$$

The mass flow rate of the entering atmospheric air is

$$\dot{m}_3 = \dot{m}_a + \dot{m}_{v3} = \dot{m}_a(1 + \omega_3) \qquad (2)$$

From Eqs. (1) and (2) it is clear that $\dot{m}_a$, ω_3, and ω_4 must be evaluated.

To find ω_3 and ω_4, write $p_{v3} = \phi_3\, p_g(T_3) = (0.35)(0.03169) = 0.01109$ bars, $p_{v4} = \phi_4\, p_g(T_4) = (0.9)(0.05628) = 0.05065$ bar. Then

$$\omega_3 = 0.622\left[\frac{0.01109}{1.01325 - 0.01109}\right] = 0.00688 \frac{\text{kg(v)}}{\text{kg(a)}}, \quad \omega_4 = 0.622\left[\frac{0.05065}{1.01325 - 0.05065}\right] = 0.0327 \frac{\text{kg(v)}}{\text{kg(a)}}$$

The mass flow rate $\dot{m}_a$ can be found from an energy rate balance which at steady state reduces to

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + [\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}] + \dot{m}_1 h_1 + \dot{m}_5 h_5 - \dot{m}_2 h_2 - [\dot{m}_a h_{a4} + \dot{m}_{v4} h_{v4}]$$

Introducing Eq. (1)

$$0 = \dot{m}_a[h_{a3} + \omega_3 h_{g3}] + \dot{m}_1[h_f(T_1) - h_f(T_2)] + \dot{m}_a(\omega_4 - \omega_3) h_f(T_5) - \dot{m}_a[h_{a4} + \omega_4 h_{g4}]$$

Solving for $\dot{m}_a$

$$\dot{m}_a = \frac{\dot{m}_1[h_f(T_2) - h_f(T_1)]}{(h_{a3} - h_{a4}) + \omega_3 h_{g3} - \omega_4 h_{g4} + (\omega_4 - \omega_3) h_f(T_5)}$$

$$= \frac{10{,}000[84 - 167.6]}{[298.18 - 308.23] + (0.00688)(2547.2) - (0.0327)(2565.3) + (0.0327 - 0.00688)(83.96)} = 11{,}261 \text{ kg(a)/s}$$

With ω_3, ω_4, and $\dot{m}_a$, Eq. (1) gives

$$\dot{m}_5 = \dot{m}_a(\omega_4 - \omega_3) = 11{,}261(0.0327 - 0.00688) = 290.8 \text{ kg/s}$$ ← Makeup

And Eq. (2) gives

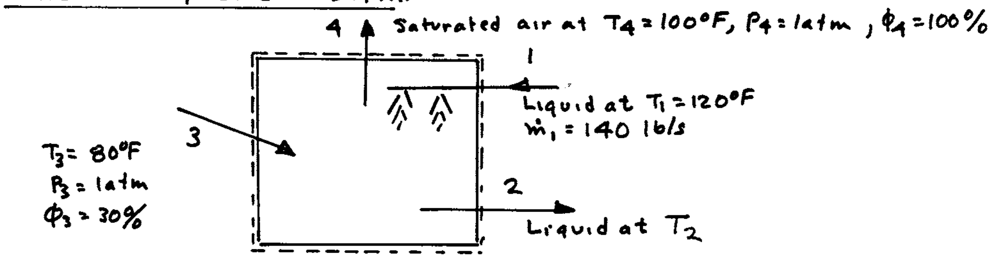
$$\dot{m}_3 = \dot{m}_a(1 + \omega_3) = 11{,}261[1 + .00688] = 11{,}338.5 \text{ kg/s}$$ ← Atm. air

PROBLEM 12.98

KNOWN: Steady state operating data are provided for a cooling tower.

FIND: Plot the mass flow rate of dry air versus the temperature at which cooled water exits the tower.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) For streams 1 and 2 $h = h_f(T)$.

ANALYSIS: At steady state, mass rate balances give $\dot{m}_{a3} = \dot{m}_{a4} \equiv \dot{m}_a$ and

$$\dot{m}_1 + \dot{m}_{v3} = \dot{m}_2 + \dot{m}_{v4} \Rightarrow \dot{m}_2 = \dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4) \qquad (1)$$

To find ω_3 and ω_4, write $p_{v3} = \phi_3 p_g(T_3) = (0.30)(0.5073) = 0.15219$ lbf/in², $p_{v4} = p_g(100°F) = 0.9503$ lbf/in². Then

$$\omega_3 = 0.622\left[\frac{0.15219}{14.696 - 0.15219}\right] = 0.0065 \frac{lb(v)}{lb(a)}, \quad \omega_4 = 0.622\left[\frac{0.9503}{14.696 - 0.9503}\right] = 0.0430 \frac{lb(v)}{lb(a)}$$

The mass flow rate $\dot{m}_a$ can be found from an energy rate balance which at steady state reduces to

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + [\dot{m}_a h_{a3} + \dot{m}_{v3} h_{v3}] + \dot{m}_1 h_1 - \dot{m}_2 h_2 - [\dot{m}_a h_{a4} + \dot{m}_{v4} h_{v4}]$$

Introducing Eq. (1)

$$0 = \dot{m}_a[h_{a3} + \omega_3 h_{g3}] + \dot{m}_1 h_f(T_1) - [\dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4)]h_f(T_2) - \dot{m}_a(h_{a4} + \omega_4 h_{g4})$$

Solving for $\dot{m}_a$

$$\dot{m}_a = \frac{\dot{m}_1[h_f(T_1) - h_f(T_2)]}{(h_{a4} - h_{a3}) + \omega_4 h_{g4} - \omega_3 h_{g3} + (\omega_3 - \omega_4)h_f(T_2)}$$

$$= \frac{140[88 - h_f(T_2)]}{0.24(100-80) + (0.043)(1105) - (0.0065)(1096.4) + (0.043 - 0.0065)h_f(T_2)}$$

$$= \frac{140[88 - h_f(T_2)]}{45.188 + (0.0365)h_f(T_2)}\left(\frac{lb}{s}\right)\left|\frac{3600 s}{1 h}\right| \qquad (1)$$

Sample Calculation: When $T_2 = 80°F$, $h_f(T_2) = 48.09$ Btu/lb. Eq. (1) gives $\dot{m}_a = 4.28 \times 10^5$ lb/h.

the data for the required plot are obtained using IT, as follows:

PROBLEM 12.98 (Cont'd.)

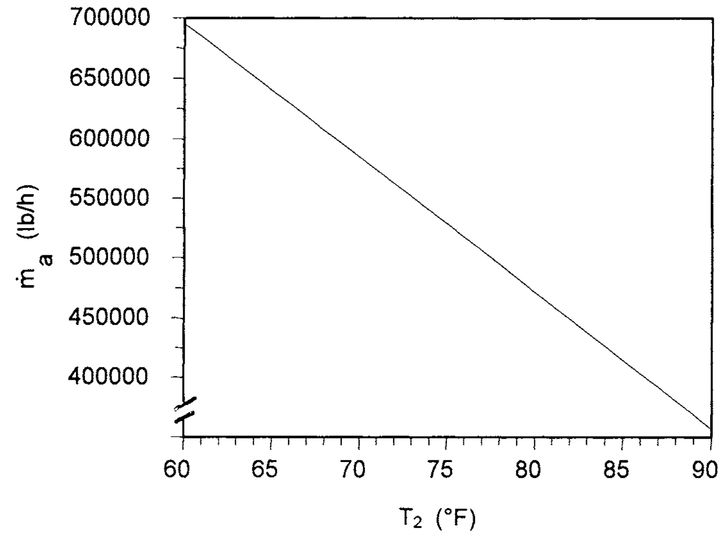
IT Code

```
T1 = 120  // °F
mdot1 = 140  // lb/s
T2 = 80  // °F
T3 = 80  // °F
p3 = 14.696  // lbf/in²
phi3 = 0.3
T4 = 100  // °F
p4 = p3
phi4 = 1.0

h1 = hsat_Px("Water/Steam", psat1, 0)
psat1 = Psat_T("Water/Steam", T1)
h2 = hsat_Px("Water/Steam", psat2, 0)
psat2 = Psat_T("Water/Steam", T2)
w3 = w_Tphi(T3,phi3,p3)
h3 = ha_Tw(T3,w3)
w4 = w_Tphi(T4,phi4,p4)
h4 = ha_Tw(T4,w4)

mdota3 = mdota4
mdot1 + w3 * mdota3 = mdot2 + w4 * mdota4
0 = mdota3 * h3 + mdot1 * h1 - mdota4 * h4 -
mdot2 * h2
mdota = mdota3 * 3600  // lb/h
```

IT Results for T_2 = 80°F

ω_3 = 0.006503 lb(v)/lb(a)
ω_4 = 0.04265 lb(v)/lb(a)
$\dot{m}_2$ = 135.3 lb/s
$\dot{m}_a$ = 4.72E5 lb/h

PLOT:



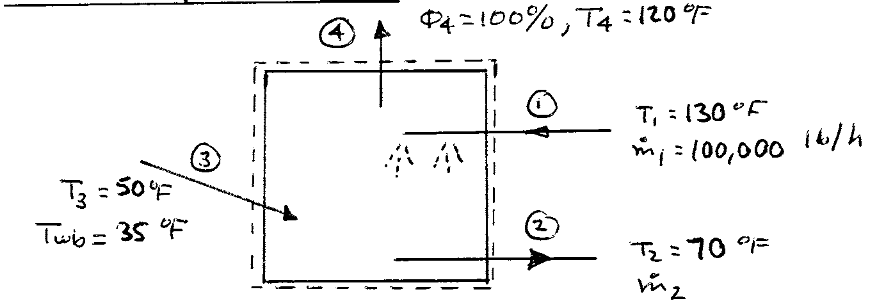
To achieve more cooling (lower liquid temperature exiting the tower), requires increased air flow, as expected.

PROBLEM 12.99*

KNOWN: Operating data are provided for a cooling tower at steady state,

FIND: Determine the mass flow rate of the cooled water stream exiting.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state with $\dot Q_{cv} = \dot W_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) For streams 1 and 2, $h = h_f(T)$. (3) The pressure is constant at 1 atm.

ANALYSIS: Conservation of mass at steady state:

(air) $\dot m_{a3} = \dot m_{a4} \equiv \dot m_a$

(water) $\dot m_1 + \dot m_{v3} = \dot m_2 + \dot m_{v4}$

Combining gives, $\dot m_2 = \dot m_1 + \dot m_a(\omega_3 - \omega_4)$.

At steady state, an energy rate balance reduces to

$$0 = \dot m_1 h_1 - \dot m_2 h_2 + \dot m_a\left[(h_a + \omega h_v)_3 - (h_a + \omega h_v)_4\right]$$

Collecting results, and using $h_1 = h_f(T_1)$, $h_2 = h_f(T_2)$,

$$0 = \dot m_1\left[h_f(T_1) - h_f(T_2)\right] + \dot m_a\left[h_{a3} - h_{a4} + \omega_3 h_{g3} - \omega_4 h_{g4} - (\omega_3 - \omega_4) h_f(T_2)\right]$$

Solving

$$\dot m_a = \frac{\dot m_1\left(h_f(T_2) - h_f(T_1)\right)}{\underbrace{h_{a3} - h_{a4}}_{c_{pa}(T_3 - T_4)} + \omega_3 h_{g3} - \omega_4 h_{g4} - (\omega_3 - \omega_4) h_f(T_2)}$$

From Fig. A-9E, $\omega_3 = 0.0010$ lb(v)/lb(a). State 4 is off the chart, so with $p_g(T_4)$

$$\omega_4 = 0.622\left[\frac{1.695}{14.7 - 1.695}\right] = 0.1303$$

Then, with steam table data

$$\dot m_a = \frac{(100{,}000)(38.09 - 97.98)}{0.24[50 - 120] + 0.0010(1083.3) - 0.1303(1113.5) + 0.1293(38.09)}$$

$$= 38{,}420 \text{ lb(a)/h}$$

Accordingly

$$\dot m_2 = \dot m_1 + \dot m_a(\omega_3 - \omega_4)$$

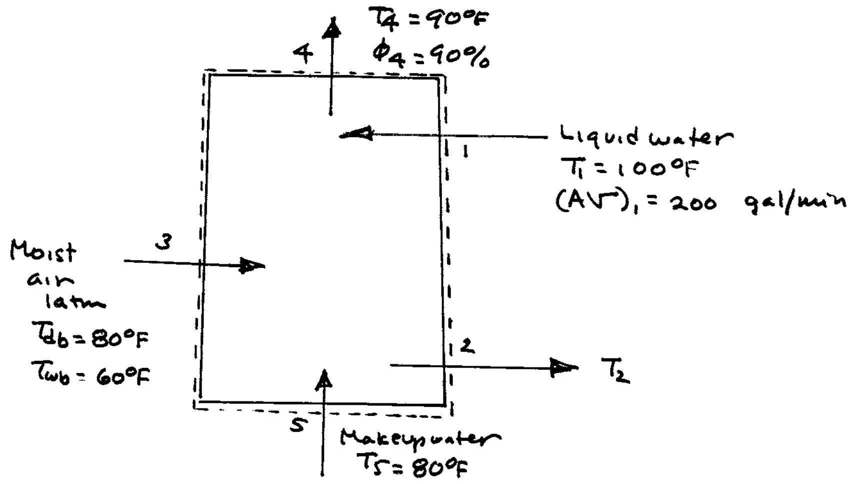
$$= 100{,}000 + 38{,}420(0.0010 - 0.1303)$$

$$= 95{,}032 \ \frac{\text{lb}}{\text{h}} \quad \longleftarrow$$

PROBLEM 12.100

KNOWN: Steady state operating data are provided for a cooling tower.

FIND: Plot the mass flow rates of the dry air and makeup water versus T_2.



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv}$, $\dot{W}_{cv}$, and kinetic/potential energy effects are negligible. (3) Each liquid stream is regarded as a saturated liquid at the corresponding temperature. (4) The moist air streams are modeled as ideal gas mixtures at 1atm. (5) The temperature T_2 is treated as a parameter, ranging $80°F \leq T_2 < 100°F$.

ANALYSIS: Following the analysis of Example 12.17, the basic relations are

$$\dot{m}_5 = \dot{m}_a(\omega_4 - \omega_3) \tag{1}$$

$$\dot{m}_a = \frac{\dot{m}_1[h_{f1} - h_{f2}]}{(h_a + \omega h_g)_4 - (h_a + \omega h_g)_3 - (\omega_4 - \omega_3)h_{f5}} \tag{2}$$

Using T_{wb}, T_{db} at 3, the psychrometric chart gives $\omega_3 \approx .0066$, $(h_a + \omega h_g)_3 \approx 26.4$ Btu/lb(a). With T_4, ϕ_4 the chart gives $\omega_4 \approx 0.028$. The value of $(h_a + \omega h_g)_4 = (0.24)(90) + (0.028)(1100.7) = 52.4$ Btu/lb(a).

With $(AV)_1$ and $v_f(100°F)$,

$$\dot{m}_1 = \frac{\left(200 \frac{\text{gal}}{\text{min}}\right)\left(0.13368 \frac{\text{ft}^3}{\text{gal}}\right)}{0.01613 \text{ ft}^3/\text{lb}} = 1657.5 \frac{\text{lb}}{\text{min}}$$

With these data, Eq. (1) becomes $\dot{m}_5 = \dot{m}_a(0.028 - 0.0066) = 0.0214\,\dot{m}_a$ (3)
and Eq. (2) becomes

$$\dot{m}_a = \frac{\left(1657.5 \frac{\text{lb}}{\text{min}}\right)[68.05 - h_{f2}]\ \text{Btu/lb}}{(52.4 - 26.4) - (0.028 - 0.0066)(48.1)} = 66.38[68.05 - h_{f2}] \tag{4}$$

Sample Calculation: $T_2 = 80°F$. Then, $h_{f2} = 48.09$ Btu/lb. Eq. (4) gives $\dot{m}_a = 66.38(68.05 - 48.09) = 1324.9$ lb(a)/min. Eq. (3) gives $\dot{m}_5 = 28.35$ lb/min.

Data for the required plots are obtained using IT, as follows:

PROBLEM 12.100 (Cont'd.)

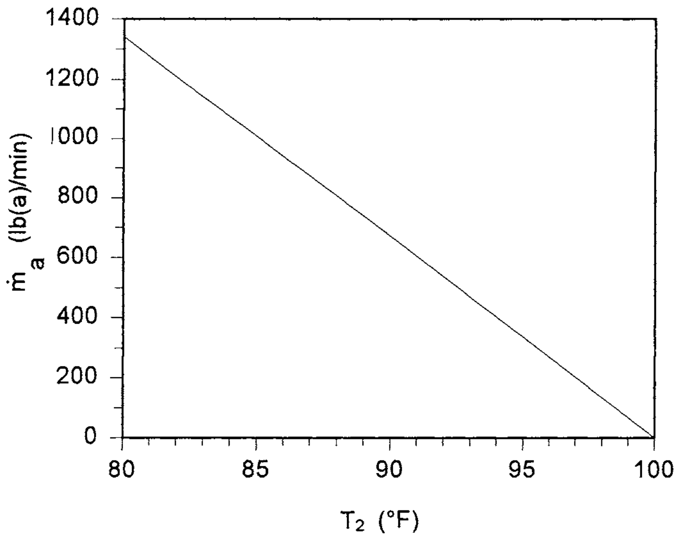
IT Code

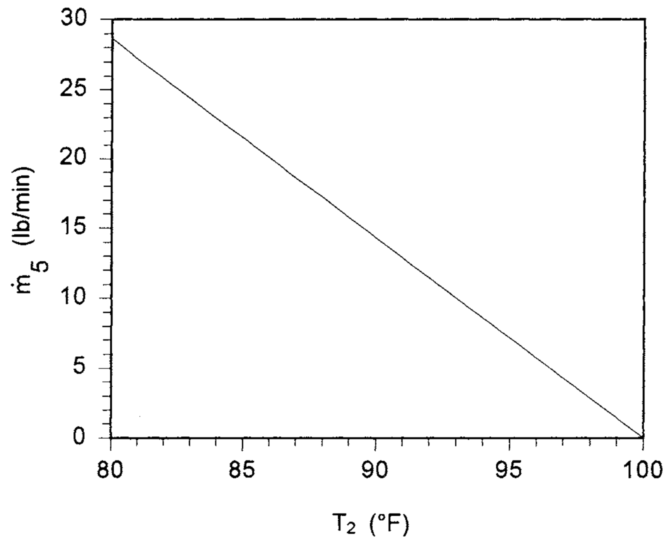
```
T1 = 100  // °F
AV1 = 200  // gal/min
T2 = 80  // °F
T3 = 80  // °F
Twb3 = 60  // °F
T4 = 90  // °F
phi4 = 0.9
T5 = 80  // °F
p = 14.696  // lbf/in²

h1 = hsat_Px("Water/Steam", psat1, 0)
psat1 = Psat_T("Water/Steam", T1)
v1 = vsat_Px("Water/Steam", psat1, 0)
h2 = hsat_Px("Water/Steam", psat2, 0)
psat2 = Psat_T("Water/Steam", T2)
w3 = w_TTwb(T3,Twb3,p)
h3 = ha_Tw(T3,w3)
w4 = w_Tphi(T4,phi4,p)
h4 = ha_Tw(T4,w4)
h5 = hsat_Px("Water/Steam", psat5, 0)
psat5 = Psat_T("Water/Steam", T5)

mdot5 = mdota * (w4 - w3)
mdot1 = AV1 * 0.13368 / v1  // lb/min
0 = mdota * (h3 - h4) + mdot5 * h5 + mdot1 * (h1 - h2)
```

IT Results for T_2 = 80°F

ω_3 = 0.006453 lb(v)/lb(a)
ω_4 = 0.02778 lb(v)/lb(a)
T_2 = 80°F
$\dot{m}_a$ = 1343 lb(a)/min
$\dot{m}_5$ = 28.64 lb/min

PLOTS:





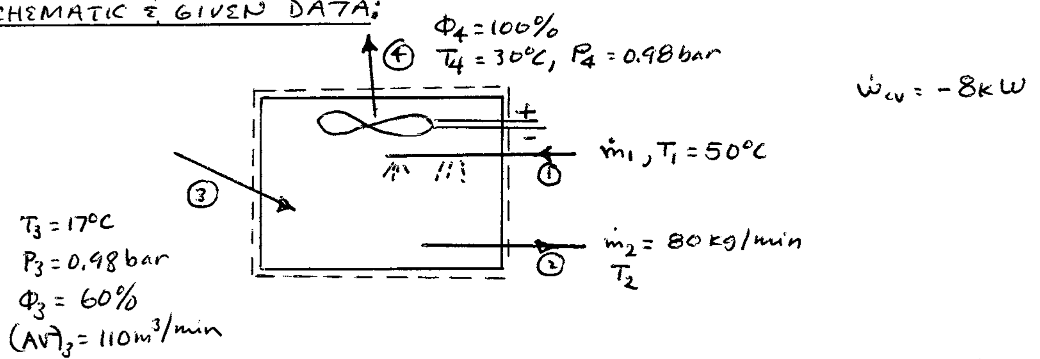
From the plots, we see that to achieve lower liquid temperature exiting the tower, we need increased air flow. Also, since there is more evaporation, greater make-up water flow is required. Both of these results are as expected.

PROBLEM 12.101

KNOWN: Steady state operating data are provided for a forced draft cooling tower.

FIND: Determine the mass flow rate of the entering liquid stream and the temperature of the cooled liquid stream exiting

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state with $\dot{Q}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) For the streams at 1 and 2, $h = h_f(T)$.

ANALYSIS: Mass rate balances at steady state give, $\dot{m}_{a3} = \dot{m}_{a4} \equiv \dot{m}_a$ and

$$\dot{m}_1 + \dot{m}_{v3} = \dot{m}_2 + \dot{m}_{v4} \Rightarrow \dot{m}_1 = \dot{m}_2 + \dot{m}_a(\omega_4 - \omega_3).$$

Calculating

$$\omega_3 = 0.622\left[\frac{(0.6)(0.01938)}{0.98-(0.6)(0.01938)}\right] = 0.0075\ \frac{\text{kg(v)}}{\text{kg(a)}}, \quad \omega_4 = 0.622\left[\frac{0.04246}{0.98-0.04246}\right] = 0.0282$$

$$\dot{m}_a = \frac{(AV)_3}{v_{a3}} = \frac{(AV)_3}{\left(\frac{\bar{R}}{M_a}\right)\left(\frac{T_3}{p_{a3}}\right)} = \frac{(110\ \text{m}^3/\text{min})(0.968372\times10^5\ \text{N/m}^2)}{\left(\frac{8314}{29}\ \frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)(290\ \text{K})} = 128.12\ \frac{\text{kg(a)}}{\text{min}}$$

Thus

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_a(\omega_4 - \omega_3) = 80\ \frac{\text{kg}}{\text{min}} + 128.12\ \frac{\text{kg(a)}}{\text{min}}\left(0.0282 - 0.0075\right)\frac{\text{kg}}{\text{kg(a)}}$$

$$= 82.65\ \frac{\text{kg}}{\text{min}} \quad \longleftarrow$$

An energy rate balance reduces at steady state to give

$$0 = -\dot{W}_{cv} + \dot{m}_a\left[(h_a + \omega h_v)_3 - (h_a + \omega h_v)_4\right] + \dot{m}_1 h_1 - \dot{m}_2 h_2$$

Or, with $h = h_f(T)$ at 1, 2

$$h_f(T_2) = \frac{-\dot{W}_{cv} + \dot{m}_a\left[(h_a + \omega h_g)_3 - (h_a + \omega h_g)_4\right] + \dot{m}_1 h_f(T_1)}{\dot{m}_2}$$

$$= \frac{-\dot{W}_{cv} + \dot{m}_a\left[c_{pa}(T_3 - T_4) + \omega_3 h_{g3} - \omega_4 h_{g4}\right] + \dot{m}_1 h_f(T_1)}{\dot{m}_2}$$

$$= \frac{-\left(-8\ \frac{\text{kJ}}{\text{s}}\right)\left(\frac{60\ \text{s}}{\text{min}}\right) + 128.12\left[1.005(17-30) + 0.0075(2532.6) - 0.0282(2556.3)\right] + 82.65(209.3)}{80\ \text{kg/min}}$$

$$= 116.3\ \frac{\text{kJ}}{\text{kg}}$$

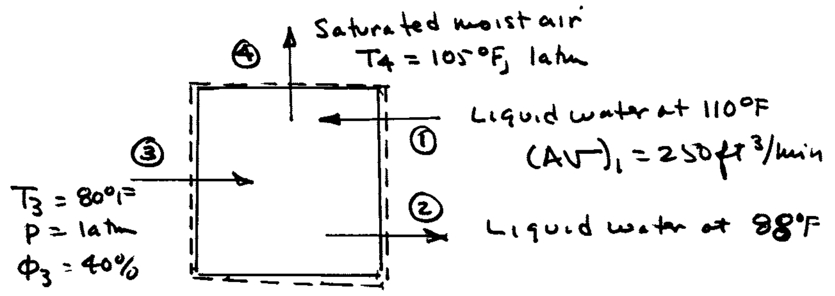
Interpolating in Table A-2 gives $T_2 \approx 28°C$. $\longleftarrow$

PROBLEM 12.102

KNOWN: Steady state operating data are provided for a cooling tower.

FIND: Determine (a) the mass flow rates of the dry air and cooled water, (b) the tower rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv}$, $\dot{W}_{cv}$, and kinetic/potential energy effects are negligible. (3) Each liquid stream is regarded as a saturated liquid at the corresponding temperature. (4) The moist air streams are treated as ideal gas mixtures. (5). For the exergy reference environment, $T_0 = 537°R$.

ANALYSIS: (a) Mass rate balances at steady state read $\dot{m}_{a3} = \dot{m}_{a4} \equiv \dot{m}_a$,

$$\dot{m}_2 = \dot{m}_1 + \dot{m}_{v3} - \dot{m}_{v4} = \dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4) \qquad (1)$$

An energy rate balance reads

$$0 = \dot{m}_1 h_{f1} - \dot{m}_2 h_{f2} + \dot{m}_a[h_a + \omega h_g]_3 - \dot{m}_a[h_a + \omega h_g]_4$$

Or, with Eq(1)

$$0 = \dot{m}_1 h_{f1} - [\dot{m}_1 + \dot{m}_a(\omega_3 - \omega_4)]h_{f2} + \dot{m}_a[h_a + \omega h_g]_3 - \dot{m}_a[h_a + \omega h_g]_4$$

or, solving for $\dot{m}_a$

$$\dot{m}_a = \frac{\dot{m}_1(h_{f1} - h_{f2})}{[h_a + \omega h_g]_4 - [h_a + \omega h_g]_3 + (\omega_3 - \omega_4)h_{f2}}$$

$$= \frac{\dot{m}_1(h_{f1} - h_{f2})}{c_{pa}(T_4 - T_3) + \omega_4 h_{g4} - \omega_3 h_{g3} + (\omega_3 - \omega_4)h_{f2}} \qquad (2)$$

where

$$\omega_3 = \frac{0.622(\phi_3 p_{g3})}{p - \phi_3 p_{g3}} = \frac{0.622(0.4)(0.5073)}{14.7 - (0.4)(0.5073)} = 0.00871$$

$$\omega_4 = \frac{0.622(\phi_4 p_{g4})}{p - \phi_4 p_{g4}} = \frac{(0.622)(1)(1.103)}{14.7 - 1.103} = 0.05046$$

$$\dot{m}_1 = \frac{(AV)_1}{v_{f1}} = \frac{250 \text{ ft}^3/\text{min}}{.01617 \text{ ft}^3/\text{lb}} = 15461 \frac{\text{lb}}{\text{min}}$$

Inserting values in Eq.(2)

$$\dot{m}_a = \frac{15461[78.02 - 56.07]}{0.24[105 - 80] + (0.05046)(1107.2) - 0.00871(1096.4) + (-0.04175)(56.07)}$$

$$= 6790 \frac{\text{lb(a)}}{\text{min}} \longleftarrow \dot{m}_a$$

PROBLEM 12.102 (Continued)

Then, with Eq. (1)

$$\dot{m}_2 = 15461 + 6790(.00871 - 0.05046)$$

$$= 15,178 \ \frac{\text{lb}}{\text{min}} \longleftarrow\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!- \ \dot{m}_2$$

(b) The rate of exergy destruction can be evaluated in terms of the entropy production rate: $\dot{E}_d = T_0 \dot{\sigma}_{cv}$, where from an entropy rate balance

$$\dot{\sigma}_{cv} = \dot{m}_2 s_f(T_2) - \dot{m}_1 s_f(T_1) + \dot{m}_a\left[s_a(T_4, \overset{p_4 - p_{g4}}{p_{a4}}) + \omega_4 s_g(T_4)\right] - \dot{m}_a\left[s_a(T_3, p_{a3}) + \omega_3 \underbrace{s_v(T_3, p_{v3})}_{= s_g(T_3) - R\ln\phi_3 \text{ (Sec. 12.5.2)}}\right]$$

Steam Table Data: (Btu/lb·°R)

$s_f(T_1) = 0.1473$ $\qquad$ $s_g(T_3) = 2.0356$, $s_v(T_3, p_{v3}) = s_g(T_3) - R\ln\phi_3$

$s_f(T_2) = 0.10801$ $\qquad$ $s_g(T_4) = 1.9697$ $\qquad = 2.0356 - \frac{1.986}{18}\ln 0.4$

$\qquad\qquad\qquad\qquad\qquad\qquad = 2.1367$

Also

$$s_a(T_4, p_{a4}) - s_a(T_3, p_{a3}) = c_p \ln\frac{T_4}{T_3} - \frac{\bar{R}}{M_a}\ln\frac{p_{a4}}{p_{a3}} = 0.24\ln\left(\frac{565}{540}\right) - \frac{1.986}{28.97}\ln\frac{[14.7 - 1.103]}{[14.7 - (0.4)(.5073)]}$$

$$= 0.01526 \text{ Btu/lb·°R}$$

Then

① $$\dot{\sigma}_{cv} = 15,178(0.10801) - 15,461(0.1473) + 6790\left[0.01526 + 0.05046(1.9697) - 0.00871(2.1367)\right]$$

$$= 14.01 \text{ Btu/min·°R}$$

Finally,

$$\dot{E}_d = T_0 \dot{\sigma}_{cv} = (537°R)\left(14.01 \frac{\text{Btu}}{\text{min·°R}}\right)$$

$$= 7523 \ \frac{\text{Btu}}{\text{min}}$$

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1. Entropy balance calculations tend to be sensitive to roundoff.

CHAPTER THIRTEEN

REACTING MIXTURES AND COMBUSTION

PROBLEM 13.1

KNOWN: A vessel contains a mixture of 60% O_2 and 40% CO on a mass basis.

FIND: Determine the percent excess or per cent deficiency of O_2, as appropriate.

ANALYSIS: For complete combustion of CO with the minimum amount of O_2, $CO + \frac{1}{2} O_2 \rightarrow CO_2$. Consider a typical unit mass of mixture, then

	mf_i	m_i	M_i	$n_i = m_i/M_i$
O_2	0.6	0.6	32	0.01875
CO	0.4	0.4	28.01	0.01428
				0.03303

$$\left(\frac{n_{O_2}}{n_{CO_2}}\right)_{act} = 1.31 \qquad \left(\frac{n_{O_2}}{n_{CO_2}}\right)_{theo} = 0.5$$

$$\%\,\text{excess}\,O_2 = \left(\frac{1.31 - 0.5}{0.5}\right)(100) = 162$$

PROBLEM 13.2

KNOWN: A typical automobile burns gasoline.

FIND: Estimate the amount of CO_2 produced, in lb, per gallon of gasoline burned. Also, estimate the amount of CO_2 produced, in lb, annually.

ASSUMPTIONS: (1) For the combustion reaction, gasoline is taken as $C_8H_{18}(\ell)$, combustion is complete, N_2 is inert, and 3.76 moles of N_2 accompany each mole of O_2. (2) The automobile burns gas at the rate of 25 miles/gal and travels 12,000 miles per year.

ANALYSIS: Complete combustion of $C_8H_{18}(\ell)$ with the theoretical amount of air is described by

$$C_8H_{18} + 12.5\,[O_2 + 3.76\,N_2] \rightarrow 8\,CO_2 + 9\,H_2O + 47\,N_2$$

The amount of CO_2 produced is then

$$\left(\frac{8\ \text{lbmol}\,(CO_2)}{1\ \text{lbmol}\,(\text{fuel})}\right)\left(\frac{44.01\ \text{lb/lbmol}}{114.22\ \text{lb/lbmol}}\right) = 3.08\ \frac{\text{lb}\,(CO_2)}{\text{lb}\,(\text{fuel})}$$

① The specific gravity of gasoline is in the range 0.72–0.78, giving a density in the range 45–49 lb/ft³. Taking $\rho = 47$ lb/ft³, the mass of 1 gallon is

$$\text{mass(fuel)} = \left(0.13368\ \frac{\text{ft}^3}{\text{gal}}\right)\left(47\ \frac{\text{lb}}{\text{ft}^3}\right) = 6.28\ \frac{\text{lb(fuel)}}{\text{gal}}$$

Accordingly, the mass of CO_2 produced per gallon of fuel burned is

$$\text{Mass of } CO_2 = 3.08\ \frac{\text{lb}\,(CO_2)}{\text{lb}\,(\text{fuel})} \cdot 6.28\ \frac{\text{lb(fuel)}}{\text{gal}} = 19.3\ \frac{\text{lb}\,(CO_2)}{\text{gal}} \longleftarrow$$

With assumptions (2)

② $$\text{Mass of } CO_2 \text{ per year} = \left(19.3\ \frac{\text{lb}(CO_2)}{\text{gal}}\right)\left(12{,}000\ \frac{\text{miles}}{\text{year}}\right)\left(\frac{\text{gal}}{25\ \text{miles}}\right)$$

$$= 9264\ \text{lb}(CO_2)/\text{yr} \quad \left(= 4.63\ \frac{\text{tons}}{\text{yr}}\right) \longleftarrow$$

---

1. See, for example, Table D.4 of Internal Combustion Engine Fundamentals, by J.B. Heywood, McGraw-Hill, New York, 1988

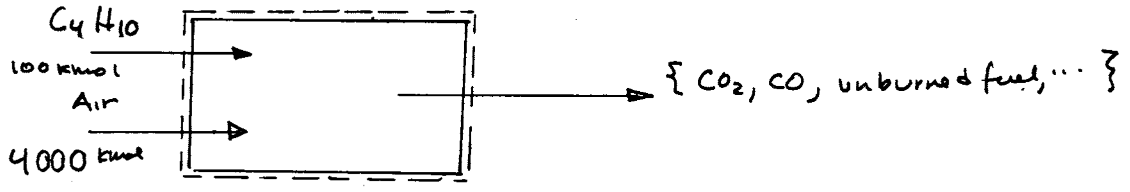
2. The accumulation of "greenhouse gases" in the atmosphere, including CO_2, CH_4, ozone, and chlorofluorocarbons is thought by many to contribute to global warming: the rise in the earth's temperature.

PROBLEM 13.3*

KNOWN: 100 kmole of C_4H_{10} and 4000 kmol of air enter a furnace. CO_2, CO and unburned fuel are in the products of combustion.

FIND: Determine the percent excess or percent deficiency of air, as appropriate.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: 3.76 kmol of N_2 accompany each kmol of O_2 in air. N_2 is inert.

ANALYSIS: The data provided are for actual operation. Thus

$$\overline{AF} = \frac{4000}{100} = 40.00 \ \frac{\text{kmol (a)}}{\text{kmol (fuel)}}$$

The balance equation for complete combustion with the theoretical amount of air is

$$C_4H_{10} + 6.5\,(O_2 + 3.76\,N_2) \rightarrow 4\,CO_2 + 5\,H_2O + 24.44\,N_2$$

The theoretical air/fuel ratio is

$$(\overline{AF})_{theo} = \frac{6.5(4.76)}{1} = 30.94 \ \frac{\text{kmol (a)}}{\text{kmol (fuel)}}$$

Accordingly

$$\%\,\text{excess} = \left(\frac{40.00 - 30.94}{30.94}\right)(100) = 29.3\,\% \quad \longleftarrow$$

PROBLEM 13.4

KNOWN: Three cases where C_3H_8 is burned with air.

FIND: Obtain the balanced chemical reaction for complete combustion.

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: (a) Complete combustion with the theoretical amount of air.

$$C_3H_8 + 5(O_2+3.76N_2) \rightarrow 3\,CO_2 + 4\,H_2O + 18.8\,N_2$$

(b) Complete combustion with 20% excess air.

$$C_3H_8 + (5)(1.2)(O_2+3.76N_2) \rightarrow 3CO_2 + 4\,H_2O + \alpha\,O_2 + (5)(1.2)(3.76\,N_2)$$

$$O:\ (6)(2) = 6+4+2\alpha \Rightarrow \alpha = 1$$

$$\therefore\ C_3H_8 + 6(O_2+3.76N_2) \rightarrow 3CO_2 + 4H_2O + 1\,O_2 + 22.56\,N_2$$

(c) Combustion with 20% excess air. 90% of the fuel is burned.

$$C_3H_8 + 6(O_2+3.76\,N_2) \rightarrow 0.1\,C_3H_8 + \alpha\,CO_2 + \beta\,H_2O + \gamma\,O_2 + 22.56\,N_2$$

$$C:\ 3 = 0.3+\alpha \Rightarrow \alpha = 2.7 \qquad O:\ 12 = 2(2.7)+(3.6)+2\gamma \Rightarrow \gamma = 1.5$$

$$H:\ 8 = 0.8+2\beta \Rightarrow \beta = 3.6$$

$$\therefore\ C_3H_8 + 6(O_2+3.76N_2) \rightarrow 0.1\,C_3H_8 + 2.7\,CO_2 + 3.6H_2O + 1.5\,O_2 + 22.56N_2$$

PROBLEM 13.5

KNOWN: Methane burns completely with the stoichometric amount of H_2O_2.

FIND: Determine the balanced reaction equation.

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air and N_2 is inert. (2) The amount of O_2 provided is the minimum amount for complete combustion.

ANALYSIS: On a per mole of fuel basis

$$CH_4 + \alpha\,H_2O_2 \rightarrow \beta\,CO_2 + \gamma\,H_2O$$

$$\left.\begin{array}{ll} C: & 1 = \beta \\ H: & 4+2\alpha = 2\gamma \\ O: & 2\alpha = 2\beta + \gamma \end{array}\right]\ \text{solving,}\ \begin{array}{l}\alpha = 4\\ \gamma = 6\end{array}$$

Accordingly, the balanced reaction equation is

$$CH_4 + 4\,H_2O_2 \rightarrow CO_2 + 6\,H_2O \qquad \longleftarrow$$

PROBLEM 13.6*

KNOWN: A fuel mixture with a specified molar analysis burns completely with 33 % excess air.

FIND: Determine (a) the balanced reaction equation, (b) the air-fuel ratio on a molar and a mass basis.

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: On the basis of 1 mole of fuel mixture, the reaction equation for complete combustion with the theoretical amount of air is

$$(0.4\,CH_3OH + 0.5\,C_2H_5OH + 0.1\,N_2) + \alpha(O_2 + 3.76\,N_2)$$
$$\rightarrow \beta\,CO_2 + \gamma\,H_2O + (3.76\alpha + 0.1)\,N_2$$

$C: 0.4 + (2)(0.5) = \beta, \quad \beta = 1.4 \qquad O: 0.4 + 0.5 + 2\alpha = 2(1.4) + 2.3, \quad \alpha = 2.1$

$H: (4)(0.4) + (6)(0.5) = 2\gamma, \quad \gamma = 2.3$

Thus

$$(0.4\,CH_3OH + 0.5\,C_2H_5OH + 0.1\,N_2) + 2.1(O_2 + 3.76\,N_2) \rightarrow 1.4\,CO_2 + 2.3\,H_2O$$
$$+ 7.996\,N_2$$

Thus, the reaction for complete combustion with 33 % excess air is

$$(0.4\,CH_3OH + 0.5\,C_2H_5OH + 0.1\,N_2) + (1.33)(2.1)(O_2 + 3.76\,N_2) \rightarrow 1.4\,CO_2 + 2.3\,H_2O$$
$$+ \alpha\,O_2 + [(2.793)(3.76) + 0.01]\,N_2$$

$O: \quad 0.4 + 0.5 + 2(2.793) = 2.8 + 2.3 + 2\alpha, \quad \alpha = 0.693$

Accordingly

$$(0.4\,CH_3OH + 0.5\,C_2H_5OH + 0.1\,N_2) + 2.793(O_2 + 3.76\,N_2) \rightarrow 1.4\,CO_2 + 2.3\,H_2O +$$
$$0.693\,O_2 + 10.60\,N_2 \qquad \leftarrow$$

(b) The air-fuel ratio is

$$\overline{AF} = \frac{(2.793)(4.76)}{1} = 13.29\ \frac{\text{kmol (air)}}{\text{kmol (fuel)}} \qquad \leftarrow$$

The fuel molecular weight is

$$M_f = (0.4)(32.04) + (0.5)(46.07) + (0.1)(28.01)$$
$$= 38.652$$

So

$$AF = (\overline{AF})\left(\frac{M_a}{M_f}\right) = (13.29)\left(\frac{28.97}{38.652}\right) = 9.961\ \frac{\text{kg (air)}}{\text{kg (fuel)}} \qquad \leftarrow$$

PROBLEM 13.7*

KNOWN: A specified fuel mixture burns completely with 40% excess air.

FIND: Determine the molar flow rate of the air, if the molar flow rate of the fuel is 0.5 kmol/h.

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: On the basis of 1 mole of fuel mixture, complete combustion with the theoretical amount of air is

$$[0.78\,CH_4 + 0.13\,C_2H_6 + 0.06\,C_3H_8 + 0.017\,C_4H_{10} + 0.013\,N_2] +$$

$$\alpha[O_2 + 3.76N_2] \rightarrow \beta CO_2 + \gamma H_2O + [3.76\alpha + 0.013]N_2$$

$$C:\ 0.78 + 2(0.13) + 3(0.06) + 4(0.017) = \beta, \quad \beta = 1.288$$

$$H:\ 4(0.78) + 6(0.13) + 8(0.06) + 10(0.017) = 2\gamma, \quad \gamma = 2.275$$

$$O:\ 2\alpha = 2(1.288) + 2.275, \quad \alpha = 4.851$$

Combustion with 40% excess air means that the molar flow rate is

$$\overline{AF} = \frac{1.4(4.851)(4.76)}{1} = 32.33\ \frac{kmol(air)}{kmol(fuel)}$$

Then, with $\dot{n}_{fuel} = 0.5\,kmol/h$

$$\dot{n}_{air} = 16.17\ kmol(air)/h \qquad \longleftarrow$$

PROBLEM 13.8

KNOWN: A fuel mixture having the molar analysis below burns completely with 150% of theoretical oxygen.

$$\{20\%\ CH_4,\ 40\%\ H_2,\ 40\%\ NH_3\}$$

FIND: Determine the balanced reaction equation.

ASSUMPTION: Combustion is complete and N_2 is inert.

ANALYSIS: The first step is to determine the theoretical amount of O_2. Based on one mole of fuel mixture

$$(0.2\,CH_4 + 0.4\,H_2 + 0.4\,NH_3) + \alpha O_2 \rightarrow \beta CO_2 + \gamma H_2O + \omega N_2$$

$$C:\ 0.2 = \beta$$

$$H:\ (0.2)(4) + (0.4)(2) + (0.4)(3) = 2\gamma, \quad \gamma = 1.4$$

$$O:\ 2\alpha = 2\beta + \gamma = 2(0.2) + 1.4 = 1.8, \quad \alpha = 0.9$$

Then, for 150% of theoretical O_2

$$(0.2\,CH_4 + 0.4\,H_2 + 0.4\,NH_3) + \underbrace{(1.5)(0.9)}_{1.35}O_2 \rightarrow 0.2\,O_2 + 1.4\,H_2O + 0.45\,O_2 + 0.2\,N_2 \qquad \longleftarrow$$

PROBLEM 13.9

KNOWN: Coal with a specified mass analysis burns completely with 120% of theoretical air.

FIND: Determine (a) the balanced reaction equation, (b) the amount of SO_2 produced, in kg per kg of coal.

ASSUMPTIONS: 3.76 kmol of N_2 accompany each kmol of O_2 in the combustion air. N_2 is inert.

ANALYSIS: (a) On the basis of one kg of coal mixture, the corresponding molar analysis is

$$C = \frac{0.7754}{12} = 0.06462 \qquad O = \frac{0.0772}{16} = 0.00483$$

$$H = \frac{0.0428}{1} = 0.04280 \qquad N = \frac{0.0134}{14} = 0.00096$$

$$S = \frac{0.0146}{32} = 0.00046 \qquad ASH = \text{balance}$$

Complete combustion with the theoretical amount of air is

$$[0.06462\,C + 0.04280\,H + 0.00046\,S + 0.00483\,O + 0.00096\,N] + \alpha[O_2 + 3.76N_2] \rightarrow \beta CO_2 + \gamma H_2O + \omega SO_2 + \delta N_2$$

C: $0.06462 = \beta$

H: $0.04280 = 2\gamma, \; \gamma = 0.02140$

S: $0.00046 = \omega$

O: $0.00483 + 2\alpha = 2(.06462) + 0.02140 + 2(0.00046), \; \alpha = 0.07337$

For comple combustion with 120% of theoretical air

$$[0.06462\,C + 0.04250\,H + 0.00046\,S + 0.00487\,O + 0.00096\,N] + 0.08804[O_2 + 3.76N_2] \rightarrow 0.06462\,CO_2 + 0.02140\,H_2O + 0.00046\,SO_2 + \lambda O_2 + \delta N_2$$ (a)

O: $0.00483 + 2(0.08804) = 2(0.06462) + 0.02140 + 2(0.00046) + 2\lambda, \; \lambda = 0.01468$

N: $0.00096 + (0.08804)(2)(3.76) = 2\delta, \; \delta = 0.33151$

(b) For each 1 kg of coal burned, 0.00046 kmol of SO_2 is formed. Thus

$$\begin{bmatrix} \text{mass of } SO_2 \text{ formed} \\ \text{per kg of coal} \\ \text{burned} \end{bmatrix} = 0.00046 \text{ kmol} \left[64.06 \frac{kg(SO_2)}{kmol(SO_2)}\right]$$

$$= 0.029 \frac{kg(SO_2)}{kg(coal)}$$ (b)

PROBLEM 13.10*

KNOWN: Coal having the mass analysis given below burns completely with air.

FIND: For complete combustion with 120% of the theoretical amount of air, determine the air-fuel ratio on a mass basis,

ASSUMPTIONS: 1. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. 2. The ash is noncombustible.

ANALYSIS: On the basis of 100 kg of coal, the amounts of substances present, in kmol, are

	%	m_i	M_i	$n_i = m_i/M_i$
C	80.4	80.4	12.01	6.694
H_2	3.9	3.9	2.016	1.935
O_2	5.0	5.0	32.00	0.156
N_2	1.1	1.1	28.01	0.039
S	1.1	1.1	32.06	0.043
ASH	8.5	8.5	---	---

For complete combustion with the theoretical amount of air

$$[6.694\,C + 1.935\,H_2 + 0.156\,O_2 + 0.039\,N_2 + 0.043\,S + \text{ASH}]$$
$$+ \alpha[O_2 + 3.76N_2] \rightarrow \beta\,CO_2 + \gamma\,SO_2 + \omega\,H_2O + \lambda\,N_2 + \text{ASH}$$

Balancing:

C: $6.694 = \beta$

H_2: $1.935 = \omega$

S: $0.043 = \gamma$

O: $2(0.156) + 2\alpha = 2\beta + 2\gamma + \omega$

$2\alpha = 2(6.649) + 2(0.043) + 1.935 - 2(0.156)$

$\alpha = 7.549$

Accordingly, the reaction requires $(7.549)(4.76)$ kmol of air per 100 kg of coal

$$(AF)_{theo} = \frac{(7.549)(4.76)(28.97)}{100} = 10.410\ \frac{kg(a)}{kg(coal)}$$

For 120 % of the theoretical amount of air

$$(AF) = (1.2)(10.410) = 12.49\ \frac{kg(a)}{kg(coal)} \longleftarrow$$

PROBLEM 13.11

KNOWN: Dried feedlot manure with the mass analysis below burns completely with 120% of theoretical air.

$\{ 42.7\% C,\ 5.5\% H_2,\ 31.3\% O_2,\ 2.4\% N_2,\ 0.3\% S,\ 17.8\% \text{ Ash} \}$

FIND: Determine (a) the balanced reaction equation, and (b) the air-fuel ratio on a mass basis.

ASSUMPTIONS: (a) 3.76 kmol of N_2 accompany each kmol of O_2 in the air, and the N_2 is inert. (2) The ash is non-combustible.

ANALYSIS: (a) On the basis of 100 kg of manure, the amounts of the substances present, are respectively

	m_i (kg)	M_i	$n_i = m_i/M_i$ (kmol)
C:	42.7	12	3.558
H_2:	5.5	2.016	2.728
O_2:	31.3	32	0.978
N_2:	2.4	28.01	0.0857
S:	0.3	32.06	0.00936
Ash:	17.8	---	---
	100.0 kg		

For complete combustion with the theoretical amount of air

$$(3.558\,C + 2.728\,H_2 + 0.978\,O_2 + 0.0857\,N_2 + 0.00936\,S + \text{Ash}) + \alpha_{th}(O_2 + 3.76\,N_2)$$
$$\rightarrow \beta\,CO_2 + \gamma\,H_2O + \delta\,SO_2 + \epsilon\,N_2 + \text{Ash}$$

C: $3.558 = \beta$

H: $2(2.728) = 2\gamma \Rightarrow \gamma = 2.728$

S: $0.00936 = \delta$

O: $2(0.978) + 2\alpha_{th} = 2(3.558) + 2.728 + 2(0.00936) \Rightarrow \alpha_{th} = 3.958$

Then, for complete combustion with 120% of theoretical air

$$(3.558C + 2.728H_2 + 0.978\,O_2 + 0.0857N_2 + 0.00936S + \text{Ash}) + (1.2)(3.958)(O_2 + 3.76N_2)$$
$$\rightarrow 3.558\,CO_2 + 2.728\,H_2O + 0.00936\,SO_2 + 17.944\,N_2 + \text{Ash} \longleftarrow \text{(a)}$$

(b) The actual reaction requires (1.2)(3.958)(4.76) kmol of air per 100 kg of manure. Thus

$$AF = \frac{(1.2)(3.958)(4.76)(28.97)}{100} = 6.55\ \frac{\text{kg(air)}}{\text{kg(manure)}} \longleftarrow \text{AF}$$

PROBLEM 13.12

KNOWN: Coal with the mass analysis below burns completely with the theoretical amount of air.

$$\{71.1\%\,C,\ 5.1\%\,H_2,\ 9.0\%\,O_2,\ 1.4\%\,N_2,\ 5.8\%\,S,\ 7.6\%\,Ash\}$$

FIND: Determine (a) the amount of SO_2 produced, in kg per kg of coal, and (b) the air-fuel ratio on a mass basis.

ASSUMPTIONS: (1) 3.76 kmol of N_2 accompany each kmol of O_2 in the air, and the N_2 is inert. (2) The ash is non-combustible.

ANALYSIS: On the basis of 100 kg of coal, the amounts of the substances present are, respectively

	m_i (kg)	M_i	$n_i = m_i/M_i$ (kmol)
C:	71.1	12	5.925
H_2:	5.1	2.016	2.530
O_2:	9.0	32	0.281
N_2:	1.4	28.01	0.050
S:	5.8	32.06	0.181
Ash:	7.6	----	----
	100 kg		

For complete combustion with the theoretical amount of air

$$(5.925C + 2.530H_2 + 0.281O_2 + 0.050N_2 + 0.181S + Ash) + \alpha_{th}(O_2 + 3.76N_2) \rightarrow \beta CO_2 + \gamma H_2O + \delta SO_2 + \epsilon N_2$$

$C:\ 5.925 = \beta$

$H_2:\ \gamma = 2.530$

$S:\ 0.181 = \delta$

$O:\ (0.281)2 + \alpha_{th}(2) = (5.925)2 + 2.530 + (0.181)2 \Rightarrow \alpha_{th} = 7.09$

$N:\ (0.05)2 + (7.09)(3.76)(2) = \epsilon \cdot 2 \Rightarrow \epsilon = 26.71$

(a) The amount of SO_2 produced is 0.181 kmol per 100 kg of coal. Thus

$$m_{SO_2} = \frac{(0.181)(64.06)}{100} = 0.116\ \frac{kg(SO_2)}{kg(coal)} \longleftarrow m_{SO_2}$$

(b) The amount of air required is (7.09)(4.76) kmol per 100 kg of coal. Thus

$$AF = \frac{(7.09)(4.76)(28.97)}{100} = 9.777\ \frac{kg(air)}{kg(coal)} \longleftarrow AF$$

PROBLEM 13.13 *

KNOWN: C_8H_{18} burns completely with 120% of theoretical air.

FIND: Determine (a) the air-fuel ratio on a molar and mass basis.
(b) the dew point temperature at 1atm.

ASSUMPTIONS: 3.76 kmol of N_2 accompany each kmol of O_2 in the combustion air.
N_2 is inert.

ANALYSIS: (a) For complete combustion with the theoretical amount of air

$$C_8H_{18} + \alpha(O_2 + 3.76N_2) \rightarrow \beta CO_2 + \gamma H_2O + 3.76\alpha N_2$$

C: $8 = \beta$ O: $2\alpha = 2(8) + 9$, $\alpha = 12.5$

H: $18 = 2\gamma$, $\gamma = 9$

Then, complete combustion with 120 % of the theoretical amount of air

$$C_8H_{18} + (1.2)(12.5)[O_2 + 3.76N_2] \rightarrow 8CO_2 + 9H_2O + \delta O_2 + \underbrace{(1.2)(12.5)(3.76)}_{56.4}N_2$$

$$O: 2(1.2)(12.5) = 16 + 9 + 2\delta, \quad \delta = 2.5$$

Accordingly,

$$\overline{AF} = \frac{(1.2)(12.5)(4.76)}{1} = 71.4 \ \frac{\text{kmol (air)}}{\text{kmol (fuel)}}$$ (a)

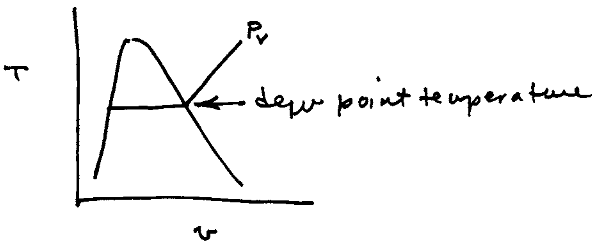
$$AF = \overline{AF}\left(\frac{M_{air}}{M_{fuel}}\right) = \frac{(71.4)(28.97)}{\underbrace{(8)(12.01) + 18(1.009)}_{114.152}} = 18.12 \ \frac{\text{kg (air)}}{\text{kg (fuel)}}$$

(b) The partial pressure of the water in the combustion products is
$p_v = y_v\, p$, where

$$y_v = \frac{9}{8 + 9 + 2.5 + 56.4} = \frac{9}{75.9} = 0.1186$$

So,

$$p_v = (0.1186)(1.01325 \text{ bar}) = 0.12015 \text{ bar}$$



Then, from Table A-2

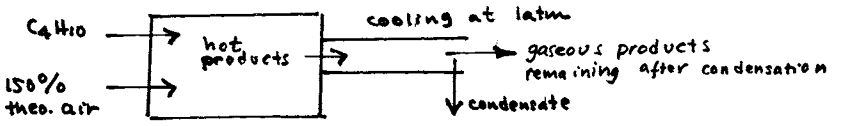
$T_{\text{dew pt}} \approx 49.4$ °C ← (b)

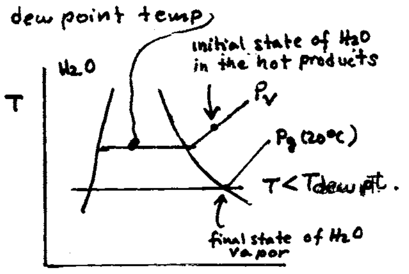
PROBLEM 13.14

KNOWN: C_4H_{10} burns completely with 150% of theoretical air. The products of combustion are cooled to temperature T at 1atm, $20 \le T \le 60\,°C$.

FIND: Plot the amount of water vapor condensed, in kmol per kmol of fuel.

SCHEMATIC & GIVEN DATA:




dew point temp
initial state of H_2O in the hot products
H_2O
T
p_v
$p_g(20°C)$
$T < T_{dew\,pt.}$
final state of H_2O vapor
v

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The gaseous products of combustion can be modeled as an ideal gas.

ANALYSIS: Complete combustion of C_4H_{10} with the theoretical amount of air is described by

$$C_4H_{10} + 6.5(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + (6.5)(3.76)\,N_2$$

Complete combustion with 150% of theoretical air is then

$$C_4H_{10} + (1.5)(6.5)(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 3.25\,O_2 + \underbrace{(1.5)(6.5)(3.76)}_{36.66}\,N_2$$

Condensation occurs when the combustion products are cooled below the dew point temperature. First, evaluate the partial pressure of water in the combustion products:

$$p_v = y_v\,p = \left(\frac{5}{4+5+3.25+36.66}\right)(1.01325\,\text{bar}) = 0.10358\ \text{bar}$$

Interpolation in Table A-2 gives $T_{\substack{dew\\pt}} \approx 46.5°C$. Accordingly, the amount of condensate is zero until the temperature T is less than 46.5°C.

According to the model introduced in Chap. 12, the gaseous products remaining after the products have been cooled to $T < T_{dew\,pt.}$ would have saturated water vapor at temperature T. That is, the partial pressure of watervapor in this mixture would equal $p_g(T)$. The partial pressure is given by

$$p_v = \left(\frac{n_v}{n_v + n_{dry}}\right)p \qquad (1)$$

where n_{dry} accounts for the moles of "dry" products: CO_2, O_2, and N_2. That is, $n_{dry} = 4 + 3.25 + 36.66 = 43.91$. Accordingly, with p = 1atm and $p_v = p_g(T)$, Eq. (1) becomes

$$p_g(T) = \left[\frac{n_v}{n_v + 43.91}\right](1.01325) \Rightarrow n_v = \frac{p_g(T)(43.91)}{1.01325 - p_g(T)}\ . \quad \text{Amount condensed} = 5 - n_v\ . \qquad (2)$$

Sample Calculation: T = 20°C, $p_g = 0.02339$ bar, $n_v = 1.038$, Amt. Condensed = 3.962,

The data for the required plots are obtained using IT, as follows:

PROBLEM 13.14 (Cont'd.)

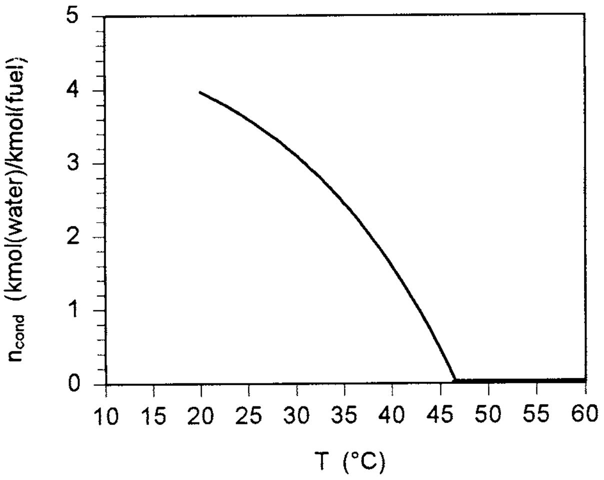
IT Code

```
T = 20  // °C
pv = 0.10358  // bar
Tdp = Tsat_P("Water/Steam", pv)
ncond = 5 - nv
nv  = pg * 43.41 / (1.01325 - pg)
pg = Psat_T("Water/Steam", T)
```

IT Results for T = 20°C

T_{dp} = 46.5°C
n_v = 1.026 bar
pg = 0.02339 bar
n_{cond} = 3.974 kmol(water)/kmol(fuel)

PLOT:



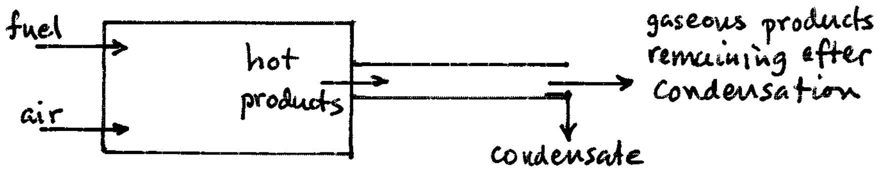
Note that condensation begins at the dew point temperature and increases as temperature is lowered further, as expected.

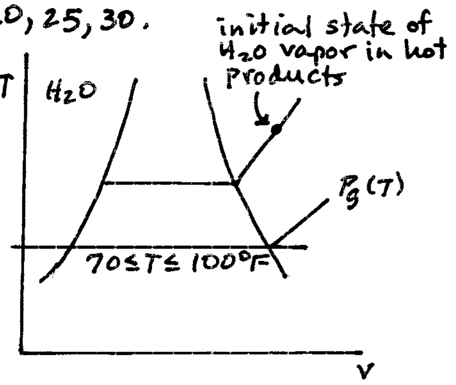
PROBLEM 13.15

KNOWN: C_2H_4 burns completely with air. The air-fuel ratio on a mass basis is AF, and the products of combustion are cooled to temperature T at 1 atm.

FIND: (a) Determine, for AF = 15 and T = 70°F, the percent excess air and the amount of water vapor condensed. (b) Plot the amount of water vapor condensed versus T ranging from 70 to 100°F for AF = 15, 20, 25, 30.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) 3.76 lbmol of N_2 accompany each lbmol of O_2 in the air, and the N_2 is inert. (2) The gaseous products of combustion can be modeled as an ideal gas mixture.

ANALYSIS: (a) Complete combustion with the theoretical amount of air is described by

$$C_2H_4 + \alpha_{th}(O_2 + 3.76\ N_2) \rightarrow 2\ CO_2 + 2H_2O + \alpha_{th}(3.76)N_2 \Rightarrow \alpha_{th} = 3 \qquad (1)$$

For complete combustion with excess air

$$C_2H_4 + \alpha(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + (\alpha - \alpha_{th})O_2 + \alpha(3.76)N_2 \qquad (2)$$

For AF = 15

$$\overline{AF} = AF\left(\frac{M_{fuel}}{M_{air}}\right) = (15)\left(\frac{28.05}{28.97}\right) = 14.524 \ \frac{\text{lbmol (air)}}{\text{lbmol (fuel)}}$$

Thus

$$\alpha = \frac{\overline{AF}}{4.76} = \frac{14.524}{4.76} = 3.0513$$

Accordingly

$$\%\ \text{excess} = \left(\frac{\alpha - \alpha_{th}}{\alpha_{th}}\right)100 = \left(\frac{3.0153 - 3}{3}\right)100 = 1.71\% \longleftarrow \%\ \text{Excess air}$$

Now, the actual reaction equation is

$$C_2H_4 + 3.0513(O_2 + 3.76N_2) \rightarrow 2CO_2 + 2H_2O + 0.0513\,O_2 + 11.473\,N_2$$

According to the model introduced in Chap. 12, the gaseous products remaining after the products have been below their dew point would have saturated vapor at T present. In this case, the partial pressure of the water vapor remaining would be $p_g(T) = p_g(70°F) = 0.3632$ lbf/in². This pressure can be expressed as

$$p_v = \left(\frac{n_v}{n_v + n_{dry}}\right)p \qquad (3)$$

where "dry" refers to the products CO_2, O_2, and N_2. That is $n_{dry} = 2 + 0.0513 + 11.473 = 13.524$. With these results, Eq. (3) becomes

$$0.3632 = \left(\frac{n_v}{n_v + 13.524}\right)(14.696) \Rightarrow n_v = 0.3427 \text{ lbmol (vapor)/ lbmol (fuel)}$$

Since 2 lbmol of H_2O are formed on combustion, the amount condensed is

$$m_{cond} = (2 - 0.3427)\text{ lbmol}\ \frac{18.02 \text{ lb}}{\text{lb mol}} = 29.86 \text{ lb (water) per lbmol of fuel} \longleftarrow m_{cond}$$

PROBLEM 13.15 (Cont'd.)

(b) The data for the required plot are obtained using IT, as follows:

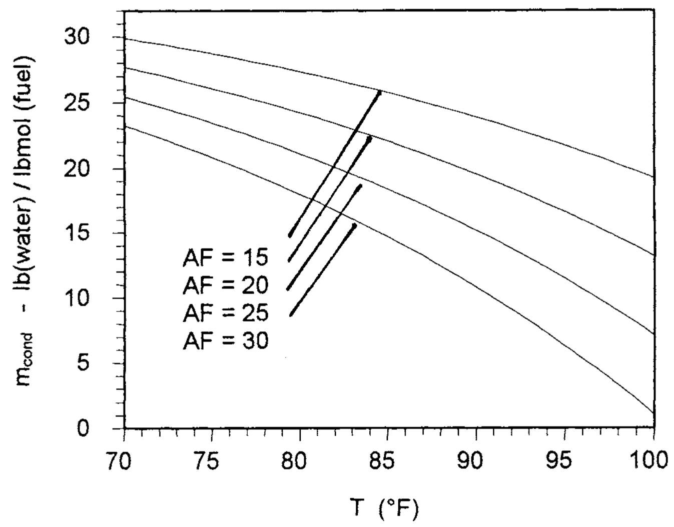
IT Code

```
T = 70  // °F
AF = 15  // kg(air) / kg(fuel)
alpha = AF * (28.05 / 28.97) / 4.76
pctXS = ((alpha - 3) / 3) * 100
ndry = 2 + (alpha - 3) + alpha * 3.76
pv = Psat_T("Water/Steam", T)
pv = (nv / (nv + ndry)) * 14.696
mcond = (2 - nv) * 18.02  // lb(water) / lbmol(fuel)
```

IT Results for AF = 15, T = 70°F

α = 3.051
% excess air = 1.706 %
n_{dry} = 13.52 lbmol(dry prod.)/ lbmol(fuel)
p_v = 0.3632 lbf/in.^{2}
n_v 0.3427 lbmol(water vap.)/lbmol(fuel)
m_{cond} = 29.86 lb(water)/lbmol(fuel)

PLOT:



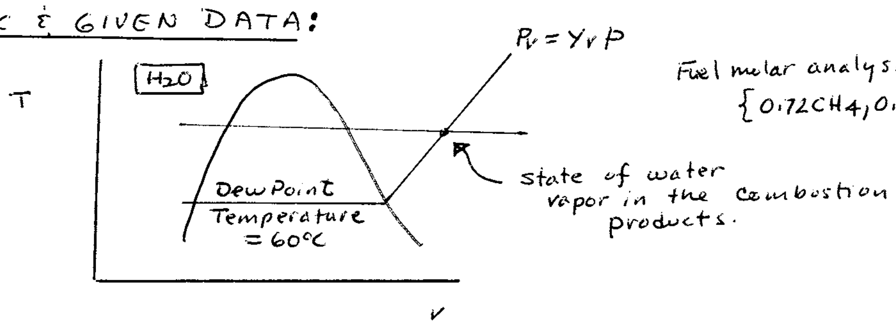
With higherAF (more air), there is more "dry" gas in the products. By Eq.(3) we see that n_v must increase as well. Thus, for fixed T there is less condensate formed as AF increases. For fixed AF, cooling to lower values of T causes more condensate to be formed, as expected.

PROBLEM 13.16

KNOWN: A fuel mixture with the molar analysis below burns completely with moist air to form gaseous products at 1atm. The dew point temperature of the products is 60°C.

FIND: Determine the amount of water vapor present in the combustion air, in kmol/kmol(fuel).

SCHEMATIC & GIVEN DATA:



$p_v = y_v p$
T
H_2O
Dew Point Temperature = 60°C
v
state of water vapor in the combustion products.

Fuel molar analysis:
$\{0.72CH_4, 0.09H_2, 0.14N_2, 0.02O_2, 0.03CO_2\}$

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air and N_2 is inert. (2) The gaseous combustion products can be modeled as an ideal gas mixture.

ANALYSIS: First, the balanced reaction equation is obtained on the basis of one kmol of fuel mixture.

$$[0.72\,CH_4 + 0.09\,H_2 + 0.14\,N_2 + 0.02\,O_2 + 0.03\,CO_2] + \underbrace{[\alpha O_2 + 3.76\alpha N_2 + \beta H_2O]}_{\text{(moist air)}}$$

$$\longrightarrow \gamma CO_2 + \delta H_2O + \omega N_2$$

$C: \quad 0.72 + 0.03 = \gamma, \qquad \gamma = 0.75$

$H: \quad 4(0.72) + 2(0.09) + 2\beta = 2\delta \implies \delta = 1.53 + \beta$

$O: \quad 2(0.02) + 2(0.03) + 2\alpha + \beta = 2\gamma + \delta \implies 2\alpha + \beta = 1.4 + \delta$

$\implies 2\alpha + \beta = 1.4 + (1.53 + \beta)$

$\alpha = 1.465$

$N_2: \quad 0.14 + 3.76\alpha = \omega \implies \omega = 5.648$

The combustion products form an ideal gas mixture in which the partial pressure p_v is $p_v = y_v p$, where

$$y_v = \frac{\delta}{\gamma + \delta + \omega} = \frac{\delta}{\delta + 0.75 + 5.648} = \frac{\delta}{\delta + 6.398}$$

Since the dewpoint temperature is known: $p_v = p_g(60°C) = 0.1994$ bar.

Collecting results,

$$y_v = \frac{p_v}{p} = \frac{0.1994\ \text{bar}}{1.01325\ \text{bar}} = \frac{\delta}{\delta + 6.398} \implies \delta = 1.568$$

Returning to the H balance, $\delta = 1.53 + \beta$, or

$$\beta = 1.568 - 1.53 = 0.038\ \frac{\text{kmol}(H_2O)}{\text{kmol (fuel)}}$$ ◄

PROBLEM 13.17

KNOWN: The volumetric analysis of a producer gas is provided.
FIND: If the gas burns completely with theoretical amount of air determine (a) the molar analysis of the dry products, (b) the amount of water vapor condensed if the products are cooled to 70°F at 1atm.
ASSUMPTIONS: 1. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. 2. Ideal gas mixture principles apply to the products.
ANALYSIS: Basing the calculation on 100 moles of fuel mixture

$$\left[3.8\,CH_4 + 0.1\,C_2H_6 + 4.8\,CO_2 + 11.7\,H_2 + 0.6\,O_2 + 23.2\,CO + 55.8\,N_2\right]$$
$$+ \alpha\left[O_2 + 3.76\,N_2\right] \rightarrow \beta\,CO_2 + \gamma\,H_2O + \omega\,N_2$$

$C:\ 3.8 + 2(0.1) + 4.8 + 23.2 = \beta \quad , \beta = 32$

$H:\ (3.8)(4) + 6(0.1) + 2(11.7) = 2\gamma \quad , \gamma = 19.6$

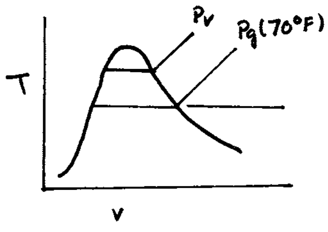
$O:\ (4.8)(2) + (0.6)(2) + 23.2 + 2\alpha = 2(32) + 19.6 \quad , \alpha = 24.8$

$N_2:\ (55.8) + (24.8)(3.76) = \omega \quad , \omega = 149.05$

(a) The total amount of dry products is $32 + 149.05 = 181.05$. Thus, the molar analysis of the dry products is

$$\% CO_2 = \frac{32}{181.05} = 0.177\ (17.7\%)\ ,\ \% N_2 = \frac{149.05}{181.05} = 0.823\ (82.3\%) \longleftarrow \text{(a)}$$

(b)



The partial pressure of the water vapor in the products is $P_v = \left(\frac{19.6}{32+149.05+19.6}\right)\left(14.7\frac{lbf}{in^2}\right) = 1.44\ lbf/in^2$

Since $P_v > P_g(70°F) = 0.3632\ lbf/in^2$, condensation would occur as the products are cooled to 70°F at $p = 1atm$.

The gas phase would consist of 181.05 moles of dry products plus n_v, which is the water vapor present. The partial pressure of the water vapor would be $P_g(70°F)$. Thus,

$$0.3632\frac{lbf}{in^2} = \left[\frac{n_v}{181.05+n_v}\right]\left(14.7\frac{lbf}{in^2}\right) \Rightarrow n_v = 4.587$$

Accordingly, the amount of water that condenses per kmol of fuel is

$$\frac{(19.6-4.587)\,lbmol(H_2O)}{100\,lbmol\,(fuel)} = 0.15\ \frac{lbmol(H_2O)}{lbmol(fuel)} \longleftarrow \text{(b)}$$

PROBLEM 13.18*

KNOWN: C_2H_2 burns completely with 110 % of theoretical air entering at 25°C, 1atm, $\phi = 50\%$.

FIND: Obtain the balanced reaction equation and the dew point temperature of the products.

ASSUMPTIONS: 1. 3.76 moles of N_2 accompany each mole of O_2 in the air. 2. N_2 and H_2O are inert. 3. Air and combustion products, each at 1atm, are modeled as ideal gases.

ANALYSIS: Complete combustion of C_2H_2 with the theoretical amount of dry air is

$$C_2H_2 + 2.5(O_2 + 3.76N_2) \rightarrow 2CO_2 + H_2O + 9.4\,N_2$$

Complete combustion with 110 % of theoretical air is

$$C_2H_2 + 2.75(O_2 + 3.76N_2) \rightarrow 2CO_2 + H_2O + 0.25O_2 + 10.34\,N_2 \qquad (1)$$

Consider next a moist air mixture at 25 °C, 1atm, $\phi = 50\%$ in which there is 2.75(4.76) = 13.09 kmol of dry air and n_v kmol of water vapor. The partial pressure of the water vapor is then

$p_v = \left(\frac{n_v}{n_a + n_v}\right)p = \left(\frac{n_v}{13.09 + n_v}\right)(1.01325\text{ bar})$. With $p_v = \phi p_g = (0.50)(0.03169\text{ bar}) = 0.01585\text{ bar}$, we get $n_v = 0.2080$ kmol. When water vapor is present in the combustion air, Eq.(1) becomes

$$C_2H_2 + 2.75(O_2 + 3.76N_2) + 0.2080H_2O \rightarrow 2CO_2 + 1.2080\,H_2O + 0.25O_2 + 10.34\,N_2$$

Considering the combustion products, the mole fraction of water vapor in the products is

$$y_v = \frac{1.2080}{13.798} = 0.08755 \Rightarrow p_v = (0.08755)(1.01325\text{ bar}) = 0.08871\text{ bar}$$

Interpolating with $p_v = p_g$ in Table A-3, $T_{\substack{dew\\pt}} = 43.4°C$

PROBLEM 13.19

KNOWN: C_4H_{10} burns completely with 150% of theoretical air at 68°F, 1atm, $\phi = 75\%$.

FIND: (a) Obtain the balanced reaction equation. (b) Determine the amount of water condensed, in lbmol per lbmol of fuel, if the combustion products are cooled to 68°F at 1atm.

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 and H_2O in the air are inert. (3) The air is modeled as an ideal gas.

ANALYSIS: (a) Complete combustion of C_4H_{10} with the theoretical amount of dry air is described by

$$C_4H_{10} + 6.5[O_2 + 3.76N_2] \rightarrow 4CO_2 + 5H_2O + 24.44\,N_2$$

Complete combustion with 150% of theoretical dry air is then

$$C_4H_{10} + 9.75[O_2 + 3.76N_2] \rightarrow 4CO_2 + 5H_2O + 3.25\,O_2 + 36.66\,N_2 \qquad (1)$$

Consider next a moist air mixture at 68°F, 1atm, $\phi = 75\%$ in which there is $9.75(4.76) = 46.41$ lbmol of dry air and n_v lbmol of water vapor. The partial pressure of the water vapor is

$$p_v = \left(\frac{n_v}{n_a + n_v}\right)p = \left(\frac{n_v}{46.41 + n_v}\right)(14.696)$$

Also, $p_v = \phi\, p_g(68°F) = 0.75\,(0.3391\ \mathrm{lbf/in^2}) = 0.2543\ \mathrm{lbf/in^2}$. Collecting results

$$0.2543 = \left(\frac{n_v}{46.41 + n_v}\right)(14.696) \Rightarrow n_v = 0.8172\ \mathrm{lbmol}.$$

When water vapor is present in the combustion air, Eq. (1) reads

$$C_4H_{10} + 9.75(O_2 + 3.76N_2) + 0.8172H_2O \rightarrow 4CO_2 + 5.8172H_2O + 3.25O_2 + 36.66N_2 \longleftarrow \text{(a)}$$

(b) To determine if condensation occurs, determine the dew point temperature:

$$p_v = \left(\frac{5.8172}{4 + 5.8172 + 3.25 + 36.66}\right)\left(14.7\,\frac{\mathrm{lbf}}{\mathrm{in^2}}\right) = 1.7196\,\frac{\mathrm{lbf}}{\mathrm{in^2}} \overset{\text{Table A-2E}}{\Longrightarrow} T_{\substack{\text{dew}\\\text{point}}} \approx 120°F$$

Thus, cooling to 68°F will result in some condensation.

According to the model introduced in Chap. 12, the gas products remaining after the combustion products have been cooled to 68°F would have saturated water vapor present. That is, the partial pressure of the water vapor would equal $p_g(68°F)$. That is,

$$\boxed{\begin{matrix}\text{Table A-2E}\\ 0.3391\,\frac{\mathrm{lbf}}{\mathrm{in^2}}\end{matrix}} \rightarrow p_g(68°F) = \left(\frac{n_v}{n_v + n_{dry}}\right)\left(14.7\,\frac{\mathrm{lbf}}{\mathrm{in^2}}\right) = \left(\frac{n_v}{n_v + 43.91}\right)(14.7)$$

where $n_{dry} = 4 + 3.25 + 36.66 = 43.91$ kmol. Solving, $n_v = 1.037$ kmol. Then

$$\begin{bmatrix}\text{amount of water condensed}\\ \text{per kmol of fuel}\end{bmatrix} = (5.8172 - 1.037) = 4.78\ \text{kmol} \longleftarrow \text{(b)}$$

PROBLEM 13.20*

KNOWN: C_2H_6 burns completely with 130% of theoretical air at 25°C, 0.85 bar, $\phi = 50\%$.

FIND: Obtain (a) the balanced reaction equation, (b) the dewpoint temperature.

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 and H_2O in the air are inert. (3) The air is modeled as an ideal gas.

ANALYSIS: (a) The complete combustion of C_2H_6 with 130% of theoretical dry air is described by

$$C_2H_6 + \underbrace{(1.3)(3.5)}_{4.55}(O_2 + 3.76N_2) \rightarrow 2CO_2 + 3H_2O + 1.05O_2 + 17.11N_2 \quad (1)$$

Consider next a moist air mixture at 25°C, 0.85 bar, $\phi = 50\%$ in which there is $4.55(4.76) = 21.66$ kmol of dry air and n_v kmol of water vapor. The partial pressure of the water vapor is

$$p_v = \left(\frac{n_v}{n_a + n_v}\right)p = \left(\frac{n_v}{21.66 + n_v}\right)(0.85)$$

Also, $p_v = \phi\, p_g(25°C) = 0.5\,(0.03169 \text{ bar}) = 0.01585$ bar. Collecting results

$$0.01585 = \left(\frac{n_v}{21.66 + n_v}\right)(0.85) \Rightarrow n_v = 0.412 \text{ kmol}$$

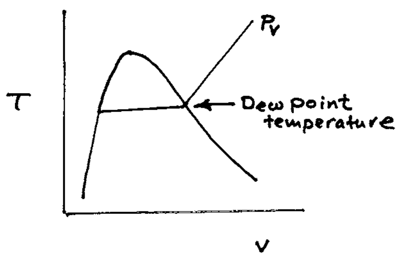
When water vapor is present in the combustion air, Eq. (1) reads

$$C_2H_6 + 4.55(O_2 + 3.76N_2) + 0.412H_2O \rightarrow 2CO_2 + 3.412H_2O + 1.05O_2 + 17.11N_2 \leftarrow (a)$$

(b) To find the dew point temperature,

$$p_v = \left(\frac{n_v}{n_{prods}}\right)p$$

$$= \left(\frac{3.412}{23.572}\right)(0.85 \text{ bar}) = 0.1230 \text{ bar}$$



Interpolation in Table A-2, $T_{\text{dew point}} \approx 50°C \leftarrow$ (b)

PROBLEM 13.21

KNOWN: C_5H_{12} burns completely with the theoretical amount of air at 75°F, 1atm, $\phi = 75\%$

FIND: Determine (a) the balanced reaction equation, (b) the dew point temperature of the combustion products, (c) the amount of water condensed if the products are cooled to 75°F

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. (2) The N_2 and H_2O in the air are inert. (3) The air and products are modeled as ideal gases.

ANALYSIS: (a) Complete combustion of C_5H_{12} with the theoretical amount of dry air is described by

$$C_5H_{12} + 8(O_2 + 3.76 N_2) \rightarrow 5 CO_2 + 6 H_2O + 30.08 N_2 \qquad (1)$$

Consider next a moist air mixture at 75°F, 1atm, $\phi = 75°F$ in which there is $8(4.76) = 38.08$ lbmol of dry air and n_v lbmol of water vapor. The partial pressure of the water vapor is

$$P_v = \left(\frac{n_v}{n_a + n_v}\right) p = \left(\frac{n_v}{38.08 + n_v}\right)(14.696 \text{ lbf/in}^2)$$

Also, $P_v = \phi P_g(75°F) = 0.75(0.4302) = 0.3227 \text{ lbf/in}^2$. Collecting results

$$0.3227 = \left(\frac{n_v}{38.08 + n_v}\right)(14.696) \Rightarrow n_v = 0.855 \text{ lbmol}$$

When water vapor is present in the combustion air, Eq.(1) reads

$$C_5H_{12} + 8(O_2 + 3.76 N_2) + 0.855 H_2O \rightarrow 5 CO_2 + 6.855 H_2O + 30.08 N_2 \longleftarrow (a)$$

(b) The partial pressure of the water vapor in the products is

$$P_v = \left(\frac{6.855}{5 + 6.855 + 30.08}\right)(14.696) = 2.402 \text{ lbf/in}^2$$

Interpolating in Table 3E with $P_v = P_g$, the dew point temperature is 132.2°F ← (b)

(c) Since 75°F is below the dew point temperature, condensation would occur for cooling to 75°F. At 75°F, the vapor phase would consist of n_v moles of water vapor and $n_{dry} = 5 + 30.08 = 35.08$ moles of dry products. The partial pressure of the vapor would equal $P_g(75°F)$. Thus

$$P_g(75°F) = \left(\frac{n_v}{n_v + n_{dry}}\right) p \quad , \quad \underset{\uparrow\ \text{Table 2E}}{0.4302 \frac{\text{lb}}{\text{in}^2}} = \left(\frac{n_v}{n_v + 35.08}\right)\left(14.7 \frac{\text{lbf}}{\text{in}^2}\right)$$

Solving this equation, $n_v = 1.058$ lbmol. The amount of water condensed per lbmol of fuel consumed is then

$$\begin{bmatrix}\text{amount of water} \\ \text{condensed per} \\ \text{lbmol of fuel} \\ \text{consumed}\end{bmatrix} = 6.855 - 1.058 = 5.8 \frac{\text{kmol (water)}}{\text{kmol (fuel)}} \longleftarrow (c)$$

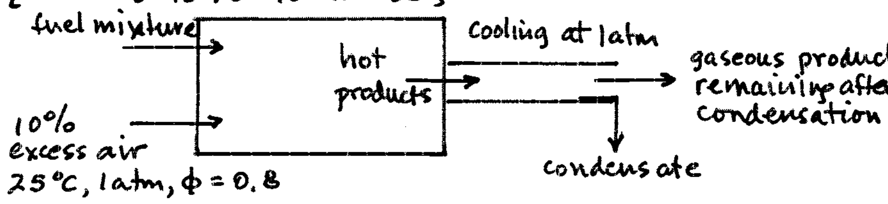
PROBLEM 13.22

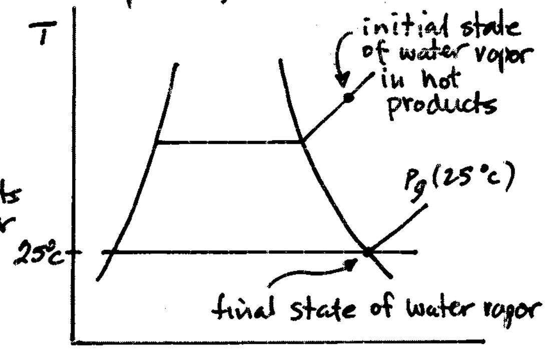
KNOWN: A liquid fuel mixture with a known mass analysis is burned completely with excess air at a specified temperature and relative humidity.

FIND: (a) Determine the equivalent hydrocarbon composition, C_aH_b, with the same carbon-hydrogen ratio on a mass basis as the mixture. (b) If the products are cooled below their dew point, determine the amount of water vapor condensed.

SCHEMATIC & GIVEN DATA:

{40% C_8H_{18}, 60% $C_{10}H_{22}$}





T
initial state of water vapor in hot products
$p_g(25°C)$
25°C
final state of water vapor
v

ASSUMPTIONS: (1) 3.76 kmol of N_2 accompany each kmol of O_2 in the air. (2) The N_2 and H_2O in the supply air are inert. (3) The air and products are both modeled as ideal gas mixtures.

ANALYSIS: (a) Assuming 100 kg of liquid fuel

	m_i (kg)	M_i	$n_i = m_i/M_i$ (kmol)
C_8H_{18}:	40	114	0.3509
$C_{10}H_{20}$:	60	142	0.4225

Thus, in 100 kg of fuel, there are the following amounts of C and H, respectively

$$C: \left[0.3509 \text{ kmol}(C_8H_{18})\right]\left[8 \frac{\text{kmol}(C)}{\text{kmol}(C_8H_{18})}\right] + [0.4225][10] = 7.0322 \text{ kmol}(C)$$

$$H: [0.3509][18] + [0.4225][22] = 15.61 \text{ kmol}(H)$$

The "equivalent" hydrocarbon composition is

$$C_{7.0322}H_{15.61} \longleftarrow \text{(a)}$$

(b) Complete combustion with the theoretical amount of air is described by

$$(0.3509\,C_8H_{18} + 0.4225\,C_{10}H_{22}) + \alpha_{th}(O_2 + 3.76N_2) \rightarrow 7.0322\,CO_2 + 7.805\,H_2O + \alpha_{th}3.76N_2$$

$$\alpha_{th} = 7.0322 + \frac{7.805}{2} = 10.935$$

Complete combustion with 10% excess is then

$$(0.3509\,C_8H_{18} + 0.4225\,C_{10}H_{22}) + 12.0285(O_2 + 3.76N_2)$$

$$\longrightarrow 7.0322\,CO_2 + 7.805\,H_2O + 1.0935\,O_2 + 45.227\,N_2 \quad (1)$$

Consider next a moist air mixture at 25°C, 1 atm, φ = 80% in which there is 12.0285(4.76) = 57.256 kmol of dry air and n_v kmol of water vapor. The partial pressure of water vapor is

$$p_v = \left(\frac{n_v}{n_a + n_v}\right)p = \left(\frac{n_v}{57.256 + n_v}\right)(1.01325 \text{ bar})$$

Also, $p_v = \phi\, p_g(25°C) = 0.8(0.03169) = 0.02535$ bar. Collecting results

$$0.02535 = \left(\frac{n_v}{57.256 + n_v}\right)(1.01325) \Rightarrow n_v = 1.4693 \text{ kmol}$$

PROBLEM 13.22 (Cont'd.)

When water vapor is present in the combustion air, Eq.(1) reads

$$(0.3509\,C_8H_{18} + 0.4225\,C_{10}H_{22}) + 12.0285\,(O_2 + 3.76N_2) + 1.4693\,H_2O$$

$$\rightarrow 7.0322\,CO_2 + 9.2743\,H_2O + 1.0935\,O_2 + 45.227\,N_2$$

According to the model introduced in chap. 12, the gaseous products remaining after the products have been cooled to 25°C would contain saturated water vapor at 25°C. That is, the partial pressure of the water vapor in this mixture would be $p_g(25°C) = 0.03169$ bar. The partial pressure is expressed as

$$p_v = \left(\frac{n_v'}{n_{dry} + n_v'}\right) p \qquad (2)$$

where $n_{dry} = 7.0322 + 1.0935 + 45.227 = 53.353$. Accordingly, from Eq. (2)

$$0.03169 = \left(\frac{n_v'}{53.353 + n_v'}\right)(1.01325) \Rightarrow n_v' = 1.7225 \text{ kmol}$$

Since there is 9.2743 kmol of H_2O per 100 kg of fuel in the products, the amount of water that condenses is $9.2743 - 1.7225 = 7.5518$ kmol (H_2O) per 100 kg of fuel. Thus

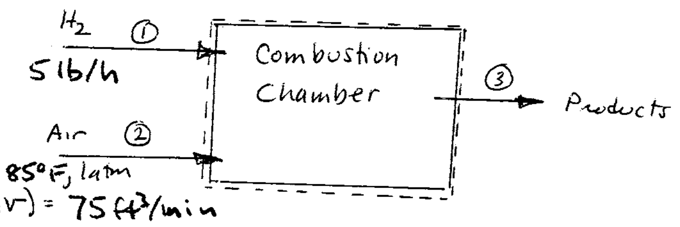
$$m_{cond} = \left[7.5518 \frac{\text{kmol}(H_2O)}{100\text{ kg(fuel)}}\right]\left[\frac{18.02\text{ kg}(H_2O)}{1\text{ kmol}(H_2O)}\right] = 1.361 \frac{\text{kg}(H_2O)}{\text{kg(fuel)}} \longleftarrow \text{(b)}$$

PROBLEM 13.23*

KNOWN: H_2 enters a combustion chamber at a rate of 5 lb/h and burns with air entering at 85°F, 1atm and a volumetric flow rate of 75 ft³/min.

FIND: Determine the percent of theoretical air used.

SCHEMATIC & GIVEN DATA



ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of N_2 in the entering air. N_2 is inert and the air is dry. (2) The air can be modeled as an ideal gas.

ANALYSIS: The percent theoretical air can be found from

$$\% \text{ theo air} = \frac{(\overline{AF})}{(\overline{AF})_{\text{theo}}} \times 100 \qquad (1)$$

For the complete combustion of H_2 with the theoretical amount of air,

$$H_2 + \tfrac{1}{2}(O_2 + 3.76N_2) \rightarrow H_2O + 1.88\,N_2$$

So, $(\overline{AF})_{\text{THEO}} = 2.38$.

For the actual reaction, the mass flow rate of the air is

$$\dot{m}_{AIR} = \frac{(AV)_2}{v_2} = \frac{(AV)_2 p_2}{(\overline{R}/M)T_2} = \frac{(75)\frac{\text{ft}^3}{\text{min}}(14.7)(144)\frac{\text{lbf}}{\text{ft}^2}}{\left(\frac{1545}{28.97}\right)\frac{\text{ft·lbf}}{\text{lb·°R}}(545)\text{°R}}\left|\frac{60\text{ min}}{1\text{ h}}\right| = 327.73\frac{\text{lb(air)}}{\text{h}}$$

Thus,

$$AF = \frac{327.73\text{ lb(air)/h}}{5\text{ lb(fuel)/h}} = 65.55\frac{\text{lb(air)}}{\text{lb(fuel)}}$$

on a molar basis

$$\overline{AF} = 65.55\frac{\text{lb(air)}}{\text{lb(fuel)}}\left[\frac{2.018\text{ lb(fuel)/lbmol(fuel)}}{28.97\text{ lb(air)/lbmol(air)}}\right]$$

$$= 4.57\frac{\text{lbmol(air)}}{\text{lbmol(fuel)}}$$

Finally, returning to Eq. (1)

$$\% \text{ theo air} = \left(\frac{4.57}{2.38}\right)(100) = 191.8 \qquad \longleftarrow$$

PROBLEM 13.24

KNOWN: Carbon burns with 80% theoretical air yielding CO_2, CO, and N_2 only.

FIND: Determine (a) the balanced reaction equation, (b) AF, (c) the analysis of the products on a molar basis.

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the air, and N_2 is inert.

ANALYSIS: (a) Complete combustion of C with the theoretical amount of air is described by

$$C + (O_2 + 3.76 N_2) \rightarrow CO_2 + 3.76 N_2$$

Thus, combustion with 80% theoretical air to produce CO_2, CO, and N_2 is

$$C + 0.80(O_2 + 3.76 N_2) \rightarrow a\, CO_2 + b\, CO + (0.80)(3.76) N_2$$

C: $1 = a + b \Rightarrow b = 1 - a$

O: $(.80)(2) = 2a + b \Rightarrow 1.6 = 2a + (1-a) \Rightarrow a = 0.6,\ b = 0.4$

Accordingly, the balanced reaction equation is

$$C + 0.80(O_2 + 3.76 N_2) \rightarrow 0.6 CO_2 + 0.4 CO + 3.008\ N_2 \longleftarrow (a)$$

(b) The air fuel ratio is

$$\overline{AF} = 0.80(4.76)/1 = 3.808 \text{ kmol (air)/kmol (fuel)}$$

Then with Eq. 13-2

$$AF = \overline{AF}\left(\frac{M_{air}}{M_{fuel}}\right) = (3.808)\left(\frac{28.97}{12.01}\right) = 9.19 \frac{\text{kg (air)}}{\text{kg (fuel)}} \longleftarrow (b)$$

(c) The molar analysis of the products is

$$\%\ CO_2 = \left(\frac{0.6}{4.008}\right)(100) = 15\%,\quad \%\ CO = \frac{0.4}{4.008} = 10\%,\quad \%\ N_2 = 75\% \longleftarrow (c)$$

PROBLEM 13.25

KNOWN: C_3H_8 reacts with 80% of theoretical air yielding CO_2, CO, H_2O and N_2 only.

FIND: Determine (a) the balanced reaction equation, (b) AF, (c) the analysis of the products on a dry molar basis.

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the air, and N_2 is inert.

ANALYSIS: (a) Complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8 + 5(O_2 + 3.76 N_2) \rightarrow 3 CO_2 + 4 H_2O + 18.8 N_2$$

Combustion with 80% of theoretical air is then

$$C_3H_8 + 4(O_2 + 3.76 N_2) \rightarrow x\, CO_2 + y\, CO + z\, H_2O + 15.04 N_2$$

C: $3 = x + y \Rightarrow y = 3 - x$

H: $8 = 2z \Rightarrow z = 4$

O: $8 = 2x + y + 4 \Rightarrow 4 = 2x + (3 - x) \Rightarrow x = 1,\ y = 2$

Accordingly

$$C_3H_8 + 4(O_2 + 3.76 N_2) \rightarrow CO_2 + 2 CO + 4 H_2O + 15.04 N_2 \longleftarrow (a)$$

(b) $$AF = \overline{AF}\left(\frac{M_{air}}{M_{fuel}}\right) = (4)(4.76)\left[\frac{28.97}{44.09}\right] = 12.51 \frac{\text{kg (air)}}{\text{kg (fuel)}} \longleftarrow (b)$$

(c)

The amount of dry products per mole of fuel is $n_{dry} = 1 + 2 + 15.04 = 18.04$. Thus, the analysis of the products on a dry basis is $\longleftarrow$ (c)

$$y_{CO_2} = \frac{1}{18.04} = 0.0554\ (5.54\%),\quad y_{CO} = \frac{2}{18.04} = 0.1109\ (11.09\%),\quad y_{N_2} = \frac{15.04}{18.04} = 0.8337\ (83.37\%)$$

PROBLEM 13.26*

KNOWN: C_6H_{14} burns with air to give products with the dry molar analysis
{11.5% CO_2, 2.4% CO, 2.0% O_2, 1.6% H_2, 82.5% N_2}

FIND: Determine the air-fuel ratio on a molar basis.

ASSUMPTIONS: Each mole of O_2 in the combustion air is accompanied by 3.76 moles of N_2 which is inert.

ANALYSIS: The analysis is on the basis of 100 moles of dry products:

$$a\,C_6H_{14} + b\,(O_2+3.76N_2) \rightarrow 11.5CO_2 + 2.4CO + 2.0O_2 + 1.6H_2 + 82.5N_2 + c\,H_2O$$

C: $6a = 11.5 + 2.4$, $a = 2.317$

H: $14a = 2(1.6) + 2c$, $c = 14.619$

O: $2b = 2(11.5) + 2.4 + 2(2.0) + c$, $b = 22.010$

① Check for closure:

N_2: $(22.010)(3.76) = 82.8 \approx 82.5$ (OK)

Then

$$\overline{AF} = \frac{22.010(4.76)}{2.317} = 45.22\ \frac{\text{kmol (air)}}{\text{kmol (fuel)}} \longleftarrow$$

---

1. Although there are three unknowns: a, b, c, four balances can be written. It is good practice to perform all balances to check for closure, as shown above.

PROBLEM 13.27

KNOWN: The products of combustion of C_8H_{17} have the following dry molar analysis: 8.7% CO_2, 8.9% CO, 0.3% O_2, 3.7% H_2, 0.3% CH_4, 78.1% N_2.

FIND: Determine the equivalence ratio

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the air, N_2 is inert.

ANALYSIS: Basing the calculation on 100 moles of dry products

$$\alpha\, C_8H_{17} + \beta(O_2 + 3.76N_2) \rightarrow 8.7\,CO_2 + 8.9\,CO + 0.3\,O_2 + 3.7\,H_2 + 0.3\,CH_4 + 78.1\,N_2 + \gamma\, H_2O$$

C: $8\alpha = 8.7 + 8.9 + 0.3 \Rightarrow \alpha = 2.2375$

H: $17(2.2375) = 2(3.7) + 4(0.3) + 2\gamma \Rightarrow \gamma = 14.7188$

N_2: $3.76\beta = 78.1 \Rightarrow \beta = 20.7713$

① Check O: $(2)(20.7713) \overset{?}{=} (8.7)(2) + 8.9 + 2(.3) + 14.7188$

$41.54 \doteq 41.62$

The fuel air ratio is $FA = \dfrac{2.2375}{(20.7713)(4.76)} = 0.0226\ \dfrac{\text{kmole(fuel)}}{\text{kmole (air)}}$

The reaction equation for complete combustion with the theoretical amount of air is

$$C_8H_{17} + \alpha(O_2 + 3.76N_2) \rightarrow \beta\, CO_2 + \gamma\, H_2O + 3.76\alpha\, N_2$$

C: $8 = \beta$

H: $17 = 2\gamma,\ \gamma = 8.5$

O: $2\alpha = 2\beta + \gamma = 16 + 8.5,\ \alpha = 12.25$

The fuel air ratio is $(FA)_{theo} = \dfrac{1}{(12.25)(4.76)} = 0.0171\ \dfrac{\text{kmol (fuel)}}{\text{kmol (air)}}$

The equivalence ratio:

② $$\left(\begin{matrix}\text{equivalence}\\ \text{ratio}\end{matrix}\right) = \frac{FA}{(FA)_{theo}} = \frac{0.0226}{0.0171} = 1.32$$

---

1. When using experimental data, an exact closure cannot be expected.
2. The reactants form a rich mixture.

PROBLEM 13.28

KNOWN: $C_{10}H_{22}$ burns with 95% of theoretical air, producing CO_2, CO, H_2O, N_2.

FIND: Determine (a) $\overline{AF}$, (b) the analysis of the products on a dry basis.

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the air, N_2 is inert.

ANALYSIS: The complete combustion of $C_{10}H_{22}$ with the theoretical amount of air is

$$C_{10}H_{22} + \alpha(O_2 + 3.76N_2) \rightarrow \beta CO_2 + \gamma H_2O + 3.76\alpha N_2$$

$C: 10 = \beta$

$H: 22 = 2\gamma \Rightarrow \gamma = 11$

$O: 2\alpha = 2\beta + \gamma = 31, \quad \alpha = 15.5$

Combustion with 95% of theoretical air is then

$$C_{10}H_{22} + 14.725(O_2 + 3.76N_2) \rightarrow \alpha CO_2 + \beta CO + \gamma H_2O + 55.366 N_2$$

$C: 10 = \alpha + \beta, \quad \beta = 10 - \alpha$

$H: 22 = 2\gamma, \quad \gamma = 11$

$O: (14.725)(2) = 2\alpha + (10 - \alpha) + 11 \Rightarrow \alpha = 8.45$

The balanced reaction equation is then

$$C_{10}H_{22} + 14.725(O_2 + 3.76N_2) \rightarrow 8.45\, CO_2 + 1.55\, CO + 11\, H_2O + 55.366\, N_2$$

(a) $$\overline{AF} = \frac{14.725(4.76)}{1} = 70.09 \frac{\text{kmol(air)}}{\text{kmol(fuel)}}$$

(b) The amount of dry products is

$$n_{dry} = 8.45 + 1.55 + 55.366 = 65.366$$

Thus, the analysis on a dry basis is

$$y_{CO_2} = \frac{8.45}{65.366} = 0.129 \quad (12.9\%)$$

$$y_{CO} = \frac{1.55}{65.366} = 0.024 \quad (2.4\%)$$

$$y_{N_2} = \frac{55.366}{65.366} = 0.847 \quad (84.7\%)$$

PROBLEM 13.29

KNOWN: C_4H_{10} burns with air to form products with the dry molar analysis:

$$\{11\%\ CO,\ 1\%\ CO,\ 3.5\%\ O_2,\ 84.5\%\ N_2\}$$

FIND: Determine (a) the percent theoretical air, (b) the dewpoint temperature.

ASSUMPTIONS: Each mole of O_2 in the combustion air is accompanied by 3.76 moles of N_2. N_2 is inert.

ANALYSIS: (a) On the basis of 100 moles of dry products, the reaction is

$$a\,C_4H_{10} + b(O_2 + 3.76N_2) \rightarrow 11\,CO_2 + 1\,CO + 3.5\,O_2 + 84.5\,N_2 + c\,H_2O$$

C: $4a = 11 + 1$, $a = 3$

H: $(10)(3) = 2c$, $c = 15$

O: $2b = 22 + 1 + 7.0 + 15$, $b = 22.5$

Check N: $(3.76)(22.5) \overset{?}{=} 84.5$

$84.6 \neq 84.5$ OK, close

For this reaction,

$$\overline{AF} = \frac{22.5(4.76)}{3} = 35.7$$

For complete combustion with the theoretical amount of air

$$C_4H_{10} + 6.5(O_2 + 3.76N_2) \rightarrow 4\,CO_2 + 5\,H_2O + (3.76)(6.5)\,N_2$$

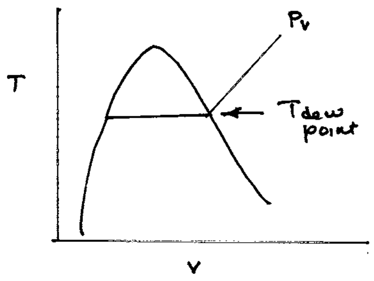
For this reaction

$$(\overline{AF})_{theo} = \frac{(6.5)(4.76)}{1} = 30.94$$

Then

$$\%\ \text{theoretical air} = \left(\frac{35.7}{30.94}\right)(100) = 115\% \longleftarrow \quad (a)$$

(b) To find the dew point temperature for p = 1 bar



$$p_v = \left(\frac{n_v}{n_{prod}}\right)p$$

$$= \left(\frac{15}{115}\right)(1\ \text{bar})$$

$$= 0.1304\ \text{bar}$$

Table 2E, $T_{dew\ pt} \approx 51°C \longleftarrow \quad (b)$

PROBLEM 13.30

KNOWN: A natural gas with a known volumetric analysis burns with air to give products for which a dry molar analysis is provided.

FIND: (a) Determine the percent of theoretical air used, (b) the dew point temperature.

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: Basing the calculation on one mole of fuel mixture, complete combustion with the theoretical amount of air is

$$[0.973\,CH_4 + 0.023\,CO_2 + 0.004\,N_2] + \alpha[O_2 + 3.76N_2] \rightarrow \beta CO_2 + \gamma H_2O + \lambda N_2$$

$$C:\ 0.973 + 0.023 = \beta \Rightarrow \beta = 0.996$$
$$H:\ 4(0.973) = 2\gamma,\quad \gamma = 1.946$$
$$O:\ 2(.023) + 2\alpha = 2(0.996) + 1.946,\quad \alpha = 1.946$$

The actual reaction equation takes the form

$$[0.973\,CH_4 + 0.023\,CO_2 + 0.004\,N_2] + \alpha[O_2 + 3.76N_2] \rightarrow \beta[9.2CO_2 + 3.84O_2 + 0.64\,CO + 86.32N_2] + \gamma H_2O$$

$$C:\ (0.973 + 0.023) = \beta[9.2 + 0.64],\quad \beta = 0.1012$$
$$H:\ 4(0.973) = 2\gamma,\quad \gamma = 1.946$$
$$O:\ 2[0.023] + 2\alpha = \beta[(9.2)(2) + (3.84)(2) + 0.64] + 1.946,\quad \alpha = 2.302$$

CHECK!

$$N_2:\ (0.004) + (2.302)(3.76) = (0.1012)(86.32)$$
① $$8.66 \overset{?}{=} 8.74$$

(a) The actual $\overline{AF}$ value is obtained as

$$\overline{AF} = \frac{(2.302)(4.76)}{1}$$

whereas the theoretical value is

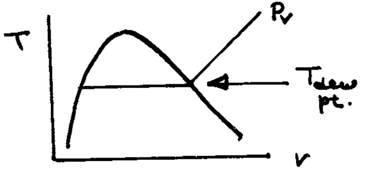
$$(\overline{AF})_{theo} = \frac{1.946(4.76)}{1}$$

$$\%\text{theo. air} = \left[\frac{\overline{AF}}{(\overline{AF})_{theo}}\right]100 = \left(\frac{2.302}{1.946}\right)(100) = 118$$

(b) The partial pressure of the water vapor is $p_v = y_v\,p$, where

$$y_v = \frac{1.946}{0.1012(100) + 1.946} = 0.1613$$

Then, with $p = 14.7$ lbf/in², $p_v = 2.37$ lbf/in². Interpolation in Table A-3E with $p_g = 2.37$ lbf/in² gives $T_{dew\ point} = 132°F$



PROBLEM 13.31

KNOWN: A fuel oil having the mass analysis {85.7% C, 14.2% H, 0.1% inert} burns with air to give products with a dry molar analysis {12.29% CO_2, 3.76% O_2, 83.95% N_2}.

FIND: Determine the air-fuel ratio on a mass basis.

ASSUMPTIONS: (1) 3.76 kmole of N_2 accompany the combustion air and the N_2 is inert. (2) The inert matter in the fuel is ignored in writing the reaction equation below.

ANALYSIS: Based on 1 kg of fuel oil, the mass analysis of the fuel allows the molar amounts of the combustibles to be evaluated:

$$C = \frac{0.857 \text{ kg(C)/kg(fuel)}}{12.01 \text{ kg(C)/kmol(C)}} = 0.07136 \frac{\text{kmol (C)}}{\text{kg (fuel)}}$$

$$H = \frac{0.142 \text{ kg(H)/kg(fuel)}}{1.009 \text{ kg(H)/kmol(H)}} = 0.14073 \frac{\text{kmol (H)}}{\text{kg (fuel)}}$$

On the basis of 1 kg of fuel oil, the reaction equation reads

$$\{0.07136\ C + 0.14073\ H\} + a[O_2 + 3.76 N_2] \rightarrow b[0.1229\ CO_2 + 0.0376\ O_2 + 0.8395 N_2] + c\,H_2O$$

C: $0.07136 = 0.1229\ b \Rightarrow b = 0.5806$

H: $0.14073 = 2c \Rightarrow c = 0.0704$

N_2: $3.76a = 0.8395\ b \Rightarrow a = 0.1296$

Check for closure:

O : $2a \overset{?}{=} b[(2)(0.1229) + 2(0.0376)] + 2c$

$2(0.1296) \overset{?}{=} 2(0.5806)[0.1229 + 0.0376] + (0.0704)$

$0.2592 \overset{?}{=} 0.2568$ (good)

The air-fuel ratio is then

$$AF = \frac{(0.1296)(4.76)(28.97)}{1}$$

$$= 17.87$$ ←

PROBLEM 13.32*

KNOWN: C_2H_5OH burns with air. A laboratory report gives only these percentages on a dry molar basis: 6.9% CO_2, 1.4% CO, 0.5% C_2H_5OH.

FIND: Determine (a) the percentages of O_2 and N_2 in the dry molar analysis, (b) percent excess air.

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (2) The dry products include only CO_2, CO, C_2H_5OH, O_2 and N_2.

ANALYSIS: (a) On the basis of 100 moles of dry products

$$y\,C_2H_5OH + \alpha(O_2 + 3.76N_2) \rightarrow 6.9\,CO_2 + 1.4\,CO + 0.5\,C_2H_5OH + x\,N_2 + (91.2 - x)O_2 + w\,H_2O$$

Note: For the dry products, $6.9 + 1.4 + 0.5 + \underset{N_2}{x} + \underset{O_2}{z} = 100 \Rightarrow x + z = 91.2$ or $z = 91.2 - x$

$C:\ 2y = 6.9 + 1.4 + 2(0.5)$, $y = 4.65$

$H:\ 6(4.65) = (0.5)(6) + 2w$, $w = 12.45$

$O:\ (4.65) + 2\alpha = (6.9)(2) + 1.4 + 0.5 + 2(91.2 - x) + 12.45$

$\Rightarrow \alpha = 102.95 - x$

$N:\ (\alpha)(2)(3.76) = 2x$

$\Rightarrow 3.76\alpha = x$

$\alpha = 102.95 - 3.76\alpha$

$\alpha = 21.628$

$x = 81.322$

and $91.2 - x = 9.88$

Accordingly,

$\%N_2 = 81.32$, $\%O_2 = 9.88 \longleftarrow$ (a)

(b) For complete combustion with the theoretical amount of air

$$C_2H_5OH + \alpha(O_2 + 3.76N_2) \rightarrow \beta CO_2 + \gamma H_2O + 3.76\alpha N_2$$

$C:\ 2 = \beta$

$H:\ 6 = 2\gamma$, $\gamma = 3$

$O:\ 1 + 2\alpha = 2(2) + 3$

$\alpha = 3$

$$\left(\%\ \text{Theoretical Air}\right) = \left(\frac{21.628\,(4.76)/4.65}{3\,(4.76)/1}(100)\right) = 155$$

% excess air = 55 $\longleftarrow$ (b)

PROBLEM 13.33

KNOWN: A fuel oil with the mass analysis {87% C, 11% H, 1.4% S, 0.6% inert} burns with 120% of theoretical air. H and S are fully oxidized, but 95% of the carbon is oxidized to CO_2 and the balance to CO.

FIND: Determine (a) the balanced reaction equation, (b) for CO and SO_2 the amounts in kmol per 10^6 kmol of combustion products.

ASSUMPTIONS: There are 3.76 kmoles of N_2 accompanying each kmol of O_2 in the combustion air. N_2 is inert.

ANALYSIS: (a) Based on 100 kg of fuel oil, the molar analysis is

$$C : \frac{87}{12.01} = 7.24, \quad H: \frac{11}{1.01} = 10.89, \quad S: \frac{1.4}{32.06} = 0.04 \text{ kmol}$$

To find the theoretical amount of air, the reaction equation for 100 kg of fuel oil is then

① $$(7.24\,C + 10.89\,H + 0.04\,S) + \alpha(O_2 + 3.76N_2) \rightarrow \beta\,CO_2 + \gamma H_2O + \omega\,SO_2 + 3.76\alpha N_2$$

C: $7.24 = \beta$

H: $10.89 = 2\gamma, \; \gamma = 5.45$

S: $0.04 = \omega$

O: $2\alpha = 2(7.24) + 5.45 + 2(0.04), \; \alpha = 10.01$

Then, for the actual reaction

$$(7.24C + 10.89H + 0.04S) + (1.2)(10.01)[O_2 + 3.76N_2] \rightarrow (0.95)(7.24)\,CO_2 + (0.05)(7.24)\,CO + \alpha\,O_2 + \gamma H_2O + \omega\,SO_2 + (1.2)(10.01)(3.76)\,N_2$$

H: $10.89 = 2\gamma, \; \gamma = 5.45$ (as above)

S: $0.04 = \omega$ (as above)

O: $\underbrace{(1.2)(10.01)(2)}_{24.024} = 13.756 + 0.362 + 2\alpha + 5.45 + 2(0.04), \; \alpha = 2.19$

The balanced reaction equation for 100 kg of fuel oil is (a)

$$(7.24C + 10.89H + 0.04S) + 12.01[O_2 + 3.76N_2] \rightarrow 6.88CO_2 + 0.36\,CO + 2.19\,O_2 + 5.45H_2O + 0.04\,SO_2 + 45.17N_2$$

(b) Total moles of products = 60.09 kmols/100 kg of fuel oil.

$$\text{ppm CO} = \left(\frac{0.36}{60.09}\right)(10^6) = 5991$$ (b)

$$\text{ppm } SO_2 = \left(\frac{0.04}{60.09}\right)(10^6) = 666$$

---

1. The presence of the inert matter is not shown in the reaction equation of part (a). Also, in part (b) only the gaseous products are considered in the ppm calculations.

PROBLEM 13.34

KNOWN: C_5H_{12} burns with air so that a fraction x of the carbon is converted to CO_2 and the rest appears as CO. No free O_2 appears in the products.

FIND: Plot $\overline{AF}$ and % theoretical air versus x.

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert.

ANALYSIS: The reaction equation takes the form

$$C_5H_{12} + a(O_2 + 3.76N_2) \rightarrow 5x\,CO_2 + 5(1-x)\,CO + b\,H_2O + 3.76a\,N_2$$

H: $12 = 2b,\ b = 6$

O: $2a = 10x + 5(1-x) + 6 \Rightarrow 2a = 5x + 11 \Rightarrow a = (5x+11)/2$.

The complete combustion of C_5H_{12} with the theoretical amount of air is described by

$$C_5H_{12} + 8(O_2 + 3.76N_2) \rightarrow 5CO_2 + 6H_2O + (8)(3.76)N_2$$

Accordingly, the percent of theoretical air required is

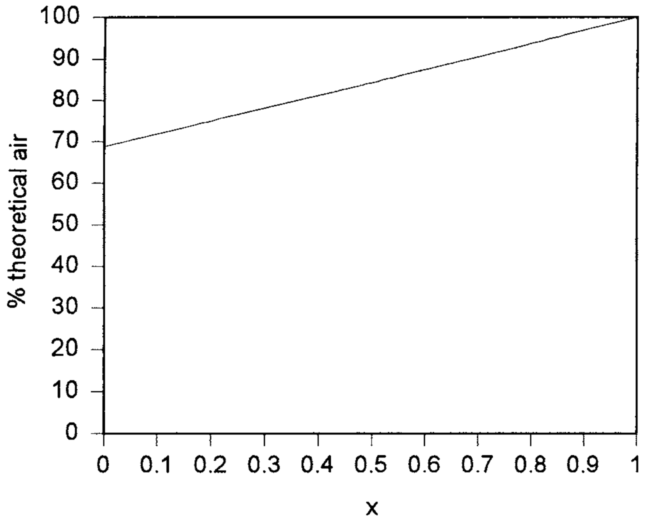
$$\%\text{ theo. air} = \left(\frac{\overline{AF}}{(\overline{AF})_{theo}}\right)(100) = \left(\frac{(5x+11)/2}{8}\right)100 = \left(\frac{5x+11}{16}\right)(100)$$

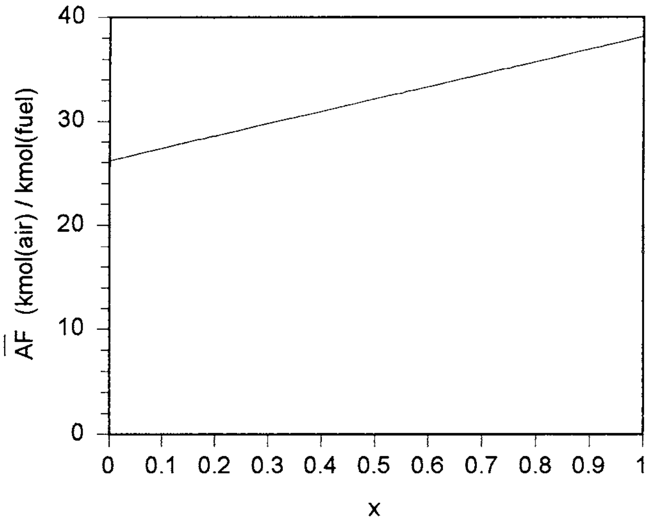
$$= 31.25x + 68.75 \qquad (1)$$

The air-fuel ratio is

$$\overline{AF} = \frac{[(5x+11)/2](4.76)}{1} = (2.5x + 5.5)\,4.76 \quad \text{or} \quad \overline{AF} = 11.9x + 26.18 \qquad (2)$$

Eqs. (1) and (2) are simple linear relations that can be plotted readily by hand or using computer software. The following plots are obtained using IT:





Note that with a deficiency of combustion air, the combustion is less complete, i.e. less CO_2 and more CO produced.

PROBLEM 13.35*

KNOWN: Fuel-air mixtures are specified.

FIND: Determine the equivalence ratio for each case and indicate if the mixture is lean or rich

ASSUMPTION: Accompanying each mole of O_2 in the air is 3.76 moles of N_2 which is inert.

ANALYSIS: (a) {1 kmol C_4H_{10}, 32 kmol air}

The reaction equation for the theoretical amount of air reads

$$C_4H_{10} + 6.5[O_2 + 3.76N_2] \rightarrow 4CO_2 + 5H_2O + 24.44N_2$$

$$\Rightarrow \quad (\overline{AF})_{THEO} = \frac{6.5(4.76)}{1} = 30.94$$

Then, the equivalence ratio is

① $$\text{Equivalence ratio} = \frac{(\overline{AF})_{THEO}}{(\overline{AF})_{ACT}} = \frac{30.94}{(32/1)} = 0.967 \quad \text{(Lean)}$$

(b) {1 lb C_3H_8, 14.5 lb air}

The reaction equation for the theoretical amount of air reads

$$C_3H_8 + 5[O_2 + 3.76N_2] \rightarrow 3CO_2 + 4H_2O + 18.8N_2$$

$$\Rightarrow \quad (AF)_{THEO} = \frac{(5)(4.76)}{1}\left[\frac{28.97}{44.09}\right] = 15.64$$

Then, the equivalence ratio is

$$\text{Equivalence ratio} = \frac{(AF)_{THEO}}{(AF)_{ACT}} = \frac{15.64}{(14.5/1)} = 1.078 \quad \text{(Rich)} \quad \longleftarrow$$

---

1. Alternatively,

$$\left(\text{equivalence ratio}\right) = \frac{(\overline{FA})_{act}}{(\overline{FA})_{theo}}$$

PROBLEM 13.36

KNOWN: C_8H_{18} burns with air to give products with the dry molar analysis: {10.5% CO_2, 5.8% CO, 0.9% CH_4, 2.6% H_2, 0.3% O_2, 79.9% N_2}

FIND: Determine the equivalence ratio.

ASSUMPTION: Each mole of O_2 in the combustion air is accompanied by 3.76 moles of N_2 which is inert.

ANALYSIS: On the basis of 100 moles of dry products, the reaction equation is

$$x\,C_8H_{18} + y[O_2 + 3.76N_2] \rightarrow [10.5\,CO_2 + 5.8CO + 0.9CH_4 + 2.6H_2 + 0.3\,O_2 + 79.9\,N_2] + z\,H_2O$$

C: $8x = 10.5 + 5.8 + 0.9$, $x = 2.15$

H: $(18)(2.15) = 4(0.9) + 2(2.6) + 2z$, $z = 14.95$

N_2: $3.76y = 79.9$, $y = 21.25$

Check for closure

O: $2y = 2(10.5) + 5.8 + 2(.3) + 14.95$, $y = 21.18$ (good)

The air-fuel ratio is then

$$\overline{AF} = \frac{(21.18)(4.76)}{2.15} = 46.89$$

The balanced equation for the theoretical amount of air reads

$$C_8H_{18} + 12.5[O_2 + 3.76N_2] \rightarrow 8CO_2 + 9H_2O + 47N_2$$

Thus

$$(\overline{AF})_{THEO} = \frac{(12.5)(4.76)}{1} = 59.5$$

The equivalence ratio is

① $$\text{Equivalence ratio} = \frac{(\overline{AF})_{THEO}}{(\overline{AF})_{ACT}} = \frac{59.5}{46.89} = 1.269 \quad (\text{Rich})$$ ←

---

1. Alternatively

$$\begin{pmatrix}\text{equivalence}\\ \text{ratio}\end{pmatrix} = \frac{(\overline{FA})_{act}}{(\overline{FA})_{theo}}$$

PROBLEM 13.37

KNOWN: CH_4 burns with air to form products consisting of $\{CO_2, CO, H_2O, N_2\}$. The equivalence ratio is 1.25.

FIND: Determine the balanced reaction equation.

ASSUMPTIONS: 3.76 moles of N_2 accompany each mole of O_2 in the combustion air and N_2 is inert.

ANALYSIS: The balanced reaction equation for combustion with the theoretical amount of air gives $(\overline{AF})_{THEO} = 9.52$ (Eq. 13.4). For the actual reaction, the equivalence ratio is

① $$\text{Equivalence ratio} = \frac{(\overline{AF})_{THEO}}{(\overline{AF})_{ACT}} = 1.25$$

Accordingly

$$(\overline{AF})_{ACT} = \frac{9.52}{1.25} = 7.616$$

The amount of O_2 in the reaction is then $7.616/4.76 = 1.6$.

Thus, the reaction equation has the form

$$CH_4 + 1.6[O_2 + 3.76 N_2] \rightarrow a\, CO_2 + b\, CO + d\, H_2O + 6.016\, N_2$$

$$\left.\begin{array}{ll} C: & 1 = a + b \\ H: & 4 = 2d, \quad d = 2 \\ O: & 2(1.6) = 2a + b + 2 \\ & 1.2 = 2a + b \end{array}\right\} \longrightarrow \begin{array}{l} a = 0.2 \\ b = 0.8 \end{array}$$

Finally

$$CH_4 + 1.6[O_2 + 3.76 N_2] \rightarrow 0.2\, CO_2 + 0.8\, CO + 2\, H_2O + 6.016\, N_2 \quad \longleftarrow$$

---

1. Alternatively,

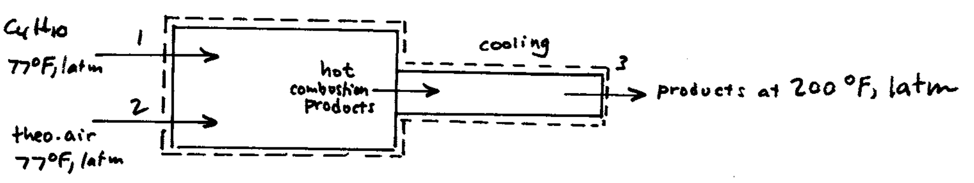
$$\left(\text{equivalence ratio}\right) = \frac{(\overline{FA})_{act}}{(\overline{FA})_{theo}}$$

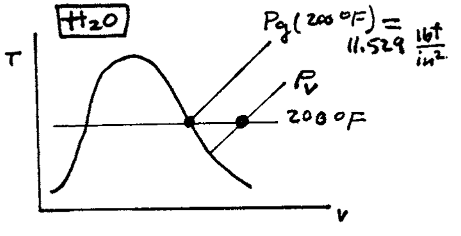
PROBLEM 13.38*

KNOWN: C_4H_{10} at 77°F, 1atm enters a combustion chamber operating at steady state and burns completely with the theoretical amount of air entering at 77°F, 1atm.

FIND: Determine the rate of heat transfer, in Btu per lbmol of fuel.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of N_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the combustion air and the gaseous products of combustion.

ANALYSIS: Complete combustion of C_4H_{10} with the theoretical amount of air is described by

$$C_4H_{10} + 6.5(O_2 + 3.76 N_2) \rightarrow 4CO_2 + 5H_2O + 24.44 N_2 \qquad (1)$$

Before applying the energy balance it is necessary to determine whether any of the water formed during combustion condenses as the products are cooled to 200°F. If the combustion products remain a gas throughout, the partial pressure of the water vapor is

$P_v = \left(\frac{5}{4+5+24.44}\right)\left(14.7\ \frac{lbf}{in^2}\right) = 2.20\ \frac{lbf}{in^2}$. Since $P_v < P_g(200\,°F)$, the water formed remains a vapor as the products are cooled.

An energy rate balance reduces to give

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{fuel}}}^{\,0} + \left(\bar{h}_{C_4H_{10}}\right)_1 + 6.5\left[\bar{h}_{O_2} + 3.76\,\bar{h}_{N_2}\right]_2 - \left[4\bar{h}_{CO_2} + 5\bar{h}_{H_2O} + 24.44\,\bar{h}_{N_2}\right]_3$$

with $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = \left[4[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 5[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O(g)} + 24.44[\cancel{\bar{h}_f^\circ}^{\,0} + \Delta\bar{h}]_{N_2}\right]_3 - \left(\bar{h}_f^\circ\right)_{C_4H_{10}}$$

With data from Tables A-23E, 25E

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = \Big\{4[(-169{,}300) + (5165 - 4028)] + 5[(-104{,}040) + (5250 - 4258)] + 24.44[4586 - 3730]\Big\}$$

$$- (-54{,}270)$$

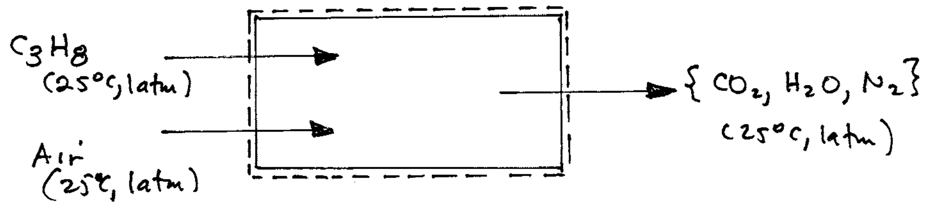
$$= -1{,}112{,}701 \text{ Btu/lbmol(fuel)}$$

PROBLEM 13.39

KNOWN: C_3H_8 enters a combustion chamber operating at steady state and burns completely with the theoretical amount of air. The reactants and products are each at 25°C, 1atm.

FIND: Determine the rate of heat transfer from the combustion chamber, in kJ per kmol of fuel.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) Combustion is complete with the theoretical amount of air. (3) 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (4) The air and gaseous products can each be modeled as ideal gas mixtures. (5) Kinetic/potential energy effects can be ignored.

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8 + 5[O_2 + 3.76N_2] \longrightarrow 3\,CO_2 + 4\,H_2O + 18.8\,N_2$$

At 25°C, 1atm some of the water formed on combustion would be a liquid. The gas phase of CO_2, N_2, and H_2O vapor would be saturated: the partial pressure of the water vapor would be $p_g(25°C)$:

$$\underbrace{p_g(25°C)}_{\substack{\text{Table A-2}\\ = 0.03169\,\text{bar}}} = \left(\frac{n_v}{n_v + \underbrace{n_{DRY}}_{=3+18.8}}\right)(1.01325\text{ bar}) \Rightarrow n_v = 0.704 \text{ kmole}$$

where n_v is the amount of water vapor in the products. The amount of liquid water in the products is then $(4 - 0.704) = 3.296$ kmol.

An energy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_F} - \cancelto{0}{\frac{\dot{W}_{cv}}{\dot{n}_F}} + (\bar{h}_f^\circ)_{C_3H_8} + 5\left[\cancelto{0}{(\bar{h}_f^\circ)_{O_2}} + 3.76\cancelto{0}{(\bar{h}_f^\circ)_{N_2}}\right]$$
$$- \Big[3(\bar{h}_f^\circ)_{CO_2} + 18.8\cancelto{0}{(\bar{h}_f^\circ)_{N_2}} + 0.704(\bar{h}_f^\circ)_{H_2O(g)} +$$

① $$3.296(\bar{h}_f^\circ)_{H_2O(\ell)}\Big]$$

With data from Table A-25

$$\frac{\dot{Q}_{cv}}{\dot{n}_F} = 3(-393{,}520) + 0.704(-241{,}820) + 3.296(-285{,}830) - (-103{,}850)$$
$$= -2{,}189{,}047 \frac{\text{kJ}}{\text{kmol(fuel)}} \quad \triangleleft$$

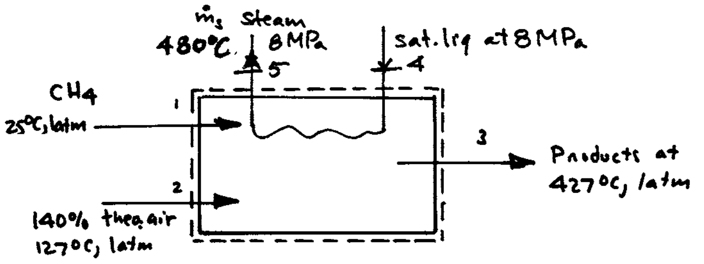
---

1. For the exiting liquid water we use $(\bar{h}_f^\circ)_{H_2O(\ell)}$ from Table A-25, ignoring the slight difference between the enthalpy of saturated liquid water at 25°C and liquid water at 25°C, 1atm.

PROBLEM 13.40

KNOWN: Steady state operating data are provided for a boiler fueled by methane.

FIND: Determine the volumetric flow rate of the fuel.



$\dot{m}_s = 3.7 \times 10^5 \frac{kg}{h}$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: Complete combustion of CH_4 with 140% of theoretical air is described by

$$CH_4 + 2.8(O_2 + 3.76 N_2) \rightarrow CO_2 + 2H_2O + 0.8\, O_2 + 10.528\, N_2 \qquad (1)$$

An energy rate balance reduces to give

$$0 = \frac{\dot{m}_s}{\dot{n}_F}[h_4 - h_5] + (\bar{h}_{CH_4})_1 + [2.8\bar{h}_{O_2} + 10.528\,\bar{h}_{N_2}]_2 - [\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 0.8\bar{h}_{O_2} + 10.528\,\bar{h}_{N_2}]_3$$

with $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$\frac{\dot{m}_s}{\dot{n}_F}[h_4 - h_5] = [\bar{h}_f^\circ + \bar{h}(700) - \bar{h}(298)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(700) - \bar{h}(298)]_{H_2O(g)} + 0.8[\bar{h}(700) - \bar{h}(298)]_{O_2} + 10.528\,[\bar{h}(700) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{CH_4} - 2.8[\bar{h}(400) - \bar{h}(298)]_{O_2} - 10.528\,[\bar{h}(400) - \bar{h}(298)]_{N_2}$$

Inserting data from the steam tables and the ideal gas tables

$$\frac{\dot{m}_s}{\dot{n}_F}[316.6 - 3348.4] = [-393{,}520 + 27{,}125 - 9{,}364] + 2[-241{,}820 + 24{,}088 - 9{,}904] + 0.8[21{,}184 - 8{,}682] + 10.528\,[20{,}604 - 11{,}640] - (-74{,}850) - 2.8[11{,}711 - 8{,}682]$$

$$= -660{,}287 \text{ kJ/kmol (CH4)}$$

Thus

$$\frac{\dot{m}_s}{\dot{n}_F} = \frac{660{,}287}{2031.8} = 324.98 \frac{kg(steam)}{kmol\,(fuel)}$$

or

$$\dot{n}_F = \frac{3.7 \times 10^5\, kg/h}{324.98\, kg/kmol(fuel)} = 1138.5 \frac{kmol}{h}$$

Since

$$\dot{n}_F = \frac{(AV)_F}{\bar{v}_F} = \frac{(AV)_F}{\bar{R}T_F/p_F} \Rightarrow (AV)_F = \dot{n}_F\left[\frac{\bar{R}T_F}{p_F}\right]$$

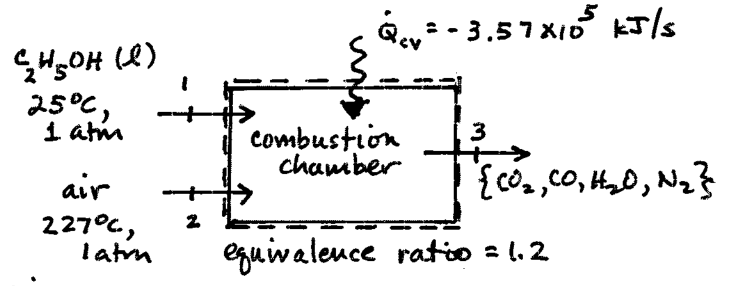
$$= \left(1138.5 \frac{kmol}{h}\right)\left(\frac{\left(8314 \frac{N\cdot m}{kmol\cdot K}\right)(298K)}{1.01325 \times 10^5 N/m^2}\right) = 27{,}838 \frac{m^3}{h} \longleftarrow$$

PROBLEM 13.41

KNOWN: Liquid ethanol enters a combustion chamber and burns with entering separately with an equivalence ratio of 1.2. The mass flow rate of fuel and the heat transfer rate are specified as are the combustion products.

FIND: Determine (a) the exit temperature, and (b) the air-fuel ratio on a mass basis.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state, and $\dot W_{cv} = 0$ with negligible kinetic and potential energy effects. (2) 3.76 kmol of N_2 accompany each kmol of O_2 in the air and the N_2 is inert. (3) The combustion air and product gases behave as ideal gas mixtures.

ANALYSIS: (a) For complete combustion of $C_2H_5OH(l)$ with theoretical air

$$C_2H_5OH + \alpha_{th}(O_2 + 3.76N_2) \rightarrow 2CO_2 + 3H_2O + \alpha_{th}(3.76)N_2 \Rightarrow \alpha_{th} = 3$$

thus $\quad (\bar{AF})_{theo} = \frac{(3)(4.76)}{1} = 14.28$

Now $\quad \text{equiv. ratio} = \frac{(\bar{FA})_{act}}{(\bar{FA})_{theo}} = \frac{(\bar{AF})_{theo}}{(\bar{AF})_{act}} \Rightarrow (\bar{AF})_{act} = \frac{(\bar{AF})_{theo}}{\text{equiv. ratio}} = 11.9$

and $\quad \alpha_{act} = \frac{(\bar{AF})_{act}}{(4.76)} = 2.5$

Thus, the actual combustion reaction is

$$C_2H_5OH + 2.5(O_2 + 3.76N_2) \rightarrow \beta CO_2 + \gamma CO + 3H_2O + 9.4N_2$$

$$\left.\begin{aligned} C:&\ 2 = \beta + \gamma \\ O:&\ 1 + 2.5(2) = \beta(2) + \gamma + 3 \end{aligned}\right\} \begin{aligned}\beta &= 1\\ \gamma &= 1\end{aligned}$$

$$C_2H_5OH + 2.5(O_2 + 3.76N_2) \rightarrow CO_2 + CO + 3H_2O + 9.4N_2 \tag{1}$$

(a) An energy rate balance at steady state reduces to

$$0 = \frac{Q_{cv}}{\dot n_F} - \cancel{\frac{\dot W_{cv}}{\dot n_F}}^{0} + \bar h_{fuel} + [2.5\bar h_{O_2} + 9.4\bar h_{N_2}]_2 - [\bar h_{CO_2} + \bar h_{CO} + 3\bar h_{H_2O} + 9.4\bar h_{N_2}]_3$$

With $\bar h = \bar h_f^\circ + \Delta\bar h$, and noting that $\bar h_f^\circ = 0$ for O_2 and N_2, this becomes

$$\begin{aligned} 0 = \frac{Q_{cv}}{\dot n_F} &+ (\bar h_f^\circ)_{C_2H_5OH(l)} + 2.5[\bar h(500) - \bar h(298)]_{O_2} + 9.4[\bar h(500) - \bar h(298)]_{N_2} \\ &- [\bar h_f^\circ + \bar h(T_3) - h(298)]_{CO_2} - [\bar h_f^\circ + \bar h(T_3) - \bar h(298)]_{CO} - 3[\bar h_f^\circ + \bar h(T_3) - \bar h(298)]_{H_2O(g)} \\ &- 9.4[\bar h(T_3) - \bar h(298)]_{N_2} \end{aligned} \tag{2}$$

The molar flow rate of fuel is $\dot n_F = \dot m_F / M_{fuel} = \frac{25 \text{ kg/s}}{46.07 \text{ kg/kmol}} = 0.5427 \frac{\text{kmol}}{\text{s}}$.

solving for $[\bar h(T_3)]_{CO_2} + [\bar h(T_3)]_{CO} + 3[\bar h(T_3)]_{H_2O} + 9.4[\bar h(T_3)]_{N_2}$ and inserting data from Tables A-23E and A-25E

$$\begin{aligned} &[\bar h(T_3)]_{CO_2} + [\bar h(T_3)]_{CO} + 3[\bar h(T_3)]_{H_2O} + 9.4[\bar h(T_3)]_{N_2} \\ &\quad = \frac{Q_{cv}}{\dot n_F} + (\bar h_f^\circ)_{C_2H_5OH(l)} + 2.5[\bar h(500) - \bar h(298)]_{O_2} + 9.4[\bar h(500) - \bar h(298)]_{N_2} \\ &\quad - [\bar h_f^\circ - h(298)]_{CO_2} - [\bar h_f^\circ - h(298)]_{CO} - 3[\bar h_f^\circ - h(298)]_{H_2O(g)} - 9.4[-\bar h(298)]_{N_2} \end{aligned}$$

PROBLEM 13.41 (Cont'd.)

$$= \frac{(-3.75 \times 10^5)}{(0.5427)} + 2.5[14770 - 8682] + 9.4[14581 - 8669] - [-393520 - 9364]$$
$$- [-110530 - 8669] - 3[-241820 - 9904] - 9.4[-8669] = 460{,}857 \frac{\text{kJ}}{\text{kmol(fuel)}}$$

This equation can be solved iteratively for T_3 using table data. The result is

① $T_3 \approx 994$ K. $\longleftarrow T_3$

(b) Using Eq. (13.2), the air-fuel ratio on a mass basis is

$$AF = \overline{AF}\left(\frac{M_{air}}{M_{fuel}}\right) = 11.9\left(\frac{28.97}{46.07}\right) = 7.483 \frac{\text{kg(air)}}{\text{kg(fuel)}} \longleftarrow AF$$

---

1. To avoid iteration using table data, IT can be used. The IT solution follows.

IT Code

```
T1 = 25 + 273  // K
T2 = 227 + 273  // K
hfo_fuel = -277690  // from Table A-25
Qdot = -3.75E05
ndot_fuel = 25 / 46.07  // kmol/s

0 = Qdot / ndot_fuel + hfuel + 2.5 * hO2_2 +
        9.4 * hN2_2 - hCO2_3 - hCO_3 - 3 * hH2O_3 - 9.4 * hN2_3
hfuel = hfo_fuel
hO2_2 = h_T("O2", T2)
hN2_2 = h_T("N2", T2)
hCO2_3 = h_T("CO2", T3)
hCO_3 = h_T("CO", T3)
hH2O_3 = h_T("H2O", T3)
hN2_3 = h_T("N2", T3)
```

IT Result

```
T3 = 993.7 K
```

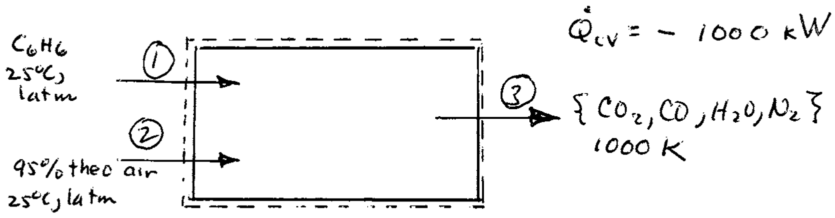
Note that the IT solution agrees favorably with the iterative solution. Also, note that IT evaluates the specific enthalpies in the bracketed terms in Eq. (c2) directly.

PROBLEM 13.42

KNOWN: Steady state operating data are provided for a combustion chamber in which C_6H_6 burns with 95% theoretical air.

FIND: Determine the fuel mass flow rate for a heat transfer rate of 1000 kW.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic/potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the combustion air and N_2 is inert. (3) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: The balanced reaction equation for combustion with the theoretical amount of air is

$$C_6H_6 + 7.5[O_2 + 3.76N_2] \rightarrow 6\,CO_2 + 3H_2O + 28.2N_2$$

The balanced reaction equation for combustion with 95% theoretical air is

$$C_6H_6 + 7.125[O_2 + 3.76N_2] \rightarrow 5.25\,CO_2 + 0.75CO + 3H_2O + 26.79\,N_2$$

An energy rate balance reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \overset{0}{\cancel{\frac{\dot{W}_{cv}}{\dot{n}_{fuel}}}} + (\bar{h}_{fuel})_1 + [7.125\,\bar{h}_{O_2} + 26.79\,\bar{h}_{N_2}]_2 - [5.25\,\bar{h}_{CO_2} + 0.75\,\bar{h}_{CO} + 3\bar{h}_{H_2O} + 26.79\,\bar{h}_{N_2}]_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 5.25[\bar{h}^\circ + \Delta\bar{h}]_{CO_2} + 0.75[\bar{h}_f^\circ + \Delta\bar{h}]_{CO} + 3[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 26.79[\overset{0}{\cancel{\bar{h}_f^\circ}} + \Delta\bar{h}]_{N_2} - (\bar{h}_f^\circ)_{fuel}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 5.25[-393520 + 42769 - 9364] + 0.75[-110{,}530 + 30355 - 8669] + 3[-241{,}820 + 35882 - 9904]$$
$$+ 26.79[30129 - 8669] - (82930)$$

$$= -1{,}890{,}604 - 66{,}633 - 647{,}526 + 574{,}913 - 82{,}930$$

$$= -2{,}112{,}780 \text{ kJ/kmol(fuel)}$$

Then, for $\dot{Q}_{cv} = -1000$ kJ/s

$$\dot{n}_{fuel} = \frac{-10^3 \text{ kJ/s}}{-2{,}112{,}780 \text{ kJ/kmol(fuel)}} = 4.73 \times 10^{-4} \frac{\text{kmol(fuel)}}{\text{s}}$$

With M = 78.11 from Table A-1

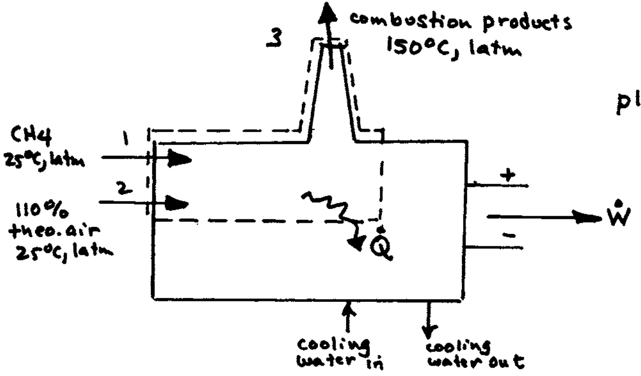
$$\dot{m}_{fuel} = \left(4.73 \times 10^{-4} \frac{\text{kmol(fuel)}}{\text{s}}\right)\left(78.11 \frac{\text{kg(fuel)}}{\text{kmol(fuel)}}\right) = 0.037 \frac{\text{kg(fuel)}}{\text{s}} \longleftarrow$$

PROBLEM 13.43

KNOWN: Steady state operating data are provided for a simple vapor plant.

FIND: Plot the mass flow rate of fuel required, in kg/h per MW of power developed, versus the plant thermal efficiency, η.

SCHEMATIC & GIVEN DATA:



plant thermal efficiency: $30 \le \eta \le 40\%$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure by the dashed line operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and products of combustion are modeled as ideal gases.

ANALYSIS: By definition, $\eta = \dot{W}/\dot{Q} \Rightarrow \dot{Q} = \dot{W}/\eta$. For N MW of power ~~developed~~ generated,

$$\dot{Q} = \frac{N \times 10^3 \text{ kJ/s}}{\eta} \qquad (1)$$

where η is expressed as a decimal.

To evaluate the mass flow rate of the fuel requires an energy rate balance, the chemical reaction equation, and this value for $\dot{Q}$.

Complete combustion of CH_4 with the theoretical amount of air is described by Eq. 13.4. Complete combustion with 110% of theoretical air is then

$$CH_4 + 2.2(O_2 + 3.76 N_2) \rightarrow CO_2 + 2H_2O + 0.2O_2 + 8.272 N_2$$

Noting that $\dot{Q}$ is positive in the direction of the arrow on the above figure, an energy rate balance at steady state reduces to give

$$\frac{\dot{Q}}{\dot{n}_{CH_4}} = (\bar{h}_{CH_4})_1 + (2.2\bar{h}_{O_2} + 8.272\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 0.2\bar{h}_{O_2} + 8.272\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$\frac{\dot{Q}}{\dot{n}_{CH_4}} = (\bar{h}_f^\circ)_{CH_4} + (2.2\cancel{\bar{h}_f^\circ})_{O_2} + 8.272\cancel{\bar{h}_f^\circ})_{N_2}) - [(\bar{h}_f^\circ + \bar{h}(423) - \bar{h}(298))_{CO_2} + 2(\bar{h}_f^\circ + \bar{h}(423) - \bar{h}(298))_{H_2O(g)}$$
$$+ 0.2[\bar{h}(423) - \bar{h}(298)]_{O_2} + 8.272(\bar{h}(423) - \bar{h}(298))_{N_2}]$$

Then, with data from the ideal gas tables

$$\frac{\dot{Q}}{\dot{n}_{CH_4}} = (-74,850) - [-393,520 + 14333 - 9364] - 2[-241,820 + 14,147 - 9,904] - 0.2[12,405 - 8,682]$$
$$- 8.272[12,313 - 8,669]$$
$$= 757,967 \text{ kJ/kmol}(CH_4)$$

Combining this with (1)

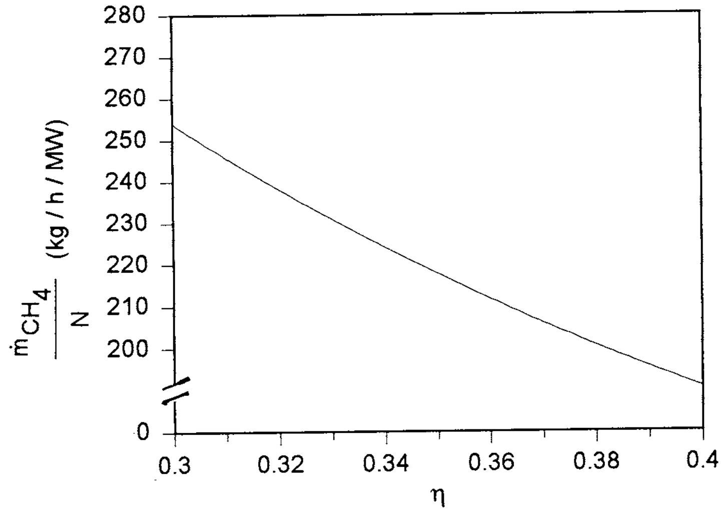
$$\dot{n}_{CH_4} = \frac{N \times 10^3 \text{ kJ/s}}{\eta\,(757,967 \text{ kJ/kmol})}$$

PROBLEM 13.43 (Continued)

$$\dot{m}_{CH_4} = \frac{N \times 10^3 \text{ kJ/s}}{\eta [757{,}967 \text{ kJ/kmol}]} \left[\frac{16.04 \text{ kg}}{\text{kmol}}\right] \left|\frac{3600 \text{ s}}{\text{h}}\right|$$

$$\frac{\dot{m}_{CH_4}}{N} = \frac{76.18}{\eta} \left(\frac{\text{kg/h}}{\text{MW}}\right) \qquad (2)$$

Eq. (2) is a simple mathematical relation that can be plotted readily by hand or using computer software. The following plot is obtained using IT:



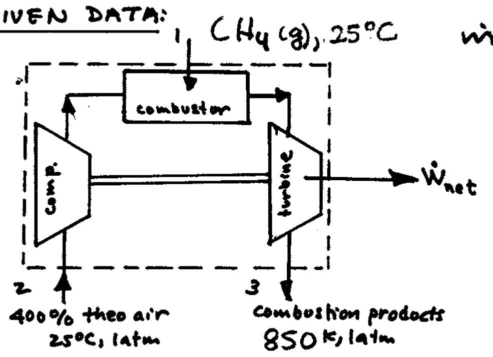
Note that as the thermal efficiency increases, less fuel is needed per MW of power developed, as expected.

PROBLEM 13.44*

KNOWN: Steady state operating data are provided for a simple gas turbine power plant.

FIND: Determine the net power output for a given fuel mass flow rate.

SCHEMATIC & GIVEN DATA:



$\dot{m}_f = 1200$ kg/h

$\dot{Q}_{cv} = -0.1\ \dot{W}_{net}$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the fuel, combustion air, and the products of combustion.

ANALYSIS: Complete combustion of CH_4 with the theoretical amount of air is described by

$$CH_4 + 2(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 7.52N_2$$

Complete combustion with 400% of theoretical air is then

$$CH_4 + 8(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 6\ O_2 + 30.08N_2$$

An energy rate balance reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{CH_4})_1 + (2\bar{h}_{O_2} + 30.08\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 6\bar{h}_{O_2} + 30.08\bar{h}_{N_2})_3$$

Then, with $\dot{Q}_{cv} = -0.1\ \dot{W}_{cv}$ and $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

① $$1.1\frac{\dot{W}_{cv}}{\dot{n}_{fuel}} = (\bar{h}_f^\circ)_{CH_4} + (2\cancel{\bar{h}_{f,O_2}^\circ}^{\,0} + 30.08\cancel{\bar{h}_{f,N_2}^\circ}^{\,0}) - [\bar{h}_f^\circ + \bar{h}(850) - \bar{h}(298)]_{CO_2} - 2[\bar{h}_f^\circ + \bar{h}(850) - \bar{h}(298)]_{H_2O}$$
$$- 6[\bar{h}(850) - \bar{h}(298)]_{O_2} - 30.08[\bar{h}(850) - \bar{h}(298)]_{N_2}$$

With data from the ideal gas tables and Table A-25

$$1.1\frac{\dot{W}_{cv}}{\dot{n}_{fuel}} = -74{,}850 - [-393{,}520 + 34{,}773 - 9{,}364] - 2[-241{,}820 + 29{,}846 - 9{,}904]$$
$$-6[26{,}218 - 8{,}682] - 30.08[25{,}292 - 8{,}669]$$
$$= 131{,}781 \text{ kJ/kmol fuel}$$

$$\dot{n}_{fuel} = \frac{(1200)\text{ kg/h}}{(16.04)\text{ kg/kmol}}\left|\frac{1\text{ h}}{3600\text{ s}}\right| = 0.02078\ \frac{\text{kmol fuel}}{\text{s}}$$

$$\dot{W}_{cv} = \frac{(0.02078)\text{ kmol/s}}{1.1}(131{,}781)\frac{\text{kJ}}{\text{kmol}}\left|\frac{1\text{ MW}}{1000\text{ kJ/s}}\right| = 2.49\text{ MW} \longleftarrow$$

---

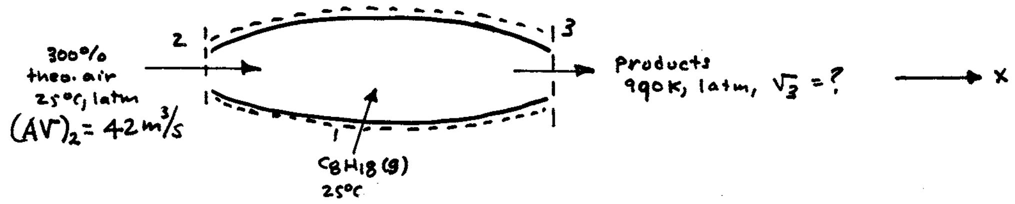
1. The fuel must enter the combustor at an elevated pressure because the combustion air pressure has been increased in passing through the compressor. However, with the ideal gas model the fuel enthalpy is determined by the temperature only.

PROBLEM 13.45

KNOWN: $C_8H_{18}(g)$ at 25°C, 1atm enters a jet engine and burns completely with 300% of theoretical air entering at 25°C, 1atm. Products exit at 990 K, 1atm.

FIND: Determine the thrust produced by the engine.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$. (2) The kinetic energies of the incoming air and fuel can be neglected. Potential energy can be neglected throughout. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The ideal gas model is applicable to the combustion air and the products of combustion.

ANALYSIS: Complete combustion of C_8H_{18} with the theoretical amount of air is described by

$$C_8H_{18} + 12.5(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 47N_2$$

Complete combustion with 300% of theoretical air is then

$$C_8H_{18} + 37.5O_2 + 141N_2 \rightarrow 8CO_2 + 9H_2O + 25O_2 + 141N_2 \qquad (1)$$

An energy rate balance at steady state reduces to give

$$0 = \cancel{\dot{Q}_{cv}}^{0} - \cancel{\dot{W}_{cv}}^{0} + \underbrace{[\dot{n}_{C_8H_{18}}\bar{h}_{C_8H_{18}}]_1}_{(fuel)} + \underbrace{[\dot{n}_{O_2}\bar{h}_{O_2} + \dot{n}_{N_2}\bar{h}_{N_2}]_2}_{(air)} - \underbrace{\left\{[\dot{n}_{CO_2}\bar{h}_{CO_2} + \dot{n}_{H_2O}\bar{h}_{H_2O} + \dot{n}_{O_2}\bar{h}_{O_2} + \dot{n}_{N_2}\bar{h}_{N_2}] + \dot{m}_3\frac{V_3^2}{2}\right\}}_{(products)} \qquad (2)$$

where $\dot{n}$ denotes a molar flow rate and $\dot{m}_3$ is the mass flow rate of the exiting products:

$$\dot{m}_3 = (\dot{m}_{CO_2} + \dot{m}_{H_2O} + \dot{m}_{O_2} + \dot{m}_{N_2})_3 = (M_{CO_2}\dot{n}_{CO_2} + M_{H_2O}\dot{n}_{H_2O} + M_{O_2}\dot{n}_{O_2} + M_{N_2}\dot{n}_{N_2})_3 \qquad (3)$$

Dividing throughout by the molar flow rate of the fuel and solving for $V_3^2/2$, Eqs. (1) - (3) give

$$\frac{V_3^2}{2} = \frac{(\bar{h}_{C_8H_{18}})_1 + [37.5\bar{h}_{O_2} + 141\bar{h}_{N_2}]_2 - [8\bar{h}_{CO_2} + 9\bar{h}_{H_2O} + 25\bar{h}_{O_2} + 141\bar{h}_{N_2}]}{(8M_{CO_2} + 9M_{H_2O} + 25M_{O_2} + 141M_{N_2})}$$

With $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$ and noting that $\bar{h}_f^o = 0$ for O_2 and N_2

$$\frac{V_3^2}{2} = \frac{(\bar{h}_f^o)_{C_8H_{18}(g)} - 8[\bar{h}_f^o + \bar{h}(990) - \bar{h}(298)]_{CO_2} - 9[\bar{h}_f^o + \bar{h}(990) - \bar{h}(298)]_{H_2O} - 25[\bar{h}(990) - \bar{h}(298)]_{O_2} - 141[\bar{h}(990) - \bar{h}(298)]_{N_2}}{(8M_{CO_2} + 9M_{H_2O} + 25M_{O_2} + 141M_{N_2})}$$

With data from the ideal gas tables

$$\frac{V_3^2}{2} = \frac{(-208{,}450) - 8[-393{,}520 + 42{,}226 - 9{,}364] - 9[-241{,}820 + 35{,}472 - 9{,}904] - 25(31{,}041 - 8{,}682) - 141[29{,}803 - 8{,}669]}{(8(44.01) + 9(18.02) + 25(32.00) + 141(28.01))}$$

$$= \frac{-208{,}450 + 2{,}885{,}264 + 1{,}946{,}268 - 558{,}975 - 2{,}979{,}894}{5263.67}$$

$$= 205.98\ \frac{kJ}{kg(prods)}$$

PROBLEM 13.45 (Contd.)

Solving for V_3, and introducing appropriate conversion factors

$$V_3 = \sqrt{2\left(205.98\ \frac{kJ}{kg}\right)\left|\frac{10^3\ N\cdot m}{kJ}\right|\left|\frac{1\ kg\cdot m/s^2}{1\ N}\right|} = 641.84\ m/s$$

Ignoring the momentum transfer associated with the fuel and incoming combustion air, Eq. 9.31 gives the magnitude of the resultant force in the x direction acting on the control volume

$$F_x = \dot{m}_3 V_3 \qquad (4)$$

where

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = \dot{m}_1\left[1 + \overbrace{(\dot{m}_2/\dot{m}_1)}^{(FA)}\right] = \dot{m}_1\left[1 + (FA)\right]$$

$$\dot{m}_1 = \frac{(AV)_1}{RT_1/P_1} = \frac{(42\ m^3/s)(1.01325 \times 10^5\ N/m^2)}{\left(\frac{8314}{28.97}\ \frac{N\cdot m}{kg\cdot K}\right)(298 K)} = 49.8\ \frac{kg}{s}$$

From the reaction equation the fuel-air ratio on a mass basis is

$$FA = \frac{[1\ kmol\,(fuel)][114.22\ kg/kmol]}{[(37.5 + 141)\ kmol\,(air)][28.97\ kg/kmol]} = 0.0221\ \frac{kg\,(fuel)}{kg\,(air)}$$

$$\Rightarrow \quad \dot{m}_3 = (1.0221)(49.8\ \tfrac{kg}{s}) = 50.9\ \frac{kg}{s}$$

Accordingly, Eq. (4) gives

$$F_x = \left(50.9\ \frac{kg}{s}\right)\left(641.84\ \frac{m}{s}\right)\left|\frac{1\ N}{1\ kg\cdot m/s^2}\right|$$

$$= 32{,}670\ N$$

The thrust is the force developed by the control volume:

$$\text{Thrust} = -F_x = -32{,}670\ N$$

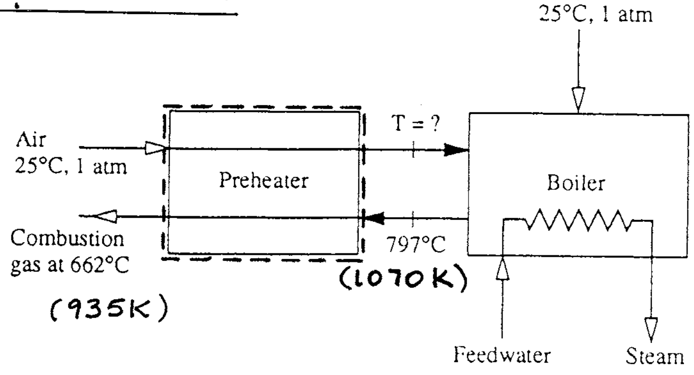
↑ acts to the left (see figure)

PROBLEM 13.46

KNOWN: Steady state operating data are provided for a boiler and preheater.

FIND: Determine the temperature of the combustion air entering the boiler from the preheater.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume, $\dot{Q}_{cv}$ and kinetic/potential energy effects are negligible. (3) Combustion is complete with 170% of theoretical air. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The air and the combustion products can each be modeled as an ideal gas mixture.

ANALYSIS: Complete combustion with 170% of theoretical air is described by

$$CH_4 + 3.4(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 1.4O_2 + 12.78N_2$$

An energy rate balance for the preheater reads

$$0 = \left[3.4\bar{h}_{O_2} + 12.78\bar{h}_{N_2}\right]_1 - \left[3.4\bar{h}_{O_2} + 12.78\bar{h}_{N_2}\right]_2 + \left[\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 1.4\bar{h}_{O_2} + 12.78\bar{h}_{N_2}\right]_3$$
$$- \left[\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 1.4\bar{h}_{O_2} + 12.78\bar{h}_{N_2}\right]_4$$

With data from the ideal gas tables

$$3.4\left[\bar{h}(T_2) - 8682\right]_{O_2} + 12.78\left[\bar{h}(T_2) - 8669\right]_{N_2} = 12.78\left[32432 - 28016\right] +$$
$$\left[46602 - 39268\right]_{CO_2} + 2\left[38802 - 33234\right]_{H_2O} + 1.4\left[33842 - 29133\right]_{O_2}$$

$$\Rightarrow \quad 3.4\bar{h}_{O_2}(T_2) + 12.78\bar{h}_{N_2}(T_2) = 221{,}808 \text{ kJ/kmol(fuel)}$$

① Solving iteratively using data from Table A-23, $T_2 = 197°C$ ◀

---

1. Iteration using table data can be avoided by using IT, as follows:

IT Code

```
T1 = 25  // °C
T3 = 797  // °C
T4 = 662  // °C

hO2_1 = h_T("O2",T1)
hN2_1 = h_T("N2",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hH2O_3 = h_T("H2O",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
hCO2_4 = h_T("CO2",T4)
hH2O_4 = h_T("H2O",T4)
hO2_4 = h_T("O2",T4)
hN2_4 = h_T("N2",T4)

0 = (3.4 * hO2_1+ 12.78 * hN2_1) - (3.4 * hO2_2 + 12.78 * hN2_2) +
      ( hCO2_3 + 2 * hH2O_3 + 1.4 * hO2_3 + 12.78 * hN2_3) -
      ( hCO2_4 + 2 * hH2O_4 + 1.4 * hO2_4 + 12.78 * hN2_4)
```

IT Result

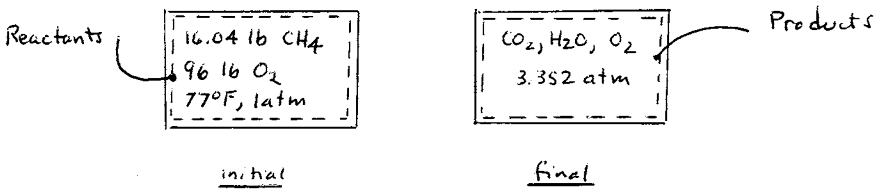
$T_2 = 196.4°C$

PROBLEM 13.47

KNOWN: A rigid tank initially contains CH_4 and O_2 at a known condition. Complete combustion occurs and the final pressure is specified.

FIND: Determine the heat transfer in Btu.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is indicated in the above figure. (2) For the system, $W = 0$ and kinetic/potential energy effects are absent. (3) The ideal gas model is applicable.

ANALYSIS: An energy balance reduces to give $Q = \Delta U$. To evaluate ΔU the final temperature is required. Using the ideal gas equation of state

$$P_R = \frac{n_R \bar{R} T_R}{V}, \quad P_P = \frac{n_P \bar{R} T_P}{V} \Rightarrow \frac{P_P}{P_R} = \underbrace{\frac{n_P}{n_R}}_{=1} \frac{T_P}{T_R} \Rightarrow \frac{P_P}{P_R} = \frac{T_P}{T_R}$$

where $n_R = \frac{16.04}{16.04} + \frac{96}{32} = 1 + 3 = 4$ and for complete combustion

$$CH_4 + 3O_2 \rightarrow CO_2 + 2H_2O + O_2 \quad \Rightarrow n_P = 1 + 2 + 1 = 4$$

Accordingly, $T_P = \left(\frac{3.352}{1}\right)(537°R) = 1800°R$.

Then, $Q = U_P - U_R$ or

$$Q = [\bar{u}_{CO_2} + 2\bar{u}_{H_2O} + \bar{u}_{O_2}]_P - [\bar{u}_{CH_4} + 3\bar{u}_{O_2}]_R$$

With $\bar{u} = \bar{h} - \bar{R}T$

$$Q = [(\bar{h}_{CO_2} - \bar{R}T_P) + 2(\bar{h}_{H_2O} - \bar{R}T_P) + (\bar{h}_{O_2} - \bar{R}T_P)]_P - [(\bar{h}_{CH_4} - \bar{R}T_R) + 3(\bar{h}_{O_2} - \bar{R}T_R)]_R$$

$$= [\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + \bar{h}_{O_2}]_P - [\bar{h}_{CH_4} + 3\bar{h}_{O_2}]_R + 4\bar{R}[T_R - T_P]$$

Then, using $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$Q = \{[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 2[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + [\Delta\bar{h}]_{O_2}\}_P - [(\bar{h}_f^\circ)_{CH_4}]_R + 4\bar{R}[T_R - T_P]$$

With data from the ideal gas tables

$$Q = [-169{,}300 + (18392 - 4028)] + 2[-104{,}040 + (15433 - 4258)] + [13486 - 3725] - [-32{,}210] + 4(1.986)[537 - 1800]$$

$$= -154{,}936 - 185{,}730 + 9761 + 32{,}210 - 10{,}033$$

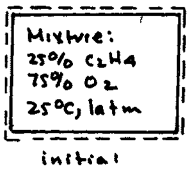
$$= -308{,}728 \text{ Btu} \quad \leftarrow$$

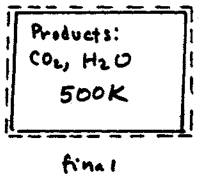
PROBLEM 13.48

KNOWN: A closed, rigid vessel contains a gaseous mixture at 25°C, 1atm with the molar analysis {25% C_2H_4, 75% O_2}. The mixture burns completely and the products are cooled to 500K.

FIND: Determine (a) the heat transfer from the vessel per kmol of C_2H_4, and (b) the final pressure, in atm.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The system is shown in the accompanying figure. (2) For the system W = 0 and there are no effects of kinetic and potential energy. (3) The ideal gas model is applicable to the initial and final mixtures. (4) Combustion is complete.

ANALYSIS: Complete combustion of C_2H_4 with the theoretical amount of O_2 is described by

$$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O \qquad (1)$$

For these reactants on a molar basis: C_2H_4 : 25%, O_2 : 75%. Accordingly, it can be concluded that there is no excess O_2, and that the reaction undergone by the system is described by Eq. (1).

(a) An energy balance reduces to $\Delta U = Q - \overset{0}{\cancel{W}}$ or $Q = \Delta U = U_2 - U_1$. When expressed on a per mole of C_2H_4 basis, this is

$$\frac{Q}{n_{C_2H_4}} = \frac{U_2 - U_1}{n_{C_2H_4}} = [2\bar{u}_{CO_2} + 2\bar{u}_{H_2O(g)}]_2 - [\bar{u}_{C_2H_4} + 3\bar{u}_{O_2}], \qquad (2)$$

For an ideal gas $u(T) = h(T) - \bar{R}T$. Accordingly, Eq. (2) becomes

$$\frac{Q}{n_{C_2H_4}} = \left[2(\bar{h}_{CO_2}(T_2) - \bar{R}T_2) + 2(\bar{h}_{H_2O}(T_2) - \bar{R}T_2)\right] - \left[(\bar{h}_{C_2H_4}(T_1) - \bar{R}T_1) + 3(\bar{h}_{O_2}(T_1) - \bar{R}T_1)\right]$$

$$= \{2\bar{h}_{CO_2}(T_2) + 2\bar{h}_{H_2O}(T_2) - \bar{h}_{C_2H_4}(T_1) - 3\bar{h}_{O_2}(T_1)\} - 4\bar{R}T_2 + 4\bar{R}T_1$$

or, with $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$

$$\frac{Q}{n_{C_2H_4}} = 2[\bar{h}_f^o + \bar{h}(500) - \bar{h}(298)]_{CO_2} + 2[\bar{h}_f^o + \bar{h}(500) - \bar{h}(298)]_{H_2O} - (\bar{h}_f^o)_{C_2H_4} - 3\overset{0}{\cancel{(\bar{h}_f^o)_{O_2}}} - 4\bar{R}(T_2 - T_1)$$

With data from the ideal gas tables

$$\frac{Q}{n_{C_2H_4}} = 2[-393,520 + 17,678 - 9,364] + 2[-241,820 + 16,828 - 9,904] - (52,280) - 4(8.314)[500 - 298]$$

$$= -770\,412 - 469\,792 - 52,280 - 6718$$

$$= -1,299,202 \text{ kJ/kmol}(C_2H_4) \longleftarrow (Q/n_{C_2H_4})$$

(b) Using the ideal gas equation of state

$$\left.\begin{aligned} P_R &= \frac{n_R \bar{R} T_R}{V} \\ P_P &= \frac{n_P \bar{R} T_P}{V} \end{aligned}\right\rangle \quad \frac{P_P}{P_R} = \frac{n_P T_P}{n_R T_R} = \left(\frac{4}{4}\right)\left(\frac{500K}{298K}\right) = 1.68$$

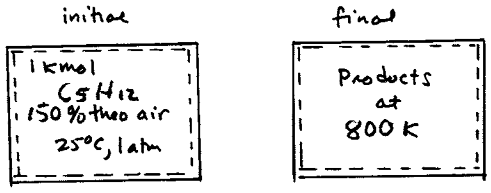
$$\Rightarrow P_R = 1.68 \text{ atm.} \longleftarrow P_P$$

PROBLEM 13.49*

KNOWN: 1 kmol of C_5H_{12} and 150 % of theoretical air are contained in a closed, rigid vessel at 25°C, 1atm. The mixture burns completely forming products at 800 K.

FIND: Determine (a) the heat transfer and (b) the final pressure.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is the contents of the vessel. (2) Combustion is complete. 3.67 kmol of N_2 accompany each kmol of O_2 in the combustion air. N_2 is inert. (3) The ideal gas model applies to both the initial and final gas mixtures.

ANALYSIS: The complete reaction of C_5H_{12} with the theoretical amount of air is

$$C_5H_{12} + 8\,[O_2 + 3.76 N_2] \rightarrow 5CO_2 + 6H_2O + (8)(3.76)N_2$$

Complete reaction with 150 % of theoretical air is

$$C_5H_{12} + 12[O_2 + 3.76N_2] \rightarrow 5CO_2 + 6H_2O + 4\,O_2 + 45.12\,N_2$$

Using the ideal gas model

$$p_R = \frac{n_R \bar{R} T_R}{V}, \qquad p_P = \frac{n_P \bar{R} T_P}{V} \quad \Big> \quad \frac{p_P}{p_R} = \left(\frac{n_P}{n_R}\right)\left(\frac{T_P}{T_R}\right) = \left(\frac{60.12}{58.12}\right)\left(\frac{800\text{ K}}{298\text{ K}}\right) = 2.777$$

$$\Rightarrow \quad p_P = 2.777 \text{ atm} \longleftarrow \text{(b)}$$

Following the discussion of Sec. 13.2.2, an energy balance reduces to read

$$Q - \cancel{W} = \sum_P n(\bar{h}_f^\circ + \Delta\bar{h}) - \sum_R n(\bar{h}_f^\circ + \Delta\bar{h}) - \bar{R}T_P \sum_P n + \bar{R}T_R \sum_R n \qquad (13.17b)$$

or

$$Q = 5[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 6[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 4[\overset{0}{\cancel{\bar{h}_f^\circ}} + \Delta\bar{h}]_{O_2} + 45.12[\overset{0}{\cancel{\bar{h}_f^\circ}} + \Delta\bar{h}]_{N_2}$$
$$- (\bar{h}_f^\circ)_{C_5H_{12}} - \bar{R}T_P \underbrace{\sum_P n}_{60.12} + \bar{R}T_R \underbrace{\sum_R n}_{58.12}$$

With ideal gas table data and $(\bar{h}_f^\circ)_{C_5H_{12}}$ from Table A-25

$$Q = 5[-393{,}520 + 32{,}179 - 9364] + 6[-241{,}820 + 27{,}896 - 9904] + 4[24{,}523 - 8682]$$
$$+ 45.12[23{,}714 - 8669] - (-146{,}440) - 8.314[(800)(60.12) - (298)(58.12)]$$

$$= -2{,}563{,}732 \text{ kJ} \longleftarrow \text{(a)}$$

PROBLEM 13.50

KNOWN: $C_4H_{10}(g)$ reacts completely with air. The reactants and products are at 25°C, 1 atm.

FIND: Determine the enthalpy of combustion for (a) $H_2O(g)$ in the products, (b) $H_2O(l)$ in the products, express each in kJ per kmol of fuel and kJ per kg of fuel.

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) Combustion is with the theoretical amount of air. (3) The combustion air and the combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of C_4H_{10} with the theoretical amount of air is described by

$$C_4H_{10} + 6.5(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 24.44\,N_2$$

The enthalpy of combustion is

$$\bar{h}_{RP} = 4\bar{h}_{CO_2} + 5\bar{h}_{H_2O} - \bar{h}_{C_4H_{10}} - 6.5\bar{h}_{O_2}$$

Since the reactants and products are at 25°C, 1 atm this reduces simply to

$$\bar{h}^{o}_{RP} = 4(\bar{h}^{o}_f)_{CO_2} + 5(\bar{h}^{o}_f)_{H_2O} - (\bar{h}^{o}_f)_{C_4H_{10}}$$

(a) H₂O is a vapor. Using data from Table A-25

$$\bar{h}^{o}_{RP} = 4(-393{,}520) + 5(-241{,}820) - (-126{,}150) = -2.657\times10^{6}\ \text{kJ/kmol}(C_4H_{10})$$

For C_4H_{10}, M = 58.12, so

$$h^{o}_{RP} = -\frac{2.657\times10^{6}}{58.12} = -45{,}716\ \frac{kJ}{kg}$$

← (a)

(b) H₂O is a liquid. Using data from Table A-25

$$\bar{h}^{o}_{RP} = 4(-393{,}520) + 5(-285{,}830) - (126{,}150) = -2.877\times10^{6}\ \text{kJ/kmol}\ (C_4H_{10})$$

and

$$h^{o}_{RP} = -\frac{2.877\times10^{6}}{58.12} = -49{,}501\ \frac{kJ}{kg}$$

← (b)

PROBLEM 13.51

KNOWN: $C_3H_8(g)$ reacts completely with air. The reactants and products are each at 1atm and temperature T. For $C_3H_8(g)$, $c_p = 0.41$ Btu/lb·°R.

FIND: Plot the enthalpy of combustion versus T for $77 \le T \le 500$ °F.

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) Combustion is with theoretical amount of air. (3) In the products, H_2O is a vapor. (4) The combustion air and combustion products can be modeled as ① ideal gases. (5) For $C_3H_8(g)$, c_p is constant.

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8(g) + 5(O_2 + 3.76 N_2) \rightarrow 3CO_2 + 4H_2O(g) + 18.8 N_2$$

The enthalpy of combustion is

$$\bar{h}_{RP} = 3\bar{h}_{CO_2} + 4\bar{h}_{H_2O(g)} - \bar{h}_{C_3H_8(g)} - 5\bar{h}_{O_2} \qquad (1)$$

$$= 3[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} + 4[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O(g)}$$

$$- [\bar{h}_f^\circ + \Delta\bar{h}]_{C_3H_8(g)} - 5[\overset{0}{\cancel{\bar{h}_f^\circ}} + \bar{h}(T) - \bar{h}(537)]_{O_2}$$

$$= \left\{3(\bar{h}_f^\circ)_{CO_2} + 4(\bar{h}_f^\circ)_{H_2O(g)} - (\bar{h}_f^\circ)_{C_3H_8(g)}\right\} + 3[\bar{h}(T) - \bar{h}(537)]_{CO_2}$$

$$+ 4[\bar{h}(T) - \bar{h}(537)]_{H_2O(g)} - [\bar{c}_p(T-537)]_{C_3H_8(g)} - 5[\bar{h}(T) - \bar{h}(537)]_{O_2}$$

Sample Calculation: T = 960°R (500°F). With data from Table A-23E and A-25E

$$\bar{h}_{RP} = \{3(-169{,}300) + 4(-104{,}040) - (-44680)\} + 3[\bar{h}(T) - 4028]_{CO_2} +$$

$$4[\bar{h}(T) - 4258]_{H_2O(g)} - (0.41)(44.09)[T-537] - 5[\bar{h}(T) - 3725]_{O_2}$$

$$= -879{,}830 \frac{kJ}{kmol(fuel)} + 3[\bar{h}(T) - 4028]_{CO_2} + 4[\bar{h}(T) - 4258]_{H_2O(g)}$$

$$- \underbrace{(0.41)(44.09)}_{18.08}[T-537] - 5[\bar{h}(T) - 3725]_{O_2}$$

$$= -879{,}830 + 3[8244 - 4028] + 4[7738 - 4258] - 18.08[960-537] - 5[6786 - 3725]$$

$$= -875{,}765 \text{ Btu/lbmol (fuel)}$$

The data for the required plot are obtained using IT, as follows:

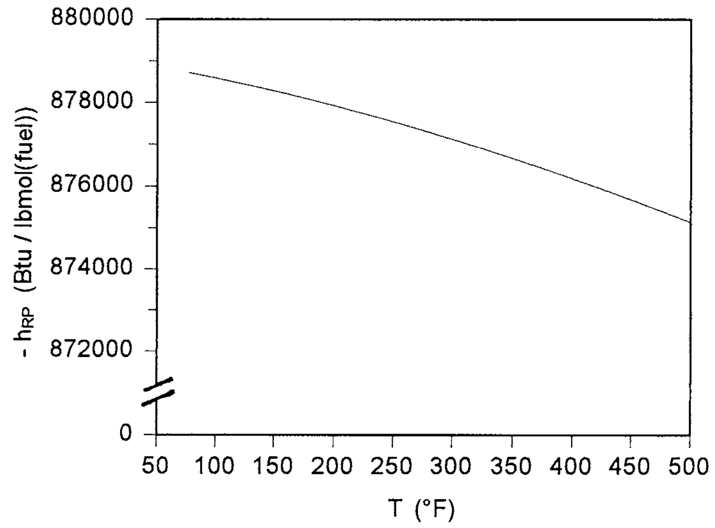
IT Code

```
T = 500 // °F

hCO2 = h_T("CO2",T)
hH2O = h_T("H2O",T)
hC3H8 = - 44680 + (0.41) * (44.04) * (T - 77)
hO2 = h_T("O2",T)

-hRP = 3 * hCO2 + 4 * hH2O - hC3H8 - 5 * hO2
```

IT Reults for T = 500°F

- h_{RP} = 8.751 x 10^5 Btu / lbmol (fuel)

PLOT:



Note that h_{RP} varies only slightly with temperature in the range shown.

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1. The results would differ only slightly if the variation of c_p with temperature were considered. IT includes a function for C_3H_8 that accounts for such variation.

PROBLEM 13.52

KNOWN: $CH_4(g)$ reacts completely with air. The reactants and products are each at 1 atm and temperature T. For CH_4,

$$\bar{c}_p = 4.52 + 7.37(T/1000) \text{ Btu/lbmol}\cdot{}^\circ R, \quad T \text{ in } {}^\circ R.$$

FIND: Plot $\bar{h}_{RP}$ versus T for $537 \le T \le 1800^\circ R$.

ASSUMPTIONS: (1) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) Combustion is with the theoretical amount of air. N_2 is inert. (3) In the products, H_2O is a vapor. (4) The CH_4, combustion air, and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of CH_4 with the theoretical amount of air is described by

$$CH_4 + 2O_2 + 7.52N_2 \rightarrow CO_2 + 2H_2O(g) + 7.52N_2$$

The enthalpy of combustion is

$$\begin{aligned}\bar{h}_{RP} &= \bar{h}_{CO_2} + 2\bar{h}_{H_2O(g)} - \bar{h}_{CH_4} - 2\bar{h}_{O_2}\\ &= [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O(g)} - [\bar{h}_f^\circ + \Delta\bar{h}]_{CH_4} - 2[\overset{0}{\cancel{\bar{h}_f^\circ}} + \bar{h}(T) - \bar{h}(298)]_{O_2}\\ &= \{(\bar{h}_f^\circ)_{CO_2} + 2(\bar{h}_f^\circ)_{H_2O(g)} - (\bar{h}^\circ)_{CH_4}\} + [\bar{h}(T) - \bar{h}(537)]_{CO_2} + 2[\bar{h}(T) - \bar{h}(537)]_{H_2O(g)}\\ &\qquad - (\Delta\bar{h})_{CH_4} - 2[\bar{h}(T) - \bar{h}(298)]_{O_2} \qquad (1)\end{aligned}$$

where

$$(\Delta\bar{h})_{CH_4} = \int_{537^\circ R}^{T} \left[4.52 + 7.37\left(\frac{T}{1000}\right)\right] dT = 4.52[T - 537] + \frac{7.37}{2}\left[\frac{T^2 - (537)^2}{1000}\right] \qquad (2)$$

Sample Calculation: $T = 900^\circ R$. With data for ideal gases from Table A-23E and $(\bar{h}_f^\circ)_{CH_4}$ from Table A-25E

$$\begin{aligned}\bar{h}_{RP} &= \{(-169{,}300) + 2(-104{,}040) - (-32{,}210)\} + [7598 - 4028] + 2[7231 - 4258]\\ &\quad - \left\{4.52[900 - 537] + \frac{7.37}{2}\left[\frac{900^2 - 537^2}{1000}\right]\right\} - 2[6338 - 3725]\\ &= -344{,}446 \text{ Btu/lbmol (fuel)}\end{aligned}$$

The data for the required plot are obtained using IT, as follows:

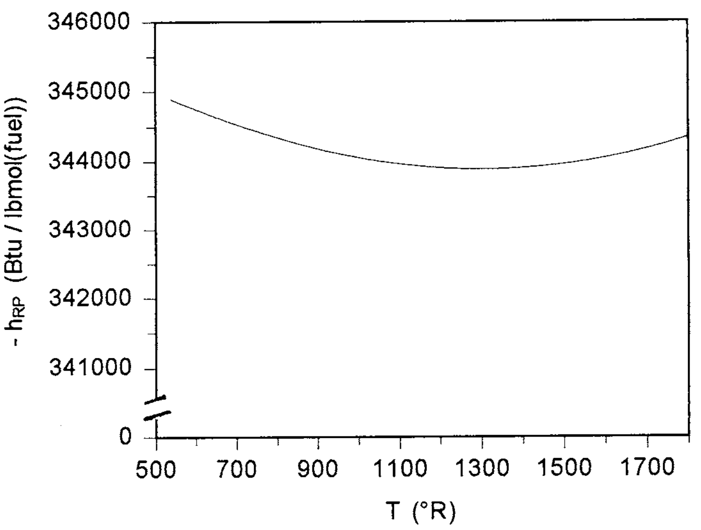
IT Code

```
T = 900  // °R

hCO2 = h_T("CO2", T)
hH2O = h_T("H2O", T)
hCH4 = -32210 + 4.52 * (T - 537) +
       (7.37 / 2000) * (T^2 - 537^2)
hO2 = h_T("O2", T)

-hRP = hCO2 + 2 * hH2O - hCH4 - 2 * hO2
```

IT result for T = 900°R

$-h_{RP} = 3.442 \times 10^5$ Btu / lbmol(fuel)

PLOT:



From the plot we see that h_{RP} varies only slightly with temperature over the range from 537 to 1800°R. Interestingly, the curve exhibits a minimum within this range.

PROBLEM 13.53

① KNOWN: The producer gas of Prob. 13.17 reacts completely with air. The reactants and products are at 77°F, 1 atm.

FIND: Determine the enthalpy of combustion for $H_2O(g)$ in the products.

ASSUMPTIONS: (1) 3.76 lbmol of N_2 accompany each lbmol of O_2 in the air, and the N_2 is inert. (2) Combustion is with the theoretical amount of air. (3) The combustion air and products can be modeled as ideal gases.

ANALYSIS: Complete combustion of the producer gas with the theoretical amount of air is described by (based on 100 lbmol of gas)

$$[3.8CH_4 + 0.1C_2H_6 + 4.8CO_2 + 11.7H_2 + 0.6O_2 + 23.2CO + 55.8N_2] + 24.8[O_2 + 3.76N_2]$$
$$\rightarrow 32CO_2 + 19.6H_2O + 149.05N_2$$

The enthalpy of combustion is

$$\bar{h}_{RP} = \sum_P n_e \bar{h}_e - \sum_R n_i \bar{h}_i$$

For combustion at $T = 77°F = 537°R$, $\bar{h} = \bar{h}_f^\circ + \cancelto{0}{\Delta\bar{h}}$. Also, for O_2, H_2, and N_2, $\bar{h}_f^\circ = 0$. Thus

$$\bar{h}_{RP}^\circ = 32(\bar{h}_f^\circ)_{CO_2} + 19.6(\bar{h}_f^\circ)_{H_2O(g)} - 3.8(\bar{h}_f^\circ)_{CH_4} - 0.1(\bar{h}_f^\circ)_{C_2H_6} - 4.8(\bar{h}_f^\circ)_{CO_2} - 23.2(\bar{h}_f^\circ)_{CO}$$

With data from Table A-25E

$$\bar{h}_{RP}^\circ = 32(-169{,}300) + 19.6(-104{,}040) - 3.8(-32{,}210) - 0.1(-36{,}420)$$
$$- 4.8(-169{,}300) - 23.2(-47{,}540)$$
$$= -5{,}415{,}176 \text{ Btu / 100 lbmol fuel}$$
$$= -54{,}152 \text{ Btu / lbmol fuel} \longleftarrow \bar{h}_{RP}^\circ$$

PROBLEM 13.54

KNOWN: Specified fuels react completely with air. The reactants and products are each at 25°C, 1atm.

FIND: Determine the higher heating value for (a) $C_8H_{18}(l)$, (b) $H_2(g)$, (c) $CH_3OH(l)$, (d) $C_4H_{10}(g)$, in kJ/kmol and kJ/kg. Compare with Table A-25 data.

ASSUMPTIONS: (1) Combustion is with the theoretical amount of air. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. (3) The combustion air and gaseous products of combustion can be modeled as ideal gases. The water formed on combustion is a liquid.

ANALYSIS: (a) $C_8H_{18}(l)$. The reaction equation for complete combustion with the theoretical amount of air is

$$C_8H_{18} + 12.5O_2 + 47N_2 \rightarrow 8CO_2 + 9H_2O + 47N_2$$

The enthalpy of combustion at 25°C, 1atm is

$$\bar{h}_{RP}^{o} = 8(\bar{h}_f^o)_{CO_2} + 9(\bar{h}_f^o)_{H_2O(l)} - (\bar{h}_f^o)_{C_8H_{18}}$$

$$= 8(-393,520) + 9(-285,830) - (-249,910) = -5,470,720 \text{ kJ/kmol}(C_8H_{18}).$$

Accordingly

$$\overline{HHV} = 5,470,720 \text{ kJ/kmol } (C_8H_{18}) \qquad (a)$$

$$HHV = \frac{5,470,720 \text{ kJ/kmol}(C_8H_{18})}{114.22 \text{ kg/kmol}(C_8H_{18})} = 47,896 \frac{\text{kJ}}{\text{kg}(C_8H_{18})}$$

This value agrees with the Table A-25 entry.

(b) $H_2(g)$. The reaction equation is

$$H_2 + \tfrac{1}{2}O_2 + 1.88N_2 \rightarrow H_2O + 1.88N_2$$

The enthalpy of combustion at 25°C, 1atm is

$$\bar{h}_{RP}^{o} = (\bar{h}_f^o)_{H_2O(l)} = -285,830 \text{ kJ/kmol}(H_2)$$

Accordingly

$$\overline{HHV} = 285,830 \text{ J/kmol }(H_2) \qquad (b)$$

$$HHV = \frac{285,830}{2.016} = 141,781 \frac{\text{kJ}}{\text{kg}(H_2)}$$

This value agrees with the Table A-25 entry.

(c) $CH_3OH(l)$. The reaction equation is

$$CH_3OH + 1.5(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 5.64N_2$$

The enthalpy of combustion at 25°C, 1atm is

$$\bar{h}_{RP}^{o} = 2(\bar{h}_f^o)_{H_2O(l)} + (\bar{h}_f^o)_{CO_2} - (\bar{h}_f^o)_{CH_3OH(l)}$$

$$= 2(-285,830) + (-393,520) - (-238,810) = -726,370 \text{ kJ/kmol}(CH_3OH)$$

Accordingly

$$\overline{HHV} = 726,370 \text{ kJ/kmol}(CH_3OH) \qquad (c)$$

$$HHV = \frac{726,370}{32.05} = 22,664 \frac{\text{kJ}}{\text{kg}(CH_3OH)}$$

This value agrees with the Table A-25 entry.

PROBLEM 13.54 (Contd.)

(d) $C_4H_{10}(g)$. The reaction equation is

$$C_4H_{10} + 6.5(O_2 + 3.76\,N_2) \rightarrow 4\,CO_2 + 5\,H_2O(l) + 24.44\,N_2$$

The enthalpy of combustion at 25°C, 1atm is

$$\bar{h}_{RP}^{o} = 4(\bar{h}_f^{o})_{CO_2} + 5(\bar{h}_f^{o})_{H_2O(l)} - (\bar{h}_f^{o})_{C_4H_{10}(g)}$$

$$= 4(-393{,}520) + 5(-285{,}830) - (-126{,}150)$$

$$= -2{,}877{,}080 \text{ kJ/kmol } (C_4H_{10})$$

Accordingly

$$\overline{HHV} = 2{,}877{,}080 \text{ kJ/kmol } (C_4H_{10}) \longleftarrow \text{(d)}$$

$$HHV = \frac{2{,}877{,}080}{58.12} = 49{,}502\ \frac{kJ}{kg\,(C_4H_{10})} \longleftarrow$$

This value agrees with the Table A-25 entry.

PROBLEM 13.55

KNOWN: A natural gas at 25°C, 1 atm has a molar analysis $\{86.5\%\ CH_4,\ 8\%\ C_2H_6,\ 2\%\ C_3H_8,\ 3.5\%\ N_2\}$

FIND: Determine $\overline{LHV}$ and LHV at 25°C, 1 atm.

ASSUMPTION: (1) Combustion is with the theoretical amount of air. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (2) The fuel, combustion air, and combustion products can be modeled as ideal gas mixtures. (3) The water formed on combustion is a vapor.

ANALYSIS: On the basis of 1 kmol of natural gas

$$(0.865\,CH_4 + 0.08\,C_2H_6 + 0.02\,C_3H_8 + 0.035\,N_2) + \alpha(O_2 + 3.76N_2) \longrightarrow \beta CO_2 + \gamma H_2O + \omega N_2$$

C: $0.865 + 2(0.08) + 3(0.02) = \beta$, $\beta = 1.085$.

H: $4(0.865) + 6(0.08) + 8(0.02) = 2\gamma$, $\gamma = 2.05$

$$\overline{LHV} = 0.865(\bar{h}_f^\circ)_{CH_4} + 0.08(\bar{h}_f^\circ)_{C_2H_6} + 0.02(\bar{h}_f^\circ)_{C_3H_8} - 1.085(\bar{h}_f^\circ)_{CO_2} - 2.05(\bar{h}_f^\circ)_{H_2O(g)}$$

$$= 0.865[-74,850] + 0.08[-84,680] + 0.02[-103,850] - 1.085[-393,520] - 2.05[-241,820]$$

$$= 849,104\ \frac{kJ}{kmol}$$ ← $\overline{LHV}$

The molecular weight of the fuel mixture is

$$M = 0.865(16.04) + 0.08(30.07) + 0.02(44.09) + 0.035(28.01) = 18.14$$

Then

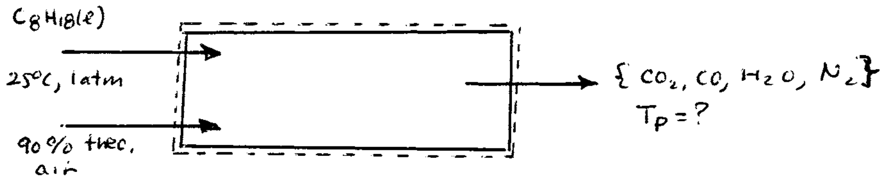
$$LHV = \frac{849,104}{18.14} = 46,808\ \frac{kJ}{kg}$$ ← LHV

PROBLEM 13.56

KNOWN: Operating data are provided for an insulated reactor in which $C_8H_{18}(l)$ is burned with 90% of theoretical air.

FIND: Determine the temperature of the exiting products.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and kinetic/potential energy effects can be ignored. (3) The ideal gas model is applicable to the combustion air and the combustion products. (4) 3.76 moles of inert N_2 accompany each mole of O_2 in the combustion air. N_2 is inert.

ANALYSIS: The balanced reaction equation for complete combustion with the theoretical amount of air is

$$C_8H_{18} + 12.5\,[O_2 + 3.76N_2] \rightarrow 8CO_2 + 9H_2O + 47N_2$$

For combustion with 90% of theoretical air

$$C_8H_{18} + 11.25O_2 + 42.3N_2 \rightarrow 5.5CO_2 + 2.5CO + 9H_2O + 42.3N_2$$

An energy rate balance reduces to read $\bar{h}_P = \bar{h}_R$. With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, this takes the form

$$5.5\,[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 2.5\,[\bar{h}_f^\circ + \Delta\bar{h}]_{CO} + 9\,[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 42.3\,[\Delta\bar{h}]_{N_2} = [\bar{h}_f^\circ]_{C_8H_{18}(l)}$$

or

$$5.5[\Delta\bar{h}]_{CO_2} + 2.5[\Delta\bar{h}]_{CO} + 9[\Delta\bar{h}]_{H_2O} + 42.3[\Delta\bar{h}]_{N_2} = [\bar{h}_f^\circ]_{C_8H_{18}(l)} - 5.5[\bar{h}_f^\circ]_{CO_2} - 2.5[\bar{h}_f^\circ]_{CO} - 9[\bar{h}_f^\circ]_{H_2O}$$

With data from the ideal gas tables

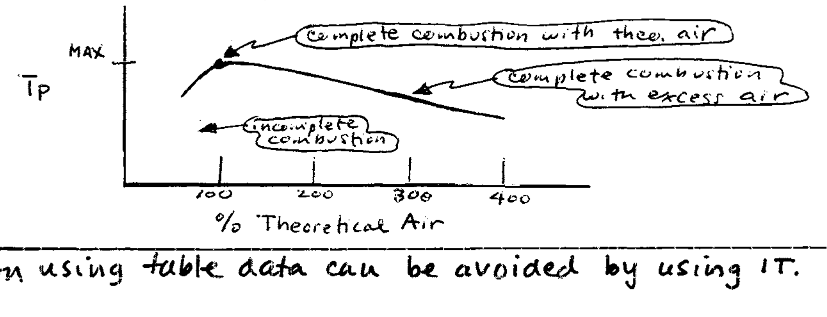
$$5.5\,[\bar{h}(T_p) - 9364]_{CO_2} + 2.5\,[\bar{h}(T_p) - 8669]_{CO} + 9\,[\bar{h}(T_p) - 9904]_{H_2O} + 42.3\,[\bar{h}(T_p) - 8669]_{N_2}$$

$$= [-249{,}910] - 5.5[-393{,}520] - 2.5[-110{,}530] - 9[-241{,}820]$$

$$= 4{,}367{,}155 \text{ kJ/kmol(fuel)}$$

① The above expression is a single equation in just one unknown: T_p. Solving iteratively using table data, $T_p = 2286$ K

Discussion. In Example 13.8, for the case of complete combustion with the theoretical amount of air $T_p = 2395$ K, and for complete combustion with 400% theoretical air $T_p = 962$ K. Accordingly, in accordance with the discussion at the close of Sec. 13.3



---

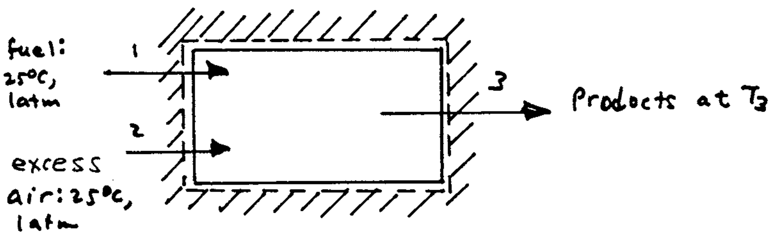
1. Iteration using table data can be avoided by using IT.

PROBLEM 13.57

KNOWN: Fuel at 25°C, 1 atm enters a reactor and burns completely with x percent excess air entering at 25°C, 1atm.

FIND: Plot the adiabatic flame temperature versus x for (a) C, (b) H_2, and (c) $C_8H_{18}(l)$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) For the control volume shown in the accompanying figure $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and the effects of kinetic and potential energy are negligible. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The ideal gas model is applicable to the combustion air and products.

ANALYSIS: To find the adiabatic flame temperature, use Eq. 13.21a

$$\sum_P n_e \bar{h}_e = \sum_R n_i \bar{h}_i \qquad (1)$$

When using IT, the enthalpies are evaluated directly at the respective inlet and exit temperatures. When using table data, Eq. (1) takes the form

$$\sum_P n_e(\bar{h}_f^\circ + \Delta\bar{h})_e = \sum_R n_i(\bar{h}_f^\circ + \Delta\bar{h})_i$$

or

$$\sum_P n_e(\Delta\bar{h})_e = \sum_R n_i(\Delta\bar{h})_i + \sum_R n_i \bar{h}_{fi}^\circ - \sum_P n_e \bar{h}_{fe}^\circ \qquad (2)$$

(a) Fuel is C: For complete combustion with x percent excess air

$$C + (1 + x/100)[O_2 + 3.76N_2] \rightarrow CO_2 + (x/100)O_2 + (1 + x/100)(3.76)N_2$$

Applying Eq. (2) with $\Delta\bar{h}_i = 0$ and $\bar{h}_f^\circ = 0$ for O_2 and N_2 and C.

$$[\bar{h}_{CO_2}(T_3) - \bar{h}_{CO_2}(298)] + (x/100)[\bar{h}_{O_2}(T_3) - \bar{h}_{O_2}(298)] + (1 + x/100)(3.76)[\bar{h}_{N_2}(T_3) - \bar{h}_{N_2}(298)] = 0$$

For $x = 100\%$; we can solve iteratively using table data to get $T_3 = 1506$ K.

(b) Fuel is H_2. $H_2 + \frac{1}{2}(1 + x/100)[O_2 + 3.76N_2] \rightarrow H_2O + \frac{x}{200}O_2 + \frac{1}{2}(1 + x/100)(3.76)N_2$

and

$$[\bar{h}(T_3) - \bar{h}(298)]_{H_2O} + (\tfrac{x}{200})[\bar{h}(T_3) - \bar{h}(298)]_{O_2} + \tfrac{1}{2}(1 + x/100)(3.76)[\bar{h}(T_3) - \bar{h}(298)]_{N_2} = -(\bar{h}_f^\circ)_{H_2O(g)}$$

For $x = 100\%$, we get $T_3 = 1647$ K

(c) Fuel is $C_8H_{18}(l)$: $C_8H_{18} + 12.5(1 + \frac{x}{100})[O_2 + 3.76N_2] \rightarrow 8CO_2 + 9H_2O + \frac{12.5x}{100}O_2 + 12.5(1 + \frac{x}{100})(3.76)N_2$

and

$$8[\bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 9[\bar{h}(T_3) - \bar{h}(298)]_{H_2O} + \frac{12.5x}{100}[\bar{h}(T_3) - \bar{h}(298)]_{O_2}$$
$$+ 12.5(1 + \tfrac{x}{100})(3.76)[\bar{h}(T_3) - \bar{h}(298)]_{N_2} = (\bar{h}_f^\circ)_{C_8H_{18}(l)} - 8(\bar{h}_f^\circ)_{CO_2} - 9(\bar{h}_f^\circ)_{H_2O(g)}$$

Solving for $x = 100\%$; $T_3 = 1507$ K.

PROBLEM 13.51 (Contd.)

The data for the required plots are obtained using IT, as follows:

IT Code

```
T = 25 + 273.15  // K
x = 1
PCT = x * 100
```

Part (a)

```
0 = - hCO2_3 - x * hO2_3 - (1+x) * 3.76 * hN2_3
hCO2_3 = h_T("CO2",Tp)
hO2_3 = h_T("O2", Tp)
hN2_3 = h_T("N2", Tp)
```

Result for x = 100%: Tp = 1507 K

Part (b)

```
0 = - hH2O_3 - (x/2) * hO2_3 - ((1+x) / 2) * 3.76 * hN2_3
hH2O_3 = h_T("H2O",Tp)
hO2_3 = h_T("O2",Tp)
hN2_3 = h_T("N2",Tp)
```

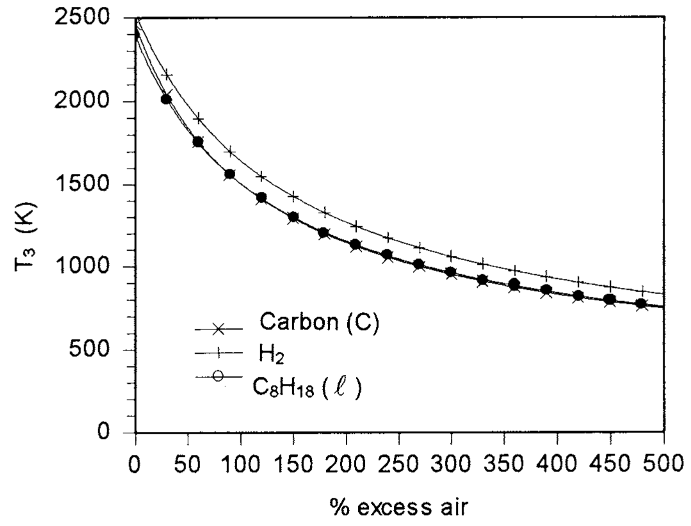
Result for x = 100%: Tp = 1647 K part (b)

Part (c)

```
0 = hC8H18_1 - 8 * hCO2_3 - 9 * hH2O_3 - 12.5 * x * hO2_3 - 12.5 * (1+x) * 3.76 * hN2_3
hC8H18_1 = -249910  // Table A-25
hCO2_3 = h_T("CO2",Tp)
hH2O_3 = h_T("H2O",Tp)
hO2_3 = h_T("O2",Tp)
hN2_3 = h_T("N2",Tp)
```

Result for x = 100%: Tp = 1507 K part(c)

PLOTS:



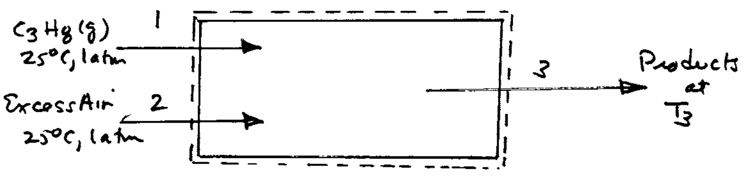
The excess air leads to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

PROBLEM 13.58

KNOWN: $C_3H_8(g)$ at 25°C, 1atm enters an insulated reactor and burns completely with air entering at 25°C, 1atm.

FIND: Plot the adiabatic flame temperature versus the percent of theoretical air varying from 100 to 400%

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{W}_{cv} = 0$, $\dot{Q}_{cv}$ and kinetic/potential energy effects are negligible. (3) Combustion is complete. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. The combustion air and combustion products can each be modeled as ideal gas mixtures

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8 + 5[O_2 + 3.76] \rightarrow 3\,CO_2 + 4H_2O + (5)(3.76)\,N_2$$

For n times the theoretical amount of air

$$C_3H_8 + 5n[O_2 + 3.76\,N_2] \rightarrow 3CO_2 + 4H_2O + 5(n-1)\,O_2 + 18.8n\ N_2$$

An energy rate balance reduces with listed assumptions to read

$$0 = (\bar{h}^\circ_f)_{C_3H_8} - 3[\bar{h}^\circ_f + \Delta h]_{CO_2} - 4[\bar{h}^\circ_f + \Delta\bar{h}]_{H_2O} - 5(n-1)[\Delta\bar{h}]_{O_2} - [\Delta\bar{h}]_{N_2} \qquad (1)$$

or

$$0 = (\bar{h}^\circ_f)_{C_3H_8} - 3[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} - 4[\bar{h}^\circ_f + \bar{h}(T_3) - \bar{h}(298)]_{H_2O}$$
$$- 5(n-1)[\bar{h}(T_3) - \bar{h}(298)]_{O_2} - 18.8n[\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

or

$$3\,\bar{h}_{CO_2}(T_3) + 4\,\bar{h}_{H_2O}(T_3) + 5(n-1)\,\bar{h}_{O_2}(T_3) + 18.8n\bar{h}_{N_2}(T_3) = (\bar{h}^\circ_f)_{C_3H_8} - 3[\bar{h}^\circ_f - \bar{h}(298)]_{CO_2}$$
$$- 4[\bar{h}^\circ_f - \bar{h}(298)]_{H_2O} + 5(n-1)\,\bar{h}_{O_2}(298) + 18.8n\,\bar{h}_{N_2}(298)$$

with ideal gas table data and $(\bar{h}^\circ_f)_{C_3H_8}$ from Table A-25

$$3\bar{h}_{CO_2}(T_3) + 4\bar{h}_{H_2O}(T_3) + 5(n-1)\bar{h}_{O_2}(T_3) + 18.8n\,\bar{h}_{N_2}(T_3) = -103,850 - 3[-393,520 - 9364]$$
$$- 4[-241,820 - 9904] - 5(n-1)(8682) - 18.8n(8669)$$

$$3\bar{h}_{CO_2}(T_3) + 4\bar{h}_{H_2O}(T_3) + 5(n-1)\bar{h}_{O_2}(T_3) + 18.8n\bar{h}_{N_2}(T_3) = 2,111,698 + 43,410(n-1) + 162,977n$$

Sample Calculation. Using Table A-23 data for the case $n = 1.2$, we get $T_3 = 2125\,K$.

The data for the required plot are obtained using IT, as follows:

PROBLEM 13.58 (Cont'd.)

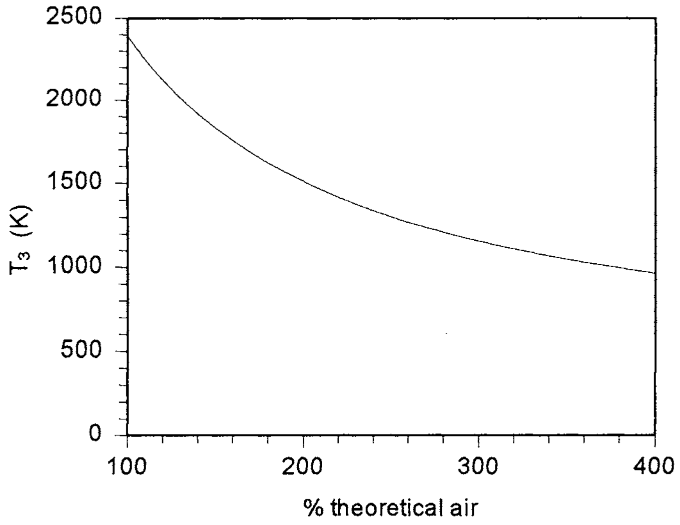
IT Code

```
n = 120  // % theoretical air

0 = hfo_C3H8 - 3 * hCO2_3 - 4 * hH2O_3 - 5*(n/100 - 1) * hO2_3 - 18.8 * n/100 * hN2_3
hfo_C3H8 = -103850  // Table A-25
hCO2_3 = h_T("CO2", T3)
hH2O_3 = h_T("H2O",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
```

IT Result for n = 120%

T_3 = 2124 K

PLOT:



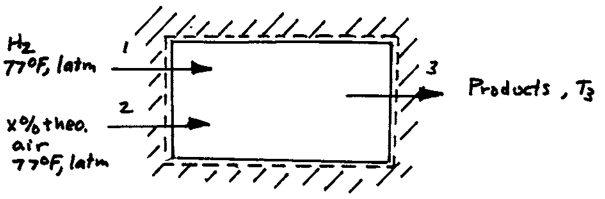
The excess air leads to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

PROBLEM 13.59

KNOWN: H_2 at 77°F, 1atm enters an insulated reactor and burns completely with x% of theoretical air entering at 77°F, 1atm.

FIND: Plot the adiabatic flame temperature for x ranging from 100 to 400 %.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of H_2 with the theoretical amount of air is described by

$$H_2 + 0.5O_2 + 1.88N_2 \rightarrow H_2O + 1.88N_2$$

Complete combustion with x percent of the theoretical amount of air is then

$$H_2 + 0.5x\,O_2 + 1.88x\,N_2 \rightarrow H_2O + 0.5(x-1)O_2 + 1.88x\,N_2$$

An energy rate balance at steady state reduces to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{H_2}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{H_2}}}^{0} + (\bar{h}_{H_2})_1 + (0.5x\,\bar{h}_{O_2} + 1.88x\,\bar{h}_{N_2})_2 - (\bar{h}_{H_2O} + 0.5(x-1)\bar{h}_{O_2} + 1.88x\,N_2)_3$$

With $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$, and noting that $\bar{h}_f^o = 0$ for H_2, O_2, and N_2, this becomes

$$0 = [\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 0.5(x-1)[\bar{h}(T_3) - \bar{h}(537)]_{O_2} + 1.88x[\bar{h}(T_3) - \bar{h}(537)]_{N_2}$$

with data from the ideal gas tables

$$0 = [-104{,}040 + \bar{h}_{H_2O}(T_3) - 4258] + 0.5(x-1)[\bar{h}_{O_2}(T_3) - 3725.1] + 1.88x[\bar{h}_{N_2}(T_3) - 3729.5]$$

or

$$\bar{h}_{H_2O}(T_3) + 0.5(x-1)\bar{h}_{O_2}(T_3) + 1.88x\,\bar{h}_{N_2}(T_3) = 108298 + (x-1)(1862.55) + x(7011.46) \qquad (1)$$

Sample Calculation: with data from Table A-23 for the case x = 2, we get $T_3 \approx 2965°R$

The data for the required plot are obtained using IT, as follows:

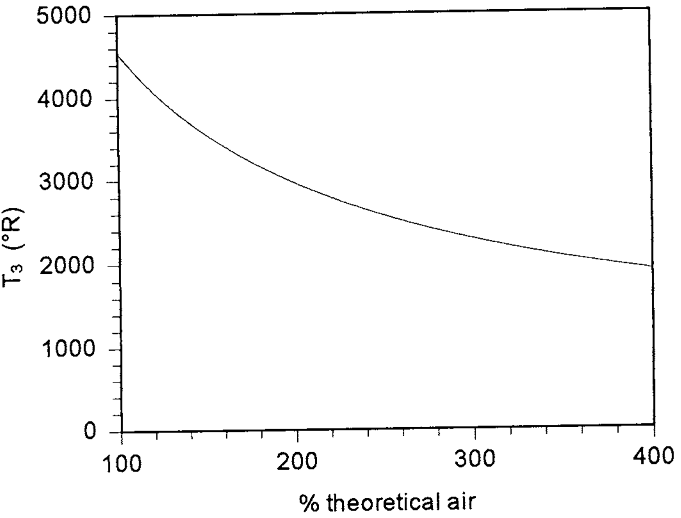
IT Code

```
x = 200  // % theoretical air

0 = hH2O_3 + 0.5 * (x/100 - 1) * hO2_3
     + 1.88 * x/100 * hN2_3
hH2O_3 = h_T("H2O",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
```

IT Result for x = 200%

T_3 = 2964 °R

PLOT:



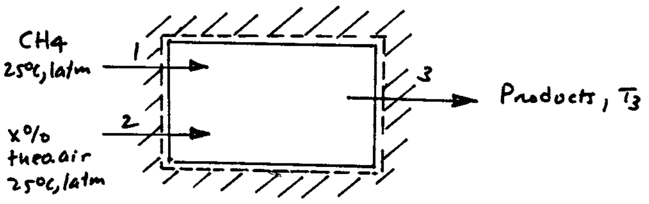
The excess air leads to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

PROBLEM 13.60

KNOWN: CH_4 at 25°C, 1atm enters an insulated reactor and burns completely with x % of theoretical air entering at 25°C, 1atm.

FIND: Plot the adiabatic flame temperature for x ranging from 100 to 400 %

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of CH_4 with the theoretical amount of air is given by Eq. 13.4. Complete combustion with x % of the theoretical amount of air is then

$$CH_4 + 2x\,O_2 + 7.52x\,N_2 \rightarrow CO_2 + 2H_2O + 2(x-1)O_2 + 7.52x\,N_2$$

An energy rate balance at steady state reduces to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{0} + (\bar{h}_{CH_4})_1 + (2x\bar{h}_{O_2} + 7.52x\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2(x-1)\bar{h}_{O_2} + 7.52x\bar{h}_{N_2}]_3$$

With $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$, and noting that $\bar{h}_f^o = 0$ for O_2 and N_2, this becomes

$$0 = (\bar{h}_f^o)_{CH_4} + (0) - \Big[[\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 2[\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(298)]_{H_2O} + 2(x-1)[\bar{h}(T_3) - \bar{h}(298)]_{O_2} + 7.52x[\bar{h}(T_3) - \bar{h}(298)]_{N_2}\Big]$$

or

$$\bar{h}_{CO_2}(T_3) + 2\bar{h}_{H_2O}(T_3) + 2(x-1)\bar{h}_{O_2}(T_3) + 7.52x\bar{h}_{N_2}(T_3) = (\bar{h}_f^o)_{CH_4} - [\bar{h}_f^o - \bar{h}(298)]_{CO_2} - 2[\bar{h}_f^o - \bar{h}(298)]_{H_2O} + 2(x-1)\bar{h}_{O_2}(298) + 7.52x\bar{h}_{N_2}(298)$$

With data from Tables A-23, A-25

$$\bar{h}_{CO_2}(T_3) + 2\bar{h}_{H_2O}(T_3) + 2(x-1)\bar{h}_{O_2}(T_3) + 7.52x\bar{h}_{N_2}(T_3) = -74{,}850 - [-393{,}520 - 9{,}364] - 2[-241{,}820 - 9{,}904] + 2(x-1)[8682] + 7.52x[8669]$$

$$= 831{,}482 + 17364(x-1) + 65191x$$

Sample calculation: when $x = 2$, $T_3 \approx 1481$ K

The data for the required plot are obtained using IT, as follows:

PROBLEM 13.60 (Cont'd.)

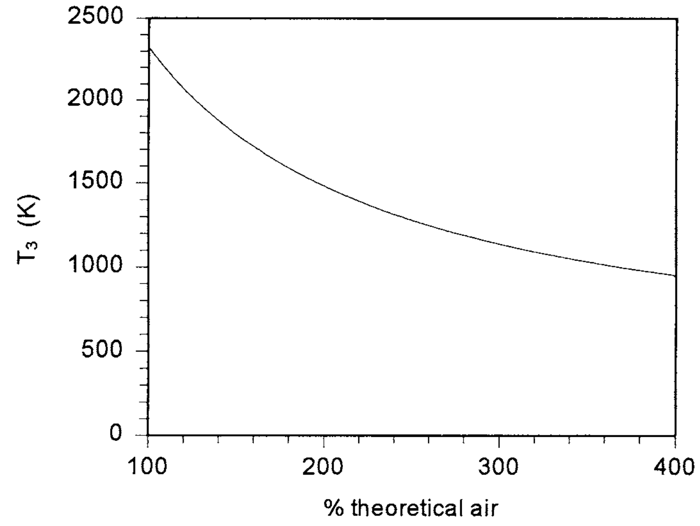
IT Code

```
T1 = 25 + 273.15  // K
x = 200  // % theoretical air

0 = hCH4_1 - (hCO2_3 + 2 * hH2O_3 + 2 * (x/100 - 1) * hO2_3 + 7.52 * x/100 * hN2_3)
hCH4_1 = h_T("CH4",T1)
hCO2_3 = h_T("CO2",T3)
hH2O_3 = h_T("H2O",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results for x = 200%

$T_3 = 1481$ K

PLOT:



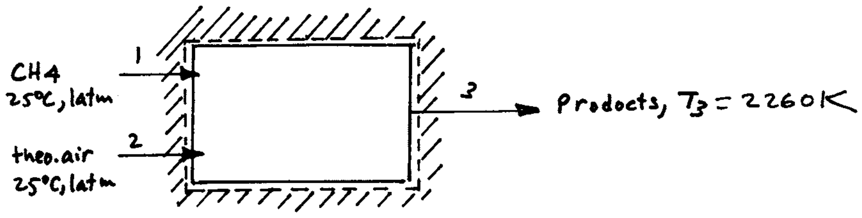
The excess air lead to more O_2 and N_2 in the products, thereby lowering the adiabatic flame temperature, as expected.

PROBLEM 13.61

KNOWN: CH_4 at 25°C, 1atm enters an insulated reactor operating at steady state and burns with the theoretical amount of air entering at 25°C, 1atm. The products, which contain CO_2, CO, H_2O, O_2, and N_2, exit at 2260K.

FIND: Determine the fractions of the entering carbon that burn to CO_2, CO.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Combustion is with the theoretical amount of air. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: The reaction equation takes the form

$$CH_4 + 2\,O_2 + 7.52\,N_2 \rightarrow x\,CO_2 + (1-x)\,CO + 2H_2O + \left(\frac{1-x}{2}\right) O_2 + 7.52\,N_2$$

At steady state, an energy rate balance reduces to read

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}}^{\,0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{\,0} + (\bar{h}_{CH_4})_1 + [2\bar{h}_{O_2} + 7.52\bar{h}_{N_2}]_2 - [x\bar{h}_{CO_2} + (1-x)\bar{h}_{CO} + 2\bar{h}_{H_2O} + \left(\tfrac{1-x}{2}\right)\bar{h}_{O_2} + 7.52\bar{h}_{N_2}]_3$$

with $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$, and noting that $\bar{h}_f^o = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^o)_{CH_4} + [0] - x[\bar{h}_f^o + \Delta\bar{h}]_{CO_2} - (1-x)[\bar{h}_f^o + \Delta\bar{h}]_{CO} - 2[\bar{h}_f^o + \Delta\bar{h}]_{H_2O} - \left(\tfrac{1-x}{2}\right)(\Delta\bar{h})_{O_2} - 7.52(\Delta\bar{h})_{N_2}$$

With data from Tables A-23, 25

$$0 = (-74{,}850) - x[-393{,}520 + (116{,}594 - 9364)] - (1-x)[-110{,}530 + (74{,}882 - 8669)] - 2[-241{,}820 + (96089 - 9904)] - \left(\tfrac{1-x}{2}\right)[77781 - 8682] - 7.52[74220 - 8669]$$

or

$$0 = (-74{,}850) - x[-286{,}290] - (1-x)[-44{,}317] - 2[-155{,}635] - \left(\tfrac{1-x}{2}\right)(69{,}099) - 492{,}944$$

$$0 = (-74{,}850) - (-44{,}317) - 2(-155{,}635) - \frac{69{,}099}{2} - 492{,}944 - x\left(-286{,}290 + 44{,}317 - \frac{69{,}099}{2}\right)$$

$$x = \frac{(-74{,}850) + 44{,}317 + 2(155{,}635) - 69{,}099/2 - 492{,}944}{(-286{,}290 + 44{,}317 - 69{,}099/2)}$$

$$= \frac{-246{,}757}{-276{,}523} = 0.892 \Rightarrow$$

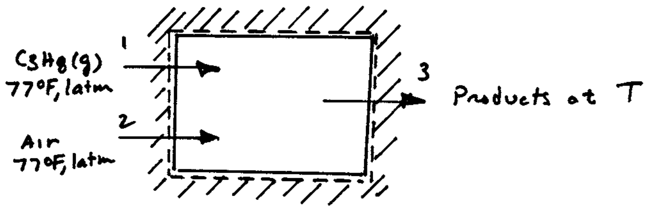
89.2% of carbon to CO_2
10.8% of carbon to CO ←

PROBLEM 13.62

KNOWN: $C_3H_8(g)$ at 77°F, 1atm enters an insulated reactor and burns completely with air entering at 77°F, 1atm.

FIND: Determine the percent of theoretical air if products exit at (a) 1140°F, (b) 2240°F.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Complete combustion occurs. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8 + 5(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 18.8N_2$$

Complete combustion with x times the theoretical amount of air is described by

$$C_3H_8 + 5x(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 5(x-1)O_2 + 18.8x\,N_2$$

An energy rate balance at steady state reduces to

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{\,0}}{\dot{n}_{fuel}} - \frac{\cancel{\dot{W}_{cv}}^{\,0}}{\dot{n}_{fuel}} + (\bar{h}_{C_3H_8})_1 + (5x\bar{h}_{O_2} + 18.8x\bar{h}_{N_2})_2 - (3\bar{h}_{CO_2} + 4\bar{h}_{H_2O} + 5(x-1)\bar{h}_{O_2} + 18.8x\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{C_3H_8} + (0) - 3[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} - 4[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O}$$
$$- 5(x-1)[\bar{h}(T) - \bar{h}(537)]_{O_2} - 18.8x[\bar{h}(T) - \bar{h}(537)]_{N_2}$$

Accordingly

$$x = \frac{(\bar{h}_f^\circ)_{C_3H_8} - 3[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{CO_2} - 4[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)]_{H_2O} + 5[\bar{h}(T) - \bar{h}(537)]_{O_2}}{5[\bar{h}(T) - \bar{h}(537)]_{O_2} + 18.8[\bar{h}(T) - \bar{h}(537)]_{N_2}}.$$

(a) T = 1600°R (1140°F). With data from the ideal gas tables and Table A-25

$$x = \frac{-44680 - 3[-169,300 + 15829 - 4028] - 4[-104040 + 13494 - 4258] + 5[11833 - 3725]}{5[11833 - 3725] + 18.8[11410 - 3730]}$$

$= 4.58$ (458% of theoretical air)

(b) T = 2700°R (2240°F)

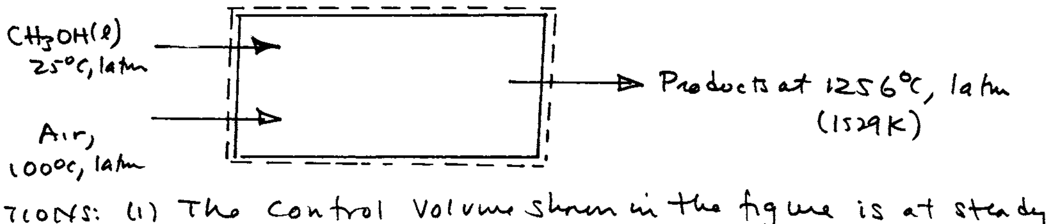
$$x = \frac{-44,680 - 3[-169,300 + 30,581 - 4028] - 4[-104,040 + 24957 - 4258] + 5[21,183 - 3725]}{5(21,183 - 3725) + 18.8(20,246 - 3730)}$$

$= 2.02$ (202% of theoretical air)

PROBLEM 13.63

KNOWN: $CH_3OH(l)$ at 25°C, 1 atm enters an insulated reactor operating at steady state and burns completely with air entering at 100°C, 1atm.

FIND: If combustion products exit at 1256°C, determine the percent excess air used.



ASSUMPTIONS: (1) The control volume shown in the figure is at steady state. (2) For the control volume $\dot{Q}_{cv}=0$, $\dot{W}_{cv}=0$, and kinetic/potential energy effects can be ignored. (3) Combustion is complete. 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (4) The combustion air and the combustion products can be modeled as ideal gas mixtures.

ANALYSIS: Complete combustion of CH_3OH with the theoretical amount of air is described by

$$CH_3OH + \tfrac{3}{2}(O_2 + 3.76 N_2) \rightarrow CO_2 + 2H_2O + (\tfrac{3}{2})(3.76) N_2$$

Complete combustion with x times the theoretical amount of air is

$$CH_3OH + \tfrac{3}{2}x[O_2 + 3.76 N_2] \rightarrow CO_2 + 2H_2O + \tfrac{3}{2}(x-1)O_2 + 5.64x\, N_2$$

With the given assumptions, an energy rate balance reduces to read

$$0 = (\bar{h}_f^o)_{fuel} + \tfrac{3}{2}x[\Delta\bar{h}]_{O_2} + 5.64[\Delta\bar{h}_{N_2}] - \Big[(\bar{h}_f^o + \Delta\bar{h})_{CO_2} + 2(\bar{h}_f^o + \Delta\bar{h})_{H_2O} + \tfrac{3}{2}(x-1)(\Delta\bar{h})_{O_2} + 5.64x(\Delta\bar{h})_{N_2}\Big]$$

where $(\bar{h}_f^o)_{O_2} = (\bar{h}_f^o)_{N_2} = 0$ are not shown. That is

$$0 = (\bar{h}_f^o)_{fuel} + \tfrac{3}{2}x[\bar{h}(373) - \bar{h}(298)]_{O_2} + 5.64[\bar{h}(373) - \bar{h}(298)]_{N_2} - [\bar{h}_f^o + \bar{h}(1529) - \bar{h}(298)]_{CO_2}$$
$$- 2[\bar{h}_f^o + \bar{h}(1529) - \bar{h}(298)]_{H_2O} - \tfrac{3}{2}(x-1)[\bar{h}(1529) - \bar{h}(298)]_{O_2} - 5.64x[\bar{h}(1529) - \bar{h}(298)]_{N_2}$$

$$0 = (\bar{h}_f^o)_{fuel} - [\bar{h}_f^o + \bar{h}(1529) - \bar{h}(298)]_{CO_2} - 2[\bar{h}_f^o + \bar{h}(1529) - \bar{h}(298)]_{H_2O} - \tfrac{3}{2}x[\bar{h}(1529) - \bar{h}(373)]_{O_2}$$
$$- 5.64x[\bar{h}(1529) - \bar{h}(373)]_{N_2} + \tfrac{3}{2}[\bar{h}(1529) - \bar{h}(298)]_{O_2}$$

Solving for x

$$x = \frac{(\bar{h}_f^o)_{fuel} - [\bar{h}_f^o + h(1529) - \bar{h}(298)]_{CO_2} - 2[\bar{h}_f^o + \bar{h}(1529) - \bar{h}(298)]_{H_2O} + \tfrac{3}{2}[\bar{h}(1529) - \bar{h}(298)]_{CO_2}}{5.64[\bar{h}(1529) - \bar{h}(373)]_{N_2} + \tfrac{3}{2}[\bar{h}(1529) - \bar{h}(373)]_{O_2}}$$

$$= \frac{-238{,}810 - [-393520 + 72773 - 9364] - 2[-241{,}820 + 59368 - 9904] + \tfrac{3}{2}[50353 - 8682]}{5.64[48{,}086 \quad 10{,}851] + 3/2\,[50353 - 10899]}$$

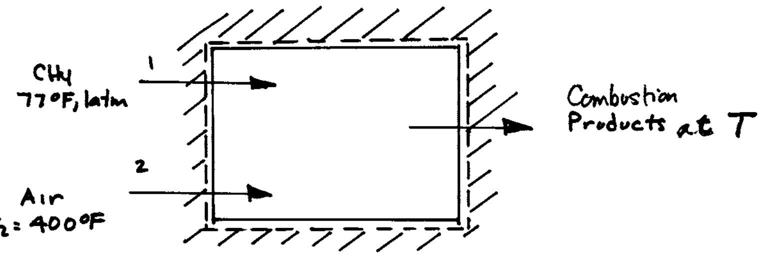
$= 2.0$ (100% excess air) ◄

PROBLEM 13.64

KNOWN: Steady state operating data are provided for the combustor of a gas turbine power plant.

FIND: Plot the percent excess air versus the combustion product temperature for $1400 \le T \le 1800\,°F$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Complete combustion with excess air occurs. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and combustion products can be modeled as ideal gases.

ANALYSIS: Complete combustion of CH_4 with excess air is described by

$$CH_4 + 2x(O_2 + 3.76) \rightarrow CO_2 + 2H_2O + 2(x-1)O_2 + 7.52x\,N_2$$

For combustion with the theoretical amount of air, x=1.

An energy rate balance at steady state reduces to

$$0 = \frac{\cancelto{0}{\dot{Q}_{cv}}}{\cancel{\dot{n}_{CH_4}}} - \frac{\cancelto{0}{\dot{W}_{cv}}}{\cancel{\dot{n}_{CH_4}}} + (\bar{h}_{CH_4})_1 + (2x\bar{h}_{O_2} + 7.52x\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2(x-1)\bar{h}_{O_2} + 7.52x\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}^\circ_f + \Delta\bar{h}$, and noting that $\bar{h}^\circ_f = 0$ for O_2 and N_2, this becomes

$$\begin{aligned} 0 = {} & (\bar{h}^\circ_f)_{CH_4} + 2x[\bar{h}(860) - \bar{h}(537)]_{O_2} + 7.52x[\bar{h}(860) - \bar{h}(537)]_{N_2} - (\bar{h}^\circ_f + \bar{h}(T) - \bar{h}(537))_{CO_2} \\ & - 2[\bar{h}^\circ_f + \bar{h}(T) - \bar{h}(537)]_{H_2O} - 2(x-1)[\bar{h}(T) - \bar{h}(537)]_{O_2} - 7.52x(\bar{h}(T) - \bar{h}(537))_{N_2} \\ = {} & (\bar{h}^\circ_f)_{CH_4} - (\bar{h}^\circ_f + \bar{h}(T) - \bar{h}(537))_{CO_2} - 2(\bar{h}^\circ_f + \bar{h}(T) - \bar{h}(537))_{H_2O} - 2x[\bar{h}(T) - \bar{h}(860)]_{O_2} \\ & + 2[\bar{h}(T) - \bar{h}(537)]_{O_2} - 7.52x[\bar{h}(T) - \bar{h}(860)] \end{aligned}$$

Introducing data from the ideal gas tables and Table A-25, and solving for x

$$x = \frac{(\bar{h}^\circ_f)_{CH_4} - [\bar{h}^\circ_f + \bar{h}(T) - \bar{h}(537)]_{CO_2} - 2[\bar{h}^\circ_f + \bar{h}(T) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(T) - \bar{h}(537)]_{O_2}}{2[\bar{h}(T) - \bar{h}(860)]_{O_2} + 7.52[\bar{h}(T) - \bar{h}(860)]_{N_2}}$$

$$= \frac{(-32{,}210) - [-169{,}300 + \bar{h}(T) - 4028]_{CO_2} - 2[-104{,}040 + \bar{h}(T) - 4258] + 2[\bar{h}(T) - 3725]_{O_2}}{2[\bar{h}(T) - 6042]_{O_2} + 7.52[\bar{h}(T) - 5986]_{N_2}}$$

$$x = \frac{-\bar{h}_{CO_2}(T) - 2\bar{h}_{H_2O}(T) + 2\bar{h}_{O_2}(T) + 350{,}264}{2\bar{h}_{O_2}(T) + 7.52\bar{h}_{N_2}(T) - 57099}$$

Sample Calculation: T = 1860°R (1400°F):

$$x = \frac{-19173 - 2(16028) + 2(13987) + 350{,}264}{2(13987) + 7.52(13427) - 57099} = 4.55 \qquad \text{(355\% excess air)}$$

PROBLEM 13.64 (Cont'd.)

The data for the required plot are obtained using IT, as follows:

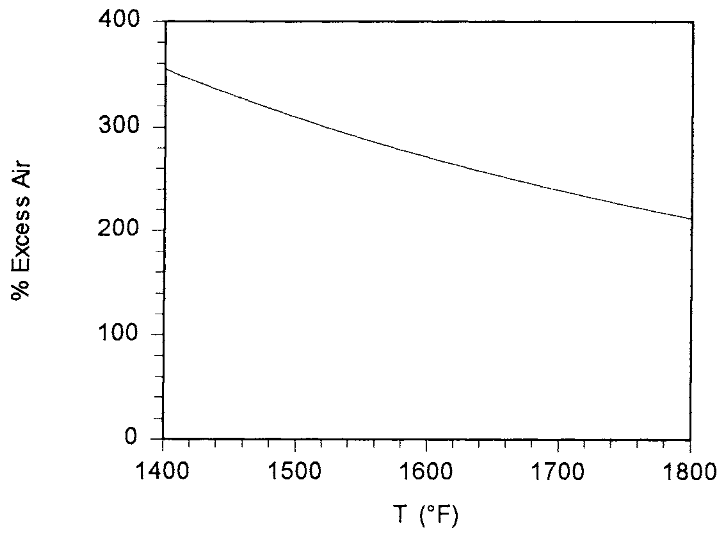
IT Code

```
T1 = 77  // °F
T2 = 400  // °F
T = 1400  // °F

0 = hCH4_1 + 2 *x*(hO2_2 + 3.76 * hN2_2) -
        (hCO2_3 + 2 * hH2O_3 + 2*(x - 1)*hO2_3 + 7.52*x*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T)
hH2O_3 = h_T("H2O",T)
hO2_3 = h_T("O2",T)
hN2_3 = h_T("N2",T)
PCTexcess = (x - 1) * 100
```

IT Results for T = 1400°F

% Excess = 354.7%

PLOT:



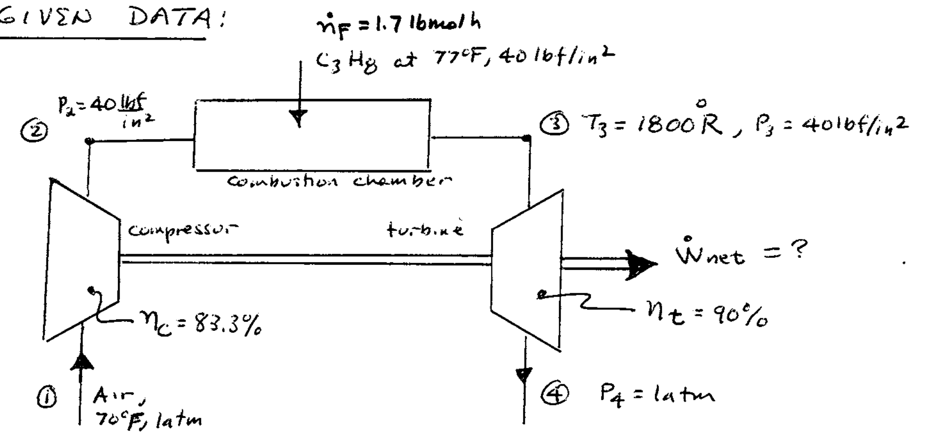
Notice that excess air leads to a decrease in the temperature of the combustion products, as expected.

PROBLEM 13.65

KNOWN: Operating data are provided for a simple gas turbine operating at steady state. C_3H_8 burns completely in the combustion chamber.

FIND: For a fuel consumption rate of 1.7 lbmol/h, determine (a) the percent of theoretical air, (b) the power developed, in hp.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) Control volumes enclosing various components, as required, are at steady state. (2) The compressor, turbine, and combustion chamber are adiabatic. (3) The fuel, air, and combustion products are modeled as ideal gases. (4) Kinetic and potential energy effects are negligible. (5) 3.76 moles of inert N_2 accompany each mole of O_2 in the combustion air. (6) Combustion is complete.

ANALYSIS: (a) Letting n denote the multiple of the theoretical amount of air, the balanced reaction equation reads

$$C_3H_8 + 5n[O_2 + 3.76N_2] \rightarrow 3CO_2 + 4H_2O + 5(n-1)O_2 + 18.8n\,N_2 \qquad (1)$$

An energy rate balance for the combustion chamber reads $\bar{h}_P = \bar{h}_R$. Solving this gives n, but T_2 is required. To find T_2, use the given compressor efficiency: $\eta_c = 0.833 = (h_{2s}-h_1)/(h_2-h_1)$. To determine T_{2s}, consider an isentropic compression of the assumed air mixture from $T_1 = 530°R$, P_1 = 1 atm to T_{2s}, 40 lbf/in². Then,

$$0 = \bar{s}_2 - \bar{s}_1 = 1\left[\bar{s}^\circ_{O_2}(T_{2s}) - \bar{s}^\circ_{O_2}(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right] + 3.76\left[\bar{s}^\circ_{N_2}(T_{2s}) - \bar{s}^\circ_{N_2}(T_1) - \bar{R}\ln\frac{P_2}{P_1}\right]$$

$$\Rightarrow \quad \bar{s}^\circ_{O_2}(T_{2s}) + 3.76\,\bar{s}^\circ(T_{2s}) = \bar{s}^\circ_{O_2}(T_1) + 3.76\,\bar{s}^\circ_{N_2}(T_1) + 4.76\bar{R}\ln\frac{P_2}{P_1}$$

$$= 48.889 + 3.76(45.65) + (4.76)(1.986)\ln\frac{40}{14.7}$$

$$= 230 \text{ Btu/lb·°R}$$

① Solving iteratively, T_{2s} = 705°R. Accordingly,

②
$$\bar{h}_2 - \bar{h}_1 = \frac{\bar{h}_{2s} - \bar{h}_1}{\eta_c} = \frac{1(\bar{h}_{O_2}(705) - \bar{h}_{O_2}(530)) + 3.76(\bar{h}_{N_2}(705) - \bar{h}_{N_2}(530))}{0.833}$$

$$= \frac{1(4915 - 3676) + 3.76(4900 - 3681)}{0.833} = 6990 \frac{\text{Btu}}{\text{lbmol}(O_2)}$$

note

Then, with $\bar{h}_2 - \bar{h}_1 = 6990$, or

$$1[\bar{h}_{O_2}(T_2) - h_{O_2}(T_1)] + 3.76[\bar{h}_{N_2}(T_2) - \bar{h}_{N_2}(T_1)] = 6990$$

$$\bar{h}_{O_2}(T_2) + 3.76\,h_{N_2}(T_2) = 6990 + 3676 + 3.76(3681) = 24507 \frac{\text{Btu}}{\text{lbmol}(O_2)}$$

② Solving for T_2, $T_2 \approx 740°R$.

Returning to the energy rate balance $\bar{h}_P = \bar{h}_R$, and using $\bar{h} = \bar{h}^\circ_f + \Delta\bar{h}$

$$3[\bar{h}^\circ_f + \Delta\bar{h}]_{CO_2} + 4[\bar{h}^\circ_f + \Delta\bar{h}]_{H_2O} + 5(n-1)[\Delta\bar{h}]_{O_2} + 18.8n[\Delta\bar{h}]_{N_2} = [\bar{h}^\circ_f]_{C_3H_8} + 5n[\Delta\bar{h}]_{O_2} + 18.8n[\Delta\bar{h}]_{N_2}$$

Then with data from the ideal gas tables and Table A-25

PROBLEM 13.65 (Continued)

$$3[-169{,}300+[18392-4028]]+4[-104{,}040+(15433-4258)]+5(n-1)[13486-3725]+18.8n[12956-3730]$$

$$=-44{,}680+5n[5167-3725]+18.8n\,[5144-3730]$$

Solving, $n = 4.46 \Rightarrow 446\%$ of theoretical air. $\longleftarrow$ (a)

(b) The net power developed is the difference between the turbine power output and the compressor power input. To find the turbine output

$$\eta_t = \frac{\dot{W}_t/\dot{n}_{fuel}}{(\dot{W}_t/\dot{n}_{fuel})_s} \Rightarrow \frac{\dot{W}_t}{\dot{n}_{fuel}} = \eta_t\left(\frac{\dot{W}_t}{\dot{n}_{fuel}}\right)_s \qquad (2)$$

Accordingly, consider next an isentropic expansion of the combustion products from (1800°R, 40 lbf/in²) to (T_{4s}, 1 atm): $\bar{s}_3 - \bar{s}_{4s} = 0$, That is

$$0 = 3\left[\bar{s}^\circ(1800)-\bar{s}^\circ(T_{4s})-\bar{R}\ln\frac{40}{14.7}\right]_{CO_2} + 4\left[\bar{s}^\circ(1800)-\bar{s}^\circ(T_{4s})-\bar{R}\ln\frac{40}{14.7}\right] +$$
$$17.3\left[\bar{s}^\circ(1800)-\bar{s}^\circ(T_{4s})-\bar{R}\ln\frac{40}{14.7}\right]_{O_2} + 83.848\left[\bar{s}^\circ(1800)-\bar{s}^\circ(T_{2s})-\bar{R}\ln\frac{40}{14.7}\right]_{N_2}$$

Or, on rewriting this as

$$3\,\bar{s}^\circ_{CO_2}(T_{4s}) + 4\,\bar{s}^\circ_{H_2O}(T_{4s}) + 17.3\,\bar{s}^\circ_{O_2}(T_{4s}) + 83.848\,\bar{s}^\circ_{N_2}(T_{4s})$$

$$= 3\,\bar{s}^\circ_{CO_2}(1800) + 4\,\bar{s}^\circ_{H_2O}(1800) + 17.3\,\bar{s}^\circ_{O_2}(1800) + 83.848\,\bar{s}^\circ_{N_2}(1800) - 108.148\bar{R}\ln\frac{40}{14.7}$$

$$= 3(64.292)+4(55.559)+17.3(58.155)+83.848(54.472) - 215$$

$$= 5773.6\ \frac{\text{Btu}}{\text{lbmol}\cdot{}^\circ\text{R}}$$

② Solving $T_{4s} \approx 1400$°R. The power that would be developed, per mole of fuel consumed, in an isentropic expansion is then

$$\left[\frac{\dot{W}_t}{\dot{n}_{fuel}}\right]_s = 3[\bar{h}(1800)-\bar{h}(1400)]_{CO_2} + 4[\bar{h}(1800)-\bar{h}(1400)]_{H_2O} + 17.3[\bar{h}(1800)-\bar{h}(1400)]_{O_2}$$
$$+ 83.848[\bar{h}(1800)-\bar{h}(1400)]_{N_2}$$

$$= 3[18392-13345]+4[15433-11625]+17.3[13486-10210]+83.848[12956-9897]$$

$$= 343{,}539\ \frac{\text{Btu}}{\text{lbmol (fuel)}}$$

Eq. (2) gives

$$\left(\frac{\dot{W}_t}{\dot{n}_{fuel}}\right) = 0.9(343{,}539) = 309{,}185\ \frac{\text{Btu}}{\text{lbmol (fuel)}}$$

For the compressor

$$\frac{\dot{W}_c}{\dot{n}_{fuel}} = (\bar{h}_2-\bar{h}_1)\left(\frac{\text{lbmol}(O_2)}{\text{lbmol (fuel)}}\right) = (6990)\left((5)(4.46)\right) = 155{,}877\ \frac{\text{Btu}}{\text{lbmol (fuel)}}$$

Finally, the net power output is

$$\dot{W}_{net} = \dot{n}_{fuel}\left[\frac{\dot{W}_t}{\dot{n}_{fuel}} - \frac{\dot{W}_c}{\dot{n}_{fuel}}\right]$$

$$= 1.7\ \frac{\text{lbmol (fuel)}}{\text{h}}\left[309{,}185-155{,}877\right]\left(\frac{\text{Btu}}{\text{lbmol (fuel)}}\right)\left|\frac{1\ \text{hp}}{2545\ \text{Btu/h}}\right| = 102.4\ \text{hp} \longleftarrow \text{(b)}$$

---

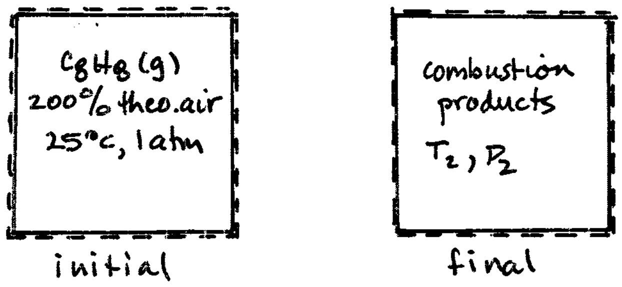
1. By ignoring the slight difference between the combustion air model and the air of Table A-22, Table A-22 data can be used to analyze the compressor. Thus, for example, $p_{r,2s} = (p_2/p_1)p_{r1} = (40/14.7)(1.3) = 3.537 \Rightarrow T_{2s} = 705$°R.
2. An iterative solution using table data can be avoided by using IT.

PROBLEM 13.66

KNOWN: A mixture of gaseous C_8H_{18} and 200% of theoretical air, initially at 25°C, 1 atm, reacts completely in a rigid vessel.

FIND: (a) If the vessel is well-insulated, determine the temperature and pressure of the combustion products. (b) If the combustion products are cooled to 25°C, determine the final pressure and the heat transfer per kmol of fuel.

SCHEMATIC & GIVEN DATA:



(a) $Q = 0$; $T_2 = ?$, $P_2 = ?$
(b) $T_2 = T_1$, $P_2 = ?$, $Q/n_F = ?$

ASSUMPTIONS: (1) For the closed system shown, (a) $Q = 0$, (b) $T_2 = T_1$, and $W = 0$. There are no effects of kinetic or potential energy. (2) Combustion is complete. (3) 3.76 kmol of N_2 accompany each kmol of O_2, and the nitrogen is inert. (4) The initial fuel-air mixture and the gaseous products of combustion can be modeled as ideal gas mixtures.

ANALYSIS: (a) Assuming 1 kmol of fuel initially present, the reaction is described by

$$C_8H_{18} + 25\,(O_2 + 3.76N_2) \rightarrow 8CO_2 + 9H_2O + 12.5O_2 + 94N_2 \qquad (1)$$

The energy balance reduces to $\Delta U + \cancel{\Delta KE}^{\,0} + \cancel{\Delta PE}^{\,0} = \cancel{Q}^{\,0} - \cancel{W}^{\,0} \Rightarrow \Delta U = 0$. Thus

$$0 = \left(8\bar{u}_{CO_2} + 9\bar{u}_{H_2O} + 125\,\bar{u}_{O_2} + 94\,\bar{u}_{N_2}\right)_2 - \left(\bar{u}_{C_8H_{18}} + 25\,\bar{u}_{O_2} + 94\,\bar{u}_{N_2}\right)_1$$

With $\bar{u} = \bar{h} - \bar{R}T$ this becomes

$$0 = \left(8\bar{h}_{CO_2} + 9\bar{h}_{H_2O} + 12.5\bar{h}_{O_2} + 94\bar{h}_{N_2}\right)_2 - \left(\bar{h}_{C_8H_{18}} + 25\bar{h}_{O_2} + 94\bar{h}_{N_2}\right)_1 - (8+9+12.5+94)\bar{R}T_2 + (1+25+94)\bar{R}T_1$$

Introducing $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = 8\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)\right]_{CO_2} + 9\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)\right]_{H_2O(g)} + 12.5\left[\bar{h}(T_2) - \bar{h}(298)\right]_{O_2}$$
$$+ 94\left[\bar{h}(T_2) - \bar{h}(298)\right]_{N_2} - \left(\bar{h}_f^\circ\right)_{C_8H_{18}(g)} - \bar{R}\left[123.5\,T_2 - 120\,T_1\right]$$

With data from Tables A-23 and A-25

$$8\bar{h}_{CO_2}(T_2) + 9\bar{h}_{H_2O}(T_2) + 12.5\bar{h}_{O_2}(T_2) + 94\bar{h}_{N_2}(T_2) - (123.5)(8.314)T_2 = (-8)(-393{,}520 - 9364)$$
$$-9(-241{,}820 - 9904) + 12.5(8682) + 94(8669) - 208{,}450 - (120)(8.314)(298)$$
$$= 5{,}906{,}240$$

① Solving iteratively for T_2, $T_2 = 1859$ K. ← (a) T_2

Using the ideal gas equation of state

$$\left.\begin{array}{l} P_1V = n_1\bar{R}T_1 \\ P_2V = n_2\bar{R}T_2 \end{array}\right\} \frac{P_2}{P_1} = \frac{n_2}{n_1}\cdot\frac{T_2}{T_1} \Rightarrow P_2 = \frac{n_2}{n_1}\cdot\frac{T_2}{T_1}\cdot P_1 = \left(\frac{123.5}{120}\right)\left(\frac{1859}{298}\right)(1\text{ atm})$$

$= 6.42$ atm ← (a) P_2

PROBLEM 13.66 (Contd.)

(b) The reaction is described by Eq. (1). For 1 kmol of fuel, the products contain 9 kmol of H_2O and $n_{dry} = 114.5$ kmol. When the products cool to 25°C, condensation occurs. Thus, if n_v is the amount of water vapor and n_ℓ the amount of saturated liquid water at the final state

$$n_v + n_\ell = 9 \text{ kmol}$$

Further, the partial pressure of the water vapor is

$$p_v = p_g(25°C) = 0.03169 \text{ bar} = \left(\frac{n_v}{114.5+n_v}\right) p_2 \qquad (2)$$

Both p_2 and n_v are unknown. Another relation is obtained based on the assumption that the volume occupied by the liquid phase is negligible. Then

$$\frac{n_1 \bar{R} T}{p_1} = \frac{(n_{dry}+n_v)\bar{R}T}{p_2} \Rightarrow p_2 = \left(\frac{n_{dry}+n_v}{n_1}\right) p_1$$

$$= \left(\frac{114.5+n_v}{120}\right)(1.01325 \text{ bar}) \qquad (3)$$

Solving (2) and (3) simultaneously

$$n_v = 3.753 \text{ kmol} \quad , \quad p_2 = 0.9985 \text{ bar} \longleftarrow \text{(b) } p_2$$

$$n_\ell = 9 - n_v = 5.247 \text{ kmol}$$

Therefore

$$C_8H_{18}(g) + 25(O_2 + 3.76N_2) \rightarrow 8CO_2 + (3.753\, H_2O(g) + 5.247\, H_2O(\ell)) + 12.5O_2 + 94N_2$$

The energy balance reduces to $\Delta U = Q$. Thus

$$Q = (8\bar{u}_{CO_2} + 3.753\bar{u}_{H_2O(g)} + 5.247\bar{u}_{H_2O(\ell)} + 12.5\bar{u}_{O_2} + 94\bar{u}_{N_2})_2 - (\bar{u}_{C_8H_{18}} + 25\bar{u}_{O_2} + 94\bar{u}_{N_2})_1$$

With $\bar{u} = \bar{h} - \bar{R}T$ for the gas components and $\bar{u} = \bar{h} - \cancel{p\bar{v}}^{\,0}$ for the liquid

$$Q = (8\bar{h}_{CO_2} + 3.753\bar{h}_{H_2O(g)} + 12.5\bar{h}_{O_2} + 94\bar{h}_{N_2})_2 - (\bar{h}_{C_8H_{18}} + 25\bar{h}_{O_2} + 94\bar{h}_{N_2})_1$$

$$+ 5.247\,\bar{h}_{H_2O(\ell)} - (123.5 - 5.247 - 120)(8.314)(298)$$

Introducing $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2, and setting the $\Delta\bar{h}$ terms to zero since $T_2 = T_1$

$$Q = 8(\bar{h}_f^\circ)_{CO_2} + 3.753(\bar{h}_f^\circ)_{H_2O(g)} - (\bar{h}_f^\circ)_{C_8H_{18}(g)} + 5.247(\bar{h}_f^\circ)_{H_2O(\ell)} - (-1.747)(8.314)(298)$$

$$= 8(-393{,}520) + 3.753(-241{,}820) - (-208{,}450) + 5.247(-285{,}830) + 4328.3$$

$$= -5.343 \times 10^6 \text{ kJ/kmol (fuel)} \longleftarrow \text{(b) } Q$$

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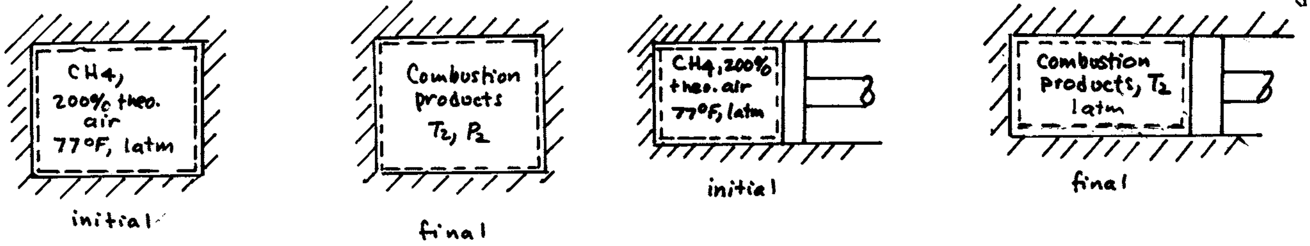
1. Iteration using table data can be avoided by using IT.

PROBLEM 13.67

KNOWN: A mixture of CH_4 and 200% of theoretical air, initially at 77°F, 1atm, reacts completely in an insulated vessel. Two cases are considered: (a) reaction at constant volume, (b) reaction at constant pressure.

FIND: Determine the temperature of the combustion products.

SCHEMATIC & GIVEN DATA:


(a) Constant Volume

(b) Constant Pressure

CH_4, 200% theo. air 77°F, 1atm
initial
Combustion products T_2, P_2
final
CH_4, 200% theo. air 77°F, 1atm
initial
Combustion products, T_2 1atm
final

ASSUMPTIONS: (1) For the systems shown in the accompanying figure $Q = 0$ and there are no effects of kinetic or potential energy. (2) Pressure remains constant. (3) Combustion is complete. (4) 3.76 moles of N_2 accompany each mole of O_2 in the air. (5) The initial fuel-air mixture and the products of combustion can be modeled as ideal gases.

ANALYSIS: Basing the calculation on one mole of fuel initially present, the reaction is described by

$$CH_4 + 4(O_2 + 3.76 N_2) \rightarrow CO_2 + 2H_2O + 2O_2 + 15.04 N_2$$

(a) Volume is constant. Since volume is constant, $W = 0$, and an energy balance reduces with assumption (1) to read $\Delta U = 0$. Thus

$$0 = (\bar{u}_{CO_2} + 2\bar{u}_{H_2O} + 2\bar{u}_{O_2} + 15.04\bar{u}_{N_2})_2 - (\bar{u}_{CH_4} + 4\bar{u}_{O_2} + 15.04\bar{u}_{N_2}),$$

With $\bar{u} = \bar{h} - \bar{R}T$

$$0 = (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_2 - (\bar{h}_{CH_4} + 4\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_1 - 20.04\bar{R}T_2 + 20.04\bar{R}T_1$$

Introducing $\bar{h} = \bar{h}_f^{\circ} + \Delta\bar{h}$, and noting that $\bar{h}_f^{\circ} = 0$ for O_2 and N_2

$$0 = [\bar{h}_f^{\circ} + \bar{h}(T_2) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^{\circ} + \bar{h}(T_2) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(T_2) - \bar{h}(537)]_{O_2} +$$
$$15.04[\bar{h}(T_2) - \bar{h}(537)]_{N_2} - (\bar{h}_f^{\circ})_{CH_4} - 20.04\bar{R}T_2 + 20.04\bar{R}T_1$$

With data from the ideal gas tables and Table A-25E

$$\bar{h}_{CO_2}(T_2) + 2\bar{h}_{H_2O}(T_2) + 2\bar{h}_{O_2}(T_2) + 15.04\bar{h}_{N_2}(T_2) - 20.04\bar{R}T_2 = -[-169,300 - 4028] - 2[-104,040 - 4258] + 2(3725) + 15.04(3730) - 32,210 - (20.04)(1.986)(537)$$

$$= 399,891$$

① Solving $T_2 \approx 3260°R$. ◄ (a)

PROBLEM 13.67 (Continued)

(b) Pressure is constant. Since pressure is constant, $W = \int p\,dV = p\,\Delta V$. So, with assumption (1) an energy balance reads $\Delta U = \cancel{Q}^{0} - p\Delta V$, or $\Delta H = 0$. That is

$$(\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2\bar{h}_{O_2} + 15.04\,\bar{h}_{N_2})_2 - (\bar{h}_{CH_4} + 4\bar{h}_{O_2} + 15.04\,\bar{h}_{N_2})_1 = 0$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(T_2) - \bar{h}(537)]_{O_2} +$$
$$15.04\,[\bar{h}(T_2) - \bar{h}(537)]_{N_2} - (\bar{h}_f^\circ)_{CH_4} = 0$$

With data from the ideal gas tables and Table A-25E

$$\bar{h}_{CO_2}(T_2) + 2\bar{h}_{H_2O}(T_2) + 2\bar{h}_{O_2}(T_2) + 15.04\,\bar{h}_{N_2}(T_2) = -[\bar{h}_f^\circ - \bar{h}(537)]_{CO_2} -$$
$$2[\bar{h}_f^\circ - \bar{h}(537)]_{H_2O} +$$
$$2\bar{h}_{O_2}(537) + 15.04\,\bar{h}_{N_2}(537) +$$
$$(\bar{h}_f^\circ)_{CH_4}$$

$$= -[-169{,}300 - 4028] - 2[-104{,}040 - 4258]$$
$$+ 2(3725) + 15.04(3730) - 32{,}210$$

$$= 421{,}263$$

② ③ Solving, $T_2 \approx 2666\,^\circ R \longleftarrow$ (b)

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1. Note that with the ideal equation of state the final pressure can be obtained:

$$\left.\begin{array}{l} P_1 V = n_1 R T_1 \\ P_2 V = n_2 R T_2 \end{array}\right\} \Rightarrow \frac{P_2}{P_1} = \frac{n_2}{n_1}\cdot\frac{T_2}{T_1} = \left(\frac{20.04}{20.04}\right)\left(\frac{3260}{537}\right) = 6.07$$

Thus, $P_2 = 6.07$ atm.

2. Note that with the ideal gas equation of state, the volume ratio can be obtained:

$$\begin{array}{l} pV_1 = n_1 \bar{R} T_1 \\ pV_2 = n_2 \bar{R} T_2 \end{array} > \quad \frac{V_2}{V_1} = \left(\frac{n_2}{n_1}\right)\left(\frac{T_2}{T_1}\right) = \left(\frac{20.04}{20.04}\right)\left(\frac{2666}{537}\right) = 4.96$$

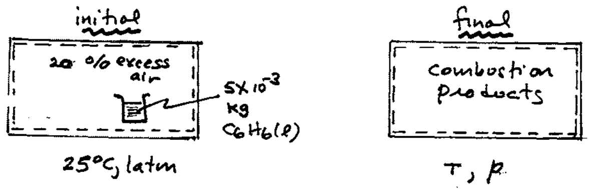
3. For both parts (a) and (b), iteration with table data can be avoided by using IT.

PROBLEM 13. 68

KNOWN: 5×10^{-3} kg of liquid C_6H_6 and 20% excess air, initially at 25°C, 1atm, reacts completely in a rigid insulated vessel.

FIND: Determine the final temperature and final pressure.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is shown in the figure above. (2) For the system, $Q = 0$, $W = 0$, and kinetic/potential energy effects are absent. (3) The reaction of the fuel and air is complete. 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (4) The volume occupied by the liquid fuel initially is much less than the volume of the gas phase. (5) The air and the combustion products can each be modeled as an ideal gas.

ANALYSIS: The reaction is described by

$$C_6H_6(\ell) + 9\,(O_2 + 3.76 N_2) \rightarrow 6\,CO_2 + 3 H_2O + 1.5\,O_2 + 33.84\,N_2 \qquad (1)$$

With assumption (1), the energy balance reduces to $\Delta U = 0$. Or

$$0 = (6\bar{u}_{CO_2} + 3\bar{u}_{H_2O} + 1.5\bar{u}_{O_2} + 33.84\,\bar{u}_{N_2}) - [\bar{u}_{C_6H_6} + 9\,\bar{u}_{O_2} + 33.84\,\bar{u}_{N_2}] \qquad (2)$$

For the gases, the ideal gas model gives $\bar{u} = \bar{h} - \bar{R}T$. For the liquid fuel, $pv << u$; so $\bar{h} \approx \bar{u}$. Thus Eq. (2) becomes

$$0 = 6[\bar{h} - \bar{R}T]_{CO_2} + 3[\bar{h} - \bar{R}T]_{H_2O} + 1.5[\bar{h} - RT]_{O_2} + 33.84[\bar{h} - \bar{R}T]_{N_2}$$
$$- \bar{h}_{C_6H_6} - 9[\bar{h} - \bar{R}T_1]_{O_2} - 33.84[\bar{h} - \bar{R}T_1]_{N_2}$$

or

$$0 = 6[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 3[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 1.5[\Delta\bar{h}]_{O_2} + 33.84[\Delta\bar{h}]_{N_2}$$
$$- (\bar{h}_f^\circ)_{C_6H_6(\ell)} - 9[\cancelto{0}{\bar{h}_f^\circ + \Delta\bar{h}}]_{O_2} - 33.84[\cancelto{0}{\bar{h}_f^\circ + \Delta\bar{h}}]_{N_2}$$
$$+ 42.84\,\bar{R}T_1 - 44.34\,\bar{R}T$$

With ideal gas table data and data from the literature for $(\bar{h}_f^\circ)_{C_6H_6(\ell)} = 49{,}100$ kJ/kmol

$$0 = 6[-393{,}520 + \bar{h}_{CO_2}(T) - 9364] + 3[-241{,}820 + \bar{h}_{H_2O}(T) - 9904]$$
$$+ 1.5[\bar{h}_{O_2}(T) - 8682] + 33.84[\bar{h}_{N_2}(T) - 8669] - 49{,}100 +$$
$$(42.84)(8.314)(298) - (44.34)(8.314)\,T$$

$$\Rightarrow 6\bar{h}_{CO_2}(T) + 3\bar{h}_{H_2O}(T) + 1.5\bar{h}_{O_2}(T) + 33.84\,\bar{h}_{N_2}(T) - 368.6\,T = 3{,}421{,}819 \text{ kJ/kmol(fuel)}$$

① Solving iteratively with table data $T \approx 2700\,K$ $(T = 2427°C)$ ←

Using the ideal gas equation of state for the gas phase only

$pV = n_P \bar{R} T$
$P_1 V = n_R \bar{R} T_1$
← gas phase only
↑ ignore volume occupied by the liquid

$$\Rightarrow \frac{p}{p_1} = \left(\frac{n_P}{n_R}\right)\left(\frac{T}{T_1}\right) = \left(\frac{44.34}{42.84}\right)\left(\frac{2700}{298}\right) = 9.38$$

$\Rightarrow p = 9.38$ atm ←

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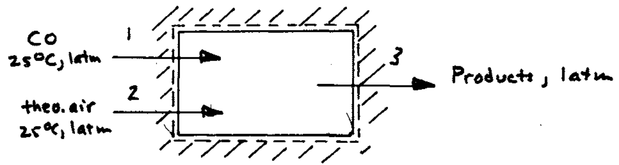
1. Iteration with table data can be avoided by using IT.

PROBLEM 13.69

KNOWN: CO at 25°C, 1 atm enters an insulated reactor and reacts completely with the theoretical amount of air entering in a separate stream at 25°C, 1atm. The products exit at 1 atm.

FIND: Determine the rate of entropy production per kmol of CO entering.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The reaction is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases.

ANALYSIS: The reaction is described by

$$CO + \tfrac{1}{2} O_2 + 1.88 N_2 \rightarrow CO_2 + 1.88 N_2$$

It is necessary to determine the temperature of the combustion products. Thus, an energy rate balance at steady state reduces to give

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{co}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{co}}}^{0} + (\bar{h}_{co})_1 + (\tfrac{1}{2}\bar{h}_{o_2} + 1.88\bar{h}_{N_2})_2 \rightarrow (\bar{h}_{co_2} + 1.88\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$, and noting that $\bar{h}_f^o = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^o)_{co} + (0) - (\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(298))_{CO_2} - 1.88\,[\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

or

$$\bar{h}_{co_2}(T_3) + 1.88\bar{h}_{N_2}(T_3) = (\bar{h}_f^o)_{co} - (\bar{h}_f^o - \bar{h}(298))_{co_2} + 1.88\,\bar{h}_{N_2}(298)$$

with data from the ideal gas tables

$$\bar{h}_{co_2}(T_3) + 1.88\bar{h}_{N_2}(T_3) = -110{,}530 - (-393{,}520 - 9364) + 1.88(8669)$$
$$= 308{,}652$$

① Solving for T_3s, $T_3 \approx 2665$ K.

An entropy rate balance at steady state takes the form

$$0 = \cancel{\sum \frac{\dot{Q}_j/T_j}{\dot{n}_{co}}}^{0} + (\bar{s}_{co})_1 + (\tfrac{1}{2}\bar{s}_{O_2} + 1.88\bar{s}_{N_2})_2 - (\bar{s}_{co_2} + 1.88\bar{s}_{N_2})_3 + \dot{\sigma}_{cv}/\dot{n}_{co}$$

or

$$\dot{\sigma}_{cv}/\dot{n}_{co} = (\bar{s}_{co_2} + 1.88\,\bar{s}_{N_2})_3 - (\bar{s}_{co})_1 - (\tfrac{1}{2}\bar{s}_{O_2} + 1.88\,\bar{s}_{N_2})_2 \qquad (1)$$

The CO enters separately at 298K, 1atm, so from Table A-24, $\bar{s}_{co} = 197.54$ kJ/kmol·K. The O_2 and N_2 in the combustion air enters as a mixture at $P_{ref} = 1$atm. Thus, with Eq. 13.23 and absolute entropy data from Table A-25

$$\bar{s}_{O_2} = \bar{s}_{O_2}^o(298) - \bar{R}\ln\frac{y_{O_2}P_{ref}}{P_{ref}} = 205.03 - 8.314\ln .21 = 218.01 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}_{N_2}^o(298) - R\ln\frac{y_{N_2}P_{ref}}{P_{pref}} = 191.5 - 8.314\ln .79 = 193.46 \text{ kJ/kmol·K}$$

The products exit as an ideal gas mixture at 1 atm, 2665K with the following composition $y_{co_2} = 1/2.88$, $y_{N_2} = 1.88/2.88$. Then, with absolute entropy data from Tables A-23 and 27

$$\bar{s}_{co_2} = \bar{s}_{co_2}^o(2665) - \bar{R}\ln\frac{y_{co_2}P_{ref}}{P_{ref}} = 326.74 - 8.314\ln\frac{1}{2.88} = 335.53 \text{ kJ/kmol·K}$$

$$s_{N_2} = \bar{s}_{N_2}^o(2665) - \bar{R}\ln\frac{y_{N_2}P_{ref}}{P_{ref}} = 262.42 - 8.314\ln\frac{1.88}{2.88} = 265.97 \text{ kJ/kmol·K}$$

Substituting values into Eq. (1)

$$\dot{\sigma}_{cv}/\dot{n}_{co} = 335.53 + 1.88(265.97) - 197.54 - \tfrac{1}{2}(218.01) - 1.88(193.46) = 165.3\,\frac{\text{kJ}}{\text{kmol(CO)·K}} \longleftarrow \dot{\sigma}_{cv}/\dot{n}_{co}$$

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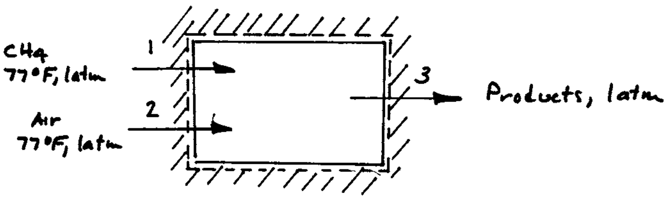
1. Iteration with table data can be avoided by using IT.

PROBLEM 13.70

KNOWN: CH_4 at 77°F, 1atm enters an insulated reactor and burns completely with air entering in a separate stream at 77°F, 1atm. The products exit at 1atm.

FIND: Determine the rate of entropy production, per lbmol of CH_4, for combustion with (a) the theoretical amount of air, (b) 200% of theoretical air.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The reaction is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases.

ANALYSIS: (a) Complete combustion of CH_4 with the theoretical amount of air is described by

$$CH_4 + (2O_2 + 7.52N_2) \rightarrow CO_2 + 2H_2O + 7.52N_2$$

To determine T_3, write an energy rate balance at steady state:

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{0} + (\bar{h}_{CH_4})_1 + (2h_{O_2} + 7.52\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 7.52\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{CH_4} + (0) - \{[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 7.52[\bar{h}(T_3) - \bar{h}(537)]_{N_2}\}$$

With data from the ideal gas tables and Table A-25E

$$\bar{h}_{CO_2}(T_3) + 2\bar{h}_{H_2O}(T_3) + 7.52\bar{h}_{N_2}(T_3) = (\bar{h}_f^\circ)_{CH_4} - [\bar{h}_f^\circ - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^\circ - \bar{h}(537)]_{H_2O} + 7.52\bar{h}_{N_2}(537)$$

$$= -32{,}210 - [-169{,}300 - 4028] - 2[-104{,}040 - 4258] + 7.52[3730]$$

$$= 385{,}764$$

① Solving for T_3, $T_3 \approx 4187$°R.

An entropy rate balance at steady state takes the form

$$0 = \cancel{\sum \frac{\dot{Q}_j/T_j}{\dot{n}_{CH_4}}}^{0} + (\bar{s}_{CH_4})_1 + (2\bar{s}_{O_2} + 7.52\bar{s}_{N_2})_2 - (\bar{s}_{CO_2} + 2\bar{s}_{H_2O} + 7.52\bar{s}_{N_2})_3 + \dot{\sigma}_{cv}/\dot{n}_{CH_4}$$

or

$$\dot{\sigma}_{cv}/\dot{n}_{CH_4} = (\bar{s}_{CO_2} + 2\bar{s}_{H_2O} + 7.52\bar{s}_{N_2})_3 - (\bar{s}_{CH_4})_1 - (2\bar{s}_{O_2} + 7.52\bar{s}_{N_2})_2 \qquad (1)$$

The CH_4 enters separately at 537°R, 1atm, so from Table A-25E $\bar{s}_{CH_4} = 44.49$ Btu/lbmol·°R. The O_2 and N_2 enters as a mixture at p_{ref} = 1atm. With Eq. 13.23 and $\bar{s}^\circ$ from Table A-23E

$$\bar{s}_{O_2} = \bar{s}_{O_2}^\circ(537) - \bar{R}\ln\frac{y_{O_2}p_{ref}}{p_{ref}} = 48.98 - 1.986\ln 0.21 = 52.079 \text{ Btu/lbmol}\cdot°R$$

$$\bar{s}_{N_2} = \bar{s}_{N_2}^\circ(537) - \bar{R}\ln\frac{y_{N_2}p_{ref}}{p_{ref}} = 45.74 - 1.986\ln 0.79 = 46.208 \text{ Btu/lbmol}\cdot°R$$

The products exit as an ideal gas mixture at 1atm, 4187°R with the following composition $y_{CO_2} = 1/10.52$, $y_{H_2O} = 2/10.52$, $y_{N_2} = 7.52/10.52$. Then, with $\bar{s}$ data from the ideal gas tables

$$\bar{s}_{CO_2} = \bar{s}_{CO_2}^\circ(4187) - \bar{R}\ln\frac{y_{CO_2}p_{ref}}{p_{ref}} = 76.673 - 1.986\ln\frac{1}{10.52} = 80.747 \text{ Btu/lbmol}\cdot°R$$

$$\bar{s}_{H_2O} = \bar{s}_{H_2O}^\circ(4187) - \bar{R}\ln y_{H_2O} = 65.105 - 1.986\ln\frac{2}{10.52} = 68.402 \text{ Btu/lbmol}\cdot°R$$

$$\bar{s}_{N_2} = \bar{s}_{N_2}^\circ(4187) - \bar{R}\ln y_{N_2} = 61.494 - 1.986\ln\frac{7.52}{10.52} = 62.161 \text{ Btu/lbmol}\cdot°R$$

Inserting values into Eq. (1)

$$\dot{\sigma}_{cv}/\dot{n}_{CH_4} = [80.747 + 2(68.402) + 7.52(62.161)] - 44.49 - [2(52.079) + 7.52(46.208)]$$

$$= 188.87 \text{ Btu/lbmol(CH}_4)\cdot{}^\circ\text{R} \longleftarrow \text{(a)}$$

(b) Complete combustion of CH_4 with 200 percent of the theoretical amount of air is

$$CH_4 + (4O_2 + 15.04\, N_2) \longrightarrow CO_2 + 2H_2O + 2O_2 + 15.04\, N_2$$

To determine T_3, write an energy rate balance at steady state:

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{\,0}}{\cancel{\dot{n}_{CH_4}}} - \frac{\cancel{\dot{W}_{cv}}^{\,0}}{\cancel{\dot{n}_{CH_4}}} + (\bar{h}_{CH_4})_1 + (4\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_2 - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2\bar{h}_{O_2} + 15.04\bar{h}_{N_2})_3$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$

$$0 = (\bar{h}_f^\circ)_{CH_4} + (0) - \{[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 2[\bar{h}(T_3) - \bar{h}(537)]_{O_2} + 15.04[\bar{h}(T_3) - \bar{h}(537)]_{N_2}$$

With data from the ideal gas tables and Table A-25E

$$\bar{h}_{CO_2}(T_3) + 2\bar{h}_{H_2O}(T_3) + 2\bar{h}_{O_2}(T_3) + 15.04\bar{h}_{N_2}(T_3) = (\bar{h}_f^\circ)_{CH_4} - [\bar{h}_f^\circ - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^\circ - \bar{h}(537)]_{H_2O} + 2\bar{h}_{O_2}(537) + 15.04\,\bar{h}_{N_2}(537)$$

$$= -32{,}210 - [-169{,}300 - 4028] - 2[-104{,}040 - 4258] + 2(3725) + 15.04(3730)$$

$$= 421{,}264$$

① Solving for T_3, $T_3 \approx 2666^\circ R$.

At steady state an entropy rate balance reduces to give

$$\dot{\sigma}_{cv}/\dot{n}_{CH_4} = (\bar{s}_{CO_2} + 2\bar{s}_{H_2O} + 2\bar{s}_{O_2} + 15.04\bar{s}_{N_2}) - \bar{s}_{CH_4} - (4\bar{s}_{O_2} + 15.04\bar{s}_{N_2}) \qquad (2)$$

The values of $\bar{s}_{CH_4}$, $\bar{s}_{O_2}$ and $\bar{s}_{N_2}$ of the reactants are the same as in part (a). The products exit as an ideal gas mixture at 1atm, 2666°R with the following composition $y_{CO_2} = 1/20.04$, $y_{H_2O} = y_{O_2} = 2/20.04$, $y_{N_2} = 15.04/20.04$. Then with $\bar{s}^\circ$ from the ideal gas tables

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(2666) - \bar{R}\ln y_{CO_2} = 69.594 - 1.986 \ln\frac{1}{20.04} = 75.547 \text{ Btu/lbmol}\cdot{}^\circ\text{R}$$

$$\bar{s}_{H_2O} = \bar{s}^\circ_{H_2O}(2666) - \bar{R}\ln y_{H_2O} = 59.694 - 1.986\ln\frac{2}{20.04} = 64.271 \text{ Btu/lbmol}\cdot{}^\circ\text{R}$$

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(2666) - \bar{R}\ln y_{O_2} = 61.506 - 1.986\ln\frac{2}{20.04} = 66.083 \text{ Btu/lbmol}\cdot{}^\circ\text{R}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(2666) - \bar{R}\ln y_{N_2} = 57.644 - 1.986\ln\frac{15.04}{20.04} = 58.214 \text{ Btu/lbmol}\cdot{}^\circ\text{R}$$

Inserting values into Eq. (2)

$$\dot{\sigma}_{cv}/\dot{n}_{CH_4} = 75.547 + 2(64.271) + 2(66.083) + 15.04(58.214) - 44.49 - 4(52.079) - 15.04(46.208)$$

$$= 263.97 \text{ Btu/lbmol}\cdot{}^\circ\text{R} \longleftarrow \text{(b)}$$

---

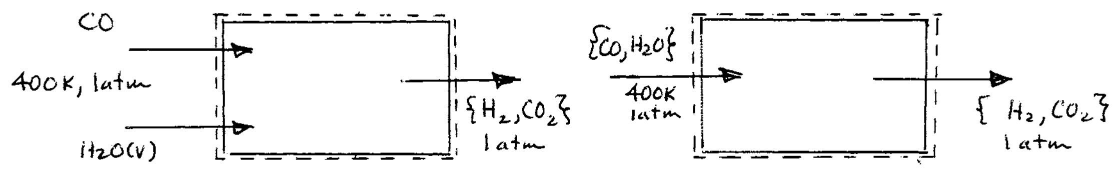
1. Iteration with table data can be avoided by using IT.

PROBLEM 13.71

KNOWN: Two cases are under consideration involving an insulated reactor at steady state: (a) carbon monoxide and water vapor enter in separate streams and form H_2, CO_2, (b) CO and $H_2O(v)$ enter as a mixture and form H_2, CO_2.

FIND: For each case determine the rate of entropy production.

SCHEMATIC & GIVEN DATA:



(a) (b)

ASSUMPTIONS: (1) The control volumes shown above are at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic/potential energy effects. (2) The ideal gas model is applicable to incoming and exiting streams.

ANALYSIS: To evaluate the entropy production rates, the temperature of the exiting mixture is required. To determine this, write an energy balance for the reaction

$$CO + H_2O \longrightarrow CO_2 + H_2$$

That is

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{0}}{\cancel{\dot{n}_{CO}}} - \frac{\cancel{\dot{W}_{cv}}^{0}}{\cancel{\dot{n}_{CO}}} + [\bar{h}_{CO} + \bar{h}_{H_2O}]_R - [\bar{h}_{CO_2} + \bar{h}_{H_2}]_P \qquad (1)$$

① Eq. 1 is solved using IT, as follows:

```
Tin = 400  // K

hCO_in = h_T("CO",Tin)
hH2O_in = h_T("H2O",Tin)
hH2_out = h_T("H2",Tp)
hCO2_out = h_T("CO2",Tp)

0 = hCO_in + hH2O_in - hH2_out - hCO2_out
```

IT result: T_p = 922.5 K ← T_p

Now, an entropy balance reduces to

$$0 = \frac{1}{\dot{n}_{CO}} \cancel{\sum_j \left(\dot{Q}/T\right)_j}^{0} + \bar{s}_{CO} + \bar{s}_{H_2O(v)} - \bar{s}_{CO_2} - \bar{s}_{H_2} + \dot{\sigma}_{cv}/\dot{n}_{CO}$$

or

$$\dot{\sigma}_{cv}/\dot{n}_{CO} = \bar{s}_{CO_2} + \bar{s}_{H_2} - \bar{s}_{CO} - \bar{s}_{H_2O(v)} \qquad (2)$$

In terms of temperatures and pressures

$$\dot{\sigma}_{cv}/\dot{n}_{CO} = \bar{s}_{CO_2}(T_P, p_{CO_2\text{-}out}) + \bar{s}_{H_2}(T_P, p_{H_2\text{-}out}) - \bar{s}_{CO}(T_{in}, p_{CO\text{-}in}) - \bar{s}_{H_2O(v)}(T_{in}, p_{H_2O(v)\text{-}in})$$

with IT:

```
sCO_in = s_TP("CO",Tin,pCO_in)
sH2O_in = s_TP("H2O",Tin,pH2O_in)
sCO2_out = s_TP("CO2",Tp,pCO2_out)
sH2_out = s_TP("H2",Tp,pH2_out)

sigmadot = sCO2_out + sH2_out - sCO_in - sH2O_in
```

PROBLEM 3.71 (Contd.)

(a) The CO and $H_2O(v)$ enter separately, each at 400K, 1atm = 1.01325 bar. The products exit as an ideal gas mixture at 922.5K, 1.01325 bar, with the molar analysis $y_{H_2} = y_{CO_2} = 0.5$. Thus, with IT:

```
pCO_in = 1.01325
pH2O_in = 1.01325
pCO2_out = 0.5 * 1.01325
pH2_out = 0.5 * 1.01325
```

and the result is: $\dot{\sigma}_{cv}/\dot{n}_{cv}$ = 35.29 kJ/kmol(CO)·K $\longleftarrow$ (a)

(b) In this case, the CO and $H_2O(v)$ enter as a mixture, with $y_{CO} = y_{H_2O} = 0.5$. Thus, with IT:

```
pCO_in = 0.5 * 1.01325
pH2O_in = 0.5 * 1.01325
```

and the result is: $\dot{\sigma}_{cv}/\dot{n}_{cv}$ = 23.77 kJ/kmol(CO)·K $\longleftarrow$ (b)

Discussion: The entropy production rate in case (b) is less than for part (a) because the mixture has already been formed prior to entering the reactor. In case (a), the mixture forms within the reactor and so there is an accompanying entropy production with this type of irreversibility.

---

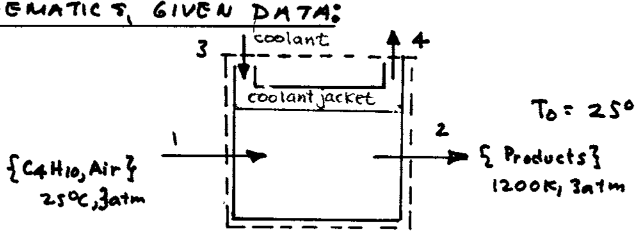
1. Only an IT solution is presented here, as Tables A-23 do not include data for H_2. Data for H_2 can be found in the same source as Table A-23.

PROBLEM 13.72

KNOWN: A gaseous mixture of C_4H_{10} and 80% excess air enters a coolant-jacketed reactor at 25°C, 3atm. Complete combustion occurs and products exit at 1200 K, 3atm. Saturated liquid coolant enters the jacket and saturated vapor exits.

FIND: Determine (a) the mass flow rate of the coolant, in kg per kmol of fuel, (b) the rate of entropy production, (c) and the rate of exergy destruction. Consider two coolants: water at 1 bar and ammonia at 10 bar.

SCHEMATIC & GIVEN DATA:



$T_0 = 25°C$

Coolants:
- water at 1 bar
- ammonia at 10 bar.

ASSUMPTIONS: (1) The control volume shown in the accompany figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The incoming fuel-air mixture and exiting combustion products can be modeled as ideal gases.

ANALYSIS: The complete combustion of C_4H_{10} with the theoretical amount of air is

$$C_4H_{10} + 6.5(O_2 + 3.76\,N_2) \rightarrow 4\,CO_2 + 5H_2O + 24.44\,N_2$$

Complete combustion with 80% excess air is then

$$C_4H_{10} + 11.7(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 5.2\,O_2 + 43.992\,N_2$$

(a) An energy rate balance at steady reduces to the form

$$0 = \overset{0}{\cancel{\dot{Q}_{cv}}} - \overset{0}{\cancel{\dot{W}_{cv}}} + \dot{m}_{cw}[h_3 - h_4] + \dot{n}_{C_4H_{10}}\Big[(\bar{h}_{C_4H_{10}} + 11.7\,\bar{h}_{O_2} + 43.992\,\bar{h}_{N_2})_1 - (4\bar{h}_{CO_2} + 5\bar{h}_{H_2O} + 5.2\,\bar{h}_{O_2} + 43.992\,\bar{h}_{N_2})_2\Big]$$

where $\dot{m}_{cw}$ is the mass flow rate of the coolant and $\dot{n}_{C_4H_{10}}$ is the molar flow rate of the C_4H_{10}. Accordingly

$$\frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = \frac{(\bar{h}_{C_4H_{10}} + 11.7\,\bar{h}_{O_2} + 43.992\,\bar{h}_{N_2})_1 - (4\,\bar{h}_{CO_2} + 5\,\bar{h}_{H_2O} + 5.2\,\bar{h}_{O_2} + 43.992\,\bar{h}_{N_2})_1}{h_4 - h_3}$$

With $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$ and noting that $\bar{h}_f^o = 0$ for O_2 and N_2

$$\frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = \frac{(\bar{h}_f^o)_{C_4H_{10}} - \{4[\bar{h}_f^o + \bar{h}(1200) - \bar{h}(298)]_{CO_2} + 5[\bar{h}_f^o + \bar{h}(1200) - \bar{h}(298)]_{H_2O} + 5.2[\bar{h}(1200) - \bar{h}(298)]_{O_2} + 43.992[\bar{h}(1200) - \bar{h}(298)]_{N_2}\}}{h_g - h_f}$$

$$= \frac{-126{,}150 - 4[-393{,}520 + 53{,}848 - 9364] - 5[-241{,}820 + 44{,}380 - 9904] - 5.2[38{,}447 - 8682] - 43.992[36{,}777 - 8669]}{h_g - h_f}$$

$$= \frac{915{,}409\ \text{kJ/kmol(fuel)}}{h_g - h_f} \qquad (1)$$

- For water at 1bar, $h_{fg} = 2258\ \frac{kJ}{kg} \Rightarrow \frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = \frac{915{,}409}{2258} = 405.4\ \frac{kg}{kmol(fuel)}$

(a)

- For ammonia at 10 bar, $h_{fg} = 1165.4\ \frac{kJ}{kg} \Rightarrow \frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}} = 785.5\ \frac{kg}{kmol(fuel)}$

PROBLEM 13.72 (Continued)

(b) An entropy rate balance at steady state reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = \left(4\,\bar{s}_{CO_2} + 5\,\bar{s}_{H_2O} + 5.2\,\bar{s}_{O_2} + 43.992\,\bar{s}_{N_2}\right) - \left(\bar{s}_{C_4H_{10}} + 11.7\,\bar{s}_{O_2} + 43.992\,\bar{s}_{N_2}\right) + \frac{\dot{m}_{cw}}{\dot{n}_{C_4H_{10}}}(s_4 - s_3) \quad (1)$$

The fuel and air enter as a mixture at 25°C, 3 atm with the composition, $y_{C_4H_{10}} = 1/56.692$, $y_{O_2} = 11.7/56.692$, $y_{N_2} = 43.992/56.692$. Accordingly

$$\bar{s}_{C_4H_{10}} = \bar{s}^{\circ}_{C_4H_{10}}(298) - \bar{R}\ln\frac{y_{C_4H_{10}}p}{p_{ref}} = 310.03 - 8.314\ln\frac{3}{56.692} = 334.46 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(298) - \bar{R}\ln\frac{y_{O_2}p}{p_{ref}} = 205.03 - 8.314\ln\frac{(11.7)(3)}{56.692} = 209.02 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{N_2} = \bar{s}^{\circ}_{N_2}(298) - \bar{R}\ln\frac{y_{N_2}p}{p_{ref}} = 191.5 - 8.314\ln\frac{(43.992)(3)}{56.692} = 184.47 \text{ kJ/kmol}\cdot\text{K}$$

The products exit as a mixture at 1200 K, 3 atm with the composition, $y_{CO_2} = 4/58.192$, $y_{H_2O} = 5/58.192$, $y_{O_2} = 5.2/58.192$, $y_{N_2} = 43.992/58.192$. Accordingly

$$\bar{s}_{CO_2} = \bar{s}^{\circ}_{CO_2}(1200) - \bar{R}\ln\frac{y_{CO_2}p}{p_{ref}} = 279.307 - 8.314\ln\frac{(4)(3)}{58.192} = 292.43 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{H_2O} = \bar{s}^{\circ}_{H_2O}(1200) - \bar{R}\ln\frac{y_{H_2O}p}{p_{ref}} = 240.333 - 8.314\ln\frac{(5)(3)}{58.192} = 251.60 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(1200) - \bar{R}\ln\frac{y_{O_2}p}{p_{ref}} = 249.906 - 8.314\ln\frac{(5.2)(3)}{58.192} = 260.85 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{N_2} = \bar{s}^{\circ}_{N_2}(1200) - \bar{R}\ln\frac{y_{N_2}p}{p_{ref}} = 234.115 - 8.314\ln\frac{(43.992)(3)}{58.192} = 227.31 \text{ kJ/kmol}\cdot\text{K}$$

From Table A-3, $s_4 - s_3 = (7.3559 - 1.306) = 6.0499$ kJ/kg·K.

Substituting values into Eq. (1)

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = 4(292.43) + 5(251.60) + 5.2(260.85) + 43.992(227.31) - 334.46 - 11.7(209.02) - 43.992(184.47) + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}}(s_4 - s_3)$$

$$= 2888.77 + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}}(s_4 - s_3)$$

$$= 2888.77 + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}}\,s_{fg}$$

The value for s_{fg} can be obtained directly from table data, but it is more instructive to rewrite the last expression using $s_{fg} = h_{fg}/T_{sat}$ from Sec. 6.3. Then

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = 2888.77 + \frac{\dot{m}_{cw}}{\dot{n}_{fuel}}\cdot\frac{h_{fg}}{T_{sat}} = 2888.77 + \left[\underset{\nwarrow \text{Eq. (1)}}{\frac{915{,}409}{h_{fg}}}\right]\left[\frac{h_{fg}}{T_{sat}}\right]$$

$$= 2888.77 + \frac{915{,}409}{T_{sat}} \quad (2)$$

①

- Water at 1 bar, $T_{sat} = 373\text{K} \Rightarrow \dfrac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = 2888.77 + \dfrac{915{,}409}{373} = 5342.9 \dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}$
- Ammonia at 10 bar, $T_{sat} = 298\text{K} \Rightarrow \dfrac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}} = 2888.77 + \dfrac{915{,}409}{298} = 5960.6 \dfrac{\text{kJ}}{\text{kmol}\cdot\text{K}}$

(b)

PROBLEM 13.72 (Continued)

(c) The exergy destruction is obtained using:

$$\frac{\dot{E}_d}{\dot{n}_{C_4H_{10}}} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{n}_{C_4H_{10}}}$$

- water at 1 bar:

$$\frac{\dot{E}_d}{\dot{n}_{C_4H_{10}}} = (298\,K)(5342.9) = 1.592 \times 10^6 \frac{kJ}{kmol\,(fuel)}$$

- ammonia at 10 bars:

$$\frac{\dot{E}_d}{\dot{n}_{C_4H_{10}}} = (298)(5960.6) = 1.776 \times 10^6 \frac{kJ}{kmol\,(fuel)}$$

(c)

---

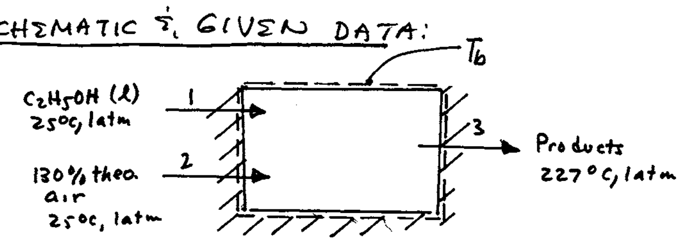
1. The difference between the entropy production rates is not due to the natures of the two coolants, but only to the temperature T_{sat}, as shown by Eq. (2). If each coolant were at the same temperature, the same entropy production rate would be determined for each. The choice of the coolant is determined by other considerations. For example, at a temperature $T_{sat} = 50°C$ the pressure of saturated water is just 0.1235 bar, whereas the pressure of saturated ammonia is 20.331 bar. Such extreme pressures are likely to be avoided in applications of the type under consideration, however. And as shown by part (a) the temperature T_{sat} also affects the mass flow rate.

PROBLEM 13.73

KNOWN: $C_2H_5OH(l)$ at 25°C, 1atm enters a reactor and burns completely with 130% of theoretical air entering separately at 25°C, 1atm. Products exit at 227°C, 1atm. Heat transfer from the reactor occurs at temperature T_b.

FIND: For T_b ranging from 25°C to 200°C, determine the rate of exergy destruction within the reactor.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv}=0$, negligible kinetic and potential energy effects, and heat transfer occuring at temperature T_b (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases. (5) For the exergy reference environment, $T_0 = 25°C$ (298K)

ANALYSIS: The rate of exergy destruction is given by $\dot{E}_d = T_0\dot{\sigma}_{cv}$ where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance. The plan then is to evaluate $\dot{\sigma}_{cv}$. Complete combustion of C_2H_5OH with the theoretical amount of air is described by

$$C_2H_5OH + 3(O_2+3.76N_2) \rightarrow 2CO_2 + 3H_2O + 11.28N_2$$

Complete combustion with 130% of the theoretical amount of air is thus

$$C_2H_5OH + 3.9(O_2+3.76N_2) \rightarrow 2CO_2 + 3H_2O + 0.9O_2 + 14.664N_2$$

With assumption 1, an entropy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (\bar{s}_{fuel})_1 + (3.9\bar{s}_{O_2} + 14.664\bar{s}_{N_2})_2 - (2\bar{s}_{CO_2} + 3H_2O + 0.9\bar{s}_{O_2} + 14.664\bar{s}_{N_2})_3 + \frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}}$$

Accordingly

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = -\frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (2\bar{s}_{CO_2} + 3\bar{s}_{H_2O} + 0.9\bar{s}_{O_2} + 14.664\bar{s}_{N_2})_3 - (\bar{s}_{fuel})_1 - (3.9\bar{s}_{O_2} + 14.664\bar{s}_{N_2})_2 \quad (1)$$

The term $\dot{Q}_{cv}/\dot{n}_{fuel}$ can be obtained from an energy rate balance which reduces with assumption 1 to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = (2\bar{h}_{CO_2} + 3\bar{h}_{H_2O} + 0.9\bar{h}_{O_2} + 14.664\bar{h}_{N_2})_3 - (\bar{h}_{fuel})_1 - (3.9\bar{h}_{O_2} + 14.664\bar{h}_{N_2})_2$$

$$= 2[\bar{h}_f^o + \bar{h}(500) - \bar{h}(298)]_{CO_2} + 3[\bar{h}_f^o + \bar{h}(500) - \bar{h}(298)]_{H_2O} + 0.9[\bar{h}(500) - \bar{h}(298)]_{O_2}$$
$$+ 14.664[\bar{h}(500) - \bar{h}(298)]_{N_2} - (\bar{h}_f^o)_{fuel} - (0)$$

With data from the ideal gas tables and Table. A-25

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 2[-393{,}520 + 17{,}678 - 9364] + 3[-241{,}820 + 16{,}828 - 9904] + 0.9[14770 - 8682]$$
$$+ 14.664[14{,}581 - 8669] - (-277{,}690)$$

$$= -1{,}105{,}237 \text{ kJ/kmol(fuel)}$$

PROBLEM 13.73 (Continued)

Next, the specific entropies appearing in Eq. (1) are evaluated. Since the fuel enters at 25°C, 1 atm, $\bar{s}_{fuel}$ is obtained from Table A-25: $\bar{s}_{fuel} = 160.7$ kJ/kmol·K. The combustion air enters as a mixture with $y_{O_2} = 0.21$, $y_{N_2} = 0.79$. Thus, with $p = p_{ref}$ and $T = 298$K

$$\bar{s}_{O_2} = \bar{s}^o_{O_2}(298) - \bar{R}\ln\frac{y_{O_2}p_{ref}}{p_{ref}} = 205.03 - 8.314\ln 0.21 = 218.01 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^o_{N_2}(298) - \bar{R}\ln\frac{y_{N_2}p_{ref}}{p_{ref}} = 191.502 - 8.314\ln 0.79 = 193.462 \text{ kJ/kmol·K}$$

The combustion products exit as a mixture at 500K, 1atm with the composition $y_{CO_2} = 2/20.564$, $y_{H_2O} = 3/20.564$, $y_{O_2} = .9/20.564$, $y_{N_2} = 14.664/20.564$. Thus, with $\bar{s}^o$ data from the ideal gas tables

$$\bar{s}_{CO_2} = \bar{s}^o_{CO_2}(500) - \bar{R}\ln\frac{y_{CO_2}p_{ref}}{p_{ref}} = 234.81 - 8.314\ln\frac{2}{20.564} = 254.185 \text{ kJ/kmol·K}$$

$$\bar{s}_{H_2O} = \bar{s}^o_{H_2O}(500) - \bar{R}\ln\frac{y_{H_2O}p_{ref}}{p_{ref}} = 206.413 - 8.314\ln\frac{3}{20.564} = 222.417 \text{ kJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^o_{O_2}(500) - \bar{R}\ln\frac{y_{O_2}p_{ref}}{p_{ref}} = 220.589 - 8.314\ln\frac{0.9}{20.564} = 246.603 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^o_{N_2}(500) - \bar{R}\ln\frac{y_{N_2}p_{ref}}{p_{ref}} = 206.630 - 8.314\ln\frac{14.664}{20.564} = 209.441 \text{ kJ/kmol·K}$$

Collecting results

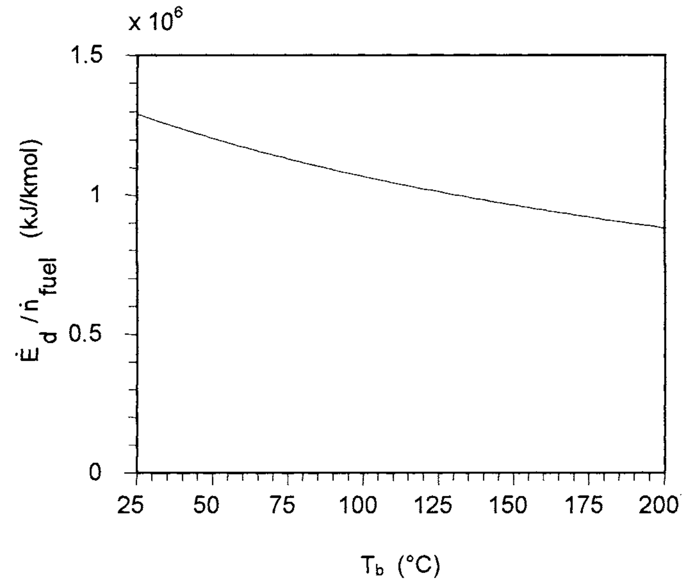
$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = \frac{-(-1{,}105{,}237)}{T_b} + 2(254.185) + 3(222.417) + 0.9(246.603) + 14.664(209.441) - 160.7 - 3.9(218.01) - 14.664(193.402)$$

$$= \frac{1{,}105{,}237}{T_b} + 620.9 \frac{\text{kJ}}{\text{kmol(fuel)}\cdot\text{K}} \Rightarrow \frac{\dot{E}_d}{\dot{n}_{fuel}} = T_0\left[\frac{1{,}105{,}237}{T_b} + 620.9\right]\frac{\text{kJ}}{\text{kmol(fuel)}}$$

Sample Calculation: When $T_b = 298$K, $(\dot{E}_d/\dot{n}_{fuel}) = 1.29 \times 10^6$ kJ/kmol(fuel)

PLOT:

①



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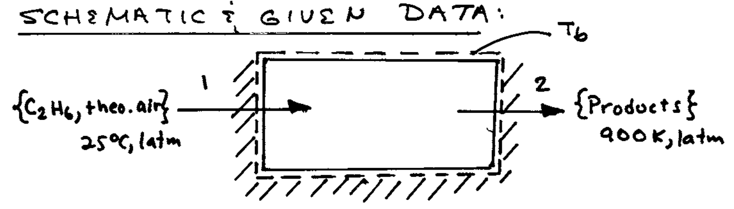
① At steady state, the net exergy carried into the reactor is a fixed value equal to the exergy destroyed within the reactor plus the exergy carried out accompanying heat transfer. Since the exergy carried out accompanying heat transfer increases as T_b increases: $[1 - T_0/T_b]\dot{Q}_{cv}$, the rate of exergy destruction within the reactor decreases as T_b increases, as shown in the figure.

PROBLEM 13.74

KNOWN: A gaseous mixture of C_2H_6 and the theoretical amount of air at 25°C, 1atm enters a reactor and burns completely. Combustion products exit at 627°C, 1atm. Heat transfer from the reactor takes place at T_b.

FIND: Determine the rate of exergy destruction, in kJ per kmol of C_2H_6, for T_b ranging from 25 to 600°C.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$, negligible effects of kinetic and potential energy, and heat transfer occuring at temperature T_b. (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The incoming and exiting mixtures can be modeled as ideal gases. (5) For the exergy reference environment, $T_0 = 298K$.

ANALYSIS: The rate of exergy destruction is given by $\dot{E}_d = T_0 \dot{\sigma}_{cv}$ where $\dot{\sigma}_{cv}$ is the rate of entropy production from an entropy rate balance. The plan then is to evaluate $\dot{\sigma}_{cv}$. Complete combustion of C_2H_6 with the theoretical amount of air is described by

$$C_2H_6 + 3.5 O_2 + 13.16 N_2 \rightarrow 2CO_2 + 3H_2O + 13.16 N_2$$

With assumption 1, an entropy rate balance at steady state reduces to

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (\bar{s}_{fuel} + 3.5\,\bar{s}_{O_2} + 13.16\,\bar{s}_{N_2})_1 - (2\,\bar{s}_{CO_2} + 3\,\bar{s}_{H_2O} + 13.16\,\bar{s}_{N_2})_2 + \frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}}$$

Accordingly

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = -\frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + (2\,\bar{s}_{CO_2} + 3\,\bar{s}_{H_2O} + 13.16\,\bar{s}_{N_2})_2 - (\bar{s}_{fuel} + 3.5\,\bar{s}_{O_2} + 13.16\,\bar{s}_{N_2})_1 \qquad (1)$$

The term $\dot{Q}_{cv}/\dot{n}_{fuel}$ can be obtained from an energy rate balance which reduces with assumption 1 to give

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = (2\bar{h}_{CO_2} + 3\bar{h}_{H_2O} + 13.16\bar{h}_{N_2})_2 - (\bar{h}_{fuel} + 3.5\bar{h}_{O_2} + 13.16\bar{h}_{N_2})_1$$

$$= 2[\bar{h}_f^\circ + \bar{h}(900) - \bar{h}(298)]_{CO_2} + 3[\bar{h}_f^\circ + \bar{h}(900) - \bar{h}(298)]_{H_2O} + 13.16[\bar{h}(900) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{fuel}$$

With data from the ideal gas tables and Table A-25

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 2[-393{,}520 + 37{,}405 - 9364] + 3[-241{,}820 + 31{,}828 - 9904] + 13.16[26{,}890 - 8669] - (-84{,}680)$$

$$= -1{,}066{,}178 \text{ kJ/kmol } (C_2H_6)$$

Next, the specific entropies appearing in Eq. (1) are evaluated. The fuel and air enter as a mixture at $p = p_{ref}$ and $T = 298$ K with the composition $y_{fuel} = 1/17.66$, $y_{O_2} = 3.5/17.66$, $y_{N_2} = 13.16/17.66$. Thus

$$\bar{s}_{fuel} = \bar{s}^\circ_{fuel}(298) - \bar{R}\ln\frac{y_{fuel}\,p_{ref}}{p_{ref}} = 229.49 - 8.314\ln\frac{1}{17.66} = 253.36 \text{ kJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(298) - \bar{R}\ln y_{O_2} = 205.03 - 8.314\ln\frac{3.5}{17.66} = 218.49 \text{ kJ/kmol·K}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(298) - \bar{R}\ln y_{N_2} = 191.5 - 8.314\ln\frac{13.16}{17.66} = 193.95 \text{ kJ/kmol·K}$$

The combustion products exit as a mixture at 900 K, 1 atm with the composition $y_{CO_2} = 2/18.16$, $y_{H_2O} = 3/18.16$, $y_{N_2} = 13.16/18.16$. Thus

$$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(900) - \bar{R} \ln \frac{y_{CO_2} p_{ref}}{p_{ref}} = 263.559 - 8.314 \ln \frac{2}{18.16} = 281.90 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{H_2O} = \bar{s}^\circ_{H_2O}(900) - \bar{R} \ln y_{H_2O} = 228.321 - 8.314 \ln \frac{3}{18.16} = 243.29 \text{ kJ/kmol}\cdot\text{K}$$

$$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(900) - \bar{R} \ln y_{N_2} = 224.647 - 8.314 \ln \frac{13.16}{18.16} = 227.32 \text{ kJ/kmol}\cdot\text{K}$$

Collecting results

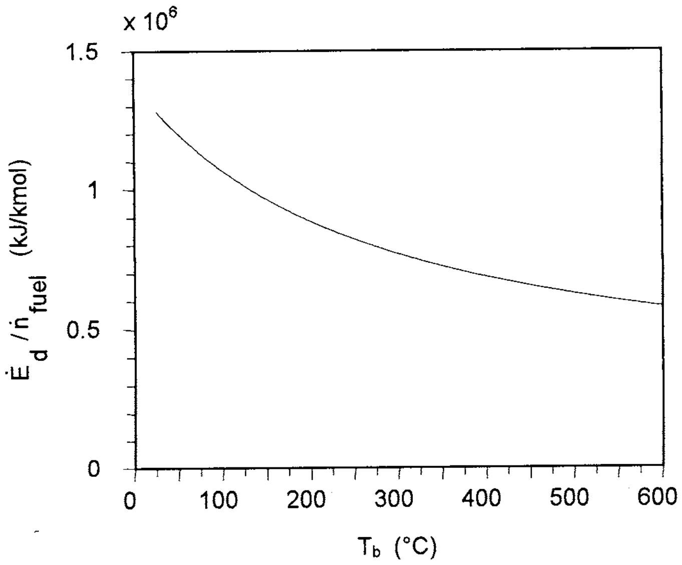
$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = -\frac{(-1{,}066{,}178)}{T_b} + 2(281.90) + 3(243.29) + 13.16(227.32) - 257.36 - 3.5(218.49) - 13.16(193.95) = \frac{1{,}066{,}178}{T_b} + 714.7 \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$$

$$\Rightarrow (\dot{E}_d/\dot{n}_{fuel}) = T_0 \left[\frac{1{,}066{,}178}{T_b} + 714.7\right] \frac{\text{kJ}}{\text{kmol(fuel)}}$$

Sample Calculation: when $T_b = 298$ K, $(\dot{E}_d/\dot{n}_{fuel}) = 1.28 \times 10^6$ kJ/kmol(fuel)

PLOT:

①



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1. At steady state, the net exergy carried into the reactor is a fixed value equal to the exergy destroyed within the reactor plus the exergy carried out accompanying heat transfer. Since the exergy carried out accompanying heat transfer increases as T_b increases: $[1 - T_0/T_b]\dot{Q}_{cv}$, the rate of exergy destruction within the reactor decreases as T_b increases, as shown in the figure.

PROBLEM 13.75

KNOWN: The reaction is $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$. Reactants and products are at 25°C, 1 atm.

FIND: Determine ΔG using (a) Gibbs function of formation data, (b) enthalpy of formation and absolute entropy data.

ANALYSIS: For $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$, ΔG is

$$\Delta G = \bar{g}_{CO_2} + 2\bar{g}_{H_2O} - \bar{g}_{CH_4} - 2\bar{g}_{O_2} \qquad (1)$$

where each $\bar{g}$ is at 25°C, 1 atm

(a) With $\bar{g}_f^\circ$ data from Table A-25, for H_2O as a gas, Eq. (1) gives

$$\Delta G = -394{,}380 + (2)(-228{,}590) - (-50{,}790) - (2)(0) = -800{,}770 \text{ kJ/kmol } CH_4 \leftarrow \text{(a)}$$

(b) With $\bar{g} = \bar{h} - T\bar{s}$, Eq. (1) becomes

$$\Delta G = (\bar{h} - T\bar{s})_{CO_2} + 2(\bar{h} - T\bar{s})_{H_2O} - (\bar{h} - T\bar{s})_{CH_4} - 2(\bar{h} - T\bar{s})_{O_2}$$

$$= (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} - \bar{h}_{CH_4} - 2\bar{h}_{O_2}) - T(\bar{s}_{CO_2} + 2\bar{s}_{H_2O} - \bar{s}_{CH_4} - 2\bar{s}_{O_2})$$

Inserting $\bar{h}_f^\circ$ and $\bar{s}^\circ$ data from Table A-25

$$\Delta G = [-393{,}520 + (2)(-241{,}820) - (-74{,}850) - (2)(0)] - 298.15[213.69 + (2)(188.72) - 186.16 - (2)(205.03)]$$

$$= -802{,}310 - 298.15(-5.09)$$

① $$= \underline{-800{,}792 \text{ kJ/kmol } CH_4} \leftarrow \text{(b)}$$

---

1. This is in agreement with the answer of part (a). The difference is due to round off.

PROBLEM 13.76

KNOWN: The reaction is $H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g)$. Reactants and products are at 77°F, 1 atm.

FIND: Determine ΔG using (a) Gibbs function of formation data, (b) enthapy of formation and absolute entropy data.

ANALYSIS: For $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$, ΔG is

$$\Delta G = \bar{g}_{H_2O(g)} - \bar{g}_{H_2} - \frac{1}{2}\bar{g}_{O_2} \qquad (1)$$

where each of the $\bar{g}$'s is eva uated at 77°F, 1 atm.

(a) With $\bar{g}_f^\circ$ data from Table A-25, Eq.(1) becomes

$$\Delta G = (\bar{g}_f^\circ)_{H_2O(g)} - (\bar{g}_f^\circ)_{H_2} - \frac{1}{2}(\bar{g}_f^\circ)_{O_2}$$

$$= -98,350 - (0) - \frac{1}{2}(0) = -98,350 \text{ Btu/lbmol} \longleftarrow \text{(a)}$$

(b) With $\bar{g} = \bar{h} - T\bar{s}$, Eq(1) becomes

$$\Delta G = (\bar{h} - T\bar{s})_{H_2O(g)} - (\bar{h} - T\bar{s})_{H_2} - \frac{1}{2}(\bar{h} - T\bar{s})_{O_2}$$

$$= (\bar{h}_{H_2O(g)} - \bar{h}_{H_2} - \frac{1}{2}\bar{h}_{O_2}) - T(\bar{s}_{H_2O(g)} - \bar{s}_{H_2} - \frac{1}{2}\bar{s}_{O_2})$$

Using $\bar{h}_f^\circ$ and $\bar{s}^\circ$ data from Table A-25

$$\Delta G = [-104,040 - 0 - \frac{1}{2}(0)] - 537[45.08 - 31.19 - \frac{1}{2}(49.981)]$$

① $$= -98,348 \text{ Btu/lbmol} \longleftarrow \text{(b)}$$

---

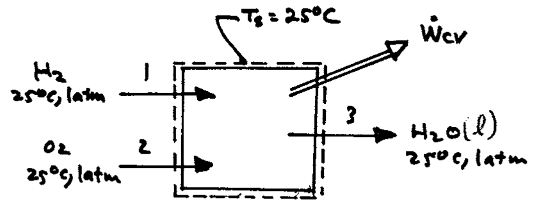
1. This agrees very closely with the result of part(a). The slight difference is due to round off.

PROBLEM 13.77

KNOWN: Separate streams of H_2 and O_2 at 25°C, 1 atm enter a fuel cell operating at steady state. A stream of liquid water exits at 25°C, 1atm.

FIND: Determine the maximum theoretical power the cell can develop for a specified fuel flow rate and the accompanying rate of heat transfer.

SCHEMATIC & GIVEN DATA:



$\dot{n}_{H_2} = 2 \times 10^{-4}\ \frac{kmol}{s}$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) The cell temperature is 25°C.

ANALYSIS: The overall cell reaction is $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$. At steady state an energy rate balance reduces to the form

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} - \frac{\dot{W}_{cv}}{\dot{n}_{H_2}} + (\bar{h}_{H_2})_1 + \frac{1}{2}(\bar{h}_{O_2})_2 - (\bar{h}_{H_2O(\ell)})_3$$

or

$$\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} + (\bar{h}_{H_2})_1 + \frac{1}{2}(\bar{h}_{O_2})_2 - (\bar{h}_{H_2O(\ell)})_3 \qquad (1)$$

At steady state an entropy rate balance takes the form

$$0 = \frac{(\dot{Q}_{cv}/\dot{n}_{H_2})}{T_s} + (\bar{s}_{H_2})_1 + \frac{1}{2}(\bar{s}_{O_2})_2 - (\bar{s}_{H_2O(\ell)})_3 + \frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} \qquad (2)$$

where $\dot{\sigma}_{cv}$ accounts for entropy production within the cell.

Combining Eqs. (1) and (2)

$$\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} = \underline{[\bar{h}_{H_2} + \frac{1}{2}\bar{h}_{O_2} - \bar{h}_{H_2O(\ell)}](25°C, 1atm) - T_s[\bar{s}_{H_2} + \frac{1}{2}\bar{s}_{O_2} - \bar{s}_{H_2O(\ell)}](25°C, 1atm)} - T_s\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}}$$

As the value of the underlined term is fixed by 25°C, 1atm and $\dot{\sigma}_{cv} \geq 0$, it is evident that the maximum theoretical value is obtained when $\dot{\sigma}_{cv} = 0$:

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2}}\right)_{MAX} = [\bar{h}_{H_2} + \frac{1}{2}\bar{h}_{O_2} - \bar{h}_{H_2O(\ell)}](25°C, 1atm) - T_s(\bar{s}_{H_2} + \frac{1}{2}\bar{s}_{O_2} - \bar{s}_{H_2O(\ell)}) \qquad (3)$$

This can be evaluated using enthalpy of formation data together with absolute entropy data. Alternatively, since $T_s = 25°C$ and $\bar{g} = \bar{h} - T\bar{s}$, Eq. (3) can be expressed as

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2}}\right)_{MAX} = [\bar{g}_{H_2} + \frac{1}{2}\bar{g}_{O_2} - \bar{g}_{H_2O(\ell)}](25°C, 1atm)$$

Then, with $\bar{g}_f$ data from Table A-25

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2}}\right)_{MAX} = 0 + \frac{1}{2}(0) - (-237{,}180)$$

$$= 237{,}180\ kJ/kmol(H_2)$$

PROBLEM 13.77 (Continued)

With the specified fuel flowrate

$$(\dot{W}_{cv})_{MAX} = \left[237{,}180\ \frac{kJ}{kmol(H_2)}\right]\left[2\times10^{-4}\ \frac{kmol(H_2)}{s}\right]\left|\frac{1\ kW}{1\ kJ/s}\right|$$

$$= 47.44\ kW \quad \longleftarrow \quad \dot{W}_{cv}$$

Returning to Eq. (1)

$$(\dot{Q}_{cv})_{MIN} = (\dot{W}_{cv})_{MAX} + \dot{n}_{H_2}\left[(\bar{h}_{H_2O(\ell)})_3 - \cancelto{0}{(\bar{h}_{H_2})_1} - \frac{1}{2}\cancelto{0}{(\bar{h}_{O_2})_2}\right]$$

$$= 47.44\ kW + \left(2\times10^{-4}\ \frac{kmol(H_2)}{s}\right)\left[-285{,}830\ \frac{kJ}{kmol}\right]\left|\frac{1\ kW}{1\ kJ/s}\right|$$

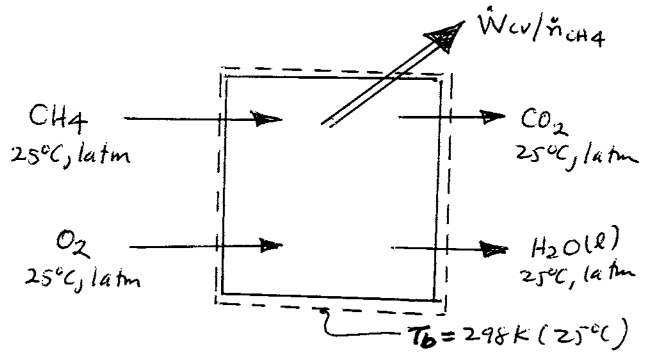
$$= -9.73\ kW \quad \longleftarrow \quad \dot{Q}_{cv}$$

PROBLEM 13.78

KNOWN: Operating data are provided for a CH_4/O_2 fuel cell at steady state.

FIND: Determine the maximum theoretical work that can be developed.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) Kinetic and potential energy effects are negligible. (3) Heat transfer takes place at $T_b = 298K$ (25°C). (4) At the condition specified, H_2O exits as a liquid.

ANALYSIS: The cell reaction is

$$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

Energy and entropy balances at steady state read, respectively.

(energy) $$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} - \frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} + (\bar{h}_{CH_4} + 2\bar{h}_{O_2} - \bar{h}_{CO_2} - 2\bar{h}_{H_2O})(298K, 1atm)$$

(entropy) $$0 = \frac{\dot{Q}_{cv}/\dot{n}_{CH_4}}{T_b} + (\bar{s}_{CH_4} + 2\bar{s}_{O_2} - \bar{s}_{CO_2} - 2\bar{s}_{H_2O})(298K, 1atm) + \dot{\sigma}_{cv}/\dot{n}_{CH_4}$$

Eliminating the heat transfer term between these gives

$$\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} = (\bar{h}_{CH_4} + 2\bar{h}_{O_2} - \bar{h}_{CO_2} - 2\bar{h}_{H_2O})(298K, 1atm) - T_b[\bar{s}_{CH_4} + 2\bar{s}_{O_2} - \bar{s}_{CO_2} - 2\bar{s}_{H_2O}](298K, 1atm) - T_b\,\dot{\sigma}_{cv}/\dot{n}_{CH_4}$$

In terms of the Gibbs function

$$\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} = [\bar{g}_{CH_4} + 2\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O(l)}](298, 1atm) - T_b\,\dot{\sigma}_{cv}/\dot{n}_{CH_4}$$

Accordingly, with data from Table A-25, and noting that the maximum value corresponds to the case of no entropy production,

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}\right)_{MAX} = -50{,}790 + 2(0) - (-394{,}380) - 2(-237{,}180)$$

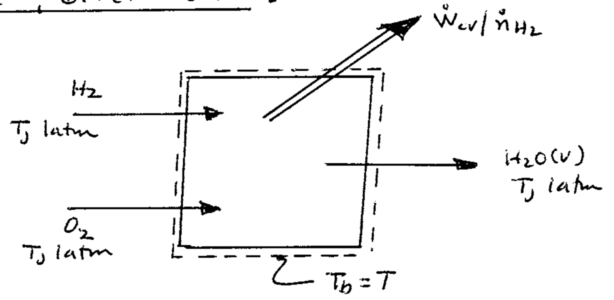
$$= 817{,}950 \ \frac{kJ}{kmol(CH_4)} \qquad \longleftarrow$$

PROBLEM 13.79

KNOWN: Operating data are provided for a H_2/O_2 fuel cell at steady state.

FIND: Determine the maximum theoretical work that can be developed if the cell operates isothermally at (a) 300K, (b) 400K, (c) 500K.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) Kinetic and potential energy effects are negligible. (3) Heat transfer occurs at the cell temperature.

ANALYSIS: The cell reaction is $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$. Energy and entropy balances read respectively

(energy)
$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2}} - \frac{\dot{W}_{cv}}{\dot{n}_{H_2}} + \left[\bar{h}_{H_2} + \tfrac{1}{2}\bar{h}_{O_2} - \bar{h}_{H_2O(v)}\right](T, 1atm)$$

(entropy)
$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{H_2}}{T_b} + \left[\bar{s}_{H_2} + \tfrac{1}{2}\bar{s}_{O_2} - \bar{s}_{H_2O(v)}\right](T, 1atm) + \frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}}$$

Eliminating $\dot{Q}_{cv}$ from these expressions

$$\frac{\dot{W}_{cv}}{\dot{n}_{H_2}} = \left[\bar{h}_{H_2} + \tfrac{1}{2}\bar{h}_{O_2} - \bar{h}_{H_2O(v)}\right] - T\left[\bar{s}_{H_2} + \tfrac{1}{2}\bar{s}_{O_2} - \bar{s}_{H_2O(v)}\right] - T\dot{\sigma}_{cv}/\dot{n}_{H_2}$$

Accordingly, with $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and $\bar{s} = \bar{s}^\circ(T, 1atm)$, and noting that the maximum value corresponds to the case of no entropy production

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2}}\right)_{MAX} = \left[\cancelto{0}{(\bar{h}_f^\circ)_{H_2}} + \tfrac{1}{2}\cancelto{0}{(\bar{h}_f^\circ)_{O_2}} - (\bar{h}_f^\circ)_{H_2O(v)}\right] + [\Delta\bar{h}]_{H_2} + \tfrac{1}{2}[\Delta\bar{h}]_{O_2} - [\Delta\bar{h}]_{H_2O}$$
$$- T\left[\bar{s}^\circ_{H_2} + \tfrac{1}{2}\bar{s}^\circ_{O_2} - \bar{s}^\circ_{H_2O(v)}\right](T)$$

$$= -[\bar{h}_f^\circ]_{H_2O(v)} + [\Delta\bar{h}]_{H_2} + \tfrac{1}{2}[\Delta\bar{h}]_{O_2} - [\Delta\bar{h}]_{H_2O} - T\left[\bar{s}^\circ_{H_2} + \tfrac{1}{2}\bar{s}^\circ_{O_2} - \bar{s}^\circ_{H_2O(v)}\right](T) \quad (1)$$

Eq. (1) is evaluated using the following IT code:

```
T = 300  // K
p = 1.01325  // bar

Wdot_max = - hfo_H2O + delh_H2 + .5 * delh_O2 - delh_H2O - T * (so_H2 + .5 * so_O2 -so_H2O)
hfo_H2O = h_T("H2O",298.15)
delh_H2 = h_T("H2",T) - h_T("H2",298.15)
delh_O2 =  h_T("O2",T) - h_T("O2",298.15)
delh_H2O =  h_T("H2O",T) - h_T("H2O",298.15)
so_H2 = s_TP("H2",T,p)
so_O2 = s_TP("O2",T,p)
so_H2O = s_TP("H2O",T,p)
```

①

The results are

PROBLEM 13.79 (Cont'd.)

	T = 300 K	T = 400 K	T = 500 K
$[\Delta\bar{h}]_{H_2}$ (kJ/kmol)	53.39	2955	5877
$[\Delta\bar{h}]_{H_2O}$ (kJ/kmol)	62.04	3449	6918
$[\Delta\bar{h}]_{O_2}$ (kJ/kmol)	54.36	3030	6094
$\left[\bar{h}_f^\circ\right]_{H_2O}$ (kJ/kmol)	-2.418E5	-2.418E5	-2.418E5
$\bar{s}^\circ_{H_2}$ (kJ/kmol·K)	130.7	139.1	145.6
$\bar{s}^\circ_{H_2O}$ (kJ/kmol·K)	188.9	198.7	206.4
$\bar{s}^\circ_{O_2}$ (kJ/kmol·K)	205.2	213.8	220.6
$\left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2}}\right)_{max}$ (kJ/kmol)	2.285E5	2.239E5	2.191E5

As T increases, $(\dot{W}_{cv}/\dot{n}_{H_2})_{MAX}$ decreases and the accompanying heat transfer increases. In principle, additional work could be developed from the heat transfer. The maximum obtainable corresponds to the case where the heat transfer is provided to a reversible power cycle operating between the cell at temperature T and the surroundings at T_0: the maximum would equal

$$\left[1 - \frac{T_0}{T}\right](\dot{Q}_{cv}/\dot{n}_{H_2})$$

---

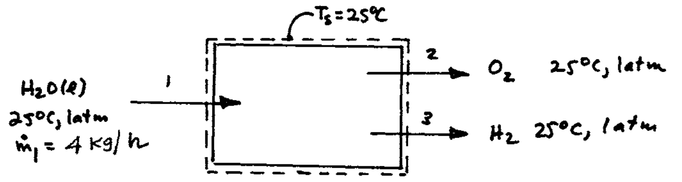
1. Only an IT solution is presented here, as Tables A-23 do not include data for H_2. Data for H_2 can be found in the same source as Table A-23.

PROBLEM 13.80

KNOWN: At steady state a device takes in liquid water at 25°C, 1atm with a mass flow rate of 5 kg/h and produces separate streams of H_2 and O_2, each at 25°C, 1atm. The device operates isothermally at 25°C. It is claimed that a power input of 16 kW is required.

FIND: Evaluate this claim

SCHEMATIC & GIVEN DATA:



$H_2O(\ell) \rightarrow H_2 + \frac{1}{2}O_2$

$\dot{W}_{cv} = -14.6\text{kW}$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) The device operates isothermally at 25°C.

ANALYSIS: The plan is to calculate the rate of entropy production on the basis that the given value for $\dot{W}_{cv}$ is correct. Thus, at steady state an entropy rate balance reduces to

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{H_2O}}{T_s} + [\bar{s}_{H_2O(\ell)}]_1 - [\bar{s}_{H_2}]_3 - \tfrac{1}{2}[\bar{s}_{O_2}]_2 + \dot{\sigma}_{cv}/\dot{n}_{H_2O} \qquad (1)$$

The heat tranfer rate can be evaluated from an energy rate balance at steady state:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}} - \frac{\dot{W}_{cv}}{\dot{n}_{H_2O}} + [\bar{h}_{H_2O(\ell)}]_1 - [\bar{h}_{H_2}]_3 - \tfrac{1}{2}[\bar{h}_{O_2}]_2 \qquad (2)$$

Combining Eqs. (1) and (2)

$$T_s\left(\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2O}}\right) = -\frac{\dot{W}_{cv}}{\dot{n}_{H_2O}} - \left\{[\bar{h}_{H_2}]_3 + \tfrac{1}{2}[\bar{h}_{O_2}]_2 - [\bar{h}_{H_2O(\ell)}]_1\right\} + T_s\left\{[\bar{s}_{H_2}]_3 + \tfrac{1}{2}[\bar{s}_{O_2}]_2 - [\bar{s}_{H_2O(\ell)}]_1\right\}$$

Each of the $\bar{h}$ and $\bar{s}$ terms in this expression is evaluated at 25°C, 1atm. Then, since $T_s = 25°C$ and $\bar{g} = \bar{h} - T\bar{s}$, it can be rewritten as

$$T_s\left(\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2O}}\right) = -\frac{\dot{W}_{cv}}{\dot{n}_{H_2O}} - \left[(\bar{g}_{H_2})_3 + \tfrac{1}{2}(\bar{g}_{O_2})_2 - (\bar{g}_{H_2O(\ell)})_1\right](25°C, 1atm) \qquad (3)$$

Using $M = 18.02$ for water, the molar flow rate of H_2O is

$$\dot{n}_{H_2O} = \frac{\dot{m}_1}{M} = \frac{4 \text{ kg/h}}{18.02 \text{ kg/kmol}} = 0.222 \frac{\text{kmol}}{\text{h}}$$

Accordingly

$$\frac{\dot{W}_{cv}}{\dot{n}_{H_2O}} = \frac{-(14.6 \text{ kJ/s})\left|3600\text{s/h}\right|}{(0.222 \text{ kmol/h})} = -236{,}757 \frac{\text{kJ}}{\text{kmol}}$$

Then, with $\bar{g}_f^\circ$ data from Table A-25

$$T_s\left(\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2O}}\right) = 236{,}757 - \left[0 + \tfrac{1}{2}(0) - (-237{,}180)\right]$$

$$= -423 \frac{\text{kJ}}{\text{kmol}(H_2O)}$$

As $T_s > 0$, this implies $\dot{\sigma}_{cv}/\dot{n}_{H_2O} < 0$ which is impossible. The value claimed for $\dot{W}_{cv}$ cannot be correct. ◄

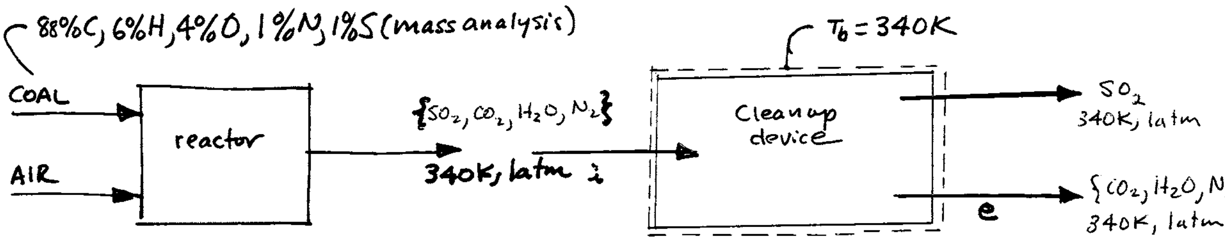
PROBLEM 13.81

KNOWN: Coal with a specified mass analysis burns completely with the theoretical amount of air.

FIND: Determine (a) the amount of SO_2 produced, (b) the air-fuel ratio on a mass basis. Also determine (c) the minimum theoretical power required to remove SO_2 from the combustion products via a specified device.

ASSUMPTIONS: 1. 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. 2. The cleanup device operates at steady state at 340 K with negligible kinetic & potential energy effects. 3. Ideal gas principles apply.

SCHEMATIC & GIVEN DATA:



ANALYSIS: On the basis of 100 kg of coal, the amounts of the substances present, in kmol, are

	m_i	M_i^*	$n_i = m_i/M_i$
C	88	12	7.33
H	6	1	6.00
O	4	16	0.25
N	1	14	0.07
S	1	32	0.03

* rounded to nearest integer

$$\Rightarrow (7.33\,C + 6H + 0.25\,O + 0.07\,N + 0.03\,S) + \alpha(O_2 + 3.76N_2) \rightarrow \beta CO_2 + \gamma H_2O + \omega SO_2 + \left[3.76\alpha + \frac{0.07}{2}\right]N_2$$

$C:\ 7.33 = \beta,\quad H:\ 6 = 2\gamma,\ \gamma = 3,\quad S:\ 0.03 = \omega,$

$O:\ 0.25 + 2\alpha = 2(7.33) + 3 + 2(0.03) \Rightarrow \alpha = 8.735$

(a) The amount of SO_2 produced is 0.03 kmol per 100 kg of coal. Thus, with $M_{SO_2} = 64$

$$m_{SO_2} = \frac{(0.03)(64)}{100} = 0.0192\ \frac{kg(SO_2)}{kg(coal)}$$

(b) The amount of air required is (8.735)(4.76) kmol per 100 kg of coal. Thus

$$AF = \frac{(8.735)(4.76)(28.97)}{100} = 12.05\ \frac{kg(air)}{kg(coal)}$$

(c) Referring to the reaction equation, for a coal burn rate of 10 kg/s, the molar flow rates of the combustion products are

Based on a coal burn rate of 10 kg/s		
CO_2	0.733	kmol/s
H_2O	0.300	kmol/s
N_2	3.284	kmol/s
SO_2	0.003	kmol/s
$\dot{n}_i =$	4.320	kmol/s

For the cleanup device, the molar analyses of the incoming and outgoing streams are

	i entering, y	e exiting, y'
CO_2	0.1697	0.1698
H_2O	0.0694	0.0695
N_2	0.7602	0.7607
SO_2	0.0007	—

PROBLEM 13.81 (Contd)

An energy rate balance reduces at steady state to $0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum \dot{n}_i h_i - \sum \dot{n}_e h_e$. Then, since ideal gas principles apply, the enthalpy terms cancel because the entering and exiting streams are at the same temperature. Accordingly, $\dot{W}_{cv} = \dot{Q}_{cv}$.

An entropy rate balance reduces at steady state to

$$0 = \frac{\dot{Q}_{cv}}{T_b} + \left[\dot{n}_{CO_2}\bar{s}_{CO_2}(T, y_{CO_2}p) + \dot{n}_{H_2O}\bar{s}_{H_2O}(T, y_{H_2O}p) + \dot{n}_{N_2}\bar{s}_{N_2}(T, y_{N_2}p) + \dot{n}_{SO_2}\bar{s}_{SO_2}(T, y_{SO_2}p)\right]$$

$$- \left[\dot{n}_{CO_2}\bar{s}_{CO_2}(T, y'_{CO_2}p) + \dot{n}_{H_2O}\bar{s}_{H_2O}(T, y'_{H_2O}p) + \dot{n}_{N_2}\bar{s}_{N_2}(T, y'_{N_2}p)\right] - \dot{n}_{SO_2}\bar{s}_{SO_2}(T, p) + \dot{\sigma}_{cv}$$

Collecting results, and using Eq. 13.23: $\bar{s}_i(T, y_i p) = \bar{s}_i^{\circ}(T) - \bar{R}\ln y_i p/p_{ref}$ and $p = p_{ref}$

$$\dot{W}_{cv} = T_b\left[\dot{n}_{CO_2}\left(-\bar{R}\ln\frac{y'_{CO_2}}{y_{CO_2}}\right) + \dot{n}_{H_2O}\left(-\bar{R}\ln\frac{y'_{H_2O}}{y_{H_2O}}\right) + \dot{n}_{N_2}\left(-\bar{R}\ln\frac{y'_{N_2}}{y_{N_2}}\right) + \dot{n}_{SO_2}\left(-\bar{R}\ln\frac{1}{y_{SO_2}}\right) - \dot{\sigma}_{cv}\right]$$

The minimum theoretical power input corresponds to $\dot{\sigma}_{cv} = 0$. Then, with y and y′ from above

$$(\dot{W}_{cv})_{min} = -\left(8.314\,\frac{kJ}{kmol\cdot K}\right)(340\,K)\left[\left(0.733\,\frac{kmol}{s}\right)\ln\frac{0.1698}{0.1697} + 0.3\ln\frac{0.0695}{0.0694} + 3.284\ln\frac{0.7607}{0.7602} + 0.003\ln\frac{1}{0.007}\right]$$

$$= -70.2\,\frac{kJ}{s}$$

$$= -70.2\ kW \qquad \longleftarrow$$

PROBLEM 13.82

KNOWN: An environment is specified in which the gas phase obeys the ideal gas model.

FIND: For each of several specified substances, determine the chemical exergy.

ASSUMPTIONS: The environment consists of an ideal gas mixture having the following analysis

Environment

T_0 = 298.15 K (25°C), p_0 = 1 atm

Gas Phase:	Component	y^e (%)
	N_2	75.67
	O_2	20.35
	$H_2O(g)$	3.12
	CO_2	0.03
	Other	0.83

ANALYSIS: (a) Carbon: $C + O_2 \rightarrow CO_2$. Eq. 13.36 takes the form

$$\bar{e}^{CH} = (\bar{g}_C + \bar{g}_{O_2} - \bar{g}_{CO_2})(T_0, p_0) + \bar{R}T_0 \ln\left[\frac{y^e_{O_2}}{y^e_{CO_2}}\right]$$

Then, with $\bar{g}^\circ_f$ data from Table A-25

$$\bar{e}^{CH} = (0 + 0 - (-394,380)) + (8.314)(298.15)\ln\left[\frac{0.2035}{0.0003}\right]$$

$$= 410,541 \text{ kJ/kmol}$$

With M = 12 from Table A-1

$$e^{CH} = \frac{410,541}{12} = 34,212 \frac{kJ}{kg} \longleftarrow \text{(a)}$$

(b) Hydrogen: $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$. Eq. 13.36 takes the form

$$\bar{e}^{CH} = (\bar{g}_{H_2} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{H_2O})(T_0, p_0) + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{1/2}}{y^e_{H_2O}}\right]$$

Then, with $\bar{g}^\circ_f$ data from Table A-25

$$\bar{e}^{CH} = (0 + \tfrac{1}{2}(0) - (-228,590)) + (8.314)(298.15)\ln\left[\frac{(0.2035)^{1/2}}{0.0312}\right]$$

$$= 235,212 \text{ kJ/kmol}$$

With M = 2.018

$$e^{CH} = \frac{235,212}{2.018} = 116,557 \text{ kJ/kg} \longleftarrow \text{(b)}$$

(c) Methane: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$. Eq. 13.36 takes the form

$$\bar{e}^{CH} = [\bar{g}_{CH_4} + 2\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O}] + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^2}{(y^e_{CO_2})(y^e_{H_2O})^2}\right]$$

Then, with $\bar{g}^\circ_f$ data from Table A-25

$$\bar{e}^{CH} = [-50,790 + 2(0) - (-394,380) - 2(-228,590)] + (8.314)(298.15)\ln\left[\frac{(0.2035)^2}{(0.0003)(0.0312)^2}\right]$$

$$= 830,174 \text{ kJ/kmol}$$

With M = 16.04

$$e^{CH} = \frac{830,174}{16.04} = 51,756 \frac{kJ}{kg} \longleftarrow \text{(c)}$$

PROBLEM 13.82 (Contd.)

(d) Carbon Monoxide: $CO + \frac{1}{2} O_2 \rightarrow CO_2$. Eq. 13.38 takes the form

$$\bar{e}^{CH} = \bar{g}_{CO} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2})(T_0, P_0) + \bar{R} T_0 \ln\left[\frac{(y^e_{O_2})^{1/2}}{y^e_{CO_2}}\right]$$

With $\bar{g}^o_f$ data from Table A-25

$$\bar{e}^{CH} \; \left((-137{,}150) + \tfrac{1}{2}(0) - (-394{,}380)\right) + (8.314)(298.15)\ln\left[\frac{(0.2035)^{1/2}}{0.0003}\right]$$

$$= 275{,}364 \text{ kJ/kmol}$$

With M = 28.01

$$e^{CH} = \frac{275{,}364}{28.01} = 9831 \frac{kJ}{kg} \quad \longleftarrow\!\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!- \text{(d)}$$

(e) Liquid Methanol: $CH_3OH + 1.5 O_2 \rightarrow CO_2 + 2H_2O$

$$\bar{e}^{CH} = \left(\bar{g}_{CH_3OH} + 1.5\,\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\,\bar{g}_{H_2O}\right) + \bar{R} T_0 \ln\left[\frac{(y^e_{O_2})^{1.5}}{y^e_{CO_2}(y^e_{H_2O})^2}\right]$$

$$= \left((-166{,}290) + 1.5(0) - (-394{,}380) - 2(-228{,}590)\right) + (8.314)(298.15)\ln\left[\frac{(0.2035)^{1.5}}{(0.0003)(0.0312)^2}\right]$$

$$= 716{,}647 \text{ kJ/kmol}$$

With M = 32.05

$$e^{CH} = \frac{716{,}647}{32.05} = 22{,}360 \frac{kJ}{kg} \quad \longleftarrow\!\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!- \text{(e)}$$

(f) Nitrogen: With Eq. 13.40

$$\bar{e}^{CH} = \bar{R} T_0 \ln\left(\frac{1}{y^e_{N_2}}\right) = (8.314)(298.15)\ln\left(\frac{1}{0.7567}\right) = 691.1 \text{ kJ/kmol}$$

With M = 28.01

$$e^{CH} = \frac{691.1}{28.01} = 24.7 \frac{kJ}{kg} \quad \longleftarrow\!\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!- \text{(f)}$$

(g) Oxygen. With Eq. 13.40

$$\bar{e}^{CH} = \bar{R} T_0 \ln\left(\frac{1}{y^e_{O_2}}\right) = (8.314)(298.15)\ln\left(\frac{1}{0.2035}\right) = 3947 \frac{kJ}{kmol}$$

With M = 32

$$e^{CH} = \frac{3947}{32} = 123.3 \frac{kJ}{kg} \quad \longleftarrow\!\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!- \text{(g)}$$

(h) Carbon dioxide. With Eq. 13.40

$$\bar{e}^{CH} = \bar{R} T_0 \ln\left(\frac{1}{y^e_{CO_2}}\right) = (8.314)(298.15)\ln\left(\frac{1}{0.0003}\right) = 20{,}108 \text{ kJ/kmol}$$

With M = 44.01

$$e^{CH} = \frac{20{,}108}{44.01} = 456.9 \text{ kJ/kg} \quad \longleftarrow\!\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!- \text{(h)}$$

PROBLEM 13.82 (Contd.)

(i) water. With Eq. 13.39

$$\bar{e}^{CH} = \left[\bar{g}_{H_2O(l)} - \bar{g}_{H_2O(g)}\right](T_0, p_0) + \bar{R}T_0 \ln\left(\frac{1}{y^e_{H_2O}}\right)$$

With $\bar{g}^\circ_f$ data from Table A-30

① $$\bar{e}^{CH} = \left((-237{,}180) - (-228{,}590)\right) + (8.314)(298.15) \ln\left(\frac{1}{0.0312}\right) = 4.9 \text{ kJ/kmol}$$

---

1. This is a negligible amount that would be zero were the table values for $\bar{g}^\circ_f$ not rounded, suggesting that when $y^e_{H_2O} = 0.312$ liquid water at T_0, p_0 could also be included within the environment with no inconsistency. This point is explored in Problem 13.83.

PROBLEM 13.83

KNOWN: An environment is specified in which there is a gas phase and a condensed water phase.

FIND: (a) Show that the chemical exergy of C_aH_b can be determined as

$$\bar{e}^{CH} = \left[\bar{g}_F + \left(a + \frac{b}{4}\right)\bar{g}_{O_2} - a\bar{g}_{CO_2} - \frac{b}{2}\bar{g}_{H_2O(l)}\right] + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{a+b/4}}{(y^e_{CO_2})^a}\right]$$

(b) Using the result of part (a), repeat parts (a)-(c) of Problem 13.82

ASSUMPTIONS: The environment consists of a gas phase obeying the ideal gas model and a condensed water phase:

Environment

T_0 = 298.15 K (25°C), p_0 = 1 atm

Condensed Phase: $H_2O(l)$ at T_0, p_0

Gas Phase:	Component	y^e (%)
	N_2	75.67
	O_2	20.35
	H_2O	3.12
	CO_2	0.03
	Other	0.83

ANALYSIS: The derivation proceeds as in Sec. 13.6 except we now think of the water formed in the cell reaction as being released to the environment as liquid water at T_0, p_0. Accordingly, in Eqs. 13.30 through 13.33 $\bar{h}_{H_2O}$ and $\bar{s}_{H_2O}$ refer to liquid water at T_0, p_0. Then, Eq. 13.34 applies only to the O_2 and CO_2, giving in place of Eq 13.35 the following expression

$$\bar{e}^{CH} = \left[\bar{h}_F + \left(a + \frac{b}{4}\right)\bar{h}_{O_2} - a\bar{h}_{CO_2} - \frac{b}{2}\bar{h}_{H_2O}\right](T_0, p_0) - T_0\left[\bar{s}_F + \left(a + \frac{b}{4}\right)\bar{s}_{O_2} - a\bar{s}_{CO_2} - \frac{b}{2}\bar{s}_{H_2O}\right](T_0, p_0) + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{a+b/4}}{(y^e_{CO_2})^a}\right]$$

With $\bar{g} = \bar{h} - T\bar{s}$ this can be expressed in the form indicated above under FIND.

Thus, two expressions for $\bar{e}^{CH}$ are obtained. One is given by Eq 13.36:

$$\bar{e}^{CH} = \left[\bar{g}_F + \left(a + \frac{b}{4}\right)\bar{g}_{O_2} - a\bar{g}_{CO_2} - \frac{b}{2}\bar{g}_{H_2O(g)}\right](T_0, p_0) + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{a+b/4}}{(y^e_{CO_2})^a (y^e_{H_2O})^{b/2}}\right]$$

The other is the result obtained here:

$$\bar{e}^{CH} = \left[\bar{g}_F + \left(a + \frac{b}{4}\right)\bar{g}_{O_2} - a\bar{g}_{CO_2} - \frac{b}{2}\bar{g}_{H_2O(l)}\right](T_0, p_0) + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{a+b/4}}{(y^e_{CO_2})^a}\right]$$

PROBLEM 13.83 (Contd.)

Upon comparison, it is evident that the same numerical value for $\bar{e}^{CH}$ will be obtained only if y^e_{H2O} is specified so that

$$\bar{g}_{H_2O(g)} + \bar{R}T_0 \ln y^e_{H_2O} = \bar{g}_{H_2O(l)}$$

or

$$\ln y^e_{H_2O} = \frac{\bar{g}_{H_2O(l)} - \bar{g}_{H_2O(g)}}{\bar{R}T_0}$$

where each of the $\bar{g}$'s is evaluated at T_0, P_0. Since $T_0 = 298.15$ K and $P_0 = 1$ atm, Table A-25 gives

$$\ln y^e_{H_2O} = \frac{-237{,}180 - (-228{,}590)}{(8.314)(298.15)}$$

This gives, closely, $y^e_{H_2O} = 0.03126$ which departs from the table value only slightly owing to round off in the values used for $\bar{g}^\circ_f$.

(b) Using the result of part (a), repeat parts (a)-(c) of Problem 13.82

(a) Carbon: $C + O_2 \rightarrow CO_2$. The result of part (a) takes the form

$$\bar{a}^{ch} = (\bar{g}_C + \bar{g}_{O_2} - \bar{g}_{CO_2}) + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})}{y^e_{CO_2}}\right]$$

This clearly gives the same answer as in Problem 13.82(a): $a^{ch} = 34{,}212$ kJ/kg. $\longleftarrow$ (a)

(b) Hydrogen: $H_2 + \frac{1}{2}O_2 \rightarrow H_2O$. The result of part (a) takes the form

$$\bar{e}^{CH} = \left[\bar{g}_{H_2} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{H_2O(l)}\right] + \bar{R}T_0 \ln\left((y^e_{O_2})^{1/2}\right)$$

With $\bar{g}^\circ_f$ data from Table A-25

$$\bar{e}^{CH} = \left[0 + \tfrac{1}{2}(0) - (-237{,}180)\right] + (8.314)(298.15)\ln\left((0.2035)^{1/2}\right)$$
$$= 235{,}207 \text{ kJ/kmol}$$

or with M = 2.018

$$e^{CH} = 116{,}554 \text{ kJ/kg}$$

$\longleftarrow$ (b)

which is in excellent agreement with the result of Problem 13.82(b)

(c) Methane: $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$. The result of part (a) takes the form

$$\bar{e}^{CH} \cdot \left[\bar{g}_{CH_4} + 2\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O(l)}\right] + \bar{R}T_0 \ln\left(\frac{(y^e_{O_2})^2}{(y^e_{CO_2})}\right)$$

With $\bar{g}^\circ_f$ data from Table A-25

$$\bar{e}^{CH} = \left[(-50{,}790) + 2(0) - (-394{,}380) - 2(-237{,}180)\right] + (8.314)(298.15)\ln\left[\frac{(.2035)^2}{0.0003}\right]$$
$$= 830{,}165 \text{ kJ/kmol}$$

or with M = 16.04

$$\bar{e}^{CH} = 51756 \frac{kJ}{kg}$$

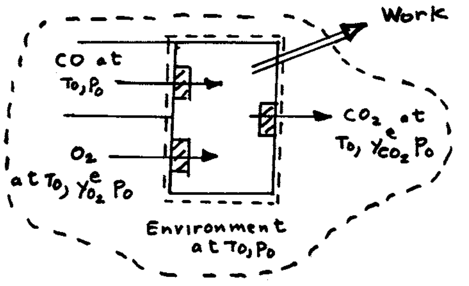
$\longleftarrow$ (c)

which is the result obtained in Problem 13.82(c).

PROBLEM 13.84

FIND: Showing all important steps, derive (a) Eq. 13.38, (b) Eq. 13.39, (c) Eq. 13.40, (d) Eq. 13.41a,b, (e) Eq. 13.44 a, b.

ANALYSIS: (a) Eq. 13.38. Paralleling the derivation of Sec. 13.6 consider this device operating at steady state


Work
CO at T_0, P_0
O_2 at $T_0, y^e_{O_2} P_0$
CO_2 at $T_0, y^e_{CO_2} P_0$
Environment at T_0, P_0

$$CO + \tfrac{1}{2} O_2 \rightarrow CO_2$$

energy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CO}} - \frac{\dot{W}_{cv}}{\dot{n}_{CO}} + \bar{h}_{CO} + \tfrac{1}{2}\bar{h}_{O_2} - \bar{h}_{CO_2}$$

entropy rate balance:

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{CO}}{T_0} + \bar{s}_{CO}(T_0, P_0) + \tfrac{1}{2}\bar{s}_{O_2}(T_0, y^e_{O_2} P_0) - \bar{s}_{CO_2}(T_0, y^e_{CO_2} P_0) + \frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}}$$

Combining these two equations

$$\frac{\dot{W}_{cv}}{\dot{n}_{CO}} = \left\{\bar{h}_{CO}(T_0) + \tfrac{1}{2}\bar{h}_{O_2}(T_0) - \bar{h}_{CO_2}(T_0)\right\} - T_0\left\{\bar{s}_{CO}(T_0, P_0) + \tfrac{1}{2}\bar{s}_{O_2}(T_0, y^e_{O_2} P_0) - \bar{s}_{CO_2}(T_0, y^e_{CO_2} P_0)\right\} - T_0 \dot{\sigma}_{cv}/\dot{n}_{CO}$$

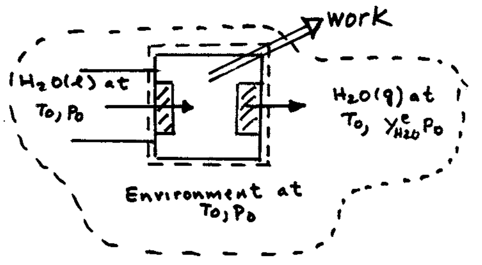
The maximum value for $\dot{W}_{cv}/\dot{n}_{CO}$ corresponds to the case $\dot{\sigma}_{cv} = 0$. Then, using Eq. 13.34 for the $\bar{s}_{O_2}$ and $\bar{s}_{CO_2}$ terms

$$\bar{a}^{CH} = \left\{\bar{h}_{CO}(T_0) + \tfrac{1}{2}\bar{h}_{O_2}(T_0) - \bar{h}_{CO_2}(T_0)\right\} - T_0\left\{\bar{s}_{CO}(T_0, P_0) + \tfrac{1}{2}\bar{s}_{O_2}(T_0, P_0) - \bar{s}_{CO_2}(T_0, P_0)\right\} + \bar{R} T_0 \ln\left[\frac{(y^e_{O_2})^{1/2}}{y^e_{CO_2}}\right]$$

Finally, using $\bar{g} = \bar{h} - T\bar{s}$

$$\bar{e}^{CH} = (\bar{g}_{CO} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2})(T_0, P_0) + \bar{R} T_0 \ln\left[\frac{(x^{o}_{O_2})^{1/2}}{x^{o}_{CO_2}}\right] \longleftarrow$$

(b) Eq. 13.39. Consider this device operating at steady state


work
$H_2O(\ell)$ at T_0, P_0
$H_2O(g)$ at $T_0, y^e_{H_2O} P_0$
Environment at T_0, P_0

energy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}} - \frac{\dot{W}_{cv}}{\dot{n}_{H_2O}} + \bar{h}_{H_2O(\ell)} - \bar{h}_{H_2O(g)}$$

entropy rate balance:

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{H_2O}}{T_0} + \bar{s}_{H_2O(\ell)}(T_0, P_0) - \bar{s}_{H_2O(g)}(T_0, y^e_{H_2O} P_0) + \cancel{\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2O}}}^{\,0}$$

Collecting these two equations

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_{H_2O}}\right)_{MAX} = \left\{h_{H_2O(\ell)} - \bar{h}_{H_2O(g)}\right\} - T_0\left\{\bar{s}_{H_2O(\ell)}(T_0, P_0) - s_{H_2O(g)}(T_0, y^e_{H_2O} P_0)\right\}$$

Using Eq. 13.34 for the water vapor

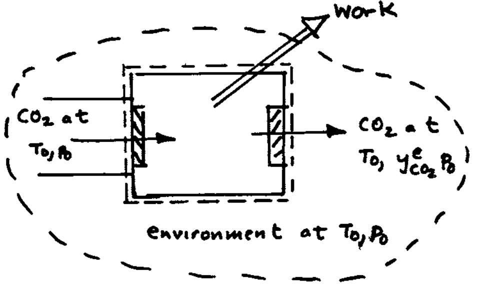
$$\bar{e}^{CH} = \left\{\bar{h}_{H_2O}(\ell) - \bar{h}_{H_2O}(g)\right\} - T_0\left\{\bar{s}_{H_2O(\ell)}(T_0, P_0) - \bar{s}_{H_2O(g)}(T_0, P_0)\right\} - \bar{R} T_0 \ln y^e_{H_2O}$$

Or with $\bar{g} = \bar{h} - T\bar{s}$

$$\bar{e}^{CH} \quad \left[\bar{g}_{H_2O}(\ell) - \bar{g}_{H_2O(g)}\right](T_0, P_0) + \bar{R} T_0 \ln 1/y^e_{H_2O} \longleftarrow$$

PROBLEM 13.84 (Contd.)

(c) Eq. 13.40 . Taking CO_2 as an example, consider this device operating at steady state



energy rate balance:

$$0 = \frac{\dot Q_{cv}}{\dot n_{CO_2}} - \frac{\dot W_{cv}}{\dot n_{CO_2}} + \underbrace{(\bar h_{CO_2})_{in} - (\bar h_{CO_2})_{out}}_{\text{= 0 since } \bar h \text{ depends on T alone and the temperature is } T_0 \text{ at inlet and exit}}$$

entropy rate balance:

$$0 = \frac{\dot Q_{cv}/\dot n_{CO_2}}{T_0} + \bar s_{CO_2}(T_0, p_0) - \bar s_{CO_2}(T_0, y^e_{CO_2} p_0) - \overset{0}{\cancel{\frac{\dot\sigma_{cv}}{\dot n_{CO_2}}}}$$

Combining these two equations

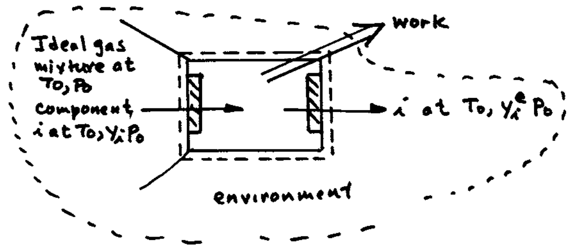
$$\left(\frac{\dot W_{cv}}{\dot n_{CO_2}}\right)_{MAX} = -T_0\left[\bar s_{CO_2}(T_0, P_0) - \bar s_{CO_2}(T_0, y^e_{CO_2} P_0)\right]$$

Using Eq. 13.34

$$\bar e^{CH} = -T_0\left[s_{CO_2}(T_0, P_0) - \left(s_{CO_2}(T_0, P_0) - \bar R \ln y^e_{CO_2}\right)\right]$$

$$= \bar R T_0 \ln\left(1/y^e_{CO_2}\right) \longleftarrow$$

(d) Eq. 13.41a For each component i of an ideal gas mixture, consider a cell into which i passes at T_0 and the partial pressure $y_i P_0$ and from which i exits at T_0 and the partial pressure $y_i^e P_0$.


work
Ideal gas mixture at T_0, P_0
component i at $T_0, y_i P_0$
i at $T_0, y_i^e P_0$
environment

Energy rate balance:

$$0 = \frac{\dot Q_{cv}}{\dot n_i} - \frac{\dot W_{cv}}{\dot n_i} + \underbrace{(\bar h_i)_{in} - (\bar h_i)_{out}}_{\text{= 0 since } \bar h \text{ depends on T alone and the temperature is } T_0 \text{ at the inlet and exit}}$$

Entropy rate balance:

$$0 = \frac{\dot Q_{cv}/\dot n_i}{T_0} + \bar s_i(T_0, y_i P_0) - \bar s_i(T_0, y_i^e P_0) + \overset{0}{\cancel{\frac{\dot\sigma_{cv}}{\dot n_i}}}$$

Combining these equations

$$\left(\frac{\dot W_{cv}}{\dot n_i}\right)_{MAX} = -T_0\left(\bar s_i(T_0, y_i P_0) - \bar s_i(T_0, y_i^e P_0)\right)$$

Using Eq. 13.34, the chemical exergy per mole of i is

$$\bar e_i^{CH} = -T_0\left[\left(\bar s_i(T_0, P_0) - \bar R \ln y_i\right) - \left(\bar s_i(T_0, P_0) - \bar R \ln y_i^e\right)\right]$$

$$= \bar R T_0 \ln \frac{y_i}{y_i^e}$$

Summing all such expressions for the mixture, the chemical exergy per mole of mixture is

$$\bar e^{CH} = \sum_i y_i \bar R T_0 \ln \frac{y_i}{y_i^e}$$

$$= \bar R T_0 \sum_i y_i \ln \frac{y_i}{y_i^e} \longleftarrow$$

PROBLEM 13.84 (Contd.)

Eq. 13.41b. Expanding the last expression

$$\bar{e}^{CH} = \bar{R}T_0 \sum_i \left(y_i \ln y_i - y_i \ln y_i^e \right)$$

$$= \bar{R}T_0 \sum_i y_i \ln y_i + \sum_i y_i \underbrace{\left(\bar{R}T_0 \ln\left(\frac{1}{y_i^e}\right)\right)}_{\bar{e}_i^{CH} \ (\text{Eq. } 13.40)}$$

$$= \sum_i y_i \bar{e}_i^{CH} + \bar{R}T_0 \sum_i y_i \ln y_i \quad \longleftarrow$$

(e) Eq. 13.44a. Rewrite the underlined term of Eq. 13.43 as $\overline{HHV}$. Introducing this into Eq. 13.43, gives Eq. 13.44a:

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_F}\right)_{\substack{int\\rev}} = \underbrace{\overline{HHV}(T_0, p_0) - T_0\left[\bar{s}_F + \left(a + \frac{b}{4}\right)\bar{s}_{O_2} - a\,\bar{s}_{CO_2} - \frac{b}{2}\bar{s}_{H_2O(\ell)}\right]}(T_0, p_0)$$

Then

$$\bar{e}_F^{CH} = \{\cdots\} + a\,\bar{a}_{CO_2}^{CH} + \frac{b}{2}\bar{a}_{H_2O(\ell)}^{CH} - \left(a + \frac{b}{2}\right)\bar{a}_{O_2}^{CH} \quad \longleftarrow$$

Eq. 13.44b. Using $\bar{g}(T_0, s_0) = \bar{h}(T_0, s_0) - T_0\,\bar{s}(T_0, s_0)$, Eq. 13.43 becomes

$$\left(\frac{\dot{W}_{cv}}{\dot{n}_F}\right)_{\substack{int\\rev}} = \underbrace{\left[\bar{g}_F + \left(a + \frac{b}{4}\right)\bar{g}_{O_2} - a\,\bar{g}_{CO_2} - \frac{b}{2}\bar{g}_{H_2O(\ell)}\right]}(T_0, p_0)$$

Then, Eq. 13.42 becomes

$$\bar{e}_F^{CH} = \{\cdots\} + a\,\bar{a}_{CO_2}^{CH} + \frac{b}{2}\bar{a}_{H_2O(\ell)}^{CH} - \left(a + \frac{b}{2}\right)\bar{a}_{O_2}^{CH} \quad \longleftarrow$$

PROBLEM 13.85

KNOWN: Data from Tables A-25 and A-26

FIND: Using Eq. 13.45, determine the standard molar chemical exergy, in kJ/kmol, for (a) $NH_3(g)$, (b) C_3H_8.

ASSUMPTION: Model I of Table A-26 applies.

ANALYSIS:

NH_3. Consider the reaction

$$NH_3(g) + \frac{3}{4}O_2 \rightarrow \frac{1}{2}N_2 + \frac{3}{2}H_2O(g)$$

For this reaction, Eq. 13.45 takes the form

$$\bar{e}^{CH} = \left[\bar{g}_{NH_3(g)} + \frac{3}{4}\bar{g}_{O_2} - \frac{1}{2}\bar{g}_{N_2} - \frac{3}{2}\bar{g}_{H_2O(g)}\right](T_0, p_0)$$
$$+\frac{1}{2}\bar{e}^{CH}_{N_2} + \frac{3}{2}\bar{e}^{CH}_{H_2O(\ell)} - \frac{3}{4}\bar{e}^{CH}_{O_2}$$

① With $\bar{g}$'s from Table A-25 and $\bar{e}^{CH}$'s from Table A-26 (Model I)

$$\bar{e}^{CH} = \left[-16{,}590 + \frac{3}{4}(0) - \frac{1}{2}(0) - \frac{3}{2}(-228{,}590)\right]$$
$$+\left[\left(\frac{1}{2}\right)(640) + \left(\frac{3}{2}\right)(8635) - (3/4)(3950)\right]$$
$$= 326{,}295 + 10{,}310 = 336{,}605 \text{ kJ/kmol} \longleftarrow \text{(a)}$$

This value agrees very closely with the Table A-26 entry.

C_3H_8. Consider the reaction

$$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O(g)$$

For this reaction, Eq. 13.45 takes the form

$$\bar{e}^{CH} = \left[\bar{g}_{C_3H_8} + 5\bar{g}_{O_2} - 3\bar{g}_{CO_2} - 4\bar{g}_{H_2O}\right](T_0, p_0) + 3\bar{e}^{CH}_{CO_2} + 4\bar{e}^{CH}_{H_2O(g)} - 5\bar{e}^{CH}_{O_2}$$

With $\bar{g}$'s from Table A-25 and $\bar{e}^{CH}$'s from Table A-26 (Model I)

$$\bar{e}^{CH} = [-23{,}490 + (5)(0) - (3)(-394{,}380) - (4)(-228{,}590)] + [(3)(14{,}175) + (4)(8{,}635) - (5)(3950)]$$
$$= 2{,}074{,}010 + 57{,}315 = 2{,}131{,}325 \text{ kJ/kmol} \longleftarrow \text{(b)}$$

The value of $\bar{e}^{CH}$ for $H_2O_2(g)$ is not provided in Table A-26. Nevertheless, such a calculated result would be a plausible value to use in the absence of other information.

---

1. Here we ignore the very slight difference in pressure of the two tables: Table A-25 gives data at 1atm whereas Table A-26 (Model I) is based on 1.019 atm.

PROBLEM 13.86

FIND: Express the chemical exergies of several (a) gaseous hydrocarbons, C_aH_b, and (b) liquid hydrocarbons, C_aH_b, by an expression of the form

① $$\frac{\bar{e}^{CH}}{(\overline{LHV})} = c_1 + c_2\left[\frac{b}{a}\right] + \frac{c_3}{a} \qquad (1)$$

where c_1, c_2, and c_3 are constants, and $(\overline{LHV})$ is the lower heating value.

ASSUMPTION: The environment is described as in Problem 13.82.

ANALYSIS: The required values of $\bar{e}^{ch}$ can be evaluated from Eq. 13.36. Following the discussion of Sec. 13.2.3,

$$\overline{LHV} = \bar{h}_{C_aH_b} + \left(a+\frac{b}{4}\right)\bar{h}_{O_2} - a\,\bar{h}_{CO_2} - \frac{b}{2}\,\bar{h}_{H_2O(g)}$$

where each $\bar{h}$ is evaluated at T_0, p_0.

For each family of hydrocarbons, c_1, c_2, and c_3 can be obtained from these data using a computer, together with suitable software. Expressions having the form of Eq. (1) for fairly large families of gaseous and liquid hydrocarbons obtained in this manner are presented on p. 174 (Problem 7-8) of the text: M.J. Moran, Availability Analysis: A Guide to Efficient Energy Use, ASME PRESS, New York, 1989. These expressions are determined, however, for a slightly different specification of the environment than described in Problem 13.82.

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1. This expression provides a plausible means for estimating $\bar{e}^{CH}$ for hydrocarbon mixtures represented as C_aH_b from measured values of the lower heating value.

PROBLEM 13.87

KNOWN: N_2 is at 200°F, 4 atm

① FIND: Determine the specific flow exergy using (a) the environment of Problem 13.82, (b) data from Table A-26.

ASSUMPTIONS: (1) N_2 is modeled as an ideal gas. (2) The effects of motion and gravity can be ignored.

ANALYSIS: Introducing ideal gas relations for h and s, Eq. 13.47 becomes

$$e_f = h - h_0 - T_0(s - s_0) + \underbrace{\frac{V^2}{2} + gz}_{\text{ignore by assumption (2)}} + e^{ch}$$

$$= h(T) - h(T_0) - T_0\left(s^\circ(T) - s^\circ(T_0) - R \ln \frac{p}{p_0}\right) + e^{ch}$$

With data from Table A-23, T = 660°R, T_0 = 537°R, p = 4 atm and p_0 = 1 atm

$$e_f = \frac{4585.8 - 3729.5 - 537[47.178 - 45.743 - 1.986 \ln 4]}{28.01} + e^{ch}$$

$$= 55.84 \frac{\text{Btu}}{\text{lb}} + e^{ch} \qquad (1)$$

(a) Evaluating e^{ch} relative to the environment of Problem 13.82, use Eq. 13.40

$$e^{ch} = R T_0 \ln\left(\frac{1}{y^e}\right) = \left(\frac{1.986}{28.01}\right)(537) \ln\left(\frac{1}{0.7567}\right)$$

$$= 10.61 \frac{\text{Btu}}{\text{lb}}$$

So, with Eq. (1)

$$e_f = 66.45 \frac{\text{Btu}}{\text{lb}} \longleftarrow \text{(a)}$$

(b) Using data from Table A-26 for N_2,

② Model I: $\bar{e}^{CH} = 640 \frac{\text{kJ}}{\text{kmol}} \Rightarrow e^{CH} = \left(640 \frac{\text{kJ}}{\text{kmol}}\right)\left(\frac{0.42992 \text{ Btu/lbmol}}{1 \text{ kJ/kmol}}\right)\left(\frac{1 \text{ lbmol}}{28.01 \text{ lb}}\right) = 9.82 \frac{\text{Btu}}{\text{lb}}$

Model II: $\bar{e}^{CH} = 720 \frac{\text{kJ}}{\text{kmol}} \Rightarrow e^{CH} = (720)(0.42992)\left(\frac{1}{28.01}\right) = 11.05$

So with Eq. (1)

Model I: $e_f = 65.66 \frac{\text{Btu}}{\text{lb}}$ ⟵ (b)

Model II: $e_f = 66.89 \frac{\text{Btu}}{\text{lb}}$

---

1. In any particular analysis, only one environmental model should be used.
2. We ignore the difference between p_0 = 1 atm used in obtaining Eq. (1) and the Table A-26 value of p_0 for Model I.

PROBLEM 13.88

KNOWN: Water vapor is at 200°C, 1 bar

① FIND: Determine the specific flow exergy using (a) the environment of Problem 13.82, (b) data from Table A-26

ASSUMPTIONS: (1) For liquid water $h \sim h_f(T)$, $s \sim s_f(T)$. (2) the effects of motion and gravity can be ignored.

ANALYSIS: Equation 13.47 becomes with assumption (2)

$$e_f = h - h_o - T_o(s - s_o) + e^{CH}$$

Using Steam Table data with assumption (1)

$$\begin{aligned} e_f &= (2875.3 - 104.89) - 298.15(7.8343 - 0.3674) + e^{ch} \\ &= 544.2 + e^{ch} \ \frac{kJ}{kg} \qquad (1) \end{aligned}$$

(a) Evaluating e^{ch} relative to the environment of Problem 13.82, use Eq. 13.39, together with $\bar{g}$'s from Table A-25

$$\begin{aligned} \bar{e}^{ch} &= \left[\bar{g}_{H_2O(l)} - \bar{g}_{H_2O(g)}\right](T_o, P_o) + \bar{R}T_o \ln \frac{1}{y^e_{H_2O}} \\ &= (-237{,}180) - (-228{,}590) + (8.314)(298.15) \ln \frac{1}{0.0312} \\ &= 4.9 \ \frac{kJ}{kmol} \end{aligned}$$

or

$$e^{ch} = \frac{4.9}{18.02} = 0.27 \ \frac{kJ}{kg}$$

This quantity is a negligible amount that could be ignored owing to round-off of table data. But, including it with Eq. (1) gives

$$e_f = 544.5 \ \frac{kJ}{kg} \longleftarrow \text{(a)}$$

(b) Using data from Table A-26 for $H_2O(g)$

② Model I: $\bar{e}^{CH} = 45 \ \frac{kJ}{kmol} \Rightarrow e^{CH} = \frac{45}{18.02} = 2.5 \ \frac{kJ}{kg}$

Model II: $\bar{e}^{CH} = 900 \ \frac{kJ}{kmol} \Rightarrow e^{CH} = \frac{900}{18.02} = 49.9 \ \frac{kJ}{kg}$

So, with Eq. (1)

Model I: $e_f = 546.7$ kJ/kg $\longleftarrow$ (b)

③ Model II: $e_f = 594.1$ kJ/kg

---

1. In any particular analysis, only one environmental model should be used.

2. We ignore the difference between $P_o = 1$ atm used in the other calculations and the Table A-26 value of P_o for Model I.

3. The Model I and part (a) values are in agreement. The Model II result is about 9% higher.

PROBLEM 13.89

KNOWN: An equimolar mixture of O_2 and N_2 is at 20°C, 1atm.

① FIND: Determine the specific flow exergy using (a) the environment of Problem 13.82, (b) data from Table A-26.

ASSUMPTIONS: (1) The ideal gas model is applicable. (2) Ignore the effects of motion and gravity.

ANALYSIS: With assumption (2), Equation 13.47 becomes

$$e_f = h - h_o - T_o(s - s_o) + e^{CH} \qquad (1)$$

Using ideal gas mixture principles

$$\bar{h} - \bar{h}_o - T_o(\bar{s} - \bar{s}_o) = \left[y_{O_2}(\bar{h}_{O_2}(T) - \bar{h}_{O_2}(T_o)) + y_{N_2}(\bar{h}_{N_2}(T) - \bar{h}_{N_2}(T_o)) \right]$$
$$- T_o \left[y_{O_2}\left(\bar{s}^o_{O_2}(T) - \bar{s}^o_{O_2}(T_o) - \bar{R} \ln \frac{y_{O_2} p}{y_{O_2} p_o} \right) + y_{N_2}\left(\bar{s}^o_{N_2}(T) - \bar{s}^o_{N_2}(T_o) - \bar{R} \ln \frac{y_{N_2} p}{y_{N_2} p_o} \right) \right]$$

Since $p = p_o$ the ln terms drop out, leaving

$$\bar{h} - \bar{h}_o - T_o(\bar{s} - \bar{s}_o) = y_{O_2}(\bar{h}_{O_2}(T) - \bar{h}_{O_2}(T_o)) + y_{N_2}(\bar{h}_{N_2}(T) - \bar{h}_{N_2}(T_o))$$
$$- T_o \left[y_{O_2}(\bar{s}^o_{O_2}(T) - \bar{s}^o_{O_2}(T_o)) + y_{N_2}(\bar{s}^o_{N_2}(T) - \bar{s}^o_{N_2}(T_o)) \right]$$

Then, with $y_{O_2} = y_{N_2} = 0.5$ and ideal gas table data

$$\bar{h} - \bar{h}_o - T_o(\bar{s} - \bar{s}_o) = 0.5\left[\{ [8521 - 8669] - 298[190.998 - 191.502] \} + \{ [8533 - 8682] - 298[204.52 - 205.03] \} \right]$$
$$= 2.6 \text{ kJ/kmol} \qquad (2)$$

The mixture molecular weight is

$$M = y_{O_2} M_{O_2} + y_{N_2} M_{N_2} = 0.5[32 + 28.01] = 30.01 \qquad (3)$$

(a) Evaluating e^{CH} relative to the environment of Problem 13.82, using Eq. 13.41a

$$\bar{e}^{CH} = \bar{R} T_o \left[y_{O_2} \ln \frac{y_{O_2}}{y^e_{O_2}} + y_{N_2} \ln \frac{y_{N_2}}{y^e_{N_2}} \right]$$

With $y_{O_2} = y_{N_2} = 0.5$ and $y^e_{O_2}$ and $y^e_{N_2}$ from the table of Problem 13.82

$$\bar{e}^{CH} = (8.314)(298)(0.5)\left[\ln \frac{0.5}{0.2035} + \ln \frac{0.5}{0.7567} \right] = 600.3 \ \frac{\text{kJ}}{\text{kmol}} \qquad (4)$$

Then, with Eqs 2-4, Eq. 1 gives

② $$e_f = \frac{2.6 + 600.3}{30.01} = 20.1 \ \frac{\text{kJ}}{\text{kg}} \longleftarrow \qquad (a)$$

(b) Using data from Table A-26 and Eq. 13.41 b

$$\bar{e}^{CH} = y_{O_2} \bar{e}^{CH}_{O_2} + y_{N_2} \bar{e}^{CH}_{N_2} + \bar{R} T_o (y_{O_2} \ln y_{O_2} + y_{N_2} \ln y_{N_2})$$
$$= 0.5 \left[\bar{e}^{CH}_{O_2} + \bar{e}^{CH}_{N_2} \right] + \underbrace{(8.314)(298.15) \ln 0.5}_{-1718.2 \text{ kJ/kmol}} \qquad (5)$$

Model I:

③ $\bar{e}^{CH}_{O_2} = 3950$ kJ/kmol, $\bar{e}^{CH}_{N_2} = 640$ kJ/kmol $\Big\}$ $0.5(\bar{e}^{CH}_{O_2} + \bar{e}^{CH}_{N_2}) = 0.5(3950 + 640) = 2295 \frac{kJ}{kmol}$

Model II

$\bar{e}^{CH}_{O_2} = 3970$ kJ/kmol, $\bar{e}^{CH}_{N_2} = 720$ kJ/kmol $\Big\}$ $0.5(\bar{e}^{CH}_{O_2} + \bar{e}^{CH}_{N_2}) = 0.5(3970 + 720) = 2345 \frac{kJ}{kmol}$

Thus, combining these results with Eqs. (1), (2), (3), (5)

Model I:

$$e_f = \frac{2.6 + (2295 - 1718.2)}{30.01} = 19.3 \frac{kJ}{kg}$$

Model II:

$$e_f = \frac{2.6 + (2345 - 1718.2)}{30.01} = 21.0 \frac{kJ}{kg} \longleftarrow$$

---

1. In any particular analysis, only one environmental model should be used.

2. In this application, the chemical exergy is the largest contributor to e_f.

3. We ignore the difference between $p_0 = 1$ atm used in the other calculations and the Table A-26 value of p_0 for Model I.

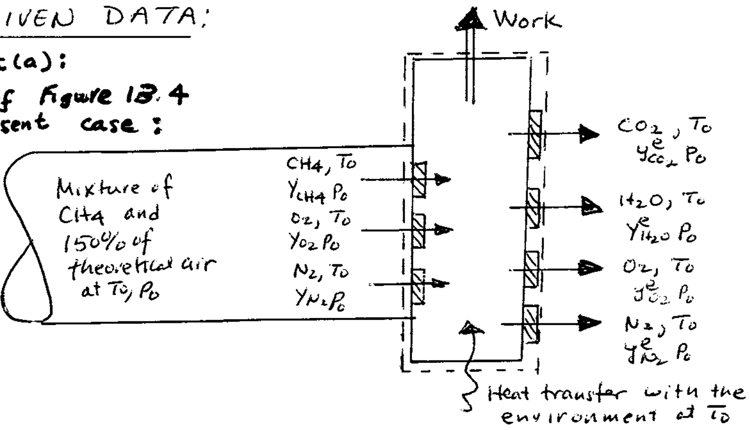
PROBLEM 13.90

KNOWN: A mixture of CH_4 and 150% of theoretical air is under consideration.

FIND: Determine the specific flow exergy using (a) the environment of Problem 13.82, (b) data from Table A-26.

SCHEMATIC & GIVEN DATA:

Figure for Part (a):
Counterpart of Figure 13.4
for the present case:



ASSUMPTIONS: (1) In evaluating the chemical exergy, the idealizations of Sec. 13.6 apply. (2) The effects of motion and gravity can be ignored (3) Ideal gas principles apply.

ANALYSIS: (a) The plan is to parallel the development of Sec. 13.6:
The cell reaction is (Eq. 13.5)

$$CH_4 + 3O_2 + 11.28 N_2 \rightarrow CO_2 + 2H_2O + O_2 + 11.28 N_2$$

The mole fractions of the mixture of CH_4, O_2, N_2 are

$$y_{CH_4} = \frac{1}{15.28} = 0.0654, \quad y_{O_2} = \frac{3}{15.28} = 0.1963, \quad y_{N_2} = \frac{11.28}{15.28} = 0.7382$$

An energy rate balance reduces to

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} - \frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} + \underset{(\text{entering})}{\left[\bar{h}_{CH_4} + 3\bar{h}_{O_2} + 11.28\bar{h}_{N_2}\right]} - \underset{(\text{exiting})}{\left[\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + \bar{h}_{O_2} + 11.28\bar{h}_{N_2}\right]}$$

or

$$\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} + \left(\bar{h}_{CH_4} + 3\bar{h}_{O_2} + 11.28\bar{h}_{N_2}\right) - \left(\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + \bar{h}_{O_2} + 11.28\bar{h}_{N_2}\right) \quad (1)$$

An entropy rate balance reads

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{CH_4}}{T_0} + \left[\bar{s}_{CH_4}(T_0, y_{CH_4}P_0) + 3\bar{s}_{O_2}(T_0, y_{O_2}P_0) + 11.28\,\bar{s}_{N_2}(T_0, y_{N_2}P_0)\right] - \left[\bar{s}_{CO_2}(T_0, y^e_{CO_2}P_0) + 2\bar{s}_{H_2O}(T_0, y^e_{H_2O}P_0) + \bar{s}_{O_2}(T_0, y^e_{O_2}P_0) + 11.28\,\bar{s}_{N_2}(T_0, y^e_{N_2}P_0)\right] + \dot{\sigma}_{cv}/\dot{n}_{CH_4}$$

Using Eq 13.34, as appropriate

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = -T_0\Big[\left(\bar{s}_{CH_4}(T_0,P_0) - \bar{R}\ln y_{CH_4}\right) + 3\left(\bar{s}_{O_2}(T_0,P_0) - \bar{R}\ln y_{O_2}\right) + 11.28\left(\bar{s}_{N_2}(T_0,P_0) - \bar{R}\ln y_{N_2}\right) - \left[\bar{s}_{CO_2}(T_0,P_0) - \bar{R}\ln y^e_{CO_2}\right] - 2\left[\bar{s}_{H_2O}(T_0,P_0) - \bar{R}\ln y^e_{H_2O}\right] - \left[\bar{s}_{O_2}(T_0,P_0) - \bar{R}\ln y^e_{O_2}\right] - 11.28\left[\bar{s}_{N_2}(T_0,P_0) - \bar{R}\ln y^e_{N_2}\right]\Big] - T_0\,\dot{\sigma}_{cv}/\dot{n}_{CH_4} \quad (2)$$

Combining Eqs. (1), (2) gives the counterpart to Eq. 13.36.

PROBLEM 13.90 (Contd.)

$$\bar{e}^{CH} = \left[\bar{g}_{CH_4} + 3\bar{g}_{O_2} + \cancel{11.28\bar{g}_{N_2}} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O} - \bar{g}_{O_2} - \cancel{11.28\bar{g}_{N_2}} \right](T_0, P_0)$$
$$+ \bar{R}T_0 \ln \left\{ \frac{y_{CH_4}(y_{O_2})^3}{y^e_{CO_2}(y^e_{H_2O})^2 y^e_{O_2}} \left[\frac{y_{N_2}}{y^e_{N_2}}\right]^{11.28} \right\}$$

Since $T_0 = 77°F$, $P_0 = 1$ atm, the $\bar{g}$'s can be read directly from Table A-30E. Accordingly,

$$\bar{e}^{CH} = [-21,860 + 3(0) - (-169,680) - 2(-98,350) - (0)] +$$
$$(1.986)(537) \ln \left\{ \frac{(0.0654)(0.1963)^3}{(0.0003)(0.0312)^2(0.2035)} \left[\frac{0.7382}{0.7567}\right]^{11.28} \right\}$$

① $= 344520 + 9329 = 353849 \ \frac{\text{Btu}}{\text{lbmol}(CH_4)}$ ← (a)

Since the mixture is at T_0, P_0, and the effects of motion/gravity can be ignored, Equation **13.47** reduces to $\bar{a}_f = \bar{a}^{CH}$.

(b) For use with data from Table A-26, Eq. 13.41b may be invoked:

$$\bar{e}^{CH} = y_{CH_4}\bar{e}^{CH}_{CH_4} + y_{O_2}\bar{e}^{CH}_{O_2} + y_{N_2}\bar{e}^{CH}_{N_2} + \bar{R}T_0\underline{[y_{CH_4}\ln y_{CH_4} + y_{O_2}\ln y_{O_2} + y_{N_2}\ln y_{N_2}]}$$

The underlined term is

$$= (8.314)(298.15)[0.0654 \ln(0.0654) + 0.1963 \ln(0.1963) + 0.7382 \ln(0.7382)]$$
$$= -1789.8 \ \frac{\text{kJ}}{\text{kmol(mix)}}.$$

So

② $\bar{e}^{CH} = (0.0654)\bar{e}^{CH}_{CH_4} + (0.1963)\bar{e}^{CH}_{O_2} + (0.7382)\bar{e}^{CH}_{N_2} - 1789.8 \ \frac{\text{kJ}}{\text{kmol(mix)}}.$

Model I:

$$\bar{e}^{CH} = (0.0654)(824,350) + (0.1963)(3950) + (0.7382)(640) - 1789.8$$
$$= 53371 \ \text{kJ/kmol(mix)}$$

or, on a per mole of CH_4 basis,

$$\bar{e}^{CH} = \frac{53371 \ \text{kJ/kmol(mix)}}{0.0654 \ \text{kmol}(CH_4)/\text{kmol(mix)}} \left| \frac{0.42992 \ \text{Btu/lbmol}}{1 \ \text{kJ/kmol}} \right| = 350,845 \ \frac{\text{Btu}}{\text{lbmol}(CH_4)}$$ ← (b)

Model II: $\bar{e}^{CH} = 354,395$ Btu/lbmol(CH_4)

---

1. The term involving Gibbs functions accounts for over 97% of the sum.
2. By comparing this calculation to the considerations of part (a), we see the relative simplicity of using standard chemical exergies.

PROBLEM 13.91

KNOWN: A mixture having an analysis on a molar basis {85% dry air, 15% CO} enters a device at 125°C, 2.1 atm, 250 m/s with a mass flow rate of 1.0 kg/s.

FIND: Determine the rate that exergy enters using (a) the environment of Problem 13.82, (b) data from Table A-26.

SCHEMATIC & GIVEN DATA:

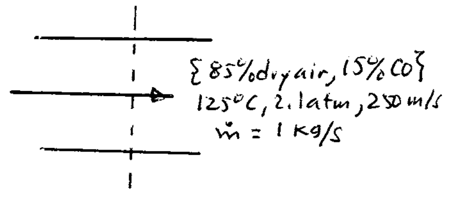
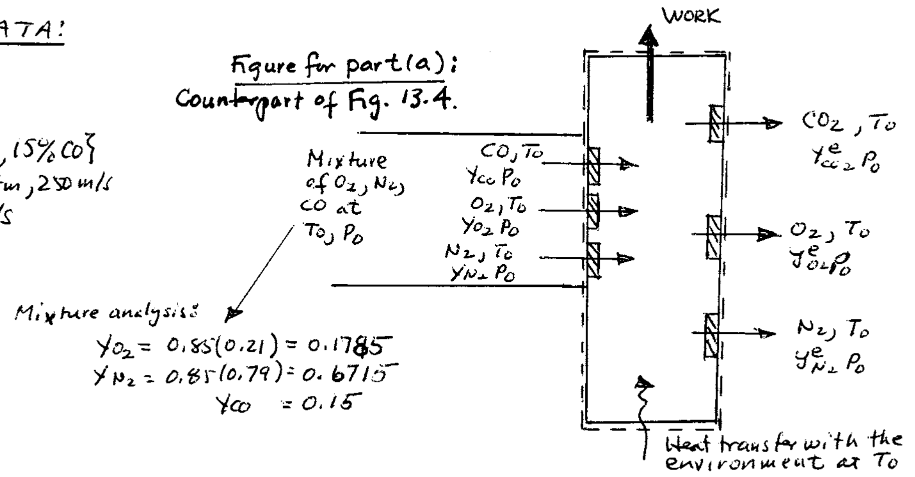


Figure for part (a): Counterpart of Fig. 13.4.



ASSUMPTIONS: (1) In evaluating the chemical exergy the idealizations of Sec. 13.6 apply. (2) Dry air is modeled as 21% O₂, 79% N₂. (3) The effect of gravity can be ignored. (4) Ideal gas principles apply.

ANALYSIS: With assumption (3), Equation 13.47 reduces to

$$e_f = \underline{h - h_0 - T_0(s - s_0) + \frac{V^2}{2}} + a^{CH}$$

where the underlined term is the thermomechanical contribution, e_f^{TM}. On a per kmol of mixture basis

$$\bar{e}_f^{TM} = \Big[0.15\,[\bar{h}_{CO}(398\text{K}) - \bar{h}_{CO}(298\text{K})] + 0.6715\,[\bar{h}_{N_2}(398) - \bar{h}_{N_2}(298)] + 0.1785\,[\bar{h}_{O_2}(398) - \bar{h}_{O_2}(298)]\Big]$$
$$- 298\Big[0.15\Big(\bar{s}^\circ_{CO}(398) - \bar{s}^\circ_{CO}(298) - \bar{R}\ln\frac{p}{p_0}\Big) + 0.6715\Big(\bar{s}^\circ_{N_2}(398) - \bar{s}^\circ_{N_2}(298) - \bar{R}\ln\frac{p}{p_0}\Big)$$
$$+ 0.1785\Big(\bar{s}^\circ_{O_2}(398) - \bar{s}^\circ_{O_2}(298) - \bar{R}\ln\frac{p}{p_0}\Big)\Big] + M\frac{V^2}{2}$$

where M denotes the mixture molecular weight:

$$M = (0.1785)(32) + (0.6715)(28.01) + (0.15)(28.01) = 28.72.$$

Then, with data from the ideal gas tables

$$\bar{e}_f^{TM} = \Big[0.15(11585 - 8669] + 0.6715[11581 - 8669] + 0.1785[11651 - 8682]\Big]$$
$$- 298\Big[0.15\Big(205.98 - 197.54 - 8.314\ln\frac{2.1}{1}\Big) + 0.6715\Big[199.92 - 191.50 - 8.314\ln\frac{2.1}{1}\Big]$$
$$+ 0.1785\Big[213.61 - 205.03 - 8.314\ln\frac{2.1}{1}\Big] + \left(\frac{28.72}{2}\frac{\text{kg}}{\text{kmol}}\right)\left(250\frac{\text{m}}{\text{s}}\right)^2\left|\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= \Big[0.15[2916] + 0.6715[2912] + 0.1785[2969]\Big] - 298\Big[0.15(2.2715) + 0.6715(2.2515)$$
$$+ 0.1785(2.4115)\Big] + 897.5$$

$$= 3140\ \frac{\text{kJ}}{\text{kmol(mix)}}$$

Or

$$e_f^{TM} = \frac{3140}{28.72} = 109.3\ \frac{\text{kJ}}{\text{kg(mix)}}$$

PROBLEM 13.91 (Continued)

(a) To find the chemical exergy relative to the environment of Problem 13.82, a procedure that parallels the one employed in Sec. 13.6 is used. The cell reaction is $CO + \frac{1}{2}O_2 \rightarrow CO_2$. Accordingly, for every mole of mixture, 0.15 moles of CO enters the cell and reacts with $0.15/2 = 0.075$ moles of O_2, producing 0.15 moles of CO_2. An energy balance on the cell reads

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{MIX}} - \frac{\dot{W}_{cv}}{\dot{n}_{MIX}} + [0.15\,\bar{h}_{CO} + 0.1785\,\bar{h}_{O_2} + \cancel{0.6715\,\bar{h}_{N_2}}] - [0.15\,\bar{h}_{CO_2} + 0.1035\,\bar{h}_{O_2} + \cancel{0.6715\,\bar{h}_{N_2}}]$$

An entropy balance reads

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{MIX}}{T_0} + (0.15\,\bar{s}_{CO}(T_0, y_{CO}p_0) + 0.1785\,\bar{s}_{O_2}(T_0, y_{O_2}p_0) + 0.6715\,\bar{s}_{N_2}(T_0, y_{N_2}p_0)]$$
$$-[0.15\,\bar{s}_{CO_2}(T_0, y^e_{CO_2}p_0) + 0.1035\,\bar{s}_{O_2}(T_0, y^e_{O_2}p_0) + 0.6715\,\bar{s}_{N_2}(T_0, y^e_{N_2}p_0)] + \frac{\dot{\sigma}_{cv}}{\dot{n}_{MIX}}$$

Using Eq 13.34 as appropriate

$$-\frac{\dot{Q}_{cv}/\dot{n}_{MIX}}{T_0} = 0.15[\bar{s}_{CO}(T_0,p_0) - \bar{R}\ln y_{CO}] + 0.1785(\bar{s}_{O_2}(T_0,p_0) - \bar{R}\ln y_{O_2}) + 0.6715(\bar{s}_{N_2}(T_0,p_0) - \bar{R}\ln y_{N_2})]$$
$$-[0.15[\bar{s}_{CO_2}(T_0,p_0) - \bar{R}\ln y^e_{CO_2}] + 0.1035(\bar{s}_{O_2}(T_0,p_0) - \bar{R}\ln y^e_{O_2}) + 0.6715(\bar{s}_{N_2}(T_0,p_0) - \bar{R}\ln y^e_{N_2})]$$
$$+ \dot{\sigma}_{cv}/\dot{n}_{MIX}$$

Combining these results gives the counterpart to Eq 13.36

$$\bar{e}^{CH} = 0.15[\bar{g}_{CO} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2}](T_0,p_0) + \bar{R}T_0\Big[0.15\ln y_{CO} + 0.1785\ln y_{O_2} - 0.15\ln y^e_{CO_2}$$
$$- 0.1035\ln y^e_{O_2} + 0.6715\ln\frac{y_{N_2}}{y^e_{N_2}}\Big]$$

$$= 0.15[\bar{g}_{CO} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2}](T_0,p_0) + \bar{R}T_0\Big[0.15\ln\Big(\frac{y_{CO}}{y^e_{CO_2}}\Big) + 0.6715\ln\frac{y_{N_2}}{y^e_{N_2}} + 0.1785\ln y_{O_2}$$
$$- 0.1035\ln y^e_{O_2}\Big]$$

Since $T_0 = 25°C$, $p_0 = 1\,atm$, the $\bar{g}$'s can be read directly from Table A-25. Accordingly, the chemical exergy per kmole of mixture is

① $$\bar{e}^{CH} = 0.15[(-137{,}150) + \tfrac{1}{2}(0) - (-394{,}380)]$$
$$+ (8.314)(298)\Big[0.15\ln\Big(\frac{0.15}{0.0003}\Big) + 0.6715\ln\Big(\frac{0.6715}{0.7567}\Big) + 0.1785\ln(0.1785) - 0.1035\ln(0.2035)\Big]$$

$$= 40{,}342 \text{ kJ/kmol(mix)}$$

or on a unit mass of mixture basis

$$e^{CH} = \frac{40{,}342}{28.72} = 1404.7 \frac{\text{kJ}}{\text{kg(mix)}}$$

Adding the e_f^{TM} and e^{CH} contributions

② $$e_f = 109.3 + 1404.7 = 1514 \text{ kJ/kg}$$

For a mass flow rate of 1 kg/s

$$\dot{E}_f = \Big(1\frac{\text{kg}}{\text{s}}\Big)\Big(1514\frac{\text{kJ}}{\text{kg}}\Big)\Big(\frac{1\text{ kW}}{1\text{ kJ/s}}\Big) = 1514\text{ kW} \longleftarrow (a)$$

PROBLEM 13.91 (Continued)

(b) For use with data from Table A-26, Eq. 13.41b may be invoked,

$$\bar{e}^{CH} = y_{CO}\bar{e}^{CH}_{CO} + y_{O_2}\bar{e}^{CH}_{O_2} + y_{N_2}\bar{e}^{CH}_{N_2} + \underline{\bar{R}T_0\left[y_{CO}\ln y_{CO} + y_{O_2}\ln y_{O_2} + y_{N_2}\ln y_{N_2}\right]}$$

The underlined term is

$$= (8.314)(298.15)\left[0.15\ln 0.15 + 0.1785\ln 0.1785 + 0.6715\ln 0.6715\right]$$

$$= -2130.8\ \frac{kJ}{kmol(mix)}$$

So

③ $$\bar{e}^{CH} = 0.15\,\bar{e}^{CH}_{CO} + 0.1785\,\bar{e}^{CH}_{O_2} + 0.6715\,\bar{e}^{CH}_{N_2} - 2130.8$$

Model I

$$\bar{e}^{CH} = 0.15(269{,}410) + 0.1785(3950) + 0.6715(640) - 2130.8$$

$$= 39416\ kJ/kmol(mix)$$

or

$$e^{CH} = \frac{39416}{28.72} = 1372.4\ \frac{kJ}{kg(mix)}$$

The sum is

$$e_f = 109.3 + 1372.4 = 1481.7\ \frac{kJ}{kg(mix)}$$

For a mass flow rate of 1 kg/s

$$\dot{E}_f = 1481.7\ kW \qquad \longleftarrow \ (6)$$

Model II

$$\bar{e}^{CH} = 40326\ kJ/kmol.$$

$$e^{CH} = 1404.1\ kJ/kg$$

$$\dot{E}_f = 1513.4\ kW \qquad \longleftarrow \ (6)$$

---

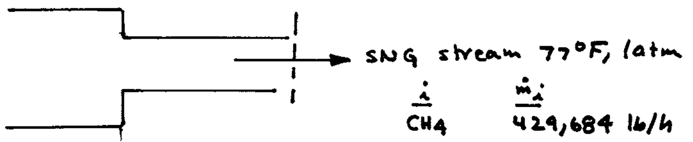
1. The term involving Gibbs functions accounts for over 95% of the sum.
2. In the present application, the chemical contribution dominates.
3. By comparing this calculation to the considerations of part (a), we see the relative simplicity of using standard chemical exergies.

PROBLEM 13.92

KNOWN: An SNG stream is flowing at 77°F, 1atm. The mass flowrates of the components are specified.

FIND: Determine the flow exergy rate.

SCHEMATIC & GIVEN DATA:



i	$\dot{m}_i$
CH_4	429,684 lb/h
CO_2	9,093 "
N_2	3,741 "
H_2	576 "
CO	204 "
H_2O	60 "

ASSUMPTIONS: (1) The SNG stream can be modeled as an ideal gas. (2) The effects of motion and gravity can be neglected. (3) The environment is described in Problem 13.82.

ANALYSIS: Since the stream is at 77°F, 1atm the thermomechanical contribution to the flow exergy vanishes, leaving only the chemical contribution. Thus

$$\dot{E}_f = \sum_i \dot{n}_i \bar{e}_i^{CH}$$

where $\dot{n}_i$ is the molar flow rate of component i and $\bar{e}_i^{CH}$ is its chemical exergy. For CO_2, N_2, H_2O, which appear in the environment,

$$\bar{e}_i^{CH} = \bar{R}T_0 \ln \frac{y_i}{y_i^e} \qquad (1)$$

(See Problem 13.84d for elaboration.) As carbon monoxide does not appear in the environment, its contribution is

$$\bar{e}_{CO}^{ch} = (\bar{g}_{CO} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2})(T_0, p_0) + \bar{R}T_0 \ln\left[\frac{y_{CO}(y_{O_2}^e)^{1/2}}{y_{CO_2}^e}\right]$$

$$= \underline{(\bar{g}_{CO} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2})(T_0, p_0) + \bar{R}T_0 \ln\left[\frac{(y_{O_2}^e)^{1/2}}{y_{CO_2}^e}\right]} + \bar{R}T_0 \ln y_{CO} \qquad (2)$$

(See Sec. 13.6 for discussion.) with data from Table A-25E

$$\bar{e}_{CO}^{ch} = 118{,}384 \frac{\text{Btu}}{\text{lbmol(CO)}} + \bar{R}T_0 \ln y_{CO} \qquad (3)$$

where y_{CO} is the mole fraction of CO in the SNG stream. With similar reasoning

$$\bar{e}_{H_2}^{ch} = 101{,}122 \frac{\text{Btu}}{\text{lbmol}(H_2)} + \bar{R}T_0 \ln y_{H_2} \qquad (4)$$

$$\bar{e}_{CH_4}^{ch} = 356{,}908 \frac{\text{Btu}}{\text{lbmol}(CH_4)} + \bar{R}T_0 \ln y_{CH_4} \qquad (5)$$

The required molar flow rates and mole fractions are obtained using the mass flow rate data provided:

i	M_i	$\dot{n}_i$ (lbmol/h)	$y_i = \dot{n}_i/\dot{n}$
CH_4	16.04	26788.28	0.97680
CO_2	44.01	206.61	0.00753
N_2	28.01	133.56	0.00487
H_2	2.018	285.43	0.01041
CO	28.01	7.28	0.00027
H_2O	18.02	3.33	0.00012

PROBLEM 13.92 (Contd.)

Using known data, Eq. (1) gives the following for CO_2, N_2, H_2O

$$\bar{e}^{CH}_{CO_2} = (1.986)(537) \ln \frac{0.00753}{0.0003} = 3437 \text{ Btu/lbmol}(CO_2)$$

$$\bar{e}^{CH}_{N_2} = (1.986)(537) \ln \frac{0.00487}{0.7567} = -5381 \text{ Btu/lbmol}(N_2)$$

$$\bar{e}^{CH}_{H_2O} = (1.986)(537) \ln \frac{0.00012}{0.00312} = -3475 \text{ Btu/lbmol}(H_2O)$$

Also, Eqs. (3), (4), and (5) give the following for CO, H_2, CH_4

$$\bar{e}^{CH}_{CO} = 118{,}384 + (1.986)(537) \ln(0.00027) = 109{,}621 \text{ Btu/lbmol}(CO)$$

$$\bar{e}^{CH}_{H_2} = 101{,}122 + (1.986)(537) \ln(0.01041) = 96{,}254 \text{ Btu/lbmol}(H_2)$$

$$\bar{e}^{CH}_{CH_4} = 356{,}908 + (1.986)(537) \ln(0.97680) = 356{,}883 \text{ Btu/lbmol}(CH_4)$$

In summary

①

i	$(\dot{n}_i \bar{e}^{ch}_i) \times 10^{-5}$ (Btu/h)
CH_4	95602
CO_2	7
N_2	-7
H_2	275
CO	8
H_2O	---
	95,885

Accordingly, $\dot{E}_f = 95{,}885 \times 10^5$ Btu/h

---

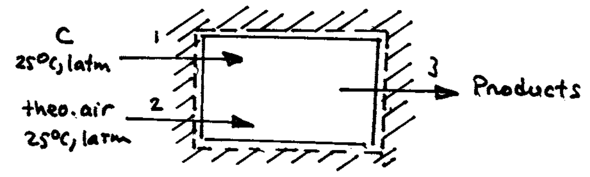
1. The CH_4 contribution clearly dominates.

PROBLEM 13.93

KNOWN: C at 25°C, 1atm enters an insulated reactor and burns completely with the theoretical amount of air entering at 25°C, 1atm.

FIND: Determine (a) the rate of exergy destruction, and (b) evaluate an exergetic efficiency.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases. (4) The environment is described in Problem 13.82. (5) The combustion air enters at the condition of the environment. (6) The products exit at 1 atm.

ANALYSIS: As a first step, an energy rate balance is applied to determine the temperature of the products of the reaction

$$C + O_2 + 3.76 N_2 \rightarrow CO_2 + 3.76 N_2$$

That is,

$$0 = \overset{0}{\cancel{\frac{\dot{Q}_{cv}}{\dot{n}_C}}} - \overset{0}{\cancel{\frac{\dot{W}_{cv}}{\dot{n}_C}}} + (\bar{h}_C)_1 + [\bar{h}_{O_2} + 3.76\bar{h}_{N_2}]_2 - [\bar{h}_{CO_2} + 3.76\bar{h}_{N_2}]_3$$

Then, with $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$, and noting that $\bar{h}_f^o = 0$ for C, N_2, and O_2

$$0 = [\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 3.76[\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

or

$$\bar{h}_{CO_2}(T_3) + 3.76\,\bar{h}_{N_2}(T_3) = -[\bar{h}_f^o - \bar{h}(298)]_{CO_2} + 3.76\,\bar{h}_{N_2}(298)$$

With data from Table A-23

$$\bar{h}_{CO_2}(T_3) + 3.76\,\bar{h}_{N_2}(T_3) = -[-393{,}520 - 9364] + 3.76[8669] = 435{,}479 \Rightarrow T_3 \cong 2460\,K.$$

(a) In this case where there is no heat transfer or work, the rate of exergy destruction is the difference between the exergy carried in with the fuel and the exergy carried out with the combustion products:

$$\dot{E}_d = \dot{E}_F - \dot{E}_{products}$$

① Also note that the combustion air enters at the condition of the environment, and thus with no exergy.

As the fuel enters at 25°C, 1atm, there is no thermomechanical contribution, so $(\dot{E}_F/\dot{n}_C) = \bar{e}^{CH} = 410{,}541$ kJ/kmol(C), which is calculated in Problem 13.82a.

The exergy of the products can be evaluated using Eq. 13.47. Since CO_2 and N_2 are present in the environment, Eq. 13.41a can be used to obtain $\bar{e}^{CH}$. Thus, with ideal gas principles

$$\frac{\dot{E}_{products}}{\dot{n}_C} = 1[\bar{h}(2460) - \bar{h}(298)]_{CO_2} + 3.76[\bar{h}(2460) - \bar{h}(298)]_{N_2}$$

$$- T_0\left\{1\left[\bar{s}^o_{CO_2}(2460) - \bar{s}^o_{CO_2}(298) - \bar{R}\ln\frac{y_{CO_2}p}{y_{CO_2}p_0}\right] + 3.76\left[\bar{s}^o_{N_2}(2460) - \bar{s}^o_{N_2}(298) - \bar{R}\ln\frac{y_{N_2}p}{y_{N_2}p_0}\right]\right\}$$

$$+ \left\{RT_0\left[1\ln\frac{y_{CO_2}}{y^e_{CO_2}} + 3.76\ln\frac{y_{N_2}}{y^e_{N_2}}\right]\right\}$$

PROBLEM 13.93 (Contd.)

Since $p = p_0$, the log terms drop out. Then, with data from Table A-23

$$\frac{\dot{E}_{products}}{\dot{n}_C} = 1[128,833 - 9364] + 3.76[81,515 - 8669] - 298\{1[321.814 - 213.685] + 3.76[259.480 - 191.502]\} + 8.314(298)\left[1 \ln \frac{(1/4.76)}{0.0003} + 3.76 \ln \frac{(3.76/4.76)}{0.7567}\right]$$

$$= 301,612 \text{ kJ/kmol(C)}$$

The rate of exergy destruction per unit mole of carbon is

$$\frac{\dot{E}_d}{\dot{n}_C} = 410,541 - 301,612 = 108,929 \frac{\text{kJ}}{\text{kmol(C)}} \longleftarrow$$

(b) An exergetic efficiency can be written as

② $$\epsilon = \frac{\dot{E}_{products}}{\dot{E}_F} = \frac{301,612}{410,541} = 0.735 \; (73.5\%) \longleftarrow$$

---

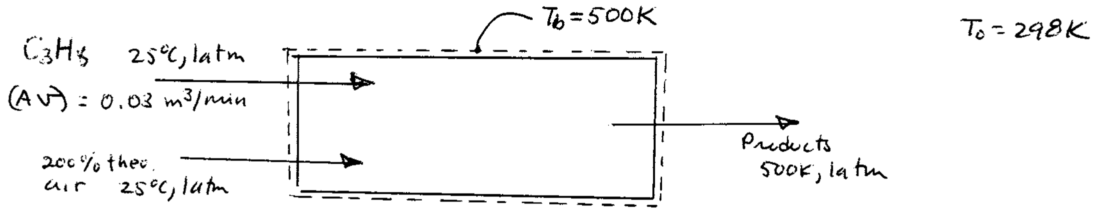
1. Throughout Chap. 13 combustion air is modeled as a binary mixture: 79% N_2, 21% O_2. As these mole fractions differ from those listed in Problem 13.82 for N_2 and O_2, a small nonzero value for the exergy of the combustion air can be calculated using Eq. 13.41a; this is ≈ 479 kJ/kmol(C).
2. Combustion is a significant source of irreversibility.

PROBLEM 13.94

KNOWN: Steady state operating data are provided for a furnace fueled by propane gas.

FIND: Compare the rate of exergy transfer accompany heat transfer from the furnace with the rate of exergy destruction within the furnace.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible kinetic/potential energy effects. (2) 3.76 moles of inert N_2 accompany each mole of O_2 in the combustion air. (3) The fuel, air, and combustion products can be modeled as ideal gases. (4) For the environment, $T_0 = 298K$.

ANALYSIS: The balanced reaction equation for complete combustion of C_3H_8 with 200% of the theoretical amount of air is

$$C_3H_8 + 10(O_2 + 3.76N_2) \rightarrow 3CO_2 + 4H_2O + 5O_2 + 37.6N_2$$

An energy rate balance for the furnace reads

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} + \left(\bar{h}_{fuel} + 10\bar{h}_{O_2} + 37.6\bar{h}_{N_2}\right) - \left(3\bar{h}_{CO_2} + 4\bar{h}_{H_2O} + 5\bar{h}_{O_2} + 37.6\bar{h}_{N_2}\right)$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and data from Table A-23

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = 3[\bar{h}_f^\circ + \Delta\bar{h}]_{CO_2} + 4[\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 5[\overset{0}{\cancel{\bar{h}_f^\circ}} + \Delta\bar{h}]_{O_2} + 37.6[\overset{0}{\cancel{\bar{h}_f^\circ}} + \Delta\bar{h}]_{N_2} - (\bar{h}_f^\circ)_{fuel}$$

$$= 3[-393{,}520 + (17678 - 9364)] + 4[-241{,}820 + (16828 - 9904)]$$

$$+ 5[14770 - 8682] + 37.6[14{,}581 - 8669] - (-103{,}850)$$

$$= -1{,}738{,}621\ \text{kJ/kmol(fuel)}$$

Using the ideal gas equation of state, the fuel molar flow rate is

$$\dot{n}_{fuel} = \frac{(AV)}{\bar{v}} = \frac{(0.03\ m^3/min)(1.01325\times10^5\ N/m^2)}{\left(8314\ \frac{N\cdot m}{kmol\cdot K}\right)(298K)} = 0.00123\ \frac{kmol(fuel)}{min}$$

So,

$$\dot{Q}_{cv} = -(1{,}738{,}621)(0.00123) = -2138.5\ \frac{kJ}{min}$$

The accompanying exergy transfer is then

$$\begin{bmatrix}\text{Rate of exergy transfer}\\ \text{accompanying heat transfer}\end{bmatrix} = \left[1 - \frac{T_0}{T_b}\right]\dot{Q}_{cv}$$

$$= \left[1 - \frac{298}{500}\right]\left(-2138.5\ \frac{kJ}{min}\right)$$

$$= -864\ \frac{kJ}{min} \qquad \longleftarrow$$

PROBLEM 13.94 Contd.)

The rate of exergy destruction can be found in terms of the entropy production rate as $\dot{E}_d = T_0\,\sigma_{cv}$ The entropy production rate is obtained from an entropy balance:

$$0 = \frac{\dot{Q}_{cv}/\dot{n}_{fuel}}{T_b} + \bar{s}_{fuel}(298K, \overset{\text{1atm}}{p}) + 10\,\bar{s}_{O_2}(298, 0.21\overset{\text{1atm}}{p}) + 37.6(298, 0.79\overset{\text{1atm}}{p}) -$$
$$[3\,\bar{s}_{CO_2}(500K, y_{CO_2}p) + 4\,\bar{s}_{H_2O}(500K, y_{H_2O}p) + 5\,\bar{s}_{O_2}(500K, y_{O_2}p)$$
$$+ 37.6\,\bar{s}_{N_2}(500K, y_{N_2}p)] + \dot{\sigma}_{cv}/\dot{n}_{fuel}$$

Reactants:

$\bar{s}_{fuel}(298K, 1atm) = 269.91\ kJ/kmol\cdot K$

$\bar{s}_{O_2}(298, 0.21p) = \bar{s}^{o}(298K) - \bar{R}\ln 0.21 = 205.03 - 8.314\ln(0.21) = 218.01$

$\bar{s}_{N_2}(298, 0.79p) = 191.5 - 8.314\ln(0.79) = 193.46$

Products: $y_{CO_2} = 3/44.6$, $y_{O_2} = 5/49.6$, $y_{H_2O} = 4/49.6$, $y_{N_2} = 37.6/49.6$

$\bar{s}_{CO_2}(500K, y_{CO_2}p) = \bar{s}^{o}_{CO_2}(500K) - \bar{R}\ln y_{CO_2} = 234.814 - 8.314\ln(3/49.6) = 258.14\ \frac{kJ}{kmol\cdot K}$

$\bar{s}_{H_2O}(500, y_{H_2O}p) = 206.413 - 8.314\ln(4/49.6) = 227.35$

$\bar{s}_{O_2}(500, y_{O_2}p) = 220.589 - 8.314\ln(5/49.6) = 239.67$

$\bar{s}_{N_2}(500, y_{N_2}p) = 206.63 - 8.314\ln(37.6/49.6) = 208.93$

Inserting values into the entropy rate balance

$$0 = \frac{-1{,}738{,}621\ kJ/kmol(fuel)}{500\ K} + 269.91 + 10(218.01) + 37.6(193.46) \cdot$$
$$- 3(258.14) - 4(227.35) - 5(239.67) - 37.6(208.93) + \frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}}$$

$$\Rightarrow \quad \frac{\dot{\sigma}_{cv}}{\dot{n}_{fuel}} = 4491\ \frac{kJ/kmol(fuel)}{K}$$

$$\Rightarrow \quad \dot{E}_d = (298K)\left(1.23\times10^{-3}\ \frac{kmol(fuel)}{min}\right)\left(4491\ \frac{kJ}{kmol(fuel)}\right)$$
$$= 1646\ \frac{kJ}{min} \qquad \longleftarrow$$

Comparing the rates of exergy transfer and destruction, we get

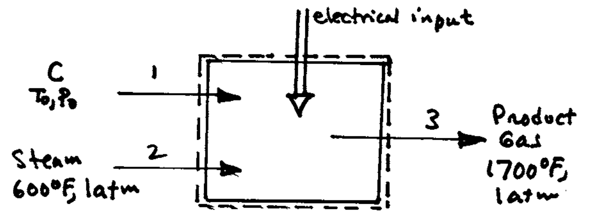
$$\frac{\dot{E}_d}{|\dot{E}_q|} = \frac{1646}{864} = 1.9 \quad (\approx 2) \qquad \longleftarrow$$

PROBLEM 13.95

KNOWN: Steady-state operating data are provided for a coal gasification reactor making use of the carbon-steam process.

FIND: Determine per lbmol of carbon entering (a) the required electrical input, (b) the exergy entering with the carbon, (c) the exergy entering with the steam, (d) the exergy exiting with the product gas, and (e) the exergy destruction.

SCHEMATIC & GIVEN DATA:



$C + 1.25\,H_2O(g) \rightarrow CO + H_2 + 0.25\,H_2O(g)$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The combustion products can be modeled as an ideal gas mixture. (3) The environment is described in Problem 13.82

ANALYSIS: (a) An energy rate balance reduces at steady state to

$$0 = \overset{0}{\cancel{\frac{\dot{Q}_{cv}}{\dot{n}_C}}} - \frac{\dot{W}_{cv}}{\dot{n}_C} + (\bar{h}_C)_1 + 1.25(\bar{h}_{H_2O})_2 - (\bar{h}_{CO} + \bar{h}_{H_2} + 0.25\bar{h}_{H_2O})_3$$

Noting that $(\bar{h}_C)_1 = (\bar{h}_f^\circ)_C(77°F, 1\text{ atm}) = 0$, and using IT to evaluate the other enthapy values, $\dot{W}_{cv}/\dot{n}_C$ is found as follows:

```
T0 = 77  // °F
p0 = 1  // atm
T2 = 600  // °F
p2 = 1  // atm
T3 = 1700  // °F
p3 = 1  // atm

hC_in = 0
hH2O_in = h_T("H2O",T2)
hCO_out = h_T("CO",T3)
hH2_out = h_T("H2",T3)
hH2O_out = h_T("H2O",T3)

ndotC = 1
Wdotcv = ndotC * (hC_in + 1.25 * hH2O_in - (hCO_out + hH2_out + 0.25 * hH2O_out))
```

IT result:

$\dot{W}_{cv}/\dot{n}_C = -7.848 \times 10^4$ Btu/lbmol (C) ← (a) $\dot{W}_{cv}/\dot{n}_C$

(b) Since the carbon enters a T_0, p_0, there is no thermomechanical contribution. The chemical exergy has been evaluated in Prob. 13.82 a: $e^{CH} = 176{,}500$ Btu/lbmol (C), where the units have been converted from SI to English units. ← (b)

(c) The exergy of the entering steam is given by Eq. 13.47. However, since water is a liquid at T_0, p_0 the chemical exergy of water relative to the environment of Problem 13.82 is zero (for discussion, see Problem 13.82(i).) Thus, with steam table data, using $h_0 \approx h_f(T_0)$, $s_0 \approx s_f(T_0)$, we get

$$(e_f)_2 = (h_2 - h_0) - T_0(s - s_0) = (1335.2 - 45.1) - 537(1.9787 - 0.08775)$$
$$= 277.34 \text{ Btu/lb(steam)}$$

PROBLEM 13.95 (Contd.) - page 2

Since 1.25 lbmol (steam) enters per lbmol (c)

$$\frac{\dot{E}_{steam}}{\dot{n}_c} = \left(277.34 \frac{\text{Btu}}{\text{lb(steam)}}\right)\left(\frac{18.02 \text{ lb}}{1 \text{ lbmol}}\right)\left(1.25 \frac{\text{lbmol (steam)}}{\text{lbmol (c)}}\right) = 6247 \frac{\text{Btu}}{\text{lbmol (c)}} \longleftarrow \text{(c)}$$

(d) The combustion products exit as an ideal gas mixture at 2160°R, 1atm. The thermomechanical contribution, per lbmol of C, is

$$\bar{h}-\bar{h}_0 - T_0(\bar{s}-\bar{s}_0) = 1\left(\bar{h}(2160) - \bar{h}(537) - 537\left(\bar{s}^\circ(2160) - \bar{s}^\circ(537) - 537 \cancelto{0}{\ln \frac{y_{CO}\,p}{y_{CO}\,p_0}}\right)_{CO} + \right.$$
$$1\left(\bar{h}(2160) - \bar{h}(537) - 537\left(\bar{s}^\circ(2160) - \bar{s}^\circ(537) - 537 \cancelto{0}{\ln \frac{y_{H_2}\,p}{y_{H_2}\,p_0}}\right)_{H_2} + \right.$$
$$0.25\left(\bar{h}(2160) - \bar{h}(537) - 537\left(\bar{s}^\circ(2160) - \bar{s}^\circ(537) - 537 \cancelto{0}{\ln \frac{y_{H_2O}\,p}{y_{H_2O}\,p_0}}\right)_{H_2O}\right.$$

Using IT to evaluate the properties:

```
efCO_out = h_T("CO",T3) - h_T("CO",77) - 537* (s_TP("CO",T3,1) - s_TP("CO",77, 1))
efH2_out = h_T("H2",T3) - h_T("H2",77) - 537* (s_TP("H2",T3,1) - s_TP("H2",77, 1))
efH2O_out = h_T("H2O",T3) - h_T("H2O",77) - 537* (s_TP("H2O",T3,1) - s_TP("H2O",77, 1))
ef3 = efCO_out + efH2_out + 0.25 * efH2O_out
```

The result is : e_{f3} = 15,000 Btu/lbmol (C) ←

The chemical contribution to the exergy of the products is obtained by summing the contribution of each of the three components. First, for the water vapor,

$$\bar{e}^{ch}_{H_2O} = \bar{R}T_0 \ln \frac{y_{H_2O}}{y^e_{H_2O}} = (1.986)(537) \ln\left(\frac{0.25/2.25}{0.0312}\right) = 1354.6 \text{ Btu/lbmol}(H_2O)$$

For CO as a member of an ideal gas mixture

$$\bar{e}^{ch}_{CO} = \left[\bar{g}_{CO} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{CO_2}\right](T_0,p_0) + \bar{R}T_0 \ln\left[\frac{y_{CO}(y^e_{O_2})^{1/2}}{y^e_{CO_2}}\right]$$

With data from Table A-25

$$\bar{e}^{ch}_{CO} = \left[-59{,}010 + \tfrac{1}{2}(0) - (-169{,}680)\right] + (1.986)(537)\ln\left[\frac{(1/2.25)(.2035)^{1/2}}{0.0003}\right]$$
$$= 117{,}607.2 \text{ Btu/lbmol (CO)}$$

Similarly, for H_2 as a member of an ideal gas mixture

$$\bar{e}^{ch}_{H_2} = \left(\bar{g}_{H_2} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}_{H_2O(g)}\right)(T_0,p_0) + \bar{R}T_0 \ln\left[\frac{y_{H_2}(y^e_{O_2})^{1/2}}{y^e_{H_2O}}\right]$$
$$= \left(0 + \tfrac{1}{2}(0) - (-98{,}350)\right) + (1.986)(537)\ln\left(\frac{(1/2.25)(.2035)^{1/2}}{0.0312}\right)$$
$$= 100{,}334.0 \text{ Btu/lbmol}(H_2)$$

Accordingly, the chemical exergy of the mixture is

$$\bar{e}^{ch} = (1)(117{,}607.2) + (1)(100{,}334.0) + 0.25(1354.6) = 218{,}280 \text{ Btu/lbmol (C)}$$

Collecting results the exergy of the exiting products is

$$(\dot{E}_{products}/\dot{n}_C) = 15000 + 218{,}280 = 233{,}280 \text{ Btu/lbmol (C)}$$

PROBLEM 13.95 (Cont'd.) - page 3

(e) The rate of exergy destruction is the difference between the exergy entering and exiting. That is

① $$(\dot{E}_d/\dot{n}_c) = \underset{\left(\substack{\text{electrical}\\ \text{work}}\right)}{78,480} + \underset{(\text{carbon})}{176,500} + \underset{(\text{steam})}{6247} - \underset{(\text{products})}{233,280} = 27,947 \ \frac{\text{Btu}}{\text{lbmol(C)}} \longleftarrow (e)$$

---

1. An exergetic efficiency can be written as

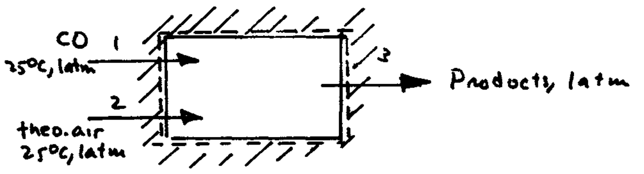
$$\epsilon = \frac{\dot{E}_{\text{products}}}{\dot{E}_{\text{in}}} = \frac{233,280}{(78,480+176,500+6247)} = 0.893 \ (89.3\ \%)$$

PROBLEM 13.96

KNOWN: CO at 25°C, 1atm enters an insulated reactor and burns with the theoretical amount of air entering separately at 25°C, 1atm. Products exit at 1atm.

FIND: Determine per mole of CO entering (a) the exergy entering with CO, (b) the exergy exiting with the products, (c) the rate of exergy destruction. (d) Evaluate an exergetic efficiency for the reactor.

SCHEMATIC & GIVEN DATA:



$$CO + \tfrac{1}{2}O_2 + 1.88N_2 \rightarrow CO_2 + 1.88N_2$$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) The reaction is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and products of combustion can be modeled as ideal gases. (5) The environment is as described in Problem 13.82. 6) The combustion air enters at the condition of the environment.

ANALYSIS: The solution to Problem 13.69 gives $T_P = 2665\,K$ and $\dot{\sigma}_{cv}/\dot{n}_{CO} = 165.3$ kJ/kmol(CO)·K. The rate of exergy destruction is then

$$\frac{\dot{E}_d}{\dot{n}_{CO}} = T_0\frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = (298)(165.3) = 49{,}259\ \frac{kJ}{kmol(CO)} \longleftarrow (c)$$

Since CO enters at T_0, p_0, there is no thermomechanical contribution to its exergy. The chemical contribution, evaluated in Problem 13.82(d), is

$$\frac{\dot{E}_{CO}}{\dot{n}_{CO}} = 275{,}364\ \frac{kJ}{kmol(CO)} \longleftarrow (a)$$

① Regarding the combustion air as entering with no exergy, the exergy of the products is the difference between the exergy entering with CO and the exergy destroyed:

② $$\frac{\dot{E}_{products}}{\dot{n}_{CO}} = \frac{\dot{E}_{CO}}{\dot{n}_{CO}} - \frac{\dot{E}_d}{\dot{n}_{CO}} = (275{,}364 - 49{,}259) = 226{,}105\ \frac{kJ}{kmol(CO)} \longleftarrow (b)$$

An exergetic efficiency can be written as

$$\epsilon = \frac{\dot{E}_{products}}{\dot{E}_{CO}} = \frac{226{,}105}{275{,}364} = 0.821\ (82.1\%) \longleftarrow (d)$$

---

1. Throughout Chap. 13, combustion air is modeled as a binary mixture of N_2 and O_2: 79% N_2, 21% O_2. As these mole fractions differ from those for N_2, O_2 in Problem 13.82, a small nonzero value for the exergy of combustion air can be calculated using Eq. 13.41a: ≈ 240 kJ/kmol(CO). This has been ignored in the analysis given.

2. Alternatively, the exergy of the products can be found using Eq. 13.47, as follows: The thermomechanical contribution is

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = 1\left[\bar{h}(T_3) - \bar{h}(298) - T_0\left(\bar{s}^\circ(T_3) - \bar{s}^\circ(298) - \bar{R}\ln\overset{0}{\cancel{\frac{y_{CO_2}p}{y_{CO_2}p_0}}}\right)\right]_{CO} +$$
$$1.88\left[\bar{h}(T_3) - \bar{h}(298) - T_0\left(\bar{s}^\circ(T_3) - \bar{s}^\circ(298) - \bar{R}\ln\overset{0}{\cancel{\frac{y_{N_2}p}{y_{N_2}p_0}}}\right)\right]_{N_2}$$

PROBLEM 13.96 (Contd.)

With data from Table A-23

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = 1\left[141{,}459 - 9364 - 298(326.742 - 213.685)\right]$$

$$1.88\left[89040 - 8669 - 298(262.420 - 191.502)\right]$$

$$= 209{,}770 \text{ kJ/kmol (CO)}$$

The chemical contribution, found with Eq. 13.41, is

$$\bar{e}^{ch} = \bar{R}T_0\left[1 \ln \frac{y_{CO_2}}{y^e_{CO_2}} + 1.88 \ln \frac{y_{N_2}}{y^e_{N_2}}\right]$$

$$= (8.314)(298)\left[1 \ln\left(\frac{1/2.88}{0.0003}\right) + 1.88 \ln\left(\frac{1.88/2.88}{0.7567}\right)\right.$$

$$= 16789 \text{ kJ/kmol (CO)}$$

Adding these contributions

$$\frac{\dot{E}_{products}}{\dot{n}_{CO}} = 209{,}770 + 16789 = 226{,}559 \text{ kJ/kmol (CO)}$$

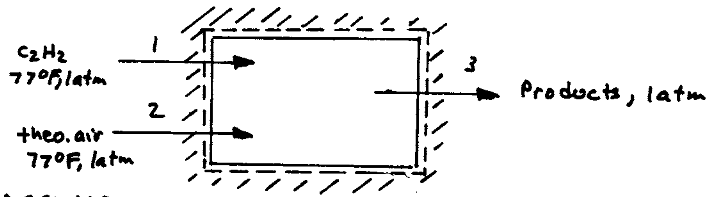
This calculation shows that, for the present application, the thermomechanical exergy is the dominant contribution to the exergy of the combustion products.

PROBLEM 13.97

KNOWN: C_2H_2 at 77°F, 1atm enters an insulated reactor and burns completely with 180% of theoretical air at 77°F, 1atm. Products at 1atm exit.

FIND: Determine, per mole of fuel, (a) the exergy of the fuel entering the reactor, (b) the exergy exiting with the products, (c) the rate of exergy destruction. Also, evaluate an exergetic efficiency for the reactor.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases. (4) The environment is as described in Problem 13.82. (5) The combustion air enters at the condition of the environment.

ANALYSIS: Complete combustion of C_2H_2 with 180% of the theoretical amount of air is described by

$$C_2H_2 + 4.5(O_2 + 3.76N_2) \rightarrow 2CO_2 + 1H_2O + 2O_2 + 16.92N_2$$

The first step is to determine the temperature of the exiting products using an energy rate balance at steady state:

$$0 = \cancelto{0}{\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}}} - \cancelto{0}{\frac{\dot{W}_{cv}}{\dot{n}_{fuel}}} + (\bar{h}_{fuel})_1 + [4.5\bar{h}_{O_2} + 16.92\bar{h}_{N_2}]_2 - [2\bar{h}_{CO_2} + \bar{h}_{H_2O} + 2\bar{h}_{O_2} + 16.92\bar{h}_{N_2}]_3$$

Then, using $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$ and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{C_2H_2} + [0] - 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} - [\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} - 2[\bar{h}(T_3) - \bar{h}(537)]_{O_2} - 16.92[\bar{h}(T_3) - \bar{h}(537)]_{N_2}$$

or with data from the ideal gas tables and Table A-25E

$$2\bar{h}_{CO_2}(T_3) + \bar{h}_{H_2O}(T_3) + 2\bar{h}_{O_2}(T_3) + 16.92\bar{h}_{N_2}(T_3) = (\bar{h}_f^\circ)_{C_2H_2} - 2[\bar{h}_f^\circ - \bar{h}(537)]_{CO_2} - [\bar{h}_f^\circ - \bar{h}(537)]_{H_2O} + 2\bar{h}_{O_2}(537) + 16.92\bar{h}_{N_2}(537)$$

$$= 97{,}540 - 2[-169{,}300 - 4028] - [-104{,}040 - 4258] + 2[3725] + 16.92[3730] = 623056 \Rightarrow T_3 = 3456°R$$

(a) Since C_2H_2 enters at T_0, p_0, the exergy entering with the fuel is chemical exergy, obtained using Eq. 13.36 as follows:

$$\bar{e}^{CH}_{C_2H_2} = [\bar{g}_{C_2H_2} + 2.5\bar{g}_{O_2} - 2\bar{g}_{CO_2} - \bar{g}_{H_2O(g)}](T_0, p_0) + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^{2.5}}{(y^e_{CO_2})^2(y^e_{H_2O})}\right]$$

$$= [87{,}990 + 2.5(0) - 2(-169{,}680) - (-98{,}350)] + (1.986)(537)\ln\left[\frac{(.2035)^{2.5}}{(.0003)^2(0.0312)}\right]$$

$$= 542{,}455 \text{ Btu/lbmol}(C_2H_2) \longleftarrow \text{(a)}$$

(b) The exergy exiting with the products can be evaluated using Eq. 13.47 with $\bar{e}^{CH}$ obtained from Eq. 13.41. Accordingly, when expressed on a per lbmol of fuel basis, we get

$$\bar{e}^{ch} = \bar{R}T_0\left[2\ln\frac{y_{CO_2}}{y^e_{CO_2}} + \ln\frac{y_{H_2O}}{y^e_{H_2O}} + 2\ln\frac{y_{O_2}}{y^e_{O_2}} + 16.92\ln\frac{y_{N_2}}{y^e_{N_2}}\right]$$

$$= (1.986)(537)\left[2\ln\left(\frac{2/21.92}{0.0003}\right) + \ln\left(\frac{1/21.92}{0.0312}\right) + 2\ln\left(\frac{2/21.92}{0.2035}\right) + 16.92\ln\left(\frac{16.92/21.92}{0.7567}\right)\right]$$

$$= 11{,}248 \text{ Btu/lbmol}(C_2H_2)$$

And

$$\bar{h}-\bar{h}_0-T_0(\bar{s}-\bar{s}_0) = 2\left[\bar{h}(3456)-\bar{h}(537)-537\left(\bar{s}^\circ(3456)-\bar{s}^\circ(537)-\bar{R}\ln\cancelto{0}{\frac{y_{CO_2}p}{y_{CO_2}p_0}}\right)\right]_{CO_2}+$$

$$\left[\bar{h}(3456)-\bar{h}(537)-537\left(\bar{s}^\circ(3456)-\bar{s}^\circ(537)-\bar{R}\ln\cancelto{0}{\frac{y_{H_2O}p}{y_{H_2O}p_0}}\right)\right]_{H_2O}+$$

$$2\left[\bar{h}(3456)-\bar{h}(537)-537\left(\bar{s}^\circ(3456)-\bar{s}^\circ(537)-\bar{R}\ln\cancelto{0}{\frac{y_{O_2}p}{y_{O_2}p_0}}\right)\right]_{O_2}+$$

$$16.92\left[\bar{h}(3456)-\bar{h}(537)-537\left(\bar{s}^\circ(3456)-\bar{s}^\circ(537)-\bar{R}\ln\cancelto{0}{\frac{y_{N_2}p}{y_{N_2}p_0}}\right)\right]_{N_2}$$

$$= 2[41330-4028-537(73.280-51.032)] + [33791-4258-537(62.724-45.079)] +$$

$$2[27879-3725-537(63.801-48.982)] + 16.92[26639-3730-537(59.836-45.743)]$$

$$= 362{,}730 \text{ Btu/lbmol}(C_2H_2)$$

Adding these contributions

$$\dot{E}_{products}/\dot{n}_{C_2H_2} = 362{,}730 + 11{,}248 = 373{,}978 \text{ Btu/lbmol}(C_2H_2) \longleftarrow \text{(b)}$$

(c) The rate of exergy destruction equals the difference between the exergy entering and the exergy exiting. As the combustion air enters ① at the condition of the environment, and thus with zero exergy, the exergy entering is just the result of part (a). The exergy exiting is the result of part (b). Accordingly

$$\frac{\dot{E}_d}{\dot{n}_{C_2H_2}} = 542{,}455 - 373{,}978 = 168477 \text{ Btu/lbmol}(C_2H_2) \longleftarrow \text{(c)}$$

An exergetic efficiency can be expressed as

$$\epsilon = \frac{\dot{E}_{products}}{\dot{E}_{fuel}} = \frac{373{,}978}{542{,}455} = 0.689 \quad (68.9)\% \longleftarrow \%$$

---

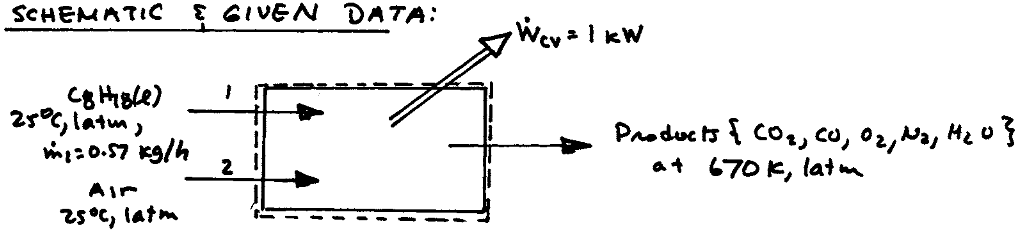
1. Throughout Chap. 13 combustion air is modeled as 79% N_2, 21% O_2. As these values differ from the percentages listed for N_2, O_2 in Problem 13.82, a small nonzero value for the chemical exergy of the combustion air can be calculated via Eq. 13.41a. This is ignored in the present analysis.

PROBLEM 13.98

KNOWN: $C_8H_{18}(l)$ at 25°C, 1atm and a mass flow rate of 0.57 kg/h enters an engine and burns with air entering at 25°C, 1atm. Products exit at 670 K, 1atm with a dry molar analysis: 11.4% CO_2, 2.9% CO, 1.6% O_2, 84.1% N_2. The engine develops power at a rate of 1 kW.

FIND: Determine (a) the rate of heat transfer, (b) Evaluate an exergetic efficiency for the engine.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (3) The combustion air and combustion products can be modeled as ideal gases. (4) The environment is described as in Problem 13.8a. (5) The combustion air enters at the condition of the environment.

ANALYSIS: The reaction equation takes the form

$$C_8H_{18}(l) + a[O_2 + 3.76N_2] \rightarrow b[0.114\,CO_2 + 0.029\,CO + 0.016\,O_2 + 0.841\,N_2) + c\,H_2O$$

From a carbon balance $b = 55.9$. From a hydrogen balance $c = 9$. From an oxygen balance $a = 12.58$. (Checking the nitrogen balance for closure: $3.76a = 0.841b$ or $47.3 \approx 47.01$.) Accordingly, the balance reaction equation is

$$C_8H_{18}(l) + 12.58[O_2 + 3.76N_2] \rightarrow 55.9[0.114\,CO_2 + 0.029\,CO + 0.016\,O_2 + 0.841\,N_2) + 9H_2O$$

(a) From an energy rate balance at steady state

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} - \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} + (\bar{h}_{fuel})_1 + [12.58\,\bar{h}_{O_2} + (12.58)(3.76)\bar{h}_{N_2}]_2 - \Big\{55.9\,(0.114\,\bar{h}_{CO_2} + 0.029\,\bar{h}_{CO}$$
$$+ 0.016\,\bar{h}_{O_2} + 0.841\,\bar{h}_{N_2})\}_3 + 9(\bar{h}_{H_2O})_3\Big\}$$

Introducing $\bar{h} = \bar{h}_f^o + \Delta\bar{h}$, and noting that $\bar{h}_f^o = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{fuel}} = \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} - (\bar{h}_f^o)_{C_8H_{18}(l)} + 55.9\Big[0.114\,[\bar{h}_f^o + \bar{h}(670) - \bar{h}(298)]_{CO_2} +$$
$$0.029\,[\bar{h}_f^o + \bar{h}(670) - \bar{h}(298)]_{CO} + 0.016\,[\bar{h}(670) - \bar{h}(298)]_{O_2} +$$
$$0.841\,[\bar{h}(670) - \bar{h}(298)]_{N_2}\Big] + 9\,[\bar{h}_f^o + \bar{h}(670) - \bar{h}(298)]_{H_2O}$$

$$= \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} - (-249{,}910) + 55.9\Big[0.114\,[-393{,}520 + 25648 - 9364] +$$
$$0.029\,[-110{,}530 + 19758 - 8669] + 0.016\,[20{,}197 - 8682] +$$
$$0.841\,[19{,}685 - 8669]\Big] + 9\,[-241{,}820 + 22{,}970 - 9904]$$

$$= \frac{\dot{W}_{cv}}{\dot{n}_{fuel}} - 3{,}845{,}872\ \frac{\text{kJ}}{\text{kmol}\,(C_8H_{18})}$$

PROBLEM 13.98 (Contd.)

The molar flow rate of the fuel is $\dot{n}_{fuel} = \dot{m}_{fuel}/M_{fuel}$. Thus

$$\dot{Q}_{cv} = 1\,\text{kW} - \left(\frac{0.57\ \text{kg/h}}{114.22\ \text{kg/kmol}}\right)\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\left(3{,}845{,}872\ \frac{\text{kJ}}{\text{kmol}}\right)\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|$$

$$= -4.33\ \text{kW}$$ ◄——

(b) Paralleling the analysis of Example 13.15, an exergetic efficiency takes the form

$$\epsilon = \frac{\dot{W}_{cv}}{\dot{E}_F}$$

where $\dot{E}_F$ is the rate exergy enters the engine with the fuel.

Since the fuel enters at T_0, p_0, there is no thermomechanical contribution to its exergy. The chemical exergy is evaluated in Example 13.12(a) as $\bar{e}^{CH} = 5{,}407{,}843$ kJ/kmol (C_8H_{18}). Then

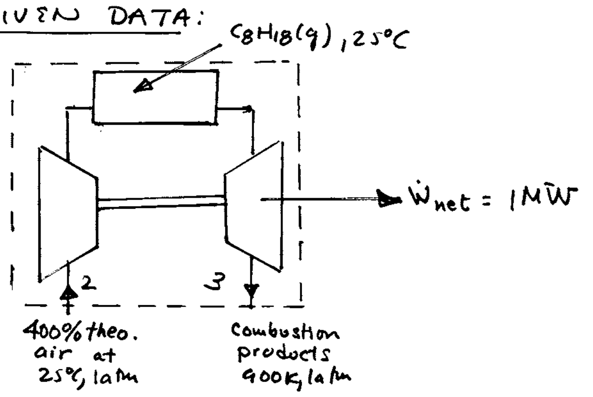
$$\epsilon = \frac{1\ \text{kW}}{\left(5{,}407{,}843\ \frac{\text{kJ}}{\text{kmol}}\right)\left(\frac{0.57}{114.22}\ \frac{\text{kmol}}{\text{h}}\right)\left|\frac{1\ \text{h}}{3600\ \text{s}}\right|\left|\frac{1\ \text{kW}}{1\ \text{kJ/s}}\right|} = 0.133\ \ (13.3\%)$$ ◄——

PROBLEM 13.99

KNOWN: Steady state operating data are provided for a gas turbine power plant.

FIND: Evaluate an exergetic efficiency for the power plant.

SCHEMATIC & GIVEN DATA:



$\dot{Q}_{cv} = -0.15$ MW

From solution to Problem 13.44,

$\dot{n}_{fuel} = 7.57 \frac{kmol}{h}$

$C_8H_{18} + 50(O_2 + 3.76 N_2) \rightarrow$
$8CO_2 + 9H_2O + 37.5O_2 + 188N_2$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with negligible effects of kinetic and potential energy. (2) 3.76 moles of N_2 accompany each mole of O_2 in the combustion air. N_2 is inert. (3) The ideal gas model is applicable to the fuel, combustion air, and the products of combustion. (4) The exergy of the air entering the control volume at 2 can be ignored. (5) Exergy values are evaluated relative to the environment of Problem 13.82.

ANALYSIS: An exergy rate balance for the control volume reads

$$0 = \dot{E}_{fuel} + \dot{E}_{air,2} - \dot{E}_{prods,3} - \dot{W}_{net} - \sum[1 - T_0/T_j]|\dot{Q}_j| - \dot{E}_d$$

Rearranging

$$\dot{E}_{fuel} + \dot{E}_{air,2} = \underset{(i)}{\dot{W}_{net}} + \underset{(ii)}{\dot{E}_{prods,3}} + \underset{(iii)}{\sum[1 - T_0/T_j]|\dot{Q}_j|} + \dot{E}_d$$

That is, the exergy entering the control volume with the fuel and combustion air exits as (i) net power out, (ii) with the combustion products, (iii) with heat transfer or is destroyed by irreversibilities within the control volume. An exergetic efficiency can be written as

$$\epsilon = \frac{\dot{W}_{net} + \dot{E}_{prods,3}}{\dot{E}_{fuel}} \qquad (1)$$

① In writing this, $\dot{E}_{air,2}$ has been ignored (assumption 4) and the exergy exiting with heat transfer has been taken as a loss. The exergy of the the products $\dot{E}_{prods,3}$ has been included in the numerator, together with $\dot{W}_{net}$, in recognition of its considerable utility (see Secs. 9.7, 9.10 for discussion).

The value of $\bar{e}^{CH}_{fuel}$ is found as in Example 13.12(a), except here the octane is considered to be a gas. The result is

$$\bar{e}^{CH}_{fuel} = 5{,}418{,}553 \text{ kJ/kmol(fuel)}$$

Since the fuel enters at 25°C and ideal gas principles are assumed to apply, Equation 13.47 reads

PROBLEM 13.99 (Continued)

$$\bar{e}_{f,\,fuel} = \underbrace{h(T_0) - h(T_0)}_{=0} - T_0[\underbrace{\bar{s}^\circ(T_0) - \bar{s}^\circ(T_0)}_{=0} - \bar{R}\ln\frac{p}{p_0} + \bar{e}^{ch}$$

That is, only the term $\bar{R}T_0 \ln P/P_0$ remains of the thermomechanical contribution. This term is ignored relative to $\bar{e}^{CH}$ (sample calculations indicate that the term would be on the order of 0.1 to 0.2% of $\bar{e}^{CH}$).

For the combustion gases exiting at 3, $\bar{e}_{f3}$ can be evaluated as in Example 13.14. First, the molar analysis of the products is found:

CO_2: $\frac{8}{242.5} = 0.033$ O_2: $\frac{37.5}{242.5} = 0.155$

H_2O: $\frac{9}{242.5} = 0.037$ N_2: $\frac{188}{242.5} = 0.775$

Then

$$\bar{e}_3^{CH} = (8.314)(298)\left[8\ln\left(\frac{3.3}{0.03}\right) + 9\ln\left(\frac{3.7}{3.12}\right) + 37.5\ln\left(\frac{15.5}{20.35}\right) + 188\ln\left(\frac{77.5}{75.67}\right)\right]$$

per mole of fuel

$$= 82,800 \text{ kJ/kmol (fuel)}$$

And

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = \big[8[37,405 - 9364 - 298(263.559 - 213.685)] + 9[31,828 - 9904 - 298(228.321 - 188.720)] + 37.5[27,928 - 8682 - 298(239.823 - 205.033)] + 188[26,890 - 8669 - 298(224.647 - 191.502)]$$

$$= 2,098,114 \text{ kJ/kmol (fuel)}$$

Summing

$$\bar{e}_{f,3} = 2,098,114 + 82,800 = 2,180,914 \text{ kJ/kmol(fuel)}$$

Returning to Eq. (1)

$$\epsilon = \frac{10^3 \text{ kJ/s} + \left(\frac{7.57}{3600}\right)(2,180,914)}{\left(\frac{7.57}{3600}\right)(5,418,553)} = \frac{5586}{11390.}$$

$$= 0.49 \quad (49\%) \quad \longleftarrow$$

---

1. If all heat transfer from the turbine takes place at temperature T_b, the rate exergy exits with heat transfer is evaluated as

$$\left[1 - \frac{T_0}{T_b}\right]|\dot{Q}_{cv}| = \left[1 - \frac{T_0}{T_b}\right]\left(150 \frac{kJ}{s}\right)$$

As a sample calculation, consider $T_b = 500K$ then

$$\left[1 - \frac{298}{500}\right]\left(150 \frac{kJ}{s}\right) = 61 \text{ kJ/s}$$

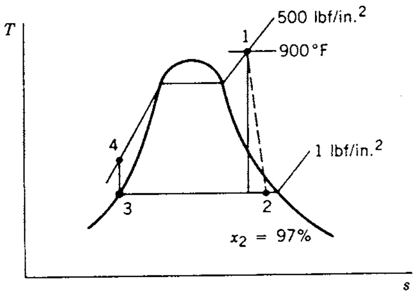
If this amount would be included with the numerator of Eq. (1), the value of ϵ would be ≈ 49.6%.

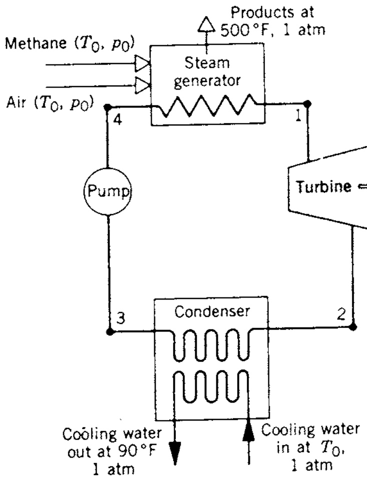
PROBLEM 13.100

KNOWN: CH_4 enters the steam generator of a vapor power plant and burns completely with 200% of theoretical air at 77°F, 1 atm. Additional steady-state operating data are provided.

FIND: Determine (a) the balanced reaction equation, (b) the vapor mass flow rate and (c) the cooling water mass flow rate, each per mole of fuel. (d) Also, find as a percentage of the exergy entering with the fuel each of the following: exergy exiting with the stack gases, exergy destroyed in the steam generator, power developed by the turbine, exergy destroyed in the turbine, exergy exiting with the cooling water, exergy destroyed in the condenser.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component can be modeled as a control volume at steady state with negligible energy loss by heat transfer. Kinetic and potential energy effects can be ignored. (2) For the vapor power plant, pump work can be neglected and saturated liquid exits the condenser. (3) Combustion is complete. (4) 3.76 moles of N_2 accompany each mole of air. N_2 is inert. (5) The environment is described as in Problem 13.86. (6) The combustion air and combustion products can be modeled as ideal gases. (7) The air enters at the condition of the environment.

ANALYSIS: (a) The balanced reaction equation for complete combustion with 200% of the theoretical amount of air is

$$CH_4 + 4(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 2O_2 + 15.04N_2 \longleftarrow \text{(a)}$$

(b) An energy rate balance at steady state for a control volume enclosing the steam generator reads

$$0 = \cancelto{0}{\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}}} - \cancelto{0}{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}} + \bar{h}_{CH_4} + (4\bar{h}_{O_2} + 15.04\bar{h}_{N_2}) - [\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2\bar{h}_{O_2} + 15.04\bar{h}_{N_2}] + \frac{\dot{m}_v}{\dot{n}_{CH_4}}(h_4 - h_1)$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{CH_4} + (0) - [\bar{h}_f^\circ + \bar{h}(960) - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^\circ + \bar{h}(960) - \bar{h}(537)]_{H_2O} - 2[\bar{h}(960) - \bar{h}(537)]_{O_2}$$
$$- 15.04[\bar{h}(960) - \bar{h}(537)]_{N_2} + \frac{\dot{m}_v}{\dot{n}_{CH_4}}(h_4 - h_1)$$

or

$$\frac{\dot{m}_v}{\dot{n}_{CH_4}} = \frac{(\bar{h}_f^\circ)_{CH_4} - [\bar{h}_f^\circ + \bar{h}(960) - \bar{h}(537)]_{CO_2} - 2[\bar{h}_f^\circ + \bar{h}(960) - \bar{h}(537)]_{H_2O} - 2[\bar{h}(960) - \bar{h}(537)]_{O_2} - 15.04[\bar{h}(960) - \bar{h}(537)]_{N_2}}{h_1 - h_4}$$

PROBLEM 13.100 (Contd.)

From Table A-4E, $h_1 = 1466.5$ Btu/lb, $s_1 = 1.6987$ Btu/lb·°R. From Table A-3E, $h_3 = 69.74$ Btu/lb, $s_3 = 0.1327$ Btu/lb·°R. Then, with $s_4 = s_3$ and $p_4 = 500$ lb/in², Table A-5E gives $h_4 = 71.33$ Btu/lb. Then, with data from the ideal gas tables and Table A-25E

$$\frac{\dot{m}_v}{\dot{n}_{CH_4}} = \frac{-32{,}210 - [-169{,}300 + 8244 - 4028] - 2[-104{,}040 + 7738 - 4258] - 2[6786 - 3725] - 15.04[6693 - 3730]}{(1466.5 - 71.33)}$$

$$= 203.1 \text{ lb(vapor)/lbmol}(CH_4) \longleftarrow \text{(b)}$$

(c) An energy rate balance at steady state for a control volume enclosing the condenser reads

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + \dot{m}_v[h_2 - h_3] + \dot{m}_{cw}[h_{cw,in} - h_{cw,out}]$$

To find h_2 write

$$h_2 = h_f + x_2(h_g - h_f) = 69.74 + (0.97)(1036) = 1074.66 \text{ Btu/lb}$$

Then, evaluating $h_{cw} \approx h_f(T)$

$$\frac{\dot{m}_{cw}}{\dot{n}_{CH_4}} = \left(\frac{\dot{m}_v}{\dot{n}_{CH_4}}\right)\left[\frac{h_2 - h_3}{h_{cw,out} - h_{cw,in}}\right] = (203.1)\left[\frac{1074.66 - 69.74}{58.07 - 45.09}\right] = 15{,}724 \frac{\text{lb(cw)}}{\text{lbmol}(CH_4)} \longleftarrow \text{(c)}$$

(d) As the fuel enters at T_0, p_0, the exergy entering with the fuel is its chemical exergy, found from Eq. 13.36:

$$\bar{e}^{ch}_{CH_4} = (\bar{g}_{CH_4} + 2\bar{g}_{O_2} - \bar{g}_{CO_2} - 2\bar{g}_{H_2O})(T_0, p_0) + \bar{R}T_0 \ln\left[\frac{(y^e_{O_2})^2}{(y^e_{CO_2})(y^e_{H_2O})^2}\right]$$

$$= (-21{,}860 + 2(0) - (-169{,}680) - 2(-98{,}350)] + (1.986)(537)\ln\left[\frac{(0.2035)^2}{(0.0003)(0.0312)^2}\right]$$

$$= 357{,}171 \text{ Btu/lbmol}$$

Stack Gases: The exergy exiting with the stack gases can be found using Eq. 13.47 where $\bar{e}^{ch}$ is obtained with Eq. 13.41a. Thus, on a per lbmol of fuel basis

$$\bar{e}^{ch} = (1.986)(537)\left[\ln\left(\frac{1/20.04}{0.0003}\right) + 2\ln\left(\frac{2/20.04}{0.0312}\right) + 2\ln\left(\frac{2/20.04}{.2035}\right) + 15.04\ln\left(\frac{15.04/20.04}{.7567}\right)\right]$$

$$= 6282.4 \text{ Btu/lbmol}(CH_4)$$

And

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = [9243.8 - 4027.5 - 537(56.765 - 51.032)]_{CO_2} + 2[7738 - 4258 - 537(49.843 - 45.079)]_{H_2O}$$
$$+ 2[6786 - 3725.1 - 537(53.170 - 48.982)]_{O_2} + 15.04[6693.1 - 3729.5 - 537(49.808 - 45.743)]_{N_2}$$
$$= 16{,}046.8 \text{ Btu/lbmol}(CH_4)$$

Adding these contributions, $\dot{E}_{\substack{stack\\gas}}/\dot{n}_{CH_4} = 22{,}329$ Btu/lbmol(CH$_4$). Thus

$$\begin{pmatrix}\text{\% of fuel}\\ \text{exergy exiting}\\ \text{with stack gases}\end{pmatrix} = \left(\frac{22{,}329}{357{,}171}\right)(100) = 6.25\ \%$$

Steam Generator: The exergy destroyed in the steam generator equals the difference between the total exergy entering and the total exergy exiting. Regarding the exergy of the entering combustion air as negligible

$$\frac{\dot{E}_d}{\dot{n}_{CH_4}} = \bar{e}^{ch}_{CH_4} + \frac{\dot{m}_v}{\dot{n}_{CH_4}}[(h_4 - h_1) - T_0(s_4 - s_1)] - \frac{\dot{E}_{\text{stack gas}}}{\dot{n}_{CH_4}}$$

Thus

$$\frac{\dot{E}_d}{\dot{n}_{CH_4}} = 357{,}171 - 203.1[1466.5 - 71.33 - 537(1.6987 - 0.1327)] - 22{,}329$$

$$= 222{,}278 \text{ Btu/lbmol (CH}_4\text{)}$$

And

① $$\begin{bmatrix}\text{% of fuel exergy} \\ \text{destroyed in the} \\ \text{steam generator}\end{bmatrix} = \left(\frac{222{,}278}{357{,}171}\right)(100) = 62.23\%$$

Turbine. The power developed by the turbine is obtained from an energy rate balance at steady state for a control volume enclosing the turbine:

$$\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}} = \left(\frac{\dot{m}_v}{\dot{n}_{CH_4}}\right)(h_1 - h_2) = (203.1)(1466.5 - 1074.66) = 79{,}583 \text{ Btu/lbmol (CH}_4\text{)}$$

The rate of exergy destruction is

$$\frac{\dot{E}_d}{\dot{n}_{CH_4}} = T_0\left(\frac{\dot{m}_v}{\dot{n}_{CH_4}}\right)(s_2 - s_1) = (537)(203.1)\left[[0.1327 + 0.97(1.8453)] - 1.6987\right]$$

$$= 24{,}424 \text{ Btu/lbmol (CH}_4\text{)}$$

Accordingly

$$\begin{bmatrix}\text{% of fuel exergy} \\ \text{exiting as developed power}\end{bmatrix} = \left(\frac{79{,}583}{357{,}171}\right)(100) = 22.28$$

$$\begin{bmatrix}\text{% of fuel exergy} \\ \text{destroyed in the turbine}\end{bmatrix} = \left(\frac{24{,}424}{357{,}171}\right)(100) = 6.84$$

Condenser. The net exergy exiting with the cooling water is

$$\frac{\dot{E}_{cw}}{\dot{n}_{CH_4}} = \frac{\dot{m}_{cw}}{\dot{n}_{CH_4}}\left[(h_{cw,out} - h_{cw,in}) - T_0(s_{cw,out} - s_{cw,in})\right]$$

$$= 15{,}724\left[(58.07 - 45.09) - 537(0.1117 - 0.08775)\right]$$

$$= 1869 \text{ Btu/lbmol (CH}_4\text{)}$$

The exergy destroyed in the condenser is

$$\frac{\dot{E}_d}{\dot{n}_{CH_4}} = T_0\left[\frac{\dot{m}_{cw}}{\dot{n}_{CH_4}}(s_{cw,out} - s_{cw,in}) + \frac{\dot{m}_v}{\dot{n}_{CH_4}}(s_3 - s_2)\right]$$

$$= 537\left[15{,}724(.1117 - 0.08775) + 203.1(.1327 - 1.9226)\right]$$

$$= 7013 \text{ Btu/lbmol (CH}_4\text{)}$$

Accordingly

$$\begin{bmatrix}\text{% of fuel exergy} \\ \text{carried out with cooling water}\end{bmatrix} = \left(\frac{1869}{357{,}171}\right)(100) = 0.52$$

$$\begin{bmatrix}\text{% of fuel exergy} \\ \text{destroyed in the condenser}\end{bmatrix} = \left(\frac{7013}{357{,}171}\right)(100) = 1.96$$

---

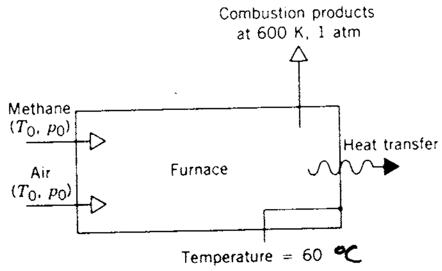
1. The steam generator is, by far, the most significant site of irreversibility.

PROBLEM 13.101

KNOWN: CH_4 at 25°C, 1atm enters a furnace and burns completely with 200% theoretical air entering at 25°C, 1atm. Combustion products exit at 600K, 1atm and the furnace delivers energy by heat transfer at $T_s = 333\,K$ (60°C).

FIND: Determine in kJ per kmol of fuel (a) the exergy entering with the fuel, (b) the exergy exiting with the products, (c) the rate of exergy destruction. Also, evaluate an exergetic efficiency for the furnace.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) Combustion is complete. (3) 3.76 moles of N_2 accompany each mole of O_2 in the air. N_2 is inert. (4) The combustion air and combustion products can be modeled as ideal gases. (5) The environment is as described in Problem 13.82. (6) The combustion air enters at the condition of the environment.

ANALYSIS: The balanced reaction equation for complete combustion with 200% theoretical air is

$$CH_4 + 4(O_2 + 3.76\,N_2) \rightarrow CO_2 + 2H_2O + 2O_2 + 15.04\,N_2.$$

(a) The fuel enters at T_0, p_0. Accordingly, there is no thermomechanical contribution, and the exergy entering with the fuel is $\bar{e}^{ch}_{j}$ which is determined in Problem 13.82(c) as $\bar{e}^{ch}_{CH_4} = 830{,}174$ kJ/kmol. ⟵ (a)

(b) The exergy exiting with the combustion products can be evaluated using Eq. 13.47 where $\bar{e}^{ch}$ is obtained with Eq. 13.41a. Thus, on a per kmol of CH_4 basis

$$\bar{e}^{ch} = \bar{R}T_0\left[\ln\frac{y_{CO_2}}{y^e_{CO_2}} + 2\ln\frac{y_{H_2O}}{y^e_{H_2O}} + 2\ln\frac{y_{O_2}}{y^e_{O_2}} + 15.04\ln\frac{y_{N_2}}{y^e_{N_2}}\right]$$

$$= (8.314)(298)\left[\ln\left(\frac{1/20.04}{.0003}\right) + 2\ln\left(\frac{2/20.04}{0.0312}\right) + 2\ln\left(\frac{2/20.04}{0.2035}\right) + 15.04\ln\left(\frac{15.04/20.04}{0.7567}\right)\right]$$

$$= 14595\ \text{kJ/kmol}(CH_4)$$

PROBLEM 13.101 (Contd.)

And

$$\bar{h}-\bar{h}_0-T_0(\bar{s}-\bar{s}_0) = [22{,}280-9364-298(243.199-213.685)]_{CO_2} + 2[20{,}402-9904-298(212.920-188.720)]_{H_2O} + 2[17{,}929-8682-298(226.346-205.033)]_{O_2} + 15.04[17{,}563-8669-298(212.066-191.502)]_{N_2}$$

$$= 58085 \text{ kJ/kmol(CH}_4)$$

Adding these contributions

$$\frac{\dot{E}_{products}}{\dot{n}_{CH_4}} = 58085 + 14595 = 72{,}680 \text{ kJ/kmol(CH}_4) \longleftarrow \text{(b)}$$

(c) At steady state, the rate exergy is destroyed equals the difference between the rate exergy enters and the rate exergy exits. As the air enters at the condition of the environment, and thus with zero exergy, exergy enters only with the fuel. Exergy exits with the combustion products and accompanying the heat transfer. The heat transfer can be evaluated using an energy rate balance:

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{\,0} + \bar{h}_{CH_4} + (4\bar{h}_{O_2} + 15.04\bar{h}_{N_2}) - (\bar{h}_{CO_2} + 2\bar{h}_{H_2O} + 2\bar{h}_{O_2} + 15.04\bar{h}_{N_2})$$

With $\bar{h} = \bar{h}_f^\circ + \Delta\bar{h}$, and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = [\bar{h}_f^\circ + \bar{h}(600) - \bar{h}(298)]_{CO_2} + 2[\bar{h}_f^\circ + \bar{h}(600) - \bar{h}(298)]_{H_2O} + 2[\bar{h}(600) - \bar{h}(298)]_{O_2} + 15.04[\bar{h}(600) - \bar{h}(298)]_{N_2} - (\bar{h}_f^\circ)_{CH_4}$$

$$= [-393{,}520 + 22{,}280 - 9364] + 2[-241{,}820 + 20{,}402 - 9904] + 2[17929 - 8682] + 15.04[17{,}563 - 8669] - (-74{,}850)$$

$$= -616{,}138 \text{ kJ/kmol(CH}_4)$$

The accompanying exergy transfer is

$$\frac{\dot{E}_q}{\dot{n}_{CH_4}} = \left[1-\frac{T_0}{T_s}\right]\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = \left[1-\frac{298}{333}\right](-616{,}138) = -64{,}759 \text{ kJ/kmol(CH}_4)$$

where the minus sign denotes a transfer from the furnace.

In summary

$$\dot{E}_d/\dot{n}_{CH_4} = \underset{\text{(fuel)}}{830{,}174} - \underset{\text{(heat transfer)}}{64{,}759} - \underset{\text{(products)}}{72{,}680} = 692{,}735 \text{ kJ/kmol(CH}_4) \longleftarrow \text{(c)}$$

As the object of the furnace is to provide an energy transfer by heat, an exergetic efficiency that gauges the extent to which the exergy entering the furnace with the fuel is obtained as exergy accompanying heat transfer is

① $$\epsilon = \frac{|\dot{E}_q/\dot{n}_{CH_4}|}{\bar{e}^{ch}_{CH_4}} = \frac{64{,}759}{830{,}174} = 0.078 \quad (7.8\%) \longleftarrow \%$$

---

1. Owing to the significant destruction of exergy during combustion, such a furnace does not make an effective use of the exergy supplied with the fuel.

PROBLEM 13.102

KNOWN: Coal enters a combustor with the mass analysis:

C: 49.8%, H: 3.5%, O: 6.8%, S: 6.4%, H_2O: 14.1%, Ash: 19.4%
The higher heating value of the coal, as received, is HHV = 21,220 kJ/kg. The lower heating value on a dry basis is $(LHV)_d = 20{,}450$ kJ/kg.

FIND: (a) Using the following expression, estimate the chemical exergy

$$e^{ch} = (LHV)_d\left[1.0438 + 0.0013\frac{h}{c} + 0.1083\frac{o}{c} + 0.0549\frac{n}{c}\right] + 6740s$$

where h/c, o/c, and n/c denote, respectively, the mass ratio of hydrogen to carbon, oxygen to carbon, and nitrogen to carbon. s is the mass fraction of sulfur in kg per kg of fuel. (b) Compare this estimate with the values that would result by approximating the chemical exergy with the given heating values.
(c) If the chemical exergy of the coal were evaluated using the method of Sec. 13.6, what data would be required?

ANALYSIS: (a) Using the expression provided, together with given data

$$e^{ch} = (20{,}450)\left[1.0438 + 0.0013\left(\frac{3.5}{49.8}\right) + 0.1083\left(\frac{6.8}{49.8}\right) + 0.0549\left(\frac{0}{49.8}\right)\right] + 6740\left(\frac{6.4}{100}\right)$$

$$= (20{,}450)\left[1.0438 + 0.00009 + 0.01479 + 0\right] + 431$$

$$= 22{,}081 \text{ kJ/kg(coal)} \longleftarrow \text{(a)}$$

(b) With $e^{CH} \approx$ HHV, a value about 4% less than found in part (a) is obtained. With $e^{CH} \approx (LHV)_d$, a value about 8% less than found in part (a) is obtained.

(c) Although the coal under present consideration is not represented as C_aH_b, as in Sec. 13.6., the same approach to finding $\bar{e}^{CH}$ can be applied, in principle. However, to implement the procedures, it would be necessary to know the absolute entropy of the fuel at T_0, P_0 as well as either the enthalpy of formation of the fuel or one of the heating values, $\overline{LHV}$ or $\overline{HHV}$. As the heating values can be measured, the principal stumbling block in implementing the procedures usually is lack of knowledge concerning the absolute entropy. Accordingly, it is often necessary to resort to approximate methods for determining e^{CH}. A reasonable approximation often can be had by taking the chemical exergy to equal an experimentally determined lower (or higher) heating value. Moreover, as illustrated in part (a), special expressions for this purpose also have been developed.

PROBLEM 13.103

KNOWN: A moist air stream is at temperature T, pressure p, with dry air and water vapor mole fractions y_a and y_v, respectively.

FIND: Show that the flow exergy can be expressed as

$$\bar{e}_f = T_0\left\{(y_a\bar{c}_{pa} + y_v\bar{c}_{pv})\left[\left(\frac{T}{T_0}\right) - 1 - \ln\left(\frac{T}{T_0}\right)\right] + \bar{R}\ln\left(\frac{p}{p_0}\right)\right\} + \bar{R}T_0\left[y_a\ln\left(\frac{y_a}{y_a^e}\right) + y_v\ln\left(\frac{y_v}{y_v^e}\right)\right]$$

relative to an environment modeled as an ideal gas mixture of water vapor and dry air at T_0, P_0, and mole fractions of dry air and water vapor, respectively, of y_a^e, y_v^e.

ANALYSIS: The flow exergy can be determined using Eq. 13.47, where $\bar{e}^{CH}$ is obtained using Eq. 13.41a.

When expressed on a per mole of mixture basis

$$\bar{e}^{ch} = \bar{R}T_0\left[y_a \ln\frac{y_a}{y_a^e} + y_v \ln\frac{y_v}{y_v^e}\right]$$

And

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = y_a\left[\bar{h}_a(T) - \bar{h}_a(T_0) - T_0\left(\bar{s}_a^\circ(T) - \bar{s}_a^\circ(T_0) - \bar{R}\ln\frac{\cancel{y_a}p}{\cancel{y_a}p_0}\right)\right] +$$
$$y_v\left[\bar{h}_v(T) - \bar{h}_v(T_0) - T_0\left(\bar{s}_v^\circ(T) - \bar{s}_v^\circ(T_0) - \bar{R}\ln\frac{\cancel{y_v}p}{\cancel{y_v}p_0}\right)\right]$$

Evaluating the enthalpy and entropy terms on the basis of mean values for the specific heats $\bar{c}_{pa}$ and $\bar{c}_{pv}$, this expression becomes

$$\bar{h} - \bar{h}_0 - T_0(\bar{s} - \bar{s}_0) = y_a\left[\bar{c}_{pa}[T - T_0] - T_0\left(\bar{c}_{pa}\ln T/T_0 - \bar{R}\ln p/p_0\right)\right] +$$
$$y_v\left[\bar{c}_{pv}[T - T_0] - T_0\left(\bar{c}_{pv}\ln T/T_0 - \bar{R}\ln p/p_0\right)\right]$$
$$= T_0\left\{(y_a\bar{c}_{pa} + y_v\bar{c}_{pv})\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right] + \bar{R}\underbrace{(y_a + y_v)}_{=1}\ln p/p_0\right\}$$

Adding these contributions

$$\bar{e}_f = T_0\left\{(y_a\bar{c}_{pa} + y_v\bar{c}_{pv})\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right] + \bar{R}\ln\frac{p}{p_0}\right\} + \bar{R}T_0\left[y_a\ln\frac{y_a}{y_a^e} + y_v\ln\frac{y_v}{y_v^e}\right]$$

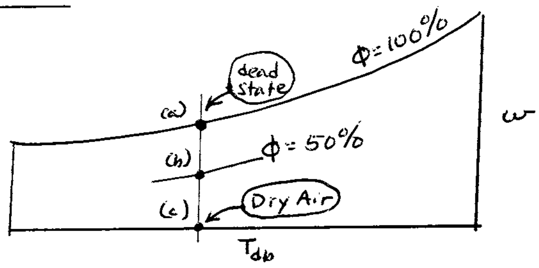
which is the result to be obtained.

PROBLEM 13.104

KNOWN: Three cases involving air at 20°C, 1atm are specified: (a) saturated air, (b) $\phi = 50\%$, (c) dry air.

FIND: Using the result of Problem 13.103, evaluate e_f relative to an environment consisting of moist air at 20°C, 1atm, $\phi = 100\%$.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The idealizations of Problem 13.103 apply. (2) The effects of motion and gravity can be ignored.

ANALYSIS: Since $T = T_0$ and $P = P_0$, the result of Problem 13.103 becomes, simply

$$\bar{e}_f = \bar{R} T_0 \left[y_a \ln\left(\frac{y_a}{y_a^e}\right) + y_v \ln\left(\frac{y_v}{y_v^e}\right) \right]$$

Also, for moist air at T, p, ϕ

$$\begin{cases} y_a = \frac{p_a}{p} = \frac{p - p_v}{p} = 1 - \frac{p_v}{p} = 1 - \phi\left[\frac{p_g}{p}\right] \\ y_v = 1 - y_a = \phi\left[\frac{p_g}{p}\right] \end{cases}$$

$$\phi^e = 1 \Rightarrow \begin{cases} y_a^e = 1 - \frac{p_g(T_0)}{p_0} \\ y_v^e = \frac{p_g(T_0)}{p_0} \end{cases}$$

(a) $\phi = 1 \Rightarrow y_a = y_a^e,\ y_v = y_v^e \Rightarrow \bar{e}_f = 0$ ← (a

(b) $\phi = .50$, $p_g = 0.02339$ bar at $T_0 = 20°C$.

$$y_v^e = \frac{0.02339}{1.01325} = 0.0231, \quad y_a^e = 0.9769$$

$$y_v = 0.5[0.0231] = 0.01155, \quad y_a = 0.98845$$

$$\bar{e}_f = \left(8.314 \frac{kJ}{kmol(mix)\,K}\right)(293K)\left[0.98845 \ln\left(\frac{0.98845}{0.9769}\right) + 0.01155 \ln\left(\frac{0.01155}{0.0231}\right)\right]$$

$$= 8.8 \frac{kJ}{kmol\,(mix)}$$

For the mixture, $M = y_a M_a + y_v M_v = (0.98845)(28.97) + (0.01155)(18.02) = 28.84$

$$\Rightarrow e_f = (8.8/28.84) = 0.305\ kJ/kg(mix)$$ ← (b)

(c) Dry air, $y_v = 0$, $y_a = 1.0$, and a special case of the form Eq (13.40) applies:

$$e_f = R T_0 \ln\left[\frac{1}{y_a^e}\right] = \left(\frac{8.314}{28.97}\right)(293) \ln\left[\frac{1}{0.9769}\right]$$

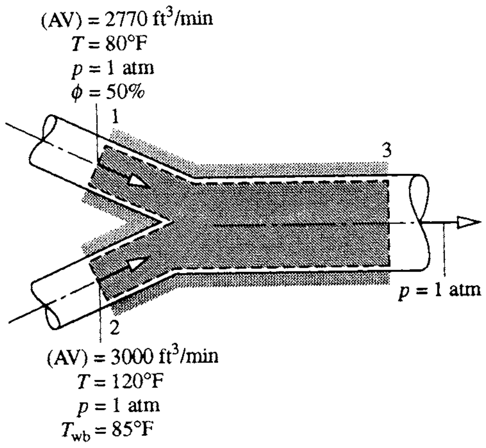
$$= 1.965\ kJ/kg(a)$$ ← (c

PROBLEM 13.105

KNOWN: Steady-state operating data are provided for a device in which two moist air streams are mixed to form a single exiting moist air stream

FIND: Using the result of Problem 13.103, determine the flow exergy at 1, 2, and 3 (see figure below), and the rate of exergy destruction.

SCHEMATIC & GIVEN DATA:



From the Solution to Problem 12.96

$T_3 = 100.1°C$

$\dot{m}_{a1} = 200$ lb/min

$\dot{m}_{a2} = 199.5$ lb/min

$\dot{m}_{a3} = 399.5$ lb/min

$\omega_1 = 0.01092$

$\omega_2 = 0.01803 \qquad \phi_2 = 24.4\%$

$\omega_3 = 0.01447$

Assumptions: 1. See solution to Problem 12.96. 2. The environment is a mixture of dry air and water vapor at 95°F, 1atm with $y_v^e = 0.022$, $y_a^e = 0.978$, $c_{pa} = 0.24$ Btu/lb·°R, $c_{pv} = 0.44$ Btu/lb·°R.

ANALYSIS: Since $p = p_0$ for each stream, $\ln(p/p_0)$ drops out of the expression of Problem 13.103. Moreover, the expression can be written on a unit mass of dry air basis as

$$① \quad e_f = T_0\left\{[c_{pa} + \omega c_{pv}]\left[\frac{T}{T_0} - 1 - \ln\frac{T}{T_0}\right] + \frac{\bar{R}}{M_a}\left[(1+\tilde{\omega})\ln\left(\frac{1+\tilde{\omega}^e}{1+\tilde{\omega}}\right) + \tilde{\omega}\ln\left(\frac{\tilde{\omega}}{\tilde{\omega}^e}\right)\right]\right\} \quad (1)$$

where $\tilde{\omega} = (M_a/M_v)\,\omega = 1.6078\,\omega$. Also

$$\omega = \frac{m_v}{m_a} = \frac{M_v y_v}{M_a y_a} = 0.622\,\frac{y_v}{y_a}$$

Thus, $\tilde{\omega}^e = y_v^e/y_a^e = 0.02249$.

Using Eq.(1)

$$e_{f1} = 555\left\{[0.24 + 0.01092(0.44)]\left[\frac{540}{555} - 1 - \ln\frac{540}{555}\right] + \left(\frac{1.986}{28.97}\right)\left[1.01756\ln\left(\frac{1.02249}{1.01756}\right) + 0.01756\ln\left(\frac{0.01756}{0.02249}\right)\right]\right\}$$

$= 0.0723$ Btu/lb(a)

$$e_{f2} = 555\left\{[0.24 + 0.01803(0.44)]\left[\frac{580}{555} - 1 - \ln\frac{580}{555}\right] + \left(\frac{1.986}{28.97}\right)\left[(1.02899)\ln\left(\frac{1.02249}{1.02899}\right) + 0.02899\ln\left(\frac{0.02899}{0.02249}\right)\right]\right\}$$

$= 0.1675$ Btu/lb(a)

$$e_{f3} = 555\left\{[0.24 + 0.01447(0.44)]\left[\frac{560}{555} - 1 - \ln\frac{560}{555}\right] + \left(\frac{1.986}{28.97}\right)\left[1.02326\ln\left(\frac{1.02249}{1.02326}\right) + 0.02326\ln\left(\frac{0.02326}{0.02249}\right)\right]\right\}$$

$= 0.00552$ Btu/lb(a)

An exergy rate balances reduces to give, $\dot{E}_d = (200)(0.0723) + (199.5)(0.1675) - (399.5)(0.00552) = 45.67$ Btu/min, which agrees with the result of Problem 12.96.

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1. See the development of Eq. (6-5b), p.140 of M.J. Moran, Availability Analysis, ASME Press, New York, 1989.

PROBLEM 13.106

A full discussion of this class of problems including a solution to Problem 13.106 is provided in

M.J. Moran, Availability Analysis, ASME Press, New York, 1989

See pp. 139 – 143.

CHAPTER FOURTEEN

CHEMICAL AND PHASE EQULIBRIUM

Ideal Gas Properties of Hydrogen, H_2 : Special Note

In several problems of this chapter, it is necessary to evaluate the specific enthalpy of hydrogen, H_2, modeled as an ideal gas. This can be done using data from the two following tables, obtained from the same source as Tables A-23, or by means of Interactive Thermodynamics : IT.

SI . Ideal Gas Properties of Hydrogen, H_2

T(K), $\bar{h}$ and $\bar{u}$(kJ/kmol), $\bar{s}°$(kJ/kmol·K)

[$\bar{h}_f° = 0$ kJ/kmol]

T	$\bar{h}$	$\bar{u}$	$\bar{s}°$	T	$\bar{h}$	$\bar{u}$	$\bar{s}°$
0	0	0	0	1440	42,808	30,835	177.410
260	7,370	5,209	126.636	1480	44,091	31,786	178.291
270	7,657	5,412	127.719	1520	45,384	32,746	179.153
280	7,945	5,617	128.765	1560	46,683	33,713	179.995
290	8,233	5,822	129.775	1600	47,990	34,687	180.820
298	8,468	5,989	130.574	1640	49,303	35,668	181.632
300	8,522	6,027	130.754	1680	50,662	36,654	182.428
320	9,100	6,440	132.621	1720	51,947	37,646	183.208
340	9,680	6,853	134.378	1760	53,279	38,645	183.973
360	10,262	7,268	136.039	1800	54,618	39,652	184.724
380	10,843	7,684	137.612	1840	55,962	40,663	185.463
400	11,426	8,100	139.106	1880	57,311	41,680	186.190
420	12,010	8,518	140.529	1920	58,668	42,705	186.904
440	12,594	8,936	141.888	1960	60,031	43,735	187.607
460	13,179	9,355	143.187	2000	61,400	44,771	188.297
480	13,764	9,773	144.432	2050	63,119	46,074	189.148
500	14,350	10,193	145.628	2100	64,847	47,386	189.979
520	14,935	10,611	146.775	2150	66,584	48,708	190.796
560	16,107	11,451	148.945	2200	68,328	50,037	191.598
600	17,280	12,291	150.968	2250	70,080	51,373	192.385
640	18,453	13,133	152.863	2300	71,839	52,716	193.159
680	19,630	13,976	154.645	2350	73,608	54,069	193.921
720	20,807	14,821	156.328	2400	75,383	55,429	194.669
760	21,988	15,669	157.923	2450	77,168	56,798	195.403
800	23,171	16,520	159.440	2500	78,960	58,175	196.125
840	24,359	17,375	160.891	2550	80,755	59,554	196.837
880	25,551	18,235	162.277	2600	82,558	60,941	197.539
920	26,747	19,098	163.607	2650	84,386	62,335	198.229
960	27,948	19,966	164.884	2700	86,186	63,737	198.907
1000	29,154	20,839	166.114	2750	88,008	65,144	199.575
1040	30,364	21,717	167.300	2800	89,838	66,558	200.234
1080	31,580	22,601	168.449	2850	91,671	67,976	200.885
1120	32,802	23,490	169.560	2900	93,512	69,401	201.527
1160	34,028	24,384	170.636	2950	95,358	70,831	202.157
1200	35,262	25,284	171.682	3000	97,211	72,268	202.778
1240	36,502	26,192	172.698	3050	99,065	73,707	203.391
1280	37,749	27,106	173.687	3100	100,926	75,152	203.995
1320	39,002	28,027	174.652	3150	102,793	76,604	204.592
1360	40,263	28,955	175.593	3200	104,667	78,061	205.181
1400	41,530	29,889	176.510	3250	106,545	79,523	205.765

English. Ideal Gas Properties of Hydrogen, H_2

T(°R), $\bar{h}$ and $\bar{u}$(Btu/lbmol), $\bar{s}^\circ$(Btu/lbmol·°R)

[$\bar{h}_f^\circ$ = 0 Btu/lbmol]

T	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$	T	$\bar{h}$	$\bar{u}$	$\bar{s}^\circ$
300	2063.5	1467.7	27.337	1400	9673.8	6893.6	37.883
320	2189.4	1553.9	27.742	1500	10381.5	7402.7	38.372
340	2317.2	1642.0	28.130	1600	11092.5	7915.1	38.830
360	2446.8	1731.9	28.501	1700	11807.4	8431.4	39.264
380	2577.8	1823.2	28.856	1800	12526.8	8952.2	39.675
400	2710.2	1915.8	29.195	1900	13250.9	9477.8	40.067
420	2843.7	2009.6	29.520	2000	13980.1	10008.4	40.441
440	2978.1	2104.3	29.833	2100	14714.5	10544.2	40.799
460	3113.5	2200.0	30.133	2200	15454.4	11085.5	41.143
480	3249.4	2296.2	30.424	2300	16199.8	11632.3	41.475
500	3386.1	2393.2	30.703	2400	16950.6	12184.5	41.794
520	3523.2	2490.6	30.972	2500	17707.3	12742.6	42.104
537	3640.3	2573.9	31.194	2600	18469.7	13306.4	42.403
540	3660.9	2588.5	31.232	2700	19237.8	13876.0	42.692
560	3798.8	2686.7	31.482	2800	20011.8	14451.4	42.973
580	3937.1	2785.3	31.724	2900	20791.5	15032.5	43.247
600	4075.6	2884.1	31.959	3000	21576.9	15619.3	43.514
620	4214.3	2983.1	32.187	3100	22367.7	16211.5	43.773
640	4353.1	3082.1	32.407	3200	23164.1	16809.3	44.026
660	4492.1	3181.4	32.621	3300	23965.5	17412.1	44.273
680	4631.1	3280.7	32.829	3400	24771.9	18019.9	44.513
700	4770.2	3380.1	33.031	3500	25582.9	18632.4	44.748
720	4909.5	3479.6	33.226	3600	26398.5	19249.4	44.978
740	5048.8	3579.2	33.417	3700	27218.5	19870.8	45.203
760	5188.1	3678.8	33.603	3800	28042.8	20496.5	45.423
780	5327.6	3778.6	33.784	3900	28871.1	21126.2	45.638
800	5467.1	3878.4	33.961	4000	29703.5	21760.0	45.849
820	5606.7	3978.3	34.134	4100	30539.8	22397.7	46.056
840	5746.3	4078.2	34.302	4200	31379.8	23039.2	46.257
860	5885.9	4178.0	34.466	4300	32223.5	23684.3	46.456
880	6025.6	4278.0	34.627	4400	33070.9	24333.1	46.651
900	6165.3	4378.0	34.784	4500	33921.6	24985.2	46.842
920	6305.1	4478.1	34.938	4600	34775.7	25640.7	47.030
940	6444.9	4578.1	35.087	4700	35633.0	26299.4	47.215
960	6584.7	4678.3	35.235	4800	36493.4	26961.2	47.396
980	6724.6	4778.4	35.379	4900	35356.9	27626.1	47.574
1000	6864.5	4878.6	35.520	5000	38223.3	28294.0	47.749
1100	7564.6	5380.1	36.188	5100	39092.8	28964.9	47.921
1200	8265.8	5882.8	36.798	5200	39965.1	29638.6	48.090
1300	8968.7	6387.1	37.360	5300	40840.2	30315.1	48.257

PROBLEM 14.1

KNOWN: The reaction is

$$CH_4(g) + 2\,O_2 \rightleftarrows CO_2 + 2\,H_2O(g)$$

FIND: Determine the change in Gibbs function ΔG° at 25°C using (a) Gibbs function of formation data, (b) enthalpy of formation and absolute entropy data.

ANALYSIS: ΔG° is given by

$$\Delta G^\circ = \bar{g}^\circ_{CO_2} + 2\,\bar{g}^\circ_{H_2O(g)} - \bar{g}^\circ_{CH_4(g)} - 2\,\bar{g}^\circ_{O_2} \qquad (1)$$

(a) With data from Table A-25

$$\Delta G^\circ = [(-394{,}380) + 2(-228{,}590) - (-50{,}790) - 2(0)]\ \text{kJ/kmol}$$

$$= -800{,}770\ \text{kJ/kmol} \longleftarrow (a)$$

(b) Using $\bar{g} = \bar{h} - T\bar{s}$ Eq. (1) becomes

$$\Delta G^\circ = [\bar{h} - T\bar{s}]^\circ_{CO_2} + 2[\bar{h} - T\bar{s}]^\circ_{H_2O(g)} - [\bar{h} - T\bar{s}]^\circ_{CH_4(g)} - 2[\bar{h} - T\bar{s}^\circ]_{O_2}$$

$$= \left[\bar{h}^\circ_{CO_2} + 2\,\bar{h}^\circ_{H_2O(g)} - \bar{h}^\circ_{CH_4(g)} - 2\,\bar{h}^\circ_{O_2}\right] - (298.15\,\text{K})\left[\bar{s}^\circ_{CO_2} + 2\,\bar{s}^\circ_{H_2O(g)} - \bar{s}^\circ_{CH_4(g)} - 2\,\bar{s}^\circ_{O_2}\right]$$

With data from Table A-25

$$\Delta G^\circ = [(-393{,}520) + 2(-241{,}820) - (-74{,}850) - 2(0)]$$
$$-(298.15)(213.69 + 2(188.72) - 186.16 - 2(205.03))]$$

① $$= -800{,}792\ \text{kJ/kmol} \longleftarrow (b)$$

---

1. The slight difference in values calculated results from round-off in data values listed in Table A-25.

PROBLEM 14.2

KNOWN: The reaction is

$$CO_2 \rightleftarrows CO + \tfrac{1}{2}\,O_2$$

FIND: Determine $\log_{10} K$ at (a) 500 K (b) 1800°R

ASSUMPTION: Ideal gas principles apply.

ANALYSIS: From Eq. 14.31 $\ln K = -\Delta G^\circ/\bar{R}T$ where

$$\Delta G^\circ = \bar{g}^\circ_{CO} + \tfrac{1}{2}\,\bar{g}^\circ_{O_2} - \bar{g}^\circ_{CO_2}$$

With $\bar{g} = \bar{h} - T\bar{s}$ where $\bar{h} = \bar{h}^\circ_f + \Delta\bar{h}$

$$\Delta G^\circ = \{[\bar{h}^\circ_f + \Delta\bar{h}]_{CO} + \tfrac{1}{2}[\bar{h}^\circ_f + \Delta\bar{h}]_{O_2} - [\bar{h}^\circ_f + \Delta\bar{h}]_{CO_2}\} - T[\bar{s}^\circ_{CO} + \tfrac{1}{2}\,\bar{s}^\circ_{O_2} - \bar{s}^\circ_{CO_2}] \quad (1)$$

(a) T = 500K with data from ideal gas tables

$$\Delta G^\circ = \{[(-110{,}530) + 14{,}600 - 8669] + \tfrac{1}{2}[0 + 14{,}770 - 8682] - [(-393{,}520) + 17{,}678 - 9364]$$
$$-500[212.719 + \tfrac{1}{2}(220.589) - 254.814]\} = 239{,}551\ \text{kJ/kmol}$$

Then

$$\ln K = -239551/(8.314)(500) \Rightarrow \log_{10} K = -25.027$$

Table Value:
$\log_{10} K = -25.025$

(b) T = 1800°R = 1000K. With data from ideal gas tables

$$\Delta G^\circ = \{[(-110{,}530) + [30{,}355 - 8669]] + \tfrac{1}{2}[0 + [31{,}389 - 8682]] - [(-393{,}520) + [42{,}769 - 9364]]\}$$
$$-1000[234.421 + \tfrac{1}{2}(243.471) - 269.215] = 195{,}683\ \text{kJ/kmol}$$

Then

$$\ln K = \frac{-195{,}683}{(8.314)(1000)} \Rightarrow \log_{10} K = -10.222$$

Table Value:
$\log_{10} K = -10.221.$

PROBLEM 14.3

KNOWN: The reaction is

$$CO + H_2O(g) \rightleftarrows CO_2 + H_2 \qquad (1)$$

FIND: Determine $\log_{10} K$ at (a) 298K, (b) 1000K. Compare with Table A-27 data.

ASSUMPTION: Ideal gas principles apply.

ANALYSIS: From Eq. 14.31 $\ln K = -\Delta G^\circ / \bar{R}T$ where

$$\Delta G^\circ = \bar{g}^\circ_{CO_2} + \bar{g}^\circ_{H_2} - \bar{g}^\circ_{CO} - \bar{g}^\circ_{H_2O(g)}$$

(a) 298K. With data from Table A-25

$$\Delta G^\circ = (-394{,}380) + (0) - (-137{,}150) - (-228{,}590)$$

$$= -28{,}640 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{28{,}640}{(8.314)(298.15)} \Rightarrow \log_{10} K = 5.018 \quad \longleftarrow$$

Table A-27 gives $\log_{10} K$ for the reaction

$$CO_2 + H_2 \rightleftarrows CO + H_2O(g)$$

as $\log_{10} K = -5.018$, which in light of Eq. 14.34 and the accompanying discussion is in agreement with the calculated value.

(b) 1000K. With data from the ideal gas tables

①
$$\Delta G^\circ = \Big[[\bar{h}^\circ_f + \Delta\bar{h}]_{CO_2} + [\bar{h}^\circ_f + \Delta\bar{h}]_{H_2} - [\bar{h}^\circ_f + \Delta\bar{h}]_{CO} - [\bar{h}^\circ_f + \Delta\bar{h}]_{H_2O(g)}$$
$$- T[\bar{s}^\circ_{CO_2} + \bar{s}^\circ_{H_2} - \bar{s}^\circ_{CO} - \bar{s}^\circ_{H_2O(g)}]$$

$$= \Big[[-393{,}520 + 42{,}769 - 9364] + [0 + 29{,}154 - 8468] - [-110\,530 + 30{,}355 - 8669]$$
$$- [-241{,}820 + 35882 - 9904]\Big] - 1000[269.215 + 166.114 - 234.421 - 232.597]\Big]$$

$$= -3054 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{3054}{(8.314)(1000)} \Rightarrow \log_{10} K = 0.1595$$

Table A-27 at 1000K for $CO_2 + H_2 \rightleftarrows CO + H_2O$, $\log_{10} K = -0.159$.
Thus, for $CO + H_2O \rightleftarrows CO_2 + H_2$, $\log_{10} K = +0.159$.

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1. IT can be used alternatively.

PROBLEM 14.4

KNOWN: The reaction is

$$H_2O \rightleftarrows H_2 + \tfrac{1}{2} O_2$$

FIND: Calculate $\log_{10} K$ at (a) 298 K, (b) 3600°R and compare with values from Table A-27.

ASSUMPTION: Ideal gas principles apply.

ANALYSIS: From Eq. 14.31 $\ln K = -\Delta G^\circ / \bar{R} T$ where

$$\Delta G^\circ = \bar{g}^\circ_{H_2} + \tfrac{1}{2} \bar{g}^\circ_{O_2} - \bar{g}^\circ_{H_2O(g)} \qquad (1)$$

(a) At 298.15 K, Table A-25 data give

$$\Delta G^\circ = 0 + \tfrac{1}{2}(0) - (-228{,}590) = 228{,}590 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{-228{,}590}{(8.314)(298.15)} \Rightarrow \log_{10} K = -40.05 \qquad \longleftarrow \text{(a)}$$

Table A-27 gives $\log_{10} K = -40.048$.

(b) At 3600°R = 2000 K

$$\Delta G^\circ = \{(\bar{h}^\circ_f + \Delta\bar{h})_{H_2} + \tfrac{1}{2}(\bar{h}^\circ_f + \Delta\bar{h})_{O_2} - (\bar{h}^\circ_f + \Delta\bar{h})_{H_2O(g)}\} - T\left[\bar{s}^\circ_{H_2} + \tfrac{1}{2}\bar{s}^\circ_{O_2} - \bar{s}^\circ_{H_2O(g)}\right]$$

Then, with ideal gas table data (or using IT)

$$\Delta G^\circ = \{[0 + (61{,}400 - 8468)] + \tfrac{1}{2}[0 + (67{,}881 - 8682)] - [-241{,}820 + (82{,}593 - 9904)]\} - 2000\left[188.297 + \tfrac{1}{2}(268.655) - 264.571\right]$$

$$= 135{,}555.5 \text{ kJ/kmol}$$

Then

$$\ln K = \frac{-135{,}555.5}{(8.314)(2000)} \Rightarrow \log_{10} K = -3.5405 \qquad \longleftarrow \text{(b)}$$

which checks the value from Table A-27.

PROBLEM 14.5

KNOWN: Reactions are

(a) $H_2O \rightleftarrows H_2 + \tfrac{1}{2} O_2$

(b) $H_2 + \tfrac{1}{2} O_2 \rightleftarrows H_2O$

(c) $2H_2O \rightleftarrows 2H_2 + O_2$

FIND: Using Table A-27, determine $\log_{10} K$ at 2500 K

ASSUMPTION: Ideal gas principles apply.

ANALYSIS: For reaction (a) Table A-27 gives $\log_{10} K = -2.224$. $\longleftarrow$ (a)

Using Eq. 14.34, the accompany discussion indicates that for reaction (b)

$\log_{10} K_{(b)} = -\log_{10} K_{(a)} = +2.224$. $\longleftarrow$ (b)

For reaction (c), $(\Delta G^\circ)_{(c)} = 2(\Delta G^\circ)_{(a)}$. Then, with Eq. 14.31

$$\ln K_{(c)} = \frac{-\Delta G^\circ_{(c)}}{\bar{R} T} = -\frac{2(\Delta G^\circ)_{(a)}}{\bar{R} T}$$

or

$$\ln K_{(c)} = 2 \ln K_{(a)} \Rightarrow \log_{10} K_{(c)} = 2 \log_{10} K_{(a)} = -4.448 \longleftarrow \text{(c)}$$

PROBLEM 14.6

KNOWN: For interpolation in Table A-27, $\log_{10} K$ is nearly linear in $1/T$:

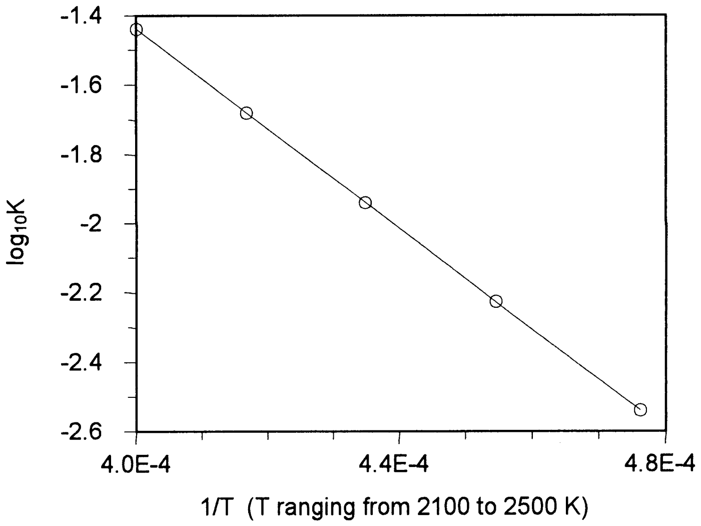
$$\log_{10} K = C_1 + C_2/T$$

where C_1 and C_2 are constants.

FIND: (a) Verify the interpolation rule by plotting $\log_{10} K$ vs $1/T$ for $2000 \le T \le 2500$ K.

(b) Evaluate C_1, C_2 for any pair of adjacent table entries in the temperature interval of (a)

ANALYSIS: (a) For $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$ as an example



(b) The linearity of $\log_{10} K$ vs. $1/T$ suggests a more accurate scheme for interpolating in Table A-27 than simple linear interpolation. First, let us determine C_1 and C_2 based on $T = 2400$ and $T = 2500$ K and the corresponding $\log_{10} K$ data for $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$ from Table A-27:

$$\left.\begin{aligned} T = 2400\text{K}: \quad -1.679 &= C_1 + C_2/2400 \\ T = 2500\text{K}: \quad -1.440 &= C_1 + C_2/2500 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} C_1 &= 4.296 \\ C_2 &= -14340 \end{aligned}$$

Thus, $\log_{10} K = 4.296 - 14340/T \quad (2400 \le T \le 2500\text{K}) \longleftarrow$ (b)

Let us use this result to estimate $\log_{10} K$ at $T = 2450$ K. That is

$$\log_{10} K = 4.296 - 14340/2450 = -1.5571$$

For comparison, the value obtain by direct linear interpolation is -1.5595, which differs from the curve fit value by about 0.15 %.

PROBLEM 14.7

KNOWN: Two alternative ways of expressing the ammonia synthesis reaction:

1. $\frac{1}{2}N_2 + \frac{3}{2}H_2 \rightleftarrows NH_3$
2. $N_2 + 3H_2 \rightleftarrows 2NH_3$

FIND: Determine the relationship between the ideal gas equilibrium constants K_1 and K_2.

ASSUMPTIONS: (1) Ideal gas principles apply. (2) The temperature is T.

ANALYSIS: For the two reactions the corresponding ΔG° expressions are

$$(\Delta G^\circ)_{(1)} = \bar{g}^\circ_{NH_3} - \frac{1}{2}\bar{g}^\circ_{N_2} - \frac{3}{2}\bar{g}^\circ_{H_2}$$

$$(\Delta G^\circ)_{(2)} = 2\bar{g}^\circ_{NH_3} - \bar{g}^\circ_{N_2} - 3\bar{g}^\circ_{H_2}$$

That is,

$$(\Delta G^\circ)_{(2)} = 2(\Delta G^\circ)_{(1)} \tag{1}$$

Using Eq. 14.31

$$\ln K_{(1)} = -\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} \tag{2}$$

$$\ln K_{(2)} = -\frac{(\Delta G^\circ)_{(2)}}{\bar{R}T} \tag{3}$$

Inserting Eq. (1) into Eq. (3) and using Eq. (2)

$$\ln K_{(2)} = -\frac{2(\Delta G^\circ)_{(1)}}{\bar{R}T}$$

$$= 2 \ln K_{(1)}$$

$$\Rightarrow \qquad K_{(2)} = K_{(1)}^2 \qquad \longleftarrow$$

PROBLEM 14.8

KNOWN: Reactions are specified as

1. $CO + H_2O \rightleftarrows H_2 + CO_2$
2. $2CO_2 \rightleftarrows 2CO + O_2$
3. $2H_2O \rightleftarrows 2H_2 + O_2$

FIND: Show that the equilibrium constants are related by the expression $K_1 = (K_3/K_2)^{1/2}$.

ASSUMPTIONS: (1) Ideal gas principles apply. (2) The temperature is T.

ANALYSIS: Using Eq. 14.31 the equilibrium constants corresponding to the three reactions have the forms

$$\ln K_1 = -\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} \quad , \quad \ln K_2 = -\frac{(\Delta G^\circ)_{(2)}}{\bar{R}T} \quad , \quad \ln K_3 = -\frac{(\Delta G^\circ)_{(3)}}{\bar{R}T}$$

where

$$\left\{\begin{aligned} (\Delta G^\circ)_{(1)} &= \bar{g}^\circ_{H_2} + \bar{g}^\circ_{CO_2} - \bar{g}^\circ_{CO} - \bar{g}^\circ_{H_2O} \\ (\Delta G^\circ)_{(2)} &= 2\bar{g}^\circ_{CO} + \bar{g}^\circ_{O_2} - 2\bar{g}^\circ_{CO_2} \\ (\Delta G^\circ)_{(3)} &= 2\bar{g}^\circ_{H_2} + \bar{g}^\circ_{O_2} - 2\bar{g}^\circ_{H_2O} \end{aligned}\right. \qquad (1)$$

From Eqs. (1) it follows that

$$(\Delta G^\circ)_{(1)} = \frac{1}{2}\left[(\Delta G^\circ)_{(3)} - (\Delta G^\circ)_{(2)}\right]$$

Accordingly

$$\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} = \frac{1}{2}\left[\frac{(\Delta G^\circ)_{(3)}}{\bar{R}T} - \frac{(\Delta G^\circ)_{(2)}}{\bar{R}T}\right]$$

or

$$\ln K_1 = \frac{1}{2}\left[\ln K_3 - \ln K_2\right]$$

Finally

$$K_1 = \left(\frac{K_3}{K_2}\right)^{1/2} \longleftarrow$$

PROBLEM 14.9

KNOWN: Reactions are specified as

1. $CO_2 + H_2 \rightleftarrows CO + H_2O$
2. $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$
3. $H_2O \rightleftarrows H_2 + \frac{1}{2} O_2$

FIND: (a) Show that $K_1 = K_2 / K_3$. (b) Use the result from part (a) and data from Table A-27 to evaluate $\log_{10} K_1$ at 298K. (c) Check the result of part (b) using Eq. 14.31 together with $\bar{g}_f^\circ$ data from Table A-25.

ASSUMPTIONS: (1) Ideal gas principles apply. (2) The temperature is T.

ANALYSIS: (a) For these reactions the respective ΔG° expressions are

$$(\Delta G^\circ)_{(1)} = \bar{g}^\circ_{CO} + \bar{g}^\circ_{H_2O(g)} \quad \bar{g}^\circ_{CO_2} - \bar{g}^\circ_{H_2}$$

$$(\Delta G^\circ)_{(2)} = \bar{g}^\circ_{CO} + \tfrac{1}{2}\bar{g}^\circ_{O_2} - \bar{g}^\circ_{CO_2}$$

$$(\Delta G^\circ)_{(3)} = \bar{g}^\circ_{H_2} + \tfrac{1}{2}\bar{g}_{O_2} - \bar{g}^\circ_{H_2O(g)}$$

Thus

$$(\Delta G^\circ)_{(1)} = (\Delta G^\circ)_{(2)} - (\Delta G^\circ)_{(3)}$$

$$\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} = \frac{(\Delta G^\circ)_{(2)}}{\bar{R}T} - \frac{(\Delta G^\circ)_{(3)}}{\bar{R}T} \Rightarrow -\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} = -\left[\frac{\Delta G^\circ_{(2)}}{\bar{R}T} - \frac{(\Delta G^\circ)_{(3)}}{\bar{R}T}\right]$$

With Eq. 14.31

$$\ln K_1 = \ln K_2 - \ln K_3 \Rightarrow K_1 = \frac{K_2}{K_3} \longleftarrow (a)$$

(b) From part (a)

$$\log_{10} K_1 = \log_{10} K_2 - \log_{10} K_3$$

With values from Table A-27 at 298 K for reactions 2 and 3

$$\log_{10} K_1 = (-45.066) - (-40.048) = -5.018 \longleftarrow (b)$$

(c) With $\bar{g}_f^\circ$ data from Table A-25.

$$(\Delta G^\circ)_{(1)} = (-137{,}150) + (-228{,}590) - (-394{,}380) - (0)$$
$$= 28{,}640 \text{ kJ/kmol}$$

Then

$$\ln K_1 = -\frac{(\Delta G^\circ)_{(1)}}{\bar{R}T} = \frac{-28{,}640}{(8.314)(298.15)} \Rightarrow \log_{10} K_1 = -5.018 \longleftarrow (c)$$

PROBLEM 14.10

KNOWN: At 2000 K

(1) $C + \frac{1}{2}O_2 \rightleftarrows CO$, $\log_{10} K_1 = 7.469$

(2) $C + 2H_2 \rightleftarrows CH_4$, $\log_{10} K_2 = -3.408$

FIND: Evaluate K at 2000 K for

(3) $CH_4 + H_2O \rightleftarrows 3H_2 + CO$ $\quad (\Delta G^\circ = 3\bar{g}^\circ_{H_2} + \bar{g}^\circ_{CO} - \bar{g}^\circ_{CH_4} - \bar{g}^\circ_{H_2O})$

ASSUMPTIONS: The principles of Sec. 14.3 apply.

ANALYSIS: For (1), (2)

(1)′ $(\Delta G^\circ)_{(1)} = \bar{g}^\circ_{CO} - \bar{g}^\circ_{C} - \frac{1}{2}\bar{g}^\circ_{O_2}$

(2)′ $(\Delta G^\circ)_{(2)} = \bar{g}^\circ_{CH_4} - \bar{g}^\circ_{C} - 2\bar{g}^\circ_{H_2}$

Also, for the following reaction Table A-27 gives

(4) $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$, $\log_{10} K_4 = -3.54$

and for (4)

(4)′ $(\Delta G^\circ)_{(4)} = \bar{g}^\circ_{H_2} + \frac{1}{2}\bar{g}^\circ_{O_2} - \bar{g}^\circ_{H_2O}$

Subtracting (2)′ from (1)′

$$(\Delta G^\circ)_{(1)} - (\Delta G^\circ)_{(2)} = \bar{g}^\circ_{CO} - \frac{1}{2}\bar{g}^\circ_{O_2} - \bar{g}^\circ_{CH_4} + 2\bar{g}^\circ_{H_2}$$

adding (4)′

$$(\Delta G^\circ)_{(1)} - (\Delta G^\circ)_{(2)} + (\Delta G^\circ)_{(4)} = \underbrace{\bar{g}^\circ_{CO} + 3\bar{g}^\circ_{H_2} - \bar{g}^\circ_{CH_4} - \bar{g}^\circ_{H_2O}}_{= \Delta G^\circ_{(3)}}$$

Accordingly,

$$\frac{\Delta G^\circ_{(3)}}{\bar{R}T} = \frac{\Delta G^\circ_{(1)}}{\bar{R}T} - \frac{\Delta G^\circ_{(2)}}{\bar{R}T} + \frac{\Delta G^\circ_{(4)}}{\bar{R}T}$$

$$\Rightarrow \quad \ln K_3 = \ln K_1 - \ln K_2 + \ln K_4$$

or

$$\log_{10} K_3 = \log_{10} K_1 - \log_{10} K_2 + \log_{10} K_4$$

$$= 7.469 - (-3.408) + (-3.54) = 7.337$$

$$\Rightarrow \quad K_3 = 2.173 \times 10^7 \longleftarrow$$

PROBLEM 14.11

KNOWN (a) One kmol of N_2O_4 dissociates to form an equilibrium mixture of N_2O_4 and NO_2 at 25°C, 2atm. For $N_2O_4 \rightleftarrows 2NO_2$, $\Delta G^\circ = 5400$ kJ/kmol at 25°C.

(b) One kmol of CH_4 dissociates to form an equilibrium mixture at 1000K, 5 atm. For $C + 2H_2 \rightleftarrows CH_4$, $\log_{10} K = 1.011$ at 1000 K.

FIND: In each case determine the equilibrium composition.

ASSUMPTION: Ideal gas mixture principles apply.

ANALYSIS: (a) Applying conservation of mass

$1\,N_2O_4 \rightarrow (1-x)\,N_2O_4 + y\,NO_2 \Rightarrow$ N: $2(1-x)+y=2$, or $y=2x$ where $(1-x)$ is the amount of N_2O_4 present in the mixture. The total number of moles of mixture is $n=(1-x)+2x=1+x$. At equilibrium $N_2O_4 \rightleftarrows 2NO_2$, so Eq. 14.35 takes the form

$$K = \frac{[2x]^2}{1-x}\left[\frac{P/P_{ref}}{1+x}\right]^{2-1}$$

$$= \frac{2[2x]^2}{(1-x)(1+x)} = \frac{8x^2}{1-x^2} \qquad (1)$$

Using Eq. 14.31 and the known ΔG° value

$$\ln K = \frac{-5400}{(8.314)(298.15)} \Rightarrow K = 0.11322 \qquad (2)$$

Combining Eqs. (1) and (2)

$$\frac{8x^2}{1-x^2} = 0.11322$$

$$\Rightarrow \quad 8.11322\,x^2 = 0.11322 \Rightarrow x = 0.118$$

Finally, the equilibrium mixture is $\{0.882\ N_2O_4,\ 0.236\ NO_2\}$ $\longleftarrow$ (a)

(b) Mass balance: $1\,CH_4 \rightarrow (1-x)\,CH_4 + 2x\,H_2 + x\,C$, where $n = 1+2x$. Then Eq 14.35 reads for $C+2H_2 \rightleftarrows CH_4$

$$K = \frac{(1-x)}{(x)(2x)^2}\left(\frac{P/P_{ref}}{n}\right)^{1-1-2} = \frac{(1-x)}{(x)(2x)^2}\left(\frac{5}{1+2x}\right)^{-2}$$

or

$$K = \frac{(1-x)}{x}\left(\frac{1+2x}{2x}\right)^2 \frac{1}{25} \qquad \text{where } \log_{10}K = 1.011$$

$$\Rightarrow \quad 256.413 = \left(\frac{1-x}{x}\right)\left(\frac{1+2x}{2x}\right)^2$$

Using an equation solver or iteration with a hand calculator, $x = 0.1088$, and the equilibrium mixture is

$\{0.8912\ CH_4,\ 0.2176\ H_2,\ 0.1088\ C\}$ $\longleftarrow$ (b)

PROBLEM 14.12

KNOWN (a) One lbmol of H_2O dissociates to form an equilibrium mixture of H_2O, H_2, O_2 at 5200°R, 1.25 atm.

(b) One lbmol of CO_2 dissociates to form an equilibrium mixture of CO_2, CO, O_2 at 5200°R, 1.25 atm.

FIND: For each case determine the extent to which dissociation occurs.

ASSUMPTION: Ideal gas mixture principles apply.

(a) $1\,H_2O \rightarrow (1-x)\,H_2O + y\,H_2 + z\,O_2$

H: $2 = 2(1-x) + 2y \Rightarrow y = x$

O: $1 = (1-x) + 2z \Rightarrow z = x/2$

Thus

$$1\,H_2O \rightarrow (1-x)\,H_2O + x\,H_2 + x/2\,O_2$$

where $1-x$ is the amount of H_2O, in kmol, present in the mixture. The amount of mixture is $n = (1-x) + x + x/2 = (2+x)/2$.

At equilibrium $H_2O \rightleftarrows H_2 + 1/2\,O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][x/2]^{\frac{1}{2}}}{[1-x]}\left[\frac{p/p_{ref}}{(2+x)/2}\right]^{1+\frac{1}{2}-1}$$

$$= \left(\frac{x}{1-x}\right)\left(\frac{x}{2+x}\right)^{1/2}\left(p/p_{ref}\right)^{1/2}$$

or

$$K^2 = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right)\left(p/p_{ref}\right) \qquad (1)$$

① At 5200°R, Table A-27 gives for $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$, $\log_{10}K = -1.51311$ or $K = 0.030682$. Thus, Eq.(1) becomes with $p/p_{ref} = 1.25$

$$\Rightarrow \quad 7.5313 \times 10^{-4} = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right)$$

. Using an equation solver or iteration with a hand calculator, $x = 0.108$. Thus, the extent of dissociation is 0.892. ← (a)

(b) $CO_2 \rightarrow (1-x)\,CO_2 + y\,CO + z\,O_2$

C: $1 = (1-x) + y \Rightarrow y = x$

O: $2 = 2(1-x) + y + 2z \Rightarrow z = x/2$

Thus

$$CO_2 \rightarrow (1-x)\,CO_2 + x\,CO + \frac{x}{2}\,O_2$$

where $1-x$ is the amount of CO_2, in kmol, present in the mixture. The number of moles of mixture is $n = (1-x) + x + \frac{x}{2} = (2+x)/2$

At equilibrium, $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][\frac{x}{2}]^{\frac{1}{2}}}{[1-x]}\left[\frac{p/p_{ref}}{(2+x)/2}\right]^{1+\frac{1}{2}-1}$$

$$= \left(\frac{x}{1-x}\right)\left(\frac{x}{2+x}\right)^{1/2}\left(p/p_{ref}\right)^{1/2} \Rightarrow K^2 = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right)\left(p/p_{ref}\right) \qquad (1)$$

At 5200°R, Table A-27 gives for $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$, $\log_{10}K = -0.668556$. Thus Eq.(1) gives

① $$3.6811 \times 10^{-2} = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right)$$

. Using an equation solver or iteration with a hand calculator, $x = 0.336$. The extent of dissociation is 0.664. ← (b)

---

1. $\log_{10}K$ is obtained from Table A-27 using linear interpolation. Alternatively, the interpolation procedure of Problem 14.6 can be used.

PROBLEM 14.13

KNOWN: One lbmol of C reacts with 2 lbmol of O_2 to form an equilibrium mixture of CO_2, CO, and O_2 at 5400 °R, 1atm.

FIND: Determine the equilibrium composition.

ASSUMPTION: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The reaction takes the form

$$C + 2O_2 \rightarrow x\,CO + y\,CO_2 + z\,O_2$$

Balancing

$$C:\quad 1 = x + y \Rightarrow y = 1 - x$$

$$O:\quad 4 = x + 2y + 2z \Rightarrow 4 = x + 2(1-x) + 2z \Rightarrow z = 1 + \frac{x}{2} = \frac{2+x}{2}$$

Thus

$$C + 2O_2 \rightarrow x\,CO + (1-x)CO_2 + \left(1 + \frac{x}{2}\right)O_2$$

The amount of mixture, in lbmol, is $n = x + (1-x) + \left(1 + \frac{x}{2}\right) = (4+x)/2$.
At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x]\left[\frac{2+x}{2}\right]^{1/2}}{[1-x]}\left[\frac{P/P_{ref}}{(4+x)/2}\right]^{1+1/2-1}$$

$$= \frac{x}{1-x}\left[\frac{2+x}{4+x}\right]^{1/2}$$

At 5400°R, Table A-27 gives $\log_{10} K = -0.485 \Rightarrow K = 0.32734$. Thus

$$0.32734 = \frac{x}{1-x}\left[\frac{2+x}{4+x}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $x = 0.309$.
The equilibrium mixture is

$$\{0.309\,CO,\ 0.691\,CO_2,\ 1.1545\,O_2\}$$

PROBLEM 14.14

KNOWN: Three cases are provided involving the formation of oxides of nitrogen.

FIND: Provide information about the equilibrium mixture in each case.

ASSUMPTION: Ideal gas mixture principles apply

ANALYSIS: (a) One kmol of N_2O_4 dissociates at 25°C, 1atm to form an equilibrium mixture of $\{N_2O_4, NO_2\}$ in which the amount of N_2O_4 present is 0.8154. Determine the amount of N_2O_4 that would be present in an equilibrium mixture at 25°C, 0.5 atm.

$$N_2O_4 \rightarrow (1-x)N_2O_4 + 2x\,NO_2$$

The amount of mixture is $n = (1-x) + 2x = 1+x$.

At equilibrium $N_2O_4 \rightleftarrows 2NO_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[2x]^2}{[1-x]}\left[\frac{P/P_{ref}}{1+x}\right]^{2-1}$$

$$K = \frac{4x^2}{1-x^2}\left(P/P_{ref}\right)$$

As K depends on T which is 25°C in both cases, $K_1 = K_2$ or

$$\frac{4x_1^2}{1-x_1^2}\left[\frac{P_1}{P_{ref}}\right] = \frac{4x_2^2}{1-x_2^2}\left[\frac{P_2}{P_{ref}}\right]$$

where 1 denotes the case $P_1 = 1atm$ and 2 denotes the case $P_2 = 0.5$. Thus

$$\frac{x_1^2}{1-x_1^2} = \frac{0.5x_2^2}{1-x_2^2}$$

From the given information, $1-x_1 = 0.8154 \Rightarrow x_1 = 0.1846$. Accordingly

$$\frac{x_2^2}{1-x_2^2} = 0.070559$$. Solving this quadratic equation, $x_2 = 0.2567$.

The amount of N_2O_4 present is then 0.7433 kmoles. $\longleftarrow$ (a)

(b) 1 kmol NO, 10 kmol O_2, 40 kmol N_2 react to form an equilibrium mixture of $\{NO_2, NO, O_2\}$ at 500K, 0.1atm. If for $NO + \frac{1}{2}O_2 \rightleftarrows NO_2$ K=120 at 500K, determine the equilibrium mixture composition. The reaction is described by

$$1\,NO + 10\,O_2 + 40N_2 \rightarrow (1-x)\,NO + x\,NO_2 + (10-0.5x)\,O_2 + 40\,N_2$$

where $n = 51-0.5x$. Equation 14.35 takes the form

$$K = \frac{x}{(1-x)[10-0.5x]^{1/2}}\left[\frac{P/P_{ref}}{51-0.5x}\right]^{1-1-1/2} = \left(\frac{x}{1-x}\right)\left[\frac{51-0.5x}{10-0.5x}\right]^{1/2}\left(\frac{1}{0.1}\right)^{1/2}$$

The value of K is given as 120. So

$$37.9473 = \left(\frac{x}{1-x}\right)\left(\frac{51-0.5x}{10-0.5x}\right)^{1/2}$$. Using an equation solver or iteration with a hand calculator, $x = 0.943$. The equilibrium mixture is $\{0.057\,NO, 0.943\,NO_2, 9.5285\,O_2, 40\,N_2\}$

In this analysis, N_2 has been regarded as inert. If N_2 also dissociated, the procedures of Section 14.4.4 would be applied.

PROBLEM 14.14 (Continued)

(c) An equimolar mixture of O_2 and N_2 reacts to form an equilibrium mixture of O_2, N_2, NO. Plot the mole fraction of NO versus T ranging from 1200 to 2000 K.

The reaction is described by

$$1\,N_2 + 1\,O_2 \rightarrow 2x\,NO + (1-x)N_2 + (1-x)\,O_2$$

The amount of mixture is $n = 2x + (1-x) + (1-x) = 2$.

Thus, the mole fraction of NO in the equilibrium mixture is

$$y_{NO} = \frac{2x}{2} = x$$

For $\frac{1}{2}N_2 + \frac{1}{2}O_2 \rightleftarrows NO$, Equation 14.35 takes the form

$$K = \frac{2x}{[(1-x)(1-x)]^{1/2}}\left(\frac{p/p_{ref}}{n}\right)^{1-\frac{1}{2}-\frac{1}{2}} = \frac{2x}{(1-x)}$$

Here the term $\left(\frac{p/p_{ref}}{n}\right)$ drops out of the evaluation since $1-\frac{1}{2}-\frac{1}{2}=0$.

Solving,

$$x = \frac{K(T)}{2+K(T)}$$

<u>Sample calculation</u>: $T = 1200\,K$. From Table A-27, $\log_{10}K = -3.275 \Rightarrow k = 0.0005309$ and $x = 2.654 \times 10^{-4}$.

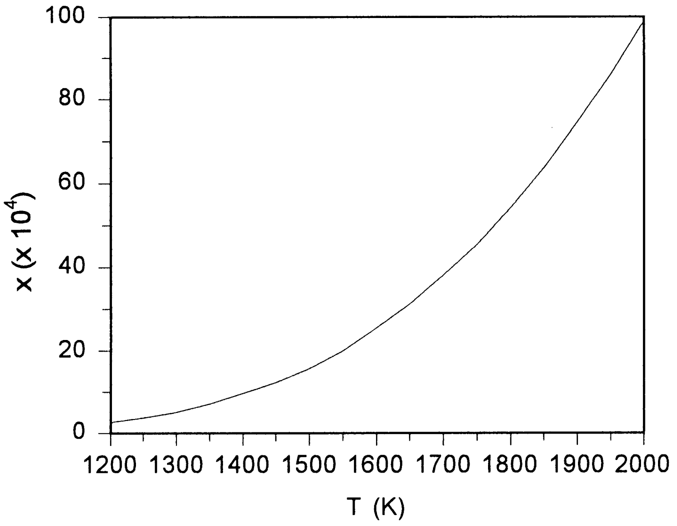
To obtain data for the required plot, use IT, as follows:

IT Code

```
T = 1200  // K
p = 1  // atm

// 1 N2 + 1 O2  ===>  2x NO + (1 - x) N2 + (1 - x) O2
nNO = 2 * x
nN2 = 1 - x
nO2 = 1 - x
ntot = 2 * x + 2 * (1 - x)
yNO = nNO / ntot
yN2 = nN2 / ntot
yO2 = nO2 / ntot
pref = 1  // atm

// For the reaction 1/2 N2 + 1/2 O2  <===> NO
K = (yNO / (yN2 * yO2)^0.5) * (p / pref)^0
// Data from Table A-27 are stored in EQNO.LUT.
log(K) = LOOKUPVAL(EQNO,1,T,3)
```

IT Results

T (K)	K	x
1200	.0005309	0.0002654
1300	.001015	0.0005073
1400	.001941	0.0009695
1500	.003155	0.001575
1600	.005129	0.002558
1700	.007656	0.003813
1800	.01091	0.005428
1900	.01503	0.007460
2000	.02000	0.009900



Note that at T=1200k, very little NO is formed, whereas at T=2000K, nearly all of the N_2 has combined with oxygen to form NO.

PROBLEM 14.15

KNOWN: One kmol of CO dissociates to form an equilibrium mixture of $\{CO_2, CO, O_2\}$ at T, p

FIND: (a) For $T = 3000\,K$, plot the amount of CO present, in kmol, versus pressure for $1 \le p \le 10$ atm.
(b) For $p = 1$ atm, plot the amount of CO present, in kmol, versus temperature for $2000 \le T \le 3500\,K$

ASSUMPTION: The ideal gas model applies.

ANALYSIS: The reaction takes the form

$$1\,CO_2 \rightarrow x\,CO + (1-x)\,CO_2 + \frac{x}{2}\,O_2$$

The amount of mixture is $n = x + (1-x) + \frac{x}{2} = 1 + \frac{x}{2}$

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$, so

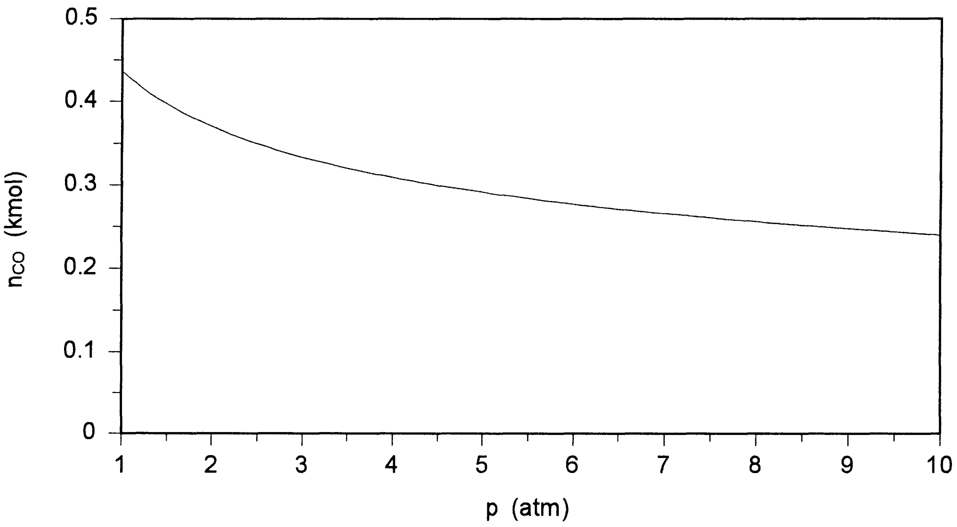
① $$K = \frac{x\,(x/2)^{1/2}}{1-x}\left[\frac{p/p_{ref}}{1+x/2}\right]^{1+\frac{1}{2}-1} = \frac{x}{1-x}\left[\frac{x}{2+x}\right]^{1/2}(p/p_{ref})^{1/2} \qquad (1)$$

(a) $T = 3000\,K$. Table A-27 gives $\log_{10}K = -0.485$ or $K = 0.32734$. Then Eq. (1) becomes

$$\frac{x}{1-x}\left[\frac{x}{2+x}\right]^{1/2} = \frac{0.32734}{[p/p_{ref}]^{1/2}}$$

Solving and plotting, we get

p(atm)	n_{CO}
1.000	0.436
2.000	0.370
3.000	0.333
4.000	0.309
5.000	0.291
6.000	0.277
7.000	0.265
8.000	0.256
9.000	0.247
10.000	0.240



Thus, at fixed temperature the amount of CO in the equilibrium mixture decreases as pressure increases.

PROBLEM 14.15 (Cont'd.)

(b) <u>p = 1 atm</u>. Eq. (1) becomes $\frac{x}{1-x}\left[\frac{x}{2+x}\right]^{1/2} = K(T)$.

<u>Sample calculation</u>. From Table A-27 at T= 000 K, $\log_{10} K = -0.485$. Thus K = 0.32734. Solving, x = 0.4362.

The data for the required plot are obtained using IT, as follows:

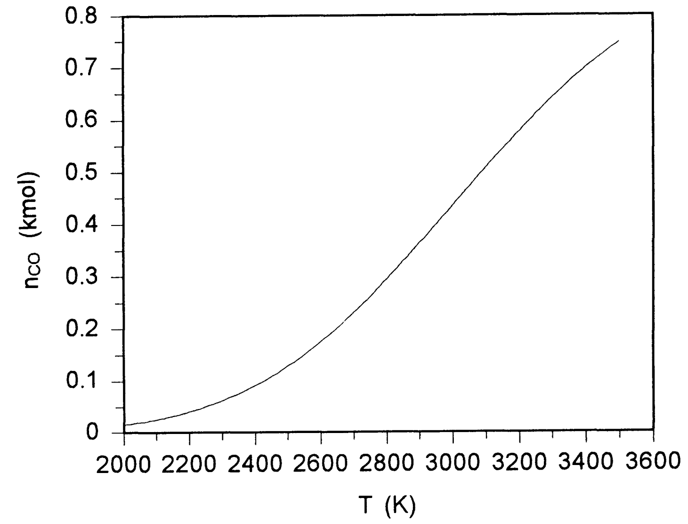
IT Code

```
T = 3000  // K
p = 1  // atm

// CO2 ===> x CO + (1 - x) CO2 + x/2 O2
nCO = x
nCO2 = 1 - x
nO2 = x / 2
ntot = x + (1 - x) + x / 2
yCO = nCO / ntot
yCO2 = nCO2 / ntot
yO2 = nO2 / ntot
pref = 1  // atm

K = ((yCO * yO2^0.5) / yCO2) * (p / pref)^0.5
// Data from Table A-27 are stored in
EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T,3)
```

IT Results

T(K)	n_{CO} (kmol)
2000	0.01494
2100	0.02524
2200	0.04048
2300	0.06207
2400	0.09116
2500	0.1287
2600	0.1754
2700	0.2309
2800	0.2946
2900	0.3638
3000	0.4362
3100	0.5084
3200	0.5775
3300	0.6418
3400	0.6983
3500	0.7480

PLOT:



When pressure is held constant, the amount of CO present at equilibrium increases greatly with T over the range of values in this problem.

---

1. Eq. (1) can be written as

$$\underbrace{\frac{K^2}{P/P_{ref}}}_{\equiv \alpha} = \left(\frac{x}{1-x}\right)^2\left(\frac{x}{2+x}\right) \Rightarrow x^3 + 3\underbrace{\left[\frac{\alpha}{1-\alpha}\right]}_{\equiv r} x - 2\left[\frac{\alpha}{1-\alpha}\right] = 0$$

The real root of this cubic equation is

$$x = \left[r + r[1+r]^{1/2}\right]^{1/3} + \left[r - r[1+r]^{1/2}\right]^{1/3} \quad \text{where } r \equiv \frac{\alpha}{1-\alpha}$$

PROBLEM 14.16

KNOWN: 1 lbmol of H_2O dissociates to form an equilibrium mixture of H_2O, H_2, O_2 at T, p.

FIND: (a) For T = 5400°R, plot the amount of H_2 present in the mixture versus pressure for $1 \le p \le 10$ atm.
(b) For p = 1 atm, plot the amount of H_2 present versus temperature, $3600°R \le T \le 6300°R$.

ASSUMPTION: Ideal gas principles apply.

ANALYSIS:

The reaction takes the form

$$1\,H_2O \rightarrow (1-x)\,H_2O + x\,H_2 + x/2\,O_2$$

The amount of mixture is $n = (1-x) + x + x/2 = (2+x)/2$.

At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$. Then Eq. 14.35 takes the form

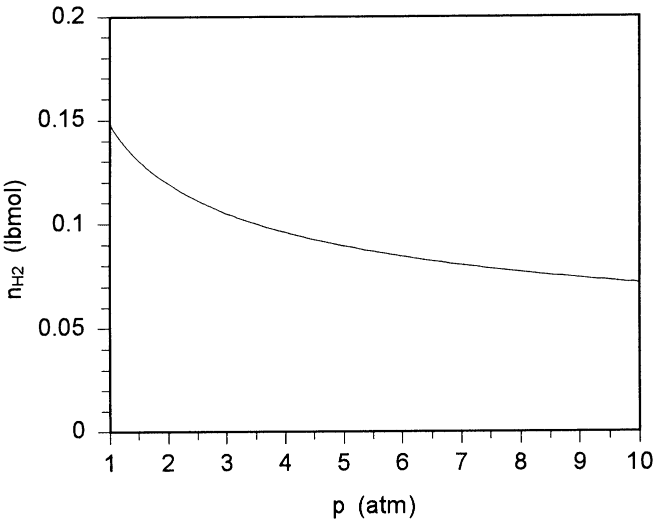
① $$K = \frac{x\,(x/2)^{1/2}}{(1-x)}\left[\frac{p/p_{ref}}{(2+x)/2}\right]^{1+1/2-1} = \left(\frac{x}{1-x}\right)\left[\frac{x}{2+x}\right]^{1/2}\left[p/p_{ref}\right]^{1/2} \qquad (1)$$

(a) T = 5400°R. Table A-27 gives $\log_{10}K = -1.343$, K = 0.04539. So, Eq (1) becomes

$$\left(\frac{x}{1-x}\right)\left(\frac{x}{2+x}\right)^{1/2} = \frac{0.04539}{[p/p_{ref}]^{1/2}}$$

Solving and plotting

p (atm)	n_{H2} (lbmol)
1	0.1476
2	0.1192
3	0.1050
4	0.09592
5	0.08938
6	0.08435
7	0.08031
8	0.07696
9	0.07411
10	0.07165



Thus, at fixed temperature the amount of H_2 in the equilibrium mixture decreases as pressure increases.

---

1. See note 1 of the solution to problem 14.15

PROBLEM 14.16 (Continued)

(b) p = 1 atm

Equation (1) takes the form

$$K(T) = \frac{x}{1-x}\left[\frac{x}{2+x}\right]^{1/2}$$

Sample calculation. From Table A-27 at T = 5400 °R, $\log_{10} K = -1.343 \Rightarrow K = 0.04539$.
Solving, x = 0.1476.

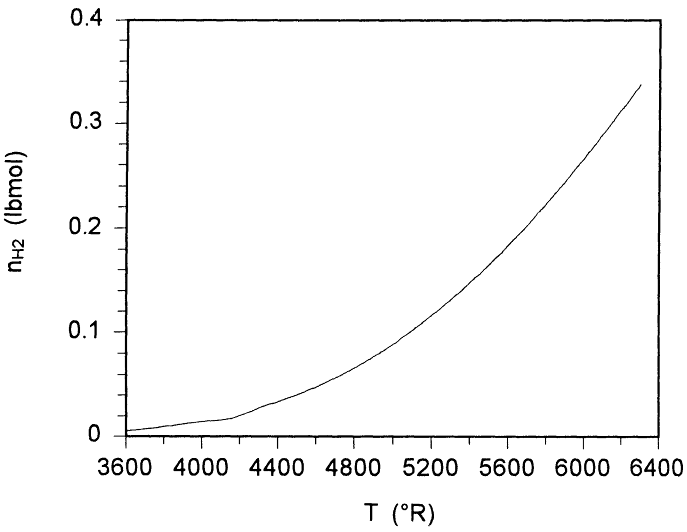
The data for the required plot are obtained using IT, as follows:

IT Code

```
T = 5400  // °R
p = 1  // atm

// H2O  ↔  (1 - x) H2O + x H2 + x/2 O2
nH2O = 1 - x
nH2 = x
nO2 = x / 2
ntot = (1 - x) + x + x/2
yH2O = nH2O / ntot
yH2 = nH2 / ntot
yO2 = nO2 / ntot
pref = 1  // atm

//  For the reaction H2O  ↔  H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p/pref)^0.5
//Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,2,T,3)
```

T (°R)	K	n_{H2} (lbmol)
3600	0.0002884	0.005485
3750	0.0005258	0.008174
3900	0.0009183	0.01183
4050	0.001332	0.01514
4200	0.002056	0.02016
4350	0.003922	0.03084
4500	0.005970	0.04061
4650	0.008814	0.05232
4800	0.01272	0.06629
4950	0.01797	0.08271
5100	0.02491	0.1017
5250	0.03391	0.1233
5400	0.04539	0.1476
5550	0.05961	0.1740
5700	0.07733	0.2030
5850	0.09897	0.2341
6000	0.1251	0.2671
6150	0.1566	0.3018
6300	0.1941	0.3380

PLOT:



Thus, at fixed pressure the amount of H_2 present in the equilibrium mixture increases as temperature increases.

PROBLEM 14.17

KNOWN: 1 lbmol of H_2O plus x lbmol of N_2 forms an equilibrium mixture at 5400°R and 1 atm consisting of H_2O, H_2, O_2 and N_2

FIND: Plot the amount of H_2 present in the mixture versus x, $0 \le x \le 2$.

ASSUMPTIONS: (1) Ideal gas principles apply. (2) N_2 is inert.

ANALYSIS: The reaction is

$$1\,H_2O + x\,N_2 \rightarrow (1-\alpha)H_2O + \alpha H_2 + \frac{\alpha}{2} O_2 + x\,N_2$$

The amount of mixture is $n = (1-\alpha) + \alpha + \alpha/2 + x = 1 + \frac{\alpha}{2} + x$

For $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$, Eq. 14.35 takes the form

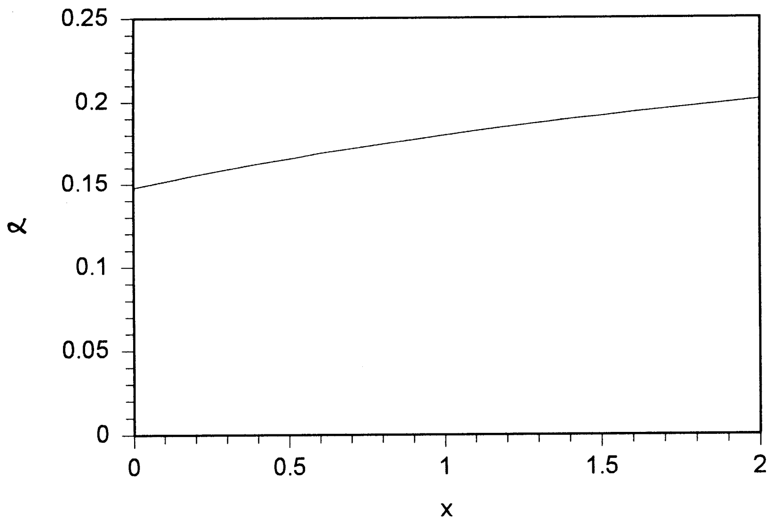
$$K = \frac{\alpha\,[\alpha/2]^{1/2}}{1-\alpha}\left[\frac{P/P_{ref}}{1+\frac{\alpha}{2}+x}\right]^{1+1/2-1} = \frac{\alpha}{1-\alpha}\left[\frac{\alpha/2}{1+\frac{\alpha}{2}+x}\right]^{1/2}\underbrace{(P/P_{ref})}_{=1}{}^{1/2}$$

From Table A-27 at 5400°R, $\log_{10}K = -1.343$, $K = 0.04539$. So

$$\frac{\alpha}{1-\alpha}\left[\frac{\alpha/2}{1+\frac{\alpha}{2}+x}\right]^{1/2} = 0.04539$$

Solving and plotting

x	α
0.000	0.148
0.100	0.152
0.200	0.155
0.300	0.159
0.400	0.162
0.500	0.166
0.600	0.169
0.700	0.172
0.800	0.174
0.900	0.177
1.000	0.180
1.100	0.182
1.200	0.185
1.300	0.187
1.400	0.189
1.500	0.191
1.600	0.193
1.700	0.195
1.800	0.197
1.900	0.199
2.000	0.201



Thus, at 5400°R, 1 atm the amount of H_2 in the equilibrium mixture increases as the amount of the inert N_2 increases.

PROBLEM 14.18

KNOWN: An equimolar mixture of CO and O_2 reacts to form an equilibrium mixture of CO_2, CO, and O_2 at 3000 K and pressure p.

FIND: Determine the effect of pressure on the composition of the mixture

ASSUMPTION: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The reaction takes the form

$$CO + O_2 \rightarrow x\,CO_2 + \alpha\,CO + \beta\,O_2$$

Balancing

C: $1 = x + \alpha \quad , \alpha = 1 - x$

O: $3 = 2x + (1-x) + 2\beta \quad , \beta = 1 - \frac{x}{2} = \frac{2-x}{2}$

Thus

$$CO + O_2 \rightarrow x\,CO_2 + (1-x)\,CO + \left(1 - \frac{x}{2}\right) O_2$$

The amount of mixture is $n = x + (1-x) + \left(1 - \frac{x}{2}\right) = 2 - \frac{x}{2} = \frac{4-x}{2}$

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$. Accordingly, Eq. 14.35 takes the form

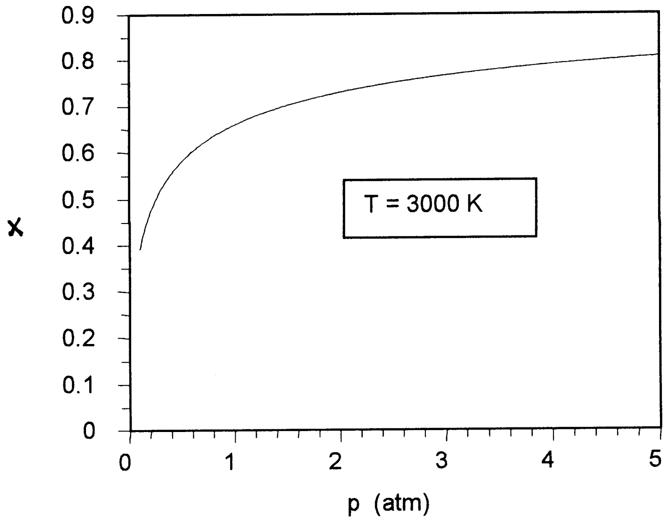
$$K = \frac{[1-x]\left[\frac{2-x}{2}\right]^{\frac{1}{2}}}{x} \left[\frac{p/p_{ref}}{\frac{4-x}{2}}\right]^{1+\frac{1}{2}-1}$$

$$K = \left[\frac{1-x}{x}\right]\left[\frac{2-x}{4-x}\right]^{1/2}\left[\frac{p}{p_{ref}}\right]^{1/2}$$

At 3000K, Table A-27 gives $\log_{10} K = -0.485 \Rightarrow K = 0.32724$. Thus

$$0.32724 = \left[\frac{1-x}{x}\right]\left[\frac{2-x}{4-x}\right]^{1/2}\left[\frac{p}{p_{ref}}\right]^{1/2} \quad (1)$$

The following plot is obtained using IT, with $p_{ref} = 1$ atm:



From the plot we conclude that the amount of CO_2 present at equilibrium decrease as pressure decreases at T=3000K.

PROBLEM 14.19

KNOWN: An equimolar mixture of CO and $H_2O(g)$ reacts to form an equilibrium mixture of CO_2, CO, H_2O, and H_2 at 2000 K, 1 atm.

FIND: Determine whether the amount of H_2 formed increases or decreases as temperature decreases. Also determine the affect of pressure on the amount of H_2 formed.

ASSUMPTION: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The reaction takes the form

$$CO + H_2O \rightarrow \alpha\, CO_2 + \beta\, H_2O + \gamma\, CO + x\, H_2$$

$$C: \quad 1 = \alpha + \gamma \quad \Rightarrow \gamma = 1 - \alpha$$

$$O: \quad 2 = 2\alpha + \beta + \gamma$$

$$H: \quad 2 = 2\beta + 2x \quad \Rightarrow \beta = 1 - x$$

Finally, $\alpha = x$, $\gamma = 1-x$, $\beta = 1-x$. Thus

$$CO + H_2O \rightarrow x\, CO_2 + (1-x)\, H_2O + (1-x)\, CO + x\, H_2$$

The amount of mixture is $n = x + (1-x) + (1-x) + x = 2$

At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[1-x][1-x]}{[x][x]} \left[\frac{P/P_{ref}}{n}\right]^{1+1-1-1}$$

or

$$K = \left(\frac{1-x}{x}\right)^2$$

Thus

$$\sqrt{K} = \frac{1-x}{x} \qquad (1)$$

Solving Eq. (1)

$$x = \frac{1}{1+\sqrt{K}} \qquad (2)$$

Observe that pressure drops out of the final expression. Accordingly, pressure has no effect on the amount of H_2 formed. ← (b)

Referring to Table A-27 at 2000 K, $\log_{10} K = 0.656 \Rightarrow K = 4.52898$, whereas at 1900 K $\log_{10} K = 0.619 \Rightarrow 4.15911$. It follows from Eq. (2), therefore, that x increases as T decreases. ← (a)

PROBLEM 14.20

KNOWN: Nine percent of H_2 dissociates into H at $p = 10$ atm, T.

FIND: Determine T.

ASSUMPTION: The equilibrium mixture is modeled as an ideal gas.

ANALYSIS: The reaction is

$$1\, H_2 \rightarrow \alpha\, H_2 + \beta\, H$$

where $\alpha = 0.91$. Then

$$H: \quad 2 = 2(0.91) + \beta \Rightarrow \beta = 0.18$$

So

$$1\, H_2 \rightarrow 0.91\, H_2 + 0.18\, H$$

The amount of mixture is $n = 0.91 + 0.18 = 1.09$

At equilibrium $H_2 \rightleftarrows 2H$, so Eq. 14.35 takes the form

$$K = \frac{(0.18)^2}{(0.91)}\left[\frac{10}{1.09}\right]^{2-1} = 0.3266$$

① Then $\log_{10} K = -0.486$. Interpolating in Table A-27, $T = 3488$ K. ←

Consider the case of 10% dissociation: $1 H_2 \rightarrow 0.9 H_2 + 0.2 H$. Then

$$K = \frac{(0.2)^2}{(0.9)}\left[\frac{10}{1.10}\right]^{2-1} = 0.404 \Rightarrow \log_{10} K = -0.394 \Rightarrow T > 3488K$$

By inspection of Table A-27, T increases as the percentage of H_2 dissociating increases (i.e., as the amount of H_2 in the equilibrium mixture decreases).

---

1. Linear interpolation is used here. Alternatively, the approach of Problem 14.6 can be invoked.

PROBLEM 14.21

<u>KNOWN:</u> 2 kmol of CO_2 dissociate to form an equilibrium mixture of CO_2, CO, and O_2 in which 1.8 kmol is present at temperature T and pressure p.

<u>FIND:</u> Plot T versus p for $0.5 \le p \le 10$ atm.

<u>ASSUMPTION:</u> The equilibrium mixture is modeled as an ideal gas mixture.

<u>ANALYSIS:</u> The reaction takes the form

$$2\ CO_2 \rightarrow 1.8\,CO_2 + \alpha\ CO + \beta\ O_2$$

$$C:\quad 2 = 1.8 + \alpha \Rightarrow \alpha = 0.2$$

$$O:\quad 4 = (1.8)(2) + 0.2 + 2\beta \Rightarrow \beta = 0.1$$

Thus

$$2\,CO_2 \rightarrow 1.8\,CO_2 + 0.2\,CO + 0.1\,O_2$$

The amount of mixture is $n = 2.1$.

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[0.2][0.1]^{1/2}}{1.8}\left[\frac{p/p_{ref}}{2.1}\right]^{1 + 1/2 - 1} \qquad (1)$$

<u>Sample calculation</u>. T = 2400 K. From Table A-27, $\log_{10}K = -1.679 \Rightarrow K = 0.02094$.

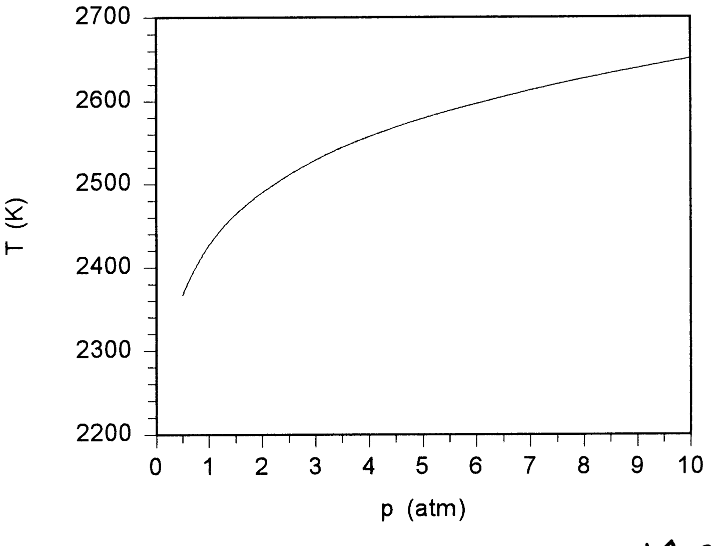
Soving for p, with $p_{ref} = 1$

$$p = \left(\frac{2.1}{0.1}\right)\left[\frac{(0.02094)(1.8)}{0.2}\right]^2 = 0.7459 \text{ atm}$$

The data for the required plot are obtained using IT, as follows:

<u>*IT* Code</u>

```
p = 0.5  // atm

// 2 CO2 → 1.8 CO2 + .2 CO + 0.1 O2
nCO2 = 1.8
nCO = 2 - nCO2
nO2 = 2 - nCO2 - nCO / 2
ntot = nCO2 + .2 + 0.1
yCO2 = nCO2 / ntot
yCO = nCO / ntot
yO2 = nO2 / ntot
pref = 1  // atm

// For the reaction CO2 ↔ CO + 1/2 O2
K = ((yCO * yO2^0.5 ) /  yCO2) * (p / pref)^0.5
//Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T,3)
```



<u>*IT* Results</u>

K	T (K)	p (atm)
0.01714	2367	0.5
0.02425	2427	1.0
0.02970	2463	1.5
0.03429	2490	2.0
0.03834	2511	2.5
0.04200	2529	3.0
0.04536	2544	3.5
0.04849	2557	4.0
0.05143	2568	4.5
0.05422	2579	5.0
0.05686	2588	5.5
0.05939	2597	6.0
0.06182	2605	6.5
0.06415	2613	7.0
0.06640	2620	7.5
0.06858	2627	8.0
0.07069	2634	8.5
0.07274	2640	9.0
0.07473	2645	9.5
0.07667	2651	10.0

For fixed composition, temperature increases as pressure increases.

PROBLEM 14.22

KNOWN: One kmol of $H_2O(g)$ dissociates to form an equilibrium mixture of $H_2O(g)$, H_2, and O_2 in which the amount of water vapor present is 0.95 kmol at temperature T and pressure p.

FIND: Plot T versus p for $1 \le p \le 10$ atm.

ASSUMPTION: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The reaction takes the form

$$1\,H_2O \rightarrow 0.95\,H_2O + \alpha\,H_2 + \beta\,O_2$$

$$H:\ 2 = 1.9 + 2\alpha\ ,\ \alpha = 0.05$$
$$O:\ 1 = 0.95 + 2\beta\ ,\ \beta = 0.025$$

Thus

$$1\,H_2O \rightarrow 0.95\,H_2O + 0.05\,H_2 + 0.025\,O_2$$

The amount of mixture is 1.025 kmol.

At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[0.05][0.025]^{1/2}}{0.95}\left[\frac{p/p_{ref}}{1.025}\right]^{1+1/2-1}$$

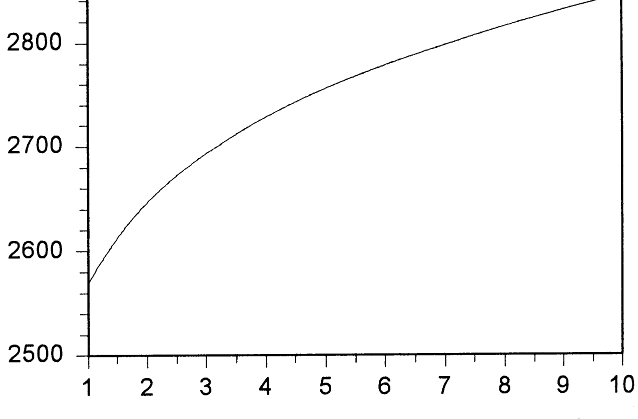
Sample calculation. T = 2600 K. From Table A-27, $\log_{10}K = -2.038 \Rightarrow K = 0.0091622$. Solving for p, with $p_{ref} = 1$ atm

$$p = \left(\frac{1.025}{0.025}\right)\left[\frac{(0.0091622)(0.95)}{0.05}\right]^2 = 1.242 \text{ atm}$$

The data for the required plot are obtained using IT, as follows:

IT Code

```
p = 1  // atm

// H2O → 0.95 H2O + (1 - 0.95) H2 + ((1 - 0.95) / 2) O2
nH2O = 0.95
nH2 = 1 - nH2O
nO2 = (1 - nH2O) / 2
ntot = 0.95 + (1 - 0.95) + (1 - 0.95)/2
yH2O = nH2O / ntot
yH2 = nH2 / ntot
yO2 = nO2 / ntot
pref = 1  // atm

// For the reaction H2O ↔ H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
//Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,1,T,3)
```



IT Results

T (K)	K	p (atm)
2568	0.00822	1.0
2613	0.01007	1.5
2646	0.01162	2.0
2672	0.01300	2.5
2693	0.01424	3.0
2711	0.01538	3.5
2728	0.01644	4.0
2743	0.01744	4.5
2756	0.01838	5.0
2767	0.01928	5.5
2778	0.02013	6.0
2788	0.02096	6.5
2797	0.02175	7.0
2806	0.02251	7.5
2815	0.02325	8.0
2823	0.02396	8.5
2831	0.02466	9.0
2838	0.02533	9.5
2845	0.02599	10.0

For fixed composition, temperature increases as pressure increases.

PROBLEM 14.23

KNOWN: A vessel initially containing 1 kmol of $H_2O(g)$ and x kmol of N_2 forms an equilibrium mixture at 1 atm and temperature T consisting of $H_2O(g)$, H_2, O_2, N_2 in which 0.5 kmol of $H_2O(g)$ is present.

FIND: Plot x versus T for $3000 \le T \le 3500$ K.

ASSUMPTIONS: (1) The equilibrium mixture is modeled as an ideal gas mixture. (2) N_2 is inert.

ANALYSIS: The reaction takes the form

$$1\ H_2O(g) + x\ N_2 \rightarrow 0.5\ H_2O(g) + \alpha H_2 + \beta O_2 + x N_2$$

$$\text{H: } 2 = (0.5)(2) + 2\alpha \Rightarrow \alpha = 0.5$$

$$\text{O: } 1 = 0.5 + 2\beta \Rightarrow \beta = 0.25$$

Thus

$$1\ H_2O(g) + x\ N_2 \rightarrow 0.5 H_2O(g) + 0.5 H_2 + 0.25 O_2 + x N_2$$

The amount of mixture is $n = 0.5 + 0.5 + 0.25 + x = 1.25 + x$.

At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2} O_2$. Accordingly Eq 14.35 takes the form

$$K = \frac{[0.5][0.25]^{1/2}}{[0.5]} \left[\frac{p/p_{ref}}{1.25+x}\right]^{1/2}$$

Since $p/p_{ref} = 1$, this reduces to

$$K = \frac{0.5}{(1.25+x)^{1/2}} \Rightarrow x = \left[\frac{0.25}{\{K(T)\}^2} - 1.25\right]$$

Sample calculation. From Table A-27 at T = 3000 K, $\log_{10} K = -1.343 \Rightarrow K = 0.045394$. Thus $X = 120.07$.

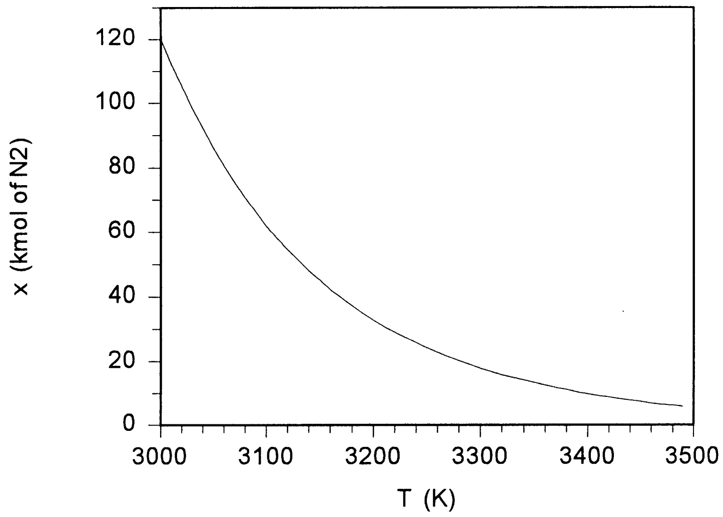
The data for the required plot are obtained using IT, as follows:

IT Code

```
p = 1  // atm
T = 3000  // K

// H2O + x N2 → 0.5 H2O + alpha H2 + beta O2 + x N2
nH2O = 0.5
alpha = 1 - 0.5
nH2 = alpha
beta = (1 - 0.5) / 2
nO2 = beta
ntot = 0.5 + alpha + beta + x
yH2O = nH2O / ntot
yH2 = nH2 / ntot
yO2 = nO2/ntot
pref = 1  // atm

// For the reaction H2O ↔ H2 + 1/2 O2
K  =  ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
// Data from Table A-27are stored in EQH2O.LUT.
log(K)  =  LOOKUPVAL(EQH2O,1,T,3)
```

IT Results

T (K)	K	x (kmol of N_2)
3000	0.04539	120.1
3050	0.05346	86.24
3100	0.06295	61.84
3150	0.07345	45.09
3200	0.08570	32.79
3250	0.09897	24.27
3300	0.1143	17.89
3350	0.1309	13.34
3400	0.1500	9.866
3450	0.1706	7.339



For fixed composition in terms of H_2O, H_2, and O_2, the amount of inert N_2 required decreases as the temperature increases at fixed pressure.

PROBLEM 14.24

KNOWN: A vessel initially contains 2 lbmol of N_2 and 1 lbmol of O_2. An equilibrium mixture of N_2, O_2 and NO forms at 1 atm, T.

FIND: Plot the amount of NO formed versus T.

ASSUMPTION: The ideal gas model applies.

ANALYSIS: The reaction takes the form

$$2N_2 + 1O_2 \rightarrow x\,N_2 + y\,O_2 + z\,NO$$

$$N:\quad 4 = 2x + z \;\Rightarrow\; z = 4 - 2x$$

$$O:\quad 2 = 2y + z \;\Rightarrow\; 2 = 2y + (4-2x) \;\Rightarrow\; y = x - 1$$

At equilibrium $\frac{1}{2}O_2 + \frac{1}{2}N_2 \rightleftarrows NO$. Accordingly

$$K = \frac{[4-2x]}{[x-1]^{1/2}[x]^{1/2}}\left[\frac{p/p_{ref}}{n}\right]^{\overbrace{1-\frac{1}{2}-\frac{1}{2}}^{=0}} \Rightarrow K(T) = \frac{[4-2x]}{[x(x-1)]^{1/2}} \qquad (1)$$

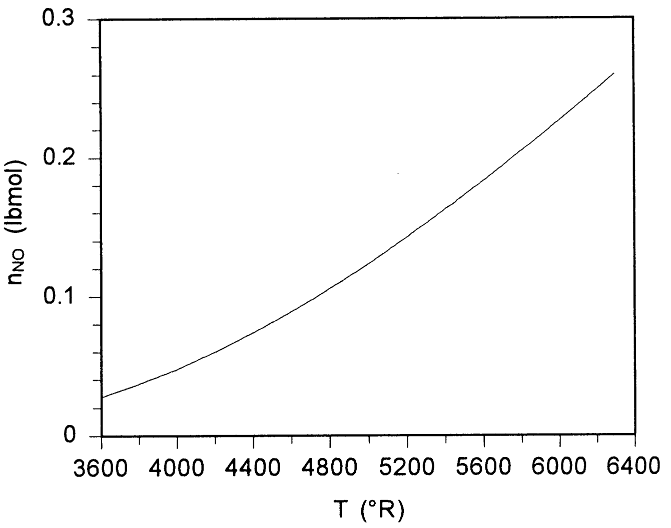
This expression is a quadratic equation that can be solved analytically for x as a function of K(T). Then, x can be calculated for given values of T using data for K(T) from Table A-27. Alternatively, data for the required plot are obtained using IT, as follows:

It Code

```
p = 1  // atm
T = 5400  // °R

// 2 N2 + 1 O2  ===>  x N2 + (x - 1) O2 + 2 * (2 - x) NO
nN2 = x
nO2 = x - 1
nNO = 2 * (2 - x)
ntot = x + (x - 1) + 2 * (2 - x)
yN2 = nN2 / ntot
yO2 = nO2 / ntot
yNO = nNO / ntot
pref = 1  // atm

//  For the reaction  1/2 N2 + 1/2 O2  <===>  NO
K  =  (yNO / (yN2^0.5 * yO2^0.5)) * (p / pref)^0
// Data from Table A-27 are stored in EQNO.LUT.
log(K) = LOOKUPVAL(EQNO,2,T,3)
```

IT results

T (°R)	K	n_{NO} (lbmol
3600	0.02000	0.02799
3800	0.02663	0.03713
4000	0.03441	0.04779
4200	0.04342	0.06002
4400	0.05366	0.07379
4600	0.06510	0.08898
4800	0.07774	0.1056
5000	0.09146	0.1233
5200	0.1063	0.1423
5400	0.1222	0.1622
5600	0.1388	0.1828
5800	0.1566	0.2044
6000	0.1750	0.2264
6200	0.1942	0.2489



As temperature increases at fixed pressure, the amount of NO in the equilibrium mixture increases.

PROBLEM 14.25

KNOWN: A vessel initially containing 1 kmol of CO and 4.76 kmols of dry air forms an equilibrium mixture of CO_2, CO, O_2, and N_2 at 3000K, 1atm.

FIND: Determine the equilibrium composition.

ASSUMPTIONS: (1) The equilibrium mixture is modeled as an ideal gas mixture. (2) The N_2 is inert.

ANALYSIS: The reaction takes the form

$$1\,CO + 1O_2 + 3.76\,N_2 \rightarrow \alpha\,CO_2 + \beta CO + \gamma O_2 + 3.76 N_2$$

C: $1 = \alpha + \beta \Rightarrow \beta = 1 - \alpha$

O: $3 = 2\alpha + \cancel{\beta} + 2\gamma \Rightarrow 3 = 2\alpha + (1-\alpha) + 2\gamma \Rightarrow \gamma = 1 - \frac{\alpha}{2}$

Thus

$$1\,CO + 1\,O_2 + 3.76\,N_2 \rightarrow \alpha\,CO_2 + (1-\alpha)\,CO + \left(1-\frac{\alpha}{2}\right)O_2 + 3.76\,N_2$$

The amount of mixture is $n = \alpha + (1-\alpha) + \left(1 - \frac{\alpha}{2}\right) + 3.76 = 5.76 - \frac{\alpha}{2}$.

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly Eq 14.35 takes the form

$$K = \frac{[1-\alpha]\left[\frac{2-\alpha}{2}\right]^{1/2}}{\alpha}\left[\frac{P/P_{ref}}{(11.52-\alpha)/2}\right]^{1/2} \Rightarrow K = \frac{1-\alpha}{\alpha}\left[\frac{2-\alpha}{11.52-\alpha}\right]^{1/2}$$

From Table A-27 at 3000K, $\log_{10} K = -0.485 \Rightarrow K = 0.32784$. Thus

$$0.32734 = \frac{1-\alpha}{\alpha}\left[\frac{2-\alpha}{11.52-\alpha}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $\alpha = 0.528$.
The equilibrium composition is

$$\{0.528\,CO_2,\ 0.472\,CO,\ 0.736\,O_2,\ 3.76\,N_2\}$$

PROBLEM 14.26

KNOWN: A vessel initially containing 1 kmol O_2, 2 kmol N_2, 1 kmol Ar forms an equilibrium mixture of O_2, N_2, NO, and Ar at 3000K, 1atm.

FIND: Determine the equilibrium composition.

ASSUMPTIONS: (1) The equilibrium mixture is modeled as an ideal gas. (2) Argon is inert.

ANALYSIS: The reaction takes the form

$$1\,O_2 + 2\,N_2 + 1\,Ar \rightarrow \alpha\,O_2 + \beta N_2 + x\,NO + 1\,Ar$$

O: $2 = 2\alpha + x$, $\alpha = 1 - x/2$

N: $4 = 2\beta + x$, $\beta = 2 - x/2$

Thus

$$1O_2 + 2N_2 + 1\,Ar \rightarrow \left(1-\frac{x}{2}\right)O_2 + \left(2-\frac{x}{2}\right)N_2 + x\,NO + 1\,Ar$$

The amount of mixture is $n = \left(1-\frac{x}{2}\right) + \left(2-\frac{x}{2}\right) + x + 1 = 4$.

At equilibrium $\frac{1}{2}O_2 + \frac{1}{2}N_2 \rightleftarrows NO$. Accordingly, Eq. 14.35 takes the form

① $$K = \frac{x}{\left[1-\frac{x}{2}\right]^{1/2}\left[2-\frac{x}{2}\right]^{1/2}}\left[\frac{P/P_{ref}}{4}\right]^{1-1/2-1/2} \Rightarrow K = \frac{x}{\left[1-\frac{x}{2}\right]^{1/2}\left[2-\frac{x}{2}\right]^{1/2}}$$

At 3000 K, Table A-27 gives $\log_{10} K = -0.913 \Rightarrow K = 0.12218$. Thus

$$0.12218 = \frac{x}{\left[\left(1-\frac{x}{2}\right)\left(2-\frac{x}{2}\right)\right]^{1/2}}$$

Using an equation solver or iteration with a hand calculator, $x = 0.162$.
The equilibrium composition is

$$\{0.919\,O_2,\ 1.919\,N_2,\ 0.162\,NO,\ 1\,Ar\}$$

---

1. Note that (P/P_{ref}) drops out of this expression.

PROBLEM 14.27

KNOWN: One kmol CO and 0.5 kmol O_2 react to form $\{CO_2, O_2, CO\}$ at T, p. When p = 1 atm, 0.35 kmol of CO is present in the equilibrium mixture.

FIND: Determine the amount of CO present in an equilibrium mixture at T, p = 10 atm.

ASSUMPTION: The equilibrium mixture is modeled as an ideal gas.

ANALYSIS: The reaction takes the form

$$CO + 0.5\ O_2 \rightarrow \alpha\ CO_2 + \beta\ CO + \gamma\ O_2$$

$$C: \quad 1 = \alpha + \beta \Rightarrow \alpha = 1 - \beta$$

$$O: \quad 2 = 2(1-\beta) + \beta + 2\gamma$$

$$\Rightarrow \gamma = \beta/2$$

For this case, $n = (1-\beta) + \beta + \beta/2 = 1 + (\beta/2)$. For $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$ Eq. 14.35 reads

$$K = \frac{\beta\,(\beta/2)^{1/2}}{(1-\beta)}\left[\frac{p/p_{ref}}{1+(\beta/2)}\right]^{1/2} \qquad (1)$$

Thus, when $\beta = 0.35$, and $p = p_{ref}$, we get

$$K = \frac{(0.35)(0.175)^{1/2}}{(0.65)}\left[\frac{1}{1.175}\right]^{1/2} = 0.2078$$

Since T remains unchanged, K remains unchanged. Then, with p = 10 atm, Eq. (1) reads

$$0.2078 = \frac{\beta\,(\beta/2)^{1/2}}{1-\beta}\left[\frac{10}{1+(\beta/2)}\right]^{1/2}$$

$$= \frac{\beta}{1-\beta}\left[\frac{10\beta}{2+\beta}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator,

① $\beta = 0.1844$ ←

---

1. Keeping temperature constant and increasing pressure results in a greater amount of CO_2 formed.

PROBLEM 14.28

KNOWN: A vessel initially contains 1 kmol H_2 and 4 kmol N_2. An equilibrium mixture of H_2, H, and N_2 forms at 3000K, 1atm.

FIND: Determine the equilibrium composition. If pressure is increased while keeping temperature fixed, determine if the amount of H would increase or decrease.

ASSUMPTIONS: (1) The equilibrium mixture is modeled as an ideal gas. (2) N_2 is inert.

ANALYSIS: The reaction takes the form

$$1\,H_2 + 4\,N_2 \rightarrow x\,H + \alpha\,H_2 + 4\,N_2$$

H: $2 = x + 2\alpha \Rightarrow \alpha = 1 - \frac{x}{2}$

Thus

$$1\,H_2 + 4\,N_2 \rightarrow x\,H + \left(1 - \frac{x}{2}\right) H_2 + 4\,N_2$$

The amount of mixture is $n = x + \left(1 - \frac{x}{2}\right) + 4 = 5 + \frac{x}{2}$.

At equilibrium $H_2 \rightleftarrows 2H$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x]^2}{\left[1 - \frac{x}{2}\right]}\left[\frac{p/p_{ref}}{\frac{10+x}{2}}\right]^{2-1} \Rightarrow K = \frac{4x^2\,(p/p_{ref})}{(2-x)(10+x)} \qquad (1)$$

Eq (1) gives upon rearrangement

$$(\gamma+1)x^2 + 8x - 20 = 0$$

where $\gamma = \left[4\,(p/p_{ref})/K\right]$. Solving this quadratic equation

$$x = \frac{-8 + \sqrt{64 + 80(\gamma+1)}}{2(\gamma+1)} \qquad (2)$$

From Table A-27 at 3000 K $\log_{10} K = -1.606 \Rightarrow K = 0.024774$. With $p/p_{ref} = 1$, $\gamma = 161.46$. Thus, Eq. (2) gives $x = 0.3271$. The equilibrium mixture is then

$$\{0.3271\,H,\ 0.8365\,H_2,\ 4\,N_2\}$$ ←

To determine the effect on x owing to an increase in pressure at fixed temperature, consider $p/p_{ref} = 1.1$. Then $\gamma = 177.61$ and Eq. (2) gives $x = 0.313$. Accordingly x tends to decrease as p increases. ←

PROBLEM 14.29

KNOWN: An equilibrium mixture of N_2, O_2, and NO at 4342 °R, 1atm forms from dry air.

FIND: Determine the mole fraction of NO in the equilibrium mixture. Determine if the amount of NO increases or decreases as T decreases at fixed p.

ASSUMPTION: (1) The equilibrium mixture is modeled as an ideal gas. (2) The analysis is based on 1 lbmol of dry air initially.

ANALYSIS: The reaction takes the form

$$0.79\,N_2 + 0.21\,O_2 \rightarrow x\,NO + \alpha\,N_2 + \beta\,O_2$$

N: $(0.79)(2) = x + 2\alpha \Rightarrow \alpha = (1.58 - x)/2$

O: $(0.21)(2) = x + 2\beta \Rightarrow \beta = (0.42 - x)/2$

Thus

$$0.79\,N_2 + 0.21\,O_2 \rightarrow x\,NO + \left[\frac{1.58-x}{2}\right]N_2 + \left[\frac{0.42-x}{2}\right]O_2$$

The amount of mixture is $n = x + \left[\frac{1.58-x}{2}\right] + \left[\frac{0.42-x}{2}\right] = 1$

At equilibrium $\frac{1}{2}O_2 + \frac{1}{2}N_2 \rightleftarrows NO$. According Eq. 14.35 takes the form

① $$K = \frac{x}{\left[\frac{1.58-x}{2}\right]^{1/2}\left[\frac{0.42-x}{2}\right]^{1/2}}\left[\frac{P/P_{ref}}{1}\right]^{1-1/2-1/2} = \frac{2x}{[1.58-x]^{1/2}[0.42-x]^{1/2}}$$

or

$$K^2 = \frac{4x^2}{(1.58-x)(0.42-x)} \Rightarrow \gamma x^2 + 2x - 0.6636 = 0 \qquad (1)$$

where $\gamma = \frac{4}{K^2} - 1$. Solving this quadratic equation

$$x = \frac{-2 + \sqrt{4 + 2.6544\gamma}}{2\gamma} \qquad (2)$$

At 4342°R, Table A-27 gives $\log_{10} K = -1.2956 \Rightarrow K = 0.05062$. Thus $\gamma = 1560$ and Eq. (2) gives $x = 0.02$. The mole fraction of NO in the equilibrium mixture is then

$$y_{NO} = \frac{0.02}{1} = 0.02 \qquad \longleftarrow$$

To determine the effect of a higher temperature on x, consider T = 4500°R. Then Table A-27 gives $\log_{10} K = -1.227 \Rightarrow K = 0.05929$. Thus $\gamma = 1136.9$ and Eq. (2) gives $x = 0.0233$. Thus, as temperature decreases the amount of NO also decreases. $\longleftarrow$

---

1. Note that (P/P_{ref}) drops out of this expression, and thus of the calculation of the equilibrium composition.

PROBLEM 14.30

KNOWN: A gaseous mixture with a specified molar analysis is heated at a pressure of 1.5 bar and forms an equilibrium mixture of CO_2, CO, and O_2 at 3000 K.

FIND: Determine the molar analysis of the equilibrium mixture.

ASSUMPTIONS: (1) The analysis is based on 1 kmol of initial mixture. (2) The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: On the basis of 1 kmol of initial mixture, the reaction is

$$0.2\,CO_2 + 0.4\,CO + 0.4\,O_2 \longrightarrow \alpha\,CO_2 + \beta\,CO + \gamma\,O_2$$

$$C: \quad 0.2 + 0.4 = \alpha + \beta \Rightarrow \beta = .6 - \alpha$$

$$O: \quad 2(0.2) + 0.4 + 2(0.4) = 2\alpha + \beta + 2\gamma$$

$$1.6 = 2\alpha + (.6 - \alpha) + 2\gamma \Rightarrow \gamma = \frac{1-\alpha}{2}$$

Then

$$0.2\,CO_2 + 0.4\,CO + 0.4\,O_2 \longrightarrow \alpha\,CO_2 + (0.6-\alpha)\,CO + \left(\frac{1-\alpha}{2}\right) O_2$$

The amount of mixture is $n = \alpha + (0.6-\alpha) + \left(\frac{1-\alpha}{2}\right) = (2.2-\alpha)/2$.

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly Eq 14.35 takes the form

$$K = \frac{[0.6-\alpha]\left[\frac{1-\alpha}{2}\right]^{1/2}}{[\alpha]}\left[\frac{P/P_{ref}}{(2.2-\alpha)/2}\right]^{1+\frac{1}{2}-1}$$

$P/P_{ref} = 1.5/1.01325 \Rightarrow$

$$K = \left[\frac{0.6-\alpha}{\alpha}\right]\left[\frac{1-\alpha}{2.2-\alpha}\right]^{1/2}[1.21671]$$

From Table A-27 at 3000 K, $\log_{10} K = -0.485 \Rightarrow K = 0.32734$. Thus

$$0.27127 = \left[\frac{0.6-\alpha}{\alpha}\right]\left[\frac{1-\alpha}{2.2-\alpha}\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $\alpha = 0.408$.

The analysis of the equilibrium mixture in terms of mole fractions is

$$\begin{cases} y_{CO_2} = \dfrac{0.408}{0.896} = 0.4554 & (45.54\%) \\ y_{CO} = \dfrac{0.192}{0.896} = 0.2143 & (21.43\%) \\ y_{O_2} = \dfrac{0.296}{0.896} = 0.3304 & (33.04\%) \end{cases}$$

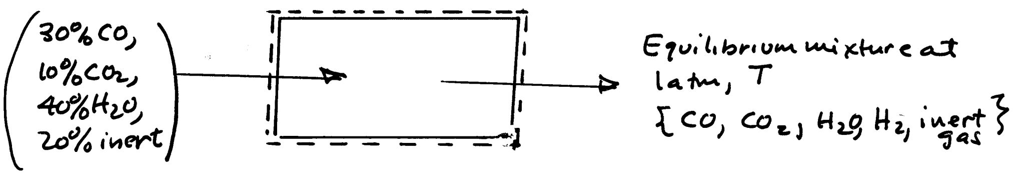
PROBLEM 14.31

KNOWN: An ideal gas mixture with a molar analysis {30% CO, 10% CO_2, 40% H_2O, 20% inert} enters a reactor operating at steady state. An equilibrium mixture of CO, CO_2, H_2O, H_2, and inert gas exits the reactor at 1atm.

FIND: (a) If the exiting mixture is at 1200K, determine the ratio of moles of H_2 exiting to moles of H_2O entering.

(b) If $y_{CO} = 0.075$ in the exiting mixture, determine the mixture temperature.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown is at steady state. (2) The exiting mixture is modeled as an ideal gas mixture.

ANALYSIS: Based on 100kmol of entering gas, at steady state

$$30\,CO + 10\,CO_2 + 40\,H_2O + 20\,\text{inert} \rightarrow (30-x)CO + (10+x)CO_2 + (40-x)H_2O + x\,H_2 + 20\,\text{inert}$$

For the exiting mixture, $n = (30-x) + (10+x) + (40-x) + x + 20 = 100\,\text{kmol}$

At equilibrium for $CO + H_2O \rightleftarrows CO_2 + H_2$, and so

$$① \quad K = \frac{(10+x)(x)}{(30-x)(40-x)}\left[\frac{P/P_{ref}}{n}\right]^{\overset{=0}{\overbrace{1+1-1-1}}} = \frac{(10+x)(x)}{(30-x)(40-x)} \qquad (1)$$

(a) When T = 1200K, Table A-27 gives $\log_{10}K = -0.135 \Rightarrow K = 0.7328$.
Rearranging Eq.(1), it can be expressed as a quadratic equation:
$x^2 + 229.401x - 3291.02 = 0$. Solving with the quadratic formula, $x = 13.547$

Thus, $\dfrac{13.547\ \text{kmol}\ H_2\ \text{(produced)}}{40\ \text{kmol}\ H_2O\ \text{(feed)}} = 0.339 \longleftarrow$ (a)

(b) When $y_{CO} = 0.075$, then

$$y_{CO} = \left(\frac{30-x}{100}\right) = 0.075 \Rightarrow x = 22.5 \xrightarrow[\text{Eq.(1)}]{} \quad K = \frac{(32.5)(22.5)}{(7.5)(17.5)} \Rightarrow \log_{10}K = 0.74597.$$

For use with Table A-27, $\log_{10}K = -0.74597$. Inspection shows that $500 < T < 1000K$. Using the interpolation rule of Problem 14.6:

$$\log_{10}K = C_1 + \frac{C_2}{T}$$

500K: $-2.139 = C_1 + \dfrac{C_2}{500}$

1000K: $-0.159 = C_1 + \dfrac{C_2}{1000}$

$\Big\}$ $C_1 = 1.821$, $C_2 = -1980\,K$

Thus, $-0.74597 = 1.821 - \dfrac{1980}{T} \Rightarrow \dfrac{1980}{T} = 2.56697 \Rightarrow T = 771K \longleftarrow$ (b)

---

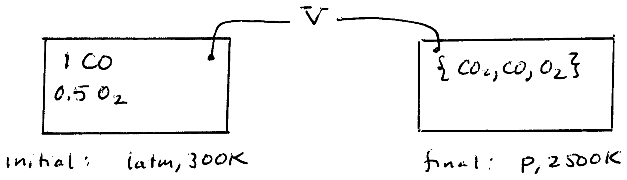
1. Note that (P/P_{ref}) drops out of this expression, and thus plays no role in the subsequent evaluations.

PROBLEM 14.32

KNOWN: One kmol CO and 0.5 kmol O_2, initially at 1atm and 300K, reacts to form an equilibrium mixture of $\{CO_2, CO, O_2\}$ at 2500K and the same total volume

FIND: Determine the final pressure

SCHEMATIC & GIVEN DATA:



ASSUMPTION: The ideal gas model applies.

ANALYSIS: The reaction has the form

$$1\,CO + 0.5\,O_2 \rightarrow \alpha\,CO_2 + \beta\,CO + \gamma\,O_2$$

$C: 1 = \alpha + \beta \Rightarrow \alpha = 1-\beta$, $O: 2 = 2(1-\beta) + \beta + 2\gamma$, $\gamma = \beta/2$

The amount of mixture is $n = (1-\beta) + \beta + \beta/2 = 1 + \beta/2$.

The ideal gas equation of state for the initial mixture gives $p'V = n'\bar{R}T'$, where the prime indicates the initial values. For the final mixture $pV = n\bar{R}T$. Accordingly

$$\frac{p}{p'} = \frac{nT}{n'T'} \Rightarrow \frac{p}{n} = \frac{p'}{n'}\frac{T}{T'} = \left(\frac{1\text{ atm}}{1.5\text{ kmol}}\right)\left(\frac{2500\text{ K}}{300\text{ K}}\right) = 5.556\ \frac{\text{atm}}{\text{kmol}} \qquad (1)$$

For $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$, Eq. 14.35 takes the form

$$K = \frac{\beta[\beta/2]^{1/2}}{1-\beta}\left[\underset{\uparrow\ 5.556\ \frac{\text{atm}}{\text{kmol}}\ (\text{Eq. }(1))}{\frac{p}{n}}\right]^{1/2}$$

With $K = 0.0363$ from Table A-27 at 2500K

$$0.0363 = \frac{\beta}{1-\beta}\left[\frac{5.556}{2}\beta\right]^{1/2}$$

Using an equation solver or iteration with a hand calculator, $\beta = 0.074$. Thus,

$$n = 1 + \frac{0.074}{2} = 1.037\text{ kmol}$$

Returning to Eq. (1)

$$p = (5.556)(1.037)$$
$$= 5.761\text{ atm}$$

PROBLEM 14.33

KNOWN: CH_4 burns with 90% of theoretical air to form an equilibrium mixture of CO_2, CO, $H_2O(g)$, H_2, N_2 at 1000 K, 1 atm

FIND: Determine the composition of the equilibrium mixture, per kmol of mixture.

ASSUMPTIONS: (1) The ideal gas model applies. (2) N_2 is inert.

ANALYSIS: The reaction takes the form

$$CH_4 + (0.9)(2)[O_2 + 3.76N_2] \rightarrow \alpha CO_2 + \beta CO + \gamma H_2O + \omega H_2 + 6.768N_2$$

$C: \quad 1 = \alpha + \beta \Rightarrow \alpha = 1 - \beta$

$H: \quad 4 = 2\gamma + 2\omega, \quad 2 = \gamma + \omega$

$O: \quad 3.6 = 2(1-\beta) + \beta + \gamma \Rightarrow \gamma = 1.6 + \beta \quad \Rightarrow \quad \omega = 0.4 - \beta$

At equilibrium, $CO_2 + H_2 \rightleftarrows CO + H_2O$. Table A-27 gives for T = 1000 K, K = 0.693.
And therefore

① $$0.693 = \frac{[\beta][1.6+\beta]}{[1-\beta][0.4-\beta]}\left[\frac{P/P_{ref}}{n}\right]^{\overbrace{1+1-1-1}^{0}}$$

Using an equation solver or iteration with a hand calculator, $\beta = 0.1065$. So, the composition of the equilibrium mixture per kmol of CH_4 is

$$\{0.8935\,CO_2,\ 0.1065\,CO,\ 1.7065\,H_2O,\ 0.2935\,H_2,\ 6.768\,N_2\}$$

The amount of mixture per kmol of CH_4 is 9.768 kmol. So, the molar analysis of the equilibrium mixture is

9.15% CO_2, 1.09% CO, 17.47% $H_2O(g)$, 3% H_2, 69.29% N_2 ◄

---

1. Note that (P/P_{ref}) drops out of this expression and thus out of the subsequent evaluations.

PROBLEM 14.34

KNOWN: C_8H_{18} burns with air to form an equilibrium mixture of $\{CO_2, H_2, CO, H_2O(g), N_2\}$ at 1700 K, 1 atm.

FIND: If the equivalence ratio is 1.2, determine the composition of the equilibrium mixture in kmol per kmol of fuel.

ASSUMPTIONS: (1) The ideal gas model applies. (2) N_2 is inert.

ANALYSIS: With $(\overline{AF})_{theo} = 12.5(4.76)$ from Example 13.4

$$\text{Equivalence ratio} = \frac{(\overline{AF})_{theo}}{\overline{AF}} \Rightarrow \quad AF = \frac{12.5(4.76)}{1.2} = (10.417)(4.76)$$

Thus

$$C_8H_{18} + 10.417[O_2 + 3.76N_2] \rightarrow a\,CO_2 + b\,H_2O + c\,CO + d\,H_2 + 39.168\,N_2$$

C: $8 = a + c \Rightarrow a = 8 - c$

H: $18 = 2b + 2d \Rightarrow d = 9 - b$

O: $20.834 = 2a + b + c$

$= 2(8-c) + b + c \Rightarrow b = 4.834 + c$ and $d = 9 - [4.834 + c]$

$= 4.166 - c$

At equilibrium, $CO_2 + H_2 \rightleftarrows CO + H_2O$. Table A-27 gives for T = 1700 K, K = 3.388. And therefore

① $$3.388 = \frac{[c][4.834+c]}{[8-c][4.166-c]}\left[\frac{p/p_{ref}}{n}\right]^{\overbrace{1+1-1-1}^{0}}$$

This can be solved analytically by expressing it as a quadratic equation and using the quadratic formula. Alternatively, an equation solver can be used, or iteration with a hand calculator can be employed. We get c = 2.883.

The composition of the equilibrium mixture, per kmol of C_8H_{18}, is then

$$\{5.117\,CO_2,\ 7.717\,H_2O(g),\ 2.883\,CO,\ 1.283\,H_2,\ 39.168\,N_2\} \leftarrow$$

---

1. Note that (p/p_{ref}) drops out of this expression and thus from the following evaluations.

PROBLEM 14.35

KNOWN: C_2H_2 at 25°C, 1atm burns with 40% excess air at 25°C, 1atm, $\phi = 80\%$. An equilibrium mixture of $\{CO_2, H_2O, O_2, NO, N_2\}$ is formed at 2200K, 0.9atm.

FIND: Determine the composition of the equilibrium mixture, per kmol of C_2H_2.

ASSUMPTION: The ideal gas model applies.

ANALTSIS: The balanced reaction equation for complete combustion of C_2H_2 with the theoretical amount of dry air is $C_2H_2 + \frac{5}{2}[O_2 + 3.76N_2] \rightarrow 2CO_2 + H_2O + (\frac{5}{2})3.76N_2$. The amount of dry air for combustion with 40% excess air is, then $(1.4)(2.5)(4.76)$ per kmol of C_2H_2. That is, $(3.5)(4.76) = 16.66$ kmol(air)/kmol(fuel). The amount of water vapor accompanying the dry air is found using Chap. 12 concepts: $P_v = \phi\, P_g(25°C) = (0.8)(0.03169\text{ bar}) = 0.025352\text{bar}$. $P_a = P - P_v = 1.01325 - 0.02535 = 0.987898$ bar. Then, referring to the development of Eqs. 12.41, we get

$$\frac{n_v}{n_a} = \frac{P_v}{P_a} \Rightarrow n_v = \left(\frac{0.025352}{0.987898}\right)\left(16.66\,\frac{\text{kmol(dry air)}}{\text{kmol(fuel)}}\right) = 0.428\,\frac{\text{kmol}(H_2O)}{\text{kmol(fuel)}}$$

The reaction equation then has the form

$$C_2H_2 + 3.5(O_2 + 3.76N_2) + 0.428H_2O \rightarrow 2CO_2 + 1.428H_2O + \gamma NO + [1 - \tfrac{\gamma}{2}]O_2 + [13.16 - \tfrac{\gamma}{2}]N_2$$

At equilibrium, $\frac{1}{2}O_2 + \frac{1}{2}N_2 \rightleftarrows NO$. Table A-27 gives for 2200K, K = 0.0328. And therefore

① $$0.0328 = \frac{\gamma}{[1-\frac{\gamma}{2}]^{1/2}[13.16-\frac{\gamma}{2}]^{1/2}}\left[\frac{P/P_{ref}}{n}\right]^{\overbrace{1-\frac{1}{2}-\frac{1}{2}}^{0}}$$

$$0.0328 = \frac{\gamma}{\left[(1-\frac{\gamma}{2})(13.16-\frac{\gamma}{2})\right]^{1/2}}$$

Squaring both sides this can be expressed as a quadratic equation and solved using the quadratic formula. Alternatively, an equation solver can be used, or iteration with a hand calculator employed. We get $\gamma = 0.1134$.

The composition of the equilibrium mixture, per kmol of C_2H_2, is

$\{2CO_2, 1.428H_2O, 0.1134NO, 0.9433O_2, 13.1033N_2\}$

---

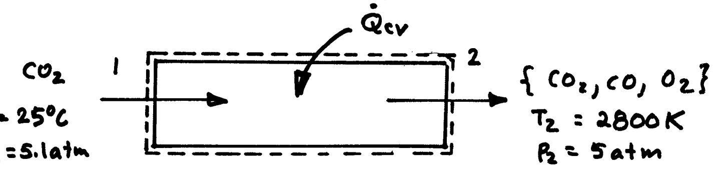
1. Note that (P/P_{ref}) drops out of this expression.

PROBLEM 14.36

KNOWN: CO_2 gas at 25°C, 5.1atm enters a heat exchanger operating at steady state. An equilibrium mixture of CO_2, CO, and O_2 exits at 2800 K, 5atm.

FIND: Determine the composition of the exiting mixture and the heat transfer, each per kmol of CO_2 entering.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ideal gas model applies to the incoming CO_2 and the exiting equilibrium mixture.

ANALYSIS: The reaction has the form

$$1\,CO_2 \rightarrow (1-x)\,CO_2 + x\,CO + \frac{x}{2}\,O_2$$

The amount of mixture is $n = (1-x) + x + \frac{x}{2} = (2+x)/2$. At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x][x/2]^{1/2}}{[1-x]}\left[\frac{P/P_{ref}}{(x+2)/2}\right]^{1/2} = \left[\frac{x}{1-x}\right]\left[\frac{5x}{x+2}\right]^{\frac{1}{2}}$$

From Table A-27 at 2800 K, $\log_{10}K = -0.825 \Rightarrow K = 0.14962$. Solving, $x = 0.1867$. The equilibrium mixture has the following composition in kmol per kmol of CO_2 entering

$$\{0.8133\,CO_2,\ 0.1867\,CO,\ 0.09335\,O_2\} \longleftarrow$$

An energy rate balance reduces at steady state to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} - \overset{0}{\cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CO_2}}}} + 1\cdot\bar{h}_{CO_2}(T_1) - \left[0.8133\,\bar{h}_{CO_2}(T_2) + 0.1867\,\bar{h}_{CO}(T_2) + 0.09335\,\bar{h}_{O_2}(T_2)\right]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} = 0.8133\left[\bar{h}_f^o + \bar{h}(T_2) - h(298)\right]_{CO_2} + 0.1867\left[\bar{h}_f^o + \bar{h}(T_2) - \bar{h}(298)\right]_{CO} + 0.09335\left[\overset{0}{\cancel{\bar{h}_f^o}} + \bar{h}(T_2) - \bar{h}(298)\right]_{O_2} - \left[\bar{h}_f^o\right]_{CO_2}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} = 0.8133\left[-393{,}520 + 149{,}808 - 9{,}364\right] + 0.1867\left[-110{,}530 + 94{,}784 - 8{,}669\right] + 0.09335\left[98{,}826 - 8{,}682\right] - \left[-393{,}520\right]$$

$$= 191{,}550 \text{ kJ/kmol}(CO_2) \longleftarrow$$

PROBLEM 14.36 (Cont'd.)

IT can be used as an alternative to iteration using a hand calculator and to using table data in the energy balance. The IT program follows.

<u>*IT* Code</u>

```
T1 = 25 + 273.15  // K
p1 = 5.1  // atm
T2 = 2800  // K
p2 = 5  // atm

// CO2  ===>  (1 - x) CO2 + x CO + x/2 O2
ndot1 = 1  // per kmol entering
ndotCO2_2 = 1 - x
ndotCO_2 = x
ndotO2_2 = x/2
ndot2 = ndotCO2_2 + ndotCO_2 + ndotO2_2
yCO2 = ndotCO2_2 / ndot2
yCO = ndotCO_2 / ndot2
yO2 = ndotO2_2 / ndot2
pref = 1  // atm

// For the reaction CO2  <===>  CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p2 / pref)^0.5
//Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T2,3)

0 = Qdot + ndot1*hCO2_1 - ndotCO2_2*hCO2_2 - ndotCO_2*hCO_2 -
ndotO2_2*hO2_2
hCO2_1 = h_T("CO2",T1)
hCO2_2 = h_T("CO2",T2)
hCO_2 = h_T("CO",T2)
hO2_2 = h_T("O2",T2)
```

<u>*IT* Results</u>

K = 0.1496

$\dot{n}_{CO_2,2} / \dot{n}_{CO_2,1}$ = 0.8136 kmol / kmol (CO_2 entering)

$\dot{n}_{CO,2} / \dot{n}_{CO_2,1}$ = 0.1864 kmol / kmol (CO_2 entering)

$\dot{n}_{O_2,2} / \dot{n}_{CO_2,1}$ = 0.09322 kmol / kmol (CO_2 entering)

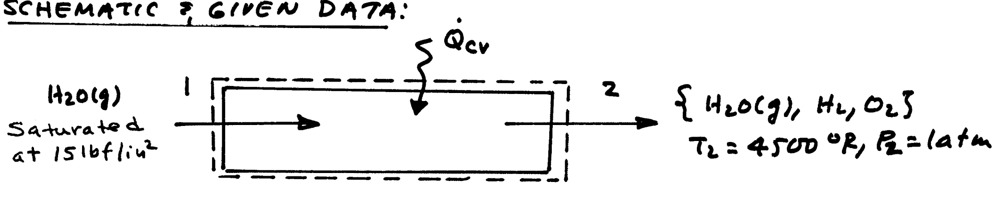
$\dot{Q}_{cv} / \dot{n}_{CO_2,1}$ = 1.916E5 kJ/kmol (CO_2 entering)

PROBLEM 14.37

KNOWN: Saturated water vapor at 15 lbf/in² enters a heat exchanger operating at steady state. An equilibrium mixture of $H_2O(g)$, H_2, and O_2 exits at 4500°R, 1 atm

FIND: Determine the composition of the exiting mixture and the heat transfer, each per lbmol of steam entering.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The incoming steam and exiting equilibrium mixture can be modeled as ideal gases.

ANALYSIS: The reaction has the form

$$1\,H_2O \longrightarrow (1-x)\,H_2O + x\,H_2 + \frac{x}{2}\,O_2$$

The amount of mixture is $n = (1-x) + x + \frac{x}{2} = (x+2)/2$. At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[x]\left[\frac{x}{2}\right]^{1/2}}{[1-x]}\left[\frac{P/P_{ref}}{(x+2)/2}\right]^{1/2} = \left[\frac{x}{1-x}\right]\left[\frac{x}{x+2}\right]^{1/2}$$

From Table A-27 at 4500°R $\log_{10} K = -2.224 \Rightarrow K = 0.00597$. Solving $x = 0.0406$. The equilibrium mixture has the following composition in lbmol per lbmol of steam entering

$$\{0.9594\ H_2O,\ 0.0406\ H_2,\ 0.0203\ O_2\}$$ ←————

An energy rate balance reduces at steady state to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{H_2O}}}^{\,0} + \bar{h}_{H_2O}(T_1) - \left[0.9594\,\bar{h}_{H_2O}(T_2) + 0.0406\,\bar{h}_{H_2}(T_2) + 0.0203\,\bar{h}_{O_2}(T_2)\right]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}} = 0.9594\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)\right]_{H_2O} + 0.0406\left[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_2) - \bar{h}(537)\right]_{H_2} +$$
$$0.0203\left[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_2) - \bar{h}(537)\right]_{O_2} - \left[\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(537)\right]_{H_2O}$$

With ideal gas table data ← $T_1 = T_{sat}$ at 15 lbf/in² = 213 °F = 213°F (673°R)

$$\frac{\dot{Q}_{cv}}{\dot{n}_{H_2O}} = 0.9594\left[-104{,}040 + 46{,}836 - 4258\right] + 0.0406\left[33921.6 - 3640.3\right] +$$
$$0.0203\left[37{,}412 - 3725.1\right] - \left[-104{,}040 + 5356 - 4258\right]$$

$$= 45{,}888 \text{ Btu/lbmol } (H_2O)$$ ←————

PROBLEM 14.37 (Cont'd.) Alternative IT solution.

IT Code

```
p1 = 15  // lbf/in², saturated vapor
T1 = Tsat_P("Steam",p1)
T2 = 4500  // °R
p2 = 1  // atm

// H2O  →  (1 - x) H2O + x H2 + x/2 O2
ndot1 = 1  //  Do calculations on the basis of 1 lbmol of H2O entering.
ndotH2O_2 = 1 - x
ndotH2_2 = x
ndotO2_2 = x/2
ndot2 = ndotH2O_2 + ndotH2_2 + ndotO2_2
yH2O = ndotH2O_2 / ndot2
yH2 = ndotH2_2 / ndot2
yO2 = ndotO2_2 / ndot2
pref = 1  // atm

//  For the reaction H2O  ↔  H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p2 / pref)^0.5
//Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,2,T2,3)

0 = Qdot + ndot1 * hH2O_1 - (ndotH2O_2 * hH2O_2 + ndotH2_2 * hH2_2 + ndotO2_2 * hO2_2)
//  Note:  Use the ideal gas function for H2O in order to get the correct
//  reference value for H2O in the calculations.  This results in only a slight error.
hH2O_1 = h_T("H2O", T1)
hH2O_2 = h_T("H2O",T2)
hH2_2 = h_T("H2",T2)
hO2_2 = h_T("O2",T2)
```

IT Results

K = 0.00597

$\dot{n}_{H_2O,2}/\dot{n}_{H_2O,1}$ = 0.9594 lbmol/lbmol(H_2O entering)

$\dot{n}_{H_2,2}/\dot{n}_{H_2O,1}$ = 0.04061 lbmol/lbmol(H_2O entering)

$\dot{n}_{O_2,2}/\dot{n}_{H_2O,1}$ = 0.0203 lbmol/lbmol(H_2O entering)

$\dot{Q}_{cv}/\dot{n}_{H_2O,1}$ = 4.582×10^4 Btu/lbmol(H_2O entering)

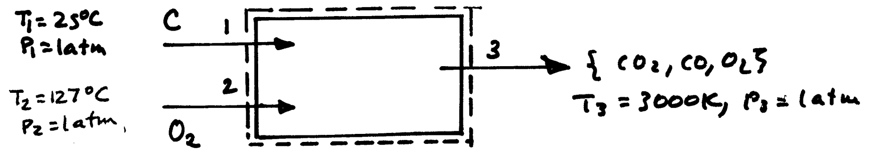
PROBLEM 14.38

KNOWN: Carbon at 25°C, 1atm and O_2 at 127°C, 1atm enter a reactor operating at steady state with equal molar flow rates. An equilibrium mixture of CO_2, CO, and O_2 exits at 3000K, 1atm.

FIND: Determine the composition of the exiting mixture and the heat transfer, each per kmol of carbon entering.

SCHEMATIC & GIVEN DATA:

$\dot{n}_C = \dot{n}_{O_2}$



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ideal gas model applies to the equilibrium mixture.

ANALYSIS: The reaction takes the form

$$1C + 1O_2 \rightarrow x\,O_2 + 2x\,CO + (1-2x)\,CO_2$$

The amount of mixture is $n = x + 2x + (1-2x) = 1+x$. At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[2x][x]^{1/2}}{[1-2x]}\left[\frac{P/P_{ref}}{1+x}\right]^{1/2} = \frac{2x}{1-2x}\left[\frac{x}{1+x}\right]^{1/2}$$

At 3000K, Table A-27 gives $\log_{10}K = -0.485 \Rightarrow K = 0.3273$. Solving $x = 0.218$. The equilibrium mixture has the following composition in kmol per kmol of carbon entering

$$\{0.218\,O_2,\ 0.436\,CO,\ 0.564\,CO_2\} \longleftarrow$$

An energy rate balance reduces at steady state to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_C} - \overset{0}{\cancel{\frac{\dot{W}_{cv}}{\dot{n}_C}}} + \overset{0}{\cancel{\bar{h}_C(T_1,P_1)}} + \bar{h}_{O_2}(T_2,P_2) - [0.218\,\bar{h}_{O_2}(T_3) + 0.436\,\bar{h}_{CO}(T_3) + 0.564\,\bar{h}_{CO_2}(T_3)]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_C} = 0.218[\overset{0}{\cancel{\bar{h}_f^\circ}} + \bar{h}(T_3) - \bar{h}(298)]_{O_2} + 0.436[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO} + 0.564(\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} - [\bar{h}(T_2) - \bar{h}(298)]_{O_2}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_C} = 0.218[106{,}780 - 8682] + 0.436[-110{,}530 + 102{,}210 - 8669] + 0.564[-393{,}520 + 162{,}226 - 9364] - [11{,}711 - 8682]$$

$$= -124{,}782 \text{ kJ/kmol(C)} \longleftarrow$$

IT can be used as an alternative to iteration using a hand calculator and to using table data in the energy balance. The IT program follows.

PROBLEM 14.38 (Cont'd.) Alternative IT solution.

IT Code

```
p1 = 1  // atm
T1 = 25 + 273.15 // K
p2 = 1  // atm
T2 = 127 + 273.15  // K
ndot1 = 1  //  Do all calculations on the basis of 1 kmol of carbon entering.
ndot2 = ndot1  //  Entering streams have equal molar flow rates.
T3 = 2727 + 273.15 // K
p3 = 1  // atm

//  C + O2  → x O2 + 2x CO + (1 - 2x) CO2
ndotO2_3 = x
ndotCO_3 = 2*x
ndotCO2_3 = 1 - 2 * x
ndot3 = ndotO2_3 + ndotCO_3 + ndotCO2_3
yO2 = ndotO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yCO2 = ndotCO2_3/ ndot3
pref = 1  // atm

// For the reaction CO2  ↔  CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQCO2a.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)

0 = Qdot + ndot1*hC + ndot2*hO2_2 - (ndotCO2_3*hCO2_3 +
ndotCO_3*hCO_3 + ndotO2_3*hO2_3)
hC = 0  //  See Table A-25.
hO2_2 = h_T("O2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
```

IT Results

K = 0.3275

$\dot{n}_{CO_2,3}/\dot{n}_1$ = 0.5637 kmol/kmol(C)

$\dot{n}_{CO,3}/\dot{n}_1$ = 0.4363 kmol/kmol(C)

$\dot{n}_{O_2,3}/\dot{n}_1$ = 0.2181 kmol/kmol(C)

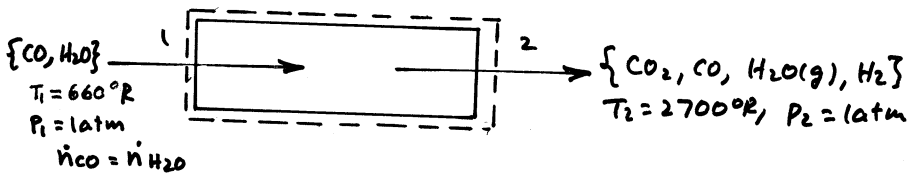
$\dot{Q}_{cv}/\dot{n}_1$ = -1.246×10^5 kJ/kmol(C)

PROBLEM 14.39

KNOWN: An equimolar mixture of CO and $H_2O(g)$ at 200°F, 1atm enters a reactor operating at steady state. An equilibrium mixture of CO_2, CO, $H_2O(g)$, and H_2 exits at 2700°R, 1atm.

FIND: Determine the heat transfer per lbmol of CO entering.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv}=0$ and negligible effects of kinetic and potential energy. (2) The ideal gas model applies to the incoming and outgoing mixtures.

ANALYSIS: The reaction takes the form

$$1\,CO + 1\,H_2O \rightarrow x\,CO_2 + (1-x)\,CO + (1-x)\,H_2O + x\,H_2$$

The amount of mixture is $n = x + (1-x) + (1-x) + x = 2$. At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq. 14.35 takes the form

$$K = \frac{[1-x][1-x]}{[x][x]}\left[\frac{p/p_{ref}}{2}\right]^{1+1-1-1} = \left(\frac{1-x}{x}\right)^2 \Rightarrow x = \frac{1}{1+\sqrt{K}}$$

At 2700°R, Table A-27 gives $\log_{10} K = 0.4035 \Rightarrow K = 2.5322$. Solving, $x = 0.386$.

At steady state an energy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{co}} - \frac{\cancelto{0}{\dot{W}_{cv}}}{\dot{n}_{co}} + h_{co}(T_1) + \bar{h}_{H_2O}(T_1) - \Big[0.386\,\bar{h}_{CO_2}(T_2) + 0.614\,\bar{h}_{co}(T_2) + 0.614\,\bar{h}_{H_2O}(T_2) + 0.386\,\bar{h}_{H_2}(T_2)\Big]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_{co}} = 0.386[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{CO_2} + 0.614[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{CO} + 0.614[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(537)]_{H_2O} + 0.386[\cancelto{0}{\bar{h}_f^\circ} + \bar{h}(T_2) - \bar{h}(537)]_{H_2} - \{1[\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(537)]_{CO} + 1[\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(537)]_{H_2O}\}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{co}} = 0.386[-169{,}300 + 30{,}581 - 4027.5] + 0.614[-47{,}540 + 20{,}434 - 3725.1] + 0.614[-104{,}040 + 24{,}957 - 4258] + 0.386[19237.8 - 3640.3] - \{1[-47{,}540 + 4586.6 - 3725.1] + 1[-104{,}040 + 5250 - 4258]\}$$

$$= 30{,}547 \text{ Btu/lbmol(CO)} \longleftarrow$$

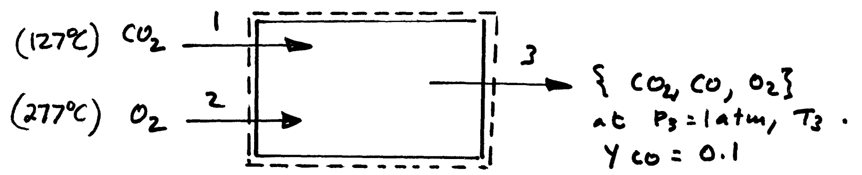
Alternatively, a solution using IT can be developed.

PROBLEM 14.40

KNOWN: CO_2 and O_2 in a 1:2 molar ratio enter a reactor operating at steady state in separate streams at 1atm and 127°C, 277°C respectively. An equilibrium mixture of CO_2, CO, and O_2 exits at 1atm. The mole fraction of CO in the exiting mixture is 0.1

FIND: Determine the heat transfer per kmol of CO_2 entering.

SCHEMATIC & GIVEN DATA:



$P_1 = P_2 = 1atm$

$\dot{n}_{O_2}/\dot{n}_{CO_2} = 2$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas.

ANALYSIS: The first step is to determine T_3 using equilibrium principles. The reaction takes the form

$$1\,CO_2 + 2O_2 \rightarrow x\,CO + (1-x)\,CO_2 + \left(\frac{4+x}{2}\right)O_2$$

The amount of mixture is $n = x + (1-x) + \left(\frac{4+x}{2}\right) = \frac{6+x}{2}$. From the given data $y_{co} = 0.1$. Thus

$$0.1 = \frac{x}{(6+x)/2} \Rightarrow x = 0.316$$

Thus

$$1\,CO_2 + 2O_2 \rightarrow 0.316\,CO + 0.684\,CO_2 + 2.158\,O_2$$

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[0.316][2.158]^{1/2}}{0.684}\left[\frac{P/P_{ref}}{3.158}\right]^{1/2} = \left(\frac{0.316}{0.684}\right)\left(\frac{2.158}{3.158}\right)^{1/2} = 0.3819$$

Thus $\log_{10} K = -0.418$. Interpolation in Table A-27 gives $T_3 = 3044\,K$.

At steady state an energy rate balance reduces to give

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CO_2}}}^{0} + \bar{h}_{CO_2}(T_1) + 2\bar{h}_{O_2}(T_1) - \left[0.316\,\bar{h}_{CO}(T_3) + 0.684\,\bar{h}_{CO_2}(T_3) + 2.158\,\bar{h}_{O_2}(T_3)\right]$$

or

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} = 0.316\left[\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(298)\right]_{CO} + 0.684\left[\bar{h}_f^o + \bar{h}(T_3) - \bar{h}(298)\right]_{CO_2} + 2.158\left[\cancel{\bar{h}_f^o}^{0} + \bar{h}(T_3) - \bar{h}(298)\right]_{O_2} - \left[\bar{h}_f^o + \bar{h}(T_1) - \bar{h}(298)\right]_{CO_2} - 2\left[\bar{h}(T_2) - \bar{h}(298)\right]_{O_2}$$

With data from ~~the ideal~~ gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CO_2}} = 0.316\left[-110{,}530 + 103{,}849 - 8669\right] + 0.684\left[-393{,}520 + 164{,}967 - 9364\right] + 2.158\left[108{,}538 - 8682\right] - \left[(-393{,}520) + 13{,}372 - 9364\right] - 2\left[14{,}770 - 8682\right]$$

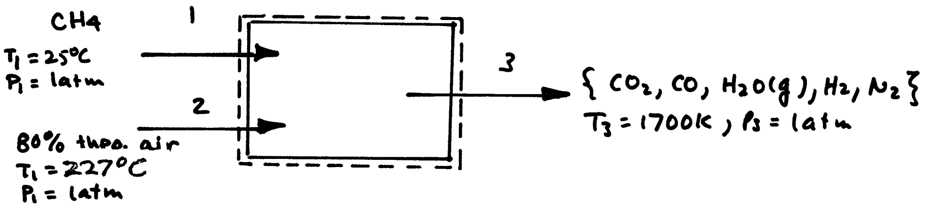
$$= 425{,}239 \text{ kJ/kmol}(CO_2) \longleftarrow$$

PROBLEM 14.41

KNOWN: CH_4 at 25°C, 1atm enters a reactor operating at steady state and burns with 80% of theoretical air entering at 227°C, 1atm. An equilibrium mixture of CO_2, CO, $H_2O(g)$, H_2 and N_2 exits at 1700K, 1atm.

FIND: Determine the composition of the exiting mixture and the heat transfer, each per kmol of CH_4 entering.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas. (3) N_2 is inert.

ANALYSIS: The balanced reaction equation for complete combustion of CH_4 with the theoretical amount of air is given by Eq. 13.4. Accordingly, combustion with 80% of theoretical air to form CO_2, CO, $H_2O(g)$, H_2, and N_2 takes the form

$$CH_4 + (0.8)2[O_2 + 3.76N_2] \rightarrow x\,CO_2 + (1-x)CO + (2.2-x)H_2O + (x-0.2)H_2 + 6.016N_2$$

The amount of mixture is $n = x + (1-x) + (2.2-x) + (x-0.2) + 6.016 = 9.016$.

At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq. 14.35 takes the form

① $$K = \frac{[1-x][2.2-x]}{[x][x-0.2]}\left[\frac{P/P_{ref}}{9.016}\right]^{1+1-1-1} = \left(\frac{1-x}{x}\right)\left(\frac{2.2-x}{x-0.2}\right)$$

At 1700K, Table A-27 gives $\log_{10}K = 0.530 \Rightarrow K = 3.3884$. Solving, $x = 0.5674$. Thus, the composition of the exiting equilibrium mixture in kmol per kmol of CH_4 is

$\{0.5674\,CO_2,\ 0.4326\,CO,\ 1.6326\,H_2O,\ 0.3674\,H_2,\ 6.016\,N_2\}$ ←

At steady state an energy rate balance reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH_4}}}^{\,0} + \bar{h}_{CH_4}(T_1) + [1.6\bar{h}_{O_2}(T_2) + 6.016\bar{h}_{N_2}(T_2)] - [0.5674\bar{h}_{CO_2} + 0.4326\bar{h}_{CO} + 1.6326\bar{h}_{H_2O} + 0.3674\bar{h}_{H_2} + 6.016\bar{h}_{N_2}]$$

Thus

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = 0.5674[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 0.4326[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO} + 1.6326[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{H_2O} + 0.7674[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_3) - \bar{h}(298)]_{H_2} + 6.016[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_3) - \bar{h}(298)]_{N_2} - [\bar{h}_f^\circ]_{CH_4} - 1.6[\bar{h}(T_2) - \bar{h}(298)]_{O_2} - 6.016[\bar{h}(T_2) - \bar{h}(298)]_{N_2}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{CH_4}} = 0.5674[-393{,}520 + 82{,}856 - 9364] + 0.4326[-110{,}530 + 54{,}609 - 8669] + 1.6326[-241{,}820 + 67{,}589 - 9904] + 0.3674[51305 - 8468] + 6.016[54{,}099 - 8669] - [-74{,}850] - 1.6[14{,}770 - 8682] - 6.016[14{,}581 - 8669]$$

$$= -191{,}558 \text{ kJ/kmol } (CH_4)$$ ←

---

1. (P/P_{ref}) drops out of this expression.

PROBLEM 14.41 (Cont'd.) Alternative IT solution.

IT Code

```
T1 = 25 + 273.15  // K
p1 = 1  // atm
T2 = 227 + 273.15  // K
p2 = 1  // atm
T3 = 1427 + 273.15  // K
p3 = 1  // atm
ndotCH4 = 1  //  Do calculations on the basis of 1 kmol of CH4 entering.

// CH4 + 1.6 (O2 + 3.76 N2)  →  x CO2 + (1 - x) CO + (2.2 - x) H2O + (x - 0.2) H2 + 6.016 N2
ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotH2O_3 = 2.2 - x
ndotH2_3 = x - 0.2
ndotN2_3 = 6.016
ndot3 = ndotCO2_3 + ndotCO_3 + ndotH2O_3 + ndotH2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
pref = 1  // atm

// For the reaction CO2 + H2  ↔  CO + H2O
K = (yCO * yH2O) / (yCO2 * yH2) * (p3 / pref)^0
//Data from Table A-27are stored in EQWATGAS.LUT.
log(K) = LOOKUPVAL(EQWATGAS,1,T3,3)

0 = Qdot + ndotCH4*hCH4_1 + 1.6*hO2_2 + 6.016*hN2_2 - (ndotCO2_3*hCO2_3 +
          ndotCO_3*hCO_3 + ndotH2O_3*hH2O_3 + ndotH2_3*hH2_3 + 6.016*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

$K = 3.389$

$\dot{n}_{CO,3}/\dot{n}_{CH_4} = 0.4326$ kmol/kmol(CH_4)

$\dot{n}_{CO_2,3}/\dot{n}_{CH_4} = 0.5674$ kmol/kmol(CH_4)

$\dot{n}_{H_2,3}/\dot{n}_{CH_4} = 0.3674$ kmol/kmol(CH_4)

$\dot{n}_{H_2O,3}/\dot{n}_{CH_4} = 1.633$ kmol/kmol(CH_4)

$\dot{n}_{N_2,3}/\dot{n}_{CH_4} = 6.016$ kmol/kmol(CH_4)

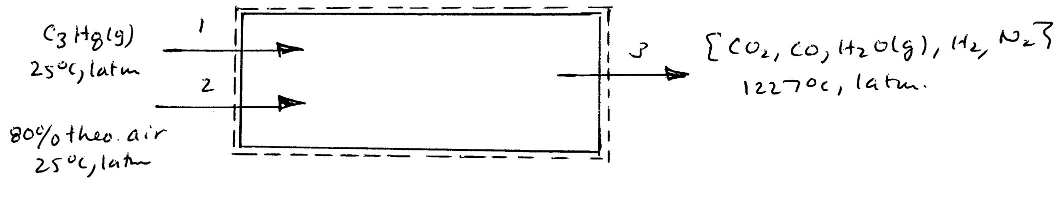
Qdot = -1.917×10^5 kJ/kmol(CH_4)

PROBLEM 14.42

KNOWN: C_3H_8 at 25°C, 1atm enters a reactor at steady state and burns with 80% of theoretical air entering separately at 25°C, 1atm. An equilibrium mixture of $\{CO_2, CO, H_2O(g), H_2, N_2\}$ exits at 1227°C, 1atm.

FIND: Determine the heat transfer, in kJ per kmol of C_3H_8.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv}=0$ and negligible effects of kinetic and potential energy. (2) The exiting equilibrium mixture is modeled as an ideal gas mixture. (3) N_2 is inert.

ANALYSIS: The complete combustion of C_3H_8 with the theoretical amount of air is described by

$$C_3H_8 + 5[O_2+3.76N_2] \rightarrow 3CO_2 + 4H_2O + 18.8N_2$$

Accordingly, the reaction of C_3H_8 with 80% of theoretical air to form the specified mixture is

$$C_3H_8 + 4[O_2+3.76N_2] \rightarrow x\,CO_2 + a\,CO + b\,H_2O + c\,H_2 + 15.04N_2$$

C: $3 = x + a \Rightarrow a = 3 - x$

H: $8 = 2b + 2c$

O: $8 = 2x + a + b$

$= 2x + (3-x) + b \Rightarrow b = 5 - x$

$\Rightarrow c = 4 - b = 4-(5-x) = x - 1$

That is

$$C_3H_8 + 4[O_2+3.76N_2] \rightarrow x\,CO_2 + (3-x)\,CO + (5-x)\,H_2O + (x-1)\,H_2 + 15.04\,N_2$$

At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[3-x][5-x]}{[x][x-1]}\left[\frac{p/p_{ref}}{n}\right]^0$$

At 1500 K, Table A-27 gives $\log_{10}K = 0.4035 \Rightarrow K = 2.5322$. Solving $x = 1.818$. Accordingly

$$C_3H_8 + 4[O_2+3.76N_2] \rightarrow 1.818\,CO_2 + 1.182\,CO + 3.182\,H_2O + 0.818\,H_2 + 15.04\,N_2$$

An energy rate balance at steady state reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{C_3H_8}}}^{\,0} + \bar{h}_{C_3H_8}(T_1) + [4\bar{h}_{O_2}(T_2) + 15.04\bar{h}_{N_2}(T_2)] - [1.818\bar{h}_{CO_2}(T_3) + 1.182\bar{h}_{CO}(T_3) + 3.182\bar{h}_{H_2O}(T_3) + 0.818\bar{h}_{H_2}(T_3) + 15.04\bar{h}_{N_2}]$$

Rearranging and noting that $\bar{h}_f^\circ = 0$ for H_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 1.818[\bar{h}_f^\circ + \bar{h}(T_3) - h(298)]_{CO_2} + 1.182[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO} + 3.182[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{H_2O} + 0.818[\bar{h}(T_3) - \bar{h}(298)]_{H_2} + 15.04[\bar{h}(T_3) - \bar{h}(298)]_{N_2} - [\bar{h}_f^\circ]_{C_3H_8}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 1.818[-393,520 + 71,078 - 9364] + 1.182[-110,530 + 47,517 - 8669] + 3.182[-241,820 + 57,999 - 9904] + 0.818[44,738 - 8468] + 15.04[47,073 - 8669] - (-103,850)$$

$$= -593,269\ \frac{kJ}{kmol(C_3H_8)}$$

PROBLEM 14.42 (Cont'd.) Alternative IT solution.

IT can be used as an alternative to iteration with a hand calculator and to using table data in the energy balance. The IT program follows.

IT Code

```
T1 = 25 + 273.15  // K
T2 = 25 + 273.15  // K
T3 = 1227 + 273.15  // K
p3 = 1 // atm
ndotfuel = 1  //  Do calculations on the basis of 1 kmol of C3H8 entering.

// C3H8 + 4 (O2 + 3.76 N2)  →  x CO2 + (3 - x) CO + (5 - x) H2O + (x - 1) H2 + 15.04 N2
ndotCO2_3 = x
ndotCO_3 = 3 - x
ndotH2O_3 = 5 - x
ndotH2_3 = x - 1
ndotN2_3 = 15.04
ndot3 = ndotCO2_3 + ndotCO_3 + ndotH2O_3 + ndotH2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
pref = 1 // atm

// For the reaction CO2 + H2  ↔  CO + H2O
K = ((yCO * yH2O) / (yCO2 * yH2)) * (p3/ pref)^0
// Data from Table A-27 are stored in EQWATGAS.LUT.
log(K) = LOOKUPVAL(EQWATGAS,1,T3,3)

0 = Qdot + ndotfuel*hC3H8 + 4*hO2_2 + 15.04*hN2_2 - (ndotCO2_3*hCO2_3 +
ndotCO_3*hCO_3 + ndotH2O_3*hH2O_3 + ndotH2_3*hH2_3 + ndotN2_3*hN2_3)
hC3H8 = h_T("C3H8",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

K = 2.533

$\dot{n}_{CO,3} / \dot{n}_{fuel}$ = 1.182 kmol/lmol(C_3H_8)

$\dot{n}_{CO_2,3} / \dot{n}_{fuel}$ = 1.818 kmol/lmol(C_3H_8)

$\dot{n}_{H_2,3} / \dot{n}_{fuel}$ = 0.8175 kmol/lmol(C_3H_8)

$\dot{n}_{H_2O,3} / \dot{n}_{fuel}$ = 3.182 kmol/lmol(C_3H_8)

$\dot{n}_{N_2,3} / \dot{n}_{fuel}$ = 15.04 kmol/lmol(C_3H_8)

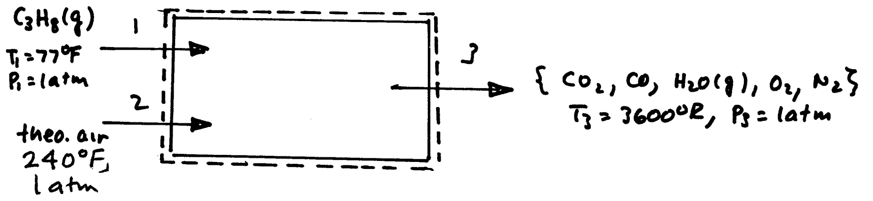
$\dot{Q}_{cv} / \dot{n}_{fuel}$ = -5.933×10^5 kJ/kmol(C_3H_8)

PROBLEM 14.43

KNOWN: $C_3H_8(g)$ at 77°F, 1atm enters a reactor operating at steady state and burns with the theoretical amount of air entering at 240°F, 1atm. An equilibrium mixture of CO_2, CO, $H_2O(g)$, O_2, and N_2 exits at 3600°R, 1atm.

FIND: Determine the heat transfer per lbmol of fuel entering.

SCHEMATIC & GIVEN DATA:



$C_3H_8(g)$
1
$T_1 = 77°F$
$P_1 = 1atm$
2
theo. air
240°F, 1atm
3
$\{CO_2, CO, H_2O(g), O_2, N_2\}$
$T_3 = 3600°R$, $P_3 = 1atm$

ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The exiting equilibrium mixture is modeled as an ideal gas mixture. (3) N_2 is inert.

ANALYSIS: The reaction of $C_3H_8(g)$ with the theoretical amount of air to form CO_2, CO, $H_2O(g)$, O_2, and N_2 is given by

$$C_3H_8 + 5O_2 + 18.8N_2 \rightarrow (3-x)CO_2 + x\,CO + \frac{x}{2}O_2 + 4H_2O + 18.8N_2 \qquad (1)$$

The amount of mixture is $n = (3-x) + x + \frac{x}{2} + 4 + 18.8 = 25.8 + \frac{x}{2}$

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[x][\frac{x}{2}]^{1/2}}{[3-x]}\left[\frac{P/P_{ref}}{51.6+\frac{x}{2}}\right]^{1/2} = \frac{x}{3-x}\left[\frac{x}{51.6+x}\right]^{1/2}$$

At 3600°R, Table A-27 gives $\log_{10}K = -2.884 \Rightarrow K =$ $K = 0.001306$. Solving, $x = 0.091$. Accordingly, Eq (1) becomes

$$C_3H_8 + 5O_2 + 18.8N_2 \rightarrow 2.909CO_2 + 0.091CO + 0.0455O_2 + 4H_2O + 18.8N_2$$

An energy rate balance at steady state reduces to read

$$0 = \frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{C_3H_8}}}^{0} + \bar{h}_{C_3H_8}(T_1) + [5\bar{h}_{O_2}(T_1) + 18.8\bar{h}_{N_2}(T_1)] - [2.909\,\bar{h}_{CO_2}(T_3) +$$
$$0.091\,\bar{h}_{CO}(T_3) + 0.0455\,\bar{h}_{O_2}(T_3) + 4\,\bar{h}_{H_2O}(T_3) + 18.8\,\bar{h}_{N_2}(T_3)]$$

Rearranging and noting that $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 2.909[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + 0.091[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO} + 0.0455[\bar{h}(T_3) - \bar{h}(537)]_{O_2}$$
$$+ 4[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 18.8[\bar{h}(T_3) - h(537)]_{N_2} - [\bar{h}_f^\circ]_{C_3H_8}$$
$$- 5[\bar{h}(T_2) - \bar{h}(537)]_{O_2} - 18.8[\bar{h}(T_2) - \bar{h}(537)]_{N_2}$$

With data from the ideal gas tables

$$\frac{\dot{Q}_{cv}}{\dot{n}_{C_3H_8}} = 2.909[-169{,}300 + 43{,}411 - 4027.5] + 0.091[-47{,}540 + 28127 - 3725.1] +$$
$$0.0455[29{,}174 - 3725.1] + 4[-104{,}040 + 35541 - 4258] +$$
$$18.8[27875 - 3729.5] - [-44{,}680] - 5[4879 - 3725] - 18.8[4865 - 3730]$$

$$= -198{,}396 \text{ Btu/lbmol}(C_3H_8) \longleftarrow$$

PROBLEM 14.43 (Cont'd.)   Alternative IT solution.

IT can be used as an alternative to iteration with a hand calculator and to using table data in the energy balance. The IT program follows.

<u>*IT* Code</u>

```
T1 = 77 + 459.67  // °R
T2 = 240 + 459.67 // °R
T3 = 3140 + 460 // °R
p3 = 1  // atm
ndotfuel = 1 // Do calculations on the basis of 1 lbmol of C3H8 entering.

// C3H8 + 5 (O2 + 3.76 N2)  →  (3 - x) CO2 + x CO + x/2 O2 + 4 H2O + 18.8 N2
ndot_CO2 = 3 - x
ndot_CO = x
ndot_O2 = x / 2
ndot_H2O = 4
ndot_N2 = 18.8
ndot3 = ndot_CO2 + ndot_CO + ndot_O2 + ndot_H2O + ndot_N2
yCO2 = ndot_CO2 / ndot3
yCO = ndot_CO / ndot3
yO2 = ndot_O2 / ndot3
pref = 1  // atm

// For the reaction  CO2  ↔  CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^.5
// Data from Table A-27are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T3,3)

0 = Qdot + ndotfuel*hC3H8 + 5*hO2_2 + 18.8*hN2_2 - (ndot_CO2*hCO2_3 +
ndot_CO*hCO_3 + ndot_O2*hO2_3 + ndot_H2O*hH2O_3 + 18.8*hN2_3)
hC3H8 = h_T("C3H8",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
hN2_3 = h_T("N2",T3)
```

<u>*IT* Results</u>

K = 0.001306

$\dot{n}_{CO,3} / \dot{n}_{fuel}$ = 0.09071 lbmol/lbmol(C_3H_8)

$\dot{n}_{CO_2,3} / \dot{n}_{fuel}$ = 2.909 lbmol/lbmol(C_3H_8)

$\dot{n}_{O_2,3} / \dot{n}_{fuel}$ = 0.04536 lbmol/lbmol(C_3H_8)

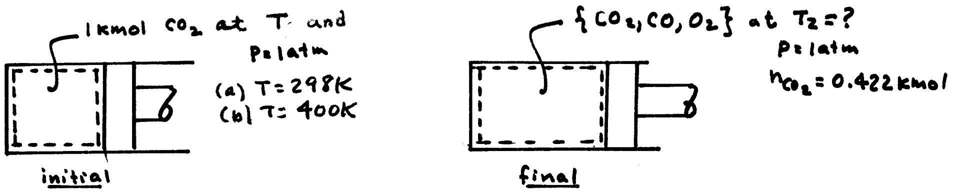
$\dot{Q}_{cv} / \dot{n}_{fuel}$ = -1.985E5 Btu/lbmol(C_3H_8)

PROBLEM 14.44

KNOWN: One kmol of CO_2 initially at T, 1atm is heated at constant pressure until a final state is attained consisting of an equilibrium mixture of CO_2, CO, an O_2 in which 0.422 kmol of CO_2 is present.

FIND: Determine the heat transfer and the work for (a) T= 298K, (b) T=400K.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is shown in the accompanying figure by the dashed line. (2) Changes in kinetic and potential energy are negligible. (3) The final mixture is an equilibrium mixture. (4) Pressure remains constant during the process. (4) Ideal gas model applies to the CO_2 initially present and to the final equilibrium mixture.

ANALYSIS: The first step is to determine T_2: The reaction takes the form

$$1\,CO_2 \rightarrow 0.422\,CO_2 + \alpha\,CO + \beta\,O_2$$

C: $1 = 0.422 + \alpha \Rightarrow \alpha = 0.578$

O: $2 = 2(0.422) + 0.578 + 2\beta \Rightarrow \beta = 0.289$

Thus

$$1\,CO_2 \rightarrow 0.422\,CO_2 + 0.578\,CO + 0.289\,O_2$$

At equilibrium $CO_2 \rightleftarrows CO + 1/2\,O_2$. Accordingly Eq. 14.35 takes the form

$$K = \frac{[0.578][0.289]^{1/2}}{0.422}\left[\frac{p/p_{ref}}{1.299}\right]^{1/2} \Rightarrow K = \left[\frac{0.578}{0.422}\right]\left[\frac{0.289}{1.289}\right]^{1/2} \Rightarrow \log_{10}K = -0.1881$$

Interpolating in Table A-27, $T_2 \approx 3200K$.

(a) T=298K

The work is given by $W = \int_1^2 p\,dV = p(V_2 - V_1)$. Using the ideal gas equation of state, $pV_1 = n_1\bar{R}T_1$, $pV_2 = n_2\bar{R}T_2$. Thus

$$W = \bar{R}[n_2T_2 - n_1T_1] = \left(8.314\,\frac{kJ}{kmol\cdot K}\right)\left((1.289)(3200) - (1)(298)\right)kmol\cdot K$$

$$= 31\,816\ kJ \longleftarrow W$$

An energy balance gives $\Delta U = Q - W$. With $W = p(V_2 - V_1)$ this yields

$$Q = (U_2 - U_1) + p(V_2 - V_1) = (U_2 + p_2V_2) - (U_1 + p_1V_1) = H_2 - H_1$$

Accordingly

$$Q = [0.422\,\bar{h}_{CO_2}(T_2) + 0.578\,\bar{h}_{CO}(T_2) + 0.289\,\bar{h}_{O_2}] - 1\,\bar{h}_{CO_2}(T_1)$$

$$= 4.22[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{CO_2} + 0.578[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)]_{CO} +$$

$$0.289[\cancel{\bar{h}_f^\circ}^{\,0} + \bar{h}(T_2) - \bar{h}(298)]_{O_2} - (\bar{h}_f^\circ)_{CO_2}$$

with data from the ideal gas tables

$$Q = 0.422[-393,520 + 174,695 - 9364] + 0.578[-110,530 + 109,667 - 8669] +$$

$$0.289[114,809 - 8682] - [-393,520] = 322,386\,kJ \longleftarrow Q$$

PROBLEM 14.44 (Continued)

(b) T = 400K

As in part (a)

$$W = \bar{R}\,(n_2 T_2 - n_1 T_1)$$

$$= (8314)((1.289)(3200) - (1)(400))$$

$$= 30{,}968\ \text{kJ} \longleftarrow\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\ W$$

And as in part (a), $Q = H_2 - H_1$. The only difference is the evaluation of the initial CO_2 enthalpy:

$$[\bar{h}^\circ_f + \bar{h}(400) - \bar{h}(298)]_{CO_2} = [-393{,}520 + 13{,}372 - 9364]$$

$$= -389{,}512\ \text{kJ/kmol}(CO_2)$$

Then

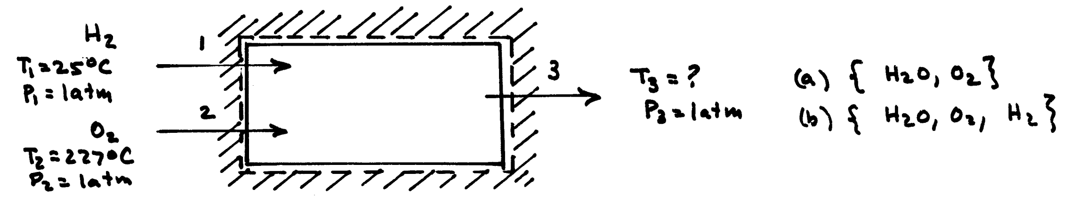
$$Q = 318{,}378\ \text{kJ} \longleftarrow\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\ Q$$

PROBLEM 14.45

KNOWN: H_2 at 25°C, 1atm enters an insulated reactor operating at steady state and reacts with 250% excess oxygen entering at 227°C, 1atm. Products of combustion exit at 1 atm.

FIND: Determine the temperature if (a) combustion is complete, (b) an equilibrium mixture of H_2O, H_2, and O_2 exits.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible kinetic and potential energy effects. (2) In part (a), combustion is complete. In part (b), an equilibrium mixture exits. (3) The ideal gas model is applicable.

ANALYSIS: (a) For complete combustion with 250% excess O_2, the reaction equation is

$$H_2 + 1.75O_2 \rightarrow H_2O + 1.25O_2$$

An energy rate balance at steady state reduces to give

$$0 = \frac{\cancelto{0}{\dot{Q}_{cv}}}{\dot{n}_{H_2}} - \frac{\cancelto{0}{\dot{W}_{cv}}}{\dot{n}_{H_2}} + [\bar{h}_{H_2}]_1 + [1.75\bar{h}_{O_2}]_2 - [\bar{h}_{H_2O} + 1.25\bar{h}_{O_2}]_3$$

Thus

$$0 = [\cancelto{0}{\bar{h}_f^o}]_{H_2} + 1.75[\cancelto{0}{\bar{h}_f^o} + \bar{h}(T_2) - \bar{h}(298)]_{O_2} - [\bar{h}_f^o + \bar{h}(T_3) - h(298)]_{H_2O} - 1.25[\cancelto{0}{\bar{h}_f^o} + \bar{h}(T_3) - \bar{h}(298)]_{O_2}$$

or

$$\bar{h}_{H_2O}(T_3) + 1.25\,\bar{h}_{O_2}(T_3) = -[\bar{h}_f^o - \bar{h}(298)]_{H_2O} + 1.75\,[\bar{h}(T_2) - \bar{h}(298)]_{O_2} + 1.25\,\bar{h}_{O_2}(298)$$

$$= -[-241{,}820 - 9904] + 1.75\,[14{,}770 - 8682] + 1.25\,(8682)$$

$$= 273{,}231 \text{ kJ/kmol}$$

Solving iteratively with data from Table A-23, $T_3 = 3033$ K.

(b) For this case the reaction takes the form

$$H_2 + 1.75O_2 \rightarrow x\,H_2O + (1-x)H_2 + \left(\frac{3.5-x}{2}\right)O_2$$

The amount of product mixture is $n = x + (1-x) + \left(\frac{3.5-x}{2}\right) = \left(\frac{5.5-x}{2}\right)$. At equilibrium $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[1-x]\left[\frac{3.5-x}{2}\right]^{1/2}}{x}\left[\frac{P_3/P_{ref}}{(5.5-x)/2}\right]^{1/2} = \frac{1-x}{x}\left[\frac{3.5-x}{5.5-x}\right]^{1/2} \qquad (1)$$

Another equation relating x and T_3 is obtained from an energy rate equation at steady state which reduces to give

$$0 = \cancelto{0}{\dot{Q}_{cv}} - \cancelto{0}{\dot{W}_{cv}} + [\bar{h}_{H_2}]_1 + 1.75[\bar{h}_{O_2}]_1 - \left[x\,\bar{h}_{H_2O} + (1-x)\,\bar{h}_{H_2} + \left(\frac{3.5-x}{2}\right)\bar{h}_{O_2}\right]_3$$

Noting that $\bar{h}_f^o = 0$ for H_2 and O_2 this becomes

$$0 = 1.75[\bar{h}_{O_2}(500) - \bar{h}_{O_2}(298)] - \Big\{x[(\bar{h}_f^o)_{H_2O} + \bar{h}_{H_2O}(T_3) - \bar{h}_{H_2O}(298)] + (1-x)[\bar{h}_{H_2}(T_3) - \bar{h}_{H_2}(298)] + \left(\frac{3.5-x}{2}\right)[\bar{h}_{O_2}(T_3) - \bar{h}_{O_2}(298)]\Big\}$$

or

$$x\,[-241{,}820 + \bar{h}_{H_2O}(T_3) - 9904] + (1-x)[\bar{h}_{H_2}(T_3) - 8468] + \left(\frac{3.5-x}{2}\right)[\bar{h}_{O_2}(T_3) - 8682]$$

$$= \underbrace{1.75[14{,}770 - 8682]}_{10{,}654} \qquad (2)$$

Equations (1), (2) are simultaneous equations in x and T_3. Solving iteratively with table data, we get

$$x = 0.9953, \quad T_3 = 2926 \text{ K}.$$

PROBLEM 14.45 (Cont'd.) Alternative IT solution.

The following IT solution provides an alternative to iteration using table data by hand, as shown previously :

IT Code

```
T1 = 25 + 273.15  // K
T2 = 227 + 273.15  // K
p3 = 1  // atm
ndot1 = 1  //  Do calculations on the basis of 1 kmol of H2 entering.

/* Use this section for part (a), comment out part (b).
// For part (a):   H2 + 1.75 O2  →  H2O + 1.25 O2
hr = h_T("H2",T1) + 1.75 * h_T("O2",T2)
hp = h_T("H2O",T3) + 1.25 * h_T("O2",T3)
hp = hr
*/

// For part (b)  H2 + 1.75 O2  →  x H2O + (1 - x) H2 + ((3.5 - x)/2) O2
ndotH2O_3 = x
ndotH2_3 = 1 - x
ndotO2_3 = (3.5 - x) / 2
ndot3 = (5.5 - x) / 2
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

// For the reaction  H2O  ↔  H2 + 1/2 O2
K = ((yH2 * yO2^0.5) / yH2O) * (p3 / pref)^0.5
//  Data from Table A- 27are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,1,T3,3)

hr = h_T("H2",T1) + 1.75 * h_T("O2",T2)
hp = ndotH2O_3 * hH2O_3 + ndotH2_3 * hH2_3 + ndotO2_3 * hO2_3
hp = hr
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hO2_3 = h_T("O2",T3)
```

IT Results

Part (a): T_3 = 3033 K

Part (b): x = 0.9533, T_3 = 2926 K

PROBLEM 14.46

KNOWN: H_2 at 25°C, 1atm enters an insulated reactor operating at steady state and reacts with 250% excess oxygen entering at 227°C, 1atm. Products of combustion exit at 1atm.

FIND: Determine the rate of entropy production per kmol of H_2 entering if (a) combustion is complete, (b) an equilibrium mixture of H_2O, H_2, and O_2 exits.

SCHEMATIC & GIVEN DATA: See Problem 14.45 solution.

ASSUMPTIONS: See Problem 14.45 solution.

ANALYSIS: (a) Complete combustion of H_2 with 250% excess O_2 is described by

$$H_2 + 1.75\,O_2 \rightarrow H_2O + 1.25\,O_2$$

The solution to Problem 14.45(a) gives 3033 K as the temperature of the products.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} = (\bar{s}_{H_2O} + 1.25\,\bar{s}_{O_2})_2 - (\bar{s}_{H_2})_1 - 1.75(\bar{s}_{O_2})_2 \qquad (1)$$

As H_2 enters at 25°C, 1atm, Table A-23 gives $\bar{s}_{H_2} = 130.57$ kJ/kmol·K. O_2 enters at 500 K, 1atm. Thus, with Eq. 13.22 and data from Table A-23 $\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(500) = 220.589$ kJ/kmol·K. The products exit as a mixture at 3033 K, 1atm with composition $y_{O_2} = 1.25/2.25$, $y_{H_2O} = 1/2.25$. Thus, with Eq. 13.23 and data from Table A-23

$$\bar{s}_{H_2O} = \bar{s}^{\circ}_{H_2O}(3033) - \bar{R}\ln\frac{y_{H_2O}\,p_{ref}}{p_{ref}} = 286.88 - 8.314\ln\frac{1}{2.25} = 293.62 \text{ kJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(3033) - \bar{R}\ln y_{O_2} = 284.84 - 8.314\ln\frac{1.25}{2.25} = 289.73 \text{ kJ/kmol·K}$$

Inserting values into Eq. (1)

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} = (293.62) + (1.25)(289.73) - 130.57 - 1.75(220.589) = 139.18 \frac{\text{kJ}}{\text{kmol}(H_2)\cdot\text{K}} \longleftarrow (a)$$

(b) The reaction is described by

$$H_2 + 1.75\,O_2 \longrightarrow 0.955\,H_2O + 0.045\,H_2 + 1.2725\,O_2$$

The temperature of the products is obtained in the solution to Problem 14.45(b) as 2926 K.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} = (0.955\,\bar{s}_{H_2O} + 0.045\,\bar{s}_{H_2} + 1.2725\,\bar{s}_{O_2})_3 - (\bar{s}_{H_2})_1 - (\bar{s}_{O_2})_2 \qquad (2)$$

The values of $(\bar{s}_{H_2})_1$ and $(\bar{s}_{O_2})_2$ are the same as in part (a) above. The products exit as a mixture at 2926 K, 1atm with the composition $y_{H_2O} = 0.955/2.2725$, $y_{H_2} = 0.045/2.2725$, $y_{O_2} = 1.2725/2.2725$. Thus, with Eq. 13.23 and data from the ideal gas tables

$$\bar{s}_{H_2O} = \bar{s}^{\circ}_{H_2O}(2926) - \bar{R}\ln y_{H_2O} = 284.88 - 8.314\ln\frac{0.955}{2.2725} = 292.09 \text{ kJ/kmol·K}$$

$$\bar{s}_{H_2} = \bar{s}^{\circ}_{H_2}(2926) - \bar{R}\ln y_{H_2} = 201.85 - 8.314\ln\frac{0.045}{2.2725} = 234.46 \text{ kJ/kmol·K}$$

$$\bar{s}_{O_2} = \bar{s}^{\circ}_{O_2}(2926) - \bar{R}\ln y_{O_2} = 283.40 - 8.314\ln\frac{1.2725}{2.2725} = 288.22 \text{ kJ/kmol·K}$$

Inserting values into Eq. (2)

① $$\frac{\dot{\sigma}_{cv}}{\dot{n}_{H_2}} = (.955)(292.09) + (0.045)(234.46) + (1.2725)(288.22) - 130.57 - 1.75(220.589) = 139.66 \frac{\text{kJ}}{\text{kmol}(H_2)\cdot\text{K}} \longleftarrow (b)$$

---

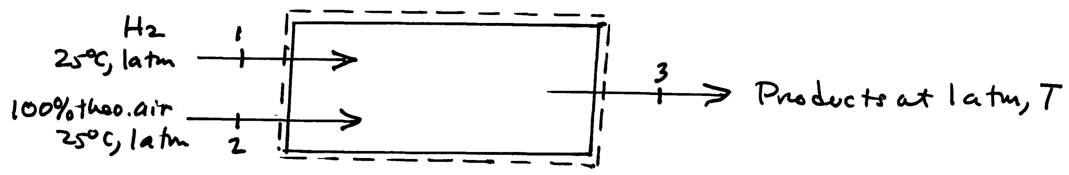
1. The greater entropy production of part (b) suggests that complete combustion would not occur under the specified conditions.

PROBLEM 14.47

KNOWN: H_2 at 25°C, 1atm enters an insulated reactor operating at steady state and reacts with 100% of theoretical air entering at 25°C, 1atm. The products of combustion exit at 1atm and temperature T.

FIND: Determine T if (a) combustion is complete, (b) an equilibrium mixture of H_2O, H_2, O_2, and N_2 exits.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown above is at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic/potential energy effects are negligible. (3) The ideal gas model is applicable. (4) In part (a) combustion is complete. In part (b) an equilibrium mixture is formed.

ANALYSIS: (a) The complete reaction with 100% theoretical air is

$$H_2 + 0.5O_2 + 1.88N_2 \rightarrow H_2O + 1.88N_2$$

An energy rate balance reduces with given assumptions to read $H_R = H_P$, or

$$0 = [\bar{h}_f^\circ + \Delta\bar{h}]_{H_2O} + 1.88[\cancel{\bar{h}_f^\circ}^{0} + \Delta\bar{h}]_{N_2} - [\cancel{\bar{h}_f^\circ}^{0}]_{H_2} - 1.88[\cancel{\bar{h}_f^\circ}^{0}]$$

Inserting ideal gas table data

$$0 = [-241{,}820 + \bar{h}_{H_2O}(T) - 9904] + 1.88[\bar{h}_{N_2}(T) - 8669] = 0$$

or

$$\bar{h}_{H_2O}(T) + 1.88\bar{h}_{N_2}(T) = 268{,}022 \text{ kJ/kmol}.$$ Solving iteratively with table data, T = 2525 K. ←

(b) If an equilibrium mixture is formed, the reaction equation reads

$$H_2 + 0.5O_2 + 1.88N_2 \rightarrow (1-x)H_2O + xH_2 + \frac{x}{2}O_2 + 1.88N_2$$

where $n = (1-x) + x + x/2 + 1.88 = 2.88 + x/2$. For $H_2O \rightleftarrows \frac{1}{2}O_2 + H_2$ and $p/p_{ref} = 1$

$$K = \frac{(x/2)^{1/2}(x)}{(1-x)}\left[\frac{p/p_{ref}}{n}\right]^{\frac{1}{2}+1-1} = \left(\frac{x}{1-x}\right)\left(\frac{x/2}{2.88+x/2}\right)^{1/2} \qquad (1)$$

An energy rate balance reduces to $H_R = H_P$, or

$$0 = (1-x)[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{H_2O} + x[\bar{h}(T) - \bar{h}(298)]_{H_2} + \frac{x}{2}[\bar{h}(T) - \bar{h}(298)]_{O_2} + 1.88[\bar{h}(T) - \bar{h}(298)]_{N_2}$$

with ideal gas table data

$$0 = (1-x)[-241{,}820 + \bar{h}_{H_2O}(T) - 9904] + x[\bar{h}_{H_2}(T) - 8468] + \frac{x}{2}[\bar{h}_{O_2}(T) - 8682] + 1.88[\bar{h}_{N_2}(T) - 8669] \qquad (2)$$

Equations (1), (2) are simultaneous equations for x, T. Solving iteratively

① with table data, x = 0.045, T = 2433 K.

---

1. Because of dissociation – case (b) – the temperature of the products is less than in the complete combustion case (a).

PROBLEM 14.47 (Cont'd.)

Alternative Solution Using IT

IT Code

```
T1 = 25 + 273.15  // K
T2 = 25 + 273.15  // K
p3 = 1  // atm
ndot1 = 1  // Do all calcualtions on the basis of I kmol of H2 entering.


// For part (a):  H2 + 0.5 O2 + 1.88 N2  →  H2O + 1.88 N2
hr = h_T("H2",T1) + 0.5 * h_T("O2",T2) + 1.88 * h_T("N2",T2)
hp = h_T("H2O",Tp) + 1.88 * h_T("N2",Tp)
hp =hr

/* Use this section fcr part (b) and comment out part (a).
// For part (b):  H2 + 0.5 O2 + 1.88 N2  →  (1 - x) H2O + x H2 + x/2 O2 +
1.88 N2
ndotH2O_3 = 1 - x
ndotH2_3 = x
ndotO2_3 = x/2
ndotN2_3 = 1.88
ndot3 = ndotH2O_3 + ndotH2_3 + ndotO2_3 + ndotN2_3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

// For the reaction H2O  ↔  H2 + 1/2 O2

K = ((yH2 * yO2^0.5) / yH2O) * (p3 / pref)^0.5
// Data from Table A-27 are stored in EQH2O.LUT.
log(K) = LOOKUPVAL(EQH2O,1,T3,3)

hr = h_T("H2",T1) + 0.5 * h_T("O2",T2) + 1.88 * h_T("N2",T2)
hp = (1-x)*hH2O_3 + x*hH2_3 + (x/2)*hO2_3 + 1.88*hN2_3
hp = hr
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
*/
```

IT Results

Part (a): T_3 = 2526 K

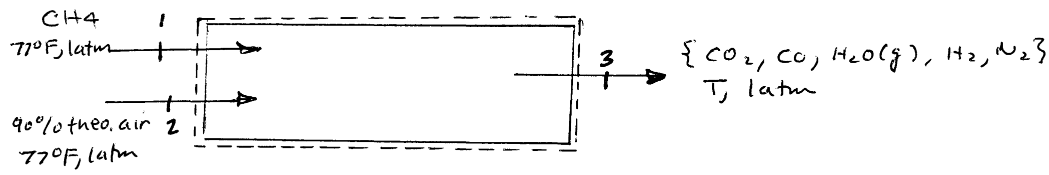
Part(b): x = 0.04563, K = 0.004239, T_3 = 2432 K

PROBLEM 14.48

KNOWN: CH_4 at 77°F, 1atm enters an insulated reactor and burns with 90% of theoretical air entering separately at 77°F, 1atm. An equilibrium mixture $\{CO_2, CO, H_2O(g), H_2, N_2\}$ exits at 1atm.

FIND: Determine the temperature of the exiting mixture.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure operates at steady state with $\dot{W}_{cv} = \dot{Q}_{cv} = 0$ and negligible kinetic/potential energy effects. (2) The exiting equilibrium mixture is modeled as an ideal gas mixture. (3) N_2 is inert.

ANALYSIS: The complete combustion of CH_4 with the theoretical amount of air is given by Eq. 13.4. Accordingly, combustion with 90% of theoretical air to form the specified mixture is described by

$$CH_4 + 1.8O_2 + 6.768N_2 \rightarrow (1-x)CO_2 + xCO + (1.6+x)H_2O + (0.4-x)H_2 + 6.768N_2$$

At equilibrium $CO_2 + H_2 \rightleftarrows CO + H_2O$. Accordingly, Eq 14.35 takes the form

$$K = \frac{[x][1.6+x]}{[1-x][0.4-x]}\left[\frac{p/p_{ref}}{n}\right]^0 = \frac{x(1.6+x)}{(1-x)(0.4-x)} \qquad (1)$$

An energy rate balance at steady state reduces to

$$[1-x]\left[\bar{h}_f^\circ + \bar{h}(T) - h(537)\right]_{CO_2} + x\left[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)\right]_{CO} + (1.6+x)\left[\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(537)\right]_{H_2O}$$
$$+ [0.4-x]\left[\bar{h}(T) - \bar{h}(537)\right]_{H_2} + 6.768\left[\bar{h}(T) - \bar{h}(537)\right]_{N_2} - \left(\bar{h}_f^\circ\right)_{CH_4}$$

Inserting table data

$$0 = [1-x][-169{,}300 + \bar{h}_{CO_2} - 4028] + x[-47{,}540 + \bar{h}_{CO} - 3725] + (1.6+x)[-104{,}040 + \bar{h}_{H_2O}(T) - 4258]$$
$$+ [0.4-x][\bar{h}_{H_2} - 3640] + 6.768[\bar{h}_{N_2} - 3730] - (-32{,}210)$$

or

$$0 = [1-x][-173{,}328 + \bar{h}_{CO_2}] + x[-51{,}265 + \bar{h}_{CO}] + [1.6+x][-108{,}298 + \bar{h}_{H_2O}] +$$
$$[0.4-x][\bar{h}_{H_2} - 3640] + 6.768[\bar{h}_{N_2} - 3730] + 32{,}210 \qquad (2)$$

Equations (1), (2) are simultaneous in x, T. When solved iteratively
① using Table data, $T = 4000°R$, $x = 0.2693$.

---

1. For the products to include both CO and H_2, inspecting the reaction equation shows that x must be in the interval $0 < x < 0.4$.

PROBLEM 14.48 (Cont'd.)

Alternative Solution Using IT

IT Code

```
T1 = 77 + 459.67  // °R
T2 = 77 + 459.67  // °R
p3 = 1  // atm
ndot_CH4 = 1  // Do calculations on the basis of 1 kmol of CH4 entering.

// CH4 + 1.8 O2 + 6.768 N2  →  (1 - x) CO2 + x CO + (1.6 + x) H2O + (0.4 - x) H2 + 6.768 N2
ndotCO2_3 = 1 - x
ndotCO_3 = x
ndotH2O_3 = 1.6 + x
ndotH2_3 = 0.4 - x
ndotN2_3 = 6.768
ndotO2_3 = 1.8
ndot3 = ndotCO2_3 + ndotCO_3 + ndotH2O_3 + ndotH2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yH2 = ndotH2_3 / ndot3
pref = 1  // atm

//  For the reaction: CO2 + H2  ↔  CO + H2O
K = (yCO * yH2O) / (yCO2 * yH2)
//  Data from Table A-27 are stored in EQWATGAS.LUT.
log(K) = LOOKUPVAL(EQWATGAS,2,T3,3)

0 = hCH4_1 + 1.8*hO2_2 + 6.768*hN2_2 - (1-x)*hCO2_3 - x*hCO_3 - (1.6+x)*hH2O_3 -
        (0.4-x)*hH2_3 -6.768*hN2_3
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hH2O_3 = h_T("H2O",T3)
hH2_3 = h_T("H2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

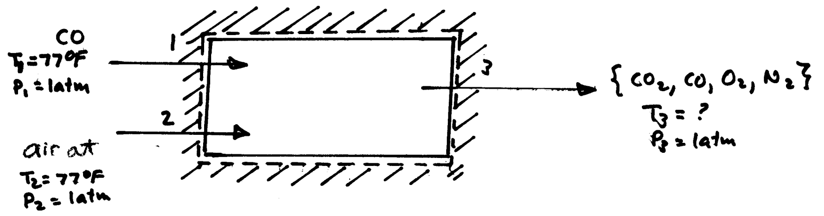
K = 5.276
x = 0.2694
T_3 = 4003°R

PROBLEM 14.49

KNOWN: CO at 77°F, 1atm enters an insulated reactor operating at steady state and burns with air entering at 77°F, 1atm. An equilibrium mixture of CO_2, CO, O_2, N_2 exits at 1atm, T.

FIND: Determine T if the combustion occurs with (a) 80% theoretical air, (b) 100% theoretical air.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas as is the entering combustion air. (3) N_2 is inert.

ANALYSIS: (a) The reaction of CO with 80% of theoretical air to form $\{CO_2, CO, O_2, N_2\}$ is described by

$$CO + 0.4O_2 + 1.504N_2 \rightarrow x\,CO_2 + (1-x)\,CO + \left(\frac{0.8-x}{2}\right)O_2 + 1.504\,N_2$$

The amount of mixture is $n = x + (1-x) + \left(\frac{0.8-x}{2}\right) + 1.504 = (5.808-x)/2$. At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[1-x]\left[\frac{0.8-x}{2}\right]^{1/2}}{x}\left[\frac{p_3/p_{ref}}{(5.808-x)/2}\right]^{1/2} = \frac{1-x}{x}\left[\frac{0.8-x}{5.808-x}\right]^{1/2} \qquad (1)$$

Another equation relating x and T_3 is obtained from an energy rate equation at steady state which reduces to give

$$0 = \frac{\cancelto{0}{\dot{Q}_{cv}}}{\dot{n}_{CO}} - \frac{\cancelto{0}{\dot{W}_{cv}}}{\dot{n}_{CO}} + [h_{CO}]_1 + [0.4\bar{h}_{O_2} + 1.504\bar{h}_{N_2}]_2 - \left[x\bar{h}_{CO_2} + (1-x)\bar{h}_{CO} + \left(\frac{0.8-x}{2}\right)\bar{h}_{O_2} + 1.504\bar{h}_{N_2}\right]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O_2 and N_2 this becomes

$$0 = [\bar{h}_f^\circ]_{CO} - \left\{x[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + (1-x)[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO} + \left(\frac{0.8-x}{2}\right)[\bar{h}(T_3) - \bar{h}(537)]_{O_2} + 1.504[\bar{h}(T_3) - \bar{h}(537)]_{N_2}\right.$$

or with data from Table A-23E

$$x[-169{,}300 + \bar{h}_{CO_2}(T_3) - 4027.5] + (1-x)[-47{,}540 + \bar{h}_{CO}(T_3) - 3725.1] +$$
$$\left(\frac{0.8-x}{2}\right)[\bar{h}_{O_2}(T_3) - 3725.1] + 1.504[\bar{h}_{N_2}(T_3) - 3729.3] = -47{,}540 \qquad (2)$$

For the products to include both CO_2 and O_2, inspection of the reaction equation shows that x must be in the interval $0 < x < 0.8$. Eqs. (1), (2) are simultaneous in x and T_3. Solving iteratively with table data, $T_3 = 4380°R$, $x = 0.766$.

(b) The reaction of CO with the theoretical amount of air to produce CO_2, CO, O_2, and N_2 is

$$CO + \frac{1}{2}O_2 + 1.88N_2 \rightarrow x\,CO + (1-x)CO_2 + \frac{x}{2}O_2 + 1.88N_2$$

For the products to include both CO and CO_2, it is necessary for x to be in the interval $0 < x < 1$.

The amount of mixture is $n = x + (1-x) + \frac{x}{2} + 1.88 = (5.76+x)/2$. At equilibrium, $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Thus, Eq. 14.35 takes the form

$$K(T_3) = \frac{[x][x/2]^{1/2}}{[1-x]}\left[\frac{p_3/p_{ref}}{(5.76+x)/2}\right]^{1/2} = \left[\frac{x}{1-x}\right]\left[\frac{x}{5.76+x}\right]^{1/2} \qquad (1)$$

PROBLEM 14.49 (Contd.)

Another equation involving T_3 and x can be obtained from an energy rate balance which reduces at steady state to

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{0}}{\cancel{\dot{n}}_{CO}} - \frac{\cancel{\dot{W}_{cv}}^{0}}{\cancel{\dot{n}}_{CO}} + \left[0.5\bar{h}_{O_2} + 1.88\bar{h}_{N_2}\right]_2 + \left[\bar{h}_{CO}\right]_1 - \left[x\bar{h}_{CO} + (1-x)\bar{h}_{CO_2} + \frac{x}{2}\bar{h}_{O_2} + 1.88\bar{h}_{N_2}\right]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O_2 and N_2 this becomes

$$x\left[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)\right]_{CO} + (1-x)\left[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)\right]_{CO_2} + \frac{x}{2}\left[\bar{h}(T_3) - h(298)\right]_{O_2}$$
$$+ 1.88\left[\bar{h}(T_3) - \bar{h}(298)\right]_{N_2} = \left(\bar{h}_f^\circ\right)_{CO}$$

Noting that 77°F corresponds to 25°C, we use data from the SI table Table A-23 to obtain

$$x\left[-110{,}530 + \bar{h}(T_3) - 8669\right]_{CO} + (1-x)\left[-393{,}520 + \bar{h}(T_3) - 9364\right]_{CO_2} + \frac{x}{2}\left[\bar{h}(T_3) - 8682\right]_{O_2} +$$
$$1.88\left[\bar{h}(T_3) - 8669\right]_{N_2} = -110{,}530$$

or

$$x\left[\bar{h}_{CO}(T_3) - 119{,}199\right] + (1-x)\left[\bar{h}_{CO_2}(T_3) - 402{,}884\right] + \frac{x}{2}\left[\bar{h}_{O_2}(T_3) - 8682\right] + 1.88\left[\bar{h}_{N_2}(T_3) - 8669\right]$$
$$= -110{,}530 \quad (2)$$

Equations (1), (2) are simultaneous equations for x, T_3. Solving ① iteratively with table data, $x = 0.125$, $T_3 = 2399\ K$ (4318°R)

---

1. As expected, the greater amount of air used in part (b) results in a lower combustion product temperature T_3 and a greater amount of CO_2 formed.

PROBLEM 14.49 (Cont'd.)

IT can be used as an alternative to an iterative solution by hand using table data. The program follows.

IT Code

```
T1 = 77 + 459.67  // °R
T2 = 77 + 459.67  // °R
p3 = 1  // atm
ndotCO_1 = 1  // Do calculations on the basis of 1 lbmol of CO entering.

//  CO + a (0.5) (O2 + 3.76 N2)  → x CO2 + (1 - x) CO + ((a - x) / 2) O2 + a 1.88 N2

ndotCO2_3  = x
ndotCO_3 = 1 - x
ndotO2_3 = (a - x) / 2
ndotN2_3 = a * 1.88
ndot_3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot_3
yO2 = ndotO2_3 / ndot_3
yCO = ndotCO_3 / ndot_3
pref = 1  // atm
//  Part (a):
a = 0.8
//  For Part(b): Set a = 1.

//  For the reaction CO2  <===>  CO + 1/2 O2
K = (((yCO) * (yO2)^0.5) / (yCO2)) * (p3 / pref)^.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T3,3)

0 = hCO_1 + a*0.5*hO2_2 + a*1.88*hN2_2 - x*hCO2_3 - (1-x)*hCO_3 -
        ((a-x)/2)*hO2_3 - a*1.88*hN2_3
hCO_1 = h_T("CO",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

Part (a): 80% theoretical air: x = 0.7658, T_3 = 4380°R

Part (b): Theoretical air: x = 0.8751 (lbmol of CO out per lbmol of CO in)
T3 = 4318°R

PROBLEM 14.50

KNOWN: See solution to Problem 14.49.

FIND: For each case, determine the exergy destruction per kmol of CO entering.

SCHEMATIC & GIVEN DATA: See solution to Problem 14.49.

ASSUMPTIONS: 1. See solution to Problem 14.49. 2. $T_0 = 537°R$.

ANALYSIS: (a) From the solution to Problem 14.49, the reaction is

$$CO + 0.4O_2 + 1.504N_2 \longrightarrow 0.766\,CO_2 + 0.234\,CO + 0.017\,O_2 + 1.504\,N_2$$

where the products are at 4380°R.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = (0.766\,\bar{s}_{CO_2} + 0.234\,\bar{s}_{CO} + 0.017\,\bar{s}_{O_2} + 1.504\,\bar{s}_{N_2})_3 - (\bar{s}_{CO})_1 - (0.4\,\bar{s}_{O_2} + 1.504\,\bar{s}_{N_2})_2 \quad (1)$$

The carbon monoxide enters at 77°F, 1atm. Thus from Table A-23E $(\bar{s}_{CO})_1 = \bar{s}^{o}_{CO}(537°R) = 47.27$ Btu/lbmol·°R. The combustion air enters at 77°F, 1atm, so with Eq. 13.23

$$\bar{s}_{O_2} = \bar{s}^{o}_{O_2}(537) - \bar{R}\ln y_{O_2} = 48.98 - 1.986\ln 0.21 = 52.08 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^{o}_{N_2}(537) - \bar{R}\ln y_{N_2} = 45.74 - 1.986\ln 0.79 = 46.21 \text{ Btu/lbmol·°R}$$

The products exit at 4380°R, 1atm with the composition, $y_{CO_2} = \frac{0.766}{2.521}$, $y_{CO} = \frac{0.234}{2.521}$, $y_{O_2} = \frac{0.017}{2.521}$, $y_{N_2} = \frac{1.504}{2.521}$. Thus, with Eq. 13.23 and data from Table A-23E

$$\bar{s}_{CO_2} = \bar{s}^{o}_{CO_2}(4380) - \bar{R}\ln y_{CO_2} = 76.736 - 1.986\ln\frac{0.766}{2.521} = 79.10 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{CO} = \bar{s}^{o}_{CO}(4380) - \bar{R}\ln y_{CO} = 63.567 - 1.986\ln\frac{0.234}{2.521} = 68.29 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{O_2} = \bar{s}^{o}_{O_2}(4380) - \bar{R}\ln y_{O_2} = 65.958 - 1.986\ln\frac{0.017}{2.521} = 75.89 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^{o}_{N_2}(4380) - \bar{R}\ln y_{N_2} = 61.887 - 1.986\ln\frac{1.504}{2.521} = 62.91 \text{ Btu/lbmol·°R}$$

Inserting values into Eq. (1)

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = (0.766)(79.10) + (0.234)(68.29) + (0.017)(75.89) + (1.504)(62.91) - (47.27) - (0.4)(52.08) - (1.504)(46.21)$$

$$= 34.88 \text{ Btu/lbmol·°R}$$

Calculating the exergy destruction in SI units, as required

$$\frac{\dot{E}_d}{\dot{n}_{CO}} = T_0\frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = (537°R)\left(34.88\,\frac{\text{Btu}}{\text{lbmol·°R}}\right)\left|\frac{1\text{ kJ}}{0.94788\text{ Btu}}\right|\left|\frac{2.2046\text{ lb}}{1\text{ kg}}\right|$$

$$= 43{,}568\ \frac{\text{kJ}}{\text{kmol(CO)}} \longleftarrow (a)$$

PROBLEM 14.50 (Contd.)

(b) From the solution to Problem 14.49(b)

$$CO + \frac{1}{2}O_2 + 1.88 N_2 \rightarrow 0.125\, CO + 0.875\, CO_2 + 0.0625\, O_2 + 1.88\, N_2$$

where the combustion products are at 2399 K.

At steady state an entropy rate balance reduces to give

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = \left(0.125\,\bar{s}_{CO} + 0.875\,\bar{s}_{CO_2} + 0.0625\,\bar{s}_{O_2} + 1.88\,\bar{s}_{N_2}\right)_3 - (\bar{s}_{CO})_1 - \left(\tfrac{1}{2}\bar{s}_{O_2} + 1.88\,\bar{s}_{N_2}\right)_2 \quad (1)$$

① The CO enters at 25°C, 1 atm. Thus $(\bar{s}_{CO})_1 = \bar{s}^\circ_{CO}(298) = 197.54$ kJ/kmol·K from Table A-23. The air enters at 25°C, 1 atm, so with Eq. 13.23 and data from Table A-23

$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(298) - \bar{R}\ln y_{O_2} = 205.03 - 8.314 \ln 0.21 = 218.01$ kJ/kmol·K

$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(298) - \bar{R}\ln y_{N_2} = 191.5 - 8.314 \ln 0.79 = 193.46$ kJ/kmol·K

The combustion products exit at 2399 K, 1 atm with composition $y_{CO} = \frac{0.125}{2.9425}$, $y_{CO_2} = \frac{0.875}{2.9425}$, $y_{O_2} = \frac{0.0625}{2.9425}$, $y_{N_2} = \frac{1.88}{2.9425}$). Thus with Eq. 13.23 and data from Table A-23

$\bar{s}_{CO} = \bar{s}^\circ_{CO}(2399) - \bar{R}\ln y_{CO} = 265.238 - \bar{R}\ln\frac{0.125}{2.9425} = 291.499$ kJ/kmol·K

$\bar{s}_{CO_2} = \bar{s}^\circ_{CO_2}(2399) - \bar{R}\ln y_{CO_2} = 320.276 - \bar{R}\ln\frac{0.875}{2.9425} = 330.359$ kJ/kmol·K

$\bar{s}_{O_2} = \bar{s}^\circ_{O_2}(2399) - \bar{R}\ln y_{O_2} = 275.609 - \bar{R}\ln\frac{0.0625}{2.9425} = 307.633$ kJ/kmol·K

$\bar{s}_{N_2} = \bar{s}^\circ_{N_2}(2399) - \bar{R}\ln y_{N_2} = 258.565 - \bar{R}\ln\frac{1.88}{2.9425} = 262.290$ kJ/kmol·K

Inserting values in Eq. (1)

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = (0.125)(291.499) + (0.875)(330.359) + (0.0625)(307.633) + (1.88)(262.290) - (197.54) - (0.5)(218.01) - 1.88(193.46)$$

$$= 167.583 \text{ kJ/kmol(CO)}\cdot\text{K}$$

Then

② $$\frac{\dot{E}_d}{\dot{n}_{CO}} = T_0 \frac{\dot{\sigma}_{cv}}{\dot{n}_{CO}} = (298\,\text{K})\left(167.583 \frac{\text{kJ}}{\text{kmol(CO)}\cdot\text{K}}\right) = 49{,}940 \frac{\text{kJ}}{\text{kmol(CO)}} \longleftarrow \text{(b)}$$

---

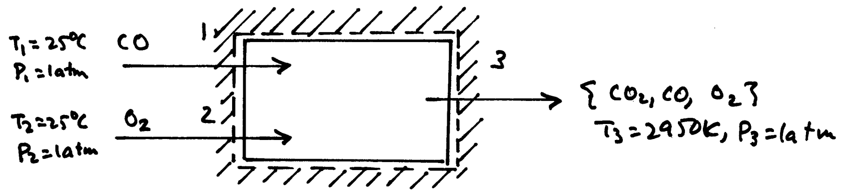
1. Working in SI units as in Problem 14.49(b)
2. The exergy destruction of part (b) is greater than in part (a) because more air is used, and this entails more mixing within the control volume.

PROBLEM 14.51

KNOWN: CO at 25°C, 1atm enters an insulated reactor and burns with excess O_2 entering at 25°C, 1atm. An equilibrium mixture of CO_2, CO, and O_2 exits at 1atm, 2950K.

FIND: Determine the percent excess O_2.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture is modeled as an ideal gas.

ANALYSIS: The reaction of CO with O_2 to produce CO_2, CO, and O_2 is

$$CO + \alpha O_2 \longrightarrow x\, CO_2 + (1-x)\, CO + \left(\frac{2\alpha - x}{2}\right) O_2$$

The amount of mixture is $n = x + (1-x) + \left(\frac{2\alpha - x}{2}\right) = \frac{2(1+\alpha) - x}{2}$. At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$. Accordingly Eq. 14.35 takes the form

$$K = \frac{[1-x]\left[\frac{2\alpha - x}{2}\right]^{1/2}}{x} \left[\frac{P_3/P_{ref}}{(2(1+\alpha) - x)/2}\right]^{1/2} = \left(\frac{1-x}{x}\right)\left(\frac{2\alpha - x}{2(1+\alpha) - x}\right)^{1/2}$$

At 2950 K Table A-27 gives $\log_{10} K = -0.567 \Rightarrow K = 0.271019$. Thus

$$0.07345 = \left[\frac{1-x}{x}\right]^2 \left(\frac{2\alpha - x}{2(1+\alpha) - x}\right) \qquad (1)$$

Another equation relating α and x can be obtained from an energy rate equation which reduces at steady state to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CO}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CO}}}^{0} + [\bar{h}_{CO}]_1 + \alpha [\bar{h}_{O_2}]_2 - \left[x \bar{h}_{CO_2} + (1-x)\bar{h}_{CO} + \left(\frac{2\alpha - x}{2}\right)\bar{h}_{O_2}\right]_3$$

Then, with $\bar{h}^\circ_f = 0$ for O_2 this gives

$$x[\bar{h}^\circ_f + \bar{h}_{CO_2}(2950) - \bar{h}_{CO_2}(298)]_{CO_2} + (1-x)[\bar{h}^\circ_f + \bar{h}_{CO}(2950) - \bar{h}_{CO}(298)]_{CO} + \left(\frac{2\alpha - x}{2}\right)[\bar{h}_{O_2}(2950) - \bar{h}_{O_2}(298)] = (\bar{h}^\circ_f)_{CO}$$

or with table data

$$x[-393{,}520 + 159{,}117 - 9364] + (1-x)[-110{,}530 + 100{,}352 - 8669] + \left(\frac{2\alpha - x}{2}\right)[104{,}785 - 8682] = -110{,}530$$

$$x[-243{,}767] + (1-x)[-18{,}847] + \left(\frac{2\alpha - x}{2}\right)(96{,}103) = -110{,}530$$

On further reduction

$$\alpha = 2.8404x - 0.954 \qquad (2)$$

Combining Eqs. (1) and (2)

$$0.07345 = \left[\frac{1-x}{x}\right]^2 \left[\frac{4.6808x - 1.908}{4.6808x + 0.092}\right] \Rightarrow x = 0.702$$

Using this value for x, Eq (2) gives $\alpha = 1.04$. The theoretical amount of O_2 required is 0.5 kmol/kmol(CO). Accordingly, the percent excess O_2 is 108%. ←

PROBLEM 14.51 (Cont'd.)

Alternative Solution Using IT

IT Code

```
T1 = 25 + 273.15  // K
T2 = 25 + 273.15  // K
T3 = 2950  // K
p3 = 1  // atm
ndotCO_1 = 1  // Do calculations on the basis of 1kmol of CO entering.

//  CO + (1+XS) (0.5) O2  →  x CO2 + (1 - x) CO + ((1 + XS - x) / 2) O2
ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotO2_3 = (1 + XS - x) / 2
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

//  For the reaction  CO2  ↔  CO + 1/2 O2
K = (((yCO) * (yO2)^0.5) / (yCO2)) * (p3 / pref)^0.5
//  Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)

0 = hCO_1 + (1+XS)*(0.5)*hO2_2 - x*hCO2_3 - (1-x)*hCO_3 - ((1+XS-x)/2)*hO2_3
hCO_1 = h_T("CO",T1)
hO2_2 = h_T("O2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
PCTXS = XS * 100
```

IT Results

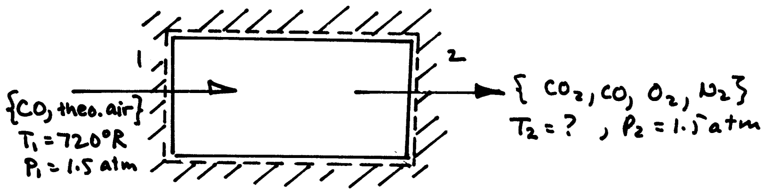
```
x = 0.7019
% excess oxygen = 107.6%
```

PROBLEM 14.52

KNOWN: A mixture of CO and the theoretical amount of air at 720 °R, 1.5 atm enters an insulated reactor operating at steady state. An equilibrium mixture of CO_2, CO, O_2, and N_2 exits at 1.5 atm.

FIND: Determine the temperature of the exiting mixture.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The ingoing and outgoing mixtures are modeled as ideal gas mixtures. (3) N_2 is inert.

ANALYSIS: The reaction is described by

$$CO + 0.5\,O_2 + 1.88\,N_2 \rightarrow x\,CO + (1-x)\,CO_2 + \frac{x}{2}\,O_2 + 1.88\,N_2$$

The amount of mixture is $n = (5.76 + x)/2$. At equilibrium $CO_2 \rightleftarrows CO + 1/2\,O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_2) = \frac{[x][x/2]}{[1-x]}\left[\frac{P_2/P_{ref}}{(5.76+x)/2}\right]^{1/2} = \left[\frac{x}{1-x}\right]\left[\frac{1.5x}{5.76+x}\right]^{1/2} \qquad (1)$$

Another equation involving T_2 and x can be obtained from an energy rate balance which reduces at steady state to

$$0 = \frac{\cancel{\dot{Q}_{cv}}^{\,0}}{\cancel{\dot{n}_{CO}}} - \frac{\cancel{\dot{W}_{cv}}^{\,0}}{\cancel{\dot{n}_{CO}}} + \left[0.5\bar{h}_{O_2} + 1.88\bar{h}_{N_2} + \bar{h}_{CO}\right]_1 - \left[x\bar{h}_{CO} + (1-x)\bar{h}_{CO_2} + \frac{x}{2}\bar{h}_{O_2} + 1.88\bar{h}_{N_2}\right]_2$$

or, with $\bar{h}_f^{\circ} = 0$ for O_2 and N_2, and data from the ideal gas tables

$$x[-47{,}540 + \bar{h}_{CO}(T_2) - 3725.1] + (1-x)[-169{,}300 + \bar{h}_{CO_2}(T_2) - 4027.5] + \frac{x}{2}[\bar{h}_{O_2}(T_2) - 3725.1]$$
$$+ 1.88[\bar{h}_{N_2}(T_2) - 3729.5] = 0.5[5022.9 - 3725.1] + 1.88[5004.5 - 3729.5] + [-47{,}540 + 5006.1 - 3725.1]$$

or

$$x[\bar{h}_{CO}(T_2) - 51{,}265] + (1-x)[\bar{h}_{CO_2}(T_2) - 173{,}328] + \frac{x}{2}[\bar{h}_{O_2}(T_2) - 3725.1] + 1.88[\bar{h}_{N_2}(T_2) - 3729.5]$$
$$= -43{,}213 \qquad (2)$$

Equations (1), (2) are simultaneous in X and T_2. Inspection of the reaction equation shows that for CO and CO_2 to be in the products X must be in the interval $0 < x < 1$. Solving (1), (2) iteratively using table data, $x = 0.1335$, $T_2 = 4420°R$. $\longleftarrow$ T_2

The solution presented on the next page use IT to avoid the iterative solution of simultaneous equations involving table data.

PROBLEM 14.52 (Cont'd.)

Alternative Solution Using IT:

IT Code

```
T1 = 260 + 459.67  // °R
p1 = 1.5  // atm
p2 = 1.5  // atm
ndotCO_1 = 1  //  Do calculations on the basis of 1 lbmol of CO entering.

//  CO + 0.5 O2 + 1.88 N2  ===>  x CO2 + (1 - x) CO + (1 - x)/2 O2 + 1.88 N2
ndotCO2_2 = x
ndotCO_2 = 1 - x
ndotO2_2 = (1 - x) / 2
ndotN2_2 = 1.88
ndot2 = ndotCO2_2 + ndotCO_2 + ndotO2_2 + ndotN2_2
yCO2 = ndotCO2_2 / ndot2
yCO = ndotCO_2 / ndot2
yO2 = ndotO2_2 / ndot2
pref = 1  // atm

// For the reaction  CO2  <===>  CO + 1/2 O2
K = (((yCO)*(yO2^0.5)) / (yCO2)) * (p2 / pref)^0.5
//  Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T2,3)

0 = hCO_1 + 0.5*hO2_1 + 1.88*hN2_1 - x*hCO2_2 - (1-x)*hCO_2 - ((1-x)/2)*hO2_2 - 1.88*hN2_2
hCO_1 = h_T("CO",T1)
hO2_1 = h_T("O2",T1)
hN2_1 = h_T("N2",T1)
hCO2_2 = h_T("CO2",T2)
hCO_2 = h_T("CO",T2)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
```

IT Results

```
K = 0.02862
x = 0.8659
```

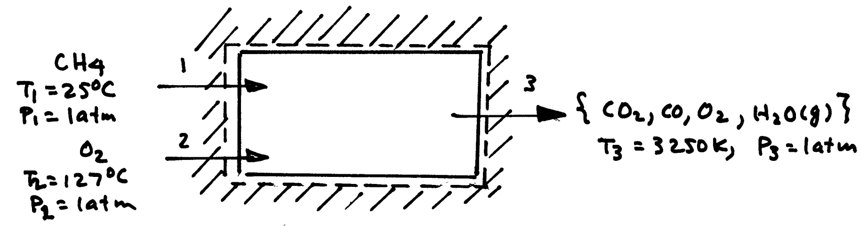
T_2 = 4422°R

PROBLEM 14.53

KNOWN: CH_4 at 25°C, 1atm enters an insulated reactor operating at steady state and burns with O_2 entering at 127°C, 1atm. An equilibrium mixture of CO_2, CO, O_2, and $H_2O(g)$ exits at 3250K, 1atm.

FIND: Determine the rate O_2 enters, in kmol per kmol of CH_4.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown on the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The mixture at 3 and incoming O_2 can be modeled as ideal gases.

ANALYSIS: The equation describing the reaction of CH_4 with O_2 to form CO_2, CO, O_2, H_2O is

$$CH_4 + \alpha O_2 \longrightarrow (1-2x)CO_2 + 2x\,CO + (\alpha+x-2)O_2 + 2H_2O$$

The amount of mixture is $n = (1-2x) + 2x + (\alpha+x-2) + 2 = 1+\alpha+x$.
At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[2x][\alpha+x-2]^{1/2}}{[1-2x]}\left[\frac{P_3/P_{ref}}{1+\alpha+x}\right]^{1/2} = \left[\frac{2x}{1-2x}\right]\left[\frac{\alpha+x-2}{\alpha+x+1}\right]^{1/2} \qquad (1)$$

At 3250K, Table A-27 gives $\log_{10}K = -0.1215 \Rightarrow K = 0.75596$. Squaring both sides of Eq. (1) and solving for α

$$\alpha = \frac{[1-f(x)]x - (f(x)+2)}{f(x)-1} \qquad \text{where } f(x) \equiv \left[\frac{(1-2x)K}{2x}\right]^2 \qquad (2)$$

Another equation involving α and x can be obtained from an energy rate balance which reduces at steady state to give

$$0 = \frac{\cancelto{0}{\dot{Q}_{cv}}}{\dot{n}_{CH_4}} - \frac{\cancelto{0}{\dot{W}_{cv}}}{\dot{n}_{CH_4}} + [\bar{h}_{CH_4}]_1 + \alpha[\bar{h}_{O_2}]_2 - [(1-2x)\bar{h}_{CO_2} + 2x\bar{h}_{CO} + (\alpha+x-2)\bar{h}_{O_2} + 2\bar{h}_{H_2O}]_3$$

Then with $\bar{h}_f^\circ = 0$ for O_2 and data from the ideal gas tables

$$0 = (\bar{h}_f^\circ)_{CH_4} + \alpha(\bar{h}(T_2) - \bar{h}(298))_{O_2} - \Big[(1-2x)[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO_2} + 2x[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO}$$
$$(\alpha+x-2)[\bar{h}(T_3) - \bar{h}(298)]_{O_2} + 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{H_2O}\Big]$$

$$0 = [-74,850] + \alpha[11,711 - 8,682] - (1-2x)[-393,520 + 177,822 - 9364] - 2x[-110,530 + 111,534 - 8669]$$
$$-(\alpha+x-2)[116,827 - 8682] - 2[-241,820 + 150,272 - 9904]$$

Upon reduction

$$105,116\,\alpha + 542,939\,x = 569,406 \qquad (3)$$

Equations (2), (3) are simultaneous solutions in x and α. By inspecting the reaction equation, for the products to contain CO and CO_2 x must be in the interval $0 < x < 0.5$. For O_2 to be in the products, $\alpha > 2$. Solving iteratively using table data
$x = 0.2671$, $\alpha = 4.0373$. kmol(O_2)/kmol(CH_4) ←

PROBLEM 14.53 (Cont'd.)

Iterative solution of simulaneous equations involving table is avoided by using IT, as follows:

IT Code

```
T1 = 25 + 273.15  // K
T2 = 127 + 273.15  // K
T3 = 3250  // K
p3 = 1  // atm
ndotCH4_1 = 1  //  Do calculations on the basis of 1 kmol of CH4 entering.

//  CH4 + A O2  ===>  x CO2 + (1 - x) CO + (A - (3 + x) / 2) O2 + 2 H2O
ndotCO2_3 = x
ndotCO_3 = 1 - x
ndotO2_3 = A - (3 + x)/2
ndotH2O_3 = 2
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotH2O_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

// For the reaction  CO2  <===>  CO + 1/2 O2
K = (((yCO) * (yO2^0.5)) / (yCO2)) * (p3 / pref)^0.5
//  Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)

0 = hCH4_1 + A*hO2_2 - (x*hCO2_3 + (1-x)*hCO_3 + (A-(3+x)/2)*hO2_3 + 2*hH2O_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
```

IT Result

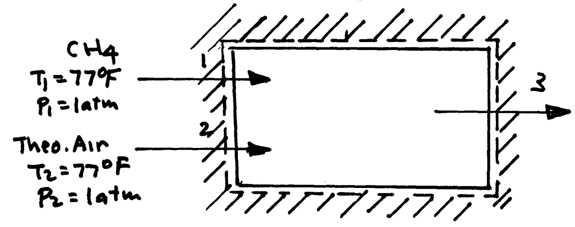
$\dot{n}_{O_2,2}/\dot{n}_{CH_4}$ = 4.037 kmol/kmol

PROBLEM 14.54

KNOWN: CH_4 at 77°F, 1atm enters an insulated reactor operating at steady state and burns with the theoretical amount of air entering at 77°F, 1atm. An equilibrium mixture of CO_2, CO, O_2, $H_2O(g)$, and N_2 exits at 1 atm.

FIND: Determine (a) the temperature of the exiting mixture, (b) the exergy destruction in Btu per lbmol of CH_4 entering.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The incoming air and exiting mixture can be modeled as ideal gases. (3) N_2 is inert. (4) $T_0 = 537°R$.

ANALYSIS: (a) Combustion of methane with the theoretical amount of air to produce CO_2, CO, O_2, $H_2O(g)$, and N_2 is described by

$$CH_4 + 2O_2 + 7.52 N_2 \rightarrow (1-x)CO_2 + xCO + \frac{x}{2}O_2 + 2H_2O + 7.52N_2$$

The amount of mixture is $n = (1-x) + x + \frac{x}{2} + 2 + 7.52 = (21.04+x)/2$. At equilibrium $CO_2 \rightleftarrows CO + 1/2\, O_2$. Accordingly, Eq. 14.35 takes the form

$$K(T_3) = \frac{[x][x/2]^{1/2}}{[1-x]}\left[\frac{P_3/P_{ref}}{(21.04+x)/2}\right]^{1/2} = \frac{x}{1-x}\left[\frac{x}{21.04+x}\right]^{1/2} \qquad (1)$$

Another equation in x and T_3 can be obtained from an energy rate equation which reduces at steady state to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH4}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH4}}}^{0} + (\bar{h}_{CH4})_1 + (2\bar{h}_{O_2} + 7.52\bar{h}_{N_2})_2 - [(1-x)\bar{h}_{CO_2} + x\bar{h}_{CO} + \frac{x}{2}\bar{h}_{O_2} + 2\bar{h}_{H_2O} + 7.52\bar{h}_{N_2}]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$(1-x)[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO_2} + x[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{CO} + \frac{x}{2}[\bar{h}(T_3) - \bar{h}(537)]_{O_2}$$
$$+ 2[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(537)]_{H_2O} + 7.52[\bar{h}(T_3) - \bar{h}(537)]_{N_2} = (\bar{h}_f^\circ)_{CH4}$$

With data from the ideal gas tables

$$(1-x)[-169{,}300 + \bar{h}_{CO_2}(T_3) - 4027.5] + x[-47{,}540 + \bar{h}_{CO}(T_3) - 3725.1] + \frac{x}{2}[\bar{h}_{O_2}(T_3) - 3725.1]$$
$$+ 2[-104{,}040 + \bar{h}_{H_2O}(T_3) - 4258] + 7.52[\bar{h}_{N_2}(T_3) - 3729.5] = -32{,}210$$

$$(1-x)[\bar{h}_{CO_2}(T_3) - 173{,}328] + x[\bar{h}_{CO}(T_3) - 51{,}265] + \frac{x}{2}[\bar{h}_{O_2}(T_3) - 3725] + 2[\bar{h}_{H_2O}(T_3) - 108{,}298]$$
$$+ 7.52[\bar{h}_{N_2}(T_3) - 3730] = -32{,}210 \qquad (2)$$

Eqs. (1), (2) are simultaneous in x and T_3. By inspection of the reaction equation, for CO and CO_2 to be in the products x must be in the interval $0 < x < 1$. Solving iteratively using table data $x = 0.1088$, $T_3 = 4066°R$.

PROBLEM 14.54 (Contd.)

(b) From the solution to part (a), the reaction is described by

$$CH_4 + 2O_2 + 7.52\,N_2 \rightarrow 0.8912\,CO_2 + 0.1088\,CO + 0.0544\,O_2 + 2H_2O + 7.52\,N_2$$

where the combustion products are at 4066 °R.

The rate of exersy destruction is $\dot{E}_d = T_0\,\dot{\sigma}_{cv}$, where $\dot{\sigma}_{cv}$ is obtained from an entropy rate balance:

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CH_4}} = \left[0.8912\,\bar{s}_{CO_2} + 0.1088\,\bar{s}_{CO} + 0.0544\,\bar{s}_{O_2} + 2\,\bar{s}_{H_2O} + 7.52\,\bar{s}_{N_2}\right]_3 - \left[\bar{s}_{CH_4}\right]_1 - \left[2\bar{s}_{O_2} + 7.52\,\bar{s}_{N_2}\right]_2$$

The methane enters at 77°F, 1atm. Thus, from Table A-25, $\bar{s}_{CH_4} = 44.49$ Btu/lbmol·°R. The air enters at 25°C, 1atm, so with Eq. 13.23 and data from Table A-25

$$\bar{s}_{O_2} = \bar{s}^o_{O_2}(537) - \bar{R}\ln y_{O_2} = 48.98 - 1.986\ln 0.21 = 52.08 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^o_{N_2}(537) - \bar{R}\ln y_{N_2} = 45.74 - 1.986\ln 0.79 = 46.21 \text{ Btu/lbmol·°R}$$

The combostion products exit at 4066°R, 1atm with composition $y_{CO_2} = (0.8912/10.5744)$, $y_{CO} = (0.1088/10.5744)$, $y_{O_2} = (0.0544/10.5744)$, $y_{H_2O} = (2/10.5744)$, $y_{N_2} = (7.52/10.5744)$. Thus, with data from the ideal gas tables

$$\bar{s}_{CO_2} = \bar{s}^o_{CO_2}(4066) - \bar{R}\ln y_{CO_2} = 75.643 - 1.986\ln(0.8912/10.5744) = 80.556 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{CO} = \bar{s}^o_{CO}(4066) - \bar{R}\ln y_{CO} = 62.915 - 1.986\ln(0.1088/10.5744) = 72.004 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{O_2} = \bar{s}^o_{O_2}(4066) - \bar{R}\ln y_{O_2} = 65.274 - 1.986\ln(0.0544/10.5744) = 75.740 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{H_2O} = \bar{s}^o_{H_2O}(4066) - \bar{R}\ln y_{H_2O} = 64.784 - 1.986\ln(2/10.5744) = 68.041 \text{ Btu/lbmol·°R}$$

$$\bar{s}_{N_2} = \bar{s}^o_{N_2}(4066) - \bar{R}\ln y_{N_2} = 61.238 - 1.986\ln(7.52/10.5744) = 61.915 \text{ Btu/lbmol·°R}$$

Using calculated data

$$\frac{\dot{\sigma}_{cv}}{\dot{n}_{CH_4}} = \{(0.8912)(80.556) + (0.1088)(72.004) + (0.0544)(75.740) + 2(68.041) + 7.52(61.915)\} - 44.49 - ((2)(52.08) + (7.52)(46.21)) = 189.28 \text{ Btu/lbmol·°R}$$

Finally, the exergy destruction is

$$\frac{\dot{E}_d}{\dot{n}_{CH_4}} = T_0\left(\frac{\dot{\sigma}_{cv}}{\dot{n}_{CH_4}}\right) = (537°R)\left(189.28\,\frac{\text{Btu}}{\text{lbmol·°R}}\right) = 101{,}643\,\frac{\text{Btu}}{\text{lbmol}\,(CH_4)} \longleftarrow$$

An iterative solution of simutaneous equations with table data can be avoided using IT, as shown on the next page.

PROBLEM 14.54 (Cont'd.)

Alternative Solution Using IT.

IT Code

```
T1 = 77 + 459.67  // °R
p1 = 1 // atm
T2 = 77 + 459.67  // °R
p2 = 1  // atm
p3 = 1  // atm
To = 537  // °R
ndotCH4_1 = 1  //  Do calculations on the basis of 1 lbmol of CH4 entering.

// CH4 + 2 * (O2 + 3.76 N2)  →  (1 - x) CO2 + x CO + x/2 O2 + 2 H2O + 7.52 N2
ndotCO2_3 = 1 - x
ndotCO_3 = x
ndotO2_3 = x/2
ndotH2O_3 = 2
ndotN2_3 = 7.52
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotH2O_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
yH2O = ndotH2O_3 / ndot3
yN2 = ndotN2_3 / ndot3
pref = 1  // atm

//  For the reaction CO2  ↔  CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^0.5
//  Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,2,T3,3)

0 = hCH4_1 + 2*hO2_2 + 7.52*hN2_2 - ((1-x)*hCO2_3 + x*hCO_3 + (x/2)*hO2_3 + 2*hH2O_3
+ 7.52*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
hN2_3 = h_T("N2",T3)

Edotd = To * (sigmadot / ndotCH4_1)
sigmadot / ndotCH4_1 = (1-x)*sCO2_3 + x*sCO_3 + (x/2)*sO2_3 + 2*sH2O_3 + 7.52*sN2_3 -
                                        sCH4_1 - 2*sO2_2 - 7.52*sN2_2
sCH4_1 = s_Tp("CH4",T1,p1)
pO2_2 = .21 * p2
sO2_2 = s_Tp("O2",T2,pO2_2)
pN2_2 = .79 * p2
sN2_2 = s_Tp("N2",T2,pN2_2)
pCO2_3 = yCO2 * p3
sCO2_3 = s_Tp("CO2",T3,pCO2_3)
pCO_3 = yCO * p3
sCO_3 = s_Tp("CO",T3,pCO_3)
pO2_3 = yO2 * p3
sO2_3 = s_Tp("O2",T3,pO2_3)
pH2O_3 = yH2O * p3
sH2O_3 = s_Tp("H2O",T3,pH2O_3)
pN2_3 = yN2 * p3
sN2_3 = s_Tp("N2",T3,pN2_3)
```

IT Results

$x = 0.109$

$T_3 = 4067°R$

$\sigma_{cv} / \dot{n}_{CH_4}$ = 189.3 Btu/lbmol(CH_4)·°R

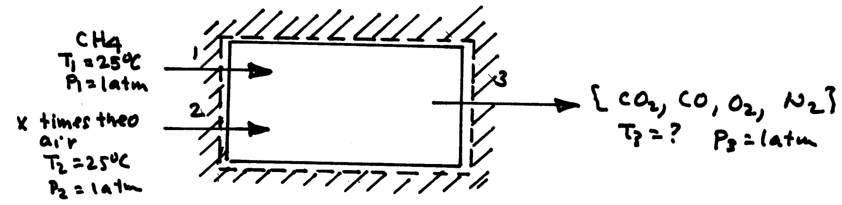
$\dot{E}_d / \dot{n}_{CH_4}$ = 1.016×10^5 Btu/lbmol(CH_4)

PROBLEM 14.55

KNOWN: $CH_4(g)$ at 25°C, 1atm enters an insulated reactor operating at steady state and burns with x times the theoretical amount of air entering at 25°C, 1atm. An equilibrium mixture of CO_2, CO, O_2 and N_2 exits at 1atm.

FIND: For x ranging from 1 to 4, determine the temperature of the exiting equilibrium mixture.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The control volume shown in the accompanying figure is at steady state with $\dot{Q}_{cv} = \dot{W}_{cv} = 0$ and negligible effects of kinetic and potential energy. (2) The equilibrium mixture can be modeled as an ideal gas. (3) N_2 is inert. (4) $x \geq 1$.

ANALYSIS: The reaction of CH_4 with x times the theoretical amount of air to produce CO_2, CO, O_2, and N_2 is described by

$$CH_4 + 2xO_2 + 7.52x\,N_2 \rightarrow (1-\alpha)\,CO_2 + \alpha CO + \left(2x + \frac{\alpha}{2} - 2\right)O_2 + 2H_2O + 7.52x\,N_2$$

For CO and CO_2 to appear in the products, it is necessary for α to be in the interval $0 < \alpha < 1$. The amount of mixture, n, is

$$n = (1-\alpha) + \alpha + \left(2x + \frac{\alpha}{2} - 2\right) + 2 + 7.52x = (2 + 19.04x + \alpha)/2$$

At equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly, Eq. 14.35 becomes

$$K(T_3) = \frac{[\alpha]\left[\frac{4x+\alpha-4}{2}\right]^{1/2}}{[1-\alpha]}\left[\frac{P/P_{ref}}{(2+19.04x+\alpha)/2}\right]^{1/2} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{4x+\alpha-4}{2+19.04x+\alpha}\right)^{1/2} \qquad (1)$$

Another equation relating T_3 and α for specified x is obtained using the energy rate balance which at steady state reduces to

$$0 = \cancel{\frac{\dot{Q}_{cv}}{\dot{n}_{CH4}}}^{0} - \cancel{\frac{\dot{W}_{cv}}{\dot{n}_{CH4}}}^{0} + (\bar{h}_{CH4})_1 + [2x\bar{h}_{O_2} + 7.52x\bar{h}_{N_2}]_2 - \left[(1-\alpha)\bar{h}_{CO_2} + \alpha\bar{h}_{CO} + \left(2x + \frac{\alpha}{2} - 2\right)\bar{h}_{O_2} + 2\bar{h}_{H_2O} + 7.52x\bar{h}_{N_2}\right]_3$$

Then, with $\bar{h}_f^\circ = 0$ for O_2 and N_2

$$0 = (\bar{h}_f^\circ)_{CH_4} - (1-\alpha)[\bar{h}_f^\circ + h(T_3) - h(298)]_{CO_2} - \alpha[\bar{h}_f^\circ + \bar{h}(T_3) - \bar{h}(298)]_{CO}$$
$$- \left(2x + \frac{\alpha}{2} - 2\right)[\bar{h}(T_3) - h(298)]_{O_2} - 2[\bar{h}_f^\circ + \bar{h}(T_3) - h(298)]_{H_2O} - 7.52x[\bar{h}(T_3) - \bar{h}(298)]_{N_2}$$

With data from the ideal gas tables

$$0 = (-74850) - (1-\alpha)[-393{,}520 + \bar{h}_{CO_2}(T_3) - 9364] - \alpha[-110{,}530 + \bar{h}_{CO}(T_3) - 8669]$$
$$- \left(2x + \frac{\alpha}{2} - 2\right)[\bar{h}_{O_2}(T_3) - 8682] - 2[-241{,}820 + \bar{h}_{H_2O}(T_3) - 9904] + 7.52x[\bar{h}_{N_2}(T_3) - 8669]$$

or

$$0 = (-74850) - (1-\alpha)[\bar{h}_{CO_2}(T_3) - 402{,}884] - \alpha[\bar{h}_{CO}(T_3) - 119{,}199] - \left(2x + \frac{\alpha}{2} - 2\right)[\bar{h}_{O_2}(T_3) - 8682]$$
$$- [\bar{h}_{H_2O}(T_3) - 251{,}724] - 7.52x[\bar{h}_{N_2}(T_3) - 8669] \qquad (2)$$

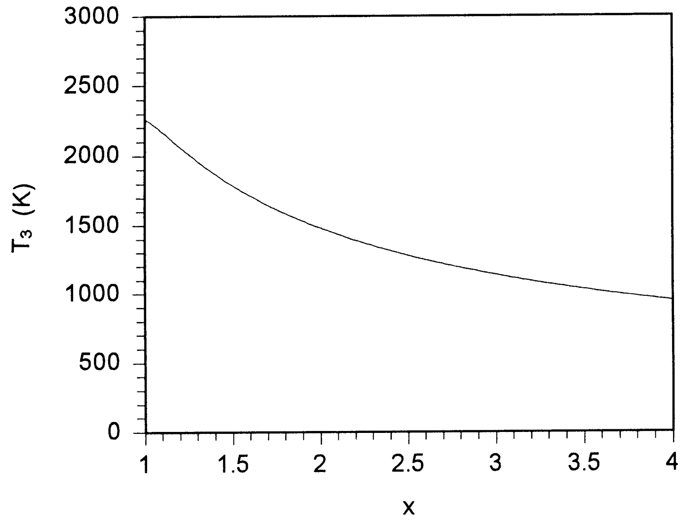
Eqs. (1), (2) are simultaneous in T_3, α, and x. For each specified value for x: $1 \leq x \leq 4$, the equations can be solved for T_3 and α, as shown in the table following the IT solution

PROBLEM 14.55 (Cont'd.)

```
T1 = 25 + 273.15  // K
T2 = 25 + 273.15  // K
p3 = 1  // atm
x = 1
ndotCH4_1 = 1  //  Do calculations on the basis of 1 kmol of CH4 entering.


// CH4 + (2 * x) O2 + (7.52 * x) N2  ===>
//       (1- a) CO2 + a CO + (2 * x + a / 2 - 2) O2 + 2 H2O + (7.52 * x) N2
ndotCO2_3 = 1 - a
ndotCO_3 = a
ndotO2_3 = 2 * x + a / 2 - 2
ndotH2O_3 = 2
ndotN2_3 = 7.52 * x
ndot3 = ndotCO2_3 + ndotCO_3 + ndotO2_3 + ndotH2O_3 + ndotN2_3
yCO2 = ndotCO2_3 / ndot3
yCO = ndotCO_3 / ndot3
yO2 = ndotO2_3 / ndot3
pref = 1  // atm

// For the reaction:  CO2  <===>  CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p3 / pref)^0.5
//  Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = LOOKUPVAL(EQCO2A,1,T3,3)

0 = hCH4_1 + 2*x*hO2_2 + 7.52*x*hN2_2 - ((1-a)*hCO2_3 + a*hCO_3 + (2*x+a/2-2)*hO2_3 +
                   2*hH2O_3 + 7.52*x*hN2_3)
hCH4_1 = h_T("CH4",T1)
hO2_2 = h_T("O2",T2)
hN2_2 = h_T("N2",T2)
hCO2_3 = h_T("CO2",T3)
hCO_3 = h_T("CO",T3)
hO2_3 = h_T("O2",T3)
hH2O_3 = h_T("H2O",T3)
hN2_3 = h_T("N2",T3)
```

IT Results

x	a	T_3 (K)
1.0	0.109	2259
1.5	7.025×10^{-4}	1789
2.0	1.036×10^{-5}	1481
2.5	2.471×10^{-7}	1280
3.0	8.101×10^{-9}	1138
3.5	3.934×10^{-10}	1032
4.0	5.312×10^{-12}	950.7



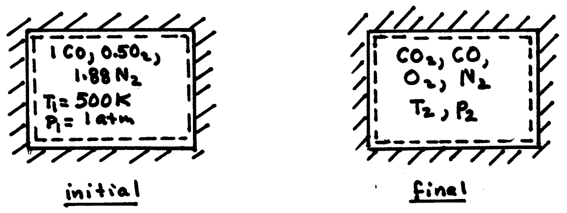
As the amount of air supplied increases, the temperature of the combustion products and the amount of CO decrease.

PROBLEM 14.56

KNOWN: A mixture of 1 kmol CO, 0.5 kmol O_2, 1.88 kmol N_2 is initially at 227°C, 1atm in a closed, rigid, insulated vessel. An equilibrium mixture of CO_2, CO, O_2, and N_2 eventually forms.

FIND: Determine the final pressure.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is shown in the accompanying figure. (2) The ideal gas model applies at the initial and final states. (3) For the system Q = W = 0 and kinetic and potential energy effects are negligible. (4) N_2 is inert.

ANALYSIS: The reaction takes the form

$$CO + 0.5O_2 + 1.88\,N_2 \rightarrow x\,CO + (1-x)\,CO_2 + \frac{x}{2}\,O_2 + 1.88\,N_2$$

The initial amount of mixture is $n_1 = 1 + 0.5 + 1.88 = 3.38$ kmol. The final amount of mixture is $n_2 = x + (1-x) + \frac{x}{2} + 1.88 = (5.76 + x)/2$.

The ideal gas equation of state gives

$$\left.\begin{aligned} P_1 V &= n_1 \bar{R} T_1 \\ P_2 V &= n_2 \bar{R} T_2 \end{aligned}\right\} : \quad \frac{P_2}{P_1} = \frac{n_2 T_2}{n_1 T_1} \qquad (1)$$

As P_1, T_1, and n_1 are known, P_2 can be determined once n_2 and T_2 are known.

At equilibrium, $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$. Accordingly Eq. 14.35 takes the form

$$K(T_2) = \frac{[x][\frac{x}{2}]^{1/2}}{[1-x]}\left[\frac{P_2/P_{ref}}{n_2}\right]^{1/2} \qquad (2)$$

Since $P_1 = P_{ref}$, use of Eq.(1) in Eq.(2) gives

$$K(T_2) = \left[\frac{x}{1-x}\right]\left[\frac{x\,T_2}{2\,n_1 T_1}\right]^{1/2} \qquad (3)$$

Another equation involving x and T_2 can be obtained from an energy balance which reduces to $\Delta U = \overset{0}{\cancel{Q}} - \overset{0}{\cancel{W}}$. That is,

$$\left[x\bar{u}_{CO} + (1-x)\bar{u}_{CO_2} + \frac{x}{2}\bar{u}_{O_2} + 1.88\bar{u}_{N_2}\right]_2 - \left[\bar{u}_{CO} + 0.5\bar{u}_{O_2} + 1.88\bar{u}_{N_2}\right]_1 = 0$$

With $\bar{h} = \bar{u} + \bar{R}T$, $\bar{u} = \bar{h} - \bar{R}T$

$$\left[x[\bar{h}_{CO} - \bar{R}T] + (1-x)[\bar{h}_{CO_2} - \bar{R}T] + \frac{x}{2}[\bar{h}_{O_2} - \bar{R}T]\right]_2 - \left[(\bar{h}_{CO} - \bar{R}T) + 0.5(\bar{h}_{O_2} - \bar{R}T)\right]_1 + 1.88\left[\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_1)\right] = 0$$

or

$$\left[x\bar{h}_{CO} + (1-x)\bar{h}_{CO_2} + \frac{x}{2}\bar{h}_{O_2}\right]_2 - \left[\bar{h}_{CO} + 0.5\bar{h}_{O_2}\right]_1 + 1.80\left[\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_1)\right] + \frac{3}{2}\bar{R}T_1 - \left(1 + \frac{x}{2}\right)\bar{R}T_2 = 0$$

For CO, CO_2, and O_2, $h = \bar{h}_f^\circ + \Delta\bar{h}$. Thus

$$x\left[\bar{h}_f^\circ + h(T_2) - h(298)\right]_{CO} + (1-x)\left[\bar{h}_f^\circ + \bar{h}(T_2) - \bar{h}(298)\right]_{CO_2} + \frac{x}{2}\left[\bar{h}(T_2) - \bar{h}(298)\right]_{O_2}$$
$$- \left[\bar{h}_f^\circ + \bar{h}(T_1) - \bar{h}(298)\right]_{CO} - 0.5\left[\bar{h}(T_1) - \bar{h}(298)\right]_{O_2} + 1.88\left[\bar{u}_{N_2}(T_2) - \bar{u}_{N_2}(T_1)\right] + \frac{3}{2}\bar{R}T_1 - \left(1 + \frac{x}{2}\right)\bar{R}T_2 = 0$$

With data from Table A-23

$$x[-110{,}530 + \bar{h}_{CO}(T_2) - 8669] + (1-x)[-393{,}520 + \bar{h}_{CO_2}(T_2) - 9364] +$$
$$\frac{x}{2}[\bar{h}_{O_2}(T_2) - 8682] - [-110{,}530 + 14{,}600 - 8669] - 0.5[14{,}770 - 8682] +$$
$$1.88[\bar{u}_{N_2}(T_2) - 10{,}423] + \frac{3}{2}(8.314)(500) - \left(1 + \frac{x}{2}\right)\bar{R}T_2 = 0$$

PROBLEM 14.56 (Contd.)

Thus

$$x\left[\bar{h}_{CO}(T_2) - 119{,}199\right] + (1-x)\left[\bar{h}_{CO_2}(T_2) - 402{,}884\right] + \frac{x}{2}\left[\bar{h}_{O_2}(T_2) - 8682\right] +$$
$$1.88\,\bar{u}_{N_2}(T_2) - \left(1+\frac{x}{2}\right)\bar{R}T_2 + 88195.3 = 0 \qquad (4)$$

Eqs. (3), (4) are simultaneous in x and T_2. Inspection of the reaction equation shows that for CO and CO_2 to be in the products x must be in the interval $0 < x < 1$. Solving iteratively using table data, $x = 0.2262$, $T_2 = 2761\,K$. Thus

$$n_2 = \frac{5.76 + 0.2262}{2} = 2.993$$

Eq. (1) then gives

$$p_2 = \left(\frac{2.993}{3.38}\right)\left(\frac{2761}{500}\right)(1\text{ atm}) = 4.89\text{ atm} \longleftarrow$$

The following IT solution provides an alternative to the iterative approach involving the use of table data:

IT Code

```
nCO_1 = 1  // kmol
nO2_1 = 0.5  // kmol
T1 = 227 + 273.15  // K
p1 = 1  // atm
n1 = nCO_1 + nO2_1 + nN2

// CO + 0.5 O2 + 1.88 N2 → x CO + (1 - x) CO2 + x/2 O2 + 1.88 N2
nCO = x
nCO2 = 1 - x
nO2 = x/2
nN2 = 1.88
n2 = nCO + nCO2 + nO2 + nN2
yCO = nCO / n2
yO2 = nO2 / n2
yCO2 = nCO2 / n2
pref = 1  // atm

p2 / p1 = (n2 / n1) * (T2 / T1)

// For the reaction  CO2 ↔ CO + 1/2 O2
K = ((yCO * yO2^0.5) / yCO2) * (p2 / pref)^0.5
// Data from Table A-27 are stored in EQCO2A.LUT.
log(K) = Lookupval(EQCO2A,1,T2,3)

U2/ nCO_1 = (x * uCO_2 + (1 - x) * uCO2_2 + (x / 2) * uO2_2 + 1.88 * uN2_2)
U1/ nCO_1 = (uCO_1 + 0.5 * uO2_1 + 1.88 * uN2_1)
U2 = U1
uCO_1 = u_T("CO",T1)
uO2_1 = u_T("O2",T1)
uN2_1 = u_T("N2",T1)
uCO_2 = u_T("CO",T2)
uCO2_2 = u_T("CO2",T2)
uO2_2 = u_T("O2",T2)
uN2_2 = u_T("N2",T2)
```

IT Results

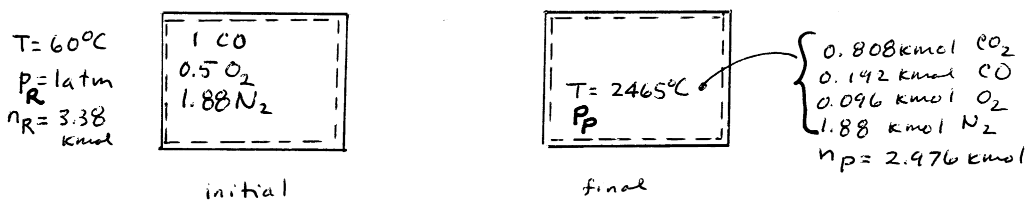
n_2 = 2.993 kmol
x = 0.2262
T_2 = 2760 K
p_2 = 4.887atm

PROBLEM 14.57

KNOWN: A mixture of CO and the theoretical amount of air reacts in an insulated vessel to form an equilibrium mixture of CO_2, CO, O_2 for which a molar analysis is provided. A measured value for the final temperature is indicated as well.

FIND: Check the consistency of the given data.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is shown in the figure above. (2) For the system Q = W = 0 and kinetic/potential energy effects are negligible. (3) Ideal gas principles apply. (4) N_2 is inert.

ANALYSIS: Using the data above the balanced reaction equation is

$$1\,CO + 0.5\,O_2 + 1.88\,N_2 \rightarrow 0.808\,CO_2 + 0.192\,CO + 0.096\,O_2 + 1.88\,N_2$$

Carbon and oxygen balances verify this expression is correct.

Next, an energy balance reads $\Delta U = \cancel{Q} - \cancel{W}$. Paralleling the development of Eq 13.17b, this can be rewritten as

$$0 = \sum_P n(\bar{h}_f^\circ + \Delta\bar{h}) - \sum_R n(\bar{h}_f^\circ + \Delta\bar{h}) - \bar{R}T_P \sum_P n + \bar{R}T_R \sum_R n .$$ With table data this becomes,

$$0 \overset{?}{=} 0.808[-393{,}520 + (145{,}971 - 9364)] + 0.192[-110{,}530 + (92485 - 8669)] + 0.096[96{,}379 - 8682]$$
$$+ 1.88[91{,}729 - 9684] - 1[-110{,}530 + (9685 - 8669)] - 0.5[9709 - 8682]$$
$$- 8.314[(2738)(2.976) - (333)(3.38)] = 561 \text{ kJ}$$

Accordingly, the energy balance requirement is closely satisfied.

Finally, the equilibrium constraint has to be considered: This requires the final mixture pressure, obtained using the ideal gas equation of state at fixed volume: $p\bar{V} = n\bar{R}T$. Thus

$$\frac{p_P}{p_R} = \frac{n_P T_P}{n_R T_R} \Rightarrow \frac{p_P}{n_P} = \frac{p_R}{n_R}\frac{T_P}{T_R} = \frac{(1\text{ atm})}{(3.38\text{ kmol})}\left[\frac{2738\text{ K}}{333\text{ K}}\right] = 2.433$$
$$\Rightarrow p_P = 2.433 \text{ atm}.$$

Then, at equilibrium $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$, so

$$K(2738\text{ K}) = \frac{[0.192][0.096]^{1/2}}{[0.808]}\left[2.433\right]^{1+\frac{1}{2}-1}$$

$$= \left[\frac{0.192}{0.808}\right]\left[(0.096)(2.433)\right]^{1/2} = 0.1148$$

Then, from Table A-27 at 2738 K, $\log_{10} K = -0.9428$, $K = 0.1141$. Thus, the equilibrium condition is also closely satisfied.

Accordingly, the given data are consistent. $\longleftarrow$

PROBLEM 14.58

KNOWN: At 2000 K, $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$

FIND: Estimate the enthalpy of reaction using the van't Hoff equation and equilibrium constant data. Compare with the value calculated using enthalpy data.

ASSUMPTION: The ideal gas model is applicable.

ANALYSIS: Rearranging Eq 14.43b and using $\ln K = 2.303 \log_{10} K$

$$\Delta H = \bar{R}T^2 \frac{d \ln K}{dT} = 2.303 \bar{R} T^2 \frac{d \log_{10} K}{dT}$$

With the interpolation rule of Problem 14.6

$$\log_{10} K = C_1 + \frac{C_2}{T} \Rightarrow \frac{d \log_{10} K}{dT} = -\frac{C_2}{T^2}$$

Collecting results

$$\Delta H = -2.303 \bar{R} C_2 \qquad (1)$$

Then, in the interval $1900 \le T \le 2000K$, using data from Table A-27

$$\log_{10} K = C_1 + \frac{C_2}{T}$$

@ 1900K: $-3.267 = C_1 + C_2/1900$

@ 2100K: $-2.539 = C_1 + C_2/2100$ } $C_2 = -14\,523$ K

Using this value for C_2, Eq. (1) gives

$$\Delta H = -2.303 \left(8.314 \frac{kJ}{kmol \cdot K}\right)(-14\,523 K)$$

$$= 278{,}074 \text{ kJ/kmol} \qquad (2)$$

With data from the ideal gas tables

$$\Delta H = \bar{h}_{CO} + \tfrac{1}{2}\bar{h}_{O_2} - \bar{h}_{CO_2}$$

$$= [\bar{h}_f^\circ + \bar{h}(2000) - \bar{h}(298)]_{CO} + \tfrac{1}{2}[\cancelto{0}{\bar{h}_f^\circ} + \bar{h}(2000) - \bar{h}(298)]_{O_2} - [\bar{h}_f^\circ + \bar{h}(2000) - \bar{h}(298)]_{CO_2}$$

$$= [-110{,}530 + 65{,}408 - 8669] + \tfrac{1}{2}[67{,}881 - 8682] - [-393{,}520 + 100{,}804 - 9364]$$

$$= 277{,}889 \text{ kJ/kmol} \qquad (3)$$

The results given by Eqs (2), (3) differ by less than 0.1%

PROBLEM 14.59

KNOWN: At 2000 K, $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$

FIND: Estimate the enthalpy of reaction using the van't Hoff and equilibrium constant data; compare with the value calculated using enthalpy data.

ASSUMPTION: The ideal gas model is applicable.

ANALYSIS: Rearranging Eq. 14.43b and using $\ln K = 2.303 \log_{10} K$

$$\Delta T = \bar{R}T^2 \frac{d \ln K}{dT} = 2.303\, \bar{R}\, T^2 \frac{d \log_{10} K}{dT}$$

With the interpolation rule of Problem 14.6

$$\log_{10} = c_1 + \frac{c_2}{T}, \quad \frac{d \log_{10}}{dT} = -\frac{c_2}{T^2}$$

Collecting results

$$\Delta H = -2.303\, \bar{R}\, C_2 \qquad (1)$$

Then, in the interval $1900 \leq T \leq 2100\,K$, using data from Table A-27

$$\left.\begin{array}{ll} @\ 1900K: & -3.886 = c_1 + c_2/1900 \\ @\ 2100K: & -3.227 = c_1 + c_2/2100 \end{array}\right\} \quad c_2 = -13{,}147\,K$$

Using this value for c_2, Eq. (1) gives

$$\Delta H = -2.303\left(8.314 \frac{kJ}{kmol\cdot K}\right)(-13{,}147\,K) = 251{,}727 \frac{kJ}{kmol} \qquad (2)$$

With data from the ideal gas tables

$$\Delta H = \bar{h}_{H_2} + \frac{1}{2}\bar{h}_{O_2} - \bar{h}_{H_2O}$$

$$= [\overset{0}{\cancel{\bar{h}_f^\circ}} + \bar{h}(2000) - \bar{h}(298)]_{H_2} + \frac{1}{2}[\overset{0}{\cancel{\bar{h}_f^\circ}} + \bar{h}(2000) - \bar{h}(298)]_{O_2}$$

$$- [\bar{h}_f^\circ + \bar{h}(2000) - \bar{h}(298)]_{H_2O}$$

$$= [61{,}400 - 8468] + \frac{1}{2}[67{,}881 - 8682] - [-241{,}820 + 82{,}593 - 9904]$$

$$= 251{,}663 \frac{kJ}{kmol} \qquad (3)$$

The results given by Eqs. (2), (3) differ by less than 0.03 %.

PROBLEM 14.60

KNOWN: At 2800K, $CO_2 \rightleftarrows CO + \frac{1}{2} O_2$

FIND: Estimate the equilibrium constant at 2800K using K at 2000 K from Table A-27, together with the Van't Hoff equation and enthalpy data. Compare with the value obtained from Table A-27 at 2800K.

ASSUMPTION: The ideal gas model is applicable.

ANALYSIS: The van't Hoff equation, Eq 14.43b, gives upon rearrangement

$$\frac{d \ln K}{dT} = \frac{\Delta H}{\bar{R} T^2} \tag{1}$$

Evaluating ΔH as the average of the values at 2000K and 3000K, Eq. (1) gives upon integration

$$\ln \frac{K_2}{K_1} = -\frac{(\Delta H)_{ave}}{\bar{R}} \left[\frac{1}{T_2} - \frac{1}{T_1} \right] \tag{2}$$

For $CO_2 \rightarrow CO + \frac{1}{2} O_2$, data from the ideal gas tables gives

$$\Delta H = \bar{h}_{CO} + \frac{1}{2} \bar{h}_{O_2} - h_{CO_2}$$

$$= [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{CO} + \frac{1}{2} [\overset{0}{\cancel{\bar{h}_f^\circ}} + \bar{h}(T) - \bar{h}(298)]_{O_2} - [\bar{h}_f^\circ + \bar{h}(T) - \bar{h}(298)]_{CO_2}$$

$$= [\bar{h}_{CO}(T) + \frac{1}{2} \bar{h}_{O_2}(T) - \bar{h}_{CO_2}(T)] + [\bar{h}_f^\circ - \bar{h}(298)]_{CO} - [\frac{\bar{h}(298)}{2}]_{O_2} - [\bar{h}_f^\circ - \bar{h}(298)]_{CO_2}$$

$$= (\bar{h}_{CO}(T) + \frac{1}{2} \bar{h}_{O_2}(T) - \bar{h}_{CO_2}(T)] + [-110{,}530 - 8669] - 4341 - [-393{,}520 - 9364]$$

$$= [\bar{h}_{CO}(T) + \frac{1}{2} \bar{h}_{O_2}(T) - \bar{h}_{CO_2}(T)] + 279{,}344$$

At T = 2000K:

$$\Delta H = [65{,}408 + \frac{67{,}881}{2} - 100{,}804] + 279{,}344 = 277{,}889 \text{ kJ/kmol}$$

At T = 2800 K:

$$\Delta H = [94{,}784 + \frac{98826}{2} - 149808] + 279{,}344 = 273{,}733 \text{ kJ/kmol}$$

Then $(\Delta H)_{ave}$ = 275,811 kJ/kmol, and Eq. (2) gives

$$\ln \frac{K_2}{K_1} = \frac{-275{,}811 \text{ kJ/kmol}}{8.314 \text{ kJ/kmol} \cdot \text{K}} \left[\frac{1}{2800} - \frac{1}{2000} \right] \text{K}^{-1}$$

$$\Rightarrow \quad \frac{K_2}{K_1} = 114.34$$

From Table A-27, at 2000K, $\log_{10} K_1 = -2.884$, $K_1 = 1.3061 \times 10^{-3}$. Thus

$$K_2 = 114.34 (1.3061 \times 10^{-3}) = 0.1493$$

For comparison, Table A-27, at 2800 K, $\log_{10} K_2 = -0.825$, so $K_2 = 0.1496$. The estimated value is within 0.2 % of the table value.

PROBLEM 14.61

KNOWN: At 2800 K, $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$

FIND: Estimate the equilibrium constant at 2800 K using K at 2500 K from Table A-27, together with the van't ~~Hoff~~ equation and enthalpy data. Compare with the value obtained from Table A-27 at 2800 K.

ASSUMPTION: The ideal gas equation is applicable.

ANALYSIS: The van't Hoff equation, Eq. 14.43b, gives upon rearrangement

$$\frac{d\ln K}{dT} = \frac{\Delta H}{\bar{R}T^2} \qquad (1)$$

Evaluating ΔH as the average of the values at 2500 K and 2800 K, Eq. (1) gives upon integration

$$\ln\frac{K_2}{K_1} = -\frac{(\Delta H)_{ave}}{\bar{R}}\left[\frac{1}{T_2}-\frac{1}{T_1}\right] \qquad (2)$$

For $H_2O \rightarrow H_2 + \frac{1}{2}O_2$, data from the ideal gas tables

$$\Delta H = \bar{h}_{H_2} + \frac{1}{2}\bar{h}_{O_2} - \bar{h}_{H_2O}$$

$$= \left[\cancel{\bar{h}_f^{\circ}}^{\,0} + \bar{h}(T) - h(298)\right]_{H_2} + \frac{1}{2}\left[\cancel{\bar{h}_f^{\circ}}^{\,0} + h(T) - h(298)\right]_{O_2} - \left[\bar{h}_f^{\circ} + \bar{h}(T) - \bar{h}(298)\right]_{H_2O}$$

$$= \bar{h}_{H_2}(T) + \frac{1}{2}\bar{h}_{O_2}(T) - \bar{h}_{H_2O}(T) - \bar{h}_{H_2}(298) - \frac{1}{2}\bar{h}_{O_2}(298) - (\bar{h}_f^{\circ} - \bar{h}(298))_{H_2O}$$

$$= \bar{h}_{H_2}(T) + \frac{1}{2}\bar{h}_{O_2}(T) - \bar{h}_{H_2O}(T) - 8468 - 4341 - (-241{,}820 - 9904)$$

$$= \bar{h}_{H_2}(T) + \frac{1}{2}\bar{h}_{O_2}(T) - \bar{h}_{H_2O}(T) + 238{,}915$$

At 2500 K:

$$\Delta H = 78{,}960 + \frac{1}{2}(87{,}057) - 108{,}868 + 238{,}915 = 252{,}536 \text{ kJ/kmol}$$

At 2800 K:

$$\Delta H = 89838 + \frac{1}{2}(98{,}826) - 125{,}198 + 238{,}915 = 252{,}968 \text{ kJ/kmol}$$

Thus, $(\Delta H)_{ave} = 252{,}752$ kJ/kmol, and Eq. (2) gives

$$\ln\frac{K_2}{K_1} = -\frac{252{,}752}{8.314}\left[\frac{1}{2800}-\frac{1}{2500}\right] \Rightarrow \frac{K_2}{K_1} = 3.6799$$

From Table A-27, at 2500 K, $\log_{10}K_1 = -2.2224$, $K_1 = 5.992\times10^{-4}$. Thus

$$K_2 = (5.992\times10^{-4})(3.6799) = 0.02205$$

For comparison, Table A-27, at 2800 K, $\log_{10}K_2 = -1.658$, so $K_2 = 0.02198$. The estimated value is within about 0.3% of the table value.

PROBLEM 14.62

KNOWN: For $C + 2H_2 \rightleftarrows CH_4$ at 25°C $\log_{10} K = 8.9$.

FIND: Estimate $\log_{10} K$ at 500°C.

ASSUMPTIONS: (1) The van't Hoff equation is applicable. (2) ΔH is constant.

ANALYSIS: The van't Hoff equation is given by

$$\frac{d \ln K}{dT} = \frac{\Delta H}{\bar{R}T^2}$$

If ΔH is constant

$$\ln \frac{K_2}{K_1} = -\frac{\Delta H}{\bar{R}}\left[\frac{1}{T_2} - \frac{1}{T_1}\right] \qquad (1)$$

For $C + 2H_2 \rightleftarrows CH_4$, ΔH is given by

$$\Delta H = \bar{h}_{CH_4} - \bar{h}_C - 2\bar{h}_{O_2}$$

At 298 K, Table A-25 gives

$$\Delta H = (-74{,}850) - (0) - 2(0) = -74{,}850 \text{ kJ/kmol}$$

Substituting values into Eq. (1),

$$\ln \frac{K_2}{K_1} = \frac{74850}{8.314}\left[\frac{1}{773} - \frac{1}{298}\right]$$

$$\Rightarrow$$

$$\log_{10} \frac{K_2}{K_1} = -8.06$$

or

① $$\log_{10} K_2 = 8.9 - 8.06 = 0.84 \longleftarrow$$

---

1. For a discussion of the equilibrium constant and the heterogeneous system in which the pure solid or liquid phase of one (or more) of the substances is present in the equilibrium mixture, see Concepts of Thermodynamics by E.F. Obert, McGraw-Hill, 1960, Sec. 14-13.

PROBLEM 14.63

KNOWN: For $Cs \rightleftarrows Cs^+ + e^-$, $K = 0.78$ and 15.63 at 1600 K, 2000 K, respectively.

FIND: Estimate ΔH at 1800 K.

ASSUMPTIONS: (1) The van't Hoff equation is applicable. (2) ΔH varies only gradually with temperature from 1600 to 2000 K.

ANALYSIS: The van't Hoff equation is

$$\frac{d \ln K}{dT} = \frac{\Delta H}{\bar{R}T^2}$$

Then

$$\ln \frac{K_2}{K_1} = -\frac{(\Delta H)_{ave}}{\bar{R}}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

where $(\Delta H)_{ave}$ is the average value of the enthalpy of ionization over the temperature range. Thus

$$(\Delta H)_{ave} = \frac{\bar{R} \ln \frac{K_2}{K_1}}{\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

$$= \frac{(8.314)\left(\ln \frac{15.63}{0.78}\right)}{\left(\frac{1}{1600} - \frac{1}{2000}\right)} = 199{,}380 \ \frac{\text{kJ}}{\text{kmol}} \longleftarrow$$

PROBLEM 14.64

KNOWN: An equilibrium mixture at 2000K, 1atm consists of Cs, Cs^+, e^-. K = 15.63

FIND: If 1 kmol of Cs is present initially, determine the percent ionization of Cs.

ASSUMPTION: Equilibrium can be treated using ideal gas mixture equilibrium concepts.

ANALYSIS: The ionization of Cs to form a mixture of Cs, Cs^+, and e^- is described by

$$Cs \rightarrow (1-z)\,Cs + z\,Cs^+ + z\,e^-$$

The amount of mixture is $n = (1-z) + z + z = 1+z$

At equilibrium $Cs \rightleftarrows Cs^+ + e^-$, so Eq. 14.35 takes the form

$$K = \frac{[z][z]}{[1-z]}\left[\frac{p/p_{ref}}{1+z}\right]^{1+1-1} = \frac{z^2}{1-z^2} \Rightarrow z = \left[\frac{K}{1+K}\right]^{1/2}$$

With K = 15.6, this becomes

$$z = \left[\frac{15.6}{16.6}\right]^{1/2} = 0.969$$

Thus, the amount of Cs in the mixture is 0.031 kmol. Cs is 96.9% ← dissociated.

PROBLEM 14.65

KNOWN: At 18,000 °R and pressure p, an equilibrium mixture consists of Ar, Ar^+, e^-. For $Ar \rightleftarrows Ar^+ + e^-$, $K = 4.2 \times 10^{-4}$ at 18,000°R.

FIND: Plot the percent ionization of Ar for $0.01 \le p \le 0.05$ atm.

ASSUMPTION: Equilibrium can be treated using ideal gas mixture concepts.

ANALYSIS: The ionization of Ar to form a mixture of Ar, Ar^+, and e^- is described by

$$Ar \rightarrow (1-z)\,Ar + z\,Ar^+ + z\,e^-$$

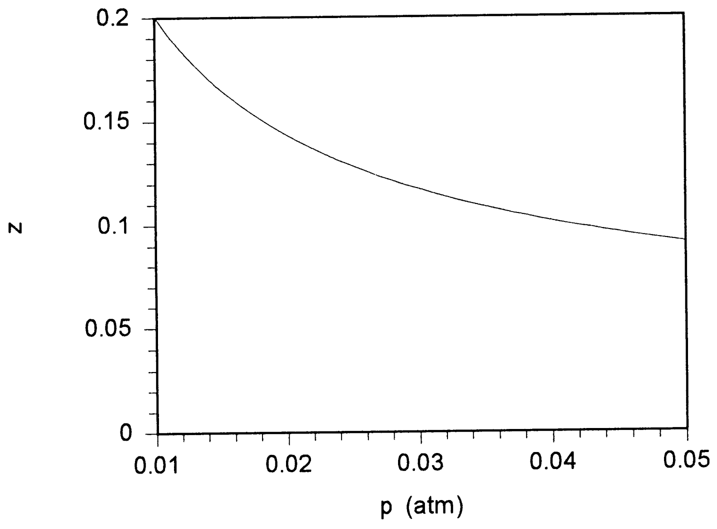
The amount of mixture is $n = (1-z) + z + z = 1+z$.

At equilibrium $Ar \rightleftarrows Ar^+ + e^-$, so Eq. 14.35 takes the form

$$K = \frac{[z][z]}{[1-z]}\left[\frac{p/p_{ref}}{1+z}\right]^{1+1-1} = \left(\frac{z^2}{1-z}\right)\left(p/p_{ref}\right)$$

Thus

$$z = \left[\frac{K/(p/p_{ref})}{1+K/(p/p_{ref})}\right]^{1/2} = \left[\frac{1}{\frac{(p/p_{ref})}{\underset{\uparrow\,(=4.2\times10^{-4})}{K}}+1}\right]^{\frac{1}{2}}$$



PROBLEM 14.66

KNOWN: 1 kmol of Na ionizes to form an equilibrium mixture of Na, Na^+, e^- at 2000K and pressure p in which the amount of Na present is x kmol. For $Na \rightleftarrows Na^+ + e^-$ at 2000K, K = 0.668.

FIND: Plot p versus x for $0.2 \le x \le 0.3$ kmol.

ASSUMPTION: Equilibrium can be treated using ideal gas mixture concepts.

ANALYSIS: The ionization of Na to form a mixture of Na, Na^+, e^- is described by

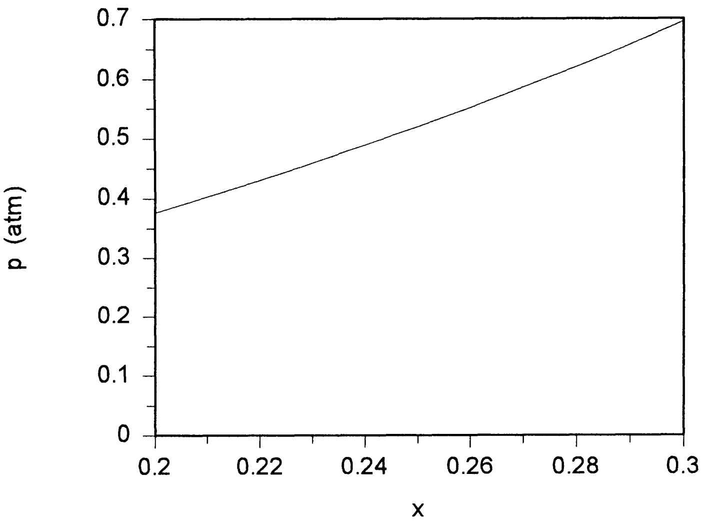
$$1\,Na \rightarrow x\,Na + (1-x)\,Na^+ + (1-x)\,e^-$$

The amount of mixture is $n = x + (1-x) + (1-x) = 2-x$.

At equilibrium $Na \rightleftarrows Na^+ + e^-$, so Eq. 14.35 takes the form

$$K = \frac{(1-x)(1-x)}{x}\left[\frac{p/p_{ref}}{2-x}\right]^{1+1-1} = \frac{(1-x)^2}{x(2-x)}(p/p_{ref})$$

Since K = 0.668, $\frac{p}{p_{ref}} = 0.668\,\frac{x(2-x)}{(1-x)^2}$. Thus for p_{ref} = 1 atm



PROBLEM 14.67

KNOWN: 1 kmol of N ionizes to form an equilibrium mixture of N, N^+, and e^- at 12,000 K, 6 atm in which the the amount of N is 0.95 kmol.

FIND: For $N \rightleftarrows N^+ + e^-$, determine K

ASSUMPTION: Equilibrium can be treated using ideal gas mixture concepts.

ANALYSIS: The ionization of N to form an equilibrium mixture of N, N^+, and e^- is described by

$$1\,N \rightarrow (1-z)\,N + z\,N^+ + z\,e^-$$

Since $1-z = 0.95$ kmol, $z = .05$ kmol. Accordingly

$$1N \rightarrow 0.95\,N + 0.05\,N^+ + 0.05\,e^-$$

The amount of mixture is n = 1.05 kmol.

At equilibrium $N \rightleftarrows N^+ + e^-$, so Eq. 14.35 takes the form

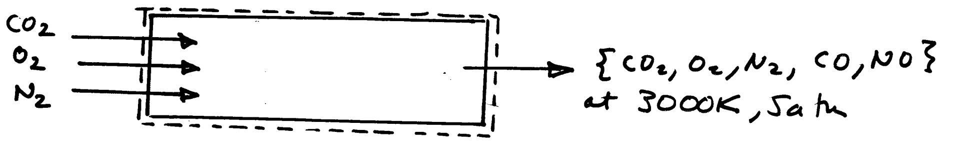
$$K = \frac{[0.05][0.05]}{[0.95]}\left[\frac{6}{1.05}\right]^{1+1-1} = 0.01504 \longleftarrow$$

PROBLEM 14.68

KNOWN: CO_2, O_2, N_2 enter a reactor operating at steady state with equal molar flow rates. An equilibrium mixture of $\{CO_2, O_2, N_2, CO, NO\}$ exits at 3000K, 5atm.

FIND: Determine the molar analysis of the equilibrium mixture.

SCHEMATIC & GIVEN DATA



ASSUMPTIONS: (1) The control volume shown above is at steady state. (2) Ideal gas principles apply.

ANALYSIS: On the basis of 1 mole each of CO, O_2, N_2 entering, at steady state the balanced reaction equation has the form

$$1\,CO_2 + 1\,O_2 + 1\,N_2 \rightarrow (1-B)\,CO_2 + B\,CO + \left(1+\frac{B}{2}-\frac{E}{2}\right)O_2 + E\,NO + \left(1-\frac{E}{2}\right)N_2 \qquad (1)$$

where B, N are unknowns. For the products, $n = 3 + B/2$.

At equilibrium, these independent reactions must be satisfied

$$(I)\quad CO_2 \rightleftarrows CO + \tfrac{1}{2}O_2, \qquad II\quad \tfrac{1}{2}O_2 + \tfrac{1}{2}N_2 \rightleftarrows NO$$

For reaction (I) with $K_1 = 0.3273$ from Table A-27 at 3000 K,

$$0.3273 = \frac{B\left(1+\frac{B}{2}-\frac{E}{2}\right)^{1/2}}{(1-B)}\left[\frac{p/p_{ref}}{n}\right]^{1/2} = \left(\frac{B}{1-B}\right)\left[\frac{1+\frac{B}{2}-\frac{E}{2}}{3+\frac{B}{2}}\right]^{1/2}(5)^{1/2} \qquad (2)$$

For reaction (II) with $K_2 = 0.1222$ from Table A-27. at 3000K

$$0.1222 = \frac{E}{\left[1+\frac{B}{2}-\frac{E}{2}\right]^{1/2}\left[1-\frac{E}{2}\right]^{1/2}}\left[\frac{p/p_{ref}}{n}\right]^{0} = \frac{2E}{\left[[2+B-E][2-E]\right]^{1/2}} \qquad (3)$$

Equations (2), (3) are two simultaneous equations for B and E. Solving, $B = 0.2017$, $E = 0.1208$. Then, referring to Eq. (1)

$$\left.\begin{aligned} (1-B)\,CO_2 &: 0.7983 \\ B\,CO &: 0.2017 \\ \left(1+\tfrac{B}{2}-\tfrac{E}{2}\right)O_2 &: 1.0405 \\ E\,NO &: 0.1208 \\ \left(1-\tfrac{E}{2}\right)N_2 &: \underline{0.9396} \\ &\quad 3.1009 \end{aligned}\right\} : \qquad \begin{aligned} y_{CO_2} &= 0.257 \\ y_{CO} &= 0.065 \\ y_{O_2} &= 0.336 \\ y_{NO} &= 0.039 \\ y_{N_2} &= 0.303 \end{aligned} \qquad \longleftarrow$$

PROBLEM 14.68 (Cont'd.)

Alternative Solution Using IT.

IT Code

```
T = 3000  // K
p = 5  // atm

// CO2 + O2 + N2 →  (1 - b) CO2 + b CO + (1 + b/2 - e/2) O2 + e NO + (1 - e/2) N
nCO2 = 1 - b
nCO = b
nO2 = 1 + b/2 - e/2
nNO = e
nN2 = 1 - e/2
ntot = nCO2 + nCO + nO2 + nNO + nN2
yCO = nCO / ntot
yO2  = nO2 / ntot
yCO2 = nCO2 / ntot
yNO = nNO / ntot
yN2 = nN2 / ntot
pref = 1  // atm

// For CO2 ↔ CO + 0.5 O2
K1 = (yCO * yO2^0.5 / yCO2) * (p / pref)^0.5

// For 1/2 O2 + 1/2 N2 ↔ NO
K2  = (yNO / (yO2^0.5 * yN2^0.5))

// Look up log10(K1) in EQCO2A.LUT.
log(K1) = LOOKUPVAL(EQCO2A,1,T,3)

// Look up log10(K2) in EQNO.LUT.
log(K2) = LOOKUPVAL(EQNO,1,T,3)
```

IT Results

$K_1 = 0.3273$
$K_2 = 0.1222$
$y_{CO} = 0.06506$
$y_{CO2} = 0.2574$
$y_{N2} = 0.303$
$y_{NO} = 0.03896$
$y_{O2} = 0.3355$

PROBLEM 14.69

KNOWN: An equimolar mixture of CO and $H_2O(g)$ enters a heat exchanger. An equilibrium mixture of CO, CO_2, O_2, H_2O, and H_2 exits at 2500 K, 1atm.

FIND: Determine the molar analysis of the equilibrium mixture.

ASSUMPTION: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The overall reaction has the form

$$CO + H_2O \rightarrow x\,CO_2 + y\,H_2O + (1-x)CO + \tfrac{1}{2}(1-x-y)O_2 + (1-y)H_2$$

The total number of moles n in the product mixture is

$$n = x + y + (1-x) + \tfrac{1}{2}(1-x-y) + (1-y) = (5-x-y)/2$$

At equilibrium, two independent reactions relate the components of the product mixture:

1. $CO_2 \rightleftarrows CO + \tfrac{1}{2}O_2$
2. $CO_2 + H_2 \rightleftarrows CO + H_2O$

From Table A-27 at 2500 K the respective values of $\log_{10} K$ are

$$\log_{10} K_1 = -1.44 \Rightarrow K_1 = 0.03631$$

$$\log_{10} K_2 = +0.784 \Rightarrow K_2 = 6.08135$$

The forms taken by Eq 14.35 when $P/P_{ref} = 1$ are, respectively

$$K_1 = \frac{[1-x]\left[\frac{1-x-y}{2}\right]^{1/2}}{x}\left[\frac{P/P_{ref}}{(5-x-y)/2}\right]^{1/2} = \frac{(1-x)}{x}\left[\frac{1-x-y}{5-x-y}\right]^{1/2}$$

$$K_2 = \frac{[1-x][y]}{[x][1-y]}\left[\frac{P/P_{ref}}{n}\right]^{0} = \left[\frac{1-x}{x}\right]\left[\frac{y}{1-y}\right]$$

Solving this pair of simultaneous equations for x and y

$x = 0.28808$

$y = 0.71105$

The equilibrium mixture on a molar basis is then

$$\{0.28808\,CO_2,\ 0.71105\,H_2O,\ 0.71192\,CO,\ 0.000435\,O_2,\ 0.28895\,H_2\} \longleftarrow$$

PROBLEM 14.69 (Cont'd.)

Altenative Solution Using IT.

IT Code

```
T = 2227 + 273.15  // K
p = 1  // atm

// CO2 + H2O  →  x CO2 + y H2O + (1 - x) CO + 1/2 (1 - x - y) O2 + (1 - y) H2

nCO2 = x
nH2O = y
nCO = 1 - x
nO2 = (1/2) * (1 - x - y)
nH2 = 1 - y
ntot = nCO2 + nH2O + nCO + nO2 + nH2
yCO2 = nCO2 / ntot
yH2O = nH2O / ntot
yCO = nCO / ntot
yO2 = nO2 / ntot
yH2 = nH2 / ntot
pref = 1  // atm

//  For the reaction:  CO2  ↔ CO + 1/2 O2
K1 = ((yCO * yO2^0.5) / yCO2) * (p / pref)^0.5

//  Look up log10(K2) in EQCO2A.LUT.
log(K1) = LOOKUPVAL(EQCO2A,1,T,3)

// For the reaction:  CO2 + H2  ↔  CO + H2O
K2= (yCO * yH2O) / (yCO2 * yH2)

//  Look up log10(K1) in EQWATGAS.LUT.
log(K2) = LOOKUPVAL(EQWATGAS,1,T,3)
```

IT Results

K_1 = 0.03634
K_2 =6.082
n_{CO} = 0.7119 kmol
n_{CO2} = 0.2881 kmol
n_{H2} = 0.2889 kmol
n_{H2O} = 0.7111 kmol
n_{O2} = 0.0004324 kmol
x = 0.2881
y = 0.7111

PROBLEM 14.70

KNOWN: A closed vessel initially contains 3 lbmol CO_2, 6 lbmol CO, and 1 lbmol H_2. An equilibrium mixture is formed at 4680°R, 1atm consisting of CO_2, CO, H_2O, H_2, O_2.

FIND: Determine the composition of the equilibrium mixture.

ASSUMPTION: The equilibrium mixture is modeled as an ideal gas mixture.

ANALTSIS: The overall reaction has the form

$$3\,CO_2 + 6\,CO + 1\,H_2 \rightarrow x\,CO_2 + (9-x)\,CO + (3-x-2y)\,H_2O + (x+2y-2)\,H_2 + y\,O_2$$

The total number of moles n in the product mixture is

$$n = x + (9-x) + (3-x-2y) + (x+2y-2) + y = 10+y$$

At equilibrium, two independent reactions relate the components of the product mixture:

1. $CO_2 \rightleftarrows CO + \frac{1}{2}O_2$
2. $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$

From Table A-27 at 4680°R the respective values of $\log_{10}K$ are

$$\log_{10}K_1 = -1.219 \Rightarrow K_1 = 0.06039$$

$$\log_{10}K_2 = -2.021 \Rightarrow K_1 = 0.00953$$

The forms taken by Eq. 14.35 when $P/P_{ref} = 1$ are, respectively

$$K_1 = \frac{[9-x][y]^{1/2}}{[x]}\left[\frac{P/P_{ref}}{10+y}\right]^{1/2} = \left(\frac{9-x}{x}\right)\left(\frac{y}{10+y}\right)^{1/2}$$

$$K_2 = \frac{[x+2y-2][y]^{1/2}}{[3-x-2y]}\left[\frac{P/P_{ref}}{10+y}\right]^{1/2} = \left[\frac{x+2y-2}{3-x-2y}\right]\left[\frac{y}{10+y}\right]^{1/2}$$

Solving this pair of simultaneous equations for and y

$$x = 2.30559$$
$$y = 0.00433$$

The equilibrium mixture on a molar basis is then

$$\{\,2.30559\,CO_2,\ 6.69441\,CO,\ 0.68575\,H_2O,\ 0.31425\,H_2,\ 0.00433\,H_2\,\}$$

PROBLEM 14.70 (Cont'd.)

Alternative Solution Using IT.

IT Code

```
T = 4220 + 459.67  // °R
p = 1  // atm

// 3 CO2 + 6 CO + 1 H2 → x CO2 + (9 - x) CO + (3 - x - 2y) H2O + (x + 2y - 2) H2 + y O2
nCO2 = x
nCO = 9 - x
nH2O = 3 - x - 2*y
nH2 = x + 2*y - 2
nO2 = y
ntot = nCO2 + nCO + nH2O + nH2 + nO2
yCO2 = nCO2 / ntot
yCO = nCO / ntot
yH2O = nH2O / ntot
yH2 = nH2 / ntot
yO2 = nO2 / ntot
pref = 1  // atm

// For the reaction:  CO2 ↔ CO + 1/2 O2
K1 = ((yCO * yO2^0.5) / yCO2) * (p / pref)^0.5

// Look up log10(K1) in EQCO2A.LUT.
log(K1) = LOOKUPVAL(EQCO2A,2,T,3)

// For the reaction:  H2O ↔ H2 + 1/2 O2
K2 = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5

// Look up log10(K2) in EQH2O.LUT.
log(K2) = LOOKUPVAL(EQH2O,2,T,3)
```

IT Results

$K_1 = 0.06034$
$K_2 = 0.00952$
$n_{CO} = 6.694$ lbmol
$n_{CO2} = 2.306$ lbmol
$n_{H2} = 0.3142$ lbmol
$n_{H2O} = 0.6858$
$n_{O2} = 0.00432$
$x = 2.306$
$y = 0432$

PROBLEM **14.71**

KNOWN: C_4H_{10} burns with 100% excess air to form an equilibrium mixture at 1400K, 20 atm consisting of CO_2, O_2, $H_2O(g)$, N_2, NO, NO_2. For the reaction $N_2 + 2O_2 \rightleftarrows 2NO_2$, at 1400K, $K = 8.4 \times 10^{-10}$

FIND: Determine the balanced reaction equation.

① ASSUMPTIONS: The ideal gas model applies to the equilibrium mixture.

ANALYSIS: The balanced reaction equation for complete combustion with the theoretical amount of air is

$$C_4H_{10} + 6.5(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 24.44\,N_2$$

For combustion with 100% excess air

$$C_4H_{10} + 13(O_2 + 3.76N_2) \rightarrow \alpha\,CO_2 + \beta\,H_2O + 2a\,NO + 2b\,NO_2 + \gamma\,O_2 + \delta\,N_2$$

C: $4 = \alpha$. H: $10 = 2\beta$, $\beta = 5$.

O: $26 = 8 + 5 + 2a + 4b + 2\gamma \Rightarrow \gamma = 6.5 - a - 2b$

N: $2(48.88) = 2a + 2b + 2\delta \Rightarrow \delta = 48.88 - a - b$

The total amount of mixture is $n = 64.38 - b$.

At equilibrium there are two reactions applicable:

1. $\frac{1}{2}N_2 + \frac{1}{2}O_2 \rightleftarrows NO$, $K_1 = 1.941 \times 10^{-3}$ **(Table A-27)**

2. $N_2 + 2O_2 \rightleftarrows 2NO_2$, $K_2 = 84 \times 10^{-10}$ **(Given)**

Accordingly, there are two constraints that determine a and b:

$$1.941 \times 10^{-3} = \frac{2a}{\left[[48.88 - a - b][6.5 - a - 2b]\right]^{1/2}} \left[\frac{p/p_{ref}}{n}\right]^{\overbrace{1-\frac{1}{2}-\frac{1}{2}}^{0}}$$

$$= \frac{2a}{\left[[48.88 - a - b][6.5 - a - 2b]\right]^{1/2}} \qquad (1)$$

$$8.4 \times 10^{-10} = \frac{[2b]^2}{[48.88 - a - b][6.5 - a - 2b]^2} \left[\frac{20/1}{64.38 - b}\right]^{2-1-2}$$

$$= \left[\frac{2b}{6.5 - a - 2b}\right]^2 \left[\frac{64.38 - b}{48.88 - a - b}\right] \left[\frac{1}{20}\right] \qquad (2)$$

Solving Eqs. (1), (2) simultaneously results in $a = 1.727 \times 10^{-2}$; $b = 3.660 \times 10^{-4}$

The reaction equation is then

$$C_4H_{10} + 13(O_2 + 3.76N_2) \rightarrow 4CO_2 + 5H_2O + 3.454 \times 10^{-2}\,NO + 7.32 \times 10^{-4}\,NO_2 + 6.482\,O_2 + 48.862\,N_2 \quad \longleftarrow$$

---

1. Using Kay's rule (Sec 11.8) to evaluate T_c for the mixture, it can be confirmed **that, because of** the relatively high mixture temperature: **1400K**, T_R for the mixture is large enough for the ideal gas model to be applicable.

PROBLEM 14.71 (Cont'd.)

Alternative Solution Using IT.

IT Code

```
T = 1400  // K
p = 20  // atm
K2 = 8.4E-10

//  C4H10 + 13 (O2 + 3.76 N2) →
          4 CO2 + 5 H2O + x NO + y NO2 + (6.5 - x/2 - y) O2 + (48.88 - x/2 - y/2) N2
nCO2 = 4
nH2O = 5
nNO = x
nNO2 = y
nO2 = 6.5 - x/2 - y
nN2 = 48.88 - x/2 - y/2
ntot = nCO2 + nH2O + nNO + nNO2 + nO2 + nN2
yNO = nNO / ntot
yO2 = nO2 / ntot
yNO2 = nNO2 / ntot
yN2 = nN2 / ntot
pref = 1  // atm

// For the reaction:  1/2 N2 + 1/2 O2  ↔  NO
K1 = (yNO / (yN2^0.5 * yO2^0.5))
//  Data from Table A-27 are stored in EQNO.LUT.
log(K1) = LOOKUPVAL(EQNO,1,T,3)

//  For the reaction:  N2 + 2 O2  ↔  2 NO2
K2  =  (yNO2^2 / (yN2 * yO2^2)) * (p / pref)^-1
```

IT Results

$K_1 = 0.001941$
$n_{CO2} = 4$ kmol/kmol(C_4H_{10})
$n_{H2O} = 5$ kmol/kmol(C_4H_{10})
$n_{NO} = 0.03454$ kmol/kmol(C_4H_{10})
$n_{NO2} = 0.0007319$ kmol/kmol(C_4H_{10})
$n_{O2} = 6.482$ kmol/kmol(C_4H_{10})
$n_{N2} = 48.86$ kmol/kmol(C_4H_{10})

PROBLEM 14.72

KNOWN: One lbmol of $H_2O(g)$ dissociates to form an equilibrium mixture at 5000°R, 1atm consisting of $H_2O(g)$, H_2, O_2, OH.

FIND: Determine the equilibrium composition.

ASSUMPTION: The ideal gas model applies to the equilibrium mixture.

ANALYSIS: The dissociation reaction has the form

$$1\,H_2O \Rightarrow \alpha H_2 + \beta O_2 + \gamma OH + \omega H_2O$$

$$H:\quad 2 = 2\alpha + \gamma + 2\omega \Rightarrow \alpha = 1-\omega-\gamma/2 \qquad \alpha = 1-(1-2\beta-\gamma)-\gamma/2$$

$$O:\quad 1 = 2\beta+\gamma+\omega \Rightarrow \omega = 1-2\beta-\gamma \qquad = 2\beta+\gamma/2$$

The total amount of mixture is $n = (2\beta+\gamma/2)+\beta+\gamma+(1-2\beta-\gamma) = 1+\beta+\frac{\gamma}{2}$.

At equilibrium there are two applicable reactions

1. $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$, $K_1 = 0.0201$ (Table A-27)
2. $H_2O \rightleftarrows OH + \frac{1}{2}H_2$, $K_2 = 0.0215$ (Table A-27)

Accordingly, there are two constraints that determine β and γ:

$$0.0201 = \frac{[2\beta+\gamma/2][\beta]^{1/2}}{[1-2\beta-\gamma]}\left[\frac{p/p_{ref}}{1+\beta+\frac{\gamma}{2}}\right]^{1+\frac{1}{2}-1}$$

$$= \left[\frac{2\beta+\gamma/2}{1-2\beta-\gamma}\right]\left[\frac{\beta}{1+\beta+\gamma/2}\right]^{1/2} \qquad (1)$$

$$0.0215 = \frac{[\gamma][2\beta+\gamma/2]^{1/2}}{[1-2\beta-\gamma]}\left[\frac{p/p_{ref}}{1+\beta+\frac{\gamma}{2}}\right]^{1+\frac{1}{2}-1}$$

$$= \left[\frac{\gamma}{1-2\beta-\gamma}\right]\left[\frac{2\beta+\gamma/2}{1+\beta+\gamma/2}\right]^{1/2} \qquad (2)$$

Solving Eqs (1), (2) simultaneously results in $\beta = 0.0341$, $\gamma = 0.06$. The equilibrium composition is then, per lbmol of H_2O initially present

$\{0.0982\ H_2,\ 0.0341\ O_2,\ 0.06\ OH,\ 0.8718\ H_2O(g)\}$ ←

PROBLEM 14.72 (Cont'd.)

Alternative Solution Using IT.

IT Code

```
T = 5000  // °R
p = 1  // atm

// H2O → a H2 + b O2 + c OH + d H2O
1 = a + c/2 + d  // H2 balance
1/2 =  b + c/2 + d/2  // O2 balance
ntot = a + b + c + d
yH2 = a / ntot
yO2 = b / ntot
yOH = c / ntot
yH2O = d / ntot
pref = 1  // atm

// For the reaction:  H2O ↔ H2 + 1/2 O2
K1 = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
// Data from Table A-27 are stored in EQH2O.LUT
log(K1) = LOOKUPVAL(EQH2O,2,T,3)

// For the reaction:  H2O ↔ OH + 1/2 H2
K2 = ((yOH * yH2^0.5) / yH2O) * (p / pref)^0.5
// Data from Table A-27 are stored in  EQH2O_OH.LUT.
log(K2) = LOOKUPVAL(EQH2O_OH,2,T,3)
```

IT Results

$K_1 = 0.0201$
$K_2 = 0.02147$
a = 0.09828 lbmol
b = 0.03376 lbmol
c = 0.06153 lbmol
d = 0.8709 lbmol

PROBLEM 14.73

KNOWN: Steam enters a heat exchanger. An equilibrium mixture of H_2O, H_2, O_2, H, and OH exits at 1 atm and (a) 2800 K, (b) 3000 K.

FIND: Determine the molar analysis of the equilibrium mixture.

ASSUMPTION: The equilibrium mixture is modeled as an ideal gas mixture.

ANALYSIS: The overall reaction has the form

$$1\,H_2O \rightarrow (1-a-b)\,H_2O + \left(a+\tfrac{b}{2}-c\right)H_2 + \tfrac{a}{2}\,O_2 + 2c\,H + b\,OH$$

The total number of moles n in the product mixture is

$$n = (1-a-b) + \left(a+\tfrac{b}{2}-c\right) + \tfrac{a}{2} + 2c + b = 1 + \tfrac{a}{2} + \tfrac{b}{2} + c$$

At equilibrium, three independent reactions relate the components of the product mixture:

1. $H_2O \rightleftarrows H_2 + \frac{1}{2}O_2$
2. $H_2O \rightleftarrows \frac{1}{2}H_2 + OH$
3. $H_2 \rightleftarrows 2H$

(a) From Table A-27 at 2800 K the respective values of $\log_{10}K$ are

$\log_{10} K_1 = -1.658 \Rightarrow K_1 = 0.02198$

$\log_{10} K_2 = -1.624 \Rightarrow K_2 = 0.02377$

$\log_{10} K_3 = -2.178 \Rightarrow K_3 = 0.00664$

The forms taken by Eq. 14.35 when $p/p_{ref} = 1$ are, respectively

$$K_1 = \frac{\left(a+\frac{b}{2}-c\right)\left(\frac{a}{2}\right)^{1/2}}{(1-a-b)}\left(\frac{p/p_{ref}}{1+\frac{a}{2}+\frac{b}{2}+c}\right)^{1/2} = \left(\frac{a+\frac{b}{2}-c}{1-a-b}\right)\left(\frac{a/2}{1+\frac{a}{2}+\frac{b}{2}+c}\right)^{1/2}$$

$$K_2 = \frac{\left(a+\frac{b}{2}-c\right)^{1/2}(b)}{1-a-b}\left(\frac{p/p_{ref}}{1+\frac{a}{2}+\frac{b}{2}+c}\right)^{1/2} = \left(\frac{b}{1-a-b}\right)\left(\frac{a+\frac{b}{2}-c}{1+\frac{a}{2}+\frac{b}{2}+c}\right)^{1/2}$$

$$K_3 = \frac{[2c]^2}{\left(a-\frac{b}{2}-c\right)}\left[\frac{p/p_{ref}}{\left(1+\frac{a}{2}+\frac{b}{2}+c\right)}\right]^{1/2} = \frac{4c^2}{\left(a+\frac{b}{2}-c\right)\left(1+\frac{a}{2}+\frac{b}{2}+c\right)}$$

Solving this set of simultaneous equations for a, b, and c:

a = 0.076754, b = 0.0679, c = 0.013334

The equilibrium mixture on a molar basis is then

$\{0.855346\,H_2O,\ 0.09737\,H_2,\ 0.038377\,O_2,\ 0.026668\,H,\ 0.0679\,OH\}$ $\longleftarrow$

PROBLEM 14.73 (Contd.)

(b) From Table A-27 at 3000 K the respective values of $\log_{10} K$ are

$\log_{10} K_1 = -1.343 \Rightarrow K_1 = 0.04539$

$\log_{10} K_2 = -1.265 \Rightarrow K_2 = 0.05433$

$\log_{10} K_3 = -1.606 \Rightarrow K_3 = 0.02477$

Solving the set of equations considered in part (a) gives

$a = 0.12568$, $b = 0.11232$, $c = 0.03282$

The equilibrium mixture on a molar basis is then

$\{ 0.762\ H_2O,\ 0.1490\ 2\ H_2,\ 0.06284\ O_2,\ 0.06564\ H,\ 0.11232\ OH \}$ ◄——

Alternative Solution Using IT.

IT Code

```
T = 3000  // K
p = 1  // atm

// H2O → a H2O + b H2 + c O2 + d H + e OH
1 = a + b + d/2  // H2 balance
1/2 = a/2 + c + e/2   // O2 balance
ntot = a + b + c + d + e
yH2O = a / ntot
yH2 = b / ntot
yO2 = c / ntot
yH = d / ntot
yOH = e / ntot
pref = 1  // atm

//  H2O ↔ H2 + 1/2 O2          // Reaction 1
//  H2O ↔ 1/2 H2 + OH          // Reaction 2
//  H2 ↔ 2 H                   // Reaction 3

K1 = ((yH2 * yO2^0.5) / yH2O) * (p / pref)^0.5
K2 = ((yH2^0.5 * yOH) / yH2O) * (p / pref)^0.5
K3 = (yH^2 / yH2) * (p / pref)^1

// Data from Table A-27 are stored in EQH2O.LUT, EQH2O_OH.LUT, and EQH2.LUT.
// Look up log10(K) in the appropriate tables.
log(K1) = LOOKUPVAL(EQH2O,1,T,3)
log(K2) = LOOKUPVAL(EQH2O_OH,1,T,3)
log(K3) = LOOKUPVAL(EQH2,1,T,3)
```

IT Results

Part (a): T = 2800 K
n_{H2O} = 0.8719 kmol
n_{H2} = 0.1136 kmol
n_{O2} = 0.03162 kmol
n_{H} = 0.02894 kmol
n_{OH} = 0.06481 kmol

Part (b): T = 3000 K
n_{H2O} = 0.7884 kmol
n_{H2} = 0.1755 kmol
n_{O2} = 0.04983 kmol
n_{H} = 0.07216 kmol
n_{OH} = 0.1119 kmol

PROBLEM 14.74

KNOWN: The system is a two-phase liquid-vapor mixture of (a) water at 100°C, (b) R134a at 20°C.

FIND: Using tabulated property data show that $g_f = g_g$.

ANALYSIS: (a) At 100°C, Table A-2 gives

$h_f = 419.04$ kJ/kg $\qquad h_g = 2676.1$ kJ/kg

$s_f = 1.3069$ kJ/kg·K $\qquad s_g = 7.3549$ kJ/kg·K

Then, with $g = h - Ts$

①
$$g_f = h_f - Ts_f = 419.04 - (373.15)(1.3069) = -68.63 \text{ kJ/kg}$$
$$g_g = h_g - Ts_g = 2676.1 - (373.15)(7.3549) = -68.38 \text{ kJ/kg}$$

(b) At 20°C, Table A-10 gives

$h_f = 77.26$ kJ/kg $\qquad h_g = 258.36$ kJ/kg

$s_f = 0.2924$ kJ/kg·K $\qquad s_g = 0.9102$ kJ/kg·K

Then, with $g = h - Ts$

$$g_f = 77.26 - (293.15)(0.2924) = -8.457 \text{ kJ/kg}$$
$$g_g = 258.36 - (293.15)(0.9102) = -8.465 \text{ kJ/kg}$$

---

1. As developed in Sec. 6.3.1, $s_g - s_f = (h_g - h_f)/T$ for a change in phase from saturated liquid to saturated vapor at T. Thus $h_g - Ts_g = h_f - Ts_f$, or $g_g = g_f$. The slight difference in the calculated values of g_g and g_f owes to table roundoff and signifies a slight inconsistency in table values.

PROBLEM 14.75

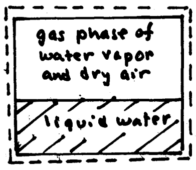
See the respective problems in Chapter 11 for solutions.

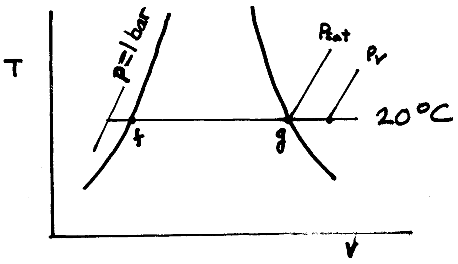
PROBLEM 14.76

KNOWN: A phase of liquid water only is in equilibrium with moist air at 20°C, 1 bar.

FIND: Determine the departure, in percent, of the partial pressure of the water vapor from the saturation pressure of pure water at 20°C.

SCHEMATIC & GIVEN DATA





ASSUMPTIONS: (1) The gas phase can be modeled as an ideal gas. (2) The liquid phase involves water only. ①

ANALYSIS: Proceeding as in Example 14.10, the following result is obtained

$$\frac{p_v}{p_{sat}} = \exp \frac{\bar{v}_f (p - p_{sat})}{\bar{R}T} \qquad (1)$$

With data from Table A-2 at 20°C, $v_f = 1.0018 \times 10^{-3}$ m³/kg, $p_{sat} = 0.02339$ bar, so

$$\frac{\bar{v}_f (p - p_{sat})}{\bar{R}T} = \frac{[1.0018 \times 10^{-3} \text{ m}^3/\text{kg}](18.02 \text{ kg/kmol})(1 - 0.02339) \times 10^5 \text{ N/m}^2}{[8314 \text{ N}\cdot\text{m/kmol}\cdot\text{K}][293.15 \text{ K}]}$$

$$= 7.23 \times 10^{-4}$$

Equation (1) then gives

$$\frac{p_v}{p_{sat}} = \exp(7.23 \times 10^{-4})$$

$$= 1.00072$$

When expressed as a percentage the departure of p_v from p_{sat} is

② $$\left(\frac{p_v - p_{sat}}{p_{sat}}\right)(100) = (1.00072 - 1)(100) = 0.072\%$$

---

1. Review note 1 of Example 14.10
2. Review note 2 of Example 14.10

PROBLEM 14.77

KNOWN: Graphite and diamond exist in equilibrium at 25°C, p.

FIND: Derive an expression for estimating the pressure p in terms of $\bar{v}$, $\bar{g}$ and the isothermal compressibility κ of each phase at 25°C, 1atm.

ASSUMPTION: The isothermal compressibility κ is constant with pressure.

ANALYSIS: By Eq 14.59, at equilibrium $\bar{g}(25°C, p) = \bar{g}'(25°C, p)$ where $\bar{g}$ and $\bar{g}'$ denote the molar Gibbs functions of graphite and diamond, respectively.

To evaluate the molar Gibbs functions relative to $T_{ref} = 25°C$, $p_{ref} = 1\text{atm}$, begin with Eq. 11.23. Since T is fixed

$$d\bar{g} = \bar{v}\,dp \qquad (1)$$

Also, at fixed T the differential of $\bar{v}(\underset{0}{T}, p)$ reduces to

$$d\bar{v} = \underbrace{\left(\frac{\partial \bar{v}}{\partial p}\right)_T}_{= -\kappa\bar{v}\ (\text{Eq. }11.63)} dp + \cancel{\left(\frac{\partial \bar{v}}{\partial T}\right)_p dT} = -\kappa \bar{v}\,dp$$

So

$$\frac{d\bar{v}}{\bar{v}} = -\kappa\,dp \quad \xrightarrow[\text{T and constant }\kappa]{\text{Integration at fixed}}, \quad \ln\left[\frac{\bar{v}(T_{ref}, p)}{\bar{v}(T_{ref}, p_{ref})}\right] = -\kappa(p - p_{ref})$$

or

$$\bar{v}(T_{ref}, p) = \bar{v}(T_{ref}, p_{ref}) \exp(-\kappa(p - p_{ref})) \qquad (2)$$

Combining Eqs (1), (2), and integrating

$$\bar{g}(T_{ref}, p) - \bar{g}(T_{ref}, p_{ref}) = \bar{v}(T_{ref}, p_{ref}) \frac{\exp(\kappa p_{ref})}{-\kappa}\Big[\exp(-\kappa p)\Big]_{p_{ref}}^{p}$$

$$= -\frac{\bar{v}(T_{ref}, p_{ref})}{\kappa}\Big[\exp(-\kappa(p - p_{ref})) - 1\Big]$$

Then, with $T_{ref} = 25°C$, $p_{ref} = 1\text{atm}$ and the condition $\bar{g} = \bar{g}'$

$$\bar{g}(25°C, 1\text{atm}) - \frac{\bar{v}(25°C, 1\text{atm})}{\kappa}\Big[\exp(-\kappa(p - 1\text{atm})) - 1\Big] =$$

$$\bar{g}'(25°C, 1\text{atm}) - \frac{\bar{v}'(25°C, 1\text{atm})}{\kappa'}\Big[\exp(-\kappa'(p - 1\text{atm})) - 1\Big] \qquad (3)$$

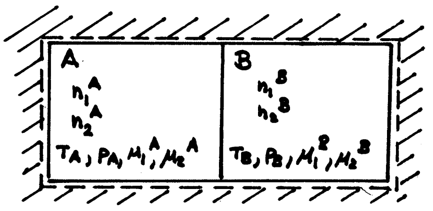
Equation (3) has a single unknown: the pressure p. Thus Eq (3) gives p implicitly in terms of $\bar{g}$, $\bar{v}$ and κ for each phase at 25°C, 1atm. ←

PROBLEM 14.78

KNOWN: An isolated system has two phases, A and B. Each phase consists of the same two substances, 1 and 2.

FIND: Show that conditions for equilibrium are 1: $T_A = T_B$, 2: $p_A = p_B$, 3: $\mu_1^A = \mu_1^B$, $\mu_2^A = \mu_2^B$.

SCHEMATIC & GIVEN DATA:



ASSUMPTION: (1) The system shown in the accompanying figure is isolated. (2) Eq. 11.114(a) is applicable to each phase. (3) A and B do not react with one another.

ANALYSIS: According to Eq. 14.3 any process of an isolated system must satisfy $dS]_{U,V} \geq 0$. This indicates that the entropy of an isolated increases during an irreversible process. Each step of the process results in an increase in the entropy of the system and brings the system closer to equilibrium. The equilibrium state is the one having the maximum value of S. Therefore, when

$$dS]_{U,V} = 0 \qquad (1)$$

there is equilibrium.

For the isolated system under present consideration, $dS = dS_A + dS_B$. Using Eq. 11.114a to evaluate dS_A and dS_B, respectively, we get

$$dS = \left[\frac{dU_A}{T_A} + \frac{p_A}{T_A}dV_A - \frac{\mu_1^A}{T_A}dn_1^A - \frac{\mu_2^A}{T_A}dn_2^A\right] + \left[\frac{dU_B}{T_B} + \frac{p_B}{T_B}dV_B - \frac{\mu_1^B}{T_B}dn_1^B - \frac{\mu_2^B}{T_B}dn_2^B\right] \qquad (2)$$

For the isolated system, the total energy, total volume, and total mass for each substance are constant:

$$dU_A + dU_B = 0, \quad dV_A + dV_B = 0, \quad dn_1^A + dn_1^B = 0, \quad dn_2^A + dn_2^B = 0 \qquad (3)$$

Introducing Eqs. (3) into Eq. (2)

$$dS = \left[\frac{1}{T_A} - \frac{1}{T_B}\right]dU_A + \left[\frac{p_A}{T_A} - \frac{p_B}{T_B}\right]dV_A - \left[\frac{\mu_1^A}{T_A} - \frac{\mu_1^B}{T_B}\right]dn_1^A - \left[\frac{\mu_2^A}{T_A} - \frac{\mu_2^B}{T_B}\right]dn_2^A \qquad (4)$$

Eq. (4) shows that in seeking the maximum in S the independent variables are U_A, V_B, n_1^A, and n_2^A. Necessary conditions for a maximum are

$$\frac{\partial S}{\partial U_A} = \frac{\partial S}{\partial V_A} = \frac{\partial S}{\partial n_1^A} = \frac{\partial S}{\partial n_2^A} = 0$$

The first of these conditions requires

$$\frac{1}{T_A} - \frac{1}{T_B} = 0 \Rightarrow T_A = T_B \qquad \longleftarrow$$

From the second

$$\frac{p_A}{T_A} - \frac{p_B}{T_B} = 0 \Rightarrow p_A = p_B \qquad \longleftarrow$$

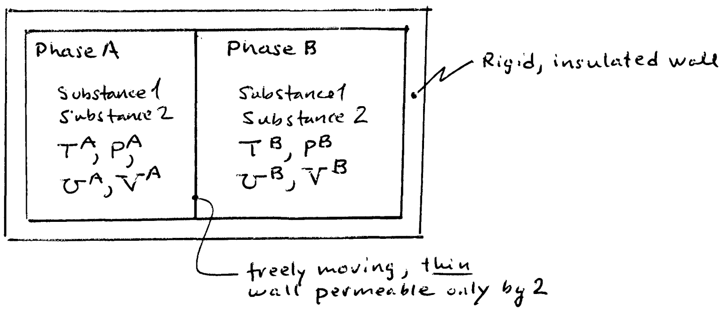
With the remaining two conditions

$$\frac{\mu_1^A}{T_A} - \frac{\mu_1^B}{T_B} = 0 \Rightarrow \mu_1^A = \mu_1^B, \quad \frac{\mu_2^A}{T_A} - \frac{\mu_2^B}{T_B} = 0 \Rightarrow \mu_2^A = \mu_2^B \qquad \longleftarrow$$

PROBLEM 14.79

KNOWN: An isolated system has two phases, A and B, each of which consists of the same two substances, 1 and 2. The phases are separated by a freely moving, thin wall permeable only by substance 2.

FIND: Determine the necessary conditions for equilibrium.



ASSUMPTION: (1) The system consists of phase A plus phase B (2) For the system, $Q = W = 0$ and there are no kinetic/potential energy effects. (3) The phases are separated by a freely moving, thin wall permeable only by 2.

ANALYSIS: The appropriate equilibrium criteria for this case is obtained via Equation 14.3 as $dS]_{U,V} = 0$, where $dS = dS^A + dS^B$ Invoking Eq 11.114a

$$dU^A = T^A dS^A - p^A dV^A + \cancel{\mu_1^A dn_1^A} + \mu_2^A dn_2^A$$
$$dU^B = T^B dS^B - p^B dV^B + \cancel{\mu_1^B dn_1^B} + \mu_2^B dn_2^B$$

Since the wall is permeable only by substance 2, the amount of substance 1 present in each of the phases cannot change, and so these terms have been dropped in the expression above.

Since the container is rigid and insulated,

total volume is constant : $dV^B = -dV^A$

total energy is constant : $dU^B = -dU^A$

Since B is conserved, $dn_2^B = -dn_2^A$,

Collecting results:

$$dS]_{U,V} = \left[\frac{dU^A}{T^A} + \frac{p^A}{T^A}dV^A - \frac{\mu_2^A}{T^A}dn_2^A\right] + \left[\frac{dU^B}{T^B} + \frac{p^B}{T^B}dV^B - \frac{\mu_2^B}{T^B}dn_2^B\right]$$

$$= dU^A\left[\frac{1}{T^A} - \frac{1}{T^B}\right] + dV^A\left[\frac{p^A}{T^A} - \frac{p^B}{T^B}\right] - dn_2^A\left[\frac{\mu_2^A}{T^A} - \frac{\mu_2^B}{T^B}\right]$$

Since U^A, V^A, n_2^A can be varied independently, it follows that when $dS]_{U,V} = 0$ the terms in the parentheses must vanish. Therefore, the criteria of equilibrium are

$T^A = T^B$ (temperature is the same in each phase)

① $p^A = p^B$ (pressure is the same in each phase)

$\mu_2^A = \mu_2^B$ (chemical potential of substance 2 is the same in each phase)

---

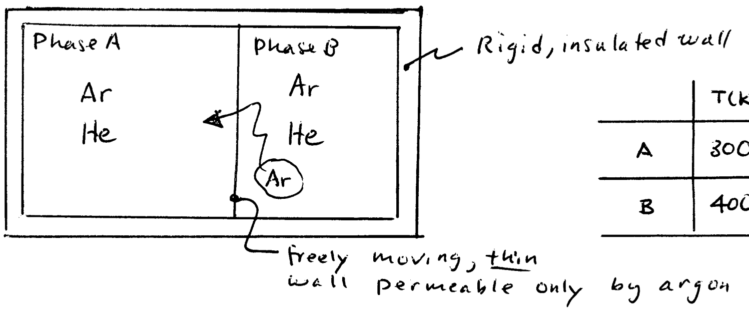
1. There is no restriction on μ_1^A or μ_1^B.

PROBLEM 14.80

KNOWN: An isolated system has two gas phases, A and B, each of which consists of argon and helium. The phases are separated by a freely moving, thin wall permeable only to argon. Initial data is provided.

FIND: Using the result of Problem 14.79, determine the final equilibrium temperature, pressure, and composition in the two phases.

SCHEMATIC & GIVEN DATA:



	T(K)	p(MPa)	n(kmol)	y_{AR}	y_{He}
A	300	0.2	6	0.5	0.5
B	400	0.1	5	0.8	0.2

ASSUMPTIONS: (1) The idealizations of Problem 14.79 apply (2) Ideal gas principles apply to the gas mixtures.

ANALYSIS: An energy balance requires $\Delta U = \Delta U^A + \Delta U^B = 0$, where

$$\Delta U^A = [n^A_{a,f}\,\bar{u}_a(T_f) + n^A_h\,\bar{u}_h(T_f)] - [n^A_{a,i}\,\bar{u}_a(T^A) + n^A_h\,\bar{u}_h(T^A)]$$

$$\Delta U^B = (n^B_{a,f}\,\bar{u}_a(T_f) + n^B_h\,\bar{u}_h(T_f)] - [n^B_{a,i}\,\bar{u}_a(T^B) + n^B_h\,\bar{u}_h(T^B)]$$

So,

$$0 = [n^A_{a,f} + n^B_{a,f}]\,\bar{u}_a(T_f) - n^A_{a,i}\,\bar{u}_a(T^A) - n^B_{a,i}\,u_a(T^B) + n^A_h[\bar{u}_h(T_f) - \bar{u}_h(T^A)] + n^B_h[\bar{u}_h(T_f) - \bar{u}_h(T^B)]$$

With $n^A_{a,f} + n^B_{a,f} = n^A_{a,i} + n^B_{a,i}$, where $n^A_{a,i} = 3$, $n^B_{a,i} = 4$, $n_a^{TOT} = 7$.

$$0 = n^A_{a,i}[\bar{u}_a(T_f) - \bar{u}_a(T^A)] + n^B_{a,i}[\bar{u}_a(T_f) - \bar{u}_a(T^B)] + n^A_h[\bar{u}_h(T_f) - \bar{u}_h(T^A)] + n^B_h[\bar{u}_h(T_f) - \bar{u}_h(T^B)]$$

For monatomic gases $\Delta\bar{u} = c_v\Delta T$, where c_v is a constant. So, this becomes after arrangement

$$T_f = \frac{(n^A_{a,i} + n^A_h)T^A + (n^B_{a,i} + n^B_h)T^B}{(n^A_{a,i} + n^A_h) + (n^B_{a,i} + n^B_h)} = \frac{(6)(300\text{K}) + 5(400\text{K})}{11} = 345.5\text{K} \longleftarrow T_f$$

The total volume is unchanged. Thus $(V^A + V^B)_f = (V^A + V^B)_i$:

$$n_{TOT}\frac{\bar{R}T_f}{p_f} = \frac{n^A\bar{R}T^A}{p^A} + \frac{n^B\bar{R}T^B}{p^B} \Rightarrow p_f = \frac{n_{TOT}\,T_f}{\frac{n^AT^A}{p^A} + \frac{n^BT^B}{p^B}} = \frac{(11)(345.5)}{\frac{(6)(300)}{0.2} + \frac{(5)(400)}{0.1}} = 0.131\,\text{MPa} \nwarrow p_f$$

The final constraint is $\mu^A_a = \mu^B_a$ at the final state. With Eq. 14.17, this becomes $y^A_{a,f} = y^B_{a,f}$. That is

$$\frac{n^A_{a,f}}{n^A_{a,f} + n^A_h} = \frac{n^B_{a,f}}{n^B_{a,f} + n^B_h} \Rightarrow \frac{n^A_{a,f}}{n^A_{a,f} + 3} = \frac{(7 - n^A_{a,f})}{(7 - n^A_{a,f}) + 1} \Rightarrow 4n^A_{a,f} = 21 \Rightarrow n^A_{a,f} = 5.25$$

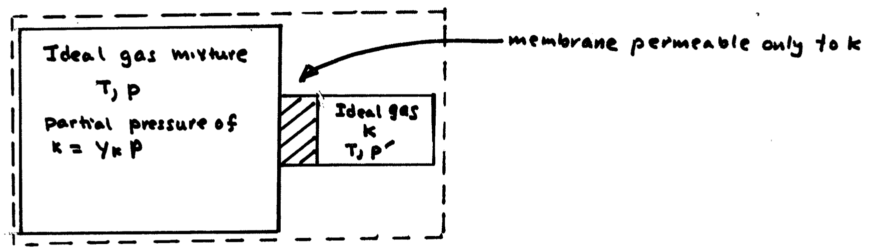
Accordingly at the final equilibrium state Phase A contains 5.25 kmol of Argon and 3.0 kmol of Helium. Phase B contains 1.75 kmol of Argon and 1.0 kmol of Helium. ⟵

PROBLEM 14.81

KNOWN: An ideal gas mixture at T and p containing substance k is separated by a semipermeable membrane from a gas phase of pure k at T and p'.

FIND: Determine the relationship between p and p' for there to be no net transfer through the membrane.

SCHEMATIC & GIVEN DATA:



ASSUMPTIONS: (1) The system is shown in the accompanying figure. (2) The gas mixture and the pure phase can be modeled as ideal gases. (3) The membrane allows only k to pass.

ANALYSIS: Following the discussion of Sec. 14.6.1, there would be no net transfer of k when the chemical potential of k in the mixture equals the chemical potential of pure k:

$$\mu_{k,\text{mixture}} = \mu_{k,\text{pure}} \qquad (1)$$

With Eq. 14.12, $\mu_{k,\text{pure}} = \bar{g}_k(T, p')$, and with Eq. 14.16, $\mu_{k,\text{mixture}} = \bar{g}_k(T, y_k p)$. Thus Eq. (1) becomes

$$\bar{g}_k(T, y_k p) = \bar{g}_k(T, p') \Rightarrow p' = y_k p \longleftarrow$$

That is, the pressure p' of the pure phase must equal the partial pressure of k in the mixture.

PROBLEM 14.82

KNOWN: A system involves (a) one component, (b) two components, (c) three components.

FIND: Determine the maximum number of homogeneous phases that can exist at equilibrium.

ASSUMPTION: No reaction takes place.

ANALYSIS: Equation 14.68 summarizes Gibbs' phase rule. On rearrangement

$$P = 2 + N - F$$

where N is the number of components and P is the number of phases. The value of P is greatest when F, the degrees of freedom is least: zero. Thus, when $F = 0$

$$P = 2 + N \qquad (1)$$

(a) $N = 1 \Rightarrow P = 3$. (b) $N = 2 \Rightarrow P = 4$. (c) $N = 3$, $P = 5$. $\longleftarrow$

PROBLEM 14.83

KNOWN: Various systems are specified.
FIND: Determine the number of degrees of freedom.
ASSUMPTION: No reaction occurs.
ANALYSIS: Equation 14.68 summarizes Gibbs' phase rule:

$$F = 2 + N - P \qquad (1)$$

where F is the number of degrees of freedom, N is the number of components, and P is the number of phases.

(a) ice and liquid water : $N=1$, $P=2$

$$F = 2+1-2 = 1$$

(b) ice, liquid water, water vapor : $N=1$, $P=3$

$$F = 2+1-3 = 0$$

(c) liquid water, water vapor : $N=1$, $P=2$

$$F = 2+1-2 = 1$$

(d) water vapor only: $N=1$, $P=1$

$$F = 2+1-1 = 2$$

(e) water vapor, dry air: $N=2$, $P=1$ (dry air regarded as a single component)

$$F = 2+2-1 = 3$$

(f) liquid water, water vapor, dry air: $N=2$, $P=2$ (dry air regarded as a single component)

$$F = 2+2-2 = 2$$

(g) ice, water vapor, dry air: $N=2$, $P=2$ (dry air regarded as a single component)

$$F = 2+2-2 = 2$$

(h) N_2 and O_2 at 20°C, 1 atm : $N=2$, $P=1$.

$$F = 2+2-1 = 3$$

(i) liquid and vapor phases, each containing ammonia and water: $N=2$. $P=2$.

$$F = 2+2-2 = 2$$

(j) liquid mercury, liquid water, vapor phase of mercury and water: $N=2$, $P=3$

$$F = 2+2-3 = 1$$

Note: Water and mercury are immiscible, and thus form two liquid phases.

(k) liquid acetone and a vapor phase of acetone and N_2 : $N=2$, $P=2$

$$F = 2+2-2 = 2$$

PROBLEM 14.84

FIND: Develop the phase rule for chemically reacting systems.

ANALYSIS: If there is a chemical reaction within the system, the constituents are not completely independent. For example, if the constituents A, B, C, D undergo the reaction

$$\nu_A A + \nu_B B \rightleftarrows \nu_C C + \nu_D D$$

there is an additional independent equation. Accordingly, the number of independent equations is $\{N(P-1)+1\}$. The number of variables is $P[N-1]+2$, as on p. 685. Accordingly, the number of degrees of freedom is

$$F = [P(N-1)+2] - [N(P-1)+1]$$

$$= (N-1) - P + 2$$

If a number of independent reactions occur, say r reactions, the phase rule takes the form

$$F = (N-r) - P + 2.$$

PROBLEM 14.85

KNOWN: The gas phase reaction: $CH_4 + H_2O \rightleftarrows CO + 3H_2$

FIND: Determine the number of degrees of freedom.

ASSUMPTION: There is a single independent reaction.

ANALYSIS: Applying the result of Problem 14.84, the number of degrees of freedom is

$$F = (\underbrace{N}_{\text{No. of components} = 4} - \underbrace{r}_{\text{No. of independent reactions} = 1}) - \underbrace{P}_{\text{No. of phases} = 1} + 2$$

Thus

① $$F = (4-1) - 1 + 2 = 4$$

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1. To specify this system, one might give values for T, p and the mole fractions of any two of the four components.

PROBLEM 14.86

KNOWN: Two cases are identified involving a two-phase liquid-vapor ammonia-water system at temperature T and pressure p:
(a) T = 20°C, mole fraction of NH_3 in the liquid phase is 80%
(b) T = 40°C, p = 12 bar.

FIND: (a) Determine p, in bar, and the mole fraction of NH_3 in the vapor phase. (b) Determine the mole fraction of NH_3 in each of the phases.

ASSUMPTION: Raoult's law is applicable.

ANALYSIS: (a) For the liquid phase $y_{NH_3} = 0.80$. Thus, it follows that $y_{wv} = 0.20$. By Raoult's law, the partial pressures in the vapor phase are

$$p_{NH_3} = 0.8\ p_{sat,NH_3}(20°C) = 0.8\,(\underset{\nwarrow \text{Table A-13}}{8.5762\text{ bars}}) = 6.861\text{ bar}$$

$$p_{wv} = 0.2\ p_{sat,wv}(20°C) = 0.2\,(\underset{\nwarrow \text{Table A-2}}{0.02339\text{ bars}}) = 0.005\text{ bar}$$

Then for the vapor phase

$$p = p_{NH_3} + p_{wv} = 6.861 + 0.005 = 6.866\text{ bar}. \quad \longleftarrow p$$

Also, for the vapor phase, $p_{NH_3} = x_{NH_3}\,p$. So

① $$x_{NH_3} = \frac{p_{NH_3}}{p} = \frac{6.861}{6.866} = 0.999 \quad \longleftarrow x_{NH_3}$$

(b) The partial pressures of the vapor phase sum to give the system pressure:

$$12\text{ bars} = p_{NH_3} + p_{wv} \qquad (1)$$

According to Raoult's law

$$p_{NH_3} = y_{NH_3}\,p_{sat,NH_3}(40°C) = y_{NH_3}\,(\underset{\swarrow \text{Table A-13}}{15.549\text{ bar}}) \qquad (2)$$

$$p_{wv} = y_{wv}\,p_{sat,wv}(40°C) = y_{wv}\,(\underset{\nwarrow \text{Table A-2}}{0.07384\text{ bar}}) \qquad (3)$$

Also,

$$1 = y_{NH_3} + y_{wv} \qquad (4)$$

Eqs. (1)-(4) give

$$12\text{ bar} = y_{NH_3}(15.549\text{ bars}) + (1 - y_{NH_3})(0.07384\text{ bar})$$

Solving, $y_{NH_3} = 0.771$. $\longleftarrow y_{NH_3}$

For the gas phase, $p_{NH_3} = x_{NH_3}\,p$. Thus, Eq. (2) with $y_{NH_3} = 0.771$ gives

$$(0.771)(15.549\text{ bar}) = x_{NH_3}\,(12\text{ bar})$$

① $$\Rightarrow x_{NH_3} = \frac{(0.771)(15.549)}{12} = 0.999 \quad \longleftarrow x_{NH_3}$$

---

1. In each case the vapor phase is nearly all ammonia. The small amount of water vapor present in the gas phase is important owing to the possibility of freezing at some points of the cycle.